Analysis of Local Impact of Rooftop Photovoltaic Panels

by

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ABSTRACT

Rooftop photovoltaic (PV) systems are becoming increasingly common as the efficiency of solar panels increase, the cost decreases, and worries about climate change increase and become increasingly prevalent. An under explored aspect of rooftop solar systems is the thermal effects that the systems have on the local area. These effects are investigated in this paper to determine the overall impact that solar systems have on the heating and cooling demands of a building as well as on the efficiency losses of the solar panels due to the increased temperature on the panels themselves. The specific building studied in this paper is the Goldwater Center for Science and Engineering located in the Tempe campus of Arizona State University. The ambient conditions were modeled from a typical July day in Tempe. A numerical model of a simple flat roof was also created to find the average rooftop temperature throughout the day. Through this study it was determined that solar panels cause a decrease in the maximum temperature of the rooftop during the day, while reducing the ability of the roof to be cooled during the night. The solar panels also saw a high temperature during the day during the most productive time of day for solar panels, which saw a decrease in total energy production for the panels.

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NOMENCLATURE

a	=	temperature coefficient
A	=	area
C_p	=	specific heat capacity
dt	=	differential with respect to time
D	=	distance
h	=	heat transfer coefficient
k	=	thermal conductivity
K	=	turbulent kinetic energy
K_T	=	efficiency correction coefficient
Nu	=	Nusselt number
Pr	=	Prandtl number
Q	=	heat transfer
Re	=	Reynolds number
T	=	temperature
u	=	velocity
W	=	work
z	=	height
α	=	absorptivity
β	=	thermal Expansion Coefficient
σ	=	Stefan-Boltzmann constant
ϵ	=	rate of dissipation of turbulent kinetic energy
ε	=	emissivity

NOMENCLATURE

μ	=	dynamic viscosity
ρ	=	density
ω	=	specific dissipation rate

CHAPTER 1

INTRODUCTION

1.1 Background

The field of energy production for use in buildings is one of the great challenges of the 21^{st} century. Most experts agree that the need to transition away from non-renewable resources is a necessity in order to avert the greatest dangers of climate change and to create a resilient energy grid [6]. For this reason, solar photovoltaic panels are becoming increasing common, both in large, centralized solar farms, and in smaller rooftop insulations. These smaller insulations are the focus of this paper. The affect that solar panels have on local heat transfer is an important consideration in installing rooftop solar. This is important for two reasons. First, rooftops are one of the largest sources of heat transfer from a building to the ambient [7]. Because of this, if solar panels noticeably increase or decrease the temperature of the rooftop, the heating and cooling demands of the buildings may be impacted one way or the other to a significant extent. This could potentially increase energy usage and reduce the effectiveness of solar panels, or decrease energy usage, making solar panels an even better deal for building owners. Second, solar panels see decreasing efficiency as their temperature increases [5]. Because of this, if solar panels reduce the ability of a building to cool itself, the overall efficiency may decrease, causing lower than expected energy outputs. Because of these two factors, if the local conditions around a rooftop solar installation become hotter, then energy production will be decreased while also increasing the energy draw for cooling the building. This is important to consider both from a personal perspective of the financial costs and benefits of a solar system and from a societal perspective since these effects may increase or decrease energy production and use in future projections.

Both of these considerations are of special importance in Phoenix, Arizona, which reliably sees temperatures higher than one hundred Fahrenheit (38 Celsius) during summer days. During these summer months energy demands are at their highest for cooling buildings. This necessitates cheap, renewable energy sources to meet the demand without excessive greenhouse gas emissions. Due to the extreme high temperatures the effects on heat transfer from a rooftop becomes especially pronounced. This will be the central focus of the research. However, similar situations during the winter are also of importance and should be investigated further in the future.

1.2 Literature Review

There is a solid body of research into computational solutions to the flow of mass and heat around solar panels. In a paper by Jubayer, et al, a CFD analysis of ground mounted solar panels was conducted in order to determine the convective heat transfer coefficient at different wind speeds and directions [3]. This paper used a steady solar panel temperature of 70°C and disregarded radiative cooling. The researchers used three wind speeds, 1, 5, and 10 m/s, directed towards either the front face or back face of the panel. The paper's findings were a comparison of the convective heat transfer coefficient on the upper and lower surface of the solar panels as a function of Reynolds number. This information was used as a point of comparison for determining the heat transfer coefficient that was used in the Fluent model of this research. However, this comparison should be taken with some considerations for the differences in the research. The primary difference of notes here is that the wind in the current research came at the side face of the solar panel, 90 degrees different from the research conducted by Jubayer. Because of the difference in relative geometry, it is important to consider this paper as a reference for trends and not for one-to-one comparisons of computed convective heat transfer coefficients.

In a paper by Brown, et al, a situation very similar to this paper's focus was studied through the use of direct data collection as opposed to computational models [2]. The researchers placed a test building onto the rooftop of a building. The test building had a white rooftop, an air conditioning system, and nine solar panels in a three-by-three array. The researchers measured the temperature of the rooftop and solar panels to determine heat flux from the rooftop. Setups of the building were without solar panels, solar panels placed 10 cm from the rooftop and 28 cm from the rooftop. The paper found that the solar panels reduced heat flux from the system and resulted in higher average temperatures. They also found that the solar panels resulted in increased electrical demands from the air conditioning system to maintain the building's constant temperature. This paper used a very similar experimental set up to the one in the current paper and was used as a reference as a point of comparison for results. One limitation of the paper was a (presumed) lack of insulation in the experimental data collection. This may lead to exaggerated results in the temperature profiles.

A paper by Dominguez, et al, had an experimental setup most similar to the present paper [1]. This research used a functioning building that featured solar panels. Using a wide angled infrared camera, they were able to monitor the long wave radiation coming off of the rooftop in the area covered and uncovered by the solar panels. They also monitored the temperatures of the ceiling beneath the rooftop. Using this data, they found that the solar panels reduced the average temperature of the covered rooftop, which led to a slight reduction in the temperature of the ceiling surfaces beneath the covered rooftop. Their analysis showed that the solar panels reduced both heating and cooling loads of the building during all months. This is in contrast to the paper by Brown [2]. The most likely reason for the marked difference in the two papers is the inclusion of insulation in the Dominguez paper. The rooftop of the Brown paper, which was highly reflective, may not have been similar to the Dominguez paper's rooftop. This would have increased the effectiveness of radiative cooling, which would be dramatically hampered by a hot solar panel surface over the top of the rooftop. The small scale of the model rooftop may have also been a contributing factor in the differences between the two papers. For instance, the small scale of the test building in the work by Brown may have substantially reduced the Reynolds number of the test environment, causing a decrease in the effectiveness of convective heat transfer, and overstating the importance of conductive and radiative cooling. This paper, being the most similar to the setup of the current research, was used extensively for data comparison.

1.3 Computational Solutions

This paper relies on a computational solution in order to draw conclusions. This approach was chosen because, while an experimental setup would allow for real world results, a computational approach allows for complete control over every variable: humidity, temperature, time of day and year, weather conditions, as well as allowing for hypothetical situations to be considered and tested very quickly. Furthermore, this research was primarily conducted during the winter months. This is an issue because the most interesting time for the purposes of this paper is during the summer months. This is because the summer months in Tempe is when the greatest energy costs are incurred due to cooling building.

Having stated the benefits of computational solutions, it is important to also understand the importance of experimental results. Experimental results allow for validation of computational results which may be affected by improper computational setups or wrong assumptions. Furthermore, experimental results can be much more compelling to audiences, as well as being harder to refute lacking experimenter error. For these reasons it is important to have research that focuses on both computational and experimental solutions.

1.4 Objective

The objectives of this research are largely focused around understanding the thermal effects due to rooftop solar panels. The first major objective is to understand what temperature differences are experienced on the rooftop during the day when solar panels are placed on the roof. Next, the effect that the solar panels have on the rooftop during the nighttime are to be explored. Finally, the efficiency of the solar panels during this time will be studied briefly.

This research will be focused on a flat rooftop with angled solar panels facing south at an inclination of about 14 degrees. This is a common configuration on non-residential buildings that are common throughout the ASU campus in Tempe. Because of this, the research will not have a large connection to residential solar panels that are commonly placed on a slanted roof and placed flat against the roof. In fact, if similar research were carried out on such a setup very different results would likely be found for reasons that will be explained within in paper. However, non-residential buildings, such as warehouses, big box stores, and manufacturing sites, are ideal for solar implementation due to their large footprint, high energy demand, and lack of visual appeal in need of protection. Because of these factors, solar panel use may become increasingly common in such buildings in the coming decades.

The paper will not make use of original experimental data from either model buildings or functional buildings. Although data collection from such structures can be important and useful, it does offer some drawbacks that limit their usefulness. First, the data collected is highly season dependent and therefore the data that is gathered would be very different than the data generated in this study. Due to the weather and climate in Arizona, it was desired for the data to be representative of a normal summer day; however, during the summer months this research was not being carried out. Another limitation of experimental data is the cost associated with it. Monitoring temperature and air speed near a rooftop would require equipment that would quickly add cost to the research. On the other hand, computed data allows for accurate information to be gathered for little to no cost. Finally, computational methods allow for extreme control of environmental variables. If experimental data was used, then wind speed and direction would not be able to be controlled in order for quick and accurate comparisons to similar studies. Furthermore, CFD allows for additional studies to be carried out by simply changing a few inputs. Experimental data would require a full additional study to be carried out, including time to set up equipment, time to gather data, and time to analyze the data. Using a program such as Fluent allows for all of these steps to be carried out in one step, potentially decreasing the time to create new data from weeks to hours.

This paper will also make use of MATLAB for data generation. This will be used for creating data about a flat rooftop without solar panels. The flat rooftop design is a much simpler design than the rooftop with solar panels and therefore can be modeled using a simple MATLAB code that models the heat transfer without also analyzing the fluid flow in a large domain near the building. This enables incredibly fast analysis that creates robust data. As will be discussed, this method does require some simplifications to reduce code complexity, but these simplifications would not cause the data to be erroneous. In fact, despite the relative simplicity of the code, the data seems to corroborate the data gathered from Fluent and mirrors the findings in many ways. Despite this, there are some areas of disagreement in the data that will also be explored.

CHAPTER 2

COMPUTATIONAL SETUP

2.1 Geometry

The first consideration when using Fluent is to design a robust CAD model that can represent the target of the research. For this research the Goldwater Center for Science and Engineering (GWC) was modeled as the only building in a large bounding box. This is a large simplification to the system since in reality the GWC is located in a dense urban environment with many buildings around it. However, the GWC was chosen because it is the tallest building within a relatively large area, and so the other buildings will have limited impact on the fluid flow near the high rooftop and solar panels. Having said that, this does not mean that this simplification will have no impacts on the result. For instance, even buildings below the roofline of GWC can have a significant effect on the fluid flow at the roof level. With a minimum of analysis, it is clear that the buildings may in fact increase the wind speed at the roof level of GWC.

Another simplification was to the GWC. The building was modeled without windows and some other finer details that would impact the thermal properties of the building and the flow properties of the air. This is expected to have less impact on the relevant results since these changes are on the sides of the building and would largely impact the temperature of the sides of the building.

