Examining Mathematical Knowledge for Teaching

In The Mathematics Teaching Cycle:

A Multiple Case Study

by

Margaret (Meg) Kathleen Burke

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Finbarr Sloane, Co-Chair
James Middleton, Co-Chair
Daniel Battey

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ABSTRACT

The research indicated effective mathematics teaching to be more complex than assuming the best predictor of student achievement in mathematics is the mathematical content knowledge of a teacher. This dissertation took a novel approach to addressing the idea of what it means to examine how a teacher's knowledge of mathematics impacts student achievement in elementary schools. Using a multiple case study design, the researcher investigated teacher knowledge as a function of the Mathematics Teaching Cycle (NCTM, 2007). Three cases (of two teachers each) were selected using a compilation of Learning Mathematics for Teaching (LMT) measures (LMT, 2006) and Developing Mathematical Ideas (DMI) measures (Higgins, Bell, Wilson, McCoach, & Oh, 2007; Bell, Wilson, Higgins, & McCoach, 2010) and student scores on the Arizona Assessment Collaborative (AzAC). The cases included teachers with: a) high knowledge & low student achievement v low knowledge & high student achievement, b) high knowledge & average achievement v low knowledge & average achievement, c) average knowledge & high achievement v average knowledge & low achievement, d) two teachers with average achievement & very high student achievement. In the end, my data suggested that MKT was only partially utilized across the contrasting teacher cases during the planning process, the delivery of mathematics instruction, and subsequent reflection. Mathematical Knowledge for Teaching was utilized differently by teachers with high student gains than those with low student gains. Because of this insight, I also found that MKT was not uniformly predictive of student gains across my cases, nor was it predictive of the quality of instruction provided to students in these classrooms.

To My Mom,

May she rest in peace knowing she is my inspiration

To My Dad,

The strongest, kindest, and most loving man I know

To My Brother,

For being the voice of reason in tough times

To My Sister-in-Law,

For keeping me motivated when I wanted to quit

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CHAPTER ONE: INTRODUCTION

When thinking about what makes an effective elementary school mathematics teacher, one intuitively thinks teacher content knowledge (Ball, Lubienski, & Mewborn, 2001). Continuing this line of argument, one might consider a teacher who demonstrates sufficient understanding of the mathematics being taught to be able to produce high achievement in her students. Interestingly, empirical support for such an hypothesis has been very elusive. Demonstrating direct links between teachers' mathematical content knowledge and student outcomes is a tricky business (Shechtman, Roschelle, Haertel, & Knudsen, 2010). In fact, Nye, Konstantopoulos, & Hedges (2004) found that only 11% of the total variability in student achievement gains in mathematics over a year could be attributable to teacher effects. In addition, while Hill et al (2005) reported significant statistical findings mapping teachers' mathematical knowledge and student achievement gains across two grade levels, the standardized regression coefficients were quite low at .05 and .06 respectively (Shechtman et al, 2010). So, while we intuitively think that content knowledge should be highly correlated to student achievement gains at the elementary school level, it seems that when other factors are held constant, teachers' having high content knowledge have lower impact on student gains scores we anticipate.

In 1986, at the annual meeting of the American Educational Research Association, Lee Shulman introduced the theoretical idea of Pedagogical Content Knowledge. He argued that as a community of researchers we have missed a critical component in a teacher's repertoire: The blending of pedagogy and content that resulted in knowing *why* we do something in a particular content area beyond *how* to engage it procedurally. He noted that this knowledge separated the teacher from the content expert.

Since Shulman's original insight, a multitude of studies have been conducted examining specific aspects of teacher knowledge and classroom instruction that impact student achievement. Many studies employed case studies to look at expert/novice comparisons within classroom instruction (Borko & Livingston, 1989; Hill, Blunk, Charalambous, Lewis, Phelps, Sleep, Ball, 2008; Lehrer & Franke, 1992; Leinhardt & Greeno, 1989; Stein, Baxter, Leinhardt, 1990; Thompson & Thompson, 1994 & 1996). Others used proxy variables such as courses taken or certification tests as a direct measure of teacher knowledge (Begle, 1979; Monk 1994). Recently, researchers have begun using quantitative measures to assess the linkages between a teachers' mathematical knowledge, classroom instruction, and student achievement (Baumert, Kunter, Blum, Brunner, Vos, Jordan, Klusmann, Krauss, Neubrand, & Tsai, 2010; Hill, Rowan, & Ball, 2005; Hill, Blunk, Charalambous, Lewis, Phelps, Sleep, & Ball, 2008; Shechtman, Roschelle, Haertel, & Knudsen, 2010).

Despite three decades of research in this area: linkages between teacher knowledge and student learning, the field still struggles to determine how teacher' knowledge fosters learning in students and what aspects of teacher knowledge can be developed in teacher training. This dissertation provides insight into what knowledge teachers' use when planning, implementing, and reflecting on classroom practice in elementary mathematics. I constructed cases of teachers whose knowledge level and student gain scores either has yet to be examined or are scarce in the current research. In addition, the observational data was collected prior to analyzing the participants' measured knowledge and post-test interview. This is an important difference between this dissertation and the past literature because during the classroom observations I was blind to the types of measured knowledge held by each teacher. The blinding reduced some of the potential biases that could occur if I had a deep understanding of the types of knowledge held by each teacher.

Purpose

The purpose of this dissertation was to use a multiple case study design (Yin, 2009) to explore the association between six teachers' Mathematical Knowledge for Teaching (MKT) (Ball et al, 2008) and the Mathematics Teaching Cycle (including: planning and implementation of instruction).

Research Question

1) How does a teacher's Mathematical Knowledge for Teaching impact planning, instruction, and student gains in elementary mathematics?

Importance of Study

This study was important for the four main reasons. First, it examined the commonplace belief that teachers with high levels of mathematics content knowledge produced high student achievement scores and that teachers with low levels level of mathematics content knowledge produced low student achievement scores (NCLB, 2000; NMAP, 2008).

Second, there is a tremendous amount of money being funneled into states and school districts to improve student achievement through improving the distribution of high quality teachers (NCLB, 2000) across schools. This study added to the discussion of what makes a high quality elementary school mathematics teacher, as well as where to funnel the government resources to best improve students understanding and achievement in mathematics.

Third, the study unveils long-term implications for building a reflective practice, for designing professional development around teacher knowledge and instructional practice, and for instructing pre-service elementary school teachers.

Fourth, this study addressed some methodological issues in the scarce number of studies that look at the associations between a teacher's MKT score, classroom practice, and student achievement

Limitations

Due to practical constraints, this dissertation cannot provide a comprehensive review of the impact of teacher knowledge on classroom practice. Teaching is a living, dynamic, and complex process. Teachers make decisions instantly, sometimes without any tacit knowledge of what they are doing or why they made a specific choice. Unfortunately, capturing the instantaneous decision-making process is currently impossible without disrupting the learning of students. Similarly, there is no method for mapping the brain during instruction so as to understand which types of knowledge were used at specific times to make specific decisions. Instead, I used interviews and teacher reflections during instruction (if they chose to talk to me during instructional times, or made explicit comments about their teaching to the class) to assess instructional decisions. I used the Mathematical Knowledge for Teaching framework and descriptions found in Ball et al (2008) to best pinpoint instances of teacher knowledge being used during instruction and planning times.

In addition, this dissertation was somewhat limited by not having formal training on the Mathematical Knowledge for Teaching categories and the Mathematics Tasks Framework (Stein, Smith, Henningsen, & Silver, 2009), which was used to assess the tasks selected and presented during each class observation. To account for the limited training, I discussed examples of each category of MKT I chose from my data with people familiar with the framework. I engaged in a similar process with the Mathematics Tasks Framework and also strictly adhered to the analysis guides provided in Stein et al (2009).

Similarly, the frameworks used to collect and analyze the data limit this dissertation. The Mathematical Knowledge for Teaching framework is a work-in-progress (Hill, 2010). Therefore, as that framework develops, my rendering of the original ideas for MKT and how I used the MKT categories might need adjusting.

Other limitations of the study pertained to access to complete data sets for the six teachers. I have incomplete data for two teachers and modified some of the data for a third teacher. The first of the two teachers with incomplete data (Teacher 1.2) disallowed audio-recordings during both the interviews and classroom observations. Due to this limitation, I could not conduct Classroom Assessment Scoring Systems (CLASS) observations and take detailed field notes of the instruction concurrently. Therefore, Teacher 1.2 does not have CLASS data. In addition, there are limitations in the analysis of Teacher 1.2 because her observational data was based on my field notes only.

The second teacher who has incomplete data (Teacher 2.1) only partook in one of the three interviews. The missing interviews were about her task selections and the follow-up to her teacher knowledge assessment.

The last teacher (Teacher 3.1) was on maternity leave for the first two months of data collection and, thus, her data was collected over a three-month period at the start of the second semester of the 2011-2012 school year. This limitation on when data could be collected might impact a discussion on her teaching habits at the start of the year versus the instruction closer to the state's standardized testing date. It is possible that her teaching methods changed as the school year drew closer to the testing date but without baseline, I cannot say so with surety.

This dissertation was also limited by the location, the access I had to teachers, and the fact that teachers could choose to participate in my study or not. Even with its limitations, this dissertation created compelling descriptions of how Mathematical Knowledge for Teaching impacts teachers throughout the teaching process.

Organization of the Dissertation

This dissertation is organized into seven chapters. The following sections provide a brief description of chapters 2 through 7.

Chapter Two

In chapter two, I carefully summarize and critique research relevant to teacher content knowledge, teacher instructional practices in mathematics, and linkages between the two as it relates to student achievement in mathematics. Following this review, the conceptual framework that undergirds the dissertation research study was presented. This dissertation used the Mathematical Knowledge for Teaching framework (Ball et al, 2008) and the Mathematics Teaching Cycle (NVTM, 2007) as the basis for exploring the research question. These two components, the review and the theoretical framing, serve as the foundation for the generation of the research questions, choice of research design, data collection and analysis, and allowed claims to be warranted in ways that lead to results that are both theoretically relevant and practically significant.

Chapter Three

Chapter three outlines the methodology used in this dissertation. This study used a multiple-case study design (Yin, 2009). Three cases were used to illustrate "similar" results and "contrasting" results (Yin, 2009, p. 54). Purposeful and convenience sampling were used to select the participants (Maxwell, 2005). The teachers in each case were selected using scores on a teacher knowledge assessment and student gain scores over the 2011-2012 school year. Two of the cases represented pairs of teachers with similar MKT scores and contrasting student gain scores, while the third case presented two teachers with low MKT scores, compared to their peers, who had the highest growth in student scores, compared to their peers.

The data for the dissertation were collected from two sources (each containing three subcomponents): Teacher interviews and classroom observations. Three interviews were conducted with the participants. The first interview reviewed the general planning process used to construct daily lessons and units. The second interviewed explored how a teacher selected specific tasks for students and what knowledge they anticipated gaining about student learning from the particular tasks. The third interview was a follow-up to the teacher knowledge assessment. During this interview, the teacher walked me through their thought-process for solving the problems on the test.

The classroom observation data consisted of information from the CLASS observation protocol, the tasks used during each observation, field notes from the observations, and audio-recordings of each class period observed. The purpose of these data was to assess the learning environment, the task selection, and the discourse in the classroom, as outlined in the Mathematics Teaching Cycle (NCTM, 2007).

To analyze the data, I employed the guidelines of Miles and Huberman (1997). According to this analytic framework, the coding of chunks of data was used to "review a set of field notes, transcribe or synthesized, and to dissect them meaningfully, while keeping the relations between the parts intact . . ." (p. 56). The data were chunked into the three categories within the Mathematics Teaching Cycle: Knowledge, Planning, and Classroom Instruction. Codes based on the MKT categories found in Ball et al (2008) were used to examine the data. The coded data were then used to construct meaning from the themes and patterns across the data sources. I also used the CLASS observation protocol, the Mathematical Tasks Framework, and Bloom's Taxonomy (Bloom, 1956) to analyze components of the data sets and triangulate the findings. From here, specific vignettes were selected from the transcripts. The vignettes illustrated specific coding schemes, or themes, that appeared across the data for a particular teacher or as a comparison across teachers within and between cases.

Chapter Four

Chapter four starts the presentation of cases. In chapter four, one learns the case of two teachers who scored similarly on the Teacher Knowledge Assessment but whose student gain scores differed greatly. Both teachers scored at least a full standard deviation above the mean of their peers, from the larger NSF-funded study in which this dissertation is housed, on the teacher knowledge assessment. One teacher had student gain scores that were the second highest of all of the student gain scores of the participants in the larger NSF-funded grant. The student gain scores for the other teacher in this case were just above the mean of the student gain scores of those in the larger NSF-funded grant.

This case demonstrated differences between the two teachers related to the types of knowledge used across all three facets of the Mathematics Teaching Cycle (knowledge test, planning, and implementation). The teacher with high student gain scores used a complex mix of Common Content Knowledge, Specialized Content Knowledge, Knowledge of Content and Students, and Knowledge of Content and Teaching. Her teaching style embraced student thinking. Both her classroom environment and her planning focused on providing activities to students based on what she learned through talking with her students about the mathematics. As she learned what the students understood, Teacher 1.1 selected new tasks to challenge the students to think one step further than the point at which their knowledge extended mathematically. She also challenged students to share their thinking at the board and was able to hear and interpret what students knew about the mathematics based on their explanations.

Across the three facets of the Mathematics Teaching Cycle, the teacher with relatively average student gain scores demonstrated reliance on Common Content Knowledge. Unlike the first teacher in this case, the second teacher imparted knowledge to her students. She gave her students the formulas or procedures repeatedly and limited the classroom discourse to answering basic recall or comprehension questions. She followed the order of the textbook and the district curriculum map to plan, regardless of what her students understood.

Chapter Five

This case examined two teachers who scored on opposite sides of the mean of the participants in the NSF-funded grant on the Teacher Knowledge Assessment. The student gain scores over the 2011-2012 school year for these two teachers also differed from each other. The first teacher in this case had a MKT score almost three quarters of a standard deviation above the rest of the participants in the larger NSF-funded grant. Her student gain scores were a half of a standard deviation below the mean of the scores for the participants in the larger NSF-funded grant. The second teacher in this case had an MKT score that fell just below the mean of the scores for all of the participants in the NSF-funded grant. Her student change scores were a half of a standard deviation above the mean of the rest of the participants' students' gain scores.

This case found that differences occurred in how the two teachers' MKT manifested in their instruction. For the most part, Common Content Knowledge (CCK) was very apparent in instruction of the teacher with relatively low student gain scores. This teacher stood at the front of the classroom and dictated the standard procedure for solving problems. She read the question to the students, showed them the steps for completing the problem, and then the students demonstrated in their independent work that they could follow her directions for solving a task. Once she was satisfied that the students could mimic the procedure she gave them, the students were given multiple problems from the textbook to solve during the remainder of the class period.

Much like the first teacher in this case, a basic level of CCK was seen in the teaching of the second teacher. What differed was that while the second teacher used the sequence of instruction laid out in the textbook and the procedures given in the textbook, she encouraged some discussion in her classroom and used her Knowledge of the Content and Students (KCS) when deciding what textbook tasks would be hard or easy for students to solve. She sometimes allowed students to tell her how they solved the problem before she gave them the procedure but usually she presented the procedure first. She also used a combination of KCS and Knowledge of Content and Teaching to determine when she could combine lessons. For example, she knew her students mastered the concept of Mode from their work on the Math Board. So she collapsed that lesson in the textbook into the lesson on other measures of central tendency. This teacher also used her knowledge of Bloom's Taxonomy to structure her questions for students. She learned about developing questions from the Taxonomy from a professional development program provided for her school, during the 2011-2012 school year.

It was possible that the differences in student gain scores for these teachers was a function of the differences in the amount of classroom discussions, the use, or lack thereof, of knowledge of student thinking when planning and implementing lessons, and the willingness of the second teacher to embrace ideas she learned in professional development courses. Because of the missing data from the first teacher in this case, it was hard to determine how MKT influenced the teacher's decisions, but it was apparent that, for the most part, her instruction and planning were based on her CCK and knowledge of standard algorithms.

Chapter Six

The final case presented two teachers with relatively low MKT scores when compared to the rest of the participants' scores in the NSF-funded grant. These two teachers, however, had the highest and third highest student change scores of all of the teachers in the NSF-funded grant.

This case found that both teachers used their pedagogical content knowledge far more often than their subject matter knowledge. They understood which resources provided them with examples of possible standardized test items and how those resources could be used to structure their daily lessons. They also used their students' thinking to modify the scope and sequence of the instruction and to know which types of tasks to give the students each day.

Chapter Seven

The last chapter presents the discussion section. In the end, my data suggested that MKT was only partially utilized across the cases during the planning process, the delivery of mathematics instruction, and subsequent reflection. Mathematical Knowledge for Teaching was utilized differently by teachers with high student gains than those with low student gains. Mathematical Knowledge for Teaching was also utilized differently by teachers within the high student gain category. Because of this insight, I also found that MKT was not uniformly predictive of student gains across my cases, nor was it predictive of the quality of instruction provided to students in these classrooms.

CHAPTER II: LITERATURE REVIEW & THEORETICAL FRAMING

In this chapter, I carefully summarize and critique research relevant to teacher content knowledge, teacher instructional practices in mathematics, and linkages between the two as it relates to student achievement in mathematics. Following this review, I develop the conceptual framework that undergirds the dissertation research study. These two components, the review and the theoretical framing, serve as the foundation for the generation of the research question, choice of research design, data collection and analysis, allowing me to warrant the claims that will lead to results that are both theoretically relevant and practically significant.

For decades, mathematics education researchers have grappled with understanding how specific aspects of teacher effectiveness and quality (i.e., teacher knowledge, teacher affect and beliefs, classroom practices, discourse, teacher qualifications, etc.) pertain to student achievement and learning (Begle, 1972; Darling-Hammond, 2000; Eisenberg, 1977; Escudero & Sanchez, 2007; Fennema & Franke, 1992; Franke, Kazemi, & Battey, 2006; Hashweh, 1987; Hill, Blunk, Charalambous, Lewis, Phelps, Sleep, & Ball, 2008; Hill, Rowan, & Ball, 2005; Hill, Sleep, Lewis, & Ball, 2006; Philipp, R, 2006; Stein, Remillard, Smith, 2006; Thompson, 1992; Walkowiak, 2010).

As my particular study pertains to teacher knowledge of mathematics and mathematical pedagogy, I used the search terms teacher knowledge, mathematics, pedagogical content knowledge, mathematics knowledge for teaching, instructional practices, elementary schools, student achievement, and learning in Google Scholar. From the list of references, I ruled out articles that only looked at content knowledge as the determinant of teacher knowledge because my definition of teacher knowledge goes far beyond basic content knowledge. I also found numerous articles documented that content knowledge alone does not necessarily equate to noticing teachable moments or lead to considerably improved pedagogical skills in the classroom (Baumert, Kunter, Blum, Brunner, Voss, Jordan, Klusmann, Krauss, Neubrand, & Tsai, 2010). In addition, I ruled out articles that only focused simply on the expert/novice relationship because I do not view the act of teaching from a deficit perspective. I am not looking to see what teachers do not know, instead I wish to examine how different levels of Mathematical Knowledge for Teaching affect teachers instructional choices, and thus inform us on how teachers utilize and draw on their individual resources to support learning in their classrooms (Cohen, Raudenbush, & Ball, 2000; Escudero & Sanchez, 2007).

For over 40 years, researchers have examined the links between teachers' knowledge, classroom practice, and student achievement (Hill et al, 2008). In 1972, Edward Begle assessed the knowledge of three hundred 9th grade algebra teachers participating in an NSF summer institute, to determine if a link existed between teachers' algebraic knowledge and student achievement. Begle (1972) used the Abstract Algebra Inventory Forms B and C to assess teacher knowledge and the Mathematics Inventories III and IV to assess their students' learning at the end of 9th grade. Using these tests, Begle concluded that there was no significant correlation between teacher understanding of algebra and student achievement in algebraic computation. He found a statistically significant correlation between a teacher's understanding of the algebra of real number systems and students' understanding of 9th grade algebra, however, the correlation was so small that Begle argued it was not educationally significant.

Begle’s (1972) study indicated early on the importance of using curriculum sensitive measures to assess teacher knowledge, the possibility that classroom practice could impact student achievement, and the issues of selection bias. While Begle (1972) found that the predictor variables in the Mathematics inventories given to the teachers were well chosen and meaningful, Begle’s assessment measured basic content knowledge of algebra with no emphasis on pedagogy or pedagogical content knowledge. In addition, the sample of teachers used in the study was not representative of the general population of teachers in the United States. The sample consisted of teachers who volunteered for a summer professional development course in mathematics. Therefore, it is possible that the effect of teacher knowledge on student achievement might be more easily seen with a different sample of participants, or if it were measured differently.

To account for the sampling bias, Eisenberg (1977) replicated Begle’s study using what he considered a “normal” sample of teachers. Using the same methodology and assessment tools, Eisenberg (1977) collected a sample of teachers from all of the junior high schools across Columbus, Ohio. The 28 participants were given the Algebra Inventory Form B during one 50-minute time period. Their students were assessed using Begle’s Mathematics Inventory during the last two weeks of the school year. Eisenberg confirmed that teacher knowledge of the subject matter had little measurable effect on student performance, using Begle’s (1972) test of algebra.

Ten years later, Hashweh (1987) used three different methods of assessing science teachers’ subject matter knowledge when investigating the relationship between subject matter knowledge and classroom practice and found dramatically different results. In his study, Hashweh defined subject matter knowledge as the “teacher’s knowledge of the discipline – knowledge of content and its organization. This included both abstract knowledge, such as knowledge of disciplinary conceptual schemas, and more specific knowledge, such as knowledge of a particular topic (Hashweh, 1987, p. 110). The researcher used card sorting, concept-map labeling, and free recall to determine the knowledge base of three biology and three physics secondary teachers had of their own disciplines and of the other discipline. In addition, in two subsequent interview sessions, Hashweh asked the teachers to design a unit within their own field of expertise (either in biology or physics) and the one they had less experience with (again, either biology or physics), and gave the six teachers simulated teaching tasks from each discipline. The purpose of the study was to measure the effects of the knowledge level on planning and teaching of two science subcategories.

Hashweh (1987) found that within their field of expertise, the teachers had "(a) more detailed topic knowledge, (b) more knowledge of other disciplinary concepts; (c) more knowledge of higher-order principles that are basic to their discipline and (d) more knowledge of ways of connecting the topic to other entities in the discipline" (p. 113). More specifically, the physics teachers expressed a detailed organization of their understanding of the relationships among topics found in physics. However, the biology teachers displayed no overarching organizational structure of relationships across biology topics. The biology teachers demonstrated clear organization of specific components related to different aspects of the field of biology but not a single, unified organization of their knowledge.

When it came to teacher planning, Hashweh (1987) found that knowledgeable teachers used their understanding of the topic to modify lessons, to add supplementary activities, and to explain why they decided upon certain activities and not others. When teachers were asked to plan a unit outside of their field of expertise, the teachers consistently followed the textbook lessons sequentially and without modification. Regarding the instructional tasks, Hashweh found that knowledgeable teachers asked higher-order thinking questions that required synthesis and analysis of the readings in the textbook. Less knowledgeable teachers asked recall and memorization questions. In addition, knowledgeable teachers identified and addressed students' misconceptions, while less knowledgeable teachers were unaware of their student misconceptions, even reinforcing student misconception at times.

> *The physics incidents indicated that the unknowledgeable (sic) teachers might actually reinforce preconceptions, incorrectly criticize correct student answers, and accept faulty laboratory results. The biology incidents indicated that in some cases even relatively knowledgeable teachers would lack the knowledge necessary to deal effectively with student difficulties (p. 118).*

His results indicated that subject-matter knowledge influenced the decisions teachers made when planning and when engaging teachable moments.

This study illuminated some critical methodological techniques that aided in assessing teacher subject matter knowledge. First, Hashweh (1987) used a broader definition of subject matter knowledge than either Begle (1972) or Eisenberg (1977). His definition aligned with that of Shulman (1986). This expansion of what constituted useable knowledge in teaching enabled Hashweh to construct meaningful tasks that explored a teacher's content and pedagogical knowledge of familiar and unfamiliar topics. Hashweh also established the importance of linking knowledge to classroom practice and planning, which is a key aspect of NCTM's Mathematics Teaching Cycle

(2007). Lastly, this study provided positive insight into the need for case study analysis when trying to understand teacher behaviors and learning. The author generated an in depth analysis of teacher knowledge across a variety of sources and illustrate the impact of teacher knowledge on practice.

Hashweh's (1987) work is not without limitations, however. First, Hashweh simulated teaching moments to assess teacher decision-making. That is, he linked teacher knowledge of content to hypothetical situations that the teachers had time to process and evaluate before discussing. He did not observe practice as the intellectual stepping-stone to understanding teachers' decision-making processes or to evaluate what teachers actually do in the moment. Second, Hashweh met with each teacher for three sessions totaling 4-6 hours (perhaps leading to fatigue on the part of the participants). Third, the fact that he used two different content areas could account for different schemas, as the structure of each subject differs greatly within each field. Finally, Hashweh did not address how teacher knowledge linked to student achievement, a central goal for mathematics education.

In a similar study, Stein, Baxter, and Leinhardt (1990) examined how subject matter knowledge of one elementary school teacher impacted his teaching of functions and graphing. Much like Hashweh (1987), these researchers defined subject matter knowledge broader than their predecessors (Begle, 1972; Eisenberg, 1977). For Stein, Baxter, and Leinhardt (1990), subject matter knowledge was defined as a combination of beliefs and knowledge of mathematics and content-specific pedagogy. Like Hashweh, the authors used an interview method and card sorting activity to assess the teacher's subject matter knowledge, specifically, of functions and graphing.

The Stein et al (1990) study differed in some critical features from Hashweh (1987). First, the authors used one teacher for their case study rather than six. Second, the participant's knowledge and classroom practices were compared to an "expert" mathematics educator selected by the researchers as a comparison, rather than another teacher in the field. Neither study linked teacher behavior to student achievement.

Much like Hashweh (1987), Stein et al (1990) found that the participant in their study lacked important components of knowledge compared to their chosen "expert." The teacher demonstrated a basic understanding of functions arithmetically and a basic internal organization of functions. His knowledge lacked the complex, multi-layered organization held by the expert. This lack of depth impacted his instruction greatly. The case teacher did not provide students with examples that demonstrated the mathematical relationships between functions and graphs. He relied heavily on textbook material, much like the teachers in Hashweh's (1987) study when they were planning lessons outside of their field of expertise. Stein et al (1990) also concluded that much of what the teacher found important to teach linked back to his beliefs and understanding about functions and graphing.

Stein et al (1990) concluded the following. First, the investigators found that the depth of a teacher's knowledge influenced the degree to which the teacher could provide groundwork for later math learning. Because the participant in their study lacked a sophisticated understanding of functions and how these functions related to graphing, his students were never exposed to opportunities to learn fundamental algebraic ideas. Second, the authors noted that a lack of rich mathematical knowledge generated misconceptions in students, poorer planning, and limited learning opportunities for students. Again, these two conclusions paralleled the findings of Hashweh (1987). These are important conclusions for teacher education research but one must consider that some of the conclusions were drawn about the participant's knowledge and teaching ability in contrast with a mathematics "expert," whose teaching practices were not evaluated. Also, there were no explicit details given about what made the participant an "excellent" teacher in the eyes of his principal or what made the "expert" the expert in this study. Consequently, one needs to be cautious of the generalizations made about teacher knowledge based on the Stein et al (1990) study.

With regard to the method of assessing teacher knowledge, Baxter and Lederman (1999) note that card sorting has yet to be established as a "literal representation of how knowledge is stored in memory" (p. 153). It is also a very restrictive technique to use when assessing an individual's knowledge base. The technique "requires either a particular format (hierarchical, static and two-dimensional) or use of particular ideas in the representation of one's conceptual schema. As a consequence, the researcher is only provided with how the research subject views the ideas presented by the researcher, or a representation that is restricted to a particular hierarchical format" (Baxter & Lederman, 1999, 152).

A second, and central, concern with this study is that there is no link to student achievement. While the teacher participant might have less knowledge than the expert, the reader cannot assess the impact of the missing knowledge on student learning. In my study, I have linked teacher knowledge with student achievement in order to select the sample of teachers as a way to alleviate this possible concern. Using this method, I was able to select teachers at two levels of teacher mathematical knowledge for teaching (high and low) and two levels of student performance (high and low). This created the opportunity for four groupings of contrasting cases.

In 2005, Hill, Rowan, and Ball assessed whether, and how, teachers' mathematical knowledge for teaching contributed to gains in students' mathematical achievement. They defined mathematical knowledge for teaching as "the mathematical knowledge used to carry out the work of teaching mathematics." (p. 373). The authors used a quantitative linear mixed-model methodology to complete their study. In order to assess teacher knowledge, Hill et al (2005) used LMT (Learning Mathematics for Teaching) items, specifically developed to assess at teacher's MKT (Mathematical Knowledge for Teaching). Students' learning was assessed using the Terra Nova mathematics test at the end of a school year. In addition, teacher practice information was gathered using a self-report log. The teachers filled out the log after each lesson. The log was used to measure the amount of time given to instruction, content covered, and instructional practices. The use of the log was piloted prior to the actual study. The inter-rater agreement between teachers and trained observers was estimated to be above .7.

The Hill et al (2005) study employed 3,000 1st and 3rd grade students and their 700 teachers from 115 elementary schools across 15 states. The study lasted 4 years. Eighty-nine of the 115 schools were participating in larger comprehensive school reform programs (i.e., America's Choice), while 26 schools were not. These 26 schools were used as a comparison group. The schools were matched in terms of community disadvantage and district setting. There was deliberate overrepresentation of high poverty schools in urban, suburban, and urban fringe areas (neighborhoods close to urban settings).

Effect size analyses indicated that their analytic model displayed a positive link between teacher knowledge and student achievement. They found that their task-sensitive tool (LMT, 2006) was positively related to student achievement in mathematics. Based on this relationship, the authors noted that,

> *Teacher's content knowledge for teaching mathematics was a significant predictor of student gains in both models at both grade levels . . . expressed as a fraction of average monthly student growth in mathematics, this translates to roughly one half of two thirds of a month of additional growth per standard deviation difference on the CKT-M variable (Hill et al, 2005, p. 396).*

Lastly, the article reported that the average length of a lesson significantly impacted the student gains in third grade and that years of teaching experience showed no relationship with the student achievement of first graders but a slightly positive relationship with third grade student achievement scores.

The findings from this study highlighted a number of gaps in the literature, however. One significant gap discussed in the conclusion of the study was centered on the instructional practices of mathematically knowledgeable and less knowledgeable teachers. Specific questions in need of exploration included:

> *How knowing mathematics affects instruction - has yet to be studied and analyzed. Does teachers' knowledge of mathematics affect the decisions they make? Their planning? How they work with students, or use their textbooks? How they manage students' confusion or insights, or how they explain concepts . . . how mathematical and everyday language is bridged, for example, or how representations are deployed or numerical examples selected? (Hill et al, 2005, p. 401).*

This dissertation begins to address, to varying degrees of depth, many of these questions. The cases were generated to provide explicit contrasts between teachers with high and low levels of measured MKT, whose students' gain scores were either high or low, relative to their peers in other classrooms. Thus, the case selection method described earlier allowed me to investigate carefully these interactive questions in situ. Others have tried to address these questions but with limited success. I review their work below.

Using a case study method with two experienced high school teachers, Escudero and Sanchez (2007) investigated how domains of knowledge integrate into classroom practices. The authors defined domains of knowledge using Shulman's (1986) definitions of pedagogical content knowledge and content knowledge in conjunction with Shoenfeld's (1998) theory of teaching in context. To link the practice and knowledge domains, the participants were interviewed during planning sessions and given pre- and post-observation interviews to assess goals and immediate reactions to their lesson. The researchers also used Leinhardt's (1989) framework to analyze each lesson for the quality of its mathematics content.

Escudero and Sanchez (2007) found that one teacher provided students with experiences that allowed for the demonstration of student knowledge and meanings. He used a lot of interconnected ideas, showed a deep understanding of the mathematical concepts, and explained how concepts related to each other. The other teacher used more traditional methods of teaching introducing basic procedures with close-ended questions. The second teacher tried to anticipate student issues when students were learning mathematical procedures.

Once again, we annotate the limitations to this study. Teacher knowledge levels were assessed from years of teaching, peer recommendations, and that they both had degrees in mathematics. Also, each teacher had complete freedom to plan the lesson at will. The authors did not control for differences in planning, which likely lead to many variables influencing differences in teaching not just knowledge levels. Finally, the authors never addressed the impact of the differing teaching styles on student achievement. While it is important to look at knowledge differences, without examining student learning one cannot say if more knowledge or what type of knowledge is needed to be a successful and high quality teacher.

In a subsequent study, Hill, Blunk, Charalambous, Lewis, Phelps, Sleep, & Ball (2008) explored the link between teachers' Mathematical Knowledge for Teaching (MKT) and the Mathematical Quality of Instruction (MQI). The authors assessed the mathematical knowledge for teaching in 10 elementary school teachers and the mathematical quality of their instruction using comparative case studies. Four types of data were collected from each of the teachers: a paper-and-pencil Mathematical Knowledge for Teaching test, nine videotaped classroom observations of teaching practices, post-observation debriefings, and interviews. The authors constructed five cases to illustrate convergent and divergent examples of how teachers' knowledge influences pedagogical decisions, beliefs about what mathematics is, and the goals for student learning.

The first case depicted a teacher with a high-degree of mathematics knowledge and a very high score on the mathematical quality of instruction index. Her instruction included a rich exploration of mathematical concepts, links across mathematical ideas, few mathematical errors, and a strong connection to student thinking (including using students' mistakes to move the instruction forward).

The second case illustrated a classroom where low mathematical knowledge aligned with low quality of mathematics instruction. The researchers reported that this teacher created misconceptions for students. Her language, superficial teaching methods, and a lack of connection among the different parts of each lesson confused students and led them to forming incorrect assumptions about the mathematics. Frequently, she dealt with students' mistakes by repeating a procedure to the students or by completing tasks for the students.

The last three cases did not easily align as convergent or divergent across MKT and MQI scores. The teacher's instruction did not necessarily match the expectations elicited from the MKT scores. For example, one of the teachers had a very high MKT score but lessons were discrete and unconnected. The teacher provided rich activities and language to students but each activity lacked a link to an overarching purpose.

Another teacher had average MKT scores but scored very low on the MQI tool. This teacher used a plethora of "fun" activities to entice kids to love math but she was unable to draw out rich mathematical discussions or concepts using these games. Also, almost one-third of every mathematics class was unrelated to mathematics at all, in contrast to the nine other participants who spent at least 90% of the time on mathematics tasks.

The last case discussed was of a teacher without a teaching certification who scored lowly on both the MKT and MQI assessments. This teacher demonstrated many mathematical errors through out her instruction. However, few of these errors seemed to be as damaging to student learning as some of the other teachers with higher MKT scores.

This study (Hill et al, 2008) provided support for my own research. The authors demonstrated "that there is a powerful relationship between what a teacher knows, how she knows it, and what she can do in the context of instruction" (p. 496). However, this study left room for other interesting contrasts between teachers. For example, the following dissertation added two supplemental comparative cases: 1) a teacher with high MKT scores and low student achievement scores with a teacher who had a low MKT score and high student achievement, 2) two teachers with modest MKT scores but the highest student achievement scores in the district, and 3) a teacher who scored highly on the teacher knowledge assessment but whose students had a miniscule amount of change from first quarter AZAC scores to third quarter. This type of teacher is rarely found in the data when looking at teacher knowledge and student achievement. Therefore, my comparisons add depth to the case-study literature examining how MKT relates to student achievement.

In the last year, two very interesting studies appeared in the literature regarding teacher knowledge of mathematics, teaching, and student achievement. One study, out of Germany, (Baumert, Kunter, Blum, Brunner, Voss, Jordan, Klusmann, Krauss, Neubrand, & Tsai, 2010) differences in Pedagogical Content Knowledge and Content Knowledge of mathematics teachers to determine how each contributed to teachers' professional knowledge for teaching. The researchers also assessed the impact of the differences in knowledge on student achievement. They defined PCK as "the knowledge that makes mathematics accessible to students" (p. 142). The authors distinguished three forms of knowledge within PCK: knowledge of tasks as instructional tools, knowledge of student thinking and understanding, and knowledge of multiple representations and explanations. Content knowledge, on the other hand, was defined as “a profound mathematical understanding of curricular content to be taught” (p. 142). The authors argued that content knowledge was based in academic research but modified and developed through practice.

The authors (Baumert et al, 2010) sampled 181 teachers with 194 classes of 4,353 students. The sample included two types of teachers: gymnasium teachers and not-gymnasium teachers. The gymnasium teachers, who majored in maths, taught higher-level mathematics and were expected to have higher content knowledge then non-gymnasium teachers, who minored in maths. The researchers assessed the content knowledge of the 5^{th} through 10^{th} grade teachers using a paper-pencil test. Observations and a single interview consisting of all open-ended questions determined the pedagogical content knowledge of the teachers. Instruction was examined at three levels: tasks, the amount of individual learning support given to students, and effective class management. The authors used a mediation model to test the extent to which CK and PCK influenced instructional quality and student learning gains in mathematics.

Baumert et al (2010) found that gymnasium teachers scored significantly higher on CK test than non-gymnasium teachers, as anticipated. However, when CK was controlled for, non-gymnasium teachers scored higher on PCK than gymnasium teachers. They found that 69% of the variance in achievement between classes was accounted for by PCK. The PCK of the teachers influenced cognition, curricular knowledge, and learning support aspects of a teacher’s instructional quality. These findings implied that a teacher's pedagogical content knowledge impacted student gains more than content knowledge. Also, a teacher who is strong in content knowledge might not have the highest achieving students over time. My study took these findings and broke them down further. My study examined what happens in the classroom and how different dimensions of mathematical knowledge for teaching influence a teacher's decision-making process and student achievement.

The second study (Shechtman, Roschelle, Haertel, & Knudsen, 2010) used Ball's notion of MKT (Ball, 1990; Ball, Hill, & Bass, 2005; Shulman, 1986) to determine the links among teacher knowledge, classroom practice, and student learning in middle-school mathematics classrooms. One hundred twenty-five seventh grade teachers and 56 eighth grade teachers volunteered to participate in this study. The teachers were randomly assigned to treatment or control groups. The treatment group received professional development on using a SimCalc intervention to teach proportionality and linear functions. The control group members received professional development on the same topics but were then asked to teach the SimCalc units using the normal curriculum. The teachers' knowledge was assessed three times throughout the study using a measure developed by the researchers. Student achievement scores and daily logs helped researchers understand the learning goals of each lesson, the use of technology in the classroom, and the daily topics covered.

The authors (Shechtman et al, 2010) found a significant relationship in the seventh grade treatment group between a teacher's MKT level and student gains. They determined that the pretest was a modest predictor of student learning gains in proportionality and linear functions. In addition, they found that high MKT scores did not necessarily equate to high student gains. Many teachers who received a low score on the MKT measure showed high student gains over the course of the study. Other teachers with high MKT scores had low student gains. Lastly, the authors determined that MKT was an independent construct and that while SimCalc teachers' added difficult topics to their daily lessons, it was not related to their MKT scores.

Like many of the other studies reviewed here, Shechtman et al (2010) displayed critical limitations. First the authors created their own measure of MKT (LMT, 2006). It is unfortunate they did not replicate their measured results with student achievement and were unable to link their findings with the already developed measure of MKT that was publically available. With a more sensitive measurement tool, the findings could have been different. Lastly, neither observations nor interviews were used to assess classroom practices. Instead, the teachers filled out a daily log and their answers in the log measured their classroom decisions.

Even so, much like the previous studies, this study (Shechtman et al, 2010) provided both information and gaps that support the need for my dissertation. For example, this study provided additional evidence that successful teaching involves more than high mathematical content scores. Teachers with a variety of levels of mathematics and pedagogical knowledge showed student gains over time. Therefore, we need to look at how the mathematical and pedagogical knowledge impacts the many components of the mathematical teaching cycle. This requires examinations beyond expert/novice comparisons and investigations of good teaching versus bad teaching. It requires a holistic view of the instructional system (Lerman, 2001; Sfard, Forman, & Kieran, 2001).

From this literature review, one can see many gaps in the area of linking teacher knowledge, classroom practices, and student achievement in mathematics education. My study addressed many of the following concerns:

- Holistic view of teaching based on the Mathematics Teaching Cycle,
- Creating a sample of teachers using statistical data regarding the teachers' Mathematical Knowledge for Teaching scores and gains in student learning over a three-quarter period,
- Constructing case-studies based on contrasting evidence against the common belief that more content knowledge directly aligns with higher achieving students,
- Observing teachers' instruction prior to delving into their mathematical knowledge in order to limit observation bias,
- Using outside observers to check rater reliability of classroom observations
- Capturing real-time decision-making process through interviewing teachers during instruction and post-instruction on a weekly basis, and
- Assessing teacher knowledge through paper-pencil assessments, open-ended interviews, and talk aloud interviews (Chapter 4),
- Assessing instructional practice using interviews, task analysis, questioning techniques, and discourse analysis.

Theoretical Framing

Understanding teacher knowledge of mathematics is a complex and dynamic area of research in the field of mathematics education. It spreads beyond counting college math courses, examining scores on teacher preparation tests, and unlocking procedural knowledge of a teacher. The research on teacher knowledge includes understanding how teachers internalize the underlying processes of mathematical ideas, link different concepts within mathematics, figure out multiple ways of representing the mathematics to students, and (Shulman, 1986) student thinking of the mathematics to best prepare instruction (Fennema and Franke, 1992).

This elusive and compound notion of deciphering what knowledge teachers hold and how they utilize that knowledge in the classroom has lead to the development of multiple theories and frameworks. These theories strive to explain the large system of integrated knowledge used by teachers in hopes of developing future professional development and strong teacher preparation programs for generations of teachers to come.

In the later half of this chapter, we examine a number of poignant theories describing teacher knowledge of mathematics to illustrate the diverse yet overlapping views within our field. We start with a general, largely influential theory, about teacher knowledge and then moved into specific theories found in mathematics education. We conclude this chapter with a framework that envelops the notion that of teacher knowledge as situated knowledge (Fennema and Franke, 1992). This theoretical framing (NCTM, 2007) sets the foundation for exploring our research questions, the data that needs to be gathered, the analytic models we will utilize, the warranting of our data based claims, and building empirical linkages back to theory (NRC, 2002).

Lee Shulman's (1986) Framework

In his 1985 Presidential Address to the members of the American Educational Research Association (Shulman, 1986), Lee Shulman argued, that teaching entails much more thought, skill, and knowledge than perceived by the public. He also argued that previous research rarely focused on content knowledge but instead on the generic process of teaching or decision-making. He structured teaching as an integration of subject matter knowledge and pedagogical knowledge, a novel idea to research on teaching up until Shulman's address in 1986.

> *. . . No one asked how subject matter was transformed from the knowledge of the teacher into the content of instruction. Nor did they ask how particular formulations of that content related to what students came to know or misconstrue . . . what we miss are questions about content of the lessons taught, the questions asked, and the explanations offered. . . . Our work does not intend to denigrate the importance of pedagogical understanding or skill in the development of a teacher or in enhancing the effectiveness of instruction. Mere content knowledge is likely to be as useless pedagogically as [a] content-free skill. But to blend properly the two aspects of a teacher's capacities requires that we pay as much attention to the content aspects of teaching as we have recently devoted to the elements of teaching process. (Shulman, 1986, pp. 6 & 8)*

What Shulman (1986) laid out for the Educational Research community was an original theoretical perspective that mapped out teacher knowledge. He "distinguish[ed] among three categories of content knowledge: (a) subject matter content knowledge, (b) pedagogical content knowledge, and (c) curricular knowledge" (p. 9).

Content knowledge. Content knowledge, "refers to the amount of organization of knowledge per se in the mind of the teacher" (Shulman, 1986, p. 9). He elaborated stating that not only does the teacher know the facts and concepts of a particular subject matter but also the structure of how the ideas and concepts within a subject matter are organized and related. Similarly, the content knowledge held by a teacher included understanding what ideas were truths and/or falsehoods within a particular discipline based on particular warrants or propositions held within the theory and practice in a subject.

> *. . . The teacher need not only understand that something is so; the teacher must further understand why it is so, on what grounds its warrant can be asserted, and under what circumstances our belief in its justification can be weakened and even denied. Moreover, we expect the teacher to understand why a given topic is particularly central to a discipline whereas another may be somewhat peripheral." (Shulman, 1986, p. 9)*

Pedagogical content knowledge. The second category, Pedagogical Content Knowledge (PCK), encompassed a deeper understanding of the content knowledge needed for teaching. Included in PCK were understandings of representing ideas in multiple fashions depending upon the skills of the learner, knowing potential conceptions and misconceptions students bring with them, strategies for recognizing misconceptions and methods for dismantling falsehoods, and ways of adjusting content to allow the learner success when tackling foreign or difficult notions (Shulman, 1986). Shulman argued that it was within the concept of PCK where research on teaching and learning concur. It was also where much of the present research on teacher knowledge and instructional practice failed.

Curricular knowledge. The final category, curricular knowledge, is:

> *represented by the full range of programs designed for the teaching of particular subjects and topics at a given level, the variety of instructional materials available in relation to those programs, and the set of characteristics that serve as both the indications and contraindications for the use of particular curriculum or program materials in particular circumstances. (Shulman, 1986, p. 10)*

He explained that curricular knowledge extended beyond knowledge of multiple curricula within one's own discipline into knowledge of what students were learning within other disciplines simultaneously. This lateral knowledge, as he called it, enabled teachers to reference and pull from the ideas in other subjects that might help clarify or pertain to ideas being taught during a specific lesson. The integration of three categories of knowledge: content, PCK, and curricular are, for Shulman (1986), the essence of his theoretical framework that depicted teacher knowledge.

Liping Ma's (1999): Profound Understanding of Fundamental Mathematics

As Ma (1999) explains, there is a clear difference between the training of teachers in China and the U.S. While most U.S. teachers complete at least a bachelor's degree, Chinese teachers complete only between two and three years of formal training following the ninth grade. How, then, could it be possible that Chinese teachers have a better understanding of elementary mathematics? Ma hypothesizes that elementary teachers in the two countries possess differently structured bodies of mathematical knowledge, where pedagogical content knowledge (i.e. knowing how to represent the content in a comprehensible way) is central. The question of teachers' mathematics subject matter knowledge – what does a teacher need to know to be well equipped to teach mathematics – has been a focus of mathematics education researchers since the late 1980's.

For the U.S. sample, Ma interviewed twenty-three "above average" (school in-service training leaders, or near the completion of a Master's degree) U.S. elementary teachers. As well, Ma interviewed seventy-two Chinese teachers from five urban/rural elementary schools "ranging from very high to very low quality" educational status (Ma, 1999, p. xxiii). All these teachers were interviewed with four questions from the TELT study. The following topics were covered: (questions paraphrased)

1. Subtraction with regrouping – how would you teach and explain this topic to a grade two class?
2. Multi-digit number multiplication – how would you respond to student mistakes when dealing with questions from this topic?
3. Division by fractions – how would you represent ("real-world" situation, story, model, etc.) this concept?
4. The relationship between area and perimeter: This content area is important and is explored in more detail below. Interestingly, this question is intended to determine how teachers may "explore new knowledge." As Ma suggests, students suggest novel ideas in math classrooms all the time. The focus of this question was on how teachers would respond to seemingly novel student claims. The question is as follows: "Imagine that one of your students comes to class very excited. She tells you that she has figured out a theory that you never told the class. She explains that she has discovered that as the perimeter of a closed figure increases, the area also increases. She shows you this picture to prove what she is doing: [a 4cm by 4 cm square and a 4cm by 8cm rectangle, with respective perimeters and areas calculated correctly, and "in support" of her claim]. How would you respond to this student?"

The perimeter-area question. Of the U.S. teachers, 2 simply accepted the claim, 18 did not pursue a mathematical investigation, and 3 investigated the claim mathematically. Of this group, only one teacher achieved a correct solution (via counterexample). Approaches included consulting a book, calling for more examples, and approaching the question mathematically. Of the Chinese teachers, about 8% simply accepted the claim to be true (similar to U.S. sample). However, 92% explored the problem mathematically. Of this 92%, 22% reached an incorrect solution due to problematic strategies, but 70% of these teachers reached the correct solution. Based on the Chinese teachers' solutions, Ma presents four levels of understanding in relation to this problem:

1. Disproving the claim – finding a viable counterexample (14 teachers).
2. Identifying the possibilities – finding examples that display the possibilities of relationships between the area and perimeter of two closed figures (8 teachers).

3. Clarifying the conditions – determining the conditions under which the possible relationships between area and perimeter held true (26 teachers).
4. Explaining the conditions – elaborating on why the area and perimeter relate as they do under particular conditions, so as to support or refute the student's claim (6 teachers).

Exploring teacher success. In examining the interview results of the teachers, Ma explains that strategy and intention were key factors in the nature of teachers' exploration of the student's claim. Strategy appears to be an obvious ingredient, which includes knowledge of appropriate formulae, their underlying rationales, and modes of mathematical thinking such as the use of examples and counterexamples. In contrast to mathematical maturity, Ma expresses her concern regarding the layperson-like attitude that some of the teachers expressed in their investigations. For instance, the idea that a mathematical statement can be proved through the use of a single example was evident in the responses of several of the U.S. and Chinese teachers.

Intention, on the other hand, is dependent on a teacher's interest in exploring a mathematical idea and their self-confidence in tackling a new problem. The teachers in the study who thoroughly explored (or unpacked) the area-perimeter problem showed a genuine interest in the problem, which fuelled their desire to reach a plausible conclusion. Interestingly, Ma suggests that teachers' confidence is a function of teacher attitudes towards the possibility of solving a novel problem. While some of the U.S. teachers showed difficulty with the strategy component of the problem, Ma suggests that their intention was their pitfall – many of the U.S. teachers knew the appropriate formulae, but lacked the necessary interest and confidence to approach the new problem mathematically.

