

Generation of Tolerance Maps for Line Profile by Primitive T-Map Elements

by

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## ABSTRACT

The objective of this research is to develop methods for generating the Tolerance-Map for a line-profile that is specified by a designer to control the geometric profile shape of a surface. After development, the aim is to find one that can be easily implemented in computer software using existing libraries. Two methods were explored: the parametric modeling method and the decomposed modeling method. The Tolerance-Map (T-Map) is a hypothetical point-space, each point of which represents one geometric variation of a feature in its tolerance-zone. T-Maps have been produced for most of the tolerance classes that are used by designers, but, prior to the work of this project, the method of construction required considerable intuitive input, rather than being based primarily on automated computer tools. Tolerances on line-profiles are used to control cross-sectional shapes of parts, such as every cross-section of a mildly twisted compressor blade. Such tolerances constrain geometric manufacturing variations within a specified two-dimensional tolerance-zone. A single profile tolerance may be used to control position, orientation, and form of the cross-section. Four independent variables capture all of the profile deviations: two independent translations in the plane of the profile, one rotation in that plane, and the size-increment necessary to identify one of the allowable parallel profiles. For the selected method of generation, the line profile is decomposed into three types of segments, a primitive T-Map is produced for each segment, and finally the T-Maps from all the segments are combined to obtain the T-Map for the given profile. The types of segments are the (straight) line-segment, circular arc-segment, and the freeform-curve segment. The primitive T-Maps are generated

analytically, and, for freeform-curves, they are built approximately with the aid of the computer. A deformation matrix is used to transform the primitive T-Maps to a single coordinate system for the whole profile. The T-Map for the whole line profile is generated by the Boolean intersection of the primitive T-Maps for the individual profile segments. This computer-implemented method can generate T-Maps for open profiles, closed ones, and those containing concave shapes.

## DEDICATION

To my parents

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## Chapter 1

### INTRODUCTION

#### 1.1 Background

Tolerances are specified by a designer to allow reasonable leeway by a manufacturer for imperfections and inherent variability without compromising performance. The object is to specify large enough tolerances so that manufacturing can be inexpensive, yet not have the dimensional variations of parts adversely influence their assembly and functionality. Two types of principles are used in industry for assigning tolerances: conventional tolerancing and geometric tolerancing.

In conventional tolerancing, a tolerance specified a limiting range on a dimension, and it applies limit to one degree of freedom to the target entity. But this method had some serious drawback concerning controls on form and location [1]. So this tolerancing method may fail to produce the desired part geometry within a high level of accuracy when it is to be used for a complicated assembly. This often results in many rejected parts and rework, and increases the cost of manufacturing. The modern and more rigorous method of tolerancing, called geometric tolerancing, solves many of these problems, but not all of them.

Geometric tolerancing refers to modern methods that are prescribed in the ANSI/ASME Y14.5 Standard [2] and the ISO 1101 standards [3]. This thesis is based on the ASME Standard, which will be introduced in section 2.1. The geometric tolerancing method permits a designer to control several degrees of freedom for a target entity in its tolerance-zone. The location of the tolerance-zone is defined with a basic dimension; and its magnitude is determined by a tolerance to control a geometric variation, such as size,

position, or orientation. Additional geometric tolerances may also be specified on the same feature to control more exact geometric form or orientation of a feature within its larger size or position tolerance-zone. The ASME Standard gives a common language on dimensioning and tolerancing, which is easily understood by professionals, and it is widely used in the engineering design and manufacturing fields. Because of the efficiency of GD&T in mass production, compared to conventional tolerancing, the number of rejected manufactured parts is reduced, as well as the manufacturing cost. However, the Standard is based on a collection of special cases and has little rigorous mathematical basis, so it causes some inefficiency or ambiguity in communications between design, manufacturing, and inspection. A mathematical model for tolerances has the potential to eliminate errors and lower cost of manufacturing.

## **1.2 Problem statement**

The tolerance-Map model, proposed by Davidson and Shah is one of the mathematical models to represent geometric tolerances. A Tolerance-Map is a hypothetical Euclidean volume of points, the shape, size, and internal subsets of which represent all possible variations in size, position, form, and orientation of a target feature.[4] In section 2.3, previous work done by several graduate students will be introduced. A T-Map can represent the tolerance-zones for size, form, position and orientation tolerances under the ASME Standard. The model can effectively represent the interaction between tolerances types and the effects of datum precedence and sequence that are important in tolerance accumulation. Of the six tolerance types built in the standard, line profile tolerances have recently been modeled [33], and surface profiles have not yet been modeled, either by T-Map, or by any other model.

The objective of this research is to develop methods for generating the Tolerance-Map for line-profiles and to identify one that can be easily implemented in computer software using existing libraries. In this thesis the T-Map is developed for any shape of profile feature used in design, such as a polygon (combination of lines), a mixture of line and arc segments, and those containing free-form curves. A profile combined from these segments can be either open or closed. Two different modeling methods are developed in this thesis for a given line profile and its specification: the parametric method and the geometric decomposition method. It is the geometric decomposition method that is more readily adaptable to computer automation. A remaining artifact from the project is the computer programming code that may be used to build T-Maps for line-profiles.

## Chapter 2

### LITERATURE REVIEW

#### 2.1 Tolerance standard

The ANSI/ASME Y14.5 Standard is used for specifying dimensions and tolerances in order to ensure that the design requirements are interpreted unambiguously. Geometric variations have been decomposed into specific types in the order of their functional effect and assembling processes. Figure 2.1 gives the standard tolerance classes. Each geometric tolerance is specified in the form of standard symbols (feature control frame) on a drawing, as the example shown in Figure 2.2. All frames contain a type symbol, and tolerance value, and optionally contain one or more datum reference frames, and modifier symbols.

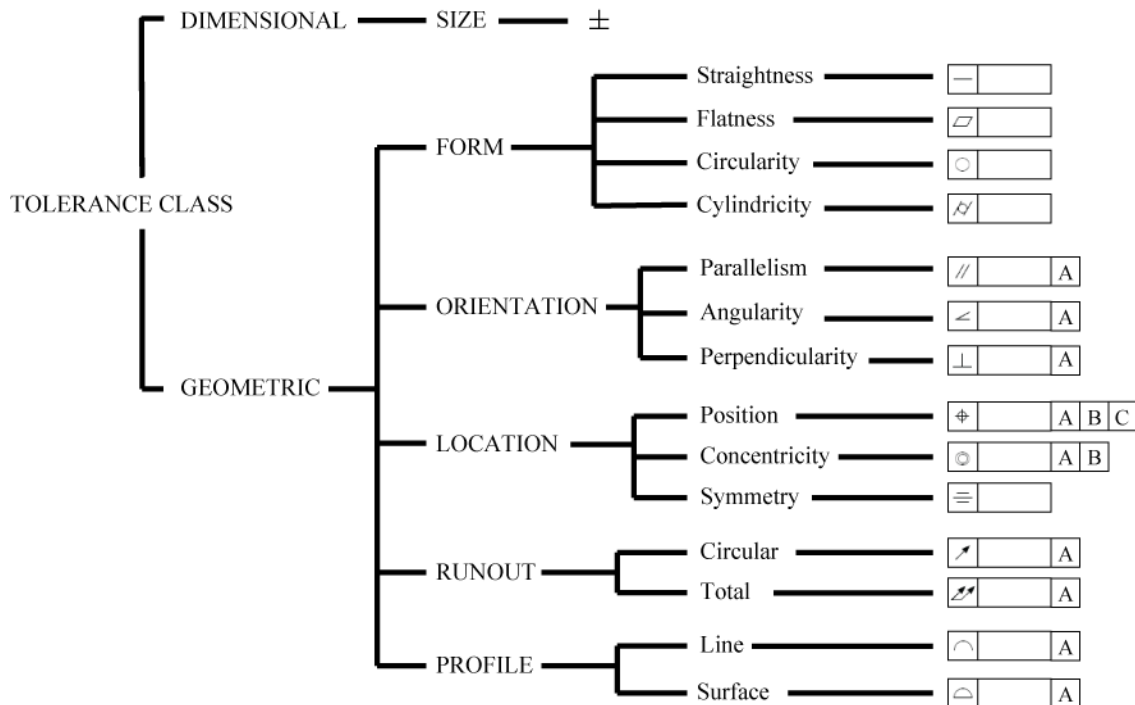


Figure 2.1: Standard tolerance classes

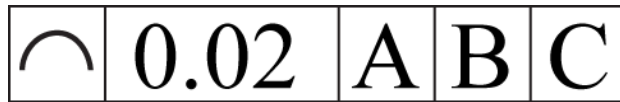


Figure 2.2: A feature control frame used to designate a line profile related to a reference frame defined by feature A, B, and C.

## 2.2 GD&T computer models

Besides the T-Map model, several attempts have been proposed to represent geometric tolerances described in the ASME Standard.

Parametric models use a set of explicit dimensions and constraints in the form of parametric equations to create geometry in parametric computer-aided design (CAD). These parametric equations can be solved to get one or more values for dependent dimensions; tolerances are obtained by allowing variations on the dependent dimension [5, 6, 7, 8]. Because equations are written and solved for vertex positions, the method is only applicable in 2-D profiles and 3-D polyhedral parts. Parametric models have trouble to deal with form tolerances, datum reference frames or directed datum-target relations. Indirect parameterization models [9, 10] have been developed to decouple model construction variables from variables for dimensioning and tolerancing.

Offset zone models represent a tolerance-zone as a Boolean subtraction of maximal and minimal object volumes that are obtained by offsetting the object by amounts equal to the tolerances on either side [11, 12, 13]. But the composite tolerance zone, which is constructed from boundary surfaces of the part, cannot present each type of variation separately, nor to study their interactions. Another problem is that some tolerances in the ASME Standard, such as location, apply to axes or mid-planes of

toleranced features. Also, it does not include the effect from precedence of datum reference frame (DRF).

Variational surfaces model uses the variational surfaces approach to calculate the surface coefficients independently, in the way of changing the values of the model variables according to the tolerance values [14, 15]. And positions of the vertices and edges are computed from the surface variations. Form tolerances can be handled by using higher-degree surfaces or surface triangulation. However, this model has some topological problems, such as maintenance of tangency and incidence conditions among vertices, edges and surfaces of a solid. And variational surfaces cannot model tolerances applied to derived features such as axes or mid-planes, feature that are used widely in practice and appear in the standard.

In kinematic models each tolerance class is represented by a combination of kinematic joints [16]. The combinations are then used to estimate geometric variations dependent on feature tolerances. This model is built upon prior work by Chase et al.[17] of using transformation matrices to analyze tolerance stack-ups in mechanisms. The developers of kinematic models have yet to show this approach can be extended to combine the interaction of geometric variations with size dimensions.

The Degrees of freedom (DOF) model represents geometric tolerances in the form of spatial degree of freedom. [18, 19] Several groups including ASME were independently working on this model. Kramer [20] used symbolic reasoning to demonstrate the determination of DOFs of parts in an assembly and to determine the feasibility of the assembly.



In technologically and topologically related surfaces TTRS models, Clément et al. [21, 22] used elementary surfaces (planes, cylinders, spheres, etc.) to model the six lower kinematic pairs, identified by Reuleaux [23], with the complete constraint of a fixed rigid body, as described by Hunt [24]. These are named the seven technologically and topologically related surfaces (TTRS). 28 different geometric relationships are created from the combination of TTRS, by using group theory and small displacement torsors. The tolerance-zone for each tolerance related to a TTRS was represented as a displacement torsor containing noninvariant rotations and translations. This model is a part of the minimum system of datum reference frames required for each type of geometric tolerance. But this model cannot distinguish between variations from form, size and location. Also, datum precedence is not considered.

The vector space models created by Giordano et al.[25] represents tolerance-zones with a point-space defined by a set of inequalities, and these inequalities are expressed by the components of a deviation torsor and mapped to a corresponding geometric deviation space. It has a similar concept to the Tolerance-Map, but the number of independent parameters limits this model to represent a line or plane. An assembly is modeled as one of two types of variations: clearances in joints between parts or deviations between features. Using Minkowski sum of deviation spaces, the model can represent the interaction of tolerances in an assembly of parts effectively. They have also modeled projected tolerance-zones and material conditions of the tolerances.

### **2.3 The Tolerance Map**

A T-Map is a hypothetical Euclidean point-space, the size and shape of which reflects all variational possibilities for a target feature.[4] It is the range of points

resulting from a one-to-one mapping from all the variational possibilities of a feature, within its tolerance-zone, to the Euclidean point-space. These variations are determined by the tolerances specified to control size, position, orientation, and form of the feature. Figure 2.3 shows the T-Map for the size dimension variation of a round bar. Every possible plane in the allowable tolerance zone is represented by a point in the T-Map. Thus, the T-Map represents the entire family of parts that are acceptable for the given size tolerance.

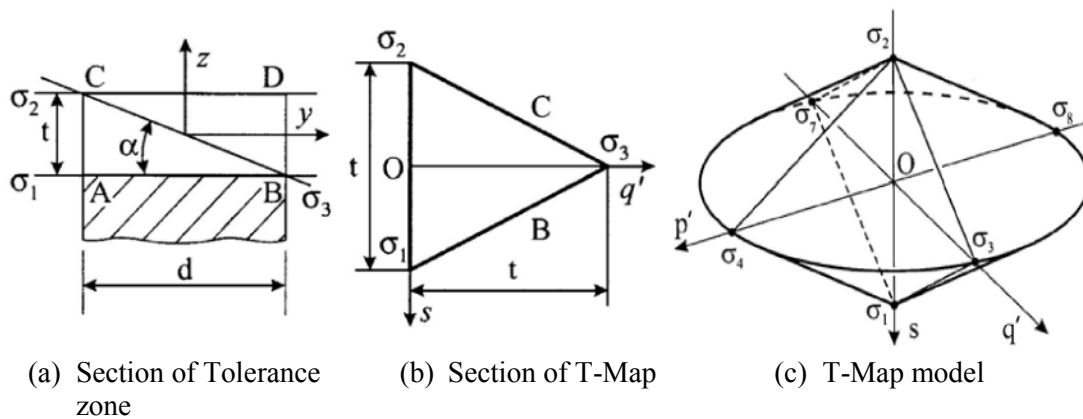


Figure 2.3: T-Map for the size tolerance of a round bar (end plane) [4]

The number of degrees of freedom or the number of unique variations modeled determines the dimension of the T-Map. For example, if ‘n’ types of variations of a feature within the tolerance zone are considered, then the T-Map will be an n-dimensional geometric entity. To model tolerance accumulation, T-Maps can be added with Minkowski sums to model the composite quantitative effect of all tolerances on a sequence of feature stock up.

T-Map models have been under development since 1998, and previous work has been done by several researchers.

Mujezinović [26] developed T-Maps for size, form and orientation tolerances of planar polyhedral and round faces. He used T-Maps for tolerance analysis for the stack-up and tolerance allocation of an assembly of rectangular and cylindrical parts. He demonstrated how T-Maps distinguished the effects of different datum sequencing on parts with orientation tolerances. He has partially developed the stack-up for an assembly of parts with an offset included.

Davidson and Shah [27] developed a T-Map for axes in accordance with a position tolerance. This 4-D T-Map is built from 5 basis features, and it represents all possible variations of the axis in the tolerance-zone. Davidson and Shah [28] also developed the T-Map for a tab and slot features and demonstrated that T-Maps can be used to represent tolerances on any part cross-section using methods of triangulation [29].

Bhide [30] expanded the T-Map for an axis of a cylindrical feature by mapping it to variations due to position, orientation and form. The T-Map for a pin or a hole having size, position and form tolerances as well as material modifiers was developed. This is a 5-D T-Map which represents 5 different variations along its axes. Bhide also gave the difference in T-Maps for sequence of datums on a cylindrical hole, and evaluated the interaction of tolerances in a pin-hole assembly.

Ameta [31] developed T-Maps for angled faces and for incident point-line feature clusters. For angled faces, tolerances for size and angularity were included. He applied the T-Maps to stack-ups of parts with angled faces, and the T-Map for the point-line cluster to a picture frame assembly.

Clasen [32] developed T-Maps for runout and straightness tolerances and for a line-plane cluster. He showed the T-Map circular and total runout specified on both surfaces of revolution or on round faces, and the interaction of T-Map for size and form tolerances with T-Map for runout. He demonstrated the usefulness of the T-Map for circular runout in minimizing runout errors in assemblies. He constructed an accumulated T-Map to demonstrate the increased variation of a part axis when the part has a runout specification in reference to an imperfect datum axis. He also constructed the T-Map for line-plane cluster, which is a pairing of two features: a plane and an axis at right angles to it.

## Chapter 3

### T-MAP FOR LINE PROFILE TOLERANCE

In this chapter, the T-Map for line profile is introduced, including tolerance specification, tolerance zone, and T-Map model. Content of this chapter is based on Davidson and Shah's research [33]. They created a method to generate the T-Maps for line profiles, but the profile shapes were limited to square, rectangle and triangle, they have not given a general method of modeling for arbitrarily shapes of line profiles.

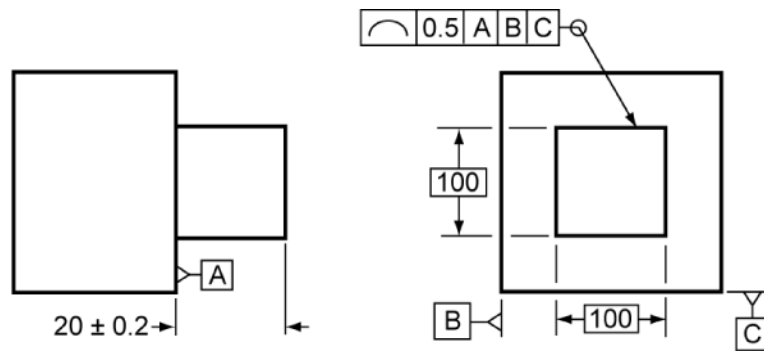


Figure 3.1: Square boss (external feature) with its shape controlled by the profile tolerance  $t = 0.5\text{mm}$  relative to Datums A, B, and C.

#### 3.1 Profile specification and tolerance zone

The profile tolerance is used to control each cross-section of a surface that may be formed by extruding a 2-D feature (curve). The specification of a profile tolerance establishes a true (theoretical) line-element and tolerance zone at each cross-section of the surface feature. The boundaries to the tolerance zone have similar geometric shape of the true profile.

The specification for a simple profile is shown in Figure 3.1. The shape of the rectangular boss is controlled by the profile tolerance  $t = 0.5\text{mm}$  relative to the Datums A, B and C. This specification gives two square boundaries. The outside one is 0.25 mm larger along every line normal to the surface, and the other is 0.25 mm smaller. According to the ASME Standard [2], measured points on the manufactured surface at every cross-section, which forms a line profile, must lie within the outer and inner boundary lines at that cross-section. These kinds of line profiles are used to control the shape of mildly twisted surfaces, such as those on compressor blades.