The final important change to the system was the size and layout of the solar panels. The surface of each individual solar panel is accurate to the ones used on the GWC, but the thickness of each panel is dramatically increased in order for Fluent to be capable of creating a mesh around the solar panel. If the thickness were accurate, it would be too thin to allow for a mesh to be created without using such a fine mesh that no results could be taken in a reasonable amount of time. This change is expected to produce changes to the results that are not too dramatic since the boundary layer of the solar panel would create a similar area of low-speed flow near the panels [8]. Finally, the layout of the solar panels was simplified. The GWC has solar panels that have occasional gaps. This level of detail has been dropped in favor of modeling the same number of rows of solar panels but placed in a consistent pattern. This change will affect the exact results of the system but not the underlying physics of the situation and therefore the results can be taken to be a generalized case. This change does result in higher coverage of the roof with solar panels, which will tend to exaggerate the results. For example, if a cooling effect is observed the large number of solar panels will cause the cooling to be greater than what is likely to be observed in reality over the building. However, this is not necessarily entirely a bad thing. The exaggerated results may allow trends to be more readily visible than if the results were reduced. Furthermore, the exaggerated results would not be non-physical results since they are simply describing a situation with more solar panels and not a situation with different physics.

Fig. 1: Google Maps Image of GWC as it is Located in Tempe

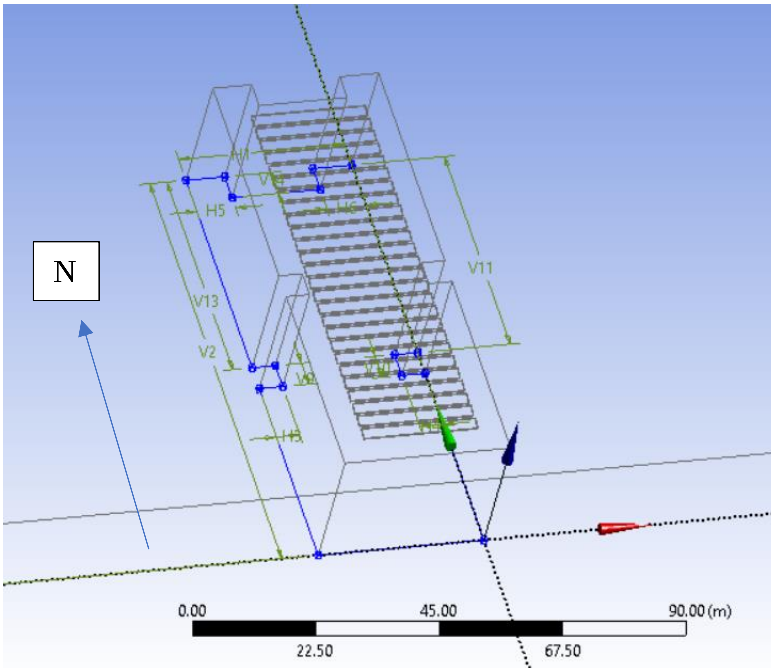

Fig 2: CAD Model of GWC

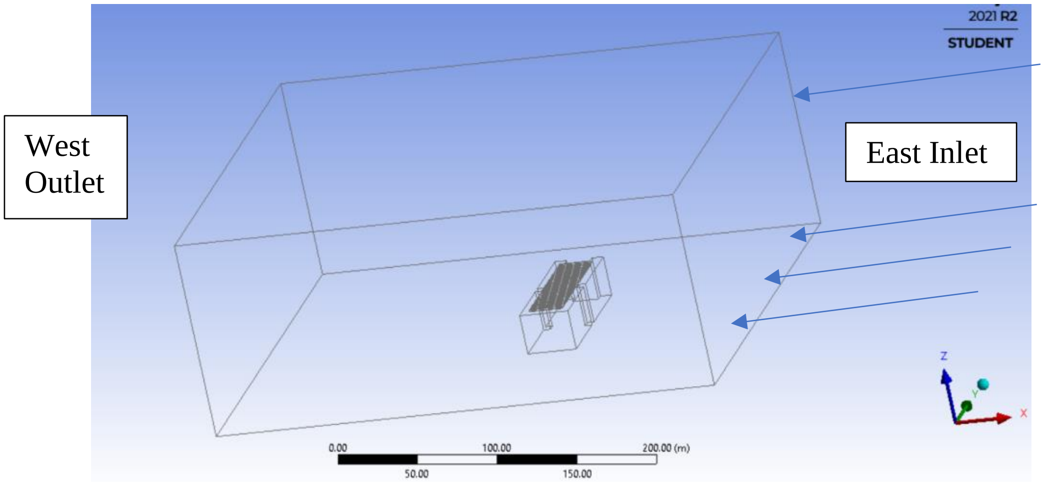

Fig. 3: Domain of the Simulation

2.2 Mesh

The next step in creating a Fluent model is to create a mesh of the geometry. This requires two factors to be balanced. A finer mesh will result in more accurate results but will require more time processing the results. This is one reason that the GWC was modeled alone instead of with other, non-target buildings. Having only one building in an otherwise empty bounding box allows a courser mesh to be used in the empty areas with the area around the building to be refined in a later step. This is a good compromise between accurate results and processing time. Data from the meshing step can be found below.

Nodes	82841
Elements	427922
Element Size	3.5 m
Bounding Box Diagonal	445.76 m
Minimum Edge Length	0.4 m

Table 1: Meshing metrics

2.3 Setup

Fluent has two options for modeling fluid flows: density-based and pressure-based. For this model, pressure-based was used because it is computationally less expensive. This leaves the problem of Fluent no longer attempting to model changes in density, an important consideration in thermal flows. To overcome this limitation, a Boussinesq approximation was used [9]. This approximation models the density change of compressible fluids as only a function of temperature. Air is modeled as an ideal gas and so the thermal expansion coefficient, β, is approximately $\frac{1}{T}$. Fluent requires one value of β and so the reference temperature of 300 K was chosen. This resulted in a β value of 0.0033 K^{-1}. This results in an approximate 12% error when the temperature reaches 340 K. As will be seen this is on the high end of the model and is almost never reached by the air and is so deemed acceptable.

The Boussinesq approximation, as used by Fluent, provides much faster convergence of the governing equations then one would see by simply making density a function of temperature. For instance, density could be input as an equation of the form:

$$\rho = \rho_0(1 - \beta(T - T_0)) \quad (1)$$

However, this would cause Fluent to use that equation to solve every equation that is required to be run in every iteration, including continuity, momentum, and energy as well as in every mesh element. This puts an additional computational load on the program. In comparison, the Boussinesq approximation causes Fluent to use the initial density value in every equation except for the buoyancy term in the momentum equation [10]. This allows for hot air to rise and be considered less dense, while reducing the load from computing additional density terms in areas that will not be significantly affected by changes in density.

The energy equation is turned on so that temperature can be modeled. Likewise, gravity is turned on and set at -9.81 m/s^2. The viscosity of the simulation is modeled by a K-ω model. This is a relatively standard model, along with K- ϵ. In comparison with K-ϵ, K-ω functions better when modeling flows near walls, which is an important consideration in this simulation. K-ω is a turbulence model that approximates the Reynolds-Averaged Navier-Stokes equations. This model predicts turbulence using two variables, K, the turbulent kinetic energy, and ω, the specific rate of dissipation of turbulent kinetic energy into internal thermal energy [11]

The major sources of energy in this model come from mass flow through the inlet and outlet, and radiation from the solar source. The inlet will be discussed momentarily along with the other boundary. For the solar thermal radiation, Fluent has a built-in solar calculator that can determine the solar load based on time of day, year, and latitude and longitude. This calculation updates as the time changes in transient simulations allowing for changes in the sunlight to be modeled across time. Furthermore, Fluent models shadows within the simulation with a simple geometric ray tracing program. This creates shadowed areas that see reduced solar flux while still receiving some solar flux from diffuse solar irradiation. For this simulation the date July 1st was chosen and the coordinates for Tempe, Arizona were input. This resulted in a direct solar irradiation of 880 W/m^2 at noon. The so called "Spectral Fraction," the percent that is visible light, was set as 0.5. This means that Fluent will model half of the 880 Watts per square meter as being infrared radiation that causes heating. Finally, the surfaces within the simulation will radiate thermal energy, which is modeled by the simple Rosseland model. This allows for radiative heating and cooling to be modeled [12]

The boundary conditions within this simulation were either walls, an inlet, or an outlet. The inlet was placed at the east end of the bounding box (positive X). The inlet allowed in air moving at a speed given by the function

$$u = -u_0 * \left(\frac{z}{130}\right)^{1/3}. \tag{2}$$

This provided a flow that approached u_0 relatively quickly and had zero velocity at the floor of the simulation. In this equation u_0 is the freestream velocity of the air that is not affected by forces from ground obstacles. A plot of the velocity as a function of the height, z, for $u_0=3.4$ is below. Two simulations were done at varying u_0 values. One simulation was done at 3.4 m/s, and another was done at 1.7 m/s. This allowed for the effects of different Reynolds numbers to be studied.

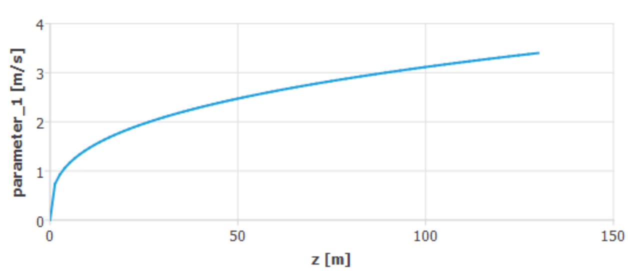

Fig 4: Velocity Distribution as a Function of Height

The west end of the simulation (negative X) is an outlet. It is modeled as a pressure outlet with zero-gauge pressure and allows backflow. When the air moves over the tall building, a low-pressure zone is created on the leeward side of the building. This creates reverse flow moving into the low-pressure zone. Allowing for backflow from the pressure outlet enables air to move backwards into this zone, which better fits the physics of the situation. This is because without reverse flow, the simulation would have to either accept a low-pressure zone without anything to sustain the low pressure, a physical impossibility, or draw in air from above the building. This would create an unphysical flow that has a powerful down draft near the building.

The remaining boundaries are created as wall. The walls of the bounding box, excluding the floor, are created with a specified shear slip condition that allows fluid to move along the wall, as opposed to a no slip condition which would stop all movement at the wall. Although in reality these walls do not exist, they simulate real world interactions better than outlets would. Outlets would enable the flow to be moved out of the bounding box. Since the flow is moving, outlets would mean a large portion of the flow leaves through the sides and top of the bounding box. In reality, flow in the simulated conditions would move largely parallel to itself. Although neither solution is physically accurate, using walls provides the better simulation. In order to reduce the impact that the walls have on the simulation, the specified shear condition was input to allow for flow along the bounding box.