Thinking mathematically. Ma notes that the U.S. teachers did not necessarily have less to say than the Chinese teachers, but their answers were, generally speaking, "less mathematically relevant and mathematically organized" (Ma, 1999, p.104). Furthermore, she suggests that the Chinese teachers' proficiency in communicating mathematics may stem from their chosen style of teaching, which involves a significant component of lecture presentation. Thus, for each new lesson, Chinese teachers spend some time preparing a lecture-style introductory, yet complete, presentation on the topic. According to Ma, these lecture-style lessons require the teachers to practice and train their mathematical communication skills. Acquiring well-developed communication skills in mathematics is possible through other teaching styles. However, since the lecture element was a salient feature of the Chinese teachers' pedagogical repertoire (and not necessarily for the U.S. teachers), Ma suggests that this may be a significant factor for the differences in communication skills between the two groups.

Knowledge packages. Throughout her investigation, Ma used the term "knowledge package" when speaking about the subject matter knowledge of teachers. Ma explains that when a teacher begins to teach a new topic, that teacher has an idea in her mind about where this idea is situated in the field of mathematics. Thus, "given a topic, a teacher tends to see other topics related to its learning," and such topics comprise the knowledge package for the topic to be taught (Ma, 1999, p.118). In knowledge packages, there are "key pieces," which include certain related mathematical topics that are viewed as being more important to the comprehension of the topic at hand. Knowledge packages for any topic can contain both procedural and conceptual elements, and Ma asserts that the two are interrelated. Ma found that teachers with a conceptual understanding of a topic viewed related procedural topics as being essential to student understanding. “In fact,” Ma emphasizes, these teachers felt that “a conceptual understanding is never separate from the corresponding procedures where the understanding ‘lives’” (Ma, 1999, p.114). Ma believes that knowledge packages are important because it is from this information that a teacher attempts to construct a cohesive and comprehensive picture of a mathematical topic. With underdeveloped knowledge packages, it can be very difficult for a teacher to plan and facilitate a course of study for their students.

Learning progressions. Knowledge packages also contain implicit sequences of student learning, where a student is expected to know and understand key related pieces before they can grasp the topic at hand. Ma (1999) explains, “teachers believe that these sequences are the main paths through which knowledge and skill about the… topic develop” (p.114). While this may seem to be a linear progression of learning (you need to know x, y, and z, before you can learn topic A), Ma clarifies that topics in a knowledge package are interdependent, and that “linear sequences, however, do not develop alone, but are supported by other topics” (p.114). Thus, the learning progressions generated through teachers’ knowledge packages are similar to the hypothetical learning trajectories.

Profound Understanding of Fundamental Mathematics (PUFM). Ma builds her analysis of teacher training in mathematics around the idea of teachers’ acquiring a profound understanding of fundamental mathematics (PUFM). Early on, Ma (1999) explains that a teacher with PUFM “goes beyond being able to compute correctly and to give a rationale for computational algorithms” (p. xxiv). A grasp of both the procedural and conceptual elements of topics in elementary mathematics is necessary. A PUFM teacher is "not only aware of the conceptual structure of mathematics inherent in elementary mathematics, but is able to teach them to students" (p. xxiv). Thus, Ma situates subject matter knowledge (concepts, procedures & attitudes) and pedagogical content knowledge (how to teach math) as both being essential to a successful elementary teacher. Ma highlights the fact that PUFM is possible at the elementary level because elementary mathematics is a field rich with "depth, breadth, and thoroughness" (p.122). It is not a superficial discipline that is easily and commonly understood in its entirety by people.

PUFM characteristics. Ma explains that a classroom led by a PUFM teacher has the following characteristics:

Multiple perspectives. PUFM teachers will stress the idea that multiple solutions are possible, but also stress the advantages and disadvantages of using certain methods in certain situations. The aim is to give the students a flexible understanding of the content.

Basic Ideas. PUFM teachers stress basic ideas about mathematics and the conduct of mathematics. For example, these include the idea that single examples cannot be used as proof.

Longitudinal Coherence. PUFM teachers are fundamentally aware of the entire elementary curriculum (and not just the grades that they are teaching or have taught). These teachers know where their students are coming from and where they are headed in the mathematics curriculum. Thus, they will take opportunities to review what they feel are "key pieces" in knowledge packages, or lay appropriate foundation for something that will be learned in the future (see, for example, The Common Core Standards).

In this study we explore the central role that a profound understanding of mathematics plays in teacher planning for instruction, their observed instruction, classroom discourse, and their assessment practices.

Mathematical Knowledge for Teaching

One of the most recent frameworks for understanding teacher knowledge in the field of mathematics education comes from Deborah Ball and her colleagues. This framework, called Mathematical Knowledge for Teaching (MKT), developed from Ball's own teaching experience in an elementary school classroom (Ball, 1990; Ball, Hill, and Bass, 2005) and the work of Lee Shulman (1986). Ball expressed in her 1999 *Contemporary Mathematics* article that, while fascinated with the notion of Pedagogical Content Knowledge, she grappled with the idea of how teachers negotiated the interplay of content with pedagogy in the classroom. When teachers look at student work, choose a text to read, design a task, or moderate a discussion, they must attend, interpret, decide, and make moves. Their thinking depends on their capacity to call into play different kids of knowledge form different domains (Ball, 1999, p. 27).

Using video of Dr. Ball's teaching, investigators examined the practice of teaching to determine the actual work teachers engaged in during math instruction (Ball, 1999). From this job analysis, Ball developed a theory of professional knowledge (MKT) used by teachers when teaching. Her theory encompassed Shulman's (1986) idea of Pedagogical Content Knowledge adding a second category called, Subject Matter Knowledge. In combination, she labeled the new theory: Mathematical Knowledge for Teaching.

MKT: pedagogical content knowledge. The pedagogical content knowledge section of MKT aligns with Shulman's theory of PCK. Contained in this portion are three strands: knowledge of content and students (KCS), Knowledge of content and teaching (KCT), and knowledge of content and curriculum.

Knowledge of content and students. Knowledge of content and students refers to the idea that "teachers must anticipate what students are likely to think and what they will find confusing. Central to these tasks is knowledge of common student conceptions and misconceptions about particular mathematical content" (Ball, Thames, and Phelps, 2008, p. 401). For instance, teachers need to be able to anticipate how students will think through a task when the teacher is preparing the lesson for the day (NCTM, 2007). Teachers must also interpret student thinking by navigating the language used by students to explain their techniques so as to reiterate the explanation using correct mathematical language for the rest of the class. Critically important to this subcategory is the idea that teachers use their understanding of how students think about a topic as a mechanism for preparing lessons (Ball et al, 2008; NCTM, 2007).

Knowledge of content and teaching. The second component of PCK is knowledge of content and teaching. This dimension "requires an interaction between specific mathematical understanding and an understanding of pedagogical issues that affect student learning" (Ball et al, 2008, p. 401). Specifically, teachers evaluate how to structure examples so that they build on both the students' prior knowledge and on each other in terms of difficulty. It is a matter of understanding sequencing problems and evaluating how to represent mathematical ideas in multiple ways to help students achieve mastery of specific concepts. For example, when looking at place value through the use of money, one might ask “what does money afford instructionally for a particular subtraction problem and how is this different from what coffee stirrers bundled with rubber bands would afford?” (Ball et al, 2008, p. 402).

Knowledge of content and curriculum. The last component of the PCK strand is knowledge of content and curriculum. This type of knowledge refers to Shulman’s notion of curricular knowledge (1986). The following paragraphs describe the second half of Ball’s theory.

MKT: Subject matter knowledge. Under the category of Subject Matter Knowledge, Ball, Thames, and Phelps (2008) include three subsets of knowledge: common content knowledge (CCK), specialized content knowledge (SCK), and horizon content knowledge.

Common content knowledge. Common content knowledge refers to “the mathematical knowledge and skill used in settings other than teaching” (Ball et al, 2008, p. 399). For example, being able to correctly solve a math problem would be viewed as common content knowledge, as would the provision of explanations for common procedures for solving the problem. Common content knowledge is not unique to teachers. It is knowledge that is used across settings and people (Hill, Ball, & Schilling, 2008).

With regard to teaching, Ball, Thames, and Phelps (2008) explained that common content knowledge was expressed through recognizing the flaw in students’ thinking when the children present incorrect solutions or when the textbook provides incorrect definitions or insufficient examples of a specific topic. In addition, teachers demonstrate this type of knowledge when writing appropriate mathematical notations or providing the correct algorithm for different problems. Through the analysis of hours of video, Ball et al (2008) noted that critical instructional time was lost when teachers could not figure out a problem mathematically or quickly decipher the flaw in their students' thinking. Thus, this type of knowledge is vital to successful real time decision-making and consequent instruction.

Specialized content knowledge. The second subset under subject matter knowledge is specialized content knowledge (SCK). This domain refers to the specific knowledge and skills employed when teaching. More specifically, SCK includes "understanding different interpretations of the operations in ways that students need not explicitly distinguish; it requires appreciating the difference between 'take-away' and 'comparison' models of subtraction and between 'measurement' and 'partitive' models of division" (Ball, Thames, & Phelps, 2008, p. 400). Furthermore, teachers need to be able to unpack mathematical ideas, such as place value in order to help students carry out operations requiring regrouping, in a way that uses appropriate mathematical language while still being understood by young children (Ball and Cohen, 1999; NCTM, 2007). Ball et al (2008) elaborated this central idea stating, "Accountants have to calculate and reconcile numbers and engineers have to mathematically model properties of materials, but neither group needs to explain why, when you multiply by 10, you 'add a zero'" (p. 401).

Horizon content knowledge. The last subset within subject matter knowledge is horizon content knowledge. This particular subset is relatively newer to the theory of Mathematical Knowledge for Teaching (Hill and Ball, 2009). This category is described as "a kind of mathematical 'peripheral vision' needed in teaching, that is, a view of the larger mathematical landscape that teaching requires" (Hill & Ball, 2009, p. 70). Furthermore, the mathematical horizon encompasses the idea that teachers must understand how topics within mathematics relate so they can best tackle the curriculum being taught. For example, in Arizona, teachers in primary grades need to understand how they are building the foundation of number sense in children and how the development of deep understanding of place value will further a child's mathematical knowledge when they engage more complex issues of rational numbers in middle school. According to Ball, Thames, and Phelps (2008), this category is still underdeveloped. They are even unsure if the concept of horizon content knowledge fits within the larger domain of Subject Matter Knowledge, but they are hoping to explore this idea in the future.

As one can see investigating teacher knowledge in mathematics is complex. We constantly strive to better understand how teacher knowledge impacts student learning and achievement. Across the three main frameworks, the researchers agreed that common content knowledge served as the tip of the iceberg for unpacking teacher knowledge. Beyond knowing facts and procedures, teachers held knowledge of general pedagogy, math-specific pedagogy, student thinking, curriculum, and how the mathematics worked.

Each of the previously explained educational frameworks examined teacher knowledge as a complex notion seemingly independent of classroom practice and student achievement. All of the frameworks included knowledge of the curriculum, of student thinking, of content, of instructional practices, and of other disciplines; however, this knowledge was measured without taking into account the influence of what the teacher learns as a consequence of being one of many components in an educational environment. This dissertation expands on the described frameworks by embracing knowledge as situated (Fennema and Franke, 1992; Forman, 1996; Van Oers, 1996). To best understand the specific framework embraced in this dissertation (NCTM, 2007), we first need to illustrate what is meant by knowledge being situated.

Conceptual Framework

Sociocultural theory of learning or Cultural-Historical Activity Theory (CHAT) originated out of the work of Vygotsky, Luria, and Leont'ev during the 1920s in the former Soviet Union (Roth & Lee, 2007) and was further developed, decades later, by Michael Cole in "Anglo-Saxon academia" (Roth & Lee, 2007, p. 190). The main premise of this theory follows that learning is a social endeavor and occurs through social participation in meaningfully structured activities within a community (Cobb, Jaworski, & Presmeg, 1996; Forman, 1996; Van Oers, 1996). In terms of education, the goal is to engage students in authentic practices of the desired cultural community so that overtime the child can internalize the practices of the culture through specific learning activities and eventually create more efficient and productive ways to complete these activities.

Rogoff (2003) defined a cultural community as including: a) a structured means of communication, b) stability of involvement, c) participants taking on different roles within the society, d) a way of resolving conflicts, e) a common history (including generations of people participating in the community), and f) a way for adapting the community as times changed. Wenger defined a community as, "a way of talking about the social configurations in which our enterprises are defined as worth pursuing and our participation is recognizable as competence" (Wenger, 2009, p. 211). Both of these definitions explain a cultural community as more than just the local experience and members. Communities include a history, a set of established norms, roles for members, a language, and a method for constructing knowledge within the group that only group members can acquire. Newcomers to such a community must apprentice into it through the help of old timers, or those already legitimately accepted in the culture (Lave and Wenger, 1991).

In the mathematics classroom, the community occurs outside or inside the school system. It includes the students, the teacher, the past and present mathematicians who have contributed to the overarching mathematics knowledge base; along with the school norms, the norms of the mathematics, the norms of the classroom, the norms of the state standards, the norms of the textbook, etc. According to Vygotsky (1978), students become enculturated in the community through the Zone of Proximal Development (ZPD). This activity setting is co-constructed between the child (the newcomer) and the adult (the old timer). Based on what learning the child is motivated to engage, and what the child can do independently. The teacher (i.e., the adult) then structures activities to engage the newcomer (the student) in the language and social practices of the desired culture. Lave and Wenger (1991) explain further that learning occurs through the power struggles, the social structure, and the conditions for legitimate participation. As teachers, in their roles as instructors and classroom leaders, model social norms, apprentices gather information about acceptable practices within the culture in order to negotiate into full participants. The teachers, thus, challenge the apprentices to explore ways of thinking just beyond the apprentices' actual level.

More specifically, and most relevant to this dissertation, knowledge is situated within the negotiations that occur as newcomers engage with old timers as they apprentice into the community through Legitimate Peripheral Participation (LPP). The LPP defines the apprentices' movement in community from marginalized to full participation with the help of the expert members. It is "proposed as a descriptor of engagement in social practice that entails learning as an integral constituent" (Lave & Wenger, 1991, p. 35). The LPP portrays how the new member moves across the ZPD. For example, the apprentice observes and internalizes the practices of the group. The old timers show the apprentice appropriate ways of interacting in the society. This illustration encourages the apprentice to move beyond his or her way of thinking and into using more of the community behaviors. Theorists watch the negotiations of the newcomer with the experienced member and how these interactions change the new participant (socially and intellectually). As the novice interacts more, the community practices evolve, and the person (here the student) gradually becomes a full participant (Lave & Wenger, 1991). I will use this framing throughout the dissertation as it theoretically grounds the NCTM framework (2007).

The Mathematics Teaching Cycle (NCTM, 2007)

The Mathematics Teaching Cycle (Figure 1), or cycle of teaching activity, consists of three components: knowledge, analysis, and implementation (NCTM, 2007; Walkowiak, 2010).

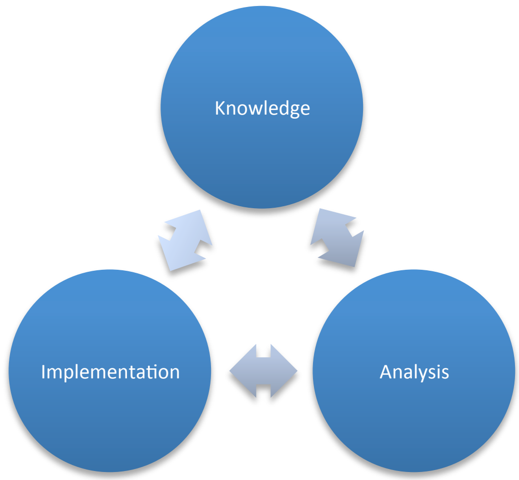

Figure 1. Mathematics teaching cycle (adapted from NCTM, 2007).

These interrelated dimensions of teaching represent the critical aspects of teaching that are central for creating a classroom environment that encourages the critical thinking and dialogue needed for children to develop a deep, conceptual understanding of mathematics (Hiebert et al, 1997; NCTM, 2000). Let us next examine each component of the Mathematics Teaching Cycle.

Knowledge. For teachers to successfully teach mathematics, they need a strong understanding of the content, methods for teaching the content, and a solid grasp of how their students think and approach different mathematical ideas (NCTM, 2007). Much like the previously described frameworks on teacher knowledge, the mathematics teaching cycle states, "teachers need a sound knowledge of the concepts, skills, and reasoning processes of mathematics to construct and achieve short- and long-term curricular goals" (NCTM, 2007, 19).

Knowledge of mathematics and general pedagogy. One key feature of the knowledge component of the Mathematics Teaching Cycle is that teachers hold a deep understanding of the mathematics, mathematical learning trajectories of students, and effective mathematics pedagogy. The deep understanding includes viewing school mathematics as interconnected and embedded in the natural world. Teachers must engage in problem solving, proving and justifying mathematical ideas, and find alternative ways of representing the mathematics. As described in both the MKT and PUFM frameworks, the Mathematics Teaching Cycle argues teachers need specialized content knowledge that spans the entire mathematics K-12 curriculum.

Knowledge of planning poignant and motivating lessons is also crucial for effective teaching. Teachers must think out multiple ways to represent the mathematical concept being taught. They need to anticipate the prior knowledge of their students and how to address misconceptions their students will likely bring to the lesson. Formative assessment affords teachers opportunities to learn about their students' knowledge and help teachers formulate future lessons (NCTM, 2007).

Knowledge of student mathematical learning. The second key feature of the knowledge strand is recognizing how students learn mathematics, knowing methods of supporting students as they grapple with complex concepts, and finding ways to build on students' prior knowledge. When listening to students explain their thinking, teachers learn common misconceptions held by students, how to best represent the mathematics to suit the needs of the students, and tailor experiences to individual students. This knowledge gives teachers a pathway for guiding students through explorations of new mathematics concepts and for questioning students to discover their misconceptions in ways that allow students to rethink their work. Further, teachers learn what tool will aid their students in discovering new ideas or comparing and connecting multiple representations of a specific topic.

Implementation. The second component of the Mathematics Teaching Cycle is implementation. According to the NCTM (2007), implementing one's mathematical knowledge in the classroom consists of three subcomponents: (1) selecting challenging and worthwhile tasks, (2) creating a learning environment that supports mathematical reasoning and opportunities for all students to demonstrate understanding and learning, and (3) mathematical discourse that encourages communication, reasoning, conjecturing, justifications, and evaluations in positive ways.

Learning Environment. When developing strong mathematics learners, teachers need to use their knowledge of the mathematics, the curriculum, and student thinking to create learning environments conducive to supporting discussion, critical thinking, problem solving, and skill mastery (Cohen, Raudenbush, & Ball, 2000; Hiebert et al, 1997; Lampert, 2001; NCTM, 2000 & 2007; NRC, 2000). To create such an environment, a teacher needs to foster collaboration, explanations, evaluation, and justification through thoughtful, guided questioning (Carpenter, Fennema, Peterson, Chiang, & Loef, 1989; Franke, Webb, Chan, Ing, Freund, & Battey, 2009; Lampert, 2001), appropriate task assignments (NCTM, 2007; Stein, Smith, Henningsen, & Silver, 2009), and student collaboration (Franke, Kazemi, & Battey, 2006; Hiebert et al, 1997; NCTM, 2007). To sum up, a teacher creates a positive learning environment when "students work independently or collaboratively to develop skills, make conjectures, and develop arguments within a mathematical community that values the contributions of all participants and defers to the authority of sound reasoning in the search for mathematical truth" (NCTM, 2007, p. 45). Two key aspects of creating such an environment are selecting worthwhile mathematical tasks and orchestrating appropriate mathematical discourse.

Task Selection. If we think about tasks as providing students with opportunities to learn specific types of mathematics, then task selection is very crucial to students' future success and to their learning of mathematics (Hiebert et al, 1997; Stein, Smith, Henningsen, & Silver, 2009). Hiebert et al (1997) emphasized the importance of giving students tasks that encourage reflection, analyzing procedures, building relationships across mathematical ideas, and comparing methods for solving problems. These types of tasks, as opposed to ones where the teacher gives direct instructions on the algorithm for completing a specific type of problem, better allow students to construct new understandings and connections between concepts. Therefore, when creating a learning environment conducive for students to engage in mathematical reasoning and success, teachers must depend upon their own knowledge of mathematics, students, and pedagogy when selecting appropriate tasks for students (NCTM, 2007).

What makes a task important or of greater learning value than another? One idea is that tasks should encourage students to use basic computational skills to solve larger mathematical problems or mathematical relationships. When rote skills are embedded within complex problems, students can more readily see how all of their mathematical skills relate and build upon each other. One example given in NCTM (2007) distinguished between two tasks regarding data analysis. The example illustrated the importance of task selection and how tasks can build upon each other while embedding skill practice. In the first task, students were asked to compute the mean, median, and mode of a data set. In the second task, students were asked to explain which method provided the best measure of central tendency given some data and a specific claim the students would like to make about the data. The first task has the students practicing basic computational skills. The second also has skill practice but it goes further to ask the students to evaluate and justify their decisions based on their arithmetic computations.

Similarly, another idea posits that teachers select tasks while keeping in mind the cognitive demand level of the task (Stein et al, 2009). In other words, not all tasks provide students with the same opportunities to learn, therefore teachers must use their understanding of student knowledge, the level of mathematics at which his/her students are working (Hiebert et al, 1997; NCTM, 2000 & 2007, Stein et al, 2009), the goal of the lesson, and the potential each task has for engaging students in a particular type of learning, when they select tasks for their students to solve. In the earlier example from NCTM (2007), the first task is considered a *Procedure without Connections Task* (Stein et al, 2009). The task is solved using an algorithm only, there is limited cognitive demand needed to solve the problem, and there is a focus on finding a correct answer rather than developing a mathematical understanding. The second task is a *Procedure with Connections Task* (Stein et al, 2009) asks the students to understand the procedure and engage metacognitively; it requires some more cognitive effort. Both of these tasks are important but they serve different purposes and tasks with higher cognitive demand generally require students to use complex thought processes in solving them.

Unfortunately, selecting a task is only half of the battle. Stein, Grover, & Henningsen (1996) found that when teachers purposefully selected high demand tasks, about 40% of the tasks selected remained at the high cognitive demand level when students worked on them. What appeared critical was how the teacher supports the students in reflecting and communicating when engaging with a task (Carpenter, Fennema, Franke, Levi, & Empson, 1999; Hiebert et al, 1997; NCTM, 2000 & 2007, Stein et al, 2009). Reflecting upon a task means students have to think and puzzle through new or complex ideas. Communication with other students, or the teacher, needs to be engaged to allow students to see mathematics in novel ways and to see mathematics as a language full of justifications and reasoning skills (Hiebert et al, 1997; Lampert, 2001).

Centrally, teachers need to know what their students are thinking, the misconceptions they bring to the lesson, the possible methods students might disclose for solving a task, and how to guide students' thinking and communicating through questioning (Franke, Webb, Chan, Ing, Freund, & Battey, 2009; Hiebert et al, 1997; Lampert, 2001; NCTM 2007; Webb, Franke, Ing, Chan, De, Freund, & Battey, 2008). With out taking these ideas into consideration during the lesson, in addition to the planning stage, a high cognitive demand task quickly morphs into a low cognitive demand task (Stein et al, 2009). Thus, task selection is more than just picking problems 1-30 on a textbook page and assigning them. One must also carefully examine the actual dialogue, or discourse, taking place around the completion of a task.

Discourse. One central tenant in building a learning environment dedicated to fostering intellectual growth and positive learning is structuring the classroom discourse so as to foster students' learning (NCTM, 2007). Students must feel comfortable to share their conjectures, their ideas about the mathematical concept being explored, argue about the validity of a claim, and feel confident in their mathematical reasoning (Franke, Kazemi, & Battey, 2006). Knowledgeable teachers structure lessons to incorporate time for students to discuss the mathematics and question what they are learning. They also facilitate discussion by selecting specific students to share their strategies with the class and by frequently expecting students to justify their strategies when solving problems.

Creating positive classroom discourse incorporates a teacher's content knowledge, knowledge of student thinking, and pedagogical knowledge. Thoughtful task selection and using probing questions require teachers to stretch their own understanding of the mathematics, reflect on ways of interpreting the mathematics, and actively listen to each student (Lampert, 2001). A teacher must also know when to ask questions and when to allow students time to explore the mathematical ideas independent of the teacher's input. The teacher must be sensitive to who is speaking, to how ideas are shared, and to modeling good mathematical reasoning. Through these discussions, students develop connections and meaning in the activities. The students learn what it means to do mathematics and how to communicate effectively in a mathematics environment. The teacher is central to fostering positive mathematics discourse.

Analysis. Central to preparing for teaching a lesson is the question "how well are the tasks, discourse, and learning environment working to foster the development of students' mathematical proficiency and understanding?" (NCTM, 2007, p. 54). Formative assessments made by teachers are crucial to creating productive learning environments for students. These assessments move beyond structured tests. They include feedback from students through discussions about solving problems, observations of how students understand a concept, discussions with parents and colleagues about the child's learning across disciplines and in mathematics, as well as portfolios, journals, and quizzes that ask students to demonstrate their insights. These formative assessments aid teachers in understanding how students retain subject matter and how they process the information being taught in school (NCTM, 2007).

In addition, to assessing the students' proficiency and understanding, teachers need to reflect upon their own teaching practices. This is an ongoing process. For teachers, it happens before, during and after the delivery of a lesson. Without self-monitoring and analysis, teachers cannot fully evaluate how well their students are learning. Moreover, they will be left unable to evaluate the quality of their chosen tasks as their students actually engage them.

The next chapter outlines the methodology used in this dissertation study to examine how MKT plays out in the Mathematics Teaching Cycle (NCTM, 2007).

CHAPTER III: METHODOLOGY

Research Design

My study conforms to Yin's (2009) multiple-case study design. A case study is defined as "an empirical study that investigates a contemporary phenomenon in depth and within its real-life context, especially when the boundaries between phenomenon and context are not clearly evident" (Yin, 2009, p. 18). This dissertation aligns with the case study design best because the study examined the phenomena of Mathematical Knowledge for Teaching (MKT) and how this phenomenon distributed across a teacher's classroom interactions with students and the curriculum she employed and her instructional planning for upcoming lessons. As seen in Chapter Two of this dissertation, it is unclear how MKT manifests in practice and, ultimately, influences student learning.

However, the research question for this dissertation required more than one case, therefore, a multiple-case study design fit best because this design employed specific cases based on either the cases' ability to "predict similar results" (p. 54) or "predict contrasting results" (p. 54). In this dissertation, three contrasting cases (i.e., pairs teachers) were selected. The specific method and reasoning for case selection can be found in the section labeled *Sample* in this chapter.

Evidence for case studies come from a variety of sources (Creswell, 2007; Yin 2009). This study collected data from the following sources: (a) semi-structured interviews with each teacher to investigate how lessons are planned, (b) semi-structured interviews with teachers, in which mini-tour questions (Spradley, 1979) were used to explore the teacher's thought process for completing the teacher knowledge assessment, and (c) classroom observations, which included a look at task selection and implementation, teacher questioning, and classroom discourse. A further description of each data source is located in the *Data Collection* section.

Analyzing case study evidence is the most difficult aspect to case study design (Yin, 2009). Creswell (2007) describes the analysis section as creating a detailed account of each case based on the data collected. In my situation, the detailed illustrations of the cases depicted a cross-case synthesis (Yin, 2009), in which each case was treated as an individual case study initially and then synthesized across the cases. The methods of analysis, and the framework for creating a cohesive story across the cases, is discussed in the *Data Analysis* section.

Situational Context

The specific situational context for my study was an urban K-8 school district in a large metropolitan city in the southwest region of the United States, where I participated in conducting research for a university grant funded by the National Science Foundation (NSF). According to the 2010 US Census, the population of the neighborhood in which the school district resides is 80.9% Hispanic or Latino, with the average age of people being 25.6 years. The median income of the people living in the district is $25,562, which is significantly lower than the median income in the United States of $56,604. Seventy-six percent of the population lives in family households. Twenty-four percent live in non-family, predominantly single, households. The district is evenly split between owner-occupied and renter-occupied homes.

According to the greatschools.org and the school district websites, the student population (of about 2,700 pupils) in the district is: 95% Latino students, 2% Caucasian students, 2% African-American, and less than 1% Native American. Ninety-one percent of the student population speaks Spanish at home, while nine percent speak English in their homes. In addition, 89% of the students receive free or reduced lunch, as the median yearly income of the families who attend the schools is about $11,000 (significantly lower than district residents overall).

Academically, the district is in Corrective Action and is closely monitored by the State Department of Education. During the 2008-2009 school year the district failed to meet Annual Yearly Progress (AYP) for the fourth year in a row. AYP is the measure used for ensuring that 100% of the students in each state Meet or Exceed the state's academic standards by 2014. Based on the No Child Left Behind Act (2000), in order for a district to meet AYP, "each group of students meets or exceeds the statewide annual objective except for: (1) the number below proficient reduced 10% from prior year, and (2) subgroup made progress on other indicators. In addition, for each group, 95% of students enrolled participate in the assessments on which AYP is based" (NCLB, 2000). When a school fails to meet AYP goals for two consecutive years, the school (or district, if the failure includes all of the schools) must create an Improvement Plan. The school receives two years to meet the goals of the Improvement plan and reach AYP. If neither goal is achieved by the end of the second year, the state takes over the school (or district) and tries to restructure the school in a manner that results in reaching AYP.

The district where my study takes place failed to meet AYP after two years of being on an Improvement Plan. During the 2009-2010 school year, the state retained control of the district. Throughout the school year, state representatives made surprise visits to classrooms in order to check teachers' lesson plans, ensure state standards were visible in the classroom, and that students' academic progress was also visibly charted in the classroom. In addition, principals and other administrative officials were redistributed across the schools in the district and a few teachers were placed into instructional coaching roles.

Also, as previously described, the majority of the students enrolled are English Language Learners (ELL). Ninety-one percent of the ELL-designated students speak Spanish at home and in the community, while they learn all academic material using English in schools (greatschools.org). What makes this situation particularly interesting and more unique is that 63% voters in Arizona passed Proposition 203 on November 7, 2000 mandating English-only instruction in the classroom (Arizona Department of Education website). According to the proposition, all children enrolled in public schools would be taught only in English. ELL students would be placed into Sheltered English Immersion (SEI) classrooms to receive special instruction in reading, writing, and conversational English over the course of one academic school year.

In 2006, The State of Arizona House of Representatives passed House Bill (HB) 2064, which expanded state laws regarding ELL students in public education. While HB 2064 included many administrative expectations for the Department of Education, the main ideas affecting public schools focused on developing a SEI classroom model. The model resulted in the following changes in the school district in this study: (1) Materials and instruction were in English, (2) A student was given an English Language Assessment test (AZELLA) upon entering school to determine his or her English language proficiency level, (3) students were grouped together according to the Arizona English Language Learner Assessment (AZELLA) results and were taught by a SEI-certified teachers. The expressed goal of HB 2064 was to have students reach academic proficiency in English at the end of one academic school year.

In the district where my study occurs both Proposition 203 and HB 2064 play important roles in structuring the daily calendar for academics. Based on their AZELLA test results, students are designated into a Pre-Emergent/Emergent SEI classroom, a Basic/Intermediate SEI classroom, or a Proficient/Mainstream classroom, on entry to school. In all classrooms, instruction is provided in English-only, further all materials and resources are aligned with the state's language and subject matter curriculum standards. What differs between classrooms is the amount of time spent on English Language Development (ELD). According to the Arizona Department of Education website, any non-mainstream SEI classroom must incorporate a 4-hour ELD block into the 6-hour school day. In our study schools, the Pre-Emergent/Emergent level, the 4-hour block is broken up as follows: 45 minutes of Conversation, 60 minutes of Grammar, 60 minutes of Reading, 60 minutes of Vocabulary, and 15 minutes of Pre-Writing. At the Basic level, the Grammar, reading, and vocabulary times remain the same. The change occurs with Conversation taking 30 minutes and instead of pre-writing, the students are writing for 30 minutes. The Intermediate level varies only slightly from the Basic level. Conversation time lasts for 15 minutes, while writing takes 45 minutes of the 4-hour block. The curricula for each of the components of the ELD program are provided by the State and school administrators and State officials closely monitor the implementation of these curricular materials. The last two hours of the school day are dedicated to lunch, specials (i.e., art, physical education, music, library, or computers), and mathematics.

At the end of each school year, or the beginning of the subsequent school year, students are reassessed using the AZELLA test. Since the goal of the ELD program is to move students into a proficient classroom after a year in an SEI classroom, students are expected to demonstrate mastery of the English language upon taking the AZELLA at the completion of the school year. Students who do not pass into a mainstream classroom are placed back into an SEI classroom the following school year.

For students in proficient/mainstream classrooms in the district, the subject matter can vary greatly between time spent on reading, language arts, mathematics, science, and social studies. However, the teachers are still required to post language and subject-matter objectives each day, as well as the students' weekly scores on each Performance Objective aligned to the state standards because the district continues to be in Corrective Action.

Sample

Purposeful and convenience sampling (Maxwell, 2005) supported my research design. Thirty-one teachers participating in the overarching NSF-grant provided a platform from which to select my dissertation teachers. The 31 teachers had all completed two years of monthly professional development and weekly classroom observations as participants in the NSF grant. I gathered data about the 31 teachers regarding: 1) the 2010 or 2011 scores on a test assessing components of Mathematical Knowledge for Teaching and 2) the four scores their students received on the Arizona Assessment Consortium (AzAC) test, over the 2011-2012 academic year. These two data sources will be explained next and then the process for using these scores to narrow down my participants follows.

Teacher Knowledge Test

In 2008, the Co-PIs and graduate students working on the NSF funded-grant created a twelve-question teacher knowledge test. This test was given to participating teachers at the end of each year that they received monthly professional development from the grant. Over the five years of the grant, the teacher cohorts (grades k-1; grades 2-3; grades 4-5) took the teacher assessment test twice: 1) end of the first year of participation and 2) end of the second year of participation, see Table 1.

Table 1

Administration of the Teacher Knowledge Assessment

Cohort	Years test taken				
(grade level)	2008	2009	2010	2011	2012 (follow-up)
K-1	Yes	Yes	--	--	--
2-3	--	Yes	Yes	--	--
4-5	--	--	Yes	Yes	--

Table 1 illustrates that each cohort participated in the NSF grant for two consecutive years. At the end of each of those years, the teacher took the Teacher Knowledge Test. The last year of the grant, the 2011-2012 school year, no professional development was provided and therefore no teacher knowledge test was given.

The test used 9 multiple-choice items from the Learning Mathematics for Teaching (LMT) project (Hill, Schilling, & Ball, 2004) and 4 open-ended items adopted from the Developing Mathematical Ideas (DMI) group (Higgins, Bell, Wilson, Oh, & McCoach, 2007; Bell, Wilson, Higgins, & McCoach, 2010). The multiple-choice items focused on numbers and operations. The open-ended questions asked teachers to formulate ideas of how students might solve a particular problem, to assess student strategies, and to evaluate misconceptions held by students around numbers and operations (Battey, Llamas-Flores, Burke, Guerra, Kang, & Kim, 2013).

Originally, the Learning Mathematics for Teaching (LMT) items were developed to measure the mathematical learning and field-tested with teachers who participated in a professional development program in California. The researchers wanted to see how much learning occurred during the professional development, in addition to assessing how "useful and usable knowledge of mathematics develops in teachers" (Hill & Ball, 2004, p. 333).

These items were "grounded in common tasks of mathematics instruction, were designed to elicit both teachers' common and specialized knowledge of content, and were drawn both from the research literature (e.g., Ball, 1993a, 1993b; Carpenter, Hiebert, & Moser, 1981; Lamon, 1999; Lampert, 2001; Ma, 1999) and from writers' experiences teaching and observing elementary classrooms" (Hill & Ball, 2004, p. 337). The authors sought to create items based upon elementary school mathematics concepts, such as number concepts, operations, patterns, functions, and algebra, as well as how students thought about these particular mathematical ideas. Subsequently, the Hill and Ball (2004) focused on writing items that also captured specific types of professional knowledge for mathematics called knowledge of content and students (Schilling & Hill, 2007).

As described in Chapter Two, Mathematical Knowledge for Teaching consists of three more components than what the LMT items measure. At this time, Schilling, Blunk, & Hill (2007) state that they have an understanding of Knowledge for Content and Teaching but no measures for this element as of yet. The last two areas, Horizon Content Knowledge and Knowledge of the Curriculum and Content, are under developed. In

2010, Heather Hill explained that MKT test encompasses only a portion of the full MKT framework,

> *. . . we elected to combine all items into one indicator, which we named mathematical knowledge for teaching (MKT). We chose to do this for several reasons. First, we did not have a sufficient number of items to return adequate person-level reliabilities for most subscales. Second, although we might have omitted the one KCS items or constructed a measure of only SCK items, this would have had the effect of decreasing the measure's accuracy and reducing the amount of information provided about various aspects of teachers' knowledge. . . . Third, the idea composition of an MKT measure is, in fact, unknown; until we have more information regarding which dimensions contribute with which weight to student outcomes, we can only guess what such a measure should look like. (p. 525)*

To account for the limitations with the LMT items, we added DMI items to the teacher knowledge test to help flush out the Knowledge of Content and Teaching and Knowledge of Content and Students components of a teacher's mathematical knowledge for teaching. The four DMI items enabled teachers to explain their thought process and have more freedom to divulge their understanding of elementary school students.

I used this test in the sample selection process in two ways. First, the assessment consisted of two validated and reliable measures of Teacher Knowledge: the LMT and DMI items. Using pre-made and valid test items meant that I did not have to construct my own items and that there was a standard by which to compare the teachers' scores. Second, the assessment allowed me to rank and compare the Mathematical Knowledge for Teaching (MKT) across all of the teachers in the NSF-funded study. Once I compared teachers of similar and different MKT scores, I used the student scores from the Arizona Assessment Consortium (AzAC) to construct comparable and contrasting cases for this dissertation, as discussed in the next section.

Arizona Assessment Consortium (AzAC)

The second measure used to select the sample were the student achievement scores from the Arizona Assessment Collaborative (AzAC). The AzAC test is a cumulative, multiple-choice test. The test was distributed four times across the school year – at the end of the 1st, 2nd, 3rd, and 4th quarters; however, only students who did not pass the AzAC in the 3rd quarter took the test in the 4th quarter.

The AzAC test highly correlated with Arizona's Instrument to Measure Standards (AIMS) test with ranges from .86 to .92, using time point three (third quarter distribution of test) for the estimated correlation. The correlation was permissible because the AzAC test aligned with the 2008 Arizona standards, it was a cumulative assessment, and a few questions were given on the AzAC and AIMS tests (the matching items were unknown to the author of this dissertation).

Upon initial selection of the teachers for this dissertation in 2010, a z score was constructed using the change in the students' scores from 1st quarter to 3rd quarter. The mean score of the student data was 0.386 with a standard deviation of 0.62147. Eighteen classes fell below the mean and 13 classes were above the mean. The data collection occurred over the 2011-2012 school year, however, and therefore, these class means and subsequent z-scores changed, although the process for finding the z-scores did not. The specific sample used, and selection criteria for the sample, will be discussed in the following section.

Selection of Sample

To create cases for this dissertation, I constructed a 2 x 2 table (Figure 2) with the help of Dr. Middleton in Fall 2010. This table created a means for comparisons to be made between teachers using their scores from the two measures described above. From this matrix, purposeful sampling (Creswell, 2007) could be used to select cases that contradicted the current research (i.e., two low MKT scoring teachers with high student gain scores) and that provided contrasts that would illuminate how different manifestations of MKT might impact instruction (i.e., a high MKT teacher who have low student gains versus a high MKT teacher with high student gains, or two average MKT teachers with contrasting student gain scores).

<table>
<tr><th></th><th colspan="3">MKT Levels (zscore)</th></tr>
<tr><td rowspan="3">Student Achievement (Q3 zscore)</td><td></td><td>Different</td><td>Same</td></tr>
<tr><td>Different</td><td></td><td></td></tr>
<tr><td>Same</td><td></td><td></td></tr>
</table>

Figure 2. Basic sampling chart.

The columns in Figure 2 refer to the Mathematical Knowledge for Teaching (MKT) Scores/Levels. The rows in Figure 2 refer to the change in student scores on the AZAC test between the first quarter and third quarter administrations. For this measure, the different and similar categories also align to teachers whose student assessment z-scores either clumped together or differed significantly around the mean (of all classrooms).

The data collection for this dissertation was anticipated to occur during the 2010-2011 school year; however, such was not the case. The data collection occurred during the 2011-2012 school year when the NSF grant. Figure 2 represents the original sample of participants used in this dissertation.

<table>
<tr><th></th><th colspan="3">MKT Levels (zscore)</th></tr>
<tr><td rowspan="3">Student Achievement (Q3 zscore)</td><td></td><td>Different</td><td>Same</td></tr>
<tr><td>Different</td><td>Case 2: -0.48/0.83
Case 2: 2.08/-0.64

A: 0.72/-1.20
B: -1.33/0.01</td><td>C: 0.38/-1.35
D: 0.38/1.21

E: -0.48/0.83
F: -0.48/-0.58</td></tr>
<tr><td>Same</td><td>Case 3: 1.23/-0.05
Case 3: -1.33/0.01</td><td>Case 1: -0.13/1.83
Case 1: 0.04/2.42

G: -0.65/-0.14
H: -0.65/-0.21</td></tr>
</table>

**The teachers willing to participate in the study are in bold and marked by "Case #." The other teachers, who were not participants either by choice or because they left the district, were assigned an arbitrary letter.

Figure 3. 2010 sample chart: adjusting for change in q1-q3 scores.

As stated, Figure 3. Depicts the initial sample of teachers for this dissertation. Multiple things happened, as a result of waiting one year to collect data impacting the selection process. The first issue was attrition. Teacher turnover was very high throughout the five years of the NSF grant. Between the 2010-2011 and 2011-2012 school years, many teachers left the district for other jobs, which narrowed the pool from which I could construct a sample. The reason for the high teacher turnover is unclear, other than a few teachers retiring. I lost one compelling case due to teachers leaving the district. The case would have been teachers C and D in Figure 3. However, when the test scores were run with the 2011-2012 data, I was able to replace the "lost" comparison with two other teachers who met my selection criteria.

Second, by waiting until the 2011-2012 school year finished, I was able to create cases that represented the most interesting findings of the six teachers willing to participate in my study using the test score data from the students being observed throughout the year. For example, although I lost a potential case due to attrition, when I used the 2011-2012 AzAC scores and the teachers' MKT scores, I found a pair of teachers who fit the same criteria as the original case I wanted.

Third, all of the teachers in this dissertation had completed the two years of professional development provided by the NSF grant by the fall of 2011. This fact is important because it means that the amount of years the teachers had of the NSF-funded grant's professional development was controlled for in this study. Had I selected teachers who had completed either one or two years of professional development in the NSF-Funded grant, it is possible that the differences in amount of professional development could have impacted their use of MKT during planning and instruction.

Figure 4 depicts the shift in scores that resulted in new cases illustrated in this dissertation to answer the research question. On a basic level, between 2010 and 2012, all six teachers fell into the category of Same/Different, Same/Same, or Different/Different. There is no longer a case where the teachers are Different/Same.

		MKT Levels (z-score)	
		Different	Same
Student Achievement Q1-Q3 Change Score (z-score)	Different	**T2.1**: 0.78 / -0.57 **T2.2**: -0.28/ 0.56	**T1.1**: 2.11 / 1.40 **T1.2**: 1.18/ -0.19
	Same		**T3.1**: -0.55 / 2.22 **T3.2**: -0.42 / 1.07

****Case & Teacher Number**: MKT z-score / Student Achievement Change scores**

Figure 4. Dissertation sampling chart.

The next sections will elaborate Figure 4 data further for a more detailed explanation of the matrix used. To start, the teachers were labeled with a "T#.#." The "T" indicated “teacher.” The first “#” indicated the case (i.e., case 1, case 2, or case 3). The “#” after the “.” distinguished between the two teachers in each case.

Case one was made up of two teachers with high MKT scores relative to the larger pool of teachers in the NSF grant. Teacher 1.1 was tied for the highest MKT score relative to the NSF grant participants at two standard deviations above the mean, while teacher 1.2 MKT score was one standard deviation above the mean relative to the NSF grant participants. One can see, though, that student achievement gain scores were significantly different for these two teachers. Specifically, Teacher 1.1’s student scores were the second highest at 1.399 standard deviations above the relative mean, while Teacher 1.2’s student scores were 0.198 standard deviations below the mean. To further explore the differences in average student achievement scores for these two teachers, let us examine the box and whisker plots that graphically display their student data.

At the first test time point (the end of Quarter 1 – early October, 2011), the students of Teacher 1.1 scored between 32.88 and 69.86 points on the cumulative AzAC assessment. The mean of the data was 51.20 points with a standard deviation of 9.37 points. Nine students tested as Falling Far Below the 4th grade standards, fourteen tested at Approaching the 4th grade standards, one tested at meeting the 4th grade standards and none of the students at the end of the 1st quarter had exceeded the 4th grade standards as tested by the AzAC assessment.

At the end of the third time point (the end of Quarter 3 – early March, 2012), the students of Teacher 1.1 scored between 56.16 and 95.89 points on the AzAC assessment. The mean of the data was 77.68 points with a standard deviation of 10.22 points. No students tested as Falling Far Below the 4th grade standards, three tested as Approaching the 4th grade standards, fifteen tested as Meeting the 4th grade standards and six tested as Exceeding the 4th grade standards as assessed by the AzAC assessment.

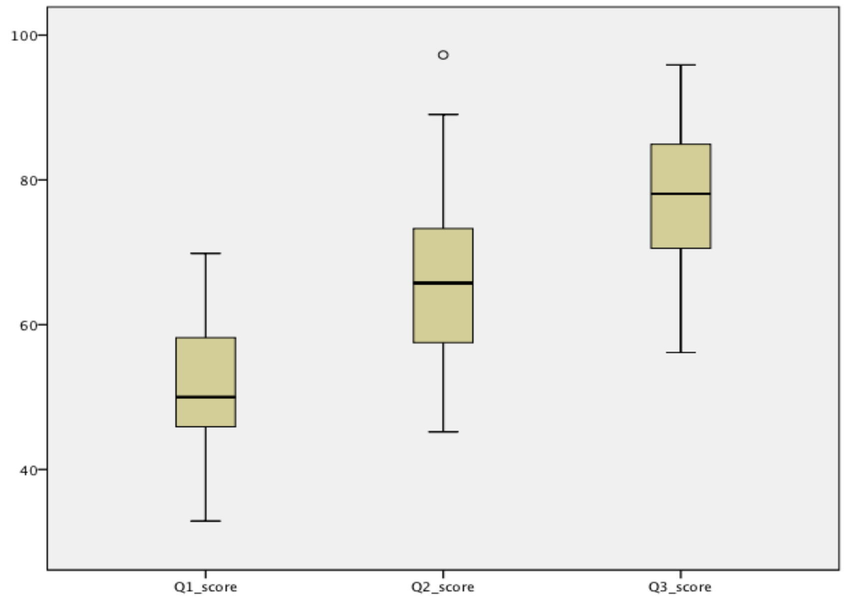

Figure 5. Teacher 1.1 student performance scores.

Teacher 1.2's student scores tell a different story. Figure 6 displays the student data in a box and whisker plot.

We see that at the first time point, Teacher 1.2's students scored between 27.94 and 83.82 points. The mean of the 1st quarter data was 53.415 points with a standard deviation of 16.398. Ten students tested as Falling Far Below the 4th grade standards, fourteen tested as Approaching the 5th grade standards, seven tested as Meeting the 4th grade standards, and none of the students tested at Exceeding the 4th grade standards as assessed by the AzAC test.

At the end of the third time point, the students of Teacher 1.2 scored between 26.47 and 95.59 points on the AzAC assessment. The mean of the data was 64.559 with a standard deviation of 19.34 points. Seven students tested as Falling Far Below the 5th

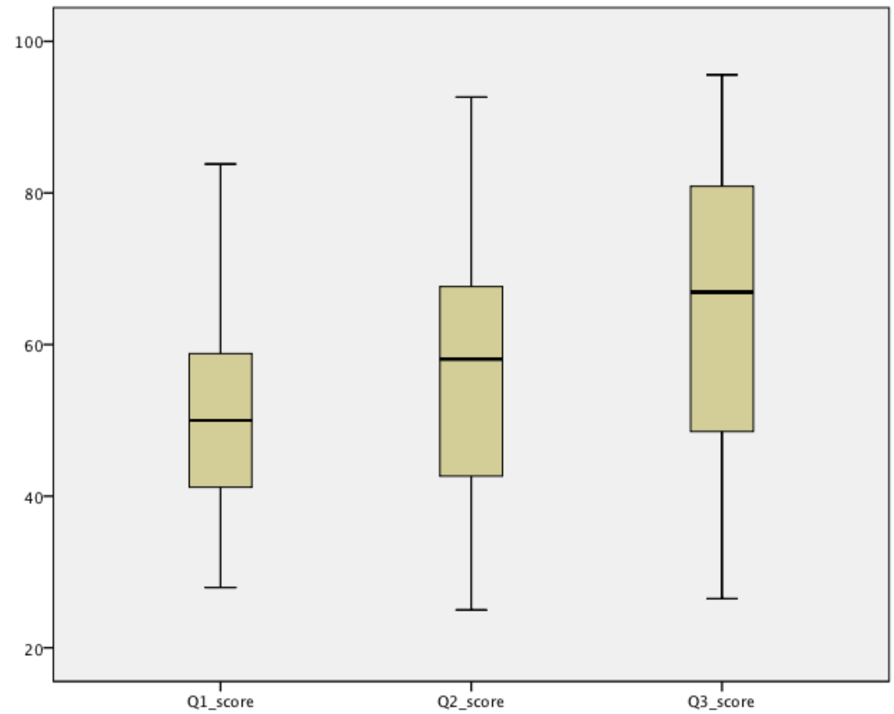

Figure 6. Teacher 1.2 student performance scores.

grade standards, five tested as Approaching the 5th grade standards, fifteen tested as Meeting the 5th grade standards, and 3 tested as Exceeding the 4th grade standards as assessed by the AzAC test. Based on these student gain differences between two high MKT teachers, the first case fell into the “same/different” quadrant of Figure 4.

In case two, the teachers had different MKT scores and different student achievement change scores. One teacher, Teacher 2.1, had a MKT score that was just above ¾ of a standard deviation above the mean relative to the NSF-grant participants. Teacher 2.2 fell just below the mean of the NSF grant participants when it came to the knowledge test score. As in the first case, Teachers 2.1 and 2.2 also had different student gain scores.

