Time-Based Subcycle Fatigue Life Model for Uniaxial and Multiaxial Loading

by

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ABSTRACT

Mechanical fatigue has been a research topic since quite a long time. It is a complex phenomenon at molecular level. The fact that fatigue failure occurs much below material's yield point, made it much interesting area for research. So, to understand the physics behind fatigue failure became an important research topic. Fatigue failure is characterized by crack initiation and then crack propagation to finally fracture the material. If this could be modelled mathematically, then it would save lot of resources and would assure the structural integrity of given component. Many such mathematical models were published to calculate fatigue crack growth for Constant Amplitude Loading, but most of the time the applied loads are variable in nature. So, to address this problem a mathematical model which will predict fatigue life in terms of time history is needed. This research study focuses on improving previously developed subcycle fatigue crack growth model also known as small time scale model which works well in Paris regime. In the first part, focus has been given on estimating threshold point using subcycle model by applying load shedding techniques. Later subcycle model has been modified to include fatigue crack growth in threshold region. In the second part of this research study, the concept of Equivalent Initial Flaw Size (EIFS) and fracture mechanics approach has been used to compute fatigue life for Constant as well as Random Amplitude Loading. Further the model has been extended to compute the fatigue life under Mixed Mode Loading (Mode I & Mode II). Standard material properties are used to calibrate the model parameters. The fatigue life results were validated using available open literature data as well as

experimental testing data. The subcycle model can be used to calculate fatigue life in case of HCF and LCF, which is suggested as a future work for this research study.

DEDICATION

To my parents who strengthen me with their love and care.

To my family who made my life colorful and joyful even in tough times.

To my teachers who molded me into a knowledgeable person.

To my friends who always cheered on me in my life journey.

To everyone who helped me reach where I am.

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TABLE OF CONTENTS

Page

CHAPTER Page

LIST OF TABLES

LIST OF FIGURES

Figure Page

Figure Page

CHAPTER 1

INTRODUCTION

1.1 Objective

The main objective of this thesis work is to develop a mathematical model using concepts in the field of Fracture Mechanics which can estimate the fatigue crack growth phenomenon and fatigue life of a given specimen under different spectrum loading including constant as well as random amplitude loading. Initially the existing model has been used to predict the threshold stress intensity factor using various loading techniques. Existing subcycle time scale model has been modified to include threshold stress intensity condition and EIFS (Equivalent Initial Flaw Size) to predict the fatigue life in case of uniaxial loading condition. Using the concepts of Equivalent stress intensity factor, two channel loadings i.e., tension/compression loading, and torque loading has been combined to get equivalent tension/compression loading to predict fatigue life. It can be summarized as

- Use existing subcycle time scale model to predict the threshold stress intensity factor using standard loading techniques.
- Integrating concept of EIFS (Equivalent Initial Flaw Size) with the time-based subcycle crack growth function and use fracture mechanics approach to compute fatigue life.

- Extend the existing subcycle crack growth model to include threshold stress intensity factor to predict the fatigue life under constant as well as random amplitude loading.
- Combine Mode I and Mode II loading to get and equivalent Mode-I loading and predict the fatigue life under mixed mode constant and random amplitude loading.

The proposed modification has been validated using various open literature test results data as well as testing data obtained by doing actual test at ASU PARA lab, thus verifying the integrity of proposed model. To begin with this study a brief background study as well as literature study has been covered in this chapter.

1.2 Background and literature Review

Mapping a mechanical fatigue phenomenon into a physical mathematical model has been a prominent research topic in the field of Fracture Mechanics. The fatigue failure phenomenon is characterized by crack or discontinuity formation or initiation in a material. If the cyclic loading is continued, then this crack tends to grow till a level where it will cause a fracture in that component. This crack initiation and propagation is very complex phenomenon which is difficult to capture in a mathematical model. Several models were presented by researchers to capture this phenomenon which we are discussed in this chapter.

In this chapter concepts related to fracture mechanics and fatigue crack growth propagation has been discussed briefly.

1.2.1 Stress intensity factor

Stresses near the crack tip can be expressed in terms of a scaling factor known as stress intensity factor, more often denoted by letter K with some additional subscripts.

$$K = Y.\sigma.\sqrt{\pi a}$$

Where, Y is geometric correction factor which depends on the type of crack, geometry of a specimen to be evaluated. σ is applied stress which can be tension/compression in case of a mode- I loading and shear in case of Mode II loading. a is current crack length. From the above K equation, we can see that K depends on the current crack length in the specimen. So, for a given constant amplitude loading, even if the applied stress is constant, still we can have an increased value of K because of increasing crack length value.

Loading stresses can be categorized into three different loading modes, named as Mode I, Mode II and Mode III, which is explained in figure 1.

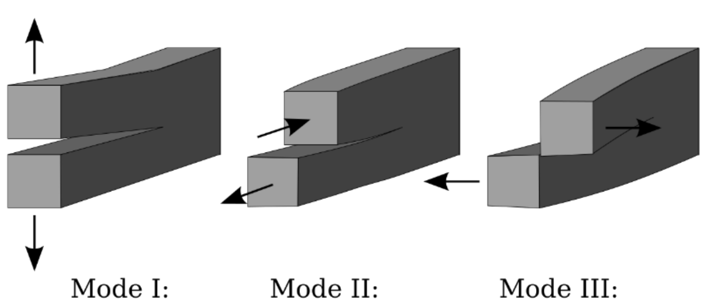


Fig 1. Fracture mode crack loading

1.2.2 Crack Tip Opening Displacement (CTOD) & Crack Tip Blunting

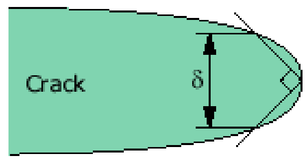


Fig 2. Crack Tip Opening Displacement (δ)

Crack Tip Opening Displacement is measured as the distance between the opposite faces of a crack tip at a 90° intercept position as shown in the figure 2. This parameter is used in fracture mechanics to characterize the stress intensity factor. We will be using this parameter to model the crack growth phenomenon later in this study.

In case of ductile material, when the tensile loading is applied on a specimen, a very high stress concentration begins to form at the end of a crack. As a result of this high stress concentration, material starts yielding at that point. When the tip starts yielding, the shape of the tip starts becoming more rounded compared to earlier sharp edge as shown in figure 3. This phenomenon is called blunting. In blunted crack, as it’s no longer a sharp corner, stresses start to relax in that area. When the material undergoes more plastic deformation, they become more brittle. After some amount of crack tip blunting the material at the bottom of the crack becomes brittle and there becomes another point at the bottom of the crack where the stresses go higher, and the material starts cracking again and this part is called tearing as shown in figure 3.

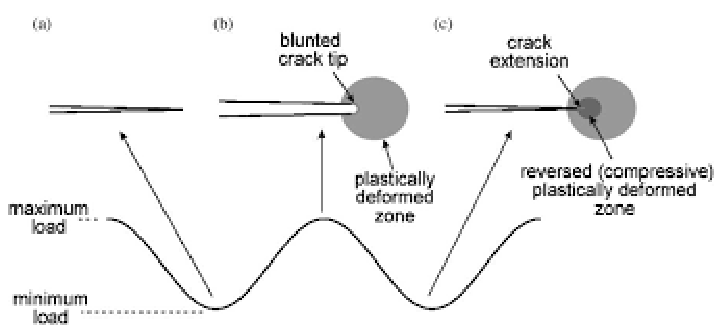


Fig.3 Crack Tip Blunting

1.2.3 Crack growth propagation rate

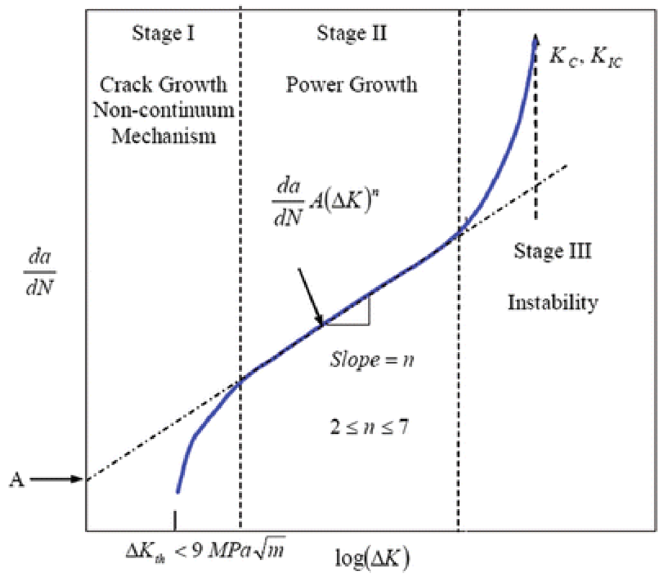


Fig.4 Crack Propagation rate

Crack growth propagation rate has mainly characterized in three stages. Detailed analysis of these 3 stages can be explained by plotting crack growth per cycle versus the applied stress intensity factor range. In this plot, stage 1 also known as threshold region where crack starts growing asymptotically to the vertical axis. For a specific material, this threshold stress intensity point is a constant which acts like a material property constant. Many researchers have proposed different methods to estimate this point which are discussed in the later part of this report. Very large number of cycles are involved in this region, to initiate the crack growth. So, to correctly estimate this point becomes an important topic of research.

Stage 2 crack growth rate is linear in nature where crack growth rate is directly proportional to the applied stress intensity factor range. This region is also known as Paris region. The slope and the y axis intercept of this straight line are constant for a specific material also known as Paris constants. It got its name after P C Paris [1] published a paper in 1961, where he introduced the idea that rate of crack growth depends on the stress intensity factor and he proposed a equation, commonly known as Paris equation as

$$\frac{da}{dN} = C(\Delta K)^m$$

In stage 3, crack growth starts accelerating as applied stress intensity factor reaches the critical stress intensity factor. It is the values, beyond which if the loading is continued material will experience failure mostly because of fracture. The crack growth behaviour in stage 3 is asymptotic with respect to vertical axis, same as in stage 1.

Stress ratio R is the most common term used to define the ratio of minimum to the maximum stress applied in each loading. Materials can be loaded to different stress condition, keeping constant stress ratio or vice versa.

1.2.4 Crack growth models

Many researchers proposed crack growth models with different focus. The most popular model which was published by Paris [1] in 1961 is widely used to calculate crack growth. As discussed in previous section Paris law represents a line with constants C and m, which means its only effective in calculating the crack growth in stage 2. Also, it does not account for change in the applied stress ratio. For different values of stress ratio, we must compute respective C and m values, to calculate the crack growth. It does not consider the threshold and critical region crack growth.

Forman [2] in 1967 modified this equation to account for different stress ratio R and crack growth propagation in region 3 considering critical stress intensity factor in to account.

$$\frac{da}{dN} = C\frac{\Delta K^n}{(1-R)K_{IC} - \Delta K}$$

Here, C and n are fitting parameters and K_{IC} is critical stress intensity factor which is a material property constant for a particular material.

Khan [3] again modified the crack growth rate function to incorporate the crack growth in threshold region by including K_{th} term in the crack growth function. This model also considers maximum stress intensity factor in a current cycle.

$$\frac{da}{dN} = A\frac{(\Delta K - K_{th})^n}{K_{IC} - K_{max}}$$

Elber [4] in 1970 proposed an hypothesis that, while loading a specimen from sigma min to sigma max, crack will not start growing until it reaches a sigma opening stress level which is in between sigma min and sigma max. This Sigma opening stress values is a function of applied stress ratio, with higher stress ratio this value tends to approach sigma min which in tern accelerate the crack growth rate. If the applied stress is less than the sigma opening level, then crack will not propagate.

