Power System Planning for Adverse Climate and Weather Events: Theoretical and

Computational Insights for Incorporating Flexible Transmission Technologies

by

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ABSTRACT

The stable and efficient operation of the transmission network is fundamental to the power system's ability to deliver electricity reliably and cheaply. As average temperatures continue to rise, the ability of the transmission network to meet demand is diminished. Higher temperatures lead to congestion by reducing thermal limits of lines while simultaneously reducing generation potential. Furthermore, they contribute to the growing frequency and ferocity of devasting weather events.

Due to prohibitive costs and limited real estate for building new lines, it is necessary to consider flexible investment options (e.g., transmission switching, capacity expansion, etc.) to improve the functionality and efficiency of the grid. Increased flexibility, however, requires many discrete choices, rendering fully accurate models intractable. This dissertation derives several classes of structural valid inequalities and employs them to accelerate the solution process for each of the proposed expansion planning problems. The valid inequalities leverage the variability of the cumulative capacityreactance products of parallel simple paths in networks with flexible topology, such as those found in transmission expansion planning problems.

Ongoing changes to the climate and weather will have vastly differing impacts a regional and local scale, yet these effects are difficult to predict. This dissertation models the long-term and short-term uncertainty of rising temperatures and severe weather events on transmission network components in both stochastic and robust mixed-integer linear programming frameworks. It develops a novel test case constructed from publicly available data on the Arizona transmission network. The models and test case are used to test the impacts of climate and weather on regional expansion decisions.

DEDICATION

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Chapter 1

INTRODUCTION

1.1 Overview

National and transnational power grids are the largest, most complex machines in the world. Their proper operation is essential to everyday living. This task is becoming more difficult as demand concentrates and expands, and new stressors including renewable generation and electric vehicles are increasingly added to the network. Because of the grid's immense size and importance, research into improving operation and planning for the future is of ongoing interest. The urgency of this research is underscored globally by changing climate and rising temperatures. It is doubly so in the United States, whose energy infrastructure received a D+ grade as recently as 2017 from the American Society of Civil Engineers [22]. On top of this, the economic impacts can be enormous: estimates of the economic costs of electricity loss, such as the northeast blackout of 2003 or the deliberate shutoff in California in 2020 are in the billions.

The work needed to update America's power grid is a mammoth task. The simplest option, building new transmission lines, is both costly and increasingly difficult. Space for such lines is running out. Individuals and their local and state governments are aggressively espousing the ideology of NIMBY: not in my backyard. Yet there is a push to merge regional power systems to help the national grid operate more efficiently. However, these regional systems were not designed to accommodate merging and, in many cases, have not been significantly updated in decades. Advancements in computing power, physical technologies, and smart grid communications have enabled new ways to improve the service provided by the grid at a fraction of the cost of building new and costly transmission lines. However, incorporating these additional options makes an already difficult problem even more so by virtue of too much flexibility. All of this leads to their currently limited inclusion in the grid.

Power systems planning and operation problems are fundamentally network problems. Many optimization methods for solving them fail to take advantage of the systems' network properties. It is essential therefore to develop techniques based on common structural characteristics which can be applied to a wide range of network problems. While there are numerous problem-specific approaches to harness the potential of a particular network's characteristics, identification, and utilization of more general insights can aid in solving a large range of difficult problems.

Even within the field of power system operation, there are a significant number of problems which share several potentially important structural properties. Transmission networks are host to a particularly interesting subset of such problems. The development of the smart grid increasingly relies on rapid communication of system states as well as a growing number of discrete choices such as whether to build new lines, expand the capacity of existing lines, harden particular lines for resilience reasons, or simply switch given lines on and off to mitigate negative effects.

While the explicit application of many of these improved optimization techniques is in transmission planning and operation, they exploit network properties present in many common optimization problems. Such structural insights can be used to produce cutting planes or valid inequalities (VIs) which can help reduce solution times by reducing the solution search space, especially for large problems. When solving linear optimization problems, the solution space is a many dimensional polyhedron. Effective VIs will cut off portions of this solution space that will not be optimal, thus reducing the amount of searching necessary. There are many methods and classes of VIs which are currently employed in power systems literature.

One common approach is Benders' decomposition, a divide-and-conquer approach to break up large linear systems into master problem and subproblem blocks, which is used for applications in system investment planning (e.g. [31], [42], [47], [106]), unit commitment problems (e.g. [63], [137]), and managing the incorporation of new or uncertain technologies (e.g. [101], [110]). However, there are other approaches to generate VIs based on structural insights into the transmission network. The collection of VIs presented in this proposed dissertation falls into the latter approach. The organization of this proposed dissertation is as follows.

Chapter 2¹ considers the most basic case of the transmission expansion planning problem, which minimizes the cost of investment in candidate transmission lines while meeting future demand. It attempts to improve solution times for this problem by exploiting a key network property of transmission networks. Specifically, this chapter presents the initial insights for VIs relating non-adjacent bus angles in transmission expansion planning. This chapter formalizes and extends some of the work from [39], including formal proofs and generalizations of some simplified VIs therein. Furthermore, it discusses the use of a data-driven low-effort heuristic to identify likely paths upon which to base those VIs. It also provides computational testing on modern benchmark instances: the GOC 500-bus system and the Polish 2383-bus system.

Chapter 3^2 generalizes the insights from Chapter 2, considering not only constructing new lines, but also reinforcing current lines to increase their capacity. New

¹Published in Annals of Operations Research [114]

²Published in Institute of Industrial and Systems Engineers Annual Conference Proceedings [116]

notation is defined to simplify the derivation of a more generalized class of VIs than those from Chapter 2, particularly to clarify the use of multiple paths as essential to their generation. An initial case study based on the transmission grid of the state of Arizona as of December 2020 is developed to test the effects of this model in a realistic test case. The grid is divided into regions based on similar climates and the effects of temperature on transmission lines are analyzed.

Chapter 4 extends the deterministic model presented in Chapter 3 to consider the uncertainty associated with rising temperatures. This results in a novel three-level robust optimization formulation for long-term expansion planning problems. A datadriven approach is taken to handling the temperature-based uncertainty which results in a tractable optimization algorithm that maintains global optimality. A k-means clustering algorithm is used to update the regions used in Chapter 3, and the Arizona transmission grid is brought up to date as of December 2021. The VIs developed in Chapter 3 are used to accelerate the solution of the three-level formulation significantly.

Chapter 5 further generalizes the structural insights from Chapters 2 and 3 by considering the problem of optimal transmission switching, in which some existing lines may be disconnected. To demonstrate the novelty of the VIs in this chapter, it is shown that the new VIs are incomparable with the cycle-based VIs for optimal transmission switching previously established in [66]. Since transmission switching is more of an operational problem than a planning problem, Chapter 5 also considers the more short-term effects of rising temperatures: high-impact, low-probability severe weather events. The result is a two-stage stochastic optimization problem with hardening investments in preparation for the weather event in the first stage and transmission switching as a method of recourse in the lower-level.

1.2 Background

This section provides some basic terminology and background for the operation of power systems. It also gives a background for the problems of optimal power flow, transmission expansion planning, and optimal transmission switching.

1.2.1 Power Flow

Two major components of the power grid are key to the delivery of electricity where and when it is needed. The *transmission network* is composed of high voltage wires used to transmit electricity over long distances. In contrast, the *distribution network* consists of lower voltage lines used to transmit electricity shorter distances and to smaller points of consumption such as houses. An overview of the interconnection between these subnetworks which comprise the power grid is provided in Figure 1. Notice that the transmission lines in the diagram operate in the voltage range from 138kV-765kV, whereas distribution lines operate in the 120V-69kV range.



Figure 1. Traditional Electricity Delivery System Source: An Assessment of Energy Technologies and Research Opportunities, Department of Energy, 2015

Of course, this figure is a considerably basic illustration. As mentioned in Section 1.1, technology and communications advancements continue to accumulate within the grid. Figure 2 provides a projection of infrastructure improvements to a future-state smart grid. Such improvements are essential to continue to meet demand, but they are difficult to model and implement effectively.



Figure 2. Smart Grid Technology Roadmap

When discussing transmission networks, it is common to abstract them as a singleline diagram such as in Figure 3. A graph node is referred to as a *bus*, and a graph edge is referred to as a *line* or *branch*. Generators represent any source of power injection to the network, and load refers to any source of demand. Often, there will be both generation and load at a given bus; as such, *net injection* refers to the difference between generation and load at that bus.

To understand how electricity flows through a network, at each bus the net injection must be known, as well as both the magnitude and bus angle of the voltage. Throughout, this *bus angle* refers to the voltage vector angle. Unlike in, say, transportation networks, flows in the transmission network are determined by these values in combination with the physical properties of the lines. Introducing additional power injection via the generator at Bus 1 in Figure 3, for example, will cause additional power to flow to both Bus 2 and Bus 3. Furthermore, the supply side and demand side of electrical networks must balance in real time, due to the conservation of energy. As such, meeting the constraints of a power network is much more complicated than those of many other physical networks.



Figure 3. Basic Single Line Diagram

The general power grid primarily operates using alternating current (AC). AC power flows are given in terms of voltage vectors which are complex numbers by the following two equations:

$$P_{k} = \sum_{j=1}^{N} |V_{k}||V_{j}| \left(G_{kj}\cos(\theta_{k} - \theta_{j}) + B_{kj}\sin(\theta_{k} - \theta_{j})\right)$$
(1.1)

$$Q_{k} = \sum_{j=1}^{N} |V_{j}| |V_{j}| \left(G_{kj} \sin(\theta_{k} - \theta_{j}) - B_{kj} \cos(\theta_{k} - \theta_{j}) \right)$$
(1.2)

where P_k represents the real component of the net power injection at bus k and Q_k represents its imaginary component. That said, direct current (DC) power flows

are still commonly used in standard industry practices for several purposes. The DC power flow approximation to AC power flows assumes that the magnitude of voltage at each bus is 1.0 per unit, that adjacent bus angle differences are sufficiently small, and that resistance is much smaller than reactance within transmission lines. This approximation allows the consideration of only the real component of power as well as a linearization of the power flow equations:

$$P_k = \sum_{j=1, j \neq k}^N B_{kj}(\theta_k - \theta_j).$$
(1.3)

Note that this simplified approximation removes nonlinearities due to both the product of voltage magnitude variables as well as the trigonometric functions. The DC approximation will be adopted throughout this entire proposed dissertation.

1.2.2 Optimal Power Flow

The determination of optimal power flow (OPF) is a notoriously difficult problem which exists in a unique overlap of the fields of electrical engineering and operations research. Most commonly, OPF problems consider either the AC power flow model (ACOPF), or a DC power flow approximation model (DCOPF). Depending on the application, one assumption is more common than the other; for general OPF problems without additional constraints or discrete decision variables, most current research concerns the ACOPF form of the problem. However, the inclusion of constraints for AC power flow based on (1.1) and (1.2) results in a nonlinear, non-convex optimization problem. A standard ACOPF formulation from [24] is presented below to demonstrate the complex mathematical structure of this problem.

$$\min \sum_{i \in N} \mathbf{c}_{2i} (\mathfrak{R}(S_i^g))^2 + \mathbf{c}_{1i} \mathfrak{R}(S_i^g) + \mathbf{c}_{0i}$$
(1.4)

s.t.
$$\mathbf{v}_i^{\mathbf{l}} \le |V_i|\mathbf{v}_i^{\mathbf{u}}$$
 $\forall i \in N$ (1.5)

$$\mathbf{S}_{i}^{\mathbf{gl}} \le S_{i}^{g} \le \mathbf{S}_{i}^{\mathbf{gu}} \qquad \qquad \forall i \in N \quad (1.6)$$

$$|S_{ij}| \le \mathbf{s}_{ij}^{\mathbf{u}} \qquad \qquad \forall (i,j) \in E \cup E^R \quad (1.7)$$

$$S_i^g - \mathbf{S}_i^d = \sum_{(i,j)\in E\cup E^R} S_{ij} \qquad \forall i\in N \quad (1.8)$$

$$S_{ij} = \mathbf{Y}_{ij}^* V_i V_i^* - \mathbf{Y}_{ij}^* V_i V_j^* \qquad \forall (i, j) \in E \cup E^R \quad (1.9)$$
$$-\theta_{ij}^{\mathbf{\Delta}} \leq \angle (V_i V_j^*) \leq -\theta_{ij}^{\mathbf{\Delta}} \qquad \forall (i, j) \in E \quad (1.10)$$

In this formulation, the decision variables are S_i^g and V_i and all norms such as V_i are the standard Euclidean norm. An asterisk such as in the term V_i^* is used to denote the complex conjugate. Then in constraint (1.10), $\angle(V_iV_j^*)$ denotes the angle between the vector V_i and the complex conjugate of the vector V_j . Notice that the objective function is quadratic, and that constraint (1.10), in particular, causes this model to be non-convex.

In comparison, a basic DCOPF formulation is presented below to illustrate some of the advantages this approximation affords. The full explanation of this model is provided in Section 2.2. From this point forward, all work in this proposed dissertation will be based on this DCOPF formulation.

$$\min \sum_{n \in B} c_n g_n$$
(1.11)

s.t.
$$\sum_{(l,i)\in\Omega} P_{li} - \sum_{(i,l)\in\Omega} P_{il} + g_n = d_n$$

$$\forall n \in B, \forall (i,j) \in \Omega$$
(1.12)

$$- \bar{P}_{ij} \leq P_{ij} \leq \bar{P}_{ij}$$

$$\forall (i,j) \in \Omega$$
(1.13)

$b_{ij}\left(\theta_i - \theta_j\right) = P_{ij}$	$\forall (i,j) \in \Omega \ (1.14)$
$0 \le g_n \le \bar{g}_n$	$\forall n \in B \ (1.15)$
$-\bar{\theta} \leq \theta_i - \theta_j \leq \bar{\theta}$	$\forall (i,j) \in \Omega \ (1.16)$
P_{ij}, θ_n, g_n	unrestricted (1.17)

The main advantage of using the DCOPF approximation is that the formulation is a linear program. When different technologies such as transmission switching or transmission expansion are added to this model, the result is a mixed-integer linear program (MILP) which is non-convex but can be solved to global optimality. In particular, the simplified power flow equations lead to a set of constraints based on Kirchoff's Second Law which states that the sum of all the voltages around any directed cycle in a circuit is equal to zero:

$$b_{ij}(\theta_i - \theta_j) = P_{ij}.$$
(1.18)

Although it results in a simplified approximation of ACOPF, DCOPF is frequently used in industry practice, just as DC power flows are used. The most common applications include expansion planning (e.g., [67], [68], [132]), market and pricing problems (e.g., [15], [59], [107], resiliency (e.g., [91], [136]), storage concerns (e.g., [28], [81], [124]), and distributed optimization for microgrids (e.g., [88], [133]). This proposed dissertation focuses on expansion planning applications of the DCOPF model.

1.2.3 Transmission Expansion Planning

Perhaps the most common application for DCOPF is the transmission expansion planning problem (TEP). While there is a growing body of research into TEP with ACOPF assumptions, these efforts are largely focused on heuristic methods. However, as noted above, the DCOPF approximation is commonly used in industry practice [66], especially as a first step, and even with this simplified assumption the problem is known to be NP-Hard [69].

The objective of TEP is to find the least costly investment options in new transmission hardware required to ensure proper power system operations into the future [51]. Optimizing this problem is important because the transmission network belongs to the so-called heavy technologies, which are both expensive and difficult to withdraw or relocate once they are installed [4]. Inadequate long-term planning can lead to low service quality, excessive oversizing, inefficient systems with high operating costs, and delays in the expansion of electricity markets. Even as new systems are growing in size and the demands imposed on them are increasing, deregulation and other challenges have made meeting those requirements ever more difficult [76]. Hence, it is critical to obtain solutions that maximize cost efficiency to enable the incorporation of more avant-garde technologies into the smart grid. For these reasons, it is necessary to devise new planning methodologies that can effectively deal with the associated combinatorial difficulties of the underlying TEP optimization models.

There are as many ways to expand transmission networks as there are purposes for doing so. Traditional TEP considers the addition of a subset of lines connecting buses when there is not already a single line connecting them. Figure 4 illustrates an example single line diagram for a network with a candidate line for TEP; this line, highlighted in green, is dashed to indicate that it has not yet been built. However, as technology has advanced, the scope of TEP has expanded to include further options. For example, the obvious extension of adding multiple lines between two buses has been studied and modeling improvements have been proposed for large numbers of lines [103]. Alternatively, line hardening describes reinforcing transmission or distribution lines to render them resistant or immune to attacks or disasters as in e.g. [17], [33], [131]. One related option is to expand the capacity in an existing line. Again, Figure 4 demonstrates an example of such a line; it is highlighted in purple. Flexible AC Transmission Systems (FACTS) devices – which can modify individual line characteristics to increase controllability of the network and are often used to manage congestion – are also being increasingly considered in planning problems as their cost decreases and feasibility increases [11], [42]. The inherent difficulty of these problems, due in no small part to an exceedingly large number of binary variables, has even inspired methods to automatically identify promising candidate lines for selection [77], [134].

1.2.4 Optimal Transmission Switching

Similar to TEP, optimal transmission switching (OTS) can be formulated as a MILP with binary decision variables, however for OTS those variables represent switching a line to be connected or disconnected. A subset of the lines in Figure 4 represent such switchable lines; they are highlighted in blue and represented as dotted lines. These lines are already built and currently connecting the buses at their endpoints but can be disconnected to not allow flow along them. OTS is a relevant problem both in planning and operation for transmission grids. The authors of [92]



Figure 4. Simple Network With Discrete Transmission Choices considered the possibility of exploiting the effects of Braess' Paradox – the idea that adding additional pathways to a network can hinder its effectiveness – in the operation of transmission networks, and the idea was formalized and refined into a disjunctive MILP by [43]. Although there have been some recent heuristic advances in solving the AC OTS problem, most research to date continues to use the DC OTS approximation (e.g. [44], [129]). Even in very simple cases, DC OTS is known to be NP-Hard [66], and its experimental difficulty is well documented, as in [58].

As with the expansion of TEP to include additional technology options, similar concerns are being raised in the context of OTS. Historically, switching decisions have been made based on operator experience rather than on mathematical optimality due to computational difficulties. Current work implementing OTS models includes real-time contingency analysis [73] and load shed prevention [16]. Just as with TEP, the large number of binary variables introduced to represent the decision of switching even a single line renders this problem NP-Hard [66]. While this at first appears to be a disadvantage, in fact it allows us to exploit similar structural characteristics of the transmission network to help solve both TEP and OTS.

Chapter 2

PATH-BASED VALID INEQUALITIES FOR TRANSMISSION EXPANSION PLANNING

2.1 Background

In its standard form, TEP consists of linear and nonlinear functions that include continuous variables (e.g., voltage angles, power flows, etc.) and integer variables (decisions, e.g., to add lines to the network). TEP can be formulated as a non-convex, mixed-integer nonlinear programming problem. It is NP-hard, which makes its solution generally intractable [69]. This is exacerbated by the fact that in large-scale systems, the number of network components and associated restrictions can number in the hundreds, thousands, or more. That is, the size and/or topology of the transmission network and the inclusion of discrete variables for representing possible transmission investments lead to a combinatorial explosion of potential solutions. In general, and due to these complications, TEP cannot practically be solved using standard optimization techniques. Different modeling techniques and algorithms have been proposed to expedite solution times (e.g., [18], [21], [26], [55], [125]). Exact methods require larger calculation times when compared to those required by metaheuristic techniques such as Tabu Search [45], [48] and Genetic Algorithms [45], [93], among others. However, the latter techniques generally do not provide formal optimality guarantees. In small- and medium-sized systems, the ideal solution can be found using methods such as branchand-bound or branch-and-cut when a disjunctive integer linear programming model approximation is utilized [6], [34], [119]. Such methods provide formal guarantees, but they are demanding computationally, such as decomposition techniques including hierarchical Benders' decomposition (e.g. [13], [54], [105]). Additionally, recent work has used Benders' decomposition techniques to solve generation and transmission expansion planning together [61]. The valid inequalities presented in this chapter can be seen as a complementary technique for solution time reduction to these exact methods.

2.1.1 Aim and Contributions

This chapter considers a DCOPF-based mixed-integer programming version of the *static* TEP problem which consists of a single investment period occurring at the beginning of the planning horizon and is a subproblem of the dynamic TEP problem. The choice of this model helps illustrate the computational intractability of TEP even for this basic context and is useful for various practical studies. Moreover, it highlights the potential of the fundamental insights introduced herein to be extended to a variety of more complex TEP models with a similar core structure (e.g. [13], [100], [123], etc.). Explicitly, this chapter derives and implements a set of theoretical contributions for detecting and including structural information on the underlying network, which is relevant to any DCOPF-based model that incorporates the linear relationship between bus angle differences and power flows (i.e., " $B - \theta$ " constraints) into the constraint set. The lemmas proved herein generalize the relationship first formalized in [13] between the shortest path problem, the longest path problem, and the big-M coefficients used in the standard TEP model. Specifically, they consider arbitrary paths – either alone or in parallel – and their derivation and implementation is independent of the longest path problem. It is important to highlight that the

techniques introduced in [13] only apply to static topologies (i.e., the big-M coefficients do not take advantage of potential topologies that can result from the addition of new lines). On the other hand, the main theorem proved herein (i.e., Theorem (3)) is the first to explicitly tighten the big-M constraints mentioned here for dynamic topologies in TEP, which permits improvements beyond considering only the static topology. The insights presented in this chapter may be applied to a variety of problem classes, since this structural information is common to many power system formulations. Such insights are captured via the concept of valid inequalities, which represent one of the most effective exact solution techniques and are a highly active research area in mathematical programming [25].

Other papers have explored structural insights based on bus angle differences, which serve as the inspiration of this chapter. In particular, in [39], a subset of the classes of valid inequalities introduced in this chapter were applied in an ad hoc manner, in particular, only those from the herein included lemmas, which are proved in this chapter for the first time. It is worthwhile to explain that, while these lemmas provide insights for the major theorems derived in this chapter, and their implementation could produce coincidental improvement in Gurobi due to the ordering of constraints, it is analytically impossible for them to reduce the linear relaxation solution space of TEP. This is because the valid inequalities presented in these lemmas are simply linear combinations of the original set of constraints. In short, the cited work lacked the systematic and theoretical depth featured in this chapter from an operations research perspective. This chapter formally establishes the validity of two classes of valid inequalities used therein via two lemmas, plus one additional class, proved via a theorem, which can in fact reduce the solution space of the linear relaxation.

In addition to these theoretical contributions, this chapter provides techniques for

applying the theory in the form of a heuristic algorithm used to help find promising candidate valid inequalities (also referred to herein as cuts). These techniques are then used to perform computational experiments that show the effectiveness of the proposed valid inequalities in reducing the solution time of two modified benchmark instances. While their effectiveness is shown herein for the static TEP context, the reduction in solution time would be amplified in, for example, stochastic programming approaches to TEP. In these approaches, many scenarios need to be solved with each using the same collection of valid inequalities, since the first-stage decisions usually involve the structure of the network. A similar argument holds for solving the multi-period TEP.

The structure of the chapter is as follows: Section 2 introduces the disjunctive model used for modeling TEP. Section 3 presents the key insights and intuition for deriving and generating the valid inequalities. Section 4 contains the main contribution of this chapter, namely the lemmas and theorem which prove the validity of the proposed cuts. Section 5 presents numerical results from testing the application of these theorems on two different test cases, and Section 6 summarizes the conclusions drawn from these results.

2.2 Modeling Framework

The nonlinear ACOPF model for TEP can be transformed into a mixed-integer linear model with bilinear equations [132]. This model is itself transformed into a disjunctive model with binary variables, which requires the incorporation of a large enough disjunctive coefficient (big-M). In the disjunctive model, a binary variable is considered for each candidate line, which converts the original mixed-integer non-linear program into a mixed-integer linear program (MILP). The DCOPF-based model is appropriate for TEP. First, it is widely used in industrial practice, especially for planning purposes [66]. Additionally, this approach is the most common classical optimization approach in the literature [76], [115]. Finally, long-term planning is primarily concerned with active power rather than reactive power, and consequently the assumption in DCOPF that active power is much larger than reactive is reasonable. The main concerns that are only captured with an AC model (e.g., stability of the network) can be incorporated in a more short-term, operational perspective [77]. The full model is as follows.

The objective function (2.1) is to minimize the joint cost of generation and investments in new lines, with investment considered to be performed at the beginning of the planning horizon:

$$\min \sum_{(i,j)\in\Omega} \sum_{k=1}^{\bar{\omega}_{ij}} c_{ij,k} y_{ij,k} + \sum_{n\in B} \sigma c_n g_n.$$
(2.1)

Here, $c_{ij,k}$ is the cost of each line in corridor (i, j) and binary variable $y_{ij,k}$ represents the decision to add the k^{th} candidate line in corridor (i, j); when $y_{ij,k} = 1$, the kth candidate line is added in corridor (i, j). Additionally, $\bar{\omega}_{ij}$ is the maximum number of candidate lines considered in corridor (i, j), and Ω is the set of expansion corridors in the expansion plan. Finally, note that the generation costs in the objective function are weighted by a factor σ to make generation costs and planning costs comparable [86]. The model's constraints are as follows:

$$\sum_{(n,i)\in\Omega} \left(\sum_{k=1}^{\omega_{ij}^0} P_{ni,k}^0 + \sum_{k=1}^{\bar{\omega}_{ij}} P_{ni,k} \right) - \sum_{(i,n)\in\Omega} \left(\sum_{k=1}^{\omega_{ij}^0} P_{in}^0 + \sum_{k=1}^{\bar{\omega}_{ij}} P_{in,k} \right) + g_n = d_{\overline{n}} \forall n \in B \quad (2.2)$$

$$-\overline{P}^{0}_{ij,k} \le P^{0}_{ij,k} \le \overline{P}^{0}_{ij,k} \qquad \qquad \forall (i,j) \in \Omega, k \in \{1 \dots \omega^{0}_{ij}\}$$
(2.3)

$$-\overline{P}_{ij,k}y_{ij,k} \le P_{ij,k} \le \overline{P}_{ij,k}y_{ij,k} \qquad \forall (i,j) \in \Omega, k \in \{1 \dots \overline{\omega}_{ij}\}$$
(2.4)

$$\frac{-1}{b_{ij,k}}P_{ij,k}^0 - (\theta_i - \theta_j) = 0 \qquad \qquad \forall (i,j) \in \Omega, k \in \{1 \dots \omega_{ij}^0\} \quad (2.5)$$

$$-M_{ij}(1 - y_{ij,k}) \le \frac{-1}{b_{ij,k}} P_{ij,k} - (\theta_i - \theta_j) \le M_{ij}(1 - y_{ij,k}) (i,j) \in \Omega, \ k \in \{1 \dots \bar{\omega}_{ij}\}$$
(2.6)

$$g_{n} \leq \overline{g}_{n} \qquad \forall n \in B \quad (2.7)$$

$$-\overline{\theta} \leq \theta_{i} - \theta_{j} \leq \overline{\theta} \qquad \forall (i, j) \in \Omega \quad (2.8)$$

$$y_{ij,k} \in \{0, 1\} \qquad \forall (i, j) \in \Omega, k \in \{1 \dots \overline{\omega}_{ij}\} \quad (2.9)$$

$$g_{n} \geq 0, \ \theta_{n} \text{ unr.} \qquad \forall n \in B \quad (2.10)$$

$$P_{ij,k}^{0}, P_{ij,k} \text{ unr.} \qquad \forall (i, j) \in \Omega, k \in \{1 \dots \overline{\omega}_{ij}\} \quad (2.11)$$

Constraint (2.2) interrelates the active power flows that arrive at and leave bus n through both existing and candidate lines and the demand and supply of active power at bus n. Constraint (2.3) represents the limit of active power flow through the current network in corridor (i, j), where $P_{ij,k}^0$ is the power flow in the k^{th} existing line. Constraint (2.4) represents the limit of active power flow through the candidate lines in corridor (i, j). Constraints (2.5) and (2.6) together represent Kirchhoff's second law, either for each existing line or each candidate line to be added to the transmission system, respectively; stated otherwise, these constraints reflect the link between incident buses i and j. Constraint (2.6) becomes active when the decision variable $y_{ij,k}$ takes the value of 1, i.e. when that candidate line is built; otherwise, a sufficiently large big-M parameter M_{ij} ensures that the constraint is extraneous for the model. Finding the best value for M_{ij} requires solving a shortest path problem in connected networks but a longest path problem in disconnected networks, which is itself an NP-hard problem [13]. Because power networks are generally connected including in the instances tested herein, except for perhaps a handful of considered buses, these problems are solved while pre-processing the networks, to use the best possible big-M parameter for each pair of buses. Constraint (2.7) presents the limits of the active power supply for generators, where a bus n with no generator is assumed to have $\bar{g}_n = 0$. Constraint (2.8) enforces the maximum bus angle difference for adjacent bus-pairs $(i, j) \in \Omega$, i.e. those bus-pairs connected by a corridor. Finally, (2.9), (2.10) and (2.11) give the variable domains.