The tolerance specifications here define tolerance-zone boundaries that are equally disposed about the true profile. But other tolerance (unilaterally outside, unilaterally inside and unequal amounts inside and outside) specifications require the boundaries to be unevenly disposed about the true profile.

When a profile tolerance is specified without any datum, it only controls form, but when datums are specified with it, the profile tolerancing provides composite control of form, orientation and location. The size variations of profiles are represented by using the true profile and the allowable array of parallel curves to it. All curves have different sizes, but they should reside in the tolerance zone.

### **3.2 T-Map modeling of a line profile**

The basic idea of modeling T-Map for line profile is mapping variations of the true profile within the tolerance zone. The variations in geometry represented are size change, translational positions (both x and y in a plane), and angular orientation  $\theta$ . Each point in the T-Map will correspond to one of the profile variations. These four freedoms for variation require a 4-D model.

Starting from a 2-D cross-section of the T-Map is an easy way to understand the mapping of line profile. Let  $a$  be the half-side length of any allowable square in the tolerance zone. The labels  $\psi_1$  and  $\psi_2$  designate the smallest and largest of these profile-squares in Figure 3.2(a). In the T-Map, shown as in Figure 3.2(b), the points on the line through  $\psi_1$  and  $\psi_2$  are mapping back to a set of non-rotated squares that have their centers at  $O$  in Figure 3.2(a). The acceptable range of size within tolerance zone is from  $a = \bar{a} - \epsilon/2$ , to  $a = \bar{a} + \epsilon/2$ , and this range corresponds to a line segment  $\psi_1\psi_2$ . Its midpoint (labeled  $\psi_{12}$ ) corresponds to the middle-size profile. Then translate the middle-sized profile (MSP) rightward by  $\epsilon/2$  to the limit of the tolerance (dotted square labeled  $\psi_3$  in Figure 3.2(a)), and the basis-point of  $\psi_3$  is placed on another direction  $e_x$  that is orthogonal to axis  $a$ , as shown in Figure 3.2(b). There is linear relation between  $a$  (the change of profile size) and  $e_x$  (the profile displacement in x direction). When size increases by some value  $\Delta F$ , the allowable range of profile translation in a direction will decrease by the same amount. So straight lines can be used to connect basis-points, such as  $\psi_1$  and  $\psi_3$ , and these lines form the boundaries of the 2-D cross-section of the T-Map.

Next, confine attention to the array of MSP, and this corresponds to a 3-D hypersection of the 4-D T-Map at the size that equals to  $\bar{a}$ . The location of a variational square is represented by components of displacement,  $e_x$ ,  $e_y$  and  $\theta$ . Basis-profile  $\psi_4$  is chosen to be the profile of this set that has been translated upward  $\epsilon/2$  to the limit of the zone (dotted square labeled  $\psi_4$  in Figure. 3.2(a)). The basis-profile  $\psi_5$  represents the clockwise rotational displacement to the limit. For the condition that tolerances are two or more orders of magnitude smaller than the corresponding dimensions, the linear relation

between rotation and translation has been proved by Davidson, Mujezinović and Shah [4]. So the 3-D hypersection of the T-Map is an octahedron, as shown in Figure 3.3.

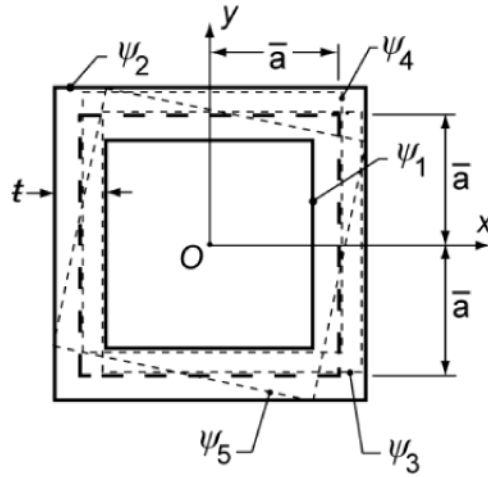


Figure 3.2(a): The MSP (dashed-lined square) in the (exaggerated) tolerance-zone that is specified with the profile tolerance  $t$ , five variational possibilities are labeled.

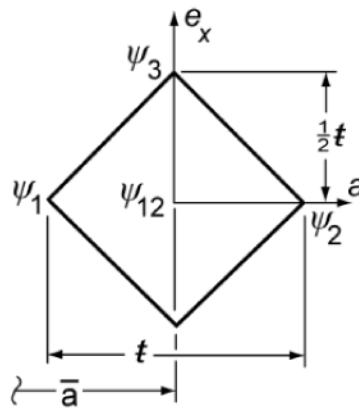


Figure 3.2 (b): One 2-D cross-section of the corresponding T-Map that is confined to all size variations and displacements  $e_x$  only in the x-direction.



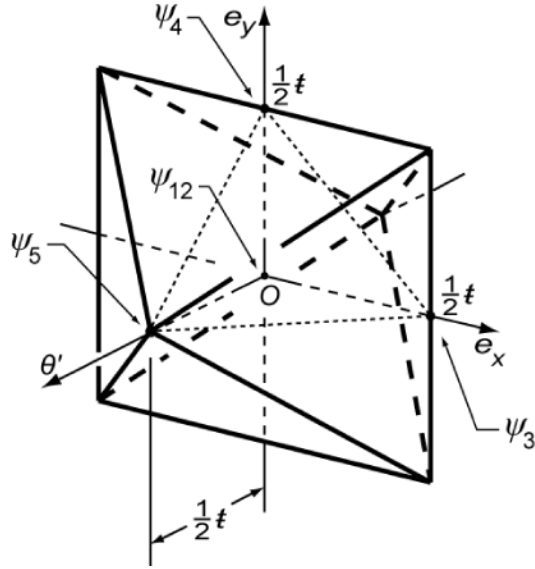


Figure 3.3: The central hypersection of the 4-D T-Map for a square line profile. This model represents all the middle-sized squares in the tolerance-zone of Figure 3.2(a).

Because the T-Map may be used for metric computations, the units along all axes should be the same. The scale of the axis for angle  $\theta$  is made  $\theta' = \bar{a} \theta$ . At the basis-point  $\psi_5$  of T-Map, the limit of rotation variations is

$$\theta' = \bar{a} \theta = \bar{a} \left( \frac{t}{2\bar{a}} \right) = \frac{t}{2}$$

The full 4-D T-Map for the square tolerance zone in Figure 3.2(a) is from the combination of Figure 3.2(b) and Figure 3.3. The result is a double hyperpyramid in 4-D that is depicted in Figure 3.4. The base for each single pyramid is the 3-D octahedron from Figure 3.3, and every other section (two are shown) at right angles to the direction of size is a smaller and geometrically similar octahedron.

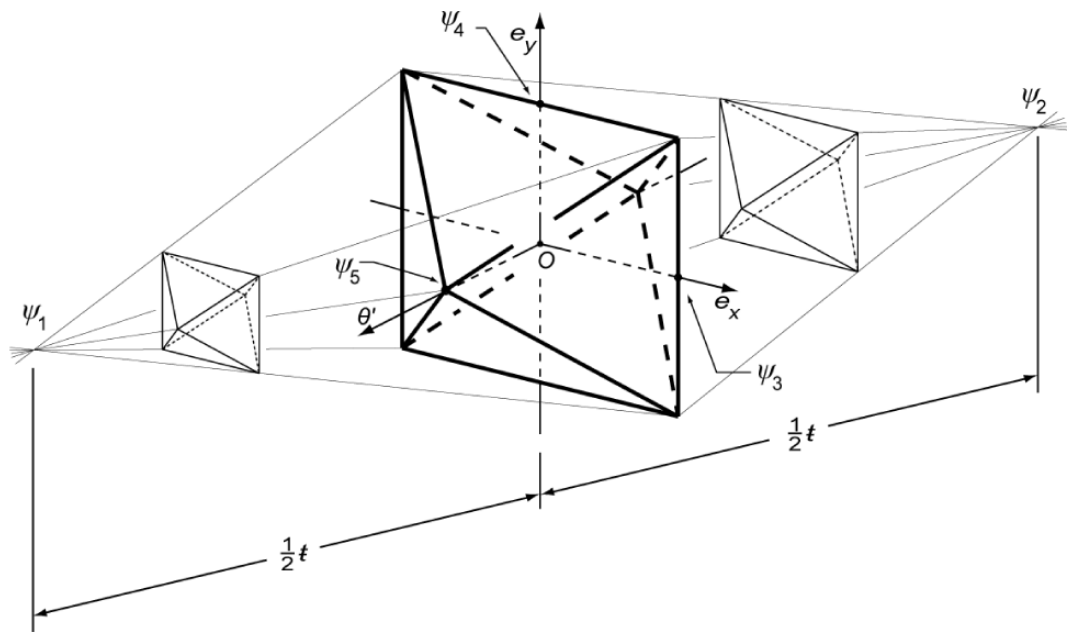


Figure 3.4: The 4-D T-Map for the square tolerance-zone in Figure 3.2(a) and showing all five basis-points  $\psi_1, \dots, \psi_5$ . For clarity of the graphics, the scale in the direction of size  $(\psi_1 \psi_2)$  is exaggerated.

## PARAMETRIC MODELING METHOD

**4.1 Parametric modeling method**

The parametric modeling method is to get the extreme positions of a profile in the extent of allowable variation, by moving the profile inside the tolerance zone. The profile here can be the middle-sized profile of parts, or it can be of allowable larger or smaller size. This method can be simulated by two laminas. One is fixed with tolerance zone, and the other carries the displaced profile. Displacement of the moving the lamina can be expressed by values of the small displacements  $e_x$ ,  $e_y$ , and  $\theta$ , and these small displacements are three of the coordinates of one point in the T-Map space. So long as the profile attached to the moving lamina lies within the fixed boundaries of the tolerance-zone, the point identified with the corresponding coordinates lies inside the T-Map. When this displaced profile touches these boundaries at one or more points, the coordinates  $e_x$ ,  $e_y$ , and  $\theta$  identify a point on the T-Map boundary. The set of all boundary points in the T-Map space gives a hypersection of T-Map in the specific size change  $\Delta F$  of profile.

To define and compute the positions analytically, both a fixed reference system and a displaced reference system are required, the displaced one being attached to the moving lamina. The fixed reference system is defined as a global fixed coordinate system, and the boundaries of tolerance zone can be expressed by coordinates or equations in this frame. The displaced reference system is bonded to the profile which can move in tolerance zone. A homogeneous coordinate transformation matrix  $[A]$  is

used to transform the coordinates in the displaced reference system to the fixed reference system. When one or more points or segments of the displaced profile touch segments or points of the tolerance-zone boundary, respectively; a point, line, or surface of the T-Map boundary may be identified by using matrix  $[A]$  to relate each contacting pair of points.

In this chapter, the example in Figure 4.1 is used to formulate the parametric modeling method for building a 4-D T-Map. The dimensions and tolerance for the square profile with a rounded corner are shown in Figure 4.1.

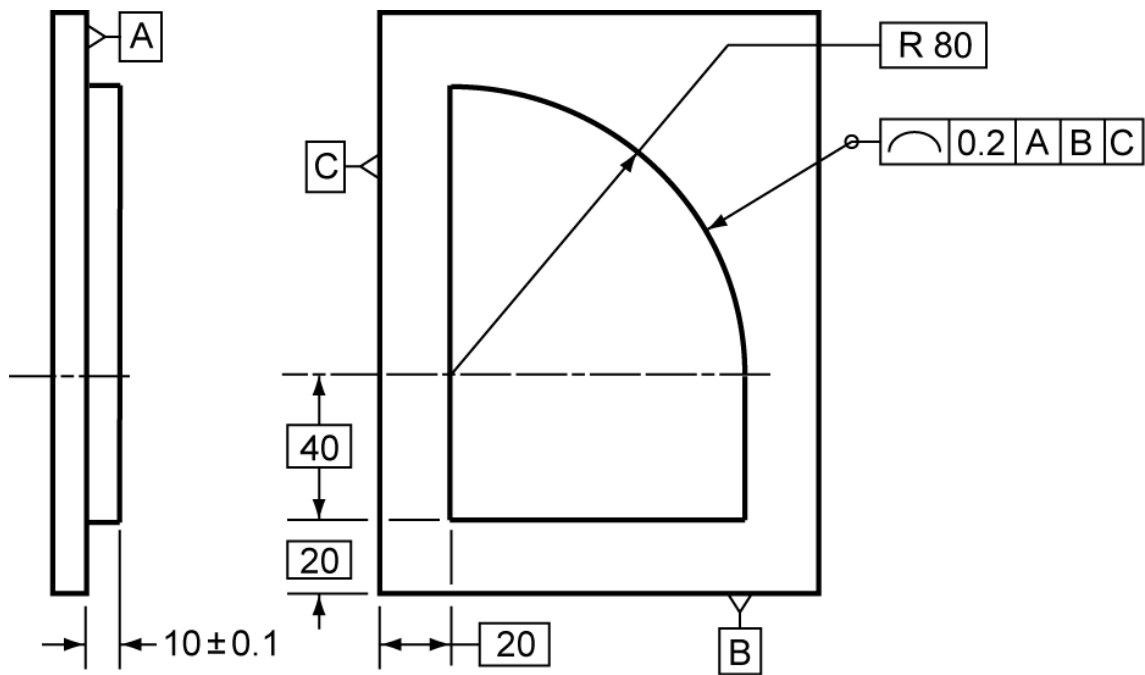


Figure 4.1: Boss with its shape controlled by the profile tolerance  $t = 0.2$  mm relative to Datums A, B, and C.

## 4.2 The Invariant Point (Pole) of the Profile

It is helpful to view the displacements  $e_x$ , and  $e_y$ , and  $\theta$  of the profile to be the same as those of a moveable lamina on which the middle-sized profile (MSP) is etched.

For square profiles, its undisplaced location is shown with the heavy dashed line in Figure 4.2(a). A fully rotated location of the displaced lamina may be used to identify the unique point that does not displace. In classic kinematics literature (see e.g. [34] or [35]) this point is called the ‘pole’ of the two locations. For the square profile in Figure 4.2(a),  $\psi_5$  represents the counterclockwise most allowable MSP, and the geometric center is the pole. For line-profiles, it is the point to which the eccentricities  $e_x$  and  $e_y$  apply in the associated T-Map.

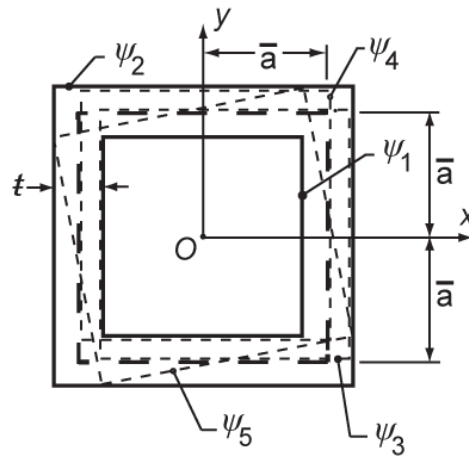


Figure 4.2 (a): The middle-sized profile (dashed-line square) for a square profile in the (exaggerated) tolerance-zone that is specified with the profile tolerance  $t$ ; five variational possibilities are labeled, three with dotted lines.

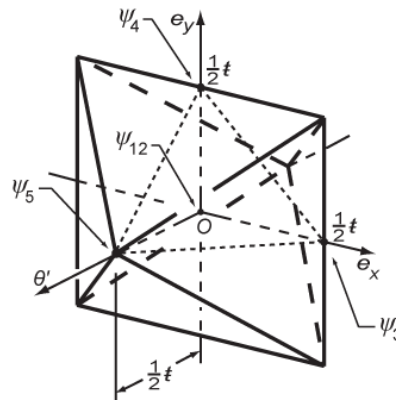


Figure 4.2(b): The 3-D T-Map for all the middle-sized squares in the sharp-cornered tolerance-zone of Figure 4.2(a). Taken from [36]

For the square and rectangular profiles in Figure 4.2(a) and Figure 4.3(a), the pole is at the geometric center  $O$ . However, for the rectangular profile, there is not one fully rotated location of the lamina that is locked in place. Of the linear array of possibilities shown in 4.3(b), we choose the one that is mid-way between the two dotted ones at the limits. Note that, for both the square and rectangular profiles, the fully rotated lamina, which is used in defining the pole and its associated origin of the required coordinate system, corresponds to one of the two points in the T-Map where the  $\theta'$ -axis pierces the boundary. See Figure 4.2(b) and Figure 4.3(b).

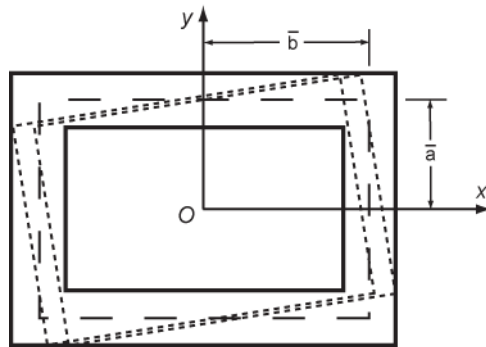


Figure 4.3(a): The middle-sized profile (dashed-lined rectangle) in the (exaggerated) tolerance-zone that is specified with the profile tolerance  $t$ , and two of its fully rotated variational possibilities (dotted lines)

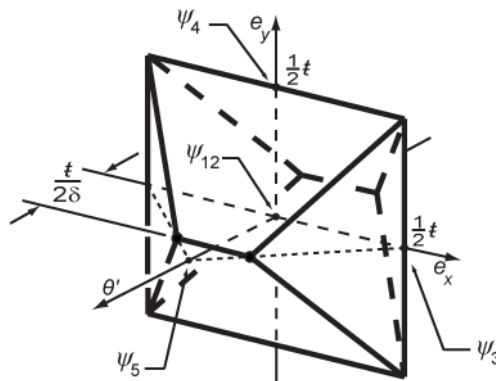


Figure 4.3(b): The 3-D T-Map for all the middle-sized rectangles in the sharp-cornered tolerance-zone of Figure 4.3(a). The two vertices at the front (with dots) correspond to the two rotated profiles shown in Figure 4.3(a). Taken from [36]

### 4.3 The base hypersection of the T-Map for the Middle-Sized Profile

The profile shape in Figure 4.1 can be seen as a rectangle with one large rounded corner. Because the pole of a rectangle is its geometric center, the pole of the profile is defined at point  $O$  in Figure 4.4. Apply a coordinate system for the whole profile having its origin at the pole. The coordinates of vertices, one discontinuity in curvature, and the arc-center of this profile are shown with dots in this coordinate system in Figure 4.4.