Newman [5] proposed that the sigma opening is function of stress level and need to be calculated for different cycles.

All above discussed model has one thing in common, that they calculate fatigue life in terms of number of cycles and gives us the crack growth values at the end of each applied loading cycle.

1.3 Need for new model.

As discussed in earlier section, there were many fatigue life models were proposed, which will given calculate fatigue life of a given specimen under applied loading in terms of loading cycles. Almost all the models will provide fatigue life in number of cycles. This works well in case of constant amplitude loading condition, where cycles are perfectly defined as shown in figure. Where one cycle is counted as distance between two consecutive points of same loading nature.

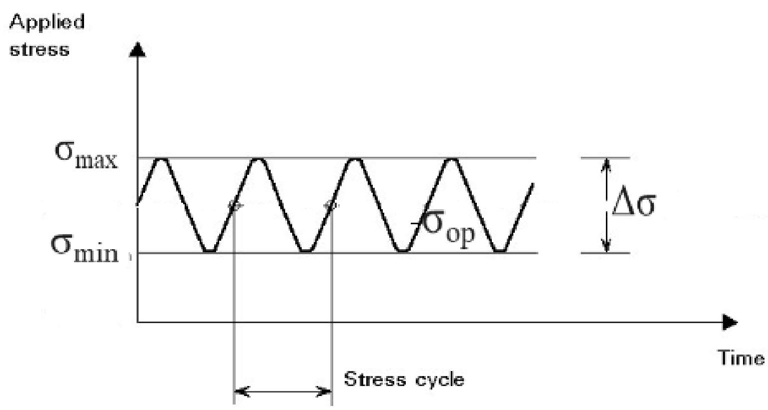


Fig. 5 Constant Amplitude Loading Spectrum

In case of random amplitude loading condition, as shown in figure. Counting fatigue life in terms of cycles is ambiguous as cycles are not defined perfectly. Most of the real-life loading conditions are random amplitude only. So, to address this problem, a time-based crack growth model was proposed by Lu and Liu [6] which calculates crack growth at any arbitrary point in loading history

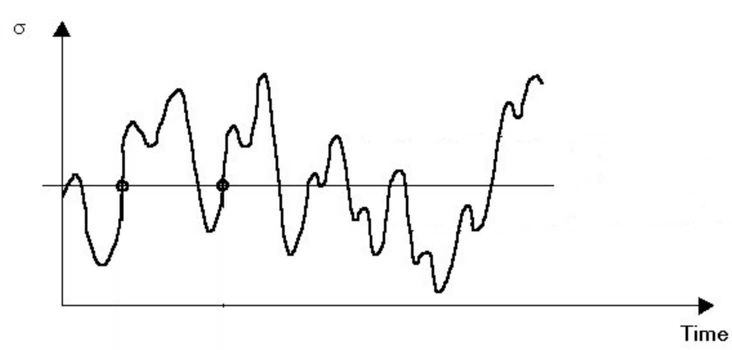


Fig. 6 Random Amplitude Loading Spectrum

1.4 Chapter Summary

We discussed main objective behind this research work, that we are proposing a time-based fatigue life model which will calculate fatigue life under various loading conditions. Later we discussed the basic terminologies and concepts from solid mechanics such as Stress intensity factor, Crack tip opening displacement, essential for this study, which we will use in later chapter to help build the mathematical model. We also discussed state of the art fatigue life model, proposed by various researchers and the advantages and disadvantages of each model. In the end we clarified the need for new model, which is best suitable to predict the fatigue life of a specimen under random amplitude loading condition. In the next chapter we will introduce new model and explain the concept of EIFS and how it can be coupled with the crack growth function to estimate the fatigue life.

CHAPTER 2

NEW TIME-BASED SUBCYCLE FATIGUE LIFE MODEL

2.1 Overview

In this chapter, first we will be using load shedding technique to estimate the threshold stress intensity factor and later we will introduce the new fatigue life model which is obtained by modifying the existing crack growth function with the threshold stress intensity factor and coupling it with Equivalent Initial Flaw size to use the fracture mechanics approach to compute the fatigue life under various loading conditions.

As discussed in the chapter 1, crack growth mechanics falls under three stages. Threshold region, Paris region and critical crack growth region. Threshold stress intensity is a point below which crack will not grow. As per the research goes, a very large number of loading cycles are involved in the threshold region. So, to estimate this point correctly, it becomes very important aspect as far as the prediction of structural reliability is concerned. Paris [1] proposed a method in early 1970s to predict threshold stress intensity factor, known as Load reduction method. A constant decreasing load cycles are applied on a specimen, so that we can get K data in the threshold region. Further this method was analyzed by Hudak et al. [7] and Bucci et al.[8] and later included in ASTM E-645 fatigue crack growth rate testing standard.

2.1.1 Load Reduction Method

In Load reduction method, a specimen is initially loaded with a cyclic loading condition with a constant stress ratio and crack growth is observed. After certain loading cycles, the max and min stress amplitude values are decreased in a manner so that stress ratio will remain constant. At this stress ratio and loading amplitude, specimen is loaded for certain loading cycles. (This certain loading number of cycles were defined as number of cycles till 0.2% crack extension. Here we initially ran simulation with 0.2% crack extension criteria and then to standardize we again ran simulation for 400 loading cycles. The final threshold estimation with both the cases showed same results, so we followed 400 loading cycles at each loading amplitude).

This Loading shedding is continued till we get zero crack growth, at that point loading is stopped to note the threshold stress intensity values. Similar load shedding method is used with different R ratios to get threshold stress intensity factor. After we get the threshold values, a increasing loading is applied in order to get the remaining da/dN vs DeltaK curve.

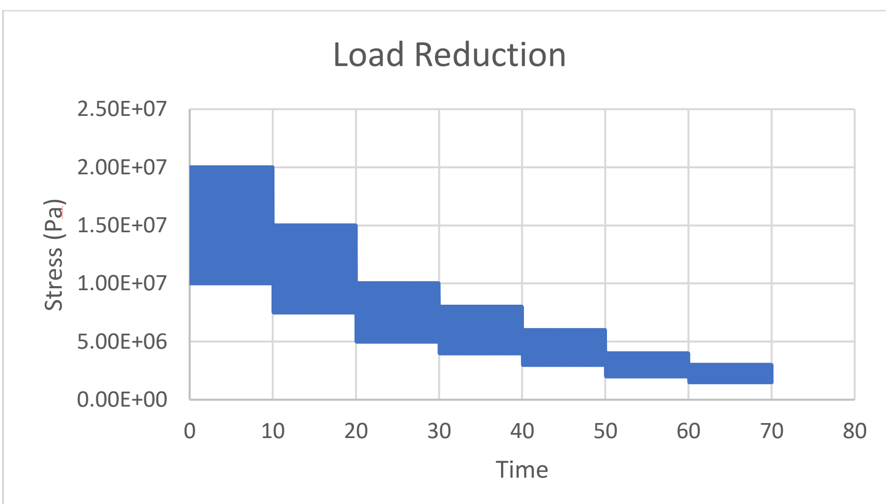


Fig.7. Load Reduction Spectrum

Researchers analyzed a key observation about the load reduction method that, with varying R values, the threshold region data points showed a large dispersion among the values. Normally in case of a da/dN vs DeltaK curve, as we change the load ratio R, the whole curve shifts upwards or downwards depending upon the change in R and the new shifted curve remains parallel to the original curve. But in case of load shedding technique by changing load ratio R, the curve was remaining parallel in the Paris region but showing large dispersion in the threshold region. This effect is called 'fanning' in threshold region. One of the major reasons for this kind of behaviour was attributed to the loading history in the load shedding method. As the load ratio is increased, more fanning was observed because of plasticity induced due to the higher loading cycles. Other reason could be formation of oxide debris because of environmental effect which causes higher crack closure level. To overcome these issues, another method was proposed which can takes care of loading history in the applied loading spectrum and the oxide formation condition known as Compression pre-cracking Constant amplitude which is discussed below.

2.1.2 CPCA and CPLR methods

In case of Load shedding technique, we are gradually decreasing the loading amplitude to arrive at threshold stress intensity value. The main drawback of this technique is the results which we are getting, they are highly influenced by the loading history. So, to overcome this problem CPCA technique were developed [9]. In this technique before applying tensile loading, compressive load cycles are applied to get rid of any time of load history. After that, a very small magnitude of tensile loading cycle is applied. If crack growth is observed, then same loading cycle is continued to be applied, if there is not any crack growth then the max loading value is increased by 10%. This process is repeated until there is a crack growth observed that value is noted as threshold stress intensity factor.

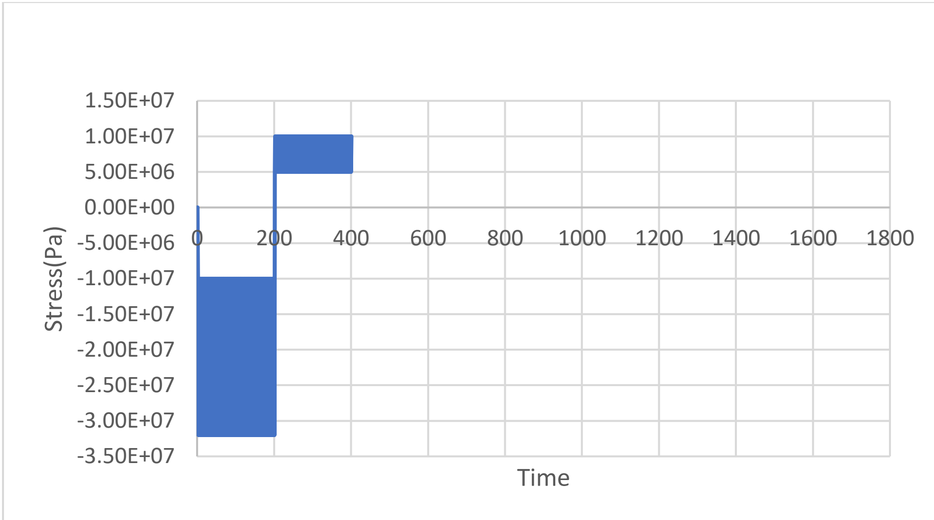


Fig. 8. CPCA Loading Spectrum

When first apply compressive loading cycles to a specimen, we can be sure of fact of Global crack closure. With this mechanics the oxide debris formed in the crack region can go under compressive loading, which will minimize its effect when the tensile loading is applied on the specimen. Aligning with CPCA method, one more method has been proposed known as CPLR which stands for compression pre-cracking load reduction method, where the first part of compressive loading is same as that of earlier proposed method but instead of increasing constant amplitude loading cycles, decreasing constant amplitude loading cycles.

Compressive Pre-cracking Load reduction is same as CPCA technique as explained earlier. Only difference is instead of constant amplitude load cycles, load shedding cycles are applied, and crack growth is observed. The point where crack growth becomes zero can be referred as threshold stress intensity factor.