2.3 Motivating the Path-based Valid Inequalities

Due to the combinatorial explosion of TEP, it is not possible to find an optimal solution for large-scale systems using standard, off-the-shelf algorithms. The computational difficulty of the problem is related directly to the size of the system to be analyzed. However, other factors increase computational difficulty, including the connectivity of the buses or how well the system is enmeshed. This is complicated by the "Braess Paradox," according to which adding more transmission lines can create a more inefficient system [92].

To solve NP-hard problems, it is often useful to investigate the structural characteristics of a particular instance. This knowledge can be highly valuable when it comes to designing effective exact solution methods [25], [127]. One key application of this knowledge is to derive valid inequalities (VIs): additional problem constraints that preserve the original solution space \mathcal{P} but may otherwise reduce an associated relaxed solution space $\mathcal{P}^R \subseteq \mathbb{R}^n$, where $\mathcal{P} \subset \mathcal{P}^R$. Formally, for the set $\mathcal{P} \subset \mathbb{R}^n$, the coefficient vector $\boldsymbol{\pi} = (\pi_1, \ldots, \pi_n) \in \mathbb{R}^n$, and the constant $\pi_0 \in \mathbb{R}$, the inequality $\boldsymbol{\pi} \mathbf{y} \leq \pi_0$ is called a *valid inequality* for \mathcal{P} if it is satisfied by all points $\mathbf{y} \in \mathcal{P}$ (i.e., \mathcal{P} is the TEP solution space). Because the solution of MILP typically proceeds by solving a sequence of linear relaxations, adding structurally useful VIs as cutting planes can reduce the number of such linear problems solved in a branch-and-bound framework, thus decreasing the computational time necessary to solve the overall problem [127]. The proposed method seeks to provide mechanisms that reduce the size of the solution space by incorporating structural information of TEP that can eliminate unpromising settings of the decision variables.

The structural insights derived in this chapter stem from the relationships between the bus angle and power flow decision variables that characterize DCOPF-based transmission system models. Specifically, if there is an existing line with index k in corridor $(i, j) \in \Omega$, with i < j, an angular VI relating the difference between θ_i and θ_j can be obtained through $P_{ij,k}$ (the flow along the line), as follows:

$$\theta_i - \theta_j = \frac{-1}{b_{ij,k}} P_{ij,k} = x_{ij,k} P_{ij,k} \le x_{ij,k} \bar{P}_{ij,k}, \qquad (2.12)$$

where b_{ij} , x_{ij} , and $\bar{P}_{ij,k}$ are the line susceptance, line reactance, and flow capacity, respectively. The right-hand side of this inequality is referred to henceforth as a capacity-reactance product and is used to improve angular VIs presented herein. Note that (2.12) is a direct result of (2.4)-(2.6). The main insight of this chapter is to leverage such adjacent-bus VIs to derive formal restrictions on *non-adjacent* buses and on buses connected via *multiple* parallel paths in the network. That is, the TEP model (and the DCOPF model, generally) provides only simple angular constraints for the buses that are directly connected via a transmission line. However, by forming a single path connecting adjacent buses in the transmission network, these VIs can be combined into potentially tighter *path-based* constraints relating the initial bus angle and the terminating bus angle of said path and the sequence of flow restrictions of each corridor along the path. Even stronger restrictions may be obtained from the combination of VIs along parallel paths—two otherwise disjoint paths which share initial and terminating buses—by taking the tighter of the separate bus angle difference restrictions or, equivalently, flow restrictions. An example application of these insights is illustrated in Figure 5 via a stylized bus-line diagram consisting

of bus set $B = \{i_0, i_1, i_2\}$, corridor set $\Omega = \{(i_0, i_1), (i_0, i_2), (i_1, i_2)\}$, and single lines between each pair of buses with reactances $x_{i_0,i_1} = x_{i_1,i_2} = x, x_{i_0,i_2} = 3x$ and capacities $\bar{P}_{i_0,i_1} = \bar{P}_{i_1,i_2} = \bar{P}_{i_0,i_2} = \bar{P}$. For this simple example, and for all future numerical examples, we assume that there can be at most one existing line and at most one candidate line per corridor. This allows us to increase visual clarity by dropping the third index of each variable.



Figure 5. The two path-based VIs adjacent to lines (i_0, i_1) and (i_1, i_2) can be combined to create a tighter VI for line (i_0, i_2) , which is provided in the bottom box.

In Figure 5, three path-based VIs (adjacent to each transmission line) are obtained by considering the capacity-reactance products of every pair of buses in the network (see (2.12)). Moreover, by combining two of these VIs, a tighter VI for bus angles θ_{i_0} and θ_{i_2} is obtained (see the boxed expression). It is important to remark that this constraint would be valid even in the absence of a direct transmission line between θ_{i_0} and θ_{i_2} , i.e. if it were an expansion corridor. In larger networks, many such VIs can be constructed, which may or may not tighten the model's simple bus angle difference constraints. In highly enmeshed electric systems, the number of parallel paths can increase exponentially, depending on the specific network properties [64]. Consequently, it may be prohibitive to identify and verify the strength of each possible VI for large-size systems. Instead, this chapter will identify the most effective of these constraints and provide data-driven insights through the use of relaxation models that are easier to solve.

We utilize the above ideas to generate a set of structurally useful VIs based on the flows in the solution to TEP that follow single and parallel paths. To this end, we make use of three relaxed models. By solving a subset of these models, each of which takes significantly less time to solve than the full MILP, we can generate a set of *structural backbones*. These are flow patterns that suggest single paths and parallel paths that are more likely to occur than others in the solution to the original problem. In particular, for any single path or parallel paths which share a common flow direction in the solution of each of a combination of relaxation models, we consider adding a VI. The technique of using these relaxation models in this way will be denoted the *low-effort heuristic*, first implemented in a non-algorithmic way in [39]. Three models are used: the *linear* model, where the restriction on the binary variables $y_{ij,k}$ is relaxed, allowing them to be continuous within the interval [0, 1]; the *transportation* model, where the restriction that flows on all lines obey (2.5) and (2.6) is relaxed; and the *hybrid* model, which is similar to the transportation model, but in which only (2.6) is relaxed.

2.4 Path-based Angular Valid Inequalities Derivation and Theorems



Figure 6. Toy Network Used to Illustrate the Main Theorem

This section will introduce the main theorem which is the fundamental contribution of this chapter. For this purpose, a graph with candidate lines (dotted edges) and existing lines (solid edges) is presented in Figure 6. An example of these lines can be seen between buses i_1 and i_2 , where there is one candidate line and one existing line. This graph will be used to illustrate an application of each lemma and theorem.

We say then that (i, j) is an established corridor of G if $\omega_{i,j}^0 > 0$ (i.e., there is an existing line along the corridor); otherwise we say that (i, j) is an expansion corridor. To better clarify instances when we must distinguish among individual lines within each corridor along a path, we introduce the vector $\hat{k}_{\rho} = \left\langle k_{i_0i_1}, \ldots, k_{i_{|\rho|-1}i_{|\rho|}} \right\rangle \subseteq \left\langle \{1, \ldots, \omega_{i_0i_1}^0\}, \ldots, \{1, \ldots, \omega_{i_{|\rho|-1}i_{|\rho|}^0}^0\} \right\rangle$ to denote any vector of valid line-indices k_{i_j} within each established corridor (i, j) along a path ρ . Then, for ease of presentation, we refer to x_{i_jk} , where k encapsulates a valid setting of element ij of vector \hat{k} , i.e. $k \in \{1, \ldots, \omega_{i_j}^0\}$. For example, if the expansion line (i_1, i_2) in Figure 6 is built, then both $\langle 1, 1 \rangle$ and $\langle 1, 2 \rangle$ are valid settings of the elements in path $(i_0, i_1), (i_1, i_2)$. Thus, in each upcoming lemma and proof, whenever k is used as a line index, it is shorthand for k_{ij} when there is no ambiguity. Additionally, because these problems traditionally specify corridors from a lower-index bus to a higher-index bus, we define $\tilde{P}_{ij,k} = sgn(j-i) \cdot P_{ij,k}$, where sgn(i-j) = 1 if i > j and sgn(i-j) = -1 if i < j.

2.4.1 Single Path over Established Corridors

Lemma 1. Let $\rho = (i_0, i_1), \ldots, (i_{|\rho|-1}, i_{|\rho|})$ represent a directed path over established corridors in G. For $(i, j) \in \rho$, set coefficient vector $\boldsymbol{\pi} = (\pi_0, \pi_1, \ldots, \pi_{|\rho|}) \in \mathbb{R}^{|\rho|+1}$ as,

$$\pi_{j} = \begin{cases} \sum_{(i,m)\in\rho} x_{im,k} \cdot \overline{P}_{im,k}^{0}, & \text{if } j=0\\ sgn(i-j)x_{ij,k}, & \text{otherwise} \end{cases},$$

$$(2.13)$$

where $k \in \{1, ..., \omega_{ij}^0\}$ is fixed for each corridor (i, j), but may vary between corridors. Then the following two-sided inequality is valid for TEP for any \hat{k}_{ρ} :

$$-\pi_0 \le \sum_{(i,j)\in\rho} \pi_j \tilde{P}^0_{ij,k} \le \pi_0.$$
(2.14)

Proof. According to (2.5), the flow along any fixed, existing line k of corridor (i, j) is given by

$$\tilde{P}_{ij,k}^0 = sgn(i-j)b_{ij,k}(\theta_i - \theta_j), \qquad (2.15)$$

or equivalently,

$$(\theta_i - \theta_j) = \pi_j \tilde{P}^0_{ij,k}, \qquad (2.16)$$

where $(i, j) \in \rho$. Hence, the bus angle difference for consecutive bus-pairs (i_0, i_1) , $(i_1, i_2), (i_2, i_3), \ldots, (i_{|\rho|-1}, i_{|\rho|})$ in ρ can be written as:

$$\begin{split} \theta_{i_1} - \theta_{i_0} &= sgn\left(i_0 - i_1\right) x_{i_0 i_1, k} \tilde{P}^0_{i_0 i_1, k} = \pi_1 \tilde{P}^0_{i_0 i_1, k}, \\ \theta_{i_2} - \theta_{i_1} &= sgn\left(i_1 - i_2\right) x_{i_1 i_2, k} \tilde{P}^0_{i_1 i_2, k} = \pi_2 \tilde{P}^0_{i_1 i_2, k}, \\ &\vdots \\ \theta_{i_{|\rho|}} - \theta_{i_{|\rho|-1}} &= sgn\left(i_{|\rho|-1} - i_{|\rho|}\right) x_{i_{|\rho|-1} i_{|\rho|}, k} \tilde{P}^0_{i_{|\rho|-1} i_{|\rho|}} \end{split}$$

$$= \pi_{|\rho|} \tilde{P}^{0}_{i_{|\rho|-1}i_{|\rho|},k}.$$

When these equations are summed, this creates a telescoping effect on the left-hand side, which yields the following bus angle difference equation for the starting and ending buses in ρ :

$$\theta_{i_{|\rho|}} - \theta_{i_0} = \sum_{(i,j)\in\rho} \pi_j \tilde{P}^0_{ij,k} \tag{2.17}$$

$$\leq \sum_{(i,j)\in\rho} \left| \pi_j \tilde{P}^0_{ij,k} \right| \tag{2.18}$$

$$\leq \sum_{(i,j)\in\rho} x_{ij,k} \overline{P}^0_{ij,k} = \pi_0, \qquad (2.19)$$

where the latter inequality is obtained by adding the rightmost inequalities from (2.3). By a similar argument we have that,

$$\sum_{(i,j)\in\rho} \pi_j \tilde{P}^0_{ij,k} \ge -\sum_{(i,j)\in\rho} \left| \pi_j \tilde{P}^0_{ij,k} \right|$$
(2.20)

$$\geq -\sum_{(i,j)\in\rho} x_{ij,k} \overline{P}^{0}_{ij,k} \tag{2.21}$$

$$= -\pi_0. \tag{2.22}$$

Since every corridor considered has at least one existing line to select and fix as k, and (2.15) holds for any line in corridor (i, j), this establishes the validity of (5.30).

As an example using Figure 6 the path $\rho^2 := (i_0, i_4), (i_4, i_5)$ creates the example two-sided VI:

$$-\overline{P}_{i_0,i_4} x_{i_0,i_4} - \overline{P}_{i_4,i_5} x_{i_4,i_5}$$

$$\leq P_{i_0,i_1} x_{i_0,i_1} + P_{i_1,i_2} x_{i_1,i_2}$$

$$\leq \overline{P}_{i_0,i_4} x_{i_0,i_4} + \overline{P}_{i_4,i_5} x_{i_4,i_5}$$
On the same note, in Figure 6, the path $\rho^1 := (i_0, i_1), (i_1, i_2), (i_2, i_5)$ is an established path, which creates the example two-sided VI:

$$-\overline{P}_{i_0,i_1} x_{i_0,i_1} - \overline{P}_{i_1,i_2} x_{i_1,i_2} - \overline{P}_{i_2,i_5} x_{i_2,i_5}$$

$$\leq P_{i_0,i_1} x_{i_0,i_1} + P_{i_1,i_2} x_{i_1,i_2} + P_{i_2,i_5} x_{i_2,i_5}$$

$$\leq \overline{P}_{i_0,i_1} x_{i_0,i_1} + \overline{P}_{i_1,i_2} x_{i_1,i_2} + \overline{P}_{i_2,i_5} x_{i_2,i_5}$$

2.4.2 Parallel Paths over Established Corridors

Lemma 2. Let ρ^1, \ldots, ρ^m represent m > 1 alternative directed paths over established corridors in G with the same starting/ending buses but with non-overlapping intermediate buses; that is, $i_0^r = i_0^{r'}$, $i_{|\rho^r|}^r = i_{|\rho^{r'}|}^{r'}$, and $\{i_k^r\}_{k=1}^{|\rho^r|-1} \cap \{i_k^{r'}\}_{k=1}^{|\rho^{r'}|-1} = \emptyset$ for $1 \le r, r' \le m$ with $r \ne r'$. Setting coefficient vectors $\pi^r = (\pi_0^r, \pi_1^r, \ldots, \pi_{|\rho^r|}^r) \in \mathbb{R}^{|\rho^r|+1}$ according to (2.13) for each path ρ^r , the following two-sided inequalities are valid for TEP for any \hat{k}_{ρ}^r :

$$-\min\{\pi_0^n\}_{n=1}^m \le \sum_{(i,j)\in\rho^r} \pi_j^r \tilde{P}_{ij,k}^0 \le \min\{\pi_0^n\}_{n=1}^m \text{ for } r=1,\dots,m.$$
(2.23)

Proof. Since paths ρ^r and $\rho^{r'}$ share the same starting/ending buses, this gives that $\theta_{i_{|\rho^r|}} - \theta_{i_0^r} = \theta_{i_0^{r'}} - \theta_{i_0^{r'}} - \theta_{i_0^{r'}}$, or equivalently, with k defined as in Lemma 1,

$$\sum_{(i,j)\in\rho^r} \pi_j^r \tilde{P}^0_{i_{j-1}^r i_j^r,k} = \sum_{(i,j)\in\rho^{r'}} \pi_j^{r'} \tilde{P}^0_{i^{r'} j^{r'},k} \quad \text{for } 1 \le r, \, r' \le m \; \text{ with } r \ne r'$$

according to the respective telescoped bus angle difference equations of the starting and ending buses associated with each path (e.g., see (2.17)). Thus, the proof is completed by joining together the two-sided inequalities,

$$-\pi_0^r \le \sum_{(i,j)\in\rho^r} \pi_j^r \tilde{P}_{ij,k}^0 \le \pi_0^r \text{ for } r = 1, \dots, m,$$

each of which is valid due to Lemma 1.

Continuing the example from subsection 4.1, in Figure 6, ρ^1 creates an established parallel path with ρ^2 . Assuming that path ρ^2 is the path with lower capacity-reactance product creates the example two-sided VI:

$$\begin{aligned} &-\bar{P}_{i_0,i_4} \, x_{i_0,i_4} - \bar{P}_{i_4,i_5} \, x_{i_4,i_5} \\ &\leq P_{i_0,i_1} \, x_{i_0,i_1} + P_{i_1,i_2} \, x_{i_1,i_2} + P_{i_2,i_5} \, x_{i_2,i_5} \\ &\leq \bar{P}_{i_0,i_4} \, x_{i_0,i_4} + \bar{P}_{i_4,i_5} \, x_{i_4,i_5} \end{aligned}$$

2.4.3 Parallel Paths over Established and Expansion Corridors

Consider two buses, θ_n and θ_m , in a network. Let \mathcal{C} denote the set of all paths starting at θ_n and ending at θ_m . For any path $\rho_r \in \mathcal{C}$, let $CR(\rho_r) = \sum_{(i,j)\in\rho_r} x_{ij}\bar{P}_{ij}$ denote the cumulative capacity-reactance product of one line from each corridor along that path. Let $\bar{\rho}$ denote a path from this set such that $\overline{CR(\rho)} = \max\{CR(\rho_r)\}$, and let ρ similarly denote a path from this set such that $\underline{CR(\rho)} = \min\{CR(\rho_r)\}$. Further, let $N_e(\rho_r)$ denote the number of expansion corridors in the path ρ_r . Note that the theorem below is stated and proved in the context of a network that meets the assumptions of all tested instances for simplicity of presentation: namely that all candidate lines for a given corridor, (i, j) have identical properties (e.g., susceptance, capacity, etc.), so that additionally we can order the candidate lines. In other words, $y_{ij,k+1} \leq y_{ij,k}$. However, the result can be easily generalized by considering *line paths*, where the path is along individual lines rather than corridors.

Theorem 3. The following are valid inequalities for TEP, for all paths $\rho_r \in C$:

$$|\theta_n - \theta_m| \le CR(\rho_r) + \left(\overline{CR(\rho)} - CR(\rho_r)\right) \left(N_e\left(\rho_r\right) - \sum_{(i,j)\in\rho_r} \mathbb{I}_{ij}y_{ij,1}\right), \quad (2.24)$$

where \mathbb{I}_{ij} is used as shorthand for the indicator function $\mathbb{I}\left(\omega_{i_{j-1},i_j}^0=0\right)$ (i.e. to identify expansion corridors).

Furthermore, let $\mathcal{C}^0 \subseteq \mathcal{C}$ denote the set of paths comprised solely of established corridors, with ρ_r^0 denoting an element of this set. Additionally, let $\underline{CR(\rho^0)} = \min\{CR(\rho_r^0)\}$. If \mathcal{C}^0 is nonempty, then the above inequalities can be strengthened as follows:

$$|\theta_n - \theta_m| \le CR(\rho_r) + \left(\underline{CR(\rho^0)} - CR(\rho_r)\right) \left(N_e(\rho_r) - \sum_{(i,j)\in\rho_r} \mathbb{I}_{ij}y_{ij,1}\right)$$
(2.25)

Before proving this theorem, it is worth noting specifically the ways in which it generalizes the basic result in [13] about the big-M coefficient in the standard TEP model. The referenced paper gives the big-M parameter as $M_{kl} \ge x_{kl}^1 C_{kl}^{\min}$, where C_{kl}^{min} is the solution to the shortest path or longest path problem between bus k and bus l when the buses are connected or disconnected, respectively. For simple networks, Theorem 3 reduces to these cases as is shown in the proof. However, the formulation in [13] does not permit multiple lines within the same corridor, and it considers only the static topology of the network. Theorem 3 improves the calculation of big-M coefficients in the TEP formulation by considering multiple lines in each corridor and many topologies that can be realized through the addition of new lines. By doing so, it is capable of producing tighter bounds by considering arbitrary paths. Note that in connected networks with one line per corridor, Lemma 1 to the shortest path problem, while the longest path problem is only relevant when adding new buses or on networks with islands. On the other hand, Theorem 3 can be applied to improve on the big-M coefficient obtained from the shortest path problem through the selection of any pair of paths which share the same initial and terminating buses. The proof of Theorem 3 follows.

Proof. The telescoped bus angle difference equation (2.17) can be written if and only if corridors $(i_0, i_1), \ldots, (i_{|\rho|-1}, i_{|\rho|})$ are each serviced by transmission lines (i.e., all consecutive bus-pairs must be connected). We then consider two cases: either there are no expansion corridors in ρ_r (or all expansion corridors in ρ_r have at least one candidate line built) or there is at least one expansion corridor in ρ_r with no candidate line built.

Case 1: There are no expansion corridors in ρ_r , or all expansion corridors in ρ_r have at least one candidate line built.

Since $\mathbb{I}_{ij,1} = 0$ indicates that there are existing lines servicing corridor (i, j), the equation $\left(N_e\left(\rho_r\right) - \sum_{(i,j)\in\rho_r}\mathbb{I}_{ij}y_{ij,1}\right) = 0$ holds if and only the path ρ_r consists entirely of serviced corridors, that is, for any expansion corridor in the path ρ_r , at least one candidate line has been built. In this case, the arguments from Lemma 1 hold for the path ρ_r , and we have that $|\theta_n - \theta_m| \leq \pi_0$. However, note that $\pi_0 = \sum_{(i,j)\in\rho_r} x_{ij}\overline{P}_{ij}^0 = CR(\rho_r)$, so in fact we have $|\theta_n - \theta_m| \leq CR(\rho)$, for all r.

Case 2: There is at least one expansion corridor in ρ_r with no candidate line built.

In this case, we have $\left(N_e(\rho_r) - \sum_{(i,j)\in\rho_r} \mathbb{I}_{ij}y_{ij,1}\right) \geq 1$. Then in all cases, the inequality $|\theta_n - \theta_m| \leq \overline{CR(\rho)}$ holds. That is, the bus angle difference between θ_n and θ_m is bounded by the largest possible cumulative capacity-reactance product along any path between those buses. Additionally, if θ_n and θ_m are connected by a path ρ_r that consists *solely of established corridors*, then by Lemma 1 we again have $|\theta_n - \theta_m| \leq \pi_0 = CR(\rho_r)$. In fact, by Lemma 2, given any collection of alternative directed paths, $\{\rho^1, \ldots, \rho^r\}$, we have $|\theta_n - \theta_m| \leq \min\{\pi_0^k\}_{k=1}^r$. Similarly to case 1, note that by selecting $\{\rho^1, \ldots, \rho^r\}$ to be all paths solely along established corridors

from θ_n to θ_m , $\min{\{\pi_0^k\}_{k=1}^r} = \underline{CR(\rho^0)}$, thus the inequality $|\theta_n - \theta_m| \leq \underline{CR(\rho^0)}$ holds. Thus the inequalities (2.24) and (2.25) are valid in all cases.

As an example of the new valid inequalities described in this theorem, consider again Figure 6. In this figure, we consider the single paths $\rho^3 :=$ $(i_0, i_1), (i_1, i_2), (i_2, i_5), (i_5, i_6), (i_6, i_3)$ and $\rho^4 := (i_0, i_4), (i_4, i_5), (i_5, i_6), (i_6, i_3)$. Additionally, a new single path, ρ^5 is created when line (i_2, i_3) is added where $\rho^5 :=$ $(i_0, i_1), (i_1, i_2), (i_2, i_3)$ which creates the following VI:

$$\begin{aligned} |\theta_0 - \theta_3| &\leq \sum_{(i,j) \in \rho^5} \bar{P}_{ij} x_{ij} + \\ & \left(\min\left\{ \sum_{(i,j) \in \rho^3} \bar{P}_{ij} x_{ij}, \sum_{(i,j) \in \rho^4} \bar{P}_{ij} x_{ij} \right\} - \sum_{(i,j) \in \rho^5} \bar{P}_{ij} x_{ij} \right) (1 - y_{i_2,i_3}) \end{aligned}$$

One important result to note about this theorem is how the coefficients on the right hand side relate to the M_{ij} values in (2.6). Case 1 can be seen as simply summing those constraints in (2.6) for each $(i, j) \in \rho_k$, using the best calculated values of big-Mas described in section 2 (that is, either by a shortest path problem if bus i is connected to bus j or a longest path problem otherwise). Case 2 allows the conditional use in this summation of the tighter big-M calculated by a shortest path problem, if enough candidate lines have been built to connect bus i and bus j. This is what permits these VIs to provide a strictly smaller solution space than their linear relaxation.

To illustrate the potential of these VIs, consider Figure 1 again but with the line connecting bus i_1 to bus i_2 as a candidate line instead of an existing line. Then one VI provided by this theorem is

$$|\theta_{i_2} - \theta_{i_0}| \le 2x\bar{P} + (3x\bar{P} - 2x\bar{P})(1 - y_{i_1i_2})$$

$$\Rightarrow |\theta_{i_2} - \theta_{i_0}| \le 2x\bar{P} + x\bar{P}(1 - y_{i_1i_2})$$
(2.26)

By comparison, the best constraints (including linear combinations of constraints) relating these two buses in the original TEP model are

$$|\theta_{i_2} - \theta_{i_0}| \le 3x\bar{P} \tag{2.27}$$

$$|\theta_{i_2} - \theta_{i_1}| \le 4x\bar{P}(1 - y_{i_1 i_2}) \tag{2.28}$$

$$|\theta_{i_1} - \theta_{i_0}| \le x\bar{P} \tag{2.29}$$

$$|\theta_{i_2} - \theta_{i_0}| \le x\bar{P} + 4x\bar{P}(1 - y_{i_1i_2}), \tag{2.30}$$

where (2.27)-(2.29) are directly from (2.5) and (2.6), and (2.30) is the sum of (2.28) and (2.29).