The walls of the building and ground are made as wall boundary conditions with a no slip condition in order to stop all movement of the flow directly touching the wall. This assumption is derived from the continuity equation which requires a mass balance between all incoming and outgoing flows. If the walls were not specified as no slip, then the flow would be allowed to move perpendicular to the wall, creating a vacuum from the outgoing flow, or along the wall, or the flow would be allowed to move along the wall. This can occur in some flows, for example water over hydrophobic walls, but does not occur in most flows. These walls are also modeled based on the materials they are made of, which is important for heat transfer.

The materials used in this simulation are air, aluminum, brick, concrete, a composite material labeled “solar panel,” and urethane foam. The aluminum is used to model the sides and bottom of the solar panels. The brick models the sides of the building. The concrete models the rooftop and the ground of the simulation. The urethane foam is used to model the insulation of the building. Below are relevant properties of each solid material [13] It might be noticed that the absorptivity of the solar panel is low given that its job is to absorb sunlight. This is to model that the solar panel converts some of the solar energy into electricity. The lowered absorptivity models this energy being removed from the system. An important note should be made that the emissivity and absorptivity values are not for the same wavelengths. The absorptivity values are for the wavelengths associated with solar heating while the emissivity values are for those associated with long-wave radiation from warm bodies. Because of Kirchhoff’s Law the emissivity and absorptivity of a material must be the same for the same wavelengths [14]. Because the oncoming and outgoing radiation are at different wavelengths, these values are able to be different.

	Aluminum	Brick	Concrete	Solar Panel	Urethane Foam
C_p (J/kg-K)	896	790	840	835	1045
ρ (kg/m^3)	2739	1920	1920	2800	70
k (W/m-K)	222	0.9	1.1	1.2	0.026
ε	0.14	0.93	0.88	0.72	-

α	0.84	0.63	0.6	0.75	-

Table 2: Solid Material Properties at 300 K [13]

The urethane foam has no radiative properties because it is modeled as being inside the building through what Fluent calls "Shell Conduction." Using this, Fluent is able to model the thickness of the walls as well as layers below the topmost layer. For this application, the top of the building is modeled as two layers, 0.1 meters of concrete and 0.2 meters of urethane foam insulation. This enables the program to model the conduction between the rooftop and the insulation inside the building. One limitation of this approach is that Fluent only models conduction in the direction normal to the boundary. This removes conduction parallel to the surface within the shell layers. This is a small simplification because it is often the case that the temperature distribution behaves smoothly across the boundary. That is to say, the temperature next to a set position is very similar to the temperature at that position [15]. This also does not simplify the final results excessively because the simplified shell conduction does not add or remove heat. Therefore, the total energy transport is the same as with a more complicated model.

The simulation chosen was a transient simulation that spanned 24 hours, starting at 5 AM. The solution was initialized at a starting temperature of 297 K. The inlet's flow and the outlet's backflow had a temperature time distribution modeled by the equation $304 - 7 * \cos(\left(\frac{t}{43200}\right)\pi)$. The resulting graph as a function of time is found below. This provides a minimum temperature of 297 K at approximately 5 AM and a maximum temperature of 311 K at approximately 5 PM. This is consistent with historic weather data [16]. In the below plot 5 AM is at the start of the time.

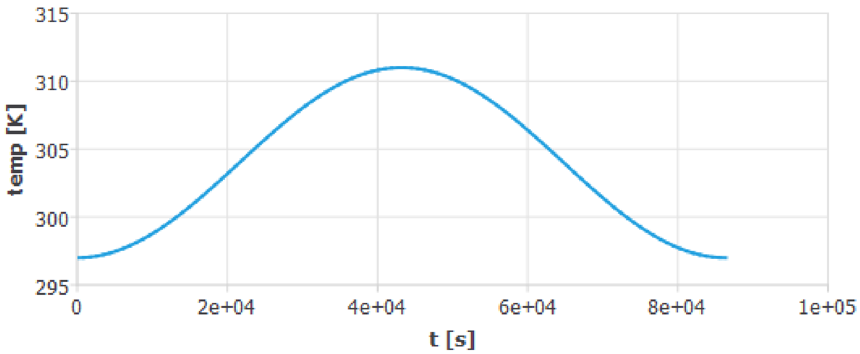

Fig. 5: Time Distribution of Inlet Temperature

A final important parameter is the convective heat transfer coefficient. This constant is found through the following sets of equations.

$$Re = \frac{\rho u L}{\mu} \quad (3)$$

$$Pr = \frac{c_p \mu}{k_a} \quad (4)$$

$$Nu = 0.037 Re^{0.8} Pr^{1/3} \quad Re > 5 * 10^5 \quad (5)$$

$$Nu = 0.664 Re^{0.5} Pr^{1/3} \quad Re < 5 * 10^5 \quad (6)$$

$$h = \frac{Nu * k_a}{L} \quad (7)$$

The convective heat transfer coefficients for the different portions of the geometry, as well as the constants used to determine them, are found below. In this chart the velocity u is taken as a function of z at 10 meters from the building's edge.

	Building Top	Solar Panel Tops	Solar Panel Bottoms	Solar Panel Sides
u (m/s)	3.0	3.4	3.2	3.2
L (m)	30	1	1	0.05

Re	$5.596*10^6$	$2.114*10^5$	$1.990*10^5$	9949
Pr	0.7280	0.7280	0.7280	0.7280
Nu	$8.3283*10^3$	274.6	266.5	59.6
h (W/m^2K)	7.18	7.01	6.80	30.40

Table 3: Thermal Properties of Flow Over Geometry

2.4 Numerical Analysis

A separate study was carried out on a similar rooftop that lacked solar installations. As opposed to running a separate Fluent simulation, this study was carried out using fundamental equations and numerically solved using MATLAB.

For this study, the flat rooftop was investigated independently of other topological disturbances. The material properties and thermal conditions were the same as were used in the Fluent study. The energy inputs were also similarly modeled.

Solar irradiance was modeled somewhat differently than in Fluent. In order to get a mathematical equation for the time distribution of solar flux the simple piecewise equation

$$sun = 700 * \sin\left((t - 18000) * \frac{\pi}{50400}\right)^{1/3} * A \tag{8}$$

between the time values of 0 seconds and 50400 seconds, and zero everywhere else. This creates solar flux between the times 5 AM and 7 PM. This represents an idealization of diurnal variation of solar flux over a day.

Net thermal radiation was modeled by the standard equation

$$-\sigma\epsilon\left({T_{roof}}^4 - {T_0}^4\right) * A. \tag{9}$$

This equation allows heat to be dissipated from the rooftop when the temperature of the roof is higher than the surrounding air and draws heat in when the temperature of the air is higher than the rooftop. Since the temperature terms are raised to the fourth power this value can quickly become significant for thermal analysis despite the very low value of σ. An important note to make is that this equation assumes the emissivity of the rooftop surface and air is the same. This is not strictly true and is a simplification of the model due to difficulties in finding gas emissivity values. Future work will need to be undertaken to better compute the value of emissivity in such a situation.

Thermal convection is modeled by the equation

$$-h(T - T_0) * A. \tag{10}$$

This equation operates very similarly to the radiative equation; however, because it is linear with temperature radiation can become a much larger term at high temperatures. The heat transfer coefficient is derived from the Nusselt number, which is itself derived from the Reynolds and Prandtl numbers. Resulting values are given below.

L_o	30 m
Re	$6.3423*10^6$
Pr	0.7280
Nu	$9.2054*10^3$
h	7.9412

Table 4: Air Flow Properties for the Flat Rooftop Case

This value of the heat transfer coefficient is derived from the average Nusselt number over a plate which is valid for flow over a flat plate for Prandtl numbers between

0.6 and 60 and Reynolds numbers between $5*10^5$ and 10^7. This scenario is not perfectly modeled by flat plate flow, but for first order analysis it will suffice.

As with the Fluent model, thermal conduction is modeled between the rooftop and the underlying insulation. In this model there is a third layer, the air within the building that is maintained at 70°F or 294 K. In order to accomplish this, the conduction from the rooftop to the insulation is modeled by the equation

$$\dot{Q}_{insulation} = k(T - T_i) * \frac{A}{D_i} \tag{11}$$

Since the layers of the conduction are taken to be lumped, the distance between the insulation and the cement are added together and halved. This creates a thermal distance between the two that exists in reality without adding in a large amount of complexity from having to model the heat conduction as it works through the materials. Next the heat flux from the insulation to the buildings air is modeled. Since the air within the building is set at a constant 294 K the equation is

$$\dot{Q}_{air} = k(T_i - 294)\frac{A}{D_i/2} \tag{12}$$

This energy is removed from the system, a modeling of the building's air conditioning system. Because of this setup it is possible to know exactly how much air is predicted to enter through the roof of the building and the resulting load on the building's air conditioning system.

These heat terms are added together in their respective environments and then converted to temperature change with the equation

$$dT = \frac{Q}{c_p \rho A D} \tag{13}$$

where Q is the sum of the heat transfer terms. This is then integrated over the time step size of the code, in this case ten seconds, and then the next time step is carried out.

CHAPTER 3

RESULTS

3.1 Numerical Results

The numerical results are split into three different graphs for the two different wind speed simulations. The first graphs show the individual contributions from each heat transfer term and the net heat transfer to the rooftop over time for both inlet velocity cases. From this information it is possible to tell that the roof begins to lose energy (cool off) at around 4 PM in both cases, although slightly later with the slower wind speed, consistent with lived experience. It is also clear that, compared to the other terms, very little heat is removed from conduction with the insulation. This is a testament to the importance of good rooftop insulation. By integrating Q over the entire time domain, it can be found that the system added a total of 4.71 MJ of heat energy in the faster wind case and 17.8 MJ of heat in the slower wind speed case. This is due to the decreased wind speed removing less energy over time. This allows the maximum temperature and the final temperature to become higher. It can also be seen that the higher temperatures cause the radiative heat transfer term to increase rapidly. In these plots the heat flux is the total heat flux over a building's rooftop that is the same size as in the Fluent model.