Teacher 2.1 student achievement z score was 0.57 standard deviations below the mean, while Teacher 2.2’s z score was 0.57 standard deviations above the mean. As you can see the scores were at opposite ends of the common z score distribution. To further explore student achievement scores for these two teachers, let us examine the box and whisker plots that graphically display their student data.

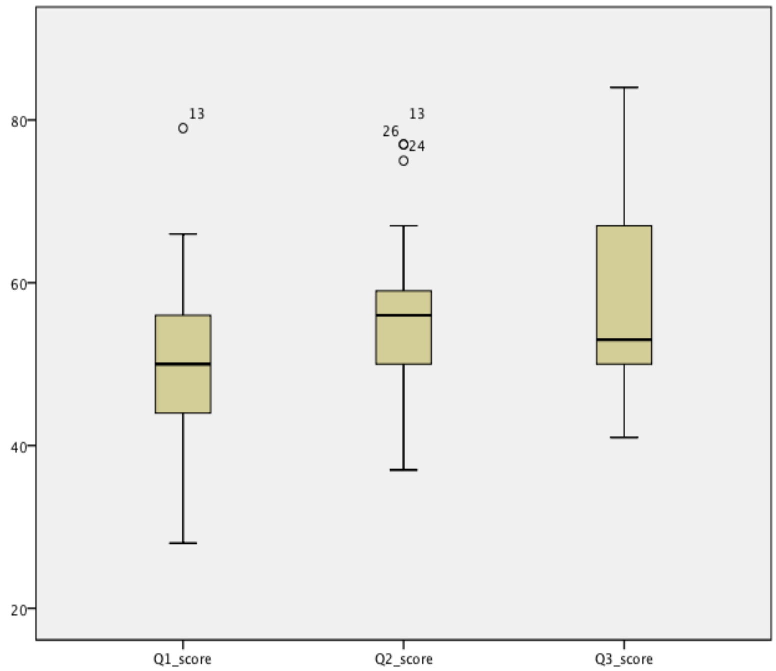

Figure 7. Teacher 2.1's student performance scores.

At the first test time point (the end of Quarter 1 – early October, 2011), the students of Teacher 2.1 scored between 27.94 and 79.45 points on the cumulative AzAC assessment. The mean of the data was 49.63 points with a standard deviation of 10.82 points. Fourteen students tested as Falling Far Below the 4th or 5th grade standards, Thirteen tested at Approaching the 4th or 5th grade standards, one tested at meeting the 4th or 5th grade standards and none of the students at the end of the 1st quarter had exceeded the 4th or 5th grade standards as tested by the AzAC assessment.

At the end of the third time point (the end of Quarter 3 – early March, 2012), the students of Teacher 2.1 scored between 41.10 and 83.56 points on the AzAC assessment. The mean of the data was 57.49 points with a standard deviation of 12.52 points. Six students tested as Falling Far Below the 4th or 5th grade standards, eleven tested as Approaching the 4th or 5th grade standards, eight tested as Meeting the 4th or 5th grade standards and zero tested as Exceeding the 4th or 5th grade standards, as assessed by the AzAC assessment.

Turning to Teacher 2.2, the following figure shows her students' performance scores for the 2011-2012 school year.

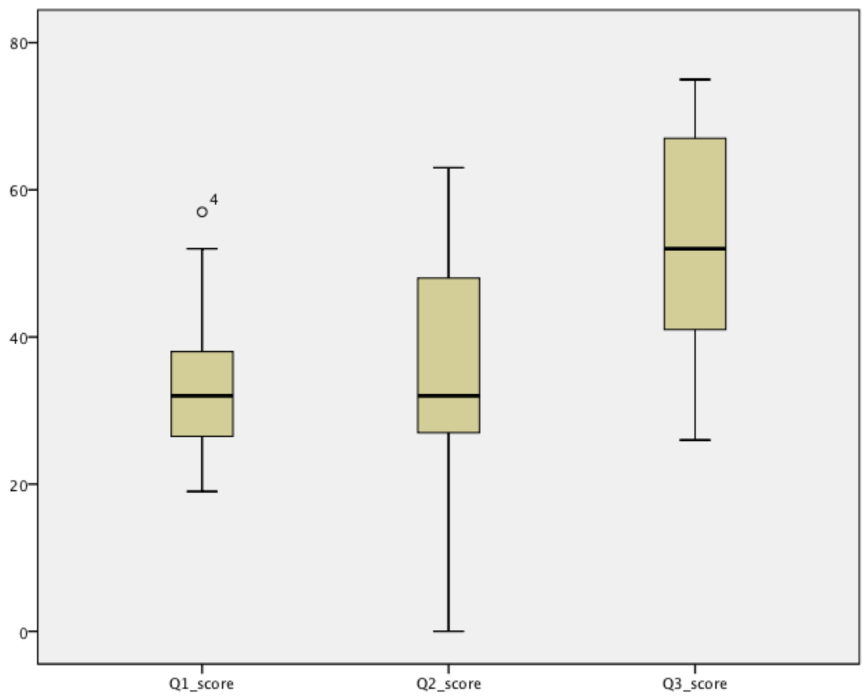

Figure 8. Teacher 2.2's student performance scores.

At the first time point, Teacher 2.2's students scored between 18.84 and 56.52 points. The mean of the 1st quarter data was 33.52 points with a standard deviation of 9.61. Nineteen students tested as Falling Far Below the 3rd or 4th grade standards, two tested as Approaching the 3rd or 4th grade standards, zero tested as Meeting the 3rd or 4th grade standards, and none of the students tested at Exceeding the 3rd or 4th grade standards, as assessed by the AzAC test.

At the end of the third time point, the students of Teacher 2.2 scored between 26.03 and 75.34 points on the AzAC assessment. The mean of the data was 52.02 with a standard deviation of 15.60 points. Eight students tested as Falling Far Below the 3^{rd} or 4^{th} grade standards, five tested as Approaching the 3^{rd} or 4^{th} grade standards, six tested as Meeting the 3^{rd} or 4^{th} grade standards, and zero tested as Exceeding the 3^{rd} or 4^{th} grade standards as assessed by the AzAC test. Based on the teacher knowledge test scores and the student gain scores, case two depicted two teachers in the "different/different" quadrant of Figure 4.

The third case in this dissertation housed the two teachers who had the same MKT score and the same student achievement change score. The two teachers fell below the mean of the NSF grant participants who took the teacher knowledge test in either May 2010 or May 2011, depending upon which year they completed two years of professional development. For these two teachers, their student change scores fell at the top of the range of scores for all 31 NSF grant participants. Teacher 3.1's students' change scores on the quarterly AzAC test were the highest of all of the participating teachers in the NSF-grant during the 2011-2012 school year. Her students' change scores were 2.22 standard deviations above the mean of her peers.

At the first test time point (the end of Quarter 1 – early October, 2011), the students of Teacher 3.1 scored between 33.33 and 69.57 points on the cumulative AzAC assessment. The mean of the data was 52.74 points with a standard deviation of 11.93 points. Five students tested as Falling Far Below the 3rd grade standards, thirteen tested at Approaching the 3rd grade standards, five tested at meeting the 3rd grade standards, and none of the students, at the end of the 1st quarter, had exceeded the 3rd grade standards as tested by the AzAC assessment.

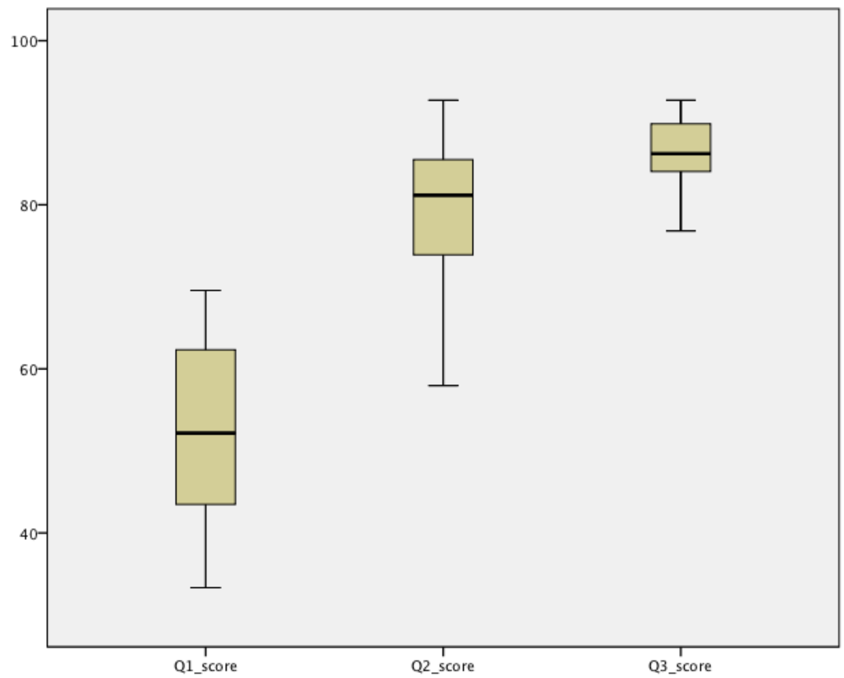

Figure 9. Teacher 3.1 student performance scores.

At the end of the third time point (the end of Quarter 3 – early March, 2012), the students of Teacher 3.1 scored between 76.81 and 92.75 points on the AzAC assessment. The mean of the data was 86.50 points with a standard deviation of 4.63 points. No students tested as Falling Far Below the 3rd grade standards, no students tested as Approaching the 3rd grade standards, thirteen tested as Meeting the 3rd grade standards and nine tested as Exceeding the 3rd grade standards as assessed by the AzAC assessment.

Teacher 3.2's students' change scores were the 4th highest at 1.07 standard deviations above the mean of the other participants in the NSF-grant. Teacher 3.2's box and whisker plot told a similar story. Figure 10 displays the student data in a box and whisker plot.

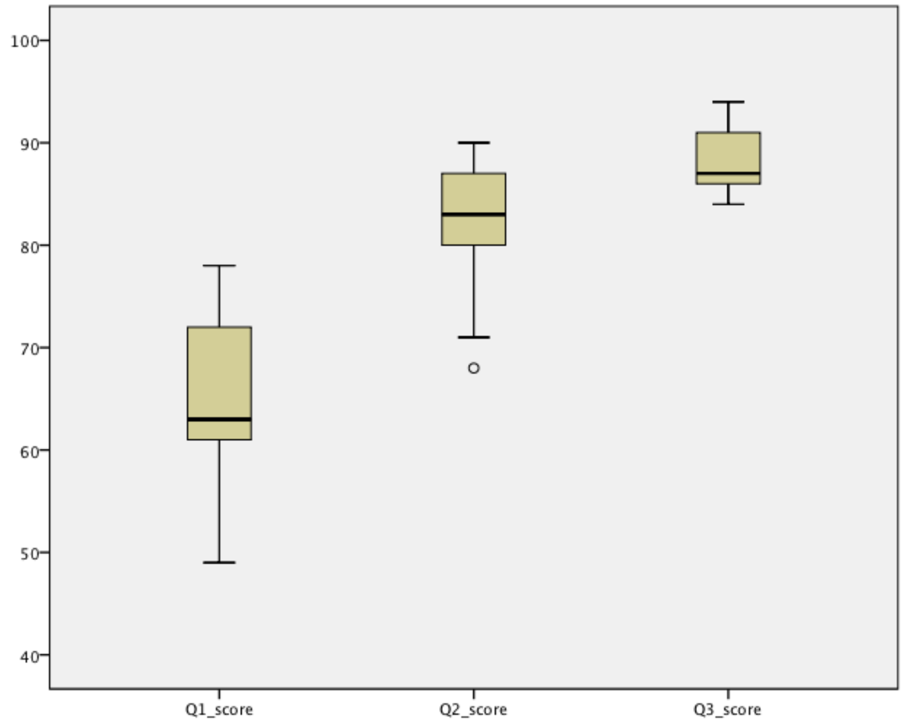

Figure 10. Teacher 3.2 student performance scores.

We see that at the first time point, Teacher 3.2's students scored between 27.54 and 78.26 points. The mean of the 1st quarter data was 63.77 points with a standard deviation of 11.22. One student tested as Falling Far Below the 2nd grade standards, thirteen tested as Approaching the 2nd grade standards, ten tested as Meeting the 2nd grade standards, and none of the students tested at Exceeding the 2nd grade standards as assessed by the AzAC test.

At the end of the third time point, the students of Teacher 3.2 scored between 55.07 and 94.20 points on the AzAC assessment. The mean of the data was 86.77 with a standard deviation of 7.61 points. No students tested as Falling Far Below the 2nd grade standards, one Approached the 2nd grade standards, twelve tested as Meeting the 2nd grade standards, and ten tested as Exceeding the 2nd grade standards as assessed by the AzAC test. Based on the MKT scores and the student gain scores Teachers 3.1 and 3.2 fit the “same/same” category presented in Figure 4.

Data was collected on the six teachers over the 2011-2012 school year. The types of data collected are described in the following section.

Data Collection

The research question posed in this dissertation is “how does a teacher’s Mathematical Knowledge for Teaching impact planning, instruction, and student gains in elementary mathematics?” To examine this question in depth data was collected from the following sources: Teacher interviews and classroom observations (including data from task analysis, the CLASS observation protocol, and classroom discussions).

Artifacts, such as student work, was not collected from the classroom was not collected for a few reasons. First, the unit of analysis was the teacher and not the student because the phenomenon being investigated was a teacher’s mathematical knowledge for teaching. Thus, the information collected corresponded to decisions made by the teacher regarding planning, implementation, task selection, and questioning. While I understand that student work should inform a teacher’s decision making process (i.e., as formative assessment or as part of reflective practice), in all but one classroom, formative assessment was not overtly taking place. Second, the student and the work constructed by students played a secondary role in the mathematical teaching cycle. Since my dissertation used that framework, I selected data sources that aligned with the components of the cycle. The lack of student work and classroom artifacts is a limitation in this study and hampered my ability to discuss student learning in detail beyond test scores and classroom discussions.

All interviews and observations (except for observations conducted in the room of Teacher 1.2) were audiotaped and transcribed. Field notes were taken during all interviews and observations as well. According to Erickson (1986) audiotaping helped to reduce bias in the research process. It allowed the researcher to revisit events that occurred at later times in the analysis process and confirm or disconfirm potential interpretations. Next, I explain each of the sources of data in more detail.

Interviews

"Conversation is an ancient form of obtaining knowledge" (Kvale, 1996, p. 8). In general, three teacher interviews were conducted with each of the six participating teachers. The first interview examined the general planning process through which the teacher follows when setting up a unit, a lesson, and the math wall. This interview employed "grand tour questions" (Spradley, 1979). Grand tour questions allowed for "a verbal description of significant features of the cultural scene" (Spradley, 1979, p. 87). These types of questions provided space for a teacher to generalize and freely talk about their planning process. The follow-up questions to the grand tour ones were dependent upon the descriptions provided by each teacher. The second interview investigated specific tasks teachers selected for their students to solve, the answer choices of these particular tasks, and the direction the teacher hoped to pursue based on how the students answered the specific tasks. The tasks discussed in this interview were specific to the particular teacher and the work being observed in the classroom. The line of questioning used in the second interview followed the structure of "mini-tour questions" (Spradley, 1979). Mini-tour questions examined specific aspects of planning, such as task selections and answer choice selections. These types of questions are identical to grand tour questions except they deal with a much smaller grain size (Spradley, 1979). The second interview also included some hypothetical-interaction questions (Spradley, 1979), where the teacher described questions they might ask a student about a particular task or anticipatory ideas of what students might do when solving problems. I did not use predetermined tasks for this interview. The final interview was used to gather more information about the teachers' MKT than what is known from the results of the one paper and pencil test. The teachers were asked to walk through their teacher knowledge assessment and explain their thinking, just as many of them do with their students in the classroom.

Teacher interview protocol. The first two interviews were semi-structured, meaning there were general questions asked to all of the teachers but the follow-up questions and overall flow of the interview was based on the teacher's responses. While using an open-ended interview leaves room for variation within the interview and subsequent answers, it allows for relevant and meaningful data to be collected pertaining to the individual knowledge of the teacher. The interview will be recorded and subsequently transcribed. It will be included in the observational component of the study.

The general questions for the Planning Interview were: a) Describe for me how you plan a lesson, b) What resources do you use when planning? And, c) How does testing impact your lesson planning?

For the Task selection interview, the general questions were: a) Describe for me how you select the tasks you give your students, b) How do you come up with answer choices for the tasks you write, c) How do you select the students who share their strategies, and d) how they differentiate tasks for students at different levels?

The second type of interview took place post-observation data collection. This interview was semi-structured. The teachers were asked to talk through their thought process for completing the teacher knowledge assessment. The teachers were asked to, "tell me how you solved this question?" and "walk me through your thinking and decision making process for answering this particular question." The interviews were audiotaped and transcribed post-interview.

Classroom Observations

Classroom observations occurred between October 2011 and May 2012 on a weekly basis. The observations lasted approximately one hour, or the length of the full mathematics lesson. The focus of the classroom observations was on the teacher-student, student-student, teacher-curriculum, and student-curriculum interactions. Also, field notes were taken on the line of questioning used by the teacher to elicit student responses, as well as the tasks completed during the instructional time. The purpose of collecting observations was to assess how a teacher's MKT impacts the classroom management style, task selection and implementation, and classroom discourse.

Procedure for classroom observations. Each observation was audiotaped on a hand-held recorder, unless the teacher stated they did not give permission for recording to occur. After each observation, the data was transferred to a laptop and saved in a data collection file using the pseudonyms for the teachers in the file name as well as the date of the lesson.

I took field notes during each observation. My role was strictly observational. I noted the types of tasks assigned by the teacher, the questions asked during the instruction, and general observations on the mathematics being taught and student thinking elicited by the teacher.

The audio-recordings were transcribed and the field notes were embedded into the transcriptions to create a vivid description of each observation. These final write-ups were saved in a data collection file using the pseudonyms for the teachers in the file name as well as the date of the lesson.

Data Analysis

Overall, the analysis of the data followed the guidelines of Miles and Huberman (1997). According to this analytic framework, the coding of chunks of data was used to "review a set of field notes, transcribe or synthesized, and to dissect them meaningfully, while keeping the relations between the parts intact…" (p. 56). The entire data set for a teacher within each case was broken down into three chunks: Knowledge, Planning, and Classroom Instruction. This separation of the data sources allowed for detailed analysis of each part to be conducted independently of the other sources of data. This is important because it enabled me to see clearly how different components of Mathematical Knowledge for Teaching (MKT) were expressed across different sections of the Mathematics Teaching Cycle and enabled comparisons between teachers in a case at both a macro- (the entire teaching cycle) and micro- (knowledge, planning, implementation) levels. Chunking the data into three sections also enabled triangulation to occur across the separate data sets.

Initial codes used to examine the three chunks of data per teacher were derived from the relevant literature relating to the topic of MKT. Specifically, I constructed a list of characteristics representative of the six components of Mathematical Knowledge for Teaching (See Appendices A & B) based on the details found in the Ball et al article (2008). After constructing the detailed category list, the MKT codes were used to analyze the teacher knowledge test and related interview, the planning interviews, the task selection episodes, as well as the entire transcript for each classroom observation per teacher. Specific methods for coding and analyzing each type of data source follow this section. One limitation of this process was that I did not have formal training in matching data to the specific category. This limitation might affect the results presented in this dissertation. However, using the thorough descriptions of the Mathematical Knowledge for Teaching (MKT) categories provided in Ball, Thames, and Phelps (2008) and the MKT chart I derived from the article, I felt confident that the codes I assigned data were true to those created by Ball et al (2008). Further studies would require the training of raters and the generation of high levels of inter-rater reliability.

The coded data were then used to construct meaning from the themes and patterns across the data sources. From here, specific vignettes were selected from the transcripts. The vignettes illustrated specific coding schemes or themes that appeared across the data for a particular teacher or as a comparison across teachers within and between cases. The process of generating themes and of coding the data was an iterative process (Miles and Huberman, 1997). The following sections describe in detail the analytic tools used, in conjunction with the MKT codes, to dissect the planning interviews and the classroom observation transcripts.

Interviews

Three interviews were conducted with each of the teachers in this dissertation. The first interview focused on the teacher knowledge assessment. The second two interviews examined the planning process for each teacher. The analysis of each type of interview follows.

I interviewed each teacher (except Teacher 2.1) about their thinking process when answering the questions from the teacher knowledge assessment. This interview was important because it allowed the participants in my study the opportunity to explain their thinking and explore their answers. One limitation regarding my interviews was that they were conducted a year, and in some cases two years, after the teacher took the Teacher Knowledge test. It is possible that teachers might have learned more about student thinking and the teaching of mathematics within the intervening time span or that they forgot what they were thinking when taking the test and were guessing based on what they knew at the time of the interview. Even so, this interview allowed teachers time to revisit the test and reassess their thinking.

To analyze the teacher knowledge interview, I read through the interview transcript and compared the responses to the answers given on the teacher knowledge test for each teacher. I noted instances when teachers said "I don't know" or "I am not sure what I was thinking" to see if there was a pattern as to when the teacher expressed a lack of knowledge (i.e., when talking about a specific mathematical concept such as fractions). I also noted instances when teachers changed their answers and were able to explain why their answer was originally incorrect. Finally, I noted whether or not teachers referenced their own students' thinking when talking through a hypothetical student answer. I selected these types of utterances to make note of because they often represented using Common Content Knowledge and Knowledge of Content and Students to think through a problem or a lack of one of the two MKT categories when working through a problem.

Next, I went back through the transcripts and coded the teacher's comments using the MKT code chart that I constructed based on Ball et al, (2008). Specifically, I coded comments paragraph by paragraph within each explanation related to the questions on the test. I selected the main category of MKT used in each utterance and then noted the specific component of the category to which the comment related (i.e., CCK – recognized when a student gave a wrong answer or KCS – Knowledge of students common conceptions and misconceptions about particular mathematical content). Once I coded all of the transcripts, I constructed a frequency table depicting the type of MKT used (correctly or incorrectly by the teacher) and the specific part of the category under which the comment fell. Using the frequency table, I was able to compare the two teachers in each case and select vignettes to illustrate the teacher's knowledge. In addition, I selected two tasks from the Teacher Knowledge Assessment to use as an illustration of how each teacher answered the tasks. It was important to use the same two tasks across the six teachers when comparing and contrasting the teachers' approaches to solving the tasks. One task was multiple-choice and the other task was open-ended. The two different types of tasks allowed for a comprehensive look at how the teachers answered comparatively.

The second type of interview conducted was the planning interview. To analyze these interviews, I transferred both of the transcripts into an excel file so that the documents were parsed paragraph by paragraph. I removed all of the questions I posed to the participants and put them in a new column. Then I chunked the response related to each question in groups in a second column. I removed any utterances such as “um” or “uh huh” or anything discussing non-teaching topics and put them in a separate excel filed labeled “irrelevant.” Next, I read through each paragraph and created a column listing to which question the paragraph corresponded. I reread the paragraphs and highlighted the main focus of each response. Using the highlighted sections, I generated summaries of each “chunk” (Miles & Huberman, 1997, p. 57) of data in a third column. As with the knowledge interview, I went through each original paragraph and assigned MKT codes that best captured the knowledge used throughout each paragraph. Some paragraphs represented one MKT code, while others used multiple MKT codes. For paragraphs with multiple codes, I noted the order in which the codes appeared to see if a pattern occurred over time. Any pieces that discussed planning issues but did not fit into a MKT code were placed to the side and labeled “Miscellaneous” and a brief description was written in place of a code. Once the coding was completed, I printed out and cut apart the excel file so that each code was its own strip. I then clumped together the pieces based on their MKT code. Once all of the pieces were grouped, I constructed a concept map for each teacher based on their interviews. The concept map illustrated how the teacher thought through the planning process. I then color-coded the components based on which MKT code corresponded to it. I also created a frequency table, similar to the one constructed for the knowledge interviews to see how often codes were used. Further, I used the table to help select vignettes to illustrate critical or unique aspects of a teacher’s planning. Lastly, I related the codes found in the planning interviews to those found in the knowledge test and interview. Later I linked them with the codes found in a teacher's classroom data. I did this as part of the triangulation of data and to look for patterns in the data.

Part of the Mathematics Teaching Cycle (NCTM, 2007) is the concept of "Analysis." A teacher should analyze what aspects of instruction help foster student learning and which aspects hamper student learning over time (NCTM, 2007). The interviews dedicated to understanding how teachers plan were critical in assessing how a teacher reflects and analyzes what occurs during instructional times. I asked each teacher to give a general overview of their planning process and then narrowed the focus down to specific task selections, answer choice selections, and how their students interacted with the various tasks. Through this line of questioning, teachers provided information about their use of formative assessment and their knowledge of the relationship among the content, the students, and teaching practices. This information was used to create a detailed picture of how MKT was used or not used by teachers when planning.

This process of analyzing the planning interviews and constructing a concept map was limited because the teachers were never given a chance to review the concept map or the coding scheme used to categorize their interviews. If this study were replicated, I would return to the teachers in the study and ask them to verify their statements and the outline of their thinking. In addition, as stated earlier, I never received training on the MKT coding scheme, other than reading the many articles published by the Learning Mathematics for Teaching project. The lack of training might have impacted my final results and my ability to examine the validity of the claim that LMT items were predictive of student achievement or teacher knowledge.

Classroom Observations

The classroom observations were analyzed using four different methods: Classroom Assessment Scoring System (CLASS) (Pianta, La Paro, & Hamre, 2008), The Mathematical Tasks Framework (Stein, Smith, Henningsen, & Silver, 2009), the MKT framework codes (Ball, Thames, & Phelps, 2008), and Bloom's Taxonomy (Bloom, 1956). Each method of analysis will be examined in the following sections.

Classroom Assessment Scoring System (CLASS). The CLASS observation protocol "is an observation instrument developed to assess classroom quality..." (Pianta et al, 2008, p.1). This protocol aligned nicely with the part of the Mathematics Teaching Cycle called "learning environment." As described in Chapter Two, the learning environment subcategory falls under the category of "Implementation" and includes eight criteria deemed important when creating a classroom that fosters mathematical learning described in the NCTM's Principles and Standards document (NCTM, 2000). These criteria included:

> *time necessary to explore sound mathematics and deal with significant ideas and problems; a physical space and appropriate materials that facilitate students' learning of mathematics, access and encouragement to use appropriate technology, a context that encourages the development of mathematical skill and proficiency; an atmosphere of respect and value for students' ideas and ways of thinking; an opportunity to work independently or collaboratively to make sense of mathematics; a climate for students to take intellectual risks in raising questions and formulating conjectures; and encouragement for the student to display a sense of mathematical competence by validating and supporting ideas with a mathematical argument (NCTM, 2007, pp. 39-40).*

The CLASS observation protocol scoring system examined all criteria present in the NCTM (2007) category "learning environment," as illustrated in the following section on the Domains of the scoring protocol. The foundation for the scoring system was based on the research stating, “that interactions between students and adults are the primary mechanism of student development in learning” (p. 1). There are three domains within the scoring system: Emotional Support, Classroom Organization, and Instructional Support. Each domain contains three to four dimensions. The dimensions “are based solely on interactions between teachers and students in classrooms; this system does not evaluate the presence of materials, the physical environment or safety, or the adoption of a specific curriculum” (p. 1).

The domains. Emotional Support examines a teacher’s ability to create a learning environment that supports social and emotional functioning. There are four dimensions under Emotional Support: Positive Climate, Negative Climate, Teacher Sensitivity, and Regard for Student Perspectives (p. 3). Under positive climate, researchers look for “warmth, respect, and enjoyment communicated by verbal and nonverbal interactions” (Pianta et al, 2008) in ways like smiling, enthusiasm, positive expectations, warm, calm voice, eye contact, cooperation and sharing, and peer assistance. A negative climate, on the other hand, includes negativity expressed in “frequency, quality, and intensity” (p. 28) of yelling, threats, peer aggression, teasing, sarcasm, humiliation, and bullying. Teacher Sensitivity contains “the teacher’s awareness of and responsively to students’ academic and emotional needs; high levels of sensitivity facilitate students’ ability to actively explore and learn because the teacher consistently provides comfort, reassurance, and encouragement” (p. 32). This dimension is manifested through anticipating problems, noticing students who are struggling with an idea, students show comfort in participating and taking risks, and providing individualized support to the students. The final dimension under Emotional Support is Regard for Student Perspectives. This dimension “captures the degree to which the teacher’s interactions with students and classroom activities place an emphasis on students’ interests, motivations, and points of view and encourage student responsibility and autonomy” (p. 38). Respect for student perspectives includes showing flexibility for students’ ideas, providing choices and students to lead the lessons, and allowing movement around the classroom.

Classroom Organization is the second domain. It includes Behavior Management, Productivity, and Instructional Learning Formats. Behavior Management “encompasses the teacher’s ability to provide clear behavioral expectations and use effective methods to prevent and redirect misbehavior” (Pianta et al, 2008, p. 44). A researcher looks for items like clear expectations that are enforced consistently, effective redirection of misbehavior, and little aggression or defiance. Under productivity, one looks for “how well the teacher manages instructional time and routines and provides activities for students so that they have the opportunity to be involved in learning activities” (p. 51). Examples of this would be clear instructions, brief transition times, having materials ready and the lesson prepared. Instructional Learning Formats “focuses on the ways in which the teacher maximizes students’ interest, engagement, and ability to learn from lessons and activities” (p. 57). Activities in this dimension are effective questioning, active student participation, using interesting and creative materials, focused attention, and using advanced organizers.

The final domain is Instructional Support. Concept Development, Quality of Feedback, and Language Modeling comprise this final domain. Concept development “measures the teacher’s use of instructional discussions and activities to promote students’ higher-order thinking skills and cognition and the teacher’s focus on understanding rather than on rote instruction" (Pianta et al, 2008, p. 63). This dimension includes integrating new knowledge with previous knowledge, relating topics to the students' lives and the real world, asking why and how questions, having the students create things, and using predictions when experimenting. Quality of Feedback "assess the degree to which the teacher provides feedback that expands learning and understanding and encourages continued participation" (p. 72) through activities like using following-up questions, asking kids to explain their thinking, expanding on students' ideas, and providing recognition for students' persistence. The last part of the domain is Language Modeling: language modeling "captures the quality and amount of the teacher's use of language-stimulation and language-facilitation techniques" (p. 79). It can be seen through peer conversations, revoicing student comments, elaborating on students' thinking, self-talk, using a variety of words, and questioning students in a way that requires more than one-word answers.

CLASS procedure. During my time as a graduate research assistant on the NSF-funded grant, I was trained on the CLASS observation protocol tool. The 16-hour training program "prepared participants to take the CLASS reliability Test and become certified CLASS observers" (CLASS Observation Training Certificate, received April 2, 2010). I successfully completed the training and passed the CLASS Reliability Test on April 2, 2010. The following is a description of the CLASS procedure and the variations I engaged in when using the protocol to assess the learning environment of each classroom during my dissertation.

Pianta et al (2008) designed the CLASS instrument to capture at least two hours of observation minimum, for a total of four cycles minimum, to determine overall scores for a teacher. A cycle is a 30-minute observational period that includes 20-minutes of observation and 10-minutes for scoring. A description of when the observations were conducted for each teacher is described within each case because variations were made on a teacher-by-teacher basis. For example, Teacher 1.2 refused audiotaping of her classroom instruction. Therefore, CLASS data was not collected for Teacher 1.2 because I felt that gathering detailed field notes of what was happening during each observation was more important for the study than constructing an overview of her learning environment. Teacher 3.1 had limited CLASS observations because she was on maternity leave for half of the time that I observed other teachers in my study. Rather than collect the CLASS observations over the course of 6 months, all of her CLASS observations were conducted over three months in the spring. This compressed time frame might have limited the reliability of her composite scores across the cycles because the scores were collected so close together. Other variations of data collection for the CLASS protocol were explained in the cases.

At the end of the observations, observers construct a composite score across the cycles. The composite score is in average for each dimension (the dimensions are the subcategories found within each of the three domains) across the number of cycles observed. The scores range from 1 to 7, with 1 being the lowest possible score and 7 being the highest score. Pianta et al (2008) described each score using both a rubric system (see Appendix C). A score of a 1 or 2 is in the low range. A score of 3, 4, or 5 is in the middle range. A score of 6 or 7 is in the high range. Once all of the composite scores for the dimensions are calculated, the average dimension scores are then used to construct a composite domain score. The composite scores for the three domains provide insight into the level and types of interactions occurring in a classroom.

For this dissertation, six observation cycles were completed for five of the six teachers. As stated earlier, one teacher opted not to be audiotaped. This meant that CLASS observations were impossible to conduct, as the researcher had to simultaneously take detailed field notes throughout the lesson. Composite and Dimension scores were calculated using the rubrics provided by Pianta et al (2008). For the most part, the composite scores for the teachers were used to provide a general comparison among the six teachers. The dimension scores were, instead, used to elaborate on how MKT impacted the learning environment of each classroom because these scores were more detailed and relevant to the eight bullet points in the Mathematics Teaching Cycle (NCTM, 2007). These scores were then matched to the MKT framework codes, if applicable. To match the MKT codes to the CLASS observation protocol, I examined the descriptions of each domain. From the descriptions, I coded the relevant aspects of MKT within the domains. From there, I looked at each teacher's score for each part of the domain and extrapolated how the various MKT categories were used or not used during interactions with students. This matching process is further explained in the cases themselves.

Using the CLASS protocol in this dissertation allowed me to examine how student to teacher interactions might be impacted by a teacher's mathematical knowledge for teaching. For example, do teachers with higher MKT scores have higher scores in Classroom Organization or Instructional Support than teachers with lower MKT scores? Are there specific aspects of MKT that appear more often than other categories? If so, is there a pattern to these appearances or are they specific to each teacher? These and other similar questions might speak to the opportunities to learn provided to students in each classroom. The next section examines the analysis of the tasks used in the classroom instruction.

The mathematical task framework (Stein et al, 2008). The Mathematical Tasks Framework (Stein et al, 2009) examines the level of cognitive demand tasks used in a mathematics lesson. Stein et al define cognitive demand as "…the kind and level of thinking required of students in order to successfully engage with and solve the task" (p. 1). There are four levels of cognitive demand (Stein & Smith, 1998): memorization, procedures without connections, procedures with connections, and doing mathematics. Because Stein et al (2009) clumped the four levels of cognitive demand together in their description of the levels I have done so in the following description of the four levels. Part of my limitation from not being formally trained on this framework is that all of my knowledge of the levels came from reading the articles and books published on the framework. Without a full description from the creators of the framework, I am limited on what I can disclose and explain in my dissertation.

Thus, the first two levels, memorization and procedures without connections, fall under the category of low-level cognitive demand. These tasks often require students to memorize a fact or perform a task using standard algorithms in the absence of context or meaning. Many times, students are required to complete 10 to 30 recall questions in one sitting or class period. The other two levels, procedures with connections and doing mathematics, are considered high-level cognitive demand tasks (Stein et al, 2009). These tasks might also use procedures but ask students "to think about the relationships [among ideas]...in a way that builds connections to underlying concepts and meaning" (Stein et al, 2009, p. 2). Students usually complete only a few of these types of problems at one time.

Cognitive demand is measured in two phases: the written task and the evolution of the task during the lesson. To decide the cognitive demand of a written task, the Task Analysis Guide found in Stein et al (2008) was used. The guide provided characteristics of the various levels of demand. One limitation of this dissertation was that the researcher was not trained in this framework, other than reading the book on the task analysis process.

The first phase in understanding the cognitive demand level of a task was examining the written task. One important factor in deciding the cognitive demand level of each written task is ensuring that the task is designed to fit the needs of the students. Tasks might look initially meet the high level of demand, however, if the task is above the students' capability of completing without teacher-direction (other than guidance) or if it just requires students to repeat a procedure, it would fall under low-level cognitive demand, as a written task.

At the implementation phase, or what is known as part of the second phase in assessing the cognitive demand level of a task, the level of cognitive demand might change. This second phase includes the teacher's set up of the task (i.e., explanation of the task, directions for completion) and the "enactment" of the task (Stein et al, 2009, p. 15). Many times, the demand level of the task is changed during the "set up" portion of the implementation phase. During this time, teachers often inadvertently provide step-by-step algorithms for solving a problem or complete most of the task for the students when presenting the task. During the "enactment" part of the implementation phase, high-level tasks might become low-level tasks "a teacher 'takes over' the thinking and reasoning and tells students how to do the problem" (Stein et al, 2009, p. 16), or if the focus of the tasks switches to the correct answer rather than the meaning or understanding of what the task requires. A task might also be at an inappropriately high level for the students, in which case students do not engage with the task and the teacher "takes over." Or, students might not be held accountable to engage in high-level thinking with the task (i.e., provide justifications for thinking or be thorough with their explanation). At the same time, a task maintains a high cognitive demand level if a teacher: scaffolds student thinking and reasoning, the teacher or students model high-level thinking, there is a "sustained press for justifications, explanations, and/or meaning through teacher questioning, comments, and/or feedback" (Stein et al, 2009, p. 16). In addition, prior knowledge is built upon by the task, or sufficient time is given for the full completion and exploration of the connections within the task. Too assess what happened during the set up and implementation phase; I used the guidelines found in Stein et al (2008) on page 16.

The importance of assessing the cognitive demand of tasks was twofold. First, it is critical to understand that the selection of tasks impacts the opportunities to learn mathematics in the classroom. Each task sets up a different learning objective, such as practicing a procedure or justifying one's thinking or making connections among mathematical ideas. These differences not only provide multiple learning consequences but also affect how students view the subject of mathematics and their identity as math doers (Stein et al, 2009). Second, it is important to investigate how the selection and implementation of tasks is impacted by a teacher's mathematical knowledge for teaching. For example, does task selection or implementation differ based on a teacher's knowledge level? If yes, then how? Or, what are the implications of these differences on student learning and performance?

Procedure for assessing cognitive demand. To analyze the tasks given to students, I first created "episodes." The episodes included the written task and the set up and implementation phases of the instruction. I cut and pasted the task, the dialogue from the transcripts, and important information from the field notes into a word document. The word document included all of the episodes for each observation per teacher. I reviewed each written task and assigned a cognitive demand level using the Task Analysis Guide (see Appendix C). After which I read each episode in its entirety. Using the Implementation Phase guidelines, (see Appendix D) from Stein et al (2009), I noted any changes in the cognitive demand of the task throughout the episode and notes that supported the label. This procedure was used to assess the tasks of each teacher in the study.

Once the cognitive demand was selected for each phase, I used the Mathematical Knowledge for Teaching codes (described earlier in this chapter) to assess the linkages between cognitive demand and MKT scores. I went through the transcripts for each episode and parsed out the specific representations of each MKT code. The compilation of codes for each episode were used to also compare the knowledge used in the task selections to the knowledge used in planning, CLASS protocol, and discourse for each teacher and within each case. This comparison method was completed for each teacher and each of the six observations. The final tool used to analyze the data was Bloom's Taxonomy (Krathwohl, 2002). Background information and the use of Bloom's Taxonomy are presented in the next section.

Bloom's taxonomy. This dissertation used the original taxonomy (Krathwohl, 2002) to analyze the participants' questions to students in the observations. The original taxonomy included sufficient details and relevant components to analyze the level of questioning presented by the teachers.

Bloom's Taxonomy was originally constructed to assist in the construction of a bank of test items (Krathwohl, 2002). The original taxonomy consisted of a hierarchy of six categories: knowledge, comprehension, application, analysis, synthesis, and evaluation. These six levels moved from concrete ideas to abstract concepts. Each level assumes mastery of the previous level's content. The levels were originally framed behaviorally. However, there application here is more cognitive in its orientation. The most basic level, Knowledge, is classified as having students recall or recognize a simple fact or procedure. The knowledge level is the most frequently used in classrooms. This is unfortunate because the remaining levels are "considered the most important goals of education" (p. 213). The second level, Comprehension, includes restating one's ideas, explaining meaning, or interpreting ideas from a scenario. Application implies being able to use a procedure in a problem. Analyzing information includes being able to take apart a method or theory and figuring out how the pieces relate to the overall theory. The fifth category is Synthesis. This includes the creation of something new. The last category in the original taxonomy is Evaluation. At this level, students are able to make judgments about a particular situation based on specific criteria. While the taxonomy is often used to assign items to categories in a priori manner, this is not how the taxonomy was employed in this dissertation.

Examining questioning techniques of the teachers enabled me to examine how teachers facilitated discourse or stifled discourse, and understood student thinking (NCTM, 2007). One of the main components of the Mathematics Teaching Cycle's discourse category was the notion that teachers needed to "provoke students' reasoning about mathematics" (p. 46) using thoughtful tasks and lines of questioning. Through the use of Bloom's Taxonomy, I assessed how teachers with different levels of MKT scores questioned students, used the information they obtained through the questions, and adjusted or did not adjust curriculum accordingly. In addition, the questioning techniques impacted the implementation of the written tasks, which in turn, influenced student learning.

Procedure for analyzing data using Bloom's Taxonomy. The questions asked by teachers were analyzed using Bloom's Taxonomy. Each of the six observations for each teacher was broken into episodes (see section on "The Mathematical Tasks Framework" for further details). These cognitive episodes were read and coded using Bloom's Taxonomy. After each episode was coded, I compared the types of questions asked with the MKT codes to see if any patterns emerged. I also used the taxonomy codes to assess how the written task was implemented regarding the cognitive demand level. This process was completed for each of the participants in the dissertation. Lastly, I used the constructed vignettes representative of teacher questioning and subsequent discourse for each classroom. These vignettes were used to illustrate the appearance of MKT during classroom discourse with the teacher's questions as the entry point for students in the discussion.

General Overview of the Cases

In the next three chapters the individual cases are presented. In Chapter Four, one finds a case of two teachers with high MKT scores, relative to their peers in the NSF-grant, who had different student gain scores. In Chapter Five, the case includes two teachers who had mid-range MKT scores, relative to their peers in the NSF-grant, who also had different student gain scores. In this case, the teacher with the higher MKT score of the two teachers in the case had lower student gain scores, while the teacher with the lower MKT score had larger student gain scores. In Chapter Six, the case includes two teachers with relatively low MKT scores. Their student gain scores are the highest and third highest scores among the participants in the NSF-grant.

The cases are outlined according to the Mathematics Teaching Cycle (NCTM, 2007). The first section of each case examined the Knowledge held by each teacher. The second section of each case illustrated the each teacher's Analysis of instruction and student learning. This was accomplished through an investigation into the teacher's planning process. The final section in each case looked at the Implementation of instruction. It included an examination of the learning environment, using the CLASS protocol, the task selection and implementation, using Stein et al's (2009) Mathematical Task Framework, and an assessment of teacher questioning and student responses using Bloom's Taxonomy (1956).

CHAPTER FOUR: CASE ONE

A Case of High MKT Scores and Different Student Gain Scores

This case examined two teachers who scored similarly on the Teacher Knowledge Assessment but whose average student gain scores differed greatly. To reiterate the information presented about the sampling process in Chapter Three, both teachers in this case scored at least one standard deviation above the mean of the measure of Teacher Knowledge of the participants in the larger NSF-study. Their average student gain scores were very different, however. Teacher 1.1's students' gains were 1.2 standard deviations above the classroom mean student gain for the other NSF-grant participants. Teacher 1.2's students' gains were -.2 standard deviations below this mean. This case presented the first layer for understanding how MKT might link to student gain scores through classroom instruction.

General Descriptions of Teacher 1.1 and Teacher 1.2

Teacher 1.1

Teacher 1.1 is a fourth year teacher. Her teacher training occurred at a large university in the Midwest. According to Teacher 1.1, the program focused on the development of deep conceptual understanding of mathematics in the pre-service teachers. She explained that in her pre-requisite math courses for elementary education teachers she was expected to provide 8-10 page explanations of how she solved whole number operation problems in bases other than base 10. She felt that explaining her thinking and experiencing learning outside of the "normal" base 10 number-system strengthened her understanding of whole number operations (Teacher 1.1, 1st interview). During her second interview, she explained that the research in the larger NSF-grant complimented her undergraduate teacher training very well and helped her to push her knowledge of mathematics and the teaching of mathematics further (Teacher 1.1, 2nd interview). Since graduating from college, Teacher 1.1 has been at her current school where she taught 4^{th} grade all but one year when she looped with her students to 5^{th} grade.

Over the 2011-2012 school year, Teacher 1.1 taught approximately 24 fourth grade students. The classroom was set up so that four to five students sat at a table. There were three tables in two columns facing the front white board. The math instructional time was separated into two time periods in the morning. When the students first arrived at school, Teacher 1.1 expected them to try to complete the Math Mastery Wall (explained later on in the description of her instruction) independently in their math journals. Then, during the designated math lesson time, the students either showed her their work on personal white boards or the students discussed their findings with their tablemates and the students presented their solution and strategies on butcher paper, as a group.

Teacher 1.2

Teacher 1.2 has been at the district for five year. She taught 5th grade for the entire duration. She is a graduate large university in the Southwest, where she went through a typical pre-service teaching program. Unfortunately, this was all of the data that I could gather from Teacher 1.2 regarding her educational and career history.

Teacher 1.2 taught approximately 26 fifth grade students over the 2011-2012 school year. The students sat in desks grouped into pods of four to five desks with a few students seated independently at desks scattered around the classroom. The math instructional time was divided up into three sections, according to the mandate put forth by the principal at the school. The one-hour allotted for mathematics each school day was spent in the following manner:

Table 2

Lesson Structure for Teacher 1.2

Lesson component	Allotted time	Purpose
Math wall	15 minutes	Spiral state standards throughout the year
Daily lesson	30 minutes	Objective provided by the district curriculum map
Problem solving (Otter Creek)	10 minutes	Adopted curriculum to help with building problem solving skills
Math facts (Otter Creek)	5 minutes	Adopted curriculum to help build math fact fluency

Teacher 1.2 followed the instructional break down shown in Table 2, as best she could. She explained that re-teaching time was built into the curriculum map, which made it easier to follow (Teacher 1.2, 1st interview).

Teacher Knowledge

One component of the Mathematics Teaching Cycle (NCTM, 2007) was Knowledge. According to the Teaching Principle (NCTM, 2000), an effective teacher needs knowledge in: “mathematical content, pedagogy, assessment strategies, and an understanding of students as learners” (NCTM, 2007, p. 19). These four criteria for “an effective teacher” aligned with Ball et al (2008) MKT components of Common Content Knowledge (CCK), Specialized Content Knowledge (SCK), Knowledge of Content and Teaching (KCT), and Knowledge of Content and Students (KCS). In the following section, I used the data representative of the entire data set gathered from the Teacher Knowledge Assessment and follow-up interview to illustrate the degree to which the components outlined in the Teaching Principle and the MKT framework presented in

Teacher 1.1 and Teacher 1.2 and how the presence or lack of some criteria might account for different student gain scores.

Example 1 From the Teacher Knowledge Assessment: A Multiple Choice Item

Because the LMT items are not released to the public, the following is a description of a task given to the teachers participating in the NSF-funded Grant. The task was a multi-digit subtraction problem that included regrouping of the minuend. The digit “0” was in the tens place of the minuend as well. The teachers were given three hypothetical students’ responses to examine. The task asked the teachers to state whether or not each response was acceptable evidence indicating that the child knew why the procedure worked. The teacher had the option to say that they were unsure if the answer was acceptable or not.

The first response (a) was a direct description of the steps taken to solve the problem. The response was void of place value or explanation regarding regrouping. The second response (b) indicated the child could decompose numbers and regroup fluidly across place value positions. The final response (c) was tricky for most of the teachers. The student indicated some understanding of place value at a superficial level.

Teacher 1.1’s response. According to her written test answers, Teacher 1.1 disagreed with answer choice (a), agreed with answer choice (b), and was unsure about answer choice (c). The follow-up interview provided insight into her thinking and the role MKT components played in her thinking.

In her follow-up interview, Teacher 1.1 started to work through the answer choices for this multi-digit subtraction problem systematically. She began at answer choice (a). Immediately she demonstrated aspects of Common Content Knowledge (Ball et al, 2008): “What does that even mean you can’t take 7 away from 6? Because technically you can take 7 away from 6. You owe a, like if it’s money, you owe a dollar” (Teacher 1.1, 3rd interview). In this except of Teacher 1.1 assessing answer choice (a), she expressed an understanding of the real numbers and operations. In addition, she evaluated the student’s understanding of the mathematics and drew a conclusion that “he doesn’t understand why the procedure works, I mean he knows the procedure but the reason why? I, but yeah that’s why I picked no, because..it doesn’t show evidence that they understand why it works” (3rd interview). Additionally, this excerpt illustrated Teacher 1.1’s ability to solve the problems “assigned” to the students and evaluate whether or not the students’ responses were correct (“correct” in this sense meaning “providing evidence that the student understands why the procedure works” as stated in the directions of the test item). These are other examples of Common Content Knowledge rooted in her evaluation of student thinking.

Rather than systematically move to the next answer choice, Teacher 1.1 provided a general description of the type of answer she would find acceptable:

> *I have three, this is 3 one-hundreds, I have zero tens, and I have six ones. I want seven ones but I only have 6 ones right now. I would accept that they said okay, so I am going to take 100 and I am going to break it up into, that’s 3 one-hundreds, now I have two one-hundreds left, if I take, like do you see what I am saying? If he broke it up and he said now I have ten tens but I need ones so I am going to break up those ten tens, I am going to take away one ten. I have nine tens left. Now I have ten ones so I have 16 ones (Teacher 1.1, 3rd Interview).*

I pointed out to her that her general description aligned with answer choice (b) at which point she stated “oh yeah! I didn’t even look at that.” This movement from looking specifically at a student’s answer choice and into her own understanding of the problem was typical of Teacher 1.1's responses. She was able to unpack the problems into rich mathematical descriptions of her own understanding of the mathematics (i.e., decomposition of numbers, the relationship between place value, regrouping, etc.).

Teacher 1.2's response. Teacher 1.2 gave a short and concise answer in her interview about this particular problem. She explained the following for the example above,

> *For the first answer choice, when you cross out the zero it doesn't become a 9, you must go to the hundreds. That is wrong. The second one works. You go to the hundreds column to get enough tens to borrow and have enough ones. The last one says can't borrow from tens, nothing there so it is wrong (Teacher 1.2, 3rd interview).*

Based on this response, deciphering what Teacher 1.2 understood about the mathematics and student thinking was difficult. It was clear that Teacher 1.2 focused on her finding responses "acceptable" or "unacceptable" based on whether or not the student demonstrated an ability to articulate the need for "borrowing" or demonstrated "borrowing." Her reasoning for selecting the use of borrowing as the benchmark for right or wrong was unclear. She did not provide any indication as to her understanding of decomposing numbers and the relationship between the place value of the particular number being described.