If \mathcal{P}_{LR} is the polytope of the linear relaxation of the original TEP model for this simple network, and \mathcal{P}'_{LR} is the polytope of the linear relaxation of the original TEP model together with (2.26), then obviously $\mathcal{P}'_{LR} \subseteq \mathcal{P}_{LR}$. In fact, it can be demonstrated that $\mathcal{P}'_{LR} \subset \mathcal{P}_{LR}$. To find a solution in \mathcal{P}'_{LR} , but not in \mathcal{P}_{LR} , assume without loss of generality that $\theta_{i_1} \geq \theta_{i_2}$, then the following system of inequalities relating bus angles θ_{i_0} and θ_{i_2} (which represents a point satisfying (2.27) and (2.30) but violating (2.26)) must be satisfied:

$$\theta_{i_2} - \theta_{i_0} > 2x\bar{P} + x\bar{P}(1 - y_{i_1 i_2}) \tag{2.31}$$

$$\theta_{i_2} - \theta_{i_0} \le 3x\bar{P} \tag{2.32}$$

$$\theta_{i_2} - \theta_{i_0} \le x\bar{P} + 4x\bar{P}(1 - y_{i_1 i_2}). \tag{2.33}$$

By joining the right hand sides of (2.31) with (2.33) and (2.32) and (2.33), this system must then satisfy

$$2x\bar{P} + x\bar{P}(1 - y_{i_1i_2}) < 3x\bar{P} \tag{2.34}$$

$$2x\bar{P} + x\bar{P}(1 - y_{i_1i_2}) < x\bar{P} + 4x\bar{P}(1 - y_{i_1i_2})$$
(2.35)

It is easy to see that (2.34) is true when $y_{i_1i_2} > 0$ and (2.35) is true when

$$x\bar{P} < 3x\bar{P}(1 - y_{i_1i_2})$$
$$\Rightarrow \quad 1 < 3(1 - y_{i_1i_2})$$
$$\Rightarrow y_{i_1i_2} < 2/3.$$

That is, (2.34) and (2.35) are both satisfied when $0 < y_{i_1i_2} < 2/3$. For example, the point $y_{i_1i_2} = 0.5$, $\theta_{i_2} = 2.75x\bar{P}$, $\theta_{i_0} = 0$ satisfies inequalities (2.31) - (2.33), i.e., it is in \mathcal{P}_{LR} but not \mathcal{P}'_{LR} .

2.5 Computational Tests and Results

The experiments consider two TEP instances: the GOC 500-bus instance and the Polish 2383-bus instance. These are first solved without adding any valid inequalities. They are then solved with valid inequalities according to the low-effort heuristic described in Section 2.3. The details of the tested instances and the results associated with each are contained in Section 2.5.1 and Section 2.5.2, respectively. The algorithmic steps of the experiment are as follows:

- 1. The low-effort heuristic method, explained in Section 2.3, is applied. This includes solving a selection of the three relaxations, i.e., a given subset of the linear relaxation, the transportation relaxation, and the hybrid relaxation.
- 2. The solution flows from the chosen relaxations are overlaid onto the same graph. Simple single paths of same-direction flows are found connecting each pair of buses in the network using a breadth-first search. For larger instances, the number of such paths for any fixed pair of buses is capped at 200 to prevent memory issues.

- 3. All single, simple paths with the same initial and final bus are combined to form pairs of parallel paths.
- 4. The original problem is passed to Gurobi for optimization. Each time a new incumbent solution is found, a random pair of parallel paths is used to generate two VIs, which are added as lazy constraints. If all paths have been used, the optimization simply continues.

It should be noted that in each of the tested instances, all candidate lines for a given corridor, (i, j), have identical properties (e.g., susceptance, capacity, cost, etc.). Because of this property, we enforce the additional set of symmetry-breaking constraints $y_{ij,k+1} \leq y_{ij,k}$, $\forall k \in \{1 \dots \bar{\omega}_{ij} - 1\}$, since each line is interchangeable. First, testing is performed on the GOC 500-bus instance from [14] to showcase the potential for the effectiveness of the proposed path-based VIs in a relatively small size instance. This instance is used to test the algorithm in both high- and low-demand scenarios. Then, testing is performed on the Polish 2383-bus system in order to show their effectiveness in a more realistically sized and designed instance. The algorithm is implemented in Python, and the disjunctive model is solved using Gurobi version 9.0.2. All tests are run on the ASU High Performance Computing Agave Cluster, which has compute nodes with two Intel Xeon E5-2680 v4 CPUs running at 2.40 GHz.

2.5.1 GOC 500-Bus System

This is a synthetic system designed for the ARPA-e Grid Optimization Competition (GOC), developed in [14]. The instance is composed of 500 buses, 224 generators, and 732 transmission corridors. Of those, 193 are designated as candidates for expansion (i.e., these corridors have no existing lines). We tested this system with two different

demand and congestion profiles, representing a scenario of peak demand and one of off-peak demand. Congestion scaling was done as in [132] to account for the effect of high temperatures on transmission line capacity, coinciding with the peak demand scenario. In both cases, the algorithm specified at the beginning of this section was used to solve the system 50 times for all eight possible combinations of the relaxation models.

Relaxation Models	Average Computational Times (secs)
TR	5.99
HR	4.34
LR	5.50
$\mathrm{TR} \oplus \mathrm{HR}$	5.61
$TR \oplus LR$	5.51
$\mathrm{HR}\oplus\mathrm{LR}$	5.49
$\mathrm{TR}\oplus\mathrm{HR}\oplus\mathrm{LR}$	5.45
N/A	48.16

Table 1. Low Demand GOC 500-Bus Results

Table 1 summarizes the results from the low demand, low congestion scenario. For this table and for future tables, "N/A" refers to the time spent solving the model with no VIs added (i.e., with none of the relaxation models solved). The table entries report the average runtimes in seconds. For each combination of relaxation models, at least two and no more than forty valid inequalities were added to the model, which results in a minimal increase in computational effort. Solving solely the hybrid relaxation model to generate the VIs produced the fastest average computation time of 4.34 seconds, but any combination of relaxation models resulted in at least an 8x improvement in solution time over the model with no VIs added, which took 48.16 seconds.

Table 2 summarizes the results from the high demand, high congestion scenario.

Relaxation Models	Average Computational Times (secs)
TR	95.46
HR	116.59
LR	84.33
$TR \oplus HR$	96.71
$TR \oplus LR$	92.98
$HR \oplus LR$	104.87
$TR \oplus HR \oplus LR$	115.59
N/A	1916.01

Table 2. High Demand GOC 500-Bus Results

Note that all solution times for this scenario are larger than those for the low demand, low congestion scenario. In this case, solving the hybrid relaxation alone or with both the linear and transport relaxations resulted in the longest average computation time (116.59 and 115.59 seconds, respectively). However, as the original model with no VIs added took 1916.01 seconds to solve to optimality, even these slower solves are a 16x improvement in solution speed. Solving only the linear relaxation produced the greatest improvement in computation time. This took only 84.33 seconds on average, for a roughly 22x improvement in solution speed.

2.5.2 Polish 2383-Bus System

We use the Polish 2383-bus system adapted for TEP in [86], which consists of 2383 buses, 327 generators, and 2896 total corridors. The system has been modified as follows: while the original 2383-bus system has candidate lines along established corridors, these options were removed and 120 of the existing lines have been removed and replaced with one candidate line each, while the remaining 2776 corridors do not allow for any expansion. This modified instance is available upon request from the corresponding author. Due to the size of this instance, solution times were dramatically increased when compared to the GOC 500-bus instance. Table 3 thus reports computation times in *minutes* instead of seconds to help clarify the difference in performance for this model.

Relaxation Models	Average Computational Times (mins)
TR	495.36
HR	262.94
LR	263.71
$\mathrm{TR} \oplus \mathrm{HR}$	455.27
$\mathrm{TR}\oplus\mathrm{LR}$	450.53
$\mathrm{HR}\oplus\mathrm{LR}$	262.60
$TR \oplus HR \oplus LR$	312.45
N/A	8055.27

Table 3. Polish 2383-Bus Results

The box and whisker plot in Figure 7 helps demonstrate most strongly the scale of the improvements afforded by the VIs. The three best improvements in computation time are provided when solving solely the linear relaxation, solely the hybrid relaxation, or the two relaxations in conjunction to generate VIs. Each of these cases solves in approximately 263 minutes (4.4 hours), which is a 30x improvement in speed over solving the 2383-bus model without any VIs (144 hours). Although using the transportation model improves solution times, even in combination with other models, the overall improvement is much lower than in the cases which do not solve the transportation model. This suggests that both the hybrid and linear relaxations provide better VIs than the transportation model.



Figure 7. Box and Whisker Plot of 2383-Bus Improvements 2.6 Conclusions

This chapter presents a new mathematical framework and an algorithm that uses a mixed-integer linear programming model, valid inequalities, and a low-effort heuristic method for solving TEP. The objective is to reduce the total computational effort of planning. It is a significant improvement of the preliminary studies carried out in [39], in which the solutions were found after manual analysis of the test system, creation of cuts using two classes of the valid inequalities introduced in this chapter (specifically from Lemmas 1 and 2), which at that time had been implemented without proof, and tests made with different cut combinations. However, in this chapter, each step of the process is automated and formally establishes the validity of three types of valid inequalities.

Computational tests show the effectiveness of the presented theorem in generating valid inequalities which reduce the solution time of TEP, up to an 8x improvement. They also suggest how to best apply the theorem for use in solving multiple test cases, as well as how they may be of use in the solution of larger scale problems. Additionally, the results demonstrate different options for the implementation of these valid inequalities that offer distinct trade-offs in efficiency in the various stages of the solution process, which provides options for approaching instances of varying sizes and expected computational effort.

Chapter 3

DETERMINISTIC TRANSMISSION EXPANSION PLANNING WITH CAPACITY EXPANSION PLANNING

3.1 Background

Environmental stressors are among the main causes of power system disturbances worldwide [95]. High temperatures, for instance, can limit the transfer capability of transmission lines by increasing energy losses and line sagging [95]. Rising temperatures also disrupt demand patterns, as they have been proved to be positively correlated with extreme temperatures during summertime [83]. Preparing power systems for adverse weather is a challenging task since the frequency of these events is increasing, but their precise occurrence in time and space is not known ahead of time. Increases in power system robustness and reliability are among the most common long-term methods to guarantee the operation of the network during extreme circumstances. Generation Expansion Planning (GEP) and Transmission Expansion Planning (TEP) have been proposed as possible pathways to adapt the system to new conditions [72]. TEP seeks to reinforce the transmission network and provide a stable supply even under worst-case scenarios. TEP has both economic and engineering reliability objectives; this makes the problem a complex case of multi-objective optimization [46]. A common approach to solving the TEP problem is the use of the DC optimal power flow (DCOPF) approximation. In this formulation, both economic dispatch and optimal power flow are considered in modeling the problem. The DC formulation offers a good approximation of the AC power flow for planning purposes, especially since it is faster and easier to solve [128]. The impacts of climate change, and specifically rising ambient temperature, have been studied from the generation and demand perspective in generation and transmission expansion problems [80], [102], [111]. TEP has also been solved using a decentralized approach in which the electricity network was divided into regions to account for differences in demand and supply sides [27], [56]. On the other hand, the effects of rising temperatures on transmission lines themselves were analyzed and estimated to account for capacity reduction in [7]. None of these studies combines regional temperature modeling together with temperature's effect on transmission lines. This chapter intends to bridge this gap.

3.1.1 Aims and Contribution

To solve the TEP problem with regional temperature considerations, this chapter proposes a DCOPF-based formulation with discrete transmission decisions including transmission capacity expansion. Other works (e.g., [95], [98]) have considered the resilience of power networks when modeling the short-term effects of weather at the regional level. These works focus on modeling the probabilistic failure of transmission components effected by windstorms. Windstorms were assumed to act homogenously across each of eight arbitrary regions based on historical time-series profiles. The failure function of each component type was shared by all such components. Separately, the effects of temperature on transmission lines and on capacity expansion have been studied [68], in the context of appropriate modeling choices for high-temperature, low-sag conductors. The authors allow both transmission expansion and transmission capacity expansion as options to counter long-term degradation of transmission components. Monte Carlo simulations are conducted to account for line failure due to degradation, but the optimization model is deterministic. This chapter aims to use combine the temperature modeling of [68] with an improvement of the regional modeling of [95], [98]. Furthermore, this chapter endeavors to establish a modern test case already embedded in extant geography (i.e., not an IEEE test instance superimposed on a location) and based entirely on publicly available information.

There are three main contributions in this chapter. The first is the inclusion of a capacity reduction factor to model the effect of high temperatures on transmission lines. The second is the division of the electricity network into climate regions motivated by EPA designated ecoregions to serve as a bridge between traditional approaches for TEP and those studying the impact of climate change on transmission networks. The final contribution is the creation of a case study on the Arizona transmission network. The analysis on this test network covers 16 different discrete scenarios that account for differences in ambient temperature projections across regions determined from an analysis of historical data.

3.2 Methodology

For the purpose of this chapter, the methodology first characterizes temperaturedriven planning scenarios. This involves estimating lower and upper bounds for ambient temperature, dividing Arizona into climate regions, and determining the implications of rising temperature on the capacity of transmission lines. These data are used as inputs for the featured TEP model, which considers two types of transmission investment options: expansion via new lines and expanding the capacity of existing lines. Data collection was performed using ambient temperature historical registers provided by the National Oceanographic and Atmospheric Administration (NOAA) and its National Centers for Environmental Information [38]. This consists of 70 years' worth of data in daily increments, reporting maximum, minimum, and average daily temperature for selected locations across the state of Arizona. This chapter includes many simplifying assumptions and serves as a base case for later chapters. In particular, Chapter 4 relaxes these assumptions and uses a principled approach for characterizing uncertainty.

3.2.1 Definition of Climate Regions

The climate regions utilized in this chapter are based on the level II ecoregions defined by the Environmental Protection Agency; note the explanation for the development of these regions is provided in [94]. Arizona was then divided into four major regions, with each county being assigned to a region based on the ecoregion which comprises the largest area within that county. Figures 8a-8c demonstrate the transmission network, climate regions and corresponding county designations respectively.







(a) Transmission Network (b) EPA Designated Ecore-(c) County Region Definitions gions

Figure 8. Climate Region Definitions

3.2.2 Estimation of Temperature Bounds

Datasets incorporating daily temperature records for the period 1950-2019 were downloaded from NOAA for four representative urban centers (Phoenix, Tucson, Flagstaff, and Douglas). The 10 highest temperatures of each of these years were averaged to avoid outliers, and then plotted to analyze the behavior of maximum temperatures over time. The time series also provided enough data to perform regression analysis on the trend for peak annual temperature. This value corresponds to the expected maximum temperature for a given year. The highest maximum temperature per year was determined by selecting the data points above the mean trend line; such values and their associated years used as inputs to another regression analysis which provided a linear equation for the overall maximum temperature trend. Both equations serve as a linear estimation for the future trend (30 years ahead), following the approach used in [49]. An example of these regressions for the city of Phoenix is presented in Figure 9.



Figure 9. Phoenix Temperature Bound Linear Regressions

These values were validated by comparison to the expected increase predicted by NOAA, often used as the reference forecast temperatures in other studies. NOAA forecasts temperature increases impacted by CO_2 emissions. The low emissions scenario projects increases in temperature of 2.7°F, 3.6°F, and 4.7°F by 2035, 2055 and 2085, respectively. The expected values due to high emissions are 3°F, 4.8°F, and 8°F for the same years [57]. By comparison, the projected trends computed from our regression analysis for Phoenix, AZ, suggest an increase of approximately 2.6°F by 2035, 3.60°F by 2055 and 5.1°F by 2085. These align closely with the intervals for low and high emissions scenarios provided by NOAA, hence validating their suitability for this study.

3.2.3 Impact of Temperature on Transmission Line Capacity

The main consequence of rising temperatures on bulk power systems is the reduction in both transmission and generation capacity and the corresponding increase in demand [7], [8]. Considering the scope of this study, the estimation of the impact of rising temperatures is focused on electricity transmission. Following the Institute of Electrical and Electronics Engineers (IEEE) Standard 738-2006, the formula to calculate the reduction in the ampacity of the transmission lines also used in [1], [7], [62], [82], will be applied in this paper. The formula, derived from the energy balance equation, is as follows:

$$I = \sqrt{\frac{\pi \cdot \overline{h} \cdot D \cdot (T_{cond} - T_{amb}) + \pi \cdot \epsilon \cdot \sigma \cdot D \cdot (T_{cond}^4 - T_{amb}^4) - \delta \cdot D \cdot a_s}{R \cdot T_{cond}}} \qquad (3.1)$$

Where I is the fractional multiplier to the rated capacity of conductor (ampacity), \overline{h} is the average heat transfer coefficient, D is the diameter of the conductor, T_{cond} and T_{amb} are the average conductor temperature and ambient temperature, respectively. This first set of terms corresponds to losses due to convection. The second set of terms, corresponding to the loss due to radiation, includes the emissivity of the conductor surface (ϵ), the Stefan-Boltzmann constant (σ), and the diameter (D), plus the conductor temperature, and the ambient temperature. In the last set of terms δ is the maximum solar radiation, and a_s is the absorptivity of the conductor surface. The diameter of the conductor is also included to account for the heat gain from solar radiation. The dividing term R corresponds to the AC resistance of the conductor.

3.3 Model Formulation

One major change to the TEP model introduced in Section 2.2 is the inclusion of the ability to perform capacity expansion along established corridors. Capacity expansion refers to increasing the capacity of transmission or distribution lines as in [78], [113] via technologies such as high temperature low sag conductors (HTLS) [68]. It can also be used as an approximation of other changes in the network, such as adding additional lines to a corridor or increasing the susceptance of lines (e.g. [138]). It is assumed here that the capacity of any existing line can be expanded without restricting the specific technology used for capacity expansion. In order to simplify the discussions and proofs in Section 3.4, the further assumption that there can only exist one line within any corridor is introduced. The model presented in this section is a deterministic model such as the one presented in Chapter 2; it is helpful to examine this model before considering the more complicated robust uncertainty model in Chapter 4.

$$\min\sum_{(i,j)\in\Omega} \left(c_{ij}y_{ij} + h_{ij}z_{ij}\right) + \sum_{n\in B} \sigma c_n g_n \tag{3.2}$$

$$s.t. \sum_{(n,i)\in\Omega} (P_{ni}^{0} + P_{ni}) - \sum_{(i,n)\in\Omega} (P_{in}^{0} + P_{in}) + g_n = d_n \qquad \forall n \in B \quad (3.3)$$

$$- \overline{P}_{ij}^{0} - \overline{P}_{ij}^{1} z_{ij} \leq P_{ij}^{0} \leq \overline{P}_{ij}^{0} + \overline{P}_{ij}^{1} z_{ij} \qquad \forall (i,j) \in \Omega \setminus \Psi \quad (3.4)$$

$$- \overline{P}_{ij} y_{ij} \leq P_{ij} \leq \overline{P}_{ij} y_{ij} \qquad \forall (i,j) \in \Psi \quad (3.5)$$

$$\frac{-1}{b_{ij}} P_{ij}^{0} - (\theta_i - \theta_j) = 0 \qquad \forall (i,j) \in \Omega \setminus \Psi \quad (3.6)$$

$$- M_{ij}(1 - y_{ij}) \leq \frac{-1}{b_{ij}} P_{ij} - (\theta_i - \theta_j) \leq M_{ij}(1 - y_{ij}) \qquad \forall (i,j) \in \Psi \quad (3.7)$$

$$0 \leq g_n \leq \overline{g}_n \qquad \forall n \in B \quad (3.8)$$

$$- \overline{\theta} \leq \theta_i - \theta_j \leq \overline{\theta} \qquad \forall (i,j) \in \Omega \setminus \Psi \quad (3.10)$$

$$y_{ij} \in \{0,1\} \qquad \forall (i,j) \in \Psi \quad (3.11)$$

$$z_{ij} \in \{0,1\} \qquad \forall (i,j) \in \Omega \setminus \Psi \quad (3.12)$$

$$g_n, \theta_n \text{ unr.} \qquad \forall n \in B \quad (3.13)$$

The objective function of TEP considering capacity expansion, (3.2), is similar to (2.1), with the additional costs of investing in capacity expansion included. As before, y_{ij} represents the decision to build a candidate line in corridor (i, j). Similarly, z_{ij} represents the decision to replace the line in corridor (i, j) with one which is expanded, increasing its capacity for active power flow.

Constraints (3.3),(3.5) - (3.9) correspond to (2.2), (2.4) - (2.8) respectively from the TEP model in Chapter 2. (3.4) represents the limit on active power flow along the existing line (i, j), which is increased by \overline{P}_{ij}^1 if $z_{ij} = 1$, i.e. if the line is expanded. The remaining constraints represent domain limits on the decision variables.

3.3.1 Grid Instantiation

The case study presented in this chapter is an approximation of the transmission grid contained within the borders of Arizona; the basic description of this network is provided by the U.S. Energy Information Administration (EIA) [121]. This data is maintained and regularly updated by the EIA; the grid tested in this chapter is based on data that was current as of December, 2020. The EIA dataset includes information on transmission line rated voltage, substation locations, and generation capacities and fuel types. Notably, it does not have information on transmission line capacity. Transmission lines rated at 69kV and above are included in the network as are all power plants rated to produce two or more MW/hr. Additional corridors for transmission expansion are approximated by connecting high generation areas to high demand areas and connecting substations with few adjacent lines to more dense areas of the grid. All listed substations are used as buses to connect transmission lines and meet aggregated demand for nearby areas.

For the purposes of the experiments presented in Section 3.5, the featured test case makes the following additional assumptions. Hourly demand for the test instance is based on peak summer demand for the full state. As the state level is the smallest resolution data available for load, this value is disaggregated by assigning load to substations according to the relative population in the nearest census block. Generation costs are approximated by the total statewide costs for generating each category of fuel (natural gas, coal, petroleum, hydro, wind, and solar), divided according to the rated MW of the corresponding plant. All existing lines are assumed to be permissible for capacity expansion. Candidate expansion lines are chosen based on lines that are designated not in use (i.e., disused or not yet constructed) as well as a number of "promising" candidate lines. The additional candidate lines can comprise no more than 5% of the total number of existing lines in the network. Transmission expansion and capacity expansion costs are approximated based on the rated voltage and length of lines, using cost estimates from [87], which account for corridor acquisition, number of needed lines, and physical properties of appropriate transmission lines and towers. For replicability, the dataset is included in Appendix A.1 in MATPOWER format.

3.4 Valid Inequalities

The results presented in this section build on the work in Chapter 2. For convenience, we first present the notation from that work then update it for capacity expansion. The initial result from there, Lemma 1, is restated here to illustrate the updated notation, and because the central argument is essential to the new results derived below.

The valid inequalities proposed below leverage the relationship between the bus angle and power flow variables that characterize DCOPF-based transmission system models similarly to those presented in Chapter 2. To refresh some of the notation, consider two buses, θ_n and θ_m , in a network. Let C_{nm} denote the set of all paths starting at θ_n and ending at θ_m . For any path $\rho \in C_{nm}$, let $CR(\rho) = \sum_{(i,j)\in\rho} x_{ij}\overline{P}_{ij}$ denote the cumulative capacity-reactance product of each corridor along that path. $CR(\rho)$ includes the capacity-reactance product of candidate lines but not the expanded capacity-reactance product of existing lines along ρ . Let $\overline{\rho}$ denote a path from this set such that $\overline{CR(\rho)} = \max\{CR(\rho)\}$. Furthermore, let $C_{nm}^0 \subseteq C_{nm}$ denote the set of paths from θ_n to θ_m comprised solely of established corridors, with ρ^0 denoting an element of this set. Analagously, let $\overline{CR(\rho^0)} = \max\{CR(\rho^0)\}$. Let $CR^r(\rho) = \sum_{(i,j)\in\rho} x_{ij} \left(\bar{P}^0_{ij} + \bar{P}^1_{ij}\right)$ denote the cumulative capacity-reactance product of each line along ρ with each line also expanded for increased capacity if it can be. Let $CR^r(\rho \setminus (i,j)) = CR^r(\rho) - \bar{P}^1_{ij}x_{ij}$, i.e. every line along ρ except (i,j) has been expanded for increased capacity.

Lemma 4. Let $\rho \in C_{nm}$ represent a directed path from θ_n to θ_m over established corridors in G. Then the following two-sided inequality is valid for TEP:

$$-CR(\rho) \le \theta_n - \theta_m \le CR(\rho), \tag{3.15}$$

or equivalently

$$|\theta_n - \theta_m| \le CR(\rho). \tag{3.16}$$

Proof. We provide an abbreviated sketch of this proof based on the proof of Lemma 2, highlighting the portion that is referenced throughout this chapter.

Let $\rho := (i_0, i_1), \ldots, (i_{|\rho|-1}, i_{|\rho|})$. We can then write the telescoping sum:

÷

÷

$$-\overline{P}_{i_1i_2}x_{i_1i_2} \le \theta_{i_1} - \theta_{i_2} \le \overline{P}_{i_1i_2}x_{i_1i_2} \tag{3.17}$$

$$-\overline{P}_{i_{j-1}i_j}x_{i_{j-1}i_j} \le \theta_{i_{j-1}} - \theta_{i_j} \le \overline{P}_{i_{j-1}i_j}x_{i_{j-1}i_j}$$
(3.19)

(3.20)

$$-\overline{P}_{i_{k-1}i_k}x_{i_{k-1}i_k} \leq \theta_{k-1} - \theta_k \leq \overline{P}_{i_{k-1}i_k}x_{i_{k-1}i_k}.$$
(3.21)

Then, since every corridor of ρ is an established corridor, we have that $-CR(\rho) \leq \theta_n - \theta_m \leq CR(\rho)$ is valid.

3.4.1 Derivation

This section proceeds to derive VIs similar to those from Section 2.4 for the problem of TEP with capacity expansion. Two theorems are presented to generalize those results. The first only considers two paths along established corridors, as was done for the simpler TEP problem in Lemma 2, while the second allows one path to include expansion corridors, similar to Theorem 3. Notably, the second path in both of these theorems is automatically assumed to be the longest possible path along solely established corridors, explicitly through the terms $\overline{CR^r(\rho^0)}$ and $\overline{CR^r(\rho)}$.

Theorem 5. For an established path $\rho \in C_{nm}$ from θ_n to θ_m , the following are valid inequalities for TEP with capacity expansion:

$$|\theta_n - \theta_m| \le CR^r \left(\rho \setminus (i, j)\right) + \left(\overline{CR^r(\rho^0)} - CR^r \left(\rho \setminus (i, j)\right)\right) z_{ij} \qquad \forall (i, j) \in \rho$$

Furthermore, let \mathcal{P} be the polytope of the linear relaxation of (3.2) - (3.14). For an individual VI that defines a half-space V, if

$$\overline{P}_{ij}^1 > \overline{CR^r(\rho^0)} - CR^r\left(\rho \setminus (i,j)\right),$$

then $V \cap P \subset P$.