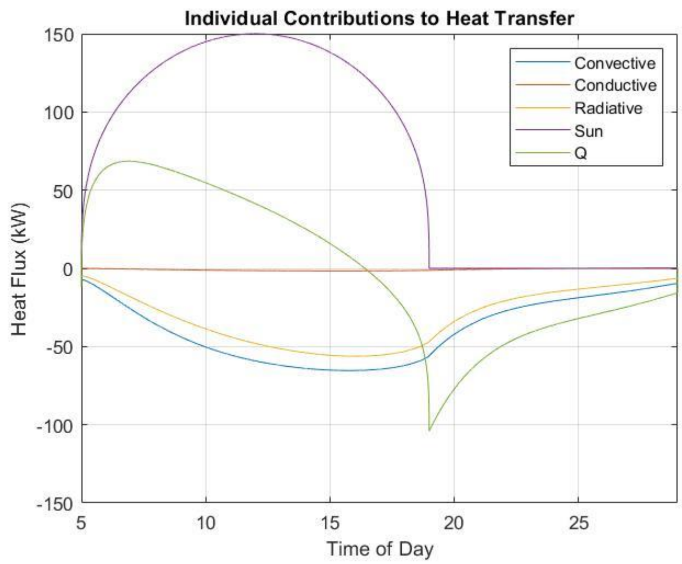

Fig. 6: Heat Transfer Contributions for u_0=3.4 m/s

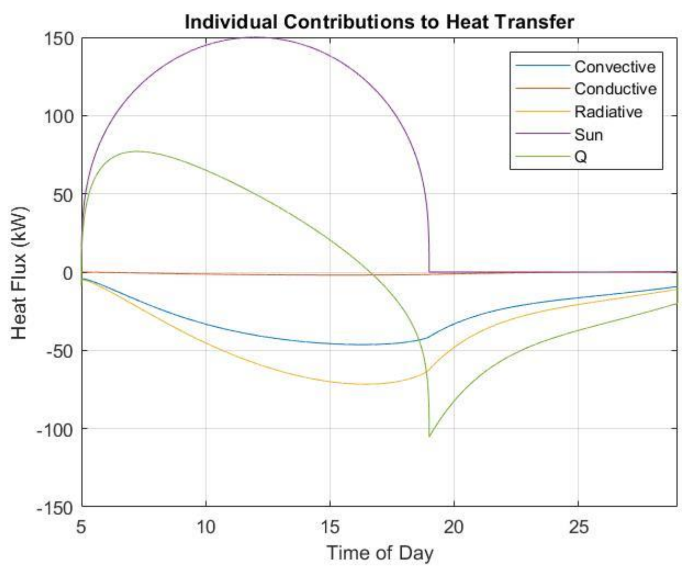

Fig. 7: Heat Transfer Contributions for u_0=1.7 m/s

The next graph shows the temperature of the rooftop, insulation, and the air over time. Using this information, the maximum temperature of the roof is 65.3°C and 71.8°C, with an average temperature over the course of the day of 47.5°C and 51.8°C for the 3.4 m/s and 1.7 m/s cases, respectively. These values are exceptionally high, but also in line with what is expected on a hot day in the sunlight. In comparison the insulation never exceeds 36.7°C and 38.7°C.

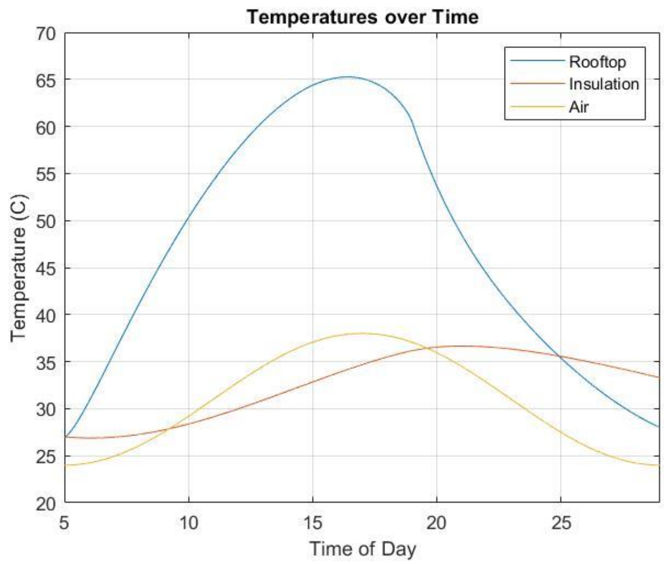

Fig. 8: Temperature Distribution for u_0=3.4 m/s

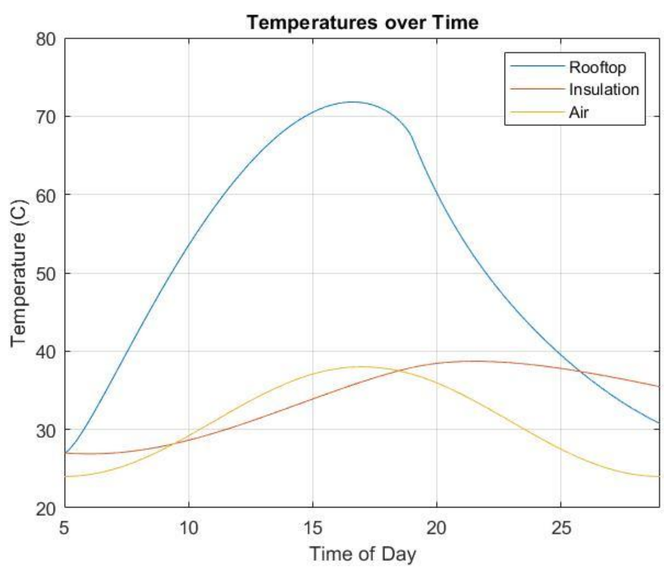

Fig. 9: Temperature Distribution for u_0=1.7 m/s

The final graph shows the heat flux into and out of the insulation. It is clearly visible the scale of the heat flux is drastically different from the scale on the roof. It is also clear that the insulation maintains some additional energy by the end of the day. The value of this stored energy is 2.76 MJ and 3.71MJ for the 3.4 m/s and 1.7 m/s cases, respectively. This value is enough to raise the temperature of the insulation by 6.3°C and 8.4°C. The total heat flux into the building is likewise computed to be 3.92 MJ and 4.35 MJ, equivalent to 1.09 kWh and 1.21 kWh. Because the entire system started at 300 K but ended the day at higher temperatures there is some error associated with beginning the day at a higher temperature. In reality the rooftop and insulation would need to start the day at different temperatures to get an idea of the exact heat inputs and outputs. However, these results are generally accurate and give a good idea of how the system responds to the temperature. It also exactly reflects what was simulated in Fluent.

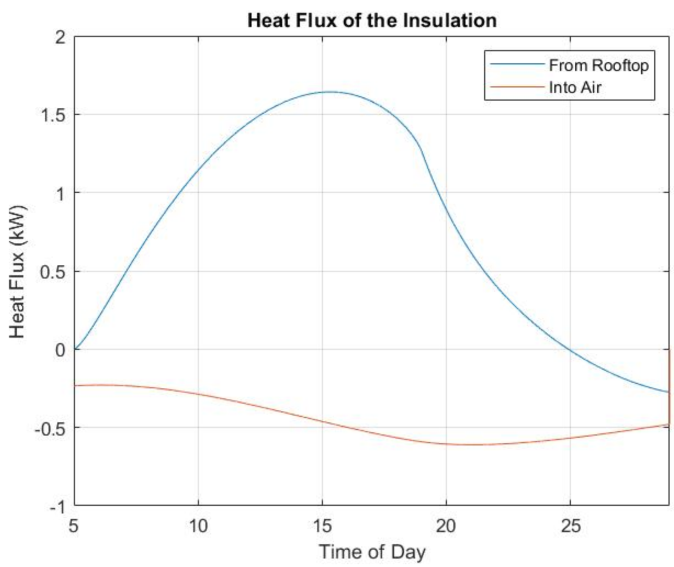

Fig. 10: Insulation Heat Flux for u_0=3.4 m/s

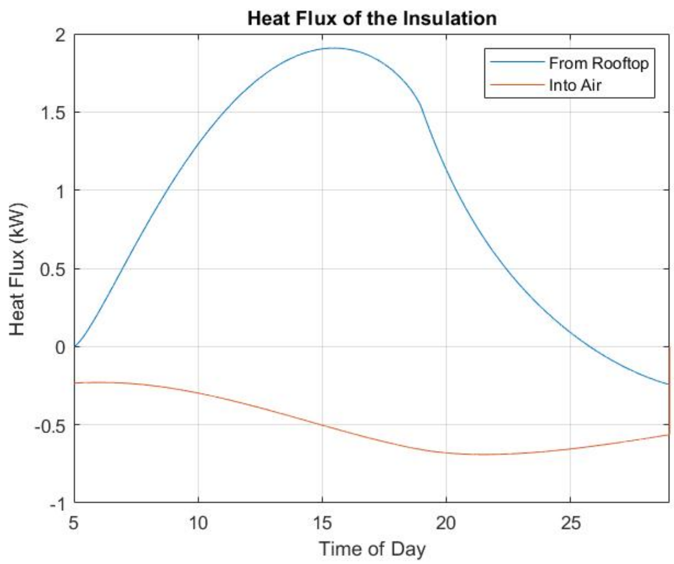

Fig. 11: Insulation Heat Flux for u_0=1.7 m/s

3.2 Fluent Fluid Results

The below plots show the fluid effects that the solar panels and thermal simulation has on the simulation. Figures 12 and 13 show that the solar panels create a larger area past the building of low-speed flow. This may be a result of the solar panels reducing the flow speed over the building, reducing the pressure different below and above the building. This would cause reversed flow to not come into the simulation as quickly. This effect becomes even greater when the solar panels with thermal effects are considered. However, when the solar panels are removed, and the thermal effects remain the flow on the far side of the building becomes much quicker than in other simulations. This is likely because of strong buoyant forces moving upwards to enter a low-pressure zone caused by the high speed both in the Z and X directions. Again, this would cause highly turbulent flow, resulting in high convection heat transfer.