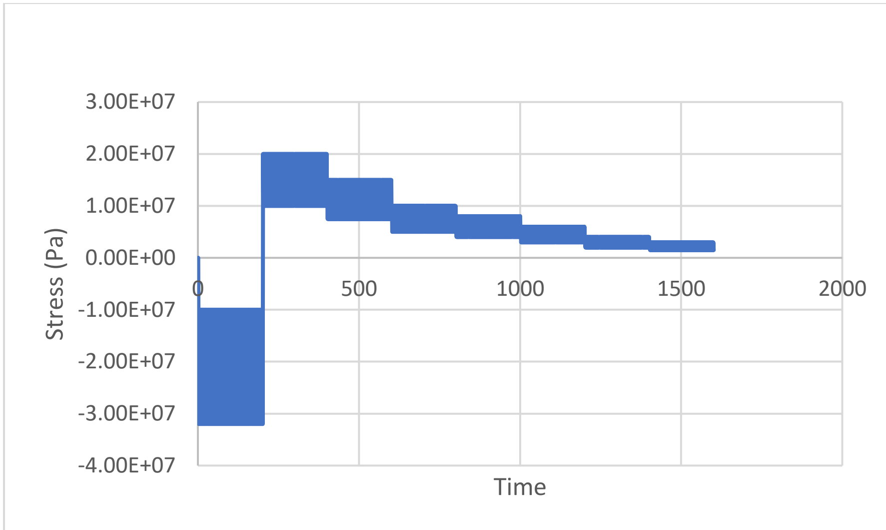


Fig.9. CPLR Loading Spectrum

2.2 Equivalent Initial Flaw Size (EIFS)

No material has perfectly aligned crystalline structure, at molecular level there are always some imperfections are present. If we must measure the length of the smallest possible discontinuity, then it will be distance between the adjacent atoms which is in the order of 0.1-0.2 nanometers. To calculate fatigue life using fracture mechanics approach for a smooth polished specimen, we need initial crack length. If we use the atomic spacing as initial crack size, then with fracture mechanics approach, we will mostly get infinite life which is not the case always.

Equivalent initial flaw size (EIFS) concept was developed in 1980s by El Haddad et al. [10] in an attempt to use it as a initial crack length in case of calculating the fatigue life using the fracture mechanics approach. It uses the standard material constants as well as back extrapolation method for fitting the parameters. One important thing about to be understood about the EIFS is that it is not an actual quantity or actual initial crack length present in the material. Its that values, with which if we performed our fracture mechanics life calculation, we should get the fatigue life of a component which agrees with the experimental data for similar specimen. EIFS is determined by matching the infinite life of a specimen as stress level at infinite life. Yongming Liu and Sankaran Mahadevan [11] proposed a methodology in 2009 which uses fatigue limit data and fatigue threshold stress intensity factor to determine the EIFS for a specific specimen. This method does not use the back extrapolation to calculate the initial flaw size.

2.2.1 EIFS Calculation

One way to predict the fatigue life of a specimen is to perform crack growth analysis starting from initial crack size. Most of the initial flaws are on the range of microns or even less than that. If we look at microstructure of a material, initial flaw size may be less than the average grain size of that specific material. If we decided to measure the initial crack size with the help of some non-destructive techniques, many NDT have limitations below which they can not measure the actual size. If we consider the lower limit of NDT as our initial crack length, then it would be a conservative approach to get to the initial crack length and may give large deviations in the calculation of fatigue life. Microstructural behaviour of small crack growth and large crack growth has a large deviation.

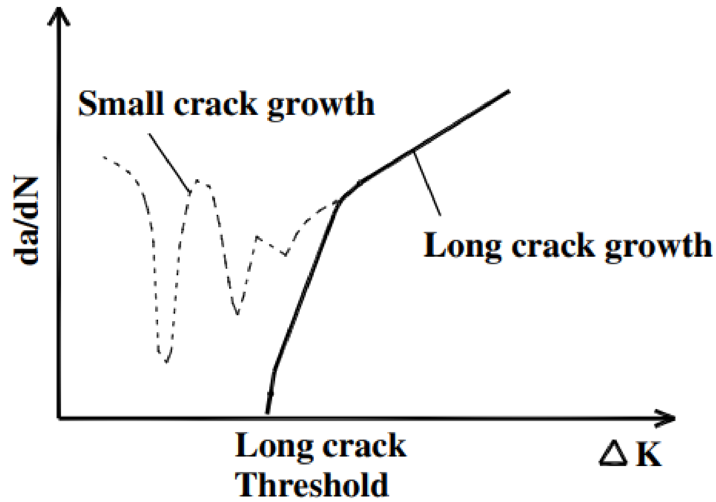


Fig. 10 Schematic illustration of small and long crack growth.

If we look at the small crack growth mechanism, it shows some wavy nature with respect to the applied stress intensity factor, this is mainly because when a small crack grows it initially starts within a particular grain as the crack reaches the grain boundary it faces crack growth retardation because of a grain boundary. As the applied stress intensity factor increases, crack growth enters a new grain and starts accelerating. This phenomenon continues and gives unsteady crack growth mechanism which when grown to a certain limit starts following long crack growth mechanism. The detail study of small crack growth mechanism is beyond the scope of this study. We will be focusing on long crack growth to formulate initial flaw size and use it to calculate the fatigue life of a specimen.

If we have to use the fracture mechanics approach, then the fatigue life of a specimen can be found out by integrating the fatigue crack growth rate cure which uses initial and final crack length as the limits of the integration function. A general fatigue life model can be written as

$$\frac{da}{dN} = g(a)$$

Here, g(a) is a function which describes the relationship between crack growth rate and applied loading. In this study we are proposing a modified subcycle time-based model as our crack growth rate and applied loading function which is covered in the next section. Here want to derive the relation between the initial crack length and fatigue life. By using variable separation and integrating above equation we can write

Fatigue life, $N = \int_{a_i}^{a_c} \frac{1}{g(a)} da$

Here a_i is an initial crack length present in the specimen and ac is the critical crack length at failure which can be computed using critical stress intensity factor, which is a material constant for a specific material. Now if we consider actual initial crack size as initial crack length, then we must use small crack growth rate function $g_s(a)$ and fatigue life can be written as

$N = \int_{IFS}^{a_c} \frac{1}{g_s(a)} da$

If we use Equivalent initial crack length and use long crack growth rate function, $g_l(a)$ then fatigue life of a specimen can be computed as

$$N = \int_{EIFS}^{a_c} \frac{1}{g_l(a)} da$$

If we choose EIFS value properly then both the approaches should give us the same fatigue life. The reason of choosing EIFS over actual initial flaw size is that small crack growth rate function is much more complex and dependent on the microstructure of the material.

There are higher chances to get the wrong initial size if we were to follow small crack growth rate function. The physical interpretation using above mentioned two functions can be seen in the fig. X where underlying areas for the two functions are same. We use fatigue limit of a specimen to calculate the EIFS value. Fatigue limit for various materials can be obtained from open literature S-N curve data.

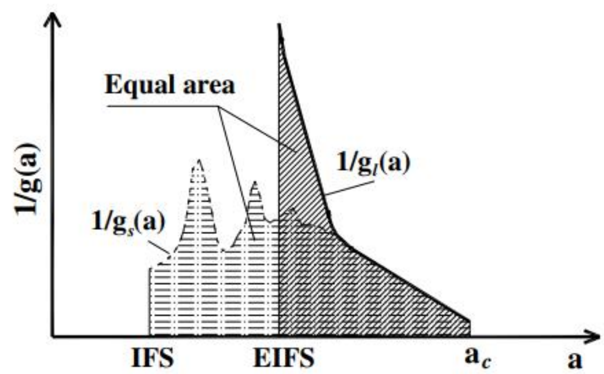


Fig. 11 Schematic illustration of actual and equivalent initial flaw size

Fatigue limit or endurance limit is a parameter widely used while designing structures or any other component which is subjected to cyclic loading. It is the limit value of applied stress below which a specimen will have infinite fatigue life. Most of the time infinite fatigue life terms refer to fatigue life of 1e7 to 1e9 cycles. Materials like steel have a specific fatigue limit value. In case of aluminum, it does not show a specific fatigue limit value, instead to get the fatigue limit values we average the applied stress values which gives us the fatigue life in the range of 1e7 to 1e9.

Fatigue limit for these materials can be found out by smooth specimen testing. El Haddad et al. [10] proposed a model to express the fatigue limit, $\Delta\sigma_f$ focusing the fatigue threshold stress intensity factor, ΔK_{th} and a fictional crack length *a*

$$\Delta K_{th} = \Delta\sigma_f Y\sqrt{\pi a}$$

Y is a geometry correction factor which depends upon the specimen type, crack length, thickness of specimen etc. A specimen with infinite length and a crack length of 2*a*, Y is 1. In a similar manner Y for any specimen can be found out using Geometry Correction factor handbook. Equation can be rewritten to get *a* as EIFS as.

$$EIFS = \frac{1}{\pi}\left(\frac{\Delta K_{th}}{\Delta\sigma_f\ Y}\right)^2$$

We will be using this equation to compute the initial crack length size and use the fracture mechanics approach to calculate the fatigue life. Another important element for using fracture mechanics approach to calculate fatigue life is a crack growth with respect to loading function. We will be using small time scale crack growth function with a modification to include threshold stress intensity factor which is discussed in the next section.

If we were to calculate the structural reliability of a specimen or any other component subjected to the cycling loading with fracture mechanics approach, we must consider the largest crack present in the specimen. These largest cracks are the results of manufacturing method defects, machining defects or erosion due to environmental conditions. The current

EIFS methodology applies to a smooth specimen only. Proper modification is needed if the material is pre-cracked, considering the geometry and shape of crack.

2.3 Time Based Subcycle Fatigue Crack Growth Function

Various cycle-based fatigue crack growth models were developed for constant amplitude loading condition. But estimating crack propagation phenomenon in number of cycles for random amplitude loading condition is ambiguous, as cycles cannot be properly defined in case of random amplitude loading. To address this problem, small time scale model has been developed by Lu and Liu [12] later named as Subcycle Fatigue Crack Growth model. The main concept behind developing a time-based crack growth model is that we can calculate instantaneous crack growth at any arbitrary point during the loading history. In case of conventional cycle-based model, we can calculate crack growth at the end of respective cycle not at any arbitrary point in the loading history. In case of time-based model, we can not only compute the crack growth at any arbitrary point but also, we can get the crack growth over an entire cycle by integrating over time domain. Crack increment at any arbitrary point can be calculated as

$$a + \Delta a = \int_{t}^{t+\Delta t} \frac{da}{dt}(\sigma, a, E, \sigma_y, \dots)\, dt$$

The function in the integration is a generic time-based function which depends on the material constants such as E and σ_y and on the applied loading condition σ . Δa is a crack increment in Δt because of applied load.

Zhang and Liu [13] performed in situ SEM testing for aluminum alloy and noted that crack growth has some correlation with the CTOD (crack tip opening displacement) and also depends on the maximum applied stress intensity factor K_{max} and proposed a crack growth function as

a = $\mathrm{A}K_{max}^{B}\delta^{D}$

where a is crack extension and A, B, D are fitting parameters. For Al 7075 series D was found out to be ½. S, we can say that crack extension is proportional to the square root of CTOD at an instantaneous point in loading history. If we differentiate the above equation with respect to time, we get.

$$da = \frac{\mathrm{A}K_{max}^{B}}{2\sqrt{\delta}}d\delta$$

As discussed in the earlier section, we modified the crack growth function to include the threshold stress intensity factor. This modification was done to make this crack growth function to behave asymptotic to the vertical axis at threshold point, which was observed in the testing data. The modified form of crack growth function can be written as

$$da = \frac{\mathrm{A}(K_{max} - K_{th})^{B}}{2\sqrt{\delta}}d\delta$$

Following are the hypotheses which are being followed in this time-based crack growth function: (1) Crack does not grow in the unloading part of the loading spectrum; it will only grow for loading part of it. (2) While loading, crack will not grow until it reaches a crack opening stress level, σ_{op}. (3) If the applied ΔK is less than K_{th} value, then the crack will not grow.