This brief response was representative of all of the responses Teacher 1.2 gave during her follow-up interview, even when probed to expand on her responses. There might be a few reasons for the lack of depth in Teacher 1.2's responses. First, she might have been nervous about sharing her thoughts. She was the one teacher in my study who did not want her classroom or interviews audio taped. I never received an explanation for why this was the case. Second, it is possible that her concise answers seemed sufficient to her. They were to the point and logical. Third, her depth of knowledge might have been lacking. It is possible that in her own mathematical development she was not pushed beyond procedural knowledge and therefore cannot move beyond the procedures. All of these ideas, and others, are plausible explanations for the short answers given by Teacher 1.2.

Example 2 From the Teacher Knowledge Assessment: An Open-Ended Item

The following problem was given to the teachers to assess their knowledge of relational thinking in students. The teachers were provided with half of a page to write or illustrate their responses. To receive total points for the problem, the participants had to answer all three sub-questions (Carpenter et al, 2003).

> *Assume we gave this problem to some elementary school students:*
> *What number can you put in the box to make this number sentence true?*
> *8 + 15 = ___ + 16*
> *What strategies would you expect students to use to solve this problem?*
> *What answers do you expect them to come up with?*

Teacher 1.1's response. Teacher 1.1 answered on the test that she would see her students give two answers, "23" and "7," when solving this type of problem. I focused on her response to students giving the answer of "23" in this section because it illustrated best her understanding of the mathematics and her knowledge of her students' thinking.

> *Okay, they would put 23. [I: how come?] because they are just going to add right here and it's like they don't even see this as like, the reason they would do that is because a lot of times they don't see this as like meaning something. Like they just see like it as, when they see the equal sign they think that they are giving an answer right there, they don't see it like as being like a, what am I trying to say? Like equal quantities or value (Teacher 1.1, 3rd interview).*

Teacher 1.1 focused on the notion that the students did not understand the meaning of the equal sign. She explained that they viewed the equal sign as "answer comes next" rather than "equal quantities or value" (Carpenter, Franke, & Levi, 2003).

This excerpt characterized Knowledge of Content and Students. Teacher 1.1 anticipated what the students would answer: "23." She anticipated what they would do with the task: view the equal sign as indicating an answer followed. In addition, she showed familiarity with common errors and an awareness of common confusion around the equal sign: "they don't see, always necessarily see that like 8 is equal to 8…they just didn't grasp the fact that what goes on either side of that [the equal sign], they represent the same thing" (3rd interview).

She added to her response with an explanation of a task she gave her students to help them develop an understanding of the equal sign: "Oh! We did something like this, 9 = 4 + 5 or something…and they'd be like yeah that's true, and, no because they are not used to, they want to see like operations on the left side and the answer on the other" (3rd interview). The task of 9 = 4 +5 confused children who have yet to develop a relational understanding of the equal sign because 1) it is an unfamiliar format, and 2) if a child has an understanding of the equal sign that "answer comes next," they only see the four and they know four is not equal to 9 (Carpenter, Franke, & Levi, 2003).

This second excerpt demonstrated Knowledge of Content and Teaching that resulted from Teacher 1.1's understanding of her students' misconceptions and her own knowledge of the equal sign. Teacher 1.1 provided her students with true and false sentences (Carpenter, Franke, & Levi, 2003) that challenged their understanding of the equal sign: "Interviewer [me]: so you wrote true/false sentences with them? Teacher 1.1:

yeah and we did all kinds of stuff" (3^{rd} interview), such as "9 = 4 +5." Her goal was to get her students to gain flexibility when working around an equal sign.

The response given by Teacher 1.1 to this second task again demonstrated the rich descriptions she provided of not only her own mathematical understanding but also of how she used her knowledge of student thinking to construct her mathematics lessons.

Teacher 1.2's response. On the Teacher Knowledge Assessment, Teacher 1.2 gave the answer of "7." She wrote on her test that her students would use algebra to solve for the unknown. Her follow-up interview response was a completely different answer. In the follow-up interview, Teacher 1.2 focused on the alternative answer choice her students might provide. She explained that "the kids would not acknowledge the first number, the number 8, and would put -1 in the box or just 15 in the box" (3rd Interview). In both of her responses, Teacher 1.2 assumed that her 5^{th} grade students knew about algebra, solving for an unknown, and integers. Her follow-up interview response also indicated that she believed her students would not know what to do with the two numbers on the left side of the equation and therefore would ignore the first number (the "8") and then would solve the right side of the equation where they would have to know that they are subtracting a positive one from sixteen in order to arrive at an answer of fifteen. Students would also have to have an understanding of the equal sign beyond "answer comes next."

Clearly, Teacher 1.2 demonstrated her ability to solve the problem in a variety of ways and potential alternative methods used by students when answering the question. This would indicate a presence of Common Content Knowledge and some Knowledge of Content and Students in her thought process. Teacher 1.2 demonstrated Common

Content Knowledge showing that she could solve 15 = ____ + 16 by putting a "-1" in the blank. She also showed me during her interview how she would solve the equality statement using algebra. She first "added the 8 and 15 to get 23 and then subtracted the 16 from both sides to get 7" (3rd interview). She demonstrated her ability to anticipate what students might do to solve the problem (Knowledge of Content and Students): use algebra or ignore the "8" and solve for the blank.

One puzzling piece in the response of Teacher 1.2 was the notion that she believed her 5th grade students had a complex, procedural understanding of algebra, variables, and the equal sign but they would arbitrarily leave out the first number in the equation. It is unclear whether or not she thought they would just not see the 8 in the problem or if they would just not think it necessary to take it into account when solving the equality statement (Carpenter, Franke, & Levi, 2003). Another puzzling idea was the amount of middle school and high school level mathematics Teacher 1.2 thought elementary school students knew. The test item explicitly stated the question was given to elementary school students and not middle or high school students. Based on her lack of response, it was difficult to understand if she held this belief because of her own personal experience as a math learner or from previous teaching experiences or for an entirely different reason. As with the multiple-choice item response, Teacher 1.2 demonstrated procedural knowledge of the mathematics needed to solve this problem and her answer was, again, short and concise.

Comparison of the Teachers' Responses Across the Test Items

Teacher 1.1 and Teacher 1.2 demonstrated Common Content Knowledge when discussing their responses to the teacher knowledge assessment items. First, they were able to complete the elementary school-level tasks, such as indicating the correct number in an equality statement and decomposing multiple-digit numbers to complete a subtraction problem that included regrouping. Both teachers demonstrated knowledge of comparing 5/9 and 3/7 using benchmark fractions of one-half. They were also able to demonstrate multiple methods for solving 61-36 other than the standard algorithm.

Where they differed in their knowledge was when it related to student thinking or non-standard methods for problem solving. For example, questions in the teacher knowledge assessment that asked the test taker to decide if a non-standard method of solving a problem was a method that would work for all whole numbers (i.e.: solving 35 x 25 as (5 groups of 25) + (30 groups of 25) instead of (5 groups of 35) + (20 groups of 25) or when using compensation as a means for solving 61-36 rather than the standard algorithm) stumped Teacher 1.2. When asked to think of her own non-standard methods for solving 61-36, she provided answers: "models – drawing base ten blocks and crossing out 36 of them. Drawing 61 pictures and crossing out 36 of them." (Teacher Knowledge Assessment Test for Teacher 1.2).

Teacher 1.1, on the other hand, received full credit on those same questions. The only question she did not receive full credit on that dealt with alternative problem solving methods was the question: *I've got 24 balloons that I am going to give out to my friends in bunches of 4. How many of my friends will get a bunch of balloons?* For this question, Teacher 1.1 received two points. During her interview, she explained that the problem as written was a measurement problem and therefore a drawing (or illustration) of a fair sharing type of method would be incorrect:

> *I think there are other ways that they would solve it; I think that this is the right way to do it as long as you are dealing with measurement. Like other kids might try to do fair sharing, some of them might be like just make four boxes and they are going to put 6 in each box, which is good if you just have 24 divided by four, that's fine. But in this problem, this is what I would want to see and expect to see based on, because I teach a lot of measurement division, so I think they would probably go into the four boxes and stuff . . . (Teacher 1.1., 3rd interview)*

It is clear in this excerpt that Teacher 1.1 not only understood the mathematics behind solving the division word problem but also that the wording of the problem was critical when deciding how she ultimately answered the question. It was also clear that the rubric used to grade the responses did not take into account the type of division problem being asked and how that might impact a teacher who understood differences between measurement and partitive division.

Looking at the responses from the teacher knowledge assessment alone, it was apparent that similar MKT scores were anything but similar when the teachers' answers were closely examined. Whether or not these differences in MKT plays out in the Analysis and Implementation components of the Mathematics Teaching Cycle (NCTM, 2007) are examined in the next two sections. It is possible that strong Common Content Knowledge but less use of Knowledge of Content and Students or Knowledge of Content and Teaching might hamper a teacher's ability to plan activities best suited for the strengths and weaknesses of the students, as well as impede a teacher's competence in adjusting instruction or task selection based on student thinking in the moment.

Analysis, or Planning, of Instruction

Teacher 1.1

In general, Teacher 1.1's planning was multi-faceted and complex. To best describe her complex planning system, the following sections will first walk through the general structure of how Teacher 1.1 planned, and then the main components of Mathematical Knowledge for Teaching that seem to be illustrated throughout her planning process are described.

Teacher 1.1 described a tri-layered structure to her planning. At the top level, she started with the state's Common Core Mathematics Standards Document for the 4th grade. After picking the concept to teach, she found all of the standards related to the concept. From there, she decided what performance objectives had been covered, which ones still needed to be covered, and whether or not students had mastered the objective. Once she has determined these three items, she developed a scope and sequence for the un-mastered standards and selects tasks to use with the students. For example, when planning a fractions unit, she explained that she ". . . looked at all of the standards that dealt with fractions and then I came up with a plan, like okay, this week I am going to target on, like you said, how many different ways can you make one whole? How many different ways can you make one half? And then, like we talked about, like adding fractions with like denominators . . ." (Teacher 1.1, 2nd Interview).

Once she established the scope and sequence, she examined the released practice items for the end of the year state standardized test and other practice booklets to determine "exactly what they [the students] are going to be tested on...I literally go through all of the books so I can see all different types of problems that they are going to need to know for planning and then I group them in order" (Teacher 1.1, 2nd Interview).

Using the revised Bloom's Taxonomy, she decided, "okay these are the types of questions [tasks] I am going to answer, ask on Monday, these are they types of questions I am going to ask on Tuesday and so on and so on. Monday, if they don't get through it, it's Monday, Tuesday, and Wednesday . . ." (Teacher 1.1, 2nd Interview). She gives an example of this sequence from a week where the focus was on ratios, ". . . say like ratios was my focus, you know? I'll start off like with like a basic ratio problem, like you know, what is this a ratio of and I'll show something. And, you know, by the end of the week they'll be dissecting word problems or they'll be like writing a word problem like using ratios, you know what I mean?" (Teacher 1.1, 1st Interview, 1/17/12).

Another example of something she did was "like when I was doing division, you know, I would start off like strand one would just be a division problem and as the week would go on by like Friday they were, and then they would have to identify, like by the middle of the week, identify if it was measurement or fair sharing problem, and then by Friday they were creating a measurement or fair sharing problems . . ." (Teacher 1.1, 1st interview, 1/17/12).

Once the sequence of the tasks has been selected for the week, she constructs the Math Mastery Wall. The Math Mastery Wall is a set of five open-ended questions that relate to the five strands in the Common Core Standards (one question per strand), with the understanding that the sixth strand, Mathematical Practices, is embedded within the other five. Teacher 1.1 sets up her Math Mastery Wall so that the students solve all five questions each day, but the focus during the lesson pertains to one specific pre-selected strand on the board. Usually, the selected strand and performance objective are the focus for the week, while the other four strands are usually reviews of previously learned topics or pre-assessments for future topics.

The last part of Teacher 1.1's planning process, the one that tied everything together, was the use of formative assessment within her planning. She indicated that formative assessment happened in two ways. First, at the end of a week of focusing on one particular task, she

> *gives the students an assessment to see how they do but then in a couple of weeks I'll bring it back again and I wanna see that by Monday, I'll give 'em that word problem and I want to see if they've got it, and if they got it then I won't spend time, I'll change it and on Tuesday I'll switch gears and I'll go to something that, something new or whatever. If they don't have it, then I already have problems written for the rest of the week that's gonna kinda back up and go through it again (Teacher 1.1, 1st interview, 1/17/12).*

Teacher 1.1 changed her instruction based on whether or not she saw the students mastered a particular concept. If they demonstrated mastery, she moved onto a different concept. She always wrote tasks for the next concept at the end of the week. If the concept was not mastered, she had alternative problems to help the students revisit the un-mastered concept.

Her second type of formative assessment happened at the beginning of teaching a new concept. While Teacher 1.1 had her week of tasks planned out ahead of time, she often put a problem on the board that mapped to where she wanted the kids to end up. She used this challenging problem as a pre-assessment and as an indication of whether or not her sequence of tasks aligned to a level just above what her students already knew about the standard or if changes need to be made to her sequence. "And sometimes I'll take the hardest one and I'll just put it up there just to see what they will do, you know? Just to see what they already know and like where they need to go . . ."(Teacher 1.1, 2nd Interview). Next, I examined how Mathematical Knowledge for Teaching manifests itself throughout the planning process of Teacher 1.1.

Across the two interviews conducted with Teacher 1.1, 46 excerpts pertained specifically to planning. Table 3 shows how the MKT codes were expressed throughout the planning interviews.

In addition to the codes found independently throughout the planning interviews with Teacher 1.1, she also expressed overlapping MKT codes. Eleven different excerpts expressed multiple MKT categories. Table 4 describes the complexity of the overlapping MKT codes found within her planning process.

As noted in the previous section, Teacher 1.1 had one of the highest MKT scores relative to the NSF-participants from which the sample in this dissertation was selected. It would not be unusual then to think that this high test score would mean her planning highlighted many facets of Mathematical Knowledge for Teaching. Such was the case with Teacher 1.1. During her interviews, a critical number of the ideas presented of how Teacher 1.1 planned lessons illustrated an extensive reliance on Pedagogical Content Knowledge, which included Knowledge of Content and Teaching (KCT), Knowledge of Content and Students (KCS), and Knowledge of the Content and Curriculum (KCC). It is noted that underlying all of these categories is a knowledge of the common content being presented to the students and without that common content knowledge, pedagogical content knowledge might not be as explicit or important (Bruner et al 2010). Next we will examine how MKT presented itself during the planning interviews with Teacher 1.2, who also earned one of the highest MKT scores, relative to the participants in the larger NSF-funded grant, in order to later compare the similarities and differences between the two high knowledge teachers in my sample.

Table 3

MKT Codes Related to Teacher 1.1's Planning

MKT category	Number of excerpts	Description of excerpts
Knowledge of Content and Teaching	17 total excerpts	• 8 about sequencing instruction • 2 about posing a new task • 1 about each: when to use a students' remark to make a mathematical point, choosing examples, evaluating instructional advantages and disadvantages of a particular method of representation, deciding which remarks to pursue and when to pose a new task based on those remarks, when to ask for clarification, and deciding which remarks to ignore and which to save.
Knowledge of Content and Students	9 total excerpts	• 3 about hearing and interpreting students' emerging and incomplete thinking • 3 about familiarity with common errors • 1 about each: knowing what kids will find confusing, and anticipating when students will find a task hard and predict examples students will find motivating
Knowledge of Content and Curriculum	4 total excerpts	4 about knowledge of available resources
Specialized Content Knowledge	3 total excerpts	• 2 about being able to unpack mathematical knowledge in ways that are not necessary in settings other than teaching • 1 about knowledge of how to make features of particular content visible to and learnable by students
Horizon Content Knowledge	1 total excerpt	1 about the vision useful in seeing connections to much later mathematical ideas
Common Content Knowledge	1 total excerpt (although this category was expressed throughout the interviews indirectly)	1 showed explicit understanding of the mathematics in the curriculum

Table 4

Multiple MKT Codes in the Planning Interviews

MKT categories	Number of excerpts	Description of excerpts
Knowledge of Content and Curriculum – Knowledge of Content and Teaching – Specialized Content Knowledge	2 total excerpts	In these excerpts, Teacher 1.1 expressed knowledge of available resources (KCC) that she used to sequence examples (KCT) based on her "decompressed mathematics knowledge" (SCK)
Knowledge of Content and Students – Knowledge of Content and Teaching	2 total excerpts	• One expressed an ability to anticipate what students are likely to think and what they will find confusing (KCS) and use that knowledge to evaluate the instructional advantages and disadvantages of representations (KCT) to determine which representations would be assist her student's learning. • The second excerpt demonstrated her ability to anticipate how students will solve a problem (KCS) and use that to sequence her instruction (KCT)
Knowledge of Content and Teaching – Knowledge of Content and Students	2 total excerpts	In these excerpts, Teacher 1.1 sequenced instruction (KCT) from most difficult to least difficult through anticipating what tasks students might find difficult (KCS). The point of this was to use a difficult question as a pre-assessment to the unit on equivalent fractions
Knowledge of Content and Teaching – Knowledge Content and Students – Knowledge of Content and Teaching – Knowledge of Content and Curriculum	1 excerpt	This excerpt showed how Teacher 1.1 sequenced instruction (KCT) based on her ability to unpack mathematical knowledge embedded in a particular concept like fraction development (SCK). Once she sequenced the instruction, she was able to choose and sequence examples (KCT) based on her extensive knowledge of available resources (KCC).
Knowledge of Content and Teaching – Knowledge of Content and Students – Knowledge of Content and Teaching	1 excerpt	In this excerpt, Teacher 1.1 knew when to pose a new task (KCT), anticipate what the students would find difficult in the task (KCS), and then evaluate instructional advantages and disadvantages of specific representations (KCT) to determine which representation would assist her teaching best.
Specialized Content Knowledge – Knowledge of Content and Teaching	1 excerpt	In this example, Teacher 1.1 was able to decide between problems (SCK) that represented the particular mathematical concept and use that knowledge to choose examples to use in her teaching (KCT)
Knowledge of Content and Teaching – Knowledge of Content and Students – Specialized Content Knowledge	1 excerpt	This excerpt demonstrated Teacher 1.1's ability to determine when to pose and new task (KCT) and her ability to hear and interpret students' emerging and incomplete thinking (KCS). During her explanation of their thinking, she was able to break down what the student did mathematically (SCK).
Horizon Content Knowledge – Specialized Content Knowledge	1 excerpt	This excerpt demonstrated Teacher 1.1's knowledge of connections to much later mathematical ideas (HCK) and her ability to unpack the mathematics in ways unnecessary to professions other than teaching

Teacher 1.2

In general, the planning process described by Teacher 1.2 focused on achieving the district and school administrative directives. For example, on a broad level, Teacher 1.2 planned lessons and structured her time according the district curriculum map given to the teachers at the start of each school year. On a more limited level, meaning the level of classroom instruction, Teacher 1.2 planned lessons based on the specific time allotments and prescribed programs given to the teachers by her school administration. The following sections will first walk through the general structure of how Teacher 1.1 plans and then we will describe the main components of Mathematical Knowledge for Teaching that seem to be illustrated throughout her planning process.

There are two levels to how Teacher 1.2 planned her lessons. At the top level is the District Curriculum Map. This map was designed by employees of the district and is based on what students need to know by the end of each quarter before taking the district quarterly assessment. The idea was that by the time the students take the state standardized test at the end of the year, all of the state's standards will have been mastered by the students at each grade level, if a teacher follows the district curriculum map. Teacher 1.2 uses this map as her guide for what to teach each day.

Since Teacher 1.2 refused audio-recordings during either the interviews or instructional time, all of her statements presented here are imperfect quotes. The researcher did her best to ensure quotes were written down exactly and confirmed by the participant. This is a limitation to the study and might influence the interpretation of the data. With that said, Teacher 1.2 stated that the district plans for them. They were given the topics for each day but there was time built into the schedule to reteach concepts that students did not learn the first time through (Teacher 1.2, 1st interview, 1/26/12).

To assess learning, Teacher 1.2 used the weekly district assessments as her guide. Employees of the district created these assessments. Each assessment contained about 5 questions related to the mathematical concepts taught during the week. The assessment questions were adapted from a computer-based program called Study Island. Upon analyzing her 3rd quarter weekly tests, Teacher 1.2 assessed that 64% of her students passed the 3rd quarter content overall. She used this knowledge to decide who was a low student and who did not know the material. These students were placed into a small group for reteaching (Teacher 1.2, 2nd interview, 2/27/12). She admitted, however, that she believed that the Study Island questions were tougher than the state standardized test questions at the 5th grade level and this made her wary of relying on the weekly tests for assessing students. For example, she said the weekly tests asked the students to multiply 3-digit by 2-digit numbers when the standard for multi-digit multiplication in 5th grade was 2-digit by 2-digit. She continued stating that it was good to have the weekly tests be harder than the quarterly test or state standardized test because then those two important tests were easier for the students (Teacher 1.2, 2nd interview, 2/27/12). She further explained that the writing and reading weekly tests were at grade level and her 83% and 74% of her students passed those on average in the 3rd quarter respectively. In addition, she explained that it was the same 10 kids that failed and the same 10 that passed each week. This knowledge helped her to structure her Math Wall and decide who needed to be in a small reteach group each week.

The second level of planning explained by Teacher 1.2 was about her daily lesson plan. Again, she explained that like the unit planning, the daily lessons were structured by the administration at her school. The teachers were allotted 15 minutes for the Math Wall (this time was the only flexible time in the hour), 10 minutes to discuss the steps of problem solving and 5 minutes for math facts (both of which were out of the Otter Creek curricula), and 30 minutes for the standard of the day (as determined by the District Curriculum Map).

The Math Wall questions were based on the AzAC test results and the weekly tests. She used the data to determine what concepts the students struggled with most and then she created questions for each strand on the Math Wall. The questions mainly focused on basic facts because that is what her students struggled with most often. She explained that each day she was supposed to review all five strands and then change the questions for the next day but she chooses to stay with similar topics each week. By staying with similar topics, she explained that she mainly just changed the numbers in the questions. She believed that changing weekly instead of daily allowed for connections to grow and stay (Teacher 1.2, 1st interview, 1/26/12). At the end of the week, she assessed learning with the weekly test. The students who failed to pass the test were placed into a small group where the students could focus better on the procedures they were taught during the week (Teacher 1.2, 1st interview, 1/26/12). Her thinking was the students did not need a new method for solving the problem just someone on top of them to keep them focused and "make them believe they can do it by letting them see their progress" (Teacher 1.2, 1st interview, 1/26/12). Now that we've investigated the general outline of how Teacher 1.2 plans, we will examine how her Mathematical Knowledge for Teaching manifests itself throughout her planning.

Across the two interviews, 9 excerpts pertained specifically to planning. In eight other excerpts, Teacher 1.2 demonstrated her common content knowledge needed to solve various algebraic problems given to her students on their weekly district assessments. While trying to explain what her students got wrong on the tests, she solved each of the problems correctly. We will look at the 9 excerpts according to Ball et al, 2008.

Teacher 1.2 had one of the highest MKT scores, relative to her peers participating in the NSF-funded grant. Like with Teacher 1.1, it would not be unusual for a person to think the high MKT score would indicate complex planning based on the high level of knowledge about the mathematics and about the teaching of mathematics shown on the knowledge test. In this case, however, the two planning interviews illustrated that planning was something for which Teacher 1.2 found already provided for her by the school level and district level administration. Her role, in planning, then, was to use the data provided by the district to manipulate the Math Board questions to function as a review and reteaching of concepts students still found difficult after the presentation of the original lesson. Beyond this, Teacher 1.2 followed the District Curriculum Map and the adopted textbook to plan her teaching.

In summary, both teachers received relatively high scores on a teacher assessment test. They both demonstrated high common content knowledge for their subject matter, knowledge of how to break down the mathematics being taught, and, for the most part,

Table 5

MKT Codes Related to Teacher 1.2's Planning

MKT category	Number of excerpts	Description of excerpts
Knowledge of Content and Curriculum	4 total excerpts	• 3 excerpts discussed the available curriculum provided by the district for her to use as a resource • 1 excerpt discussed the limitations of Study Island but also how she found the limitations beneficial for her students' learning
Knowledge of Content and Teaching	3 total excerpts	• 2 excerpts described when Teacher 1.2 thought it was wise to stay with one topic on the Math Board and when she decided to pose a new task • 1 excerpt described how she sequenced instruction on the Math Board and the benefits of concepts spiraling
Specialized Content Knowledge	1 excerpt	In this excerpt, Teacher 1.2 explained what aspect of multi-digit multiplication her students found difficult. She broke apart components in multiplication and explained which part was tricky
Knowledge of Content and Students – Knowledge of Content and Teaching	1 excerpt	In this excerpt, Teacher 1.2 anticipated whether or not a task was easy or hard for the students and how she adjusted her sequence of instruction to accommodate

were able to decipher non-standard methods of problem solving on that test. But what does that mean practically? How does this knowledge translate to what they do as teachers? In this section, we examined how these two teachers described their planning process. Both teachers discussed the resources they use, the tasks they select, the purpose of their tasks, and even some beliefs about their roles as teachers during their interviews.

While examining beliefs is not a major component of this dissertation study, it is critical to note the importance of understanding and acknowledging teacher beliefs when looking at practice (Thompson, 1992). When analyzing qualitatively, similarities and differences appeared between the two teachers. Those similarities and differences are explained next.

Mathematical Knowledge for Teaching and Planning: A Comparison Between Teachers 1.1 and 1.2

To organize a comparison, I looked across the two main domains within Mathematical Knowledge for Teaching. First, I examined how the components of Subject Matter Knowledge were expressed in the planning interviews and then I examined the Pedagogical Content Knowledge impact.

Specialized content knowledge comparison. Teacher 1.1 made explicit comments that pertained to her use of specialized content knowledge. Of the 46 excerpts, three of them pertained to some form of SCK, while in 11 other excerpts SCK overlapped with other subcategories of MKT. Teacher 1.2 only referred to SCK once in her excerpts. This does not mean that Teacher 1.2 does not utilize SCK as much as Teacher 1.1. It signified that during the interviews her remarks did not indicate the unpacking of the mathematics quite so much as Teacher 1.1. Only Teacher 1.1 expressed the last component, Horizon Content Knowledge, during the interviews. She was able to discuss how she knew where she wanted to go with the students to build the foundation for 5th grade math and higher-level math when it came to fraction development. This discussion of other grade level mathematics might have been sparked by the fact that she had just taught 5th grade the previous year. The lack of HCC examples presented in the interviews aligns with research results from the Mathematical Knowledge for Teaching study (Ball et al, 2008; Hill et al, 2010).

Pedagogical content knowledge comparison. Pedagogical Content Knowledge has three subcategories as well: Knowledge of Content and Students, Knowledge of Content and Teaching, and Knowledge of Content and Curriculum. The major differences for these two teachers regarding planning showed up when analyzing the data for PCK. As might be expected, about 90% of the excerpts for each teacher pertained to PCK. Specifically for Teacher 1.1, forty-one out of 46 comments were related to Pedagogical Content Knowledge, while for Teacher 1.2, 8 out of 9 comments exemplified aspects of PCK. The differences came when looking deeper into what parts of Pedagogical Content Knowledge were discussed. Knowledge of Content and Teaching characterized 17 excerpts for Teacher 1.1 and 10 of the 11 excerpts that housed multiple components of PCK, while for Teacher 1.2, KCT was represented in 3 excerpts and 1 combination excerpt. For Teacher 1.1, KCT manifested mostly as sequencing instruction and knowing when to pose a new task. For Teacher 1.2, all three excerpts discussed determining when to pose new tasks for students. Knowledge of Content and Students was the next highest category for Teacher 1.1. Nine of the excerpts exemplified KCS and 5 of the 11 combined excerpts related to KCS topics. It manifested predominantly through hearing and interpreting emerging and incomplete student thinking when categorized alone. For the combination excerpts, KCS was illustrated through anticipating what tasks students would find difficult or easy. For Teacher 1.2, Knowledge of Content and Students was exhibited only in the excerpt that combined KCS and KCT. It manifested as being able to anticipate when students would find a particular task hard or easy. The last part of Pedagogical Content Knowledge is Knowledge of Content and Curriculum. For Teacher 1.1, she talked extensively about the available resources that she uses outside of those provided by the district (KCC) on 4 different occasions. Teacher 1.2, however, extensively talked about the district curriculum and resources provided by the district as her go-to references for teaching (KCC). It should be noted that KCC has yet to be confirmed as its own category, according to Ball et al, 2008. It is possible that KCC is actually a part of KCT or embedded within many of the components of MKT, for now, though, it is considered its own subcategory.

Discussion of differences in MKT when planning. So, what might account for these differences in the planning of two knowledgeable teachers? A few themes that appeared across the four interviews (two per teacher) might account for the differences in how MKT impacted planning.

Teacher training. First, teacher training was talked about with both teachers. Teacher 1.1 attended a large university in the Midwest that is regarded for excellence in research on teaching and for their teacher-training program, especially in mathematics education. Teacher 1.1 attributed her wealth of knowledge of alternative methods for problems solving and how to elicit student thinking in ways that informed instruction on her teacher training. She expressed that the NSF-grant followed much of what she had learned in college and was a great extension for her own learning as a teacher during the two years she participated on the grant.

Teacher 1.2 only referenced her pre-service learning when directly asked about her training. She explained that she received a very traditional training in her methods course and that the information she was exposed to while participating on the NSF-grant was new to her. Thus, one reason there might be discrepancies in the use of components of pedagogical content knowledge might be the differences in pre-service teacher training experienced by the two teachers.

High-stakes testing. Another theme and possible explanation might be because of how the two teachers approached high stakes testing. Teacher 1.1 expressed multiple times that her goal was to build conceptual understanding of the 4th grade curriculum in her students. That conceptual foundation would assist the students as they navigated through the state standardized tests. She explained that once she understood all of the various types of problems the students might see on the test, she could decide how to best approach the teaching of the concept. She did not believe in using traditional test preparation methods, such as talking through multiple choice answers or only giving questions that were seen on the test to the students daily, instead her test preparation was based on teaching kids how to problem solve, critique their answers, solve in multiple ways, justify their answers, estimate, draw their thinking, rationalize, understand the question, etc. She also felt that the district curriculum guide did not allow for conceptual development to occur. She did not agree with the learning progression built into the map. Teacher 1.1, instead, used her own knowledge of the mathematics content and resources from the Internet, Test prep booklets, and current research to create a learning progression that suited the needs of her students. Teacher 1.2 did the opposite. She used the District Curriculum Map because she felt it mapped to the state standardized tests and the district quarterly assessments. For her, learning was the goal but having her students pass the test was the main focus. Therefore, she used the resources she was told best fit the needs of the students. It is possible that teacher 1.2 did not know of other resources to use or was not comfortable using outside resources as her evaluations in the district depended upon the administration seeing specific things happening in her classroom.

Role of teacher. The third theme that might influence the planning is how the two teachers view their roles as teachers. Teacher 1.1 expressed that she is a guide who facilitates learning. She figures out what the students know and do not know, then she finds tasks that help students figure out what they do not know, and then she reassess the students, and moves on to another topic. During the class time, she expressed that she selects students to share who have something interesting or relevant to helping other students figure out how to solve a particular problem. Again, this facilitation of selecting students is her method of guiding the class to a particular understanding of the mathematics being taught. For Teacher 1.1, the learning and teaching really happens at the hands of the students. Teacher 1.1 has a progression in mind and a goal for the lesson but each day the learning that occurs is determined by the students and what they present to her and the rest of the class. Doing this type of guiding means the teacher has to have a deep understanding of the mathematics, the possible avenues the students might travel down when problem solving, how to assist students who veer in the wrong direction, and knowledge of tasks that get at specific aspects of the standard being taught. This type of knowledge was apparent in Teacher 1.1's planning interviews.

Teacher 1.2, on the other hand, explained that she was there to help motivate kids and keep them focused on the schedule provided by the district. Her instruction followed the district map and the textbook provided by the district. When explaining how students might solve particular problems, she explained the standard algorithm and then said if they struggled she would put them in a small group so they could focus better on the method. In many ways, her role was to impart knowledge and to systematically work through the district's set plan for getting kids ready to pass the state test. It is possible that the limited exposure Teacher 1.2 has had to alternative modes of teaching mathematics concepts hampered her planning and ability to look at student thinking as a springboard for planning. It is also possible that the beliefs these teachers hold about the role of the teacher is something that needs to be further explored using a well-respected belief survey or other type of qualitative protocol. There does seem to be something related to their ideas about the role of the teacher and what it means to teach that impacts their knowledge and planning.

In the following section, we moved to explaining the classroom instruction of Teacher 1.1 and 1.2 as we move toward the final step of understanding possible reasons as to why teachers with similar MKT scores have very different student change scores over a school year. The section will examine the general look of each class if an observer walked in, the cognitive demand of the tasks (Stein et al, 2009), the types of questions asked by the teachers according to Bloom's Taxonomy, and the types of responses provided by students to both the teacher and each other.

Implementation

The Implementation Phase of the Mathematics Teaching Cycle included: the learning environment, selection of meaningful tasks, and discourse. For this section, I presented the entire Implementation Phase components for each teacher and then compared the two teachers. Using this format for discussing the instruction component seemed more comprehensive and coherent than jumping between the components and the teachers.

Teacher 1.1

The instructional practice of Teacher 1.1 was evaluated using three methods: CLASS observation protocol, the Mathematical Tasks Framework, and Bloom's Taxonomy. Across all three tools, one could see that Teacher 1.1 created a classroom environment based on student thinking and exploration of ideas, discourse, positive interactions among students and with the tasks, and conceptual learning. Teacher 1.1 also provided evidence in her instruction of high reliance on her Specialized Content Knowledge, Knowledge of Content and Students, and Knowledge of Content and Teaching. These were not discrete occurrences of these knowledge types but instead interconnected, dynamic relationships among the MKT categories throughout her observed teaching. The following sections provided the evidence to support this comprehensive summary.

The learning environment.

Classroom Assessment Scoring System (CLASS) observation. The learning environment of the classrooms in this case was assessed using the CLASS observation protocol. The interactions evaluated using the CLASS protocol were scored on a 1 to 7 scale. A score of 1 or 2 meant a low frequency of something occurring. A score of 3, 4, or 5 indicated a mid-range frequency of something occurring. A score of 6 or 7 indicated a high frequency of an interaction occurring. The importance of using this protocol and rubric system to assess the learning environment was because it aligned with much of the NCTM (2007) description of what a mathematics classroom should look like. For example, the CLASS protocol examined climate, how much flexibility students received to taken on leadership roles in the classroom, time for students to share, risk taking, and engaging in mathematical discourse. These ideas coincide with the ones described by the NCTM in the Mathematics Teaching Cycle.

In addition, the CLASS protocol allowed me to look for links between the interactions that occurred in the learning environment with MKT categories. Linking these two ideas helped with triangulation of data when making conjectures about MKT and classroom instruction.

Teacher 1.1. Six CLASS observations were conducted during the time period in which Teacher 1.1 was observed. Two were conducted in October and November 2011 and four were conducted in January and February of 2012. The following Table shows the average scores across the 10 dimensions for teacher 1.1:

Table 6
CLASS Scores for Teacher 1.1

Dimension	Average Score
Positive climate	6.5
Negative climate	1.167
Teacher sensitivity	5.83
Regard for student perspectives	5.67
Behavior management	6.16
Productivity	5.5
Instructional learning formats	6
Concept development	5.16
Quality of feedback	5
Language modeling	4.83

Except for the scores in Negative Climate and Language Modeling, Teacher 1.1 received, on average, 5s and 6s for eight out of the ten dimensions. Negative climate was below a 2, meaning very few negative interactions occurred during the observation period. Language modeling was a 4.83. This score is in the middle range but was tied for the second highest score on this dimension when compared to all of the teachers participating in this dissertation.

When composite scores were calculated for the three CLASS domains, Teacher 1.1 received a 6.20 on Emotional Support, a 5.89 on Classroom organization, and a 4.996 on Instructional Support. Again the scores based on a scale of 1 to 7, with 1 as the lowest score and 7 as the highest. As we can see by the scores for Teacher 1.1, the interactions pertaining the Emotional Support were high. This means that there was on average a high positive climate, a low range of negative interactions happening, the teacher was aware and responsive to her students' needs and students also asked questions and talked in this classroom. It also captured the fact that the teacher incorporated students' ideas into the activities, followed the students' lead much of the time, gave students responsibility and choice in the classroom, encouraged talk among and between the students, and was able to elicit students' ideas and thinking about the tasks (Pianta et al, 2008).

The Classroom Organization score was also high in this classroom. Teacher 1.1 set clear expectations and enforced them consistently. She was proactive in anticipating behavior problems and in monitoring for potential problems during activities. She used efficient redirection and subtle cues to redirect students who were getting off task. Students complied with Teacher 1.1's expectations and were infrequently defiant. The classroom environment allowed for high productivity, for the most part. Students knew what to do and transitions between activities were brief and concise. Materials were prepared ahead of time and the teacher knew how to keep a steady pace throughout the class time. In this classroom, the teacher demonstrated effective lines of questioning that kept the focus and attention of students. A wide range of auditory, visual, and movement opportunities were provided for students to complete the mathematics tasks.

Lastly, Teacher 1.1 received a middle of the range score, for Instructional support, but it was on the high end of the middle range. There was a high middle range score for items pertaining to analysis and reasoning, brainstorming, planning, connections across mathematics concepts and application to real-world situations. There were also high-middle range scores for "the degree to which the teacher provides feedback that expands learning and understanding and encourages continued participation" (Pianta et al, 2008, p. 72) and middle range scores for "the quality and amount of the teacher's use of language-stimulation and language-facilitation techniques" (Pianta et al, 2008, p. 79).

Overall, the CLASS observations captured a classroom where a large majority of the instruction encouraged positive interactions between the teacher and students and among the students themselves that aided in the learning of 4th grade math. Students were active participants in the classroom, their comments often drove the direction of the lesson, and the teacher was aware of the needs of her students. This description aligned well with how Teacher 1.1 structured her lessons, planned, and her MKT scores. One main focus in her planning was utilizing student thinking as a platform for generating tasks and direction of the lessons, which described two dimensions in the CLASS protocol: regard for student perspectives and teacher sensitivity. These two dimensions align nicely to the subcategory of Knowledge for Content and Students found in Mathematical Knowledge for Teaching. They all pertain to how a teacher anticipates what difficulties students might have with particular problems and how to plan accordingly. They also focus on a teacher understanding what motivates and interests the students regarding the content. There is also a large portion of the two dimensions and the KCS category that focus on hearing students' thinking in emerging and incomplete thoughts. In addition, teacher sensitivity and having regard for student perspectives aligns with components in Knowledge of Content and Teaching. For example, Teacher 1.1 showed high scores for regard for student perspectives, which encompassed allowing students to lead the lessons, incorporated students' ideas, and encouraged student talk. Knowledge of Content and Teaching included deciding which student contributions to pursue, which to ignore, and which to save for later, when to ask for clarification, and when to use students' ideas to make a mathematical point (Ball et al, 2008).

The middle of the range CLASS scores was mostly seen in conceptual development, quality of feedback and language modeling. These range scores aligned with her planning interviews, as well. Teacher 1.1 alluded to needing to enhance these aspects in her own teaching. She realized that she was asking mostly knowledge questions over the last few years of teaching when she needed to be asking questions that got at higher-thinking. There were also a few observations conducted where the focus of the lesson ended up being about constructing a number line accurately rather than the assigned task. While the construction of an accurate number line is an important component of mathematics development, multiple occasions of this activity inhibit productivity and the learning of other mathematical ideas about, in these instances, fractions. The change in focus of the lesson did impact her scores on sensitivity and regard for student perspectives. Again, aligning to her communication about using student thinking as a driving force for planning (Teacher 1.1, 1st interview).

Unfortunately, subcategories in MKT are harder to find in these CLASS dimensions. On a general level, these middle range scores on the CLASS protocol might match with Specialized Content Knowledge and the unpacking of mathematical ideas in ways specific to teaching because these three dimensions look at the unpacking of the mathematics content and being able to find ways to make parts of particular content visible to and learnable by students (Ball et al, 2008). Next, we examined the cognitive demand of tasks given by Teacher 1.1 (Stein et al, 2009).

Worthwhile mathematical tasks and classroom discourse.

The mathematical tasks framework (Stein et al, 2009) for Teacher 1.1. Six one-hour classroom observations were used to assess the teachers' instruction in this dissertation. During the six observations on Teacher 1.1, between one and five tasks were assigned during a single class period, for a total of 19 written tasks given to the students. Overall, there were a total of 2 memorization tasks, 11 procedures without connections tasks, 4 procedures with connections tasks, and 1 doing math task (Stein et al, 2009). The following table provides examples of the different types of tasks assigned by Teacher 1.1.

To best understand how Teacher 1.1 implemented the written tasks and how the implementation related to MKT, I used the following memorization task that was given on January 24, 2012: “plot 1/5,2/5, 3/5, 4/5, and 5/5 on a number line. On a second number line, plot 1/10, 3/10, 5/10, 9/10, and 10/10.” There was a number line drawn underneath the question with the points “0” and “2” at each end.

Table 7

Cognitive Demand Level of Tasks Given by Teacher 1.1

Classification of task	Example	Explanation
Memorization	Plot 1/5, 2/5, 3/5, 4/5, and 5/5 on the number line. Plot 1/10, 3/10, 5/10, 9/10, and 10/10 on a second number line	As written this task involves reproducing already learned facts and "have no connection to the concepts or meaning that underlie the facts, rules, formulae, or definitions being learned or reproduced" (Stein et al, 2009, p. 6).
Procedure without Connections	"87 x 56" and "The students in the sunshine club are making cards. They can each choose one sheet or construction paper: green, purple, or orange. And one shape to glue on it: a smiley face, a star, or a heart. How many different kinds of cards can be made?"	These tasks are algorithmic and require limited cognitive demand for successful completion, as written. They are "focused on producing correct answers rather than developing mathematical understanding" (Stein et al, 2009, p. 6) and they "require no explanations, or explanations that focus solely on describing the procedure that was used" (p. 6).
Procedures with Connections	"Stephanie had 37 inches of ribbon to make hair bows for her amigas. If each bow needs 7 inches to be made, how many friends get bows? How much ribbon is left?"	This task "requires some degree of cognitive effort. Although general procedures maybe followed, they cannot be followed mindlessly. Students need to engage with conceptual ideas that underlie the procedures in order to successfully complete the task and develop understanding" (Stein et al, 2009, p. 6) This task also requires students to think about what they are answering and how they will get to the answer. In addition, there are multiple ways in which a child might solve this problem and there is a demand on the children to monitor their own thinking because the answer they might initially end up with might not be the answer to the question presented.
Doing Mathematics	"Karla baked a batch of 37 brownies for a bake sale. If she places an equal amount of brownies into 7 containers, how many brownies will be left over? Show your method for solving."	This task is at the highest level of cognitive demand because "there is not a predictable, well rehearsed approach or pathway explicitly suggested by the task, task instructions, or a worked-out example"(Stein et al, 2009, p. 6). In fact, there were no examples modeled or instructions given about solving this task until after the students had tried to solve it on their own. Thus, this task "required students to access relevant knowledge and experiences and make appropriate use of them in working through the task" (p. 6).

Students worked independently on this task at the start of the school day. During math time (when my observations occurred), students worked in groups to answer the question. The students were expected to discuss their strategies and decide how best to present their thinking as a group to the class. Teacher 1.1 walked around the class during the group work time questioning students about their ideas. She used the information she gathered to select students to share the work on the board. The students were selected based on factors such as: incorrect answers but good strategies, novel strategies, having a strategy that represented what most of the class thought, or having a strategy that could help students who used a similar strategy incorrectly (Teacher 1.1, 2nd Interview).

One student selected to share his groups' thinking was Carlos. Carlos went to the board and drew a number line that had hash marks placed haphazardly from 0 to 5/5.

> *Teacher 1.1: I want to know exactly what you did because they have no idea what you are thinking, they have no idea what goes on in this head of yours so you need to tell them what was going on in your guys' brain, start with this one what was the first thing that you did?*
> *Carlos: we started*
> *Teacher 1.1: don't talk to the board, talk to the class*
> *Carlos: we started with the fifths and put them in order*
> *Teacher 1.1: how did you know the 1/5 went right there?*
> *Carlos: it's next to zero and zero is right there*
> *Teacher 1.1: so you knew, okay so you are saying that you knew that 1/5 would be closer to zero than the what?*
> *Carlos: zero*
> *Teacher 1.1: you knew 1/5 would be closer to zero than what?*
> *Carlos: then half*
> *Teacher 1.1: so did you figure out where the half would be first?*
> *Carlos: yeah*

Using a mixture of Comprehension and Analysis questions (Bloom, 1956), Teacher 1.1 encouraged Carlos to explain his thinking (see Appendix E – Maintenance of high-level cognitive demand factor 4: sustained justification and explanation through teacher questioning). She further assessed the students' learning through asking the whole class to evaluate their own work against what Carlos presented.

Teacher 1.1: why didn't you put anything over here?
Carlos: because there is no . . . ??. . . there' no 6/5, 7/5
Teacher 1.1: okay so you're right, so he's saying, I am asking him alright so the number line I did this on purpose because I am tricky. The number goes from zero to what class?
Class: two
Teacher 1.1: and he only used this side. How many of you guys did that? Only did, you only put it from zero to one? And I asked him well how come he didn't put anything over here and he's like we'll there's no 6/5, 7/5, in other words there's not more than what?
Class: 5
Teacher 1.1: one whole. Right. Actually what you did first is you figured out where 5/5 went. And 5/5 is equal to what class?
A group of kids: one whole

Again, we see Teacher 1.1 pressing the students to ensure that they understand "one whole" and what it meant to extend beyond one whole on the number line. At this point, Carlos explained that he finished his number line by plotting the rest of the points in order. Teacher 1.1 used Carlos's explanation and description to make her own number line.

Teacher 1.1: okay, so he's [Carlos] saying you need to put them in order, do you guys agree? Like 1/5, then 2/5/, then 3/5, then 4/5, do you guys all agree with that? Okay, he's right. Here's my question, if I have the number line right here, alright I'm going to do exactly what Carlos did, alright? He said zero to 5/5. He said went in order from 1/5, 2/5, 3/5, 4/5, right? Because one-fifth is closer to zero, if you only have 1/5 that's closer to not having one candy bar than having a whole one, right? Right? So he said you had to go in order, right?
Kids: right
Teacher 1.1: okay, here's my question, I am going to do exactly what he said you guys tell me what you think . . . [she draws her number line. The distances between points varies] tah dah! Cool? Does that look right to you guys?
Kids: no
Teacher 1.1: pretty cool. How many of you guys think that looks right? You agree with me? Nobody agrees with me? Why not? You know I could fail

you, right? Nobody agrees with me? Why not? What's wrong with it? I did exactly what Carlos said. I made my number line, 0 to 5/5 and I went in order just like a candy bar goes in order. Here's my whole, 1/5, 2/5, 3/5, 4/5, 5/5, it's in order. Right? Cause from here to here is one fifth [points to her number line]. This is a 1/5 piece, this is a 1/5 piece [the point is right next to the first "1/5," this is a 1/5 piece [much further away], 1/5 piece [at the far right of the number line, almost on top of the last fifth], 1/5 piece, so from here to here that's 2/5 right? so I went in order, so why is that wrong? You're right that's not right! Why is this wrong?

Teacher 1.1 drew a number line similar to ones found on the papers of other groups in the classroom (SCK – recognizes error patterns in students' thinking). She used this knowledge (KCT – using a students' remark to make a mathematical point) to expose the misconceptions the students had about fractions (KCS – knowledge of common misconceptions in student thinking) and then engage in a conversation about what a fraction represented (see Appendix E – Maintaining high level cognitive demand #1 – scaffolding of student thinking).

Teacher 1.1: . . . so why can't I do it? [Can't hear her] huh? [Can't hear her] the 5/5 doesn't go there? But that's my whole.
Boy: because the 1/5, the 2/5, the 3/5, and the 4/5 are not separate.
Teacher 1.1: what do you mean by that?
Same boy: like, the 1/5 it has to be like a little bit separately
Teacher 1.1: why do they need to be more separate?
Same boy: because it's like um if it's lie half or like um
Teacher 1.1: well, like technically guys I want you to look at what 4/5 looks like. Okay? Does that look like closer to zero, half or one whole?
Kids: one whole
Teacher 1.1: it looks closer to one whole. Do you see where 4/5 is right there? What does that look like on a number line? That looks like it is closest to half right? Basically what I just did on this number line, I am saying the way that I did it. If I were to draw it like on a candy bar, it would look like this. Here's 1/5, here's 2/5, here's 3, so this would be like a 1/5 piece, this is a fifth piece, this is a fifth piece and this is a fifth piece. What's wrong with that? Joy?
Joy: four pieces are
Teacher 1.1: are what?
Joy: aren't all the same

Through scaffolding, Teacher 1.1 lead her students to the realization that the on-fifth pieces needed to be equal distances on the number line (SCK – being able to unpack a mathematical concept in a way that is transparent and learnable by students). Teacher 1.1 never told the students that the fractional pieces had to be equidistant, instead she used what the students knew in conjunction with her own understanding of fractions to create a learning environment where student thinking propelled learning and the acquisition of a specific concept. Teacher 1.1 started with a memorization task and raised the level of cognitive demand through her questioning, use of student thinking, and her own content knowledge.

In another example from the same observation, the students were given a task of selecting a number sentence that represented a word problem about a student who broke three dishes when unloading the dishwasher. As written, the task was a procedure without connections. The students solved this problem and selected their answer at the start of the day. When they reached this question during math time, Teacher 1.1 asked the students to reexamine their answers and assigned them three things to complete when reexamining: a) write the letter that represented the answer choice that they selected from the three possibilities, b) write the number sentence that corresponded with the answer choice, and c) label each part of the number sentence with what it represented in the problem. She gave the students a few minutes of class time to make sure they answered all three components of what she asked. She proceeded asking, "how many of you guys changed your answer? How many of you guys started to write out your answer and label it and then you realized, uh oh! I need to change my answer? Did that happen to you guys? Good, that's okay, that's what you should be doing." She waited and then refocused the children, "so I am going to ask you again, how many of you guys had your answer this morning like when you were working on your math mastery and then as you guys started writing it, you are like uh oh! I didn't do this right. You changed your answer? How many of you guys did that? Awesome!"