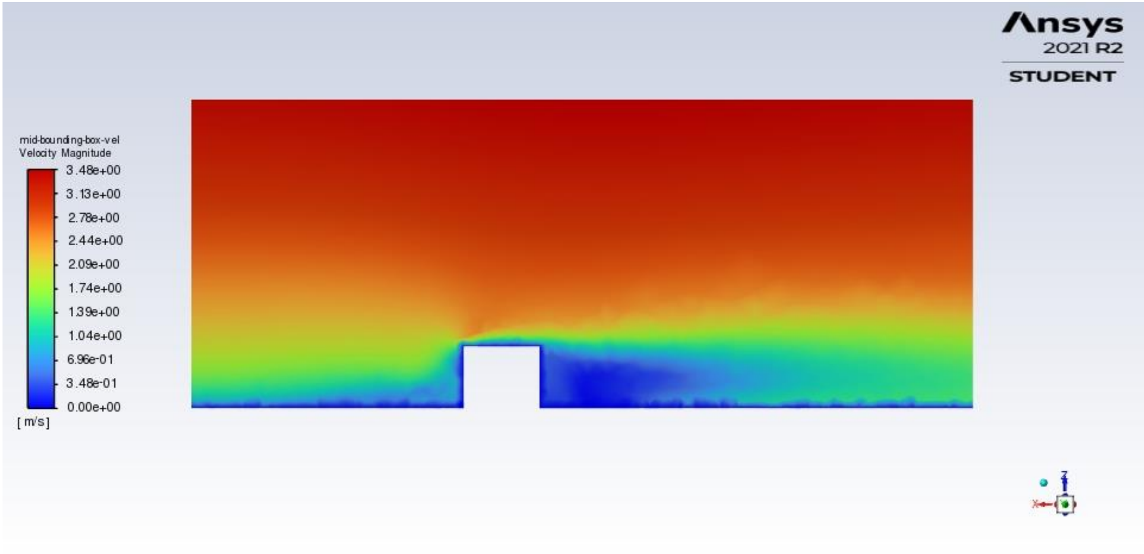

Fig. 12: No Thermal Effects and no Panels Velocity Magnitude

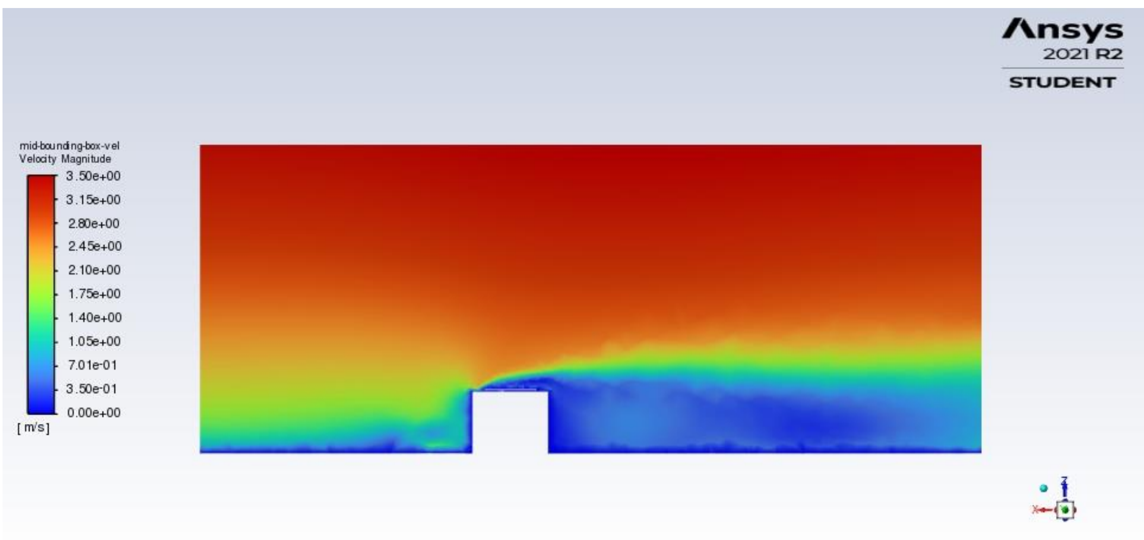

Fig. 13: No Thermal Effects with Panels Velocity Magnitude

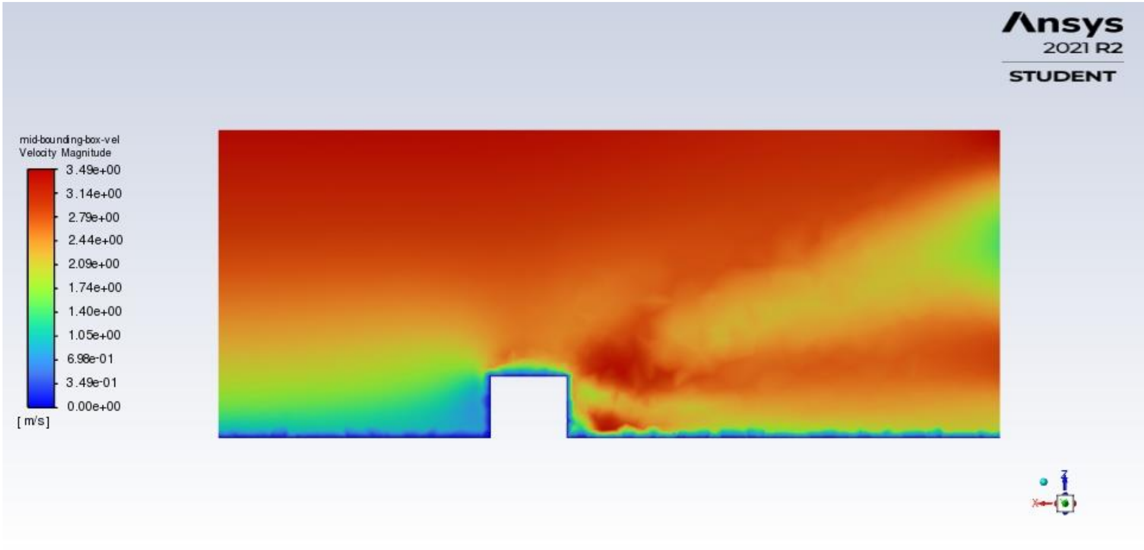

Fig. 14: Thermal Effects and no Panels Velocity Magnitude

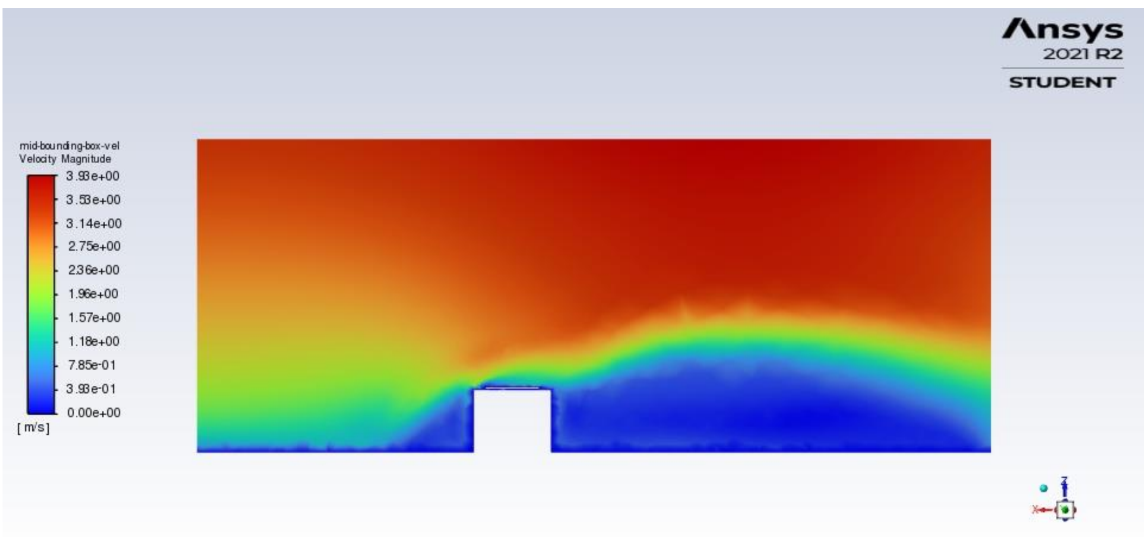

Fig. 15: Thermal Effects with Panels Velocity Magnitude

The below figures show the vertical velocity halfway through the building. As can be seen from figures 16 and 17 the solar panels seem to cause more vertical velocity over the building. This result makes sense given the reduced area for the fluid flow because of the solar panels. A similar effect is seen in the runs that include thermal effects. The combination of the two, solar panels and thermal effects, result in what appears to be mixing flow from the air diverging as it passes over the panels. This would result in more turbulent flow which would cause the solar panels to have increased convective heat transfer.