Proof. Given the two buses θ_n and θ_m and a path $\rho \in C_{nm}^0$ between them, let $(i, j) \in \rho$ be any corridor along the path. If $z_{ij} = 0$, the stated VI is given by

$$\left|\theta_{n}-\theta_{m}\right| \leq CR^{r}\left(\rho\setminus\left(i,j\right)\right).$$

This results from taking the telescoping sum as in Lemma 4.

Otherwise, if $z_{ij} = 1$, the stated VI is given by

$$|\theta_n - \theta_m| \le CR^r \left(\rho \setminus (i,j)\right) + \overline{CR^r(\rho^0)} - CR^r \left(\rho \setminus (i,j)\right) = \overline{CR^r(\rho^0)}$$

But, again by the telescoping sum as in Lemma 4,

$$|\theta_n - \theta_m| \le CR^r(\rho),$$

and since ρ is an established path and by the definition of $\overline{CR^r(\rho^0)}$, we obtain $CR^r(\rho) \leq \overline{CR^r(\rho^0)}$, so that $|\theta_n - \theta_m| \leq \overline{CR^r(\rho^0)}$, hence the stated inequality is valid.

To establish the second part of the theorem, we construct a point $\mathbf{x} \in \mathcal{P}$ which does not satisfy the given valid inequality given the additional condition. The idea is to find a point sufficiently close to a specific boundary of \mathcal{P} such that our VI is sufficiently tight to create a non-trivial gap between the point and any point satisfying the VI.

To that end, suppose that $\overline{P}_{ij}^1 > \overline{CR^r(\rho^0)} - CR^r(\rho \setminus (i, j))$. Without loss of generality, define $\rho := (i_1, i_2), (i_2, i_3), \dots, (i, j) \dots (i_{k-1}, i_k)$, where $i_1 = n, i_k = m$. Consider a point $\mathbf{x} \in \mathcal{P}$ such that $y_{ab} = 0, z_{ab} = 1$ for all (a, b) and with $\theta_n - \theta_m = CR^r(\rho \setminus (i, j)) + \overline{P}_{ij}^1 x_{ij}$. We can see such a point exists by taking the telescoping sum as above for each corridor in ρ :

$$\theta_{i_1} - \theta_{i_2} \le \overline{P}^0_{i_1 i_2} x_{i_1 i_2} + \overline{P}^1_{i_1 i_2} x_{i_1 i_2} z_{i_1 i_2}$$
(3.22)

$$\theta_i - \theta_j \le \overline{P}^0_{ij} x_{ij} + \overline{P}^1_{ij} x_{ij} z_{ij} \tag{3.24}$$

÷

$$\theta_{k-1} - \theta_k \le \overline{P}^0_{i_{k-1}i_k} x_{i_{k-1}i_k} + \overline{P}^1_{i_{k-1}i_k} x_{i_{k-1}i_k} z_{i_{k-1}i_k}.$$
(3.26)

By substituting the given values of each z (namely 1 for all z) and simplifying the sum, we get $\theta_{i_1} - \theta_{i_k} = \theta_n - \theta_m \leq CR_{ij}^r(\rho) + \overline{P}_{ij}^1 x_{ij}$. That is, we satisfy all the inequalities relevant to our specified variables in the model: namely those described in (3.4) and (3.7).

However, note that the inequality $|\theta_n - \theta_m| \leq CR^r (\rho \setminus (i, j)) + (\overline{CR^r(\rho^0)} - CR^r (\rho \setminus (i, j))) s_{ij}$ is valid, as established in the first part of this theorem. It defines a half space, V. Substituting $z_{ij} = 1$ simplifies this to $|\theta_n - \theta_m| \leq \overline{CR^r(\rho^0)}$. That is, any point $\mathbf{y} \in V$ satisfies this inequality. However, at the point \mathbf{x} , note that

$$\theta_n - \theta_m = CR^r \left(\rho \setminus (i, j)\right) + \overline{P}_{ij}^1$$

> $CR^r \left(\rho \setminus (i, j)\right) + \overline{CR^r(\rho^0)} - CR^r \left(\rho \setminus (i, j)\right)$
= $\overline{CR^r(\rho^0)},$

i.e. $\theta_n - \theta_m > \overline{CR^r(\rho^0)}$. This means that $\mathbf{x} \notin V$, and thus $\mathbf{x} \notin V \cap \mathcal{P}$ as desired. \Box

Theorem 6. For any path $\rho \in C_{nm}$ from θ_n to θ_m in a network in which θ_n to θ_m are connected, the following are valid inequalities for TEP with capacity expansion:

$$\begin{aligned} |\theta_n - \theta_m| &\leq CR^r \left(\rho \setminus (i, j)\right) + \\ & \left(\overline{CR^r(\rho)} - CR^r \left(\rho \setminus (i, j)\right)\right) \left(N_e(\rho) - \sum_{(l,k) \in \rho \cap \Psi} y_{lk}\right) + \\ & (CR^r(\rho) - CR^r \left(\rho \setminus (i, j)\right)) z_{ij} \qquad \forall (i, j) \in \rho. \end{aligned}$$

Furthermore, because the network is connected, these inequalities can be strengthened for all existing paths $\sigma^0 \in C_{nm}$ such that $CR(\sigma^0) \ge CR^r (\rho \setminus (i, j))$ as

$$\begin{aligned} |\theta_n - \theta_m| &\leq CR^r \left(\rho \setminus (i, j)\right) + \\ & \left(CR^r(\sigma^0) - CR^r \left(\rho \setminus (i, j)\right)\right) \left(N_e(\rho) - \sum_{(l,k) \in \rho \cap \Psi} y_{lk}\right) + \\ & \left(CR^r(\rho) - CR^r \left(\rho \setminus (i, j)\right)\right) z_{ij} \qquad \qquad \forall (i, j) \in \rho. \end{aligned}$$

Proof. First note that if θ_n and θ_m are disconnected buses in the existing network, then the results of Chapter 2 imply the first part of this theorem even when considering capacity expansion, since the only way to connect those buses is exclusively by building candidate lines (which cannot then be expanded, since $z + y \leq 1$).

Therefore, we proceed to prove the 'furthermore' portion of the theorem.

Given the two buses θ_n and θ_m and a path ρ between them, let $(i, j) \in \rho$ be any corridor along the path. Consider the following cases:

Case 1:

$$z_{ij} = 0, \ \sum_{(l,k) \in \rho \cap \Psi} y_{lk} = N_e(\rho)$$

This is the case in which line (i, j) is not expanded (i.e., does not have its capacity increased) and every candidate line along ρ is constructed. Then, by taking the telescoping sum as in Corollary 4, the stated VI is given by

$$|\theta_n - \theta_m| \le CR^r \left(\rho \setminus (i, j)\right)$$

Since ρ connects θ_n and θ_m along open lines without expanded capacity, this is valid.

Case 2:

$$z_{ij} = 0, \ \sum_{(l,k) \in \rho \cap \Psi} y_{lk} < N_e(\rho)$$

This is the case in which line (i, j) is not expanded (i.e., does not have its capacity increased) and some candidate line along ρ is not constructed. Then the stated VI is given by

$$\begin{aligned} |\theta_n - \theta_m| &\leq CR^r \left(\rho \setminus (i,j)\right) + \left(CR^r(\sigma^0) - CR^r \left(\rho \setminus (i,j)\right)\right) \left(N_e(\rho) - \sum_{(l,k) \in \rho \cap \Psi} y_{lk}\right) \\ &\leq CR^r(\sigma^0). \end{aligned}$$

By Theorem 3, this inequality is valid, since (i, j) is not expanded. Case 3:

$$z_{ij} = 1, \ \sum_{(l,k) \in \rho \cap \Psi} y_{lk} = N_e(\rho)$$

This is the case in which line (i, j) is expanded (i.e., has its capacity increased) and every candidate line along ρ is constructed. Then the stated VI is given by

$$|\theta_n - \theta_m| \le CR^r \left(\rho \setminus (i, j)\right) + CR^r(\rho) - CR^r \left(\rho \setminus (i, j)\right) = CR^r(\rho).$$

By the definition of $CR^r(\rho)$, this inequality is necessarily valid. Case 4:

$$z_{ij} = 1, \ \sum_{(l,k) \in \rho \cap \Psi} y_{lk} < N_e(\rho)$$

This is the case in which line (i, j) is expanded (i.e., has its capacity increased) and some candidate line along ρ is not constructed. Then the stated VI is given by

$$\begin{aligned} |\theta_n - \theta_m| &\leq CR^r \left(\rho \setminus (i, j)\right) + \left(CR^r(\sigma^0) - CR^r \left(\rho \setminus (i, j)\right)\right) \left(N_e(\rho) - \sum_{(l,k) \in \rho \in \Psi} y_{lk}\right) + \\ \overline{CR^r(\rho)} - CR^r \left(\rho \setminus (i, j)\right) \\ &= \left(CR^r(\sigma^0) - CR^r \left(\rho \setminus (i, j)\right)\right) \left(N_e(\rho) - \sum_{(l,k) \in \rho \in \Psi} y_{lk}\right) + \overline{CR^r(\rho)}. \end{aligned}$$

Because $\sum_{(l,k)\in\rho\cap\Psi} y_{lk} < N_e(\rho)$, we then have that $N_e(\rho) - \sum_{(l,k)\in\rho\cap\Psi} y_{lk} \ge 1$ so that

$$CR^{r}(\sigma^{0}) - CR^{r}(\rho \setminus (i,j)) + \overline{CR^{r}(\rho)} \leq \left(CR^{r}(\sigma^{0}) - CR^{r}(\rho \setminus (i,j))\right) \left(N_{e}(\rho) - \sum_{(l,k) \in \rho \cap \Psi} y_{lk}\right) + \overline{CR^{r}(\rho^{0})}.$$

But note that $\overline{CR^r(\rho)} - CR^r(\rho \setminus (i, j)) > 0$ by definition of $\overline{CR^r(\rho)}$. So in fact, $CR^r(\sigma^0) - CR^r(\rho \setminus (i, j)) + \overline{CR^r(\rho)} \leq CR^r(\sigma^0)$. Because θ_n and θ_m are connected, by taking the telescoping sum as in Corollary 4, we have that $|\theta_n - \theta_m| \leq \underline{CR^r(\rho^0)} \leq CR^r(\sigma^0)$ is valid. And since

$$CR^{r}(\sigma^{0}) \leq \left(CR^{r}(\sigma^{0}) - CR^{r}\left(\rho \setminus (i,j)\right)\right) \left(N_{e}(\rho) - \sum_{(l,k) \in \rho \cap \Psi} y_{lk}\right) + \overline{CR^{r}(\rho)},$$

we have that

$$\begin{aligned} \theta_n - \theta_m &| \leq CR^r \left(\rho \setminus (i, j) \right) + \\ & \left(CR^r(\sigma^0) - CR^r \left(\rho \setminus (i, j) \right) \right) \left(N_e(\rho) - \sum_{(l,k) \in \rho \cap \Psi} y_{lk} \right) + \end{aligned}$$

$$\overline{CR^{r}(\rho)} - CR^{r}\left(\rho \setminus (i,j)\right)$$

is valid as well.

3.5 Results

A MILP formulation approximating the AZ transmission network is solved for 16 scenarios: each climate zone of the network is projected to have either a large or small increase (e.g., 2–5°F for Tucson) in summer peak temperature independent from each other zone. These scenarios are encoded by an ordered quadruplet in which the temperature increase of region i is indicated by an H in position i if it is a large increase, and an L in the same position if a small increase is projected. This model is solved in Gurobi 9.0.2, and the non-zero expansion variables (and their associated objective costs) are tallied for each scenario. The results of these experiments are summarized in Table 4, with costs annualized.

Total costs increase by over 25% when comparing the scenario in which all regions experience small temperature gains to the scenario in which all regions experience large temperature gains. Since the difference in temperature changes for all regions is less than 3%, this represents a superlinear relationship between temperature increases and associated power system costs. Furthermore, larger increases in temperatures have different effects depending on the regions in which they occur. For example, when Regions 1 and 4 experience large temperature increases, the associated generation costs are much lower than when either Region 2 or 3 experiences larger temperature increases. Since Regions 2 and 3 contain Phoenix and Tucson respectively, the most

Scenario	New	Cap.	New	Cap.	Total	Gen.	Total
	Lines	Exp.	Line	Exp.	Exp.	Cost	Cost
	Built	Built	Cost	\mathbf{Cost}	Cost		
L,L,L,L	80	17	\$ 14.40B	\$ 2.79B	\$ 17.19B	\$ 7.07B	\$ 24.26B
L,L,L,H	83	19	\$ 14.94B	3.15B	\$ 18.09B	\$ 6.39B	\$ 24.48B
L,L,H,L	81	23	\$ 14.58B	\$ 3.96B	\$ 18.54B	\$ 11.27B	\$ 29.81B
L,H,L,L	89	17	\$ 16.02B	2.85B	\$ 18.87B	6.06B	\$ 24.93B
H,L,L,L	76	18	\$ 13.68B	\$ 2.94B	\$ 16.62B	\$ 7.47B	\$ 24.09B
L,L,H,H	83	25	\$ 14.94B	\$ 4.24B	\$ 19.18B	\$ 10.97B	\$ 30.15B
L,H,L,H	84	18	\$ 15.12B	\$ 3.00B	\$ 18.12B	\$ 6.77B	\$ 24.89B
H,L,L,H	85	20	\$ 15.30B	\$ 3.24B	\$ 18.54B	\$ 6.00B	\$ 24.54B
L,H,H,L	70	24	\$ 12.60B	4.09B	\$ 16.69B	\$ 13.21B	\$ 29.90B
H,L,H,L	88	24	\$ 15.84B	4.09B	\$ 19.93B	\$ 10.22B	\$ 30.15B
H,H,L,L	81	21	\$ 14.58B	\$ 3.41B	\$ 17.99B	6.98B	\$ 24.97B
L,H,H,H	83	22	\$ 14.94B	\$ 3.73B	\$ 18.67B	\$ 11.46B	\$ 30.13B
H,L,H,H	85	22	\$ 15.30B	\$ 3.79B	\$ 19.09B	\$ 10.70B	\$ 29.79B
H,H,L,H	83	18	\$ 14.94B	\$ 3.00B	\$ 17.94B	\$ 7.32B	\$ 25.26B
H,H,H,L	89	22	\$ 16.02B	\$ 3.73B	\$ 19.75B	\$ 10.16B	\$ 29.91B
H,H,H,H	78	22	\$ 14.04B	\$ 3.73B	\$ 17.77B	\$ 12.50B	\$ 30.24B

Table 4. Per Annum Projected Cost

populous metropolitan areas in the state by large margins, this is consistent with expectations. Generation costs associated with higher temperatures in these regions are nearly twice as much as those associated with higher temperatures in the less populous Regions 1 and 4.

Comparing the relative volume and costs associated with new lines rather than capacity expansion reveals further features of this network. When Region 3 (containing Tucson) experiences larger temperature increases, more lines are reinforced with higher capacity than in any other region. This suggest that Tucson has a robust infrastructure in place when considering transmission lines and generation, so the relatively less expensive option of expanding the capacity of existing lines can accommodate a large amount of increased demand in the region. In contrast, when Region 2 (containing Phoenix) experiences larger temperature increases, new lines are built at a higher rate (and correspondingly, cost) than for any other region. The increase in demand associated from higher temperatures in this region does not cause generation costs to increase very much, but it does require a large number of new lines to be built to meet this demand. This is consistent with a less robust transmission infrastructure in this region compared to Region 3, but better access to current and large sources of generation.

3.6 Conclusions

Significant temperature increases are expected by mid century: the question is the magnitude and distribution of these increases. This study considers several scenarios of temperature increase, distributed across both magnitude and location, and their effects on state-level transmission networks. A test case is built from U.S. Government provided data and publicly available data on the existing generation and transmission assets within Arizona; temperature scenarios are similarly designed based on historical data and designated ecoregions. The optimal expected annual cost of operation the power network, as well as costs associated solely with generation, transmission expansion, and capacity expansion is projected. Based on the distribution of these costs, it can be seen that regardless of the nature of temperature increases, a significant investment in transmission infrastructure is required by mid century. Furthermore, conditions associated with the existing network will cause the total cost to depend – to varying degrees – on transmission expansion, capacity expansion, or generation. Large increases in temperatures over urban areas correspond to even larger increases in generation costs in those regions. The largest overall cost differences are associated with the degree of temperature increase in these urban regions. There are also network features which are distinct to each such region that dictate whether future demand can be met primarily with less expensive capacity expansion via reconductoring or require significantly more costly transmission expansion to build new overhead lines. These results suggest that further analysis is worth performing, including cost changes due to generation or substation investments. The demand projections are also limited in scope to a fixed percentage increase based on the regional temperature changes. However, the current work demonstrates that such analysis is valuable. Further work is necessary, including an analysis of some subnetworks of the Arizona power system with more varied scenarios, in order to fully understand what network features suggest investment in certain classes of transmission asset. The joint optimization of both generation and transmission, especially with significant state- and nationally-mandated plans to expand renewable generation, also remains a question of interest.

Chapter 4

DATA-DRIVEN ROBUST TRANSMISSION EXPANSION PLANNING AND TRANSMISSION CAPACITY EXPANSION PLANNING AGAINST RISING TEMPERATURES

4.1 Background

Power system disturbances are increasing in frequency globally [3], and a major cause is rising temperatures [97]. High temperatures disrupt demand patterns, causing large increases in demand during the sustained extreme temperatures in summertime [84]. Further, they reduce the ampacity of transmission lines by inducing sagging, which increases energy loss along the line [96]. Combating the disruptions to the network associated with these environmental hazards is a challenging task; while the trend of rising temperatures has been identified, its scale and course are not known ahead of time, especially at the local level. Investments in power system infrastructure to increase its reliability and resiliency are among the most prescribed long-term methods to guarantee continued network operation during extreme temperature events. The most commonly proposed pathways to guide this investment are Generation Expansion Planning (GEP) and Transmission Expansion Planning (TEP) [71]. TEP seeks to minimize investment and operation costs for the network by reinforcing its topology and provide a stable network even under worst-case scenarios. Notably, TEP can be formulated in such a way as to allow obtaining and building new right-of-ways as well as ways to harden the existing corridors. One option for hardening is reconductoring existing lines, e.g. to expand transmission capacity, in a number of ways, such as replacing existing wires with high-temperature low-sag conductors [68].

TEP is a long-term problem: typical time frames for planning are measured in decades. It is also a large-scale problem, since transmission lines can span hundreds of miles. These features make TEP an important problem to find exact optimal values for, since investments are costly, take significant time to build, and cannot be easily moved or modified [35]. They also incentivize finding cheaper alternatives to building new transmission lines. One such alternative is reconductoring existing transmission lines to increase their capacity, for example by replacing them with high-temperature, low-sag (HTLS) conductors [68], [104]. Regardless of the exact method of expanding the grid, TEP is of growing concern, especially in the United States where the American Society of Civil Engineers rates the country's aging transmission grid infrastructure at a C- [23].

Deterministic TEP is already an NP-hard problem [69]; the increasing penetration of renewables among other flexibility requirements of the modern smart grid only make it more challenging [116]. Most grid planning problems adopt models for increased demand based on rising temperatures alongside increased renewable penetration, and some have also attempted to model the effects of temperature on decreasing potential generation [60], [89], [99]. However, there has not been much research into grid planning that models the effect of temperature on the transmission network capacity. This concern is crucially important, especially as grid modernization increases renewable penetration that may need to be delivered across long distances, which results in increased congestion. Since the transmission system spans huge geographical areas, it is important to model the effects of temperature on congestion at a regional or even local level, not simply system-wide. Nevertheless, it is necessary to consider
some of level of uncertainty when considering long-term planning [35] since global temperatures are rising at alarming but unknown rates [79].

Two major paradigms of uncertainty in optimization are stochastic optimization and robust optimization. Stochastic optimization, such as is used in [5], models uncertainty with a set of specific scenarios; this potentially allows a very focused interpretation of the unknowns. More accurate modeling arises from increasing the number of scenarios which rapidly increases the solution time associated with the corresponding problem. Therefore, stochastic uncertainty is best suited to short-term situations or ones with a limited number of scenarios. In contrast, robust optimization defines compact uncertainty sets that encompass all possible scenarios.

Since it considers worst-case realizations of uncertainty, robust optimization is the more appropriate framework for the expensive and long-term nature of power system planning problems [52], [86], [90], [134]. As technology and techniques have advanced, robust optimization has become more common for TEP in particular. [106], for example, apply an adaptive robust optimization model to solve a tri-level mixed integer formulation of TEP. Similar methods are advanced in [85] and further focused on large-scale applications in [86]. The robust approach is also extended in [47] and [29], [30] to dynamic TEP, while short- and long-term uncertainty are handled separately in [134]. Other discrete decisions than simply transmission expansion are also considered in [31], which allows energy storage expansion planning, and in [112], which allows for transmission switching to compensate for unmet demand. [74], [75] combines these ideas to allow storage and switching for real-time reliability.

The rapidly updating science on climate change makes it difficult to choose an appropriate uncertainty set for a robust model; this work proposes a data-driven approach to provide a realistic such set. However, the structure of the formulation is equally able to replace the data-driven aspects with improved predictive modeling techniques.

4.1.1 Contribution

The major contributions of this chapter are threefold. The first contribution is to provide a novel data-driven robust optimization formulation for transmission expansion planning with capacity expansion planning. The second contribution is to demonstrate an optimality-guaranteeing algorithm to solve the above formulation for TEPCE efficiently. Finally, the third contribution is to derive and prove two classes of valid inequalities to further accelerate the solution algorithm.

These contributions are contextualized as follows. While robust optimization formulations have been given for traditional TEP as well as for TEP with transmission switching or energy storage planning, to the best of the authors' knowledge, there has not been a model for robust TEP with capacity expansion. The details of this model are given in Section 4.2. Furthermore, most versions of TEP use either a robust formulation or a data-driven approach but not both. The TEP methods which are nearest to combining the two are those that share the structure of [135], which use the data-driven approach for only short-term stochastic uncertainty. An advantage of combining these approaches is that an algorithm developed for a unit commitment formulation [122] can be adapted to solve the formulation effectively. Note that the last major contribution adapts the ideas developed in Chapters 2 and 3 to iteratively add valid inequalities throughout the proposed algorithm to further accelerate the solution of TEPCE.

4.1.2 Chapter Structure

The structure of the remainder of this chapter is as follows. Section 4.2 presents the full three-stage robust optimization model for transmission expansion planning with capacity expansion. Section 4.3 provides the methodology for solving the model, including a dual column-and-constraint generation algorithm, the construction of the test network, and the uncertain consequences of temperature on the transmission elements of the network. Section 4.4 provides numerical results and, finally, Section 4.5 summarizes the conclusions drawn from those results.

4.2 Robust Transmission Expansion Planning with Capacity Expansion Model

This section extends the formulation developed in Chapter 3. That model does not account for uncertainty in temperature growth and assumes perfect knowledge. Because temperature is difficult to predict and climate change amplifies its unpredictability, it is necessary to develop a formulation that accounts for inherent uncertainty. To this end, the robust TEP with capacity expansion problem can be formulated as a min-max-min three-level optimization problem. The upper-level problem minimizes the cost of investment and worst-case operating costs, which are scaled up by a factor of σ to account for the full investment timeline. This level can be described as follows.

$$\min_{y_{ij}, z_{ij}, c^{O,wc}} \sum_{(i,j)\in\Omega} \left(c_{ij} y_{ij} + h_{ij} z_{ij} \right) + \sigma c^{O,wc}$$

$$\tag{4.1}$$

subject to:

$$\sum_{(i,j)\in\Omega} \left(c_{ij} y_{ij} + h_{ij} z_{ij} \right) \le \Pi \tag{4.2}$$

$$y_{ij}, z_{ij} \in \{0, 1\} \qquad \qquad \forall (i, j) \in \Omega \quad (4.3)$$

$$c^{O,wc} = \max_{c^O, \overline{P_{ij}^0}, \overline{P_{ij}^1}, \overline{g_n}} c^O \tag{4.4}$$

The first-level problem given by (4.1) - (4.4) represents the here-and-now investment decisions represented by the binary expansion decision variables y_{ij} and z_{ij} subject to a fixed budgetary limit Π . The objective function (4.1) represents the total cost of investment in candidate lines and in expanding capacity via reconductoring plus the operational costs given the worst-case realization of the uncertainty, $c^{O,wc}$. Note that $c^{O,wc}$ is the optimal objective value for the second-level problem described as follows.

$$\max_{c^O, \overline{P_{ij}^0, P_{ij}^1, g_n}} c^O \tag{4.5}$$

subject to:

$$\mathcal{U} = \left\{ \mathbf{U} \in \mathbb{R}^{|\Omega|} \mid \mathbf{U} = \sum_{k=1}^{K} \alpha_k \mathbf{U}_k, \sum_{k=1}^{K} \alpha_k = 1, \alpha_k \ge 0 \right\}$$
(4.6)

$$c^{O} = \min_{g_n, u_n, P_{ij}^0, P_{ij}, \theta_n} \sum_{n \in B} \left(c_n g_n + c_n^u u_n \right)$$
(4.7)

The middle-level problem (4.5) - (4.7) maximizes operational costs over the uncertainty set given the initial planning stages. Here, \mathbf{U}_k represents the congestion profile of lines from the k^{th} choice of data. The details of this mid-level problem and the generation of the uncertainty set \mathcal{U} are expanded upon in Section 4.3. c^O represents the operating costs given the investment decisions determined by the upper-level problem; it is the optimal objective value for the lower-level problem described as follows. Included in the operating cost is unmet demand, u_n , and its corresponding penalty c_n^u . This can be interpreted as part of a "social cost" associated with the transmission system failing to deliver electric service. While this is a common choice in TEP [115], other such costs could be added to the objective function at this level. Note that, for this third stage, the dual variables are stated after the colon next to each constraint.

$$\min_{g_n, u_n, P_{ij}^0, P_{ij}, \theta_n} \sum_{n \in B} \left(c_n g_n + c_n^u u_n \right) \tag{4.8}$$

subject to:

$$\sum_{(n,i)\in\Omega} \left(P_{ni}^0 + P_{ni}\right) - \sum_{(i,n)\in\Omega} \left(P_{in}^0 + P_{in}\right) + g_n + u_n = d_n : (\lambda_n) \qquad \forall n \in B \quad (4.9)$$

$$P_{ij}^0 \leq \overline{P}_{ij}^0 + \overline{P}_{ij}^1 z_{ij} : (\hat{\chi}_{ij}) \qquad \forall (i,j) \in \Omega \setminus \Psi, \quad (4.10)$$

$$P_{ij}^0 \geq -\overline{P}_{ij}^0 - \overline{P}_{ij}^1 z_{ij} : (\check{\chi}_{ij}) \qquad \forall (i,j) \in \Omega \setminus \Psi, \quad (4.11)$$

$$P_{ij} \le \overline{P}_{ij} y_{ij} : (\hat{\phi}_{ij}) \qquad \qquad \forall (i,j) \in \Psi \quad (4.12)$$

$$P_{ij} \ge -\overline{P}_{ij}y_{ij} : (\check{\phi}_{ij}) \qquad \qquad \forall (i,j) \in \Psi \quad (4.13)$$

$$\frac{-1}{b_{ij}}P_{ij}^0 - (\theta_i - \theta_j) = 0 : (\xi_{ij}) \qquad \forall (i,j) \in \Omega \setminus \Psi \quad (4.14)$$

$$\frac{-1}{b_{ij}} P_{ij} - (\theta_i - \theta_j) \le M_{ij} (1 - y_{ij}) : (\hat{\xi}_{ij}) \qquad \forall (i, j) \in \Psi \ (4.15)$$

$$\frac{-1}{b_{ij}}P_{ij} - (\theta_i - \theta_j) \ge -M_{ij}(1 - y_{ij}) : (\check{\xi}_{ij}) \qquad \forall (i,j) \in \Psi \ (4.16)$$

$$g_n \le \overline{g}_n : (\varphi_n) \qquad \qquad \forall n \in B \quad (4.17)$$

$$\theta_n \le \pi : (\hat{\varphi_n}) \qquad \qquad \forall n \in B \ (4.18)$$

$$\theta_n \ge -\pi : (\check{\varphi}_n) \qquad \qquad \forall n \in B \ (4.19)$$

$$u_n \le d_n : (v_n) \qquad \qquad \forall n \in B \ (4.20)$$

The lower-level problem (4.8) - (4.20) minimizes operational costs given the investment decisions and uncertainty realizations from the upper-level problems. Constraint (4.9) enforces flow balance at each bus. Constraints (4.10) and (4.11) enforce flow capacity limits for existing lines which can be reconductored, while constraints (4.12) and (4.13) do the same for candidate lines and require flow to be zero if the candidate line (i, j) is not built (i.e., if $y_{ij} = 0$). Note for these constraints that the variables y_{ij} and z_{ij} are binary. However, in the lower-level problem this restriction can be relaxed, since the binary nature of these variables is enforced in (4.4). Constraint (4.14) relates adjacent bus angles to power flow on existing lines, while (4.15) and (4.16) are disjunctive constraints that do the same for candidate lines given a suitably large choice of big-M values M_{ij} for each line. Finally, constraints (4.17) - (4.20) provide domain limits for generation and bus angle values. Note that this model is a robust extension, considering temperature uncertainties and their effects on transmission lines, of the deterministic model presented in Chapter 3.