2.3.1 CTOD variation

To calculate CTOD at any arbitrary point in a loading spectrum is a complex nonlinear phenomenon and usually it requires techniques like finite element modelling. Liu et. al. [14] proposed a algorithm to associate CTOD and loading parameters along with material constants in a linear approximation and it showed a good agreement with the finite element model. CTOD approximation for initial loading condition can be written as

$$\delta = \frac{K^2}{E\sigma_y}$$

$$\delta = \frac{(\sigma Y\sqrt{\pi a})^2}{E\sigma_y}$$

It depends on loading condition σ and material constants like Youngs Modulus E and yield strength σ_y. Above equation was proposed by Suresh in 1998 for monotonic loading condition. Rice and Suresh [15] made a statement about the yield strength in the above equation that material yielding can be replaced by $2\sigma_y$ while unloading it and external loading can be replaced by -ΔP

Using Dugdale's general model [16] CTOD at any point in time can be expressed as

$$\delta = \delta_{max} - \frac{(K_{max} - K)^2}{2E\sigma_y}$$

In case of a random amplitude loading there are multiple local minima and maxima points in the loading history. These loading minima and maxima can be expressed as $\sigma_{min,m}$, $\sigma_{max,m}$. Where m is a local peak and valley index. Corresponding CTOD values at these local peak and valley can be written as $\delta_{min,m}$, $\delta_{max,m}$. The tracking of CTOD variation is done by cycle removal method [17] which is described in this literature in a detail. Mathematical calculations for these CTOD are done using following equations.

$$\delta_{loading} = \begin{cases} \frac{K^2}{E\sigma_y} & K > K_{max,mem} \\ \delta_{min,m-1} + \frac{(K-K_{min,m-1})^2}{2E\sigma_y} & K_{max.mem} \geq K \geq K_{max,m-1} \\ \delta_{min,m} + \frac{(K-K_{min,m})^2}{2E\sigma_y} & K_{max,m-1} \geq K \end{cases}$$

$$\delta_{unloading} = \begin{cases} \delta_{max,m-1} + \frac{(K_{max,m-1} - K)^2}{2E\sigma_y} & K \leq K_{min,m-1} \\ \delta_{max,m} + \frac{(K_{max,m-1} - K)^2}{2E\sigma_y} & K \geq K_{min,m-1} \end{cases}$$

CTOD variation for monotonic loading followed by cyclic loading can be explained by using a loading spectrum and corresponding CTOD values as follows.

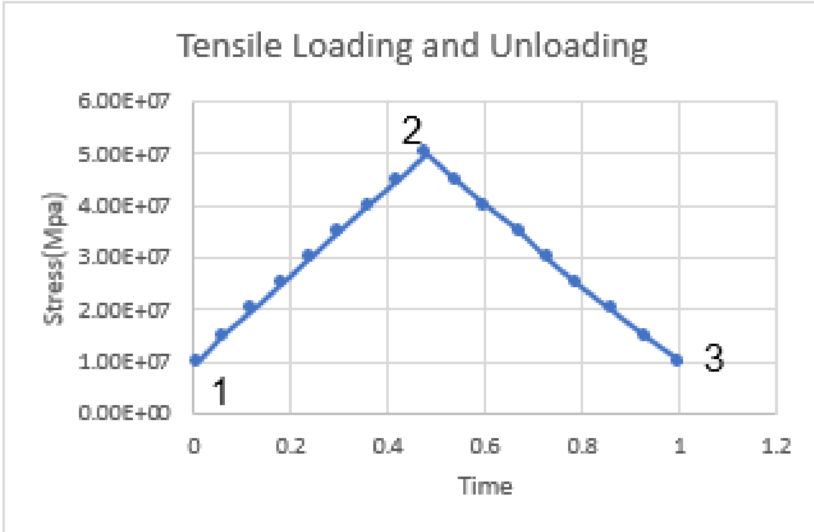


Fig.12 Tensile Loading-Unloading Path

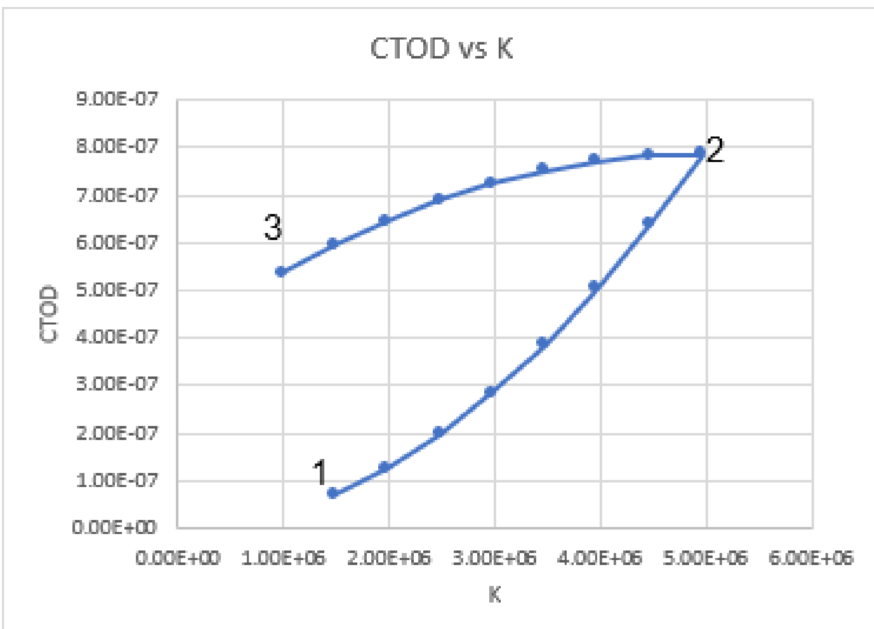


Fig.13 CTOD variation in one cycle

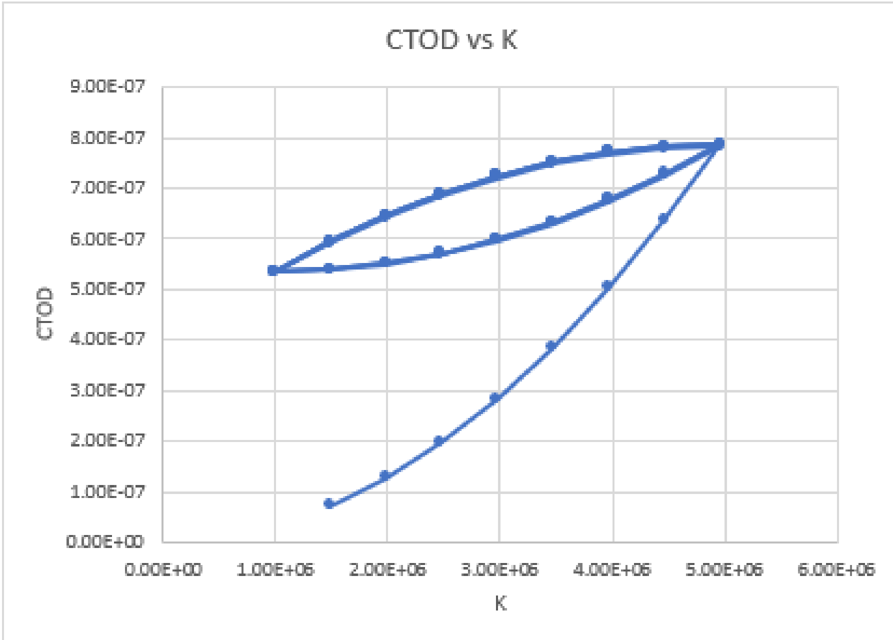


Fig.14 CTOD variation in cyclic loading

If we take a monotonic loading spectrum as shown in figure 1, in here path 1-2 is loading and path 2-3 is unloading. The corresponding CTOD variation is given in figure 2, where path 1-2 a quadratic function shows the loading path and in a similar way path 2-3 shows unloading. If we repeat the loading spectrum for multiple times a hysteresis loop as shown in figure 2 will form. Its not visible in the plot but as the cyclic constant amplitude loading continues, the hysteresis loop will move in positive x direction because K values starts going up with the crack propagation.

2.3.2 Crack Opening Stress Level

Crack opening is a complex phenomenon caused by the interaction of forward plastic zone formation in the loading path and reverse plastic zone formation in the unloading path [18].

Liu and Karthik [19] came up with the equation for crack opening stress. Detail derivation can be found in [18] this literature. It is derived by equating virtual reverse plastic zone to the addition of reverse plastic zone and forward plastic zone.

$$d_{r,virtual} = d_r + d$$

With appropriate substitution we get equation for crack opening stress level as follows:

$$\sigma_{op} = \sigma_{min} + \frac{1}{Y}\left\{\frac{1}{\pi a}\left[\frac{1}{4}(\left(\frac{r_f}{\alpha}\right)^{0.5}\sigma_y - \sigma_{min}Y\sqrt{\pi a})^2 - \frac{8\sigma_y^2 d_r}{\pi}\right]\right\}^{1/2}$$

2.4 New Time-based Subcycle Fatigue Life Model

We discussed concept of EIFS and time-based crack growth formulation in section 2.2 and 2.3. We have modified time-based crack growth formulation to include threshold stress intensity factor K_{th} in it. Using this information, we are proposing a new time-based subcycle fatigue model, which will use equivalent initial flaw size as a initial crack length in case of smooth specimen and with this we will use fracture mechanics approach to calculate fatigue life using modified time-based crack growth formulation.

2.4.1 Failure criteria

Time-based subcycle fatigue model has two failure criteria:

1. When the $K_{applied}$ exceeds K_C (Fracture toughness), then failure occurs.
2. When crack growth exceeds by 0.01 m in single cycle, then failure occurs (>0.01 m indicates unstable crack growth)

When either of the criteria is meet, then calculation loop stops indicating fatigue failure has occurred. Normally first criteria get satisfied almost all the times. Even if we suppress the first criteria then after couple of hundreds of cycles, second criteria get triggered and calculation stops indicating failure. (Mainly because when $K_{applied}$ is approaching K_C, crack growth accelerates very fast, it enters in region 3 crack growth as per da/dN vs ΔK curve.)

2.4.2 Crack Growth Criteria

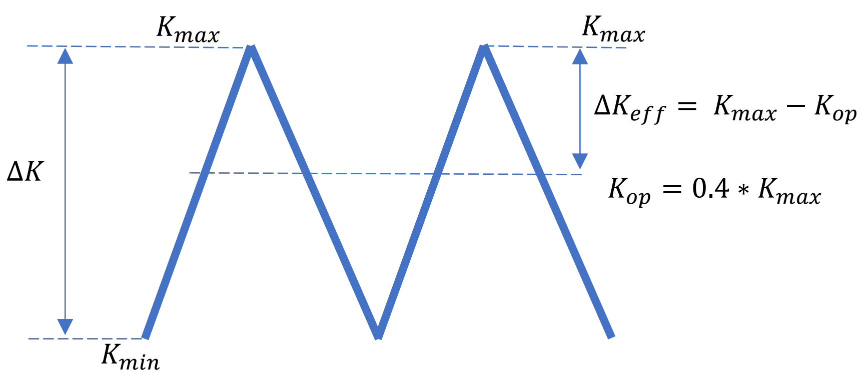


Fig. 15. Loading spectrum schematic to understand terminologies.

Fig. 10 shows the schematic of different terminologies used for crack growth criteria.

Condition 1:

$$\text{if } \Delta K > \Delta K_{th} \text{ then crack will grow}$$

$$\Delta K < \Delta K_{th} \text{ then crack won't grow}$$

Condition 2:

$$\text{if } \Delta K > \Delta K_{th} \text{ then crack will grow in } \Delta K_{op} \text{ region}$$

Condition 1 is tested first if its satisfied then crack growth is calculated for ΔK_{eff} region.

