

Coordination of Electric Vehicle Charging/Routes to Reduce Charging Time

by

Hao Yu

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Graduate Supervisory Committee:

Yang Weng, Chair  
Hongbin Yu  
Yanchao Zhang

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## ABSTRACT

Although the increasing penetration of electric vehicles (EVs) has reduced the emission of the greenhouse gas caused by vehicles, it would lead to serious congestion on-road and in charging stations. Strategic coordination of EV charging would benefit the transportation system. However, it is difficult to model a congestion game, which includes choosing charging routes and stations. Furthermore, conventional algorithms cannot balance System Optimization and User Equilibrium, which can cause a huge waste to the whole society. To solve these problems, this paper shows (1) a congestion game setup to optimize and reveal the relationship between EV users, (2) using  $\varepsilon$  - Nash Equilibrium to reduce the inefficient impact from the self-minded behavior of the EV users, and (3) finding the relatively optimal solution to approach Pareto-Optimal solution. The proposed method can reduce more total EVs charging time and most EV users' charging time than existing methods. Numerical simulations demonstrate the advantages of the new method compared to the current methods.

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## Chapter 1

### INTRODUCTION

With the increase of the greenhouse effect and serious environmental pollution, energy-saving and emission reduction is a significant direction of future industrial development. As a symbol of human civilization, vehicles made our lives convenient, however, petrol vehicles consume a lot of energy and cause worse pollution by exhaust gas. Compared with petrol vehicles, electric vehicles (EVs) improve the utilization efficiency of the energy and what's more, they are friendly to our environment. Consequently, many countries have promulgated some policies and developed several projects to promote the development of EVs and also encourage people to use them Gass *et al.* (2014). In the past ten years, the global stock of battery electric vehicles (BEVs) has increased to more than 5 million, with a growth rate of 63 percent from the previous years IEA (2020).

However, with the increase of EVs, it will consequently cause serious congestion on the road and in charging stations (CSs) Campbell (2018); Edelstein (2016); Voelcker (2013). Additionally, the huge time that EV users spend on charging is a waste to the whole society. The unreasonable strategic coordination of EV charging can lead to more congestion and increase the cost of people using electric vehicles. It is therefore of vital importance to adopt the optimal algorithm to optimize the whole charging process (including choosing charging route and charging station).

For reducing the total charging time, Shao *et al.* (2017) adopt a dynamic Dijkstra algorithm to find the shortest path to reduce the travel time. The work in Eisner *et al.* (2011) proposed speeding-up shortest path queries to speed up the Dijkstra algorithm and Schambers *et al.* (2018); Strehler *et al.* (2017); De Cauwer *et al.* (2019) solved

the trip planning problem with an approximation scheme to compute the energy-efficient shortest route for EV drivers. Yang *et al.* (2016) also developed multinomial logit-based and nested logic-based models by a stated preference survey. However, in these works, the optimization of routing and charging is performed for a single vehicle without considering multiple vehicles, missing the interaction of group behavior under the premise of limited charging resources.

To reduce the queuing time for charging stations, Xiao *et al.* (2020); Lu and Hua (2015); Qiu *et al.* (2013); Zhu *et al.* (2017) fixed largest waiting time that EV users can accept based on queuing theory to solve the problem of planning EV charging spots in charging stations. However, these works also missed the interaction of the adjunct charging stations. Therefore, a game-theoretic approach has been adopted to provide an analytical framework for the interaction between charging stations and EVs Tushar *et al.* (2012). Malandrino *et al.* (2015) also used game theory analysis, but the implementation is for selecting charging stations for EV users. The optimal solution of the charging choice has not yet been discussed in detail. In addition, some researchers deployed Wardrop equilibrium to alleviate traffic congestion Zhou *et al.* (2021); Moradipari and Alizadeh (2018). Although this concept can help EV users select a route that minimizes the time or cost incurred in its traversal, it caused the inefficiency arising from the self-minded behavior of the EV users Correa and Stier-Moses (2011).

Therefore, I propose in this paper a method that can not only inherit the nice property of game theory, e.g., sharing with limited resources but also use  $\varepsilon$  to represent toleration for each EV user to abide by the equilibrium, which means that each person can make a small and acceptable sacrifice for the general interest. Specifically, I use a congestion game to study the sharing problem under the condition of limited resources, which is highly consistent with the characteristics of the EV charging pro-



cess. Different from traditional congestion games, due to every user do not maximize their interest strictly, I significantly improve the performance of the model under the condition of relative limited resources. Additionally, I show that the congestion game is proven to have at least an  $\varepsilon$  - Nash equilibrium solution.

Different from the common method to find overlaps between the Nash equilibrium solution and Pareto-Optimal solution Monfared *et al.* (2021), in mixed strategies, the issues of finding Pareto-optimal solution have not been developed so far Zhukovskiy and Kudryavtsev (2016). In this paper, I solved the problem of the minimum whole EV users' charging time under the premise of ensuring every EV user's charging time in the range of each one does not want to deviate its choosing. This method also can be seen as a balance of System Optimization and User Equilibrium Correa and Stier-Moses (2011), which can reduce the inefficiency arising from the self-minded behavior of the EV users to a certain degree, and the solution is also close to Pareto optimality, because many EV users spend less time on their charging behaviors.

My approach is validated by simulations in limited charging resources and sufficient charging resources situations, which means different numbers of electric vehicles to charge, and the different number of charging spots in each charging station respectively. To make the experiment more persuasive, I studied the two most different congested times of a day in Tempe city, which is a typical university city in the United States and also consider the influence of EV's different charging times to our model. In addition, this paper benchmarks our method with mixed strategy Nash equilibrium Crawford (1985), Wardrop Sohet *et al.* (2020), The shortest path algorithm Shao *et al.* (2017). Promising results are observed across the numerical section.

The remaining part of this paper is organized as follows. Chapter 2 describes the management framework of EV charging Chapter 3 describes modeling methodologies and model specifications. Chapter 4 describes the calculation of the optimal solution

for EV charging. Chapter 5 presents simulations and experiments to support our study. Chapter 6 concludes the paper.

EV CHARGING MANAGEMENT SYSTEM BASED ON  $\epsilon$  - NASH EQUILIBRIUM

I consider a central controller, which can accept each requiring charging low battery EV's travel times information from the charging stations. It is shown in Fig. 2.1. In most traditional situations, EV users just find the nearest charging station without considering multiple vehicles and missing the interaction of group behavior, which can lead to a long queuing time. The huge queuing time is a waste to both individuals and society.