4.3 Solution Methodology

To solve this three-stage, min-max-min robust optimization model, the lower two levels can be combined into a single-stage optimization model. To begin, the dual of the lower-level problem is constructed and is given by

$$\max_{\Upsilon} \delta \tag{4.21}$$

subject to:

$$\lambda_{n} + \varphi_{n} \leq \sigma c_{n} \qquad \qquad \forall n \in B \quad (4.22)$$
$$\hat{\chi}_{ij} + \check{\chi}_{ij} + \lambda_{j} - \lambda_{i} - \frac{1}{b_{ij}} \xi_{ij} = 0 \qquad \qquad \forall (i, j) \in \Omega \setminus \Psi \quad (4.23)$$
$$\lambda_{j} - \lambda_{i} + \hat{\phi}_{ij} + \check{\phi}_{ij} - \frac{1}{b_{ij}} \hat{\xi}_{ij} - \frac{1}{b_{ij}} \check{\xi}_{ij} = 0 \qquad \qquad \forall (i, j) \in \Psi \quad (4.24)$$

$\sum_{(n,i)\in\Omega\backslash\Psi}\xi_{ni}+\sum_{(n,i)\in\Psi}\left(\hat{\xi}_{ni}+\check{\xi}_{ni}\right)-\sum_{(i,n)\in\Omega\backslash\Psi}\xi_{in}-$	
$\sum_{(i,n)\in\Psi} \left(\hat{\xi}_{in} + \check{\xi}_{in}\right) + \hat{\varphi} + \check{\varphi} = 0$	$\forall n \in B \ (4.25)$
$\lambda_n + \upsilon_n \le c_n^u$	$\forall n \in B \ (4.26)$
λ_n free	$\forall n \in B \ (4.27)$
$\hat{\chi}_{ij} \leq 0$	$\forall (i,j) \in \Omega \setminus \Psi \ (4.28)$
$\check{\chi}_{ij} \ge 0$	$\forall (i,j) \in \Omega \setminus \Psi \ (4.29)$
$\hat{\phi}_{ij} \leq 0$	$\forall (i,j) \in \Psi \ (4.30)$
$\check{\phi}_{ij} \geq 0$	$\forall (i,j) \in \Psi \ (4.31)$
ξ_{ij} free	$\forall (i,j) \in \Omega \setminus \Psi \ (4.32)$
$\hat{\xi}_{ij} \le 0$	$\forall (i,j) \in \Psi \ (4.33)$
$\check{\xi}_{ij} \ge 0$	$\forall (i,j) \in \Psi \ (4.34)$
$\varphi_n \le 0$	$\forall n \in B \ (4.35)$
$\hat{\varphi}_n \le 0$	$\forall n \in B \ (4.36)$
$\check{\varphi}_n \ge 0$	$\forall n \in B \ (4.37)$
$v_n \le 0$	$\forall n \in B \ (4.38)$

Where $\Upsilon = \{\lambda_n, \hat{\chi}_{ij}, \check{\chi}_{ij}, \dot{\phi}_{ij}, \check{\xi}_{ij}, \hat{\xi}_{ij}, \check{\xi}_{ij}, \varphi_n, \hat{\varphi}_n, \check{\varphi}_n\}$ is the set of dual variables defined above and the objective function is given by:

$$\delta = \sum_{n \in B} d_n \lambda_n + \sum_{(i,j) \in \Omega \setminus \Psi} \left(\overline{P}_{ij}^0 + \overline{P}_{ij}^1 z_{ij} \right) (\hat{\chi}_{ij} - \check{\chi}_{ij}) + \sum_{(i,j) \in \Psi} \left(\overline{P}_{ij} y_{ij} \right) \left(\hat{\phi}_{ij} - \check{\phi}_{ij} \right) + \sum_{(i,j) \in \Psi} \left(M_{ij} (1 - y_{ij}) \right) \left(\hat{\xi}_{ij} - \check{\xi}_{ij} \right) + \sum_{n \in B} \overline{g}_n \varphi_n + \sum_{n \in B} \pi \left(\hat{\varphi}_n - \check{\varphi}_n \right) + \sum_{n \in B} d_n v_n.$$

Since the middle-level model and the above dual of the lower-level model are both maximization problems, they can be combined. This results in the problem

$$\max_{\Upsilon, \overline{P}_{ij}^0, \overline{P}_{ij}^1, \overline{g}_n} \delta \tag{4.39}$$

subject to:

$$(4.6), (4.22) - (4.38) \tag{4.40}$$

Note that the objective function is nonlinear and not linearizable since it contains bilinear terms involving the product of two continuous variables. Namely, the expression $\sum_{(i,j)\in\Omega\setminus\Psi} \left(\overline{P}_{ij}^0 + \overline{P}_{ij}^1 z_{ij}\right) (\hat{\chi}_{ij} - \tilde{\chi}_{ij}) + \sum_{(i,j)\in\Psi} \left(\overline{P}_{ij} y_{ij}\right) \left(\hat{\phi}_{ij} - \check{\phi}_{ij}\right) + \sum_{n\in B} \overline{g}_n \varphi_n$ contains all such potentially problematic bilinear terms. However, when considering a single scenario \mathbf{U}_k , the values of \overline{P}_{ij}^0 , \overline{P}_{ij}^1 , \overline{P}_{ij} will be fixed values as $\overline{P}_{ij,k}^0$, $\overline{P}_{ij,k}^1$, $\overline{P}_{ij,k}$, respectively. This resolves the concerns about the nonlinearity of the objective function: for each scenario, the objective is linear in all remaining variable terms. Furthermore, δ is a convex function when given the outputs of the lowest level problem, and \mathcal{U} is a polyhedral set. Therefore, the optimal solution occurs at an extreme point of \mathcal{U} . In other words, the optimal solution occurs when $\alpha_k = 1$ for some k, and 0 for all other α [122] (this will be elaborated in the ensuing paragraphs). This lets us solve the the combined middle- and lower-level problem over the set of scenarios $\{\mathbf{U}_k\}_{k=1}^K$, rather than attempting to solve them over the whole uncertainty set \mathcal{U} . Hence, the lower two levels can be equivalently defined by the k subproblems given by

$$\max_{\Upsilon_k} \quad \delta_k \tag{4.41}$$

subject to:

$$(4.22) - (4.38). \tag{4.42}$$

Correspondingly, we also have the following master problem at iteration K:

$$\min_{y_{ij}, z_{ij}, c^{O,wc}} \sum_{(i,j)\in\Omega} \left(c_{ij} y_{ij} + h_{ij} z_{ij} \right) + \gamma \tag{4.43}$$

subject to:

$$\sum_{(i,j)\in\Omega} \left(c_{ij}y_{ij} + h_{ij}z_{ij}\right) \le \Pi \tag{4.44}$$

$$\gamma \ge 0 \tag{4.45}$$

$$\gamma \ge \sum_{n \in B} \sigma c_n g_{n,k} \qquad \qquad \forall U_k \text{ s.t. } k \le K$$
 (4.46)

(4.9) - (4.20)
$$\forall U_k \text{ s.t. } k \le K$$
 (4.47)

 $y_{ij}, z_{ij} \in \{0, 1\} \qquad \qquad \forall (i, j) \in \Omega \quad (4.48)$

In this formulation, constraint sets (4.46) and (4.47) are added via a column-andconstraint generation process at each iteration of Algorithm 1.

Algorithm 1: Column-and-constraint Generation Process for Robust TEPC	E

- **1:** Initialize $k \leftarrow 1$;
 - **2:** Solve master problem (4.43) (4.48);
 - **3:** Solve subproblem (4.21) (4.38);
- 4: If $k = |\mathcal{U}|$, STOP;
- 5: Else $k \leftarrow k+1$, add $\gamma \ge \sum_{n \in B} \sigma c_n g_{n,k}$ and (4.47) to the master problem and go to step 2;

It is important to address the convergence of Algorithm 1, which is guaranteed over the *sample of the uncertainty set*. A sketch of the proof for this statement is provided below. **Proposition 6.1.** Algorithm 1 guarantees convergence to optimality over the chosen sample of the uncertainty set of robust TEPCE.

Proof. Recall that the uncertainty set \mathcal{U} is defined as

$$\mathcal{U} = \left\{ \mathbf{U} \in \mathbb{R}^{|\Omega|} \mid \mathbf{U} = \sum_{k=1}^{K} \alpha_k \mathbf{U}_k, \sum_{k=1}^{K} \alpha_k = 1, \alpha_k \ge 0 \right\},\$$

where $\{\mathbf{U}_k\}_{k=1,\dots,K}$ is the set of reduced-capacity profiles. Define $\mathcal{S} = \{\mathbf{U}_k\}_{k=1,\dots,K}$ as a sampling of K days. Note then that \mathcal{U} is by definition the convex hull of \mathcal{S} . Furthermore, the vertices of the polyhedron \mathcal{U} are in \mathcal{S} [12].

The combined middle- and lower-level problem given by (4.39) - (4.40) maximizes a convex function – specifically within the polyhedral uncertainty set \mathcal{U} . This convex function is the lower-level objective function (4.8), which takes \mathbf{U}_k as an input. In fact, the function is linear. Since this is now maximizing a linear function over a polyhedron, one of the vertices of \mathcal{U} belongs to the optimal solution set [12] (note: this also means that the worst-case scenario in \mathcal{S} can only occur on the boundary of \mathcal{U}). Thus, we can replace the continuous polyhedral uncertainty set \mathcal{U} in the maximization with the set \mathcal{S} , which is the discrete set of sampled scenarios.

The result is that by iterating through each point of S in Algorithm 1, we in fact reach exact convergence over the sampled set S

However, because $S \subseteq U$, optimality over the set S does not guarantee optimality over the set U. A straightforward cardinality argument (presented in [19]) can help provide a probability-based guarantee of optimality. In particular, the samples chosen in this data-driven model function analogously to samples chosen from a Monte Carlo time-series model. Accordingly, the data-driven model presented here can be seen as a sampled version of a chance-constrained stochastic programming model. By [19], given a user-defined $\epsilon \in (0, 1)$ (which can be made arbitrarily small), choosing a sample size such that $|\mathcal{S}| \geq \frac{|R||D|}{\epsilon} - 1$ gives that the expected probability of violations (i.e., the expected probability of a potentially non-optimal solution) must be less than ϵ , where R is the set of regions and D is the full historical set of days (and thus $R \times D$ composes all ambient temperature based capacity profiles). Thus a sufficient number of samples can be chosen so that the probability of convergence of Algorithm 1 is at least $1 - \epsilon$.

4.3.1 Data-driven Uncertainty Modeling

This section describes the structure of the uncertainty set (4.7). Robust optimization is a topic of growing interest in the optimization community. Works such as [10] and [37] helped bring the possibilities of the technique to the forefront, and modern approaches such as [126] for semi-definite programming continue to grow interest in robust optimization techniques. What differentiates robust optimization from stochastic optimization is that while stochastic methods handle uncertainty by considering a finite set of scenarios, robust methods utilize an uncertainty set that contains an infinite number of points. The structure of this uncertainty set is key to capturing appropriate worst-case conditions and has significant impacts on solution methods as well as tractability. The structure of the uncertainty set for these problems typically fits into a handful of categories: box, ellipsoidal, polyhedral, cone, etc. [9], [53], [130]. For the model proposed in this chapter, a polyhedral uncertainty set proves to be a good measure of uncertainty while maintaining tractability across the tri-level problem. In particular, the uncertainty sets used are formulated using a data-driven approach. Temperature data is collected and treated as described in Section 4.3.1.2, and its effects on the network are modeled as in Section 4.3.1.1. For replicability, the dataset is included in Appendix A.2 in MATPOWER format.

4.3.1.1 Effects of Temperature on Transmission Line Ampacity

The effects of temperature on transmission line ampacity are modeled by Equation 3.1. Applying this formula to the temperature profiles described in Section 4.3.1.2 gives a sequence of transmission line capacity profiles, one for each instance of temperature across the state of Arizona. For each experiment in this chapter, 100 capacity profiles are selected; the convex hull of the selected profiles provides a polyhedral uncertainty set for the robust optimization model in Section 4.2. One concern regarding the construction of this convex hull is that while it considers the worst case loading conditions so far, it does not necessarily capture the future uncertainty especially when taking into account the effects of climate change. One option to mitigate this concern is to scale each profile by a fixed percentage to approximate a shift forward in years. For example, if we choose a congestion profile set of the peak temperature days from the last 30 years, and scale it up by an estimated 6% to approximate the effects of climate change, it could represent a congestion profile of the next 30 years leading up to 2050. While this approach reflects rising temperatures, the flat increase would tend to underestimate regional differences which are important in modeling temperature's effect on transmission ampacity.

It should be noted that constructing uncertainty sets in this fashion allows for alternate methods of profile selection. For example, traditional robust optimization methods may construct the set via Monte Carlo simulation based on estimated distribution. A forecast-driven generation of profiles through other techniques could easily construct another set of profiles as well. Thus, given a good forecasting model for future temperatures, the accuracy of these uncertainty sets can be improved without altering the model or solution methods.

4.3.1.2 Regional Clustering

The dataset used to define temperature regions for this instance includes 53 well-distributed surface weather stations over Arizona state where daily average and maximum temperature values are recorded. The weather stations were evenly distributed considering the metropolitan areas and the major cities in the state as shown in Figure 10. Since this chapter focuses on the impact of high temperatures on grid transmission elements, the data set considered only the daily average and maximum temperatures of the months May through September in the last twenty years. All raw data were retrieved from the National Oceanographic and Atmospheric Administration (NOAA). The analysis in this section summarizes the regional clustering performed in [117]; see that work for additional details.

4.3.1.2.1 Clustering Analysis

Cluster analysis is an effective statistical tool used in many climatic and atmospheric studies, especially for determining homogeneous climate regions based on meteorological variables like temperature [36]. The formation of homogeneous zones experiencing similar air temperature is the first step towards understanding the impact of high temperature on the electric grid. To this end, the approach used in this chapter



Figure 10. Weather Stations of AZ

focuses on a detailed analysis of the development of homogeneous zones based on daily average and maximum temperatures for the state of Arizona. In studies of climate and atmospheric sciences, several cluster analysis techniques are used and encompass hierarchical and non-hierarchical methods [36]. Hierarchical methods are best at clustering objects by multiple tiers of shared properties, locking cluster membership early in the clustering algorithm. Non-hierarchical methods instead permit objects to change clusters throughout the process. This makes non-hierarchical methods useful for exploratory purposes; they are preferred in many studies related to the determination of climate regions [36].

The k-means method is a widely used non-hierarchical method of clustering. k-

means is founded on the notion that a center point can symbolize a cluster [70]. The centroid is the mean or center point of a set of points and, for the most part, does not coincide with an actual data point. The basic algorithm involves: (1) selection of k points as the initial seed points or centroids, (2) assignment of all points to the closest centroid, (3) recalculation of the centroid of each cluster, and (4) repetition of previous two steps until there is no change in the centroids. These centroid points represent k sets of average and maximum temperature points that will define k clusters for representing the temperature regions. In this study, applying the k-means clustering algorithm to the temperature data, k was set to 7. This number was chosen by experimentation (values from k = 3 to k = 11 were tested) to provide a significant number of regions while maintaining logical climate similarity for each cluster.

4.3.1.2.2 Assignment Analysis

Twenty years' worth (2000-2020) of daily high temperatures were collected for each weather station and used in the analysis. To assign one cluster decision for a specific weather station, we calculate the distance between all the objects in the study – transmission lines, buses, and substations – and the weather stations. Calculating the distance between two points based on their longitude and latitude can be done following the haversine formula, which calculates the great circle distance between two points [70].

$$a = \sin \frac{\delta \phi^2}{2} + \cos \phi_1 \cos \phi_2 - \sin \frac{\delta \lambda^2}{2}$$
(4.49)

$$c = 2 \arcsin \sqrt{a} \tag{4.50}$$

$$d = Rc \qquad (4.51)$$

Here ϕ is latitude, λ is longitude, and R is earth's mean radius (6371 km). Next, we assign each object in the network to the nearest weather station. This matches all the electric grid components to weather stations. The clusters described in Section 4.3.1.2.1 are based on the full annual data for each weather station. However, for the purposes of the experiments described in Section 4.4, the concern is for rising and extreme temperature scenarios. Therefore, the subset of days corresponding to the warmest months of the year (May 1 - September 30) was used to rematch each weather station to one of the 7 clusters. This rematching is based on the distances between the high for each day and the centroid (consisting of a maximum high temperature and an average high temperature) of each cluster. Because the summer months are warmer, the relative difference between centroids shrinks. This reduces the effective number of temperature regions in Arizona to five; these regions are shown in Figure 11.

The key difference between the regions shown below and those in Figure 8c is that the regions analyzed in this chapter are defined purely based on the clustering algorithm across each weather station's high temperatures. While this causes the regions to deviate from the EPA ecoregions, it better captures areas with similar profiles of summer high temperatures. The clustering method employed in this chapter also captures improved temperature estimates for each region since they are defined by distance from weather stations.

4.4 Results

Four experiments were performed, all on the same Arizona transmission network featuring 892 buses, 376 generating units, and 1424 lines. The purpose of these



Temperature Regions by Clustering Assignment

Figure 11. Regions of AZ assigned by clustered temperatures experiments is to identify how different rising temperature scenarios affect expansion planning decisions at the regional level. Section 4.4.1 mirrors the experiment performed in Chapter 3 by allowing a low or high temperature increase to occur independently in each region; however, it uses the regions defined in this chapter. Section 4.4.2 takes full advantage of the data-driven methodology to sample high temperatures from individual days and weather stations. All experiments were solved using Gurobi version 9.1.2. One attempt to solve without employing the VIs from Chapter 3 was terminated after 72 hours. However, typical solve times when employing the VIs were between 9 and 12 hours.

4.4.1 Simple Test Case

The initial test case (Experiment 0) is structured similarly to the experiments performed in Chapter 3. Each zone can experience either a "high" temperature increase or a "low" temperature increase. These temperature values correspond to the highest high that zone reaches and the average high the zone reaches, respectively. The methodology for determining these values is presented in detail in Section 4.3.1.2. Since the clustering algorithm results in five zones, this gives 32 possible scenarios. A summary of the results for this test case is presented in Table 5. For this simple scenario, 127 new lines are built and 48 lines are reconductored. Of the allotted investment budget, 83.45% is spent on new lines rather than reconductoring. Note that because of the robust formulation as well as the updates to the Arizona transmission network recorded by EIA as of December 2021, the total number of lines invested in is greater than the number in Chapter 3. However, the ratio of investment in expansion to investment in capacity expansion remains similar. More insights into the regional differences are presented in Section 4.4.2.

Table 5. Simple Scenario Results

Expansion Lines Built	Lines Reconductored
127	48

4.4.2 Sampling Experiments

The more general test case considers both a larger number of scenarios and uses the historical data whose collection is detailed in Section 4.3.1.2. Samples are taken at random according to the following rules. For one experiment (Experiment 1), 100 days are selected uniformly at random from the full set of days. For each selected day, one weather station is selected at random for each region, and the high temperature recorded at that station is used to represent the temperature for the overall region. This corresponds to a scenario in which climate change has a smaller impact than expected on future temperatures, since it reflects a peak summer scenario only as hot as current conditions. While this experiment still samples the highest of temperatures from the summer months, those possible temperatures include cooler days in May or September from up to twenty years ago.

For a second experiment (Experiment 2), 100 days and temperatures are selected the same way as in the first experiment, but the days are sampled from only July and August. This corresponds to a middle of the road realization of climate change's effects on future temperatures. For the final experiment (Experiment 3), days and temperatures are selected exactly as in Experiment 2, but the max temperatures are increased by 20%. While this percentage increase results in potentially unrealistic high temperatures, it does represent realistic ampacity reduction due to ambient environmental changes such as humidity or windspeed. This is the most extreme effect of climate change tested on this model. Table 6 provides a summary of results for all three experiments.

Table 6. Random Sampling Results

	Experiment 1	Experiment 2	Experiment 3
Expansion Lines	127	128	126
Lines Reconductored	46	47	51

At first glance, the results for the three experiments appear very similar in that they share a ratio of investment in expansion to capacity expansion of approximately five to one. However, as the results from Chapter 3 suggest, the differences in regional temperature modeling largely manifest in the location of investment rather than quantity. Tables 7 and 8 give a quick summary of these regional investments. A line is assigned to one of the regions at its terminal buses. The region chosen is whichever of the two has a higher temperature to be consistent with the temperature modeling decisions described in 4.3.1.1 and the previous work [95]. The summary identifies that Region 2 has the largest difference in investment types, most notably between the simple case and the extreme temperature increase sampling experiment. This region is located on the western and southern portions of Arizona, but it does not include the areas around Phoenix or Yuma. It also does not include the area around Blythe, California. In fact, most of this region is sparsely populated rural desert that nevertheless contains some larger power plants.

Table 7. Expansion Investment by Region

Region	Simple Case	Experiment 1	Experiment 2	Experiment 3
1	0	0	0	0
2	1	1	1	1
3	39	38	38	43
4	8	7	8	8
5	0	0	0	0

Table 8. Reconductoring Investment by Region

Region	Simple Case	Experiment 1	Experiment 2	Experiment 3
1	1	1	1	1
2	14	9	8	8
3	102	109	109	109
4	10	8	10	8
5	0	0	0	0

To get a better sense of direct comparison between experiments, it is necessary to

illustrate the differences in investment profiles with a partial map of the whole grid. For each of the maps below, the regions are shaded with the hottest regions being darkest. Demand at each substation of the network is represented by a blue circle; demand size is proportional to the size of the circle. Likewise, fuchsia circles represent generation with larger circles representing larger generation capacity. The teal lines represent investment, either in expansion lines or in reconductoring lines as noted in the map.

For each pair of experiments, several maps are created to analyze the similarities and differences. One map shows all investment decisions common to an experiment pair (i.e., the intersection of their set of investments). Two show the set differences. Finally, one shows the symmetric difference between the pair's investment decisions, that is, the investments that are made in either experiment but not both. The most informative of these maps are analyzed below.





(a) Experiment 0 Intersect Experiment 2

(b) Experiment 0 Symmetric Difference Experiment 2

Figure 12. Symmetric Experiment 0 vs Experiment 2



(a) Experiment 0 Without Experiment 2(b) Experiment 2 Without Experiment 0Figure 13. Difference Experiment 0 vs Experiment 2

When comparing the basic 32-scenario case to the summer sampling experiment (i.e., Experiment 0 compared to Experiment 2) in Figure 12, note that the common expansion investments typically connect distant rural areas to sources of generation closest to major cities. The main exception to this are hydroelectric plants in the far west and south of Arizona, which are themselves in rural regions and are connected to rural demand in both experiments. The investments made only in one experiment are demonstrated in Figure 13. When looking at the basic experiment in these maps, the investment is focused around the outskirts of Phoenix, with generating units just outside the major metropolitan area connecting to nearby demand. Notably, in the summer sampling experiment (Experiment 2), there is instead investment to supply generation to western Arizona as well as several investments within the city of Phoenix itself.

A similar comparison appears in Figures 14 and 13, which show the common investment choices between Experiment 0 and Experiment 3 and their differences, respectively. In this case, the isolated investments are even more dissimilar. The basic experiment only invests in a single line that the scaled summer experiment (Experiment 3) does not: a short line just south of Phoenix. However, Experiment 3 invests in several long-distance transmission lines that carry generation to isolated demand as well as a number of investments within Phoenix itself. Most of these investments are for meeting demand in region 1, which is expected to experience the highest temperatures. When directly scaling the sampled temperatures, this increase is exacerbated, and thus even more investment is dedicated to delivery power in this region.



(a) Experiment 0 Intersect Experiment 3 (b) Experiment 0 Symmetric Difference Experiment 3



Figures 16 - 19 show the same comparisons as Figures 12 - 15, but the comparison here is between Experiment 1 and the summer or scaled summer sampling experiment (Experiments 2 and 3), respectively. In each case, the common investments across these experiments are distributed similarly to those in the basic experiment. However, the



(a) Experiment 0 Without Experiment 3(b) Experiment 3 Without Experiment 0Figure 15. Difference Experiment 0 vs Experiment 3

differences are much greater. Experiment 1 makes a significant number of investments connecting Phoenix to northwestern Arizona that are not made in either of the other sampling experiments. It also invests in a long-distance line up the eastern side of the state. Experiment 2 makes significantly more investments into Regions 1 and 2, which experience the highest expected temperature increases. Again, many of these terminate in the outskirts of the Phoenix greater metropolitan area, which has a notably faster increase in temperatures experienced over the last two decades than the rest of the state. Many of these same investments are made in Experiment 3, but in several cases (including between Yuma and Phoenix as well as the Lake Havasu hydroelectric plant and Phoenix) multiple nearly parallel investments are made where only a single investment occurred in Experiment 2.