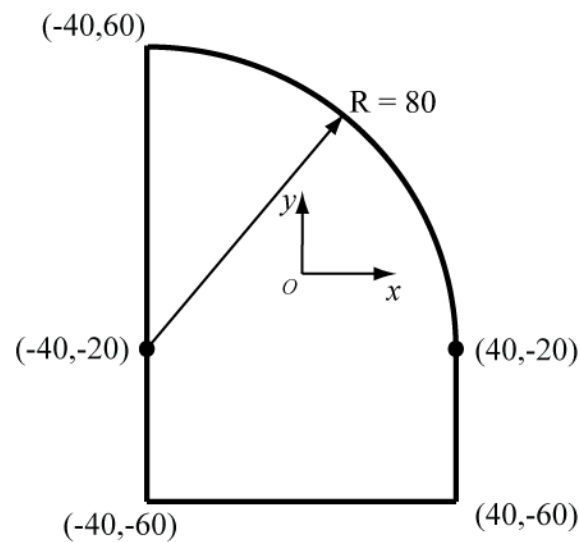


Figure 4.4: Geometry parameters of the mid-size profile identified in the fixed coordinate system.

The 3-D T-Map for the middle-sized profile in Figure 4.5 is shown in Figure 4.6. Figure 4.5 provides two displaced locations for mid-sized profile in the tolerance zone. The tolerance zone is created under the ASME Standard [36]. For each of these locations, the displaced profile contacts the boundaries at three points. It means that each of these locations is mapped to some special point in the T-Map, and, by identifying these locations, permits the building of the T-Map for the profile.

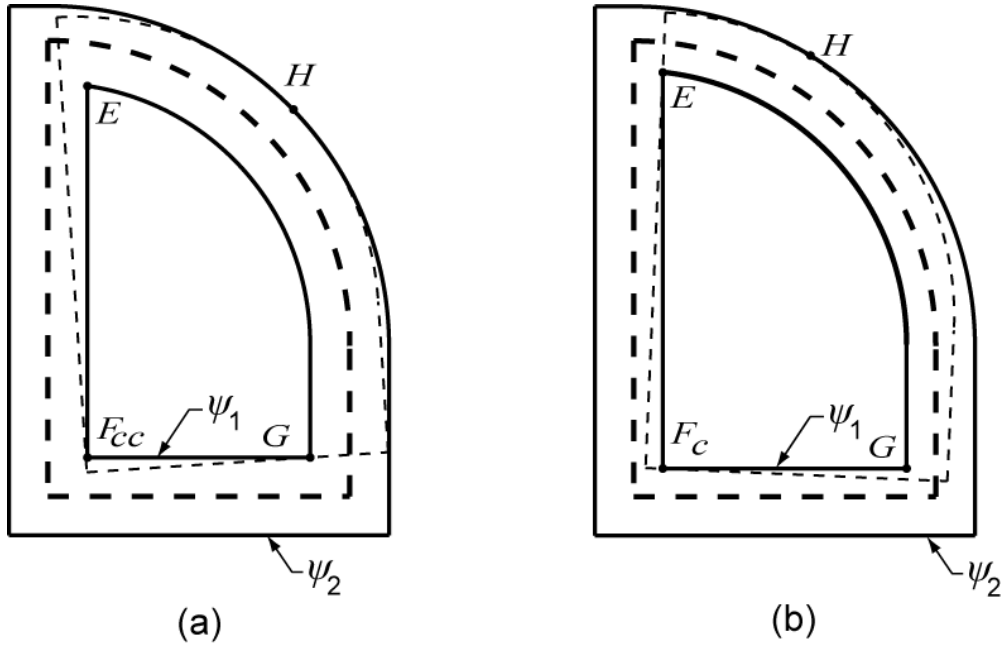


Figure 4.5: Selected displaced locations for the middle-sized profile (dashed-line) in the (exaggerated) tolerance-zone that is specified with the profile tolerance  $t = 0.2\text{mm}$  from Figure 4.1 (a) Constrained at three points of the boundary and rotated counterclockwise. (b) Also constrained, but rotated clockwise.

The shape of T-Map in Figure 4.6 may be found analytically by using the homogeneous coordinate transformation  $[A]$  that locates the displaced lamina (carrying the dotted MSP in all parts of Fig 4.3) relative to the fixed (dashed) MSP; it transforms homogeneous coordinates of points from the displaced frame to the fixed frame. From any good book on robotics, e.g. [37],

$$[A] = \begin{bmatrix} \cos \theta & -\sin \theta & e_x \\ \sin \theta & \cos \theta & e_y \\ 0 & 0 & 1 \end{bmatrix} \approx \begin{bmatrix} 1 & -\theta & e_x \\ \theta & 1 & e_y \\ 0 & 0 & 1 \end{bmatrix} \quad (4.1)$$



where the small displacements  $e_x$ , and  $e_y$ , and  $\theta$  locate the dotted frame relative to the fixed one and the origins of both frames are at the geometric centers of the unrounded rectangles. The second form of  $[A]$  in Equation 4.1 arises because angle  $\theta$  is always very small ( $<0.2/80$  for the profile in Figure 4.1) and only first-order small quantities need to be retained.

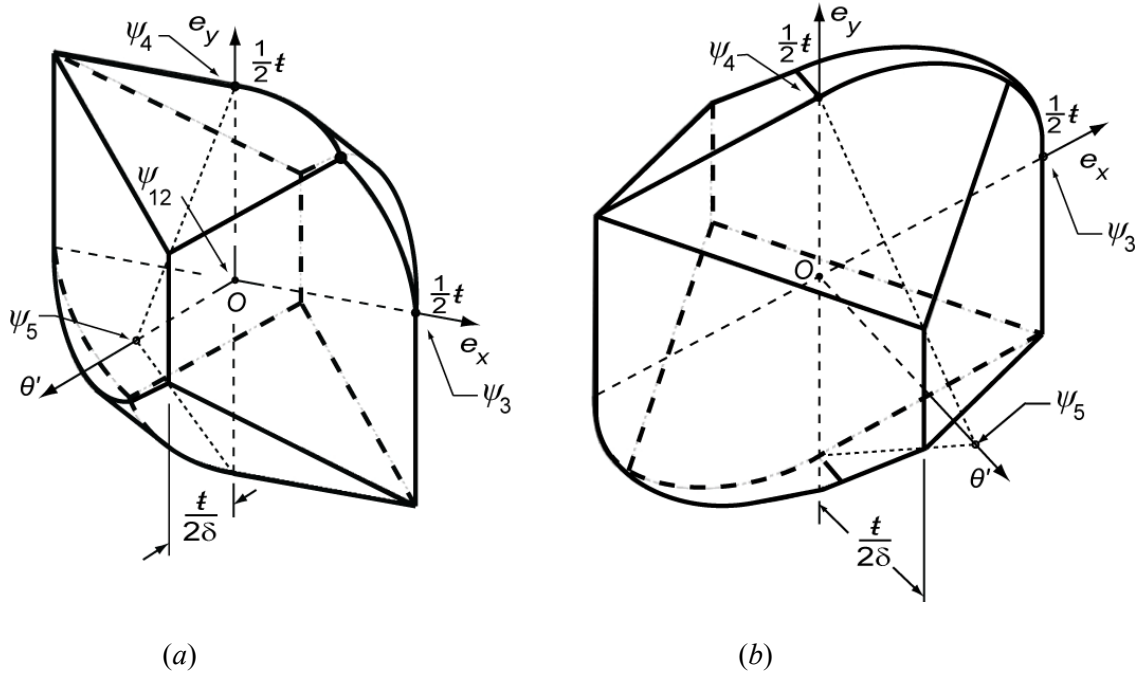


Figure 4.6: The 3-D T-Map for the middle-sized profile in Figure 4.1 in two different orientations.

Table 4.1 Coordinates of contacts in Figure 4.5(a) and 4.5(b)

Point	Coordinates (x, y), mm		Constraint equations
	In Displaced Frame	In Fixed Frame	
E	$(-40, y_E)$	$(-40 + t/2, 60 - t/2)$	$-t/2 = 1.5\theta' - e_x$
F <sub>c</sub>	$(x_F, -60)$	$(-40 + t/2, -60 + t/2)$	$-t/2 = \theta' - e_y$
F <sub>cc</sub>	$(-40, y_F)$	$(-40 + t/2, -60 - t/2)$	$t/2 = 1.5\theta' + e_x$
G	$(x_G, -60)$	$(40 - t/2, -60 + t/2)$	$t/2 = \theta' + e_y$
Arc-center (C)	$(-40, -20)$	$(x_{Hc}, y_{Hc})$	$(0.5\theta' + e_x)^2 + (-\theta' + e_y)^2 = (t/2)^2$

Table 4.1 contains the coordinates of superimposed points in both the displaced and fixed laminas; each row represents a constraint between the two laminas. For instance, the third and fourth ones constrain the left and lower line-segments to touch corners F and G, respectively, of the inner boundary (Figure 4.5(a)). The last row in the table contains the coordinates of the arc-center corresponding to the contact of a point H on the arc of the dotted MSP with the arc of the outer boundary (fixed) of the tolerance-zone (Figure 4.5(a) and Figure 4.5(b)). The coordinates in Row 5 of Table 4.1 are related by  $[x_{Hc} \ y_{Hc} \ 1]^T = [A][-40 \ -20 \ 1]^T$ . As a consequence of the contact at point H, the displaced arc-center  $C_H(x_{Hc}, y_{Hc})$  lies on a circle of radius  $t/2 = 0.1$  mm and with the fixed center  $C(x,y)=(-40, -20)$ , i.e.

$$(x_{Hc} + 40)^2 + (y_{Hc} + 20)^2 = \left(\frac{t}{2}\right)^2 \quad (4.1)$$

We now see that each of the contact constraints at points E, F, G, and H in Figure 4.5(a) and 4.5(b) may be formalized by relating the coordinates in one row of Table 4.1. These formalizations, together with displaced arc-center  $C_H$ , may be used to confirm all the surfaces that form the right half of the boundary shown in Figure 4.6(a). For the Rows 1 - 4 of Table 4.1, it is convenient to use the inverse of transformation  $[A]$ , i.e.

$$[A]^{-1} \approx \begin{bmatrix} 1 & \theta & -e_x \\ -\theta & 1 & -e_y \\ 0 & 0 & 1 \end{bmatrix} \quad (4.2)$$

in which only first-order small quantities have been retained. Taken together, the matrix equations are

$$\begin{array}{ccccc} & E & F_c & F_{cc} & G & C \\ \begin{bmatrix} -40 & x_F & -40 & x_G & -40 \\ y_E & -60 & y_F & -60 & -20 \\ 1 & 1 & 1 & 1 & 1 \end{bmatrix} & & & & & \end{array} =$$

$$\begin{array}{ccccc} & E & F_c & F_{cc} & G & C \\ \begin{bmatrix} 1 & \theta & -e_x \\ -\theta & 1 & -e_y \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} -40 + \epsilon/2 & -40 + \epsilon/2 & -40 + \epsilon/2 & 40 - \epsilon/2 & x_{Hc} \\ 60 - \epsilon/2 & -60 + \epsilon/2 & -60 + \epsilon/2 & -60 + \epsilon/2 & y_{Hc} \\ 1 & 1 & 1 & 1 & 1 \end{bmatrix} & & & & \end{array} \quad (4.3)$$

The counterclockwise displacement in Figure 4.5(a) is constrained at points  $G$  and also at point  $F$  with the coordinates in Row 3 of Table 4.1. From Equations 4.3, the constraints at  $F_{cc}$  and  $G$  lead respectively to

$$\begin{cases} -40 = (-40 + \epsilon/2) + (-60 + \epsilon/2)\theta - e_x \\ -60 = (-40 + \epsilon/2)\theta + (-60 + \epsilon/2) - e_y \end{cases}$$

Similar for clockwise displacement which is constrained at point  $E$  and  $F_c$

$$\begin{cases} -40 = (-40 + \epsilon/2) + (60 - \epsilon/2)\theta - e_x \\ -60 = (40 - \epsilon/2)\theta + (-60 + \epsilon/2) - e_y \end{cases}$$

For displaced arc-center  $H_C$

$$\begin{cases} x_{Hc} = -40 + 20\theta + e_x \\ y_{Hc} = -40\theta - 20 + e_y \end{cases} \quad (4.4)$$

Combined with Equation 4.1, the constraints for contact at point H is created

$$(20\theta + e_x)^2 + (-40\theta + e_y)^2 = (\sharp/2)^2$$

To have consistent units (length) on all those axes of the T-Map, angle  $\theta$  is multiplied by a characteristic length, 40 mm, so  $\theta' = 40\theta$ . This modified measure for angular displacement appears in the rescaled constraint equations of Table 4.1.

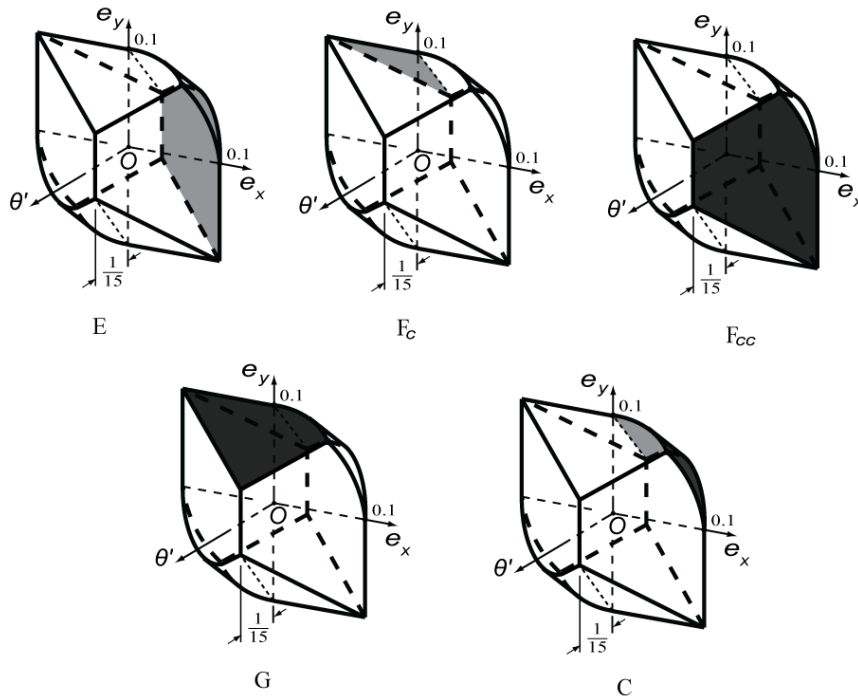


Figure 4.7: Geometric representations of the equations in Table 4.1. Each equation defines a surface of T-Map model. The dark shadow presents the surfaces on the front and the light shadow present surfaces on the back.

Figure 4.7 compares the faces of the equations in Table 4.1 and T-Map model in Figure 4.6(a). And the mapping point of displaced location in Figure 4.5(a)(b) are the intersection of faces shown in Figure 4.7. In this way, the equations of rest T-Map surfaces can be calculated

$$\left\{ \begin{array}{l} \dagger/2 = 1.5\theta' - e_x \\ \dagger/2 = \theta' - e_y \\ -\dagger/2 = 1.5\theta' + e_x \\ -\dagger/2 = \theta' + e_y \\ (0.5\theta' + e_x)^2 + (-\theta' + e_y)^2 = (\dagger/2)^2 \end{array} \right. \quad (4.5)$$

#### 4.4 Smaller 3-D Hypersections of the T-Map

Lastly in the T-Map construction, line-profiles that are larger or smaller than the middle-sized one are more limited in their allowable displacements  $e_x$ ,  $e_y$ , and  $\theta$ , and they lead to hypersections that are smaller than the base hypersection. Although some of the limits to displacement of such a profile grow linearly with either larger or smaller size, not all do. The parametric construction method for these hypersections is the same as for the base hypersection, except that now size changed is included in Equation (4.4) and (4.5).

For example, if the size is increased  $\Delta F$  (i.e. 0.05), all points of the displacing profile in Figure 4.4 will be offset outward by  $\Delta F$  mm, although the center of the arc remains unchanged. The new coordinates and dimension are shown in Figure 4.8, and the coordinates and equations in Table 4.1 are modified to be those in Table 4.2. Because the

distance between arc and outside boundary is reduced to  $\Delta F$ , the displaced arc-center  $C_H(x_{Hc}, y_{Hc})$  lies on a circle of radius  $\Delta F$  mm and with the fixed center  $C(x,y)=(-40, -20)$ .

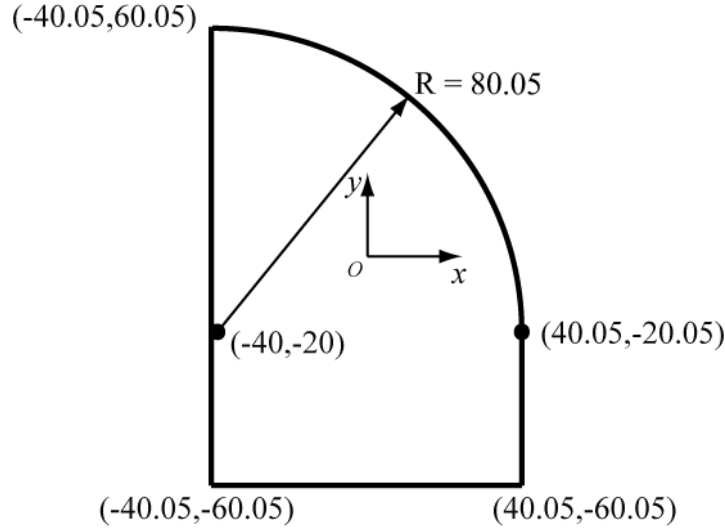


Figure 4.8: Geometry parameters of the offset profile under the coordinate system.

Table 4.2 Coordinates of contact points of size changed profile.