This snippet was another example of how characteristics of Knowledge of Content and Students and Knowledge of Content and Teaching appeared in the instruction of Teacher 1.1. First, the teacher presented the original problem to the students at the start of the day. From looking at their work and seeing what they had done on previous assessments, she modified her expectations and the requirements for answering the question when it came to math time. This represented the aspect of Knowledge of Content and Teaching where teachers know when to pose a new task to students (Ball et al, 2008). The key was that her awareness of knowing when to pose a new task or modify the original task might have been sparked by her anticipation of what students would do with the task and what errors they might have when selecting a number sentence, which is part of the Knowledge of Content and Students (Ball et al, 2008). It is possible that the intersection of these two categories enabled Teacher 1.1 to move this task to a high-level of cognitive demand.

Not all of the written tasks given by Teacher 1.1 resulted in high-level cognitive demand when implemented. As time moved closer to the State's standardized testing week, some of the low-level cognitive demand tasks maintained their level of difficulty. For example, she gave the students a picture of a triangle with the measures of 11, 22, and 13 cm. She asked them for the perimeter and to classify the triangle as scalene, isosceles, or equilateral. The students independently answered the question at the start of the day and then during math time, Teacher 1.1 asked basic Knowledge questions (Bloom, 1956) to assess if they all got the correct answer. She asked what information was needed to find the perimeter? What part of the word perimeter helped them remember what to do when solving the problem? What the tricks were that they learned to help them remember the meaning of scalene and isosceles ("Scalene was mean and all of the sides did their own thing. Isosceles had two "ees" and two sides of equal length"). These tricks helped the students remember the definitions, as seen in the observation, but they did not help build conceptual understanding.

Even in situations where the cognitive demand level was high as a written task and dropped to a low-level task during the instructional time period, components of MKT were seen. For example, when Teacher 1.1 presented the students with the task: "Stephanie has 37 inches of ribbon to make hair bows for her amigas. If each bow needs 7 inches to be made, how many friends get bows? How much ribbon is left?" she ended up diverting the class discussion away from strategies for solving this problem to a discussion about problem types (something that is not necessarily important for students to know and understand. Teachers should know and be aware of problem type differences but not necessarily students). Even so, Teacher 1.1 demonstrated Specialized Content Knowledge when she explicitly described the differences between partitive and measurement models of division to the students. She used a students' remark to help scaffold a mathematical point relevant to the differences in models, which is Knowledge of Content and Teaching. She explained the common errors she was seeing when kids tried to solve measurement problems with partitive methods and how the students might arrive at the correct numerical answer but not be able to explain what the number represented in the problem, which is a combination of Knowledge of Content and Students and Specialized Content Knowledge (Ball et al, 2008).

What is important to note is that even with the use of recall questions and basic tricks, Teacher 1.1 still made the students answer the questions and give her the information needed to solve the problems. In fact, over the six observations used for data analysis purposes of this dissertation, it was rare to see Teacher 1.1 stand and deliver instruction. If such a pedagogical method was used, it was usually because the student struggled to explain their thinking to the class and Teacher 1.1 modeled how to explain the student's method to the class. Traditional direct instruction was never seen during the six observations, however. It is possible that such methods were used during other days of the week. All of the observations conducted on Teacher 1.1 were completed on Tuesdays. Had observations been conducted on another day of the week, other methods of teaching might have been observed. However, one can only speculate at this point without such data.

In summary, the instructional practice of Teacher 1.1 was evaluated using three methods: CLASS observation protocol, the Mathematical Tasks Framework, and Bloom's Taxonomy. Across all three tools, one could see that Teacher 1.1 created a classroom environment based on student thinking and exploration of ideas, discourse, positive interactions among students and with the tasks, and conceptual learning. Teacher 1.1 also provided evidence in her instruction of high reliance on her Specialized Content Knowledge, Knowledge of Content and Students, and Knowledge of Content and Teaching. These were not discrete occurrences of these knowledge types but instead interconnected, dynamic relationships among the MKT categories throughout her observed teaching. Next, we examined Teacher 1.2 using the same analytic methods to see if any patterns or disparities were seen between the two teachers with relatively high MKT scores.

Teacher 1.2

The instructional practice of Teacher 1.2 was evaluated using two methods: the Mathematical Tasks Framework and Bloom's Taxonomy. Teacher 1.2 provided evidence in her instruction of a high reliance on Common Content Knowledge. This type of knowledge was evident in her implementation of the tasks she gave the students and in the types of questions she asked the students during class discussions. Her teaching style was very systematic and orderly. For example, during each observation, Teacher 1.2 stood at the front of the class and solved problems found in the 5th grade textbook for the students. Her instruction did not change based on students demonstrating a lack of understanding. She addressed confusion with a repeat of the procedure and additional examples from the textbook. The following sections provided the evidence to support the claims made in this introductory paragraph.

The learning environment.

CLASS observation protocol. Unfortunately, one limitation to the data collection process of Teacher 1.2 was that she did not agree to audiotaping of either her observations or interviews. This means that CLASS observations could not be collected for Teacher 1.2 because instead of conducting live CLASS observations, while the audio recording captured the specific dialogue and interactions occurring at the same time, I chose to omit the CLASS observations as a data source for this teacher.

During all observations and interviews, detailed notes and quotes were taken down in a notebook and then immediately flushed out following the observation or interview. This method of collecting data on Teacher 1.2 is another limitation to the ability to analyze and piece together how MKT influences her instruction to the same degree as the other 5 participants. Even so, because Teacher 1.2 was an interesting contrast and enough data could be collected about her instruction and planning, I chose to keep her in the sample even with the limitations.

Worthwhile mathematics tasks and classroom discourse.

The mathematical tasks framework (Stein et al, 2009) for teacher 1.2. Over the six observations selected for this dissertation analysis, between five and twelve tasks were assigned during a single class period, for a total of 29 written tasks. There were a total of 3 memorization tasks and 26 procedures without connections tasks (Stein et al, 2009). The following table provides examples of the different types of tasks assigned by Teacher 1.1. The following table provides examples of the tasks assigned by Teacher 1.2. To best understand how Teacher 1.2 implemented the written tasks and how the implementation related to MKT, I used the following memorization task that was given on October 27, 2011. The students were working on learning and applying divisibility rules to reduce fractions. Each student had a textbook open in front of them at their desks. Teacher 1.2 is stood at the front of the class at the white board.

> *Repeat after me. Put your finger on the 5. We're looking at the green table. Find where it says 5. Okay? Repeat after me. If a number ends in zero or five, it's divisible by five. The kids repeat the rule to the teacher. She turns to the board and writes 15/90 on the board. Does this [pointing at the 15] end in a zero or a five? A student asks her what she means by end. The last digit is the end. Is it a five or zero? Yes, replies the student. Now look at the bottom number. Does the last digit, or the end, end in a*

Table 8

Cognitive Demand Level of Tasks Given by Teacher 1.2

Classification of task	Example	Explanation
Memorization	"24/n = 3" or "draw an isosceles triangle" or "Start with 2 1/2. Complete the alternating pattern: adding 1, adding 1/2 --> starting at 2 1/2"	As written, this task has "no connection to the concepts or meaning that underlie the facts, rules, formulae, or definitions being learned or reproduced" (Stein et al, 2009, p. 6). It also is "not ambiguous – such tasks involve exact reproduction of previously seen material and what is to be produced is clearly and directly stated" (p. 6). Lastly, this task "involves either reproducing previously learned facts, rules, formulae, or definitions OR committing facts, rules, formulae, or definitions to memory" (p. 6).
Procedure without Connections	"10 2/4 - 4 3/4" or "2/10 + 3/10" or "4/5 + 1/10"	These problems, and all of the others like them, "are algorithmic. Use of procedure is either specifically called for or its use is evident based on prior instruction, experience, or placement of the task" (Stein et al, 2009, p. 6). Also, these problems "are focused on producing correct answers rather than developing mathematical understanding" (p. 6) and they "require limited cognitive demand for successful completion. There is little ambiguity about what needs to be done and how to do it" (p. 6).

zero or five? Yes or no? Yes. Okay so 5 is a common factor. She turns back to the board and writes
15/90 ÷ 5/5
Look at the table. We're going to read the rules. Not that you'll remember them just because you heard them once. We will memorize them later in the hour. Teacher 1.2 reads the rule for dividing by 2. The kids softly repeat the rule. Okay, I'm convinced you know where we are. Imagine we have 6/27 as the answer to a problem. We have to simplify that fraction. We have to find the little number that goes into both the top and the bottom. It will be good to know the rules to figure this out. Let's try. Can you reread the rule for 2s to see if it will apply? Raise your hand if you think you know if two will go into six, if two will go into our numerator? Is two a factor of six? The class says yes. What about 27? Is two a factor of 27? Last digit 2 or 7? Is 7 even? Can you see how these rules might help you later? Let's do another . . .

At the start of this lesson, the tasks were written at a level of Procedure without Connection. As the task was implemented the low-level cognitive demand of the task remained low-level. The task remained algorithmic and required limited cognitive demand for successful completion. The most the students had to do with the task was read the rule and apply the divisibility rule. Even so, as it was implemented during the lesson, Teacher 1.2 modeled how to do all of the problems for the students. They did not have time or room to work through the problems independently until after she modeled every type of problem they were going to encounter in their class work. Teacher 1.2 used mostly knowledge and comprehension level questions from Bloom's Taxonomy. Her questions required recall of material read in the textbook. The students had to recognize digits, as well, in this vignette.

The second vignette is from January 26, 2011. The questions asked during this observation were taken from the math wall, written by Teacher 1.2. The students worked through the five questions independently. We pick up where the whole class is working through the answers to the questions, in particular they are answering the question: Start with 2 1/2. Complete the alternating patter: adding 1, adding 1/2 --> 2 1/2, ____, _____. _____. _____, etc. Teacher 1.2 stands at the front of the room.

> *Last week, no one could add 1/2 in a patter so that's what we are practicing. She picks a name from a cup and calls on the girl whose name she selected. Angela, starting at 2 1/2 we are going, plus 1, plus 1/2, plus 1, plus 1/2. Ready? Angela answers 3 1/2. Okay, plus 1/2, Georgia? 4. Okay, plus one, Michael? Michael does not answer. You're supposed to be paying attention. What is this plus one? Michael responds. Four and one half. Hum, Teacher 1.2 says, what is 4 plus 1? Michael says 4 1/2. Teacher 1.2 turns to the board and writes 4 + 1. What is that? Michael says 5. What is plus one-half, Michael? 5 and one-half. Good. What is plus whole, Brynn? 6 and one-half. Teacher 1.2 continues asking the students to extend the pattern by giving the next step in the pattern to the child. Okay, good. Now, whose pattern matches? Did I do this right? Look at your pattern? Same or different? She walks around the class and checks each students' paper.*

The task worked on during this vignette was written at a memorization level of cognitive demand. There was no ambiguity in how to solve the problem. The steps for creating the pattern were given to the students in the written task and then during the implementation phase giving the answer was all that was required of the students. This task did not increase in cognitive demand during the implementation phase (Stein et al, 2009, p. 15).

According to Bloom's Taxonomy, the questions in this vignette aligned with knowledge question. The students had to provide an answer by following a rule she gave them in her question. The students did not have to explain, extend, show, or interpret the pattern or their thinking. They had to recall the answer to the next step in the pattern.

Overall, the instruction of Teacher 1.2 could be characterized as following traditional direct instruction methods. The teacher imparted knowledge to the students through providing them with the procedure for solving problems before the students are given time to solve subsequent problems independently. The questions asked of the students were rote questions that required very little thinking or analysis. Responses from the students were usually one word or a short phrase that directly answered a recall question.

Across the six one-hour observations, two main categories characterized the instruction of Teacher 1.2: Common Content Knowledge (mostly) and Knowledge of Content and Teaching (less often). In the first vignette, described above, Teacher 1.1 used terms and notations correctly; she accurately and correctly completed the work the assigned the students, and she demonstrated an understanding of when students were incorrect in their thinking. All of these are characteristics of Common Content Knowledge (Ball et al, 2008). In this excerpt, other forms of MKT were less notable.

In another example, Teacher 1.2 used arrays to help students visualize the area of a rectangle. Teacher 1.2 writes "Area" on the board. Turning to the class, she tells them today's lesson is a continuation of the geometry lessons they worked on during the previous week.

> *Does anyone remember learning about area last year? A few students raised their hands. Okay. Take out your notebooks and title the page "Area," just like I wrote on the board. Good. What does area mean? Well, it means how much space is inside a shape. Or in our case today, how many boxes are in a shape. She pauses and writes her definition on the board. The students copy it into their notebooks. The teacher passes out whiteboard markers. Now, kids, this was a disaster last time we used the markers so let us try to be better this time. Please, open up the marker and put the cap on the back. Okay. Everyone finish that? On your whiteboards, draw a 1 by 4 model, or 1 row of four. She draws a long, horizontal rectangle on the board. Let's everyone do that. Start on the upper left hand corner of your paper. 1 row of 4. She walks around the room checking on the students. She holds up the work of one student to demonstrate the quality she was looking for. Now draw a 2 by 4, or 2 rows of four, on your paper. She returns to the board and draws the rectangle. What I want you*

to do is outline the grid in marker and then write the 2 by 4 next to the model in pencil. Do not color in the squares, just outline them in marker. This procedure continues for another two rectangles. A young girl stops and asks why they are not supposed to outline the squares in the grid like Teacher 1.2 did on the board. Teacher 1.2 responds, yours will look like just an outline of the squares. In pencil, write the problem next to the model. Remember don't separate the squares. You are outlining them. After constructing a rectangle of 4 by 7, Teacher 1.2 turns to the class and says, repeat after me. Area means . . . The kids repeat Area means. Now, find the first model you drew. I want you to calculate the area and write it next to it. How many boxes inside that shape. Write it next to or near by. Who knows the answer? How many boxes are inside this shape? My hope is you did this. She turns to the board and writes "4 x 1 = 4" underneath her 4 by 1 rectangle.

This example is a depiction of Common Content Knowledge. The characteristics of the instruction follow the same ones from the previous vignette: used terms correctly. She was able to accurately complete the students' work and she pronounced the terms correctly. It looked as though other MKT categories could be found in this vignette but looking closely at what was happening in the classroom, there was very little mathematics being worked on that was unique to teaching. There was no explicit connection to multiplication, commutativity, what area actually means, or alternative ways to represent area. There was no attention given to students' prior knowledge, other than asking if they remembered area from the previous school year. The tasks assigned were basic and low-level. In fact, later in the lesson the students tell her they are bored and she says she will speed things up.

In summary, the instructional practice of Teacher 1.2 was evaluated using two methods: the Mathematical Tasks Framework and Bloom's Taxonomy. Teacher 1.2 provided evidence in her instruction of a high reliance on Common Content Knowledge. This type of knowledge was evident in her implementation of the tasks she gave the students and in the types of questions she asked the students during class discussions. Her teaching style was very systematic and orderly. For example, during each observation, Teacher 1.2 stood at the front of the class and solved problems found in the 5^{th} grade textbook for the students. Her instruction did not change based on students demonstrating a lack of understanding. She addressed confusion with a repeat of the procedure and additional examples from the textbook.

Discussion

Overall, Teachers 1.1 and 1.2 presented a case of two teachers with similar MKT scores relative to their peers in the NSF-funded grant but very different student gain scores. Why was this? What was it about how they drew upon their MKT that might have accounted for the differences in their scores?

One major difference between Teacher 1.1 and Teacher 1.2 was the fact that across all three facets of the Mathematics Teaching Cycle (knowledge test, planning, and implementation), Teacher 1.1 used a complex mix of Common Content Knowledge, Specialized Content Knowledge, Knowledge of Content and Students, and Knowledge of Content and Teaching. Her teaching style embraced student thinking. Both her classroom environment and her planning focused on providing activities to students based on what she learned through talking with her students about the mathematics. As she learned what the students understood, Teacher 1.1 selected new tasks to challenge the students to think one step further than the point at which they resided mathematically. She also challenged students to share their thinking at the board and was able to hear and interpret what students knew about the mathematics based on their explanations.

Across the three facets of the Mathematics Teaching Cycle, Teacher 1.2 demonstrated reliance on Common Content Knowledge. Unlike Teacher 1.1, Teacher 1.2 imparted knowledge to her students. She gave her students the formulas or procedures repeatedly and limited the classroom discourse to answering basic recall or comprehension questions. She followed the order of the textbook and the district curriculum map to plan, regardless of what her students understood.

Based on the differences outlined in this case study, it is possible that the differences in student gain scores for these teachers was a function of the differences in what types of MKT were used by each teacher throughout the Mathematics Teaching Cycle: a complex mix of MKT categories versus a heavy reliance on Common Content Knowledge. The next chapter examined the case of two teachers with different MKT scores from each other and who had opposite student gain scores from what one might think looking at their MKT scores.

CHAPTER FIVE: CASE TWO

A Case of Different MKT scores and Different Student Gain Scores

This case examined two teachers who above and below the mean of the participants in the NSF-funded grant on the Teacher Knowledge Assessment. Their student gain scores over the 2011-2012 school year also differed from each other. To reiterate the information presented about the sampling process in Chapter Three, Teacher 2.1 scored 0.78 of a standard deviation above the mean of the participants in the larger NSF-funded study, while Teacher 2.2 scored just below the mean of her peers. The change scores for the two teachers respective students presented in direct contrast to general expectations. They were exact opposite of what one might anticipate. Teacher 2.1's students' gain scores were a half of a standard deviation *below* the mean of the student scores for those participating in the NSF-funded grant, while Teacher 2.2's students' gain scores were half of a standard deviation *above* the mean of her peers' students' scores. This case added another layer for understanding how MKT might link to student gain scores through classroom instruction with the addition of a teacher who has a relatively average MKT score with above average student gain scores.

General Descriptions of Teacher 2.1 and Teacher 2.2

Both of the teachers in this case provided a limited amount of background data. What was provided is presented below.

Teacher 2.1

Teacher 2.1 had been teaching almost 30 years. She taught almost every grade level from primary to 8th grade all around the United States. She planned to retire at the end of the 2011-2012 school year.

Over the 2011-2012 school year, Teacher 2.1 taught approximately 25 fourth and fifth grade students who tested at the "Basic" level for English Language Development at the start of the school year. The desks in her classroom were pushed into little pods of five to six desks each. The pods were scattered around the classroom and faced either the front chalkboard or the side chalkboard. During instructional time, Teacher 2.1 taught each grade level separately. The group being taught sat at the front of the classroom, while the other students moved to the back of the class. In addition, the grade level not being taught worked on a review of previously learned material in the textbook or worked on a worksheet practicing a previously learned skill.

Teacher 2.2

Teacher 2.2 was a 3rd year teacher. All three of the years were spent in this district. She grew up in the district and as a child she had aspirations to return as a teacher. She was a graduate of a larger university in the Southwest.

Over the 2011-2012 school year, Teacher 2.2 taught approximately 18 third and fourth grade students who tested at the "Basic" level for English Language Development at the start of the school year. Similar to Teacher 2.1, the desks in this classroom were pushed together to make groups of four. During many of my observations, the 3rd grade students were sent to learn math from a teacher who taught only 3rd grade students. Teacher 2.2 kept the fourth grade students for math. Teacher 2.2 received a set of 3rd grade textbooks at the start of the school year and a few resource books for the 4th graders but no 4th grade textbook. The students completed assignments out of the 3rd grade textbook, however, the example problems reviewed during the lessons came from the 4th grade “Reteach” resource book. The students also received homework from the “Reteach” book.

Teacher Knowledge

As stated in Case One, one component of the Mathematics Teaching Cycle (NCTM, 2007) was Knowledge. According to the Teaching Principle (NCTM, 2000), an effective teacher needs knowledge in: “mathematical content, pedagogy, assessment strategies, and an understanding of students as learners” (NCTM, 2007, p. 19). These four criteria for “an effective teacher” aligned with Ball et al (2008) MKT components of Common Content Knowledge (CCK), Specialized Content Knowledge (SCK), Knowledge of Content and Teaching (KCT), and Knowledge of Content and Students (KCS). In the following section, I used the data representative of the entire data set gathered from the Teacher Knowledge Assessment and follow-up interview to illustrate the degree to which the components outlined in the Teaching Principle and the MKT framework presented in Teacher 2.1 and Teacher 2.2 and how the presence or lack of some criteria might account for different student gain scores.

Example 1 From the Teacher Knowledge Assessment: A Multiple Choice Item

Because the LMT items are not released to the public, the following is a description of a task given to the teachers participating in the NSF-funded Grant. The task was a multi-digit subtraction problem that included regrouping of the minuend. The digit “0” was in the tens place of the minuend as well. The teachers were given three hypothetical students’ responses to examine. The task asked the teachers to state whether or not each response was acceptable evidence indicating that the child knew why the procedure worked. The teacher had the option to say that they were unsure if the answer was acceptable or not.

The first response (a) was a direct description of the steps taken to solve the problem. The response was void of place value or explanation regarding regrouping. The second response (b) indicated the child could decompose numbers and regroup fluidly across place value positions. The final response (c) was tricky for most of the teachers. The student indicated some understanding of place value at a superficial level.

Teacher 2.1's response. Unfortunately, Teacher 2.1 never returned any correspondence regarding the follow-up interview to the teacher knowledge assessment. There are a few possibilities as to why Teacher 2.1 never participated in a follow-up interview. First, she expressed repeatedly during our first interview that she had limited time to plan or do anything school related because she only received a planning period once every six days when her students went to P.E. Second, it is possible that Teacher 2.1 did not want to talk about her answers to the teacher knowledge assessment. Lastly, Teacher 2.1 retired at the end of the school year and might have had other obligations after school that took up her time. Therefore, the only data I have for Teacher 2.1 regarding the teacher knowledge assessment was the written test and the answers she provided on that test.

Looking solely at the written test, Teacher 2.1 one point for this particular problem. She left answer choice (a) blank. She responded "Yes" to answer choice (b) and responded "Yes" to choice (c). The only correct answer was the "Yes" to answer choice (b).

Teacher 2.2's response. Teacher 2.2 received zero points for this question. She did not provide an answer for (a). She said "no" to (b) and "yes" to part (c). During the interview, she explained "I didn't know what they were talking about, hahahah!" (Teacher 2.2, 2nd Interview) when asked why she left answer choice (a) blank. I cannot say why she laughed at the end but it did not sound malicious. The laugh seemed more out of nerves or embarrassment for not understanding the response, however, this is speculation and not grounded in explanation provided by Teacher 2.2.

When looking at choices (b) and (c), Teacher 2.1 left the answer choices as written and further explained "yeah, I put I understand they understand grouping but I would also ask them why they regroup to kind of get that questioning like why? Why did you guys regroup? Why do we regroup when we subtract?" (Teacher 2.2, 2nd Interview) when she read (b) this time around. She also explained that she would accept the response in (c) but again wanted to know why they had "to borrow" (2nd Interview, 4/11/12).

The response given by Teacher 2.1 demonstrated Knowledge of Content and Teaching. Knowing when to ask a follow-up question or ask for more clarification, which is what she was doing when probing about why the children wanted to regroup, were characteristic of Knowledge of Content and Teaching. It was unclear as to what this information was going to inform or how she was going to use it but she was the first participant to want to get clarification for some of the "answers" provided by the students.

Example 2 from the Teacher Knowledge Assessment: An Open-Ended Item

The following problem was given to the teachers to assess their knowledge of relational thinking in students. The teachers were provided with half of a page to write or illustrate their responses. To receive total points for the problem, the participants had to answer all three sub-questions (Carpenter et al, 2003).

> *Assume we gave this problem to some elementary school students:*
> *What number can you put in the box to make this number sentence true?*
> *8 + 15 = ___ + 16*
> *What strategies would you expect students to use to solve this problem?*
> *What answers do you expect them to come up with?*

Teacher 2.1's response. Teacher 2.1 wrote the following response to this question on her test: "I would have the children add the 8 +15, which is 23. Either tell the children to subtract 16 from 23 for missing addend or ask children what could you add to 16 to make 23?" (Teacher 2.1, Teacher Knowledge Test response). Based on this response, her thinking was characterized as Common Content Knowledge. Teacher 2.1 demonstrated an ability to correctly solve the problem. She also used correct terminology: "missing addend." She provided two methods for solving for the unknown: a) find the difference between 16 and 23 or b) Count up from the smaller (Carpenter et al, 1999).

Teacher 2.2's response. Teacher 2.2 answered on the test that would answer the question with "7." Her written response was a little confusing. She wrote "1) My students would think of order of operations. 2) They would write PEMDAS. 3) Then figure out what to do first." During the follow-up interview, Teacher 2.2 expressed that she did not know why she wrote down order of operations. She proceeded to say, "I probably would have told them to solve it first…8+15 and then, so whatever your answer is. I did it here [points to the top of her test paper]; I don't know why I put order of operations. That has nothing to do with it." (Teacher 2.2, 2nd Interview). It is clear that Teacher 2.2 recognized that the order of operations was not necessary in this problem.

What she did provide, however, was a reliance on Knowledge of Content and Students through detailed explanations of how her students would directly model their thinking using pictures. On her test paper, she wrote "4) some would add the 8 + 15 and come up with an answer. Some will draw. My kids love to draw and it helps them a lot [she included an illustration of 8 circles + 15 circles]" (Teacher 2.2, 2nd Interview).

The written test answer to this second test question raised an issue, which I followed up with during her interview. Teacher 2.2 stated, "Some would add 8 +15 and come up with an answer." As written, it seemed like Teacher 2.2 thought the answer to 8 + 15 belonged in the blank and that she disregarded the "+ 16" on the right side of the equation. This would indicate a misunderstanding in her Common Content Knowledge. In the follow-up interview, Teacher 2.2 clarified this confusion with a description of her teaching method. "They would solve first and then so then I would say we want to know what needs to go in here, yeah, to make it true. I would have to subtract and then I like do a lot of drawing the pictures" (2nd Interview). This explanation characterized a mix of Knowledge of Content and Students and Knowledge of Content and Teaching. She anticipated that her students would solve the left side of the equation first. Then she described her understanding that this equation was a true/false type of question (Carpenter et al, 2003). Again, she anticipated that her students would model the subtraction needed to make the sentence true.

Comparison of the Teachers' Responses Across the Test Items

Due to the fact that Teacher 2.1 did not participate in a follow-up interview, this comparison was based on mostly what was written on the test paper, unless Teacher 2.2 provided extra information during her post-test interview.

Teachers 2.1 demonstrated Common Content Knowledge on the written test. She For example, she knew that "1 hundred + 119 tens + 1 one" was not equal to "391." She also knew that "3 hundreds + 9 tens 10 tenths" was equal to "391." She also knew that 5/9 was greater than 3/7 because "5/9 is greater because it is more than one-half, while 3/7 is less than one-half" (Teacher Knowledge Assessment question 3 answer choice E).

Teacher 2.2, in contrast, demonstrated limited Common Content Knowledge on this same task. She thought that "1 hundred + 119 tens + 1 one" was equal to "391." When asked about this question during the follow-up interview, Teacher 2.2 avoided parts (b), (c). She explained part A first:

> *so for A, um, she put 300 and the 90 tens, um that's when I circled it and I said well you would need 90 tens to make 90? And then I would ask them okay, well if you put 90 tens, what would that equal? To, to kind of get them to see. So in this case, I would pull out the manipulatives and have them figure it out (Teacher 2.2, 2nd Interview).*

This excerpt demonstrated her Common Content Knowledge, Knowledge of Content and Students, and Knowledge of Content and Teaching. Teacher 2.2 demonstrated that she understood that 90 tens did not equal 90. She acknowledged that her students had a misconception about place value and what 90 tens represented. She explained that she would resolve this misconception using manipulatives and having the students physically figure out what 90 tens represented.

After this explanation of part (a), Teacher 2.2 talked about one of her students using this method of demonstrating that 90 tens was more than 9 tens in class. She immediately skipped to part (d) and explained that she would have the students use manipulatives to show her ten tenths. She never responded to part (b) or (c) and due to time and courtesy, I moved forward with the interview because we have 9 questions left to address in a 20 minute lunch period. It was possible that Teacher 2.2 did not understand part (b) and (c) and therefore avoided talking about them. It was also possible that Teacher 2.2 recognized that she would use the same technique to help students understand what ten-tenths represented and, thus, jumped to part (d). What was interesting was that Teacher 2.2 did not recognize that part (d) was a correct answer. Again, demonstrating her limited Common Content Knowledge.

Teacher 2.2 struggled "sizing up" (Ball et al, 2008) whether or not a nonstandard approach to problem solving would work in general (a characteristic of Specialized Content Knowledge). For example, she marked that a method where a child solved the problem 35 x 25 by doing (5 x 25) + (30 x 25) instead of the standard method of (5 x 35) + (20 x 35) would not work for all whole numbers, when it would.

She also had difficulty with the problem that asked her assess fraction word problems. She received zero points for a question that asked: *Mrs. Wise wants to include some word problems on her fractions quiz. Which of the following problem(s) could she use as a word problem for ½ - 1/3? (Mark YES, NO, or I'M NOT SURE for each one.)* Teacher 2.2 marked yes for the incorrect word problems: *I have ½ of a pizza left. My brother comes in and eats 1/3 of my leftover pizza. How much pizza is left?* Teacher 2.2 marked no for the correct word problems: *Mom has ½ of a cup of sugar. She needs to use*

1/3 of a cup of sugar to make some brownies. How much sugar will Mom have left? Without the follow-up interview, it cannot be established what aspect of these types of problems were most difficult for Teacher 2.1 or what she understood about the word problems.

Teacher 2.2 also struggled with these problems and did not have an answer for how to solve them during her follow-up interview. Instead, Teacher 2.2 provided examples of how her students would solve the problems on the test. For example, for the fraction problem that compared 5/9 to 3/7, she drew a picture of the two fractions and knew that 5/9 was larger, as her students would do in class. She also explained that her students would use cross-multiplication to solve the problem. "I have found that easier to teach it that way [using cross-multiplication]. I mean I like for them to see it where I say draw it and check so I kind of make them do that part first and then check. But I find that most of the time, even I have trouble [drawing it]" (Teacher 2.2, 2nd Interview). These types of explanations, ones that depicted how her students would solve the problems or how she would teach the concept, were representative of how Teacher 2.2 answered most of the teacher knowledge assessment. Her main focus was on what she knew about her students and her teaching method.

Looking at the responses, it was apparent that Teacher 2.1 and Teacher 2.2 had strengths in different areas, although this assessment was limited since Teacher 2.1 never had a follow-up interview. It was apparent that Teacher 2.1 demonstrated strong Common Content Knowledge on the teacher knowledge assessment than Teacher 2.2 but Teacher 2.2 grounded all of her answers in her students' thinking and her teaching methods. Without an interview with Teacher 2.1, I cannot say if this were the case with her thinking. The next section examined the two teachers' planning process and the influence of MKT on planning.

Analysis, or Planning, of Instruction

Teacher 2.1

Teacher 2.1 presented an interesting case during her planning interviews. Even though the questions asked were the same ones as with all other participants, Teacher 2.1 focused the majority of the interview on a few themes: Time constraints and Isolation. Based on the focus during the interview, the format for this description of how Teacher 2.1 plans differed from the structure of the descriptions in the first case. After describing the general lesson structure and how it was planned, I examined the main themes in the interview.

Teacher 2.1 described her classroom structure as having two separate grade levels in one classroom. "I did try that [working with all of the students to cover one topic and then group them in mixed-grade level groups to completed problems]. In fact, I had the 5th graders divided up also so that they could be in a group so that they could be sort of the elder, elder and the more knowledgeable. That didn't work" (Interview). Since her attempts to group students in mixed grade levels were unsuccessful, she reverted back to teaching the 4th and 5th grade independent of each other. She determined the sequence of her instruction from the district curriculum map. She planned on a weekly basis. At the start of each week, she looked at the 4th grade district curriculum map to see what was expected each day and matched textbook lessons to the standard. For the 5th grade lesson plans, she received those on a weekly basis from the other 5th grade teacher.

The evidence of Mathematical Knowledge for Teaching in the planning of Teacher 2.1 was less apparent in her interview than in either Teacher 1.1 or Teacher 1.2. Instead, issues of management and survival due to time constraints and isolation saturated her responses. Teacher 2.1 explained that in the past, the school district set up the schools on a six-day schedule based on the rotation of specials (i.e., art, P.E., computers, library, music, etc.) offered at the schools. During the 2011-2012 school year, budget cuts limited the amount of specials the students attended. Of the six original specials only one was kept: Physical Education (PE). Students attended PE once every six days. The other days of the week, the teachers kept their students in their classrooms and continued teaching. As a result, the teachers' planning time was eliminated except for the one day students went to PE. "We have only one prep every six days" so "we don't have time to plan together…seriously…seriously" (Interview).

To account for the lack of planning time, the 3rd, 4th, and 5th grade teachers divided up the lesson planning. Each was responsible for the plans of their respective grade levels. The plans were dispersed among the teachers who taught multiple grade levels at the start of each week. In the twenty minutes we talked about planning, Teacher 2.1 used the following phrase, or a slightly varied version of the phrase, four different times: "yeah, so that's how we do it. We use the state standards and then we and then [another teacher] makes the plans for the 5th, I find the 4th grade, what lessons go with that PO and then…" (Interview). What happened as a result of the lack of joint planning time was isolation, both in teaching and in developing a teacher-learning community at the school.

A major drawback of the time constraints for Teacher 2.1 was the sense of isolation she felt both inside and outside of the classroom. For starters, the lack of planning time equated to less time with her colleagues. She no longer had a place to gather ideas for teaching difficult topics, such as teaching integers to the 5th graders (Teacher 2.1, interview). She also found herself less connected to the community of teachers at her school. In fact, she had no idea how one teacher was planning or instructing his combination classroom (Teacher 2.1, interview). Even during district grade level meetings, she found herself unable to utilize the time in a meaningful manner. Instead of discussing the curriculum, the teachers charted their students' performance scores. She found this unhelpful for planning her daily lessons. ". . . no but that's how we do it and the thing is we used to plan together but we don't have time so all that happens right now at our grade level collaboration is we go through the charting of our data charts, you know we get that, and you know we are not even together on that because [another teacher] is teaching 3rd and 4th grade, so I don't know what he's doing. So I don't know if he's going with the 3rd, if the 3rd, if they are doing 3-4, if he's going to the 4th grade standards or if he an [another teacher] are, I don't know" (Interview).

In the classroom, the lack of collaboration time impacted how Teacher 2.1 dealt with having 4th and 5th grade students. She found herself teaching the grade levels independent of each other. She tried multiple times to combine the two grade levels but that proved too difficult and, therefore, she taught the students in isolation. This separation resulted in one grade receiving direct instruction while the other grade level worked alone on an activity that required little assistance from her. For example, she anticipated difficulty in teaching integers to the 5th grade students, so she planned for the

4th graders to work on transformations. "So on that one I would look to see what thing I could have them review or to practice or to go over because I can't, integers is going to be hard enough for me to teach without um having to do two, see understanding? I'm putting them on transformations, so they are going to be doing, on tessellations and stuff. They are pretty good with that so I am just sort of giving them that, just giving them a very broad thingy. They have done it before and then, hopefully, they will be able to work that lesson by themselves while I go [I: is it something that they could just create their own tessellation pictures?] yeah, they can use their book. I will try to find something that will keep them occupied and busy" (Teacher 2.1, Interview).

Locating characteristics of MKT was difficult when analyzing this interview because of the overwhelming amount of discussion about the time constraints at her school. In general, there were flashes of Knowledge of Content and Students. She was able to anticipate lessons that her students could work on independently. She was able to anticipate concepts where students needed her full attention to help them succeed (i.e., as shown in the comment about teaching the 5th graders integers, while the 4th graders worked on tessellations independently). It is possible that Teacher 2.1 had limited ways to access her Knowledge of Content and Teaching or Knowledge of Content and Curriculum and that caused the difficulty in spotting MKT characteristics in her planning. It is also possible that the fact that 2011-2012 was her last year teaching before retirement impacted her drive to find ways to teach a combination class. Unfortunately, without an interview or follow-up discussion, these possibilities are only speculation and not evidence based. Next, I examined how MKT presented itself during the planning interviews with Teacher 2.2.

Teacher 2.2

In general, the planning process described by Teacher 2.2 focused on integrating the various programs adopted by her school administration in a way that best served her 3rd and 4th grade combination classroom. For example, her lesson plans were based on the district curriculum map. She deviated as little as possible from the curriculum map. She also utilized Bloom's Taxonomy when deciding the types of questions (not tasks, as was the strategy of Teacher 1.1) to ask her students when teaching. The addition of Bloom's Taxonomy in her teaching came from a new professional development program her school adopted that year. This new program taught the teachers to use student performance data to inform their teaching. In addition, the program encouraged using new strategies, like Bloom's Taxonomy, to get students to think at higher levels (Interview, 3/7/12). Teacher 2.2 also integrated her background knowledge on teaching native Spanish speakers into her classroom planning. She stressed vocabulary and using cognates (Interview, 3/7/12) to help her students understand how certain words were similar across both Spanish and English.

Teacher 2.2 taught a 3rd and 4th grade combination class. For mathematics instruction, however, the 3rd grade students attended the lesson in the classroom of the teacher who only teaches 3rd grade at the school. Therefore, Teacher 2.2 planned lessons based on the 4th grade curriculum map. One setback was that she does not have the 4th grade adopted curriculum; she only had the 3rd grade textbooks and the "reteach" book from the 4th grade supplemental materials. Using Bloom's Taxonomy and the 3rd grade materials, she figured out "how she is going to challenge the kids because they're using the 3rd grade stuff. They've already seen it so then I have to challenge them and ask them questions at a 4th grade level" (Interview, 3/7/12).

Next, Teacher 2.2 decided her examples for the "I do" and the "We Do" sections of her lesson. The "We Do" was her main focus because it was what the whole class completed together. She explained the "We Do" part of the lesson to be very helpful for her ELD students. "Yeah, so a big part of our lesson is the 'We Do'. A big part of my less is the 'We Do' and that's just you know because they are the ELD class and they have to have that extra you know reviewing and the vocabulary and the understanding it" (Interview, 3/7/12). After the "We Do," she assigned a "Ticket out the Door," which was a question (not predetermined) that reviewed the objective of the day. The name of the informal assessment was a misnomer because it actually was used as a culminating activity of the guided practice, rather than as the final activity of the math instructional time. The answers to the ticket out the door were used to determine who to pull into a small group during independent practice time, also called the "You Do" time in the lesson.

Teacher 2.2 also completed questions on the Math Wall with students on a daily basis. She based the five questions for the math wall on the 4th grade level questions from Study Island because that was where the weekly test questions were pulled from. In theory, she was supposed to work through all five questions in 15 minutes with her students; however, she chose to complete one strand question a day to ensure that her students understood the concept being reviewed. She found that over time the test scores increased. She felt that the spiraling nature of the math wall contributed to the increase

Table 9

MKT Categories in the Planning of Teacher 2.2

MKT category	Number of excerpts	Description of excerpts
Knowledge of Content and Curriculum	8 total excerpts	• 5 excerpts described her knowledge of the available resources in her district to use for planning • 3 excerpts were a critique of a program implemented across the district to help spiral the standards throughout the year
Knowledge of Content and Teaching	7 total excerpts	• 2 excerpts described when Teacher 2.2 decided to pose a new task to the students or ask a new question to the class • 1 excerpt described how she sequenced instruction on the Math Board • 1 excerpt was an evaluation of the advantages and disadvantages of using a mix of multiple choice and open-ended questions on the math board • 1 excerpt demonstrated her knowledge of when to use small groups as a method for revisiting misunderstood content • 1 excerpt demonstrated an ability to decide when to ask a child for clarification • 1 excerpt showed her method for choosing examples
Knowledge of Content and Students	1 total excerpt	Teacher 2.2 explained that she anticipates what students will do based on past teaching experiences
Horizon Content Knowledge	1 total excerpt	Teacher 2.2 described briefly how the 3rd grade curriculum matched the 4th grade curriculum
Specialized Content Knowledge	1 excerpt	In this excerpt, Teacher 1.2 explained what aspect of multi-digit multiplication her students found difficult. She broke apart components in multiplication and explained which part was tricky
Knowledge of Content and Students – Knowledge of Content and Teaching	4 total excerpts	• In one excerpt, Teacher 2.2 describes how she anticipated that a particular lesson was going to be easy for the students (KCS) so she changed the sequence of her instruction (KCT) • 1 excerpt showed how Teacher 2.2 understood common errors made by students (KCS) and structured the instruction to address the errors in a beneficial way for ELL students (KCS) • In 1 excerpt, Teacher 2.2 anticipated what students will think about a particular task (KCS) and sequences instruction to challenge the students (KCT) • The last excerpt exemplified her familiarity with language issues having been a ELL student herself (KCS) and strategies that best helped students work through the problems (KCT)
Knowledge of Content and Teaching – Knowledge of Content and Students – Specialized Content Knowledge	1 total excerpt	Teacher 2.2 described how she chose examples to teach her students using her Bloom's Taxonomy Wheel (KCT). The students were asked to analyze a graph. She anticipated that students might not understand or know how to "analyze" per se (KCS). For her teaching of this concept, she unpacked the language in the question using examples and familiar synonyms to "analyze" that the students knew (SCK)

because the kids were constantly reviewing material that they might otherwise forget how to solve.

Due of time constraints, Teacher 2.2 participated in one planning interview instead of two. In that one interview, 22 excerpts pertained specifically to planning.

Interestingly enough, the planning interviews for Teacher 2.2 reflected a dominance of Knowledge of Content and Curriculum (KCC) and Knowledge of Content and Teaching (KCT), neither of which is assessed in the Teacher Knowledge Assessment. It is possible that had such items been developed for the assessment, Teacher 2.2 might have a higher score relative to her peers in the larger study. It is also possible that her reliance on the district policies and supported teaching methods impacted the types of knowledge that appeared in her interview. It is also possible that her reliance on the district adopted curriculum and methods stems from the fact that this was her 4^{th} year teaching and is still figuring out her way among all of the changes in the state, the district, and at her school over the last three years policy- and administration-wise.

Overall, the planning of Teacher 2.2 reflected that of a new teacher who understood the district resources and what was expected of her in terms of using the resources (I.e., curriculum map, textbook, adopted professional development programs, and the Math Wall). She described the district recommended method for designing lesson plans (i.e., find the standard from the curriculum map, find the textbook page, decide the I do, the We Do, the You Do problems, decided the questions based on Bloom’s Taxonomy, etc.). She also demonstrated a willingness to try new ideas with her students, even if she was skeptical of the results. For example, at first she hesitated to state the usefulness of the Math Wall. She explained that over time she saw the benefit of the math wall for pre-teaching and reviewing purposes. She noticed that her students were ready for the mode lesson because the students had learned mode earlier in the year on the Math Wall. The previous year it took two days to teach mode but this year she was able to teach mode and range together because mode had been introduced on the Math Wall. As for reviewing, she noticed that “scores had gone up because it’s, they are constantly reviewing it, it’s just not taught you know that first quarter and then you know that’s it. That’s where it stays and then you get tested on it later on in the 4th quarter. And you know they are going to be like well I don’t remember learning this” (Interview, 3/7/12).

In summary, Teacher 2.1 and Teacher 2.2 received z-scores on a teacher assessment test above and just below the mean respectively. Teacher 2.1 demonstrated high common content knowledge for their subject matter, knowledge of how to break down the mathematics being taught, and, for the most part, was able to decipher non-standard methods of problem solving on that test. Teacher 2.2 demonstrated these skills as well, on her teacher knowledge assessment, but there were aspects of common content knowledge, breaking down the mathematics, and in deciphering students’ non-traditional methods that Teacher 2.2 did not know.

Discussion of Difference in MKT when Planning

What do these results mean practically? How does this knowledge translate to what they do as teachers? For Teacher 2.1, the impact of her high-middle range MKT score was difficult to figure out when analyzing her planning interview. The overwhelming frustration and sense of loss created by the time constraints around her planning time embodied so much of the interview, her actual knowledge of teaching or how she broke down the content for her teaching, or even how her students' knowledge base affected her planning never really became transparent. For Teacher 2.2, it was possible that her lower MKT knowledge impacted her choice to rely on the district adopted text and curriculum map for planning. Her lower knowledge might have kept her from branching out to find alternative resources for helping her teach conceptually or to be able to critique the questions in the textbook to see what would best suit the needs of her students. Unlike other teachers, Teacher 2.2 never discussed the breath of questions on the AzAC test or the state standardized test pertaining to the content she taught, such as fair sharing versus measurement division, or understanding how important it is to keep units the same throughout a fraction problem. Again, this might represent her shortage of specific types of MKT knowledge.

One interesting theme that appeared between these two teachers was how they dealt with teaching a combination classroom. Both teachers isolated the two grade levels from each other in one-way or another. They each taught the grade levels separately. In fact, Teacher 2.2 explained that she sent her 3rd grade students to another teacher for much of the math instructional time. Teacher 2.1 described how she separated the groups and planned independent activities for the grade level students not being directly taught by her at a particular time. It is interesting to note that neither one of these teachers had a degree in bilingual education. Instead, they had a Structured English Immersion (SEI) endorsement, which is required of all teachers. It is possible that part of their struggles with the combination classroom were due to a lack of knowledge of how to teach mathematics to English Language Learners. It is also possible that this lack of knowledge impacted their overall MKT scores, seeing as how MKT does not take into account the learning environment of a classroom and how teachers negotiate that aspect of teaching.

In the following section, I moved to explaining the classroom instruction of Teacher 2.1 and 2.2. I examined the interactions that occurred during instructional times (Pianta et al, 2008), the cognitive demand of the tasks (Stein et al, 2009), the types of questions asked by the teachers (Bloom, 1956), and the types of responses provided by students to both the teacher and each other.

Implementation

As seen in Case One, the Implementation Phase of the Mathematics Teaching Cycle included: the learning environment, selection of meaningful tasks, and discourse. For this section, I presented the entire Implementation Phase components for each teacher and then compared the two teachers. Using this format for discussing the instruction component seemed more comprehensive and coherent than jumping between the components and the teachers.

Teacher 2.1

The instructional practice of Teacher 2.1 was evaluated using three methods: the CLASS protocol, the Mathematical Tasks Framework, and Bloom's Taxonomy. Teacher 2.1 provided evidence in her instruction of a high reliance on Common Content Knowledge with a little use of Specialized Content Knowledge. This type of knowledge was evident in her implementation of the tasks she gave the students and in the types of questions she asked the students during class discussions. Her teaching style was very systematic and orderly. Much like Teacher 1.2, Teacher 2.1 stood at the front of the class and solved problems from the adopted textbook. Occasionally, her instruction changed based on students demonstrating a lack of understanding. Sometimes students were called to the board to show a different way to solve a problem but mostly if the children did not understand a concept, she yelled a little louder or provided another example. The following sections provided the evidence to support the claims made in this introductory paragraph.

The learning environment.

CLASS observation protocol. Six CLASS observations were conducted with Teacher 2.1 between October 2011 and February 2012. The following Table shows the average scores across the 10 dimensions for Teacher 2.1.

Table 10

CLASS Dimension Scores for Teacher 2.1

Dimension	Average Score
Positive climate	4.33
Negative climate	3.33
Teacher sensitivity	2.67
Regard for student perspectives	2.67
Behavior management	4.67
Productivity	3.83
Instructional learning formats	3.167
Concept development	1.5
Quality of feedback	2.5
Language modeling	2.67

For the most part, Teacher 2.1 received average scores ranging from 1.5 to 4.67, with most of the dimension scores being between 2 and 3. She scored a middle range score for both Behavior Management and Positive Climate. These were also her highest score. Teacher 2.1 had a relatively high Negative Climate score of 3.33. While the score of 3.33 fell in the low-middle range, it was indicative of some negative interactions in each of the six observations. Her lowest score was in Concept Development.

When composite scores were calculated for the three CLASS domains, Teacher 2.1 received the lowest scores on two of the three domains (Emotional Support and Classroom Organization) for all six participants in this dissertation. She scored a 3.59 on Emotional Support, a 3.89 on Classroom Organization, and a 2.22 on Instructional Support. Again, the scores based on a scale of 1 to 7, with 1 as the lowest score and 7 as the highest. As we can see by the scores for Teacher 2.1, the interactions pertaining to the Emotional Support were on the low end of the middle range of scores. On average, the climate in the classroom was sometimes positive and sometimes negative, the teacher was aware and responsive to her students' needs but there was not a lot of support for student leadership or autonomy or student expression. While there were many positive communications in this class between students and the teacher, there were also times where the teacher directed punitive control over the students and severe negativity. This was indicated in the Positive and Negative Climate dimensions being almost exactly the same. This classroom was also predominantly teacher-driven with little room for student expression or autonomy and leadership. Many times, the teacher was unresponsive to students who did not understand or who answered a question incorrectly. At these moments, the teacher frequently raised her voice and redirected the question to the entire class.

The CLASS data also reported that the Classroom Organization was in the middle range. The teacher was fairly proactive in this classroom and able to redirect any misbehavior quickly. She gave quite a bit of proactive praise to the students and gave clear directions during each lesson. Her lack of preparation for the lesson was seen at times when she took students down unnecessary pathways to answer a task that required students to combine fractions into wholes and find a total. She was also took a lot of time to collect homework and then scold a child for not making sure that everyone's name was on the paper when they were collected. She was very good at reviewing the lesson objective daily. She also made the vocabulary in the objective transparent to the students through examples of the word, or root word, in everyday life. The students showed a lack of interest in their work often through facial expressions, side conversations, and not following along with the instruction. There was also only one occasion where students were not taking notes or answering questions with paper and pencil. On this occasion, they used fraction tiles to compare fractions. Otherwise, there was little variety in the materials and modalities used in the teaching (Pianta et al, 2008).