Finally, Figures 20 - 21 compare Experiment 2 to Experiment 3 in order to





(a) Experiment 1 Intersect Experiment 2 (b) Experiment 1 Symmetric Difference Experiment 2





(a) Experiment 1 Without Experiment 2
 (b) Experiment 2 Without Experiment 1
 Figure 17. Difference Experiment 1 vs Experiment 2

identify the differences in investment engendered by a direct increase in summer temperatures. The same trend when comparing Experiment 0 and Experiment 1 to both summer experiments of common investments continues to hold when comparing





(a) Experiment 1 Intersect Experiment 3 (b) Experiment 1 Symmetric Difference Experiment 3

Figure 18. Symmetric Experiment 1 vs Experiment 3



(a) Experiment 1 Without Experiment 3(b) Experiment 3 Without Experiment 1Figure 19. Difference Experiment 1 vs Experiment 3

the summer sampling experiment to the summer scaling sampling experiment. In fact, looking at the investments unique to the summer scaling experiment highlights most effectively the concentration of investment for extreme global warming scenarios. These investments again focus on delivering power to region 1 over anywhere else in the state.



(a) Experiment 2 Intersect Experiment 3 (b) Experiment 2 Symmetric Difference Experiment 3

Figure 20. Symmetric Experiment 2 vs Experiment 3



(a) Experiment 2 Without Experiment 3(b) Experiment 3 Without Experiment 2Figure 21. Difference Experiment 2 vs Experiment 3

There are two key lessons that emphasize the effects and values of correct temperature modeling to be learned from these experiments, especially when considering climate change. The first is that distinct temperature models result in significantly different investment decisions, even when given identical investment budgets. The quantity of investments is similar although the costs of individual investments vary, but the localized investments reveal the importance of this modeling. The second lesson is to emphasize the tendency of climate change to have exaggerated effects in localized areas. This is illustrated in the experiments presented here by the increasing concentration of expansion investment in region one as higher temperature rise scenarios are modeled.

4.5 Conclusion

This chapter presents several major contributions. The first is a novel three-stage robust optimization model for transmission expansion planning with transmission capacity expansion (TEPCE). As part of this model, the effects of rising temperatures on transmission lines are captured with uncertainty sets, while historically the effects on generation and demand are studied. The second contribution is the generation of a realistic test case joined with a data-driven method of modeling temperature uncertainty via a regional clustering method. The test case is an approximation of the Arizona transmission grid that is based on publicly available, frequently updated data from the U.S. Energy Information Administration. The model is then tested using four different experiments for predicting the temperatures of each region of Arizona as defined by the proposed clustering method.

The four chosen experiments represent three potential climate change scenarios.

The first two experiments reflect a scenario in which temperatures do not increase significantly in the next 50 years, with one experiment based on simple temperature variations and one based on a data-driven sampling method. The third experiment reflects a more realistic expectation of temperature increase in which the average daily high temperature more closely resembles the current highs between May and September. The fourth reflects a more extreme prediction in which daily highs resemble current highs in the peak summer months of July and August only. These experiments reveal that even when an investment budget is fixed and similar numbers of investments are made, the modeling of temperature plays a significant role in the actual investment decisions. Furthermore, as temperature forecasts rise, investment concentrates in already high-temperature reasons and the major metropolitan areas in those regions. This reinforces the core result of Chapter 3 that detailed temperature prediction at a regional level is essential to efficient investment decisions in major transmission elements.

Chapter 5

GRID HARDENING AND TRANSMISSION SWITCHING FOR RELIABILITY AND RESILIENCE AGAINST HURRICANES

5.1 Background

While Chapters 3 and 4 are concerned with the ongoing low-impact, highprobability temperature increases (i.e., climate impacts), this chapter instead focuses on the more sporadic and potentially devastating natural disasters (i.e., weather impacts). Among the devastating effects of disasters is damage to transmission networks that can cause cascading failures even outside the direct impacts of the disaster. Such failures clearly prevent the proper operation of the transmission grid, which is essential to the current and increasing concern of delivering energy energy reliably. Rising temperatures are a significant threat to the functionality of the grid, yet the climate effects cascading from increased global temperatures are perhaps a more critical concern [97]. Among the most dangerous of these cascading events are *high-impact, low-probability* (HILP) events such as extraordinarily destructive hurricanes, tornadoes, and other natural disasters. The frequency and severity of such events are growing rapidly [118]. While the grid requires ongoing investment, it is inevitable that HILP events will damage the grid's infrastructure. Thus, it is necessary to have reactive options for mitigating the effects of extreme weather.

5.1.1 Aims and Contribution

This chapter considers two reactive options to be used in tandem: hardening investments and transmission switching. Hardening investments can be made in advance of a disaster to attempt to preemptively mitigate the worst effects of extreme weather, while transmission switching can be used after the initial impact of the event to enhance the system's real-time ability to return to an improved state. In particular, optimal transmission switching (OTS) is a systematic methodology for disconnecting or reconnecting existing lines, as opposed to the historical practice of switching lines on an ad hoc basis. Transmission switching can be used for multiple purposes (e.g., [16], [40], which use OTS for load shed recovery and prevention, respectively). This chapter considers it as a tool to improve network resilience. OTS shares many similarities in terms of structural network changes with TEP. Thus, a similar model for OTS is considered herein to further demonstrate the broad potential of the structural insights already discussed in previous chapters.

The contributions of this chapter are threefold. First, this chapter derives several classes of VIs for OTS. While the theoretical use of these VIs is proven herein, their experimental value is left as future work. The featured contributions share similarities with certain recent works. The closest related work is [66], which proposes a strong class of cycle-based VIs for optimal transmission switching. The VIs proved herein are more general, since they are applicable to both transmission expansion planning and optimal transmission switching. The second contribution is demonstrating that in the latter case they are incomparable to those in [66]. The third contribution is to further generalize the proposed VIs to include capacity expansion in transmission expansion planning. Finally, it is pivotal to explain that the implementation of some

of the featured OTS VIs must take into consideration the fact that the coefficients may require the solution of longest path problems, which are NP-Hard [50]. The consequences of this fact are a subject of future work.

5.2 Optimal Transmission Switching

 $(l,i) \in \Omega$

 $(i,l) \in \Omega$

Typically, OTS is an operation-level optimization problem. That is, the decisions to switch a line on or off are made in near real time. These decisions can be in response to disasters or planned outages [2], [16], [28], or to reduce congestion in the network [108], [110]. However, OTS can also be used as a subproblem in long-term planning decisions (e.g., to handle contingencies [73]) to produce improved solutions over TEP, which does not consider switching [65]. The problem is modeled here with a discrete, deterministic, DCOPF-based approach. It is formulated as a mixed integer program wthrough the use of disjunctive constraints. The full statement of the model follows:

$$\min \sum_{n \in B} c_n g_n$$
(5.1)
s.t. $\sum P_{li} - \sum P_{il} + g_n = d_n$
 $\forall n \in B, \forall (i, j) \in \Omega$ (5.2)

$$-\bar{P}_{ij}s_{ij} \le P_{ij} \le \bar{P}_{ij}s_{ij} \qquad \qquad \forall (i,j) \in \Omega \quad (5.3)$$

$$b_{ij} \left(\theta_i - \theta_j\right) - P_{ij} + (1 - s_{ij})M \ge 0 \qquad \qquad \forall (i, j) \in \Omega \quad (5.4)$$

$$b_{ij} \left(\theta_i - \theta_j\right) - P_{ij} - (1 - s_{ij})M \le 0 \qquad \qquad \forall (i, j) \in \Omega \quad (5.5)$$

$$0 \le g_n \le \bar{g}_n \qquad \qquad \forall n \in B \quad (5.6)$$

$$-\bar{\theta} \le \theta_i - \theta_j \le \bar{\theta} \qquad \qquad \forall (i,j) \in \Omega \quad (5.7)$$

$$\sum_{(i,j)\in\Omega} (1-s_{ij}) \le J \tag{5.8}$$

$$s_{ij} = 1 \qquad \qquad \forall (i,j) \in \Omega \setminus \Phi \quad (5.9)$$

$$s_{ij} \in \{0,1\} \qquad \qquad \forall (i,j) \in \Omega \ (5.10)$$

$$P_{ij}, \theta_n, g_n$$
 unrestricted (5.11)

The objective function for optimal transmission switching, (5.1), is to minimize the cost of generation needed to meet demand in the network. The constraints follow. Constraint (5.2) represents the flow-balance constraints at each bus and constraint (5.3) represents the capacity of active power flow on each line. Note that s_{ij} represents the on/off status of line (i, j), with $s_{ij} = 1$ corresponding to on, i.e. the line allows flow. Thus, this constraint enforces that when $s_{ij} = 0$, the capacity of the line (i, j) is zero. Both constraints (5.4) and (5.5) represent Kirchoff's second law governing the active power flow on line (i, j). When $s_{ij} = 1$, both constraints are activated; when $s_{ij} = 0$, a sufficiently large big-M parameter ensures they are extraneous to the model. Constraint (5.6) represents the limits on active power generation at each generator; a bus with no generator has $g_n = 0$. Constraint (5.7) enforces the maximum bus angle-difference between adjacent bus pairs $(i, j) \in \Omega$. Constraint (5.8) imposes a fixed limit on the number of lines to be switched off; this limit, J, can be specified by operators or adjusted to refine the problem as needed. Note that setting $J \geq |\Omega|$ effectively puts no limit on the number of lines switched off. Finally, the last two constraints represent the domain restrictions on the decision variables.

One concern that can arise with transmission switching, and as such was not considered in Chapter 2, is the disconnection of the network in some way. This can occur whether the network is intentionally designed to permit disconnects or whether many lines are switchable and the number of allowed switchings is large. That is, it may be possible that enough lines are switched off to disconnect one or more buses from the rest of transmission network. When planning for future operation, it is preferable to enforce that the entire network remain connected. As such, we introduce the following set of constraints:

$$\sum_{(i,j)\in\Omega, i\in S, j\in S'} s_{ij} \ge 1 \qquad \forall S \subset B, S \neq \emptyset$$
(5.12)

These enforce connectivity in the network by ensuring that any two buses i and j will have a path which allows flow between them no matter what switching decisions are made. There are an exponential number of such constraints; as such, it is not feasible to add them all to the model. Instead, they can be incorporated, as needed, whenever their violation is detected in the optimal solution of the model.

5.3 Stochastic Restoration Model

We present a two-stage stochastic programming formulation representing lastminute preparation for and restoration from the effects of a natural disaster. In particular we model the initial impacts of a hurricane on the power grid. For the preparation actions, we suppose that some entity has a rough forecast in the hours or days leading up to a hurricane's landfall. During this limited time, the entity is able to make investments in "hardening" some grid components. It may harden lines, buses, or specific generators. While the exact manner of hardening is not considered here, some examples could include piling sandbags around a substation to prevent flooding or providing a means of backup generation [17], [33]. The effect of this investment is to prevent the component from being damaged by the storm. For the restoration stage after the impact of the hurricane is realized, the system operator can redispatch generation and modify network topology with the goal of minimizing load shed. Given the proximity of the event, it can be supposed that the forecast of the storm's direction and effect is reasonably accurate but that it does not represent perfect information. Therefore, uncertainty is modeled by a set of scenarios and probabilities, and the expected value of load shed is an appropriate metric for representing costs.

Minimize

$$\left(\sum_{n\in B} C_n i_n + \sum_{(i,j)\in\Omega} C_{ij} i_{ij} + \sum_{g\in G} C_g i_g\right) + E\left[c_u^n u_n^\omega\right]$$
(5.13)

Subject to

$$\sum_{n \in B} C_n i_n + \sum_{(i,j) \in \Omega} C_{ij} i_{ij} + \sum_{g \in G} C_g i_g \le \Pi$$
(5.14)

$$\sum_{g \in G_n} P_g^{\omega} + \sum_{(n,i) \in \Omega} P_{ni}^{\omega} - \sum_{(i,n) \in \Omega} P_{in}^{\omega} = d_n - u_n^{\omega} \qquad \forall n \in B, \forall \omega \in S \quad (5.15)$$

$$p_{ij}^{\omega} = y_{ij}^{\omega} S_{ij} \left(\theta_i - \theta_j\right) \qquad \qquad \forall (i,j) \in \Omega, \forall \omega \in S \quad (5.16)$$

$$p_g^{\omega} \le \overline{P}_g \left(Y_g^{\omega} + i_g \right) \qquad \qquad \forall g \in G, \forall \omega \in S \quad (5.17)$$

$$-\overline{\theta} \le \theta_i^\omega - \theta_j^\omega \le \overline{\theta} \qquad \qquad \forall (i,j) \in \Omega, \forall \omega \in S \ (5.18)$$

$$-\overline{P_{ij}} \le p_{ij}^{\omega} \le \overline{P}_{ij} \qquad \qquad \forall (i,j) \in \Omega, \forall \omega \in S \ (5.19)$$

$$0 \le p_g^{\omega} \le P_g \qquad \qquad \forall g \in G, \forall \omega \in S \ (5.20)$$

$$0 \le u_n^{\omega} \le d_n \qquad \qquad \forall n \in B, \forall \omega \in S \quad (5.21)$$

$$y_l^{\omega} \le Y_{ij}^{\omega} + i_{ij} \qquad \qquad \forall (i,j) \in \Omega \ (5.22)$$

$$y_l^{\omega} \le Y_n^{\omega} + i_n \qquad \qquad \forall (n,j) \in \Omega \ (5.23)$$

$$y_l^{\omega} \le Y_n^{\omega} + i_n \qquad \qquad \forall (i,n) \in \Omega \ (5.24)$$

$$p_g^{\omega} \le P_g(Y_g^{\omega} + i_g) \qquad \qquad \forall g \in G \ (5.25)$$

$$u_n^{\omega} \ge d_n (1 - Y_n^{\omega} - i_n) \qquad \qquad \forall n \in B \quad (5.26)$$

The objective function (5.13) is to minimize the total cost of hardening investments and the expected value of load shed across all possible scenarios. Constraint (5.14) represents a budgetary constraint on the total possible investment. Constraint (5.15)represents standard flow-balance constraints, including demand and load shed at each bus. Constraint (5.16) represents the relationship between adjacent bus angles and the power flow on the line connecting them. It also enforces that there is no flow along line (i, j) when that line is switched off. The constraint (5.17) forces generation at q to be zero if the scenario damages it and there is no investment to harden that generator; otherwise, (5.17) is redundant. Constraint (5.18) enforces a maximum difference between adjacent bus angles, which is necessary for the use of the DCOPF approximation. Constraints (5.19) - (5.21) represent capacity limits on line flow, generation, and load shed, respectively. Constraints (5.22) - (5.24) enforce that a line must be considered switched off if the scenario dictates that the line or its incident buses are damaged and there was no investment made to harden the corresponding components; as with (5.17), these are redundant constraints otherwise. Finally, constraints (5.25) and (5.26) serve the same purpose for generation and demand, respectively. Note that, for these final constraints, the Y^{ω} variables represent the status of transmission elements in scenario ω : a value of 1 indicates that the element is undamaged by the extreme weather event while a value of 0 indicates that it would be destroyed unless a hardening investment has been made. Likewise, y_{I}^{ω} represents the status of the line l in scenario ω with a value of 1 indicating that the line is switched on.

We reformulate (5.16) with a standard big-M disjunction to linearize the constraint and transform the problem into a MILP for each scenario. This reformulation is given by

$$S_l(\theta_i - \theta_j) - P_{ij} + (1 - y_{ij}^{\omega})M \ge 0 \qquad \qquad \forall (i, j) = (i, j) \in \Omega, \omega \in S \quad (5.27)$$
$$S_l(\theta_i - \theta_j) - P_{ij} - (1 - y_{ij}^{\omega})M \le 0 \qquad \qquad \forall (i,j) = (i,j) \in \Omega, \omega \in S \quad (5.28)$$

Since there is only a finite number of scenarios considered, the objective function can be rewritten by expanding the expected value as

$$\sum_{n \in B} C_n i_n + \sum_{(i,j) \in \Omega} C_{ij} i_{ij} + \sum_{g \in G} C_g i_g + \sum_{\omega \in S} P(\omega) c_u^n u_n^{\omega},$$
(5.29)

where P^{ω} is the probability of scenario ω . While the number of scenarios is finite, it can be large. As the number of scenarios increases, so does the number of binary variables in this formulation. The ballooning number of binary variables can cause this formulation to become unsolvable for large numbers of scenarios. To counteract this, we include a set of VIs described in Section 5.4 to keep the overall problem manageable.

5.4 Optimal Transmission Switching Valid Inequalities

A parallel to Corollary 4 but for OTS is presented here. The details of the proof are similar, but because they will be referenced repeatedly throughout this chapter it is worth reinforcing them.

Corollary 6.1. Let $\rho \in C_{nm}$ represent a directed path from θ_n to θ_m over corridors in *G*. Then the following two-sided inequality is valid for OTS:

$$-CR(\rho) \le \theta_n - \theta_m \le CR(\rho). \tag{5.30}$$

Restated,

$$|\theta_n - \theta_m| \le CR(\rho). \tag{5.31}$$

Proof. Let $\rho := (i_1, i_2), (i_2, i_3), \dots, (i_{j-1}, i_j), \dots, (i_{k-1}, i_k)$, where $i_1 = \theta_n$ and $i_k = \theta_m$. By constraint (5.3), the flow along corridor (i, j) is bounded by $-\overline{P}_{ij}$ and \overline{P}_{ij} , regardless of any potential switching decisions. Further, by constraints (5.4) and (5.5), any non-zero flow along corridor (i, j) is given by $b_{ij}(\theta_i - \theta_j)$. Thus we have

$$-\overline{P}_{ij} \le b_{ij}(\theta_i - \theta_j) \le \overline{P}_{ij},$$

or equivalently

$$-x_{ij}\overline{P}_{ij} \le \theta_i - \theta_j \le x_{ij}\overline{P}_{ij}.$$

Now, taking the telescoping sum of this inequality for each corridor in ρ

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$$-\overline{P}_{i_1i_2}x_{i_1i_2} \le \theta_{i_1} - \theta_{i_2} \le \overline{P}_{i_1i_2}x_{i_1i_2} \tag{5.32}$$

$$-\overline{P}_{i_{j-1}i_j}x_{i_{j-1}i_j} \le \theta_{i_{j-1}} - \theta_{i_j} \le \overline{P}_{i_{j-1}i_j}x_{i_{j-1}i_j}$$
(5.34)

$$-\overline{P}_{i_{k-1}i_k}x_{i_{k-1}i_k} \le \theta_{k-1} - \theta_k \le \overline{P}_{i_{k-1}i_k}x_{i_{k-1}i_k}.$$
(5.36)

gives

$$-\sum_{(i,j)\in\rho} x_{ij}\overline{P}_{ij} \le \theta_{i_1} - \theta_{i_k} \le \sum_{(i,j)\in\rho} x_{ij}\overline{P}_{ij}.$$
(5.37)

But since $\sum_{(i,j)\in\rho} x_{ij}\overline{P}_{ij} = CR(\rho)$, $i_1 = n$ and $i_k = m$, we have that $-CR(\rho) \leq \theta_n - \theta_m \leq CR(\rho)$ is valid. \Box

5.4.1 Derivation

The single path VI introduced in Section 2.4 has been shown to be the basis of a number of other path-based VIs for DCOPF-based TEP. In this section, it is demonstrated how similar ideas can be extended to generate path-based VIs for DCOPF-based OTS. By selecting two parallel paths, one of which features the convenient property that none of the lines comprising the path can be switched off, we get a stepping stone lemma for generating VIs. Although it may seem mathematically restrictive to consider such a property for even one of the chosen paths, in realistic transmission systems, it is satisfied quite commonly. For example, lines in designated throughways are never considered switching options, and in general only a subset of lines is considered switchable [109].

Lemma 7. Let $\rho_1 \in C_{nm}$ and $\rho_2 \in C_{nm}$ be paths from θ_n to θ_m , with $CR(\rho_1) \leq CR(\rho_2)$ and $(i, j) \in \Omega \setminus \Phi \ \forall (i, j) \in \rho_2$, i.e. $s_{ij} = for \ each \ corridor \ (i, j)$ in the path ρ_2 (it is not switchable). Then the following are valid inequalities for OTS:

$$|\theta_n - \theta_m| \le CR(\rho_1) + (CR(\rho_2) - CR(\rho_1)) \left(|\rho_1| - \sum_{(i,j) \in \rho_1} s_{ij} \right)$$

Proof. Consider two cases:

Case 1: $s_{ij} = 1 \quad \forall (i,j) \in \rho_1$

This is the case in which all lines along ρ_1 are switched on. Then the stated VI is given by

$$|\theta_n - \theta_m| \le CR(\rho_1).$$

By direct application of Corollary 6.1, this is valid.

Case 2: $s_{ij} = 0$ for some $(i, j) \in \rho_1$

This is the case in which some line along ρ_1 is switched off, i.e. θ_n and θ_m are not connected by ρ_1 . Since θ_n and θ_m are still connected along ρ_2 , we have that $|\theta_n - \theta_m| \leq CR(\rho_2)$ is valid by Corollary 6.1. Also, since $s_{ij} = 0$ for at least one $(i, j) \in \rho_1$,

$$1 \le \left(|\rho_1| - \sum_{(i,j) \in \rho_1} s_{ij} \right),$$

hence

$$CR(\rho_2) - CR(\rho_1) \le (CR(\rho_2) - CR(\rho_1)) \left(|\rho_1| - \sum_{(i,j) \in \rho_1} s_{ij} \right).$$

Adding $CR(\rho_1)$ to both sides of this inequality gives

$$\leq CR(\rho_1) + (CR(\rho_2) - CR(\rho_1)) \left(|\rho_1| - \sum_{(i,j) \in \rho_1} s_{ij} \right).$$

Then, since $|\theta_n - \theta_m| \leq CR(\rho_2)$ is valid and

$$CR(\rho_2) \le CR(\rho_1) + (CR(\rho_2) - CR(\rho_1)) \left(|\rho_1| - \sum_{(i,j) \in \rho_1} s_{ij} \right),$$

the stated VI is valid as well.

Because of the required properties of one of the two paths and the sheer number of such pairs of parallel paths that exist in a network, it can be beneficial to have a method for generating a path-based VI solely on a single path in the network, with no requirements on the properties of such a path. **Theorem 8.** For any path $\rho \in C_{nm}$ from θ_n to θ_m , the following are valid inequalities for OTS if the network remains connected:

$$|\theta_n - \theta_m| \le CR(\rho) + \left(\overline{CR(\rho)} - CR(\rho)\right) \left(|\rho| - \sum_{(i,j)\in\rho} s_{ij}\right)$$

Proof. Again, consider two cases:

Case 1: $s_{ij} = 1 \quad \forall (i, j) \in \rho$

This represents the case that all lines in ρ are switched on; that is, ρ connects θ_n to θ_m . Then the stated VI is given by

$$|\theta_n - \theta_m| \le CR(\rho),$$

which is valid by Corollary 6.1.

Case 2: $s_{ij} = 0$ for some $(i, j) \in \rho$

Because the network remains connected, there exists some path ρ' which connects the two buses. By Corollary 6.1, $|\theta_n - \theta_m| \leq CR(\rho')$ is valid. Also, since $s_{ij} = 0$ for at least one $(i, j) \in \rho_1$,

$$1 \le \left(|\rho| - \sum_{(i,j) \in \rho} s_{ij} \right),$$

hence

$$\overline{CR(\rho)} - CR(\rho) \le \left(\overline{CR(\rho)} - CR(\rho)\right) \left(|\rho| - \sum_{(i,j) \in \rho} s_{ij} \right).$$

Adding $CR(\rho_1)$ to both sides of this inequality gives

$$\leq CR(\rho) + \left(\overline{CR(\rho)} - CR(\rho)\right) \left(|\rho| - \sum_{(i,j) \in \rho} s_{ij} \right).$$

Then, since $|\theta_n - \theta_m| \leq CR(\rho')$ is valid and $CR(\rho') \leq \overline{CR(\rho)}$ by definition, we have

$$CR(\rho') \le \overline{CR(\rho)} \le CR(\rho) + \left(\overline{CR(\rho)} - CR(\rho)\right) \left(|\rho| - \sum_{(i,j)\in\rho} s_{ij}\right),$$

so the stated VI is valid as well.

To close out this section, the previous theorem is generalized to allow the consideration of two distinct paths in the network. This most closely resembles the initial Lemma 7. The arguments presented here allow extensions to an arbitrary but finite number of parallel paths satisfying the relationship $CR(\rho_1) \leq CR(\rho_2) \leq \cdots \leq CR(\rho_n)$.

However, note that both this theorem and the previous theorem rely explicitly on the longest path in the network via the term $\overline{CR(\rho)}$. The longest path problem is NP-Hard which renders it intractable in the general case. However, transmission networks have low *tree-width* which is a measure of how tree-like a general graph is. There exists an algorithm for the longest path problem which is fixed-parameter tractable (FPT) when parameterized by tree-width. In the results presented in this chapter, it was found that weaker forms of the proven VIs (i.e., with two arbitrary paths and not using the longest path) improved computational times sufficiently to solve even large instances. Therefore, the implementation of such an FPT algorithm is left as future work.

Theorem 9. Let $\rho_1 \in C_{nm}$ and $\rho_2 \in C_{nm}$ be simple paths from θ_n to θ_m with $CR(\rho_1) \leq CR(\rho_2)$. Then the following are valid inequalities for OTS:

$$\begin{aligned} |\theta_n - \theta_m| &\leq CR(\rho_1) + (CR(\rho_2) - CR(\rho_1)) \left(|\rho_1| - \sum_{(i,j) \in \rho_1} s_{ij} \right) + \\ & \left(\overline{CR(\rho)} - CR(\rho_2) \right) \left(|\rho_2| - \sum_{(i,j) \in \rho_2} s_{ij} \right). \end{aligned}$$

Proof. Here 4 cases are considered.

Case 1: $s_{ij} = 1 \quad \forall (i, j) \in \rho_1, \rho_2$

This represents the case that all lines along both path ρ_1 and ρ_2 are switched on. Given this scenario, the stated VI simplifies to

$$|\theta_n - \theta_m| \le CR(\rho_1).$$

Since all lines along ρ_1 are switched on, this inequality is valid by Lemma 7.