Point	Coordinates $(x, y)$ , mm		Constraint equations
	In Displaced Frame	In Fixed Frame	
E	$(-40.05, y_E)$	$(-40 + t/2, 60 - t/2)$	$-(t/2 + \Delta F) = 1.5\theta' - e_x$
$F_c$	$(x_F, -60.05)$	$(-40 + t/2, -60 + t/2)$	$-(t/2 + \Delta F) = \theta' - e_y$
$F_{cc}$	$(-40.05, y_F)$	$(-40 + t/2, -60 - t/2)$	$(t/2 + \Delta F) = 1.5\theta' + e_x$
G	$(x_G, -60.05)$	$(40 - t/2, -60 + t/2)$	$(t/2 + \Delta F) = \theta' + e_y$
Arc-center (C)	$(-40 - \Delta F, -20 - \Delta F)$	$(x_{Hc}, y_{Hc})$	$(0.5\theta' + e_x)^2 + (-\theta' + e_y)^2 = (t/2 - \Delta F)^2$

Comparing the equations in Table 4.1 and Table 4.2, we will find that the faces of these equations lie outside of the 3D T-Map model, but the line-profiles that are larger or smaller than the middle-sized one are more limited in their allowable displacements  $e_x, e_y$ ,

and  $\theta$ , and the limits grow linearly with change in size. This is because the contact points E, F and G are all located on the inside boundary of the tolerance zone. When the size of the profile is increased, the displacements of the profile are controlled by more contacts with the outside boundaries. For the specific example at hand the allowable displacement is  $\Delta F$  to the undisplaced profile. Apply the coordinates changed to Equations 4.5

$$\left\{ \begin{array}{l} (\sharp/2 + \Delta F) = 1.5\theta' - e_x \\ (\sharp/2 + \Delta F) = \theta' - e_y \\ -(\sharp/2 + \Delta F) = 1.5\theta' + e_x \\ -(\sharp/2 + \Delta F) = \theta' + e_y \\ (0.5\theta' + e_x)^2 + (-\theta' + e_y)^2 = (\sharp/2 + \Delta F)^2 \end{array} \right. \quad (4.6)$$

Ten equations in Table 4.1 and Equations 4.5 represent all the surfaces of T-Map for mid-sized profile. When the size of the profile changed, these ten equations are modified in Table 4.2 and Equations 4.6. But they do not represent all the surfaces of the T-Map with size increased by  $\Delta F$ . The ten equations can be divided into five pairs, four pairs of the parallel planes, and one pair for cylinders. In each pair of surfaces, only the surface closer to the origin is used to represent a surface of the T-Map. Pick the qualified equations for the surfaces of the T-Map for size-changed profile:

$$\left\{ \begin{array}{l} (\sharp/2 + \Delta F) = 1.5\theta' - e_x \\ (\sharp/2 + \Delta F) = \theta' - e_y \\ -(\sharp/2 + \Delta F) = 1.5\theta' + e_x \\ -(\sharp/2 + \Delta F) = \theta' + e_y \\ (0.5\theta' + e_x)^2 + (-\theta' + e_y)^2 = (\sharp/2 - \Delta F)^2 \end{array} \right. \quad (4.7)$$

Beside equations in Equation 4.7, there are other equations to describe the contact between the displaced profile and the boundaries. The displaced profile with size-changed can also contact the two extension lines of the arc of outside boundary. So there are another two equations

$$\begin{cases} -(\t/2 - \Delta F) = \theta' - e_y \\ \t/2 - \Delta F = 0.5\theta' + e_x \end{cases} \quad (4.8)$$

Additional contact situation is the left bottom corner of the displaced profile contact to the right bottom edge of the outside boundary. This is a point to line contact, the equation for this contact is

$$\t/2 - \Delta F = 1.5\theta' + e_x \quad (4.9)$$

Equations 4.7, 4.8 and 4.9 are algebraic representations for all the surfaces of the T-Map for the size-changed profile and the model is shown in Figure 4.9.



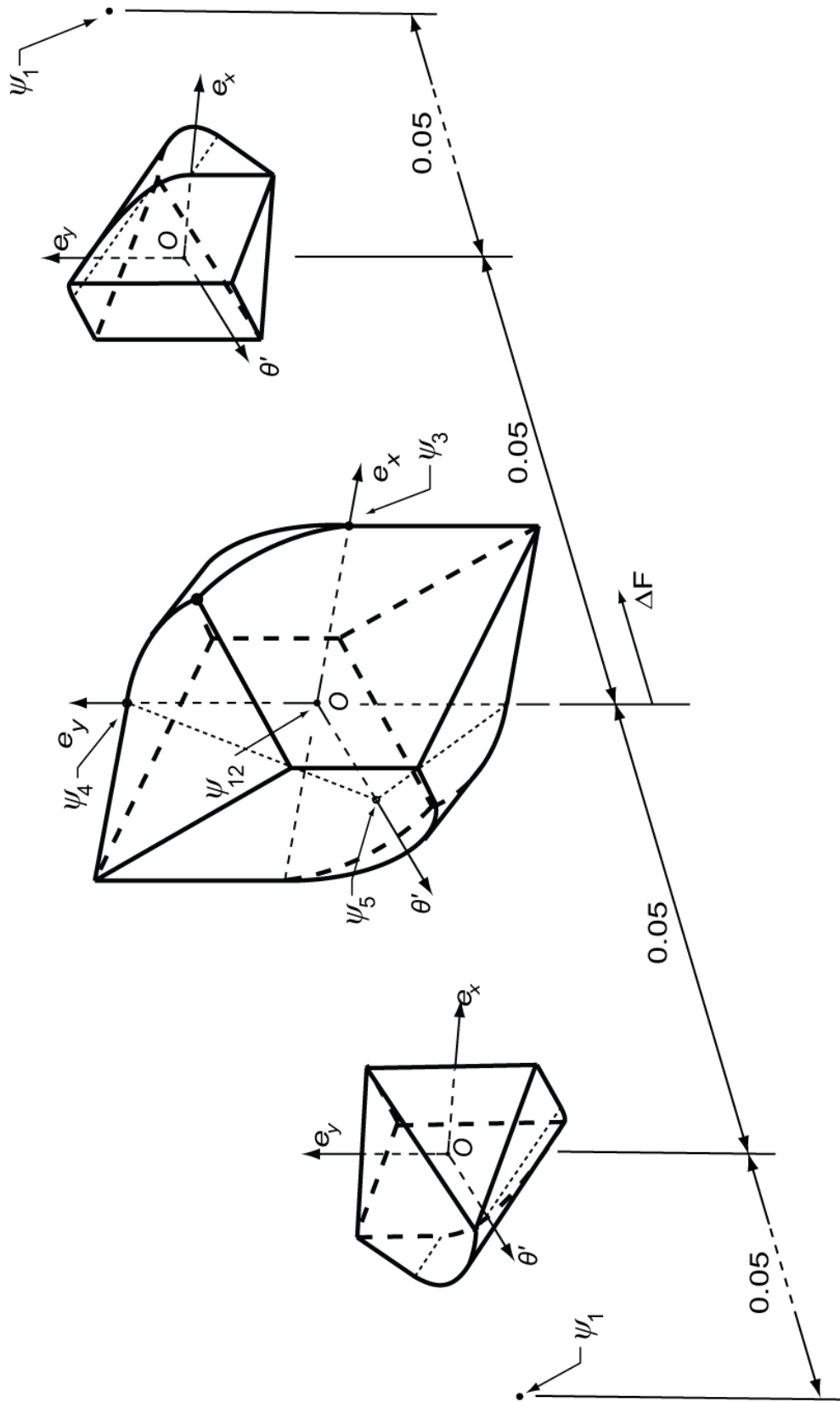


Figure 4.9: The 4-D T-Map with the 3-D hypersection in  $\Delta F = -0.05, 0, 0.05$ .

DECOMPOSED MODELING METHOD

**5.1 Decomposed modeling method**

Decomposed modeling method is to build the T-Map for line profile by intersecting T-Maps for all elements of line profile. Three basic elements of line profile are defined: Straight line, circular arc, and freeform curve.

The boundaries of tolerance-zone constrain the profile displacements which are used to represent the deviations under tolerancing, and in the corresponding T-Map, the shell of the T-Map model separates space of deviations into acceptable and unacceptable regions. The boundaries and shell have the same function, which is to present the tolerance range, in other words, there is a relation between the shell and the tolerance-zone's boundaries.

Because boundaries of tolerance-zone are fixed by theoretic profile shape and tolerance value, and the geometric properties of a point on boundaries are inherited from only one point on profile, the boundaries are not influenced by the deviation (or displacement) of the profile. And because all points on the boundaries play independent constraint roles that are not defined or influenced by other points on the boundaries, each point can constrain a series of deviation of profiles, and these deviations are mapped to parts of shell in the T-Map. Thus, there are independent, unique, one-to-one relations between the point on profile and part of the corresponding T-Map shell, so that it becomes possible to construct the shell by combining shells from the profile elements.

Divide the profile into segments, such as points or groups of points, and build a set for primitive T-Maps each of these segments. When a deviation of profile meets all

constraints, its mapped point must lie within all T-Maps for all profile segments, and the intersection of these T-Maps envelops all possible mapping points of the acceptable profile deviations.

## **5.2 Slope conservation and 2-D modeling**

In this section, researches are only focused on 2-D space, which is to map the translational displacements of profile in two directions on a plane. This 2-D model is a section of 4-D T-Map at rotational angle equal to 0 and for the size ( $\Delta F=0$ ) of the profile. A property between the shape of profile and the shape T-Map model, called slope conservation, is introduced in 5.2.1. Corresponding 2-D T-Maps are built for all basic elements of line profile, and the intersection of these operand T-Map forms the single T-Map for the whole profile.

### **5.2.1 Slope conservation**

The profile tolerance specification for a simple profile is shown in Figure 5.1, other dimensions and tolerances are not shown here. The triangle shape is controlled by the profile tolerance  $t=0.2$  mm relative to the Datums A, B and C.

The boundaries of the tolerance-zone are created by offsetting the profile in two opposite directions. The result by offsetting a curve can be defined as a curve whose points are at a constant normal distance of a given curve. In Figure 5.2(a), a boundary zone is formed between the outside and inside boundaries, which are shown in solid lines. These two boundaries are obtained by offsetting from profile, which is a triangle in dash lines. In this example, the distances from true profile to outside and inside boundaries are

equal to half tolerance. Thus, the width of tolerance-zone along a curve is equal to tolerance value  $t$  (0.2 mm).

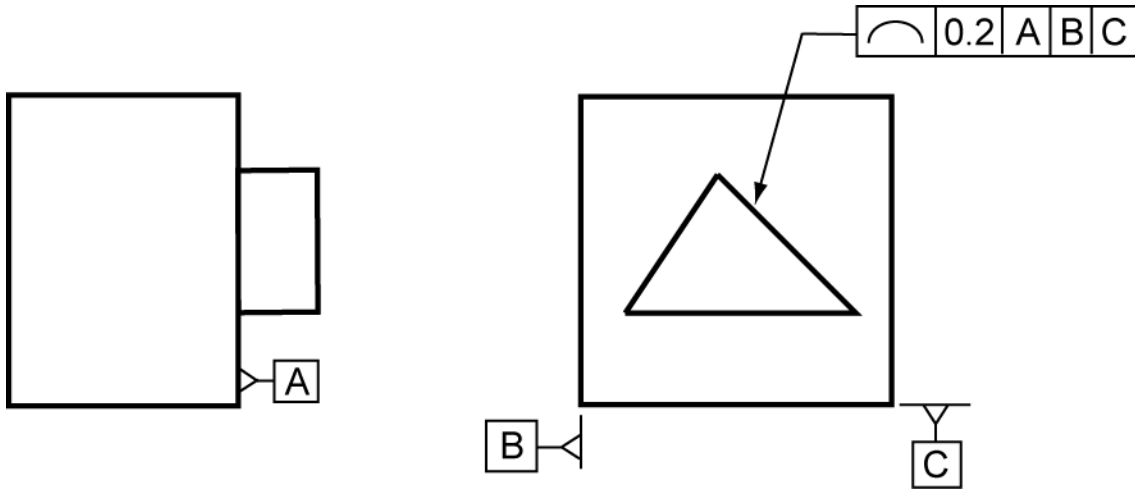


Figure 5.1: Boss with its shape controlled by the profile tolerance  $t = 0.2$  mm relative to Datums A, B, and C.

In Figure 5.2(b), the T-Map coordinates  $e_x$  and  $e_y$  represent the translational displacements of the mid-sized profile in x and y direction in Figure 5.2(a). Every point is a translated mapping of profile position. For example, the true profile  $\psi_{12}$ , which is coincident with true (theoretic) profile, has no translational displacements, and in T-Map it is the point at origin. When the mid-sized profile is moved in the direction (positive y), which is normal to bottom edge of the triangle, to the place where its bottom edge contacts with inside boundary ( $A'$ ), the profile gets its limit position, which is labeled as  $\psi_{A'}$  in Figure 5.2 (a) (b). In the limit position, the profile cannot move any more in positive y-direction, because the bottom edge of inside boundary constrains it within tolerance-zone, but it is still free in x-direction. Because the profile can only move in either of the positive or negative x-direction, the trace of these displacements mapping

points is a straight line, which has the same slope as the bottom edge of the profile, and its normal distance to origin is  $t/2$  (0.1 mm).

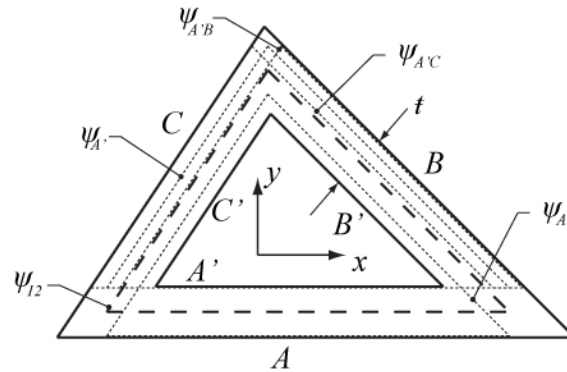


Figure 5.2(a): Exaggerated tolerance-zone (between two solid-lined triangles) for a triangular profile (dashed-lined triangle). Edges of outside boundaries is labeled in  $A, B, C$ , and inside is in  $A', B', C'$ . Five spatial positions are also labeled.

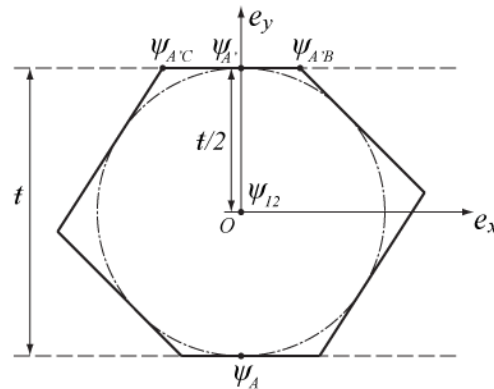


Figure 5.2(b): Corresponding 2-D T-Map that is only confined to translations.

Because the homogeneous coordinate systems of tolerance-zone and T-Map have the same scale, there must be an edge of 2-D T-Map model that has the same slope as one or more lines in tolerance-zone, and this property called slope conservation. Six edges of the hexagonal T-Map model are parallel to one of the triangular profile edges and accord with slope conservation.

A reference circle, called tolerance circle, is introduced to this 2-D T-Map. In figure 5.2(b), all edges of the hexagon are tangent lines of this circle. All points on this circle represent the displaced profile positions that are translated  $t/2$  in any planar directions. When the direction (as  $\psi_A$ ) is normal to the slope of one profile edge, the limit displace distance of the profile is  $t/2$ , that is a point on the tolerance circle. This reference circle will be used in section 5.2.3 for discussion of curves.

### 5.2.2 2-D modeling

The first step to modeling a 2-D T-Map is to decompose the triangular profile in Figure 5.1 into three line-segments, and build individual primitive models for each of them, and then combine these models to get the T-Map for the whole profile.

Take the bottom edge of the profile as an example for building a primitive model, as shown in Figure 5.3. One trace of profile movement is made by translating in the positive y-direction that is discussed in section 5.2.1. If we move the true profile in the negative y-direction, the profile will touch the bottom edge of outside boundary ( $A$ ) in the location ( $\psi_A$ ), and the other corresponding trace in the T-Map is formed. Since the constraints coming from other two triangle edges are independent of this line-segment, there is no limit in the direction parallel to this line.

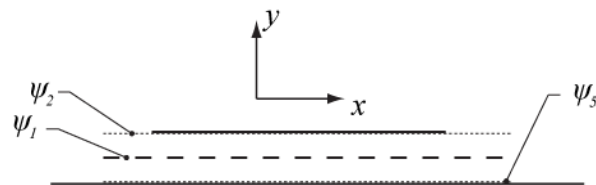


Figure 5.3: One (bottom edge of triangle) of the decomposed line-segment, and its exaggerated tolerance-zone.  $\psi_{12}$ ,  $\psi_A$ ,  $\psi_A$  are labeled.

In the T-Map model shown in Figure 5.4(a), these two trace lines (same as dash lines in Figure 5.2(b)) are both derived from the bottom edge of the triangular profile, and the region between these lines represents the acceptable deviations of the profile, so far only constrained by one side of the triangle. These two lines of T-Map obey the slope conservation property and tangent to the tolerance circle (Figure 5.2(b)).

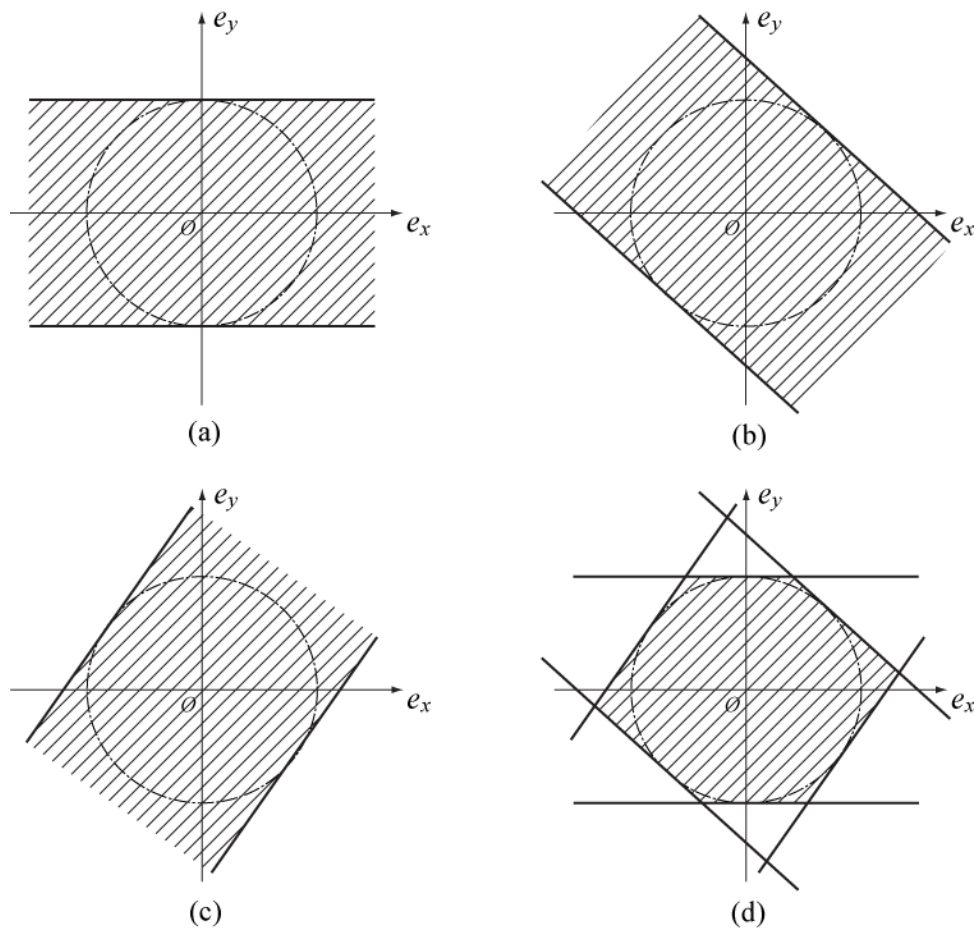


Figure 5.4: (a)(b)(c) are individual T-Maps for each portion of the triangular profile, and (d) is the intersection of these three T-Map. The shaded areas are the acceptable deviation regions of T-Maps.