To reducing the long charging waiting time, I argue that when low batteries EVs want to be charged, they can send their current locations and their destination locations to charging stations and then each charging station can calculate the EV's travel times, which include two parts, from the current location to the charging station and from the charging station to their destination. Each charging station can send every low battery EV's travel times to the central controller. The algorithm

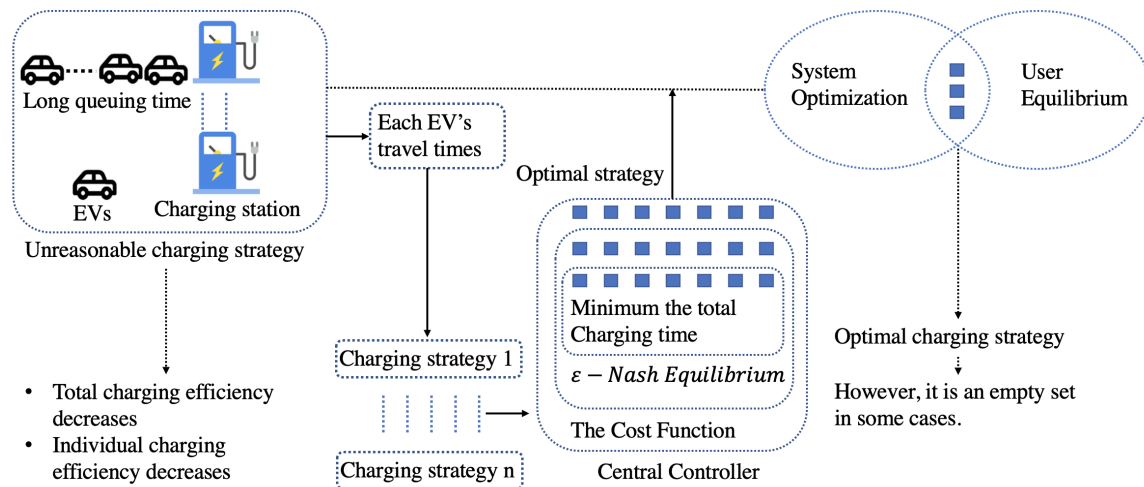


Figure 2.1: The Flowchart of the EV Charging Management Framework.

first makes sure that all the alternative strategies belong to  $\varepsilon$  - Nash Equilibrium to guarantee each user would not deviate from the strategy and then choose the one that makes the total charging time least. This way I can avoid situations where I cannot find the overlap between System Optimization and User Equilibrium and can balance them. After considering a variety of charging strategies for optimization, the central controller can indicate which charging station each electric vehicle should go to.

In conclusion, by aggregating traveling times about each electric vehicle that needs to be charged, the central controller can effectively consider both the interplay between group dynamics ,and reduce inefficiency caused by EV users' self-centered behaviors ,and gives relatively better charging strategies to every EV user.

### MODELING OF EV CHARGING PROCESS BASED ON ROUTE/CHARGING STATION CHOICE AND CHARGING TIME

When considering charging, EV drivers choose routes with charging station(s) (1) closer to their location, (2) with less charging time, and consistent with their travel direction Yang *et al.* (2016). Therefore, I consider minimizing the charging time as the optimization goal, defined as the time consumed in the traveling, waiting and charging process. It is obviously determined by the selected charging station and driving path as well as the number of vehicles that choose the same charging station at the same time. Thus, there is a clear competitive relationship between EVs during their charging process. When the charging price is low, in order to save costs, it is inevitable that congestion occurs as EVs compete for limited charging resources, resulting in a competitive relationship between EVs. Similarly, during peak charging periods, there is competition between EVs to save waiting time. This competitive relationship, or noncooperative relationship, may cause EV users to consume a substantial amount of time for charging.

The competitive relationship between EVs discussed above is not obvious when charging resources are sufficient. However, in specific rush hour periods or in circumstances of limited charging resources, the competition between EVs is particularly intense. The competition between EVs can be modeled as a congestion game, which is usually used to model noncooperative games between interactions of players who share resources. In noncooperative games, each player makes a decision on which resources to utilize. Subsequently, the individual decisions of players result in resource allocation at the population scale. Resources which are highly utilized become con-

gested so that the corresponding players incur higher losses. The characteristics of the EV charging process are thus highly consistent with congestion game research. So, I use congestion game to analyze the EV charging process.

### 3.1 EV Charging Process Model

An EV charging congestion game is a tuple  $G = (N, R, (S_i)_{i \in N}, (c_i)_{i \in N})$ , and the means of the characters are following:

- $N$  is the set of players with size  $n$ ,  $N = \{ev_1, \dots, ev_n\}$ , which corresponds to the EVs to be recharged.
- $R$  is the set of resources with size  $k$ , which is composed of all possible charging stations and road segments included in the road topology, i.e.,  $R = CS \cup L = \{cs_1, cs_2, \dots, l_1, l_2, \dots\}$ , where  $CS$  and  $L$  represent the set of all possible charging stations and the set of road segments, respectively.
- $S_i$  is the set of strategies for player  $i$  and  $S$  is the space of all possible strategy combinations in game  $G$ ,  $S = S_1 \times \dots \times S_n = \prod_{i \in N} S_i$ . In this work, each strategy in  $S_i$  is composed of two parts: one of the charging stations that EV  $i$  can reach, and the road that enables EV  $i$  to pass through the selected charging station from its current position and then reach its destination.
- $c_i$  is a cost function for player  $i \in N$ , where  $c_i : \mathbb{N} \mapsto \mathbb{R}$ . In this paper, the cost function  $c_i$  is defined as the time consumed by the charging process.

### 3.2 Building the Cost Function of Each EV

Considering the most general case, each of the  $n$  EVs may reach a selected charging station through different routes. There are also multiple routes to go from charging stations to final destinations.

