Demand for Variety Under Costly Consumer Search:

A Multi-Discrete/Continuous Approach

by

William Jacob Allender

A Dissertation Presented in Partial Fulfillment
of the Requirements for the Degree
Doctor of Philosophy

Approved May 2013 by the
Graduate Supervisory Committee:

Timothy Richards, Chair
Sungho Park
Stephen Hamilton

ARIZONA STATE UNIVERSITY

August 2013

ABSTRACT

Consumers search before making virtually any purchase. The notion that consumers engage in costly search is well-understood to have deep implications for market performance. However to date, no theoretical model allows for the observation that consumers often purchase more than a single product in an individual shopping occasion. Clothing, food, books, and music are but four important examples of goods that are purchased many items at a time. I develop a modeling approach that accounts for multi-purchase occasions in a structural way. My model shows that as preference for variety increases, so does the size of the consideration set. Search models that ignore preference for variety are, therefore, likely to under-predict the number of products searched. It is generally thought that lower search costs increase retail competition which pushes prices and assortments down. However, I show that there is an optimal number of products to offer depending on the intensity of consumer search costs. Consumers with high search costs prefer to shop at a store with a large assortment of goods and purchase multiple products, even if the prices that firm charges is higher than competing firms' prices. On the other hand, consumers with low search costs tend to purchase fewer goods and shop at the stores that have lower prices, as long as the store has a reasonable assortment offering. The implications for market performance are dramatic and pervasive. In particular, the misspecification of demand model in which search is important and/or multiple discreteness is observed will produce biased parameter estimates leading to erroneous managerial conclusions.

To my Wife and Kids, Mom and Dad.

ACKNOWLEDGMENTS

Like many graduate students I began the writings of my dissertation with very little knowledge about the process of creating scholarly work. It was only with the patience, guidance, and aid of others that I was able to complete this project. As I have drawn immense personal satisfaction from this research a debt of gratitude is owed to many people.

First and foremost I want to thank my family for the moral, financial, emotional and intellectual support. My wife is a continuous beacon of inspiration and intelligence I am honored to spend my life with. I could not have asked for more inspiration along this journey. Her continued love, care, support and patience throughout my education has helped make this project complete and allowed me to realize my potential as a researcher and a person. Spending many summers working on the family farm has sparked a life long interest in agriculture and business. My father and brother have time and again proven to be fountainheads of knowledge and a tie from the ivory tower to the farm. I have been blessed with a mother and sister that foster a commitment to education that have touched more lives than I can count, and who continue to promote an inquisitive attitude in the people they surround.

I owe a significant debt of gratitude to Timothy Richards for his selfless giving of time and knowledge throughout my entire graduate career. I have benefitted immeasurably from his intelligence, dedication, patience, and above all, willingness to go the extra mile. His clear and concise explanations have helped sharpen my understanding of business and economics, and his guidance has made me into the researcher I am today. I would like to express a deep appreciation to my committee members: Sungho Park, and Stephen Hamilton for their thoughtful comments and suggestions, and many stimulating conversations. This research has also benefited from a number of faculty in the Agribusiness, Marketing, and Economics departments including, but not limited to: Seung Ahn, Ruth Bolton, Jeff Englin, Carola Grebitus, Renee Hughner, Mark Manfredo, Andrea Morales, Yueming Qiu, Adriana Samper, Troy Schmitz, George Seperich, Rajiv Sinha, and Michael Wiles. I would also like to thank my fellow graduate students for all they have done for me over the years from providing a place to live, to helping with my experiments. While I sincerely appreciate all the time and effort each one of them provided me there are a few names in particular that made this work possible: Jessica Robinson, Sophie Theron, Koichi Yonezawa, and Jared Carlson.

Finally, I would like to gratefully acknowledge the generous financial support I received from the Marvin and June Morrison Chair of Agribusiness and Resource Management, the Arizona Board of Regents and ASU's Office of Knowledge Enterprise Development, and the AFRI-NIFA Markets and Trade award (# 2011-02763) that made this work possible. I would also like to acknowledge z-Tree code generously provided by Hong Yuan in the University of Illinois at Urbana–Champaign, and data provided by the Kilts-Nielsen Data Center at The University of Chicago Booth School of Business. Information on availability and access to the Data is available at http://research.chicagobooth.edu/nielsen/.

TABLE OF CONTENTS

LIST OF TABLES

LIST OF FIGURES

PREFACE

This dissertation is a three-paper dissertation. However, each paper deals with the subject of better understanding and modeling consumer demand in a multi product purchase environment. Chapters 1 and 5 provide a general introduction and conclusion applicable to all three papers. Chapters 2, 3, and 4 are largely self contained and independent from one another in terms of symbols and equations.

Enjoy!

CHAPTER 1.

INTRODUCTION

"In confusion there is profit" - Tony Curtis

Consumers regularly purchase multiple products on the same shopping trip, and incur some kind of cost when searching for their ideal product mix. For example, consumers regularly buy many pair of jeans, brands of cereal or bottles of wine on each trip to the store, and typically consider each purchase carefully. Despite the importance of consumer search in such multi-purchase situations, there is very little research that examines the theoretical and empirical implications. From a theoretical perspective, search is both more difficult if many products are considered on each trip, but less costly if some of the cost of search is fixed. How multi-product purchases affect search intensity, therefore, is an open question – a question that is at once interesting and important. As a subject of inquiry, search spans the fields of macroeconomics, labor, marketing, public finance, and industrial organization, among others, and touches the most important issues of the day – unemployment, inflation, the reach of technology, and market competitiveness. Empirically, multi-product purchase complicates analysis because commonly-used models are no longer valid. For example, it is well understood that applying discrete choice models such as the logit or probit to product categories in which multiple purchases are common produces incorrect estimates of the marginal effect of marketing mix elements (Dubé 2004). By extension, new models of consumer demand are necessary to study the econometrics of search. In this dissertation, I provide a better understanding of consumer demand when multiple products are regularly considered for purchase and consumers actively search to resolve uncertainty over product attributes.

Recognizing that consumers actively search for products within limited consideration sets changes the way I model demand in a fundamental way.[1] Existing models of demand assume consumers are aware of all alternatives, and search among them costlessly. Logically, this cannot be true, and the implications of costly search and limited consideration sets are dramatic. Koulayev (2010), for example, shows that a failure to account for endogenous and limited consideration sets leads to significant over-estimation of price elasticities. Similarly, Seiler (2011) and Pires (2012) use

[1] For a more comprehensive review of the effect consumer search has on demand see Ratchford (2009).

dynamic structural models of consumer demand that account for the cost of search to explain the important role played by putting items on feature or display. If the cost of search is greater than the benefit of finding a lower price, then rational retailers will put items on feature or display to lower the cost of search, and make price promotions more effective. Insights like this are only revealed by explicitly considering consumer search behavior.

Search has also gained prominence with the growth of online shopping. Indeed, the ubiquity of online search, and questions regarding marketing performance in a world of "costless" search has raised the profile of consumer search as an economic activity. While it was once thought that the ability to search for goods online would increase market competitiveness and do away with price dispersion, Brynjolffson and Smith (2000), Clay, Krishnan and Wolf (2001), and Chevalier and Goolsbee (2003) report significant price dispersion between online vendors, even in homogeneous product categories such as textbooks and music CDs. More recently, Hong and Shum (2006), Kim, Albuquerque, and Bronnenberg (2010), and de los Santos, Hortacsu, and Wildenbeest (2012) develop more formal models of search behavior and confirm that search costs are still significant, even in the most efficient of channels. What this research reveals is remarkable – the ability to shop more efficiently allows consumers to focus on finding products that meet their exact specifications in differentiated product categories (Anderson and Renault 1999; Chen and Hitt 2003; Kuksov 2004; and Cachon, Terwiesch, and Xu 2008), making demand for online goods less elastic as consumers demand goods with exact specifications. Retail price markups can rise as a result. Yet, despite the logic of this argument, there is little empirical evidence to support this outcome, and less that considers the realities of what is still the dominant form of retail purchases: Multiple goods purchased during an individual shopping trip in traditional, bricks-and-mortar retailers.

In my dissertation, I examine search behavior in brick-and-mortar retail markets to investigate whether search costs remain significant in a more general class of purchases, namely goods consumers shop for, and purchase, many items at a time. Purchases in multi-product environments rarely adhere to the unit-purchase assumption of standard discrete choice models. Rather, consumers tend to make multiple-discrete choices, and purchase continuous quantities, and to assume otherwise invites significant bias in models designed for purely single item discrete-choice environments (Dubé 2004; and Richards, Gómez, and Pofahl 2012). For example, in the household-panel data used in this study, only 41% of the trips involving ice cream purchases in the U.S. throughout 2007 and 2008 were for a single unit of ice cream.[2] In other words, existing consumer demand-search models are not applicable to 59% of ice cream purchases. Additionally, numerous categories exist in which purchases are made by weight or volume such as produce or meat, as opposed to a pre-packaged unit. Standard discrete choice demand, or demand-search models, are simply not applicable to these situations. While there are several explanations for why consumers make multiple discrete-continuous purchases, one of the prevailing arguments is that they are driven by a preference for variety.

There are a number of reasons why multi-unit purchases may reflect a demand for variety. First, consumers prefer variety as they shop in anticipation of several consumption occasions following each purchase decision (Dubé 2004). A person that has a particular taste for variety may purchase several different flavors of yogurt to avoid consuming the same one day after day. When shopping for clothes, consumers rarely leave the store with only one bag unless they anticipate wearing the same thing every day. Second, a consumer may purchase for a household with several members, each of whom prefers a different flavor of ice cream. Because demand theory describes consumption, but empirical analysis relies on purchase data, both purely discrete or continuous demand models misrepresent consumer search and purchase behavior (Hendel 1999; and Dubé 2004). Fundamental to the notion of a preference for variety is that consumers face a diminishing marginal utility across products. With uncertainty over product attributes, no single choice stands out as a clear favorite *a priori*, so utility rises the more choices are available, but at a decreasing rate. Anderson and Renault (1999) describe this as a "preference for diversity," as consumers want a diverse set of offerings in the hope that the most desirable product can be found (Moorthy, Ratchford, and Talukdar 1997). Here, I consider the preference for diversity, or variety, as the consumer's desire to consider multiple competing products due to a diminishing marginal utility across the set of products on offer.

Recognizing the role diminishing marginal utility plays in shaping a preference for variety, Bhat (2005, 2008) extends the multiple discrete-continuous demand model of Kim, Allenby, and Rossi (2002) to derive a generalized logit model that relaxes the unit purchase assumption. In Chapter 2 I develop a theoretical model that builds on this framework by recognizing consumers

[2] Calculated based on data from The Nielsen Company (US), LLC.

incur a search cost when evaluating competing alternatives. This model provides insight into the relationship between a consumer's optimal consideration set and the cost of search in the presence of a preference for variety. If a consumer's preference for variety leads to multiple purchases on the same trip (Dubé 2004; Bhat 2005, 2008), then the marginal benefit of additional search increases, directly affecting the search strategy. I show that the size of a consumer's optimal consideration set increases with their preference for variety. Single-purchase models that ignore consumers' preference for variety and the inherent endogeneity of a their consideration sets thus underpredict the size of the optimal consideration set and the elasticity of demand (Roberts and Lattin, 1991; Mehta, Rajiv, and Srinivasan 2003; Honka 2010; Koulayev 2010; and Sieler 2011; Honka 2013). My theoretical model, on the other hand, provides a framework for understanding the effect an individual's preference for variety has on his optimal consideration set, and subsequent purchase decisions.

One of the drawbacks of other theoretical consumer search models is that they are not amenable to empirical testing, or managerial application (Ratchford 2009). My theoretical model, on the other hand, can be extended in a straight forward way to be estimated, and tested, with the appropriate dataset. In Chapter 3, I develop an empirical model that is grounded in the theory of multi-product purchase developed in Chapter 2. The structural empirical model is derived from a single utility-maximizing consumer demand problem in which consumers decide the optimal number of products to search through based on their expected maximum utility. Consumers then choose optimal purchase quantities subject to their budget constraint. This model provides estimates of not only the usual set of demand parameters – brand-specific preferences, price-responses, and the importance of other element of the marketing mix – but estimates of the optimal consideration set size, and its composition based on the magnitude of product specific search costs.

Estimating consideration set size and composition is both novel, and important on a practical level. Estimates of consideration set sizes, and the extent of each consumers' preference for variety is managerially useful as retailers must make assortment planning decisions on a weekly basis. For example, if a particular category has a large number of products on display and yet consumers only consider a small proportion, managers can reduce costs by offering fewer products. Knowing the satisfaction consumers obtain from individual products also allows category managers to evaluate which products consumer prefer, and which can be eliminated from the shelf. I find that consumers search an increasing number of products as their preference for variety increases, without necessarily increasing the number of products purchased. Understanding the composition of the consideration set is critically important because the substitution between products in the set differs significantly from the substitution between products that are not considered.

Search costs are not observable, so are often ignored. In fact, one of the biggest empirical challenges to studying consumer search is that search behavior and search costs are unobservable in both retail and household purchase data (Zwick, Rapoport, Lo, and Muthukrishnan 2003). With household level purchase data, the researcher observes the products that were purchased and detailed information about those particular products, but is often unaware of the products the consumer considered, and which were ignored completely. Even within datasets that do contain explicit records of consumers' search behavior the cost of search is still unobserved (de los Santos, Hortacsu, and Wildenbeest 2012; Honka 2013; and Honka and Chintagunta 2013). In Chapter 4, I use an experimental approach to study the relationship between the number of products presented to a consumer and their subsequent search and purchase behavior. By imposing a search cost, and a preference for variety, I induce search behavior in the presence of a preference for variety that leads the participants to make both single, and multiple-purchase decisions.

My experiment reveals many important insights into consumers' search behavior as it relates to both single, and multi-product purchases. In particular, I show that as the size of the consideration set grows, consumers search less, meaning that they are more apt to purchase, but this effect is highly non-linear. Beyond a certain point, search expands as consumers become unwilling to choose. Retailers can increase the assortment available to persuade a consumer to patronize their store and avoid searching another. However, this does not increase without bound. Eventually, a consumer will be overwhelmed by the number of products offered and search the other store as well, or possibly instead. The degree to which consumers are overburdened by the variety offered exhibits significant heterogeneity among subjects, perhaps explaining why this "choice overload hypothesis" has had mixed support. The results of Chapter 4 also illustrate the importance of including variety in future consumer search studies.

Search is an equilibrium outcome. That is, if consumers search rationally, then retailers should expect search, and behave accordingly. Stigler (1961) relaxed the notion that consumers know the price each retailer is charging and showed how price dispersion can exist. Varian (1980) builds on this notion further by recognizing that stores rarely keep their prices consistently high or low all the time (Butters 1977; and Salop and Stiglitz 1977) and showed that an uninformed market segment will lead to retailers offering temporary price reduction, or sales, in an equilibrium setting. In addition to price dispersion, search has been used to explain a number of other market phenomenon. Cachon Terwiesch and Xu (2008) develop an equilibrium model that investigates the relationship between consumer search and the equilibrium number of products brought to market by suppliers. They showed that the equilibrium price can increase as consumers search across a wider range of retailers because increasingly differentiated products will be introduced. Kuksov and Villas-Boas (2010) extend this idea further by endogenizing not only the number of products offered in the market, but also the attributes of the products offered. They show that too many products increased the total cost of search to the point that consumers avoid making a purchase all together. On the other hand, if too few products are offered then consumers avoid making a purchase because they fear there is not an acceptable product (Kamenica 2008; and Norwood 2006). Recognizing that consumers consider a limited number of products has important implications for the number of products brought to market, and their attributes. Throughout the dissertation, I draw implications for optimal retailer behavior from my model of multi-product search, and purchase, behavior.

Chapter 6 offers a summary of my theoretical, empirical, and experimental findings and a comment on how my findings generalize beyond the product categories considered here. Because costly information acquisition is a fundamental to many aspects of an individual's life my results generalize beyond just multi-product consumer goods. For example, most individuals are unfamiliar with the specific coverage different insurance plans offer, and young healthy individuals have little perceived benefit to carefully considering a large number of different plans. Presenting these individuals with a large number of (complex) insurance plans may deter them from making a selection at all which could be detrimental to the entire industry as insurance plans rely on young, healthy members to maintain financial stability. In this chapter, I also provide suggestions for future work, and describe in some detail the limitations of adopting my approach to demand analysis in a multi-product, search environment.

CHAPTER 2.

CONSUMER SEARCH AND PRODUCT VARIETY: A THEORETICAL APPROACH

2.1 Introduction

It is well understood that knowledge is valuable and comes at a cost. Consumers, in particular, typically incur some kind of cost to obtain information in order to resolve *a priori* uncertainty regarding prices. Visiting websites, reading ads online or in physical media, or even travelling to different stores are all examples. Even for completely undifferentiated products, prices will vary among sellers unless the market is centralized (Stigler 1961). If search for product information were costless, consumers would consider the price of every identical product and choose the cheapest one. In which case, all firms would charge the same price, or face no demand. But, uniform prices are rarely observed, even for homogenous goods (Marvel 1976; Lach, 2002; Baye, Morgan, and Scholten, 2004; and Caglayan, Filiztekin, and Rauh 2008). Because search costs are not zero, prices will vary across firms if consumers follow a rational search procedure, comparing prices until the marginal cost of doing so exceeds the marginal benefit. Prices will also vary between firms if: (1) products are differentiated (Hortacsu and Syverson 2004; Wildenbeest 2011), (2) consumers search in non-sequential fashion (de los Santos, Hortacsu, and Wildenbeest 2012), or (3) assortments are endogenous (Anderson and Renault 1999, 2000; Cachon, Terwiesch, and Xu 2008). Search, therefore, is an inherent feature of nearly every market for products or services. Yet, there are no theoretical treatments of search relevant to where it is most common: In multi-product environments in which consumers purchase many products together, and exhibit a preference for variety. This chapter provides a theoretical examination of search under a preference for variety that is relevant to such multi-product retail environments.

Search that results in the purchase of a single product implicitly assumes the total cost of search is absorbed by that one product. Rather, the size of a consumer's optimal consideration set increases with the preference for variety and, in many cases, a consumer will spread search costs over several purchases, even within the same category. For example, consumers often purchase several pairs of jeans, or breakfast cereals at the same time, but rarely wear all of them, or eat them at the same time. If a consumer's preference for variety leads to multiple purchases on the same trip, then the marginal benefit of additional search increases, leading to more search (Anderson and Renault 1999).

Multiple purchases are more common than I expect and, as a result, more important. Purchases in multi-product environments often deviate from the standard logit or probit discrete-choice assumption (one product is purchased) because consumers tend to make multiple-discrete choices, and purchase continuous quantities (Dubé 2004; and Richards, Gómez, and Pofahl 2012). For example, in 2007 and 2008 consumers purchased single units on only 41% of the trips involving ice cream purchases.[1] Further, in many categories consumers purchase by weight or volume (e.g., produce or meat), as opposed to a pre-packaged unit so single choice discrete-choice search models are not appropriate. Such multiple discreteness is thought to be a manifestation of consumers' preference for variety (Dubé 2004; Bhat 2005, 2008). The fact that consumers have a preference for variety has been well documented (Borle, Boatwright, Nunes, and Shmueli 2005; Oppewal and Koelemeijer 2005; Richards and Hamilton 2006; Briesch, Chintagunta, and Fox 2009). However, research has also suggests that, in some cases, there can be too much variety and decrease the propensity to search (Iyengar and Lepper 2000; Chernev 2003; Shah and Wolford 2007; and Mogilner, Rudnick, and Iyengar 2008). In this chapter, I develop a model of search under a preference for variety that naturally leads to multiple-discreteness.

This chapter makes several contributions to the literature. I show that as the preference for variety increases, consideration sets become larger. Therefore, search models that ignore a preference for variety are likely to under-predict the actual size of the optimal consideration set. My results offer an explanation as to why some consumer search studies find people search too much (Zwick, Rapoport, Lo, and Muthukrishnan 2003). As an individual's preference for variety increases, so does the size of their consideration set. Ignoring preference for variety, therefore, may make it appear as though consumers search too much, instead of optimally. Like other models of search, I find that consideration set-size is inversely related to the cost of search, but directly related to expectations of the general level of prices. Larger consideration sets, in turn, imply that more of the goods in the category are direct substitutes. In particular, I show that substitution between goods, and hence the elasticity of demand, differs significantly based on which goods are, or are not, included in the consideration set. Because search costs depend on consumers' opportunity costs of time, I also show that the size of a consumer's consideration set falls in their level of income (Mehta, Rajiv, and

[1] Calculated based on data from The Nielsen Company (US), LLC.

Srinivasan 2003). My findings not only point to more useful ways of modeling the cost of search, but also inform critical retail assortment and pricing decisions.

2.1.1 Background on Consumer Search

Price dispersion, or violations of the law of one price, have long been regarded as somewhat of a theoretical puzzle. Recognizing that stores rarely keep prices consistently high or low all the time, Varian (1980) explains "temporal" price dispersion, or "sales," as resulting from the coexistence of informed and uninformed consumers. His intuition is straight forward – firms have to decide whether to extract surplus from the uninformed consumers, or sell to both informed as well as uninformed consumers by charging the lowest price. Price dispersion persists, therefore, because consumers do not search – become informed – in his model. Price dispersion instead should be consistent with the notion that consumers do indeed search, and search optimally, comparing the marginal cost and benefit of searching for another product (Stiger 1961).

Even if consumers search rationally, price dispersion may still exist if products are differentiated. In reality, firms offer goods that are differentiated in the hopes of meeting consumers' preferences better than their competitors (Chamberlin 1933). Anderson and Renault (1999, 2000) demonstrate the importance of differentiation to search behavior by incorporating search into a discrete-choice model of differentiated-product demand. In their model, search is defined over both price and attributes, and not just price alone. In order to learn the price a particular firm is charging, and the characteristics of the product, a consumer must pay a search cost and search sequentially with costless recall. They show that as a consumer's preference for variety increases, he searches more intensively, and equilibrium prices fall. As prices fall, the expected gains from price-search decrease and price settles to an equilibrium. Similarly, as the cost of search increases, the equilibrium price increases and the number of differentiated products decreases. However, Anderson and Renault (1999, 2000) do not account for the fact that consumers can obtain information on several products at a time. This simplification is not trivial as multi-product search can change the implications of the model in fundamental ways (Roberts and Lattin 1991).

The ability to gain information on several products while incurring a single cost of search, or multi-product search, lies at the core of the retailing function as retailers allow consumers to reduce their search costs by evaluating multiple items in a single location (Betancourt 2004).[2] Through retailers, consumers can learn about the attributes of multiple products at the same time while incurring a single cost of search. Stigler (1961) may have had this perspective in mind in explaining a "fixed sample-size" model of search, or one in which consumers search among a fixed set of products, and choose the one they prefer. This fixed-sample size, or non-sequential, search model predicts only the number of products searched, not their identity, and assumes products are chosen at random. Seeking a more descriptive alternative, Weitzman (1979) derived a sequential model of search in which consumers rank-order products according to their desirability and search until they find one that meets their needs. A sequential search model describes which products are searched as well as how many. However, the sequential search model has since come under scrutiny. Diel and Zauberman (2005) argue that consumers are actually better off when searching from the least preferred product to the most preferred because there are more positive experiences when going from one choice to the next. In other words, when search is not too costly, consumers are more likely to search through products in an increasing order of preference because their overall search experience will yield better outcomes.

The empirical evidence favors non-sequential search. De los Santos, Hortaçsu, and Wildenbeest (2012) test the applicability of the sequential search model against a fixed-sample size alternative using online search data in which search behavior is directly observed. They find that the pricing pattern across stores are not consistent with a sequential search model because higher prices do not induce a consumer to necessarily engage in more search. While they caution against the generalizability of their results based on a single category of data (online book sales), their research lends strong evidence to support the fixed-sample size search model. Rather, theoretical models of consumer search should not simply assume one mode of search, but rather carefully consider the context of the search at hand. Because the context for my model involves consumers searching through consumer products that are relatively well-understood, I assume a fixed-sample size model. Whether sequential or non-sequential, however, the recent literature is silent on how consumers' demand for variety should affect search. Because consumers are likely to have some preference for variety across

[2] Although arguments have been made against this point, for example Hagiu and Jullien (2011).

books, my research focuses on how consumers' preference for variety shapes their search strategy in a fixed-sample size framework.

2.2 Model of Consumer Search

In this section, I derive a model of optimal consumer search in which the demand for variety plays a prominent role. Central to the objective of the paper is to understand how heterogenous preferences for variety affect consumers' search strategy. In particular, does a high preference for variety lead consumers to change their search behavior? If so, how do changes in search behavior affect subsequent purchase behavior? Another key question this research seeks to answer is how does search behavior change when the cost of search can be spread across a number of different product purchases? Currently, the search literature has focused on single purchase models to study search. In so doing, the total cost of search must be absorbed by a single product purchase. Moreover, search behavior based on single purchase discrete choice models inherently assume consumers have a sufficient amount of income for both search and purchase and that these two expenses are independent of one another. Another goal of the model developed in this research is to explicitly recognize that total consumer income is fixed, and consumers make search decisions knowing the more spent searching the less they have to spend on product purchases. Allowing consumers to make multiple discrete/continuous purchases provides a natural framework with which to study such behavior.

Consider an industry comprised of N differentiated goods each sold by a single firm. Let the global set of products available to a consumer be $\mathbf{N} = \{1, 2, ..., N\}$. Given the set of products $\mathbf{N}$, the consumer can purchase any combination of goods from $\mathbf{N}$ in any desirable quantity conditional only on their income. The consumer's total utility is additive over all product quantities. In other words, a representative consumer from the population has a utility function given by:

$$U = \sum_i x_i, \tag{2.1}$$

where x_i is the utility obtained from product i, and consumers are allowed to purchase 1 to N products. Consumers purchase multiple products for any of a number of reasons, including product satiation, household heterogeneity, or a preference for variety (Dubé 2004). As consumers become satiated with a product, they can gain utility by consuming a different, albeit equally attractive, product. Satiation is implied by decreasing marginal returns. The satisfaction obtained from a cold beverage on a hot day is high, but will decrease as the second, and third beverages are consumed. The degree to which consumers become satiated is determined by their preference for variety. Consumers who quickly become satiated with a product have a higher preference for variety because their overall utility is increased with the consumption of a wider range of products. Preference for variety, therefore, naturally leads to multiple product purchases, even if the goods are not intended to be consumed together.

Consumers often purchase products in varying quantities. For example, grocery shoppers can purchase deli meat either in pre-weighed containers, or in bulk from the deli counter. Search models rarely include both the observation that consumers have a preference for variety that drives multiple product purchases, and continuous quantities. In single-purchase search models, search costs must be absorbed entirely by the product being purchased. However, when a consumer can spread his search cost over a number of products, the net benefit will be correspondingly higher.

I assume a flexible functional form for x_i in equation (2.1) following Kim, Allenby, and Rossi (2002), Bhat (2005), and Satomura, Kim, and Allenby (2011), and define the total utility an individual consumer obtains on a particular shopping occasion by:

$$U = \sum_i u_i \left(\mathrm{e}^{\varepsilon_i} \left(q_i + \gamma_i\right)\right)^{\alpha} \tag{2.2}$$

where q_i is the quantity purchased of good i, γ_i is the satiation rate for product i, and α is the overall preference for variety where γ_i and α are known to the consumer. A larger γ_i represents a good that provides a higher degree of inherent satisfaction, in the sense that a consumer is satisfied with a relatively small amount. The parameter α represents the consumer's preference for variety in that higher values of α suggest a higher marginal utility. The parameters γ_i and α are core to the model, so I return to their interpretation below. Finally, u_i captures product specific characteristics such as shelf location, package size, or the amount of sugar, or fat in ice cream, for example. As discussed by Kim, Allenby, and Rossi (2002) and Bhat (2005) the utility function given in (2.2) is a translated utility function that is valid so long as $u_i > 0$. To ensure that this is the case over all observations, I define $u_i = \mathrm{e}^{\phi_i}$.

Consumer search is defined over individual products. Consumers are assumed to be uncertain of each product's attributes. That is, they do not know the exact value of each, but know the distribution of an individual product's attributes, including price. The error term, e^{ε_i}, represents consumer uncertainty for product i which is resolved by incurring a search cost c_i and obtaining that product's attribute information. The error term is assumed to be independent and identically distributed (i.i.d.) Gumbel, or Extreme Value Type 1 (Fréchet 1927; Fisher and Tippett 1928; Gumbel 1958; and Kotz and Nadarajah 2000). The consumer decides on the consideration set to search from among all available consideration sets and, once search is completed, eliminates e^{ε_i}. For those products that are searched, all product attributes are revealed perfectly. Uncertainty over product attributes in the utility function affect the utility for individual goods in a fundamental way. Namely, the error term e^{ε_i} shifts the consumer's perceived utility up or down because it is an argument of the sub-utility function: $\left(e^{\varepsilon_i}\left(q_i + \gamma_i\right)\right)^{\alpha}$.

My model recognizes that search is not costless, and consumers must resolve the uncertainty for product attributes by incurring a search cost c_i for the i^{th} product searched. Consumers decide the composition of the consideration set knowing the distribution of the error, e^{ε_i} and, once the consumer incurs the search cost for product i, $\varepsilon_i = 0$. Because search is defined over products and attributes, search cost differs for each product. The utility function given in equation (2.2) is a generalized multiple-discrete version of the standard logit model (Bhat 2005).

I assume a fixed-sample search process (de los Santos, Hortaçsu, and Wildenbeest 2012). Following Mehta, Rajiv, and Srinivasan (2003), consumers choose the optimal subset of products to search, $\mathbf{K} \subseteq \mathbf{N}$, and incur a search cost for each product, so the total search cost is: $\sum_{i \in \mathbf{K}} c_i$ where $c_i > 0 \ \forall i \in \mathbf{N}$. Each product's search cost (c_i) is known prior to searching. Moreover, when deciding on the optimal set of products to search, consumers have well defined beliefs about their satiation relating to a product, γ_i, and their overall preference for variety, α. While search must be undertaken to learn the attributes of specific products, consumers do have some idea regarding the average price and attribute profile of the products, but have no other product specific information. Expectations regarding the average attribute profile is given by $\bar{\phi}$, while expectations of the average price are given by $\bar{p}$. Once search is undertaken, the true ϕ_i and p_i are revealed and the consumer makes his purchase selection.

Specific products in the optimal consideration set are determined by product-level search costs. Consumers then decide on the quantities to purchase subject to their remaining budget:

$$y - \sum_{i \in \mathbf{K}} c_i = \sum_{i \in \mathbf{I}} p_i q_i \tag{2.3}$$

where y is the total budgeted dollar value he has to spend on search and product purchases, and $\mathbf{I} \subseteq \mathbf{K}$ is the set of products chosen to purchase. The cardinality of the sets $\mathbf{I}$ and $\mathbf{K}$ are denoted I, and K respectively. Consumers undertake search and how much money to commit to the search process, while keeping in mind that the more search undertaken, the less will be available to spend on the products themselves. Once search is completed, I assume the consumer has sufficient income to purchase at least one good from the consideration set.[3]

2.2.1 Consumer's Problem

Consumers first select the optimal consideration set $\mathbf{K}^*$ to be searched in order to remove the uncertainty about product attributes, $\mathrm{e}^{\varepsilon_i}$. They then determine the optimal quantities to purchase given they are able to select multiple products in any possible quantity from the consideration set. From the researcher's perspective, consideration set formation is a complex problem as consumers select the optimal subset of products from a total of $2^N - 1$ possible subsets and then select which products to actually purchase, yielding a total of $S2_K^{(2)}$ more possibilities, where $S2_K^{(2)}$ represents the Stirling numbers of the second kind.[4] The optimal consideration set is determined by using backward induction so that the consumer first solves for the maximum possible utility that can be obtained from any subset $\mathbf{K}$, and then determines the optimal subset to choose, $\mathbf{K}^*$.

At the final step, a consumer finds the optimal quantity to purchase of the i^{th} good, given some set of purchased goods, $\mathbf{I}$ and a set of searched goods, $\mathbf{K}$ for all possible $i \in \mathbf{I} \subseteq \mathbf{K} \subseteq \mathbf{N}$. The maximum utility that can be attained is found by maximizing the utility function, subject to the budget constraint, or:

$$\max \sum_{i \in \mathbf{I}} \mathrm{e}^{\bar{\phi}} \left(\mathrm{e}^{\varepsilon_i} \left(q_i + \gamma_i\right)\right)^{\alpha} \quad \text{sub. to } \; y = \bar{p} \sum_{i \in \mathbf{I}} q_i + \sum_{i \in \mathbf{K}} c_i \tag{2.4}$$

---

[3]It is straight forward to extend this to include a "no purchase," or outside good option and is made for expositional purposes.

[4]The Stirling numbers of the second kind ($S2_K^{(N)}$) count the number of ways of partitioning a set of K elements into N unordered non-empty pairwise disjoint subsets. They satisfy the following recurrence relationship: $S2_K^{(N)} = K\left(S2_K^{(N-1)}\right) + S2_{K-1}^{(N-1)}$ and are defined as: $S2_K^{(N)} = \frac{1}{K!}\sum_{i=1}^{K}(-1)^{K-i}\binom{K}{i} i^N$. The total number of partitions of an N-set is the sum of $S2_K^{(N)}$ over N which is called the *Bell* Number (Stanley 2011, pg. 82).

by forming the Lagrangian:

$$\mathcal{L} = \sum_{i \in \mathbf{I}} \mathrm{e}^{\bar{\phi}} \left(\mathrm{e}^{\varepsilon_i} \left(q_i + \gamma_i \right) \right)^{\alpha} + \lambda \left(y - \bar{p} \sum_{i \in \mathbf{I}} q_i - \sum_{i \in \mathbf{K}} c_i \right). \tag{2.5}$$

The necessary Karush-Kuhn-Tucker conditions (FOCs) with respect to the i^{th} quantity purchased simplify to:[5]

$$\begin{aligned} \alpha \mathrm{e}^{\bar{\phi}} \left(\mathrm{e}^{\varepsilon_i} \left(q_i^* + \gamma_i \right) \right)^{\alpha - 1} - \lambda \bar{p} &= 0 \quad \text{if } q_i^* > 0 \quad \forall i & (2.6a) \\ \alpha \mathrm{e}^{\bar{\phi}} \left(\mathrm{e}^{\varepsilon_i} \left(q_i^* + \gamma_i \right) \right)^{\alpha - 1} - \lambda \bar{p} &< 0 \quad \text{if } q_i^* = 0 \quad \forall i & (2.6b) \\ y - \sum_{i \in \mathbf{K}} c_i &= \bar{p} \sum_{i \in \mathbf{I}} q_i^*. & (2.6c) \end{aligned}$$

Equations (2.6a) and (2.6b) imply that the price-normalized marginal utility is equal for only those goods that are purchased. Price-normalized marginal utilities for the non-purchased goods are strictly less than those of the purchased goods, but still greater than 0. By taking the ratio of the i^{th} and j^{th} FOCs given in (2.6a) and substituting each into the budget constraint I find the optimal quantity demanded for the purchased goods, q_i^*(derived in detail in the appendix). Rearranging terms I find:

$$q_i^* = \frac{y - \sum_{k \in \mathbf{K}} c_k + \bar{p} \sum_{k \in \mathbf{I}} \gamma_k}{\mathrm{e}^{\varepsilon_i} \bar{p} \sum_{k \in \mathbf{I}} \mathrm{e}^{-\varepsilon_k}} - \gamma_i \quad \text{if } q_i^* > 0 \quad \forall i, \tag{2.7}$$

which satisfies the conditions in equations (2.6).

2.2.2 Consideration Set Formation

Consumers choose the optimal quantity of each good to purchase q_i^* given equation (2.7) above, conditional on their expectations about product attributes. When determining the composition of their consideration set, consumers do not know which products will ultimately be purchased. Therefore, they choose the consideration set such that they expect all the searched goods to end up being purchased. In other words, the global set of goods $\mathbf{N}$ consists only of those goods the consumer would reasonably expect to purchase if searched. For ease of exposition let the products in $\mathbf{K}$ be indexed $1, 2, ..., K$. From equation (2.2) consumers' maximum possible utility for a set of

[5] This is derived in detail in appendix A.

products $\mathbf{K}$, conditional on the expectations surrounding attributes and prices, is given by:

$$\max U_{\mathbf{K}} = \mathrm{e}^{\bar{\phi}} \sum_{i=1}^{K} \left(\frac{\left(y - \sum_{k=1}^{K} c_k + \bar{p} \sum_{k=1}^{K} \gamma_k \right)}{\bar{p} \sum_{k=1}^{K} \mathrm{e}^{-\varepsilon_k}} \right)^{\alpha}. \tag{2.8}$$

Simplifying notation, I re-write the maximum utility above as:

$$\max U_{\mathbf{K}} = K \left(A_{\mathbf{K}} \right)^{\alpha} \left(\sum_{k=1}^{K} \mathrm{e}^{-\varepsilon_k} \right)^{-\alpha} \tag{2.9}$$

where $A_{\mathbf{K}} = \bar{p}^{-1} \left(y - \sum_{k=1}^{K} c_k \right) + \sum_{k=1}^{K} \gamma_k$, and $\bar{\phi}$ is normalized to 0 without loss of generality. The role of $\bar{\phi}$ is simply to shift the maximum utility up by $\mathrm{e}^{\bar{\phi}}$ and does not have any bearing on which, or how many, products are selected. Equation (2.9) is the consumer's conditional net benefit from obtaining attribute and pricing information for any subset of goods from $\mathbf{N}$. In this expression, the term $A_{\mathbf{K}}$ serves as the "netting effect" on utility, compared to the more traditional way of accounting for search costs which is to subtract them after utility is maximized. Instead, my model explicitly accounts for search costs while finding the maximum utility possible. Because consumers are allowed to make multiple product purchases, the cost of search is considered before taking the expectation of the maximum utility.