Similarly, other two primitive models are obtained from the remaining two edges as shown in Figure 5.4 (b) (c). The reference systems of these three primitive models are

same. Axes  $e_x$  and  $e_y$  represent the displacements of profile segments in x and y directions, and the scales of these axes are same. Thus, the intersection of these primitive T-Map models can be realized. The shaded area shown in Figure 5.4(d) is the 2-D T-Map build by decomposed modeling method that is same as the T-Map in Figure 5.2(b).

### 5.2.3 Slope conservation for curves

Profiles are not always combinations of straight lines, and a way to modeling primitive 2-D T-Maps for circular arcs and other curves required. The slopes at points on non-straight curves are also keeping the conservation property.

In Figure 5.5, a rounded corner is added to the profile, and this profile can be decomposed into four pieces: three straight line-segments and a curve. The primitive 2-D T-Maps of three straight lines are discussed and built in Figure 5.4 (a)(b)(c). A primitive T-Map model is required for the circular arc in order to build the T-Map for whole profile using intersection.

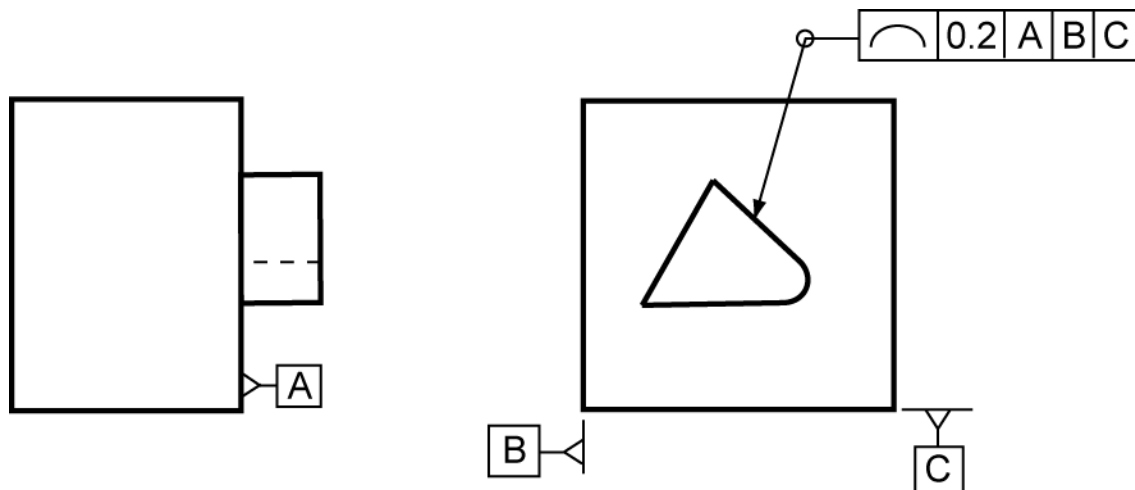


Figure 5.5: Boss with its shape controlled by the profile tolerance  $t = 0.2$  mm relative to Datums A, B, and C.



A polyline is used to approximate the curve. Divide the curve into many small pieces and use a series of line-segments to instead of these small curves. For line-segments, a way is given in section 5.2.2 to build primitive T-Map models for line-segments. The intersection of these the polyline approaches to the true shape of the curve, and primitive T-Map of polyline approaches to the T-Map of curve.

Figure 5.6(a) gives the true profile in dotted line and its approximated tolerance-zone whose boundaries are in solid lines. At the two ends of the curve, the constraints will be given by other elements of the profile, when intersection with the other primitive T-Map models is under tolerance. So in this model there are no boundaries at the ends of the profile. A polyline that is a combination of five line-segments is created to approximate the curve. All line-segments are tangent with the curved profile, and the tangent points of the first and last line-segments of polyline correspond to the two ends of the curve. In Figure 5.6(b), five pairs of parallel lines are drawn, all lines are tangent to the tolerance circle, the region between each pair of parallel lines are the primitive 2-D T-Map models for each line-segment of the polyline, and the intersection area of them is the 2-D T-Map model for polyline.

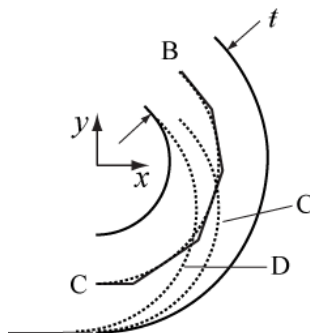


Figure 5.6(a): Exaggerated tolerance-zone for rounded curve and its approximate polyline.

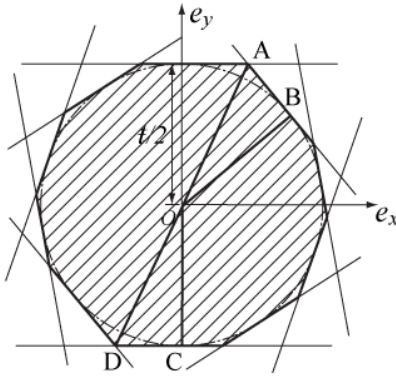


Figure 5.6(b): Primitive model for polyline. (Intersection of models of line-segments, all tangent to the tolerance circle)

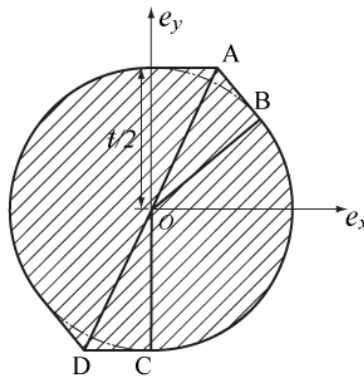


Figure 5.6(c): Primitive model for curve.

In Figure 5.6(b), line AD separates the shell of the T-Map model into two parts. The points on the left part corresponds to translational profile deviations that contact the inside boundary of the tolerance-zone, and the right part correspond to positions that contact the outside boundary. These two parts are symmetric at origin, because from the tolerance specifications the two boundaries are equally disposed about the true profile. Taking right part of shell to study, it can be divided into three portions: line-segments AB and CD, and polyline BC. AB and CD are tangent to the tolerance circle at points B and C. Because the directions of these two line-segments are the same as the tangent directions at two ends of the profile, AB and CD will not change when the number of

line-segments in polyline increases to approach the true curved shape of the profile. For polyline BC, with the increasing number of line-segments, its vertexes will close to the tolerance circle, and when the number goes to infinity, the vertexes will approach to the tolerance circle. This enables using the shell of the T-Map for polyline BC, as shown in Figure 5.6(c), and similarly for the left part of the edge.

In the process to make the T-Map in Figure 5.6(c), only one geometric property is used, that is slope. Any point on the arc-profile corresponds to a point on the tolerance circle by a tangent line through this point. For the slopes of tangent lines of the profile, a group of points is created on the tolerance circle, and these points can be expressed by one or several arcs, such as arc BC in Figure 5.6(c). Because the 2-D T-Map only depends on slopes of points on the profile, the positions of these points can vary. If a free form profile has the same slope range with a circular arc profile, both free form profile and the arc will have the same 2-D T-Map. That means this modeling process is suitable not only for arc but for more generally shapes of curves.

### **5.3 3-D modeling of polygonal profile**

The decomposition method can be used to build a T-Map model in the 3-D space for deviations that can be presented by two translations in two mutually perpendicular directions and a rotation. Use an example in Figure 5.7. The dimensions and tolerance for the triangular profile shape are shown in Figure 5.7.

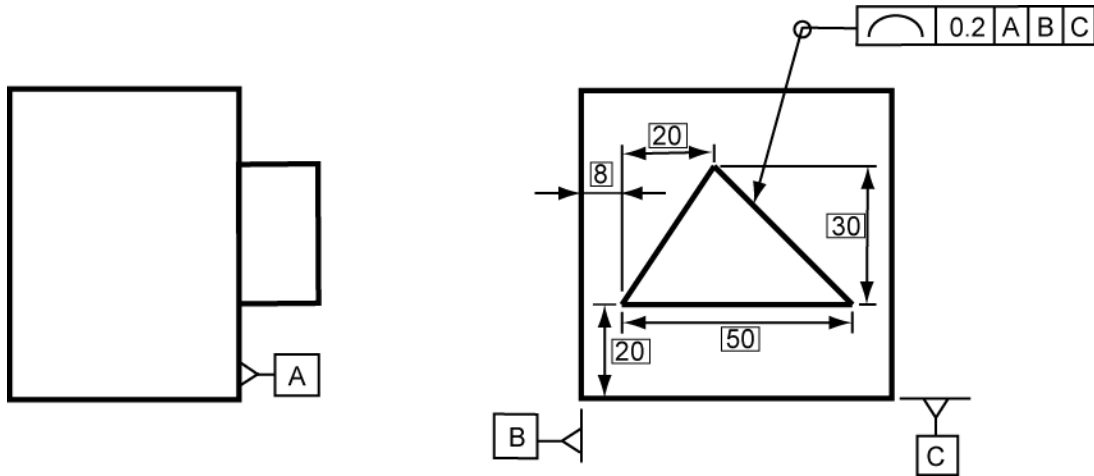


Figure 5.7: Boss with its shape controlled by the profile tolerance  $t = 0.2$  mm relative to Datums A, B, and C.

### 5.3.1 First-order approximation for rotation deviation

Similar to 2-D model, the triangular profile is decomposed into three line-segments. Use bottom edge (line-segment) of triangle as an example. Before working on the line-segment, one approximation is needed for the boundary of the tolerance-zone. In Figure 5.8, the boundaries of the line-segment are shown in solid lines. These two lines are from part of the boundaries for the whole profile, which is shown in Figure 5.2(a). One of the boundaries is shorter than the other. It is because of the influence from other adjacent line-segments. The purpose of this approximation is to eliminate the second-order influence of the length difference, and to make the line-segments independent of each other.

To get the maximum rotation deviation, rotate the line-segment of mid-sized profile to its limit rotate position, and the line-segment contacts to the boundaries at points A and B in Figure 5.8. This is the way to get the accurate limit position. The approximation is to define the position where two ends of the line-segment contact the

extended lines of the boundaries, and line AC is a limit position under this approximation. By this way, the influence from adjacent lines is eliminated.

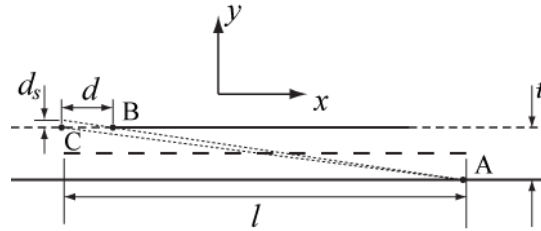


Figure 5.8: Exaggerated tolerance-zone (between two solid-lined) for a line-segment (long dashed). Extended line is in short dashed line and two limit positions of line-segment are in dotted lines.

The maximum rotation angle  $\theta_{AB}$  for line AB and  $\theta_{AC}$  for line AC are:

$$\theta_{AB} = \tan^{-1} \frac{d_s + t}{l}$$

$$\theta_{AC} = \tan^{-1} \frac{t}{l}$$

where,  $d_s$  is the distance between end of the line-segment and extended boundary,  $t$  is the tolerance value for the profile, and  $l$  is the length of the line-segment. Because the length  $l$  is much larger than tolerance  $t$ ,  $\theta = \tan \theta = \sin \theta$ . Therefore,

$$\theta_{AB} \approx \frac{d_s + t}{l}$$

$$\theta_{AC} \approx \frac{t}{l}$$

From the Figure 5.8, a relation can be found:

$$\frac{d_s}{d} = \frac{t}{l}$$

where  $d$  is the distance between points B and C.

So the angular deviation between lines AB and AC is:

$$\theta_{AB} - \theta_{AC} = \frac{d_s}{l} = \frac{dt}{l^2}$$

Because  $d$  and  $t$  are both much smaller than  $l$ , the angular deviation is a second order small value that can be ignored in T-Map building process.

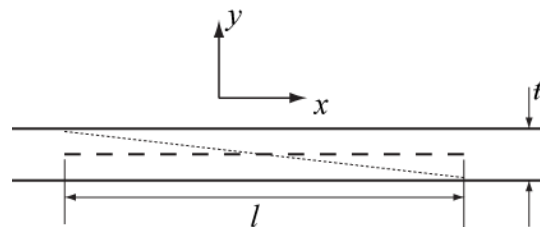


Figure 5.9: Exaggerated tolerance-zone (between two solid-lined) for a line-segment (long dashed). Line-segment (dotted line) in limit rotation position for maximum rotation deviation.

In figure 5.9, it shows the boundaries of line-segment with approximation, and the tolerance-zone is the region between the two boundaries. Boundaries and the tolerance-zone are only dependent on picked line-segment.

### 5.3.2 3-D primitive model for straight line-segment

To measure the deviation, a work coordinate system (WCS) is needed to avoid ambiguity. This reference system is bonded with the tolerance-zone. Set the origin of the coordinate at the midpoint of line-segment, point  $O$  in Figure 5.10,  $x_1$ -axis is parallel to the line-segment. Because the tolerance-zone is symmetric in  $x_1$  and  $y_1$ , the axes direction will not influence result of T-Map. All line-segments that are decomposed from line profile have the same process to set work coordinate system, so that this same process is easy to realize by a computer program.

In figure 5.10,  $\psi_{ar}$  is an arbitrary deviation. The location of this deviation can be expressed by a combination of a rotation (angle  $\theta$ ) and a translation in the  $y$  direction (distance  $y$ ) from the mid-sized position ( $\psi_{12}$ ). To get the arbitrary deviation position, the first step is to rotate the profile from  $\psi_{12}$  around origin of its WCS by  $\theta$  to  $\psi_a$ , and then translate  $\psi_a$  in  $y$ -direction to the position of arbitrary deviation by  $e_y$ .

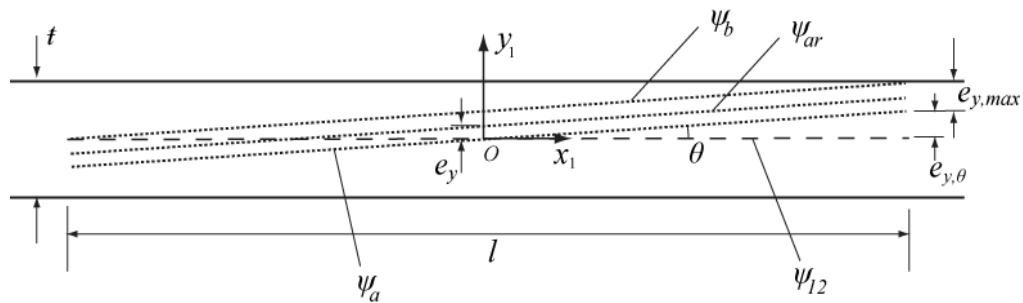


Figure 5.10: Exaggerated tolerance-zone (between the two solid-lines) for a line-segment (dashed).  $\psi_{12}$ ,  $\psi_{ar}$ ,  $\psi_a$  and  $\psi_b$  are the positions (dotted line) for the line-segment that floats inside the tolerance-zone.

When the profile is moved to limit positions, such as  $\psi_b$ , where the line-segment contacts the boundaries of tolerance-zone, the mapped points of these positions will form the surfaces of the shell of T-Map models. An equation is found in Figure 5.10:

$$e_{y,\theta} + e_{y,max} = \frac{\epsilon}{2}$$

In this equation, half tolerance  $\frac{\epsilon}{2}$  is composed by  $e_{y,\theta}$ , the distance caused by rotation, and  $e_{y,max}$ , the maximum allowable translation distance in y-direction of the rotated line-segment. Because the rotation angle  $\theta$  is a small value, so that:

$$e_{y,\theta} = \frac{l}{2} * \theta$$

Substitute  $e_{y,\theta}$ , and get the equation for making surfaces of T-Map model:

$$\frac{l}{2} * \theta + e_{y,max} = \frac{\epsilon}{2}$$

Another three equations can be created for different combinations of positive and negative rotations and translations. Four edges of the T-Map model are made from these equations, shown in Figure 5.11(a), and their envelop area is the 2-D T-Map in the dimensions  $e_y$  and  $\theta$ . For this line-segment, there is no constraint in x-direction, which is discussed and shown in Figure 5.4(a). Extrude the 2-D model in x-direction and get the 3-D primitive model. Its section in  $e_x e_y$  plane is accord with the T-Map shown in Figure 5.4(a).



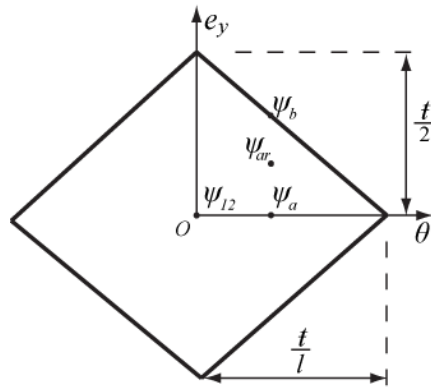


Figure 5.11(a): 2-D T-Map (model is rescaled in direction  $\theta$ ).  $\psi_{12}$ ,  $\psi_{ar}$ ,  $\psi_a$  and  $\psi_b$  are points corresponding to specific positions in Figure 5.10.

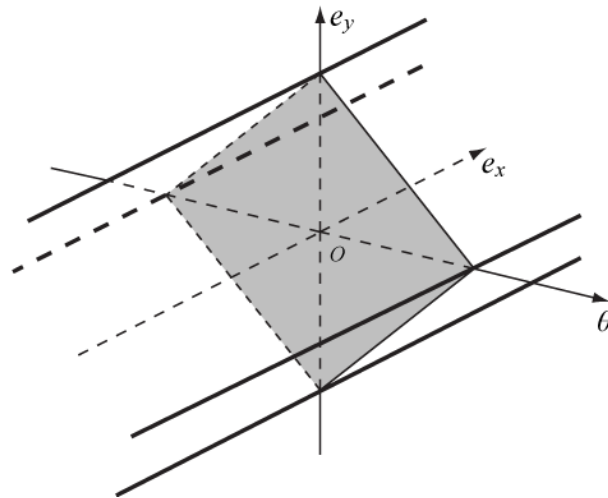


Figure 5.11(b): A 3-D primitive model is got by extruding the 2-D model (shaded face) in  $x$ -direction.

For the triangular profile shown in Figure 5.7, there are other two line-segments, and their primitive models have the same geometric shape but are different in scale and orientation. The cross-section of these primitive models are also, similar to Figure 5.11(a), but they have different aspect scales and different orientation, as given in Figure 5.4(a)(b)(c).