Case 2: $s_{ij} = 1 \quad \forall (i,j) \in \rho_1, \ s_{ij} = 0 \text{ for some } (i,j) \in \rho_2$

This represents the case where all lines along path ρ_1 are switched on but at least one line along path ρ_2 is switched off. Given this scenario, the stated VI simplifies to

$$|\theta_n - \theta_m| \le CR(\rho_1) + \left(\overline{CR(\rho)} - CR(\rho_2)\right) \left(|\rho_2| - \sum_{(i,j)\in\rho_2} s_{ij} \right).$$

By definition, $CR(\rho) \ge CR(\rho_2)$ so that $\overline{CR(\rho)} - CR(\rho_2) \ge 0$. Further, since at least one $s_{ij} = 0$ for $(i, j) \in \rho_2$, we also have $|\rho_2| - \sum_{(i,j) \in \rho_2} s_{ij} \ge 1$. That is,

$$0 \le \left(\overline{CR(\rho)} - CR(\rho_2)\right) \left(|\rho_2| - \sum_{(i,j) \in \rho_2} s_{ij} \right).$$

Then,

$$|\theta_n - \theta_m| \le CR(\rho_1) \le CR(\rho_1) + \left(\overline{CR(\rho)} - CR(\rho_2)\right) \left(|\rho_2| - \sum_{(i,j)\in\rho_2} s_{ij}\right).$$

As in case 1, since all lines along ρ_1 are switched on, the left inequality is valid by Lemma 7, so the outer inequality is valid as well.

Case 3: $s_{ij} = 1 \quad \forall (i,j) \in \rho_2, \ s_{ij} = 0 \text{ for some } (i,j) \in \rho_1$

This represents the case that all lines along path ρ_2 are switched on but at least one line along path ρ_1 is switched off. Given this scenario, the stated VI simplifies to

$$|\theta_n - \theta_m| \le CR(\rho_1) + (CR(\rho_2) - CR(\rho_1)) \left(|\rho_1| - \sum_{(i,j) \in \rho_1} s_{ij} \right)$$

Since at least one line in ρ_1 is switched off, we have that $1 \leq \left(|\rho_1| - \sum_{(i,j) \in \rho_1} s_{ij} \right)$, which in turn gives that

$$(CR(\rho_2) - CR(\rho_1)) \le (CR(\rho_2) - CR(\rho_1)) \left(|\rho_1| - \sum_{(i,j) \in \rho_1} s_{ij} \right)$$

Since every line in ρ_2 is switched on, by Lemma 7 we have that $|\theta_n - \theta_m| \leq CR(\rho_2)$ is valid. Combining these gives the chain of relations

$$\begin{aligned} \theta_n - \theta_m &| \le CR(\rho_2) \\ &= CR(\rho_1) + CR(\rho_2) - CR(\rho_1) \\ &\le CR(\rho_1) + (CR(\rho_2) - CR(\rho_1)) \left(|\rho_1| - \sum_{(i,j) \in \rho_1} s_{ij} \right), \end{aligned}$$

hence the inequality is valid in this case as well.

Case 4: $s_{ij} = 0$ for some $(i, j) \in \rho_1$ and $s_{kl} = 0$ some $(k, l) \in \rho_2$

This represents the case that some line along path ρ_1 and some line along path ρ_2 are both switched off. Since the network remains connected by constraints (5.12), there exists some path ρ' connecting θ_n and θ_m . By Lemma 7, $|\theta_n - \theta_m| \leq \overline{CR(\rho)}$; that is, the difference in bus angles is bounded by the cumulative capacity-reactance product of the longest possible path connecting the two buses, since $CR(\rho') \leq \overline{CR(\rho)}$ by definition.

By the arguments in cases 2 and 3, we have that

$$\overline{CR(\rho)} = CR(\rho_1) + (CR(\rho_2) - CR(\rho_1)) + \left(\overline{CR(\rho)} - CR(\rho_2)\right)$$
$$\leq CR(\rho_1) + (CR(\rho_2) - CR(\rho_1)) \left(|\rho_1| - \sum_{(i,j)\in\rho_1} s_{ij}\right) + \left(\overline{CR(\rho)} - CR(\rho_2)\right) \left(|\rho_2| - \sum_{(i,j)\in\rho_2} s_{ij}\right).$$

Since $|\theta_n - \theta_m| \leq \overline{CR(\rho)}$ is valid, so again is the stated inequality.

5.4.2 Incomparability with Cycle-based Valid Inequalities

Kocuk et al. ([66]) have established a set of valid inequalities for OTS problem are dubbed *cycle-based inequalities*. Since the VIs presented in Section 5.4 implicitly and explicitly depend on pairs of coterminous paths, it is necessary to establish that they are distinct from the previously defined cycle-based VIs.

We first transform the inequalities from [66] into a form similar to the one proposed herein:

$$\left| \sum_{(i,j)\in S} \overline{x}_{ij}^C P_{ij} \right| \le \Delta S(|C|-1) - \sum_S [\Delta S - w_{ij}] s_{ij} - \Delta S \sum_{C \setminus S} s_{ij}, \quad \forall S \subseteq C \quad \text{s.t.} \quad \Delta S > 0$$
(5.38)

$$|\theta_n - \theta_m| \le \Delta S(|C| - 1) - \sum_{S} [\Delta S - w_{ij}] s_{ij} - \Delta S \sum_{C \setminus S} s_{ij}, \quad \forall S \subseteq C \quad \text{s.t.} \quad \Delta S > 0$$
(5.39)

The transformation is straightforward. Inequality (5.38) is as presented in [66], and (5.39) follows directly from (2.17) in Chapter 2. In these inequalities, C is a directed cycle, |C| is the number of edges in the cycle, and $w_{ij} = \overline{x}_{ij}^C \overline{P}_{ij}$, where

$$\overline{x}_{ij}^C = \begin{cases} x_{ij} & \text{if } (i,j) \in C, (i,j) \in \Omega \\ -x_{ij} & \text{if } (i,j) \in C, (j,i) \in \Omega. \end{cases}$$
(5.40)

Further, $S \subseteq C$ and $\Delta(S) = w(S) - w(C \setminus S) = 2w(S) - w(C)$, where $w(S) = \sum_{(i,j)\in S} w_{ij}$.

Consider the simple network presented in Figure 22. Although this is clearly not representative of a full network, it is a common substructure. The capacity \overline{P} and reactance x of each line is given as in the figure without specifying values to ensure that the sample network can match up with any reasonable and realistically constructed network. Without loss of generality, let $\theta_0 \ge \theta_2$ throughout this section. Both the cycle-based and path-based VIs relating the bus angles θ_0 and θ_2 are examined with respect to the sample network shown in Figure 22.

There are two cycle-based VIs that relate θ_0 and θ_2 while satisfying the required condition that $\Delta S > 0$. To derive these, first consider the directed cycle C =



Figure 22. Sample Network

 $\{(i_0, i_2), (i_2, i_1), (i_1, i_0)\}$. Then $w(C) = 3x\overline{P} - x\overline{P} - x\overline{P} = x\overline{P}$. Consider the subset $S = \{(i_2, i_1), (i_1, i_0)\}$. Then $w(S) = -x\overline{P} - x\overline{P} = -2x\overline{P}$ and $\Delta(S) = -5x\overline{P}$, so this subset does not generate a corresponding cycle-based VI. Then consider instead the subset $S = \{(i_0, i_2)\}$. Then $w(S) = 3x\overline{P}$ and $\Delta(S) = 5x\overline{P}$. Therefore, this subset does in fact lead to a cycle-based VI. Plugging these values into (5.39) gives the first VI, (5.41), below.

To get the second cycle-based VI, consider the directed cycle which is the reverse of $C, C' = \{(i_0, i_1), (i_1, i_2), (i_2, i_0)\}$. In this case, $w(C') = x\overline{P} + x\overline{P} - 3x\overline{P} = -x\overline{P}$. Similar to the previous VI, there are two subsets to consider. The first subset $S = \{(i_2, i_0)\}$, gives $w(S) = -3x\overline{P}$ and $\Delta(S) = -7x\overline{P}$, so it does not generate a cycle-based VI. The second subset $S = \{(i_0, i_1), (i_1, i_2)\}$ gives $w(S) = 2x\overline{P}$, leading to $\Delta(S) = 5x\overline{P}$, so this subset leads to the second cycle-based VI (see (5.42)).

 $\theta_0 - \theta_2 + 2x\overline{P}s_{02} + 5x\overline{P}s_{01} + 5x\overline{P}s_{12} \le 10x\overline{P} \tag{5.41}$

$$\theta_0 - \theta_2 + 5x\overline{P}s_{02} + 4x\overline{P}s_{01} + 4x\overline{P}s_{12} \le 10x\overline{P} \tag{5.42}$$

In contrast, there is only one possible path-based VI using the most basic form of Lemma 7. To establish it, let $\rho_1 = \{(i_0, i_1), (i_1, i_2)\}$ and $\rho_2 = \{(i_0, i_2)\}$, which gives that $CR(\rho_1) = 2x\overline{P}, CR(\rho_2) = 3x\overline{P}$, and $|\rho_1| = 2$. Plugging these values into the lemma gives

$$\theta_0 - \theta_2 \le 2x\overline{P} + (3x\overline{P} - 2x\overline{P})(2 - s_{01} - s_{12}) \quad (5.43)$$

$$\Rightarrow \theta_0 - \theta_2 + 0x\overline{P}s_{02} + x\overline{P}s_{01} + x\overline{P}s_{12} \le 4x\overline{P}.$$
(5.44)

For two non-equivalent VIs $\gamma^T \mathbf{x} \leq \gamma_0$ and $\pi^T \mathbf{x} \leq \pi_0$, if there exists $\mu > 0$ such that $\gamma \geq \mu \pi$ and $\gamma_0 \leq \mu \pi_0$ then we say that $\pi^T \mathbf{x} \leq \gamma_0$ dominates $\pi^T \mathbf{x} \leq \gamma_0$. To show that a pair of VIs is incomparable, it must be demonstrated that neither dominates the other.

We begin by showing that (5.44) does not dominate either of (5.41) or (5.42). First, it is obvious that none of these inequalities is equivalent to the others. To show that (5.44) dominates (5.41), it is necessary to find $\mu > 0$ such that both of the following inequalities are satisfied:

$$\begin{bmatrix} 1 & -1 & 0x\overline{P} & x\overline{P} & x\overline{P} \end{bmatrix}^T \ge$$
(5.45)

$$\mu \cdot \begin{bmatrix} 1 & -1 & 2x\overline{P} & 5x\overline{P} & 5x\overline{P} \end{bmatrix}^T \tag{5.46}$$

and

$$4x\overline{P} \le \mu \cdot 10x\overline{P}.\tag{5.47}$$

Comparing the fourth coefficients of the first inequality gives that $\mu \leq 1/5$. However, comparing the coefficients from the second inequality gives that $\mu \geq 2/5$. Since these inequalities are inconsistent, there is no such μ implying that (5.44) does not dominate (5.41). A similar argument holds when comparing (5.44) to (5.42).

We now show the converse, that neither (5.41) nor (5.42) dominates (5.44). As before, note that none of these inequalities is equivalent to any of the others. In this case, to show that (5.41) dominates (5.44), it is necessary to find $\mu > 0$ such that both of the following inequalities are satisfied:

$$[1 \quad -1 \quad 2x\overline{P} \quad 5x\overline{P} \quad 5x\overline{P}]^T \ge$$

$$(5.48)$$

$$\mu \cdot \begin{bmatrix} 1 & -1 & 0x\overline{P} & x\overline{P} & x\overline{P} \end{bmatrix}^T$$
(5.49)

and

$$10x\overline{P} \le \mu \cdot 4x\overline{P}.\tag{5.50}$$

Comparing the third coefficients in the first inequality gives $\mu \leq 2$. However, comparing the coefficients from the second inequality gives that $\mu \geq 5/2$. As before, these inequalities are inconsistent, so (5.41) does not dominate (5.44) and a similar argument shows (5.42) likewise does not dominate (5.44).

Since neither the herein proposed path-based valid inequalities nor the previously proven in [66] cycle-based valid inequalities for OTS dominate the other, they are incomparable. This establishes that the path-based VIs derived in Section 5.4 are a novel contribution distinct from previous results in [66].

5.5 Results

All experiments for the stochastic restoration model described in Section 5.3 are tested on the GOC 2000-bus system. This is a large synthetic system designed for the ARPA-e Grid Optimization Competition (GOC), developed in [14]. To solve the model, a set of scenarios is defined. For each bus, generator, and line, a scenario is defined such that the element fails at a fixed rate independent of all other failures. All scenarios are considered equally likely. In total, 100 scenarios are generated for each experiment. Then the extensive form of the model from Section 5.3 encompassing all scenarios is generated and solved using Gurobi version 9.1.2. Throughout, the valid inequalities specified in Section 5.4 are added via a callback function whenever the optimization algorithm reaches a new incumbent solution. The list of potential VIs to be incorporated is generated by a cycle-basis, as described in [66]. All tests were completed in less than 15 minutes. Since the full formulation involves more than 1.2 million constraints and 280,000 binary variables, attempts to solve the model without adding VIs were terminated after 12 hours.



Figure 23. Expected Percent Load Shed

For the initial experiment, the constraining factor on investment is assumed to be the number of man-hours needed to harden a given transmission component. To this end, a simplifying approximation is made that hardening a bus takes one man-hour, a line takes five, and a generator takes ten. This reflects the fact that a bus failure only impacts the level of demand at a location, while transmission lines cover significant lengths and may require efforts at both ends, while hardening a generator is the most difficult or time-consuming reinforcing action. Individual components are assumed to fail at rates between 5%-30% in increments of five percentage points, with buses and generators failing at the same rate and lines failing at a distinct rate. The overall expected load shed is shown in Figure 23. Generally, as the total failure rate of all components increases, load shed expectedly increases. Of note is the bottom-right of the figure, namely when buses and generators fail at a 30% rate and lines fail at a 20% or higher rate. These combinations result in a total disconnection of the grid and represent potentially cascading failures across all elements. While hardening investments can help mitigate the effects of a disaster, these worst-case outcomes demonstrate the limits of short-term investments.



(a) Bus Hardening Investments



(b) Generator Hardening Investments



(c) Line Hardening Investments

The remaining Figures, 24a, 24b, and 24c, summarize the number of man-hours invested in bus, generator, and line hardening, respectively. In the same three extreme cases as seen in Figure 23, it can clearly be seen across each type of investment that there is no amount of hardening within the allotted budget that can keep the network connected. Hence in each case, only a single bus sees any short-term investment. Figure 24a shows clearly that if the failure rate of lines is kept fixed, then bus investment increases as bus failure rate increases. This makes sense: if a similar number of lines are damaged by the disaster but more and more buses are damaged, shifting preparation efforts toward mitigating the damage to buses and away from lines logically should minimize load shed. Conversely, Figure 24c shows the same trend when bus and generator failure rates are fixed but line failure rates increase. In fact, the pattern is demonstrated much more strongly in line investment. For minimum failure rates, both bus and line investment are allocated 205 man-hours of hardening efforts before the disaster occurs. However, bus investment only increases to 271 man-hours as bus and generator failure rates increase, while line investment increases to 370 man-hours. Because bus and generator failure rates are assumed to be the same in these cases, it is necessary to note that generator investment increases by 140 man-hours across the same scenarios.

A final key point about the bus and generation investment charts is that at low levels of line failure rates, bus hardening investments appear to peak around 15%-20% bus failure rate, then decline as the bus failure rate further increases. In large part, this is due to the generation overhead inherent in the system. Because there is more than enough possible generation in the system to meet all demand, bus investment increases quickly as bus and generator failure rates increase. However, as the failure rate continues to increase for these elements, the actual available generation begins to fall below demand. In these situations, line investment is extremely low, bus investment slightly decreases, and generator investment increases to attempt to keep available generation close to total demand.

In fact, generator investment, summarized in Figure 24b, appears to be more volatile with respect to component failure rates than either bus or line investment. There are most likely two key factors contributing to this. The first is fundamental to power system operation: in order to meet demand, generators must be online and connected to the transmission grid. Small changes in failure rates or considered scenarios can lead to outsize changes in generator investment. This is only amplified by the potentially limited pathways for delivering power during a disaster. The other factor is that a single investment in hardening a generator has been assigned a higher cost in this experiment. While this was a specific assumption, it reflects the reality of attempting to harden a generator and is likely to be a smaller factor than the standard operating principles of the transmission grid.

5.6 Conclusion

This chapter presents a new two-stage stochastic mathematical model to guide hardening investments on transmission grid elements in preparation for a natural disaster with transmission switching as a post-disaster recourse. The nature of the transmission switching decisions in the second stage of the model renders the problem exceptionally computationally difficult. To accelerate solving this problem, a set of valid inequalities for optimal transmission switching are derived and shown to be valid. These share a similar structure to those established in Chapter 2. Because another work, [66], develops a set of cycle-based VIs for OTS that seem to employ similar insights, the VIs proposed in this chapter are compared to those of [66]. The two sets of VIs are shown to be incomparable; that is, both contribute significant and distinct value to solving OTS.

The model and supporting VIs are tested on the GOC 2000-bus system to show their ability to solve realistically sized instances in a reasonable length of time. Solving the two-stage stochastic program with one hundred scenarios takes less than fifteen minutes. Based on the experiments run, the trade-offs in hardening investment decisions are clear when the relative failure rates of transmission lines compared to buses or generators are well known. Investment in hardening generators is the most sensitive to prior information and those assumptions must be considered carefully.

This chapter attempts to bridge long-term concerns of reliability from earlier chapters with short-term concerns of network resiliency. To that end, it is important to explain that *reliability* involves ensuring power is delivered, while *resiliency* refers to the network's ability to plan for, survive, and recover from an adverse shock [20]. Reliability is typically included in models as, e.g., N-k security constraints [115]; resiliency needs to be modeled over time to allow for recovery from events. The results from this chapter already help illustrate the potential for transmission switching and preemptive investment to increase the resilience of the network. However, a more detailed picture of resiliency relies on modeling more time periods (such as a few minutes after the disaster, an hour after, etc.). The valid inequalities proposed throughout this dissertation, and in this chapter in particular, can be used to accelerate the solution of such a multi-time period model that relies on binary flexible transmission options at its multiple levels. Furthermore, additional factors of resiliency such as time to repair or sequencing for repairs could be added to such a model [32], [41], [96].

Another extension of this chapter would be including fragility curves to more accu-

rately model failure probabilities of each transmission network component. Fragility curves express the probability of a component's failure as a function of the weather parameters (e.g., wind speed, temperature, rainfall, etc.) [98]. At this stage, because the model presented in this chapter only considers a most generic "high-impact, low-probability" event, specific fragility curves were not used. This choice causes all generators, buses, and lines to be equally at risk. In reality, based on age, construction, location, and the nature of the disaster event, the risk among components is distributed far from evenly. Thus, the results and practical relevance of this chapter could be strengthened by accounting for fragility curves across the network as well as improved disaster event modeling to guide such curves and better represent the true resiliency of the network.

Chapter 6

GENERAL DISCUSSION AND CONCLUSIONS

The operation of the power grid is a fundamental concern to modern society. Its construction and expansion is one of the great engineering feats of the twentieth century. Unfortunately, a number of pressing challenges threaten the grid's ongoing functionality. A large portion of the grid is comprised of heavy components, which makes it difficult, time-consuming, and expensive to invest in new components or improve existing components. Furthermore, the current power grid is already failing to meet demands with regularity, and ad hoc operating decisions are insufficient to correct this. Compounding these issues is the ongoing and ever-increasing threat of anthropogenic climate change. In fact, climate change threatens the grid in two concomitant ways. The first is simply rising temperatures. These cause increased demand, reduced generation, and reduced transmission ampacity that leads to increased congestion. However, rising temperatures lead to increased frequency and severity of severe weather events that can cause significant damage to the grid.

One effective method to partially counteract these challenges is cost-effective transmission expansion planning. This dissertation presents new models and methodologies for investing in the transmission grid specifically with the purpose of mitigating the effects of anthropogenic climate change. It does this by leveraging structural insights about the variable topologies made possible by flexible transmission decisions, including expansion, capacity expansion, and switching. The insights allow a number of classes of valid inequality which are applicable to transmission problems incorporating each of the above transmission decisions either independently or in concert with one another. These valid inequalities are then used in solution algorithms for deterministic, stochastic, and robust optimization frameworks. These improvements – and others like them – are necessary to help solve transmission problems with flexible transmission decisions because the addition of binary variables to represent those decisions renders the problems NP-hard. Valid inequalities help reduce the solution search space and are often complementary to other techniques.

The results presented in this dissertation are a first step toward incorporating the proposed VIs in a variety of flexible topology problems. However, their are clear directions along which these VIs can be extended and made more usable. An obvious direction is to combine the ideas from Chapter 4 and 5 to solve long-term planning, short-term preemptive action, and post-contingency reactive action into a single formulation. Because of the multiple stages, each of which contains binary decision variables, necessary for such a formulation, such a problem is potentially prohibitively difficult to solve. Through the inclusion of the proposed VIs at each stage of a solution algorithm alongside other modeling advances such as modern decomposition techniques, it may be possible to sufficiently accelerate the solution of such a problem so as to make it tractable.

Perhaps the more important extension is the adaptation of the proposed VIs to other problems featuring variable topology and/or non-trivial physical laws binding nodes in the network. These levers are potentially generalizable beyond the realm of power systems into many other essential network optimization problems. While the potential value of this work to fight climate change and improve power systems operation is enormous, such a general result could have untold benefits across modern optimization. By continuing to explore the application and value of the proposed VIs in the context of flexible transmission, it may be possible to shed light on these global advances.

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APPENDIX A

ARIZONA TEST CASES

The test cases used in Chapters 3 and 4 are included here for replicability; they are denoted as "AZ 2020 Test Case" and "AZ 2021 Test Case," respectively. The case study presented in this paper is an approximation of the transmission grid contained within the borders of Arizona; the basic description of this network is provided by the U.S. Energy Information Administration [120]. Transmission lines rated at 69kV and above are included in the network as are all power plants rated to produce two or more MW/Hr. Additional corridors for transmission expansion are approximated by connecting high generation areas to high demand areas and connecting substations with few adjacent lines to more dense areas of the grid. All listed substations are used as buses to connect transmission lines and meet aggregated demand for nearby areas. Hourly demand for the test instance is based on peak summer demand for the full state. As the state level is the smallest resolution data available for load, this value is disaggregated by assigning load to substations according to the relative population in the nearest census block. Generation costs are approximated by the total statewide costs of generating each category of plant (natural gas, coal, petroleum, hydro, wind, and solar), divided according to the rated MW of the corresponding plant. Transmission expansion and capacity expansion costs are approximated based on the rated voltage and length of lines, using cost estimates from [87]. The main difference between the test cases is that the "AZ 2021 Test Case" uses updated data (current to 2021) which is significantly different from the data available for "AZ 2020 Test Case," which is only accurate through December, 2019. The formatting of each test case is consistent with the MATPOWER test cases described in detail by [14].

A.1 AZ 2020 Test Case

mpc.version = '2';
mpc.baseMVA = 100.0;

```
%% bus_i type Pd Qd Gs Bs area Vm Va baseKV zone Vmax Vmin
mpc.bus = [
1 1 3 0 0 0 1 0 0 0 5 0 0;
2 1 1 0 0 0 1 0 0 0 5 0 0;
3 1 6 0 0 0 1 0 0 0 5 0 0;
4 1 4 0 0 0 1 0 0 0 5 0 0;
5 1 5 0 0 0 2 0 0 0 0 0 0;
6 1 5 0 0 0 3 0 0 0 4 0 0;
7 1 4 0 0 0 1 0 0 0 5 0 0;
8 1 7 0 0 0 3 0 0 0 4 0 0;
9 1 6 0 0 0 3 0 0 0 4 0 0;
```

```
808 1 9 0 0 0 3 0 0 0 6 0 0;
809 1 68.35666324 0 0 0 3 0 0 0 6 0 0;
810 1 130 0 0 0 2 0 0 0 0 0 0;
811 \ 1 \ 13.8359315 \ 0 \ 0 \ 3 \ 0 \ 0 \ 6 \ 0 \ 0;
812 1 21 0 0 0 2 0 0 0 4 0 0;
813 1 382.8411675 0 0 0 3 0 0 0 6 0 0;
814 1 43.55244988 0 0 0 3 0 0 0 6 0 0;
815 1 5 0 0 0 3 0 0 0 6 0 0;
816 1 18.80568918 0 0 0 3 0 0 0 6 0 0;
817 1 1070 0 0 0 3 0 0 0 6 0 0;
818 1 23.25 0 0 0 3 0 0 0 1 0 0;
819 1 64 0 0 0 2 0 0 0 0 0 0;
820 1 10 0 0 0 2 0 0 0 5 0 0;
821 1 540 0 0 0 3 0 0 6 0 0;
822 1 10 0 0 0 3 0 0 0 6 0 0;
823 1 59.71498104 0 0 0 3 0 0 0 6 0 0;
824 1 11.42599476 0 0 0 3 0 0 0 6 0 0;
825 1 7 0 0 0 3 0 0 6 0 0;
826 1 7 0 0 0 3 0 0 0 6 0 0;
827 1 5 0 0 0 3 0 0 0 1 0 0;
828 1 35.5 0 0 0 3 0 0 0 6 0 0;
829 1 3 0 0 0 3 0 0 6 0 0;
830 1 9.45999883 0 0 0 3 0 0 0 6 0 0;
831 1 7 0 0 0 2 0 0 0 5 0 0;
832 1 7 0 0 0 3 0 0 0 6 0 0;
833 1 35.5 0 0 0 3 0 0 0 6 0 0;
834 1 7 0 0 0 3 0 0 0 1 0 0;
835 1 2 0 0 0 3 0 0 0 6 0 0;
836 1 6 0 0 0 3 0 0 0 6 0 0;
837 1 21.20474167 0 0 0 3 0 0 0 4 0 0;
838 1 6.128012256 0 0 0 3 0 0 0 6 0 0;
839 1 130 0 0 0 3 0 0 0 1 0 0;
840 1 64 0 0 0 3 0 0 0 6 0 0;
841 1 10.62988682 0 0 0 3 0 0 0 6 0 0;
842 1 12.01217623 0 0 0 3 0 0 0 6 0 0;
843 1 32.72597529 0 0 0 3 0 0 0 6 0 0;
844 1 23.27476156 0 0 0 3 0 0 0 6 0 0;
845 1 13.28543047 0 0 0 3 0 0 0 1 0 0;
846 1 4 0 0 0 3 0 0 0 1 0 0;
847 1 72.01198889 0 0 0 3 0 0 0 6 0 0;
848 1 77.71348102 0 0 0 3 0 0 0 6 0 0;
849 1 64 0 0 0 3 0 0 0 6 0 0;
```