For the congestion game, the definition of the cost function  $c_i$  is the key to optimization analysis. During the charging process of EVs, the time  $c_i$  is composed of four parts: travel time  $t_{f_i}$  from the current position to the charging station, the queuing time for charging  $t_{w_i}$ , the charging time  $t_{c_i}$  and the travel time  $t_{r_i}$  from the charging station to the destination. Then, the time  $c_i$  can be written as

$$c_i = t_{f_i} + t_{w_i} + t_{c_i} + t_{r_i}. \quad (3.1)$$

Assume that there are  $l$ ,  $l \in N$  EVs choosing the charging station  $cs_j$ ,  $j \in R$  with charging spots  $k_j$  for charging at the same time. Then, the time that each EV reaches the charging station  $cs_j$  by different roads is expressed as  $t_{f_i}$ , ( $i \in l$ ). If  $k_j < l$ , congestion may occur at the charging station  $cs_j$  at this time. Therefore, the charging queuing time  $t_{w_i}$  needs to be calculated accordingly. For ease of analysis, I arrange the time  $t_{f_i}$ , ( $i \in l$ ) in ascending order, and the time  $t_{f_i}$  can be expressed as  $\{t_{f_1}, \dots, t_{f_i}, \dots, t_{f_l}\}$ ,  $t_{f_{i-1}} < t_{f_i}$ ,  $i \in l$ . This means that the EV  $i \in l$  will arrive at the charging station as the  $i^{th}$  one. Furthermore, assuming that each EV has an equal charging time  $t_u$ , it is expressed as  $t_{c_i} = t_u$ . If  $t_{f_i} < t_u$ , the charging queuing time  $t_{w_i}$  can be expressed as

$$t_{w_i} = (\lceil \frac{i}{k_j} \rceil - 1)t_u - (t_{f_i} - t_{f_{min}}), \quad (3.2)$$

where

$$t_{f_{min}} = \min\{t_{f_1}, \dots, t_{f_l}\} = t_{f_1}, \quad (3.3)$$

and the ceiling function

$$\lceil \frac{i}{k_j} \rceil = \min\{n \in \mathbb{Z} \mid \frac{i}{k_j} \leq n\}. \quad (3.4)$$

By substituting equation (3.2) and  $t_{c_i} = t_u$  into the time  $c_i$  equation, the cost function time  $c_i$  can be expressed as

$$c_i = \lceil \frac{i}{k_j} \rceil t_u + t_{f_{min}} + t_{r_i}. \quad (3.5)$$

### 3.3 Building the Cost Matrix of the EV Charging System

Building a cost matrix  $M$  to give the underlying data for further calculations is the initial stage in finding the set of  $\varepsilon$  - Nash Equilibrium. Let player  $i$  have  $m_i$  pure strategies. Then, the game's total number of pure strategy combinations is  $m = \prod_{i=1}^n m_i$ . The cost of each player can be calculated for each of the pure strategy combinations. The cost values are then constructed as a vector with  $n$  elements. When player  $i \in n$  chooses the  $j^{th}$  ( $j \in m$ ) pure strategy combination, for example, the cost of player  $i$  is  $c_i^j$ . Then, the corresponding cost vector can be expressed as  $(c_1^j \ c_2^j \ \cdots \ c_n^j)$ . In the game, the maximum number of vectors I can obtain is  $m$ . I can get the cost matrix  $M$  with a size of  $m \times n$  by combining these vectors, as illustrated below.

$$M = \begin{bmatrix} c_1^1 & c_2^1 & \cdots & c_n^1 \\ c_1^2 & c_2^2 & \cdots & c_n^2 \\ \vdots & \vdots & \vdots & \vdots \\ c_1^{m-1} & c_2^{m-1} & \cdots & c_n^{m-1} \\ c_1^m & c_2^m & \cdots & c_n^m \end{bmatrix}. \quad (3.6)$$

For the above EV charging congestion game model, its solutions of the  $\varepsilon$  - Nash equilibrium Chatterjee *et al.* (2004) is a set of all feasible solutions of EV charging and route choice. The other has been proven that every finite game has at least one mixed strategy Nash equilibrium Rosenthal (1973). So, in my EV charging game, it has at least one mixed strategy Nash Equilibrium. Therefore, it must have greater than or equal to one mixed strategy  $\varepsilon$  - Nash Equilibrium.

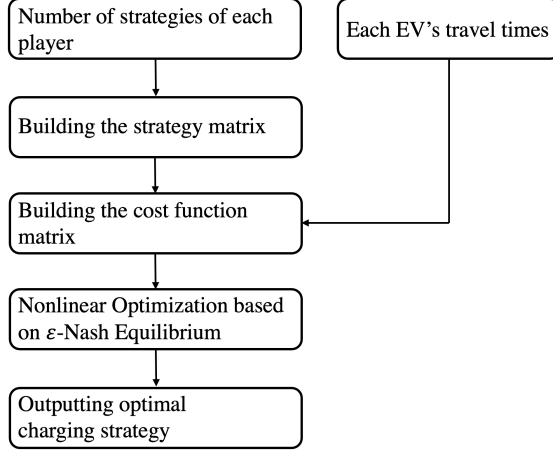


## Chapter 4

### OPTIMAL EV CHARGING STRATEGY BASED ON $\varepsilon$ - NASH EQUILIBRIUM

Characterizing the  $\varepsilon$  – Nash Equilibrium (approximate Nash Equilibrium) of the congestion game gives useful insights into the game of the EV charging process. It is different from strict Nash Equilibrium, it gives EV users more options in their charging process. At the same time, the  $\varepsilon$  – Nash Equilibrium can be a constraint of the EV charging model. EV users only can adopt these strategies, which are the solutions of the  $\varepsilon$  – Nash Equilibrium. It can guarantee that every EV user does not want to deviate from their options. Rather than the traditional method to make the approximate Nash Equilibrium closer and closer to the exact Nash Equilibrium Hazan and Krauthgamer (2011), I just want to find a set of EV user’s strategies and find which one of the strategies can minimize the total charging time. In common, this kind of problem is always turned to be a nonlinear optimization problem Krichene *et al.* (2015); Chatterjee (2009).

Because the set of strategies for the proposed EV charging model consists of finite strategies, the process of the  $\varepsilon$  – Nash Equilibrium search is a discrete optimization process. Due to the complexity of discrete optimization, I use the probability notion to transform the discrete optimization problem into a continuous optimization problem. Then, I use the optimization algorithm provided in MATLAB to find the  $\varepsilon$  – Nash Equilibrium solutions and then find which one of the solutions can minimize the total charging time, which is an optimal charging strategy. The approaches as shown in



**Figure 4.1:** Calculation Flow of the Optimal Charging Method.

Fig. 4.1 and in detail are mentioned further down.