### 5.3.3 Uniformity of reference system

When primitive models are created for individual decomposed line-segments of the line profile in Figure 5.7, the next step is to intersect these primitive models to get the 3-D T-Map for the whole profile, as we do in 2-D modeling. But in the definition of WCS, the mid-point of the line-segment is the origin of WCS, and in the process to achieve deviation position, we rotate the line-segment around its WCS origin. Since each line-segment has its own origin and orientations of coordinate axes, the primitive models cannot be intersected directly to provide the T-Map for the entire profile. For example in Figure 5.12(a), the dotted profile is a possible deviation whose position can be expressed by a pure rotation from the mid-sized profile by angle  $\theta$ , and its mapped point is  $(0, 0, \theta)$ . But for decomposed profiles, this mapped point represents positions by rotating around their own WCS origins, as shown in Figure 5.12(b). These dotted lines in Figure 5.12(b) have the same orientation and length as those in Figure 5.12(a). Rotating profiles around different points is the reason for this problem, and an additional translation is required to accompany the rotation of each line-segment. To offset the translations, the translation deviation of the line-segment  $e_x e_y$  will be changed, that means the shape of primitive T-Map models need to be modified, and this is called the deformation of T-Map. Before compose primitive T-Map models, models have to be deformed by a mount which is defined from the relation between WCS's and the point the whole profile rotates around. Section 5.4 gives a method to compute the deformation of T-Map model.

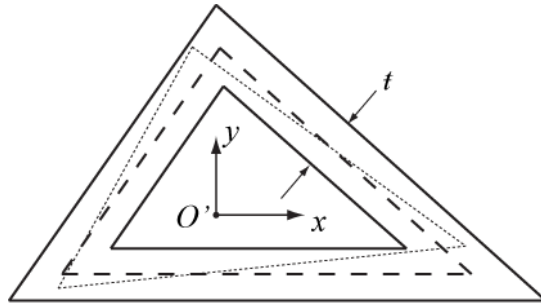


Figure 5.12(a): A deviation position (dotted lines) of the mid-sized profile in the exaggerated tolerance-zone.

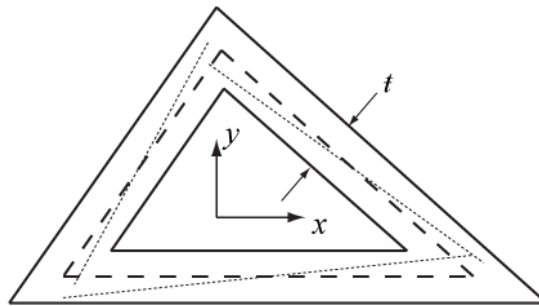


Figure 5.12(b): Deformation result (dotted lines) by rotating around individual WCS origins. An additional translation is required for each line-segment (dotted line) to help the entire profile to keep its shape as in Figure 5.12(a).

## 5.4 Deformation of T-Map

Definition of the common reference system determines the shape and location T-Map model in the point-space that maps the deviation of line profile. This section contains a description of the deformation of T-Map on different reference systems by applying transform matrixes.

### 5.4.1 Space of mapping points

By the definition of T-Map, the point-space for a line profile is a four dimensional space. Any point in this space maps a deviation of the profile from its ideal size and location. This mapping is a one to one map, and a deviation of the profile has a unique

mapping point in the space of mapped points. The model of the T-Map divides the space into two regions. The region inside of the model contains all points mapping the deviations that are acceptable under the tolerance, and the points outside of the model map those unacceptable deviations.

Four dimensions of the space are: two translations in two mutually perpendicular directions, rotation and size of the profile. To measure the deviations and map them, a common reference system is needed. This reference system is fixed with the ideal profile and tolerance-zone. The orientation of the reference system determines orientation of the T-Map model, and the origin of the reference system influence the shape of the T-Map model.

#### **5.4.2 Orientation and origin location of the reference system**

The coordinates of mapped points in the T-Map depend on the orientation of reference system. In Figure 5.2(a) the axis  $x$  is parallel to one edge of the triangle, and 2-D T-Map for the edge is shown in Figure 5.2(b). When we choose another direction  $x'$  for the axis, which has an angle  $\alpha$  to axis  $x$ , as shown in Figure 5.13(a), a different 2-D T-Map is made in Figure 5.13(b). Comparing to the 2-D model in Figure 5.2(b), the model in the new reference system is rotated by angle  $-\alpha$  from the model in Figure 5.2(b). In other words, the same point in different reference system may have different coordinate values.

The origin of reference system does not influence the whole mapping space for 2-D the T-Map model in  $e_x e_y$  plane, but it influences the 3-D mapping space in the  $\theta$  direction. The 3-D mapping space is a point space that contains all points whose coordinates can be expressed by three values of deviation  $(e_x, e_y, \theta)$ . When mapping an

arbitrary deviation of a profile, such as  $\psi_{ar}$  in Figure 5.14, two steps to determine the values of  $e_x, e_y, \theta$ . The rotational deviation value  $\theta$  can be measured between the deviated profile ( $\psi_{ar}$ ) and the mid-sized at its theoretical position ( $\psi_{12}$ ). Then rotate the profile by angle  $\theta_{ar}$  around the origin of the coordinate system, such as rotating around  $O$  to position  $\psi_a$ , and measure the displacement between rotated profile and the deviation ( $\psi_{ar}$ ) we want to measure.

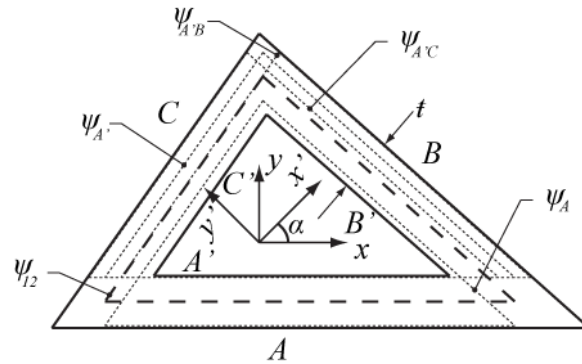


Figure 5.13(a): Choose another axes  $x'$  and  $y'$  for the reference system.

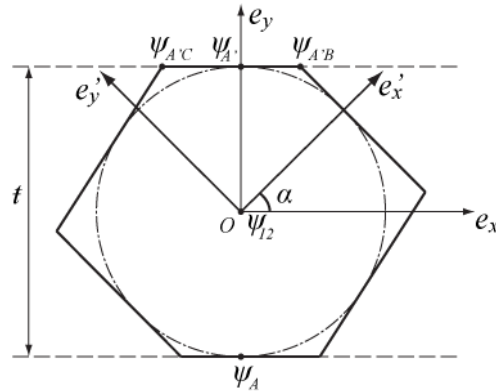


Figure 5.13(b): Corresponding 2-D T-Map for Figure 5.13(a)

Figure 5.14 gives two different reference systems which are fixed to the plane of mid-sized profile ( $\psi_{12}$ ). These two reference systems have the same orientation but two

different origins,  $O$  and  $O'$ . Using the steps talked above to measure these three values  $(e_x, e_y, \theta)$  of profile deviation, it will get two different result. In Figure 5.14, profile  $\psi_a$  and  $\psi_b$  have the same rotational deviation value  $\theta_{ar}$ , by rotating the profile  $\psi_a$  around point  $O$ , and  $\psi_b$  around point  $O'$ . Also  $\psi_{ar}$  will be expressed in two different references,  $(e_{xar}, e_{yar}, \theta_{ar})$  and  $(e_{xar}', e_{yar}', \theta_{ar})$ , due to it is from translations in the two different vectors  $\vec{e}_{ar}$  and  $\vec{e}'_{ar}$ .

However, by definition T-Map is a one to one mapping, and a point in the mapping space should have one unique corresponding deviation. Since the mapping spaces are different for using different reference systems, the 3-D T-Map models that contain all the points mapping from the acceptable deviations are vary for different reference systems. These T-Map models may have different shapes, but all of them envelop the group of acceptable points from their own mapping space. That means any of these models has all the functions of T-Map. In other words, the mapping is still keeping one to one property, but this one to one mapping result can be seen in different shape in different reference systems. When do the T-Maps operation, such as intersection of primitive models, or Minkowski sum of two T-Map models, these models must be in the same mapping space, in other words, they are using the same reference system.

### 5.4.3 Shear deformation

The description of the mapping space is sensitive to the orientation and origin location of the reference system. In this section, how to express the shape of T-Map model for different origin of reference systems will be discussed by comparing the two different ways to measure the deviations from  $\psi_a$  to  $\psi_{ar}$ . Also, a method is given to deform the T-Map models to be valid in different reference systems.

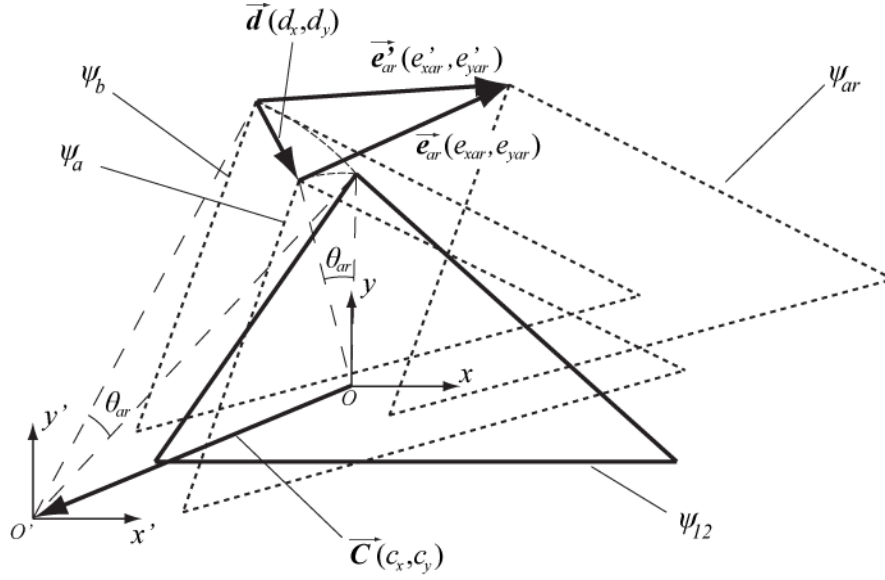


Figure 5.14: Mid-sized profile ( $\psi_{12}$ , triangle in solid line) with two different reference systems.  $\psi_{ar}$ ,  $\psi_a$  and  $\psi_b$  are the translated position from mid-sized profile.

In Figure 5.14, to the reference system  $Oxy$ , the arbitrary deviation ( $\psi_{ar}$ ) can be mapped to a point  $P_{ar}(e_{xar}, e_{yar}, \theta_{ar})$  in the T-Map model. The coordinates of point M can be gotten by rotating profile  $\psi_{12}$  around point  $O$  by  $\theta_{ar}$  and translating by the vector  $(e_{xar}, e_{yar})$  from  $\psi_a$  to  $\psi_{ar}$ . And the deviation of  $\psi_{ar}$  in the reference system  $O'x'y'$  is mapped to a point  $P_{ar}'(e_{xar}', e_{yar}', \theta_{ar})$ , by rotating profile  $\psi_{12}$  around point  $O'$  and translating by the vector  $(e_{xar}', e_{yar}')$  from  $\psi_b$  to  $\psi_{ar}$ . Vector  $\vec{d}_{ab}(d_{xab}, d_{yab})$  gives the displacement from any points in  $\psi_a$  position to  $\psi_b$  in reference system  $Oxy$ . This displacement is between the two different mapping results of  $\psi_{ar}$  in the two reference systems. The vectors have the relation  $\vec{e}'_{ar} = \vec{e}_{ar} + \vec{d}_{ab}$ , which, in the form of coordinates is:

$$\begin{cases} e'_{xar} = e_{xar} + d_{xab} \\ e'_{yar} = e_{yar} + d_{yab} \end{cases} \quad (5.1)$$

Equation 5.1 gives a relationship between the coordinates of an arbitrary point in T-Maps for the profile at two different reference systems. If the values  $d_x$  and  $d_y$  are known, the coordinates of all points in the T-Map for the profiles which are expressed with coordinates at reference system  $Oxy$  can be translated into the point-space for reference system  $O'x'y'$ , by using equation 5.1.

To compute the  $d_x$  and  $d_y$ , the relation between two reference system needs to be defined. In reference system  $Oxy$  create a vector  $\vec{C}$  from  $O$  to  $O'$  to present the relation between the origins of the two reference systems. And the value of vector  $\vec{C}$  or the coordinate of point  $O$  in reference system  $Oxy$  is  $(c_x, c_y)$ . The values  $c_x$  and  $c_y$  are measured between the coordinate origins. For different reference systems that are located at the profiles or designed parts, the displacement  $(c_x$  and  $c_y)$  between them may be vary, but they are fixed with the profiles or parts,  $c_x$  and  $c_y$  are constants in the T-Map deformation process.

To compute the values  $d_x$  and  $d_y$ , transformation matrices are used to express the movements of the profile. Assume there are two laminas in Figure 5.15. One fixed lamina contains the mid-sized profile (solid line) and reference systems  $Oxy$  and  $O'x'y'$ , and the other one, which contains a profile in mid-sized position and floats upon the fixed lamina, will be translated by matrices. For example, any point  $(x_1, y_1)$  on the triangle or on the plane of the floating lamina (such as in the position  $\psi_1$ ) can be transformed to a point  $(x_3, y_3)$  on another lamina in the position (such as  $\psi_3$ ) by multiply a matrix  $T$ :

$$\begin{bmatrix} x_3 \\ y_3 \\ 1 \end{bmatrix} = [T_{3,1}] \begin{bmatrix} x_1 \\ y_1 \\ 1 \end{bmatrix}$$



In reference system  $O'x'y'$ , use two different ways to transform points on the lamina from  $\psi_{12}$  to  $\psi_a$ , and one of them is:

$$\mathbf{T}_{a,12} = \text{Trans}(d_x, d_y) \cdot \text{Rot}(\theta_{ar}) \quad (5.2)$$

where,  $\mathbf{T}_{a,12}$  is the transform matrix from  $\psi_{12}$  to  $\psi_a$ ,  $\text{Trans}(d_x, d_y)$  is the translation by vector  $\vec{d}(d_x, d_y)$ , which are the values we want to compute, and  $\text{Rot}(\theta_{ar})$  is the rotation around point  $O'$  by angle  $\theta_{ar}$ . This way is the process to measure the deviation of profile in reference system  $O'x'y'$ , that is rotating the lamina around point  $O'$  to  $\psi_b$  and then translating from  $\psi_b$  to  $\psi_a$ . Another way to transform the profile from  $\psi_{12}$  to  $\psi_a$  is needed to simulate the pure rotation around point  $O$  in reference system  $O'x'y'$ . The rotation of the profile on the lamina in  $\psi_{12}$  around the point  $O$  to  $\psi_a$ , may be expressed as the product of three transportation matrices, i.e.

$$\mathbf{T}'_{a,12} = \text{Trans}(-c_x, -c_y) \cdot \text{Rot}(\theta_{ar}) \cdot \text{Trans}(c_x, c_y) \quad (5.3)$$

in which the elements of each transform are expressed in reference system  $O'x'y'$ .

Processing leftward through the transforms in equation (5.3), the profile translates by the vector  $\vec{C}(c_x, c_y)$  to profile  $\psi'_{12}$ , rotate around point  $O'$  by  $\theta$  to profile  $\psi'_a$ , and translate back by the vector  $-\vec{C}(-c_x, -c_y)$  to profile  $\psi_a$ .

Because  $\mathbf{T}_{a,12}$  and  $\mathbf{T}'_{a,12}$  are both the transform matrices from  $\psi_{12}$  to  $\psi_a$ , these two matrices are same, and the right sides of equation (5.2) and (5.3) are equal to each other:

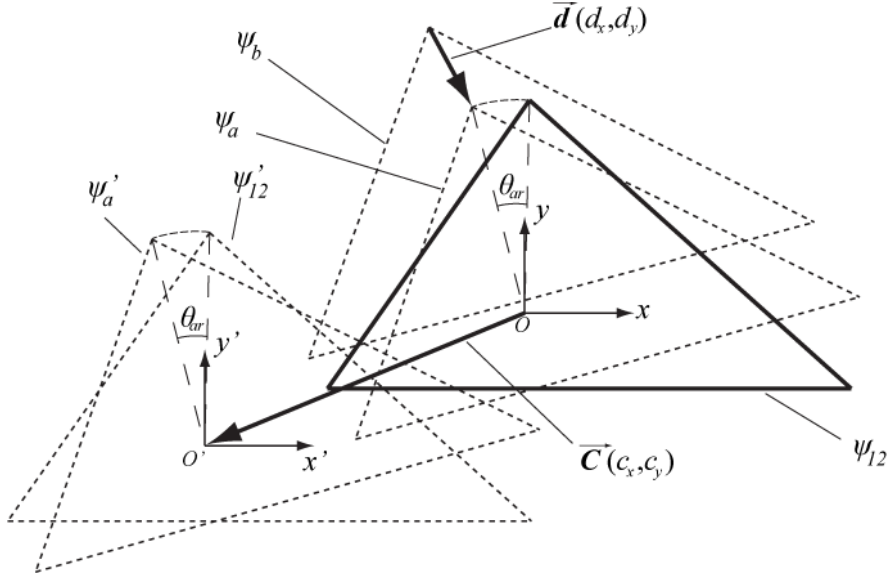


Figure 5.15: Mid-sized profile ( $\psi_{12}$ , triangle in solid line) with two different reference systems.  $\psi_{12}$ ,  $\psi_a$ ,  $\psi_b$ ,  $\psi'_{12}$  and  $\psi'_a$  are the translated position from mid-sized profile.

$$\begin{aligned}
 & Trans(d_x, d_y) \cdot Rot(\theta_{ar}) \\
 & = Trans(-c_x, -c_y) \cdot Rot(\theta_{ar}) \cdot Trans(c_x, c_y)
 \end{aligned} \tag{5.4}$$

Rewrite (5.4) in matrix form:

$$\begin{aligned}
 & \begin{bmatrix} 1 & 0 & d_x \\ 0 & 1 & d_y \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \cos \theta_{ar} & -\sin \theta_{ar} & 0 \\ \sin \theta_{ar} & \cos \theta_{ar} & 0 \\ 0 & 0 & 1 \end{bmatrix} \\
 & = \begin{bmatrix} 1 & 0 & -c_x \\ 0 & 1 & -c_y \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \cos \theta_{ar} & -\sin \theta_{ar} & 0 \\ \sin \theta_{ar} & \cos \theta_{ar} & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & c_x \\ 0 & 1 & c_y \\ 0 & 0 & 1 \end{bmatrix}
 \end{aligned} \tag{5.5}$$

And simplify (5.5):

$$\begin{aligned}
& \begin{bmatrix} \cos \theta_{ar} & -\sin \theta_{ar} & d_x \\ \sin \theta_{ar} & \cos \theta_{ar} & d_y \\ 0 & 0 & 1 \end{bmatrix} \\
& = \begin{bmatrix} \cos \theta_{ar} & -\sin \theta_{ar} & c_x \cos \theta_{ar} - c_y \sin \theta_{ar} - c_x \\ \sin \theta_{ar} & \cos \theta_{ar} & c_x \sin \theta_{ar} + c_y \cos \theta_{ar} - c_y \\ 0 & 0 & 1 \end{bmatrix} \tag{5.6}
\end{aligned}$$

Compare elements in (5.6), two equations can be got:

$$\begin{cases} d_x = c_x \cos \theta_{ar} - c_y \sin \theta_{ar} - c_x \\ d_y = c_x \sin \theta_{ar} + c_y \cos \theta_{ar} - c_y \end{cases} \tag{5.7}$$

In equation set (5.7), because  $c_x$  and  $c_y$  are measure directly between  $O$  and  $O'$ ,  $d_x$  and  $d_y$  are only depend on the rotational angle  $\theta_{ar}$ . For all T-Maps the angle  $\theta_{ar}$  is a very small value, some simplifications can be made:  $\sin \theta_{ar} \approx \theta_{ar}$ , and  $1 - \cos \theta_{ar} \approx 0$ .