2.4.3 Crack Growth formulation

As discussed in the section 2.3, in time-based crack growth function, crack increment is a function of CTOD and for Al 7075 series its proportional to the square root of CTOD. So, we can write da as

$$da = f(K) * \sqrt{\delta}$$

Where *f(K)*, we are defining as a kernel function which depends on material fitting parameters and applied loading condition. Differentiating da with respect to t to get crack growth per unit time.

$$da = \frac{A(K_{max} - K_{th})^B}{2\sqrt{\delta}} d\delta$$

$$\frac{da}{dt} = \frac{f(K) * d\delta}{2\sqrt{\delta_i}}$$

$$\frac{da}{dt} = \frac{f(K) * d\delta}{\sqrt{\delta_i} + \sqrt{\delta_{i-1}}}$$

As discussed in the section 2.3 modified form of *f(K)* given as

f(K) = $A*(K_{max} - \boldsymbol{K_{th}})^B$

Where A and B are material constants and can be found out by using Paris constants C and m as follows [19].

$$A = \frac{C(1-R)^B \sqrt{2\ E\ \sigma_y}}{0.6}$$

$$B = m - 1$$

Using this formulation and EIFS as starting crack length we can compute fatigue life of a specimen under uniaxial constant amplitude loading as well as random amplitude loading. Same methodology can be extended to get the fatigue life for multiaxial condition which is discussed in the next section.

2.5 Extending Time-based Subcycle Fatigue Model to Multiaxial Loading.

When a specimen or any mechanical component is subjected to more than one type of loading, it is known as multiaxial loading condition. It can be any combination of Mode I, II & III type of loading. In real world, most of the mechanical components are subjected to multiaxial loading. So, it becomes important to estimate the fatigue life for multiaxial loading condition. As discussed in the earlier section, a time-based subcycle model with the EIFS concept can predict the fatigue life in case of uniaxial condition. In this section, an attempt has been made to extend this method to predict the fatigue life for multiaxial condition using mixed mode fatigue crack growth model [20].

One if the key concept in case of multiaxial loading is Proportional and Non-proportional loading. When two different channels of loading acting on a component are in phase are called proportional loading. e.g., when normal stress reaches its maximum value, shear stress is also reaching its maximum value. Same in case of minimum value. This type of loading is called proportional loading. Opposite to the proportional loading in non-proportional loading normal stress and shear stress are out of phase with respect to each other.

Liu and Mahadevan [21] proposed a critical plane model based on general fatigue limit criteria, described below

$$\sqrt{\left(\frac{\sigma_c}{f_{-1}}\right) + \left(\frac{\tau_c}{t_{-1}}\right) + A\left(\frac{\sigma^H}{f_{-1}}\right)} = B$$

Where A and B are material fitting parameters. σ_c , τ_c and σ^H are the normal and shear stress range acting on critical plane. f_{-1} and t_{-1} are fully reversed normal and shear fatigue limit.

One more important material parameter we need to define which related to the material ductility and used in the critical plane orientation calculation is the ratio of shear fatigue limit to the normal fatigue limit, abbreviated as s=t_{-1} /f_{-1} . Detailed derivation for equivalent multiaxial loading can be found in [20] this literature. Brief formulation is given below.

Mode I stress intensity factor can be written as

$$K_I = \sigma\sqrt{\pi a}$$

Mode II stress intensity factor can be written as

$$K_{II} = \tau\sqrt{\pi a}$$

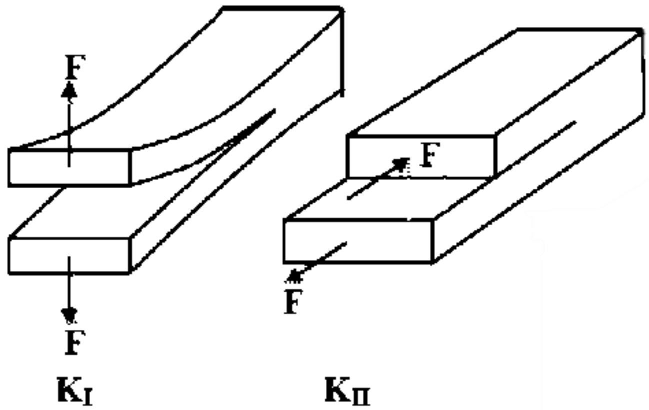


Fig. 16 Mode I and Mode II Loading

The loading related parameters at any time t during the loading history is given as

$$k_{1,t} = \frac{K_{I,t}}{2}(1 + \cos 2 \propto) + K_{II,t} \sin 2 \propto$$

$$k_{2,t} = -\frac{K_{I,t}}{2}(sin\, 2 \propto) + K_{II,t}\, cos\, 2 \propto$$

$$k_t^H = \frac{K_{I,t}}{3}$$

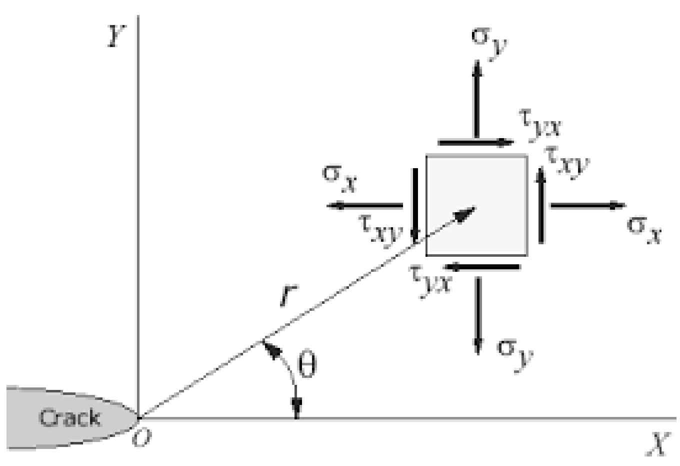


Fig. 17 Multiaxial loading element

Angle α is a critical plane angle, $\alpha = \beta + \gamma$, β is maximum normal stress amplitude plane, γ is material parameter.

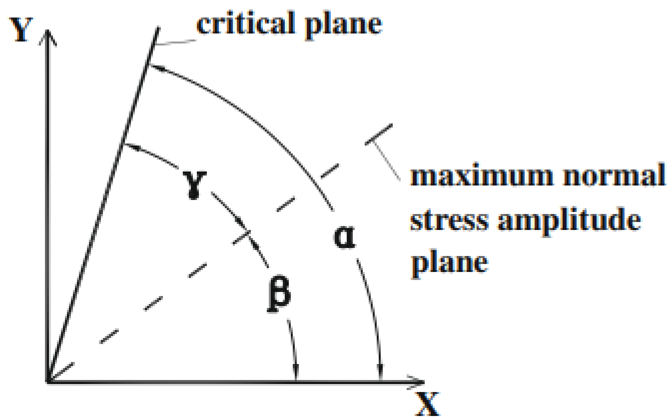


Fig. 18 Critical angle schematic

An equivalent stress intensity factor can be written as

$$K_{mixeed,eq} = \frac{1}{B}\sqrt{(k_1)^2 + \left(\frac{k_2}{s}\right)^2 + A(k^H)^2}$$

The value of s depends on the material ductility and its typically in the range of 0.55 to 0.8. The material property parameters are reported in [20] paper.

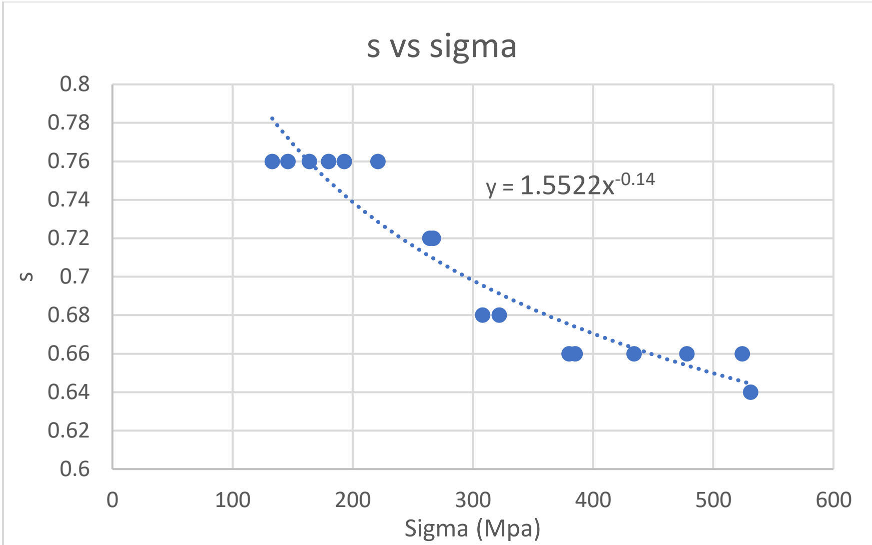


Fig.19 Fitting plot for s ratio and applied load

With this formulation we can convert mixed mode loading into an equivalent tension-compression loading and can use the same time-based subcycle fatigue life model to compute the fatigue life of a specimen.

2.5.1 Elastic Plastic correction factor

The above discussed formulation is applicable in case of elastic analysis which is normally the case in high cycle fatigue analysis. In case medium and low cycle fatigue analysis, if the applied stress crosses the material yield point, in such cases material undergoes the plastic deformation, which needs to be considered while predicting the fatigue life in low cycle fatigue analysis.

To include the effect of elastic deformation, an elastic-plastic correction factor is proposed [20] as

$$\rho = a\left(sec\frac{\pi\sigma_{max}(1-R)}{4\sigma_0} - 1\right)$$

Where ρ is plastic zone size using dislocation theory. This correction factor is added in the existing crack length to get the new crack length as

$$a' = a + \rho$$

$$a' = a + a\left(sec\frac{\pi\sigma_{max}(1-R)}{4\sigma_0} - 1\right)$$

$$a' = a * sec\frac{\pi\sigma_{max}(1-R)}{4\sigma_0}$$

From the above equation we can see that original crack length is multiplied by a secant function, value of which will approach infinity as angle approaches $\pi/2$ shown in the fig.

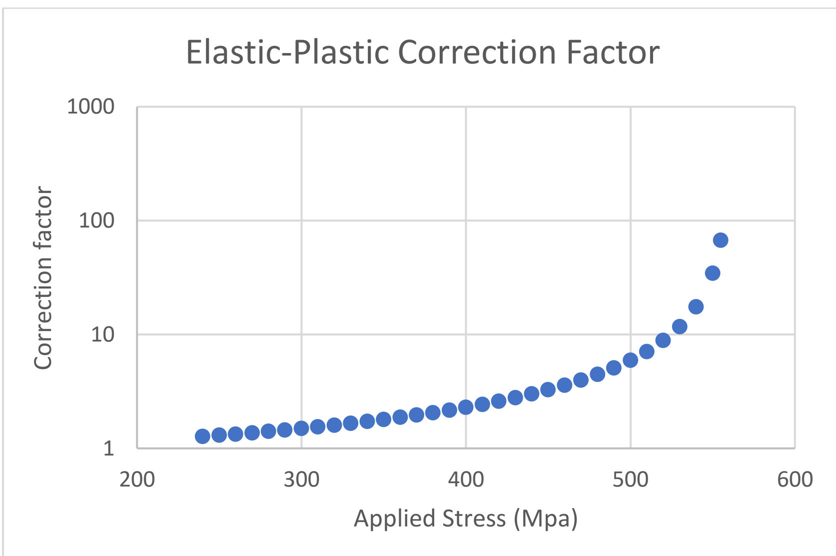


Fig.20 Correction factor behaviour with applied stress

2.6 Chapter Summary

In this chapter we have discussed in detail about the effect of threshold stress intensity factor in fatigue life prediction. We also have discussed various methods to estimate the threshold stress intensity factor and effect of each of them. In the second part of this chapter we discussed, how we can use concept of equivalent initial flaw size and use the fracture mechanics approach to estimate the fatigue life of a specimen. Further we modified the existing time-based subcycle fatigue crack growth formulation to include the threshold term in the main kernel function and integrated this formulation with EIFS to arrive at New time-based subcycle fatigue life model. We further extended this model using equivalent stress intensity factor to predict fatigue life in case of Multiaxial loading conditions. We added elastic-plastic correction factor to consider the effect of elastic deformation in case of low cycle fatigue.