Lastly, the composite domain scores indicated that there was low Instructional Support in this classroom. Instructional support included: Concept Development, Quality of Feedback, and Language Modeling. The scores showed that very little analysis and reasoning occurred over the six CLASS observations. Scaffolding of any of the mathematics happened in the form of giving the steps to a procedure, which is not exactly scaffolding as intended by Vygotsky and the Zone of Proximal Development (Vygotsky, 1978). The conversations consisted of one-word answers from the students and very little conversation. The teacher modeled how she solved the mathematics problems but it was not in the form of self-talk or language modeling. It was very procedural. The teacher rarely revoiced students' answers or extended on their thinking. The majority of the classes showed the students copying the teacher's work onto their paper and then giving brief responses when called upon (Pianta et al, 2008).

Overall, the CLASS observation protocol depicted a classroom that was very teacher-centered and procedurally driven. These scores, in a few ways, align with the planning interview and the Teacher Knowledge Assessment scores. First, Teacher 2.1 received a score that was three-quarters of a standard deviation above the mean relative to the other NSF-funded grant participants. When the test items were individually examined, Teacher 2.1 struggled with deciphering non-standard algorithms used by students and with unpacking what the student was thinking either when drawing representations of ½ +1/3 or subtracting multi-digit numbers with regrouping, or explaining the steps used to solve a multi-digit subtraction problem with regrouping. These difficulties pertaining to Knowledge of Content and Students might explain why there the scores for Regard for Student Perspectives, Concept Development, and Quality of Feedback fell into the low range. If a teacher has difficulties hearing and interpreting students' emerging and incomplete thinking as expressed in the ways that pupils use language, his or her ability to anticipate what students are likely to think and what they find confusing, and this, in turn, might impact how a teacher presents information. Not understanding or knowing how students think might either stem from teacher-centered instruction or it might prevent teachers from moving into a teaching style that encourages student exploration or autonomy. It is also possible that this restricted knowledge base might be an indicator of why Teacher 2.1 focused on time constraints and disjointed planning during her interview and neglected to respond to my in-person and email communications to set up a second and third interview. If Teacher 2.1 had little knowledge of student thinking because her teaching style did not lend itself to utilizing student thinking, she might not use much of her knowledge outside of Common Content Knowledge or basic knowledge of selecting examples and sequencing instruction (KCT). Talking about her planning based on student thinking would be quite difficult. Next, I examined the cognitive demand of tasks given by Teacher 2.1 (Stein et al, 2009).

Worthwhile mathematics tasks and classroom discourse.

The mathematical tasks framework (Stein et al, 2009) for teacher 2.1. Over six observations on Teacher 2.1, sixty-seven written tasks were given. All of the tasks were either memorization tasks or procedures without connection tasks. The range of tasks assigned was between 3 and 30. Overall, there were a total of 42 memorization tasks, and 25 procedures without connections tasks (Stein et al, 2009). Table 11 exemplifies the types of tasks assigned by Teacher 2.1.

To best understand how Teacher 2.1 implemented the written tasks and how the implementation related to MKT, I used the following memorization task that was given on November 28, 2011 to the 5^{th} grade students: *Is 3/6 greater than or less than 2/8?* This task came out of the adopted textbook. It was an "example" problem that was solved in the textbook using pictorial representations of base ten blocks.

The fifth grade students were seated at the front of the class facing the chalkboard. Teacher 2.1 stood at the board with her textbook in hand.

> *Teacher 2.1 eyes up here. Yes, up here. I'm going to do an "I do," then it's going to be a "we do" and then it's going to be a "you do" do. Okay? Ready? Eyes up here. Ready? Alright so if we have three six and we want to know if three-sixths. We are going to start easy and go from there. Is 3/6 bigger or less than 2/8? Now there's a couple of ways of doing this. Can anybody figure out one way of doing this? How would I do this? George, eyes up here.*

Table 11

Cognitive Demand Level of Tasks Given by Teacher 2.1

Classification of task	Example	Explanation
Memorization	"3/6 is greater than or less than 2/8" (classroom observation, 11/28/11) and "9 x 3 = 3 x W" (classroom observation, 10/31/11)	As written these two examples "have no connection to the concepts or meaning that underlie the facts, rules, formulae, or definitions being learned or reproduced" (Stein et al, 2009, p. 6) and "they involve either reproducing previously learned facts, rules, formulae, or definitions OR committing facts, rules, formulae, or definitions to memory" (p. 6).
Procedure without Connections	"892 x 37" (classroom observation, 11/14/11). "Find the GCF of 14 and 16."	These two tasks, as written, "are algorithmic. Use of the procedure is either specifically called for or its use is evident based on prior instruction, experience, or placement of the task" (Stein et al, 2009, p. 6). There is a limited cognitive demand required to complete these tasks and the focus was "on producing correct answers rather than developing mathematical understanding" (p. 6).

Kid: you could draw them out.
Teacher 2.1: I could draw them out. But, put them over there! But when I draw them out, I have to be very very exact and speaking of drawing out [kids says something] what? We're not waiting for them; I don't know where they are. They're in the office? Are they in the office? Does anyone know where Liz and Jonas, I mean Liz and Ariel and ah Silvia are?
Kid: Liz's right there
Teacher 2.1: Ariel. Where's Ariel? Where's Silvia?
Kid: Ariel went with Ms. Lopez
T: do you know why? Okay.

At the start of this lesson, Teacher 2.1 began to ask the students how she might solve the problem. A student told her to draw out the pictures. Teacher 2.1 explained to the child that draw pictures required careful and exact drawings. The lesson immediately got side tracked when students realized their peers were missing. Teacher 2.1 clearly did not know where the children were and demanded answers from the students in the classroom. It was evident that Teacher 2.1 worried about the students once she realized they were gone but there was a sense of awkwardness in the students reactions, especially when Teacher 2.1 asked about a child sitting in front of her. These sorts of interactions were captured in the CLASS observation protocol under "Negative Climate."

Teacher 2.1 provided the students with fraction bars from the kits that came with the adopted textbook.

> *Teacher 2.1: alright now eyes up here so we are looking at a fraction. When we look at a fraction what's this part of a fraction called? What's this called? It is called the numerator. What is it called?*
> *Kids: numerator*
> *Teacher 2.1: what is it called?*
> *Kids: numerator*
> *Teacher 2.1: what is this called?*
> *Kids: denominator*
> *Teacher 2.1: so the denominator tell you eyes up here, the denominator tells you how many parts something is divided into. So I want you to find your fraction bards that are the sixths. So go find the sixths. Open them up be careful because you are responsible for them. Okay so you have your sixths so get your sixths out . . .*

Using basic knowledge questions (Bloom, 1956), Teacher 2.1 quizzed the students' knowledge of fractional parts. She then gave the students the definition of the denominator and told them to get out sixths (CCK – using correct mathematical terms; being able to solve the problems assigned to students). She proceeded to tell the students how to set up their fraction bars to make a comparison and what a drawing of their bars might look.

> *Teacher 2.1: so get your eighths . . . okay once you get your sixths and eighths together I want you to put them in a row, nice little, so put...you know what don't worry about these. Get your sixths and your eighths together. Look back in your box to see if you can find. Did it drop? Move your backpack . . . get your eighths together and your sixths together. You're missing one? There it is. Now get your sixths together. Don't worry about these right now . . . [00:11:14.18] so your sixths and your eighths should be very close together like right underneath together. Perfect. So when, when you are drawing something, hurry up let's get them in order . . . okay now this is exactly the way that I wanted it, I wanted it separate so that you could see it away from the rest of the, ah the rest of those fraction parts. Now, look up here. Look up here. When you are. Eyes up here. When you say draw it. If you are going to draw it. That means the length of each of them as to be exactly the same. So if I would want to draw this I have to make sure that this length is the same as this length and then I would divide this into what? Sixths? And this into eights? You have already gotten that. It's already been divided because, hurry. We are using the fraction bars. Okay,*

In this monologue, Teacher 2.1 briefly explained that fractional pieces needed to be equidistant (CCK – knowledge of how to solve the mathematics of 5th grade) but there was an absence of why the length were the same and any discussion of how the pieces related to a whole unit. Teacher 2.1 maintained control of the activity and thus maintained the low-level cognitive demand of the task (see Appendix E – maintenance of low-level cognitive demand – teacher takes over the task, no time for students to puzzle through the task).

> *T: now take three-sixths down okay so there's three-sixths and there is two eighths so my question is this. Is three-sixths greater than, less than, or equal to two eighths? Look at your thing. Is 3/6 greater than, less than, or equal to two-sixths? Two-eighths? Alright I don't know what you are looking at children! Here is 3/6 and here is 2/8. My question is this is 3/6 greater than, less than, or equal to two eighths?*
> *Kids: greater*

T: it's greater than. Do you see it? There's the two eighths. There's the 3/6, which is bigger? Which is bigger? Three sixths? Soo 3/6 is greater than 2/8. Do you see that? Do you see that?
Kids: yes
T: good okay.

Teacher 2.1 directed the entire learning in this task implementation. There were a few moments in the episode where Teacher 2.1 could have drawn out conceptual understanding. For instance, instead of asking the students why drawing fractions was sometimes problematic and how to ensure they were accurate when drawing, she told them each length had to be the same length and then moved back to the fraction tiles. Even when using the fraction tiles, Teacher 2.1 never addressed the need for the two fractions to be lined up at the same starting point. They also did not examine if equivalent fractions might help them figure out this problem without tiles. It is quite possible that students understood that 3/6 was the same as 1/2 and that 2/8 was the same as 1/4 or even that 2/8 was less than one-half because they knew that 4 is half of 8 and 2 is half of four, or even that 2 is less than 4. Instead, the focus of this episode was finding the answer.

It could be argued that this was an "I do" portion of the lesson and that was why Teacher 2.1 directed the entire lesson around that task, however, a second example of task implementation based on a whole class discussion of a task will be explored next to illustrate the patterns in Teacher 2.1's teaching practices.

The second vignette comes from February 6, 2012. The class is working through the Math Wall questions from a previous week. Each child sat at his or her desk with a white board and a marker and a binder with all of the Math Wall questions. The question was written as a Procedure without Connections but possibly could be categorized as a Procedure with Connections. *An internet survey asks Web site visitors what fraction of a* *gallon of water they drink each day. The line plot displays the visitors' responses. Which column totals the greatest number of gallons drunk? [NOTE: they were given a line plot not a table]*

Fractions in a gallon	Frequency
¼	8
½	7
¾	6

a) ¾
b) ½
c) ¼

Much like the first example, Teacher 2.1 read the task to the students at the start of this example and then immediately began to question the students about the procedure for solving this task (see Appendix E – maintenance of a low-level cognitive demand task – students do not receive time to puzzle through the task).

Teacher 2.1: . . . so what bit of information do we have to know? How much what?
Kid: how many ounces in a gallon
T: how many ounces are in a gallon. Well let's start with the basics. How many ounces are in a quart?
Kids: ahhh
Kid: 8
T: nope. How many ounces are in a quart?
Kid: 4
T: how many ounces. You are guessing now. Stop and think you did this last year. How many ounces are in a quart?
Kid: 6
T: okay start over here I will give you the first. One quart equals 32 ounces. Now going from there what am I going to have to do? How many ounces in a
Kids: quart
T: NO! I already told you that! How many ounces in a
C: gallon
T: so what am I going to do? I gave you some basic knowledge so you have to know how many quarts are in a gallon? How many quarts are in a gallon?
Boy: four

T: four quarts in a gallon.

Using Knowledge questions, Teacher 2.1 walked through the procedure she thought necessary to solve this problem. She demonstrated Common Content Knowledge of converting measurements related to volume. Unfortunately, the task did not require such mathematical knowledge for solving. The task could have been solved knowing that eight one-fourth pieces made 2 gallons, seven-halves was 3.5 gallons, and six-fourths was 4.5 gallons. Instead, Teacher 2.1 proceeded to make the students convert all of the measures to ounces and then told the students that they needed to add all of the ounces together because the question asked them how many altogether. The task, as understood by Teacher 2.1, fell far beyond what the students understood (see Appendix E – maintaining low-level cognitive demand). In addition, Teacher 2.1 completed the entire task for the students. At this point, I stop the lesson and explained she was solving a different problem than the one asked. Once Teacher 2.1 realized her mistake, she walked the students through a new way to solve the problem.

Teacher 2.1: how many 8ths make one?
Kids: 2
Teacher 2.1: two-eighths make one? How many eighths make one?
Kids: four
Teacher 2.1: how many eighths make one?
Kids: one
Teacher 2.1: how many eighths make one?
Kids: four . . . two . . . four . . . eight
Teacher 2.1: eight eighths. Right? Eight eighths make one? Okay. Let's go over here. It says, oh we don't have eighths.
Teacher 2.1, I don't know why you keep seeing 8ths there. Okay fourths,
Kids: four
Teacher 2.1: Four-fourths make one. Now how many fourths do you have here? Count.
Kids: 8
Teacher 2.1: 1,2,3,4,
Kids: 8

Teacher 2.1: 5,6,7,8 . . . if you were that sick let me send you to the nurse. Which we don't have. Okay. Feeling any better. Did you get a drink of water? Good. Okay so you have 8/8 actually eight fourths. How do I change it? Divide the four into the 8?
Kids: two
Teacher 2.1: first column has two gallons so put two gallons down

Even after Teacher 2.1 realized her mistake, her teaching of the task was answer-focused, little engagement on the part of the students beyond simple one-word answers to knowledge level questions, and was a demonstration of her knowledge of a procedure for solving the task.

Teacher 2.1 did not demonstrate much in the way of Pedagogical Content Knowledge. There were no large examples of her using student thinking or her own knowledge of student thinking to inform instruction. Instead, working through procedures with the students dominated her instruction, as did working through textbook problems. As with the planning and in the CLASS observation notes, it is possible that Pedagogical Content Knowledge was rarely used because Teacher 2.1 did not know what her students thought other than what was given to her as a test score. If a teacher does not know what the students actually know, how can they plan or teach accordingly? It is possible that time constraints prevented her from utilizing student thinking because she felt that too much time would be taken if students did the work but one would not know without a method for tracking beliefs and thoughts of the teacher. It is possible that Teacher 2.1 did not have a large repertoire of activities to use, however, this seems unlikely after 30 plus years of teaching and knowing the activities the NSF-grant provided for the teachers over the two years of professional development. It is possible that accessing these activities when under stress was too much for Teacher 2.1. Again, all of these hypotheses are just that, hypotheses. Follow-up studies would have to be conducted to begin to unpack the reason why Teacher 2.1 relied so heavily on her Common Content Knowledge.

To reiterate, the instructional practice of Teacher 2.1 was evaluated using three methods: the CLASS protocol, the Mathematical Tasks Framework, and Bloom's Taxonomy. Teacher 2.1 provided evidence in her instruction of a high reliance on Common Content Knowledge with a little use of Specialized Content Knowledge. This type of knowledge was evident in her implementation of the tasks she gave the students and in the types of questions she asked the students during class discussions. Her teaching style was very systematic and orderly. Much like Teacher 1.2, Teacher 2.1 stood at the front of the class and solved problems from the adopted textbook. Occasionally, her instruction changed based on students demonstrating a lack of understanding. Sometimes students were called to the board to show a different way to solve a problem but mostly if the children did not understand a concept, she yelled a little louder or provided another example. Next, we examined Teacher 2.2 using the same analytic methods to see if any patterns or disparities were seen between the two teachers with MKT scores around the mean of the larger NSF-grant sample and different student achievement scores.

Teacher 2.2

The instructional practice of teacher 2.2 was evaluated using three methods: the CLASS protocol, the Mathematical Tasks Framework and Bloom's Taxonomy. Teacher 2.2 provided evidence of her reliance on her Common Content Knowledge and some Knowledge of Content and Teaching. These types of knowledge were evident in her procedural instruction techniques and her ability to supplement the textbook activities when students struggled to understand a concept. For the most part, Teacher 2.2 taught her 4th graders from the front of the classroom, however, she provided opportunities for students to demonstrate their ability to work through a problem using the procedure she showed them. The following sections provided the evidence to support the claims made in this introductory paragraph.

The learning environment.

CLASS observation protocol. Six CLASS observations were conducted with Teacher 2.2 between October 2011 and February 2012. Table 12 shows the average scores across the 10 dimensions for Teacher 2.2:

Table 12

CLASS Dimension Scores for Teacher 2.2

Dimension	Average score
Positive climate	6.167
Negative climate	1
Teacher sensitivity	3.83
Regard for student perspectives	2.167
Behavior management	4.83
Productivity	3.83
Instructional learning formats	3.67
Concept development	1.5
Quality of feedback	2.167
Language modeling	2.67

For the most part, Teacher 2.2 received average scores ranging from 1 to 6.167, with most of the dimension scores being between 2 and 4. Teacher 2.2 scored the highest marks for Positive Climate and the lowest scores for Negative Climate, which is what one wants to see when looking at the types of interactions occurring in the classroom. Other than Regard for Student Perspectives and Concept Development, the rest of the scores fell on the low end of the middle range.

When composite scores were calculated for the three CLASS domains, Teacher 2.2 received a score of 4.791 for Emotional Support, 4.11 for Classroom Organization, and 2.112 for Instructional Support. For both Emotional Support and Classroom Organization, Teacher 2.2 was fourth in comparison to all of the other participants in this dissertation, just above Teacher 2.1. She had the lowest score of all of the participants when it came to Instructional support but only by 0.1 of a point behind Teacher 2.2. Again, all of these scores are based on a scale of 1 to 7 points, with the lowest attainable score being a 1 and the highest being a 7.

As we can see by the scores, the interactions recorded under Emotional Support were very high. This means that, on average, Teacher 2.2 created a classroom that promoted positive interactions, such as having respect for everyone, building positive relationships with students, and having a positive affect toward the students. This score also indicated that Teacher 2.2 was mostly aware of her students' needs and was very responsive to their comfort level. She made time in her teaching to working with individual students and helped them solve problems in a timely manner. The students seemed very free to participate and take risks in this classroom for the most part. In this classroom, there was also some indication that Teacher 2.2 tried to incorporate students' ideas into her teaching and the students were allowed some choice and leadership but usually Teacher 2.2 took control of the classroom direction and focus (Pianta et al, 2008).

Classroom Organization fell in the middle range as well. Teacher 2.2 was frequently consistent with the rules and set clear expectations for the students. She often anticipated problem behavior before it escalated. She was able to use subtle cues to redirect students, either by using their name or by giving a quick look. The students were quick to comply with the teacher when she corrected their behavior for the most part. For the most part Teacher 2.2 provided activities but there were times when instructional time was lost to students goofing around or everyone getting off of task. At times, there were long transitions and some difficulty in following routines. For the most part the teacher was prepared with the materials for the class but occasionally the activities were impacted by the lack of supplies. The lowest scores for this domain came from Instructional Learning Formats. While Teacher 2.2 received middle range scores for Instructional Learning Formats, they were relatively low mid-range scores. Students participated in this classroom but occasionally their attention wandered. There were some creative and interesting materials used in this classroom, such as Smart Board Dice activities and the construction of multiplication books but the activities were not always at grade level nor expanded on what children already knew (Pianta et al, 2008).

Lastly, Teacher 2.2 received a low score for Instructional Support. Teacher 2.2 struggled to "use instructional discussions and activities to promote students' higher-order thinking skills and cognition and the teacher's focus on understanding rather than on rote instruction" (Pianta et al, 2008, p. 64). The students rarely brainstormed or problem solved in this class. There were few connections made across the concepts and few connections to real-world applications. Teacher 2.2 occasionally provided scaffolding for students and assistance but not often. Most of the feedback given to the students was positive affirmations rather than an exchange of ideas. Students were occasionally asked to explain their thinking but a lot times the teacher moved forward with the explanations on her own. Lastly, while some techniques were used to encourage language development, this protocol indicated that there were few interactions among students that promoted language development and few opportunities for students to engage in back and fourth exchanges. There were also a few moments where Teacher 2.2 used a variety of words and extended on what students were saying.

Overall, the CLASS observation depicted a very positive classroom where students were engaged for the most part and compliant with the directions of the teacher. Students were usually active participants in the classroom and were engaged in the learning that occurred. What were missing in this classroom were interactions that promoted higher-level thinking and the development of mathematical language. It is quite possible that the low scores captured by the CLASS observation protocol resulted from the below average MKT scores of Teacher 2.2. If a teacher lacks subject matter knowledge, as was depicted in the interview and test scores of Teacher 2.2, developing a learning environment that promoted higher-level thinking might be very difficult, as the teacher might not know where to take a lesson mathematically (Horizon Content Knowledge) or how to unpack the mathematics being taught with depth (SCK). Low teacher knowledge might also indicate difficulty in anticipating tasks that would challenge students, although Teacher 2.2 strove to do just that with her 4^{th} graders (KCS) and also difficulty sequencing instruction in a way that encourages creating connections across concepts and building on prior knowledge (KCT). What is interesting is that the student change scores for Teacher 2.2 were a standard deviation above the mean of the NSF-grant participants' scores. Next, we will examine the instructional strategies of Teacher 2.2 using The Mathematical Tasks Framework (Stein et al, 2009), Bloom's

Taxonomy, and MKT categories (Ball et al, 2008), in hopes of addressing this dichotomy between the knowledge scores and student scores.

Worthwhile mathematical tasks and classroom discourse.

The mathematical tasks framework (Stein et al, 2009) for teacher 2.2. Six one-hour classroom observations were used to assess the teachers' instruction in this dissertation. During the six observations on Teacher 2.2, between one and nine tasks were assigned during a single class period, for a total of 28 written tasks. Overall, there were a total of 9 memorization tasks, 12 procedures without connections tasks, 6 procedures with connections tasks, and 1 doing math task (Stein et al, 2009). Table 13 provides examples of the different types of tasks assigned by Teacher 2.2.

To best understand how Teacher 2.2 implemented the written tasks and how the implementation related to MKT, I used the following memorization task that was given on November 15, 2011 during Math Wall time. The memorization task read:

Which of these equations shows the distributive property?

a) *(4x2) +8 = 4(2+8)*
b) *23 x 2 = (20 x 2) + (3 x 2)*
c) *68 x 0 = 0*

At the start of the Math Wall time, the students solved the problems independently. After 10 minutes, Teacher 2.2 reviewed one or two of the five math wall problems with the class. For this task, she read the question to the students and then questioned them on their thinking.

Teacher 2.2: Let's do it, let's eliminate the easiest one. How do we know that it is not C?
Kid: Because it
Teacher 2.2: ohh! Thanks for raising your hand...Olivia?
Olivia: because anything times zero is zero
Teacher 2.2: zero. So that shows, which property is that one?

Table 13
Cognitive Demand Task Levels for Teacher 2.2

Classification of task	Example	Explanation
Memorization	"5 x 3" and "8 ÷ 4"	As written these task involves reproducing already learned facts and "have no connection to the concepts or meaning that underlie the facts, rules, formulae, or definitions being learned or reproduced" (Stein et al, 2009, p. 6).
Procedure without Connections	"Divide 15 counters into 5 equal groups"	This task requires limited cognitive demand for successful completion, as written. It "requires no explanations, or explanations that focus solely on describing the procedure that was used" (p. 6). Also, "use of a specific procedure is called for" (p. 6).
Procedures with Connections	"Charlie brought 25 cookies to school. He wants to give some cookies to 8 friends. How many cookies does each friend get and how many are left over?"	This task "requires some degree of cognitive effort. Although general procedures maybe followed, they cannot be followed mindlessly. Students need to engage with conceptual ideas that underlie the procedures in order to successfully complete the task and develop understanding" (Stein et al, 2009, p. 6) This task also requires students to think about what they are answering and how they will get to the answer. In addition, there are multiple ways in which a child might solve this problem and there is a demand on the children to monitor their own thinking because the answer they might initially end up with might not be the answer to the question presented.
Doing Mathematics	Kids had to estimate and then get a specific measurement of items around the classroom	This task is at the highest level of cognitive demand because "there is not a predictable, well rehearsed approach or pathway explicitly suggested by the task, task instructions, or a worked-out example"(Stein et al, 2009, p. 6). In fact, there were no examples modeled or instructions given about solving this task until after the students had tried to solve it on their own. Thus, this task "required students to access relevant knowledge and experiences and make appropriate use of them in working through the task" (p. 6).

Kids: zero
Teacher 2.2: the zero property so we know it's not C.

While Teacher 2.2 asked Olivia to explain how she knew that answer choice C was incorrect, this excerpt did not extend beyond Common Content Knowledge for Teacher 2.2. She did not have to make any mathematics transparent for students or figure out ways to represent the property being illustrated in answer choice C or use what Olivia said to make a mathematical point. Instead, the level of questioning focused on Olivia's recall of the definition of the "Zero Property" and maintained a low-level of cognitive demand.

Teacher 2.2: . . . So you guys are saying it's this one. A. Okay. Remember last time when I was telling you guys that the associative property has parentheses? And this one has parentheses too, BUT the associative prop, this one when you distribute you are distributing the problem. You're distributing it. Okay? By place value. Okay? You guys the answer is and I am going to give it to you so you guys know it and we are going to put more practice the answer is B.
Kids: yes!!
T: because I am distributing this 23. Look you guys; I am distributing this 20 times 2 and 3 times 2. That's what we've been learning in multiplication. With multi-digits remember. Remember when I say you guys what is . . . and we've been, and it's on our goal. We're using the distributive property; I am going to, what am I going to do to this? I'm going to, break it apart right? That's distributing. Okay? Are we good? We are going to do more practice. I am not going to continue with this because we are going to do this during our regular math lesson, okay? So. We're going to move on, okay? But everyone got why it was B?
C: yes
T: yes? Okay

At this point in the task implementation, Teacher 2.2 took over this problem (Stein et al, 2009, p. 16). She stopped asking the students what the different properties meant and what they were thinking. Instead, she explained the different properties at a very basic level and then showed why one answer fit versus another. Again, Teacher 2.2

demonstrated Common Content Knowledge in this excerpt, however, what is most notable is the fact that she showed incorrect Common Content Knowledge throughout this episode. First, answer choice A, as written, is incorrect. The left side of the equation equals 16, while the right side of the equation equals 40. It was apparent that Teacher 2.2 did not notice the answer choice was not only an incorrect representation of the associative property but that it also was a false statement. Second, her explanation of the distributive property was confusing. She did break apart the 23 into 20 and 3 but she did not distribute the 23 across the two, she distributed the 2 across the 23. While her method works, there left a great possibility that the students will be confused at a future point.

Beyond Common Content Knowledge, there were few examples of any other category of MKT in the first vignette. Possibly, the lack of other components stemmed from the incomplete knowledge Teacher 2.2 held about the distributive property. If one does not understand the content being taught, asking students to clarify their thinking (KCT), hearing incomplete thinking (KCS), decompressing the mathematics behind the distributive property (SCK), or choosing examples to assist in explaining the property might be very difficult. Further investigation is needed to examine what is happening during this vignette with regards to Teacher 2.2's MKT.

On January 31, 2012, Teacher 2.2's reliance on Pedagogical Content Knowledge was evident. During this episode, the students investigated the concept of measurement by Doing Math (Stein et al, 2009). The lesson started with a review of the topic discussed the previous day, which was estimating the length of an item and then measuring said items in the book such as a paper clip and an illustrated ribbon. They also recalled how to estimate the length of an item using their thumbs or their hands or their fingernails. Then Teacher 2.2 asked the students how they measured using a ruler because she anticipated difficulty in their measuring abilities based on past experiences with this activity (KCS – anticipating what students might find difficult; anticipating misconceptions students had about measuring and using a tool for measuring).

Teacher 2.2: . . . okay, so for today's activity. Okay . . . and just to go over with you how to measure something correctly, cause I know that this is a mistake that we always, we sometimes we might do, and I don't want you guys making that mistake. Okay I am waiting [phone rings] for when you are ready. 3rd grade go . . . [00:06:08.20] yeah they are on their way . . . okay. [00:06:29.05] Okay. okay. Right now? No just do your work and that's it okay. You are staying in here . . . okay let's say we have this line and we wanted to measure it, okay? Here's what I need you guys to make sure you are always doing. By looking at your ruler you can see that there are two different sides to it, okay? This side the lines are, what do we notice between this side and this side?
Kid: that the
Jorge: one says cm for centimeters and the bottom says in for inches
Teacher 2.2: there you go, so this is inches so what we are trying to find it inches so we are going to use this side, okay? And here's what I need you guys to know when you are measuring with a ruler. You don't just put it like this and you start measuring and you say oh it's like that okay? Got it?
Kid: yes
Teacher 2.2: okay. Look at my line down here. You grab the very end of it, right before the one starts, see that? [kid: yes] and you put it right at the end. And you measure it.

Teacher 2.2 demonstrated to the students how to use the ruler correctly when measuring items around the classroom. The students were given free reign to measure items around their classroom. Teacher 2.2 walked around the class helping the students. She stopped the students at one point when a disagreement occurred as two students tried to measure her.

Teacher 2.2: you said 5 inches and you said 72. [00:24:14.14] How did you get 5 and how did you get 72. Huh? Wait class?
C: yeah

T [00:24:24.16] okay, here's another great learning opportunity for all of you guys because I see a lot of you guys doing this.

In this moment, Teacher 2.2 demonstrated Knowledge of Content and Students (KCS – knowledge of common misconceptions and difficulties around the mathematics content). She also used the two answers (5 inches and 72 inches) to make a mathematical point (Knowledge of Content and Teaching) and scaffold the learning around the importance of units (Knowledge of Content and Teaching).

Teacher 2.2: how did you come up with 5?
Josue: cause I used a ruler
T: how did you come up with 5? ohhh! did I tell you to talk? Jorge, go sit down. go sit down real quick, Jorge. right now. how did you get five? so do you think he is correct?
Kids: no
Teacher 2.2: am I 5 [she demonstrates 5 inches on the ruler]?
Kids: no you are more than that!
Teacher 2.2: huh?
Kids: you are more than that!
Teacher 2.2: 5foot?
Kids: yeah
Teacher 2.2: 5?
Kids: feet!
Teacher 2.2: feet. okay. how did you figure that out?

Teacher 2.2 demonstrated Specialized Content Knowledge when she showed the students what 5 inches looked like compared to her height (SCK – making a concept transparent; knowing what representation would be impactful for students about units). She also used a student's thinking to make a mathematical point and when to ask for clarification about the units being used (Knowledge of Content and Teaching).

Teacher 2.2: okay, here's what I saw you guys were doing at first. We are making that mistake of telling, of saying that I was five inches just because when you measured me you went 1,2,3,4,4 and then 5. that's not five inches. that's tell me that I am this tall.
Kids laugh!

Teacher 2.2: okay? close but not that small, okay? you need to know that if you are trying to measure in inches how many times you went so look. I'll wait until you guys are watching me . . . cause if you don't know what, how to do you are going to continue making that mistake.

Again, Teacher 2.2 made the concept of "inches" versus "feet" transparent for students (SCK). She also stated that she knew they would continue to confuse the units unless they learned the difference (Knowledge of Content and Students). She continued by scaffolding how to convert inches to feet using the students' prior knowledge that there are 12 inches in a foot. A student explained that they could multiply 5 times 12 to figure out how many inches was 5 feet. Teacher 2.2 used the work of another student (a first grader who was in the class that day because the school did not have a substitute for his classroom) to demonstrate another method for calculating "inches" to "feet."

Teacher 2.2: Javier, what did you do when you measured me?
Javier: I was using the ruler?
Teacher 2.2: uh huh
Javier: and I was going number by number
Teacher 2.2: number by number. and what were you doing with number by number? weren't you saying numbers
Javier: I was going 12,13,14,15,16,17,1819,20, and I was going
Teacher 2.2: ohhh, did you see how he did it
Kids: yes
Boy: no
Teacher 2.2: so he went like this, this is what he did cause I was hearing him. He used his ruler and he said this is 12 and then he said okay, I don't think he still knows how to multiply but I think he knows he still has to add 12 so what he did he added all of the other number so he said, 13,14,15,16,17,18,19,20,21,22,23,24 and he did it again 24,
C: 25,26
Teacher 2.2: and all the way he got to the top of my head. that's very good thinking for a 1st grader, okay? for you 3rd and 4th graders I expected you guys to multiply it

Using Javier's work (KCT), Teacher 2.2 not only exposed the students to another way of converting units, she used this moment to ask the rest of the class how this related to the multiplication and why she had the 4th graders multiply instead of count by ones. One student yelled that they are the same. She asked him, “So I can say, 12 + 5 is the same as 12 times 5?” The students said that was not correct. Teacher 2.2 explained that 12 times 5 is the same as 12 + 12 + 12 + 12 +12 and that this was exactly what the 1st grader wrote on his paper. She further explained that the addition was a trick the students could use if they forget how to multiply.

Overall, the teaching practices of Teacher 2.2 could be characterized as a mix of traditional direct instruction and reform teaching practices. Most of the time, Teacher 2.2 stood at the front of the class and taught exactly what the textbook directed. However, many times she asked students to show her what they were thinking or solve problems at the board. She allowed kids to explore measurement through measuring items around the room and to explore vertex-edge graphs by coloring a map of Arizona. She tried new strategies that she learned in the TAP program and from the NSF-Funded grant, such as designing questions that accessed higher-levels of thinking according to Bloom’s Taxonomy.

The level at which Teacher 2.2 engaged these new strategies and reform teaching practices in her teaching seemed to map to her lower MKT score, as compared to her peers. In the second vignette, Teacher 2.2 allowed students to roam the classroom and discover why understanding differences among units of length was important. That activity also engaged students in seeing the relationship between inches and feet. During that lesson, Teacher 2.2 demonstrated Common Content Knowledge, Knowledge of Content and Students, and Knowledge of Content and Teaching. She also showed some Specialized Content Knowledge. For example, Teacher 2.2 was able to solve the conversion problem of 5 feet to 60 inches. She was able to solve it with multiplication and relate it back to the number of groups of 12 in 60 inches. She was able to assess correctly that 72 inches was 6 feet. Both are examples of Common Content Knowledge. She was able to anticipate mistakes students might have when using the ruler and when converting from inches to feet. A few times in the lesson she was able to decipher and interpret students' incomplete thoughts and scaffold the thinking of the students to help them get the correct answer. These exemplified Knowledge of Content and Students. Throughout the lesson, she was able to ask for clarification of students who misunderstood conversions and lead that student to the correct answer. This was an example of Knowledge of Content and Teaching as well as Knowledge of Content and Students. She used multiple students' thinking to make a mathematical point, which was Knowledge of Content and Teaching. She also selected specific misconceptions to clarify issues the whole class was having with measurement and units. This was another example of Knowledge of Content and Teaching.

As one can see, Teacher 2.2 utilized a variety of components of Mathematical Knowledge for Teaching in the second vignette. However, this engagement and extensive use of MKT categories was not seen often during the six observations. Instead, Teacher 2.2's teaching practices most often embrace Common Content Knowledge and occasionally Knowledge of Content and Teaching. Since Teacher 2.2 relied heavily on the textbook for her lessons, it was hard to determine whether or not her knowledge of the mathematics was representative of her own knowledge or that of the textbook writers. As seen in the first vignette, when Teacher 2.2 deviated from the textbook and veered into subjects where her common content knowledge wavered, she demonstrated fewer moments where multiple MKT categories were used. Next, I summarized the findings across the teachers in this case.

Discussion

Overall, Teachers 2.1 and 2.2 presented an interesting situation between average MKT levels relative to their peers in the larger NSF-Funded grant and gains in student achievement. Although Teacher 2.1 scored higher on the MKT test than Teacher 2.2, their students' gain scores were opposite. Teacher 2.1's gain scores were less than Teacher 2.2's gain scores. Why was this? What about how they drew upon their MKT might have accounted for the differences in these scores?

Both teachers taught combination classes based on the English Language Development level of the students as tested by the state language test. Both teachers followed the adopted textbook and the district curriculum map when planning daily lessons for each grade level. Teacher 2.1 also used the state standards document to plan her daily lessons. Although there were flashes of reliance on Knowledge of Content and Students during the planning of lessons for both Teacher 2.1 and Teacher 2.2, for the most part these teachers adhered to the predetermined curriculum map and textbook when planning. Therefore, MKT was less noticeable, and quite superficial when apparent, during the planning phase for these two teachers than others in my study.

Differences occurred in how Teacher 2.1 and Teacher 2.2 implemented the instruction. For the most part, Common Content Knowledge (CCK) was very apparent in Teacher 2.1's instruction. Teacher 2.1 stood at the front of the classroom and dictated the standard procedure for solving problems. She read the question to the students, showed them the steps for completing the problem, and then the students demonstrated they could follow her directions. Once she was satisfied that the students could mimic the procedure she gave them, the students were given multiple problems from the textbook to solve for the remainder of the class period.

Much like Teacher 2.1, a basic level of CCK was most apparent in the teaching of Teacher 2.2. What differed was that while Teacher 2.2 used the textbook and the procedures found in the textbook, she encouraged some discussion in her classroom and used her Knowledge of the Content and Students when deciding what textbook tasks would be hard or easy for students to solve. She sometimes allowed students to tell her how they solved the problem before she gave them the procedure but usually she presented the procedure first. She also used a combination of KCS and Knowledge of Content and Teaching to determine when she could combine lessons. For example, she knew her students mastered the concept of Mode from their work on the Math Board. So she collapsed that lesson in the textbook into the lesson on other measures of central tendency. Teacher 2.2 also used her knowledge of Bloom's Taxonomy to structure her questions for students. She learned this skill from a professional development program the teachers in her school participated in that year.

It was possible that the differences in student gain scores for these teachers was a function of the differences in the amount of classroom discussions, the use, or lack of use, of knowledge of student thinking when planning and implementing lessons, and the willingness of Teacher 2.2 to embrace ideas she learned in professional development courses. Because of the missing data from Teacher 2.1, it was hard to determine how MKT influenced the teacher's decisions but it was apparent that for the most part her instruction and planning was based on her CCK and knowledge of standard algorithms.

The next chapter examined the case of two teachers with relatively low MKT scores and two of the highest students' gain scores relative to the teachers in the larger NSF-Funded grant.

CHAPTER SIX: CASE THREE

A Case of Low MKT Scores and High Student Gain Scores

This final case examined two teachers who scored similarly on the Teacher Knowledge Assessment and whose student gain scores were similar. To reiterate the information presented about the sampling process in Chapter Three, both teachers in this case scored about one-half of a standard deviation below the mean of the participants in the larger NSF-funded study. In contrast, their students' gain scores were strikingly high relative to their peers. Teacher 3.1's students' gain scores were the highest of all of the teachers in the NSF-funded grant with her classroom average a little over two standard deviations above the mean of the NSF-grant participants' students. Teacher 3.2's students' average gain score was the fourth highest at one standard deviation above the mean of the NSF-grant participants' students. This case presented the last layer for understanding how MKT might link to student gain scores through classroom instruction.

General Descriptions of Teacher 3.1 and Teacher 3.2

Teacher 3.1

Teacher 3.1 was in her 7th year of teaching. All seven years were spent teaching 3rd grade in two southwest states. She graduated from a large state university in the southwest and completed her student teaching in the district in which this dissertation study took place. She immediately moved into a job in the district at the end of her undergraduate work. She left the district for a few years to live in a neighboring state. It was during that time where she was introduced to the Math Wall, a program now adopted by the district in this study. She felt very comfortable teaching the 3rd grade standards but very wary of higher-grade level mathematics. She identified herself as being bad at math growing up. She thought this assisted in her ability to teach because she understood how to work through the problems, like the kids have to do (Teacher 3.1, 2nd Interview, 3/5/12).

Over the 2011-2012 school year, Teacher 3.1 taught approximately 23 third grade students. Teacher 3.1 taught at the same school as Teacher 1.2 and had a similar break down in lesson structure as Teacher 1.2, as seen in Table 14.

Table 14

Lesson Structure for Teacher 3.1

Lesson component	Allotted time	Purpose
Math wall	30 minutes	Spiral state standards throughout the year
Daily lesson	30 minutes	Objective provided by the curriculum map
Problem solving (Otter Creek)	10 minutes	Adopted curriculum to help with building problem solving skills
Math facts (Otter Creek)	5 minutes	Adopted curriculum to help build math fact fluency

During the Math Wall time, the students gathered on the floor with their individual white boards and markers in front of the chalkboard. They sat at tables in groups of three to four during the Daily Lesson. Teacher 3.1 stood at the front of the class when providing instruction but walked among the students when they were working on problems individually or sharing ideas with their partners.

Teacher 3.2

Teacher 3.2 was also a teacher for the last seven year. Like Teacher 3.1, Teacher 3.2 graduated from a large state university in the southwest and student taught in the district where this study took place. All seven years of her teaching experience were in the 2nd grade in the same district (Teacher 3.2, 1st interview, 1/26/12).

During the 2011-2012 school year, Teacher 3.2 taught approximately 24 second-grade students. For all of my observations, the students sat on the floor with their individual white boards facing the Math Board. Like Teacher 3.1, Teacher 3.2 stood at the front of the class when discussing the task or was directly instructing the class but then moved among the students during pair-sharing time or individual think time.

Teacher Knowledge

As stated in Chapters three and four, one component of the Mathematics Teaching Cycle (NCTM, 2007) was Knowledge. According to the Teaching Principle (NCTM, 2000), an effective teacher needs knowledge in: "mathematical content, pedagogy, assessment strategies, and an understanding of students as learners" (NCTM, 2007, p. 19). These four criteria for "an effective teacher" aligned with Ball et al (2008) MKT components of Common Content Knowledge (CCK), Specialized Content Knowledge (SCK), Knowledge of Content and Teaching (KCT), and Knowledge of Content and Students (KCS). In the following section, I used the data representative of the entire data set gathered from the Teacher Knowledge Assessment and follow-up interview to illustrate the degree to which the components outlined in the Teaching Principle and the MKT framework presented in Teacher 3.1 and Teacher 3.2 and how the presence or lack of some criteria might account for different student gain scores.

Before discussing the data on the teacher knowledge test for this particular case, it must be acknowledged that during a few different administrations of the teacher knowledge test, teachers gathered together and took the test in groups. This "group work" occurred the year Teachers 3.1 and 3.2 took the test for the second time. I realize the written test scores were not necessarily indicative of what each teacher knew individually, however, the follow-up interviews were given individually and were used to parse out what information Teacher 3.1 knew versus Teacher 3.2.

Example 1 From the Teacher Knowledge Assessment: A Multiple-Choice Item

Because the LMT items are not released to the public, the following is a description of a task given to the teachers participating in the NSF-funded Grant. The task was a multi-digit subtraction problem that included regrouping of the minuend. The digit "0" was in the tens place of the minuend as well. The teachers were given three hypothetical students' responses to examine. The task asked the teachers to state whether or not each response was acceptable evidence indicating that the child knew why the procedure worked. The teacher had the option to say that they were unsure if the answer was acceptable or not.

The first response (a) was a direct description of the steps taken to solve the problem. The response was void of place value or explanation regarding regrouping. The second response (b) indicated the child could decompose numbers and regroup fluidly across place value positions. The final response (c) was tricky for most of the teachers. The student indicated some understanding of place value at a superficial level.

Teacher 3.1's response. According to the written test, Teacher 3.1 answered that both choices (a) and (c) were not acceptable and that choice (b) was acceptable. During the interview, Teacher 3.1 explained,

> *Oh no, I wouldn't accept that [answer choice A] because I would tell them that they have to make it a 10 and then borrow to make it a nine. [she reads the second answer choice] I took away 7. . . from 16 . . . and I took away 9 from 9. I mean I guess I was thinking that but I still, oh yeah, yeah, I think this one is acceptable. Why? Because they are explaining how they regrouped this one and that's why the 9 so it kind of made sense. [reads the third solution] yeah, so I would accept this because they're explaining*

each step and validating their answer. I guess. I'm sorry I am not being very helpful.

In this answer, Teacher 3.1 demonstrated that she understood the procedure for completing the subtraction problem. However, she failed to notice that answer choice (c) was also incorrect. In fact, answer choices (a) and (C) were similar in that the student tried to "borrow" from a "zero," without recognizing that they were actually "borrowing" from 30 tens. Teacher 3.1 utilized some Common Content Knowledge in this problem but was not able to unpack the underlying concepts of place value rooted in the explanations.

Teacher 3.2's response. According to the written test, Teacher 3.2 answered that both choices (a) and (c) were not acceptable and that choice (b) was acceptable. Teacher 3.2 ended up accepting answer choice (a) during the interview but explained that she wanted to hear more explanation of how the child "borrowed." Her answer was also rooted in the standard procedure for subtracting. She continued with the remainder of the problem stating,

Okay, borrowed from the tens place to make 6 a 16 but since it was a zero I had to borrow again, yeah. That's good and especially when they started with this because they don't get the three right away so who knows if they know if it's connected. So, yeah, I guess, yeah. Alright.

From this explanation, it was difficult to decipher what exactly Teacher 3.2 knew or which knowledge she might be accessing to solve this problem. She had an understanding of the traditional algorithm for solving the subtraction task but beyond that there was little indication of other MKT categories being used.

Example 2 From the Teacher Knowledge Assessment: An Open-Ended Item

The following problem was given to the teachers to assess their knowledge of relational thinking in students. The teachers were provided with half of a page to write or illustrate their responses. To receive total points for the problem, the participants had to answer all three sub-questions (Carpenter et al, 2003).

Assume we gave this problem to some elementary school students:
What number can you put in the box to make this number sentence true?
8 + 15 = ___ + 16
What strategies would you expect students to use to solve this problem?
What answers do you expect them to come up with?

Teacher 3.1's response. Teacher 3.1 wrote that her students would put "7" in the box. She wrote that her students would solve the left side of the equation first and then figure out what went into the box. She demonstrated her Common Content Knowledge for solving this problem both on the written test and during the interview. In addition, she anticipated that her students would count up from 16 to 23 to get the answer of 7. Such anticipatory understanding was characteristic of Knowledge of Content and Students. She emphasized during her interview that she would teach her students this method of problem solving.

Teacher 3.1 further explained that her students might put the answer to 8 and 15 in the box because "they wouldn't really look at the whole problem" (Teacher 3.1, 3rd Interview, 4/12/12). Teacher 3.1 was less convinced that her students would put 23 into the box. She said they have been working on these types of problems and her students were less inclined to put the sum of the left side of the equation in the box (Teacher 3.2, 3rd interview, 4/11/12).

Teacher 3.2's response. Teacher 3.2's written response was very similar to Teacher 3.1. She also said her students would put "7" in the box. She demonstrated how the students would solve the equation starting with the left side and then figure out what went into the box. The main difference in her explanation was that she anticipated that her students would subtract 16 from 23 rather than count up from 16 to 23.

Comparison of the Teachers' Responses Across the Test Items

In summary, both teachers received relatively low scores on the Mathematical Knowledge for Teaching assessment. Using the breakdown of MKT found in Ball et al (2008), I examined specific MKT categories found in the two teachers' teacher assessment interviews and test to better understood what the teachers understood and found confusing.

Teacher 3.1 demonstrated Common Content Knowledge of her grade level topics throughout the interview and test. She explained at one point that students would complete missing addend problem using the steps that she had given them in class (Teacher 3.1, 3rd Interview, 4/12/12). She struggled to work through the problems involving fractions and non-standard algorithms. For example, she received five points total out of thirteen for questions pertaining to fractional representations. She selected the correct word problem representations for the problem "1/2 – 1/3"; however, when asked to explain how she knew which were right and wrong, she said she did not know and that her sister problem knew and they worked through this test together (Teacher 3.1, 3rd Interview, 4/12/12).

Teacher 3.1 also struggled to assess whether or not a nonstandard algorithm would work in general (Ball et al, 2008). She was unable to understand how a child used the distributive property to divide a multi-digit number by a single-digit number. In addition, she had difficulty understanding how a child worked through "37-19" by "(7-9) +(30-10)." The negative number in this method was challenging for her to work through. This same difficulty was represented when asked to assess "61-36." Again, a negative number was used in the solving of the problem (Teacher 3.1, 3rd Interview, 4/12/12). These examples demonstrated a weakness in Common Content Knowledge and a lack of Specialized Content Knowledge, as assessed in this particular test. In addition, Teacher 3.1 provided an incorrect representation of the problem "*I've got 24 balloons I'm going to give out to my friend in bunches of 4. How many of my friends will get a bunch of balloons?"* The picture Teacher 3.1 drew (and the subsequent explanation) represented a partitive notion of solving the problem. The question, however, was a measurement problem. She should have drawn six bunches of four but instead she drew four bunches of six balloons. Again, this illustrated a lack of Specialized Content Knowledge (Ball et al, 2008).   Lastly, when examining the interview, I found that I explained how to complete most of the exam during the interview. Much of the interview consisted of Teacher 3.1 stating that she did not know.

Using the MKT categories from Ball et al (2008), I noticed that Teacher 3.2 demonstrated a strong aptitude toward pedagogy and how her students would think about the problems. For example, she explicitly stated how her students would solve a subtraction problem multiple times. She drew pictures of what how the students would solve the problems. This Knowledge of Content and Students was apparent throughout her interview and her test paper. She would also comment on how she didn't know how to teach certain things like when addressing the problem where the student struggled to understand the distributive property. Her focus throughout the interview was on the teaching of concepts and how the students thought through the problems rather than the procedures or methods of solving the problems.

Teacher 3.2 struggled with Common Content Knowledge and Horizon Content Knowledge. Throughout the interview, if Teacher 3.2 did not know an answer or how to solve one of the tasks, she said “I don’t know” or “We don’t teach this.” Much like the interview with Teacher 3.1, I found myself explaining how to work through a problem most of the time. For both teachers it is possible that the lack of knowledge was a product of the two-year time gap between when they took the test and the follow-up interviews. The other teachers in this dissertation took the test at the end of the previous school year. Therefore, those teachers might have an easier time recollecting their thought process. It is also possible that these teachers struggled with the concepts on the test because they teach lower grade levels, while the other teachers teach higher-grade levels or a mix of grade levels.

Analysis, or Planning, of Instruction

Teacher 3.1

In general, Teacher 3.1 planned her lessons around the school administration’s structure for teaching mathematics. Her principal mandated that math time be broken into three sections: Math Wall, Daily Lesson, and Math Facts/Problem Solving. Teacher 3.1 dedicated 30 minutes of time to each component. I explained the three factions of math time next.

Teacher 3.1 used the Math Wall as a vehicle for helping students master the state standards and pass the state’s standardized test at the end of the school year. She found that through the math board students received frequent and repeated exposure to the standards throughout the school year because she rotated the standards on a weekly basis. This rotation of standards meant that procedures remained fresh in the students' minds because the students were constantly practicing the skills needed to master each standard.