So (5.7) is simplified to:

$$\begin{cases} d_x = -c_y \theta_{ar} \\ d_y = c_x \theta_{ar} \end{cases} \tag{5.8}$$

Now we get the vector  $\vec{d}(d_x, d_y)$  which only depends on the relative displacement between origins of two reference systems  $c_x, c_y$ , and the arbitrary angle  $\theta_{ar}$ .

Displacements  $c_x$  and  $c_y$  are fixed when we are defining the reference systems. Because  $\theta_{ar}$  is a measurement to an arbitrary position of deviation, it can represent the rotational

angle of any possible deviations of the profile. In equation set (5.8),  $d_x$  and  $d_y$  have a linear relation with  $\theta_{ar}$ . And then substitute (5.8) into (5.1):

$$\begin{cases} e'_{xar} = e_{xar} - c_y \theta_{ar} \\ e'_{yar} = e_{yar} + c_x \theta_{ar} \end{cases} \quad (5.9)$$

where five variables  $e_{xar}$ ,  $e_{yar}$ ,  $e'_{xar}$ ,  $e'_{yar}$ ,  $\theta_{ar}$  can be rewritten into two coordinates ( $e_{xar}$ ,  $e_{yar}$ ,  $\theta_{ar}$ ) and ( $e'_{xar}$ ,  $e'_{yar}$ ,  $\theta_{ar}$ ), and  $c_x$ ,  $c_y$  are the displacements between the origins of reference system. In the T-Map these two coordinates represent a point that maps the deviation  $\psi_{ar}$  in two different reference systems  $Oxy$  are  $O'x'y'$ . Rewrite (5.9) in a matrix form:

$$\begin{bmatrix} e'_{xar} \\ e'_{yar} \\ \theta \end{bmatrix} = \begin{bmatrix} 1 & 0 & -c_y \\ 0 & 1 & c_x \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} e_{xar} \\ e_{yar} \\ \theta_{ar} \end{bmatrix} \quad (5.10)$$

The vector on left side contains the coordinate values of the mapping point for the deviation of profile at  $\psi_{ar}$  in the reference system  $O'x'y'$ , and the vector on right side contains the coordinate values for the same deviation at  $\psi_{ar}$  in the reference system  $Oxy$ . The matrix only contains constants, and it translates the coordinate of mapping point from  $Oxy$  to  $O'x'y'$ . Because the coordinate ( $e_{xar}$ ,  $e_{yar}$ ,  $\theta_{ar}$ ) of profile deviation  $\psi_{ar}$  can be any value, the matrix can be applied on the whole mapping point-space:

$$\begin{bmatrix} e'_x \\ e'_y \\ \theta \end{bmatrix} = \begin{bmatrix} 1 & 0 & -c_y \\ 0 & 1 & c_x \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} e_x \\ e_y \\ \theta \end{bmatrix} \quad (5.11)$$

And from the format of this matrix, its transformation type is shearing, that means when we change the origin of a reference system to measure a profile, such as from  $Oxy$  to  $O'x'y'$ , the mapping point-space will be sheared. Here name this deformation matrix shear matrix.

### 5.5 Case study for polygonal profile

This section provides the process to build a 3-D T-Map for the MSP of the example in Figure 5.7. The first step is to decompose this triangular profile into three line-segments, each of these segments has a WCS (Section 5.2.3), shown in Figure 5.16.

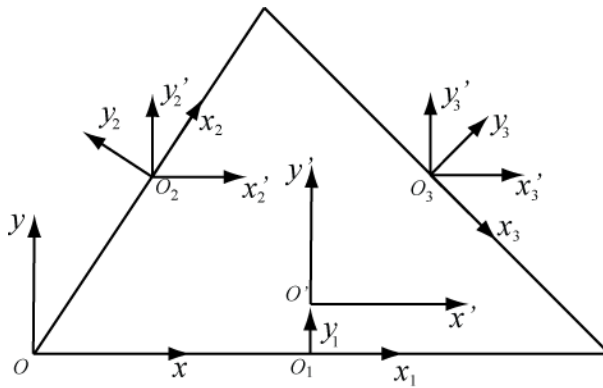


Figure 5.16: Here the triangle is the mid-sized profile. Three WCS's were created for each line-segment. A temporary reference system for the whole profile is set on a vertex of the triangle.  $O'x'y'$  is the reference system whose origin locates at invariant point.

In section 5.3.2, a primitive model is created for the line-segment which is parallel to x-direction. So we can get the primitive model for the bottom edge of the triangle from Figure 5.11. In this example, the length of bottom line  $l_{\text{bot}} = 50$ , tolerance  $\epsilon = 0.2$ , the cross-section 2-D T-Map is shown in Figure 5.17(a), and its coordinate is

rescaled in direction  $\theta$  by multiplying half-length of line-segment 25. Similar for the remaining line-segments, but they have different length,  $l_{left} = 36.06$  and  $l_{right} = 42.43$ . For these reason, they intersect with  $\theta$ -axis at different points in Figure 5.17(a). Extrude the section in x-direction, and get primitive T-Map models. In Figure 5.17(b), only show the primitive model of bottom edge.

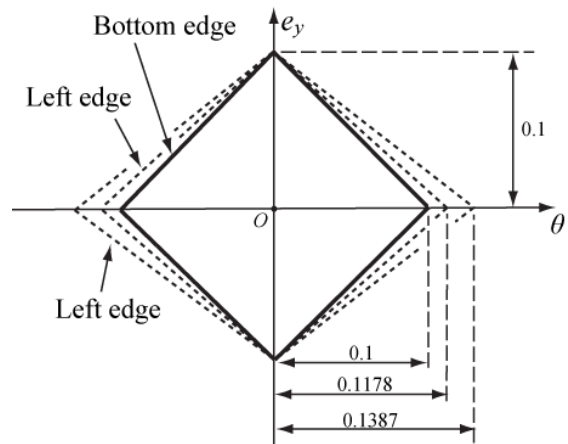


Figure 5.17(a): 2-D T-Map sections for three line-segments ( $\theta' = 25\theta$ ) at their WCS  $O_1x_1y_1$ ,  $O_2x_2y_2$  and  $O_3x_3y_3$ . Use  $\theta'$  is to make sure the height of 3-D model for mid-sized profile in  $\theta$  axis is equal to half tolerance.

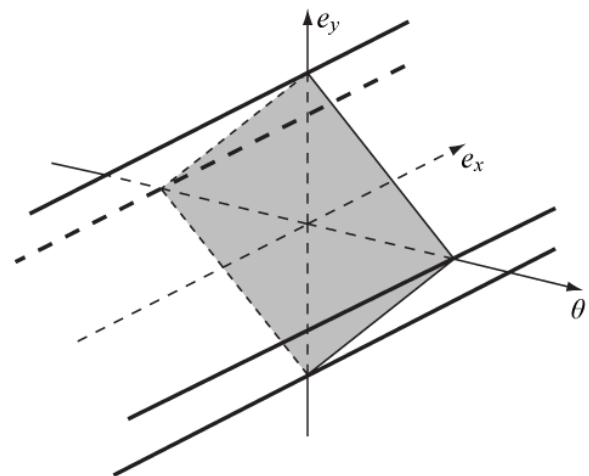


Figure 5.17(b): 3-D primitive model is got by extruding the left 2-D sections (shaded face) in x-direction. Here only show the shape of model for bottom edge, and the other two edges have the same shape.

Next step is using one reference system to present T-Map models for these line-segments. A temporary reference system is set on a vertex of the mid-sized profile. Its origin may not be at the invariant point of the profile, and in the following step the invariant point will be found. Using the reference systems  $O_2 'x_2 'y_2 '$  and  $O_3 'x_3 'y_3 '$  for left edge and right edge of the profile, the primitive model is rotated around the  $\theta'$ -axis by an angle that is equal to the angle between edge and x-axis of temporary reference system. The rotated primitive models for the other two segments are given in Figure 5.18. The cross-sections of these primitive models in Figure 5.18 are same as the 2-D T-Map in Figure 5.17(a).

After rotation, the shear deformation will be applied to the rotated model. In Table 5.1, the displacements between the origins of its WCS's and the temporary reference system are listed, and shear matrix is get from equation 5.11. Use these matrices to deform rotated primitive models in Figure 5.17(b), 5.18(a) and 5.18(b), and get the final primitive models which are ready to intersect. These models are shown in Figure 5.19.

Table 5.1 Displacements between the origins and Shear matrix

Edge	Displacement between the origins	Shear matrices
Bottom	(-25,0)	$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & -25 \\ 0 & 0 & 1 \end{bmatrix}$
Left	(-10,-15)	$\begin{bmatrix} 1 & 0 & 15 \\ 0 & 1 & -10 \\ 0 & 0 & 1 \end{bmatrix}$
Right	(-35,-15)	$\begin{bmatrix} 1 & 0 & 15 \\ 0 & 1 & -35 \\ 0 & 0 & 1 \end{bmatrix}$

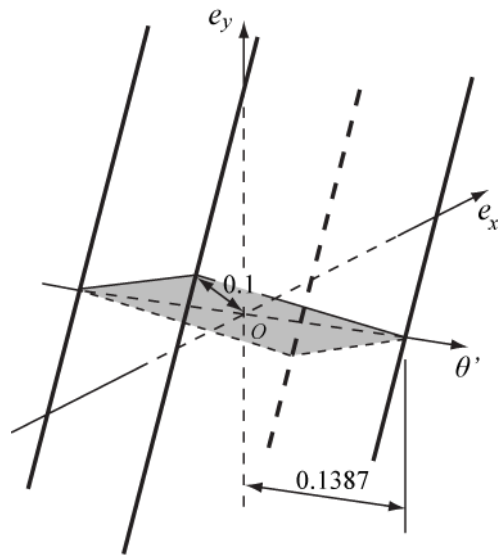


Figure 5.18(a): 3-D primitive model for left edge of triangle.

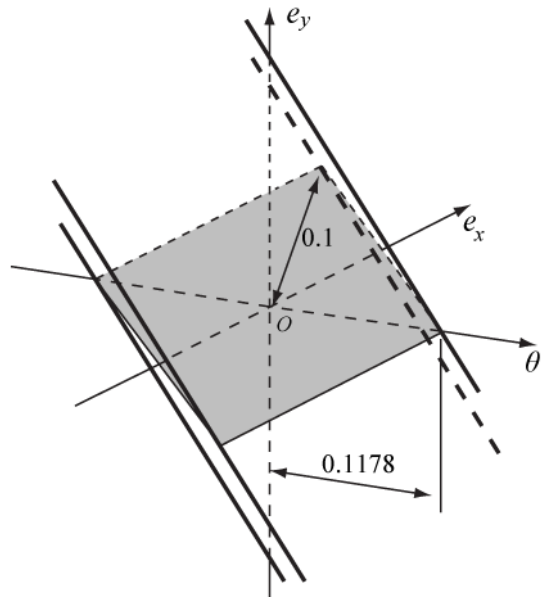


Figure 5.18(b): 3-D primitive model for right edge of triangle.



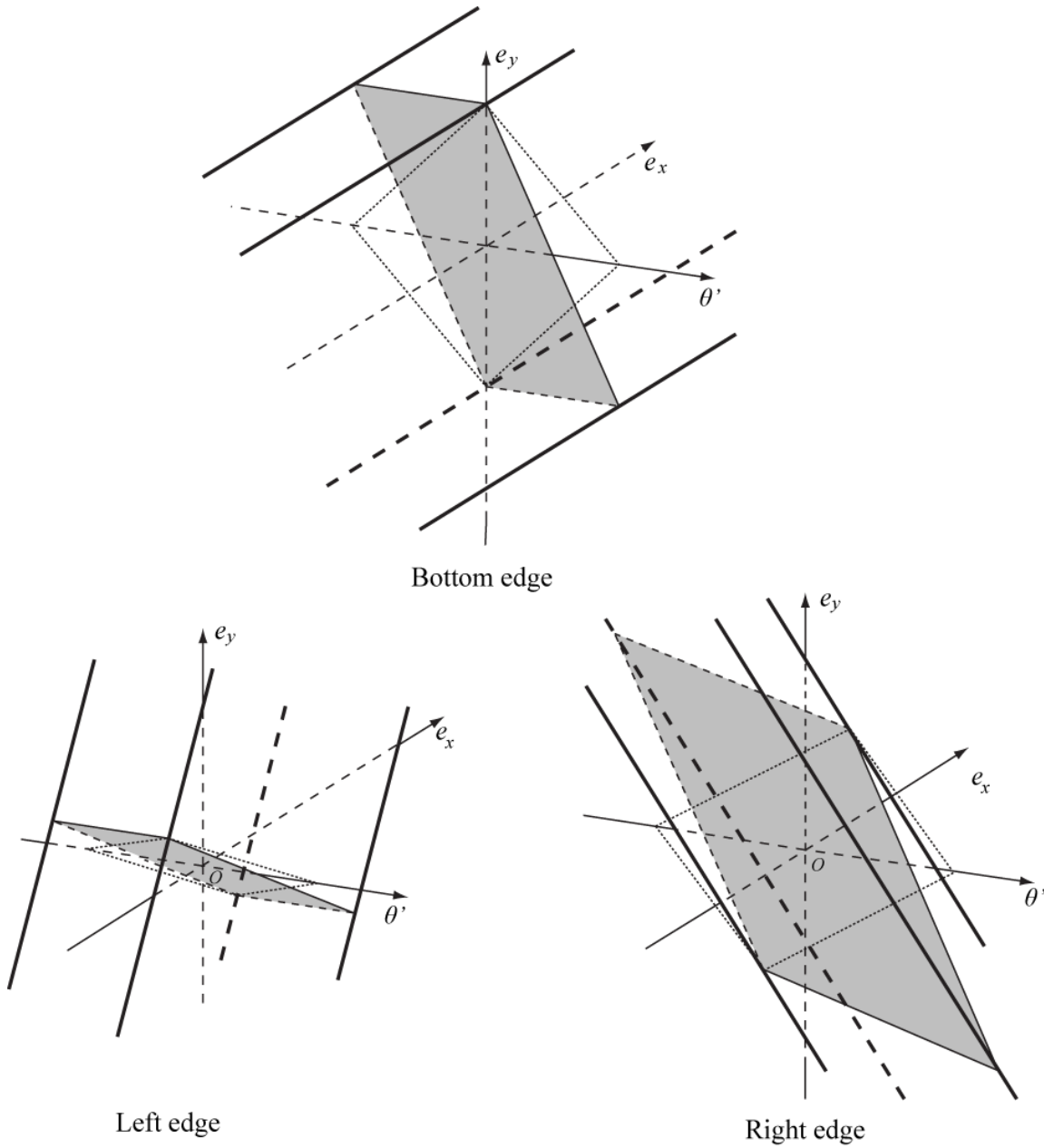


Figure 5.19: 3-D primitive models for three edges of mid-sized triangle. Dotted lines are the unsheared 2-D sections that are same as those in Figure 5.17(b), 5.18(a) and 5.18(b).

Intersect the primitive models for the decomposed line segments in Figure 5.19, and a T-Map model is created and shown in Figure 5.20. The T-Map model in 5.20 is at the temporary reference system  $Oxy$  whose origin is not located on the invariant point of

the triangle profile. Assume there is a reference system  $O'x'y'$  whose origin located at the invariant point, the shear matrix can relate the points of the T-Map as:

$$\begin{bmatrix} e'_x \\ e'_y \\ \theta' \end{bmatrix} = \begin{bmatrix} 1 & 0 & -c_y \\ 0 & 1 & c_x \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} e_x \\ e_y \\ \theta \end{bmatrix} \quad (5.12)$$

If we can calculate the values  $c_x$  and  $c_y$ , we will know the location of the origin of the reference system  $O'x'y'$  at the temporary reference system.