#### 4.1 $\varepsilon$ - Nash Equilibrium of the EV Charging System

The following is a mathematical definition of  $\varepsilon$  - Nash equilibrium. Let  $x_i$  be a strategy profile of player  $i$  and  $x_{-i}$  be a strategy profile of all players except for player  $i$ . When each player  $i \in \{1, \dots, n\}$  selects strategy  $x_i$  resulting in strategy profile  $x = \{x_1, \dots, x_n\}$ , player  $i$  obtains payoff function  $u_i(x)$ . Note that the payoff is determined by the strategy profile chosen, which includes both player  $i$ 's and all other players' strategies.  $\varepsilon$  can represent the maximum toleration of waiting time. If no unilateral deviation in strategy by any single player can obtain more than  $\varepsilon$  notion profit for that player, then a strategy profile  $x^* \in S$  is a  $\varepsilon$  - Nash equilibrium, that is

$$u_i(x_i^*, x_{-i}^*) + \varepsilon \geq u_i(x_i, x_{-i}^*), \quad \forall x_i \in S_i. \quad (4.1)$$

For the congestion game, players aim to minimize their cost, that is,  $u_i(x) = -c_i(x)$ .

Thus, the  $\varepsilon$  - Nash equilibrium for the congestion game can be expressed as

$$c_i(x_i^*, x_{-i}^*) - \varepsilon \leq c_i(x_i, x_{-i}^*), \quad \forall x_i \in S_i. \quad (4.2)$$

## 4.2 Continuous EV Charging System Model based on Mixed Strategy of Each EV

Because each player  $i$  does just one action, the  $\varepsilon$  – Nash equilibrium I described above is a pure strategy equilibrium. The  $\varepsilon$  – Nash equilibrium is then calculated using a nonlinear discrete optimization procedure based on a pure strategy equilibrium. I adapt the model to include mixed strategies to make the computation process easier. Besides, if the strategy sets are compact and the payoff functions are continuous, then a Pareto equilibrium strategy profile exists in the class of mixed strategies Zhukovskiy and Kudryavtsev (2016).

A probability distribution over the space  $R$  is regarded as a mixed strategy of player  $R$ . For player  $i$ , the probability assigned to pure strategy  $s_i^j (j \in m_i)$  is  $p_i^j$ . In this game  $G$ , the pure strategy set  $S_i$  is a finite set and the convex set generated by it can be expressed as

$$T_i = \{p_i \in \mathbb{R}^{m_i} \mid \sum_{j=1}^{m_i} p_i^j = 1\}, \quad (4.3)$$

which corresponds to the space of all mixed strategies of player  $i$ .

If a mixed strategy combination  $p$  is played, the probability that the pure strategy combination  $s = (s_1^{j_1}, s_2^{j_2}, \dots, s_n^{j_n})$  occurs is given by

$$p(s) = \prod_{i \in N} p_i^{j_i}. \quad (4.4)$$

In such a situation, the cost assigned to player  $i$  is given by

$$c_i(p) = \sum_{s \in S} p(s) c_i(s), \quad (4.5)$$

where  $c_i(s)$  is the cost to player  $i$  at the pure strategy combination  $s$ .

I can substitute the mixed strategy combination  $p$  with  $(p_i, p_{-i})$ , if  $p_{-i}$  denotes the mixed strategy vector formed by all players except player  $i$ . At this time, a mixed strategy profile  $p^*$  is called a  $\varepsilon$  – Nash equilibrium of the game  $G$  if

$$c_i(p_i^*, p_{-i}^*) - \varepsilon \leq c_i(p_i, p_{-i}^*), \quad \forall i \in N, \quad \forall p_i \in T_i. \quad (4.6)$$

In other words, for each player  $i$  modifying only its own mixed strategy and leaving all other strategies fixed would not result in a reduction of more than  $\varepsilon$  at  $\varepsilon$  – Nash equilibrium. I assume that when the range of change of charging time is within 30, these strategies are all acceptable.

### 4.3 Equivalent Nonlinear Optimization Process to Compute Optimal Charging Strategy

To obtain the optimal solution to this EV charging problem, I need to ensure all mixed strategies belong to the set of the solutions of the  $\varepsilon$  – Nash equilibrium.

A necessary and sufficient condition for  $p$  to be a  $\varepsilon$  – Nash equilibrium of the game  $G$  is

$$\begin{aligned} c_i(p_i, p_{-i}) - \varepsilon &\leq c_i(s_i^j, p_{-i}), \quad \forall i \in N, \quad \forall j = 1, \dots, m_i, \\ \sum_{j=1}^{m_i} p_i^j &= 1, \quad \forall i \in N, \\ p_i^j &\geq 0, \quad \forall j = 1, \dots, m_i, \quad \forall i \in N. \end{aligned} \quad (4.7)$$

where  $(s_i^j, p_{-i})$  denotes the mixed strategy combination in which player  $i$  plays with his  $j^{th}$  pure strategy, that is, a mixed strategy in which the  $j^{th}$  pure strategy of the  $i^{th}$  player is assigned the probability 1.

I need to minimize the total cost obtained by a combination of each player's possible mixed strategies. So, I can obtain the relative system optimum in the premise that each player is in the  $\varepsilon$  – Nash equilibrium solution sets to balance System Optimization and User Equilibrium. Then, the optimization problem is

$$\begin{aligned}
& \min_p \sum_{i \in N} c_i(p) \\
& \text{s.t. } c_i(p) - c_i(s_i^j, p_{-i}) - \varepsilon \leq 0, \forall i \in N, \forall j = 1, \dots, m_i, \\
& \sum_{j=1}^{m_i} p_i^j = 1, \forall i \in N, \\
& p_i^j \geq 0, \forall j = 1, \dots, m_i, \forall i \in N.
\end{aligned} \tag{4.8}$$

**Theorem 4.3.1.** *Although the set of mixed strategies is convex, the problem  $G$  described by equation (4.8) is non-convex.*

*Proof.* Let's consider a simple charging profile of 4 EVs and 3 charging stations, which only has 1 charging spot in each charging station with  $t_u = 30$  mins. The specific values of the driving time are shown in Table 4.1.