In the next chapter we will be discussing the results for fatigue life of Al 7075 specimen and validation with experimental data as well as open literature data.

CHAPTER 3

MODEL VALIDATION AND DEMONSTRATION

3.1 Overview

This chapter provides the information about testing data generation method, material fitting parameters, material properties used to calibrate the model and the results obtained using time-based subcycle fatigue modelling. These results are verified against the experimental results as well as the results available in open literature. In the first part, results for load shedding are presented. In the second part of this chapter, results for fatigue life in case of uniaxial constant amplitude, uniaxial random amplitude, multiaxial constant amplitude, and multiaxial random amplitude loadings are presented, respectively.

3.1.1 Material Properties

Most of the results are validated for Al-7075, mechanical properties of which are taken from open literature data [11].

Table 1. Mechanical Properties of Aluminum 7075

Materials	$\boldsymbol{\sigma_y}$ **(Mpa)**	$\boldsymbol{\sigma_u}$ **(Mpa)**	**C**	**R**	**m**	$\Delta\boldsymbol{K_{th}}$**(Mpa-**$\boldsymbol{m^{0.5}}$)	$\Delta\boldsymbol{\sigma_f}$ **(Mpa)**
7075-T6	520	575	7.29E-11	0	2.3398	0.5202	227.2
	501	569	1.62E-10	-1	2.3398	1.0034	402.5
	520	575	7.41E-11	0.1	2.3398	0.5202	227.2

The fitting parameter A and B can be found out by using Paris Constants C and m. Another approach to get the fitting parameters A and B is that we can plot da/dN vs ΔK values for the time-based subcycle model and iterate A and B values so that we get same C and m values in the da/dN vs ΔK plots as that of experimental results. Both the methods have been used, later shown more accurate agreement with the experimental fatigue life data.

3.2 Results from Load shedding

1. $\Delta \boldsymbol{K_{th}}$ v/s R by Load Shedding technique

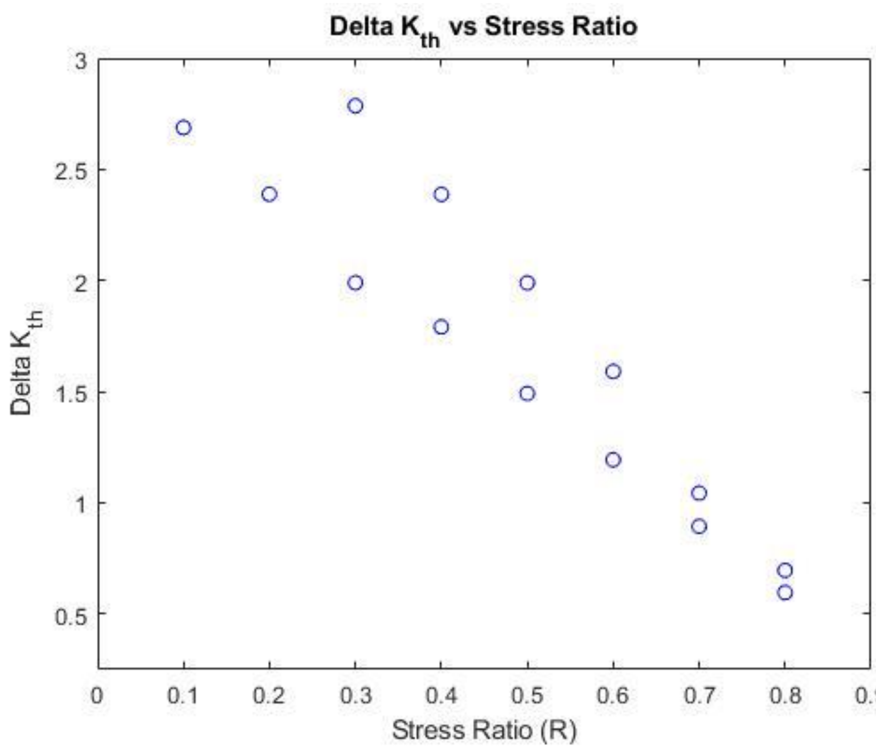


Fig. 21. ΔK vs R plot

Load shedding spectrums were used to get the threshold stress intensity factor for different stress ratio to analyze the effect of stress ratio on $\Delta \boldsymbol{K_{th}}$. Fig 6. plot shows, how $\Delta \boldsymbol{K_{th}}$(

(Threshold stress intensity factor) varies with stress Ratio R. For any stress ratio, threshold value lies in between the two points plotted vertically on the plot. With change in stress ratio, a negative correlation has been observed in the threshold stress intensity factor. Similar type of trend has been reported by Stewart [22].

2. $\boldsymbol{\Delta K_{th}}$ by varying initial loading condition in Load Shedding.

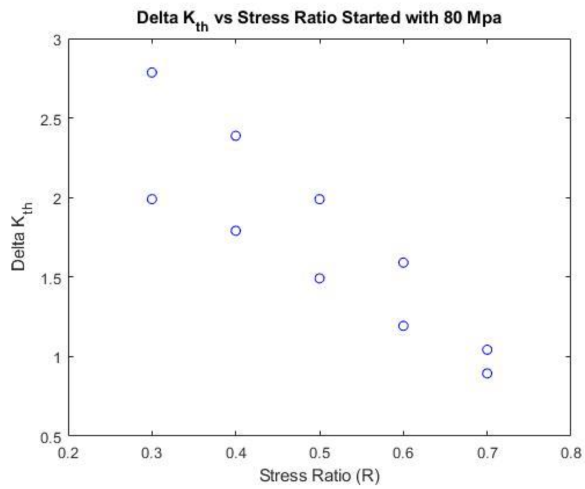


Fig.22 ΔK vs R plot when σ_1= 80 Mpa.

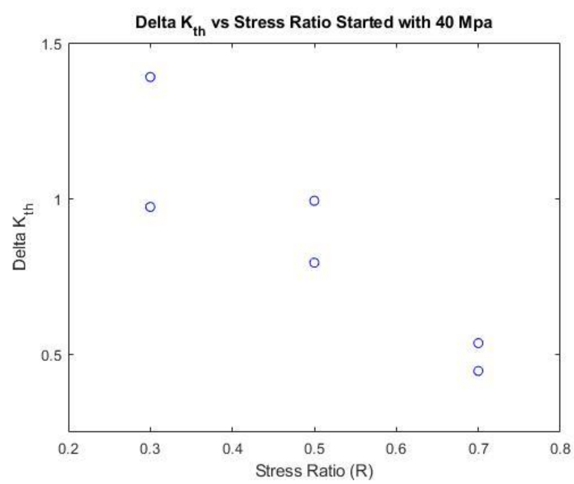


Fig.23 ΔK vs R plot when σ_1= 40 Mpa.

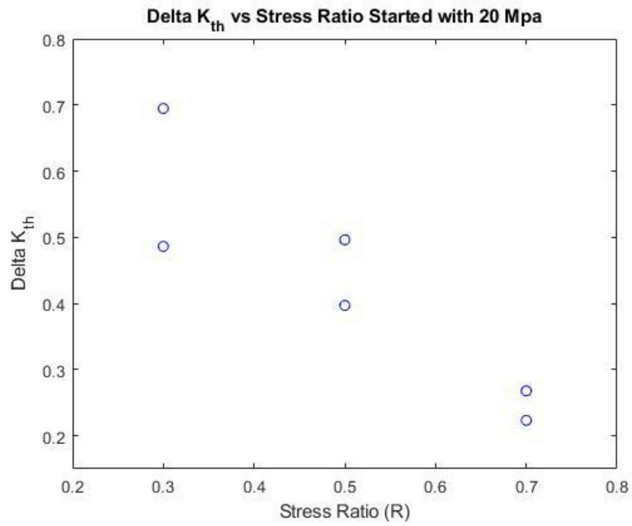
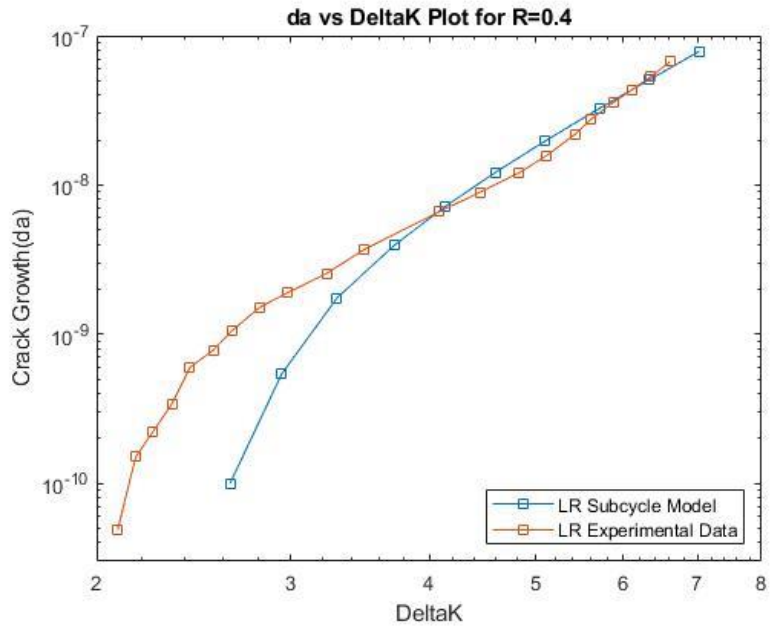


Fig.24 ΔK vs R plot when σ_1=20 Mpa

As shown in the above plots (Fig 6), we performed load shedding on a same specimen but with different starting values of applied load and results for which confirms that in load shedding technique, as the starting stress value changes the corresponding threshold stress intensity factor also changes. From the above results it shows that, there is a positive correlation between, starting value of applied stress and the threshold stress intensity factor. So, with this result we can confirm that, the threshold value which we are getting here, is not an intrinsic material property, rather it is an effect of external loading conditions.

3. da/dN v/s ΔK plot



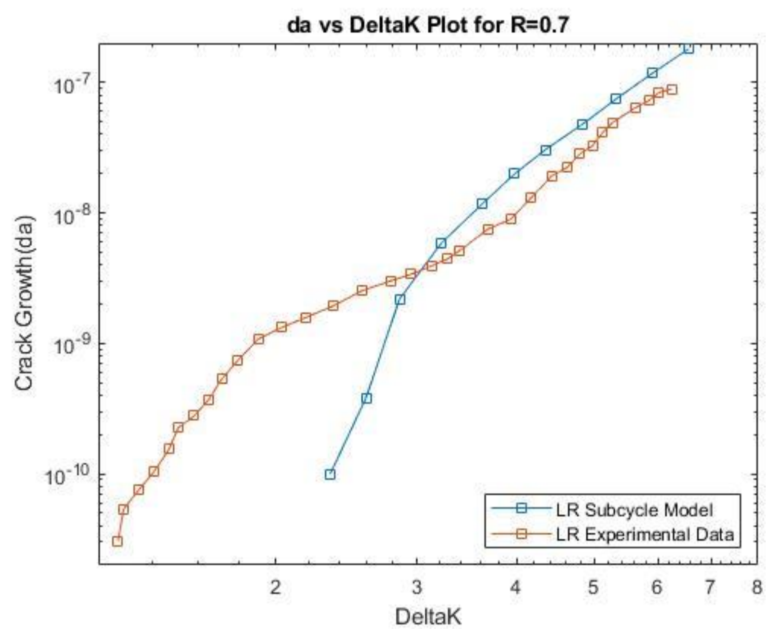


Fig. 25 *da* vs ΔK for R0.4 and R0.7

With the help of load shedding technique, we also plotted the da v/s ΔK curve in threshold and Paris region and compared with the similar load shedding, experimentally plotted da v/s ΔK curve from literature data [12]. The threshold value from literature was around 2.1 Mpa$\sqrt{m}$. stress ratio of 0.4, whereas threshold value from subcycle model was around 2.64 Mpa$\sqrt{m}$. Subcycle model was overpredicting the threshold stress intensity factor by a significant margin. In a similar way for stress ratio of 0.7, threshold value was found around 1.4 Mpa$\sqrt{m}$ from literature, whereas from subcycle model we got around 2.3 Mpa$\sqrt{m}$.