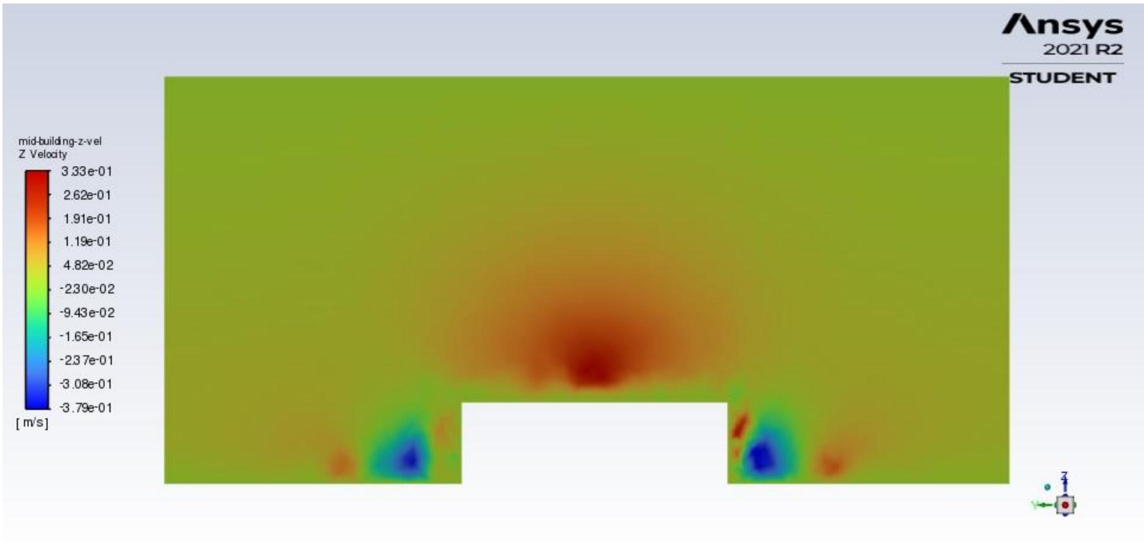

Fig. 16: No Thermal Effects and no Panels Vertical Magnitude

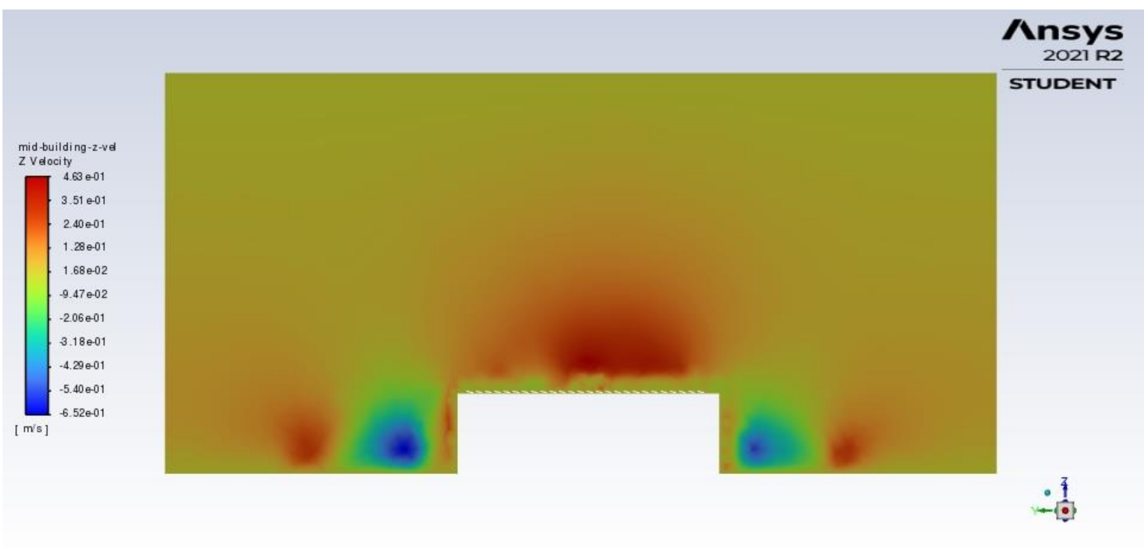

Fig. 17: No Thermal Effects with Panels Vertical Magnitude

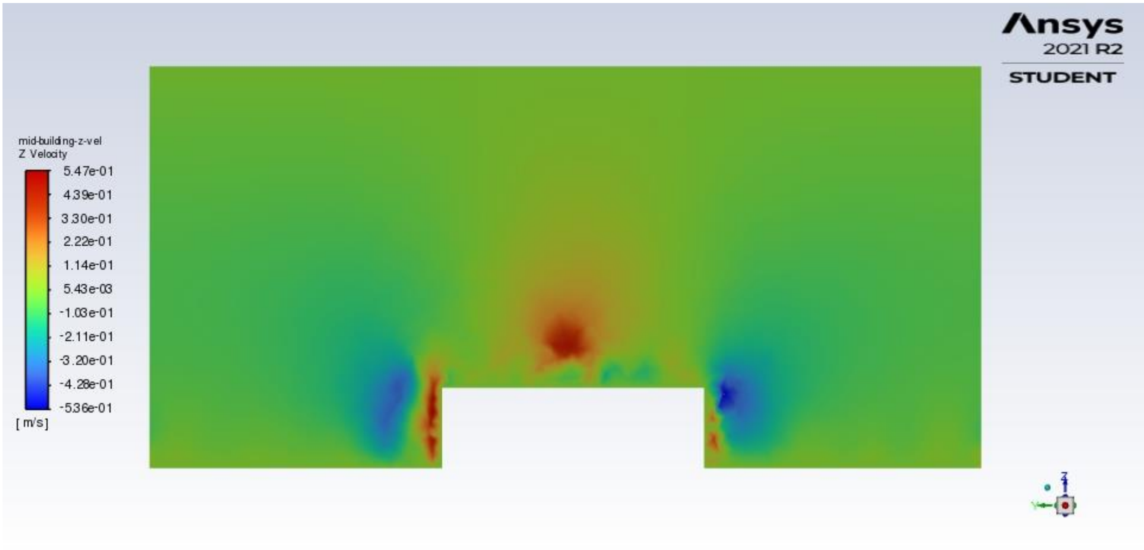

Fig. 18: Thermal Effects and no Panels Vertical Magnitude

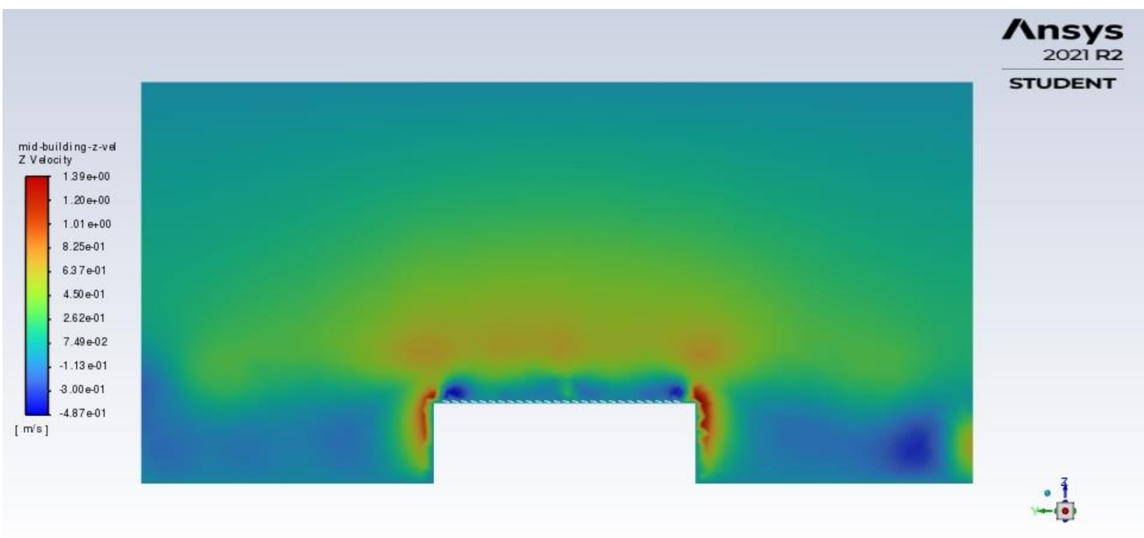

Fig. 19: Thermal Effects with Panels Vertical Magnitude

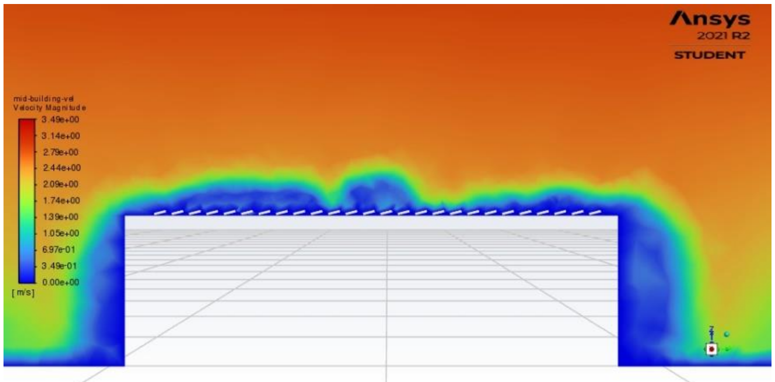

Fig. 20: Zoomed in Section of Velocity Magnitudes with no Thermal Effects

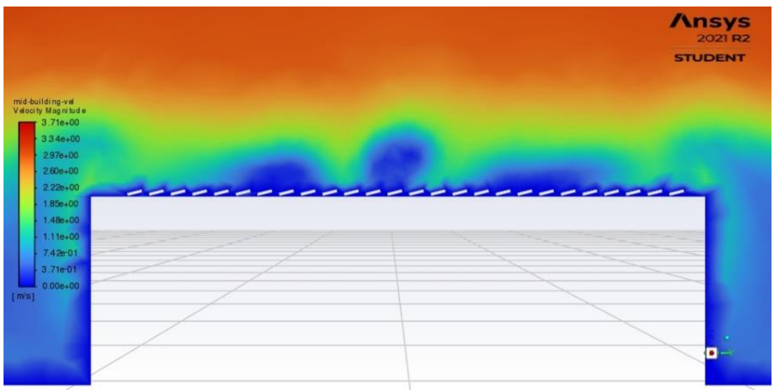

Fig. 21: Zoomed in Section of Velocity Magnitudes with Thermal Effects

3.3 Fluent Thermal Results

As was done previously, two simulations were run in Fluent, one at an oncoming wind speed of 3.4 m/s and another at 1.7 m/s. The maximum average temperatures of the rooftop were 57.9°C and 58.4°C for the 3.4 and 1.7 m/s case, respectively. These high temperatures were both recorded at approximately 4:30 PM. The half a degree rise in temperature is attributable entirely to the decreased wind speed. The average temperature of the rooftop in the two cases were 43.6°C and 43.9°C. Because these two temperatures were very nearly the same it can be imagined that wind velocity has a small impact on the overall heat transfer of the system. The tops of the solar panels reached high temperatures of 62.7°C and 66.3°C for the 3.4 and 1.7 m/s cases, respectively, at around 2:30 PM. The bottoms of the solar panels reached maximum temperatures of 45.1°C and 47.2°C at The maximum temperature of the insulation is slightly higher than expected when it is compared to the results for a flat roof. This is due to the fact that Fluent did not model air beneath the insulation, instead treating it as a vacuum. In order to remedy this post processing of the data will need to be done.

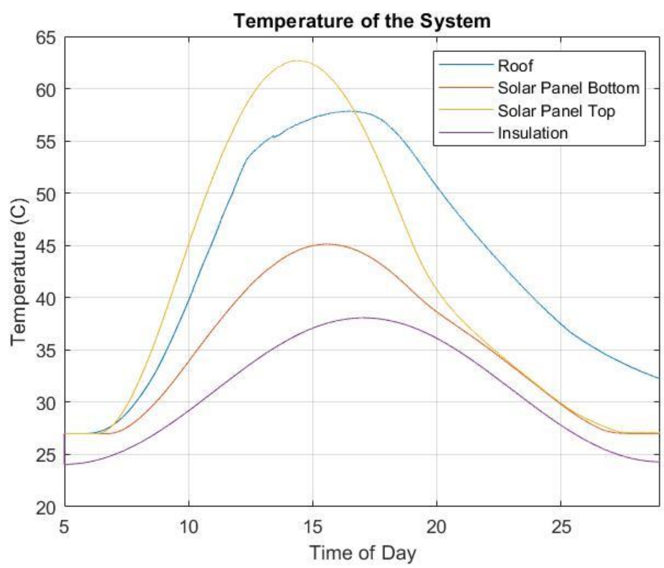

Fig. 22: Temperatures of the System for u=3.4 m/s

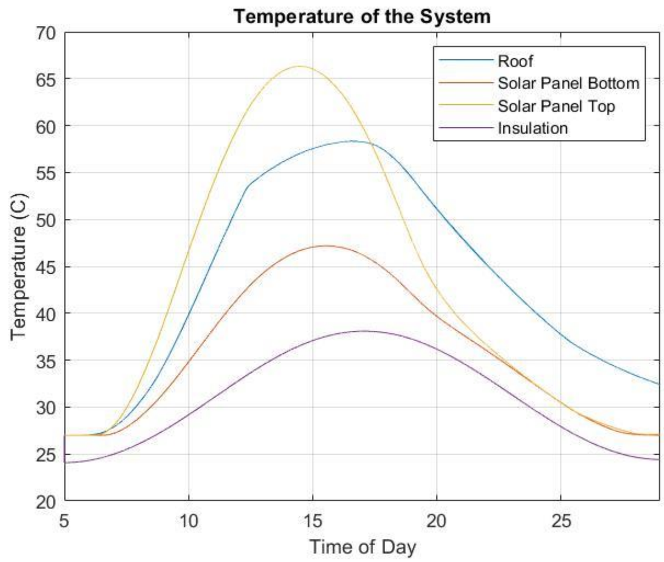

Fig. 23: Temperatures of the System for u=1.7 m/s

This post processing will be accomplished with a different method to find the heat flux out of the insulation than was used for the flat roof case. The first step is calculating the total heat flux out of the insulation given the temperature distribution found. This heat flux is then distributed over the run time based on the difference in the temperature between the insulation and the air divided by the time integral of the insulation's temperature. This heat flux is then removed from the building at that corresponding time step and all future time steps. This method provides a maximum insulation temperature of 36.8°C for both trials and a time averaged temperature of 29.8°C and 29.9°C with a final temperature of 21.7°C and 21.8°C.