**Table 4.1:** Traveling Time Data for the Selected Route

$t_{1f_1} = t_{2f_1} = 12 \text{ min}$	$t_{1f_2} = t_{2f_2} = 17 \text{ min}$	$t_{1f_3} = t_{2f_3} = 21 \text{ min}$
$t_{3f_1} = t_{4f_1} = 16 \text{ min}$	$t_{3f_2} = t_{4f_2} = 11 \text{ min}$	$t_{3f_3} = t_{4f_3} = 15 \text{ min}$
$t_{1r_1} = t_{2r_1} = 13 \text{ min}$	$t_{1r_2} = t_{2r_2} = 8 \text{ min}$	$t_{1r_3} = t_{2r_3} = 9 \text{ min}$
$t_{3r_1} = t_{4r_1} = 15 \text{ min}$	$t_{3r_2} = t_{4r_2} = 10 \text{ min}$	$t_{3r_3} = t_{4r_3} = 15 \text{ min}$

The definition of a convex function is

$$f(\lambda x + (1 - \lambda)y) \leq \lambda f(x) + (1 - \lambda)f(y), \forall \lambda \in [0, 1]. \tag{4.9}$$

For Monte Carlo verification of this problem, I choose  $\lambda = 0.5$  and the two strategies, which both belong to feasible domains, as shown in Table 4.2. Then

$$f(0.5((S)_1)^T + 0.5((S)_2)^T) > 0.5f(((S)_1)^T) + 0.5f(((S)_2)^T). \tag{4.10}$$

**Table 4.2:** Charging Strategy Selection.

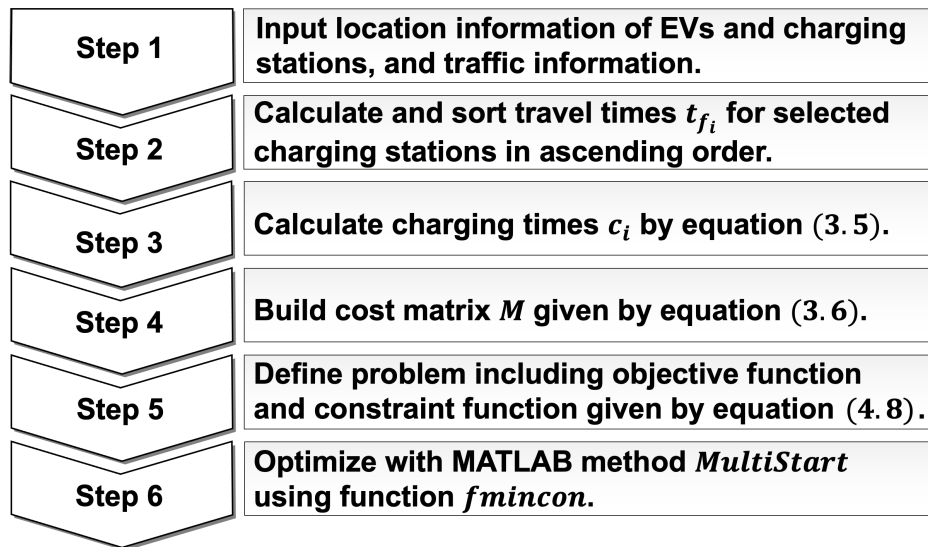
		$N$			
		$ev_1$	$ev_2$	$ev_3$	$ev_4$
$S$	Strategy	$cs_1$	$cs_2$	$cs_2$	$cs_3$
	1				
$S$	Strategy	$cs_1$	$cs_3$	$cs_2$	$cs_2$
	2				

where  $f(0.5((S)_1)^T + 0.5((S)_2)^T)$  means EV has a 0.5 probability of choosing the strategy 1 and has a 0.5 probability of choosing the strategy 2.  $f(((S)_1)^T)$  and  $f(((S)_2)^T)$  represent that EV adopts strategy 1 and Strategy 2 respectively.

This problem  $G$  violates the definition of a convex function (4.9), therefore, it is non-convex problem. □

Therefore, the problem  $G$  possesses local optimal solution. A local optimum satisfies Karush–Kuhn–Tucker (KKT) first-order necessary conditions Boyd *et al.* (2004). So, I decide to generate multiple starting points randomly for iterative calculation to be close to the global optimal point Jain and Agogino (1989).

To be specific, for this nonlinear minimization problem with nonlinear constraints, I use sequential quadratic programming (SQP) to solve the optimization with function “*fmincon*” and adopt multi-start method to be closer to the global optimum with function “*MultiStart*” in MATLAB. The calculation steps are listed below in Fig. 4.2.



**Figure 4.2:** Calculation Steps of the Optimal Charging Strategy.

## SIMULATION AND VALIDATION OF THE OPTIMAL CHARGING STRATEGY

I use simulations to show the entire optimization process in this section. Then, the characteristics of the EV charging station and route choice are studied based on the results of the simulation.

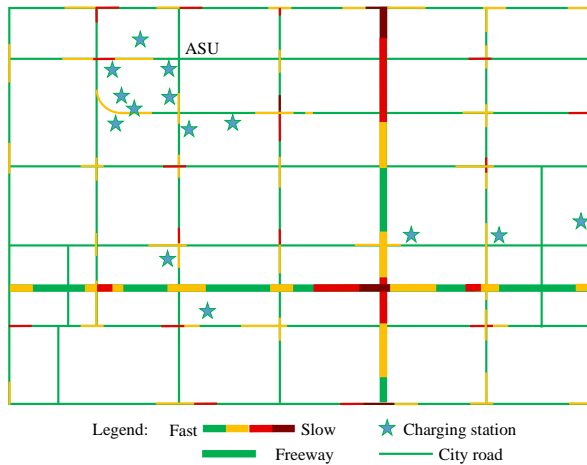
## 5.1 Experiment Setting Up

To ensure the analysis closely portrays reality, I use charginghub ref (2022) to get information about the distribution of charging stations and use Google Map to get the traffic situation in Tempe City, where Arizona State University (ASU) locates. I employ a section of the real Tempe traffic topology shown in Fig. 5.1(a), which can be seen that most charging stations are located near Arizona State University (ASU). Therefore, when a big number of EVs reach this region for charging, congestion is a crucial problem to consider.