892 1 19.21418799 0 0 0 3 0 0 0 4 0 0;

```
272 4.2 0 0 0 0 0 1 4.2 0; 0
53 160 0 0 0 0 0 1 160 0; 0
544 20 0 0 0 0 0 1 20 0; 0
57 12 0 0 0 0 0 1 12 0; 0
557 1.3 0 0 0 0 0 1 1.3 0; 0
728 10 0 0 0 0 0 1 10 0; 0
17 1.2 0 0 0 0 0 1 1.2 0; 0
606 55.3 0 0 0 0 0 1 55.3 0; 0
210 55 0 0 0 0 0 1 55 0; 0
817 30 0 0 0 0 0 1 30 0; 0
416 10 0 0 0 0 0 1 10 0; 0
378 2 0 0 0 0 0 1 2 0; 0
35 1.1 0 0 0 0 0 1 1.1 0; 0
322 2.4 0 0 0 0 0 1 2.4 0; 0
95 10 0 0 0 0 0 1 10 0; 0
71 32.5 0 0 0 0 0 1 32.5 0; 0
818 1.3 0 0 0 0 0 1 1.3 0; 0
326 10 0 0 0 0 0 1 10 0; 0
71 2409 0 0 0 0 0 1 2409 0; 0
    167 1039 0 0 0 0 0 1 1039 0; 0
    186 2 0 0 0 0 0 1 2 0; 0
    258 900 0 0 0 0 0 1 900 0; 0
    564 127 0 0 0 0 0 1 127 0; 0
    356 1678 0 0 0 0 0 1 1678 0; 0
    631 200 0 0 0 0 0 0 1 200 0; 0
    19 6 0 0 0 0 0 1 6 0; 0
    796 200 0 0 0 0 0 1 200 0; 0
    15 900 0 0 0 0 0 1 900 0; 0
    184 900 0 0 0 0 0 1 900 0; 0
    193 900 0 0 0 0 0 1 900 0; 0
    264 900 0 0 0 0 0 1 900 0; 0
    755 900 0 0 0 0 0 1 900 0; 0
    634 900 0 0 0 0 0 1 900 0; 0
];
%% generator cost data
% 2 startup shutdown n c(n-1) ... c0
mpc.gencost = [
200300;
2 0 0 3 0 186393211.2 0;
2 0 0 3 0 2211381215 0;
2 0 0 3 0 80780302.71 0;
```

```
2 0 0 3 0 119194672.8 0;
2 0 0 3 0 22802459.14 0;
2 0 0 3 0 10500501193 0;
2 0 0 3 0 9716957.022 0;
2 0 0 3 0 63484119.21 0;
2 0 0 3 0 81104201.28 0;
200300;
200300;
2 0 0 3 0 68018699.15 0;
200300;
2003000;
200300;
200300;
200300;
2 0 0 3 0 90918232.09 0;
2 0 0 3 0 6849000000 0;
2 0 0 3 0 12308145.56 0;
2 0 0 3 0 185178131.6 0;
2 0 0 3 0 7812433.445 0;
200300;
200300;
2 0 0 3 0 158969416.9 0;
2 0 0 3 0 394900871.1 0;
2 0 0 3 0 221345416.6 0;
2 0 0 3 0 6763002.087 0;
2 0 0 3 0 73848873.37 0;
2 0 0 3 0 74755789.35 0;
2 0 0 3 0 71257684.83 0;
2 0 0 3 0 75144467.63 0;
2 0 0 3 0 66723104.88 0;
2 0 0 3 0 135117526.4 0;
2 0 0 3 0 121008504.8 0;
2 0 0 3 0 80845082.42 0;
2 0 0 3 0 53119365.05 0;
2003000;
2 0 0 3 0 1075343.244 0;
2003000;
2 0 0 3 0 11660348.43 0;
200300;
200300;
2 0 0 3 0 442276243.1 0;
2 0 0 3 0 67474549.56 0;
```

200300;200300;2003000; 2003000; 200300;2003000;200300;200300;2003000;200300;2003000; 200300;2003000; 200300;2003000; 200300;2003000; 200300;2003000; 200300;200300;2003000; 2003000; 200300;2003000; 200300;2003000; 200300;2003000; 2003000; 2 0 0 3 0 2215466.201 0; 200300;200300;2003000; 200300;2003000;200300;2003000; 200300;200300;2003000; 2003000;

2 0 0 3 0 77087859.04 0; 200300;2003000; 2003000; 200300;2003000;200300;200300;2003000;200300;2003000; 2 0 0 3 0 647797.1348 0; 2 0 0 3 0 66723104.88 0; 200300;2003000; 200300;2003000;200300;2 0 0 3 0 66723104.88 0; 2003000;200300;200300;2003000; 200300;2003000;200300;2003000; 200300;200300;2003000; 2003000; 2 0 0 3 0 66723104.88 0; 200300;2003000; 200300;200300;200300;2003000; 200300;200300;200300;2003000;

```
200300;
200300;
200300;
200300;
200300;
200300;
200300;
200300;
2 0 0 3 0 158969416.9 0;
2 0 0 3 0 394900871.1 0;
2 0 0 3 0 221345416.6 0;
2 0 0 3 0 6763002.087 0;
2 0 0 3 0 73848873.37 0;
2 0 0 3 0 74755789.35 0;
2 0 0 3 0 71257684.83 0;
2 0 0 3 0 75144467.63 0;
2 0 0 3 0 66723104.88 0;
2 0 0 3 0 135117526.4 0;
2 0 0 3 0 121008504.8 0;
2 0 0 3 0 80845082.42 0;
2 0 0 3 0 53119365.05 0;
2 0 0 3 0 121008504.8 0;
2 0 0 3 0 80845082.42 0;
];
%% branch data
% fbus tbus r x b rateA rateB rateC ratio angle status angmin angmax
mpc.branch = [
3 4 0 0.013694176 0 1400000 280000 16.65 0 0 1 0 0;
4 1 0 0.000939006 0 1400000 280000 16.65 0 0 1 0 0;
5 3 0 0.239772901 0 1400000 280000 4.813287748 0 0 1 0 0;
3 7 0 0.000625165 0 1500000 300000 37.5 0 0 1 0 0;
546 569 0 0.099909127 0 1500000 300000 8.262574616 0 0 1 0 0;
722 546 0 0.075141061 0 1500000 300000 9.970430403 0 0 1 0 0;
892 81 0 0.015398315 0 1800000 360000 66 0 0 1 0 0;
722 892 0 0.025142832 0 1800000 360000 66 0 0 1 0 0;
581 722 0 0.062622462 0 1500000 300000 11.24375759 0 0 1 0 0;
569 186 0 0.422207775 0 1500000 300000 6.25 0 0 1 0 0;
240 885 0 0.019058948 0 1800000 360000 66 0 0 1 0 0;
890 886 0 0.043519245 0 1800000 360000 66 0 0 1 0 0;
890 240 0 0.003080544 0 1800000 360000 68.93814693 0 0 1 0 0;
884 886 0 0.019379787 0 1800000 360000 66 0 0 1 0 0;
```

```
539 853 0 0.00053049 0 1500000 300000 37.5 0 0 1 0 0;
804 611 0 0.004874326 0 1800000 360000 66 0 0 1 0 0;
448 474 0 0.000658664 0 1500000 300000 37.5 0 0 1 0 0;
752 493 0 0.127855434 0 1500000 300000 23 0 0 1 0 0;
376 651 0 0.019126694 0 1500000 300000 24.58178839 0 0 1 0 0;
39 293 0 0.126095479 0 1800000 360000 66 0 0 1 0 0;
155 57 0 0.233005159 0 2800000 560000 187.5 0 0 1 0 0;
281 147 0 0.09164018 0 1600000 320000 25.25 0 0 1 0 0;
744 564 0 0.14141287 0 3000000 600000 455 0 0 1 0 0;
643 267 0 0.044404608 0 1800000 360000 66 0 0 1 0 0;
803 323 0 0.550029925 0 1800000 360000 66 0 0 1 0 0;
624 854 0 0.002233978 0 1800000 360000 85.21024571 0 0 1 0 0;
825 353 0 0.014805949 0 1500000 300000 29.10397376 0 0 1 0 0;
822 537 0 0.001402083 0 1800000 360000 115.8542941 0 0 1 0 0;
607 107 0 0.033487102 0 1500000 300000 16.9901906 0 0 1 0 0;
501 804 0 0.010494257 0 1500000 300000 36.52055327 0 0 1 0 0;
363 78 0 0.124874676 0 1800000 360000 66 0 0 1 0 0;
551 323 0 0.490876762 0 1800000 360000 66 0 0 1 0 0;
261 755 0 0.023442472 0 1600000 320000 25.25 0 0 1 0 0;
481 799 0 0.181749183 0 1800000 360000 66 0 0 1 0 0;
502 236 0 0.047665245 0 1800000 360000 66 0 0 1 0 0;
114 838 0 0.006204115 0 1500000 300000 37.5 0 0 1 0 0;
17 850 0 0.012899202 0 1500000 300000 31.87416444 0 0 1 0 0;
793 223 0 0.031404738 0 1800000 360000 66 0 0 1 0 0;
746 281 0 0.033637729 0 1600000 320000 25.25 0 0 1 0 0;
828 768 0 0.00924307 0 1500000 300000 37.5 0 0 1 0 0;
656 582 0 0.008420575 0 1500000 300000 37.5 0 0 1 0 0;
314 850 0 0.007709668 0 1800000 360000 66 0 0 1 0 0;
804 768 0 0.029767982 0 1500000 300000 18.36188561 0 0 1 0 0;
620 511 0 0.274903137 0 1800000 360000 66 0 0 1 0 0;
272 626 0 0.032584476 0 1500000 300000 17.29913608 0 0 1 0 0;
244 515 0 0.117568088 0 3000000 600000 455 0 0 1 0 0;
726 120 0 0.019587626 0 1500000 300000 24.19875399 0 0 1 0 0;
250 629 0 0.016869918 0 1500000 300000 26.70385717 0 0 1 0 0;
193 244 0 0.006291898 0 3000000 600000 455 0 0 1 0 0;
588 35 0 0.082622632 0 1600000 320000 25.25 0 0 1 0 0;
796 118 0 0.465339827 0 1500000 300000 23 0 0 1 0 0;
383 200 0 0.038094588 0 1500000 300000 15.60542745 0 0 1 0 0;
446 317 0 0.020564391 0 1500000 300000 23.43446803 0 0 1 0 0;
73 145 0 0.056775465 0 1800000 360000 66 0 0 1 0 0;
787 203 0 0.458335413 0 1800000 360000 66 0 0 1 0 0;
13 638 0 0.146064103 0 1800000 360000 66 0 0 1 0 0;
```
```
606 661 0 0.028813182 0 1800000 360000 66 0 0 1 0 0;
116 69 0 0.126066261 0 1500000 300000 7.087776988 0 0 1 0 0;
440 447 0 0.019741737 0 1500000 300000 24.07400524 0 0 1 0 0;
597 682 0 0.084171061 0 1500000 300000 23 0 0 1 0 0;
585 264 0 0.370494372 0 1500000 300000 23 0 0 1 0 0;
66 20 0 0.261873889 0 3000000 600000 455 0 0 1 0 0;
734 17 0 0.019652026 0 1500000 300000 24.14642666 0 0 1 0 0;
681 356 0 0.079762389 0 1500000 300000 9.585595933 0 0 1 0 0;
43 502 0 0.056411769 0 1800000 360000 66 0 0 1 0 0;
337 696 0 0.049862036 0 1800000 360000 66 0 0 1 0 0;
648 139 0 0.005648855 0 1500000 300000 37.5 0 0 1 0 0;
141 429 0 0.034112243 0 1500000 300000 16.78420117 0 0 1 0 0;
412 848 0 0.009874379 0 1800000 360000 66 0 0 1 0 0;
735 540 0 0.009686555 0 1500000 300000 37.5 0 0 1 0 0;
598 96 0 0.45690558 0 1800000 360000 66 0 0 1 0 0;
289 240 0 0.217673819 0 1800000 360000 66 0 0 1 0 0;
83 666 0 0.03489045 0 1500000 300000 16.53636382 0 0 1 0 0;
244 20 0 0.019214603 0 3000000 600000 455 0 0 1 0 0;
111 635 0 0.230324994 0 1800000 360000 66 0 0 1 0 0;
635 337 0 0.001967562 0 1800000 360000 92.65387418 0 0 1 0 0;
744 134 0 0.292300951 0 3000000 600000 455 0 0 1 0 0;
207 634 0 0.009731206 0 1500000 300000 37.5 0 0 1 0 0;
157 438 0 0.000629558 0 1800000 360000 196.4470291 0 0 1 0 0;
215 769 0 0.154936929 0 1800000 360000 66 0 0 1 0 0;
311 380 0 0.032276717 0 1500000 300000 17.4077433 0 0 1 0 0;
152 316 0 0.026359469 0 1500000 300000 19.89515251 0 0 1 0 0;
294 462 0 0.030367056 0 1500000 300000 18.12217947 0 0 1 0 0;
641 667 0 0.319849409 0 1500000 300000 6.25 0 0 1 0 0;
809 107 0 0.06833009 0 1800000 360000 66 0 0 1 0 0;
592 676 0 0.026998425 0 1800000 360000 66 0 0 1 0 0;
540 634 0 0.009947571 0 1500000 300000 37.5 0 0 1 0 0;
591 217 0 0.022846301 0 1500000 300000 21.86331064 0 0 1 0 0;
542 673 0 0.06557819 0 1600000 320000 25.25 0 0 1 0 0;
758 21 0 0.138858415 0 1600000 320000 25.25 0 0 1 0 0;
784 162 0 0.129844661 0 1800000 360000 66 0 0 1 0 0;
852 257 0 0.036690959 0 1800000 360000 66 0 0 1 0 0;
217 793 0 0.013712675 0 1500000 300000 30.61419987 0 0 1 0 0;
564 398 0 0.404653666 0 2800000 560000 187.5 0 0 1 0 0;
118 512 0 0.133160681 0 1800000 360000 66 0 0 1 0 0;
713 224 0 0.030033108 0 1500000 300000 18.25482268 0 0 1 0 0;
595 386 0 0.001225145 0 1800000 360000 126.6338325 0 0 1 0 0;
360 620 0 0.074968377 0 1800000 360000 66 0 0 1 0 0;
```

158	695	0	0.	. 043519245	0	1800000	360000	44	0	0	1	0	0;
165	798	0	0.	.043519245	0	1800000	360000	44	0	0	1	0	0;
168	614	0	0.	.043519245	0	1800000	360000	44	0	0	1	0	0;
174	550	0	0.	.043519245	0	1800000	360000	44	0	0	1	0	0;
177	470	0	0.	.043519245	0	1800000	360000	44	0	0	1	0	0;
179	699	0	0.	.043519245	0	1800000	360000	44	0	0	1	0	0;
183	862	0	0.	.043519245	0	1800000	360000	44	0	0	1	0	0;
189	521	0	0.	.043519245	0	1800000	360000	44	0	0	1	0	0;
797	379	0	0.	.043519245	0	1800000	360000	44	0	0	1	0	0;
194	882	0	0.	.043519245	0	1800000	360000	44	0	0	1	0	0;
197	610	0	0.	. 043519245	0	1800000	360000	44	0	0	1	0	0;
198	348	0	0.	.043519245	0	1800000	360000	44	0	0	1	0	0;
205	753	0	0.	.043519245	0	1800000	360000	44	0	0	1	0	0;
209	379	0	0.	.043519245	0	1800000	360000	44	0	0	1	0	0;
211	379	0	0.	.043519245	0	1800000	360000	44	0	0	1	0	0;
226	327	0	0.	.043519245	0	1800000	360000	44	0	0	1	0	0;
228	63 () ().()43519245	0 3	1800000 3	360000 4	14 () () 1	1 () ();
245	194	0	0.	.043519245	0	1800000	360000	44	0	0	1	0	0;
867 79 0 0.043519245 0 1800000 360000 44 0 0 1 0 0;);	
259	183	0	0.	.043519245	0	1800000	360000	44	0	0	1	0	0;
260	882	0	0.	.043519245	0	1800000	360000	44	0	0	1	0	0;
262	671	0	0.	.043519245	0	1800000	360000	44	0	0	1	0	0;
277	529	0	0.	.043519245	0	1800000	360000	44	0	0	1	0	0;
282	881	0	0.	.043519245	0	1800000	360000	44	0	0	1	0	0;
284	556	0	0.	.043519245	0	1800000	360000	44	0	0	1	0	0;
287	794	0	0.	.043519245	0	1800000	360000	44	0	0	1	0	0;
291	618	0	0.	.043519245	0	1800000	360000	44	0	0	1	0	0;
300	794	0	0.	.043519245	0	1800000	360000	44	0	0	1	0	0;
303	802	0	0.	.043519245	0	1800000	360000	44	0	0	1	0	0;
324	614	0	0.	.043519245	0	1800000	360000	44	0	0	1	0	0;
306	868	0	0.	.043519245	0	1800000	360000	44	0	0	1	0	0;
307	630	0	0.	.043519245	0	1800000	360000	44	0	0	1	0	0;
312	775	0	0.	.043519245	0	1800000	360000	44	0	0	1	0	0;
327	287	0	0.	.043519245	0	1800000	360000	88	0	0	0	0	0;
332	391	0	0.	.043519245	0	1800000	360000	88	0	0	0	0	0;
333	189	0	0.	.043519245	0	1800000	360000	88	0	0	0	0	0;
348	394	0	0.	.043519245	0	1800000	360000	88	0	0	0	0	0;
353	891	0	0.	.043519245	0	1800000	360000	88	0	0	0	0	0;
364	702	0	0.	.043519245	0	1800000	360000	88	0	0	0	0	0;
371	524	0	0.	.043519245	0	1800000	360000	88	0	0	0	0	0;
379	177	0	0.	.043519245	0	1800000	360000	88	0	0	0	0	0;
390	530	0	0.	.043519245	0	1800000	360000	88	0	0	0	0	0;

834	179	0	0.	.043519245	0	1800000	360000	88	0	0	0	0	0;
835	282	0	0.	.043519245	0	1800000	360000	88	0	0	0	0	0;
860	287	0	0.	.043519245	0	1800000	360000	88	0	0	0	0	0;
862	307	0	0.	.043519245	0	1800000	360000	88	0	0	0	0	0;
863	798	0	0.	.043519245	0	1800000	360000	88	0	0	0	0	0;
864	565	0	0.	.043519245	0	1800000	360000	88	0	0	0	0	0;
865	189	0	0.	.043519245	0	1800000	360000	88	0	0	0	0	0;
868	517	0	0.	.043519245	0	1800000	360000	88	0	0	0	0	0;
869	526	0	0.	.043519245	0	1800000	360000	88	0	0	0	0	0;
870	835	0	0.	.043519245	0	1800000	360000	88	0	0	0	0	0;
875	870	0	0.	.043519245	0	1800000	360000	88	0	0	0	0	0;
879	640	0	0.	.043519245	0	1800000	360000	88	0	0	0	0	0;
880	348	0	0.	.043519245	0	1800000	360000	88	0	0	0	0	0;
881	79 () ().()43519245 (0 1	1800000 3	360000 8	38 () () () () ();
883	400	0	0.	.043519245	0	1800000	360000	88	0	0	0	0	0;
888	390	0	0.	.043519245	0	1800000	360000	88	0	0	0	0	0;
889	639	0	0.	.043519245	0	1800000	360000	88	0	0	0	0	0;
891	458	0	0.	.043519245	0	1800000	360000	88	0	0	0	0	0;
744	407	0	0.	043519245	0	1800000	360000	88	0	0	0	0	0;
744	837	0	0.	043519245	0	1800000	360000	88	0	0	0	0	0;
744	4 79 0 0.043519245 0 1800000 360000 88 0 0 0 0 0;												
744	775	0	0.	043519245	0	1800000	360000	88	0	0	0	0	0;
636	538	0	0.	.043519245	0	1800000	360000	88	0	0	0	0	0;
636	670	0	0.	.043519245	0	1800000	360000	88	0	0	0	0	0;
636	267	0	0.	.043519245	0	1800000	360000	88	0	0	0	0	0;
744	636	0	0.	043519245	0	1800000	360000	88	0	0	0	0	0;
664	276	0	0.	.043519245	0	1800000	360000	88	0	0	0	0	0;
664	724	0	0.	.043519245	0	1800000	360000	88	0	0	0	0	0;
664	3 0	0.	. 04	13519245 0	18	300000 36	60000 88	3 0	0	0	0	0;	;
664	563	0	0.	.043519245	0	1800000	360000	88	0	0	0	0	0;
664	810	0	0.	.043519245	0	1800000	360000	88	0	0	0	0	0;
664	804	0	0.	.043519245	0	1800000	360000	88	0	0	0	0	0;
664	698	0	0.	.043519245	0	1800000	360000	88	0	0	0	0	0;
664	841	0	0.	.043519245	0	1800000	360000	88	0	0	0	0	0;
664	346	0	0.	.043519245	0	1800000	360000	88	0	0	0	0	0;
817	333	0	0.	.043519245	0	1800000	360000	88	0	0	0	0	0;
817	566	0	0.	.043519245	0	1800000	360000	88	0	0	0	0	0;
817	613	0	0.	.043519245	0	1800000	360000	88	0	0	0	0	0;
817	214	0	0.	.043519245	0	1800000	360000	88	0	0	0	0	0;
817	735	0	0.	.043519245	0	1800000	360000	88	0	0	0	0	0;
817	365	0	0.	.043519245	0	1800000	360000	88	0	0	0	0	0;
817	744	0	0.	.043519245	0	1800000	360000	88	0	0	0	0	0;
```
292 570 0 0 0 0 0 1 570 0; 0
14 577 0 0 0 0 0 1 577 0; 0
731 550 0 0 0 0 0 1 550 0; 0
739 580 0 0 0 0 0 1 580 0; 0
19 515 0 0 0 0 0 1 515 0; 0
515 1042.9 0 0 0 0 0 1 1042.9 0; 0
193 934 0 0 0 0 0 1 934 0; 0
595 624 0 0 0 0 0 1 624 0; 0
821 410 0 0 0 0 0 1 410 0; 0
394 2.1 0 0 0 0 0 1 2.1 0; 0
184 8.3 0 0 0 0 0 1 8.3 0; 0
400 2.5 0 0 0 0 0 1 2.5 0; 0
473 90 0 0 0 0 0 1 90 0; 0
615 22.2 0 0 0 0 0 1 22.2 0; 0
414 295.6 0 0 0 0 0 1 295.6 0; 0
1 3 0 0 0 0 0 1 3 0; 0
402 520.8 0 0 0 0 0 1 520.8 0; 0
744 630 0 0 0 0 0 1 630 0; 0
694 20 0 0 0 0 0 1 20 0; 0
88 12.8 0 0 0 0 0 1 12.8 0; 0
813 347.7 0 0 0 0 0 1 347.7 0; 0
580 65.1 0 0 0 0 0 1 65.1 0; 0
57 1.2 0 0 0 0 0 1 1.2 0; 0
116 3.2 0 0 0 0 0 1 3.2 0; 0
154 0.9 0 0 0 0 0 1 0.9 0; 0
601 1 0 0 0 0 0 1 1 0; 0
39 19 0 0 0 0 0 1 19 0; 0
706 17 0 0 0 0 0 1 17 0; 0
706 17.6 0 0 0 0 0 1 17.6 0; 0
284 22 0 0 0 0 0 1 22 0; 0
303 25 0 0 0 0 0 1 25 0; 0
135 3.5 0 0 0 0 0 1 3.5 0; 0
193 125.4 0 0 0 0 0 1 125.4 0; 0
377 170 0 0 0 0 0 1 170 0; 0
279 4.7 0 0 0 0 0 1 4.7 0; 0
868 10.5 0 0 0 0 0 1 10.5 0; 0
123 15 0 0 0 0 0 1 15 0; 0
703 4.5 0 0 0 0 0 1 4.5 0; 0
643 2.8 0 0 0 0 0 1 2.8 0; 0
382 18.6 0 0 0 0 0 1 18.6 0; 0
300 2 0 0 0 0 0 1 2 0; 0
106 35 0 0 0 0 0 1 35 0; 0
```

2 0 0 3 0 394900871.1 0; 2 0 0 3 0 221345416.6 0; 2 0 0 3 0 6763002.087 0; 2 0 0 3 0 73848873.37 0; 2 0 0 3 0 74755789.35 0; 2 0 0 3 0 71257684.83 0; 2 0 0 3 0 75144467.63 0; 2 0 0 3 0 66723104.88 0; 2 0 0 3 0 135117526.4 0; 2 0 0 3 0 121008504.8 0; 2 0 0 3 0 80845082.42 0; 2 0 0 3 0 53119365.05 0; 200300;2 0 0 3 0 1075343.244 0; 200300;2 0 0 3 0 11660348.43 0; 2003000;200300;2 0 0 3 0 442276243.1 0; 2 0 0 3 0 67474549.56 0; 200300;200300;2003000; 200300;200300;200300;200300;200300;200300;2003000; 200300;200300;200300;200300;200300;2003000; 200300;200300;200300;200300;

2 0 0 3 0 0 0;2 0 0 3 0 0 0;

2003000;200300;2003000; 2003000; 200300;200300;200300;200300;2 0 0 3 0 2215466.201 0; 200300;2003000; 200300;200300;200300;2003000; 200300;200300;200300;2003000; 2003000;2 0 0 3 0 77087859.04 0; 200300;2003000; 200300;200300;200300;200300;200300;200300;2003000; 200300;2 0 0 3 0 647797.1348 0; 2 0 0 3 0 66723104.88 0; 2003000; 200300;200300;200300;200300;2 0 0 3 0 66723104.88 0; 200300;200300;200300;

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2 0 0 3 0 80845082.42 0;
2 0 0 3 0 80845082.42 0;
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4 1 0 0.000939006 0 1400000 280000 8.325 0 0 1 0 0;
5 3 0 0.239772901 0 1400000 280000 2.406643874 0 0 1 0 0;
3 7 0 0.000625165 0 1500000 300000 18.75 0 0 1 0 0;
546 569 0 0.099909127 0 1500000 300000 4.131287308 0 0 1 0 0;
722 546 0 0.075141061 0 1500000 300000 4.985215202 0 0 1 0 0;
892 81 0 0.015398315 0 1800000 360000 33 0 0 1 0 0;
722 892 0 0.025142832 0 1800000 360000 33 0 0 1 0 0;
581 722 0 0.062622462 0 1500000 300000 5.621878795 0 0 1 0 0;
569 186 0 0.422207775 0 1500000 300000 3.125 0 0 1 0 0;
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890 886 0 0.043519245 0 1800000 360000 33 0 0 1 0 0;
890 240 0 0.003080544 0 1800000 360000 34.46907347 0 0 1 0 0;
884 886 0 0.019379787 0 1800000 360000 33 0 0 1 0 0;
877 887 0 0.058954813 0 1600000 320000 12.625 0 0 1 0 0;
142 203 0 0.009402926 0 1500000 300000 18.75 0 0 1 0 0;
380 68 0 0.030852421 0 1500000 300000 8.966826125 0 0 1 0 0;
805 36 0 0.019645203 0 1500000 300000 12.07597857 0 0 1 0 0;
19 737 0 0.048525155 0 1800000 360000 33 0 0 1 0 0;
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265 34 0 0.056506666 0 1500000 300000 6.016103025 0 0 1 0 0;
221 115 0 0.031141731 0 1800000 360000 33 0 0 1 0 0;
297 654 0 0.048944283 0 1500000 300000 6.61403605 0 0 1 0 0;
555 411 0 0.155417826 0 1800000 360000 33 0 0 1 0 0;
636 558 0 0.313902251 0 3000000 600000 227.5 0 0 1 0 0;
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254 387 0 0.07483173 0 1500000 300000 4.99879617 0 0 1 0 0;
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308 399 0 0.025955282 0 1800000 360000 33 0 0 1 0 0;
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238 162 0 0.246954679 0 1500000 300000 11.5 0 0 1 0 0;
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% INFO : === Translation Options ===