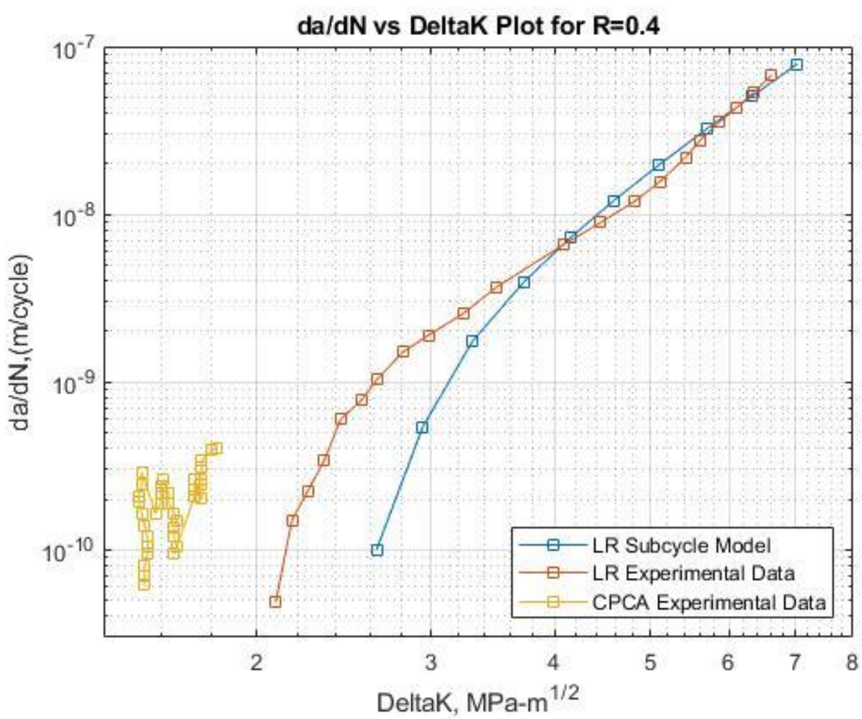


Fig. 26. *da* vs ΔK for CPCA and Load Reduction

We also tried other methods such as CPCA, CPLR mentioned earlier and compared with the literature data [2]. We performed CPLR for various stress ratios. Results for stress ratio

0.4 shown in the above plots. With CPCA loading we observed similar trend. Subcycle model tend to overpredict the threshold stress intensity factor value by a significant margin (around 0.5 Mpa$\sqrt{m}$). One main reason behind this mismatch could be that subcycle fatigue crack growth model was only calibrated in Paris region where crack growth behavior is linear with respect to applied stress intensity factor. In subcycle modelling, the threshold region line is interpolated to follow linear trend same as Paris region. But most of the experimental data shows that crack growth behavior is not linear, moreover asymptotic in threshold region.

3.3 Results for Uniaxial Loading

In this section we will be using subcycle model to estimate fatigue life of a given specimen under constant amplitude and variable amplitude uniaxial loading and compare the results with the experimental data. Following the methodology described in the previous section, fatigue life for aluminum 7075 has been calculated.

1. Constant Amplitude Uniaxial Loading Condition

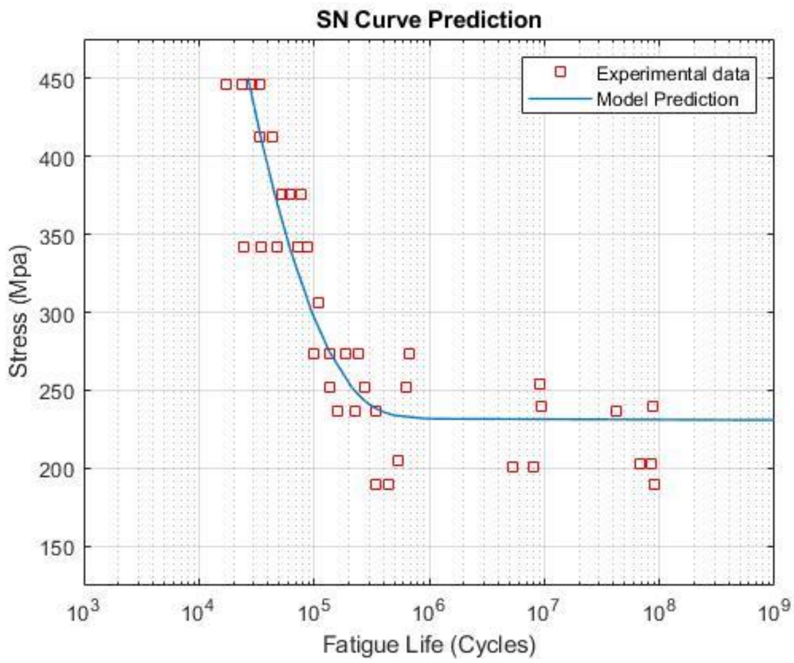


Fig. 27. Results for SN curve Prediction.

The Proposed model with the described methodology has been validated with the help of experimental data available in the literature [11]. Figure 27 plot, which indicates fatigue life prediction by subcycle model shows a good agreement with the experimental data.

2. Random Amplitude Uniaxial Loading condition

Random amplitude uniaxial loading test was carried out with MTS multipurpose elite software.

6 different types of loading spectrums were used for testing purpose. The time difference between two adjacent loading point was 0.05 sec, giving frequency of 20 Hz. The random

loading spectrum were repeated till the specimen fails. The linear non-stationary spectrum fatigue spectrum was generated using Auto regressive process. A nonlinear nonstationary fatigue spectrum was generated using Genesis 4 fatigue software. FELIX spectrum is direct experimental loading data from rotors of helicopter. Another FELIX spectrums were generated by multiplying each loading point by 1.17 and named as FELIX*1.17 and adding 35 Mpa to the maximum value known as MAXFELIX + 35. These modifications made FELIX spectrum more severe to get shorter fatigue life. Simple Linear and simplified max FELIX + 35, these two spectrums were obtained by using continuous wavelet transform method [23]. These random amplitude loading spectrums are shown in figure 28. The results for experimental fatigue life are shown in table 2.

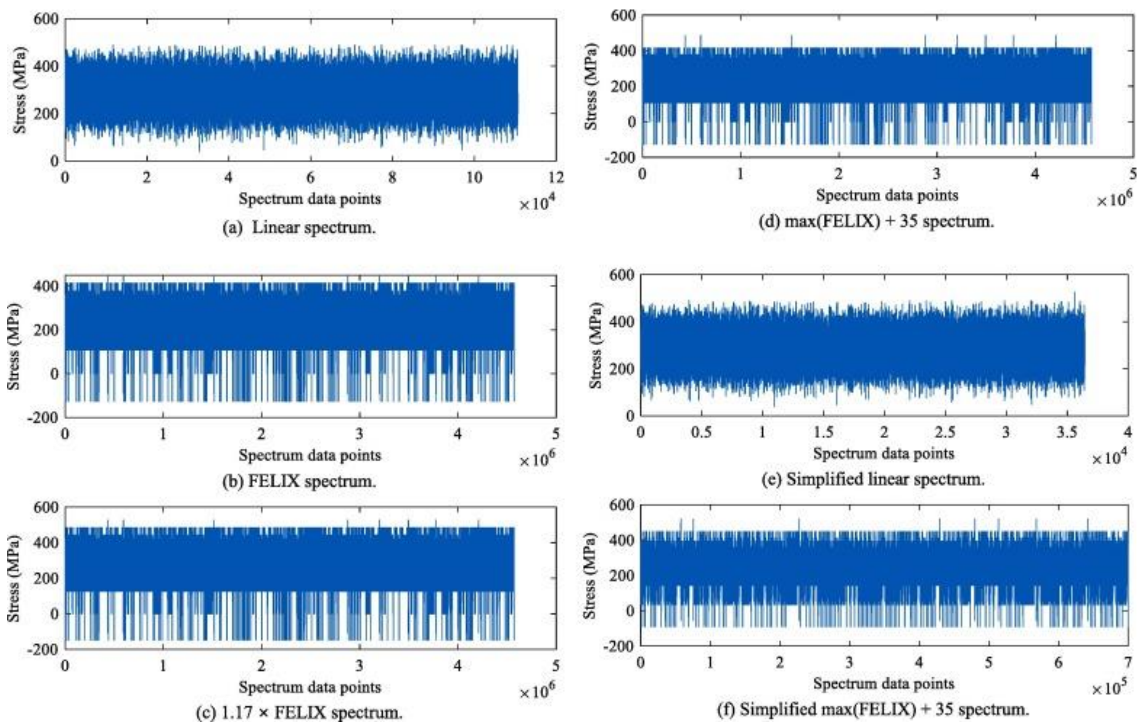


Fig. 28 Random Amplitude Loading Spectrum

Table 2. Experimental Life & Subcycle Model Life Prediction

Loading Spectrum	Exp Nf	Subcycle Nf
Linear	2.10E+06	780000
Simplified Linear	736000	300000
Felix	1.60E+07	1.28E+07
Felix*1.17	1.50E+06	1777000
Felix*1.17	674000	1777000
maxFelix+35	5.90E+06	6594000
maxFelix+35	1.90E+06	6594000
maxFelix+35	2.90E+06	6594000
simlified max felix + 35	1.10E+06	1010000

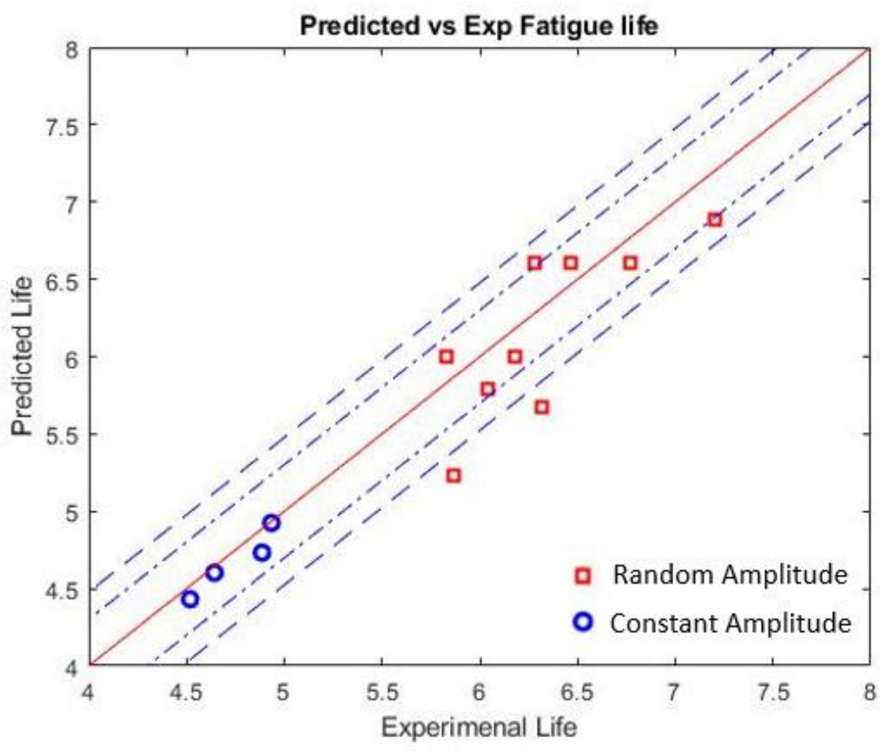


Fig. 29. Results for constant & random loading prediction.