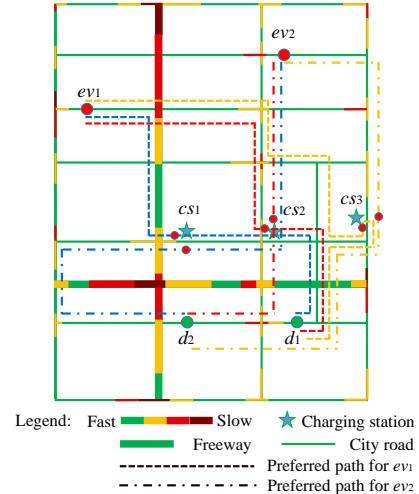
In the scenario, I assume that each charging station has the same number of charging spots for the convenience of further analysis. I randomly generate the distribution of low batteries EVs and their destinations after charging. The electric vehicles,  $ev_i$  has low batteries and can utilize one of fifteen charging stations. The  $ev_i$  has a travel time of  $t_{if_j}$  to the selected charging stations  $cs_j$ ,  $j \in (1, \dots, 15)$  and  $t_{ir_j}$  is the time it takes to travel from the charging station  $cs_j$  to its destination  $d_i$ . At the same time, I suppose that each electric car takes the same amount of time to charge  $t_u$ , which is around 30 mins Courtney (2021).

Since there are several roads to reach the selected charging station and EV's destination, I should first choose the correct road. To reduce the power consumption





(a) Transportation System Topology



(b) Traffic Topology used for Simulation

**Figure 5.1:** Transportation Network Topology near Arizona State University.

of the EVs, I choose the road that can reach the charging station and destination as quickly as possible. I calculate the driving time by Google maps and obtain an optimal route. A example of the selected roads are shown in Fig. 5.1(b).

To make the experiment more convincing, I sampled the distribution of electric vehicles in this region that needed to be charged at 9:00 am and 4:00 pm, the two most congested times of the day, respectively. In addition, I also vary the average charging time of EVs at charging stations to simulate the different charging times caused by the different power consumption conditions of EVs in different seasons. Typically, in Tempe, EVs' power consumption in the summer is greater than in the winter because of the use of car air conditioning. So, EV users may take longer time to charge in summer. The experiments are implemented by MATLAB R2020a.

## 5.2 Reduction of Total Charging Time with Respect to Interaction of Group Behavior

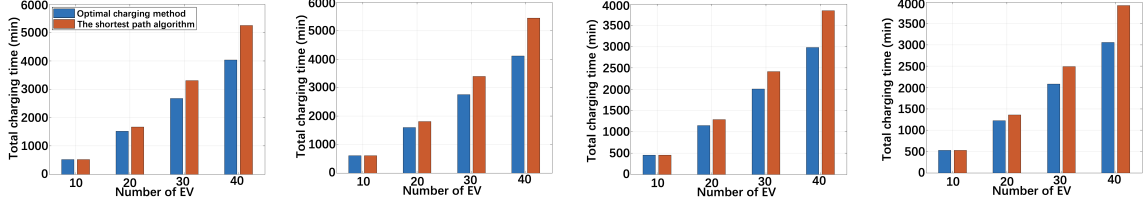
For the shortest path method, the EV user only considered which charging station was closest to him and did not consider whether other EVs would also choose this charging station and how their choice would affect him. Therefore, I compare the total charging times of our optimal charging method and the shortest path method for total EVs in two scenarios, one with progressively scarce charging resources and the other with progressively abundant charging resources.

I increase the number of EVs waiting to be charged to represent that the charging resources become more and more limited and increase the number of charging spots per charging station to represent that the charging resources become more and more sufficient. Then, I compare our method with the method at two different time points in two separate seasons.

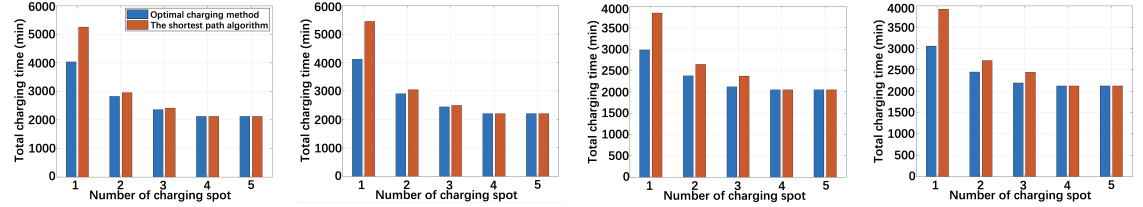
I find that as charging resources become more scarce, EVs with our optimal charging strategy save more and more time in total charging time than those with the shortest path method as shown in Fig. 5.2. The Fig. 5.3, which shows that when the charging resources become more sufficient, the total charging time that our method obtains is getting closer to the shortest path method's results, which means the effect of the interaction of group behavior becomes weaker and weaker. Despite this, our method also can obtain less total charging time in most situations.

## 5.3 Reduction of Total Charging Time with Respect to The Inefficiency Arising from The Self-minded Behavior

The common mixed Nash Equilibrium method and Wardrop method both have considered the interaction of group behavior in the charging process, but both ap-



(a) Scenery at 9 AM in Summer (b) Scenery at 4 PM in Summer (c) Scenery at 9 AM in Winter (d) Scenery at 4 PM in Winter  
**Figure 5.2:** Effect of the Interaction of Group Behavior as the Number of EVs Increase.

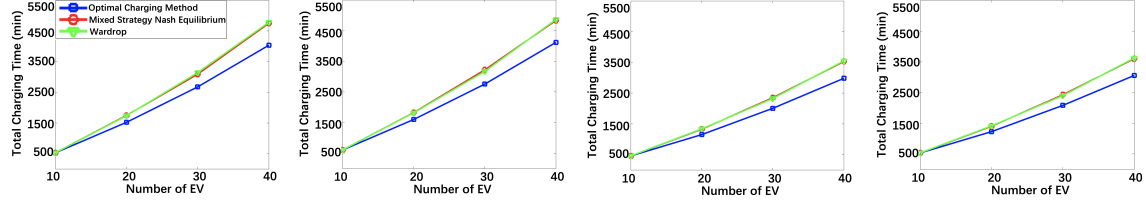


(a) Scenery at 9 AM in Summer (b) Scenery at 4 PM in Summer (c) Scenery at 9 AM in Winter (d) Scenery at 4 PM in Winter  
**Figure 5.3:** Effect of the Interaction of Group Behavior as the Number of Charging Spots in Each Charging Station Increase.

proaches have a one-sided emphasis on maximizing one’s interests in any given situation and not considering the impact of their behaviors on the group, which can cause group inefficiency due to individual selfishness.