> *So like I'll put like a problem up there and then um maybe I'll change it but I'll keep it up there until like most of the kids or like all of the kids are understanding that problem and then, um, and then once they kind of get it then like I put another type of problem and then I go back to it so they are always seeing it, you know? And I think that's like the biggest thing. I think for them I think the math board is like why they are so successful (Teacher 3.1, 1st interview, 3/1/12).*

She also found that the math board enabled her to conduct a talk aloud with the class when presenting a new procedure for solving a problem. ". . . I always do like an I Do, you know? I show them and I model them and sometimes I only need to model like two times or sometimes it's more on my part and then we try to do something together (Teacher 3.1, 1st interview, 3/1/12)." Lastly, the math board enabled her to engage the students in test prep. "I love test prep . . . they need to learn how to take a test" (Teacher 3.1, 1st interview, 3/1/12).

In this classroom, the math wall was broken into the five domains found in the 3rd grade state standards: Number Sense, Operations, Estimation, Data Analysis, Discrete Math, Patterns, Algebra, Functions, Geometry, Measurement, and Logical Reasoning. Under each concept, Teacher 3.1 posted one to two problems for the students to solve. The tasks used in the math wall were selected from both the state's standards document, sample test questions, an AIMS resource book, and a computer program called Study Island.

yeah, I've seen the problems so much but I also use this book that I have from Harcourt. It's an AIMS prep test book that I have. I used that sometimes. Sometimes like the released test questions like form the State, like I can look at those, like how that's been worded and like our curriculum assessment book. Like I pull problems from that but I kind of just like know like what type of also like just my knowledge level of like what you know, what's been on like the AzAC, like what should be teaching, you know? I also come from up there cause we do test talk with the kids and we're doing that today so it's really, makes me become more familiar with the test too and that's supposed to mirror AIMS, so hopefully it does (Teacher 3.1, 2nd interview, 3/5/12).

She explained that she pulls a lot of questions from Study Island as well because their weekly tests from the district and the AzAC questions came from that computer program. When selecting tasks from the various resources, Teacher 3.1 tried

to get something easier, you know, like scaffold it, make them feel successful at it and once they've gotten that then I'll like, you know, increase the level of it and make it like harder or easier…and so and then like once they've gotten a certain concept down then maybe I like try to word it in a different way or like change like a vocabulary word or include like which one does NOT include a multiple or something like that. So they can see it from different angles (Teacher 3.1, 2nd interview, 3/5/12).

The daily lesson followed the district curriculum map. She explained that she wrote objectives based on the standard for the day and then tried to find meaningful activities to give the children. She starts the math lesson with an I Do where she models the activity or procedure and then they complete a similar problem together. During the We DO part of the lesson, she has the students talk and explain the procedure to each other. "I have sentence frames in place that everything has a 'because, like my answer is this because' and they have to be able to verbalize it um because that way they can internalize it and it can become theirs and they can use the vocabulary. If they can use it open and freely everyday then you know you understand it" (Teacher 3.1, 1st interview, 3/1/12).

The last section of her lesson was the implementation of an adopted problem solving and math fact program called Otter Creek. Teacher 3.1 did not have to plan for this part of her math lesson because the program was scripted and the principal of the school made sure all of the teachers had the necessary handouts prior to teaching each week. She found the program helpful when teaching students how to manage solving word problems. She found the formulas useful for the students and saw her students being successful when working through the word problems.

Across the two interviews conducted with Teacher 3.1, 30 excerpts pertained specifically to planning. Table 15 shows how the MKT codes were expressed throughout the planning interviews.

In addition to the codes found independently throughout the planning interviews with Teacher 3.1, she also expressed overlapping MKT codes as seen in Table 16. Nine different excerpts expressed multiple MKT categories. The following table describes the complexity of the overlapping MKT codes found within her planning process.

During her interviews, Teacher 3.1 illustrated an extensive reliance on Pedagogical Content Knowledge, which included Knowledge of Content and Teaching (KCT), Knowledge of Content and Students (KCS), and Knowledge of the Content and Curriculum (KCC), when planning her lessons. What was fascinating about these findings was that they contradict the results of Bruner et al (2010). In that article, the authors discussed how without common content knowledge of the subject matter; pedagogical content knowledge might not be as relevant or explicit. Teacher 3.1 even discussed that she was really bad at math and was unsure of mathematics beyond the scope of what she taught her 3rd graders (Teacher 3.1, 2nd Interview, 3/5/12). It is

Table 15

MKT Codes Related to Teacher 3.1's Planning

MKT category	Number of excerpts	Description of excerpts
Knowledge of Content and Teaching	9 total excerpts	• 3 about evaluate the instructional advantages and disadvantages of representations • 2 about choosing and sequencing examples • 2 about sequencing instruction • 1 about deciding which students' contributions to pursue and when to use a students' remark to make a mathematical point • 1 about deciding when to pose a new task
Knowledge of Content and Students	4 total excerpts	• 2 anticipating what students are likely to think and what they will find confusing • 1 anticipating what students will find exciting and relevant • 1 anticipating what students are likely to think and what they will find confusing, as well as knowledge of common conceptions and misconceptions about particular mathematical content
Knowledge of Content and Curriculum	6 total excerpts	• 5 about knowledge of available resources • 1 about the uses of particular programs in specific situations
Specialized Content Knowledge	2 total excerpts	• 1 about explaining and justifying one's mathematical ideas • 1 about how to choose, make, and use mathematical representations effectively

Table 16

Multiple MKT Codes in the Planning Interviews with Teacher 3.1

MKT categories	Number of excerpts	Description of excerpts
Knowledge of Content and Students – Knowledge of Content and Teaching	4 total excerpts	• 1 expressed an ability to anticipate what students are likely find exciting and relevant (KCS) and sequence instruction accordingly (KCT). • 1 demonstrated familiarity of common errors and which most students are most likely to make (KCS) and to choose and sequence examples (KCT) • 2 expressed her ability to anticipate what students were likely to think and what they would find confusing (KCS) and then sequence instruction accordingly (KCT)
Knowledge of Content and Teaching – Knowledge of Content and Curriculum	2 total excerpts	• 1 excerpt discussed how she sequenced instruction (KCT) based on the available resources (KCC) • 1 showed how she choose and selected tasks (KCT) and used particular programs in specific situations (KCC)
Knowledge of Content and Teaching – Specialized Content Knowledge	1 excerpt	This excerpt showed how Teacher 3.1 evaluated the instructional advantages and disadvantages of representations and how she chose and selected representations effectively (KCT). She also talked about unpacking mathematical knowledge, to make features of particular content visible to and learnable by students (SCK)
Knowledge of Content and Curriculum and Knowledge of Content and Students	1 excerpt	In this excerpt, Teacher 3.1 demonstrated knowledge of available curriculum (KCC) and anticipated what tasks in the curriculum might be hard or easy for the students (SCK)
Specialized Content Knowledge – Knowledge of Content and Teaching	1 excerpt	In this example, Teacher 3.1 talked explicitly about how math language was used in her class, how to explain and justify one's mathematical ideas (SCK) and how she sequenced instruction based on this knowledge.

possible that the fact that the teacher assessment focused on content taught in 2nd through 5th grade impacted Teacher 3.1's ability to demonstrate knowledge of the mathematics she knew. It is also possible that her identity as "not a math person" might have inhibited her ability to demonstrate her mathematical knowledge on the teacher assessment. It is also possible that the high change scores on her students' assessments were a function of her focus on test prep and usage of tasks based on the AzAC test. The possibilities of her instruction influencing this discrepancy between high content knowledge and extensive pedagogical knowledge will be further explored in later sections. Next we look at the planning of Teacher 3.2.

Teacher 3.2

Math time in Teacher 3.2's classroom consisted of two different components: the Math Board and the daily lesson. Each part was planned separately and followed different objectives. For Teacher 3.2, the math board provided time for the students to review all of the new Common Core State standards before the state test in April. On the other hand, the daily lesson followed the district curriculum map. She explained that her main focus during math time was the math board because it spiraled the standards and provided time for students to review concepts they might have not learned the first time. Our interviews focused mainly on the math board and therefore the following description of her planning process centered solely on the math board preparation.

Teacher 3.2 planned the math board on a daily basis. In her classroom, the math board consisted of one to two questions per each of the four 2nd grade common core domains. For the most part, the tasks she puts on the math board were taken from the state's common core document, test prep materials, or knowledge of questions that were previously used on prior years' tests. She left tasks on the board for approximately one week or until the students demonstrated they understood the concept they were working on. "You feed a lot from the kids so like one week I might have all the same problems but just be changing the numbers or not the numbers but just change the scenario or then like you'll go through like, you'll go through different problems" (Teacher 3.2, 1st interview, 1/26/12). She further explained that her main focus was to expose the students to as many different scenarios as possible because she never knew what type of problem would be on the test.

When selecting her tasks, Teacher 3.2 started with the easiest standards at the beginning of the school year. She anticipated what aspects of the curriculum the students might understand quickly and which concepts would help the students build a foundation for the more complex concepts later in the school year. She also planned how to instruct the unfamiliar concepts using the math board. "When I first, when they don't know the concept or they are sort of unfamiliar, I can't give them all of the information. I could just do a think aloud where I am just walking through it and then I can just say I, here's why I got this answer so just talk to your partner about the three steps, or I should be hearing that I have to take or something you know? Then, you know, the next day you give a little bit more information and then the next day a little bit more and the next day none at all" (Teacher 3.2, 1st interview, 1/26/12). She explained that talk alouds during math board enabled her to scaffold the concept over a week's time. What seemed crucial was the time she gave the students to discuss how they understood her process and what it meant for them solving the problem. It was during the student talk time that Teacher 3.2 assessed what her students understood and what they needed more time learning. She also used this time to decide which students should share their ideas with the rest of the class.

The final aspect of the math wall was the notion of test preparation. Teacher 3.2 used the math board as a vehicle to help students learn how to take a test. She exposed them to multiple-choice questions that mimicked the ones on the state and district tests. She required her students to explain why certain answer choices were incorrect so they could see how their silly mistakes ended up possible answer choices. She also expressed that “people are like don’t teach to the test, but by, you’d be doing a disservice if you didn’t teach how to take a test. Cause if like, I was in college, like I have to pay, we have to pay for our education, if I’m in there and I can’t get a good grade on the test even though I am trying, I’m going to be pissed like this guy is not teaching me what I need to know and that’s kinda like, it’s our responsibility to teach them” (Teacher 3.2, 1st interview, 1/26/12). Therefore, Teacher 3.2 structured her math board to not only spiral the standards, provide time for students to discuss their thinking, but also to learn how to take a standardized test.

Across the two interviews conducted with Teacher 3.2, 65 excerpts pertained specifically to planning. Table 17 shows how the MKT codes were expressed throughout the planning interviews.

The main focus of Teacher 3.2’s planning interviews was how she taught the Common Core standards. Of the 65 comments made during her interview, 59 pertained to Pedagogical Content Knowledge categories: Knowledge of Content and Teaching, Knowledge of Content and Curriculum, and Knowledge of Content and Students (Ball et al, 2008). Her strengths showed when discussing how she used the knowledge of what

Table 17

MKT Codes Related to Teacher 3.2's Planning

MKT category	Number of excerpts	Description of excerpts
Knowledge of Content and Teaching	40 total excerpts	• 21 about how she sequenced instruction • 9 about choosing and sequencing examples • 4 about how she evaluated the instructional advantages and disadvantages of representations • 3 about deciding when to pause, ask a new question, or pose a new task • 2 about when to use a student's remark to make a mathematical point. • 1 excerpt regarding which student contributions to pursue, which to ignore, and which to save until later
Knowledge of Content and Students	4 total excerpts	• 2 anticipating what students are likely to think and what they will find confusing • 2 anticipating what students are likely to do with a task and whether they will find it easy or hard
Knowledge of Content and Curriculum	15 total excerpts	• 7 showing her knowledge of available curriculums • 5 about instructional materials • 2 discussing how she used particular programs in specific situations • 1 showing her knowledge of the new curriculum (Common Core)
Common Content Knowledge	1 total excerpt (specific demonstration of her CCK)	1 showing her specific understanding of the mathematics in the students' curriculum
Specialized Content Knowledge	3 total excerpts	• 2 demonstrating her knowledge beyond that being taught to the students • 1 showing how to explain and justify one's mathematical ideas
Horizon Content Knowledge	2 total excerpts	• 1 showing how she saw connections between the 2nd grade curriculum and much later mathematics • 1 showing her knowledge of how mathematical topics are related over the span of the mathematics included in the curriculum

her students found tricky or easy to assist her in sequencing instruction. She explained how she selected and scaffolded the Common Core standards based on her students' thinking and how she learned of her students' ideas through making them discuss and justify their work.

Another interesting point that was not captured in the table was that Teacher 3.2 really focused her teaching on what might be on the state standardized test at the end of the year. This notion came across during the interview, but was not captured when coding for Mathematical Knowledge for Teaching. What made this absence important was that the high rate of change in performance in Teacher 3.2's (as well as with Teacher 3.1) students on the AzAC test might actually be related to her knowledge of the types of questions on the test and her ability to sequence tasks similar to those on the standardized tests rather than an extensive pedagogical content knowledge, even though the coding might show otherwise. Therefore, what looked like high PCK might actually be high knowledge of the standardized test questions along with knowledge of what concepts kids find easier or harder based on years of teaching experience at the same grade level. Teacher 3.2's (and Teacher 3.1's) relatively low teacher knowledge score supported the notion that the coding misrepresented her knowledge, as did her inability to explain her answers on the teacher knowledge test about concepts she has not taught.

Mathematical Knowledge for Teaching and Planning: A Comparison Between Teachers 3.1 and 3.2

Both Teacher 3.1 and 3.2 scored relatively low on the MKT test given to the sample of teachers in the NSF-funded grant. Each of them demonstrated high reliance on categories related to Pedagogical Content Knowledge during their planning interviews.

While this seemed like an obvious place for such knowledge to appear, the discrepancies between what could be explained, or what could not be explained, during their teacher knowledge assessment interview and the extremely high prevalence of PCK categories during their planning interviews made it possible to think something else influenced their planning and their high change in student performance scores.

High Stakes Testing. It is possible that high-stakes testing heavily influenced their planning and teaching but was captured under MKT codes as Pedagogical Content Knowledge. For instance, both teachers based their instructional sequence on the state's standards documents, which has examples of high quality tasks for teaching each performance objective. The standards document is highly correlated to the state's standardized test. Also, both teachers selected tasks from sample questions released by the state. In addition, they both emphasized the importance of learning how to take a test and providing instruction that would achieve this particular learning objective. This is not to say that Teacher 3.1 and Teacher 3.2 had no or little PCK. Instead, it brings up the question of whether or not their students' dramatic increase on the AzAC test over two quarters was a function of the teachers' Mathematical Knowledge for Teaching or the teachers' knowledge of test taking skills and methods for exposing students to a plethora of possible test questions. It is hard to determine based on the planning interviews and teacher knowledge assessment alone.

Next, I examined the instruction of Teacher 3.1 and Teacher 3.2. The section will examine the general look of each class if an observer walked in, the cognitive demand of the tasks (Stein et al, 2009), the types of questions asked by the teachers according to

Bloom's Taxonomy, and the types of responses provided by students to both the teacher and each other.

Implementation

The Implementation Phase of the Mathematics Teaching Cycle included: the learning environment, selection of meaningful tasks, and discourse. For this section, I presented the entire Implementation Phase components for each teacher and then compared the two teachers. Using this format for discussing the instruction component seemed more comprehensive and coherent than jumping between the components and the teachers.

Teacher 3.1

The instructional practice of Teacher 3.1 was evaluated using three methods: CLASS observation protocol, the Mathematical Tasks Framework, and Bloom's Taxonomy. Across all three tools, one could see that positive classroom management and teacher-driven lessons characterized Teacher 3.1's teaching style. She utilized published resources that aligned with the state's standardized test to select tasks for the students. For the most part, the tasks selected were written at a low-level of cognitive demand. During the implementation of the tasks, they remained at a low-level. For the most part, Teacher 3.1 modeled test taking strategies and procedures for solving the assigned problems. Students who answer the problem incorrectly were frequently called upon to walk through a problem with the teacher. Infrequent were occasions where students explained their thinking prior to receiving assistance from Teacher 3.1.

Aside from a few occasions where blatant errors in Common Content Knowledge were made evident during instructional times, the relationship between Teacher 3.1's MKT score and her instruction were difficult to determine. For the most part, Teacher 3.1 demonstrated a keen sense of what resources would provide her with similar tasks to the state's standardized test. She also showed an ability to sequence examples in a manner that allowed her students to build upon easier skills throughout the school year to reach mastery of more complex mathematics skills. It was unclear if this knowledge of sequencing instruction and knowledge of resources stemmed from her own understanding of the mathematics or her numerous years of teaching the same grade level and having a parent in the school district who was a resource teacher for over three decades. The following sections provided the evidence to support this comprehensive summary.

The learning environment.

Classroom Assessment Scoring System (CLASS) observation. Six CLASS observations were conducted during the time period in which Teacher 3.1 was observed. Because Teacher 3.1 was on maternity leave for the entire first semester of the 2011-2012 school year, all of the CLASS observations were conducted between January 2012 and February 2012. This shortened time period was a limitation of this study. Even so, the following Table shows the average scores across the 10 dimensions for teacher 3.1:

Table 18

CLASS Scores for Teacher 3.1

Dimension	Average score
Positive climate	5.5
Negative climate	1.67
Teacher sensitivity	4.67
Regard for student perspectives	2.5
Behavior management	6.67
Productivity	5.5
Instructional learning formats	3.67
Concept development	1.67
Quality of feedback	4.5
Language modeling	4.83

The CLASS dimension scores for Teacher 3.1 ran the gamut from very low scores for Negative Climate (which is what one wants to see), Regard for Student Perspectives, and Concept Development to a very high score in Behavior Management. The rest of the dimension scores fell in the middle range.

This wide range of scores across the 10 dimensions was apparent in the composite scores for the three CLASS domains. Teacher 3.1 received a score of 4.65 on Emotional Support, 5.28 on Classroom organization, and 3.67 on Instructional Support. Again, the scores based on a scale of 1 to 7, with 1 as the lowest score and 7 as the highest. As we can see, the domain scores for Teacher 3.1 fell within the mid-level range, with her highest score being Classroom Organization. This meant that Teacher 3.1 set clear expectations and enforced them consistently. She was proactive in anticipating behavior problems and in monitoring for potential problems during activities. She used efficient redirection and subtle cues to redirect students who were getting off task. Students complied with Teacher 3.1's expectations and were infrequently defiant. The classroom environment allowed for high productivity, for the most part. Students knew what to do and transitions between activities were brief and concise. Materials were prepared ahead of time and the teacher knew how to keep a steady pace throughout the class time. The lowest scores in this domain came from the lack of variation in modalities and materials used during instruction, as well as the role of the teacher being less of a facilitator and more of a director of instruction (Pianta et al, 2008).

Teacher 3.1 received a mid-level composite score for Emotional Support. For the most part, positive interactions occurred in this classroom. Teacher 3.1 seemed to hold positive regard for the students as they did her. She frequently moved into close proximity to the students during their talk time but she mostly stood at the board to lead discussions or talk alouds. There were times during the observations where Teacher 3.1 was a little short when interacting with the students, especially when she was rushed for time or a student struggled to explain their thinking. However, these occasions were infrequent. She never yelled at the students or was sarcastic with them. Nor was there ever a moment of bullying either between the teacher and students or the students with each other during the CLASS observations. Also, Teacher 3.1 was fairly responsive to students when they were struggling with a concept but occasionally she dismissed what a student was saying and completed an activity herself, especially when she seemed pressed for time. The low score in this domain came from Regard for Student Perspectives. For the most part, Teacher 3.1 led the lessons and maintained a very structured classroom. Students were encouraged to talk to each other but only when given permission or when called upon to share ideas with the class. There were few opportunities for students to lead discussions or move about the classroom during the lesson (Pianta et al, 2008).

The lowest domain score was in Instructional Support. There was middle of the range scores for items pertaining to “the degree to which the teacher provides feedback that expands learning and understanding and encourages continued participation” (Pianta et al, 2008, p. 72) and the modeling of language in the classroom. Teacher 3.1 frequently mapped her own thinking through language and description as well as asked students a mix of open- and closed-ended questions. She also revoiced students’ thinking. There were limited conversations in the classroom, however, between students. They were given sentence frames for discussions but the peer talk time was closely monitored and controlled by Teacher 3.1. In addition, advanced language was used on a limited occurrence. The low score manifested through concept development. There was rarely a time for students to be creative and generate their own ideas for problem solving. Teacher 3.1 most often showed or provided the method for solving particular problems and then the students reproduced the method when practicing. Also, concepts were frequently shown independently from each other. Often times, the students told her when concepts related or when they had learned a relevant skill for solving a problem previously. They also told her when they used one problem on the Math board to solve another one under a different strand.

Overall, the CLASS observations depicted a classroom dominated by positive interactions, active participation, and high productivity. But, Teacher 3.1 determined the direction and focus of the instruction in the classroom. The relationship between Teacher 3.1’s CLASS scores and her MKT scores proved difficult. Teacher 3.1 demonstrated high scores for interactions related to classroom management, such as Positive Climate, Negative Climate, and Behavior Management. These aspects of teaching were not included in the Mathematical Knowledge for Teaching framework. Teacher 3.1's lowest scores in the categories of Regard for Student Perspective, Instructional Learning Formats, and Concept Development might map to her low MKT scores. These particular categories examined the flexibility in the classroom, the role of students, the variety of modalities and materials used in the classroom, effective questioning, the use of advanced organizers and summation of lessons, as well as interactions pertaining to problem solving, predictions, evaluations, brainstorming, connections among concepts and real world applications. Without a breadth of knowledge about the elementary school mathematics curriculum, content, and how to unpack the mathematics, it might be hard for a teacher to relinquish control, allow for student questions to drive the conversation, or be able to construct activities that integrate mathematical concepts across the curriculum. It is also possible that her drive to ensure her students passed the state's standardized test caused her to keep control of the discussions and the direction of the class. Teacher 3.1 moved away from the textbook, she stuck closely to the state standards document and the activities provided in resources pertaining to the state standardized test. Based on the CLASS assessment and the importance of high-stakes testing to Teacher 3.1, it was difficult to determine how Teacher 3.1's MKT pertained to interactions in the classroom as assessed by the CLASS observation protocol. Next, we examined the cognitive demand of tasks given by Teacher 3.1 (Stein et al, 2009).

Worthwhile mathematical tasks and classroom discourse.

The mathematical tasks framework (Stein et al, 2009) for Teacher 3.1. Because Teacher 3.1 was on maternity leave for the first semester of the 2011-2012 academic year, three one-hour long classroom observations were used to assess her instruction. Teacher 3.2 divided her instruction into three 30-minute sections: Math board, daily lesson, and Otter Creek. For purposes of this analysis, I analyzed the math board and daily lesson separately, even though Teacher 3.1 used the tasks on the math board as an introduction or review of the daily lesson. Over the three observations, Teacher 3.1 only taught a lesson once during the daily lesson time. The other two observations included a video to introduce fractions and the student teacher teaching the lesson. Because only one observation of the daily lesson was actual instruction, I opted not to use this portion of the lesson as part of the results. Also, Teacher 3.2 was only observed teaching the math board, therefore, it only seemed logical to only analyze Teacher 3.1's math board time in order to make a comparison across the teachers in the case. I opted not to analyze the tasks from the Otter Creek curriculum because it was not a curriculum adopted across the district. The teachers in Teacher 3.1's school were piloting Otter Creek during the 2011-2012 academic year.

In each of the three observations, 12 written tasks were given to the students during math board time. There were a total of 21 memorization tasks, 7 procedures without connections tasks, 6 procedures with connections tasks, and 2 doing math tasks assigned (Stein et al, 2009). Each task was written as a multiple-choice question, except for one of the doing math tasks. The one open-ended question was based on an Otter Creek question. Teacher 3.1 wanted to know if the students knew the Otter Creek

Table 19

Cognitive Demand Level of Tasks Given by Teacher 3.1

Classification of task	Example	Explanation
Memorization	“What is four thousand, six hundred five written in standard form?” or “Anissa has 8 quarts. How many gallons does she have?”	As written this task involves reproducing already learned facts and "have no connection to the concepts or meaning that underlie the facts, rules, formulae, or definitions being learned or reproduced" (Stein et al, 2009, p. 6).
Procedure without Connections	"Ben colored a figure with the fewest possible colors. The edges that touch must be different colors. Which could his figure be? (answer choices were drawn on the board)”	These tasks are algorithmic and require limited cognitive demand for successful completion, as written. They are "focused on producing correct answers rather than developing mathematical understanding" (Stein et al, 2009, p. 6) and they "require no explanations, or explanations that focus solely on describing the procedure that was used" (p. 6).
Procedures with Connections	"Four friends will share this pizza equally. Which fraction shows the part of the pizza that each will eat? (The answer is in eighths)” or “Brenda has 27 stuffed animals at home. She gives an equal number of stuffed animals to her three friends. Which number sentence can be used to show how many stuffed animals they each received?”	This task "requires some degree of cognitive effort. Although general procedures maybe followed, they cannot be followed mindlessly. Students need to engage with conceptual ideas that underlie the procedures in order to successfully complete the task and develop understanding" (Stein et al, 2009, p. 6) This task also requires students to think about what they are answering and how they will get to the answer. In addition, there are multiple ways in which a child might solve this problem and there is a demand on the children to monitor their own thinking because the answer they might initially end up with might not be the answer to the question presented.
Doing Mathematics	"Mr. Barbosa rode his bicycle 81 miles in 4 days. If he rode the same number of miles each day. About how many miles did he ride each day?”	This task is at the highest level of cognitive demand because "there is not a predictable, well rehearsed approach or pathway explicitly suggested by the task, task instructions, or a worked-out example"(Stein et al, 2009, p. 6). In fact, there were no examples modeled or instructions given about solving this task until after the students had tried to solve it on their own. Thus, this task "required students to access relevant knowledge and experiences and make appropriate use of them in working through the task" (p. 6).

procedure, meaning they had memorized the procedure and recognize when to use it, or if they were solving the problem other ways (Teacher 3.1, 2nd Interview, 3/5/12). Table 19 provided examples of the different types of tasks assigned by Teacher 3.1 during math board.

For the most part, all of the tasks either remained or became lower-level cognitive demand tasks upon implementation (Stein et al, 2009). For example, the following is a vignette of the implementation of a Procedure with Connections task. The observation occurred on January 30, 2012. The task given to the students was:

Cesar has 3,462 marbles, Levy has 3,211. About how many do they have in all?
a) 1,000
b) 6,000
c) 9,000
d) 7,000

To introduce the task, Teacher 3.1 read the question to the students.

T: Cesar has 3,462 marbles. Leslie has 3,211. About how many do they have in all? Something they really have to remember and the key word here, who knows the key word, one of them? Daniel?
Daniel: About
T: thank you so much. About is a key word and that means to?
C: estimate!
T; and another key word is, and what does it mean?...
Kid: in all?
T: in all most of the time means to do what?
Class (with teacher): add or multiply!
T: most of the time. K? so figure it out. think about what you have to do. do that same way we talked about, the 8th grade way. ready, ready set?
C: Go!

When introducing the task, Teacher 3.1 focused on the key words in the problem and told the students to use the "8th grade" method for solving (See Appendix E – maintaining low-level cognitive demand—students told the procedure to use; directions of what to do are clear and teacher-directed). Teacher 3.1 demonstrated that she knew the definition of the key words in the problem and that “in all” might indicate either “add or multiply” (Common Content Knowledge – knows the terminology in the problem; able to solve the assigned task).

> *T: Okay let's come together in 5,4,3,2, and 1[00:17:54.09] this is um, ???, alright so let me show you the correct. raise your hand if you think you found a goofy mistake? realizing after talking. what was your mistake, Tracy?*
> *Tracy: I put D, D, because like, um, I . . . put 3,462 and 3,211 and I forgot, I forgot to estimate. and then um I remembered and then like I like I tried four like I like I thought 4 was like a power mean, power number*

In this segment, Teacher 3.1 asked the student to explain the mistake she made when solving the problem. Teacher 3.1 frequently asked students to explain their thinking when they made a mistake. At this point in the task implementation, Teacher 3.1 took over the task and solved the problem for the class using the “8^{th} grade” method.

> *T: ohhhhh, so let's and let's make sure we are always doing the 8th grade way. I think I saw some of you that weren't. this is the way when you need to do it. you have to circle, look up here please. you're going to circle the 4. you are going to ask your self does the 4 play for the power team or the weak team [has motions for each label]?*
> *C: weak Team [make motion]*
> *T: sooo, can it push up the three?*
> *C: NO!!!*
> *T: stays the*
> *C: the same!*
> *T: put the power to the weak team*
> *C: weak team!*
> *T: it tries to put the push up the three but can it?*
> *C: NO!!*
> *T: it stays at 3,000. and then I look at in all so zero, zero, zero, six. So the answer is?*
> *C: B!!!!*
> *T: If you go D, did you, now, I saw that you put D. Did you solve it the 8th grader way or the other way?*
> *Kid: the other way?*
> *T: If you solved it the other way of first subtracting and then rounding, you're gonna sometimes get a different answer. so I WANT...everybody to solve it, this same way. every time. this will get you the correct answer all*

of the time. okay? Do not solve first. Always, always, and I got this information from the 8th grade teacher. She said that's going to get the wrong answer sometimes, so if you follow directions quickly, you solved it this way and you got it right. if you didn't you got it wrong. Don't get it wrong again. the answer, the correct answer is B.

Teacher 3.1 asked the students Knowledge questions (Bloom, 1956) when explaining how to solve the task. She did follow up with a student to find out why he selected answer choice D, but instead of allowing the child to explain *how* he solved the problem, she jump in and reemphasized exactly how the problem should be solved (maintenance of low-level cognitive demand – see Appendix E). Not once in the conversation did Teacher 3.1 explain why the 8th grade method worked or why estimation might trick students. The focus remained on selecting the correct answer choice and then moving on to the next question. It was hard to determine if the lack of depth about estimation was a function of Teacher 3.1's knowledge level or time constraints based on the amount of work that had to be accomplished in the math period.

This set of vignettes represented much of what occurred during math board time. The students received a task, then they completed the task independently and showed their answer choice using answer cards when asked. If multiple answers were given, the students discussed their work with a partner. If most answers were correct then Teacher 3.1 walked the students through the process of solving the problem. If one student gave an incorrect answer, Teacher 3.1 usually asked the child to walk through solving the problem with her. Other wise, they solved the problem as a group and then moved on to the next problem.

The second example of Teacher 3.1's teaching occurred on February 6, 2012.

The students were asked the following question:

Four friends will share this pizza equally. Which fraction shows the part of the pizza that each person will eat?

a) 1/8
b) 1/2
c) 2/8
d) 3/4

As written, this task fit the criteria for the label of a procedure with connections task (Stein et al, 2009). The problem required students to understand equivalent fractions and how to determine equivalency of fourths. This required students to fully engage with the mathematics and monitor their own thinking. The start of the implementation of this task followed the progression of most other assigned tasks, however, things changed when Teacher began to explain how to solve the problem.

> *Teacher 3.1: thank you Sam for putting it up fast...alright, hands down let's see, um, I'm going to explain the four pieces because there are four people. this one, I will eat one out of the four. this will eat one piece. this will eat one piece. and so does this one. so...whoa! what is this. the answers aren't even here. part of the piece that each people would eat. oh the answer should be here. U. . . . ..[kids are talking] that's because the thing is I was coping it . . . [kids talk] no, let's just forget this problem.*

This excerpt demonstrated Teacher 3.1's lack of Common Content Knowledge about equivalent fractions. Halfway into explaining how to divide the pieces into fourths, she realized that her answer of "1/4" was not on the board. She, later, admitted to copying the problem from another source and then proceeded to throw the problem away.

What made this interesting was that a child was making eighths during the independent work time. Teacher 3.1 did not understand what the child was doing and told him to focus on the 4 people eating the pizza. "why are you splitting that into 2,4,6,8 pieces. only 4 people."

In this episode, the teacher's lack of common content knowledge cost the children an opportunity to discuss equivalent fractions. It also cost the teacher an opportunity to ask the children to show her how they solved the problem. They all showed her their answers but she failed to ask the students to help her understand the problem.

This vignette aligns with the struggles seen in Teacher 3.1's teacher knowledge assessment and post-assessment interview. She admitted to struggling with fractions and not knowing how to teach them. It was also apparent, based on the questions in the teacher knowledge test, that Teacher 3.1 was unable to understand student thinking about fraction problems. Such struggles were demonstrated with this task. This particular weakness in Teacher 3.1's knowledge, leads one to question whether or not the students performed well or not on the AzAC test questions related to fractions. Or if the students could explain fractions, equivalent fractions, and operations with fractions beyond a procedural level at the end of the school year. Those questions are out of the scope of this dissertation, unfortunately.

In summary, positive classroom management and teacher-driven lessons characterized Teacher 3.1's instructional practice. She utilized published resources that aligned with the state's standardized test to select tasks for the students. For the most part, the tasks selected were written at a low-level of cognitive demand. During the implementation of the tasks, they remained at a low-level. For the most part, Teacher 3.1 modeled test taking strategies and procedures for solving the assigned problems. Students who answer the problem incorrectly were frequently called upon to walk through a problem with the teacher. Infrequent were occasions where students explained their thinking prior to receiving assistance from Teacher 3.1. Aside from a few occasions where blatant errors in Common Content Knowledge were made evident during instructional times, the relationship between Teacher 3.1's MKT score and her instruction were difficult to determine. For the most part, Teacher 3.1 demonstrated a keen sense of what resources would provide her with similar tasks to the state's standardized test. She also showed an ability to sequence examples in a manner that allowed her students to build upon easier skills throughout the school year to reach mastery of more complex mathematics skills. It was unclear if this knowledge of sequencing instruction and knowledge of resources stemmed from her own understanding of the mathematics or her numerous years of teaching the same grade level and having a parent in the school district who was a resource teacher for over three decades. Teacher 3.1 also demonstrated that even if a teacher used a well-crafted curriculum guide for selecting tasks and sequencing instruction, a lack of Common Content Knowledge could hamper students' opportunities to learn. Next, we examined Teacher 3.2, using the same analytic methods, to see if any patterns or disparities were seen between the two teachers with relatively low MKT scores.

Teacher 3.2

The instructional practice of Teacher 1.2 was evaluated using three methods: the CLASS protocol, the Mathematical Tasks Framework, and Bloom's Taxonomy. Overall, Teacher 3.2 constructed a classroom environment that embraced discussion, evaluation, and the exploration of the mathematics content in the second grade curriculum. Even though Teacher 3.2 directed the initial method for how students solve problems, she allowed students to present their own ideas and methods for solving problems as long as the students could explain how their method worked. Much like Teacher 1.1, Teacher 3.2

provided evidence in her instruction of high reliance on her Specialized Content Knowledge, Knowledge of Content and Students, and Knowledge of Content and Teaching. These were not discrete occurrences of these knowledge types but instead interconnected, dynamic relationships among the MKT categories throughout her observed teaching. The following sections provided the evidence to support the claims made in this introductory paragraph.

The learning environment.

CLASS observation protocol. Six CLASS observations were conducted during the time period in which Teacher 3.2 was observed. Two were conducted in October and November, 2011 and four were conducted in January and February of 2012. The following Table shows the average scores across the 10 dimensions for teacher 3.2:

Table 20
CLASS Scores for Teacher 3.2

Dimension	Average score
Positive climate	6.167
Negative climate	1.5
Teacher sensitivity	6.167
Regard for student perspectives	4.67
Behavior management	6.67
Productivity	6.67
Instructional learning formats	5.5
Concept development	3.67
Quality of feedback	5.33
Language modeling	6.33

Aside from the low score on Negative Climate (again, this low score meant there was a low occurrence of negative interactions in the classroom), Teacher 3.2 received scores averaging in the 5 or 6. The two lowest scores received were still in the middle range of scores.

When composite scores were calculated for the three CLASS domains, Teacher 3.2 received a 5.75 on Emotional Support, a 6.28 on Classroom organization, and a 5.11 on Instructional Support. Again the scores based on a scale of 1 to 7, with 1 as the lowest score and 7 as the highest. When looking across the teachers in this dissertation, Teacher 3.2 had the second highest composite score for Emotional Support and the highest composite score for Classroom Organization and Instructional Support. In this classroom, interactions related to Emotional Support reflected a setting of care, excitement, happiness, and kindheartedness where students felt capable of exploring new ideas and taking risks (Pianta et al, 2008). Teacher 3.2 added to the positive climate of the classroom by constructing an environment where the students' needs, thoughts, and interests directed the interactions, for the most part.

The Classroom Organization score was also high in this classroom. This score represented a classroom where Teacher 3.2 set expectations clearly and used successful ways of redirecting bad behavior. She also effectively constructed instructional routines that added to the productivity of the class time and many opportunities for students to express their thinking.

Lastly, Teacher 3.2 received the highest composite score for Instructional Support. While, her lowest score in this category came from the concept development dimension, Teacher 3.2 still maintained frequent use of "discussions and activities to promote students' higher-order thinking skills and cognition" (Pianta et al, 2008, p. 63) rather than on rote instruction. She also provided feedback to students that enhanced their learning and engagement in the activities. The CLASS observations captured a classroom where discussions occurred frequently between the teacher and the students and the students among themselves. There was a high occurrence of times where Teacher 3.2 used “language-stimulation and language-facilitation techniques” (Pianta et al, 2008, p. 63) to enhance learning in the classroom and to model how to use the mathematical language being taught.

Teacher 3.2 presented a very interesting contrast to Teacher 3.1, and the other teachers in the study, when examining the CLASS scores with the MKT scores. While Teacher 3.2 received the a low MKT score relative to her peers in this dissertation, Teacher 3.2 created a classroom environment with her students that encouraged discussions, risk-taking, limited behavior problems, and positive interactions that surpassed the other teachers in this study. One reason might be that in many ways, the CLASS scores for Teacher 3.2 mapped to her planning interviews more so than her teacher knowledge assessment score. For example, the planning interviews depicted a teacher who relied heavily on the interaction between her knowledge of content and teaching and her knowledge of content and students. Teacher 3.2 stressed the need for test preparation and constructing an environment where students talked through their thinking with each other in a manner that allowed them to learn how not be tricked by distractor answers on multiple choice tests. She also talked about how she used student thinking to sequence her tasks. These ideas were captured in the CLASS assessment. The lower score in concept development mapped to Teacher 3.2’s MKT score, however, the CLASS score was still in the middle range and not the lowest score on concept development of all of the teachers. Teacher 3.2 structured her instruction in a way that encouraged students to justify and explain their problem solving strategies. She also asked students higher-order thinking questions (Bloom, 1956) and frequently engaged in feedback loops with the students (Pianta et al, 2008). Next, I examined the instruction of Teacher 3.2 using the Mathematical Tasks Framework (Stein et al, 2009).

Worthwhile mathematics tasks and classroom discourse.

The mathematical tasks framework (Stein et al, 2009) for teacher 3.2. Three one-hour classroom observations were used to assess the teachers' instruction in this dissertation. Although more observations were conducted with Teacher 3.2, I kept the analysis portion to the same number used to analyze Teacher 3.1's instruction. During the three observations on Teacher 3.2, seven tasks were assigned during a single class period, for a total of 21 written tasks. All 21 tasks were written as multiple choice. Overall, there were a total of 14 memorization tasks, 3 procedures without connections tasks, and 4 procedures with connections tasks (Stein et al, 2009). The following table provides examples of the different types of tasks assigned by Teacher 3.2.

Table 21
Cognitive Demand Level of Tasks Given by Teacher 3.2

Classification of task	Example	Explanation
Memorization	"Which two numbers are even? a) 364, 324 b) 672, 675 c) 321, 324"	As written this task involves reproducing already learned facts and "have no connection to the concepts or meaning that underlie the facts, rules, formulae, or definitions being learned or reproduced" (Stein et al, 2009, p. 6). It is a task that demonstrates mastery of vocabulary.

Table 21
Cognitive Demand Level of Tasks Given by Teacher 3.2

Classification of task	Example	Explanation
Procedure without Connections	"What two numbers come next? 625, 635, 645, ___, ____? a) 646, 647 b) 655, 665 c) 745, 845"	These tasks are algorithmic and require limited cognitive demand for successful completion, as written. They are "focused on producing correct answers rather than developing mathematical understanding" (Stein et al, 2009, p. 6) and they "require no explanations, or explanations that focus solely on describing the procedure that was used" (p. 6).
Procedures with Connections	"Which person has 65¢? a) Tom has 2 quarters and 5 pennies b) Sam has 6 dimes and 5 pennies c) Mary has 6 dimes"	This task "requires some degree of cognitive effort. Although general procedures maybe followed, they cannot be followed mindlessly. Students need to engage with conceptual ideas that underlie the procedures in order to successfully complete the task and develop understanding" (Stein et al, 2009, p. 6) This task also requires students to think about what they are answering and how they will get to the answer. In addition, there are multiple ways in which a child might solve this problem and there is a demand on the children to monitor their own thinking because the answer they might initially end up with might not be the answer to the question presented.

To best understand how Teacher 3.2 implemented the written tasks and how the implementation related to MKT, I used the following memorization task (Stein et al, 2009) that was given on January 25, 2012:

Which two numbers are <u>even</u>?
a) 38, 35
b) 24, 56
c) 73, 47

The students were seated on the floor facing the math board. They each had a small white board and marker in their laps. At the start of this lesson, the task was a memorization task. It asked students to distinguish between odd and even numbers. While the task remained at a memorization level, the implementation of the task included aspects of higher cognitive demand.

Teacher 3.2: alright, everybody erase your boards and eyes up here for instruction in three, two, one. Thank you. . . . Alright, now we're looking at even and odd. Now, we have all, we only have the two numbers here so let me just talk to you about this for a second. So if we have two numbers, we know we have tens and ones. So think about the place value of the ten and the one. the one is just one little tiny square . . . is one. The tens is a stick and it has ten little squares in there, so each of these has a one in them. one one, one ten. which one is more?
Class: Ten!
Teacher 3.2: yes. because it's not one, it's one tens stick. if we throw in the hundreds, it has a huge box and it has ten ten-sticks with ten-ones in them. so it has 100. so when we say one hundred. it isn't just one it's one HUGE block of 100 little blocks. So whenever we are looking at even and odd. now that we know that. whenever we're looking at even and odds. we never look at the 10, the 100, why?
Class: cause they're already stuck together!
Teacher 3.2: they're stuck together and they all have partners in there, they all like live in there so there's no reason that it would be one left out and we never look at the 10s, why?
Class: cause they're all stuck together
Teacher 3.2: perfect. because they are in a tens stick and they all have partners.

Using comprehension and analysis level questions (Bloom's 1956), Teacher 3.2 introduced the task of deciding between odd and even numbers. The students explained to her that numbers in the 10s and 100s place already have partners and are unnecessary to look at when deciding if a number is odd or even. Teacher 3.2 demonstrated Common Content Knowledge of odd and even numbers, as well Specialized Content Knowledge upon explaining the question. She was able to break down the place value and explain what the place values meant using the representation of base 10 blocks. Using this representation, she was able to explain why they only look at the unit place when determining numbers as odd or even. This example also falls under the category of Knowledge of Content and Teaching because she was able to figure out viable models that would help students understand place value (Ball et al, 2008).

Teacher 3.2: But the ones we don't know. As soon as they get ten they are all partners and they become a stick but if they don't have ten yet they are just ones, they are all over the place so we have to even Steven them off. So what I want to see, and you're going to have to write all of these down, is you're gonna write these numbers down and I'm gonna see tens, ones just like this and I want to see 8 for this one dots, and then you are going to even Steven them off. Even Steven, even Steven, even Steven, even Steven! This one is even. is it enough? No! we have to check which two. Both of them have to be even. so don't just look at the first one, go to the next one. Can I see it?
Class: NO!
T: what is it?
Class: it's ODD!
T: odd, Todd. so this is, are these two even? no! I just did that for you. You have to decide between B and C. you want to pick the two that are even. I want to see dots. I want to see houses. Forget A. Do B and C. I already told you A.

At this point, Teacher 3.2 gave the students the procedure they should use for completing the task but then allowed students time to replicate the procedure in order to solve the problem (maintaining low-level cognitive demand – see Appendix E). She gave the students time to discuss their thinking with their partner and then selected students to share at the board.

> *T: okay, George and Cameron are going to come up to explain. You need to give them nice and respectable bodies, just like you gave me when I was explaining the problem. We'll wait for you.*
> *George: I put B because I put four dots, one, two, three, four, and even Steven they were even. Then two, they were even. But I wanted to see just to make sure. One it wasn't even. and that was wrong and three, even Steven and then {inaudible}*
> *T: Alright good.*
> *Cameron: I agreed with him because I looked at the board and I saw the question. it was wrong because one isn't even.*
> *T: what is it?*
> *Cameron: odd*
> *T: it's odd.*
> *Cameron: so if it was a two it would have been even*
> *T: but it's not. okay, good.*
> *Cameron: and I checked so that was wrong*
> *T: Cameron, why didn't you look at the tens spot?*
> *Cameron: cause this was, cause this already messed up. It already messed it up the one. So that's why I didn't chose A*
> *T: no, no. A you didn't chose. But look at, you're working, do you see that he's working on the ones, why aren't we doing dots by the tens?*
> *Boy: Cause*
> *T: no Cameron*
> *Cameron: cause they're in sticks*
> *T: and?*
> *Cameron: and, and they have partners.*
> *T: they have partners and they, is that the same reason that we're not working in the hundreds too, right?*
> *Boy: yes*
> *T: good. I like what I heard. Thank you Cameron and George.*
> *Class: thank you Cameron and George*

George and Cameron were expected to explain why they chose an answer and why the other answer choices were incorrect. In conjunction with the time allotment of solving the problem and the student discussions with their partners, the task implementation shifted from low-level to high-level. In addition, Teacher 3.2 asked a variety of questions to her students but mostly those at the level of analysis. She asked the students why they never look at the tens place when dealing with odd and even numbers. She also asked kids to analyze what their partner did and evaluate if they agreed or disagreed with each other's methods during their partner talk time. Teacher 3.2 also used George and Cameron's thinking to reiterate why only the unit place was used to solve this problem (Knowledge of Content and Teaching – using student thinking to make a mathematical point).

This vignette illustrated a typical progression of Teacher 3.2's instruction during a math board activity. Teacher 3.2 began with an introduction to the concept. In this particular case, she reviewed place value with the students and reminded the students what tens and ones looked like if using base ten blocks. The kids told her they dismissed the places other than the ones place because those units are already stuck together, meaning they are already paired up. Teacher 3.2 completed one of the answer choices for the students, as part of her talk aloud and then the students were tasked with finding the correct answer choice independently. Once Teacher 3.2 noticed enough students had finished solving the problem on their boards, the students discussed their findings with their partner. The students were given sentence frames to structure their discussions. One child started with, "I chose ____ because ___." The other child replied, "I agree (or disagree) because _____." Teacher 3.2 moved among the pairs as they talked. During that time, she decided which pair of students to call upon to share with the class. In this particular case, she selected two students who had finished quickly and explained their thinking to her well. Other times, she called upon students who had a disagreement but figured out how to help each other arrive at the correct answer. It was dependent upon what she saw troubling most of the students in the class (Teacher 3.2, 2nd interview).

A second interesting example of Teacher 3.2's instruction occurred on February 1, 2012. The written task was:

What is another way to solve five times 2?

a) 5 +2
b) 5 + 5
c) 5 - 2

As written this task was categorized as a memorization task (Stein et al, 2009). The vignette starts with Teacher 3.2 reading the task to the students:

Teacher 3.2: okay, erase your boards and bodies up here in three seconds, I can give you some. three, thank you, two, one, do your part...thank you okay now for this one, this says what is another way to solve, 5 times 2. we have not done this in a loooong time but remember when we are talking about times it's just how many times you see the number. so if it says 5 how many times are we going to add five?
Kids: two
Teacher 3.2: two so, it, it, it just is that. so what you would do, is it 5 plus two? tell me
Kid: no
Teacher 3.2: why is five plus two wrong?
Kids: cause we're doing times
Teacher 3.2: five plus two equals what?
C: 7
Teacher 3.2: 7. so if we, this is like, this is like, what this is, is saying that we have five groups or no we have the number 5 just two times. or we can say 5 dots and then another group of 5 dots. okay? so it's the number 5 in some way two times. so if we count these 1,2,3,4,56,7,8,9,10, that's not going to be the same as 7. okay? we're looking for the number five two times

In this portion of the lesson, Teacher 3.2 took over how to solve the problem. She explained that they were looking for the number 5, two times. She also worked through the first answer choice with the students to demonstrate how to think through the problem solving steps. She asked the students why the answer choice was wrong but this was introduction to the task was less about student thinking and more a "talk aloud" about how to solve the problem. Teacher 3.2 demonstrated Common Content Knowledge and

Specialized Content Knowledge in this excerpt because she was able to complete the task and break the task down into a pictorial representation that illustrated multiplication as "groups of" for students. They were able to see how "5 + 2" differed from "5 x 2."

Teacher 3.2: okay, so now we have five plus five. what is that?
Boy: no
Class: yes . . . maybe!
Teacher 3.2: okay, why do you think it's kind of a good answer? what?
Boy: because I counted by 2s five times. 2,4,6,8,10 and it make ten
Teacher 3.2: great! great job! and what else? why else do you think it's a good answer? Josue?
Josue: because um um um we got two circles and I know that two is my two, and it's two circles of 5
Teacher 3.2: two circles of five. I like that. and um job you put your hand up. Yeah, it's going to be two circles of five. and what he was saying, what Jose was saying is really good because it doesn't, do you remember me talking about this? it doesn't matter if you switch it, it can either be two circles of five or it can be five circles of what?
Kids: two
Teacher 3.2: of two, so we'll just say 2,2,2,2,2 and that's what Jose did. He counted by twos. Do it with me
C: 2,4,6,8,10
T: either way if this was a five or this they both equal ten because they are both showing um five two times or two five times. so when you see times, right now, in third grade you're going to have to know [kid walks in].

In this excerpt, Teacher 3.2 used student thinking to introduce the commutative property (without using the terminology) to the students (Knowledge of Content and Teaching – using students' thinking to make mathematical points). She heard two students giving complementary methods for solving the problem. One child counted 2s five times, while another child counted two circles of five. Teacher 3.2 also demonstrated Common Content Knowledge.