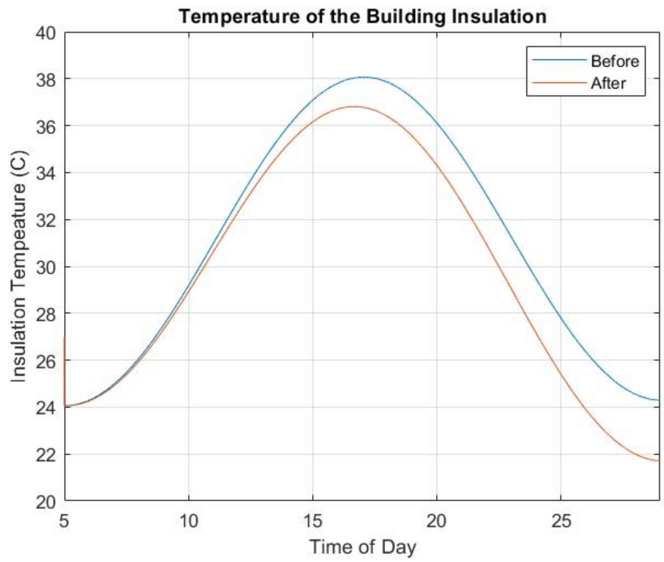

Fig. 24: Building Insulation Temperature for u=3.4 m/s

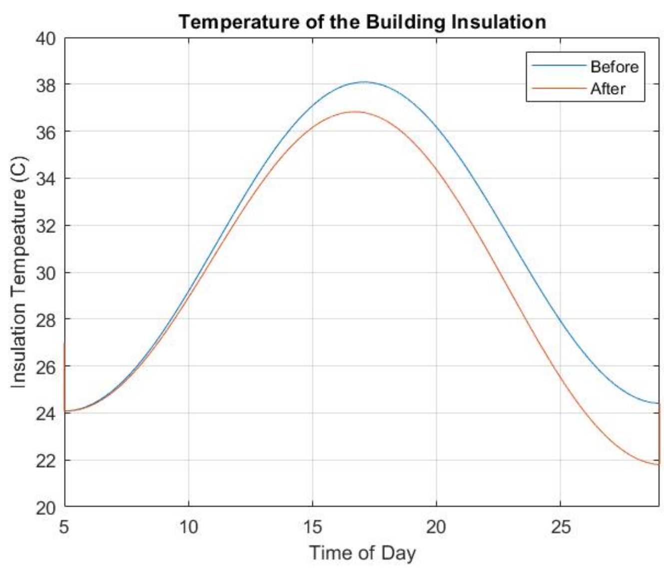

Fig. 25: Building Insulation Temperature for u=1.7 m/s

3.4 Discussion

For the sake of clarity, the results from the four simulations will be copied into a chart for reference.

	Flat Roof at 1.7 m/s	Flat Roof at 3.4 m/s	Paneled Roof at 1.7 m/s	Paneled Roof at 3.4 m/s
h over rooftop (W/m^2K)	4.56	7.94	3.68	7.18
Max Roof Temperature(°C)	71.8	65.3	58.4	57.9
Average Roof Temperature (°C)	51.8	47.5	43.9	43.6
Max Solar Panel Top Temperature (°C)	-	-	66.3	62.7
Max Solar Panel Bottom Temperature (°C)	-	-	47.2	45.1
Max Insulation Temperature (°C)	38.7	36.7	36.8	36.8
Average Insulation Temperature (°C)	33.9	32.6	29.9	29.8

Table 5: Results of all Four Cases

The most visible observation is that the maximum temperature of the rooftop decreased dramatically in the paneled simulations. Because of the reduced convective heat transfer coefficient, it is most likely that this is largely due to the reduced solar flux over the skin of the rooftop. The benefit of this is that the decreased roof temperature led to a decrease heat transfer into the building's insulation. Despite this both rooftops end the simulation at very similar temperatures, in fact a lower temperature was found at the end of the flat rooftop case. This is indicative that the higher heat transfer coefficient as well as a better ability to radiate heat through electromagnetic means allows the flat rooftop to remove more heat over time than the paneled rooftop. Despite this, the average temperature of the paneled rooftop was lower throughout the day. This could be a positive indication that solar panels are able to decrease the heat flux into a building from the rooftop.

This result agrees with Dominguez's work while disagreeing with Brown's work. The reason for the disagreement between the work is not known but could be related to the layout of the solar panels. Having a small solar array of nine 5-Watt panels may not have been enough to noticeably decrease the solar load onto the rooftop, allowing conductive heat to transfer beneath the solar panels, obscuring the results of the trial. Having noted this, it is prudent to mention that the current research may over-emphasize the effects the solar panels may have due to the high number and density of solar panels found on the rooftop. In reality there are fewer solar panels on the rooftop and many areas that are uncovered by panels. If this is the reason for the disagreement it is likely that the solar panels do decrease the average temperature of the rooftop while being highly dependent on the density of solar panels in an area.

One area in which the current research seems to have disagreement with itself is in the marked difference that the convective heat transfer coefficient has between the two different methods. The higher wind speed in the flat rooftop trial done through simple MATLAB code results in much lower temperatures due to the increased ability to carry heat in the wind. This effect is far less exaggerated in the Fluent simulation with solar panels. A simple magnitude analysis shows that radiative heat transfer is about 50 percent more important to removing heat than convective heat transfer from the flat rooftop, and about twice as important in the paneled rooftop. However, this effect alone is not enough to explain the discrepancy in the temperature differences found in the different wind speed trials. One probable guess is that the MATLAB code overemphasizes the ability of air to carry heat, resulting in more importance being given to convective heat transfer. This does not feel completely convincing, and another possibility is that the Fluent simulation did not model the convective heat transfer accurately. Based on the magnitude of the results, and the trends associated with the changing heat transfer coefficient it is believed that all the results may reflect a high-end estimate of the temperature, with natural convection playing more importance in the flat rooftop simulation and forced convection playing more importance in the paneled simulation.

This research largely agreed with the work of Gonzalez which found that the solar panels on a rooftop decreased the heat flux onto the roof while reducing the ability of the roof to cool itself. The major difference in the results is the exact magnitude of the temperature of the rooftop in both cases. Dominguez's work found temperatures that were between 10 and 20 degrees cooler than the current research found. This may be differences in methodology, Gonzalez's work used a functioning building as a test case, or be due to differences in climate and weather, San Diego, California versus Phoenix, Arizona. Again, since the current results are believed to be high end estimates it is likely that with some correction these results would be near exact matches of each other.

One point of validation of this study is the agreement between the computed values of the heat transfer coefficient and the Reynolds number with the values found in the wok by Jubayer. The agreement is quite strong and makes the assumption that the disagreement between the two methods used in this research with respects to wind speed and temperature being entirely the fault of the heat transfer coefficient harder to believe. This lends credence to the belief that the disagreement is due to some effect of Fluent's modeling that was not captured in the simple MATLAB code.

Because the efficiency of solar cells decreases with increasing temperature it is relevant to consider the effect that the thermal load has on the electricity generation of the installation. Using the data obtained from Fluent and the fact that PV cell efficiency decreases at a rate of about 0.4% per degree Celsius increase over 25°C the efficiency over the day can be found. Using the equation

$$K_T = 1 + a(T_s - 298) \quad (14)$$

Where a is the temperature coefficient with a value of -0.004, the corrected efficiency of the solar panel can be found. This analysis assumes that the solar panels operate at about 15% efficiency which is relatively standard on a new system.

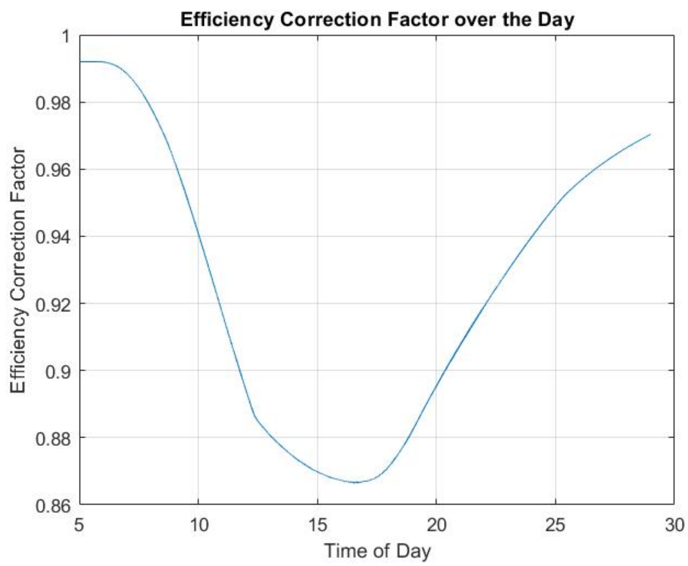

Fig. 26: Efficiency Correction Factor for u=1.7 m/s

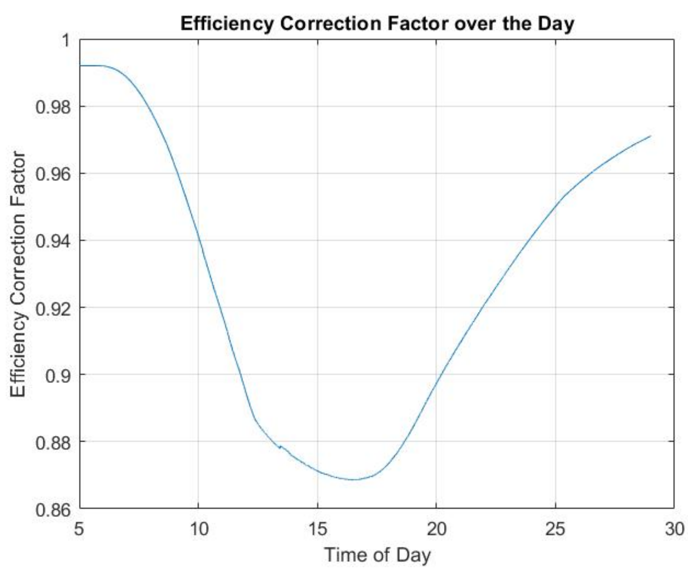

Fig. 27: Efficiency Correction Factor for u=3.4 m/s

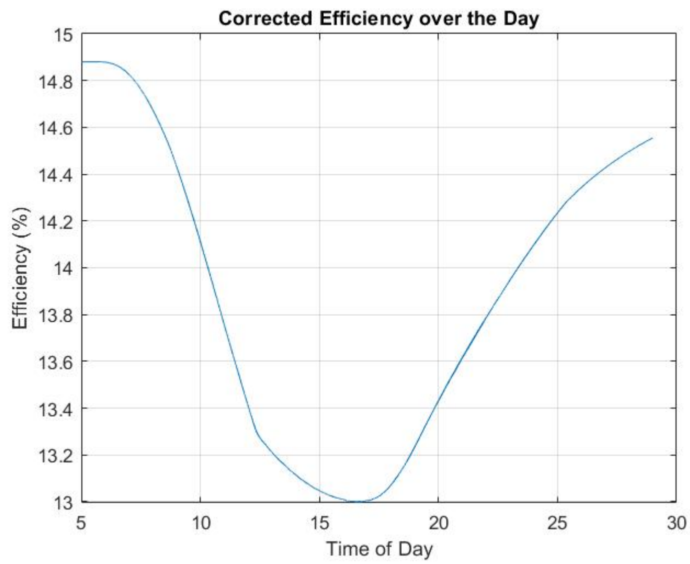

Fig. 28: Corrected Efficiency for u=1.7 m/s

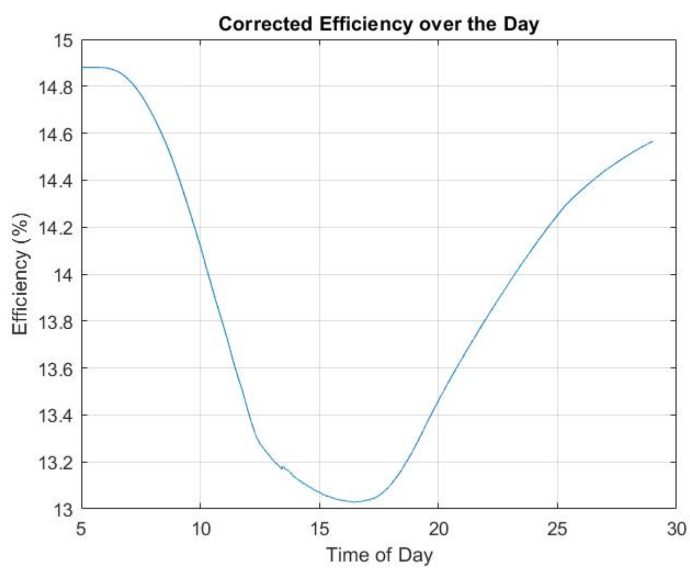

Fig. 29: Corrected Efficiency for u=3.4 m/s

As can be seen the efficiencies of the two different wind speed simulations were very similar. The primary issue with the decreasing efficiency of the solar panels is that they reach the lowest efficiencies as the solar flux becomes the greatest, causing the greatest reduction in energy generation. This is a primary concern with solar photovoltaic systems common on rooftops, as opposed to a system such as concentrating solar. By multiplying the efficiency correction factor by the solar flux throughout the day it is possible to estimate the percent of energy production that is lost due to the increased temperature.