Due to the uncertainty consumers have regarding the product attributes and prices, the expected benefit from searching any consideration set $\mathbf{K}$, is given by:

$$E[\max U_{\mathbf{K}}] = \int_{-\infty}^{\infty} \cdots \int_{-\infty}^{\infty} \left(\max U_{\mathbf{K}} \right) f(\varepsilon_1, \varepsilon_2, ..., \varepsilon_N) \mathrm{d}\varepsilon_1 \mathrm{d}\varepsilon_2 \cdots \mathrm{d}\varepsilon_N \tag{2.10a}$$

$$= \left(A_{\mathbf{K}} \right)^{\alpha} K \int_{-\infty}^{\infty} \cdots \int_{-\infty}^{\infty} \left(\sum_{k=1}^{K} \mathrm{e}^{-\varepsilon_k} \right)^{-\alpha} \prod_{j=1}^{K} \mathrm{e}^{-\varepsilon_j} \prod_{j=1}^{K} \mathrm{e}^{-\mathrm{e}^{-\varepsilon_j}} \mathrm{d}\varepsilon_1 \mathrm{d}\varepsilon_2 \cdots \mathrm{d}\varepsilon_K$$

$$= \left(\frac{y}{\bar{p}} + \sum_{k=1}^{K} \left\{ \gamma_k - \frac{c_k}{\bar{p}} \right\} \right)^{\alpha} \frac{K \; \Gamma\left[K - \alpha \right]}{\Gamma[K]} \tag{2.10b}$$

where $\Gamma\left[\bullet\right]$ is the Gamma function which reduces to $(K-1)!$ if K is an integer. Consumers choose the consideration set that provides the greatest expected benefit. The optimal composition of the consideration set is the one that provides the maximum $E[\max U_{\mathbf{K}}]$, or:

$$\mathbf{K}^{*} = \arg\max_{\mathbf{J}} \left\{ \left(\frac{y}{\bar{p}} + \sum_{k \in \mathbf{J}} \left\{ \gamma_k - \frac{c_k}{\bar{p}} \right\} \right)^{\alpha} \frac{|\mathbf{J}| \;\; \Gamma\left[|\mathbf{J}| - \alpha \right]}{\Gamma[|\mathbf{J}|]} \right\}, \tag{2.11}$$

where $|\mathbf{J}|$ represents the number of products in the set $\mathbf{J} \subseteq \mathbf{N}$. Since $y/\bar{p}$ is fixed, consumers choose products to include in $\mathbf{K}^*$ based on the difference between their satisfaction for the product, γ_k, and the price-normalized cost of searching for that product, $\frac{c_k}{\bar{p}}$. Moreover, because consumers know their price expectation $\bar{p}$, satisfaction γ_k, and search cost c_k, they can order the set of all goods in $\mathbf{N}$ based on each good's contribution to the expected utility. Define

$$G_i = \gamma_i - \frac{c_i}{\bar{p}} \tag{2.12}$$

and let the set of products in $\mathbf{N}$ be indexed as $G_1 \geq G_2 \geq \cdots \geq G_N$. Products are chosen to include in the consideration set based on this ordering because $G_i \geq G_j$ provides a higher expected benefit. Knowing the order in which products are added to the consideration set significantly reduces the total number of consideration sets that have to be compared to each other from $2^N - 1$ to N. For example, consumers know *a priori* they would not choose a consideration set composed of products $\{1, 3\}$ since the consideration set $\{1, 2\}$ yields a higher expected benefit. The key decision then becomes the size of the consideration set to choose.

Consumers choose the consideration set containing the products $i = 1, 2, ..., K$ such that:

$$G_{K+1} < \left(\frac{y}{\bar{p}} + \sum_{i=1}^{K} G_i\right)\left(\left(\frac{K}{(K+1)(K-\alpha)}\right)^{\frac{1}{\alpha}} - 1\right), \ \textit{and} \tag{2.13a}$$

$$G_K \geq \left(\frac{y}{\bar{p}} + \sum_{i=1}^{K-1} G_i\right)\left(\left(\frac{(K-1)}{K\ (K-1-\alpha)}\right)^{\frac{1}{\alpha}} - 1\right). \tag{2.13b}$$

Following Roberts and Lattin (1991), consider the case when γ_i is a decreasing linear function, or $\gamma_i = \gamma - i\tau$, $\tau > 0$ and assume $c_i = c\ \forall i$. The optimal number of products to search (K) satisfies the following inequalities:

$$c > \frac{\bar{p}\left(\left(\frac{y}{\bar{p}} + K\gamma - \frac{\tau K(K+1)}{2}\right)\left(\left(\frac{K}{(K+1)(K-\alpha)}\right)^{\frac{1}{\alpha}} - 1\right) - \gamma + (K+1)\tau\right)}{\left((K+1) - K\left(\frac{K}{(K+1)(K-\alpha)}\right)^{\frac{1}{\alpha}}\right)}, \ \textit{and} \tag{2.14a}$$

$$c \leq \frac{\bar{p}\left(\left(\frac{y}{\bar{p}} + (K-1)\gamma - \frac{\tau K(K-1)}{2}\right)\left(\left(\frac{(K-1)}{K\ (K-1-\alpha)}\right)^{\frac{1}{\alpha}} - 1\right) - \gamma + K\tau\right)}{\left(K - (K-1)\left(\frac{(K-1)}{K\ (K-1-\alpha)}\right)^{\frac{1}{\alpha}}\right)}. \tag{2.14b}$$

Even in the case of constant search costs and a linear specification for γ, an analytical solution for K is impossible to derive, due to the α term in the exponent.

After the consumer determines the optimal consideration set $\mathbf{K}^*$, the total cost of search is removed from their income and they observe the attributes of all the goods in the chosen set. The consumer then decides which goods to purchase, and their quantities. Product choices and quantities are found by solving (2.4) for prices and product attributes (see appendix A). Maximizing the utility function subject to the consumer's budget constraint yields the following FOCs:

$$\mathcal{L}_i = \alpha e^{\phi_i} \left(q_i^* + \gamma_i\right)^{\alpha-1} - \lambda p_i = 0 \qquad \text{if } q_i^* > 0 \ \ \forall i \tag{2.15a}$$

$$\mathcal{L}_i = \alpha e^{\phi_i} \left(q_i^* + \gamma_i\right)^{\alpha-1} - \lambda p_i < 0 \qquad \text{if } q_i^* = 0 \ \ \forall i \tag{2.15b}$$

$$\mathcal{L}_\lambda = y - \sum_{i \in \mathbf{K}} c_i - \sum_{i \in \mathbf{I}} p_i q_i^* = 0 \tag{2.15c}$$

where $\mathcal{L}_i = \frac{\partial \mathcal{L}}{\partial q_i}$, or the partial derivative of the Lagrangian function with respect to the i^{th} good. Solving for q_i^* as before, yields a more general solution to equation (2.7):

$$q_i^* = \frac{\tilde{y} + \sum_{i \in \mathbf{I}} p_k \gamma_k}{\sum_{i \in \mathbf{I}} p_k \left(\frac{p_i \exp[\phi_k]}{p_k \exp[\phi_i]}\right)^{\varpi}} - \gamma_i. \tag{2.16}$$

for ϕ_i and p_i more generally, where $\varpi = (\alpha - 1)^{-1}$ and $\tilde{y} = y - \sum_{i \in \mathbf{K}} c_k$. For ease of exposition I assume that the set of goods $\mathbf{I} \subseteq \mathbf{K}^*$ are the goods for which $q_i^* > 0$ and those goods in $\mathbf{K}^*$, but not in $\mathbf{I}$, are not purchased.[6] Knowing this, the consumer decides the optimal quantity of each good to purchase based on equation (2.16) once prices and attributes are observed. The consumer's maximum utility from searching only the products in $\mathbf{K}^*$ and selecting the consumption bundle $q_1^*, q_2^*, ..., q_I^*$ is given by:

$$\max U = \sum_{i=1}^{I} e^{\phi_i} \left(\frac{\tilde{y} + \sum_{k=1}^{I} p_k \gamma_k}{\sum_{k=1}^{I} p_k \left(\frac{p_i \exp[\phi_k]}{p_k \exp[\phi_i]}\right)^{\varpi}} \right)^{\alpha} + \sum_{i=I+1}^{K} e^{\phi_i} \left(\gamma_i\right)^{\alpha}. \tag{2.17}$$

The second order conditions that prove the solution q_i^* is the maximum utility is provided in appendix A. An important point here is that maximum utility is a function of both the purchased and non-purchased products. The consumer's maximum utility is, therefore, monotonically increasing in the number of products searched, K.

[6] A bit of a conundrum arrises because the solution to q_i^* is conditional on which goods are actually chosen to be purchased. Therefore, when $\mathbf{I}$ changes, so does q_i^*, so the set $\mathbf{I}$ should not be considered trivial. This is because the analytical solution to q_i^* does not apply to the non-chosen goods. However, the solution to this is described in detail in the appendix A, and follows the logic of Pinjari and Bhat (2009).

2.3 Changes in the Cost of Search and Variety

The optimal consideration set is both endogenous to the consumer's preferences, namely preference for variety, and smaller than the universe of brands available in any one category. Because of the prominence of search costs and the intensity of preference for variety, I demonstrate just how the implications of my model differ from others due to these features. Therefore, in this section, I show how a consumer's consideration set changes when his or her cost of search, and preference for variety, changes in a multi-purchase environment. My comparative static results follow from the marginal benefit of increasing the consideration set size from K to $K+1$ according to the consumer's product ordering defined by G_i in equation (2.12). I find:

$$\begin{aligned}\Delta EU &= \ln E[\max U_{\mathbf{K}\cup K+1}] - \ln E[\max U_{\mathbf{K}}] \\ &= \ln\left[\left(1+\frac{\gamma_{K+1}-\frac{c_{K+1}}{\bar{p}}}{\frac{y}{\bar{p}}+\sum_{k=1}^{K}\left\{\gamma_k-\frac{c_k}{\bar{p}}\right\}}\right)^{\alpha}\left(\frac{(K-\alpha)\left(K^2-1\right)}{K^2}\right)\right]. \end{aligned} \tag{2.18}$$

This leads to:

Proposition 1 *The marginal expected utility from searching an additional good increases as the consumer's preference for variety increases if $\alpha > 0$.*

Proof.

$$\frac{\partial \Delta EU}{\partial \alpha} = \ln\left[\frac{\bar{p}\gamma_{K+1}-c_{K+1}}{y+\bar{p}\sum_{k=1}^{K}\left\{\gamma_k-\frac{c_k}{\bar{p}}\right\}}+1\right]+\frac{1}{\alpha-K} > 0.$$

■

Proposition 1 shows that consumers' preference for variety has a significant impact on the optimal consideration set. Simply put, as the consumer's preference for variety increases, the gains from searching a larger number of products increases. A consumer who quickly becomes satiated with individual products and exhibits a higher preference for variety will search a wider range of products because he is more likely to purchase more than one product. Therefore, the marginal gains from search increase with the consumer's preference for variety. Here, Proposition 1 requires that $\alpha > 0$, however, in the next section I show that the domain of α is restricted to 0 and 1. So, requiring $\alpha > 0$ is not a restriction from the perspective of search. Proposition 1 suggests that consumer demand-search models that ignore preference for variety will under-predict the size of the consumer's optimal consideration set. Consider an individual in the market for an automobile. He may want both a BMW to commute to work, a Suburban to be able to haul the kids in and their friends to soccer practice, and a motorcycle to drive on the weekends, but his budget restricts him to purchase only one. Even though the consumer only purchases a single automobile, he still has a preference for variety that drives him to consider several different alternatives. Without accounting for preference for variety, a model of search would miss some of the vehicles the consumer actually searched.

The notion that the cost of search is critical to the number of products searched is well understood in the search literature. By allowing search costs to differ across products I am able to generalize this relationship. Namely, I find:

Proposition 2 *The marginal expected utility from searching an additional good decreases as the cost of searching an additional product increases.*

Proposition 3 *The marginal expected utility from searching an additional good increases when the total cost of search for the products already in the consideration set increases.*

Proof.

$$\begin{aligned}
\frac{\partial \Delta EU}{\partial c_{K+1}} &= -\frac{\alpha}{\bar{p}\left(\frac{y}{\bar{p}} + \sum_{k=1}^{K}\left\{\gamma_k - \frac{c_k}{\bar{p}}\right\}\right)\left(\frac{\gamma_{K+1} - \frac{c_{K+1}}{p}}{\frac{y}{\bar{p}} + \sum_{k=1}^{K}\left\{\gamma_k - \frac{c_k}{\bar{p}}\right\}} + 1\right)} < 0, \text{ and} \\
\frac{\partial \Delta EU}{\partial c_k} &= \frac{\alpha\left(\gamma_{K+1} - \frac{c_{K+1}}{\bar{p}}\right)}{\bar{p}\left(\frac{y}{\bar{p}} + \sum_{k=1}^{K}\left\{\gamma_k - \frac{c_k}{\bar{p}}\right\}\right)^2\left(\frac{\gamma_{K+1} - \frac{c_{K+1}}{\bar{p}}}{\frac{y}{\bar{p}} + \sum_{k=1}^{K}\left\{\gamma_k - \frac{c_k}{\bar{p}}\right\}} + 1\right)} > 0.
\end{aligned}$$

■

Taken together, Propositions 2 and 3 imply that an individual will search a larger number of products if c_i is decreasing in N, and a smaller set of products if c_i is increasing in N. This implies that firms would want to drive consumers' search costs as low as possible to increase the chance that their product is purchased. If firm i is one of the first few products selected the firm would want the consumer's search cost as low as possible in the hopes that the cost of searching prices and attributes of the $i+1$ firm is a little higher, thus decreasing the cardinality of the consideration set. Similarly, if firm j is close to N then they too would want consumer's search costs to be as low as possible to increase the chances that their good makes it into the final consideration set.

An implication of the second Proposition above is that the consumer recognizes that the initial investment in searching products $i = 1, 2, ..., K$ has already been exhausted and adding an additional product to the consideration set yields more information when considering their final purchase decisions. As another example of how these propositions manifest in the real world, consider advertising websites that provide firms the option of "featuring" their product on the front page, or top of the website, or next to a product that is currently being considered. In effect, the featuring aspect of the advertisement decreases the consumer's cost of searching an additional product, making it more likely that particular product will end up in the consumer's consideration set.

Intuitively, Propositions 2 and 3 suggest that firms would want to make the cost of searching their own product as low as possible, and the cost of searching other products as high as possible. In this sense, my intuition is opposite the strategic obfuscation models of Ireland (2007) and Ellison and Ellison (2009). Nonetheless, this conclusion is intuitive, and is the same as that found in single-product search models. However, some recent research has argued that it is in the firm's best interest to impose a larger search cost on consumers. Mayzlin and Shin (2011) argue that firms can use the content of their advertisements to get consumers to search, or not search for more information about the products they are offering. They show that some firms will actually develop advertisements that are devoid of any substantive information as a means of motivating consumers to search for, and learn more information about, the attributes of the product on their own. In my notation, firms are trying to influence γ_i, or the satisfaction the consumer expects to gain from the good. If, in my model, consumers only had a small fixed amount of income to spend on search, then those products that know they will be searched first would want to increase the cost of search to absorb as much of the search budget as possible, thereby reducing their competition. In other words, if consumers have a fixed budget for search and cannot necessarily spend almost all their income, y, on search then firms most likely to be in the consideration set (e.g. small i in G_i) would be better served to make their search cost high to extract as much of the consumer's search budget as possible, leaving less for the remaining products that are farther down the search list.

My model implies that consumers first decide the order of the products to include in the consideration set, and then decide how many products to search. Therefore, some firms would want to make cost of search high, while others lower down the search list would want to make the cost of searching for their product low. This intuition is also consistent with Bar-Isaac, Caruana, and Cunat (2010) who show some firms may switch away from a mass market strategy to a niche market strategy if their product is not as likely to be searched. By switching to the niche market strategy the firm is able to move up the search list, and become more likely to be searched, albeit by a smaller market.

The benefit of a firm switching away from a mass market strategy to a niche market focus can be readily seen from the following Propositions.

Proposition 4 *The marginal expected utility from searching an additional good decreases as the consumer's satisfaction for an additional product decreases.*

Proposition 5 *The marginal expected utility from searching an additional good increases when the consumer's satisfaction for products currently under consideration increases.*

Proof.

$$\begin{aligned}
\frac{\partial \Delta EU}{\partial \gamma_{K+1}} &= \frac{\alpha}{\bar{p}\left(\frac{y}{\bar{p}} + \sum_{k=1}^{K}\left\{\gamma_k - \frac{c_k}{\bar{p}}\right\}\right)\left(\frac{\gamma_{K+1} - \frac{c_{K+1}}{p}}{\frac{y}{\bar{p}} + \sum_{k=1}^{K}\left\{\gamma_k - \frac{c_k}{\bar{p}}\right\}} + 1\right)} > 0, \text{ and} \\
\frac{\partial \Delta EU}{\partial \gamma_k} &= -\frac{\alpha\left(\gamma_{K+1} - \frac{c_{K+1}}{\bar{p}}\right)}{\bar{p}\left(\frac{y}{\bar{p}} + \sum_{k=1}^{K}\left\{\gamma_k - \frac{c_k}{\bar{p}}\right\}\right)^2\left(\frac{\gamma_{K+1} - \frac{c_{K+1}}{\bar{p}}}{\frac{y}{\bar{p}} + \sum_{k=1}^{K}\left\{\gamma_k - \frac{c_k}{\bar{p}}\right\}} + 1\right)} < 0.
\end{aligned}$$

■

Propositions 4 and 5 illustrate that the number of products in the consumer's consideration set will increase as the consumer's satisfaction for the next unsearched product increases. Since the consumer knows his satisfaction for all products, he will sample a larger number of products when satisfaction is relatively high for all. Since γ_k captures the rate at which consumers become satisfied with a product, this is an intuitive result. As expected, if consumers quickly become satisfied with the products in the category, they are going to search through a wider range of products because they are more likely to benefit from purchasing several products.

By directly accounting for a consumer's budget in the search and purchase decision I find that as an individual becomes less price sensitive, he will choose consideration sets with a smaller number of products. In other words, I find:

Proposition 6 *The marginal expected utility from searching an additional good decreases as the consumer's income increases.*

Proposition 7 *The marginal expected utility from searching an additional good increases when the consumer's price expectation increases.*

Proof.

$$\frac{\partial \Delta EU}{\partial y} = -\frac{\alpha\left(\gamma_{K+1}-\frac{c_{K+1}}{\bar{p}}\right)}{\bar{p}\left(\frac{y}{\bar{p}}+\sum_{k=1}^{K}\left\{\gamma_k-\frac{c_k}{\bar{p}}\right\}\right)\left(\frac{\gamma_{K+1}-\frac{c_{K+1}}{p}}{\frac{y}{\bar{p}}+\sum_{k=1}^{K}\left\{\gamma_k-\frac{c_k}{\bar{p}}\right\}}+1\right)}<0, \text{ and}$$

$$\frac{\partial \Delta EU}{\partial \bar{p}} = \frac{\frac{\alpha}{\bar{p}^2}\left(c_{K+1}\sum_{k=1}^{K}\gamma_k+\gamma_{K+1}\left(y-\sum_{k=1}^{K}c_k\right)\right)}{\left(\frac{y}{\bar{p}}+\sum_{k=1}^{K}\left\{\gamma_k-\frac{c_k}{\bar{p}}\right\}\right)\left(\frac{y}{\bar{p}}+\sum_{k=1}^{K+1}\left\{\gamma_k-\frac{c_k}{\bar{p}}\right\}\right)}>0.$$

■

Propositions 6 and 7 illustrate that consumers are more likely to search over a large number of products when the prices for the product are high, and choose to search less intensively. In other words, a consumer with a higher opportunity cost of time is less likely to undertake extensive price comparisons of brands across stores. A consumer who plans on purchasing an expensive digital camera may still have a high opportunity cost of search, but their income is now considerably less relative to the benefit of searching for the good. In other words, his price expectation for a digital camera is much higher than that for carbonated soft drinks and would sample a larger number of cameras before making a purchase, compared to soft drinks.

Taken together, I find that the search effort will decrease as the cost of search increases, which is consistent with prior studies (Mehta, Rajiv, and Srinivasan 2003; Cachon, Terwiesch, and Xu 2008; and Kuksov and Villas-Boas 2010), and I find that search will increase as preference for variety increases. In particular, consumers opt for considerations sets with a larger number of products in them when their preference for variety increases. My research deviates from prior consumer search studies in that search is endogenous to the cost of search as well as the consumer's preference for variety. Therefore, preference for variety forms a critical component of the search decision and directly affects the size of the consideration set chosen. The consideration set, in turn, is an important driver of the final purchase decision. In particular, if a consumer exhibits little to no preference for variety and the consideration set that provides the highest expected maximum utility given in equation (2.11) only has 1 product, then a consumer will allocate their remaining budget to the one searched item. On the other hand, if a strong preference for variety drives an individual to select a consideration set of considerable size, then his optimal purchase decision becomes more flexible, and more complicated. In this case, the consideration set leaves the purchase decision much more unrestricted. In particular, therefore, intuition suggests that preference for variety will shape the optimal purchase decision through the consideration set when it is low. When there is a significant preference for variety it will shape the purchase decision more directly, since the consideration set chosen is likely to have a larger number of alternatives from which to choose. I investigate the relationship between preference for variety and consumer choice more formally next. Because choice is conditional on the consideration set and a consideration set containing only 1 product makes the optimal purchase decision trivial, it is assumed that there are at least 2 products from which to choose.

Once the optimal consideration set is determined, consumers learn the true attributes of the searched items and uncertainty is resolved (i.e. $\varepsilon_i = 0$). An individual then makes his purchase selection from within the set of searched goods $\mathbf{K}^*$ according to equation (2.16). As a first step to investigating consumer purchase behavior I confirm that demand indeed slopes downward in price. In other words:

Lemma 8 *The optimal quantity purchased of the i^{th} good decreases when the price of that good increases.*

Proof. To determine how the optimal quantity purchased, q_i^*, changes in relation to changes in its price, p_i, I differentiate the FOCs given in equation (2.15) to obtain a system in inequalities:

$$\begin{pmatrix} \mathcal{L}_{11} & 0 & \cdots & 0 & -p_1 \\ 0 & \mathcal{L}_{22} & \cdots & 0 & -p_2 \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & \cdots & 0 & \mathcal{L}_{KK} & -p_K \\ -p_1 & -p_2 & \cdots & -p_K & 0 \end{pmatrix} \begin{pmatrix} \frac{\partial q_1^*}{\partial p_i} \\ \frac{\partial q_2^*}{\partial p_i} \\ \vdots \\ \frac{\partial q_K^*}{\partial p_i} \\ \frac{\partial \lambda^*}{\partial p_i} \end{pmatrix} \leq \begin{pmatrix} 0 \\ \vdots \\ \lambda^* \\ 0 \\ q_i^* \end{pmatrix}, \tag{2.19}$$

where the strict inequalities in the system of equations above apply to the goods that are not purchased, $q_i^* = 0$, and the equalities apply to the purchased goods, $q_i^* > 0$. Since I am only interested in the relationship between the optimal quantity purchased and price for the goods that are actually purchased I am only concerned with the equalities in the above system of equations. In determining the optimal purchased quantities among the searched goods, the number of purchased products, I, is determined prior to finding the actual quantities.[7] In other words, I assume I is fixed which implies parameter changes of purchased goods will have no effect on the non-purchased goods. To simplify the analysis I also assume that $I = K$, or that the consumer will purchase a little bit of every searched good, even if it is an infinitely small amount of the good. This allows me to consider the affect one good has on all the others in the consideration set. Solving the linear system of equations above for $\frac{\partial q_i^*}{\partial p_i}$ I find:

$$\begin{aligned}\frac{\partial q_i^*}{\partial p_i} &= \frac{1}{\mathbf{D}}\begin{vmatrix} \mathcal{L}_{11} & 0 & \cdots & 0 & 0 & -p_1 \\ 0 & \mathcal{L}_{22} & 0 & \cdots & 0 & -p_2 \\ \vdots & \cdots & \ddots & \vdots & \vdots & \vdots \\ 0 & \cdots & 0 & \lambda^* & 0 & \vdots \\ 0 & \cdots & 0 & \vdots & \mathcal{L}_{KK} & -p_K \\ -p_1 & -p_2 & \cdots & q_i^* & -p_K & 0 \end{vmatrix} \\ &= -\frac{q_i^* p_i + \frac{\lambda^*}{\alpha(1-\alpha)}\sum_{k\neq i}^{K} \frac{(p_k)^2}{\mathrm{e}^{\phi_k}\left(q_k^*+\gamma_k\right)^{\alpha-2}}}{\mathrm{e}^{\phi_i}\left(q_i^*+\gamma_i\right)^{\alpha-2}\left(\sum_{k=1}^{K}\frac{(p_k)^2}{\mathrm{e}^{\phi_k}\left(q_k^*+\gamma_k\right)^{\alpha-2}}\right)} < 0\end{aligned} \tag{2.20}$$

where $\mathcal{L}_{ii} = \frac{\partial \mathcal{L}_i}{\partial q_i} = -\alpha(1-\alpha)\mathrm{e}^{\phi_i}\left(q_i^*+\gamma_i\right)^{\alpha-2}$ and $\mathbf{D}$ is the determinant of the Hessian matrix which is defined in equation (A.6) above. ■

As expected, the optimal quantity purchased decreases as the price of that good increases. Recall that equation (2.16) is only valid for those goods that are purchased, and does not apply to non-purchased goods. Since my demand analysis requires that the quantity purchased of a good decrease as the price of that good increases, or demand slopes down, this necessarily requires that $\alpha \in [0,1]$. If $\alpha < 0$ then the expression $\alpha(1-\alpha) < 0$ and $\frac{\partial q_i^*}{\partial p_i} < 0$ if and only if $q_i^* p_i < \frac{\lambda^*}{\alpha(1-\alpha)}\sum_{k\neq i}^{K}\frac{(p_k)^2}{\mathrm{e}^{\phi_k}\left(q_k^*+\gamma_k\right)^{\alpha-2}}$ or the total amount spent on the i^{th} product is less than $\frac{\lambda^*}{\alpha(1-\alpha)}\sum_{k\neq i}^{K}\frac{(p_k)^2}{\mathrm{e}^{\phi_k}\left(q_k^*+\gamma_k\right)^{\alpha-2}}$ which does not have a clear interpretation. Therefore, I assume $\alpha \in [0,1]$ which also rules out the possibility of gross compliment goods. Namely,

Lemma 9 *The consideration set is comprised of only substitute goods if and only if $\alpha \in [0,1]$.*

[7]For a discussion, see appendix A or Pinjari and Bhat (2009).

Proof. Solving the system of equations given in (2.19) for $\frac{\partial q_j^*}{\partial p_i}$ I find:

$$\frac{\partial q_j^*}{\partial p_i} = \frac{1}{\mathbf{D}} \begin{vmatrix} \mathcal{L}_{11} & 0 & \cdots & 0 & 0 & -p_1 \\ 0 & \mathcal{L}_{22} & 0 & \cdots & 0 & -p_2 \\ \vdots & \cdots & \mathcal{L}_{ii} & \lambda^* & \vdots & \vdots \\ 0 & \cdots & 0 & 0 & 0 & \vdots \\ 0 & \cdots & 0 & \vdots & \mathcal{L}_{KK} & -p_K \\ -p_1 & -p_2 & -p_i & q_i^* & -p_K & 0 \end{vmatrix}$$

$$= \frac{p_j}{\mathrm{e}^{\phi_j}\left(q_j^* + \gamma_j\right)^{\alpha-2} \sum_{k=1}^{K} \frac{(p_k)^2}{\mathrm{e}^{\phi_k}\left(q_k^* + \gamma_k\right)^{\alpha-2}}} \left(\frac{\lambda^* p_i}{\alpha\,(1-\alpha)\,\mathrm{e}^{\phi_i}\,(q_i^* + \gamma_i)^{\alpha-2}} - q_i^* \right).$$

The quantity purchased of a good will increase as the price of another good increases so long as $\frac{\lambda^* p_i}{\alpha(1-\alpha)\mathrm{e}^{\phi_i}\left(q_i^* + \gamma_i\right)^{\alpha-2}} \geq q_i^*$. In other words, the i^{th} good is a gross substitute for any other good if $\frac{\lambda^* p_i}{\alpha(1-\alpha)\mathrm{e}^{\phi_i}\left(q_i^* + \gamma_i\right)^{\alpha-2}} \geq q_i^*$ and is a gross compliment to all other goods when $\frac{\lambda^* p_i}{\alpha(1-\alpha)\mathrm{e}^{\phi_i}\left(q_i^* + \gamma_i\right)^{\alpha-2}} < q_i^*$. Using equation (2.15a) I solve for λ^* (e.g. $\lambda^* = \frac{1}{p_i}\mathrm{e}^{\phi_i}\,(q_i^* + \gamma_i)^{\alpha-1}\,,\ \forall i$) and substitute this definition into the above expression to find:

$$\frac{\partial q_j^*}{\partial p_i} = \frac{p_j}{\mathrm{e}^{\phi_j}\left(q_j^* + \gamma_j\right)^{\alpha-2} \sum_{k=1}^{K} \frac{(p_k)^2}{\mathrm{e}^{\phi_k}\left(q_k^* + \gamma_k\right)^{\alpha-2}}} \left(\left(\frac{1}{\alpha\,(1-\alpha)} - 1 \right) (q_i^* + \gamma_i) - \gamma_i \right). \tag{2.21}$$

Therefore, the i^{th} good is a compliment good if $\left(\frac{1}{\alpha(1-\alpha)} - 1\right)(q_i^* + \gamma_i) < \gamma_i$. To avoid the complementarity, α must be bound between 0 and 1. The expression $\left(\frac{1}{\alpha(1-\alpha)} - 1\right) > 0$ when $\alpha \in [0,1]$ and $\left(\frac{1}{\alpha(1-\alpha)} - 1\right) < 0$ otherwise. Furthermore, note that when α is bound, the expression $\left(\frac{1}{\alpha(1-\alpha)} - 1\right)$ attains a minimum value of 3 at $\alpha = 1/2$ ensuring $\left(\frac{1}{\alpha(1-\alpha)} - 1\right)(q_i^* + \gamma_i) > \gamma_i$, or all the goods are substitutes. ■

When consumers make a search/purchase decision in a single category setting such as carbonated soft drinks or clothing items such as jeans, where the goods under consideration are known substitutes, α must be restricted to be between 0 and 1. In other words, the reason that a consumer makes multiple purchases in a single category is largely driven by satiation and preference for variety rather than because he plans to use the products together. For example, most American teenagers purchase multiple pairs of jeans at once, but rarely wear several pairs at the same time.

The price elasticity expressions for consumer demand is defined as the cross price elasticity $\epsilon_{ij} = \frac{p_i}{q_j^*}\frac{\partial q_j^*}{\partial p_i}$ where $\frac{\partial q_j^*}{\partial p_i}$ is defined in equation (2.21), and the own price elasticity $\epsilon_{ii} = \frac{p_i}{q_i^*}\frac{\partial q_i^*}{\partial p_i}$ where

$\frac{\partial q_i^*}{\partial p_i}$ is given in equation (2.20). An interesting characteristic of the demand model considered here is that both the own and cross price elasticity formulas are directly dependent on the attributes of all the purchased goods, which are dependent on the chosen consideration set. Therefore, the consideration set the consumer decides on critically affects how the quantity purchased of the i^{th} good will change with the observed price of that good, or another good. This leads to:

Corollary 10 *The cross price elasticity for any 2 purchased goods will decrease (increase) when the average price of all the searched goods increases (decreases).*

Corollary 10 shows the importance of the consideration set to the consumer's final purchase decision. Intuitively, if a firm that sells a particular good knows that the consumer will sample products according to the ordering of G_i, and is also aware that they are high on the list to be sampled, the firm would be more likely to charge a higher price (assuming it does not affect the consumer's price expectation). If prices were decreasing according to the ordering of the average consumer's G_i, this would suggest that the average price of a smaller set of products would be higher than that of a consideration set with a large number of products. Based on this intuition, the cross price elasticities for goods in a smaller sample of products would be smaller than the cross price elasticity of a larger number of products. Said differently, the quantity purchased of a particular good would be less sensitive to price changes of other goods when the consumer only samples a small number of products. In sampling a small number of products the consumer will have spent less on search and will have more money to use for the actual purchasing of products. This somewhat counter-intuitive result stems from the fact that I consider both the cost of search and the cost of the products in the budget constraint.

The cross price elasticity is also affected by all the parameters of all the purchased goods. However, I consider these parameters' effects on the quantity purchased more directly. Namely, I find that the optimal quantity purchased of a particular good, q_i^*, will decrease as that good's satiation parameter, γ_i, increases, or the satiation parameter of another good decreases. In other words:

Proposition 11 *As consumers becomes satiated with a particular product they will switch away from that product to another.*

Proof. Differentiating the FOCs given in equation (2.15) produces the following matrix of equations:

$$\begin{pmatrix} \mathcal{L}_{11} & 0 & \cdots & 0 & -p_1 \\ 0 & \mathcal{L}_{22} & \cdots & 0 & -p_2 \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & \cdots & 0 & \mathcal{L}_{KK} & -p_K \\ -p_1 & -p_2 & \cdots & -p_K & 0 \end{pmatrix} \begin{pmatrix} \frac{\partial q_1^*}{\partial \gamma_i} \\ \frac{\partial q_2^*}{\partial \gamma_i} \\ \vdots \\ \frac{\partial q_K^*}{\partial \gamma_i} \\ \frac{\partial \lambda^*}{\partial \gamma_i} \end{pmatrix} = \begin{pmatrix} 0 \\ \vdots \\ \alpha\left(1-\alpha\right)\mathrm{e}^{\phi_i}\left(q_i^* + \gamma_i\right)^{\alpha-2} \\ 0 \\ 0 \end{pmatrix}.$$

Solving for $\frac{\partial q_i^*}{\partial \gamma_i}$ and $\frac{\partial q_j^*}{\partial \gamma_i}$ I find:

$$\begin{aligned} \frac{\partial q_i^*}{\partial \gamma_i} &= \frac{1}{\mathbf{D}} \begin{vmatrix} \mathcal{L}_{11} & 0 & \cdots & 0 & 0 & -p_1 \\ 0 & \mathcal{L}_{22} & 0 & \cdots & 0 & -p_2 \\ \vdots & \cdots & \ddots & \vdots & \vdots & \vdots \\ 0 & \cdots & 0 & \alpha\left(1-\alpha\right)\mathrm{e}^{\phi_i}\left(q_i^* + \gamma_i\right)^{\alpha-2} & 0 & \vdots \\ 0 & \cdots & 0 & \vdots & \mathcal{L}_{KK} & -p_K \\ -p_1 & -p_2 & \cdots & 0 & -p_K & 0 \end{vmatrix} \\ &= \frac{\frac{(p_i)^2}{\mathrm{e}^{\phi_i}\left(q_i^*+\gamma_i\right)^{\alpha-2}}}{\left(\sum_{j=1}^{K} \frac{(p_j)^2}{\mathrm{e}^{\phi_j}\left(q_j^*+\gamma_j\right)^{\alpha-2}}\right)} - 1 < 0, \quad \text{and} \\ \frac{\partial q_j^*}{\partial \gamma_i} &= p_i p_j \left(\mathrm{e}^{\phi_j}\left(q_j^* + \gamma_j\right)^{\alpha-2} \sum_{k=1}^{K} \frac{\left(p_k\right)^2}{\mathrm{e}^{\phi_k}\left(q_k^* + \gamma_k\right)^{\alpha-2}}\right)^{-1} > 0. \end{aligned}$$

■

Proposition 11 suggests that as the consumer's satisfaction for a particular product increases he will purchase less. This is because the price-normalized marginal utility of all the other goods is being held constant. In this case, the consumer does not need to purchase as much of product i to gain the same benefit. The price-normalized marginal utility of a particular good increases as γ_i increases (i.e. $\frac{\partial^2 U^*}{\partial q_i^* \partial \gamma_i} > 0$) suggesting that γ_i increases the consumer's satisfaction for that particular good. Similarly, I find the relationship between the optimal quantity purchased and the product's attributes also decreases, where increasing values of ϕ_i represent a product that has more appealing attributes. Namely,

Proposition 12 *As the attractiveness of a product's attributes increase the optimal quantity purchased of that good will decrease.*

Proof. The differentiated FOCs produce the following matrix of equations:

$$\begin{pmatrix} \mathcal{L}_{11} & 0 & \cdots & 0 & -p_1 \\ 0 & \mathcal{L}_{22} & \cdots & 0 & -p_2 \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & \cdots & 0 & \mathcal{L}_{KK} & -p_K \\ -p_1 & -p_2 & \cdots & -p_K & 0 \end{pmatrix} \begin{pmatrix} \frac{\partial q_1^*}{\partial \phi_i} \\ \frac{\partial q_2^*}{\partial \phi_i} \\ \vdots \\ \frac{\partial q_K^*}{\partial \phi_i} \\ \frac{\partial \lambda^*}{\partial \phi_i} \end{pmatrix} = \begin{pmatrix} 0 \\ \vdots \\ \alpha e^{\phi_i} \left(q_i^* + \gamma_i\right)^{\alpha - 1} \\ 0 \\ 0 \end{pmatrix}.$$

Solving for $\frac{\partial q_i^*}{\partial \phi_i}$ and $\frac{\partial q_j^*}{\partial \phi_i}$ I find:

$$\begin{aligned} \frac{\partial q_i^*}{\partial \phi_i} &= \frac{1}{\mathbf{D}} \begin{vmatrix} \mathcal{L}_{11} & 0 & \cdots & 0 & 0 & -p_1 \\ 0 & \mathcal{L}_{22} & 0 & \cdots & 0 & -p_2 \\ \vdots & \cdots & \ddots & \vdots & \vdots & \vdots \\ 0 & \cdots & 0 & \alpha e^{\phi_i} \left(q_i^* + \gamma_i\right)^{\alpha-1} & 0 & \vdots \\ 0 & \cdots & 0 & \vdots & \mathcal{L}_{KK} & -p_K \\ -p_1 & -p_2 & \cdots & 0 & -p_K & 0 \end{vmatrix} \\ &= \left(\frac{q_i^* + \gamma_i}{1 - \alpha} \right) \left(\frac{\frac{(p_i)^2}{e^{\phi_i}\left(q_i^* + \gamma_i\right)^{\alpha-2}}}{\sum_{j=1}^{K} \frac{(p_j)^2}{e^{\phi_j}\left(q_j^* + \gamma_j\right)^{\alpha-2}}} - 1 \right) < 0, \quad \text{and} \\ \frac{\partial q_j^*}{\partial \phi_i} &= p_i p_j \left(q_i^* + \gamma_i\right) \left((1-\alpha)\, e^{\phi_j} \left(q_j^* + \gamma_j\right)^{\alpha-2} \sum_{k=1}^{K} \frac{(p_k)^2}{e^{\phi_k}\left(q_k^* + \gamma_k\right)^{\alpha-2}} \right)^{-1} > 0. \end{aligned}$$

■

As with product satisfaction, Proposition 12 shows that the consumer will purchase less of a particular good when that good's product attributes become more attractive because the consumer attains the same level of satisfaction by purchasing less. The marginal benefit an individual good provides increases as ϕ_i increases, or $\frac{\partial^2 U^*}{\partial q_i^* \partial \phi_i} = \frac{\partial U^*}{\partial q_i^*} > 0$. The Propositions above for γ_i and ϕ_i illustrate the importance of taking the consumer's multiproduct purchase decision into consideration. Because the consumer is able to purchase multiple products, he selects those goods that provide the highest level of satisfaction, and then distributes his budget over those goods. However, because consumers can select multiple products in continuous quantities, they will purchase smaller quantities of the goods that provide the highest level of price-normalized satisfaction. From an intuitive perspective, firms want consumers to have a high degree of satisfaction for their product(s), so price products in such a way that they are more likely to fall into a consumers' consideration sets.

Consumers' satiation and preference for variety have important implications for new product introductions. When a new product is introduced a consumer will not have well defined belief regarding his satisfaction for the product (i.e. γ_i is unknown for a new product). In the search stage, therefore, the consumer is uncertain about where exactly the product falls in the ordering of the G_i's. Nonetheless, consumers do know the cost of search. This suggests that, when introducing a new product, firms want to decrease consumers' costs of both searching for, and consuming, products. This intuition explains why firms give out free samples at grocery stores and other events. A free sample essentially reduces the consumer's search cost to 0, and helps them form a measure of satisfaction for the product. This in turn helps the consumer position the new product in the ordering of the G_i's for the next shopping trip when the cost of search is not 0.