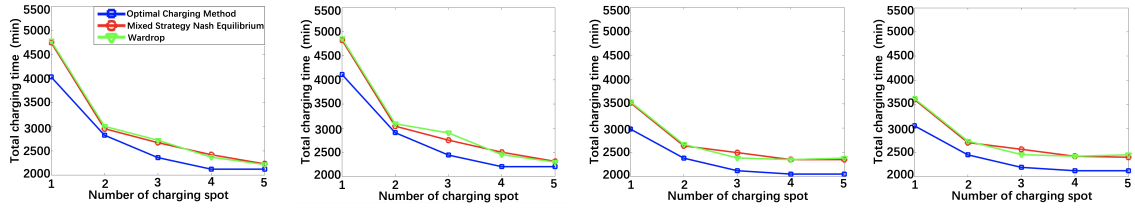
Therefore, I also compare the total charging times of our optimal charging method and the other 2 methods for total EVs in the increasing scarcity of charging resources situation and the increasing abundance of charging resources situation respectively. I find the effect of the interaction of group behavior becomes weaker and weaker as the charging resources become sufficient above. So, it is obvious that the group inefficiency due to individual selfishness decreases as the interaction of group behavior decreases in the increasing abundance of charging resources situation and vice versa.

As charging resources become more scarce, EVs with our optimal charging strategy save more and more time in total charging time than those with the other 2



(a) Scenery at 9 AM in Summer (b) Scenery at 4 PM in Summer (c) Scenery at 9 AM in Winter (d) Scenery at 4 PM in Winter

**Figure 5.4:** Effect of the Individual Self-minded Behavior as the Number of EVs Increase.



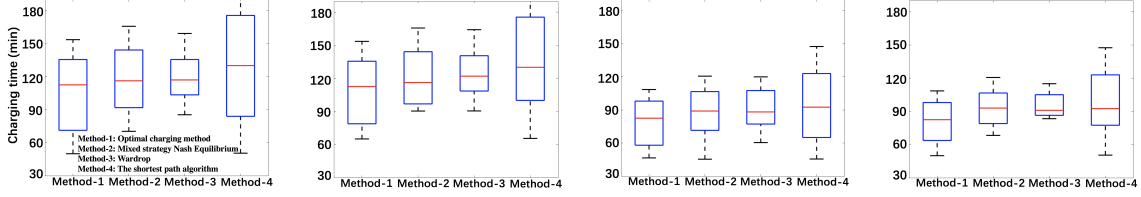
(a) Scenery at 9 AM in Summer (b) Scenery at 4 PM in Summer (c) Scenery at 9 AM in Winter (d) Scenery at 4 PM in Winter

**Figure 5.5:** Effect of the Individual Self-minded Behavior as the Number of Charging Spots in Each Charging Station Increase.

methods. The corresponding charging time is shown in Fig. 5.4. Although the total charging time that our method obtains is getting closer to Wardrop’s and the mixed Strategy Nash Equilibrium’s results in the increasing abundance of charging resources situation, our method’s results are always less than Wardrop’s and Mixed Strategy Nash Equilibrium’s. The corresponding charging time is shown in Fig. 5.5. Both of them can illustrate that I reduce the inefficient impact from the self-minded behavior of the players successfully.

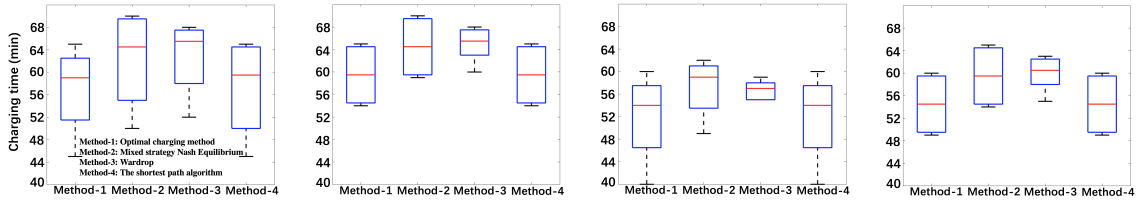
#### 5.4 Less Most EV’s Charging Time

In this section, I compare the EVs’ charging time distribution of four methods in very limited charging resources situations, which are the most representative situations. Fig. 5.6 shows most EVs which adopt the optimal charging method spend less



(a) Scenery at 9 AM in Summer (b) Scenery at 4 PM in Summer (c) Scenery at 9 AM in Winter (d) Scenery at 4 PM in Winter

**Figure 5.6:** The Distribution of EV's Charging Times in the Limited Charging Situation.



(a) Scenery at 9 AM in Summer (b) Scenery at 4 PM in Summer (c) Scenery at 9 AM in Winter (d) Scenery at 4 PM in Winter

**Figure 5.7:** The Distribution of EV's Charging Times in the Sufficient Charging Situation.

time on charging than those which adopt the other 3 methods.

Besides, in Fig. 5.7, under very sufficient charging resources, most EVs which adopt the optimal charging method also relatively spend less time on charging than those which adopt the other 3 methods, which can validate that our relatively optimal solution is also closer to the Pareto-Optimal solution.

## Chapter 6

### CONCLUSION

In this paper, I proposed a central charging management scheme for reducing charging congestion. I set up a congestion game model to analyze and optimize the EV charging process using a new optimized charging method. I discuss the new method in detail and use the solution to optimize the charging process. I not only consider the interaction of group behavior in this model but also reduce the inefficient impact from the individual self-minded behavior of the players successfully. So, I balance System Optimization and User Equilibrium. Besides, our relatively optimal solution is closer to the Pareto-Optimal solution than many traditional optimization methods' solutions. Finally, I perform numerical simulations to validate the advantages of our method. Future works will focus on the optimize the algorithm to reduce the complexity of the computation and thus the computation time.