T: okay, now in third grade you are going to have to remember all of these multiplication facts and it's going to get hard, look up, I know it's going to get hard to draw them all out like this. The little numbers it's easy but in the bigger numbers it's hard so you are going to have to remember in your *head right now you just have to know this that 5 times two is actually the number 5 two times or the number 2 five times. Okay?*

This monologue illustrated Teacher 3.2's Horizon Content Knowledge about what knowledge her students would gain in 3rd grade and how this task would help them become fluid with multiplication facts.

David: you have to go over all of them so you can make sure
Teacher 3.2: and David is right, so we stopped here and we really should go over all of them. This is just five minus two, what do you think?
Class: no.
Boy: that's a crazy one
Teacher 3.2: that's crazy. five and you take two away is how many?
Class: three!
Teacher 3.2: three we are not seeing the five two times or the two five times. this is seeing repeated numbers, seeing the numbers at different times, so um what I want you to do, I don't want you to solve this since we solved it together but you're going to talk about in multiplication when you have times or when you have to multiply you um, put the number a certain amount of times and it's okay if you put the two first and then the five, two five times or if you put five two times, whatever's easier. How about you do that and what would be cool is if you had your boards and you kind of did that. like this. 5 times two and then you could either drew five ah circles with two inside or you can do two circles and five inside. okay, so I just want you to be having conversations like that. Now wait, wait, wait, wait. Pencils um are going to start talking first and then books can start talking and I just have one question for you, who let the smart out?
Class: me! You!
Teacher 3.2: good
Kids talk to their partners
Teacher 3.2: draw the things so it's five two times or two five times...yeah, you're doing it. good, good . . . so if there are five circles how many are going to go in?. . . yeah! and we are looking for the five two times, this is saying 5 two times do you see that here? no it's only one time, do you see five two times here? that's what you want . . . okay, okay, sit down. thank you. okay, let's get back
Class: together
Teacher 3.2: Okay, so I see lots of good stuff so when it says 5 times two you can either do five two times or the two five different times so um we're going to um keep putting this up on the board and I'm going to keep trying to trick you, okay? sounds like you guys are doing a great job on that.

At the end of this lesson, David reminds Teacher 3.2 that they need to review all of the answer choices. This was a test preparation strategy that she taught her students throughout the school year. In addition, the focus of the pair sharing was an evaluation of the various methods to solve the task. Again, she was able to take a low-level cognitive demand task and add in aspects of high-level cognitive demand.

These two examples of Teacher 3.2's instruction depicted what happened through out the three observations analyzed for this dissertation. Across the lessons, Teacher 3.2 introduced a problem, asked the students to solve the problem, and then had some sort of discussion (whether that was between the pairs of students or having students present their ideas to the class) based on the students' thinking of how to solve the problem. For the most part, when using Bloom's taxonomy to analyze the questions presented to the students, Teacher 3.2 asked the students higher order thinking questions. These questions required students to think about the mathematics beyond a recall level. They had to analyze, justify, and evaluate their problem-solving methods against their fellow peers' methods to see if they solved the problem correctly.

While these were only two illustrations of Teacher 3.2's instruction, they were indicative of what happened through out the observations. Overall, Teacher 3.2 constructed a classroom environment that embraced discussion, evaluation, and the exploration of the mathematics content in the second grade curriculum. Even though Teacher 3.2 directed the initial method for how students solve problems, she allowed students to present their own ideas and methods for solving problems as long as the students could explain how their method worked. Much like Teacher 1.1, Teacher 3.2 provided evidence in her instruction of high reliance on her Specialized Content

Knowledge, Knowledge of Content and Students, and Knowledge of Content and Teaching. These were not discrete occurrences of these knowledge types but instead interconnected, dynamic relationships among the MKT categories throughout her observed teaching.

In many ways, the multiple occurrences of Specialized Content Knowledge, Knowledge of Content and Students, and Knowledge of Content and Teaching aligned with Teacher 3.2's planning and teacher knowledge assessment interviews. In her planning interviews, Teacher 3.2 demonstrated a high frequency of using students' knowledge in order to plan her lessons. She also demonstrated this on her teacher knowledge assessment test, when she explained multiple ways that her students would solve problems related to subtraction and multi-digit addition. It is possible, that if the teacher knowledge assessment test only looked at the second and third grade curriculum, Teacher 3.2 might have scored higher than she did. Where she faltered on that test was with fractions and understanding student thinking related to fractions and multi-digit multiplication. Both of these concepts were topics that she did not teach in the second grade. Next, I summarize the findings for this third case.

Discussion

For both Teacher 3.1 and Teacher 3.2, ensuring their students all passed the state's high stakes test at the end of the school year drove their instruction. They focused on mastery of the state standards document rather than the adopted textbook or the district's curriculum map. This document was constructed with the help of math educators and researchers in the state. The document included sample tasks for teaching each standard and an explanation of what the students should learn mathematically from each standard. Teacher 3.1 and Teacher 3.2 explained that this document aligned with the state test and the tasks for each performance objective imitated test questions. It was the best resource they had for planning instruction.

Similarly, Teachers 3.1 and 3.2 focused on the Math Wall as their primary means for test preparation and the mastery of standards. They constructed a classroom environment rich in high behavioral expectations and set routines. Little time was wasted with classroom management when students engaged with the math wall tasks. Students understood that during math wall the expectation was that students would solve the problem independently and then have to explain their thinking to their partner. The partner would then have to agree or disagree and justify their decision with their own mathematical thinking. Based on the discussions, Teachers 3.1 and 3.2 returned to the standards document and modified tasks to give the students following the guidelines for mastery and potential test items stated in the document.

The teachers differed in how the amount of control they had over how students solved problems. Teacher 3.1 tended to dictate exactly how she wanted problems solved. For example, with any type of fraction problem, the children had to draw "candy bars" when solving the problems. When solving division problems, Teacher 3.1 emphasized using a partitive method. The kids drew circles and then partitioned the amount into the circles evenly. She never introduced the notion of measurement division. Teacher 3.2 also showed methods for solving problems, such as "even Steven" and "odd Todd" for figuring out if a number was even or odd. They both also forced students to solve addition and subtraction problems starting with the ones place. What differed between the teachers was in the discussion time. Teacher 3.1 focused on making sure that students solved the problems using the method she showed, while Teacher 3.2 focused on making sure the students understood and explained why answer choices were incorrect. It was during this time that differences occurred in the depth and richness of the mathematics being taught.

Looking at how Teacher 3.1 and Teacher 3.2 utilized MKT during instruction, they both used a combination of Knowledge of Content and Curriculum (KCC), Knowledge of Content and Teaching (KCC), and Knowledge of Content and Students (KCS). They started with KCC. They were able to evaluate the curriculums available to them (including the state standards document, which they each made into a curriculum for their teaching) and distinguish which curriculum best suited their ultimate goal of having all students pass the state test at the end of the school year.

From there, the teachers selected tasks for their students. The tasks usually started at a basic level and then moved to more difficult tasks. The selection of tasks depended upon the teacher's knowledge of the curriculum and the prior knowledge of the topic held by the students. For example, Teacher 3.2 discussed how she began the school year using Join and Separate Result Unknown questions and moved into Join and Separate Change Unknown over time and then was going to end the year with Join and Separate Start Unknown questions (Carpenter et al, 1999) as explained in the standards document. She was unaware of the connection between the progression she used from the standards document and the research that supported the sequence. She only understood the sequence from what she read in the document (KCC) and how her students reacted to the various problem types (KCS). However, her knowledge of the content, the curriculum, and her students allowed Teacher 3.2 to determine when to pose new tasks or stay with the same task and sequence her examples (Knowledge of Content and Teaching).

The talk alouds and discussion of how students solved the tasks depended largely upon the teachers' Specialize Content Knowledge (SCK) or Common Content Knowledge (CCK) and Knowledge of Content and Teaching (KCT). For example, when Teacher 3.2 discussed even and odd numbers, she made it explicitly clear using illustrations of base ten blocks why only the unit place mattered (SCK). The kids used her explanation to determine that in other places, such as the tens or the hundreds place, all of the units making up the place had "partners." Only in the unit place was it possible for a unit to be without a "partner." Teacher 3.1 unpacked the mathematics less frequently. She predominantly showed the students a procedure (CCK) and then expected the students to mimic the process. She did, however, use whether she saw students demonstrating the procedure correctly to decide when to pose new tasks and how to sequence her instruction (KCT). She also used the answers she received for each problem to decide which students to call on and when to ask for clarification (KCT).

Overall, Teachers 3.1 and 3.2 received low MKT scores relative to their peers in the larger NSF-Funded study, but their student gain scores far exceeded those of their peers. One possibility for this is the idea that Teacher 3.1 and Teacher 3.2 used a document correlated to the state's end of the year test instead of the adopted curriculum. Their understanding of how to manipulate this document to fit the needs of their students might have helped to expose the students to the various types of questions on the state test often enough that the students were well prepared to pass. In this sense, the teachers' pedagogical content knowledge and knowledge of the benefits of using a state constructed document aligned with the state test outweighed their low subject matter knowledge.

CHAPTER SEVEN: DISCUSSION

In 2005, Hill, Rowan, and Ball found that their measure of Mathematical Knowledge for Teaching demonstrated a positive predictive relationship between teacher knowledge and student achievement. They concluded that a "teacher's content knowledge of mathematics was a significant predictor of student gains..." (p. 396). In a follow up study conducted in 2008, Hill, Blunk, Charalambous, Lewis, Phelps, Sleep, and Ball found that their MKT measures correlated positively to their measures of Mathematical Quality of Instruction (MQI). They defined MQI using literature on high-quality classroom practice. These practices included: mathematical errors, responding to students correctly, connections of classroom practices to those of the professional mathematics community, richness of the mathematics, and mathematical language. This dissertation added to this trajectory of research by carefully unpacking how MKT relates to classroom practices and student mathematical growth.

In this study, I examined the relationship among the Mathematical Knowledge for Teaching (MKT) score of six teachers, their planning for instruction, their actualized instruction and their reflective practices, as they related to the mathematical growth of students in their classrooms. I used a multiple case study method to investigate these relationships. Six 2nd to 5th grade teachers participated. The six teachers were selected using their MKT score and their student gain scores. Three comparative cases were constructed based on matching MKT scores for pairs of teachers whose students displayed approximately equal of different growth patterns in a school year. I used observations and interviews to assess how the teachers used MKT in their classroom practice. Unlike Hill et al (2008), I did not look for specific individual practices. Instead, I took a holistic view of teacher practice and to determine what aspects of MKT appeared in the teaching cycle (NCTM, 2007). The goal was to examine patterns of practice for teachers with higher student gains and see how these patterns differed from those teachers whose students had lower average gains.

My data suggested that MKT was only partially utilized across my cases during the planning process, mathematics instruction, and subsequent teacher reflection practices. Mathematical Knowledge for Teaching was utilized differently among the teachers with high student gains than those with low student gains. I also found that MKT was not predictive of student gains across my cases, nor was it predictive of the quality of instruction provided to students in these classrooms. This discussion chapter examines the variability found across the cases within the observed classrooms, students' opportunities to learn, and how my findings related back to the literature on teacher knowledge and practice. Interestingly, there seemed to be a possible relationship between the students' gains scores and the resources the teachers used for planning purposes. The chapter concludes with an examination of the limitations of the study and potential avenues for further research.

Differences in MKT Use

There was a clear distinction between the different facets of MKT that were used by teachers with high student gain scores for their students and teachers with lower average student gain scores. Before examining those differences and their implications, it should be made clear how I defined "teachers with high student gain scores" and "teachers with low student gain scores." In the first classification, teachers with high student gain scores, included four teachers whose student gain scores were higher than the mean student gain score for the larger NSF-funded grant participants. The means score for the NSF-Funded grant teachers was 12.7 points with a standard deviation of 11 points. Teachers 3.1, 1.1, 3.2, and 2.2 had student gain scores of 34 points, 26 points, 23 points, and 18 points respectively. The standard deviation for the teachers ranged from about 7 points to 12 points. The second classified group, teachers with low student gain scores, included Teachers 1.2 and 2.1. Teacher 1.2's students had an average gain score of 10.8 points with a standard deviation of 9.7 points. Teacher 2.2's had an average gain score of 7 points with a standard deviation of 9 points.

Planning. When looking across the cases, the teachers with high student gain scores utilized a complex web of Knowledge of Content and Curriculum (KCC), Knowledge of Content and Teaching (KCT), and Knowledge of Content and Students (KCS) to varying degrees to plan their lessons. Three main themes were found across the planning data. These included: how state's standards document was utilized to guide planning, student thinking, and standardized in different ways than the district's curriculum map.

First, I examined the issue of using the state's standards document versus the district's curriculum map as the foundation for lesson planning. As a reminder, educational researchers, educators, administrators, curriculum designers, and state officials in education formed the state's standards document. Teachers and instructional coaches in the district put the district's curriculum map together over one summer. Three teachers used the state's standards document as the strict guide for planning, one used a mix of the two documents, and two used only the district's curriculum map. Teachers 3.1, 3.2, and 1.1 detailed their mathematics curriculum, especially in regard to the ways it was used to support instructional tool the Math Wall. These teachers found the adopted textbook outdated and not aligned to the state's standardized test. In addition, Teacher 1.1 highlighted that the textbook and the district's curriculum map did not align with her students' thinking, or with the research on how to teach conceptually. Consequently, she relied on her own knowledge of external resources and her students' thinking to develop a curriculum that suited the needs of her students, while ensuring that the students were exposed to a sufficient number of different problem types to be prepared for what was given on the state's standardized test.

Teacher 2.2 used a mix of the state's standards document and the district's curriculum map, but mostly the curriculum map, for planning. For example, Teacher 2.2 used the adopted textbook as her curriculum and the district curriculum map for her scope and sequence. Yet, she modified the curriculum map to fit the amount of time her students needed to "master" each concept. She also supplemented the textbook with activities she learned from various professional development courses and websites she learned about from other teachers. Like Teacher 1.1, Teacher 2.2 used Bloom's Taxonomy to ensure that she provided her students with higher-order thinking questions throughout a lesson.

The two teachers in the low student gain group mostly relied on their Common Content Knowledge (CCK) and the district curriculum map for planning. Teacher 2.1 and Teacher 1.2 used the adopted textbook and the district curriculum map to plan. They followed the time allotted to each standard as dictated by the curriculum map and the textbook pages that aligned with the map. Very little of their planning took into account the prior knowledge held by their students or the use of outside resources.

Interestingly, there seemed to be a possible relationship between the students' gains scores and the resources the teachers used for planning purposes. The three teachers with the highest student gain scores did not use the adopted textbook or the district curriculum map as the foundation for planning the instruction. Instead these teachers sought outside resources and used the state's standards document to plan. These were not necessarily the teachers with the highest MKT scores, however. Teacher 1.1 had the highest MKT score relative to the participants in the NSF-funded grant, while Teachers 3.1 and 3.2 had the lowest relative to the other participants. What these three teachers had in common was an extensive use of the PCK components of Mathematical Knowledge for Teaching. All three of these teachers used their students' thinking as a guide to select new tasks, ask clarifying questions, and evaluate available curricula suited for the their teaching needs. A further break down of the nuances among the three teachers with the highest student gains scores can be found in the "implications" section of this chapter.

Teacher 2.2, who fell in the middle of the continuum of student gain scores relative to her peers, and also relied on her PCK for planning. However, she used PCK a little differently from the teachers with the highest gain scores. Teacher 2.2 used a combination of Knowledge of Content and Teaching and Knowledge of Content and Curriculum. Because Teacher 2.2 did not have the 4th grade textbook, she found resources online and in the supplemental material that she felt suited the level of her students. It would be interesting to see if her planning would have been different had she had the adopted textbook for her 4th grade students. Teacher 2.2 also engaged in occasionally asking students clarifying questions. These questions tended to help

Teacher 2.2 decide whether or not the class was ready to move forward in the 3rd grade textbook or if they needed a "reteach" activity.

To review, Teachers 1.2 and 2.1, who had the lowest student gain scores relative to their peers, used mostly CCK for their planning. Little to no reference to outside materials, use of student thinking, or development of a new scope and sequence (to name just a few components of PCK categories) was present in the planning interviews. Instead, these two teachers relied on the textbook and the district's curriculum map to set the pace and schedule of the daily lessons.

It is possible that there were a few factors that might have impacted the student gain scores for these teachers, one of which might be directly impacted by how a teacher uses their MKT. First, there were two different groups of people who developed the standards document and the district curriculum map. One major difference in the group compositions was the addition of educational researchers in the state's group. The educational researchers might have brought a depth and breath of knowledge about the teaching of mathematics, appropriate scope and sequences for the various levels of mathematics, and knowledge about student thinking that might not have been available in the group who developed the district's curriculum map. This is not to say that the teachers who developed the district's curriculum map did not have some knowledge in these three areas. It is possible, however, that the teachers had limited knowledge or a different perspective on the importance of the underlying mathematics when constructing their own scope and sequence maps for the district. The district's curriculum map was based on the state's standards document, however, without knowledge of how mathematical topics related to each other (Horizon Content Knowledge) creating a coherent map for a teaching trajectory might be anemic and not allow for the same fundamental development that might occur if following a map that better considers the progression of mathematical ideas. A further investigation on the idea of scope and sequence is discussed later in this chapter.

A second factor, directly related to a teacher's MKT and planning, that possibly created differences in student gain scores was the idea that a teacher who had limited knowledge of their students' mathematical thinking might blindly trust that the textbook, or district's curriculum map, is the best suited curriculum and teaching trajectory for her students. It is possible that this potential "blind trust" would limit student growth because students likely need more specificity in the development of topics than the textbook affords. If such is the case where the teacher uses the textbook or curriculum map without scrutinizing the sequence based on student thinking then the limited student gains might be as much a function of a teacher's knowledge as it is the textbook being problematic (Franke et al, 2006).

Next, I examined the notion of resources in more depth for the teachers who created their own scope and sequence. Teachers with high student gain scores admitted to constructing a scope and sequence for their students based on the standards document and their students' prior knowledge of specific math topics rather than the district curriculum map or textbook. Teacher 1.1 often provided her students with a task that mapped to a higher-order thinking task on Bloom's Taxonomy (Bloom, 1956) at the start of the unit. Usually, the task mapped to where she wanted her students to be at the end of the unit. Using the information she gathered from this task, Teacher 1.1 then modified the scope and sequence for the rest of the unit. In essence, Teacher 1.1 generated a formative assessment at the start of each unit to create her lesson plans based on her analysis of student thinking. She gathered ideas for tasks from the standards document, research published about the topic she was teaching, national mathematics organizations' websites, and AIMS resource books. Her assessment of student thinking did not end with the initial formative assessment. As the week progressed, Teacher 1.1 evaluated what she learned each day from students presenting their solution strategies and modified the tasks she planned to ask the following day based on her analysis and her goal for building conceptual understanding of important 4th grade topics.

Teachers 3.1 and 3.2 created a scope and sequence a little differently. They scaffolded the standards document differently. They selected specific performance objectives within a standard and scaffolded the performance objectives according to what previous students found easy or hard. Once these two teachers knew the order with which they were going to present the concept, they used the standards document, sample questions from the state test, and websites that mapped to the state's end of the year test to find tasks that exposed their students to a variety of problem types related to the topic. Tasks remained on the board until the students solved the problems without mistake. Once demonstration of "mastery" occurred, Teachers 3.1 and 3.2 either modified the problem type within a particular concept or introduced a new standard altogether. In the next section, I discuss the instruction of all the teachers and implications of these findings.

Instruction. Overall, the instruction of the teachers in the high student gain group was characterized by a combination of Knowledge of Content and Students (KCS), Knowledge of Content and Teaching (KCT), and Specialized Content Knowledge (SCK).

There was great variation in the amount and type of use of each of these three MKT components in their instruction. While all three teachers in this group allowed time to have discussions and student sharing each day, the structure and purpose of the discussions differed.

Teachers 3.1 and 3.2 assigned their students partners for discussion time and provided sentence frames for each partner to help the children structure their talking around the mathematics. The purpose of the partner talk time and sentence frames was to aid in language development (Teacher 3.1 & 3.2; 2nd interviews for each). In addition, they called on their students to share what was discussed during partner talk time. The purpose of this activity was to hold students accountable for the work and to assist in classroom management (Teacher 3.1 & 3.2; 2nd interview for each). Teacher 3.2 selected pairs of students to talk through their process for answering multiple-choice questions in front of the whole class. She selected the pairs based on "good" discussions she heard going on as she monitored the partner talk time (Teacher 3.2, 2nd Interview). For the most part, the students were explaining a process or procedure that Teacher 3.1 or 3.2 showed the class prior to the partner talk. Sometimes a child showed his or her own method for solving a problem but that was rare. Usually, the students shared a procedure that was given that day or during previous instruction. In both classrooms, Teacher 3.1 and Teacher 3.2 expected the class as a whole or the students sharing to discuss why the alternative answer choices in the multiple-choice problem were incorrect. This part of the discussion aligned to their respective beliefs about preparing students to take the standardized test through showing the students how their mistakes were usually alternative answer choices on the tests (Teachers 3.1 and 3.2, 2nd interview for each).

Teacher 1.1 used collaborative learning groups and open-ended questions to increase discussion. To ensure all students participated, she provided each group with one sheet of butcher paper and a marker. The students were expected to work together as a team to solve the assigned problem during math time. Individual work time was provided to the students at the start of the day. She selected specific students to share the group's thinking based on what she saw during the problem solving time. Unlike in Teachers' 3.1 or 3.2's classrooms, student thinking and sharing drove the whole group discussions and the direction of the lesson in Teacher 1.1's classroom. Teacher-directed methods for solving were rarely seen in this classroom. Instead, Teacher 1.1 had a goal in mind for where she wanted her students to go mathematically and used what the students brought to the class discussion as the springboard for moving the students forward in their conceptualization of a specific topic and along her trajectory. Teacher 1.1 viewed student sharing as formative assessment and used discussion time as the format for gathering student data. She used what the students showed her to manipulate her scope and sequence to follow what the students knew and where she wanted them to go.

Teacher 2.2 also used discussions in her classroom; however, the structure of the instruction exemplified a more traditional teaching environment than the other classrooms in the high student gains group. Much like in a traditional classroom, Teacher 2.2 used the methods in the textbook to teach the students and provided the procedure for solving the problems prior to allowing the kids to solve the problem. The discussion in this classroom focused on calling students to the board to show her they have mastered the provided procedure. Occasionally, a student yelled out that he or she knew the procedure prior to the instruction and she would allow the student to demonstrate what to do. What made this classroom different from traditional instruction was that Teacher 2.2 used what the data she received from both in class discussions and from the district's quarterly assessments to inform her instruction. She modified lessons and added time to her calendar based on what the data told her. In addition, she was trying to introduce alternative methods for solving problems based on what she learned in professional development courses. Lastly, Teacher 2.2 sent her 3rd grade students to another teacher for the 3rd grade mathematics lesson. When the students returned to her classroom, Teacher 2.2 reiterated the lesson she taught to the 4th graders to the 3rd graders. Thus, the 3rd grade students not only received instruction at grade level but then further instruction designed for the next grade up. If the 3rd graders remained in her classroom for all of the mathematics instruction, Teacher 2.2 modified the tasks she gave to the 4th grade. She had predetermined tasks for the 4th grade students and then ones for the 3rd grade based on grade level and competency.

The high student gain teachers also utilized the math wall/board extensively in their classrooms. The math wall was seen as a tool for spiraling the standards throughout the school year so that students had frequent exposure to all of the topics on the end of the year standardized test. For all of the teachers in this group, the math wall played an essential role in formative assessment. The teachers used the knowledge they gained from student sharing during math wall time to modify their daily lesson or future math wall problems. Teacher 2.2 used what the students showed her during math wall time to combine future textbook lessons. This compression of topics allowed her time to review concepts the students found difficult and time to add supplemental activities that encouraged higher-order thinking. Teacher 1.1 used the math wall to scaffold conceptual understanding of particular topics. Based on what the students showed her, she structured the tasks to go from very basic to abstract over a week or two time frame. Teachers 3.1 and 3.2 used the math wall to teach students how to take a test and expose students to a variety of problems types that might potentially appear on the standardized test at the end of the year.

All of the teachers in this high achieving group tended to present the students with a handful of tasks per class period. If there were more than 7 tasks given to the students, the tasks varied greatly in difficulty and type. For example, Teachers 3.1 and 3.2 often gave students one task per content strand in the standards document. However, the assigned tasks changed daily either based on problem type (such as moving from Join Result Unknown to Join Change Unknown to Join Start Unknown) or the numbers in the problem (i.e., single digit to multi-digit and single-digit to all numbers being multi-digit). The answer choices also changed. For instance, they might include a problem that asked kids what characteristic does NOT define a particular shape or pattern. Teacher 2.2 assigned the most problems per class period of the teachers in this group. She used Bloom's Taxonomy to ensure that she asked a variety of questions to the students throughout her instruction. The cognitive demand level of her tasks did not increase, as it did for some of the other teachers, during the implementation phase, it was that she asked students to evaluate and analyze the processes they used to solve problems throughout a lesson.

The teachers in the low student gain group demonstrated use of Common Content Knowledge (CCK) in their instruction. Rarely did these teachers explain mathematical processes beyond what is commonly known about specific topics in 4^{th} and 5^{th} grade mathematics. Both Teacher 1.2 and Teacher 2.1 followed the district curriculum map and the adopted textbook to plan and implement their lessons. The math wall, or other outside resources, was rarely used during the four months of weekly observations. Their instruction epitomized direct instruction and teacher-directed procedures. Teacher 1.2 often stood at the front of the class and told students to "write what you see," meaning the students needed to copy exactly what she was writing on the board when solving problems. Discussions, student sharing at the front of the board, and partner talk rarely happened, if ever, in these two classrooms. The expectation was that the students would copy the procedure directly from the textbook or shared by the teacher and then use the algorithm to solve anywhere from 15 to 30 textbook problems independently. Student thinking rarely influenced instruction and lesson design in these two classrooms. In fact, Teacher 1.2 and Teacher 2.1 struggled with anticipating what tasks might be hard for students or how students might solve problems aside from use of the standard algorithm.

Implications

One major implication of this dissertation was that Mathematical Knowledge for Teaching was not predictive of student gain scores or specific types of instruction. In 2005, Heather Hill and colleagues found that teacher's content knowledge for mathematics teaching was a significant predictor of student gains. The authors stated that for every standard deviation difference on the content knowledge for teaching mathematics variable, there was "an average months student growth in mathematics…to roughly one half of two thirds of a month of addition growth" (Hill et al, 2005, p. 396). In my dissertation, the teachers with the two lowest MKT scores relative to the sample showed the highest student gains over the course of two academic quarters. The teacher with the third lowest MKT score relative to the sample had student gains of a half standard deviation above the mean of the NSF-funded grant participants. The last teacher in the high student gains scored the highest MKT score of all of the NSF-funded grant participants but she had the second highest student gain scores among the participants in my dissertation. On the other hand, the two teachers in the low student gain score group scored a half standard deviation and a full standard deviation above the mean of the NSF-funded grant participants on the MKT measure. Therefore, aside from Teacher 1.1, who had a high MKT score and high student gain scores, the rest of the teachers in my dissertation aligned more with the results of Shechtman et al (2010) and Hill et al (2012) than the Hill et al (2005) study. In the chronologically later studies, the researchers found that many teachers in their study had low MKT scores and high student gains while other teachers had high MKT scores and low student gains (Schechtman et al, 2010) and that the predictive power of MKT scores for teachers with average MKT was less stable when looking at student gain scores (Hill et al, 2012).

A possible reason for the discrepancies between my findings and the Hill et al (2005) findings might be that I used a mix of LMT items (Hill et al, 2004) and DMI items (Higgins et al, 2007) to assess MKT. Shechtman et al (2010) also used their own measure of MKT and found that MKT scores were not necessarily predictive of student gains. It is possible that the measure I used and the measure used by Shechtman et al (2010) were not sensitive enough to MKT and thus affected the results of our studies. It is also possible that my sample size was too small to provide adequate results for the tool used to measure MKT and was too small to provide a wide range of variable MKT scores. While my sample included a range of MKT scores (all relative to the NSF-grant participants) from -0.5 standard deviations below the mean to 2.11 standard deviations above the mean. This is a spread of 2.61 standard deviations (in comparison to the larger spread of 6 standard deviation found in normally distributed data). In addition, sampled teachers did not populate the distribution evenly. A larger sample might have resulted in similar findings to Hill et al. (2005) that predicted spread in their measure of MKT. The dissertation sample aligned with Hill et al. (2012) distribution, in that 68% of the sampled teachers' MKT scores captured the mid-range of the estimated data. In this range, Hill et al. (2012) teachers' MKT scores did not predict student growth in mathematics. This result mapped to the results presented here (for a smaller sample). An important new study would measure teachers' MKT with all the teachers in the sample schools across the full complement of LMT items measure (Hill et al, 2007) and then compare student gains across MKT levels.

What did the differences in student gains mean though in regards to teaching and how MKT impacted planning, instruction, and reflection? To begin, some of my data supported the findings of Baumert et al (2010). In that study, the researchers examined differences in Pedagogical Content Knowledge (PCK) and Content Knowledge (CK) to determine how each contributed to a teacher's professional knowledge for teaching and learning. The researchers found that when content knowledge was held constant, teachers with a minor in mathematics scored higher on Pedagogical Content Knowledge than teachers with a major in mathematics. They also found that PCK influenced instructional quality on a cognitive level, curricular level, and in learning support. In fact, PCK accounted for 69% of the variance in achievement between classes. Lastly, the researchers reported that CK was essential for the development of PCK but having high content knowledge did not automatically predict a teacher had pedagogical content knowledge or higher student gains.

My data suggested that PCK contributed to the planning and instruction of teachers who had high student gains, whereas PCK was less noticeable in the planning and instruction of teachers who had low student gains. This finding related to PCK overlapped with the curriculum guide used by the teachers in this dissertation. The teachers in the high student gain group used the state's standards document as their curriculum guide,, the teachers who used more Subject Matter Knowledge with limited PCK used the textbook and district's curriculum map as a curriculum guide. This finding raises an interesting question as to whether a teacher's PCK impacted student gains, or whether the implementation of the curriculum guide increased student performance, or whether it was the interaction of PCK and curriculum guide that made a difference.

As stated previously in this dissertation, the state standard's document was constructed by a group of educational researchers, professional developers, mathematicians, teachers, administrators, and state personnel. This document included the standard, the mathematical practices aligned with a particular standard, examples of tasks that address the standard, and descriptions of how students might approach the mathematical ideas embedded in the standard. The document also included descriptions of the mathematics underlying the standard. The state standards document can be found at, http://www.azed.gov/standards-practices/mathematics-standards.

K-8 teachers in the school district in which this research took place, were appointed by the district administrators, developed the district's curriculum map. The district's curriculum map was developed over a two-week summer session. The teachers on the development committee worked in grade level bands to align the state's mathematics standards with the adopted textbook. The document included, the standard, the title of the textbook lesson that aligned with the standard, the time period indicating how long each lesson should take (i.e., a half of a class period, a whole lesson, a multiple day lesson, etc.), and possible supplementary resources (i.e., websites) to use during instruction. The curriculum maps are located on the Internet but due to identifying information in both the website and document, I decided to omit the documents and website location.

Intriguingly, the state's standards document seemed to align with aspects of PCK, such as knowing what a student is thinking or anticipating how a student might work through a problem related to the standard, or how a teacher might manipulate tasks to address various mathematical concepts found within a standard. For teachers with high PCK and high student gains, it was conceivable that as they interacted with the state's standards document over the school year, their knowledge of the content being taught increased because of the content in the state's standards document. For example, a teacher with little MKT knowledge or any measurable content knowledge (i.e., Teacher 3.1, Teacher 3.2, and Teacher 2.2) could go to the state's standards document, find a standard, and select a task aligned with the standards. The teacher could present this task to the students (as seen in the work of Teacher 3.1 and Teacher 3.2, specifically) and watch how the students interacted with the task. Through carefully observing their students solving the assigned task, the teacher could learn crucial information about what the students know, and do not know, about the standard being taught. This new information could lead the teacher to return to the standards document and find a subsequent task that would further aide the students in learning the mathematical concepts embedded in the standard. This cycle of moving from the state's standards document to the instruction and back to the state standards document might continue for the course of a few days to a week or a few weeks, depending upon the mathematical concept being taught. Regardless, through interaction with the curriculum guide (in this case the state's standards document), the teacher gains knowledge not only of the document, but how the students think about the mathematical concepts, and the mathematics itself. Over time, the knowledge in the curriculum guide becomes part of the teacher's own knowledge base (Lave & Wenger, 1991). This development occurs to the point where a teacher's mathematical knowledge for teaching grows through their practice, making their initial MKT score less relevant as a predictive measure.

One implication of viewing knowledge from a situated learning perspective (Lave & Wenger, 1991), is that it is conceivable that a teacher's initial MKT score fails to indicate a teachers' complete knowledge base because knowledge is not a static concept. Instead, knowledge continually grows based on interactions, discourse, and reflective practice with others; as well as through artifacts, and tools already established in the community of learners. Therefore, an MKT score at a fixed point in time might not be indicative of the knowledge held by a teacher over time because one's knowledge is part of a dynamic and interactive system that cannot be fully captured. Further study on the interaction between teacher knowledge and curriculum guides is necessary to better understand this phenomenon.

In addition, the teachers with high PCK and high student gain scores engaged students in critical thinking activities that included justifying and explaining different mathematical processes, and they used their knowledge of student learning to modify the curriculum. All of these practices fall under the realm of pedagogical content knowledge and align with research on practices that have positive affect on student achievement (Ball, 1993; Franke et al, 2006; Lampert, 1990; NCTM, 2007). Franke et al (2006), further, explained that the nature of the discourse in the classroom was critical. Discussion time devoted to a specific task helped teachers unpack their students' mathematical thinking in ways that foster further mathematical proficiency, especially through practices of asking questions beyond recall and requiring students to describe their strategies in detail and what makes the strategy work. Such instructional practices were noted in Chinese teachers who demonstrated Profound Understanding of Fundamental Mathematics (Ma, 1999) and in the classrooms of teachers from countries whose students outperformed U.S. students in the Third International Mathematics and Science Study (Stigler and Hiebert, 1999). These specific characteristics were present to varying degrees in all of the classrooms observed for my dissertation where students had high gain scores. These practices were not present in the two classrooms with low student gain scores, which aligns with Baumert et al's (2010) claim that having high content knowledge (alone) does not mean a teacher has well developed pedagogical knowledge or high student gains. Both Teacher 1.2 and 2.1 had relatively high content knowledge and used their content knowledge extensively during their teaching but neither demonstrated utilizing facets of pedagogical content knowledge when teaching other than knowledge of the curriculum map and how it related to the textbook. In fact, Teacher 1.2 expressed during an interview that she was sure her students could not solve specific algebraic equations because she had not taught them how to do so (albeit that she later expressed that they would address certain problem types algebraically). Also, she was unable to explain or anticipate how students might solve problems outside of her teaching method. Her questioning of students during instruction was limited to recall or comprehension questions (Bloom, 1956).

What my data did not suggest, or necessarily support, was the notion that a teacher had to have well-developed content knowledge in order to develop pedagogical content knowledge or to achieve high student gains on achievement tests. Teachers 3.1, 3.2, and 2.2 scored relatively low on the teacher assessment test. They each struggled to complete tasks outside of their grade level of teaching, such as problems with fractions or figuring out how students solved fraction problems when using non-standard algorithms. But, it seemed that these teachers knew when to pose new questions, asked for clarification, constructed a scope and sequence of activities based on student thinking, anticipated tasks that students would find difficult, understood misconceptions held by their students, and knew how to supplement their lessons with resources that helped students prepare for standardized testing. Looking across this data superficially one might think that increasing PCK and teaching students how the subcategories connect might be a key to increasing student gain scores. If we look deeper into the data, however, a different story appears and more questions arise.

Teachers 3.1 and 3.2 had two of the highest student gain scores across the teachers in our study sample and in the larger NSF-funded grant. It has been well documented in this dissertation that they also had two of the lowest MKT scores across those same two sample groups. When coding their transcripts using the MKT codes from Ball et al (2008), the teachers seemed to engage in a complex web of interrelated characteristics found in all of the PCK subcategories and some CCK and SCK. Similarly, Teacher 1.1, who tied for the highest MKT score among participants in the larger NSF-Funded grant, utilized multiple facets of the PCK subcategories as well as CCK and SCK to a greater extent. Clearly the instruction of all three teachers equated to increasing student scores. But, when I went back into the planning interviews for these three teachers and examined their goals and focus for teaching, it appeared that Teachers 3.1 and 3.2 were quite savvy in how they planned. Repeatedly, they each independently discussed the importance of high stakes testing and having their students perform well on these tests. They also talked about the fact that the state standards document aligned to the state's standardized test and how the standards document provided example questions and explanations for preparing students to master each concept. They knew researchers, educators, and people connected to the state standardized test who helped develop the document. They used the document to select initial tasks for their students and used the information in the document to find out how the tasks might be manipulated in ways that exposed students to other potential test questions around the same topic. They also used the descriptions in the state standards document of how to solve problems related to each concept to teach the students, as well as other resources they gathered during various professional development courses and from websites.

Teacher 1.1 also used the state's standards document to prepare for her lessons and used some of the tasks provided in the document. The difference was that rather than using the progression of learning stated in the document, Teacher 1.1 understood how the 4th grade curriculum mapped to future mathematics concepts and used that Horizon Content Knowledge to design a scope and sequence that lead to the foundational building of conceptual understanding of mathematics topics necessary in the future. She then presented her students with difficult tasks to assess where the students were mathematically and used the information she gathered from her students to modify her scope and sequence. In this classroom, the learning and methods for problem solving started with the students. Teacher 1.1 had a goal in mind but she facilitated the learning rather than dictated the learning (NCTM, 2007). One major question then is what do these differences in planning and instruction mean for learning? On a basic level, all students in these three classrooms met or exceeded the district test by the end of the third time period, except for roughly three students, who were approaching the standards at the end of the third time point. But two sets of students were provided opportunities to learn test taking skills throughout the year and methods for solving problems based on their teacher's understanding of the mathematics, while the other classroom of students were given opportunities to learn to puzzle through the mathematics and develop a foundation for future mathematics learning. Which of these opportunities matter? Which do we value today? All students "succeeded" in these classrooms, so is it even important to distinguish between the opportunities to learn and what it means for our students? Absolutely. However, a follow-up study would be needed to examine how these specific opportunities impacted future learning (in a lasting way). Consequently, the data collected for this study cannot fully speak to this issue.

Another implication from this dissertation is that we need to think about administrators forcing teachers to follow a set curriculum map or the adopted textbook that might not align with the students' mathematical knowledge or the best practices for teaching mathematics in elementary school. Data, evidence and warranted claims from this dissertation, demonstrate clear gain score differences among the teachers who disregarded the textbook and curriculum map, those who used a combination of the state standards document and the textbook, and the teachers who strictly followed the textbook and district curriculum map. Based on this information, it seems that we need to trust teachers who use their students' thinking as a road map for constructing their daily lesson plans and learning trajectories, to provide assistance for growth in understanding how to use student thinking as a formative assessment, to find high-quality professional development programs on resources for developing student thinking and generate tasks that carefully increase cognitive demand, and finally provide instructional coaches whose knowledge is grounded in the research on teaching mathematics as well as the mathematics content who have documented these capabilities in the classroom. Delivering on the promise of this "holy grail" will be no easy feat to accomplish.

Clearly, this dissertation added to the literature on the impact of MKT on planning, teaching, and reflection. It provided rich detail on what happens in classrooms across teachers at varying MKT levels. It showed that high student gains did not always occur with teachers who had high MKT scores but instead the gains corresponded to a high reliance on PCK (aligned with a form of test preparation) and with strategies that support standards-based reform teaching methods. It supported the literature documenting that traditional mathematics classrooms restrict student learning and growth in elementary school. It also added to the literature on what it means to educate students mathematically, especially in an age where high-stakes testing is at the forefront of everyone's mind raising interesting validity questions for the measurement community. It also brings up issues of what do we do to help teachers who utilized very little PCK?

Drastic learning differences occurred between students in Teacher 3.1's classroom and Teacher 2.1, both quantitatively and qualitatively. Based on a paper-pencil teacher knowledge test, many people would want their children in Teacher 2.1's classroom at the start of the year. But, my evidence showed that very little learning occurred (whether it was test preparation or conceptual or procedural in its essence) when contrasting the affects of these two teachers on their students' mathematical growth. One major reason for this might be how these two teachers used their MKT.

A final implication of this study was that of affective instruction and the role of a positive classroom environment on student performance. All four of the high gains teachers demonstrated high Emotional Support on the CLASS instrument, while the two low gains teachers scored quite low on this measure. Even more interesting was the case of Teacher 2.2 with regard to a positive climate and students' mathematical development.

Teacher 2.2 was selected for this study because of her relatively low MKT scores and moderately high student gain scores, as compared to the participants in the larger NSF-study. She followed the textbook for the 3rd grade students (even though she taught mostly 4th graders), used a mix of the district's and state's curriculum guides for planning and used formative assessment in her classroom. She was also a product of the school district, as she grew up attending a school in the neighborhood and always wanted to return to the district as a teacher. Her CLASS scores indicated very high Emotional Support but low Instructional Support. Her case raises a fundamental question about the impact of a positive climate and high regard for students' perspectives in a learning environment. It is possible that her belief in and high regard for her students created an environment where students thrived and wanted to learn, thus, impacting their over all performance throughout the school year. The remaining three teachers with high student gains scores (two of whom also scored relatively low on the MKT assessment) also demonstrated high Emotional Support and positive regard for students' perspectives. In light of these insights, a follow up study attending to these conditions might prove theoretically, and from a practical perspective, fruitful.

Where do we go from here? When looking at the quantitative data, it would be interesting to conduct a follow-up study using the entire MKT assessment test to see how the teachers faired and if that changed the allocation of teachers to groups for the cases. However, the reliability of the measure used (alpha = .83) suggests that this is not likely. Another quantitative follow up study would be to look at specific items on the MKT test that mapped to the various mathematics topics covered on the district quarterly assessment. As the district quarterly assessment is cumulative (over the school year) it would be easy to examine each time point measure and to see how the instruction differed across MKT levels across the school year. For example, one question might be how did students do on the district quarterly assessment on items that matched to weaknesses in teacher knowledge versus items that matched to strengths in teacher knowledge? A second study, mentioned earlier in this dissertation, might be to follow the students in these classrooms for another year or two to investigate whether or not their gains equated to actual mathematics learning, or if they learned test taking skills enabled them to pass the test without truly understanding the mathematics they learned (or without building a foundation for future mathematics learning).

Qualitative follow up studies might include a thorough examination of the mathematical discourse found in each classroom to examine the types of learning going on. Another study could examine how students at various levels of ELD labels learned in these classrooms. Was their English development increased? What mathematics was learned? How do these two types of learning intertwine over time? Another study might be to look at how students across these classrooms viewed themselves as mathematics learners. Did they identify as mathematically capable? Did they feel competent in the mathematics? One might also look at how the teachers identified themselves as mathematics learners and how that identity impacted instruction and the utilization of MKT.

One interesting fact that was learned after the data was collected for this dissertation was that Teacher 3.1 and 3.2 were promoted to learning coaches in their respective schools. This brings up a question of what is valued in mathematics teaching? These teachers produced high levels of student gains but the richness of the mathematics being taught in the classrooms was deeper in the classroom of Teacher 1.1. Therefore, what does it mean to be a leader? What is the role of High Stakes testing in selecting leaders in our schools? What characteristics of teaching should be looked at when selecting school leaders or coaches? What impact will this have on the future of our students? How will these teachers fair when it comes to assisting teachers with higher MKT than they have or what happens when these teachers are asked to present on the teaching of specific mathematics content a little outside of their knowledge base?

In conclusion, the Mathematical Knowledge for Teaching framework is a useful for looking at teacher knowledge and teaching practices, but it is not necessarily representative or predictive of what happens in all the classrooms. This dissertation, although of small scale and with limited scope, showed that the community of elementary school mathematics educators needs to continue examining what it means to teach mathematics, what knowledge is needed to teach mathematics, and what types of learning are important when educating our students. We need to explicitly examine the critical policy roles of high stakes testing, the roles of administrative mandates, the roles of textbooks, the roles of test preparation, and the roles of how we measure success in our students and in our teachers. We cannot simply state that increasing measured knowledge in our teachers will improve student mathematical knowledge. We need to carefully examine what the many types of teacher knowledge actually produce in their students and these knowledge types play out in practice. Finally, we need to be ever more clear about what we mean by the learning of mathematics in our schools. This dissertation exemplified that we are far from definitively answering the plethora of questions on teacher knowledge and its many complicated relationships with student learning. That said, it demonstrated that the Mathematical Knowledge for Teaching framework is an intellectually reasonable starting place to explore and examine how teaching influences student learning of elementary school mathematics, particularly as a lens to evaluate teacher interactions with critical artifacts including policy and standards documents, textbooks, and other curricular materials, along with the capacity of teachers to create positive classroom environments.

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APPENDIX A

OPERATIONALIZING MKT: CRITERIA FOR CODING

(adapted from Ball et al, 2008)

Subject Matter Knowledge		
Common Content Knowledge	Specialized Content Knowledge	Horizon Content Knowledge
• Recognize when students give wrong answers • Recognize when the textbook gives an inaccurate definition • Use terms and notation correctly • Be able to do the work assigned to students • Pronounce terms correctly • Calculate correctly • Understand the mathematics in the student curriculum	• Looking for patterns in student errors • Sizing up whether a nonstandard approach would work in general • "an uncanny kind of unpacking of mathematics that is not needed – or even desirable – in settings other than teaching" (p. 400) • Knowledge beyond that being taught to students • Understanding different interpretations of the operations in ways that students not need to distinguish • Figuring out which types of problems fit with which operations • Use of "decompressed mathematical knowledge" (p. 400) • Talk explicitly about how mathematical language is used • How to choose, make, and use mathematical representations effectively • How to explain and justify one's mathematical ideas	• Includes the vision useful in seeing connections to much later mathematical ideas • Can help in making decisions about how, for example, to talk about the number line • Might impact how a teacher's choices anticipate or distort later development

APPENDIX B

OPERATIONALIZING MKT: CRITERIA FOR CODING

(adapted from Ball et al, 2008)

Pedagogical Content Knowledge		
Knowledge of Content and Students	Knowledge of Content and Teaching	Knowledge of Content and Curriculum
• Anticipate what students are likely to think and what they will find confusing • Predict which examples students will find interesting and motivating • Anticipate what students are likely to do with a task and whether they will find it easy or hard • Hear and interpret students' emerging and incomplete thinking as expressed in the ways that pupils use language • Knowledge of common students conceptions and misconceptions about particular mathematical content • Familiarity with common errors and deciding which of several errors students are most likely to make	• Sequence instruction • Choose and sequence examples • Evaluate the instructional advantages and disadvantages of representations • Deciding which student contributions to pursue, which to ignore, and which to save until later • Decide when to ask for more clarification • When to use a student's remark to make a mathematical point • When to pause, ask a new question, or pose a new task	• Knowledge of the range of curricula available • Knowledge of instructional materials available • Know when to use a particular program in a specific situation

APPENDIX C

CLASS OBSERVATION PROTOCOL RUBRIC

(adapted from Pianta et al, 2008)

Low Range		Middle Range			High Range	
1	2	3	4	5	6	7
The low-range description fits the classroom and/or teacher very well. All, or almost all, of the relevant predictors are noticeable	The low-range description fits mostly with what is happening in the classroom and/or teacher, but there are one or two indicators that fall in the middle range	The middle-range description mostly fits the middle-range criteria for the classroom and/or teacher, but one or two indicators fall in the low range	The middle-range description fits the classroom and/or teacher very well. All, or almost all, of the indicators fall in the middle range	The middle-range description mostly fits the classroom and/or teacher, but one or two indicators fall in the high-range.	The high-range description mostly fits the classroom and/or teacher, but there are one or two indicators in the middle range	The high-range description fits the classroom and/or teacher very well. All, or almost all, of the indicators fall in the high-range.

APPENDIX D

THE MATHEMATICAL TASKS FRAMEWORK

(adapted from Stein et al, 2009)

Low Cognitive Demand		High Cognitive Demand	
Memorization Tasks	Procedures without Connections Tasks	Procedures with Connections Tasks	Doing Mathematics Tasks
• Involve reproducing learned facts, rule, formulae, or definitions OR committing facts, rules, formulae, or definitions to memory • Cannot be solved using a procedure because a procedure does not exist • Short time frame given to solve task • Are not ambiguous – directions on how to solve the task are clearly stated • Have no connection to underlying concepts	• Are algorithmic – use of algorithm is specifically noted or evident based on prior instruction or experience with the task • Little ambiguity on what needs to be done to solve the problem • Have no connection to underlying concepts • Are focused on producing the correct answer • Require no explanation other than describing the formula being used	• Focus students' attention on the procedures for the purpose of developing deeper levels of understanding • Suggest pathways to follow that are broad general procedures that have close connections to underlying ideas • Usually are represented in multiple ways that help develop meaning • Require some cognitive effort. Procedures cannot be followed mindlessly	• Require complex and nonalgorithmic thinking • Require students to explore and understand the nature of mathematical conceptions or relationships • Demand self-monitoring or self-regulation of one's own cognitive process • Require students to analyze the task and actively examine task constraints • Require considerable cognitive effort and may involve some level of anxiety due to the unpredictive nature of the task

APPENDIX E

THE FACTORS ASSOCIATED WITH IMPLEMENTATION OF COGNITIVE DEMAND (adapted from Stein et al, 2009)

Factors associated with the decline of high-level cognitive demands	Factors associated with the maintenance of high-level cognitive demands
1. Problematic aspects of the task become routine	1. Scaffolding of student thinking and reasoning
2. The teacher shifts emphasis to the correctness or completeness of the answer	2. Students are provided with means of monitoring their own progress
3. Not enough time allowed to figure out the demanding aspects of the task	3. Teacher or capable students model high-level performance
4. Classroom management causes problems for students to engage with the task	4. Sustained press for justifications, explanations, or meaning through teacher questioning
5. Inappropriateness of task for a given group of students	5. Tasks build on students' prior knowledge
6. Students are not held accountable for high-level products or processes	6. Teacher draws frequent conceptual connections
	7. Sufficient time to explore