Finally, I consider how changes in α affect the optimal quantity purchased. This relationship is more ambiguous than the other parameters because of the ubiquity of α in the FOCs for all K goods. Differentiating the FOCs with respect to α for all goods I obtain the matrix of equations:

$$\begin{pmatrix} \mathcal{L}_{11} & 0 & \cdots & 0 & -p_1 \\ 0 & \mathcal{L}_{22} & \cdots & 0 & -p_2 \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & \cdots & 0 & \mathcal{L}_{KK} & -p_K \\ -p_1 & -p_2 & \cdots & -p_K & 0 \end{pmatrix} \begin{pmatrix} \frac{\partial q_1^*}{\partial \alpha} \\ \frac{\partial q_2^*}{\partial \alpha} \\ \vdots \\ \frac{\partial q_K^*}{\partial \alpha} \\ \frac{\partial \lambda^*}{\partial \alpha} \end{pmatrix} = \begin{pmatrix} \frac{\partial \mathcal{L}_1}{\partial \alpha} \\ \vdots \\ \vdots \\ \frac{\partial \mathcal{L}_K}{\partial \alpha} \\ 0 \end{pmatrix} \tag{2.22}$$

where

$$\frac{\partial \mathcal{L}_i}{\partial \alpha} = \mathrm{e}^{\phi_i} \left(q_i^* + \gamma_i\right)^{\alpha - 1} \left(1 + \alpha \ln \left[q_i^* + \gamma_i\right]\right). \tag{2.23}$$

Notice in equation (2.23) above that $\frac{\partial \mathcal{L}_i}{\partial \alpha} > 0$ if $(q_i^* + \gamma_i) > \mathrm{e}^{-\frac{1}{\alpha}}$. So, the relationship between the FOCs and α begins with uncertainty. Namely, the price normalized marginal utility of a particular good may not necessary increase as α increases. However, the condition that $\frac{\partial \mathcal{L}_i}{\partial \alpha} > 0$ is not difficult to satisfy because the right hand side of the inequality attains its maximum value as α approaches 1 at approximately $\mathrm{e}^{-1} \approx 0.37$. So, if q_i^* is purchased in a unit increment of 1 the condition is satisfied, or if $q_i^* = 0$ but I find $\gamma_i > \mathrm{e}^{-1}$, the condition is satisfied. If γ_i were restricted (or normalized) to be between 0 and e^{-1} the consumer would immediately know that $q_i^* = 0$ for all i products such that $\frac{\partial \mathcal{L}_i}{\partial \alpha} < 0$ and $q_j^* > 0$ for all j when $\frac{\partial \mathcal{L}_i}{\partial \alpha} > 0$. A consumer knows exactly which products should be purchased, or belong in $\mathbf{I}$, and which should not. The consumer could then decide how many products to search with more precision since α and γ_i are known. When γ_i is left unrestricted, the contents of the set of purchased goods, $\mathbf{I}$, is more cumbersome to find. The utility maximization section of the appendix provides a discussion regarding the way consumer's determine which products to purchase, and the quantities thereof.

Looking at the relationship between the optimal quantity purchased and α more closely I solve the above matrix of equations for the i^{th} good to find:

$$\begin{aligned}
\frac{\partial q_i^*}{\partial \alpha} &= \frac{1}{\mathbf{D}} \begin{vmatrix} \mathcal{L}_{11} & 0 & \cdots & \frac{\partial \mathcal{L}_1}{\partial \alpha} & 0 & -p_1 \\ 0 & \mathcal{L}_{22} & 0 & \frac{\partial \mathcal{L}_2}{\partial \alpha} & 0 & -p_2 \\ \vdots & \cdots & \ddots & \vdots & \vdots & \vdots \\ 0 & \cdots & 0 & \vdots & 0 & \vdots \\ 0 & \cdots & 0 & \frac{\partial \mathcal{L}_K}{\partial \alpha} & \mathcal{L}_{KK} & -p_K \\ -p_1 & -p_2 & \cdots & 0 & -p_K & 0 \end{vmatrix} \\
&= -\left(\sum_{j=1}^{K} \frac{(p_j)^2}{\mathrm{e}^{\phi_j} \left(q_j^* + \gamma_j\right)^{\alpha-2}} \right)^{-1} \left(p_i \sum_{r \neq i}^{K} \frac{p_r}{\mathcal{L}_{rr}} \frac{\partial \mathcal{L}_r}{\partial \alpha} - \frac{\partial \mathcal{L}_i}{\partial \alpha} \sum_{j \neq i}^{K} \frac{(p_j)^2}{\mathcal{L}_{jj}} \right)
\end{aligned}$$

where

$$\begin{aligned}
\frac{p_r}{\mathcal{L}_{rr}} \frac{\partial \mathcal{L}_r}{\partial \alpha} &= -\frac{p_r \left(q_r^* + \gamma_r\right)}{\alpha (1-\alpha)} \left(1 + \alpha \ln \left[q_r^* + \gamma_r\right]\right), \text{ and} \\
\frac{(p_j)^2}{\mathcal{L}_{jj}} \frac{\partial \mathcal{L}_i}{\partial \alpha} &= -\frac{(p_j)^2 \mathrm{e}^{\phi_i} \left(q_i^* + \gamma_i\right)^{\alpha-1} \left(1 + \alpha \ln \left[q_i^* + \gamma_i\right]\right)}{\alpha (1-\alpha) \mathrm{e}^{\phi_j} \left(q_j^* + \gamma_j\right)^{\alpha-2}}.
\end{aligned}$$

The relationship between the optimal quantity purchased of the i^{th} good and α, therefore, depends on the relationship between that good and all the other goods in the consideration set, in a complex way. It is possible that α increases the quantity purchased for some goods, while decreasing it for the remaining goods in the consideration set. The complexity that arises between the optimal quantity purchased and α is due to the fact that α is the same for all the goods. If α was instead product specific, the derivatives would be similar to those of γ_i and ϕ_i above. In other words, relaxing the definition of $\alpha_i = \alpha \; \forall i$ actually fails to clarify the relationship between the quantity purchased and α. That said, the relationship between the marginal utility of income λ^*, and α is more definitive. Specifically:

Proposition 13 *The marginal utility of income decreases as* α *moves from* 0 *to* 1 *so long as* $(q_i^* + \gamma_i) > e^{-\frac{1}{\alpha}}$*,* $\forall i$*.*

Proof. I have:

$$\frac{\partial\lambda^*}{\partial\alpha} = -\frac{\sum_{j=1}^{K} p_j \left(q_j^* + \gamma_j\right)\left(1 + \alpha \ln\left[q_j^* + \gamma_j\right]\right)}{\sum_{k=1}^{K} \frac{(p_k)^2}{\mathrm{e}^{\phi_k}\left(q_k^* + \gamma_k\right)^{\alpha-2}}}$$

■

Proposition 13 implies that the relationship between the consumer's utility and income will decrease as α increases from 0 to 1. In other words, as α gets closer to 0, changes in the consumer's income will lead to increasingly higher levels of utility. Therefore, α can be thought of as the degree of price sensitivity. Without going through the comparative statics, this same notion is clear from the functional form of the utility function given in equation (2.2). As α increases from 0 to 1 the contribution to the total utility of any $q_i^* > 0$ will be greater, and utility rises from those i goods that are purchased. More formally, I find $\frac{\partial \max U}{\partial \alpha} > 0$ (see Appendix for proof).

Finally, the consumer's total utility is defined over all searched products, not just the purchased products. As a result, all the products searched contribute to the consumer's maximum utility because the price-normalized marginal utility of the non-purchased products, while less than the purchased products, are all greater than 0. This is also evident from the functional form of equation (2.2). The utility function is defined for all K searched goods, so unless both the quantity purchased and the satisfaction for the product are both 0 the good will make a contribution to utility just by being in the consideration set. Moreover, since search costs are assumed to be positive, it would be a contradiction to the search rule if a consumer were to choose a consideration set with a product that has $\gamma_i = 0$. If the satisfaction parameter were zero, the consumer's expected maximum utility would be lower than the same consideration set that did not contain the i^{th} product. Therefore, all searched goods make a contribution to the consumer's total utility because the consumer is able to learn about the attributes and prices of purchased and non-purchased goods.

2.4 Conclusion

In this chapter, I extend the analysis of consumer search to consider the case when consumers have a preference for variety, which may lead them to purchase multiple products. I show that when consumer preferences for variety are increasing, consumers will search a larger number of products.

More search, however, does not necessarily lead to more purchase unless a truly preferred product is found. The main implication of this chapter is that consumer search models that ignore preference for variety may, therefore, underpredict the degree of search if consumers do indeed prefer variety.

While prior experimental consumer search studies (Dickson and Sawyer 1990) found that consumers did not search enough, more recent research (Zwick, Rapoport, Lo, and Muthukrishnan 2003) finds evidence of the opposite – consumers search too much relative to the optimal stopping point. The theoretical predictions of my model are consistent with more recent research and offer an explanation as to why consumers appear to search too much. Previous research rarely considered consumers' preference for variety when predicting search, and did not allow this preference for variety to manifest in multiple-purchases. My model offers one of the first explanations as to why consumers may over-search, namely because they prefer variety, and obtain utility from searching over multiple variants, so continue to search rather than purchase. I confirm this insight using an economic experiment later in a later chapter.

My analysis does have several limitations. First, the theoretical model of consumer search and preference for variety does not consider firm response to consumer demand. Within the framework above, it is assumed that the desired number of products are available to search. While this is likely to be a reasonable assumption in many consumer goods categories, particularly mature markets, it may be less applicable to highly innovative consumer product markets. Moreover, without considering firm behavior I can not comment on how equilibrium prices will response to increasing preferences for variety.

Second, I assume that consumers know the satiation rate, or rate of satisfaction, for every product, but not its attributes. Future work may benefit by relaxing this assumption and making the consumer's satiation parameter random. This would impart some realistic uncertainty regarding the order in which products are searched and information is acquired. Finally, I use a specific functional assumption for the utility consumers obtain in order to get precise predictions about the consideration set. Assuming a specific functional form leads to a closed form expression for the integral that describes the consumer's maximum expected utility from search. Future work may benefit by relaxing the functional form and instead consider a more general utility expression.

CHAPTER 3.

DEMAND FOR VARIETY UNDER COSTLY CONSUMER SEARCH

3.1 Introduction

Interest in understanding consumer search behavior has grown with the prominence of online search. Yet, the cost of search is more important to offline purchases, and offline commerce still dominates consumer packaged good sales. SymphonyIRI's MarketPulse survey (2011), for example, found that fully 40% of consumers shop at multiple stores to obtain the lowest price possible. Simply put, in order for a good or service to be sold, it must first be in a consumer's consideration set, and to be considered, it must be searched. Relatively little is known about how the composition of the consideration set affects the final purchase decision. If a single unit of one item is purchased, then the final choice decision may not be particularly sensitive to the composition of the consideration set (Honka and Chintagunta 2013). However, if consumers purchase multiple products in differing quantities, as is common in categories such as food and clothing, then the consideration set becomes more important. In this study, I develop a structural empirical model of consideration set formation when product information is uncertain, and an individual is allowed to purchase multiple products in continuous quantities.

Existing empirical models of search behavior examine retail environments that are highly specialized, and generally do not reflect the unique nature of search for products offered by multi-product retailers. Wildenbeest (2011) derives a structural model of search and product differentiation in which he estimates search costs from differences in aggregate, store-level prices. However, by aggregating prices over individual products into representative "shopping baskets" he ignores the multi-product nature of search within supermarkets. Further, he assumes, as do others, that consumers know the number of products to search, before search is undertaken (Zwick, Rapoport, Lo, and Muthukrishnan 2003). Deciding on the number of products to consider, however, is an integral element of search. In this study, I extend Mehta, Rajiv, and Srinivasan (2003) and Koulayev (2010) by developing a structural model of endogenous consideration set formation in the presence of product information uncertainty in a multi-product retail environment.

From a multi-product retailer's perspective, the number of products consumers actively consider is an important piece of information. Both the depth and content of a retailer's assortment are perhaps the most important decisions he or she makes (Koelemeijer and Oppewal 1999; Boatwright and Nunes 2001; Borle, Boatwright, Nunes, and Shmueli 2005; Oppewal and Koelemeijer 2005; Richards and Hamilton 2006; Briesch, Chintagunta, and Fox 2009). Moreover, there is an ongoing debate whether to expand or contract the number of products offered. Empirical evidence shows that consumers are worse off from having too many choices (Iyengar and Lepper 2000; Iyengar, Huberman, and Jiang 2004; Diehl and Poynor 2010; and Kuksov and Villas-Boas 2010) while others show consumers are better off with more options (Eaton and Lipsey 1979; Chernev 2003a, 2003b; Hutchinson 2005), particularly when the product category is a familiar one. Through a number of experiments, Berger, Draganska, and Simonson (2007) show that offering more products, even with only slight modifications, increases the perception of quality. Further, they show that a brand with more variants can command a competitive advantage. In contrast to a number of empirical studies (e.g. Dreze, Hoch, and Purk 1994; and Boatwright and Nunes 2001) Borle, Boatwright, Kadane, Nunes, and Shmueli (2005) find that large-scale assortment reductions harm store level sales by reducing customer retention, shopper frequency, and purchase quantity. Therefore, understanding which products consumers are not only purchase, but also consider, is fundamentally important to assortment and promotion decisions.[1]

Researchers rarely model the preference for variety in an explicit way. Yet, demand for variety leads to multiple purchases of either the same or competing brands, which invalidates many common models of consumer demand based on the discrete-choice assumption. For example, Dubé (2004) finds that only 49% of the trips involving carbonated soft drink purchases conform to a single purchase assumption. In other words, current demand-search models are not applicable to 51% of carbonated soft drink purchases. Dubé (2004) also shows that single-purchase demand models lead to incorrect marketing mix variables when consumers purchase multiple items at the same time. However, Dubé's (2004) model is only appropriate for products that are purchased in discrete

[1] Scheibehenne, Greifeneder, Todd (2010) conduct a meta-analysis that seeks to test the "choice overload hypothesis" or the idea that too many products negatively affect consumer welfare and can deter consumers from making a purchase. Their findings indicate that the "choice overload hypothesis" was not robust. Instead, consumers prefer a large number of options when their preferences are well defined.

increments, like cans of soda or bottles of ketchup. Recognizing that consumers can often purchase products in continuous quantities, Bhat (2005, 2008) uses a more general framework (based on Hanemann (1978, 1984) and Wales and Woodland (1983)) to extend the familiar logit model to allow for the observation that consumers can purchase multiple discrete brands in continuous quantities. Bhat's (2005, 2008) multiple-discrete continuous extreme model provides a realistic framework for many CPG purchases, but still ignores a key feature of retail grocery markets.

Namely, Bhat (2005, 2008) assumes search is costless, and consumers are perfectly informed. However, consumer uncertainty and the cost of search are critical to understanding demand (Stigler 1961). Even when searching online, Hann and Terwiesch (2003) show that consumers have a perceived search cost of about \$5 for each online search activity undertaken. Even if approximately accurate, search costs of this magnitude will almost certainly lead to some products being ignored. On the other hand, while the actual monetary cost of searching non-durable goods may not be as high the cost of inspecting, thinking about, and retaining product information provide a significant barrier to searching and considering all available products (Shugan 1980; and Roberts and Lattin 1991). Such cognitive search costs restrict consumers attention to a subset of the total products available. Therefore, my model accounts for costly search and allows for multiple products to be purchased in continuous quantities. In this way, I empirically estimate consumer behavior typical of many multi-product retail environments–supermarkets, department stores, club stores and many others–that dominate consumers' shopping experiences. By allowing for both interior and exterior solutions my model is also general enough to apply categories in which a single item is regularly purchased.

This study contributes to the literature on consumer demand and search in a number of ways. First, I account for the purchase of multiple products on each shopping occasion. Second, I recognize that consumers' preference for variety and cost of search are critical variables in shaping their consideration sets, and ultimately, product purchases. Third, my model provides a framework that allows practitioners and researchers to empirically determine the number of products consumers search through before making their purchase decision. The structural model I develop combines empirical search models (Mehta, Rajiv, and Srinivasan 2003; and Koulayev 2010) with the continuous/discrete demand model literature (Hanemann 1978, 1984; Kim, Allenby, and Rossi 2002; Bhat 2005, 2008;

and Satomura, Kim, and Allenby 2011). In so doing I extend both streams of research to provide a unified framework for modeling consumer purchase behavior.

3.2 A Multi-Discrete/Continuous Demand Model with Costly Consumer Search

In this section, I describe an empirical model of demand that allows for multiple discrete / continuous purchases, and positive search costs. While I develop the model for application to household-level scanner or survey data, it is sufficiently general to apply to data generated by any costly search-and-choice process.

Assume the set of products available to the consumer is written as $\mathbf{N} = \{1, 2, ..., N\}$. Given the set of products $\mathbf{N}$, consumers can purchase any combination of the $\mathbf{N}$ in any quantity. An individual's total utility is assumed to be the additive contribution provided by each searched *and* purchased product. In addition to product attributes and market prices, consumers also have some level of satisfaction for each product, and a preference for variety across the category. Total utility, U, from a particular shopping occasion, t, from purchasing multiple products within $\mathbf{N}$ is given by, omitting the time subscript t:

$$U = \sum_i \mathrm{e}^{\phi_i} \left(\mathrm{e}^{\varepsilon_i} \left(q_i + \gamma_i\right)\right)^{\alpha} \tag{3.1}$$

where q_i is the quantity purchased of good i ($q_i \geq 0$). γ_i is the satiation, or satisfaction, the consumer obtains from product i. γ_i is assumed constant across purchase occasions t, and well defined for the consumer but unknown to the researcher (Kim, Allenby, and Rossi 2002; Bhat 2005; and Satomura, Kim, and Allenby 2011). Product attributes are unknown prior to the purchase occasion and the consumer must undertake costly search in order to resolve this uncertainty. However, the consumer's satisfaction for the products, γ_i, is known. For example, when making a purchase in a category that is well-understood, an individual will know their satisfaction for products that have been purchased and consumed many times in the past. That said, the consumer is unaware of the specific price for each product prior to the shopping trip.

A larger γ_i represents a good that provides a higher degree of satisfaction. The parameter α captures the consumer's preference for variety, and is bound between 0 and 1 to ensure all goods are substitute goods as discussed in Chapter 2. As α increases from 0 to 1 the consumer's preference for variety decreases across all products equally, holding everything else constant.[2] Finally, e^{ϕ_i} captures product-specific characteristics such as shelf location, or the amount of sugar, or fat in ice cream for example. Baseline marginal utility represents the increment in utility for the first unit consumed. As discussed by Kim, Allenby, and Rossi (2002) and Bhat (2005), the utility function in (3.1) is a translated utility function that is valid so long as $e^{\phi_i} > 0$.

The utility equation (3.1) above is very similar to Bhat (2005), except for the error term e^{ε_i}. The error term e^{ε_i} represents product uncertainty, and is assumed to be independent and identically distributed (i.i.d.) Type 1 Extreme Value (or Gumbel). The consumer's uncertainty regarding product attributes affects utility in a fundamental way. Namely, the error term e^{ε_i} shifts the consumer's perceived utility up or down because it is inside the expression $\left(e^{\varepsilon_i}\left(q_i + \gamma_i\right)\right)^{\alpha}$. Research that allows for multiple product purchases assumes search is costless and consumers search across the entire set of available products (Bhat 2005; 2008; von Haefen and Phaneuf, 2005; Pinjari, and Bhat 2010; Satomura, Kim, and Allenby 2011; and Bhat, Castro, and Pinjari 2012). Instead, the utility function above captures the notion that information comes at a cost, and uncertainty must be mitigated before a purchase decision can be made. The cost of acquiring product information is c_i for the i^{th} product searched. By engaging in search and paying c_i, the consumer resolves the error, e^{ε_i}.

The uncertainty for brand i is eliminated when the consumer incurs the search cost c_i. The search cost, c_i, is known prior to search, and is allowed to differ across individual products. It captures the total cost of learning the complete set of information on a particular product, namely its price, sugar content, package size, etc.

I follow Mehta, Rajiv, and Srinivasan (2003) and Koulayev (2010) and assume consumers follow a fixed-sample search process.[3] An individual, therefore, chooses an optimal subset of products to search, $\mathbf{K} \subseteq \mathbf{N}$, so the total search cost is $\sum_{i \in \mathbf{K}} c_i$. For those products that are searched, the product's attributes are revealed and uncertainty alleviated, $\varepsilon_i = 0$, and the consumer makes his

[2]The interpretation of γ_i and α are manifestations of the model specification. For a discussion, see Bhat (2005; 2008).

[3]See Baye, Morgan, and Scholten (2006) for a review of the fixed sample size search literature. Honka (2013) provides a discussion regarding the differences between fixed-sample search, and sequential search.

purchase decision. However, the researcher is unable to observe all product attributes, so ϕ_i represents a random variable that is assumed to be independent and identically distributed (i.i.d.) Type 1 Extreme Value (Mehta, Rajiv, and Srinivasan 2003; Bhat 2005; 2008). Assuming ϕ_i is a random variable, or $\phi_i = \phi_i + \epsilon_i$, captures the notion that consumers know the attributes of the products searched, but they are not known to the researcher. Consumers have well defined beliefs about their general satisfaction with a product, γ_i, and their overall preference for variety, α, but only have a general idea regarding the attributes of the products, which leads to search. Expectations for product attributes are denoted $\bar{\phi}$, and the expectation for price is $\bar{p}$.

Once search is undertaken, the true ϕ_i and p_i's are revealed. The consumer then makes his purchase decision from the products in the chosen consideration set. However, ϕ_i remains unobservable to the researcher, so is treated as a random variable (Mehta, Rajiv, and Srinivasan 2003; Bhat 2005, 2008; Honka 2013). The analyst is able to observe the true prices, while the consumer's γ_i and α are parameters to be estimated. Consumers select the optimal consideration set based on the cost of searching each product, and then decide on the quantities to purchase subject to their remaining budget:

$$y - \sum_{i \in \mathbf{K}} c_i = \sum_{i \in \mathbf{I}} p_i q_i > 0 \tag{3.2}$$

where y is the total budgeted dollar value and $\mathbf{I} \subseteq \mathbf{K}$ is the set of products the consumer chooses to purchase within the consideration set $\mathbf{K}$. The total number of products in the sets $\mathbf{I}$ and $\mathbf{K}$ are I, and K respectively. Consumers decide how much money to commit to search while keeping in mind that the more search undertaken, the less they will be able to spend on the products themselves. It is assumed that consumers make their search decision such that they are able to purchase some positive quantity of at least one of the goods. It is straightforward to extend the utility function in equation (3.1) to include a "no purchase," or outside good option, that is costless. The no purchase assumption is common in the search literature when a single good is purchased (Chiang, Chib, and Narasimhan, 1999; Kim, Albuqurque, and Bronnenberg 2010; de los Santos, Hortacsu, and Wildenbeest 2012; Honka 2013; and Honka and Chintagunta 2013). Since my data does not have no purchase choice occasions I use the former interpretation and assume not all of the budget will be spent on search.

3.2.1 Consumer's Problem

Consumers choose purchase quantities conditional on exhausting some of their budget on search. I solve this problem by backward induction. Consumers first solve for the maximum possible utility that can be obtained for any consideration set $\mathbf{K}$, given attribute and price expectations. With this, the size (or number of products) of the optimal consideration set, $\mathbf{K}^*$, and the specific products in it, are determined simultaneously by finding the set that provides the highest expected maximum utility. At this point, the consumer must evaluate all possible subsets to find the best one.

Assuming a fixed-sample size search process leads to a dimensionality problem, because every possible permutation must be considered, leading to $2^N - 1$ possible consideration set evaluations. The dimensionality problem is a common manifestation of the fixed-sample size search assumption and restricts the applicability of the subsequent search-demand models to situations in which only a small number of options are available (Mehta, Rajiv, Srinivasan 2003). However, the multi-product purchase nature of my model avoids the curse of dimensionality, and leads to only N subsets requiring evaluation. Once the expected maximum utility has been found for all N subsets, the optimal consideration set is selected and consumers incur search costs. After the search stage, product attributes are known and utility (e.g. eqn. 3.1) is maximized. Specific products, and the quantity of each product, are selected subject to the remaining budget.

In the first stage, I begin by finding the optimal quantity of the i^{th} good given some set of purchased goods, $\mathbf{I}$, and a set of searched goods, $\mathbf{K}$, for all possible $i \in \mathbf{I} \subseteq \mathbf{K} \subseteq \mathbf{N}$. Maximum utility is found by maximizing the utility given in equation (3.1) for q_i, conditional on attribute and price expectations $\bar{\phi}$ and $\bar{p}$ respectively, subject to the budget constraint. Namely, I solve the problem:

$$\max \sum_{i \in \mathbf{I}} \mathrm{e}^{\bar{\phi}} \left(\mathrm{e}^{\varepsilon_i} \left(q_i + \gamma_i \right) \right)^{\alpha} \quad \text{sub. to} \;\; y = \bar{p} \sum_{i \in \mathbf{I}} q_i + \sum_{i \in \mathbf{K}} c_i \tag{3.3}$$

by forming the Lagrangian:

$$\mathcal{L} = \sum_{i \in \mathbf{I}} \mathrm{e}^{\bar{\phi}} \left(\mathrm{e}^{\varepsilon_i} \left(q_i + \gamma_i \right) \right)^{\alpha} + \lambda \left(y - \bar{p} \sum_{i \in \mathbf{I}} q_i - \sum_{i \in \mathbf{K}} c_i \right). \tag{3.4}$$

Notice in the problem given in equation (3.3) that the total cost of search is over all K products, while utility is only obtained from the I purchased products. Because product attributes, including price, are assumed uncertain at each purchase occasions, obtaining the information for products not purchased does not provide any benefit. Only those products that are purchased and consumed (i.e.

$q_i > 0$) provide the consumer with any benefit. The first order Kuhn-Tucker conditions (first-order conditions, or FOC) with respect to the i^{th} quantity purchased are:

$$\alpha e^{\bar{\phi}} \left(e^{\varepsilon_i} \left(q_i^* + \gamma_i\right)\right)^{\alpha-1} - \lambda\bar{p} = 0 \qquad \text{if } q_i^* > 0 \ \ \forall i \tag{3.5a}$$

$$\alpha e^{\bar{\phi}} \left(e^{\varepsilon_i} \left(q_i^* + \gamma_i\right)\right)^{\alpha-1} - \lambda\bar{p} < 0 \qquad \text{if } q_i^* = 0 \ \ \forall i \tag{3.5b}$$

$$y - \sum_{i \in \mathbf{K}} c_i = \bar{p} \sum_{i \in \mathbf{I}} q_i^*. \tag{3.5c}$$

The conditions (3.5a) and (3.5b) suggest that the marginal utility of all the purchased products are equal, and is greater than the marginal utility of the non-purchased goods. The optimal demand for q_i^* satisfies the conditions in equations (3.5) $\forall i \in \mathbf{N}$.

The optimal quantity demanded is found by taking the ratio of the i^{th} and j^{th} conditions given in (3.5a) and substituting the result into the budget constraint, (this is derived in detail in appendix A.2). Rearranging terms I find:

$$q_i^* = \frac{y - \sum_{k \in \mathbf{K}} c_k + \bar{p} \sum_{k \in \mathbf{I}} \gamma_k}{e^{\varepsilon_i} \bar{p} \sum_{k \in \mathbf{I}} e^{-\varepsilon_k}} - \gamma_i, \quad \forall q_i^* > 0. \tag{3.6}$$

3.2.2 Consideration Set Formation

Consumers choose the optimal quantity of each good to purchase based upon expectations about product attributes and prices. Let the products in $\mathbf{K}$ be indexed $1, 2, ..., I, I+1, ..., K$ where $1, 2, ..., I$ are the products purchased, and $I+1, ..., K$ are the products searched but not purchased. The maximum utility for any set of products $\mathbf{K}$, conditional on expected attributes and prices, is given by:

$$\max U_{\mathbf{K}} = e^{\bar{\phi}} \sum_{i=1}^{I} \left(\frac{\left(y - \sum_{k=1}^{K} c_k + \bar{p} \sum_{k=1}^{I} \gamma_k\right)}{\bar{p} \sum_{k=1}^{I} e^{-\varepsilon_k}} \right)^{\alpha}. \tag{3.7}$$

In this expression, the entire search cost, $\sum_{k=1}^{K} c_k$, is absorbed by, or spread across, all the purchased goods. In contrast to single purchase search-demand models, the maximum utility above recognizes that consumers have a preference for variety that directly governs the number of different products purchased, and the number of products searched. As the preference for variety increases, both the number of products searched, and the number of products purchased increase. So, while a higher preference for variety leads to a larger total search cost, this cost is spread over a number of unique products. On the other hand, equation (3.7) above implies that consumers with little preference for variety will search and purchase fewer unique products. My model, therefore, captures choice situations in which an individual may only purchase a single product, or the behavior described by classic discrete choice models, but also allows for multi-product purchase situations. The maximum total utility is written as:

$$\max U_{\mathbf{K}} = I\left(A_{\mathbf{K}}\right)^{\alpha}\left(\sum_{k=1}^{I} \mathrm{e}^{-\varepsilon_k}\right)^{-\alpha} \tag{3.8}$$

where $A_{\mathbf{K}} = \bar{p}^{-1}\left(y - \sum_{k=1}^{K} c_k\right) + \sum_{k=1}^{I} \gamma_k$, and $\bar{\phi}$ is normalized to 0 without loss of generality as its role is simply to shift the maximum utility up or down by $\mathrm{e}^{\bar{\phi}}$. Attribute expectations, therefore, will affect the total value of the $\max U_{\mathbf{K}}$ but do not drive the consideration set decision. The error expression $\sum_{k=1}^{I}\mathrm{e}^{-\varepsilon_k}$ in the $\max U_{\mathbf{K}}$ is Erlang$(I, 1)$ distributed with a mean and variance of I.[4] distributed Equation (3.8) provides the consumer's net benefit from obtaining product attribute information for any subset of goods $\mathbf{K}$, conditional on his expectations for prices $\bar{p}$.

Knowing that the error associated with product uncertainty, ε_i, is Gumbel distributed the expected benefit from any subset of products $\mathbf{K} \subseteq \mathbf{N}$ is given by (which is derived in detail in Appendix B):

$$E[\max U_{\mathbf{K}}] = \int_{-\infty}^{\infty} \cdots \int_{-\infty}^{\infty} \left(\max U_{\mathbf{K}}\right) f(\varepsilon_1, \varepsilon_2, ..., \varepsilon_N) \mathrm{d}\varepsilon_1 \mathrm{d}\varepsilon_2 \cdots \mathrm{d}\varepsilon_N \tag{3.9a}$$

$$= \left(A_{\mathbf{K}}\right)^{\alpha} I \int_{-\infty}^{\infty} \cdots \int_{-\infty}^{\infty} \left(\sum_{k=1}^{I} \mathrm{e}^{-\varepsilon_k}\right)^{-\alpha} \prod_{j=1}^{I} \mathrm{e}^{-\varepsilon_j} \prod_{j=1}^{I} \mathrm{e}^{-\mathrm{e}^{-\varepsilon_j}} \mathrm{d}\varepsilon_1 \mathrm{d}\varepsilon_2 \cdots \mathrm{d}\varepsilon_I$$

$$= \left(\frac{y}{\bar{p}} + \sum_{k=1}^{I}\left\{\gamma_k - \frac{c_k}{\bar{p}}\right\} - \sum_{k=I+1}^{K} \frac{c_k}{\bar{p}}\right)^{\alpha} \frac{I\,\Gamma\left[I - \alpha\right]}{\Gamma\left[I\right]}, \tag{3.9b}$$

where $\Gamma[\bullet]$ is the Gamma function.[5] The expression $\max U_{\mathbf{K}}$ gives the maximum possible utility that can be attained for any income, price expectation, product satisfaction, preference for variety, *and* cost of search. An important distinguishing feature of my multi-product purchase model from a classic search-demand logit model is that the expected maximum utility expression in equation (3.9b) above is net of search costs because the budget constraint is recognized. The $\max U_{\mathbf{K}}$ is takes into account the cost of search and how the other parameters, namely preference for variety and price

[4]The Erlang distribution is a special case of the Gamma distribution because I is an integer.

[5]The Gamma function is defined as $\Gamma[z] = \int_0^{\infty} t^{z-1}\mathrm{e}^{-t}\mathrm{d}t$. When z is a positive integer $\Gamma[z]$ simplifies to $(z-1)!$.

expectation, affect the way the cost of search influences $\max U_{\mathbf{K}}$. That is, a consumer's maximum utility is determined, the expectation taken, and search costs subtracted. The optimal consideration set is then determined from the difference between the expected maximum utility and the total cost of search, or $\max\{E[\max u] - \sum_i c_i\}$, where u is the indirect utility.[6] Because my model is derived under the assumption that consumers maximize utility subject to a budget constraint, the cost search is directly taken into account when determining the maximum utility.

From the expected maximum utility equation above, it is clear that the consumer wants to select a consideration set, $\mathbf{K}$, as close to the set of goods purchased, $\mathbf{I}$, as possible. Intuitively, the consumer has a limited amount of money to spend during a particular purchase occasion and does not want to pay for product information that provides them no benefit. However, before undertaking search, it is impossible to determine which goods will ultimately be purchased since this hangs critically on the price and product information an individual obtains only after search. Therefore, when deciding which consideration set to choose, it is assumed that $I = K$, or that all possible consideration sets are comprised of goods the consumer expects to consume. This is equivalent to assuming the global set of products $\mathbf{N}$ is made up of products that are appealing on some level. Intuitively, I am assuming that a consumer will not spend money and obtain product information for a product they know *a priori* they will not possibly purchase.

Consumers choose the consideration set that provides the greatest expected benefit, or the one that provides the highest $E[\max U_{\mathbf{K}}]$ for any possible $\mathbf{K} \subseteq \mathbf{N}$. The optimal size and composition of the consideration set is, therefore, determined by:

$$\mathbf{K}^* = \arg\max_{\mathbf{J}} \left\{ \left(\frac{y}{\bar{p}} + \sum_{k \in \mathbf{J}} \left\{ \gamma_k - \frac{c_k}{\bar{p}} \right\} \right)^{\alpha} \frac{|\mathbf{J}| \;\; \Gamma\left[|\mathbf{J}| - \alpha\right]}{\Gamma[|\mathbf{J}|]} \right\}, \tag{3.10}$$

where $|\mathbf{J}|$ represents the size, or number of products in the set $\mathbf{J} \subseteq \mathbf{N}$. Since $y/\bar{p}$ is fixed, consumers choose products to include in $\mathbf{K}^*$ based on the difference between their satisfaction for the product, γ_k, and the price normalized cost of searching for that product, $\frac{c_k}{\bar{p}}$. Therefore, products are add to the consideration set based on the ordering $\gamma_i - \frac{c_i}{\bar{p}} \geq \gamma_j - \frac{c_j}{\bar{p}} \geq \cdots \geq \gamma_k - \frac{c_k}{\bar{p}}$. Let $G_i = \gamma_i - \frac{c_i}{\bar{p}}$. Because price expectation $\bar{p}$, satisfaction γ_k, and search cost c_k, are all known prior to search consumers order the global set of products $\mathbf{N}$ by G_i, so the set of products in $\mathbf{N}$ is indexed $G_1 \geq G_2 \geq \cdots \geq G_N$.

[6] Anderson, de Palma, and Thisse (1992) provide the derivation of $E[\max u]$.

Knowing the order to add products to the choice set significantly reduces the total number of possible consideration sets from $2^N - 1$ to N. However, the difference between G_i and G_i only determines the other products are added. The size of the consideration set is determined by the consideration set $\mathbf{J}$ that provides the largest $E[\max U_{\mathbf{K}}]$. The right hand side of equation (3.10), $\left(\Gamma[|\mathbf{J}|]\right)^{-1} |\mathbf{J}| \, \Gamma\left[|\mathbf{J}| - \alpha\right]$, and α are primarily responsible for determining the size of the consideration set which is driven by the consumer's preference for variety.

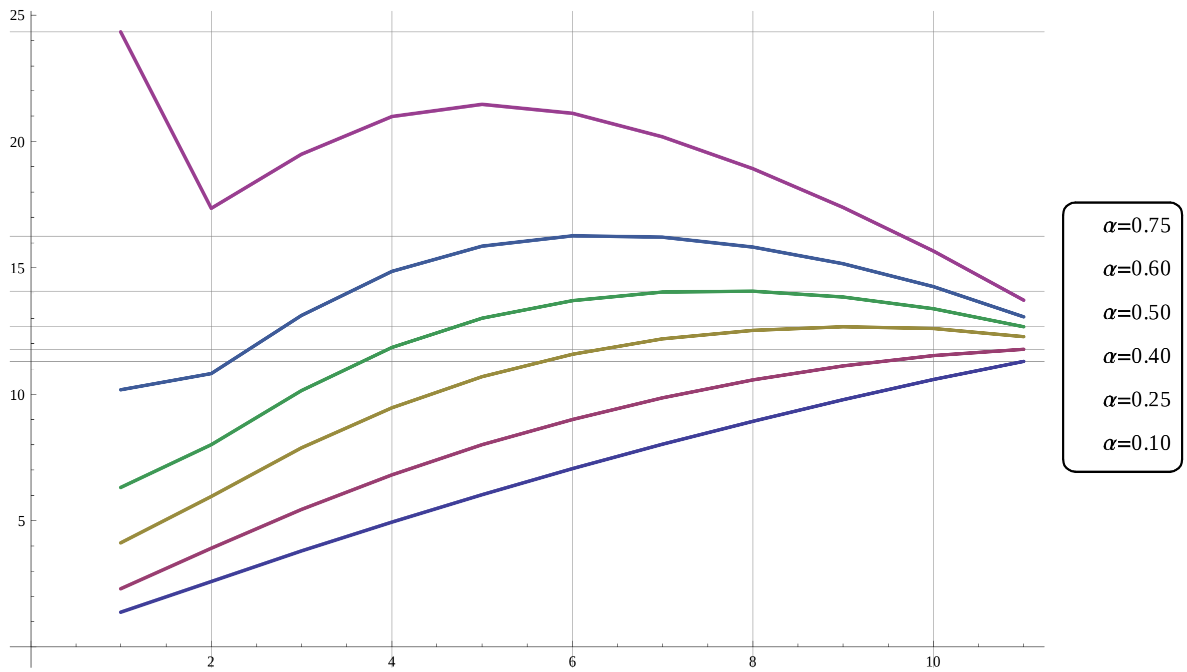

Figure 3.1

Example of $E[\max U_{\mathbf{K}}]$ for different values of α

To make help make this idea concrete consider the following example. The global set of products $\mathbf{N}$ contains 11 items ordered according to $G_1 \geq G_2 \geq \cdots \geq G_N$. The specific values for G_i, $i = 1, ..., 11$ are: {7.67, 7.67, 5.53, 2.69, 0.25, -1.37, -2.28, -2.71, -2.89, -2.96, -2.99} and let $\frac{y}{\bar{p}} = 5$. The $E[\max U_{\mathbf{K}}]$ values are shown in figure 3.1 for different values of α. From the figure it is obvious that the optimal consideration set consisting of only the first product will be selected when $\alpha \geq 0.75$. This is the result of the right hand side of the expectation because as $\Gamma\left[1 - \alpha\right] \to \infty$ as $\alpha \to 1$. As α decreases and moves toward 0 preference for variety increases and $\mathbf{K}^* = 6$ when $\alpha = 0.6$. As α gets close to 0 the specific values of G_i no longer matter and there is an overwhelming preference for variety and $\mathbf{K}^* = 11$. The non-linearity of the curves is a result of $G_i < 0$ for some of the products in $\mathbf{N}$. Both γ_i and c_i are greater than 0 however, there is no reason why $\gamma_i > c_i$ and so negative values cause a non-linear shape when $E[\max U_{\mathbf{K}}]$ is calculated over the N possible consideration sets. If $G_i > 0\ \forall i$ then the curves will slope downward, or upward depending on α, and the consideration sets at the extreme will provide the highest $E[\max U_{\mathbf{K}}]$.