## BIBLIOGRAPHY

- “Public charging for evs”, URL <https://chargehub.com/en/charging-stations-map.html> (2022).
- Boyd, S., S. P. Boyd and L. Vandenberghe, *Convex optimization* (Cambridge university press, 2004).
- Campbell, C., “Maryland’s utilities propose spending \$ 104 million on statewide electric-vehicle charging network.”, URL <https://www.baltimoresun.com/business/bs-md-electric-vehicles-20180322-story.html> (2018).
- Chatterjee, B., “An optimization formulation to compute nash equilibrium in finite games”, in “2009 Proceeding of International Conference on Methods and Models in Computer Science (ICM2CS)”, pp. 1–5 (IEEE, 2009).
- Chatterjee, K., R. Majumdar and M. Jurdziński, “On nash equilibria in stochastic games”, in “International workshop on computer science logic”, pp. 26–40 (Springer, 2004).
- Correa, J. R. and N. E. Stier-Moses, “Wardrop equilibria”, *Encyclopedia of Operations Research and Management Science*. Wiley (2011).
- Courtney, C., “How long does it take to charge a tesla?”, URL <https://blog.carvana.com/2021/07/how-long-does-it-take-to-charge-a-tesla/> (2021).
- Crawford, V. P., “Learning behavior and mixed-strategy nash equilibria”, *Journal of Economic Behavior & Organization* **6**, 1, 69–78 (1985).
- De Cauwer, C., W. Verbeke, J. Van Mierlo and T. Coosemans, “A model for range estimation and energy-efficient routing of electric vehicles in real-world conditions”, *IEEE Transactions on Intelligent Transportation Systems* **21**, 7, 2787–2800 (2019).
- Edelstein, S., “Tesla supercharger congestion worsens in peak travel periods.”, URL [https://www.greencarreports.com/news/1101675\\_tesla-supercharger-congestion-worsens-in-peak-travel-periods](https://www.greencarreports.com/news/1101675_tesla-supercharger-congestion-worsens-in-peak-travel-periods) (2016).
- Eisner, J., S. Funke and S. Storandt, “Optimal route planning for electric vehicles in large networks”, in “Twenty-Fifth AAAI Conference on Artificial Intelligence”, (2011).
- Gass, V., J. Schmidt and E. Schmid, “Analysis of alternative policy instruments to promote electric vehicles in austria”, *Renewable Energy* **61**, 96–101 (2014).
- Hazan, E. and R. Krauthgamer, “How hard is it to approximate the best nash equilibrium?”, *SIAM Journal on Computing* **40**, 1, 79–91 (2011).
- IEA, “Global ev outlook 2020”, URL <https://www.iea.org/reports/global-ev-outlook2020> (2020).

- Jain, P. and A. Agogino, “Global optimization using the multistart method”, American Society of Mechanical Engineers **3684**, 39–44 (1989).
- Krichene, W., B. Drighès and A. M. Bayen, “Online learning of nash equilibria in congestion games”, SIAM Journal on Control and Optimization **53**, 2, 1056–1081 (2015).
- Lu, F. and G. Hua, “A location-sizing model for electric vehicle charging station deployment based on queuing theory”, in “2015 International Conference on Logistics, Informatics and Service Sciences (LISS)”, pp. 1–5 (IEEE, 2015).
- Malandrino, F., C. Casetti, C.-F. Chiasserini and M. Reineri, “A game-theory analysis of charging stations selection by ev drivers”, Performance Evaluation **83**, 16–31 (2015).
- Monfared, M. S., S. E. Monabbati and A. R. Kafshgar, “Pareto-optimal equilibrium points in non-cooperative multi-objective optimization problems”, Expert Systems with Applications **178**, 114995 (2021).
- Moradipari, A. and M. Alizadeh, “Electric vehicle charging station network equilibrium models and pricing schemes”, Onikle (2018).
- Qiu, G. B., W. X. Liu and J. H. Zhang, “Equipment optimization method of electric vehicle fast charging station based on queuing theory”, in “Applied Mechanics and Materials”, vol. 291, pp. 872–877 (Trans Tech Publ, 2013).
- Rosenthal, R. W., “A class of games possessing pure-strategy nash equilibria”, International Journal of Game Theory **2**, 1, 65–67 (1973).
- Schambers, A., M. Eavis-O’Quinn, V. Roberge and M. Tarbouchi, “Route planning for electric vehicle efficiency using the bellman-ford algorithm on an embedded gpu”, in “2018 4th International Conference on Optimization and Applications (ICOA)”, pp. 1–6 (IEEE, 2018).
- Shao, S., W. Guan, B. Ran, Z. He and J. Bi, “Electric vehicle routing problem with charging time and variable travel time”, Mathematical Problems in Engineering (2017).
- Sohet, B., Y. Hayel, O. Beaude and A. Jeandin, “Coupled charging-and-driving incentives design for electric vehicles in urban networks”, IEEE Transactions on Intelligent Transportation Systems **22**, 10, 6342–6352 (2020).
- Strehler, M., S. Merting and C. Schwan, “Energy-efficient shortest routes for electric and hybrid vehicles”, Transportation Research Part B: Methodological **103**, 111–135 (2017).
- Tushar, W., W. Saad, H. V. Poor and D. B. Smith, “Economics of electric vehicle charging: A game theoretic approach”, IEEE Transactions on Smart Grid **3**, 4, 1767–1778 (2012).



- Voelcker, J., “Are california’s electric-car charging stations too congestion?”, URL <https://www.csmonitor.com/Business/In-Gear/2013/0228/Are-California-s-electric-car-charging-stations-too-congested> (2013).
- Xiao, D., S. An, H. Cai, J. Wang and H. Cai, “An optimization model for electric vehicle charging infrastructure planning considering queuing behavior with finite queue length”, *Journal of Energy Storage* **29**, 101317 (2020).
- Yang, Y., E. Yao, Z. Yang and R. Zhang, “Modeling the charging and route choice behavior of bev drivers”, *Transportation Research Part C: Emerging Technologies* **65**, 190–204 (2016).
- Zhou, Z., S. J. Moura, H. Zhang, X. Zhang, Q. Guo and H. Sun, “Power-traffic network equilibrium incorporating behavioral theory: A potential game perspective”, *Applied Energy* **289**, 116703 (2021).
- Zhu, J., Y. Li, J. Yang, X. Li, S. Zeng and Y. Chen, “Planning of electric vehicle charging station based on queuing theory”, *The Journal of Engineering* , 13, 1867–1871 (2017).
- Zhukovskiy, V. I. and K. N. Kudryavtsev, “Pareto-optimal nash equilibrium: Sufficient conditions and existence in mixed strategies”, *Automation and Remote Control* **77**, 8, 1500–1510 (2016).