$$\Delta W = \int ((1-k) * Q_{SUN})dt \,/ \int (Q_{SUN})dt \tag{15}$$

From this equation it is found that the lost in energy production is approximately 8.55% for the 1.7 m/s case and 8.47% for the 3.4 m/s case. This decrease in energy production is a difficult problem to solve from a thermodynamic perspective and can either be tackled from an angle of increase efficiency and decreasing the temperature coefficient or by managing the heat transfer over the solar panel, either by inducing a greater heat transfer coefficient, increasing the emissivity and decreasing the absorptivity in the non-productive wavelengths, or through the use of heat sinks that can reduce the temperature during the productive hours while taking longer to decrease the temperature during the nighttime.

CHAPTER 4

CONCLUSION

4.1 Findings

From this research it can be concluded that rooftop solar photovoltaic panels may be beneficial to reducing the cooling demands of buildings during warm months. This finding agrees with prior research done by Dominguez, et al. However, it is also clear that solar panels mounted on rooftops decrease the ability of the rooftop to shed heat during the nighttime which results in higher temperatures for a longer time after the sun has set. This effect does not completely negate the beneficial cooling effect of the solar panels and the end result is a net decrease in the average temperature of the rooftop over a 24-hour period.

An important note to make is that this reduced thermal load was not removed altogether. Instead, it was transferred to the solar panel installation. This caused the solar panels to increase in temperature and therefore lose efficiency. It is common for solar panels to lose about 0.4% efficiency for every degree Celsius increase over 25°C. This sees a decline in the efficiency of the solar panels of about 16% at the hottest point

4.2 Further Study

This paper focused entirely on computational methods for determining the heat flux to and from the building. Future research using experimental data collected from functioning buildings will be beneficial to validating the results of this paper. One method for gathering this data would be partnering with a building owner who plans to implement solar panels. The rooftop temperature could be monitored in the time before the solar panels were installed and then again after the installation. This would give a one-to-one comparison of the exact change due to the solar panels. The one drawback to such a setup is the uncontrollable nature of the weather which may make data gathering difficult to find an exact correlation for each day.

Further research opportunities also exist in gathering data during different seasons. This paper focused on the summer months due to the high electrical generation potential of solar in Arizona during summer as well as the high electrical demand for air conditioning. In the future, research could focus on winter months in Arizona. From this paper and prior research, it may be expected that the solar panels act as a weak insulation between the warm rooftop and the cold outside air. It may also be true that the solar panels block more incoming heat from solar radiation then they are able to keep trapped near the rooftop. However, until further research is done these are both just assumptions. Finally, and perhaps most importantly, a further study could be carried out that proposed and analyzed heat mitigation strategies for solar panels. This would have beneficial effects both for urban solar cells as well as for industrial solar farms. Both implementations can see solar panels reach exceptionally high temperatures that negatively impact the efficiency, and therefore energy output, of the photovoltaic cells. Finding an effective method to dissipate this buildup of heat could see a large increase in power generation for solar farms. It may also be implantable in urban environments to decrease the effect of urban heat islands.

A final important research area lies in active and passive ways to manage PV cell temperatures. As it stands energy production is hampered by the negative effects of the thermal load on the arrays. Finding ways to reduce the temperature of the arrays can serve as a relatively cheap and easy way to boost energy production. Ideally these methods would be passive, requiring no energy inputs and little to no maintenance, however, active cooling methods could still be beneficial provided the energy inputs are low enough to not offset the gains in efficiency. Current practices involve either misting the array or passing water or air through ducts over the back of the panels [5].

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APPENDIX A

MATLAB CODE FOR FLAT ROOF ANALYSIS

```
clear;clc
t=linspace(0,86400,8640);
dt=t(length(t))/length(t);
T0=304-7*cos((t/43200)*pi);
sigma=5.67*10^-8;
epsilon=0.88;
abs=0.6;
A=300;
Dc=0.1;
Di=0.2;

rhoa=1.164;
Cpa=1006.43;
mu=1.872*10^-5;
ka=0.02588;
beta=0.0034;
nu=mu/rhoa;
alphaa=ka/rhoa/Cpa;
g=9.81;
u=1.7;
L=30;

Cpb=820;
rhob=1920;

Cpi=1045;
rhoi=70;

Pr=Cpa*mu/ka;
Ra=rhoa*beta*L^3*g/nu/alphaa;
Re=rhoa*u*L/mu;
Pe=Re*Pr;
Nu=0.037*Re^0.8*Pr^(1/3);
h=Nu/L*ka;
k=0.026;

T=zeros(1,length(t));
T(1)=300;
Ti=zeros(1,length(t));
Ti(1)=300;

Convective=zeros(1,length(t));
Conductive=zeros(1,length(t));
Radiative=zeros(1,length(t));
Q=zeros(1,length(t));
HeatFlux=zeros(1,length(t));
Sun=zeros(1,length(t));
Sun(1:5040)=500*sin((t(1:5040))*pi/50400).^(1/3)*A; %W
for i=1:length(t)-1
    Convective(i)=-h*(T(i)-T0(i))*A; %W
    Conductive(i)=-k*(T(i)-Ti(i))/(Di/2+Dc/2)*A; %W
    Radiative(i)=-sigma*epsilon*(T(i)^4-T0(i)^4)*A; %W

    Q(i)=Convective(i)+Conductive(i)+Radiative(i)+Sun(i);
```

```
    dT=Q(i)/(Cpb*rhob*A*Dc);
    T(i+1)=T(i)+dT*dt;

    HeatFlux(i)=k*(Ti(i)-294)/(Di)*A;
    dTi=-Conductive(i)/(rhoi*Cpi*A*Di)-HeatFlux(i)/(rhoi*Cpi*Di*A);
    Ti(i+1)=Ti(i)+dTi*dt;
end
figure (1)
plot(t/60/60+5,T-273)
hold on
grid on
plot(t/60/60+5,Ti-273)
plot(t/60/60+5,T0-273)
legend("Rooftop","Insulation","Air")
title("Temperatures over Time")
xlabel("Time of Day")
ylabel("Temperature (C)")
xlim([5,29])

figure (2)
hold off
plot(t/60/60+5,Convective/1000)
hold on
plot(t/60/60+5,Conductive/1000)
plot(t/60/60+5,Radiative/1000)
plot(t/60/60+5,Sun/1000)
plot(t/60/60+5,Q/1000)
legend("Convective","Conductive","Radiative","Sun","Q")
grid on
title("Individual Contributions to Heat Transfer")
ylabel("Heat Flux (kW)")
xlabel("Time of Day")
xlim([5,29])

figure(3)
hold off
plot(t/60/60+5,-Conductive/1000)
hold on
plot(t/60/60+5,-HeatFlux/1000)
legend("From Rooftop","Into Air")
grid on
title("Heat Flux of the Insulation")
ylabel("Heat Flux (kW)")
xlabel("Time of Day")
xlim([5,29])

sum(HeatFlux)
```

APPENDIX B
MATLAB CODE TO ANALYZE FLUENT DATA

```
clear;clc

RoofData=importdata("average-temp-building-top-rfile.out").data;
TopData=importdata("average-temp-solar-panel-tops-rfile.out").data;
BottomData=importdata("average-temp-solar-panel-rfile.out").data;
InsulationData=importdata("urethane-foam-temperature-rfile.out").data;

RoofTime=RoofData(:,2);
RoofTemp=RoofData(:,3);
TopTime=TopData(:,2);
TopTemp=TopData(:,3);
BottomTime=BottomData(:,2);
BottomTemp=BottomData(:,3);
InsulationTime=InsulationData(:,3);
InsulationTemp=InsulationData(:,2);

figure (1)
plot(RoofTime/60/60+5,RoofTemp-273)
grid on
hold on
plot(BottomTime/60/60+5,BottomTemp-273)
plot(TopTime/60/60+5,TopTemp-273)
plot(InsulationTime/60/60+5,InsulationTemp-273)
legend("Roof","Solar Panel Bottom","Solar Panel Top","Insulation")
title("Temperature of the System")
xlabel("Time of Day")
ylabel("Temperature (C)")
xlim([5,29])

figure (2)
plot(InsulationTime/60/60+5,InsulationTemp-273)
hold on

k=0.026;
Di=0.2;
rhoi=70;
Cpi=1045;
dt=10;

HeatFlux=k*(InsulationTemp-294)/(Di);
HeatLoss=sum(HeatFlux)*dt;
TempIntegral=sum(InsulationTemp);
Q=(InsulationTemp-294)/TempIntegral*HeatLoss;
for i=2:8640
    InsulationTemp(i)=InsulationTemp(i)-sum(Q(1:i))/(Cpi*rhoi*Di)*dt;
end

plot(InsulationTime/60/60+5,InsulationTemp-273)
title("Temperature of the Building Insulation")
xlabel("Time of Day")
ylabel("Insulation Tempeature (C)")
legend("Before","After")
xlim([5,29])
grid on
```

```
k=1+-0.004*(RoofTemp-298);
epsilon=k*15;

figure (3)
plot(TopTime/60/60+5,k*15)
grid on
xlabel('Time of Day')
ylabel('Efficiency (%)')
title('Corrected Efficiency over the Day')

figure (4)
plot(TopTime/60/60+5,k)
grid on
xlabel('Time of Day')
ylabel('Efficiency Correction Factor')
title('Efficiency Correction Factor over the Day')

Sun=zeros(1,length(RoofTemp));
Sun(1:5040)=700*sin((RoofTime(1:5040))*pi/50400).^(1/3);
ProductionLost=sum((1-k').*Sun)/sum(Sun);
```