The size of the optimal consideration set depends not only on the comparison between each product's expected contribution, but also on the preference for variety. From the analyst's perspective the probability that the $\mathbf{K}^{\text{th}}$ consideration set is chosen is equivalent to the probability that $E[\max U_{\mathbf{K}}]$ is the largest, or $E[\max U_{\mathbf{K}}] = \max\{E[\max U_{\mathbf{1}}], ..., E[\max U_{\mathbf{N}}]\}$. Then, the cumulative distribution function for the probability that $\mathbf{K}$ is the optimal consideration set is given by:

$$\begin{aligned} & \Pr[\mathbf{K} = \mathbf{K}^*] && (3.11)\\ = \; & \Pr[E[\max U_{\mathbf{K}^*}] = \max\{E[\max U_{\mathbf{1}}], ..., E[\max U_{\mathbf{N}}]\}] \\ = \; & \Pr[E[\max U_{\mathbf{1}}] < E[\max U_{\mathbf{K}}]] \cap \cdots \cap \Pr[E[\max U_{\mathbf{N}}] < E[\max U_{\mathbf{K}}]]. \end{aligned}$$

Looking at the $\mathbf{K}^{\text{th}}$ and $\mathbf{J}^{\text{th}}$ consideration set I find:

$$\begin{aligned} & \Pr[E[\max U_{\mathbf{J}}] < E[\max U_{\mathbf{K}}]] && (3.12a)\\ = \; & \Pr\left[\left(\frac{y}{\bar{p}} + \sum_{i=1}^{J}\left\{\gamma_i - \frac{c_i}{\bar{p}}\right\}\right)^{\alpha} \frac{J\Gamma[J-\alpha]}{\Gamma[J]} < \left(\frac{y}{\bar{p}} + \sum_{i=1}^{K}\left\{\gamma_i - \frac{c_i}{\bar{p}}\right\}\right)^{\alpha} \frac{K\Gamma[K-\alpha]}{\Gamma[K]}\right] \\ = \; & \Pr\left[\begin{array}{l} \sum_{i=1}^{J}\gamma_i - \left(\frac{K\Gamma[K-\alpha]\Gamma[J]}{J\Gamma[K]\Gamma[J-\alpha]}\right)^{\alpha^{-1}} \sum_{i=1}^{K}\gamma_i \\ \quad < \left(\frac{K\Gamma[K-\alpha]\Gamma[J]}{J\Gamma[K]\Gamma[J-\alpha]}\right)^{\alpha^{-1}} \left(\frac{y}{\bar{p}} - \sum_{i=1}^{K}\frac{c_i}{\bar{p}}\right) - \left(\frac{y}{\bar{p}} - \sum_{i=1}^{J}\frac{c_i}{\bar{p}}\right) \end{array}\right]. && (3.12b) \end{aligned}$$

While the consumer is aware of their satisfaction, γ_i, for each individual product, the analyst is not. As a result, satisfaction is a random variable from the analyst's perspective. I assume that $\gamma_i \sim N[\hat{\gamma}_i, \sigma_\gamma]$ and denote $\Phi\left[\mu, \sigma^2 | \varkappa\right]$ as the cumulative distribution function of the Normal distribution whose mean and variance are μ and σ^2 respectively, evaluated at the point $\varkappa$. Notice that γ_i represents a random variable for each product, not each consideration set. While the random γ_i's govern the form of the density function of the consideration sets, and therefore $E[\max U_{\mathbf{K}}]$, the γ_i's are product specific and represent random variables across individual choices, assuming they are i.i.d. If the distribution of the consideration set, or $E[\max U_{\mathbf{K}}]$, were driven by a specific error term associated with a particular consideration set, the error terms would not be considered independent from one another. Consider the hypothetical error term associated with the $\mathbf{K}^{\text{th}}$ and $\mathbf{J}^{\text{th}}$ consideration sets e_K and e_J respectively. If $K < J$ then $\mathbf{K} \subset \mathbf{J}$ which necessarily implies that the error term associated with the $\mathbf{J}^{\text{th}}$ consideration set, e_J, is a function of e_K. In other words, associating an error component with a consideration set implies that the error would increase as the number of products in the consideration set increases. To avoid this, I follow Mehta, Rajiv, and Srinivasan (2003) and recognize the analyst's inability to perfectly observe the consumer's satisfaction for individual products. Assuming γ_i is a normally distributed random variable also accounts for different product specific satisfaction ratings across consumers. Accounting for consumer heterogeneity within the satisfaction parameters, γ_i, is particularly important as they serve a critical role in determining which products are purchased, as well as the composition of the chosen consideration set. I address how the Normality of the γ_i's are incorporated into the purchase decision in the next section.

With the assumption that the γ_i's are Normally distributed the density function of $\Pr\left[E[\max U_{\mathbf{J}}] < E[\max U_{\mathbf{K}}]\right]$ is Normally distributed with a mean and variance given by: $\mu_J = \sum_{i=1}^{J} \hat{\gamma}_i - A_J \sum_{i=1}^{K} \hat{\gamma}_i$ and $\sigma^2_{\gamma J} = \sigma^2_{\gamma}\left(J + K\left(A_J\right)^2\right)$ respectively, where A_J and $\varkappa_J$ are defined as:

$$A_J = \left(\frac{K\Gamma\left[K-\alpha\right]\Gamma[J]}{J\Gamma[K]\Gamma\left[J-\alpha\right]}\right)^{\alpha^{-1}} \tag{3.13}$$

$$\varkappa_J = \bar{p}^{-1}\left(A\left(y - \sum_{i=1}^{K} c_i\right) - \left(y - \sum_{i=1}^{J} c_i\right)\right) \tag{3.14}$$

Knowing the density of $\Pr\left[E[\max U_{\mathbf{J}}] < E[\max U_{\mathbf{K}}]\right]$ I can invoke the extreme value theorem to find the probability that the $\mathbf{K}^{\text{th}}$ consideration set provides the highest expected maximum (indirect) utility:

$$\Pr\left[\mathbf{K} = \mathbf{K}^*\right] \tag{3.15a}$$

$$\begin{aligned} &= \Pr\left[E[\max U_{\mathbf{K}}] > E[\max U_{\mathbf{1}}]\right] \cap \cdots \cap \Pr\left[E[\max U_{\mathbf{K}}] > E[\max U_{\mathbf{N}}]\right] \\ &= \prod_{i=1}^{N} \Phi\left[\mu_i, \sigma^2_{\gamma i} | \varkappa_i\right]. \end{aligned} \tag{3.15b}$$

The probability expression given in equation (3.15b) describes the probability that the $\mathbf{K}^{\text{th}}$ available consideration set is the one that yields the highest expected maximum utility, $E[\max U_{\mathbf{K}}]$. It is assumed that individuals knows their satisfaction for each product, γ_i, and search to resolve their uncertainty surrounding product attributes. Therefore, when selecting the optimal consideration set, each consumer calculates the expected maximum utility with precision. The analyst, on the other hand, does not directly observe individual product satisfaction, so γ_i are random variables that drive the shape of the density function $\Pr\left[E[\max U_{\mathbf{K}}] > E[\max U_{\mathbf{1}}]\right]$ and the subsequent cumulative distribution function $\Phi\left[\mu, \sigma^2|\varkappa\right]$. The analyst, therefore, is only able to describe the probability that an individual consideration set is optimal, which leads to equation (3.15b) above.

3.2.3 Purchase Selection

After the optimal consideration set $\mathbf{K}^*$ is chosen, consumers pay the cost of search and observe product attributes and prices. Once consumers observe prices and attributes, they choose the optimal quantity of each good based on the solution to equation (3.6) in which prices and attributes are known. The researcher, on the other hand, is still uncertain about the true attributes for the products in the consideration set. The attributes ϕ_i are random variables from the analyst's perspective and are assumed Gumbel distributed. Therefore, the researcher observes the consumer's purchases that solve the following conditional first order conditions (FOCs):

$$\mathrm{e}^{\phi_i+\epsilon_i}\alpha\left(q_i^*+\gamma_i\right)^{\alpha-1}-\lambda p_i = 0 \qquad \text{if } q_i^*>0 \;\; \forall i\in\mathbf{K} \tag{3.16a}$$

$$\mathrm{e}^{\phi_i+\epsilon_i}\alpha\left(q_i^*+\gamma_i\right)^{\alpha-1}-\lambda p_i < 0 \qquad \text{if } q_i^*=0 \;\; \forall i\in\mathbf{K} \tag{3.16b}$$

$$y-\sum_{i=1}^{K}c_i = \sum_{i=1}^{I}p_i q_i^*. \tag{3.16c}$$

Only $K-1$ of the above equations need to be solved because the optimal quantity for one of the goods is automatically determined from the budget constraint in equation (3.16b) as search costs are assumed spent. Therefore, I solve for λ_i in terms of the specific product, whose optimal quantity purchased is determined from the budget constraint. The conditional FOCs given in equations (3.16) can be re-written, after taking logarithms, as:

$$W_i+\epsilon_i = W_1+\epsilon_1 \qquad \text{if } q_i^*>0 \;\; \forall i\in\mathbf{K} \tag{3.17a}$$

$$W_i+\epsilon_i < W_1+\epsilon_1 \qquad \text{if } q_i^*=0 \;\; \forall i\in\mathbf{K}, \quad \text{where} \tag{3.17b}$$

$$W_i = \phi_i+\ln\alpha+(\alpha-1)\ln\left[q_i^*+\gamma_i\right]-\ln p_i, \quad \forall i \tag{3.17c}$$

where $\phi_i = \beta^\intercal x_i$ and x_i are product specific observations that help characterize each product's attributes. For expositional purposes let $i=1$ represent the product that is always purchased. Similar to Bhat (2005, 2008) I cannot identify a constant term for one of the products because only differences between W_i and W_1 matter.[7] Consequently the constant in β that corresponds to the first product is omitted. The definition of W_i requires $\gamma_i > 0\ \forall i$ and α to be between 0 and 1. As a result, γ_i is estimated as $\exp[\eta_i]$ while α is estimated as $\exp[\delta]/\left(1+\exp[\delta]\right)$.

Define the optimal bundle of goods purchased as $\mathbf{q}^* = \{q_1^*, q_2^*, \cdots, q_I^*, 0, \cdots, 0\}$. Following Wales and Woodland (1983), and assuming that the ϵ_i's are independent of x_i, the probability that the bundle $\mathbf{q}^*$ is purchased is:

$$\begin{aligned}\Pr[\mathbf{q}^*|\epsilon_1, \mathbf{K}^*] &= \left[|\mathbf{J}| \prod_{i=2}^{I} g\left(W_1 - W_i + \epsilon_1\right)\right]\left[\prod_{j=I+1}^{K} G\left(W_1 - W_i + \epsilon_1\right)\right] \\ &= \left(\prod_{i=2}^{I} \mathrm{e}^{-(W_1 - W_i)}\right)\left(\mathrm{e}^{-\varepsilon_1}\right)^{I-1}\left(\prod_{i=2}^{K} \mathrm{e}^{\mathrm{e}^{-(W_1 - W_i + \epsilon_1)}}\right)\end{aligned} \tag{3.18}$$

where $|\mathbf{J}|$ is the determinant of the Jacobian term whose elements are given by:

$$\begin{aligned}J_{ih} &= \frac{\partial(W_1 - W_{i+1} + \epsilon_1)}{\partial q_{h+1}^*}, \quad i, h = 1, ..., I-1 \Rightarrow \\ |\mathbf{J}| &= (1-\alpha)^{I-1}\left(\prod_{i=1}^{I} p_i\left(q_i^* + \gamma_i\right)\right)^{-1} \sum_{i=1}^{I} p_i\left(q_i^* + \gamma_i\right).\end{aligned} \tag{3.19}$$

The probability that $\mathbf{q}^*$ is purchased, conditional on $\mathbf{K}^*$, which is derived in the appendix of Bhat (2005), is:

$$\begin{aligned}\Pr[\mathbf{q}^*|\mathbf{K}^*] &= |\mathbf{J}| \prod_{i=2}^{I} \mathrm{e}^{W_i - W_1} \int_{-\infty}^{\infty} \left(\mathrm{e}^{-\varepsilon_1}\right)^{I-1}\left(\prod_{i=2}^{K} \mathrm{e}^{-\mathrm{e}^{-(W_1 - W_i + \epsilon_1)}}\right) \mathrm{e}^{-\varepsilon_1} \mathrm{e}^{\mathrm{e}^{-(W_1 - W_i + \epsilon_1)}} \mathrm{d}\varepsilon_1 \\ &= (1-\alpha)^{I-1}\left(\frac{\Gamma[I]}{p_1}\right) \frac{\left(\sum_{i=1}^{I} p_i\left(q_i^* + \gamma_i\right)\right)\left(\prod_{i=1}^{I} \mathrm{e}^{W_i}\right)}{\left(\prod_{i=1}^{I} p_i\left(q_i^* + \gamma_i\right)\right)\left(\sum_{i=1}^{K} \mathrm{e}^{W_i}\right)^{I}},\end{aligned} \tag{3.20}$$

where p_1 is the price of the outside option.

If search were assumed costless then $\Pr[\mathbf{q}^*|\mathbf{K}^*] = \Pr[\mathbf{q}^*]$ and K would be N, the total number of products available. Recognizing that consumers will incur some kind of cost to acquire product specific information, the conditional probability expression above is conditional on the subset of

[7] See Honka (2013) for a method on obtaining the parameter estimate for every product when the consideration set is observed.

products $\mathbf{K}^* \subseteq \mathbf{N}$, which is often unobserved. If the analyst was able to observe the consideration set, then $\mathbf{K}^*$ would be known and equation (3.20) could be estimated as an unconditional probability expression where the term $\sum_{i=1}^{K} \mathrm{e}^{W_i}$ is defined over the $i = 1, ..., K$ products in the consideration set. A number of studies assume the consideration set is observed, so differences between fixed-sample size and sequential search strategies can be investigated (de los Santos, Hortacsu, and Wildenbeest 2012; and Honka 2013). However, precisely observing which products a consumer considered before making a purchase is usually the exception rather than the rule. Often, only the final purchase outcome is available, which is the case with the ice cream data.

The probability expression above is implicitly conditional on $\gamma_i \sim N[\hat{\gamma}_i, \sigma_\gamma]$ because it is conditional on $\mathbf{K}^*$. As Bhat (2008) points out, it is straightforward to incorporate the error the analyst makes in observing the consumer's product satisfaction γ_i through the use of mixing distributions. In particular, I decompose the error term ϵ_i into two separate components ζ_i and ξ_i. The first term ζ_i captures the error made observing the product attributes and is i.i.d. standard Gumbel distributed. The second component ξ_i, captures the error associated with observing product satisfaction, γ_i, and is Normally distributed. The unconditional probability that the bundle of goods $\mathbf{q}^*$ is purchased from within the optimal consideration set $\mathbf{K}^*$ is given by:

$$\Pr[\mathbf{q}^*] = \int_{\boldsymbol{\gamma}} \sum_{\mathbf{K}} \left[\Pr[\mathbf{q}^*|\mathbf{K}] \Pr\left[\mathbf{K} = \mathbf{K}^*\right] \right] \Phi_{\boldsymbol{\gamma}} \mathrm{d}\boldsymbol{\gamma}, \tag{3.21}$$

where $\boldsymbol{\gamma}^\top = \{\gamma_1, ..., \gamma_K\}$ and $\Phi_{\boldsymbol{\gamma}}$ is the cumulative Normal distribution. The conditional probability expression $\Pr[\mathbf{q}^*|\mathbf{K}]$ is given in equation (3.21) and $\Pr\left[\mathbf{K} = \mathbf{K}^*\right]$ is defined in (3.15b). The probability expression above is analogous to Mehta, Rajiv, and Srinivasan (2003) except the probability that the bundle of goods $\mathbf{q}^*$ is purchased instead of the probability that an individual good is selected. The term W_i is defined as $W_i = \phi_i + \ln \frac{\alpha}{p_i} + (\alpha - 1) \ln\left[q_i^* + \gamma_i\right], \quad \forall i \in \mathbf{N}$ and is derived in equation (3.17c) above.

3.3 Estimation

I estimate the search-demand model given in equation (3.21) using simulated maximum likelihood. Given there is no closed form solution to the probability expression, simulated maximum likelihood will provide consistent parameter estimates. To obtain starting values for the ice cream application, I first estimate equation (3.21) assuming $\mathbf{K} = \mathbf{N}$ $(\Pr[\mathbf{q}^*|\mathbf{K}^* = \mathbf{N}])$, or that consumers have no search cost and search across all products. Random numbers are generated for the starting values of $\Pr[\mathbf{q}^*|\mathbf{K}^* = \mathbf{N}]$ and the model is estimated 100 times. The average parameter estimates from the 100 runs are then averaged and used as the starting values to estimate $\Pr[\mathbf{q}^*|\mathbf{K}^* = \mathbf{N}]$ one last time. The results of this model are then used as the starting values for equation (3.21).

The search-demand model also requires observing the household's income. Since I cannot observe their income perfectly, I approximate it as $\hat{y} = \mathbf{p}^\intercal\mathbf{q}+k$, where $\mathbf{p}$ and $\mathbf{q}$ are the observed prices, and quantities purchased, respectively and k is a constant that is explained in more detail below. To be consistent with my derivation above, search costs should be added to the estimate of income, as well as the total spent on the goods. The first term in the expected maximum utility given in equation (3.10) shows:

$$\left(\frac{\hat{y}}{\bar{p}} + \sum_{k\in\mathbf{J}}\left\{\gamma_k - \frac{c_k}{\bar{p}}\right\}\right)^\alpha \tag{3.22a}$$

$$\left(\frac{\mathbf{p}^\intercal\mathbf{q}}{\bar{p}} + \sum_{k\in\mathbf{J}}\gamma_k\right)^\alpha \tag{3.22b}$$

where $\check{y} = \mathbf{p}^\intercal\mathbf{q}+\sum_{k\in\mathbf{J}}\frac{c_k}{\bar{p}}$ is substituted in for the estimate of income in equation (3.22b) and search costs drop out. I estimate the model for both definitions of income given in equations (3.22) above. For equation (3.22a) the expression inside the parentheses must be great than zero because $\alpha < 1$. The constant k is defined such that it is sufficiently large to guarantee $\frac{\hat{y}}{\bar{p}} + \sum_{k\in\mathbf{J}}\left\{\gamma_k - \frac{c_k}{\bar{p}}\right\} > 0.$[8]

The cost of searching across the prices and product attributes of different brands, c_k, is the cost incurred by a consumer to ascertain prices and product information of a particular product on a given purchase occasion. The search cost captures a range of costs the consumer may have related to search, such as the time spent searching, the physical cost of searching, or the mental cost of retaining price and product information. As any of these costs increase for a particular product, so does the search cost associated with the product. Since these costs cannot be observed directly from purchase level data, I follow the convention and estimate them as $c_k = \mathbf{X}\boldsymbol{\beta}$ where $\boldsymbol{\beta}$ is a vector of parameters to be estimated, and $\mathbf{X}$ is a matrix of variables. Specifically, $\mathbf{X}$ is the brand-specific variance of prices, brand loyalty, number of products available in the market, the square of the number of products available, household income, and the variance of prices at the market level. The

[8] We set $k = 1000$.

brand-specific price variance is measured as the variance of prices for each particular brand across all retailers within the market the consumer shopped. Since Stigler (1961) a rich body of research exists that argues there is a direct relationship between the dispersion of prices in the market and the cost of search. Brand loyalty is defined as 1 if the brand was purchased in the previous period, and 0 otherwise (Mehta, Rajiv, and Srinivasan 2003). The number of products available in the market may help reduce consumer's search cost (Koelemeijer and Oppewal 1999; Boatwright and Nunes 2001; Borle, Boatwright, Nunes, and Shmueli 2005; Oppewal and Koelemeijer 2005; Richards and Hamilton 2006; Briesch, Chintagunta, and Fox 2009). However, studies have also shown that too many products can overwhelm a consumer and deter him from making a purchase at all (Iyengar and Lepper 2000; Chernev 2003; Shah and Wolford 2007; and Mogilner, Rudnick, and Iyengar 2008), so I include the number of products squared to allow for a non-linear relationship. An individual's income represents their opportunity cost of time, so higher income increases the cost of search. Finally, the variance in prices at the market level is also included. This is measured as the variance across all products in the area the household was observed making a purchase. I follow Andrews and Srinivasan (1995) and Bronnenberg and Vanhonacker (1996) and eliminate the intercept term from the definition of search as they cannot be separately identified from γ_k.[9]

3.4 Monte Carlo Experiment

In this section, I describe a Monte Carlo simulation experiment that is designed to asses the performance of the proposed model, and to determine the nature and extent of any bias associated with assuming search is costless. In particular, I want to ensure that my estimation algorithms are able to recover the true parameters used for the data generating process. I also want to see how accurate the proposed model is at predicting the correct consideration set.

I use a simplified version of the model developed above for the Monte Carlo experiment. Namely, I simulate 1000 observations of data, and $N = 8$ choices. Consumer utility is defined according to equation (3.1) above with:

$$\phi_i = \hat{\phi}_{i1} + \hat{\phi}_{i2} x_{it} + \zeta_i, \tag{3.23}$$

[9] See Mehta, Rajiv, and Srinivasan (2003) for a discussion and an alternative approach.

where $\hat{\phi}_{i1}$ are choice specific intercept terms, and $\hat{\phi}_{i2}$ is a choice specific parameter on x_{ij} which varies over choice and time and follows a uniform distribution. The analyst observes x_{it} but not $\hat{\phi}_{i2}$. Prices are generated following the exponent of a uniform distribution to ensure they are all positive. They are then rounded to 2 decimal places. Search costs are generated by starting at 4 and adding the absolute value of a normal distribution whose mean is 0 and variance is 10. A large variance is used to ensure there are both positive and negative G_i values. Search costs vary over time and choice and are not assumed to be the same for each simulated product search decision. For the $i = 1$ good, price is normalized to 1 and search costs to 0 for all observations. Additionally, $\phi_1 = \zeta_i$ and income is fixed at \$100.

A consumer determines the optimal consideration set based on their expected maximum utility defined in equation (3.9b). The consumer's price expectation is defined as the mean of the randomly generated prices. The parameter γ_k is the same over all t but differs across prices. The true γ_k form the mean of the randomly generated error term ξ_i, with standard deviation $\sigma_\gamma = 1$, so $\gamma_i = \gamma_i + \xi_i$ where ξ_i is standard Normal distributed. ζ_i is distributed according to the standard Gumbel distribution.

The consumer determines which products to include in the consideration set based on the cost of search c_k which varies over product and time. At each simulated purchase observation the products are ordered according to $\gamma_1 - \frac{c_1}{\bar{p}} \geq \gamma_2 - \frac{c_2}{\bar{p}} \geq \cdots \geq \gamma_N - \frac{c_N}{\bar{p}}$. The expected maximum utility is calculated for all N choices, and the optimal consideration set, $\mathbf{K}^*$, is the one that provides the highest $E[\max U_{\mathbf{K}}]$ at each time period, or simulated purchase observation. The consideration set $\mathbf{K}^*$ is then a matrix of 1's if the product is searched and 0's otherwise.

The constrained maximum utility problem described in equation (3.3) above for the more general case when attributes and prices are observed is summarized as:

$$\begin{aligned} &\max \sum_i \mathrm{e}^{\phi_i} \left(q_i + \gamma_i\right)^\alpha \\ &\text{sub. to } \ \tilde{y} - \sum_{i=1}^{K} p_i q_i = 0 \\ &q_i \geq 0. \end{aligned} \tag{3.24}$$

Since the consumer has already undertaken search the error term $\mathrm{e}^{\varepsilon_i}$ is no longer present and he knows all the searched product's attributes, including price. The FOCs are:

$$\begin{aligned}
\alpha e^{\phi_i} (q_i^* + \gamma_i)^{\alpha-1} - \lambda p_i &\leq 0; && (3.25a)\\
\sum_{i=1}^{K} q_i^* \left(\alpha e^{\phi_i} (q_i^* + \gamma_i)^{\alpha-1} - \lambda p_i\right) &= 0; && (3.25b)\\
\tilde{y} - \sum_{i=1}^{K} p_i q_i^* &= 0; && (3.25c)\\
q_i^* &\geq 0 && (3.25d)
\end{aligned}$$

for all i goods, where $\phi_i = \hat{\phi}_{i1} + \hat{\phi}_{i2} x_{it} + \zeta_i + \xi_i$ and ζ_i and ξ_i represent the error the analyst observes in ϕ_i and γ_i respectively. Notice that the budget constraint is satisfied for all K goods and is not restricted over only those purchased goods. I partition the set of searched goods $\mathbf{K}^*$ into a set of purchased goods $\mathbf{I}$ and non purchased goods $\mathbf{F}$ such that $\mathbf{I} \cap \mathbf{F} = \varnothing$ and $\mathbf{I} \cup \mathbf{F} = \mathbf{K}^*$. Equation (3.25b) is satisfied if, and only if, the marginal utility given in equation (3.25a) is 0 and $q_i^* > 0$ or, less than 0 and $q_i^* > 0$. Therefore, all the purchased goods, $\mathbf{I}$, satisfy equation (3.25a) with equality and all the non-purchased goods, $\mathbf{F}$, satisfy the equation with strict inequality. In other words, the KKT conditions given in equations (3.25) (or equivalently the equations 3.5) can be summarized as the price normalized marginal utility of all the purchased goods is equal to any other purchased good, and is greater than all non-purchased goods.

Given the equality for the purchased goods in $\mathbf{I}$ I solve for the analytical solution to the i^{th} good using the ratio of L_i and L_j and the budget constraint given in equation (A.2c). I have:

$$q_i^* = \frac{\tilde{y} + \sum_{k=1}^{I} p_k \gamma_k}{\sum_{k=1}^{I} p_k \left(\frac{p_i \exp[\phi_k]}{p_k \exp[\phi_i]}\right)^{\varpi}} - \gamma_i, \quad \forall i \in \mathbf{I}. \tag{3.26}$$

A key point regarding equation (3.26) above is that it does not hold for goods that are not purchased (the goods in $\mathbf{F}$) and it is conditional on the set of goods $\mathbf{I}$, or more to the point, their parameters. The specific way in which consumers choose the number of products, and their optimal quantities, is described in detail by Pinjari and Bhat (2009). To summarize: First, consider the consumer's baseline marginal utility, or the marginal utility when $q_i^* = 0$ which is $\frac{\alpha}{p_i} e^{\phi_i} \gamma_i^{\alpha-1}$. Consumers will choose products to consider for purchase such that $\frac{\alpha}{p_1} e^{\phi_1} \gamma_1^{\alpha-1} \geq \cdots \geq \frac{\alpha}{p_K} e^{\phi_K} \gamma_K^{\alpha-1}$ (see Pinjari and Bhat 2009 for the proof). Additionally, when $q_i^* = 0$ the price normalized marginal utility is less than λ^*, or $\frac{\alpha}{p_i} e^{\phi_i} \gamma_i^{\alpha-1} < \lambda^*$. Since the consumer knows all the parameters under consideration except for λ^* the consumer calculates λ^* for $I = 1$, say λ_1^*, by calculating q_1^* from equation (3.26).

If $\frac{\alpha}{p_2}\mathrm{e}^{\phi_2}\gamma_2^{\alpha-1} \geq \lambda_1^*$ then the consumer computes the solution to q_1^* and q_2^* to find λ_2^*. This process continues until $\frac{\alpha}{p_{I+1}}\mathrm{e}^{\phi_{I+1}}\gamma_{I+1}^{\alpha-1} < \lambda_I^*$, at which point consumption is set to 0 for the remaining $K-I+1$ goods. The quantity of the I consumed goods is calculated from equation (3.26) above. λ_I^* is defined as:

$$\lambda_I^* = \frac{\alpha}{p_i}\mathrm{e}^{\phi_i}\left(q_i^* + \gamma_i\right)^{\alpha-1}, \tag{3.27a}$$

where $\tilde{y}$ is the consumer's remaining income after search costs have been spent. λ_I^* is calculated for all possible I such that $1 \leq I \leq N$ and should be the same for any product i used to calculate it, so I use q_1^* as it is assumed always purchased. I represents the total number of products purchased. However, if the error term is simulated and added to the expression, as it is here, then λ_I^* may vary across $i \neq j$ choices.

The analyst observes the product attributes and the quantities the consumer purchased, but does not observe the products that were searched, $\mathbf{K}^*$. I then estimate consumer search-and-demand as in the ice cream example described above.

3.5 Data Description

The empirical estimation requires household purchase information. I use Nielsen's Homescan data for ice cream which measures household purchases on a daily basis. Homescan consists of a panel of consumers that are selected to be demographically and geographically representative. Participating households submit all food purchase information each time they visit any type of retail food outlet. The data consists of detailed demographic information from all the members of the household and product specific information regarding the purchases they make. Demographic variables include age, employment, education, and income while purchase specific information includes such things as the brand, price, unit size, and quantities purchased. However, product reporting takes place in the home so items bought and immediately consumed may not be recorded. Additionally, households that are generally hard to recruit (i.e. extremely high or low income households) may be under-represented. Nevertheless the Homescan data, in general, has been found to be at least as accurate as other commonly used (government-collected) economic data sets (Einav, Leibtag, and Nevo 2008). The Homescan data includes purchases by 38,856 households of which $8,268$ made at least one purchase in the ice cream category and resided in the markets I am considering.

Ice cream is an ideal product category to study. Unlike liquid detergent and ketchup, which many search studies use (Siddarth, Bucklin, and Morrison 1995; Andrews and Srinivasan 1995; Bronnenberg and Vanhonacker 1996; Chiang, Chib, and Narasimhan 1999; and Mehta, Rajiv, Srinivasan 2003), ice cream has a wide range of brands, flavors, and carton sizes and multiple items are regularly purchased. Tables 3.1 and 3.2 give frequency information for the number of brands the households selected and the number of cartons of ice cream purchased in a single shopping trip, respectively. The results of table 3.1 suggest that a standard discrete choice model would immediately exclude over 20% of the household's purchases due to multiple brand purchases. Additionally, the data in table 3.2 demonstrates that assuming households make only a single purchase on a single shopping trip would be an inaccurate assumption for almost 40% of the observed household purchase occasions. My dataset consists of daily purchases throughout 2007 and 2008, which is long enough to observe several category purchases, while short enough to assume consumer's preferences remain stable.

Table 3.1

Frequency of Multiple Brand Purchases of Ice Cream for 2007 and 2008.

Number of Brand Purchases	Frequency	Percent
1	651241	77.7941
2	152234	18.1851
3	22874	2.7324
4	7768	0.9279
5	1763	0.2106
6	783	0.0935
7	255	0.0304
8	147	0.0175
9	69	0.0082

$N = 837,134$

This is for the full dataset with outliers (greater than 9) removed.

Calculated based on data from The Nielsen Company (US), LLC

To reduce estimation time, I use the 19 most frequently purchased brands/flavor combinations for the analysis. Here, I define a specific consumer choice at the brand-flavor level. For example, Ben & Jerry's Vanilla would be a separate choice from Ben & Jerry's Chocolate. These 19 brands account for about 40% of the total purchases made from a total of 347 brand/flavor combinations. Due to contractual obligations I cannot display the actual brand names and flavors used in the analysis. However, the three most frequently purchased products were all Vanilla flavored. The summary statistics of the specific brand/flavor combinations used in the analysis are reported in table 3.3. The fact that ice cream is a differentiated food category is evident in the variability of prices among products. Interestingly, all of the products seem to follow a similar pricing strategy in that the minimum and maximum prices are quite different, and the standard deviation and average prices are reasonably close. However the purchase frequency differs significantly among products. From table 3.3 I see that 21.1% of all the household's purchases were for Choice 1 and drops to 8.1% for the next most frequently purchased product. This suggests that even though the products follow similar pricing strategies, the purchase frequency among products differs significantly. Honoré and Kyriazidou (2000) show that at least four purchase occasions are enough to identify the parameters of a mixed logit model. Therefore, I include only those households that made at least 4 purchases so my final sample consists of $1,137$ households and $8,028$ purchase occasions.

One of the limitations of the Nielsen Homescan data is that the product and pricing information is only available for the purchased item. So, there is no information on the other options that were available to the consumer at the time of purchase. To obtain accurate price and product information of the full spectrum of options available, I combine the Homescan data with IRI's Symphony Infoscan retail scanner data. The store-level scanner data contains weekly sales information at the UPC level for almost 10,000 retail outlets in nearly 3500 cities throughout the U.S. The data consist of dollar sales, unit volume (ounces), and product specific identifiers. Covering all the cities would have been intractable so I focus on five major markets: Chicago, IL; Los Angeles, CA; New York, NY; Atlanta, GA; and Philadelphia, PA. Infoscan data is necessary to obtain accurate information on retail price information for all other products offered in the store that the household opted to not purchase. Following previous studies that combine store-level scanner data and household purchases, I merge the two datasets together based on the household's geographic location and the store they visited (Briesch, Chintagunta, and Fox 2008; and Zhang, Gangwar, Seetharaman 2008). Specifically, the household-level purchase data provides information on the household's location via their zip code and the retail outlet visited. On the other hand, the retail-level sales data provides the sales of specific stores and their respective chain association. While the same chain store can have several different outlets in a single zip code, it is reasonable to assume that the prices across stores are maintained at least at the zip code level. By merging the data I accurately observe the number of purchases made on each purchase occasion as well as the price information for all other options in the store. When I calculate the variance of prices at the brand/flavor level I do so within markets and across all retailers. Because the Infoscan data has price and product information for a number of stores in the same area, I am able to get accurate price dispersion information, as well as the total number of products that were being sold at the time the consumer made their purchase.

Table 3.2

Frequency of the Number of Units of Ice Cream Purchased for 2007 and 2008.

Number of Carton Purchases	Frequency	Percent
1	518162	61.8971
2	243556	29.0940
3	40692	4.8608
4	22188	2.6504
5	4798	0.5731
6	3715	0.4437
7	914	0.1091
8	1077	0.1286
9	357	0.0426
10	1018	0.1216
11	201	0.0240
12	237	0.0283
13	66	0.0078
14	50	0.0059
15	39	0.0046
16	38	0.0045
17	12	0.0014
18	22	0.0026
19	9	0.0010
20	27	0.0032
21	11	0.0013
22	21	0.0025

$N = 837,134$

This is for the full dataset with outliers (greater than 23) removed

Calculated based on data from The Nielsen Company (US), LLC

Table 3.3

Ice Cream Summary Statistics for 2007 and 2008.

	Price Statistics (Cents per Ounce)				Purchase Freq.	
Brand/Flavor	Min	Max	Mean	Std. Dev.	Mean	Std. Dev.
1	0.6650	2.0389	1.2603	0.2557	0.2114	0.4083
2	0.6096	1.9633	1.2533	0.3221	0.0812	0.2732
3	0.6611	2.0846	1.3201	0.2935	0.0836	0.2768
4	0.5732	2.0767	1.2815	0.3353	0.0713	0.2573
5	0.7063	1.9633	1.3110	0.2774	0.0521	0.2222
6	0.5628	2.0767	1.3075	0.2976	0.0519	0.2219
7	0.4851	1.9664	1.2391	0.3238	0.0577	0.2331
8	0.4020	1.9633	1.3096	0.2857	0.0472	0.2121
9	0.6572	1.9633	1.2400	0.3147	0.0476	0.2129
10	0.4287	1.9633	1.2505	0.3223	0.0487	0.2153
11	0.7022	1.9835	1.2997	0.2775	0.0460	0.2094
12	0.6382	2.0928	1.2925	0.3274	0.0466	0.2108
13	0.4133	1.9633	1.3073	0.2859	0.0446	0.2064
14	0.6195	1.9633	1.2483	0.3203	0.0455	0.2083
15	0.7122	1.9910	1.2947	0.2794	0.0411	0.1985
16	0.5948	2.0719	1.2669	0.3382	0.0387	0.1930
17	0.5885	1.9633	1.3072	0.2859	0.0412	0.1988
18	0.6250	1.9633	1.2502	0.3224	0.0372	0.1894
19	0.6376	2.1003	1.2843	0.3039	0.0371	0.1891

$N = 8,028$

Calculated based on data from The Nielsen Company (US), LLC

3.6 Results and Discussion

3.6.1 Monte Carlo Simulation Results

I start out by characterizing a few of the data patterns that were generated for different parameter values. While the true parameter values chosen are provided in table 3.4, changing the values around provide some clear insight as to how the data generated changed. In particular, as the true value of α got closer to zero more products were searched and purchased, on average, as expected. However, when α got close to 0, or less than about 0.3, it became difficult for both models to identify the γ_i and σ_γ parameters. Because $\alpha < 0.30$ implies a very strong preference for variety, the generated data suggested all 8 choices would be searched, as expected, but that all, or almost all (depending on how low α was set to) would be purchased with little variation between the optimal quantities purchased over time. On the other hand, as α approached 1, or greater than about 0.7, there was no parameter identification for either model. As α approaches 1 consumers have little to no demand for variety and only search a set containing 1 product at almost every observation. Because γ_i has the largest parameter value for the first good, and the search cost was set to 0, that was the only good searched, and consequently purchased, at almost every observation. The data generated, therefore, did not provide an adequate amount of information for either of the models to be able to reasonably identify parameters. As expected, the percentage of the time each choice was in a consideration set throughout the 1000 observations generated, was largely driven their respective γ_i parameters. Search costs varied significantly across all but the first choice so there was some variation from one set of data to another, but over 1000 observations it was not significant.

Table 3.4 shows the estimation results from both the model that recognizes search based on equation (3.21) above, and a naive one that assumes search is costless based on equation (3.20) where $K = N$ in which search costs are assumed known. The results are based on the true α equal to 0.6 in which about 5.8 of a possible 8 products are included in the consideration set and 1.8 products are purchased on average. For comparison purposes I estimate both models based on $\alpha = 0.4$ in which all 8 products are searched at every observation and an average of 2.7 products are purchased.[10] When every available choice was in the consideration set both models' parameters were very close to the true parameter estimates, as expected. The naive model that assumes every

[10] I found that $\alpha = 0.4$ was about the highest value of α I could use that lead to generated data in which all 8 products were searched.

option was searched was less sensitive to the starting values used for γ_i and α and had a low standard deviation between parameter estimates over the 100 replications that were done. This is not surprising since the maintained assumption of the model that ignores search is fulfilled at every observation. However, as α increases and the severity of the maintained assumption is increasingly inaccurate, the parameter estimates of the naive model becomes increasingly biased toward 0.