The proposed framework is validated using the experiment data obtained from ASU and MERC. The random loading spectrum, the experiment fatigue life, and the predicted fatigue life using subcycle model are shown in table 1. The comparison between the prediction and experimental data are plotted in Fig.28.

3.4 Results for Multiaxial Loading

In this section we will be using time-based subcycle model to estimate fatigue life of a given specimen under constant amplitude and variable amplitude multiaxial loading and compare the results with the experimental data. Following the methodology described in the Chapter 2, fatigue life for aluminum 7075 has been calculated.

1. Results for Constant Amplitude Multiaxial Loading.

As discussed in chapter 2 section 2.5, that we are using same crack growth function which was used to get the results for uniaxial loading condition. Additionally, we are using equivalent stress intensity factor concept to combine tension loading and shear loading into equivalent tension loading and predicting fatigue life with time-based subcycle fatigue life formulation. Table 3 shows the results for experimental and predicted fatigue life results for multiaxial condition. This table also contains the equivalent sigma values for corresponding tension and shear loading. Later we introduced elastic-plastic correction factor to consider effect of plastic deformation in low cycle region. Corrected Sigma equivalent values are also reported in the table.

Table 3. Multiaxial Constant Amplitude Results

SN	Sigma	Tau	Exp_Nf	SigmaE	s	SigmaEq_s	Predicted_Nf
1	351.3	222	1967	479.8	0.65	501	5500
2	165.6	216.2	9174	349.9	0.68	380	22000
3	127.2	170.5	59194	274	0.70	286	100000
4	166.1	110.4	136646	232.2	0.72	235	340000
5	201.3	130	45500	277.7	0.70	284	100000
6	377.6	241.8	2487	518.96	0.64	544	1800
7	280.4	181.8	10191	387.55	0.67	401	17000
8	200.9	131.6	29439	279	0.70	285	100000
9	200.6	115.8	41747	263.9	0.71	268	118000

The Proposed model with the described methodology has been validated with the help of experimental data available in the literature [24]. Figure 12 plot, which indicates fatigue life prediction by subcycle model shows a good agreement with the experimental data.

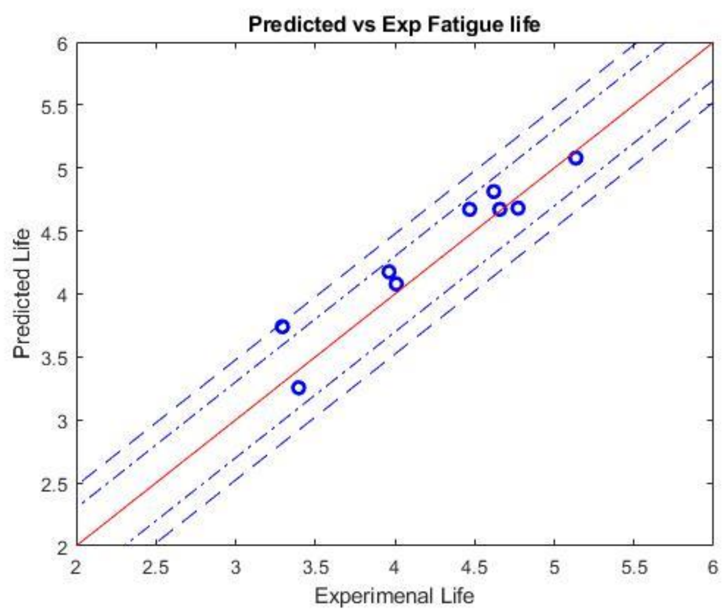


Fig. 30 Results for multiaxial loading condition.

2. Results for Variable Amplitude Multiaxial Loading.

For Multiaxial condition, limited data was available, to test the validity of the proposed methodology. The proposed framework is validated using the experiment data obtained from ASU and MERC. The random loading spectrum, the experiment fatigue life, and the predicted fatigue life using subcycle model are shown in table 4. The comparison between the prediction and experimental data are plotted in Fig.30.

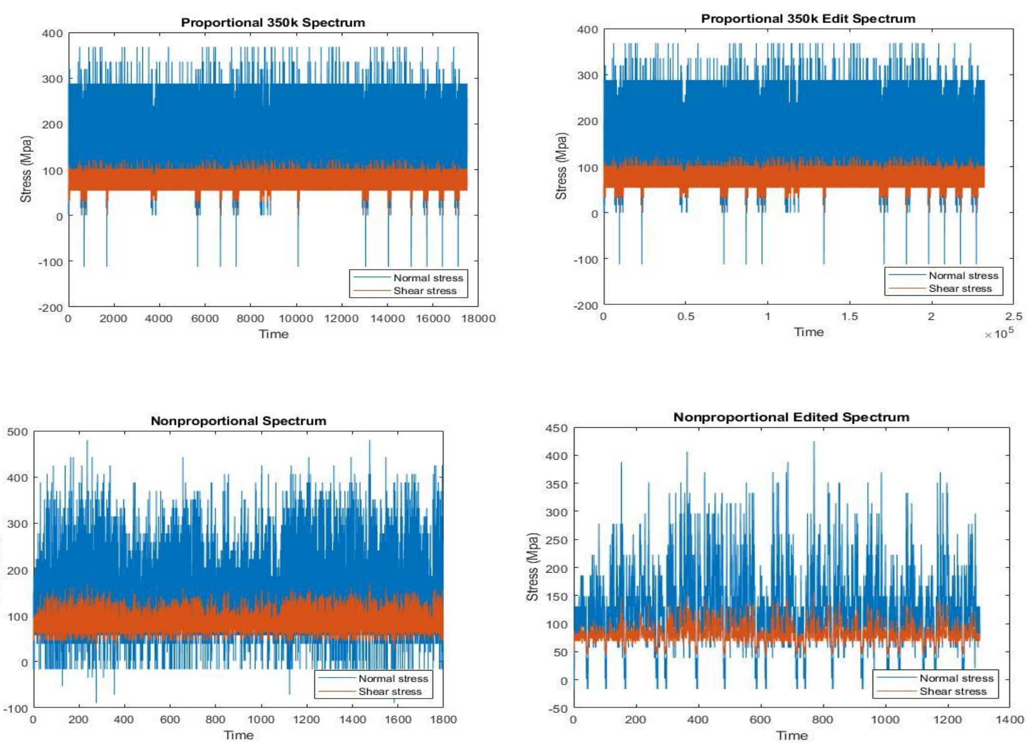


Fig. 31 Multiaxial Random amplitude loading spectrum.

Table 4. Results for Random Amplitude Multiaxial Loading

Spectrum	Experimental	Log_Exp	Subcycle	Log_Sub
Proportional_350K	1.34E+06	6.126711	1.20E+06	6.079181246
	1.75E+06	6.241954	1.20E+06	6.079181246
Proportional_350k_edit	5.11E+06	6.708098	7.80E+06	6.892094603
Non_proportional	1.19E+06	6.076102	3.00E+06	6.477121255
Non_proportional_edited	4.05E+05	5.607512	1.50E+06	6.176091259

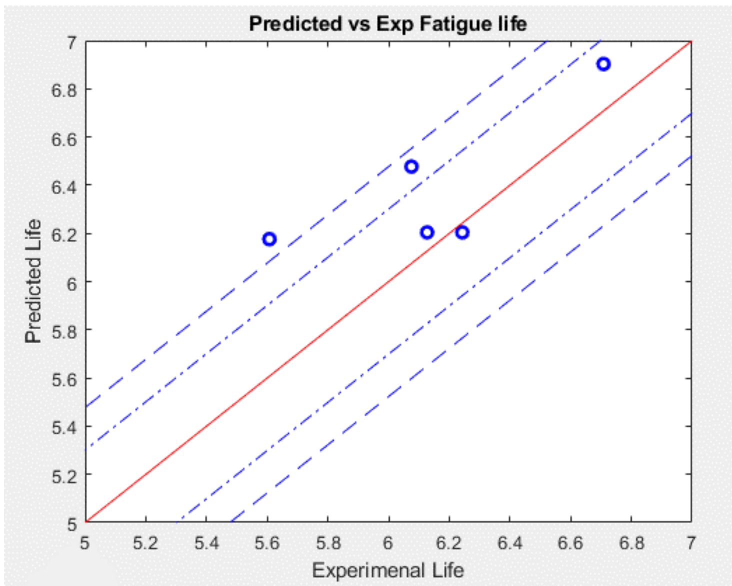


Fig. 32 Results for Random amplitude multiaxial loading condition.

CHAPTER 4

CONCLUSION AND FUTURE WORK

4.1 Conclusion

Previously developed time-based subcycle crack growth model has been modified to consider the effect of threshold stress intensity factor. Modification has been done in main kernel function for crack increment calculation by introducing ΔK_{th}. Concept of EIFS has been integrated with this modified kernel function to calculate fatigue life using fracture mechanics approach. Effect of load shedding techniques has been reproduced using proposed model and validated against the literature data which shows, in load shedding technique threshold stress intensity factor is a function of applied loading and depends on load history as well as on the load ratio R.

The goal to predict the fatigue life using proposed model has been achieved by validating results against several conditions such as constant and variable amplitude, uniaxial and multiaxial loading conditions. Most of the predicted fatigue life results lie with error factor range of 2, very few points lie within the error factor range of 3, which is assumed to be a good prediction for fatigue life.

Model fitting parameters A and B are calibrated for a specific material, here in this study for Al 7075. For high strength aluminum with different mechanical properties than used in this literature, parameters need to be calibrated again using experimental testing data. More extensive validation is required for multiaxial random amplitude condition as testing data was limited. Overall the goal to Mathematically integrate EIFS into the modified time-based subcycle fatigue crack growth function has been achieved and validated.

4.2 Future Work

The following topics can be considered as potential future work or areas that need to be studied in order improve the robustness of the model.

1. We used the concept of equivalent initial crack size, which is not the actual crack in the material, its an approximation by ignoring small crack growth in the microstructure to get the fatigue life with fracture mechanics. Microstructure of a material i.e., the way grains are oriented, effects of grain boundaries and the crack growth withing the grain need to be studied to understand the effect of microstructure on the fatigue life. Mathematical modelling of fatigue crack growth is a complex phenomenon. If we take an example of testing data. There is a large amount of variation can be seen even when used exactly same specimen, all the same material properties. Still, we get variation in the fatigue life. So, at molecular level or the way microstructure has been aligned with respect to each other, the way grains oriented with respect to each other has a big impact of crack propagation, which need to be studied and need to be mathematically modelled.

2. In this study we followed the deterministic approach, like we have considered every other material property or loading data as a constant and a specific number, which is theoretically not possible. Every loading point or every constant data point has some variation in it which can be considered by following the probabilistic approach. Where we

consider the effect of random variation in the datapoints and then calculate how this randomness is getting propagated in our results. This area can be studied further to improve the robustness of the proposed model.

3. We can couple this proposed framework with FEA simulation to improve the accuracy of the results. When we convert applied loading into a stress intensity factor, we use a geometry correction factor which depends upon the size and shape of a crack. While calculating EIFS, we are considering geometry correction factor, Y as a constant number. It is a continuous function of crack growth, which can be considered while calculating the EIFS.

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