The results in table 3.4 confirm this bias for the naive model when $\alpha = 0.6$. From an intuitive perspective a bias toward 0 is expected. Since the naive model assumes every option is considered, when in fact some are not, the model underestimates the importance of the products that are not searched. The product attributes of the non-searched goods are not being weighed against the searched products's attributes during the purchase decision, so assuming they are leads to a negative bias (bias toward zero) among the parameters. The parameter estimates for the model that recognizes search are less biased, and have a lower variance for the parameter estimates. This is because the model has to determine which of the specific products the consumer searched. The specific products searched critical affects the denominator term $\left(\sum_{i=1}^{K} \mathrm{e}^{W_i}\right)^I$ (see equation 3.20) when passed from $\Pr\left[\mathbf{K} = \mathbf{K}^*\right]$ to $\Pr[\mathbf{q}^*|\mathbf{K}]$ in equation (3.21). The more variation there is in $\hat{W}_i$, or $\hat{\phi}_{i1}$ and $\hat{\phi}_{i2}$, between the products actually searched and those estimated to have been searched, the more difficulty there is in finding the true parameter estimates. This is increasingly true as the number of products searched decreases (α increases). At the true parameter estimates the model recognizing search costs does predict the correct consideration set well, with the majority of the weight consistently given to the correct consideration set. With an average of almost 6 products being searched at every observation the difference between the estimated and actual products searched was much smaller compared to the model that assumes every product was searched. An important note here is that the estimated α critically affects the number of products searched. The estimates provided in table 3.4 use the true $\alpha = 0.6$ value as the starting value. Because α critically affects the number of products searched, starting from the true α, helped reduce estimation time and parameter bias. I found that when α was started close to the extreme of 0 or 1 the variance in the estimated parameters increased considerably and was likely to diverge to erroneous estimates, $\hat{\phi}_{i1}, \hat{\phi}_{i2} \approx \pm 500$ for example.

Table 3.4

Parameter Estimates for Monte Carlo Experiment.

		Search Model		No-Search Model	
	True Value	Parameter	Std. Dev.	Parameter	Std. Dev.
α	0.6	0.613	0.015	0.710	0.020
γ_1	5.0	5.000	0.000	5.000	0.000
γ_2	4.0	3.972	0.580	3.434	0.854
γ_3	3.0	2.939	0.655	2.498	0.718
γ_4	2.5	2.418	0.425	2.041	0.594
γ_5	2.0	1.902	0.375	1.671	0.510
γ_6	1.5	1.435	0.349	1.173	0.355
γ_7	1.5	1.453	0.371	1.249	0.355
γ_8	1.0	0.891	0.245	0.811	0.204
$\hat{\phi}_{21}$	-1.0	-1.387	0.150	-1.576	0.164
$\hat{\phi}_{31}$	-2.0	-2.567	0.154	-2.476	0.178
$\hat{\phi}_{41}$	-1.0	-1.536	0.171	-1.574	0.131
$\hat{\phi}_{51}$	-1.5	-2.028	0.123	-2.041	0.120
$\hat{\phi}_{61}$	-1.0	-1.547	0.188	-1.569	0.135
$\hat{\phi}_{71}$	-1.5	-1.997	0.153	-2.024	0.111
$\hat{\phi}_{81}$	-1.5	-2.016	0.115	-2.021	0.136
$\hat{\phi}_{22}$	1.5	1.242	0.170	1.229	0.241
$\hat{\phi}_{32}$	2.5	2.207	0.155	2.152	0.151
$\hat{\phi}_{42}$	1.5	1.230	0.230	1.247	0.191
$\hat{\phi}_{52}$	2.5	2.178	0.165	2.194	0.180
$\hat{\phi}_{62}$	1.5	1.283	0.211	1.233	0.204
$\hat{\phi}_{72}$	2.5	2.146	0.220	2.200	0.136
$\hat{\phi}_{82}$	2.5	2.154	0.159	2.203	0.218

Std. Dev. - Standard deviation across parameter estimates.

γ_1 was held fixed as it cannot be identified in the search model.

Based on 1000 observations and 100 parameter solutions

3.6.2 Ice Cream Results

In this section I present the demand estimates for equation (3.21) above under the conditions of both costless, and costly search. Table 3.5 presents the results of both models that assume (1) the full spectrum of products is considered and search costs are meaningless and, (2) consumers incur a cost to obtain price and product information. As a first step to interpreting the results I compare both models to a naive counterpart in which the parameters are zero. Using the likelihood ratio (LR) test, I reject the null hypothesis that the parameter vector from either of the models is equal to zero based on the test statistics in table 3.5, which suggests that both have at least some explanatory value in describing the demand for ice cream. In a similar fashion, I compare the models to one another to determine whether or not search costs have any bearing on consumers' ice cream decisions. The null hypothesis that search costs are not important and consumers consider all available ice cream options before making their final selection is rejected based on the LR test statistic 128.37 which is Chi-square distributed with 58 degrees of freedom. Therefore, I conclude that households do face a cost of search that limits the size of their consideration set when selecting ice cream products. So, I use the model that recognizes the importance of search costs to interpret the parameter estimates.

There are a number of parameters that are of inherent interest from both a managerial and theoretical perspective. First, the point estimates of the γ_k parameters are all statistically different from zero, and vary considerably. This rejects the linear utility structure employed in standard discrete choice models. The statistical significance and differences across the γ_k parameters suggests that the linear utility structure used in other search-demand models is not flexible enough to capture the true nature of the household's demand for goods, and by extension, may not represent an accurate assumption when deriving the consideration set households choose. The significance and variation of the γ_k parameters show that there are clear satiation effects in ice cream purchase decisions. The γ_k parameters tell us the relative satiation households, on average, get from the products relative to the outside option. The outside option is a very popular vanilla flavor that was purchased far more than any other good. Positive parameter estimates of γ_k suggest that consumers become satiated with that product faster than the outside option. Second, the parameter estimate of $\alpha - 0.81$ implies consumers have a relatively low preference for variety, on average tend to search a smaller number of options than is available. However, there is some preference for variety among the sample of consumers and so should not be ignored. These observations are consistent with the results in table 3.1.

Third, I find that several of the discount interaction parameters are statistically significant. Positive parameter estimates suggest that those particular brand/flavor combinations are highly conducive to price promotions. Parameter estimates close to 0, on the other hand, imply that promoting that particular brand/flavor does not change the consumer's propensity to purchase the product. Fourth, the brand and flavor specific intercept parameters can be thought of as a baseline measure of how likely an individual is to purchase that particular product when no other information is available. In other words, the brand and flavor specific intercepts illustrate how much utility a household obtains from a particular choice without having to consume the product. Most of the intercept parameters being negative suggests that consumers find the outside option more favorable.[11] This is consistent with the data as the outside option brand/flavor was the one purchased most often, and by the highest number of households, so presumably, that would be the population's preferred brand. Finally, the market specific binary indicator variables control for market specific effects on the part of consumers and retailers/manufacturers. For example, advertising low calorie ice cream in one market and high calorie ice cream in another. The fact that these parameter estimates are statistically significant and differ show that there are differences across markets which effect the propensity of that population to make an ice cream purchase.

[11] Contract restrictions prohibit us stating what the outside option is.

Table 3.5

Parameter Estimates for Structural Search Models.

Parameters	Search Model Estimates	Search Model t-stat	Non-Search Model Estimates	Non-Search Model t-stat
γ_1	4.5509*	28.274	0.4977*	2.636
γ_2	3.8424*	30.619	0.3364	1.515
γ_3	3.8609*	37.315	0.2865	1.440
$\vdots$	$\vdots$	$\vdots$		
γ_7	2.6049*	9.048	-1.5884	-1.162
γ_8	3.1774*	20.66	-1.9867	-1.672
γ_9	2.0373*	3.795	-2.7011	-0.319
σ_γ			2.1845	1.160
Brand_2	-2.4757*	-6.421	-1.0589*	-2.650
Brand_3	-2.5823*	-9.609	-1.0812*	-3.754
Brand_4	-0.2896*	-0.696	-1.0298*	-2.414
$\vdots$	$\vdots$		$\vdots$	
Flavor_2	-0.2309*	-0.704	-0.9673*	-2.772
Flavor_3	1.2478*	1.976	-0.9021	-1.503
Flavor_4	3.2413*	14.145	-0.9509*	-3.538
Disc*Price_2	1.6088*	13.521	1.0134*	8.202
Disc*Price_3	2.1475*	21.478	0.8905*	8.149
Disc*Price_4	6.6648*	53.698	1.0970*	8.490
$\vdots$	$\vdots$		$\vdots$	
Disc*Price_7	1.8770*	9.649	1.1037*	5.502
Disc*Price_8	6.5285*	37.823	1.5940*	12.767
Disc*Price_9	6.0557*	149.728	1.1800*	6.112
Market_2	-10.9794*	-38.400	1.0967*	3.619
Market_3	-6.1013*	-26.053	1.1222*	4.488
Market_4	-13.7262*	-58.573	1.1388*	4.614
Income	-			
BLoy	-			
Market Pr. Var.	-			
Num. Products	-			
$(\text{Num. Products})^2$	-			
Log-Likelihood[1]	-458.12		-393.94	
Likelihood Ratio				

An asterisk indicates significance at a 5.0% level.

$\vdots$ - Estimates omitted to save space

1

As Roberts and Lattin (1991) point out, there are significant retention and cognitive processing costs for frequently purchased goods that will limit the number of choices considered. My results confirm this for ice cream and illustrate that consumers only search a subset of the total number of products available. Therefore, when estimating the demand for ice cream, it is necessary to recognize the cost of search as consumers are likely to search only a subset of the products. Additionally, demand for ice cream is inherently multi-discrete/continuous as products are often purchased in multiple quantities, with at least 2 being purchased 20% of the time (see table 3.1). Prior demand models that recognize the search of cost would have to ignore full 20% of the observed ice cream purchases when estimating demand because they fundamentally assume a single product is purchased (Mehta, Rajiv, and Srinivasan 2003; Chiang, Chib, and Narasimhan, 1999; Koulayev 2010; Kim, Albuqurque, and Bronnenberg 2010; de los Santos, Hortacsu, and Wildenbeest 2012; Honak 2013; and Honka, and Chintagunta 2013). Estimating demand using only 80% of the observed purchase data is likely to yield inaccurate parameter estimates, and importantly, incorrect consideration set size predictions. Observing 2 products being purchased necessarily implies that consideration sets containing only a single product cannot be possible. Ignoring every observation in which more than 1 product is purchased will necessarily bias consideration set size estimates. Similarly, models that recognize the multi-discrete/continuous nature of ice cream, but do not account for the cost of search, or that consumers consider only a subset of the total available products, and are likely to produce biased parameter estimates, as my Monte Carlo experiment suggests (Hendel 1999; Dubé 2004; Bhat 2005; Song, and Chintagunta 2007; Bhat 2008; Richards, Gómez, and Pofahl 2012; and Bhat, Castro, and Pinjari 2012). The model developed in this paper is the only one (that I know of) that is applicable to both multi-discrete/continuous purchase environments in which a subset of the total available choices is considered.

3.7 Conclusion

Structural consumer search-demand models deal with situations in which only one alternative is chosen from a set of mutually exclusive alternatives. However, many consumer demand situations exist in which individuals' choose multiple products in continuous quantities. Until recently there has been little research on recognizing these multiple discrete situations in the context of consumer search. This paper extends the single purchase search-demand models to formalize a structural model of consumer search and multiple-continuous product purchases in a utility maximizing framework. Specifically, I extend Bhat (2005) in recognizing that people have a search cost which may limit the number of products they actually consider before purchase. In addition, the structural model also allows consumer's preference for variety to influence their search strategy, recognizing the value that more search has when an individual has a high preference for variety and may purchase multiple products, making the gains from search that much higher. I assume a translated non-linear additive utility specification that is derived through the addition of a multiplicative log-extreme value error term which results in a closed form probability expression. Consumers determine the optimal consideration set to choose based on their expected maximum utility, and then determine the number and quantity of goods to purchase. My model is a static multiple discrete/continuous extension to the consideration set formation model of Mehta, Rajiv, and Srinivasan (2003).

I find that search costs are important in a frequently-purchased consumer-good category. The implications of this finding are important in that the actual number of brands a person will consider before making a purchase will be less than the total number of choices available. My results suggest that consumers' search costs increase as their income increases due to the increased opportunity cost of time and effort spent searching. My empirical results also illustrate that the linear utility assumption of classic discrete choice models is not flexible enough is describing consumer demand, which suggests that search models derived from these assumptions may not be fully generalizable to all purchase contexts. My empirical model, on the other hand, is flexible enough to describe the search and purchase behavior in contexts of single, or multiple choice situations. Moreover, the search behavior my model describes is flexible enough to describe the optimal composition and number of products in a consideration set, but simple enough to accommodate a large number of options. In particular, unlike existing structural consideration set models, the combinatorial space of the number of considerations sets increases linearly, rather than exponentially, with the number of options available.

My findings have a number of practical implications. First, my results show that it is important to take search costs into consideration even for common, frequently purchased household products like ice cream. Roberts and Lattin (1991) argue that the cost of search in these categories is driven more by mental maintenance and cognitive processing costs rather than financial. My results confirm that households do not take every single ice cream alternative into consideration before making their final purchase decision. Recognizing the importance of search, my model can be applied by multi-product retailers to determine the products consumers consider before making a purchase. By overcoming the curse of dimensionality the model developed in this chapter can readily accommodate categories in which a large number of alternatives are offered. Using household purchase data I show that it is possible to determine the consideration set consumers choose before making a their final purchase decision, so long as a reasonable measure of search costs is available. Finally, the model provides retailers and manufacturers with valuable information regarding which products consumers get the most satisfaction from, and alleviates the bias present in models that assume search is costless and every alternative is considered. This model, therefore, provides a valuable tool for both retailers and manufacturers alike.

As with any structural model, there are several limitations my model does not address. First, it does not take into account quality uncertainty and assumes it known to the consumer before search or purchase. This assumption could be relaxed if I were to allow for a dynamic updating of brand quality learning over time. Second, Yuan and Han (2011) show that consumer's price expectations can have important implications for their search strategy. Incorporating price expectations into the formation of the current consideration set is likely to provide valuable insight into a consumer's search behavior.

CHAPTER 4.

CONSUMERS SEARCH AND THE CHOICE OVERLOAD HYPOTHESIS

4.1 Introduction

Consumers have heterogenous preferences for variety. Firms understand this observation and introduce new products that are closely aligned with the demands of particular groups of consumers. The results in a proliferation of choices that makes it difficult for the average consumer to find their ideal product. The number of new product introductions in the food and beverage market increased steadily from 1992 to 2007 (USDA-ERS 2013). However, from 2007 to 2008 the number decreased by 5% and then by a further 16% from 2008 to 2009. Surely, the recession is responsible for some of the decline, but it appears to have accelerated a trend that was already well underway. Have consumers reached their limit in their demand for variety? Consumers value deep assortments because they are able to find products that better meet their needs. Profits rise because larger assortments support higher prices. On the other hand, many retailers have increased profits by reducing the number of products on the shelf as consumers move toward familiar brands (USDA-ERS 2013). Contrary to the common wisdom, many now support the idea that less variety is preferred (Iyengar and Lepper 2000; Chernev 2003; and Iyengar, Huberman, and Jiang 2004), or that consumers have a finite optimal number of alternatives (Kuksov and Villas-Boas 2010). The objective of this research is to test these three competing ideas using experimental methods.

Experiments that consider the validity of the "choice overload hypothesis," or the notion that too many options cause consumer dissatisfaction, have had mixed results.[1] Through a number of experiments, Iyengar and Lepper (2000) found that more consumers preferred smaller choice sets compared to larger ones. Using unfamiliar products, for example exotic jams, Iyengar and Lepper (2000) gave participants the option of choosing a product from either 6 or 24 choices and found that consumers chose more often from the smaller set. In another experiment, Iyengar and Lepper (2000) gave participants the option to choose from the larger or smaller assortment or the option to forego making a selection at all and receive $1. They found that participants who were presented with

[1] There is no specific consensus on the term used to describe the phenomenon. We follow Iyengar and Lepper (2000) (among others Diehl and Poynor 2007; Mogilner, Rudnick, and Iyengar 2008; and Scheibehenne, Greifender, and Todd 2010). It has also been referred to as the "overchoice effect", "paradox of choice", "the tyranny of choice", or the "excessive-choice effect" (Schwartz 2000; Schwartz 2004; Gourville and Soman 2005; Norwood 2006; and Arunachalam, Henneberry, Lusk, and Norwood 2009).

more options were more likely to choose the monetary compensation rather than make a selection. Support for the choice overload hypothesis also lies in less exotic products such as pens, gift boxes, and coffee (Chernev 2003; Shah and Wolford 2007; and Mogilner, Rudnick, and Iyengar 2008). Arunachalam, Henneberry, Lusk, and Norwood (2009) conducted experiments very similar to those of Iyengar and Lepper (2000) using a more common product, and found no evidence in favor of the choice overload hypothesis. Clearly, the precise mechanism that leads consumers to choose fewer products is unclear.

Others find support for the more traditional notion that a consumer's overall satisfaction increases with variety. For example, Berger, Draganska, and Simonson (2007) use a number of experiments to show that manufacturers introducing new products, even with only minor characteristic differences, are able to increase consumers' perception of quality. Similarly, a number of field studies have shown that the probability of choosing a particular retailer to patronize increases with the depth of assortment. That is, a larger variety offered by a retailer provides a competitive advantage (Koelemeijer and Oppewal 1999; Boatwright and Nunes 2001; Borle, Boatwright, Nunes, and Shmueli 2005; Oppewal and Koelemeijer 2005; Richards and Hamilton 2006; Briesch, Chintagunta, and Fox 2009). One of the reasons why this question has not been resolved is that there is no precise definition for what constitutes a "large choice set" because the mechanism driving preference for variety is unclear.

Differences among purchase situations may dictate why the choice overload hypothesis is prevalent in some situations, and not others. Through a meta-analytic review Scheibehenne, Greifeneder, and Todd (2010, pg. 421) conclude that "...more choice is better with regard to consumption quantity and if decision makers had well-defined preferences prior to choice..." They also suggest that the choice overload hypothesis is likely to be more prevalent in situations where an individual is unfamiliar with the choices, and has little or no preference for the specific choices at hand. The choice overload hypothesis is more likely to apply when the consumer does not have a clear favorite among the choices, or if there are many options in which no subset of them is clearly dominant.

Differences among choice-specific situations may also be due to heterogenous search costs. When search costs are high, as would be the case when an individual is unfamiliar with a product category, the cost of searching a large number of options may deter choice. On the other hand, if someone is familiar with the products, then search costs are lower and a larger variety of differentiated goods may be preferable. Search costs, therefore, provide a more precise explanation as to why the choice overload hypothesis is prevalent in certain cases and not others.

Search costs may explain why there is a finite optimal number of products desired. Norwood (2006) develops an analytical model that explains the choice overload hypothesis as due to heterogenous consumer search costs. He shows that it is possible for markets to provide too many options, but argues that the lower revenue from losing customers due to too much variety will push variety back down to an optimal level. Similarly, Cachon, Terwiesch, and Xu (2008) find that as search costs decrease and search intensity increases, the assortment of goods offered by firms will increase, leading to higher equilibrium prices. Firms are able to raise prices because the probability that their products are searched, and purchased, increases. Recognizing the possibility that the number of products offered by the firm also affects consumers' propensity to search, Kuksov and Villas-Boas (2010) find that consumers have an optimal preference for variety based on their search costs.

These analytical models explain why researchers find support for the choice overload hypothesis in experiments where the number of choices is exogenously chosen by the experimenter, but field studies find that retailers gain a competitive advantage by offering larger varieties. Namely, in experimental studies the number of options is fixed throughout the entire experiment and does not change in response to participant's choices. In contrast, retailers adjust the number of products offered over time based on consumers' reactions and decrease the variety offered if sales fall. Witness the 2007 - 2009 experience of food and beverage manufacturers. That said, there is no empirical evidence to support these analytical models.

This chapter contributes to the literature by testing the ability of consumer search to explain the choice overload hypothesis. While numerous studies have tested the choice overload hypothesis in different contexts, and for different product categories, none have explicitly tested the ability of search costs to explain the phenomenon (Norwood 2006; and Kuksov and Villas-Boas 2010). The results of my experiment provide strong support for consumer search costs to explain the choice overload hypothesis. In particular, when search costs are low consumers want a wider range of products to choose from which is consistent with the findings of Scheibehenne, Greifeneder, and Todd (2010) who find that more products are preferred when preferences are well defined. In addition, this chapter contributes to the choice overload literature by conducting a non-hypothetical two sided experiment that allows for retailer responses to consumers. There are no experimental tests of the choice overload hypothesis, that I am aware of, that allow for retailers' dynamic reactions to consumers in adjusting the number of products they offer. Consistent with the theoretical model of Norwood (2006) my results show that retailers respond to consumers choice decisions over time and adjust the number of products offered if too many or too few are leading to poor sales.

4.2 Market Experiment

In order to test the relationship between consumer search and variety, I develop a non-hypothetical experiment that builds on the experimental design of Yuan and Han (2011).[2] My experimental design provides an equilibrium price, and an equilibrium variety, which are both dependent on the buyers' decision to search.

One of the biggest challenges to studying consumer search is that search behavior and search costs are often unobservable in both retail and household purchase data (Zwick, Rapoport, Lo, and Muthukrishnan 2003). With household level purchase data the researcher can observe the products that were purchased and detailed information about those products, but are often unaware of the other products that were available to the consumer. Moreover, when there is a large number of items available, it is unlikely that the consumer will consider every single one. Knowing which products are considered, and which are not, is a critical element to the relationship between search and variety. Therefore, I use an experimental approach so that the products considered and the consumer's search cost are clearly revealed.

I use a non-hypothetical experiment design in which the participants are rewarded based on their performance within the experiment. Hypothetical experiments are often used to investigate consumer search behavior, but participants in stated choice exercises have no real incentive to reveal their true demand, nor to put cognitive effort into the decision making process. Non-hypothetical experiments, on the other hand, provide the participants with real economic incentives to make decisions that provide the most benefit at the lowest cost. For example, List and Gallet (2001) find that participants overstate their willingness to pay by 2-20% in hypothetical experiments compared

[2] The authors extend a debt of gratitude to Yuan and Han for sharing their ztree code.

to non-hypothetical experiments. Consequently, I conduct a non-hypothetical experiment in which I use real economic incentives to motivate participants to make realistic purchase decisions.

Experimental studies of consumer search are often built around the sequential search framework that assumes products are considered one after the other, and a decision is made each time to consider another option and incur a cost, or stop searching. For example, Zwick, Rapoport, Lo, and Muthukrishnan (2003) conduct an experiment in which participants are shown an apartment and then choose whether or not to view the next one, given some exogenously determined search cost. As more and more apartments are viewed, the probability that the apartments seen earlier are still available decreases. The choice overload hypothesis is more aligned with a fixed-sample size search process in which consumers are shown a number of different alternatives at the same time, and then select a subset to consider. In addition, de los Santos, Hortaçsu, and Wildenbeest (2012) test the applicability of the sequential search model against a fixed-sample size alternative using a combination of web browsing and purchase data. Online search data allows for a direct test of the sequential versus the fixed sample-size model as search behavior is directly observed. Their analysis reveals that the pricing pattern across stores is inconsistent with a sequential search model because higher prices do not induce a consumer to necessarily undertake search. Therefore, I use a fixed-sample size experimental design. A fixed-sample design also allows for a more direct test of how the number of products offered effects consumers' decision to search.

Participants were recruited from the general public as well as students from business classes at Arizona State University. Using both public and student participants allows me to see whether there are differences between a student and public sample, and more generally, whether student choices are representative of those made by subjects drawn from the general public. Participants answered some basic demographic questions and were then randomly assigned to either a buyer or seller role.[3] Participants remained in their respective roles throughout the entire experiment. If a participant was assigned to a buyer role, they chose which product to purchase and the quantity. Participants in a seller role decided which products to offer and the prices to charge. Buyers and sellers were also randomly matched to each other such that individual buyers did not know who they were buying from, and sellers did not know who they were selling to. The aim of the random

[3] Demographic questions are available in Appendix D.

matching protocol is to minimize or eliminate reputation effects and collusion that could come into play (Yuan and Han 2011; and Amaldoss and Rapoport 2005). Participants were informed that each buyer would be randomly matched to a different seller each period.

Public participants were recruited by advertising the experiment on craigslist. The ad informed potential participants that the experiment would take place on December 8, 2013 and 4 time slots were available. Doodle.com was used to allow participants to sign up for individual time slots, and showed participants whether or not a particular time slot was full. The first experiment started at 10:00am while the last session started at 4pm, each lasting approximately 1 hour and 30 minutes which provided 30 minutes to pay the participants and get the experiment reset for the next session. Public participants were paid \$35 for coming to the experiment and a possible \$15 or \$10 bonus depending on how much profit they accumulated compared to the others in the session. A total of 76 individuals participated in the public experiment, with only one individual dropping out of the experiment half way through. Participants in each session were allowed to go at their own pace. Once an hour and a half passed, the last period was finished and the experiment stopped. In one of the four sessions, participants were only able to get through 3 periods after the initial practice periods, so that entire session was excluded from the final analysis.[4] In the end there was a total of 48 participants, 35 of which were randomly selected to be buyers.

Students were recruited from business classes ranging from freshman to senior level undergraduate classes. Students earned class credit for participating in the experiment and were not paid solely for participating. They did, however, have the opportunity to earn an additional \$10 or \$15 bonus for attaining the highest profit at the end of the experiment. The top 2 sellers and top 2 buyers that had the most Economic Credits (EC) at the end of a session were paid an additional \$15, and from the remaining participants, the top 5 buyers and top 5 sellers were paid the \$10 bonus. A total of 48 students were recruited, of which, 32 were randomly selected to be buyers. In the final analysis, therefore, there were a total of 67 buyers and $N = 1045$ search/purchase observations.

At the beginning of the experiment, general instructions were given to the participants that described how the experiment would work. This general information was followed by screen shots

[4] Since sellers had to wait for buyers to finish before moving on, and vice versa, there were several individuals in this particular session that had a difficult time moving forward, and this slowed everyone down considerably.

of each screen buyers and/or sellers would see, along with examples of how the inventory cost and preference for variety bonus subtracted from, or added to, the total EC at the end of the game. Participants then went through a practice round. At the end of the practice round, all participants were asked if they understood how the experiment worked and their individual role as either a buyer or seller. All questions that arose throughout the instructions, or the practice round, were answered publicly. After all questions were answered and everyone publicly acknowledged that they understood, each participant was again asked privately on their individual computer. Anyone that privately answered "no" to one of these questions was removed from the final analysis. Participants were otherwise asked not to talk to one another nor sit near anyone they knew. The experiment was carried out using the z-Tree software system (Fischbacher, 2007), which is an open-source software tool that allows sellers to set prices, assigns sellers to buyers, and calculate profits.

4.2.1 Experiment Design

Sellers had the option of purchasing up to 5 different products, referred to in the experiment as "Widgets," from the experimenter. Knowing the cost of each of the 5 available Widgets, sellers chose the number of products to offer - at least 1 and at most 5 - and then set the price of each product. The cost the sellers paid the experimenter for each Widget sold differed. Namely, the base cost for the cheapest Widget was 3EC. The base cost increased for each product by 1EC such that the base cost of the 5^{th} potential product, or most expensive, was 7EC. All sellers incurred the same cost for each unit of a Widget sold, and no cost for unsold Widgets. In addition to unit cost of each Widget sold, sellers paid the experimenter an ”inventory cost” for each product offered. The inventory cost was determined randomly following a uniform distribution and differed for each seller and each period. The lowest possible inventory cost per Widget offered was 1EC while the max. was 6EC. For example, if the seller's inventory cost was 2EC in a particular period, and the seller offered 3 out of 5 total products, the seller paid the experimenter 6EC regardless of the number of products sold. If the seller did not sell any products in a particular period their total profit decreased by the inventory cost.

To avoid buyers and sellers repeatedly entering the same values for each period, I induced price changes via the seller's Widget cost. All participants were informed that there would be price changes periodically throughout the experiment, but did not know when. Buyers did not see the prices sellers paid for the Widgets, and were, therefore, not aware which periods had price shocks. After several periods had passed, the sellers' Widget cost changed. These price changes were the same for all sellers, and were added to (or subtracted from) the base price. In other words, if the cost of the Widgets increased by 1EC it did so for all the sellers. Figure 4.1 provides an example of the cost sellers paid for the most expensive Widget in two different sessions. The magnitude and direction (either positive or negative) of the price changes were determined randomly following a uniform distribution. The minimum price shock was set to 1EC and the maximum price shock was 3.5EC. In the event that the price shock was lower than -3EC the experimenter sold the cheapest Widget for 0.01EC. In other words, the experimenter did not pay the sellers to sell the products. Figure 4.1 illustrates how prices changes lasted for several periods and then moved back to the base cost over several periods. The magnitude of the price changes was allowed to differ across products, but were either all positive or all negative so that all products became cheaper or more expensive. Thereafter, the prices sellers paid the experimenter for the products remained at the base cost for several periods until another series of price shocks began.

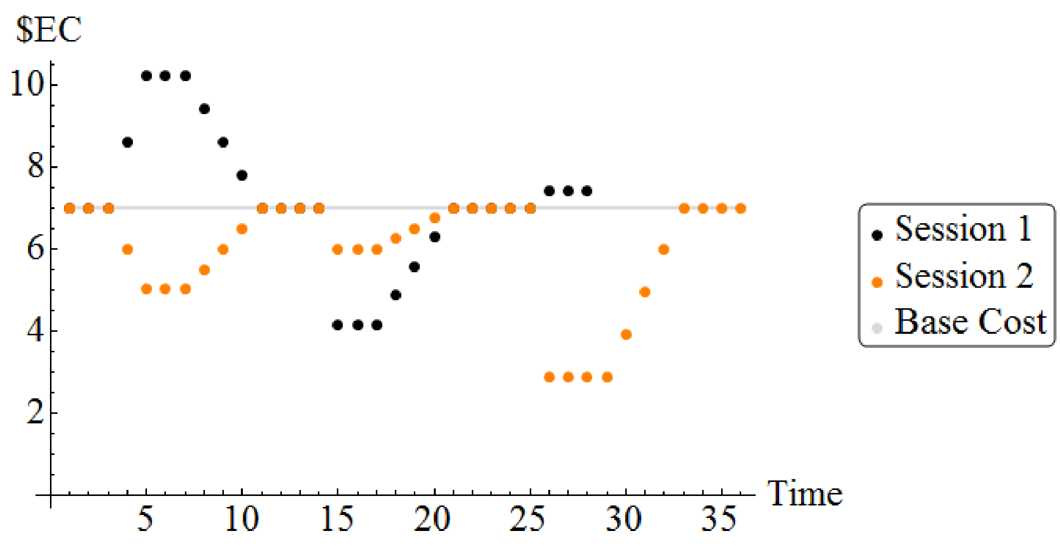

Figure 4.1

The prices Sellers paid for the most expensive Widgets.

In each period, sellers observed the cost of each Widget and their inventory cost on the same screen that they used to determine which products to offer and the prices. For the first 6 periods (including the practice period) a confirmation screen came up that displayed their product and pricing choices and confirmed the seller's selection. If a mistake had been made, they had the opportunity to go back and correct it, or confirm their original decision. The confirmation screen was limited to 6 periods, which I found to be sufficient during pre-testing. Once *all* sellers had made their final product and pricing choices, the experiment moved to the buyers.

I incorporated search into the experiment following Yuan and Han (2011) by allowing buyers to see the prices and products offered by one seller for free, and then giving them the option to pay a search cost and see another seller's products and prices. If the buyer decided to pay the search cost, they could then purchase Widgets from both sellers. Search costs were determined randomly following a uniform distribution and varied across buyers and periods. The minimum search cost was set to 0 and the maximum search cost was set to 4EC. This search cost was paid out of the buyer's final profit (or total EC) at the end of each period, and did not affect the amount the buyer had to spend on purchasing the products. In other words, if a buyer had a search cost of 3EC and they decided to search, they still had 50EC to use to purchase Widgets from sellers. Once the buyer made their search decision, his or her preference for variety was revealed.

Preference for variety was induced in a similar fashion as the sellers' inventory cost, except with each unique Widget purchased the buyer was able to increase their total profit. In this way, I obtained an equilibrium number of products offered. Sellers were aware buyers had a preference for variety that would increase their profit, and that they would, therefore, prefer a wider product selection. The degree to which the buyer preferred a wider product selection depended on the magnitude of his or her preference for variety. Widgets of the same type from different sellers were considered different. Preference for variety varied across buyers and periods and was determined randomly following a uniform distribution. The minimum preference for variety bonus was set to 0, while the maximum value attained was 3EC. At the same time preference for variety was revealed, the buyer was able to make his or her product and quantity selections.

Buyers selected the number of unique Widgets to purchase and their quantity such that a budget of 50EC was not exceeded. Buyers were alerted if their budget had been exceeded, but the amount spent was not dynamically displayed as they entered values. Buyers sold the purchased products to the experimenter with the aim of accumulating as many EC as possible each period. Product differentiation was induced by varying the value the experimenter paid for each Widget.

Specifically, the experimenter paid 10EC for the lowest valued Widget, 12EC for the second lowest Widget, 14EC, 17EC, and finally 20EC for the highest valued Widget. The value of the products were directly related to the sellers' costs. For example, the experimenter sold the lowest valued product to the seller for 3EC, and purchased it from the buyer at 10EC. Both buyers and sellers knew the price the experimenter paid the buyers for the Widgets. These prices remained constant for all buyers throughout the entire experiment. As with the sellers, the buyers saw a confirmation screen for the first 6 periods that allowed them to go back and re-submit their choices if they realized they had made an error.

Once all buyers had made their final decision, profits were tallied for all buyers and sellers and displayed to each participant. Buyers observed the quantities they had selected, their search cost, their profit in that period, and their total profit up through that period. Sellers observed the prices they had set, the total quantities sold, their total inventory cost, profit for that period, and their total profit up to that period. After the profit screen had been displayed the experiment moved on to the next period, or ended if the time was up.

Once all sessions were finished, demographic data was collected using Qualtrics.com (available in Appendix D), and was combined with the experiment data using randomly assigned subject ID numbers.

In my experiment, sellers have an incentive to offer sufficient variety and low enough prices to persuade prospective buyers not to search the other seller's product offerings, and absorb the entire $50EC budget of each buyer. Buyers, on the other hand, determine whether or not to search the additional seller's product and price offerings based on their cost of search and what the initial seller offered. A buyer's decision to search, therefore, is a signal to the seller that the expected benefit from searching the additional seller's product and price offerings outweighed the cost of search.[5] Sellers did not know how many buyers purchased from them, nor who decided to search, and adjusted their product and price decisions based on the units sold and profit made in the previous period. In other words, the number of products and prices are endogenous to the buyers decisions in the previous period. The data collected from the buyers provides information on their search decision, as determined by their cost of search, the initial seller's product and price decision, and their

[5] Recall that at the time the decision to search is made buyers know they will have some positive preference for variety, but do not know what it is.

expectation about the other seller's offerings. Observing the buyers' search decision, their search cost, and the number of products offered provides the information needed to test the relationship between consumer search and preference for variety.

4.3 Empirical Model of Search Decision

I test the effect a retailer's variety decision has on a consumer's propensity to search, while controlling for their search cost. Specifically, I test the implication of Kuksov and Villas-Boas (2010 pg. 517) that consumers have a finite, optimal number of products they will search. Support for this hypothesis implies a non-linear relationship between the propensity to search and the number of products offered. Kuksov and Villas-Boas' (2010) hypothesis is similar to the argument made for the choice overload effect. Namely, if search costs are indeed the moderating factor leading consumers to prefer fewer items when products are unfamiliar and search costs are high, then I expect the same non-linear relationship. On the other hand, if there is a strictly positive linear relationship between search and variety, then I would reject the choice overload hypothesis in favor of the more usual notion that consumers always prefer more variety to less.

I model the probability a consumer searches by extending the logistic regression model proposed by Yuan and Han (2011). Specifically, my model allows for consumer heterogeneity and controls for the endogeneity in price and the number of products offered. The indirect utility consumer i obtains from searching at time t is the sum of a deterministic and stochastic part and is written as:

$$U_{it} = \boldsymbol{\beta}_z^{\intercal}\mathbf{z}_i + \boldsymbol{\beta}_x^{\intercal}\mathbf{x}_{it} + \varepsilon_{it} \tag{4.28}$$

where $\boldsymbol{\beta}_z^{\intercal}\mathbf{z}_i+\boldsymbol{\beta}_x^{\intercal}\mathbf{x}_{it}$ is the deterministic component of utility, and ε_{it} is an independent and identically distributed error term. The deterministic utility function is made up of a vector of consumer specific attributes ($\mathbf{z}_i$) and a vector of search specific attributes ($\mathbf{x}_{it}$). Consumer-specific attributes are demographic characteristics such as income and the number of individuals in the household, as well as intrinsic preferences such as the desire to shop around, as opposed to quickly purchasing needed items. Consumer-specific attributes also include a binary indicator variable that is equal to one if the participant was in the public sample, and 0 if in the student sample. Including this binary variable permits a test of whether there is a fundamental difference between the public sample's desire to search compared to the student sample. Search specific attributes, $\mathbf{x}_{it}$, include the price of the goods at the time the search decision is made, the cost of search, and the number of products offered by the initial seller. The number of products squared is also included in the search specific attributes to test for a non-linear relationship between search and the number of products offered.

All estimated parameters are allowed to vary randomly over subjects, reflecting heterogenous preferences otherwise ignored by a fixed parameter model (Berry, Levinsohn, and Pakes 1995; Nevo 2001). I assume the parameters are normally distributed to allow for each effect to be either positive or negative. For example, I expect that as prices charged by the initial seller increase, buyers would be more likely to search the other seller's product because the gains from search increase. However, I do not want to impose this assumption *a priori* in order to test the specific relationship between search and prices. Formally, I assume the parameters are normally distributed such that:

$$\begin{pmatrix} \boldsymbol{\beta}_z \\ \boldsymbol{\beta}_x \end{pmatrix} = N\left[\begin{pmatrix} \bar{\boldsymbol{\beta}}_z \\ \bar{\boldsymbol{\beta}}_x \end{pmatrix}, \begin{pmatrix} \boldsymbol{\sigma}_z & \mathbf{0} \\ \mathbf{0} & \boldsymbol{\sigma}_z \end{pmatrix}\right] \tag{4.29}$$

where $\bar{\boldsymbol{\beta}}_z$, and $\bar{\boldsymbol{\beta}}_x$ represent the mean of the parameters and $\boldsymbol{\sigma}_z$, and $\boldsymbol{\sigma}_x$ capture consumer specific variations across parameters, or the standard deviation of the normally distributed parameter. McFadden and Train (2000) interpret the elements of equation (4.29) as an error-component model of attribute demand. Allowing the parameters to vary randomly not only accounts for consumer heterogeneity, but also defines the utility from search as being correlated according to the attributes of the decision at hand.

Given the indirect utility definition in equation (4.28) above, let $\mathbf{u}_{it1} = \boldsymbol{\beta}_z^{\intercal}\mathbf{z}_{i1} + \boldsymbol{\beta}_x^{\intercal}\mathbf{x}_{it1}$ represent the deterministic utility the consumer obtains from searching, and $\mathbf{u}_{it0} = \boldsymbol{\beta}_z^{\intercal}\mathbf{z}_{i0} + \boldsymbol{\beta}_x^{\intercal}\mathbf{x}_{it0}$ be the utility from not searching. The probability that a consumer decides to search is then given by:

$$\begin{aligned} \Pr_{it} &= \Pr\left[\mathbf{u}_{it1} + \varepsilon_{it1} > \mathbf{u}_{it0} + \varepsilon_{it0}\right] \\ &= \Pr\left[\varepsilon_{it0} - \varepsilon_{it1} < \mathbf{u}_{it1} - \mathbf{u}_{it0}\right] \end{aligned} \tag{4.30}$$

which is the cumulative distribution expression for $\varepsilon_{it0} - \varepsilon_{it1}$ evaluated at $\mathbf{u}_{it1} - \mathbf{u}_{it0}$. In order to consistently estimate equation (4.30), I address the apparent endogeneity between the price and number of product variables in $\mathbf{u}_{it}$.

Prices and variety are likely to be endogenous because the experiment is two-sided. Namely, the prices and number of products offered (or sellers' decisions) are based on the quantities sold in the previous period which are determined by, among other factors, the search decision. So, the total quantity a seller sells is a function of the error the buyers collectively make in determining whether or not to search. As a result, the prices in the current period may be correlated with the error in determining whether or not to search. I assume that the error an individual buyer makes is independent over time. Therefore, I use an instrumental variable approach to control for endogeneity in a model of the probability of search.

I use the control method approach (Petrin and Train 2010; Park and Gupta 2009), which is based on the sample-selection models of Heckman (1978) and Hausman (1978). Using simulation, Andrews and Ebbes (2013) show that when instrumental variables are available, the control function approach performs better than competing methods of addressing endogeneity (Berry, Levinsohn, and Pakes 1995; Park and Gupta 2009; Villas-Boas and Winer 1999) in estimating price elasticities. Intuitively, the control function approach derives a proxy variable that conditions on the endogenous part of the price variable, thus making the remaining variation independent of the error term. Then, the standard simulated maximum likelihood approach will be consistent.

The utility that a consumer obtains from searching is a function of both exogenous and endogenous variables. I partition the set of variables $\mathbf{x}_{it}$ into a vector of exogenous variables, $\dot{\mathbf{x}}_{it}$, and endogenous variables, $\{N_{it}, (N_{it})^2, \bar{p}_{it}\}^\top$, where N_{it} is the number of products available to buyer i at time t, and $\bar{p}_{it}$ is the average price of the N_{it} products. The decision to search is made based on the prices of the available products, the search cost, and other exogenous factors. So, the utility of searching is given by:

$$U_{it} = \boldsymbol{\beta}_z^\intercal \mathbf{z}_i + \boldsymbol{\beta}_{\dot{x}}^\intercal \dot{\mathbf{x}}_{it} + \beta_N N_{it} + \beta_{N^2} (N_{it})^2 + \beta_{\bar{p}} \bar{p}_{it} + \varepsilon_{it}. \tag{4.31}$$

Let η_{it}, and ϱ_{it} represent the errors associated with N_{it}, and $\bar{p}_{it}$, respectively, that is not independent of ε_{it}. The average price of the offered products, $\bar{p}_{it}$, is used instead of each actual price because not all products are offered each period. As a result, price information for each individual product is not available every period. Using the average price of all the offered products at time t is similar to assuming the response to price is the same for all the products (i.e. $\beta(p_{it1} + \cdots + p_{itN})$). While the amount paid by the experimenter is different for different products, it remains constant throughout the experiment and is well known by both buyers and sellers at the start of the experiment. So, while the buyers may not be as sensitive to a price change for a product that they are paid more for, sellers know this information as well, and would be expected to capitalize on it by setting slightly higher prices for higher valued goods. The variation between product prices is likely to lead to very similar, if not the same, price response for each product. Using the average price as a proxy for the effect prices have on the consumer's decision to search helps avoid the lack of price information for products that are not offered in a particular period by normalizing the sum of prices by the number of products that were offered. Using the average price also helps alleviate the endogeneity associated with each offered product by normalizing it by the total number of products available in that period.

Following Petrin and Train (2010), I assume that observed and unobserved covariates N_{it}, $(N_{it})^2$, and $\bar{p}_{it}$ are additive, or $\tilde{\eta}_{it} + \eta_{it}$ and $\tilde{\varrho}_{it} + \varrho_{it}$ respectively. Let η_{it} and ϱ_{it} represent the parts of $N_{it} + (N_{it})^2$ and $\bar{p}_{it}$, respectively, that is correlated with the error term and $\tilde{\eta}_{it}$ and $\tilde{\varrho}_{it}$ are not. I then decompose the error term associated with the decision to search, ε_{it}, into a general function of the observed and unobserved covariates of the endogenous variables leading to:

$$\varepsilon_{it} = CF\left[\eta_{it}, \varrho_{it}|\boldsymbol{\lambda}\right] + \tilde{\eta}_{it} + \tilde{\varrho}_{it} \tag{4.32}$$

where $CF\left[\eta_{it}, \varrho_{it}|\boldsymbol{\lambda}\right]$ is the control function with parameter vector $\boldsymbol{\lambda}$, and $\tilde{\eta}_{it}$, and $\tilde{\varrho}_{it}$ are the error components that are independent of ε_{it}. I approximate the control function as linear in $\tilde{\eta}_{it}$, and $\tilde{\varrho}_{it}$, or $CF\left[\eta_{it}, \varrho_{it}|\boldsymbol{\lambda}\right] = \lambda_1\eta_{it} + \lambda_2\varrho_{it}$.[6] The error terms η_{it}, and ϱ_{it} are recovered by, separately, regressing N_{it}, and $\bar{p}_{it}$ onto a set of instrumental variables.

The instrumental variables are the costs of each of the 5 available products as well as the seller margins (Berto Villas-Boas 2007; Draganska and Klapper 2007). The instrumental variables are intuitive determinants selling prices, and number of products chosen by the seller, but exogenous

[6] An interaction term was included, but provided little to no explanatory power, and had almost no effect on the parameter estimates of other variables so was excluded from the final analysis.

to an individual buyer's search decision.[7] I assume that the control function is normally distributed and that $\tilde{\eta}_{it}$, and $\tilde{\varrho}_{it}$ are defined such that $\tilde{\eta}_{it} + \tilde{\varrho}_{it}$ is Type 1 extreme value (see Bertin and Clusel (2006) for the distributional properties of $\tilde{\eta}_{it}$, and $\tilde{\varrho}_{it}$ that lead to $\tilde{\eta}_{it} + \tilde{\varrho}_{it}$ being Gumbel distributed).

Combining the utility function given in equation (4.28) with the error term in equation (4.32), indirect utility is written as:

$$U_{it} = \boldsymbol{\beta}_z^{\intercal}\mathbf{z}_i + \boldsymbol{\beta}_x^{\intercal}\mathbf{x}_{it} + \lambda_1\eta_{it} + \lambda_2\varrho_{it} + \tilde{\epsilon}_{it}, \tag{4.33}$$

where $\tilde{\epsilon}_{it} = \tilde{\eta}_{it} + \tilde{\varrho}_{it}$, and $\lambda_k \sim N(\lambda_k, \sigma_{\lambda_k})$ for $k = 1, 2$ similar to the parametric definitions given in equation (4.29) above. Estimating the models sequentially in this way can cause a compounding error problem (Cameron and Trivedi 2005; and Petrin and Train 2010). However, this is unlikely as the estimated parameters are very similar to those found without the control function.[8] Therefore, I conclude that the compounding error problem is, at most, negligible.

Empirical consumer search studies commonly assume that ε_{it} is Type 1 Extreme Value distributed and, combined with the random parameter assumption, yields the familiar mixed logit model. However, the Gumbel distribution requires all ε_{it} be independent across all choice situations. Therefore, I estimate the model twice. One estimation assumes that ε_{it} is Gumbel distributed and another assumes ε_{it} is jointly normally distributed across all choice situations. The joint normal distribution does not require an independence assumption, and generates a mixed Probit model. The mixed Probit model can accommodate random taste variation, flexible substitution patterns, and is applicable to panel data with temporally correlated errors ε_{it} (Train 2009). In general, the Gumbel distribution is nearly identical to the normal distribution except that is has slightly fatter tails which allows for more aberrant behavior. Since there is no closed form for either the mixed logit, or the mixed probit model, I use simulated maximum likelihood to estimate the probability of search. Simulated maximum likelihood provides consistent parameter estimates under general error assumptions and is readily able to accommodate complex structures regarding consumer heterogeneity. To aid in the speed and efficiency of estimation, I use a Halton draw sequence. Bhat

[7] The quantities sold in the prior period are also likely candidates for instrumental variables, but may not be exogenous, and so were not used.

[8] These results are available from the authors upon request.

(2003) provides experimental evidence that suggests Halton sequences can reduce the number of draws required to produce estimates at a given accuracy by a factor of 10. I found that $R = 500$ draws were more than sufficient to produce stable estimates.

4.4 Results and Discussion

In this section, I report the results obtained from tests of the main hypotheses of the paper, specifically how variety is related to the costs of search. Prior to presenting the parameter estimates from the formal econometric search model, I first present some summary statistics on the experimental search data. Table 4.1 presents summary statistics for the participants in both the student sample, and general-population sample that were selected to be "buyers" in the experiment. The samples appear to be generally similar, although, not surprisingly, the average age of the student sample is 22.5 years old, whereas that in the public sample is 35 years old. Moreover, the age range is much wider in the public sample compared to the student sample. The only other notable difference across the samples is the frequency with which they made a purchase online, or had something delivered to them that was purchased online. In this regard, students are more frequent online purchasers than the general public

Table 4.1

Experiment Summary Statistics.

Variable	Student Population Mean	Student Population Std. Dev.	Public Population Mean	Public Population Std. Dev.
Search Decision	0.59408	0.310798	0.54419	0.369638
Preference for Variety	1.52535	0.208929	1.45512	0.251891
Search Cost	1.91171	0.279027	1.95408	0.359910
Budget Not Spent	4.54842	3.848258	8.26910	10.212501
Number of Products Purchased	1.99567	0.754445	2.07573	0.773776
Number of Products offered - Seller 1	2.69847	0.759002	3.52750	1.223511
Number of Products offered - Seller 2[1]	1.61208	1.018545	1.76451	1.499512
Total Products Observed	4.31055	1.288343	5.29201	1.868046
Age	22.56250	3.600515	35.11429	16.795958
% of Female	0.46875	-	0.40000	-
# in the Household	2.15625	1.547305	2.54286	1.596740
Income	8.09375	5.909857	7.71429	5.344439
Education	3.12500	0.941858	3.45714	1.291211
Analytical Bach. Degree	0.75000	-	0.05714	-
% with Job in AR/AP	0.31250	-	0.40000	-
$ spent on Groceries	4.03125	2.162501	4.00000	2.029199
% that favors Prod. offered[2]	0.56250	-	0.28571	-
% that like shopping around[3]	0.40625	-	0.65714	-
Frequency of online purchases[4]	3.12500	1.263635	2.48571	1.245496
Frequency of delivered items[4]	3.06250	1.162242	2.31429	0.993255
	Min.	Max	Min.	Max
Search Decision	0.000	1.000	0.000	1.000
Preference for Variety	1.169	2.061	0.742	2.014
Search Cost	1.371	2.651	0.960	2.534
Budget Not Spent	0.272	15.404	0.373	38.625
Number of Products Purchased	1.000	4.786	1.000	4.047
Number of Products offered - Seller 1	1.556	4.462	1.227	5.000
Number of Products offered - Seller 2[1]	0.000	3.583	0.000	5.000
Total Products Observed	1.900	6.538	1.909	9.454
Age	18.000	32.000	19.000	71.000
# in the Household	1.000	6.000	1.000	6.000
Income	1.000	19.000	1.000	16.000
Education	1.000	5.000	1.000	6.000
$ spent on Groceries	2.000	9.000	1.000	9.000
Frequency of online purchases[4]	1.000	6.000	1.000	6.000
Frequency of delivered items[4]	1.000	6.000	1.000	4.000
	$H = 32$		$H = 35$	

The household statistics reported here are only for those participants who were selected to be 'buyers' in the experiment. Additionally, the average experiment variables are calculated as the average per buyer, then average over the sample. So, the averages reported here are not weighted by the number of periods each individual got through.

[1] This is conditional on the 'buyer' choosing to search.

[2] The participants had to choose whther price, or the products offered was more important when choosing a retailer.

[3] The participants had to choose whether the preferred to obtain the items they came for and leave, or shop around and look at different items offered in a retailer.

[4] 1 - Never; 2 - Less than once a Month; 3 - Once a month; 4 - 2-3 times per month; 5 - Once a week; 6 - 2-3 times a week; 7 - Daily. (No one chose 7).

Within the experiment itself, buyers included in the estimation sample were similar in terms of both the cost of searching and the preference for variety. However, sellers in the student population offered fewer products, on average, compared to the public sample. Namely, students initially had an average of 2.7 products to choose from before searching while sellers in the public sample chose to offer an average of 3.5 products.[9] Despite this, the average number of products purchased by buyers in both samples was very close to two products. So, even though buyers in the public sample saw a higher number of products for free, and had the same preference for variety incentive (not statistically different at the 5% level), the number of products purchased is consistent with the number purchased in the student sample - 2 products. When making purchase decisions, the public sample used their budget less efficiently compared to the students. On average, the public sample had 8.3EC remaining compared to 4.5EC for the student-sample. These differences are statistically significant at the 6% level of significance.[10] Finally, the difference between the two different sample's search cost and preference for variety are not statistically different. From Figure 4.2 below, the preference for variety and search costs do represent a uniform distribution reasonably well. The histograms look almost identical if split across the different samples.

In addition to buyer information, I also have information pertaining to sellers. In particular, I find that seller profit and the number of products offered are negatively correlated and statistically significant at the 1% level ($N = 475$). This relationship holds even when considering sellers' gross profit before inventory costs are subtracted, although it is no longer significant. Therefore, even though consumers' had an incentive to purchase a wider range of products, even from different sellers, sellers did not see this preference for variety reflected in their profit. Consistent with Yuan and Han (2011), there is a slightly positive correlation between price dispersion and search intensity (0.06) but this is not statistically different from 0. So, my results do not appear to support Varian (1980) in that the more consumers search, prices are less competitive and more variable. In contrast to Yuan and Han (2011), I find no correlation between buyer search intensity and seller profit. Finally, I find a negative correlation between search intensity and the buyer's search cost (-0.225)

[9] The mean is statistically different at the 1% level.

[10] If the individual purchase observations are used then the difference between the average amount of money that was not used across the two different samples is significant at the 1% level ($N = 1044$).

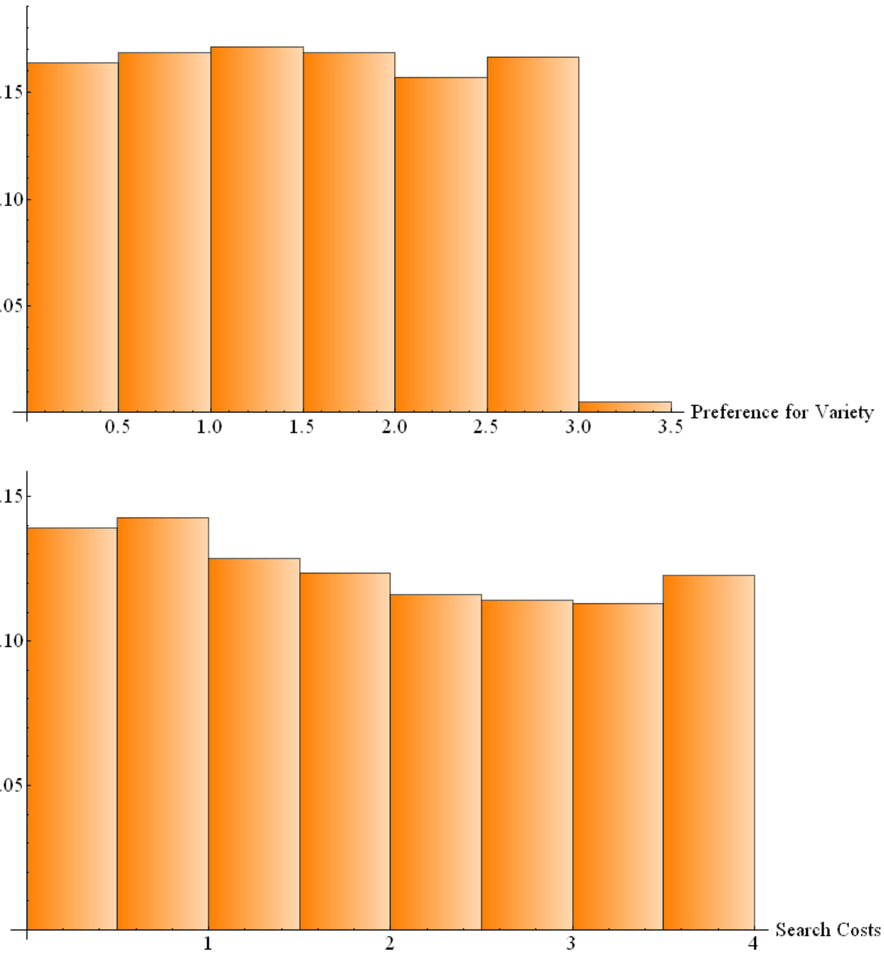

Figure 4.2

Histrogram of the Search Cost and Preference for Variety Observed by Buyers

as well as the number of products offered (-0.243), both of which are significant at the 1% level. I test these relationships more formally next.

The negative correlation between search intensity and the number of products offered does not take into account the degree to which prices may be driving the consumer's decision to search. If the buyer's search cost is reasonably high, but the prices offered by the seller are even higher, then the correlation between searching and the cost of searching may be understated. In other words, prices may be masking what is actually a stronger negative relationship between the buyer's cost of search and the decision to do so. Modeling the probability of search, as discussed above, accounts for these inter-correlations between the variables driving the consumer's search decision as well as any unobserved heterogeneity in the participant's ability to solve the problem analytically.

I test whether unobserved heterogeneity is indeed important using the likelihood ratio (LR) test. A LR test compares a model that assumes heterogeneity is not present to the model in (4.29). The LR test statistic is 127.30 for the mixed logit model, and 77.87 for the mixed probit model, both of which are Chi-square distributed and significant at the 5% level. For either model, therefore, the random parameter specification is preferred.[11]

The search model is estimated both as a mixed logit and a mixed probit. The results of both are presented in table 4.2. From a qualitative perspective, the assumption that the error terms are normally distributed versus Gumbel distributed does not seem to make a substantive difference to the conclusions of my hypothesis tests. Since the majority of empirical consumer search studies use a Gumbel distribution assumption, I interpret the mixed logit results in order to maintain comparability (Mehta, Rajiv, and Srinivasan 2003; Wildenbeest 2011; and Yuan and Han 2011).

The results presented in table 4.2 summarize the relationship between the probability of searching, and the factors affecting that decision. Consistent with the current search literature, I find that the probability a consumer searches decreases with the cost of search (Mehta, Rajiv, and Srinivasan 2003). Consistent with the experimental results of Yuan and Han (2011), I find that subjects are more likely to search when observed prices increase.[12] In addition, I find that excluding the number of products offered biases the coefficient on price towards zero. Without considering the number of products offered, the results of the mixed logit model (left columns of table 4.2) suggest that prices have less bearing on the consumer's decision to search relative to when variety is included.

The bias induced by excluding the retailer's variety decision is not surprising given the importance of variety to the subject's propensity to search. Based on the magnitude of the parameter estimates reported in table 4.2, the number of products offered by the retailer plays a more important role in guiding the consumer's search decision compared to both the average price, or the

[11] The results of the fixed parameter alternative are available from the authors upon request.

[12] Positive parameter estimates are also found on a model that estimates a different parameter for each price.

search cost (Kadiyali, Vilcassim, Chintagunta 1999; Briesch, Chintagunta, and Fox 2009). As variety increases, the probability the consumer decides to search decreases – precisely as the seller intends. Since the initial seller offers a larger variety to persuade the buyer not to search the other seller, a negative coefficient is expected. Consistent with Briesch, Chintagunta, and Fox (2009), my results suggest that retailers can use variety as a competitive tool to keep consumers from shopping at other retailers. Moreover, my results provide evidence that this relationship is non-linear.

The choice overload hypothesis states that too many options overwhelm consumers and can lead to him avoiding making a purchase at all. Kuksov and Villas-Boas (2010) formalize the concept underlying the choice overload hypothesis by explaining it in terms of consumer search costs. When the cost of search is small, consumers prefer a wider choice set, and as the cost of search increases, the demand for variety falls. I test this effect directly as it relates to the consumer's propensity to search. The positive parameter estimate on $(N_{1t})^2$ is statistically different from 0 at the 5% level for both the mixed logit and probit results, which provides support for the choice overload hypothesis. More variety causes consumers to search more, thereby reducing the probability of actually making a choice. Consistent with Scheibehenne, Greifeneder, and Todd (2010) the variance of the parameter estimate is statistically different from 0, which implies that there is considerable heterogeneity among consumers regarding the precise point at which the overload-effect begins. Therefore, the results of the mixed logit and probit models suggest that the choice overload hypothesis may have been accurate for some, but not all, experiment participants. Perhaps more important, however, is the comparison between the linear and non-linear parts of the search function. Some argue that retailers use larger product assortments to attract consumers to the store (Koelemeijer and Oppewal 1999; Boatwright and Nunes 2001; Borle, Boatwright, Nunes, and Shmueli 2005; Oppewal and Koelemeijer 2005; Richards and Hamilton 2006; and Briesch, Chintagunta, and Fox 2009), while others find that consumers are put off by too much variety (Iyengar and Lepper 2000; Chernev 2003; and Iyengar, Huberman, and Jiang 2004). I find that search is a non-linear function of the number of products offered. In addition, the number of products offered has a larger impact on the propensity of an individual to search than either the search cost, or the prices. In other words, because search is directly related to which store a participant chooses to patronize, and precedes purchase and consumption, the non-linear relationship found here suggests there is an optimal number of products that could be offered to get the consumer to avoid searching the other store. Because I use an experimental setting that offers participants, at most, 10 choices I do not compute through simulation the precise number of products a consumer would want and leave that for future field studies.

My results also highlight the importance of considering the number of products sold by a retailer in field experiments. Prices, the cost of searching, and variety can all be used as competitive tools by a retailer to persuade a consumer to shop only with them, but variety appears to be the most important.

In addition to the cost of search and prices, I find that there are a number of consumer specific demographic attributes that play an important role in the consumer's propensity to search. First, the results in table 4.2 show that the public sample is considerably less likely to search compared to the student sample. This is consistent with the probability of search reported in table 4.1. This finding suggests that a student sample may not be representative of the actual search behavior of the general public, because students are more likely to undertake search. Second, the consumer's revealed preference for variety in the previous period has no bearing on their decision to search in the current period. This result is expected and shows that the participants understood that their preference for variety was independent across periods.[13] Third, male participants were much more likely to undertake search compared to female participants. Fourth, in contrast to Bucklin (1969) there is some evidence that as the participant's income increases they are less likely to undertake search. This is an interesting result because the consumer's own income has no bearing on their profit in the experiment. However, it is consistent with the notion that the most important component of search costs is the opportunity cost of time. Fifth, the results suggest that consumer who spend more on groceries are more likely to search across different retailers. Participants' monthly grocery bill should have no effect on their decision to search, yet I find evidence that consumers who have a higher monthly grocery bill are more likely to search, perhaps because they perceive the expected benefit from search to be greater. Sixth, participants were asked whether they had a bachelors

[13]The current period's preference for variety was also found to not have any bearing on the consumer's propensity to undertake search in that period. Since the preference for variety was revealed to them after they made the search decision, this result is expected.

or advanced degree in business, or an analytical field of study.[14] The results suggest that more technically-educated participants were less likely to undertake search.

Taken together, the results begin to answer the question of whether too much variety is indeed a bad thing. I find that the variety offered by a retailer is a critical component of the consumer's decision to search. Variety not only has a larger impact on the consumer's decision to search compared to prices and the cost of search, its exclusion from the analysis actually biases the results of those parameters. Therefore, it is critically important to take variety into consideration when studying consumer search. Additionally, given the effect of the number of products has on a consumer's decision to search, firms can increase variety when search costs fall, and maintain higher prices to impede consumer search. This confirms the main hypothesis of Cachon, Terwiesch, and Xu (2008). Consumer search studies often test the relationship between search and search costs using a single purchase assumption (Kogut 1990; Sonnemans 1998; and Yuan and Han 2011). However, as my results show, the results of these studies may not be applicable to situations in which the consumer can observe multiple products at the same time. My results also show that consumers have differing intrinsic propensities to search that are not entirely due to observed heterogeneity, or differences in demographic background.

[14]In the student sample, freshman and sophmores were instructed to answer no to this question unless they had already taken all their math requirements and had 2 business classes. Juniors and Seniors will have fulfilled these requirements based on the school's curriculum and were instructed to answer yes to the question.

Table 4.2

Random Coefficient Model Estimates.

Variable	Mixed Logit		Mixed Logit		Mixed Probit	
	Estimate	t-ratio	Estimate	t-ratio	Estimate	t-ratio
Constant	-0.9071	-0.50	2.9620*	3.82	2.7874*	4.50
Public sample	-1.2686*	-5.11	-2.0919*	-7.11	-1.7088*	-7.35
Pref. for variety$_{t-1}$	0.0266	0.68	0.0518	0.63	0.0342	0.51
Female	-0.6967*	-6.08	-1.1148*	-5.99	-0.7240*	-4.91
# People in HH	-0.2419*	-2.95	-0.4193*	-6.12	-0.3908*	-6.89
Income	-0.0688*	-7.01	-0.0807*	-4.76	-0.0797*	-5.61
Education	0.1955*	6.99	0.1011	1.17	0.0256	0.36
Bachelors	-1.1082	-1.07	-1.4374*	-5.56	-1.4638*	-6.81
Grocery costs	0.3455*	7.53	0.4047*	7.01	0.3609*	7.67
Ave. Price	0.3226*	2.13	0.3766*	7.83	0.2738*	7.65
Search Cost	-0.5377*	-7.31	-0.6576*	-9.00	-0.5375*	-9.16
NPO S_{1t}†	-	-	-1.2205*	-3.66	-0.9375*	-3.48
(NPO S_{1t})^2	-	-	0.1205*	2.24	0.0870*	1.99
NPO S_{2t-1}‡	-	-	-0.0687	-1.30	-0.0561	-1.38
λ_p	-0.0185	-0.74	-0.0185*	-2.08	-0.0144*	-2.07
λ_N	-	-	-0.1901	-1.22	-0.1624	-1.34
	Std. Dev. Of Random Parameters					
Female	0.0727	0.48	0.0727*	6.36	0.9729*	8.57
# People in HH	0.0665	1.75	0.0665	0.15	0.0418	1.75
Income	0.0201	1.89	0.0201*	6.00	0.0034	0.53
Education	0.3152*	9.09	0.3152*	4.55	0.0540*	2.86
Bachelors	0.2587	1.79	0.2587	0.49	0.0895	1.01
Grocery costs	0.0358	1.58	0.0358	1.47	0.0920*	5.75
Ave. Price	0.2230*	12.47	0.2230*	12.07	0.1589*	12.29
Search Cost	0.2643*	5.66	0.2643*	6.18	0.2368*	7.13
NPO S_{1t}†	-	-	0.2785*	7.28	0.0696*	3.58
(NPO S_{1t})^2	-	-	0.0861*	8.39	0.0669*	9.73
NPO S_{2t-1}‡	-	-	0.1199*	2.38	0.0310	1.16
σ_{λ_p}	0.0438*	6.87	0.0374*	5.39	0.0167*	4.51
σ_{λ_N}	-	-	0.1063	0.91	0.0320	0.49
Log-Likelihood	-451.61		-432.43		-432.16	
LR	243.81		228.11		232.11	
LRI	0.2126		0.2170		0.2208	

An asterisk indicates significance at a 5.0

† NPO S_{1t} - Number of products offered by the seller the consumer sees for free at the time the search decision is made.

‡ NPO S_{2t-1} - Number of products offered by the searched seller in the previous period, if the participant decided to search in the previous period.

4.5 Conclusion and Future Research

In this chapter I examine the relationship between consumer search and preference for variety when consumers purchase multiple products in continuous quantities. I examine firms' incentives to offer a wider variety of products, and consider whether the "overload hypothesis," or whether retailers can offer too much variety and deter consumers from purchasing.

How consumers respond to the assortment offered by a firm remains a debate. While many studies find that variety is valued by consumers, and can be used to attract them, others find that too many options leads to consumer dissatisfaction and the avoidance of a purchase (choice overload hypothesis). This study bridges these competing schools of thought using experimental methods.

I use a two-sided experiment in which participants use the number of products, prices, and search costs to determine whether or not to search, while knowing that they will have some positive preference for variety that will motivate them to purchase multiple fictitious products. The experiment is conducted on both undergraduate college students and the general public.

My results support the theoretical models of Norwood (2006) and Kuksov and Villas-Boas (2010) who suggest that consumer search explains the choice overload hypothesis. I find that the factors affecting the propensity to search can explain both the notion that more variety is better, but too much variety can be a bad thing from a retailer's perspective. In particular, I find that consumer search is a non-linear function of variety. My results also illustrate the importance of taking variety into account in future consumer search studies as excluding variety results in serious estimation bias.

The results have a number of managerial implications for retailers. In particular, retailers can increase the assortment available to persuade a consumer to patronize their store and avoid searching another store. However, this effect does not occur without bound. My results show that, eventually, a consumer will be overwhelmed by the number of products offered and search the other store as well, or possibly, instead. The degree to which consumers are overburdened by the variety offered differs significantly across consumers and may not be as prevalent in some as in others. This finding lends further evidence as to why the choice overload hypothesis has had mixed support.

Future work may benefit from also taking into consideration consumer's price expectations in the presence of a preference for variety. Yan and Han (2011) show how consumer price expectations can also affect the decision to search. However, they assume that consumers do not have a preference for variety and only a single purchase is made. Price expectations in the presence of a heterogenous preference for variety may cause consumers to be more, or less, sensitive to the number of products offered. Future work may also benefit by considering collusion among retailers when selecting the prices and number of products available. My results show that variety is more important than prices when deciding whether or not to search. Retailers may, therefore, be able to capitalize on this information by setting higher prices and offering a wider assortment. If retailers know that other retailers are also offering a large variety, but not too much, then equilibrium prices may rise since consumers are less likely to search.

CHAPTER 5.

CONCLUSIONS AND IMPLICATIONS

In this dissertation I provide a better understanding of search behavior in the presence of heterogenous preferences for variety. By allowing for the observation that consumers purchase multiple products at each purchase occasion, I consider how a very general definition of variety is related to search behavior. There are several reasons that consumers purchase multiple products in a single shopping occasion. The most relevant to search behavior is a consumer's preference for variety. In particular, when a consumer is assumed to purchase a single product, the entire cost of search must be absorbed by the single product purchase. In contrast, as a consumer's preference for variety increases so does the perceived benefit of search because finding several products that are all equally appealing provides a higher total utility as a result of the diminishing marginal utility of each individual product. Allowing for several product purchases, therefore, not only spreads the total cost of search over several product purchases, but also fundamentally changes the optimal number of products searched which is driven by the consumer's preference for variety, as well as their cost of search.

Consistent with prior consumer search studies, I find search behavior plays an important role in shaping consumer demand, and that preference for variety has important implications for search behavior. In particular, consumers tend to choose consideration sets with a larger number of products when they have a strong preference for variety, and smaller choice sets when preference for variety is weak. Understanding a consumer population's preference for variety in general, and for particular product categories, has important implications for a retailer's decision regarding assortment depth. Knowing which products are in a majority of consumers' consideration sets helps retailers plan assortment depth and promotion planning. For example, if there are a particular set of products almost all consumers consider then these products are likely to be key category sales drivers and eliminating them from the shelf could have detrimental effects on category sales. Understanding consumer search also helps manufacturers when designing and introducing new products. In particular, understanding which products a consumer seriously considers helps guide manufacturers' new product design strategies by identifying niche consumer groups that are focused on a small subset of products with very specific characteristics. Knowing which products are seriously considered by which consumers before a purchase is made informs a critical understanding regarding the specific elements of a product category different consumer groups find particularly appealing.

I investigate the relationship between consumer search and multi-product purchases using a non-linear additive utility function analogous to Kim, Allenby, and Rossi (2002) and Bhat (2005). This allows me to develop an analytical consumer demand model in Chapter 2 that endogenizes both the consideration set and the products chosen. In this model, a consumer's decision is described in a utility maximizing framework that provides a closed form expression for the expected maximum utility from searching each consideration set. The analytical model illustrates that the gains from searching a larger consideration set can outweigh the higher cost of doing so when the consumer has a strong preference for variety. Intuitively, consumers in this case are more likely to find several products that meet their needs at a reasonable price, and can purchase some of all of them. Because the consumer is not restricted to purchasing a unit increment of a single item, their cost of search is spread over the total quantities purchased of several different products. This, in turn, allows a higher search cost to be easily off-set by the gains from search. On the other hand, when an individual has a lower preference for variety, they are more apt to purchase fewer unique brands and so the gains from a wider search are not similarly offset. My findings suggest that traditional search models tend to underpredict the number of goods that are searched if there is a strong preference for variety.

I estimate a structural model of consumer search based on this conceptual approach. With this model, I investigate how heterogenous preferences for variety affect choice. My approach provides a step forward in the analysis of consumer search behavior because it derives the optimal search decision using a more general framework than the standard discrete choice demand model. Consistent with existing studies, I find households incur a significant search cost to obtain product information. This chapter finds that a linear utility model is not flexible enough to accurately identify search, and subsequent purchase behavior, in situations where consumers exhibit a preference for variety. Consistent with Roberts and Lattin (1991) I find that search costs are important to account for in a frequently purchased household item, ice cream. Moreover, a Monte Carlo simulation experiment provides evidence that assuming every product is considered provides biased parameter estimates. Therefore, demand estimates that ignore the cost of search results in biased parameter estimates that can lead to incorrect managerial conclusions even for consumer goods that are regularly purchased like ice cream. Moreover, ignoring consumers' preference for variety in shaping the products taken into consideration can also lead to biased parameter estimates and erroneous managerial conclusion because the size of the consideration set is likely to be underrepresented. It is critically important then, to recognize the cost of acquiring information and preference for variety in shaping consumer demand.

Secondary data, however, always has limitations in the empirical analysis of search behavior, because search costs are not observed. To investigate the relationship between multi-product purchase environments and search more closely, I develop an experimental test of the relationship between search costs, variety, and purchase behavior. Using a two-sided experiment, my results show that retailers can use variety as a competitive tool to retain and attract consumers. However, the number products offered cannot increase without bound, as consumers have optimal levels of variety – too much choice can be overwhelming.

Taken together, my findings have several important implications for future research in this area, and for managers. First, discrete choice models are not sufficiently general to represent consumer search behavior in all contexts. Much of the understanding of search behavior has been built upon the logit model. While the logit, and other, linear utility models provide a number of appealing characteristics from a analytical point of view, new methods are needed to fully understand consumer search. Numerous situations exist in which the assumptions of classic discrete choice models fail, and research needs to recognize, and expand into, these areas. Focusing so much attention on search behavior from the perspective of a discrete choice model has limited our full understanding of consumer search. The first two Chapters of this dissertation are a first step to investigating search from a more general perspective. In particular I show that a consumer's search strategy is driven not only by the cost of search, but also their preference for variety. As the rate of diminishing marginal utility increases, or preference for variety increases, consumers increase the size of their consideration set because the gains from search increase.

Second, I show that there is a wide range of internal and external factors that affect a consumer's decision to search more, or to stop searching and purchase. One particularly important external factor is the number of products a firm offers. My results support the conclusion that retailing is a means of reducing consumer search costs and retailers can attract consumers to their store with the number of products offered. However, the number of unique choices demanded by consumers does not increase without bound. Instead, consumers have an optimal number of products they would like to have available to them. Providing consumers with a huge array of products increases a consumer's search cost because so many alternatives will have to be considered before finding the ones that provides the best fit with the consumer's wants. Understanding the number of products consumers want, and which in particular to provide is an important consideration for retailers and the structural model developed in Chapter 3 provides a tool for answering those questions by estimating the consideration set and the satisfaction individual products provide. This will help category and brand managers better manage their UPC offerings which will reduce costs, and increase consumer welfare through the form of more perfectly aligned product offerings and wants.

The choice overload hypothesis suggests that consumers become overwhelmed by too many choices and prefer a smaller choice set. Kuksov and Villas-Boas (2010) show how the choice overload hypothesis can be explained by consumer search costs and that consumers have a finite optimal variety to search. My experimental results support Kuksov and Villas-Boas (2010) and show that search increases with the number of products offered, but only to a point. Once there are too many options, search begins to decrease. This provides a deeper understanding of demand for variety and shows that consumer welfare does not increase infinitely with the number of products available.

My findings generalize to products and services beyond the consumer goods considered here. For example, my results suggest that individuals newly purchasing healthcare insurance may be better off with a limited number of options. Presenting every insurance package may indeed lead to poor choices, or no decisions at all. Further, the optimal number of insurance packages a consumer wants available to them will increase with their preference for variety. In other words, even though consumers are likely to only purchase a single insurance option after searching, the optimal number they will search through depends on their preference for variety. Given the importance of search to the success of the new health care legislation, understanding the relationship between search and preference for variety is critical, and this dissertation provides us with that information. By applying the models developed here to insurance search and purchase data that is readily available from online insurance websites such as eHealthInsurance.com, individual consumer characteristics can be used to determine which population segments prefer more or fewer insurance plans. Insurance, however, is typically purchased one policy at a time. In other cases, perhaps hotels for an extended vacation, or airline flights to get there, the multiple-purchase search model developed here may have similar implications.

WORKS CITED

Allenby, G. M. and P. E. Rossi. "Quality Perceptions and Asymmetric Switching Between Brands," *Marketing Science* 10 (1991): 185-204.

Amaldoss, Wilfred, and Amnon Rapoport. "Collaborative Product and Market Development: Theoretical Implications and Experimental Evidence" *Marketing Science* 24 (2005): 396-414.

Anderson, Simon P., and Régis Renault. "Pricing, Product Diversity and Search Costs: A Bertrand-Chamberlin-Diamond Model" *The RAND Journal of Economics* 30 (1999): 719-735.

Anderson, Simon P., and Régis Renault. "Consumer Information and Firm Pricing: Negative Externalities from Improved Information" *International Economic Review* 30 (2000): 721-742.

Anderson, Simon P., André de Palma, and Jacques-François Thisse. Discrete Choice Theory of Product Differentiation Massachusettes Institute of Technology, Cambridge, MA (1992).

Andrews, Rick L., and Peter Ebbs. "Properties of Instrumental Variables Estimation in Logit-Based Demand Models: Finite Sample Results" Working Paper. University of Delaware Newark, DE, 2013.

Andrews, Rick L., and T. C. Srinivasan. "Studying Consideration Effects in Empirical Choice Models Using Scanner Panel Data" *Journal of Marketing Research* 32 (1995): 30-41.

Arunachalam, Bharath, Shida R. Henneberry, Jayson L. Lusk, and F. Bailey Norwood. "An Empirical Investigation into the Excessive-Choice Effect" *American Journal of Agricultural Economics* 91 (2009): 810-825.

Banks, J., and S Moorthy. "A Model of Price Promotions with Consumer Search" *International Journal of Industrial Organization* 17 (1999) 371–398.

Bar-Isaac, Heski, Guillermo Caruana, and Vicente Cuñat. "Information Gathering and Marketing" *Journal of Economics & Management Strategy* 19 (2010): 375-401.

Barten, A.P. "Consumer demand functions under conditions of almost additive preferences" *Econometrica* 32 (1964): 1-38.

Baye, M., J. Morgan, and P. Scholten. "Information, Search, and Price Dispersion" Working Paper. Forthcoming in Handbook of Economics and Information Systems. (ed. T. Hendershott). 2005.

Berger, Jonah, Michaela Draganska and Itamar Simonson. "The Influence of Product Variety on Brand Perception and Choice" *Marketing Science* 26 (2007): 460-472.

Bertin, Eric, and Maxime Clusel. "Generalized Extreme Value Statistics and Sum of Correlated Variables" *Journal of Physics A: Mathematical and General* 39 (2006): 7607-7619.

Ben-Akiva, M. E., and S. R. Lerman. Discrete Choice Analysis: Theory and Application to Travel Demand Massachusetts Institute of Technology, Cambridge, MA (1985).

Berry, S., J. Levinsohn, and A. Pakes. “Automobile Prices in Market Equilibrium,” *Econometrica* 63 (1995): 841-890.

Berry, S., J. Levinsohn and A. Pakes. “Differentiated Products Demand System from a Combination of Micro and Macro Data: The New Car Market,” *Journal of Political Economy*, 112 (2004):68–105.

Berto Villas-Boas, Sofia. “Vertical Relationships between Manufacturers and Retailers: Inference with Limited Data” *The Review of Economic Studies*. 74 (2007):625-652.

Bertrand, J. "Théorie Mathématique de la Richesse Sociale" *Journal des Savants*, (1883): 499-508.

Besanko, D., J Dubé, and S. Gupta. "Competitive Price Discrimination Strategies in a Vertical Channel Using Aggregate Retail Data" *Management Science* 49 (2003): 1121-1138.

Betancourt, R. The Economics of Retailing and Distribution. Cheltenham, UK: Edward Elgar Publishing Ltd.

Bhat, C. R. "Simulation Estimation of Mixed Discrete Choice Models using Randomized and Scrambled Halton Sequences" *Transportation Research Part B: Methodological* 37 (2003): 837-855.

Bhat, C. R. "A multiple discrete-continuous extreme value (MDCEV) model: formulation and application to discretionary time-use decisions" *Transportation Research Part B* 39 (2005) 679–707.

Bhat, C. R. "The multiple discrete-continuous extreme value (MDCEV) model: Role of utility function parameters, identification considerations, and model extensions" *Transportation Research Part B* 42 (2008) 274–303.

Bhat, C. R. and S. Sen. "Household vehicle type holdings and usage: an application of the multiple discrete-continuous extreme value (MDCEV) model" *Transportation Research Part B* 40 (2006) 35–53.

Bhat, C. R., Marisol Castro, and Abdul Rawoof Pinjari. "Allowing for Non-Additively Separable and Flexible Utility Forms in Multiple Discrete-Continuous Models" Working Paper. The University of Texas at Austin. 2012.

Boatwright, Peter, and Joseph C. Nunes. "Reducing Assortment: An Attribute-Based Approach" *Journal of Marketing* 65 (2001): 50-63.

Borle, Sharad, Peter Boatwright, Joseph B. Kadane, Joseph C. Nunes, and Galit Shmueli. "The Effect of Product Assortment Changes on Customer Retention" *Marketing Science* 24 (2005): 616-622.

Briesch, R. A., P. K. Chintagunta, and E. J. Fox. "How Does Assortment Affect Grocery Store Choice?" *Journal of Marketing Research* 46 (2009): 176-189.

Bronnenberg, Bart J., and Wilfried R. Vanhonacker. "Limited Choice Sets, Local Price Response and Implied Measures of Price Competition" *Journal of Marketing Research* 33 (1996): 163-173.

Brynjolfsson, Erik, and Michael D. Smith. "Frictionless Commerce? A Comparison of Internet and Conventional Retailers" *Management Science* 46 (2000): 563-585.

Bucklin, Louis P. "Consumer Search, Role Enactment, and Market Efficiency" *The Journal of Business* 42 (1969) 416-438.

Butters, G. "Equilibrium Distributions of Sales and Advertising Prices" *The Review of Economic Studies* 44 (1977): 465-491.

Cachon, G., C Terwiesch, and Y. Xu. "The Effects of Consumer Search and Firm Entry in a Multiproduct Competitive Market" *Marketing Science* 27 (2008) 461-473.

Caglayan, Mustafa, Alpay Filiztekin, and Michael T. Rauh. "Inflation, price dispersion, and market structure" *European Economic Review* 52 (2008) 1187-1208.

Cameron, A. Colin, Pravin K. Trivedi. Microeconometrics: Methods and Applications Cambridge University Press, Cambridge, NY, 2005.

Cardell, N. S. "Variance Components Structures for the Extreme Value and Logistic Distributions with Applications to Models of Heterogeneity". *Econometric Theory* 13 (1997): 185–213.

Carlson, J. and R. McAfee "Discrete Equilibrium Price Dispersion" *Journal of Political Economy* 91 (1983): 357-365.

Chandra, A., and M Tappata. "Consumer Search and Dynamic Price Dispersion: An Application to Gasoline Markets" Working Paper. Sauder School of Business, University of British Columbia.

Chamberlin, E. The Theory of Monopolistic Competition. Cambridge, MA: Harvard University Press, 1933.

Chen, Y., and T. Zhang. "Equilibrium Price Dispersion with Heterogenous Searchers" *International Journal of Industrial Organization* 29 (2011) 645–654.

Chen, Pei-yu, and Lorin M. Hitt. "Understanding Price Dispersion in Internet-Enabled Markets" Working Paper. Carnegie Mellon University, 2003.

Chernev, Alexander. "When More Is Less and Less Is More: The Role of Ideal Point Availability and Assortment in Consumer Choice" *Journal of Consumer Research* 30 (2003a): 170-183.

Chernev, Alexander. "Product Assortment and Individual Decision Processes" *Journal of Personality and Social Psychology* 85 (2003b): 151-162.

Chevalier, Judith, and Austan Goolsbee. "Measuring Prices and Price Competition Online: Amazon.com and BaresandNoble.com" *Quantitative Marketing and Economics* 1 (2003): 203-222.

Chiang, Jeongwen, Siddhartha Chib, and Chakravarthi Narasimhan. "Markov Chain Monte Carlo and Models of Consideration Set and Parameter Heterogeneity" *Journal of Econometrics* 89 (1999): 223-248.

Chintagunta, P. K. "Investigating Purchase Incidence, Brand Choice and Purchase Quantity Decisions of Households" *Marketing Science* 12 (1993): 184-208.

Chintagunta, P. K. "Investigating Category Pricing Behavior at a Retail Chain"_*Journal of Marketing Research* 39 (2002): 141-154.

Chintagunta, P. K., D. C. Jain, and N. J. Vilcassim. "Investigating Heterogeneity in Brand Preferences in Logit Models for Panel Data" *Journal of Marketing Research* 28 (1991): 417-428.

Clay K., R. Krishnan, E. Wolff. "Prices and Price Dispersion on the Web: Evidence fromOnline Book Industry" National Bureau of Economic Research Working Paper# 8271, 2001.

Dahlby, B., and D. West. "Price Dispersion in an Automobile Insurance Market" *Journal of Political Economy* 94 (1986): 418-438.

De los Santos, Babur, Ali Hortaçsu, and Matthijs R. Wildenbeest. "Testing Models of Consumer Search Using Data on Web Browsing and Purchasing Behavior" *The American Economic Review* 102 (2012): 2955-2980.

Diehl, Kristin, and Cait Poynor. "Great Expectations?! Assortment Size, Expectations, and Satisfaction" *Journal of Marketing Research* 47 (2010): 312-322.

Diehl, Kristin, and Gal Zauberman. "Searching Ordered Sets: Evaluations from Sequences under Search" *Journal of Consumer Research* 31 (2005): 824-832.

de Palma, A., G. Myers, and Y. Papageorgiou. "Rational Choice Under Imperfect Ability to Choose" *American Economic Review*. 84 (1994): 419-440.

Diamond, P. "A Model of Price Adjustment" *Journal of Economic Theory* 3 (1971): 156-168.

Dickson, P. and A. Sawyer. "The Price Knowledge and Search of Supermarket Shoppers" *Journal of Marketing* 54 (1990): 42-53.

Draganska, M. and D. Klapper. 2007. "Retail Environment and Manufacturer Competitive Intensity" *Journal of Retailing*. 83:183-198.

Draganska, M. and D.C. Jain. "Product-Line Length as a Competitive Tool" *Journal of Economics and Management Strategy*. 14(2005):1-198.

Dubé, J. "Multiple Discreteness and Product Differentiation: Demand for Carbonated Soft Drinks" *Marketing Science* 23 (2004): 66-81.

Eaton, B. Curtis, and Richard G. Lipsey. "The Theory of Market Pre-emption: The Persistence of Excess Capacity and Monopoly in Growing Spatial Markets" *Economica* 49 (1979): 149-158.

Einav, Liran, Ephraim Leibtag, and Aviv Nevo. "On the Accuracy of Nielsen Homescan Data" U.S. Dept. of Agriculture, Economic Research Service. ERR-69. Dec. 2008.

Ellison, Glenn, and Sara Fisher Ellison. "Search, Obfuscation, and Price Elasticities on the Internet" *Econometrica* 77 (2009): 427-452.

Fischbacher, Urs. "z-Tree: Zurich Toolbox for Ready-Made Economic Experiments" *Experimental Economics* 10 (2007): 171-178.

Fisher, R. A., and L. H. C. Tippett. "Limiting Forms of the Frequency Distribution of the Largest or Smallest Member of a Sample" *Mathematical Proceedings of the Cambridge Philosophical Society* 24 (1928): 180-194.

Fréchet, M. "Sur La Loi Probabilité de l'écart Maximum" *Annales de la Société Polonaise de Mathématique*. 6 (1927): 93-116.

Gourville, John T., and Dilip Soman. "Overchoice and Assortment Type: When and Why Variety Backfires" *Marketing Science* 24 (2005): 382-395.

Gumbel, E. J. Statistics of Extremes Cambridge University Press, 1958.

Hagiu, A., and B. Jullien. "Why do Intermediaries Divert Search?" *RAND Journal of Economics* 42 (2011): 337-362.

Hanemann, W. Michael. "Discrete/Continuous Models of Consumer Demand" *Econometrica* 42 (1984): 541-561.

Hann, Il-Horn, and Christian Terwiesch. "Measuring the Frictional Costs of Online Transactions: The Case of a Name-Your-Own-Price Channel" *Management Science* 49 (2003): 1563-1579.

Hausman, J. A. "Specification Tests in Econometrics" *Econometrica* 49 (1978): 1251-1271.

Heckman, Jerry J. "Dummy Endogenous Variables in a Simultaneous Equation System" *Econometrica* 46 (1978): 931-959.

Hendel, Igal. "Estimating Multiple-Discrete Choice Models: An Application to Computerization Returns" *The Review of Economic Studies* 66 (1999): 423-446.

Hoch, Stephen J., Xavier Dréze and Mary E. Purk. "EDLP, Hi-Lo, and Margin Arithmetic" *Journal of Marketing* 58 (1994): 16-27.

Hong, Han, and Matthew Shum. "Using Price Distributions to Estimate Search Costs" *The RAND Journal of Economics* 37 (2006): 257-275.

Honka, Elisabeth. "Quantifying Search and Switching Costs in the U.S. Auto Insurance Industry" Working Paper. University of Texas at Dallas, 2013.

Honka, Elisabeth and Pradeep Chintagunta. "Simultaneous or Sequential? Understanding the Drivers of Search Strategies and Search in the U.S. Auto Insurance Industry" Working Paper. University of Texas at Dallas, 2013.

Honoré, B. E, and E. Kyriaziduo. "Panel Data Discrete Choice Models with Lagged Dependent Variables" *Econometrica* 68 (2000): 839-874.

Hortaçsu, Ali, and Chad Syverson. "Product Differentiation, Search Costs, and Competition in the Mutual Fund Industry: A Case Study of S&P 500 Index Funds" *The Quarterly Journal of Economics* 119 (2004): 403-456.

Ireland, Peter N. "Changes in the Federal Reserve's Inflation Target: Causes and Consequences" *Journal of Money, Credit and Banking* 38 (2007): 1851-1882.

Iyengar, Sheena S., Mark R. Lepper. "When choice is demotivating: Can one desire too much of a good thing?" *Journal of Personality and Social Psychology* 79 (2000): 995-1006.

Iyengar, Sheena S., Wei Jiang, and Gur Huberman. "How Much Choice is Too Much?: Contributions to 401(k) Retirement Plans" Pension Research Council Working Paper, 2004.

Kadiyali, Vrinda, Naufel Vilcassim, and Pradeep Chintagunta. "Product Line Extensions and Competitive Market Interactions: An Empirical Analysis" *Journal of Econometrics* 89 (1999) 339-363.

Kaiser, K. and R. Schwabe. "Preference for Variety" Working Paper, Banco de Mexico, Mexico City, Mexico, 2010.

Kamenica, Emir. "Contextual Inference in Markets: On the Informational Content of Product Lines" *The American Economic Review* 98 (2008): 2127-2149.

Katona, G. and E. Mueller (1955) "A Study of Purchasing Decisions" In L. Clark (ed.): Consumer Behavior: The Dynamics of Consumer Reaction. New York, NY: New York University Press, pg 30-87.

Kim, J., Allenby, G.M. and P.E. Rossi. "Modeling Consumer Demand for Variety" *Marketing Science* 21 (2002): 229-250.

Kim, Jun B., Paulo Albuquerque, and Bart J. Bronnenberg. "Online Demand Under Limited Consumer Search" *Marketing Science* 29 (2010): 1001-1023.

Kogurt, Carl A. "Consumer Search Behavior and Sunk Costs" 14 (1990): 381-392.

Kök, A, and Y. Xu. "Optimal and Competitive Assortments with Endogenous Pricing Under Heirarchical Consumer Choice Models" *Management Science* 57 (2011) 1546-1563.

Kotz, Samuel, and Saralees Nadarajah. *Extreme Value Distributions: Theory and Application* Imperial College Press, London, 2000.

Koulayev, Sergei. "Estimating Demand in Search Markets: The Case of Online Hotel Bookings" Working Paper, EconStor, 2010.

Kuksov, Dmitri. "Buyer Search Costs and Endogenous Product Design" *Marketing Science* 23 (2004): 490-499.

Kuksov, D., and J. M. Villas-Boas. "When More Alternatives Lead to Less Choice" *Marketing Science* 29 (2010) 507-524.

Lach, Saul. "Do R&D Subsidies Stimulate or Displace Private R&D? Evidence from Israel" *The Journal of Industrial Economics* 50 (2002): 369-390.

List, John A., and Craig A. Gallet. "What Experimental Protocol Influence Disparities Between Actual and Hypothetical Stated Values?" *Environmental and Resource Economics* 20 (2001): 241-254.

Oppewal, Harmen, and Kitty Koelemeijer. "More Choice is Better: Effects of Assortment Size and Composition on Assortment Evaluation" *International Journal of Research in Marketing* 22 (2005): 45-60.

Jain, D. C., N. J. Vilcassim, and P. K. Chintagunta. "Random-Coefficients Logit Brand-Choice Model Applied to Panel Data" *Journal of Business & Economic Statistics* 12 (1994): 317-328.

Johnson, E. and J. Russo "Product Familiarity and Learning New Information" *Journal of Consumer Search* 11 (1984): 542-550.

Marvel, Howard P. "The Economics of Information and Retail Gasoline Price Behavior: An Empirical Analysis" *The Journal of Political Economy* 84 (1976): 1033-1060.

Mayzlin, Dina, and Jiwoong Shin. "Uninformative Advertising as an Invitation to Search" *Marketing Science* 30 (2011): 666-685.

McAlister, L. and E. Pessemeier. "Variety Seeking Behavior: An Interdisciplinary Review" *Journal of Consumer Research* 9 (1982): 311-322.

McFadden, D. L. "Econometric Models for Probabilistic Choice among Products." *Journal of Business* 53 (1980): S13-29.

McFadden, D. L. and K. Train. "Mixed MNL Models for Discrete Response" *Journal of Applied Econometrics* 15 (2000): 447-470.

Mehta, Nitin, Surendra Rajiv, and Kannan Srinivasan. "Price Uncertainty and Consumer Search: A Structural Model of Consideration Set Formation" *Marketing Science* 22 (2003): 58-84.

Moorthy, S., B. Ratchford, and D. Talukdar. "Consumer Information Search Revisited: Theory and Empirical Analysis" *Journal of Consumer Research* 23 (1997): 263-277.

Morgan, P., and R. Manning. "Optimal Search" *Econometrica* 53 (1985): 923-944.

Nevo, A. “A Practitioner’s Guide to Estimation of Random-Coefficients Logit Models of Demand” *Journal of Economics & Management Strategy* 9 (2000): 513-548.

Nevo, A. “Measuring Market Power in the Ready-To-Eat Cereal Industry” *Econometrica* 69 (2001): 307-342.

Norwood, Franklin B. "Less Choice is Better, Sometimes" *Journal of Agricultural & Food Industrial Organization* 4 (2006): 1-21.

Park, Sungho, and Sachin Gupta. "Simulated Maximum Likelihood Estimator for the Random Coefficient Logit Model Using Aggregate Data" *Journal of Marketing Research* 46 (2009): 531-542.

Petrin, Amil, and Kenneth Train. "A Control Function Approach to Endogeneity in Consumer Choice Models" *Journal of Marketing Research* 47 (2010): 3-13.

Pinjari, Abdul Rawoof, and Chandra Bhat. "Computationally Efficient Forecasting Procedures for Kuhn-Tucker Consumer Demand Model Systems: Application to Residential Energy Consumption Analysis" Working Paper. University of South Florida, 2009.

Pinjari, Abdul Rawoof, and Chandra Bhat. "A Multiple Discrete-Continuous Nested Extreme Value (MDCNEV) Model: Formulation and Application to Non-Worker Activity Time-Use and Timing Behavior on Weekdays" Working Paper. University of South Florida, 2010.

Ratchford, B. "Consumer Search and Pricing" In: V. Rao (ed.): *Handbook on Pricing Research in Marketing*. Northhamption, MA: Edward Elgar Publishing, Inc. 2009.

Richards, Timothy J., Gomez, Miguel, and Geoffrey Pofahl. "A Multiple-Discrete/Continuous Model of Price Promotion" *Journal of Retailing* 88 (2012): 206-225.

Richards, T. J. and S. F. Hamilton. "Rivalry in Price and Variety among Supermarket Retailers" *American Journal of Agricultural Economics* 88 (2006): 710-726.

Richards, T. J., Pofahl, G., and S. F. Hamilton. "A Multiple-Discrete/Continuous Model of Price Promotion" Working Paper, Arizona State University, Tempe, AZ, 2011.

Salop, S. and J. Stiglitz "Bargains and Ripoffs: A Model of Monopolistically Competitive Price Dispersion" *Review of Economic Studies* 44 (1977): 492-510.

Satomura, T., J. Kim, and G. Allenby. "Multiple-Constraint Choice Models with Corner and Interior Solutions" *Marketing Science* 30 (2011): 481-490.

Scheibehenne, Benjamin, Rainer Greifeneder, and Peter M. Todd. "Can There Ever Be Too Many Options? A Meta-Analytic Review of Choice Overload" *Journal of Consumer Research* 37 (2010): 409-425.

Seiler, Stephen. "The Impact of Search Costs on Consumer Behavior: A Dynamic Approach" Working Paper. London School of Economics, Center for Economic Performance and Institute for Fiscal Studies, 2011.

Shilony, Y. "Mixed Pricing in Oligopoly" *Journal of Economic Theory* 14 (1977): 373-378.

Shugan, Steven M. "The Cost of Thinking" *Journal of Consumer Research* 7 (1980): 99-111.

Sorensen, T. "Equilibrium Price Dispersion in Retail Markets for Prescription Drugs" *Journal of Political Economy* 108 (2000): 833-850.

Stahl, D. "Optimal Pricing with Sequential Consumer Search" *American Economic Review* 79 (1989): 700-712.

Stigler, George J. "The Economics of Information" *Journal of Political Economy* 69 (1961): 213-225.

Tappata, Mariano. "Rockets and Feathers: Understanding Asymmetric Pricing" *RAND Journal of Economics*. 40 (2009): 673-687.

Theil, H. "The information approach to demand analysis" *Econometrica* 33 (1965): 67 - 87.

Train, K. E. "Halton Sequence for Mixed Logit" Working Paper No. E00-278. U. of California, Berkeley, 2000.

Train, K. E. Discrete Choice Methods with Simulation. New York: Cambridge University Press, 2009.

Train, K. E., D. L. McFadden, and M. Ben-Akiva. "The Demand for Local Telephone Service: A Fully Discrete Model of Residential Calling Patterns and Service Choices" *Rand Journal of Economics* 18 (1987): 109-123.

Van Hoomissen. "Price Dispersion and Inflation: Evidence from Israel" *Journal of Political Economy* 96 (1988): 1303-1314.

Varian, H. "A Model of Sales" *American Economic Review* 70 (1980): 651-659.

Wales, T., and A Woodland "Estimation of Consumer Demand Systems with Binding Non-negativity Constraints" *Journal of Econometrics* 21 (1983): 85-263.

Weitzman, M. "Optimal Search for the Best Alternative" *Econometrica* 47 (1979): 641-654.

Wolinsky, A. "Product Differentiation with Imperfect Information" *Quarterly Journal of Economics* 101 (1986): 493-511.

Yuan, Hong, and Song Han. "The Effects of Consumers' Price Expectations on Sellers' Dynamic Pricing Strategies" *Journal of Marketing Research* 48 (2011): 48-61.

Zhang, Q. M. Gangwar, P. B. Seetharaman. "Store Loyalty as a Category Specific Trait - What Drives it?" Working Paper, U. of Iowa, Iowa City, IA, 2008.

Zwick, Rami, Rapoport, Amnon, Lo, Alison King Chung, and A.V. Muthukrishnan. "Consumer Sequential Search: Not Enough or Too Much?" *Marketing Science* 22 (2003): 503-519.

CHAPTER A

UTILITY MAXIMIZATION

The constrained maximum utility problem described in equation (2.4) above for the more general case when attributes and prices are observed is summarized as:

$$\max \sum_{i=1}^{K} \mathrm{e}^{\phi_i} \left(q_i + \gamma_i\right)^{\alpha}$$

$$\text{sub. to } \; \tilde{y} - \sum_{i=1}^{K} p_i q_i = 0$$

$$q_i \geq 0.$$

The Lagrangian is given by:

$$\mathcal{L} = -\sum_{i \in \mathbf{I}} \mathrm{e}^{\phi_i} \left(q_i + \gamma_i\right)^{\alpha} + \lambda \left(\tilde{y} - \sum_{i=1}^{K} p_i q_i \right) - \boldsymbol{\mu}^{\intercal} \mathbf{q} \tag{A.1}$$

where $\boldsymbol{\mu} = \{\mu_1, ..., \mu_K\}^{\intercal}$, $\mathbf{q} = \{q_1^*, ..., q_K^*\}^{\intercal}$. Then the Karush-Kuhn-Tucker (KKT) conditions for the above problem are (Chong and Żak 2001 pg. 398):

$$\begin{aligned} \mu_i &\leq 0, && \forall i; \\ \alpha \mathrm{e}^{\phi_i} \left(q_i^* + \gamma_i\right)^{\alpha - 1} - \lambda p_i &= \mu_i, && \forall i; \\ \sum_{i=1}^{K} \mu_i q_i^* &= 0; && \\ \tilde{y} - \sum_{i=1}^{K} p_i q_i^* &= 0 && \\ q_i &\geq 0 && \forall i. \end{aligned}$$

If I define $\mathbf{L} = \{\frac{\partial U}{\partial q_1^*} - \bar{\lambda} p_1, ..., \frac{\partial U}{\partial q_K^*} - \bar{\lambda} p_K\}^{\intercal}$ and $\mathbf{0}$ is a vector of 0's, the above KKT conditions in matrix notation are:

$$\begin{aligned} \boldsymbol{\mu} &\leq \mathbf{0}; \\ \mathbf{L} &= \boldsymbol{\mu}; \\ \mathbf{q}^{\intercal} \boldsymbol{\mu} &= 0; \\ \tilde{y} - \mathbf{p}^{\intercal} \mathbf{q} &= 0; \\ \mathbf{q} &\geq \mathbf{0}. \end{aligned}$$

Eliminating $\boldsymbol{\mu}$ the KKT conditions become:

$$\begin{aligned}\mathbf{L} &\leq \mathbf{0};\\ \mathbf{q}^\intercal\mathbf{L} &= 0;\\ \tilde{y}-\mathbf{p}^\intercal\mathbf{q} &= 0;\\ \mathbf{q} &\geq \mathbf{0}.\end{aligned}$$

Or:

$$\alpha e^{\phi_i}\left(q_i^*+\gamma_i\right)^{\alpha-1}-\lambda p_i \leq 0; \tag{A.2a}$$

$$\sum_{i=1}^{K} q_i^*\left(\alpha e^{\phi_i}\left(q_i^*+\gamma_i\right)^{\alpha-1}-\lambda p_i\right) = 0; \tag{A.2b}$$

$$\tilde{y}-\sum_{i=1}^{K} p_i q_i^* = 0; \tag{A.2c}$$

$$q_i^* \geq 0 \tag{A.2d}$$

for all i goods. Notice that the budget constraint is satisfied for all K goods and is not restricted over only those purchased goods. Let me partition the set of searched goods $\mathbf{K}^*$ into a set of purchased goods $\mathbf{I}$ and non purchased goods $\mathbf{F}$ such that $\mathbf{I}\cap\mathbf{F}=\varnothing$ and $\mathbf{I}\cup\mathbf{F}=\mathbf{K}^*$. Now, considering the equations (A.2a) and (A.2b) above. Equation (A.2b) is satisfied when, and only when, the marginal utility given in equation (A.2a) is 0 and $q_i^*>0$ or, less than 0 and $q_i^*>0$. Therefore, all the purchased goods, $\mathbf{I}$, satisfy equation (A.2a) with equality and all the non-purchased goods $\mathbf{F}$ satisfy the equation with strict inequality. In other words, the KKT conditions given in equation (2.15) can be summarized as the price normalized marginal utility of all the purchased goods is equal to any other purchased good, and is greater than all non-purchased goods.

Given the equality for the purchased goods in $\mathbf{I}$ I solve for the analytical solution to the i^{th} good using the ratio of L_i and L_j and the budget constraint given in equation (A.2c). Namely, I find:

$$\begin{aligned}\frac{\alpha e^{\phi_i}\left(q_i^*+\gamma_i\right)^{\alpha-1}}{\alpha e^{\phi_j}\left(q_j^*+\gamma_j\right)^{\alpha-1}} &= \frac{p_i}{p_j}\Rightarrow\\ q_i^* &= \left(\frac{p_i e^{\phi_j}}{p_j e^{\phi_i}}\right)^{(\alpha-1)^{-1}}\left(q_j^*+\gamma_j\right)-\gamma_i.\end{aligned} \tag{A.3}$$

Substituting equation (A.3) into the budget constraint, and indexing the products in **I** sequentially to avoid cumbersome set notation, and letting $\varpi = (\alpha - 1)^{-1}$ the budget constraint can be written as:

$$\tilde{y} = p_j q_j^* + \left(q_j^* + \gamma_j\right) \sum_{k \neq j}^{I} p_k \left(\frac{p_i \mathrm{e}^{\phi_j}}{p_j \mathrm{e}^{\phi_k}}\right)^{\varpi} - \sum_{k \neq j}^{I} p_k \gamma_k \Rightarrow$$

$$\begin{aligned}
q_j^* &= \frac{\tilde{y} + \sum_{k \neq j}^{I} p_k \gamma_k - \gamma_j \sum_{k \neq j}^{I} p_k \left(\frac{p_k \exp[\phi_j]}{p_j \exp[\phi_k]}\right)^{\varpi}}{p_j + \sum_{k \neq j}^{I} p_k \left(\frac{p_k \exp[\phi_j]}{p_j \exp[\phi_k]}\right)^{\varpi}} \\
&= \frac{\tilde{y} + \sum_{k=1}^{I} p_k \gamma_k - p_j \gamma_j - \gamma_j \sum_{k=1}^{I} p_k \left(\frac{p_k \exp[\phi_j]}{p_j \exp[\phi_k]}\right)^{\varpi} + \gamma_j p_j \left(\frac{p_k \exp[\phi_j]}{p_j \exp[\phi_k]}\right)^{\varpi}}{\left(p_j + \sum_{k=1}^{I} p_k \left(\frac{p_k \exp[\phi_j]}{p_j \exp[\phi_k]}\right)^{\varpi} - p_j \left(\frac{p_k \exp[\phi_j]}{p_j \exp[\phi_k]}\right)^{\varpi}\right)} \\
&= \frac{\tilde{y} + \sum_{k=1}^{I} p_k \gamma_k - p_j \gamma_j + \gamma_j p_j}{\sum_{k=1}^{I} p_k \left(\frac{p_k \exp[\phi_j]}{p_j \exp[\phi_k]}\right)^{\varpi}} - \frac{\gamma_j \sum_{k=1}^{I} p_k \left(\frac{p_k \exp[\phi_j]}{p_j \exp[\phi_k]}\right)^{\varpi}}{\sum_{k=1}^{I} p_k \left(\frac{p_k \exp[\phi_j]}{p_j \exp[\phi_k]}\right)^{\varpi}} \\
&= \frac{\tilde{y} + \sum_{k=1}^{I} p_k \gamma_k}{\sum_{k=1}^{I} p_k \left(\frac{p_k \exp[\phi_j]}{p_j \exp[\phi_k]}\right)^{\varpi}} - \gamma_j. \qquad \text{(A.4)}
\end{aligned}$$

From symmetry I have:

$$q_i^* = \frac{\tilde{y} + \sum_{k=1}^{I} p_k \gamma_k}{\sum_{k=1}^{I} p_k \left(\frac{p_i \exp[\phi_k]}{p_k \exp[\phi_i]}\right)^{\varpi}} - \gamma_i, \quad \forall i \in \mathbf{I}. \qquad \text{(A.5)}$$

A key point regarding equation (A.5) above is that it does not hold for goods that are not purchased (those goods in **F**) and it is conditional on the set of goods **I**, or more to the point, their parameters. Since the products to be purchased and their quantities are determined simultaneously this posses a bit of a combinatorial problem, and suggests that continuous versions of multi-product purchase models may not hold since those solutions often describe quantity purchases for both purchased and non-purchased goods. This conditioning makes determining a firm's response to the consumer's demand in a competitive setting (i.e. extending it to an equilibrium setting) challenging since the firm needs to know what other goods were purchased, which is often unobserved. However, even in a monopolistic setting solving for the firm's price response given consumer demand is challenging due to the proliferation of price in both the numerator and the denominator of q_i^*, and most notably, inside the $(\cdot)^{\varpi}$ term.

The specific way in which consumers choose the number of product, and quantities thereof, is described in detail by Pinjari and Bhat (2009). Since their functional form is quite a bit different from that presented here, I will briefly describe the process. First, consider the consumer's baseline marginal utility, or the marginal utility when $q_i^* = 0$ which is $\alpha e^{\phi_i}\gamma_i^{\alpha-1}$. Consumers will choose products to consider to purchase such that $\frac{\alpha}{p_1}e^{\phi_1}\gamma_1^{\alpha-1} \geq \cdots \geq \frac{\alpha}{p_K}e^{\phi_K}\gamma_K^{\alpha-1}$ (see Pinjari and Bhat 2009 for the proof). Additionally, when $q_i^* = 0$ the price normalized marginal utility is less than λ^*, or $\frac{\alpha}{p_i}e^{\phi_i}\gamma_i^{\alpha-1} < \lambda^*$. Since the consumer knows all the parameters under consideration except for λ^* the consumer calculates λ^* for $I = 1$, say λ_1^*, by calculating q_1^*. If $\frac{\alpha}{p_2}e^{\phi_2}\gamma_2^{\alpha-1} \geq \lambda_1^*$ then the consumer computes the solution to q_1^* and q_2^* to find λ_2^*. This process continues until $\frac{\alpha}{p_{I+1}}e^{\phi_{I+1}}\gamma_{I+1}^{\alpha-1} < \lambda_I^*$, at which point consumption is set to 0 for the remaining $K - I + 1$ goods.

Finally, the analytical solution to the marginal utility of income, λ^*, that is not product specific, is found using equation (A.2b) to be:

$$\alpha \sum_{i=1}^{K} q_i^* e^{\phi_i} \left(q_i^* + \gamma_i\right)^{\alpha-1} = \lambda \tilde{y} \Rightarrow$$

$$\begin{aligned}
\lambda^* &= \frac{\alpha}{\tilde{y}} \sum_{i=1}^{K} e^{\phi_i} q_i^* \left(q_i^* + \gamma_i\right)^{\alpha-1} \\
\lambda^* &= \frac{\alpha}{\tilde{y}} \sum_{i=1}^{K} \left\{ e^{\phi_i} \left(1 - \frac{\gamma_i \sum_{k=1}^{I} p_k \left(\frac{p_i \exp[\phi_k]}{p_k \exp[\phi_i]}\right)^{\varpi}}{\tilde{y} + \sum_{k=1}^{I} p_k \gamma_k}\right) \left(\frac{\tilde{y} + \sum_{k=1}^{I} p_k \gamma_k}{\sum_{k=1}^{I} p_k \left(\frac{p_i \exp[\phi_k]}{p_k \exp[\phi_i]}\right)^{\varpi}}\right)^{\alpha} \right\} \\
\lambda^* &= \frac{\alpha}{\tilde{y}} \left(\tilde{y} + \sum_{k=1}^{I} p_k \gamma_k\right)^{\alpha-1} \sum_{i=1}^{K} \left\{ e^{\phi_i} \frac{\left(\tilde{y} + \sum_{k=1}^{I} p_k \gamma_k - \gamma_i \sum_{k=1}^{I} p_k \left(\frac{p_i \exp[\phi_k]}{p_k \exp[\phi_i]}\right)^{\varpi}\right)}{\left(\sum_{k=1}^{I} p_k \left(\frac{p_i \exp[\phi_k]}{p_k \exp[\phi_i]}\right)^{\varpi}\right)^{\alpha}} \right\}.
\end{aligned}$$

The solutions to the quantity purchased do in fact provide a maximum to the utility function, $\max U$. To see this, considering the FOCs that provide the solution to q_i^* give in equation (2.15) above, and define:

$$\begin{aligned}
\mathcal{L}_{ij} &= \frac{\partial \mathcal{L}_i}{\partial q_j} = -\alpha (1 - \alpha) e^{\phi_i} \left(q_i^* + \gamma_i\right)^{\alpha-2} \text{ if } i = j \text{ and 0 otherwise;} \\
\mathcal{L}_{i\lambda} &= \frac{\partial \mathcal{L}_i}{\partial \lambda} = -p_i.
\end{aligned}$$

The sufficient second-order condition for the constrained maximization problem noted in equation (3.3), once prices and attributes are revealed and uncertainty extinguished, is that the bordered Hessian be negative definite. Let $\mathbf{A}$ denote the bordered Hessian matrix, or:

$$\begin{aligned}
\mathbf{A} &= \begin{pmatrix} \mathcal{L}_{11} & \cdots & \mathcal{L}_{1K} & \mathcal{L}_{1\lambda} \\ \vdots & \ddots & \vdots & \vdots \\ \mathcal{L}_{K1} & \cdots & \mathcal{L}_{KK} & \mathcal{L}_{K\lambda} \\ \mathcal{L}_{\lambda 1} & \cdots & \mathcal{L}_{\lambda K} & \mathcal{L}_{\lambda\lambda} \end{pmatrix} \\
&= \begin{pmatrix} \mathcal{L}_{11} & 0 & \cdots & 0 & -p_1 \\ 0 & \mathcal{L}_{22} & \cdots & 0 & -p_2 \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & \cdots & 0 & \mathcal{L}_{KK} & -p_K \\ -p_1 & -p_2 & \cdots & -p_K & 0 \end{pmatrix}.
\end{aligned}$$

Let the i^{th} principal minor of $\mathbf{A}$ be denoted as $\mathbf{A}_i$, then:

$$\begin{aligned}
\mathbf{A}_1 &= \mathcal{L}_{11} < 0, \;\; \mathbf{A}_2 = \begin{vmatrix} \mathcal{L}_{11} & 0 \\ 0 & \mathcal{L}_{22} \end{vmatrix} = \mathcal{L}_{11}\mathcal{L}_{22} > 0 \; \cdots \\
\mathbf{D} &= |\mathbf{A}| = \begin{vmatrix} \mathcal{L}_{11} & 0 & \cdots & 0 & -p_1 \\ 0 & \mathcal{L}_{22} & \cdots & 0 & -p_2 \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & \cdots & 0 & \mathcal{L}_{KK} & -p_K \\ -p_1 & -p_2 & \cdots & -p_K & 0 \end{vmatrix} \\
&= -\sum_{j=1}^{K} \frac{(p_j)^2}{\mathcal{L}_{jj}} \prod_{i=1}^{K} \mathcal{L}_{ii} \\
&= \sum_{j=1}^{K} \frac{(p_j)^2}{-\alpha(1-\alpha)\,\mathrm{e}^{\phi_j}\left(q_j^* + \gamma_j\right)^{\alpha-2}} \prod_{i=1}^{K} -\alpha(1-\alpha)\,\mathrm{e}^{\phi_i}\left(q_i^* + \gamma_i\right)^{\alpha-2} \\
&= (-1)^K \left(\alpha(1-\alpha)\right)^{K-1} \left(\sum_{j=1}^{K} \frac{(p_j)^2}{\mathrm{e}^{\phi_j}\left(q_j^* + \gamma_j\right)^{\alpha-2}}\right) \left(\prod_{i=1}^{K} \mathrm{e}^{\phi_i}\left(q_i^* + \gamma_i\right)^{\alpha-2}\right).
\end{aligned} \tag{A.6}$$

Therefore, the bordered Hessian matrix is indeed negative definite and the solutions for all q_i^* represent the maximum of the utility function since the determinant of $\mathbf{A}$ is negative if K is odd and is positive if K is even as required.

Next, I consider how the maximum utility in equation (2.8) changes with changes in the parameters. To simplify notation I use the following abbreviations:

$$\varpi = (\alpha-1)^{-1} \tag{A.7a}$$

$$\Theta_j = \sum_{k=1}^{I} p_k \left(\frac{p_k \mathrm{e}^{\phi_j}}{p_j \mathrm{e}^{\phi_k}}\right)^{(\alpha-1)^{-1}} \tag{A.7b}$$

$$\check{p}\check{\gamma} = \sum_{k=1}^{I} p_k \gamma_k. \tag{A.7c}$$

The following need to be updated based on the observation that utility is summed over all K goods. However, the qualitative results will not change. I find:

$$\frac{\partial \max U}{\partial \gamma_i} = \frac{\alpha p_i}{\tilde{y} + \sum_{k=1}^{I} p_k \gamma_k} (\max U), \tag{A.8}$$

$$\begin{aligned}\frac{\partial \max U}{\partial \alpha} = {} & \frac{1}{(\alpha-1)^2} \\ & \times \sum_{i=1}^{I} \left\{ \frac{\mathrm{e}^{\phi_i}}{\Theta_i} \left(\frac{\tilde{y}+\check{p}\check{\gamma}}{\Theta_i}\right)^{\alpha} \left(\begin{array}{c} (\alpha-1)^2 \, \Theta_i \ln\left[\frac{\tilde{y}+\check{p}\check{\gamma}}{\Theta_i}\right] \\ +\alpha \sum_{k=1}^{I} \left\{ p_k \left(\frac{p_i \mathrm{e}^{\phi_k}}{p_k \mathrm{e}^{\phi_i}}\right)^{(1-\alpha)^{-1}} \ln\left[\frac{p_i \mathrm{e}^{\phi_k}}{p_k \mathrm{e}^{\phi_i}}\right] \right\} \end{array} \right) \right\},\end{aligned} \tag{A.9}$$

$$\begin{aligned}\frac{\partial \max U}{\partial \phi_i} = {} & -\frac{1}{(1-\alpha)} \\ & \times \left(\alpha p_i \sum_{k=1}^{I} \left\{ \frac{\mathrm{e}^{\phi_k}}{\Theta_k} \left(\frac{p_i \mathrm{e}^{\phi_k}}{p_k \mathrm{e}^{\phi_i}}\right)^{(1-\alpha)^{-1}} \left(\frac{\tilde{y}+\check{p}\check{\gamma}}{\Theta_i}\right)^{\alpha} \right\} - \left(\frac{\tilde{y}+\check{p}\check{\gamma}}{\Theta_i}\right)^{\alpha} \right).\end{aligned} \tag{A.10}$$

Notice that $\frac{\partial \max U}{\partial \phi_i} < 0$ which is expected since ϕ_i represents the difference between the consumer's ideal product attribute profile and the actual attributes of the product. So, as ϕ_i gets smaller and approaches 0, the consumer would be more inclined to purchase that product because it is exactly what the consumer is looking for in terms of attributes. Therefore, I expect that a smaller ϕ_i would lead a higher maximum utility value.

CHAPTER B

DERIVATION OF EXPECTED MAXIMUM UTILITY

Because I have the term $\left(\sum_{k=1}^{I}\mathrm{e}^{-\varepsilon_k}\right)^{-\alpha}$ in the integral I cannot solve this as the product of each integral. Instead, I have to solve the inner most integral for ε_1 then use that solution and solve the integral for ε_2, and so on. Looking at the inner most integral $\int_{-\infty}^{\infty}\cdots\int_{-\infty}^{\infty}\left\{\int_{-\infty}^{\infty}\bigstar \mathrm{d}\varepsilon_1\right\}\mathrm{d}\varepsilon_2\cdots\mathrm{d}\varepsilon_I$ I therefore, have:[1]

$$\begin{aligned}
&\left\{\int_{-\infty}^{\infty}\left(\sum_{k=1}^{I}\mathrm{e}^{-\varepsilon_k}\right)^{-\alpha}\exp\left[\sum_{j=1}^{I}-\varepsilon_j\right]\exp\left[-\sum_{i=1}^{I}\mathrm{e}^{-\varepsilon_i}\right]\mathrm{d}\varepsilon_1\right\} \qquad \text{(B.1)}\\
=\;&\prod_{j=2}^{I}\mathrm{e}^{-\varepsilon_j}\prod_{j=2}^{I}\mathrm{e}^{-\mathrm{e}^{-\varepsilon_j}}\int_{-\infty}^{\infty}\mathrm{e}^{-\varepsilon_1}\mathrm{e}^{-\mathrm{e}^{-\varepsilon_1}}\left(\sum_{k=1}^{I}\mathrm{e}^{-\varepsilon_k}\right)^{-\alpha}\mathrm{d}\varepsilon_1.
\end{aligned}$$

Let $u=\mathrm{e}^{-\varepsilon_1}$ so $-\mathrm{d}u=\mathrm{e}^{-\varepsilon_1}\mathrm{d}\varepsilon_1$ and define $\acute{Z}_r=\sum_{k=r}^{I}\mathrm{e}^{-\varepsilon_k}$ to find:

$$\begin{aligned}
&\int_{-\infty}^{\infty}\mathrm{e}^{-\mathrm{e}^{-\varepsilon_1}}\left(\sum_{k=1}^{I}\mathrm{e}^{-\varepsilon_k}\right)^{-\alpha}\mathrm{e}^{-\varepsilon_1}\mathrm{d}\varepsilon_1 \qquad \text{(B.2a)}\\
=\;&-\int_{\infty}^{0}\mathrm{e}^{-u}\left(u+\acute{Z}_2\right)^{-\alpha}\mathrm{d}u\\
=\;&\int_{0}^{\infty}\mathrm{e}^{-u}\left(u+\acute{Z}_2\right)^{-\alpha}\mathrm{d}u \qquad \text{(B.2b)}\\
=\;&-\mathrm{e}^{\acute{Z}_2}\Gamma\left[1-\alpha,u+\acute{Z}_2\right]\Bigg|_{u=0}^{u=\infty}\\
=\;&\mathrm{e}^{\acute{Z}_2}\Gamma\left[1-\alpha,\acute{Z}_2\right],\ \text{if } \acute{Z}_2>0, \text{ or } I>1, \qquad \text{(B.2c)}
\end{aligned}$$

where the expression $\Gamma[a,z]$ represents the upper incomplete Gamma function defined as:

$$\Gamma[a,z]=\int_{z}^{\infty}t^{a-1}\mathrm{e}^{-t}\mathrm{d}t. \qquad \text{(B.3)}$$

The integral of the first term therefore, becomes:

$$\begin{aligned}
&\prod_{j=2}^{I}\mathrm{e}^{-\varepsilon_j}\prod_{j=2}^{I}\mathrm{e}^{-\mathrm{e}^{-\varepsilon_j}}\int_{-\infty}^{\infty}\mathrm{e}^{-\varepsilon_1}\mathrm{e}^{-\mathrm{e}^{-\varepsilon_1}}\left(\sum_{k=1}^{I}\mathrm{e}^{-\varepsilon_k}\right)^{-\alpha}\mathrm{d}\varepsilon_1\\
=\;&\prod_{j=2}^{I}\mathrm{e}^{-\varepsilon_j}\prod_{j=2}^{I}\mathrm{e}^{-\mathrm{e}^{-\varepsilon_j}}\mathrm{e}^{\acute{Z}_2}\Gamma\left[1-\alpha,\acute{Z}_2\right]\\
=\;&\prod_{j=2}^{I}\mathrm{e}^{-\varepsilon_j}\prod_{j=2}^{I}\mathrm{e}^{-\mathrm{e}^{-\varepsilon_j}}\prod_{j=2}^{I}\mathrm{e}^{\mathrm{e}^{-\varepsilon_j}}\Gamma\left[1-\alpha,\acute{Z}_2\right]\\
=\;&\prod_{j=2}^{I}\mathrm{e}^{-\varepsilon_j}\Gamma\left[1-\alpha,\acute{Z}_2\right],\qquad \text{if } I>1. \qquad \text{(B.4)}
\end{aligned}$$

[1] Note that the actual order of integration, $\mathrm{d}\varepsilon_1\mathrm{d}\varepsilon_2\cdots\mathrm{d}\varepsilon_K$, is inconsequential since the probability space is continuous everywhere in its domain.

I stop here, momentarily, to look at the expected utility the consumer obtains from searching a single product i as:

$$\begin{aligned}
E[\max U_{\mathbf{1}}] &= \int_{-\infty}^{\infty} U_{\mathbf{1}} f(\varepsilon_i) \mathrm{d}\varepsilon_i & \text{(B.5a)} \\
&= (A_{\mathbf{1}})^{\alpha} \int_{-\infty}^{\infty} \left(\mathrm{e}^{-\varepsilon_i}\right)^{-\alpha} \mathrm{e}^{-\varepsilon_i} \mathrm{e}^{-\mathrm{e}^{-\varepsilon_i}} \mathrm{d}\varepsilon_i \\
&= (A_{\mathbf{1}})^{\alpha} \int_{0}^{\infty} \mathrm{e}^{-u} u^{-\alpha} \mathrm{d}u \\
&= -\left(A_{\mathbf{1}}\right)^{\alpha} \Gamma\left[1-\alpha, u\right] \Big|_{u=0}^{u=\infty} \\
&= (A_{\mathbf{1}})^{\alpha} \Gamma\left[1-\alpha\right] \\
&= \left(\frac{y}{\bar{p}} + \left\{\gamma_i - \frac{c_i}{\bar{p}}\right\}\right)^{\alpha} \Gamma\left[1-\alpha\right]. & \text{(B.5b)}
\end{aligned}$$

where $\Gamma[x]$ represents the Gamma function. Therefore, as α moves from 0 to 1 the expression $\Gamma[1-\alpha]$ goes from 0 to ∞. In the event that the expected utility of searching a single product outweighed the expected utility of searching multiple products the consumer would select the specific product i such that $\left\{\gamma_i - \frac{c_i}{\bar{p}}\right\} \geq \left\{\gamma_j - \frac{c_j}{\bar{p}}\right\} \;\; \forall j \neq i$. Notice that the larger the consumer's price expectation is, the less they would be concerned about the cost of search c_i. Turning our attention back to the case when $I > 2$ I investigate the expected utility more generally. From equation (3.9a) and (B.4) I have:

$$E[\max U_{\mathbf{K}}] = (A_{\mathbf{K}})^{\alpha}\, I \int_{-\infty}^{\infty} \cdots \int_{-\infty}^{\infty} \prod_{j=2}^{I} \mathrm{e}^{-\varepsilon_j} \Gamma\left[1-\alpha, \acute{Z}_2\right] \mathrm{d}\varepsilon_2 \mathrm{d}\varepsilon_3 \cdots \mathrm{d}\varepsilon_I \tag{B.6}$$

The inner most integral $\int_{-\infty}^{\infty} \cdots \int_{-\infty}^{\infty} \left\{\int_{-\infty}^{\infty} \bigstar \mathrm{d}\varepsilon_2\right\} \mathrm{d}\varepsilon_3 \cdots \mathrm{d}\varepsilon_I$ is:

$$\left\{\int_{-\infty}^{\infty}\prod_{j=2}^{I} e^{-\varepsilon_j}\Gamma\left[1-\alpha,\acute{Z}_2\right]\mathrm{d}\varepsilon_2\right\} \tag{B.7a}$$

$$= \prod_{j=3}^{I} e^{-\varepsilon_j}\int_{-\infty}^{\infty} e^{-\varepsilon_2}\Gamma\left[1-\alpha,\acute{Z}_2\right]\mathrm{d}\varepsilon_2, \quad \text{let } u = e^{-\varepsilon_2},$$

$$= \prod_{j=3}^{I} e^{-\varepsilon_j}\int_{0}^{\infty}\Gamma\left[1-\alpha,u+\acute{Z}_3\right]\mathrm{d}u$$

$$= \prod_{j=3}^{I} e^{-\varepsilon_j}\left(\begin{array}{c}\acute{Z}_3\Gamma\left[1-\alpha,u+\acute{Z}_3\right]+\\ u\Gamma\left[1-\alpha,u+\acute{Z}_3\right]-\Gamma\left[2-\alpha,u+\acute{Z}_3\right]\end{array}\right)\Bigg|_{u=0}^{u=\infty}$$

$$= \prod_{j=3}^{I} e^{-\varepsilon_j}\left(\Gamma\left[2-\alpha,\acute{Z}_3\right]-\acute{Z}_3\Gamma\left[1-\alpha,\acute{Z}_3\right]\right), \quad \text{if } I>2 \tag{B.7b}$$

$$= \Gamma\left[2-\alpha\right], \qquad \text{if } I=2 \text{ or } \acute{Z}_3=0. \tag{B.7c}$$

Notice that while the expected utility is solved with respect to product 1 and then 2 this is merely for convenience. So, the second term in the expected utility function that involves the multiple integrals is independent of the first two products chosen and the number of products, or integrals, in the expectation is what matters. In the event that the consumer selects $I = 2$ products I find that α plays a very small role with respect to the second term in the expectation $\Gamma[2-\alpha]$. As α goes from 0 to 1 the expression $\Gamma[2-\alpha]$ goes from 1 does to a minimum of approximately 0.886 at $\alpha \approx 0.538$ and then back up to 1. If $I > 2$ then the second integration term in the expectation is given by equations (3.9a) and (B.7b). However, to simplify notation define:

$$\Gamma_x^r = \Gamma\left[x-\alpha,\acute{Z}_r\right], \quad \text{where } x, r \in \mathbb{Z} \tag{B.8}$$

The second terms in the expectation becomes:

$$\left\{\int_{-\infty}^{\infty}\prod_{j=3}^{I}e^{-\varepsilon_j}\left(\Gamma\left[2-\alpha,\acute{Z}_3\right]-\acute{Z}_3\Gamma\left[1-\alpha,\acute{Z}_3\right]\right)\mathrm{d}\varepsilon_3\right\} \tag{B.9a}$$

$$= \prod_{j=4}^{I}e^{-\varepsilon_j}\left(\int_{-\infty}^{\infty}\mathrm{e}^{-\varepsilon_3}\Gamma\left[2-\alpha,\acute{Z}_3\right]\mathrm{d}\varepsilon_3-\int_{-\infty}^{\infty}\mathrm{e}^{-\varepsilon_3}\acute{Z}_3\Gamma\left[1-\alpha,\acute{Z}_3\right]\mathrm{d}\varepsilon_3\right),\quad \text{let } u=e^{-\varepsilon_3}$$

$$= \prod_{j=4}^{I}e^{-\varepsilon_j}\left(\int_{0}^{\infty}\Gamma\left[2-\alpha,u+\acute{Z}_4\right]\mathrm{d}u-\int_{0}^{\infty}\left(u+\acute{Z}_4\right)\Gamma\left[1-\alpha,u+\acute{Z}_4\right]\mathrm{d}u\right)$$

$$= \prod_{j=4}^{I}e^{-\varepsilon_j}\left(\frac{1}{2}\right)\left(\acute{Z}_4^2\Gamma\left[1-\alpha,u+\acute{Z}_4\right]-2\acute{Z}_4\Gamma\left[2-\alpha,u+\acute{Z}_4\right]+\Gamma\left[3-\alpha,u+\acute{Z}_4\right]\right)$$

$$= \prod_{j=4}^{I}e^{-\varepsilon_j}\left(\frac{1}{2}\right)\left(\acute{Z}_4^2\Gamma_1^4-2\acute{Z}_4\Gamma_2^4+\Gamma_3^4\right),\quad \text{if } I>3 \tag{B.9b}$$

$$= \left(\frac{1}{2}\right)\Gamma\left[3-\alpha\right],\quad \text{if } I=3. \tag{B.9c}$$

In general, the term $\left(\frac{1}{2}\right)\Gamma\left[3-\alpha\right]$ goes from 1 to 1/2 as α goes from 0 to 1. Next, using equations (3.9a) and (B.9b) I find:

$$\left\{\int_{-\infty}^{\infty}\prod_{j=4}^{I}\mathrm{e}^{-\varepsilon_j}\left(\frac{1}{2}\right)\left(\acute{Z}_4^2\Gamma_1^4-2\acute{Z}_4\Gamma_2^4+\Gamma_3^4\right)\mathrm{d}\varepsilon_4\right\} \tag{B.9d}$$

$$= \left(\frac{1}{2}\right)\prod_{j=5}^{I}\mathrm{e}^{-\varepsilon_j}\left(\int_{-\infty}^{\infty}\mathrm{e}^{-\varepsilon_4}\left(\acute{Z}_4\right)^2\Gamma_1^4\mathrm{d}\varepsilon_4-2\int_{-\infty}^{\infty}\mathrm{e}^{-\varepsilon_4}\acute{Z}_4\Gamma_2^4\mathrm{d}\varepsilon_4+\int_{-\infty}^{\infty}\mathrm{e}^{-\varepsilon_4}\Gamma_3^4\mathrm{d}\varepsilon_4\right),\quad \text{let } u=e^{-\varepsilon_4}$$

$$= \left(\frac{1}{2}\right)\prod_{j=5}^{I}\mathrm{e}^{-\varepsilon_j}\left(\begin{array}{c}\int_0^{\infty}\left(u+\acute{Z}_5\right)^2\Gamma\left[1-\alpha,u+\acute{Z}_5\right]\mathrm{d}u\\ -2\int_0^{\infty}\left(u+\acute{Z}_5\right)\Gamma\left[2-\alpha,u+\acute{Z}_5\right]\mathrm{d}u\\ +\int_0^{\infty}\Gamma\left[3-\alpha,u+\acute{Z}_5\right]\mathrm{d}u\end{array}\right)$$

$$= \left(\frac{1}{2}\right)\prod_{j=5}^{I}\mathrm{e}^{-\varepsilon_j}\left(\begin{array}{c}\left(\frac{1}{3}\right)\left(\Gamma_4^5-\left(\acute{Z}_5\right)^3\Gamma_1^5\right)\\ -2\left(\frac{1}{2}\right)\left(\Gamma_4^5-\left(\acute{Z}_5\right)^2\Gamma_2^5\right)\\ +\left(\Gamma_4^5-\left(\acute{Z}_5\right)\Gamma_3^5\right)\end{array}\right)$$

$$= \left(\frac{1}{2}\right)\left(\frac{1}{3}\right)\prod_{j=5}^{I}\mathrm{e}^{-\varepsilon_j}\left(-\left(\acute{Z}_5\right)^3\Gamma_1^5+3\left(\acute{Z}_5\right)^2\Gamma_2^5-3\left(\acute{Z}_5\right)\Gamma_3^5+\Gamma_4^5\right),\quad \text{if } I>4 \tag{B.9e}$$

$$= \left(\frac{1}{3}\right)\Gamma\left[4-\alpha\right],\quad \text{if } I=4. \tag{B.9f}$$

In general, the term $\left(\frac{1}{3}\right)\Gamma\left[4-\alpha\right]$ goes from 2 to 2/3 as α goes from 0 to 1. Next, using equations (3.9a) and (B.9e) I find:

$$\left\{\left(\frac{1}{2}\right)\left(\frac{1}{3}\right)\int_{-\infty}^{\infty}\prod_{j=5}^{I}\mathrm{e}^{-\varepsilon_j}\left(-\left(\acute{Z}_5\right)^3\Gamma_1^5+3\left(\acute{Z}_5\right)^2\Gamma_2^5-3\left(\acute{Z}_5\right)\Gamma_3^5+\Gamma_4^5\right)\mathrm{d}\varepsilon_5\right\}$$

$$=\left(\frac{1}{2}\right)\left(\frac{1}{3}\right)\prod_{j=6}^{I}\mathrm{e}^{-\varepsilon_j}\int_{-\infty}^{\infty}\mathrm{e}^{-\varepsilon_5}\left(-\left(\acute{Z}_5\right)^3\Gamma_1^5+3\left(\acute{Z}_5\right)^2\Gamma_2^5-3\left(\acute{Z}_5\right)\Gamma_3^5+\Gamma_4^5\right)\mathrm{d}\varepsilon_5,\quad \text{let } u=e^{-\varepsilon_5}$$

$$=\left(\frac{1}{2}\right)\left(\frac{1}{3}\right)\prod_{j=6}^{I}\mathrm{e}^{-\varepsilon_j}\left(\begin{array}{c}-\int_0^{\infty}\left(u+\acute{Z}_6\right)^3\Gamma\left[1-\alpha,u+\acute{Z}_6\right]\mathrm{d}u\\+3\int_0^{\infty}\left(u+\acute{Z}_6\right)^2\Gamma\left[2-\alpha,u+\acute{Z}_6\right]\mathrm{d}u\\-3\int_0^{\infty}\left(u+\acute{Z}_6\right)\Gamma\left[3-\alpha,u+\acute{Z}_6\right]\mathrm{d}u\\+\int_0^{\infty}\Gamma\left[4-\alpha,u+\acute{Z}_6\right]\mathrm{d}u\end{array}\right)$$

$$=\left(\frac{1}{2}\right)\left(\frac{1}{3}\right)\prod_{j=6}^{I}\mathrm{e}^{-\varepsilon_j}\left(\begin{array}{c}-\left(\frac{1}{4}\right)\left(\Gamma_5^6-\left(\acute{Z}_6\right)^4\Gamma_1^6\right)\\+3\left(\frac{1}{3}\right)\left(\Gamma_5^6-\left(\acute{Z}_6\right)^3\Gamma_2^6\right)\\-3\left(\frac{1}{2}\right)\left(\Gamma_5^6-\left(\acute{Z}_6\right)^2\Gamma_3^6\right)\\+\left(\Gamma_5^6-\left(\acute{Z}_6\right)\Gamma_4^6\right)\end{array}\right)$$

$$=\left(\frac{1}{2}\right)\left(\frac{1}{3}\right)\left(\frac{1}{4}\right)\prod_{j=6}^{I}\mathrm{e}^{-\varepsilon_j}\left(\left(\acute{Z}_6\right)^4\Gamma_1^6-4\left(\acute{Z}_6\right)^3\Gamma_2^6+6\left(\acute{Z}_6\right)^2\Gamma_3^6-4\left(\acute{Z}_6\right)\Gamma_4^6+\Gamma_5^6\right),\quad \text{if } I>5$$

$$=\left(\frac{1}{4}\right)\Gamma\left[5-\alpha\right],\quad \text{if } I=5.$$

The term $\left(\frac{1}{4}\right)\Gamma\left[5-\alpha\right]$ goes from 6 to 3/2 as α goes from 0 to 1. More generally, I find the I^{th} integral, where $I\geq 3$, is given by:

$$
\begin{aligned}
& \left\{\left(\frac{1}{(I-2)!}\right)\int_{-\infty}^{\infty}\mathrm{e}^{-\varepsilon_I}\left(\sum_{i=0}^{I-2}\left\{(-1)^{I-2-i}\binom{I-2}{i}\left(\mathrm{e}^{-\varepsilon_I}\right)^{I-2-i}\Gamma_{i+1}^{I}\right\}\right)\mathrm{d}\varepsilon_I\right\} \\
= & \left(\frac{1}{(I-2)!}\right)\sum_{i=0}^{I-2}\left\{(-1)^{I-2-i}\binom{I-2}{i}\int_{0}^{\infty}(u)^{I-2-i}\Gamma_{i+1}^{I}\mathrm{d}u\right\} \\
= & \left(\frac{1}{(I-2)!}\right)\sum_{i=0}^{I-2}\left\{(-1)^{I-2-i}\binom{I-2}{i}\int_{0}^{\infty}(u)^{I-2-i}\Gamma\left[(i+1)-\alpha,u\right]\mathrm{d}u\right\} \\
= & \left(\frac{1}{(I-2)!}\right)\sum_{i=0}^{I-2}\left\{(-1)^{I-2-i}\binom{I-2}{i}\left(\frac{\Gamma\left[I-\alpha\right]}{I-1-i}\right)\right\} \\
= & \left(\frac{\Gamma\left[I-\alpha\right]}{(I-2)!}\right)\sum_{i=0}^{I-2}\left\{\left(\frac{(-1)^{I-2-i}}{I-1-i}\right)\binom{I-2}{i}\right\} \\
= & \left(\frac{\Gamma\left[I-\alpha\right]}{(I-2)!}\right)\sum_{i=0}^{I-2}\left\{\left(\frac{(-1)^{I-2-i}}{I-1-i}\right)\frac{(I-2)!}{i!\,(I-2-i)!}\right\} \\
= & \Gamma\left[I-\alpha\right]\sum_{i=0}^{I-2}\left\{\left(\frac{(-1)^{I-2-i}}{i!\,(I-1-i)!}\right)\right\} \\
= & \frac{\Gamma\left[I-\alpha\right]}{\Gamma\left[I\right]}.
\end{aligned}
$$

If I allowed the utility function to be a function of all the searched products, even those that are not purchased equation (3.7) would become:

$$
\begin{aligned}
\max U_{\mathbf{K}} &= \sum_{i=1}^{I}\left(\frac{\left(y-\sum_{k=1}^{K}c_k+\bar{p}\sum_{k=1}^{I}\gamma_k\right)}{\bar{p}\sum_{k=1}^{I}\left(\varepsilon_k\right)^{-1}}\right)^{\alpha}+\sum_{i=I+1}^{K}\left(\gamma_i\varepsilon_i\right)^{\alpha} \\
&= U_{\mathbf{K}}^{1}+U_{\mathbf{K}}^{2}.
\end{aligned}
$$

The $E\left[U_{\mathbf{K}}^{1}\right]$ is given above and would be unchanged, while the $E\left[U_{\mathbf{K}}^{2}\right]$ would be:

$$\begin{aligned}
E\left[U_{\mathbf{K}}^{2}\right] &= \int_{-\infty}^{\infty}\cdots\int_{-\infty}^{\infty} U_{\mathbf{K}}^{2} f(\varepsilon_{I+1},...,\varepsilon_{K})\mathrm{d}\varepsilon_{I+1}\cdots\mathrm{d}\varepsilon_{K} \\
&= \int_{-\infty}^{\infty}\cdots\int_{-\infty}^{\infty}\sum_{i=I+1}^{K}\gamma_{i}^{\alpha}\mathrm{e}^{\alpha\varepsilon_{i}}\prod_{j=I+1}^{K}\mathrm{e}^{-\varepsilon_{j}}\prod_{j=I+1}^{K}\mathrm{e}^{-\mathrm{e}^{-\varepsilon_{j}}}\mathrm{d}\varepsilon_{I+1}\cdots\mathrm{d}\varepsilon_{K} \\
&= \gamma_{I+1}^{\alpha}\int_{-\infty}^{\infty}\cdots\int_{-\infty}^{\infty}\mathrm{e}^{\alpha\varepsilon_{I+1}}\prod_{j=I+1}^{K}\mathrm{e}^{-\varepsilon_{j}}\prod_{j=I+1}^{K}\mathrm{e}^{-\mathrm{e}^{-\varepsilon_{j}}}\mathrm{d}\varepsilon_{I+1}\cdots\mathrm{d}\varepsilon_{K} \\
&\quad +\gamma_{I+2}^{\alpha}\int_{-\infty}^{\infty}\cdots\int_{-\infty}^{\infty}\mathrm{e}^{\alpha\varepsilon_{I+2}}\prod_{j=I+1}^{K}\mathrm{e}^{-\varepsilon_{j}}\prod_{j=I+1}^{K}\mathrm{e}^{-\mathrm{e}^{-\varepsilon_{j}}}\mathrm{d}\varepsilon_{I+1}\cdots\mathrm{d}\varepsilon_{K} \\
&\quad +\cdots \\
&\quad +\gamma_{K}^{\alpha}\int_{-\infty}^{\infty}\cdots\int_{-\infty}^{\infty}\mathrm{e}^{\alpha\varepsilon_{K}}\prod_{j=I+1}^{K}\mathrm{e}^{-\varepsilon_{j}}\prod_{j=I+1}^{K}\mathrm{e}^{-\mathrm{e}^{-\varepsilon_{j}}}\mathrm{d}\varepsilon_{I+1}\cdots\mathrm{d}\varepsilon_{K}.
\end{aligned}$$

Looking at the first multiple integral in the broken out sum I have:

$$\int_{-\infty}^{\infty}\cdots\int_{-\infty}^{\infty}\left\{\int_{-\infty}^{\infty}\gamma_{I+1}^{\alpha}\mathrm{e}^{\alpha\varepsilon_{I+1}}\prod_{j=I+1}^{K}\mathrm{e}^{-\varepsilon_{j}}\prod_{j=I+1}^{K}\mathrm{e}^{-\mathrm{e}^{-\varepsilon_{j}}}\mathrm{d}\varepsilon_{I+1}\right\}\cdots\mathrm{d}\varepsilon_{K}\Rightarrow$$

Letting $u=\mathrm{e}^{-\varepsilon_{I+1}}$ so $-\mathrm{d}u=\mathrm{e}^{-\varepsilon_{I+1}}\mathrm{d}\varepsilon_{I+1}$ to get:

$$\begin{aligned}
&\gamma_{I+1}^{\alpha}\left\{\int_{-\infty}^{\infty}\mathrm{e}^{\alpha\varepsilon_{I+1}}\prod_{j=I+1}^{K}\mathrm{e}^{-\varepsilon_{j}}\prod_{j=I+1}^{K}\mathrm{e}^{-\mathrm{e}^{-\varepsilon_{j}}}\mathrm{d}\varepsilon_{I+1}\right\} \\
=\;& \gamma_{I+1}^{\alpha}\prod_{j=I+2}^{K}\mathrm{e}^{-\varepsilon_{j}}\prod_{j=I+2}^{K}\mathrm{e}^{-\mathrm{e}^{-\varepsilon_{j}}}\int_{0}^{\infty}\mathrm{e}^{\alpha\varepsilon_{I+1}}\mathrm{e}^{-\varepsilon_{I+1}}\mathrm{e}^{-\mathrm{e}^{-\varepsilon_{I+1}}}\mathrm{d}\varepsilon_{I+1} \\
=\;& \gamma_{I+1}^{\alpha}\prod_{j=I+2}^{K}\mathrm{e}^{-\varepsilon_{j}}\prod_{j=I+2}^{K}\mathrm{e}^{-\mathrm{e}^{-\varepsilon_{j}}}\int_{0}^{\infty}u^{-\alpha}\mathrm{e}^{-u}\mathrm{d}u \\
=\;& \gamma_{I+1}^{\alpha}\prod_{j=I+2}^{K}\mathrm{e}^{-\varepsilon_{j}}\prod_{j=I+2}^{K}\mathrm{e}^{-\mathrm{e}^{-\varepsilon_{j}}}\Gamma\left[1-\alpha\right].
\end{aligned}$$

I therefore, have:

$$\begin{aligned}
&\int_{-\infty}^{\infty}\cdots\int_{-\infty}^{\infty}\gamma_{I+1}^{\alpha}\Gamma\left[1-\alpha\right]\prod_{j=I+2}^{K}\mathrm{e}^{-\varepsilon_{j}}\prod_{j=I+2}^{K}\mathrm{e}^{-\mathrm{e}^{-\varepsilon_{j}}}\mathrm{d}\varepsilon_{I+2}\cdots\mathrm{d}\varepsilon_{K} \\
=\;& \gamma_{I+1}^{\alpha}\Gamma\left[1-\alpha\right]\prod_{j=I+2}^{K}\int_{-\infty}^{\infty}\mathrm{e}^{-\varepsilon_{j}}\mathrm{e}^{-\mathrm{e}^{-\varepsilon_{j}}}\mathrm{d}\varepsilon_{j} \\
=\;& \gamma_{I+1}^{\alpha}\Gamma\left[1-\alpha\right]\prod_{j=I+2}^{K}\int_{0}^{\infty}\mathrm{e}^{-u_{j}}\mathrm{d}u_{j} \\
=\;& \gamma_{I+1}^{\alpha}\Gamma\left[1-\alpha\right].
\end{aligned}$$

Extending the above derivation to the $I - r^{\text{th}}$ integral it is clear that

$$
\begin{aligned}
E\left[U_{\mathbf{K}}^2\right] &= \int_{-\infty}^{\infty} \cdots \int_{-\infty}^{\infty} U_{\mathbf{K}}^2 f(\varepsilon_{I+1}, ..., \varepsilon_K) \mathrm{d}\varepsilon_{I+1} \cdots \mathrm{d}\varepsilon_K \\
&= \Gamma\left[1 - \alpha\right] \sum_{i=I+1}^{K} \left(\gamma_i\right)^{\alpha}.
\end{aligned}
$$

CHAPTER C

DERIVATION OF THE JACOBIAN MATRIX

From equation (3.19) the i, h elements of the Jacobian are given by:

$$\mathbf{J} = \frac{\partial(W_1 - W_{i+1} + \varepsilon_1)}{\partial q^*_{h+1}}, \quad \forall i, h = 1, ..., K^*$$

and from equation (3.17c) I have:

$$W_i = \phi_i + \ln\alpha + (\alpha - 1)\ln[q^*_i + \gamma_i] - \ln[p_i] \quad \text{for } i = 1, 2, ..., K^*. \tag{C.1}$$

The i, h element of the Jacobian matrix is:

$$\begin{aligned} J_{ih} &= \frac{\partial(d_1 + (\alpha-1)\ln[q^*_1 + \gamma_1] - \left(d_{i+1} + (\alpha-1)\ln\left[q^*_{i+1,} + \gamma_{i+1}\right]\right))}{\partial q^*_{h+1,}} \\ &= \frac{\partial((\alpha-1)\ln\left(\frac{1}{p_1}\left(\tilde{y} - \sum_{i=2}^{I} p_i q_i\right) + \gamma_1\right) - (\alpha-1)\ln\left[q^*_{i+1,} + \gamma_{i+1}\right])}{\partial q^*_{h+1,}} \\ &= (1-\alpha)\left(\frac{p_i}{q^*_1 + \gamma_1} - z_{ih}\frac{1}{q^*_i + \gamma_i}\right) \end{aligned}$$

where $d_i = \phi_i + \ln(\alpha) - \ln[p_i] + \varepsilon_1$ and $\tilde{y} = y - \sum_{i=1}^{K^*} c_i$. The determinant of the Jacobian matrix becomes:

$$|\mathbf{J}| = \frac{1}{p_1}(1-\alpha)^{I-1}\left(\sum_{i=1}^{I} p_i\left(q^*_i + \gamma_i\right)\right)\left(\prod_{i=1}^{I}\left(q^*_i + \gamma_i\right)\right)^{-1} \tag{C.2}$$

which is the same as Bhat (2005) except $\alpha_i = \alpha$.

CHAPTER D

DEMOGRAPHIC QUESTIONNAIRE ADMINISTERED WITH THE EXPERIMENT

Please take your time and answer all the questions as accurately as possible. If you have any questions, please do not hesitate to ask.

- What is your current age? ________

- Are you male or female?
  - ◊ Male
  - ◊ Female

- How many people live in your household? Include yourself, your spouse and any dependents. Do not include your parents or roommates unless you claim them as dependents.
  - ◊ 1
  - ◊ 2
  - ◊ 3
  - ◊ 4
  - ◊ 5
  - ◊ 6
  - ◊ 7
  - ◊ 8+

- Please indicate the category below that describes the total amount of INCOME earned last year by the people in your household.
  - ◊ Under $5000
  - ◊ $5000-$7999
  - ◊ $8000-$9999
  - ◊ $10,000-$11,999
  - ◊ $12,000-$14,999
  - ◊ $15,000-$19,999
  - ◊ $20,000-$24,999
  - ◊ $25,000-$29,999
  - ◊ $30,000-$34,999
  - ◊ $35,000-$39,999
  - ◊ $40,000-$44,999
  - ◊ $45,000-$49,999
  - ◊ $50,000-$59,999
  - ◊ $60,000-$69,999
  - ◊ $70,000-$99,999
  - ◊ $100,000 - $124,999
  - ◊ $125,000 - $149,999
  - ◊ $150,000 - $199,999
  - ◊ $200,000 +

- What is the highest level of education you attained?
  - ◊ Some High School
  - ◊ High School / GED
  - ◊ Some college
  - ◊ Associates degree
  - ◊ Bachelor's degree
  - ◊ Master's degree
  - ◊ Some Doctorate education
  - ◊ PhD or MD

- Do you have, or will you obtain, a Bachelor's degree, or advanced degree in any of the following areas: - Business Administration (or a major related to business - marketing, accounting, etc.) - Economics - Mathematics - Statistics - Engineering - Physics - Computer Programming?
  ◊ Yes
  ◊ No

- Have you ever, or do you currently, work at a job whose duties involves handling the accounts receivable or accounts payable? In other words, have you ever, or do you currently, work at a job in which you handle some or all of the financial aspects of the company?
  ◊ Yes
  ◊ No

- Please choose the category below that describes the total amount of money your household spends on groceries in an average month.
  ◊ $0 - $100
  ◊ $101 - $200
  ◊ $201 - $300
  ◊ $301 - $400
  ◊ $401 - $500
  ◊ $501 - $600
  ◊ $601 - $700
  ◊ $700 +
  ◊ I don't know

- When doing day to day grocery shopping, which of the following selections is most likely accurate?
  ◊ I shop around at multiple stores to obtain the best price on different products.
  ◊ I check store coupons and advertisements and shop at the grocery store that offers the best deal on the products I plan to purchase.
  ◊ I check the prices of the products I plan to purchase online and go to the grocery store that makes my total purchase the cheapest.
  ◊ I shop at the grocery store that is most convenient.

- Which of the following attributes is more important when making a grocery store selection?
  ◊ Price
  ◊ Product selection

- Please rate the following categories in terms of how likely you would be to make a purchase on-line, rather than at a local retailer.

	Not Applicable	Unlikely - I almost always purchase this at local retailers - 1	Unlikely 2	···	Likely 6	Very likely - I almost always purchase this online 7
Baby products	◊	◊	◊	···	◊	◊
Beauty and fragrances	◊	◊	◊	···	◊	◊
Books and magazines	◊	◊	◊	···	◊	◊
Clothing, accessories, and shoes	◊	◊	◊	···	◊	◊
Gifts and flowers	◊	◊	◊	···	◊	◊
Health and personal care	◊	◊	◊	···	◊	◊
Computers, accessories, and services	◊	◊	◊	···	◊	◊
Education	◊	◊	◊	···	◊	◊
Electronics and telecom	◊	◊	◊	···	◊	◊
Entertainment and media	◊	◊	◊	···	◊	◊
Food retail and service	◊	◊	◊	···	◊	◊
Sports and outdoors	◊	◊	◊	···	◊	◊
Toys and hobbies	◊	◊	◊	···	◊	◊
Home and garden	◊	◊	◊	···	◊	◊

- In general, when you go shopping, which of the following is more accurate in general, not necessarily for groceries?
  - ◊ I like to get in, obtain what I was looking for, and leave.
  - ◊ I like to shop around a bit and see the different products the retailer carries.

- How often do you make purchases online?
  - ◊ Never
  - ◊ Less than Once a Month
  - ◊ Once a Month
  - ◊ 2-3 Times a Month
  - ◊ Once a Week
  - ◊ 2-3 Times a Week
  - ◊ Daily

- How often do you have something delivered to your home that was purchased online?
  - ◊ Never
  - ◊ Less than Once a Month
  - ◊ Once a Month
  - ◊ 2-3 Times a Month
  - ◊ Once a Week
  - ◊ 2-3 Times a Week
  - ◊ Daily

- If you were thinking about getting into a new hobby and were going to make a purchase, where is the first place you would go for information? For example, if you were going to get into photography, what is the first information source you would use to obtain information on different cameras?

- ◊ Recommendation of friends
- ◊ Online retailer's product information
- ◊ Online manufacturers product information
- ◊ Local retailer
- ◊ Other

- What is your Subject ID number? ____________________

You have completed the demographic portion of the questionnaire. I will begin the actual experiment in a moment. Please wait for further instructions.

CHAPTER E
IRB APPROVAL LETTER

ASU Knowledge Enterprise Development

Office of Research Integrity and Assurance

To:	Timothy Richards 1226 S. La
From:	Mark Roosa, Chair Soc Beh IRB
Date:	09/06/2011
Committee Action:	**Exemption Granted**
IRB Action Date:	09/06/2011
IRB Protocol #:	1108006772
Study Title:	Consumer Search and Retail Pass-Through

The above-referenced protocol is considered exempt after review by the Institutional Review Board pursuant to Federal regulations, 45 CFR Part 46.101(b)(2) .

This part of the federal regulations requires that the information be recorded by investigators in such a manner that subjects cannot be identified, directly or through identifiers linked to the subjects. It is necessary that the information obtained not be such that if disclosed outside the research, it could reasonably place the subjects at risk of criminal or civil liability, or be damaging to the subjects' financial standing, employability, or reputation.

You should retain a copy of this letter for your records.