In the T-Map the coordinates of mapped points for a special location (fully rotated location) can be found to help calculate the values  $c_x$  and  $c_y$ . By the definition of the invariant point in Chapter 4, fully rotate the lamina around a unique point that does not displace, and this point is the invariant point. In the reference system  $O'x'y'$  the fully rotated location means in the T-Map the rotation angle  $\theta$  at its maximum value  $\theta_{max}$ . Another property of invariant point is the profile to fully rotated location without displacement in x or y direction, in other words, the values of  $e'_x$  and  $e'_y$  should be equal to zero. The special point at reference system  $O'x'y'$  is  $(0, 0, \theta_{max})$ . In the T-Map of Figure 5.20, the points have the maximum  $\theta'$  are on two lines, both for CCW and CW. In this situation, we usually choose the mid-point of the line as the point which is mapping fully rotated location. And in this example the points for special locations are  $(0.02, -0.1, 0.004)$  and  $(-0.02, 0.1, -0.004)$  for CCW and CW. Because the shear deformation is on the plane  $Oxy$ , the value of maximum rotation angle is not changed by shear deformation, so that  $\theta_{max} = 0.1$  or  $-0.1$ . Substitute the coordinates of the points at special locations:

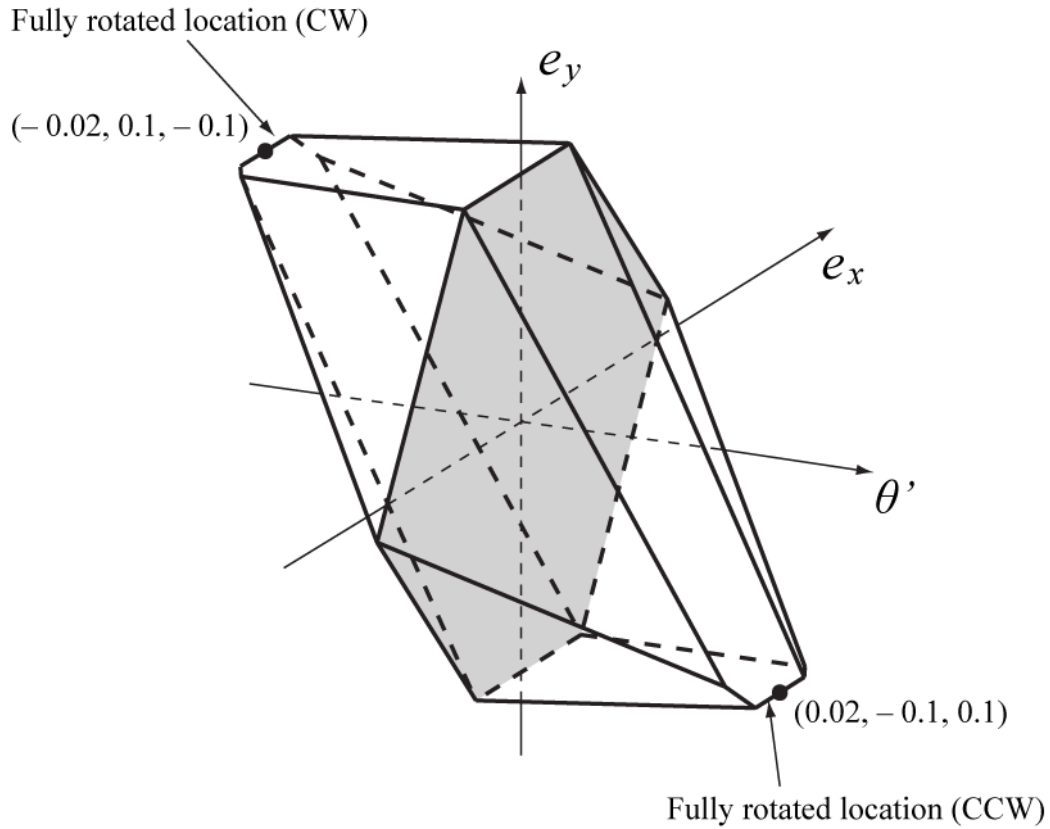


Figure 5.20: Intersection of primitive models. Shaded area is the intersection with plane  $Oe_xe_y$  and it is same as the shape in Figure 5.2(b). Points for fully rotated locations lie on two lines.

$$\begin{bmatrix} 0 \\ 0 \\ 0.004 \end{bmatrix} = \begin{bmatrix} 1 & 0 & -c_y \\ 0 & 1 & c_x \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0.02 \\ -0.1 \\ 0.004 \end{bmatrix} \quad (5.13)$$

$$\begin{bmatrix} 0 \\ 0 \\ -0.004 \end{bmatrix} = \begin{bmatrix} 1 & 0 & -c_y \\ 0 & 1 & c_x \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} -0.02 \\ 0.1 \\ -0.004 \end{bmatrix} \quad (5.14)$$

Solve equation 5.13 or 5.14, we can get  $c_x = 25$  and  $c_y = 5$ . The invariant point  $O'$  is shown in Figure 5.16. Deform the T-Map in Figure 5.20 by the matrix

$$\begin{bmatrix} 1 & 0 & -5 \\ 0 & 1 & 25 \\ 0 & 0 & 1 \end{bmatrix}$$

and then get the final T-Map for the profile shown in Figure 5.7. The final T-Map is at the reference system  $O'x'y'$ , and its 3-D model is shown in Figure 5.21. The final model has twelve surfaces, and each primitive model provides four surfaces to the final model.

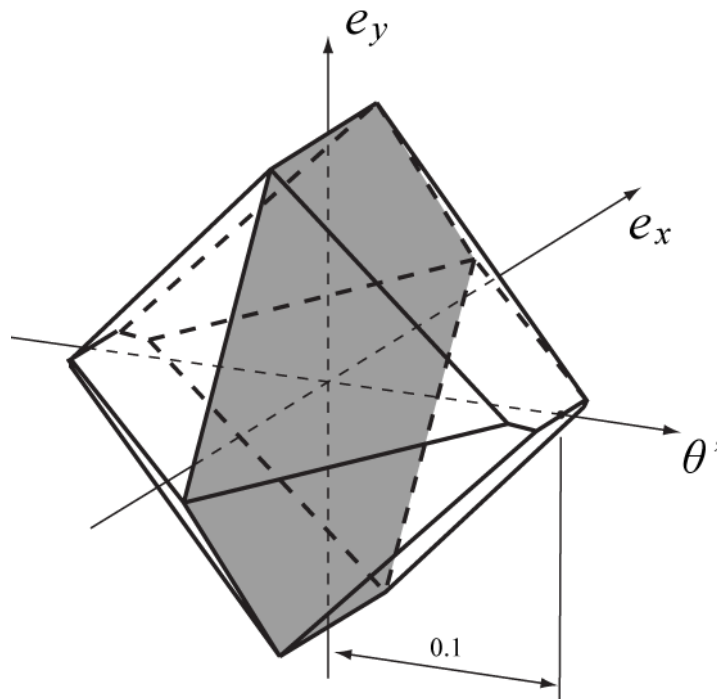


Figure 5.21: The final 3-D T-Map. . Shaded area is the intersection with plane  $Oe_xe_y$  and it is same as the shape in Figure 5.2(b).

### 5.6 3-D primitive model for Circular Arc Segment

The tolerance-zone for circular arc segment is shown in Figure 5.6(a), and its corresponding 2-D primitive model (section 5.2.3) lies in the  $e_xe_y$  plane and is shown in Figure 5.22(b). If we set the origin of the WCS for this segment at the center of the arc, as

in Figure 5.22(a), the lamina with mid-sized profile (arc) can rotate by any angle around its center. Therefore, the 3-D model in Figure 5.22(a) is open-ended and formed by extruding the 2-D T-Map in the  $\theta$  direction.

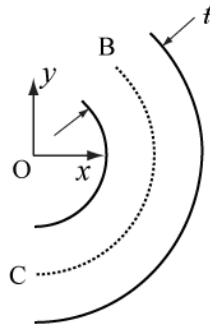


Figure 5.22(a): The tolerance-zone same as in Figure 5.6(a). The origin of WCS at the center of the arc.

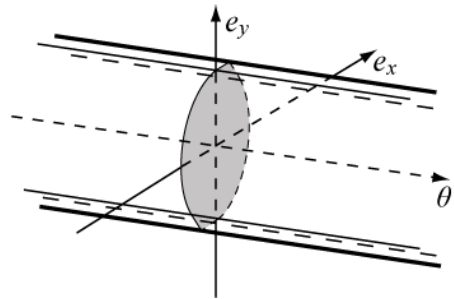


Figure 5.22(b): 3-D primitive model for circular arc segment, the shaded section is same with Figure 5.6(c).

Add a rounded corner to the example of triangle, and get the profile shown in Figure 5.23. The origin of the profile segments are shown in Table 5.2. Comparing to the Table 5.1, the length and the mid-points of the line-segments may be changed. And follow the procedure for building a 3-D T-Map. A 3-D T-Map for the profile with circular arc is in Figure 5.24.

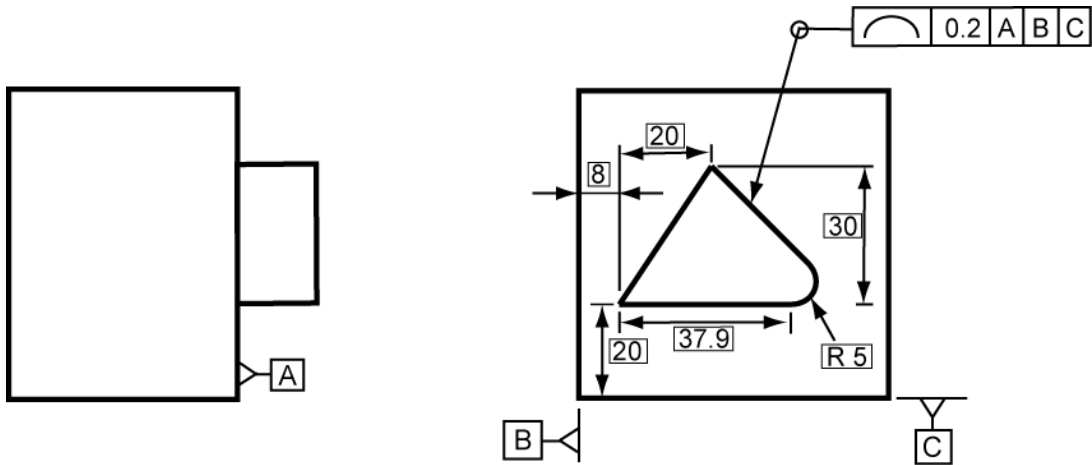


Figure 5.23: Boss with its shape controlled by the profile tolerance  $t = 0.2$  mm relative to Datums A, B, and C. A rounded corner is added to the right corner.

Table 5.2 Displacements between the origins and Shear matrix for the profile with arc

Edge	Displacement between the origins	Shear matrices
Bottom	(-18.965,0)	$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & -18.965 \\ 0 & 0 & 1 \end{bmatrix}$
Left	(-10,-15)	$\begin{bmatrix} 1 & 0 & 15 \\ 0 & 1 & -10 \\ 0 & 0 & 1 \end{bmatrix}$
Right	(-30.732,-19.268)	$\begin{bmatrix} 1 & 0 & 19.268 \\ 0 & 1 & -30.732 \\ 0 & 0 & 1 \end{bmatrix}$
Arc	(-37.929,-5)	$\begin{bmatrix} 1 & 0 & 5 \\ 0 & 1 & -37.929 \\ 0 & 0 & 1 \end{bmatrix}$

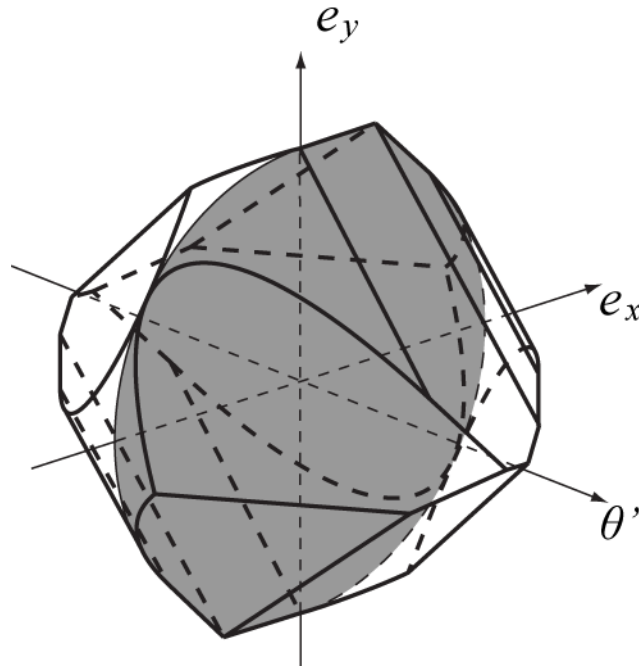


Figure 5.24: 3-D T-Map model for the profile tolerance in Figure 5.23. The shaded area is the section in  $e_x e_y$  plane.

### 5.7 3-D primitive Models for free curve element

This section gives a method to create an approximate 3-D T-Map model for the free curve element. This method needs the computer programming to create many open ended primitives to approximate the real model.

Because the straights or arcs have the constant tangents or curvatures, the shapes of 3-D T-Maps for straight lines are extruded from rhombus, and the shapes of 3-D T-Maps for circular arcs are extruded from a plane shape by truncating a rhombus with two congruent, but opposite circular arcs that are tangent to it. However, the free curve elements don't have the constant tangents or curvatures. Unlike straights and arcs, there is no similar shape between primitive models for free curves. Primitive models for free curve elements are created based on the geometric properties of the profile shapes, but

every point on a curve may have different geometric properties. If all the points on a curve can be decomposed into independent point elements, accurate models can be created for these points. But, on the curve, the number of points is infinite. It is impossible to intersect infinite number of the models. Only an approximate T-Map model can be created.

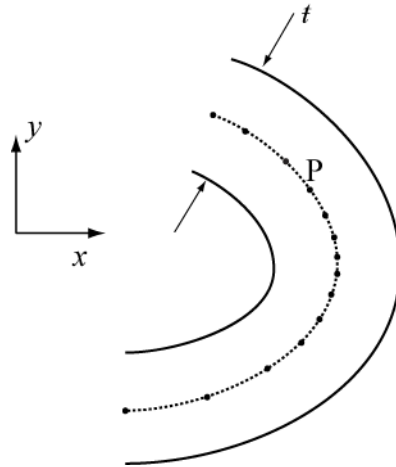


Figure 5.25(a): The tolerance-zone, and sampling points.

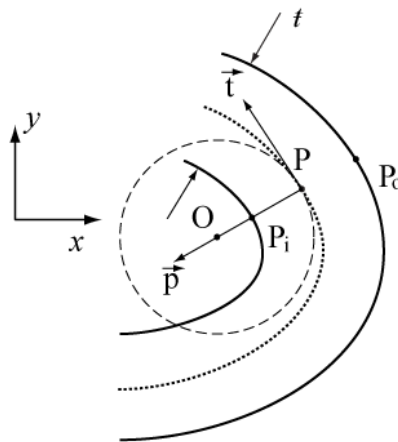


Figure 5.25(b): Tangent direction and osculating circle at point P.

A group of sampling points is picked from the curve. In the sampling, the turnings of tangents at two sampling points are same. The number of sampling points defines the



accuracy of the approximation, but more sampling points will take more time to compute the result. In the Figure 5.25(a), point P is one of the sampling points on the mid-sized profile. Take this point as an example to create a primitive 3-D sub-model.

Here the point P is considered as a small arc segment with length that is close to zero, and the arc is on the osculating circle that has center O and passes through the point P. On the inside and outside boundaries of the tolerance zone,  $P_i$  and  $P_o$  are the closest points to P. The direction ( $\vec{p}$ ) of the vector passing through these points is normal to the tangent of the curve at P.  $P_i$  and  $P_o$  are defined from P in the processing of offsetting mid-sized curve. These two points constrain the displacement of P in the direction  $\vec{p}$ . In the T-Map space, the 2-D model of translation in x and y direction is shown in Figure 5.26(a). It is a 2-D model for an arc element with the arc length close to 0. To compute the rotational displacement of the profile, the rotation center needs to be defined first. Use the center of osculating circle of point P as the rotation center. Here the point P is considered as a small arc segment with length close to zero, and the arc is on the osculating circle at the point P. Because P can be rotated around O without any constraint in the tolerance zone, in the T-Map, the 3-D model is built by extruding the 2-D model of allowable translations in the x and y directions along z direction to infinite. Figure 5.26(b) is the 3-D T-Map for one of the sub-models of the mid-sized free curve. The acceptable region of the T-Map is a volume between two planes.

The primitive sub-models for all sampling points are sheared by the shear matrixes. The matrixes are created by the displacements between the centers of the osculating circles and the pole. After shearing all primitive sub-models are represented in

the same reference system, and they are ready to intersect. The intersection of these sub-models is the approximate 3-D primitive model for the whole free curve.

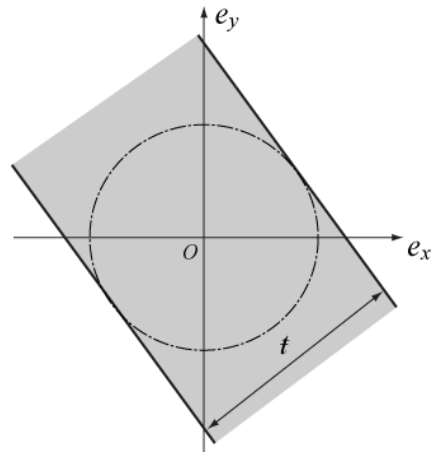


Figure 5.26(a): The 2-D T-Map model of allowable translations in x and y directions.

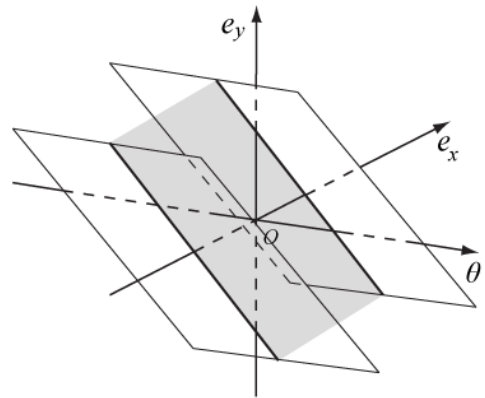


Figure 5.26(b): 3-D primitive sub-model for point P, the shaded section is same with Figure 5.26(a).

Take the profile example in Figure 5.23, and use a free curve to represent the arc in the right corner of the triangle. The 3-D T-Map of this mid-sized profile is similar to the one in Figure 5.24, but curved faces are replaced by groups of small faces which are used to approach to the real shapes.

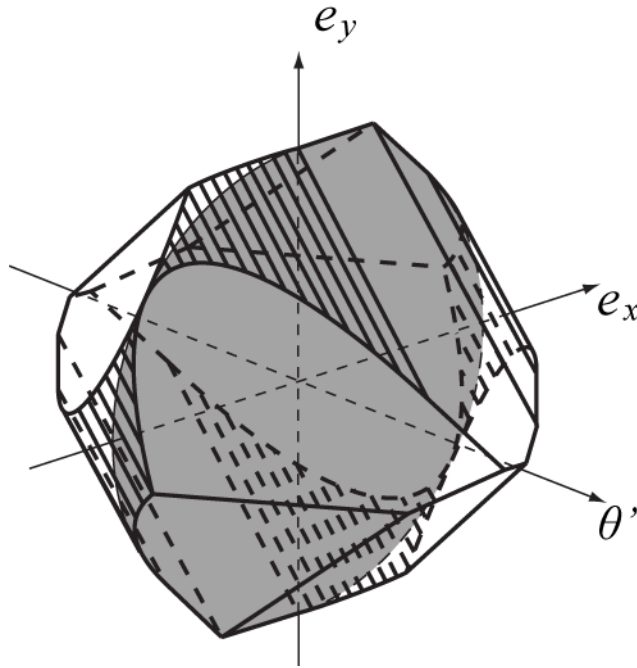


Figure 5.27: 3-D T-Map model for the profile tolerance in Figure 5.23. The shaded area is the section in  $e_x e_y$  plane. The curved faces are replaced by groups of small faces.

## 5.8 3-D model with size change and unequally disposed profile tolerance

From section 5.3 to 5.7, the primitive T-Map models for mid-sized line segment, circle arc and free form curve are created. This section explains how to create the 3-D primitive models with size changes for these elements. In Figure 5.28, the size of the profile is changed. The dot profile is the profile offset outward by the value  $\Delta F$  from the true position. This size change will deform the primitive models of all the decomposed profile portions. And the way to modify the primitives is discussed for each type of these three elements.

### 5.8.1 Size change for line element

Because of the size change, the locations of the profile with zero translations and rotation are not in the middle of the tolerance-zone. Figure 5.29 is a line segment and its

tolerance zone. The dot line is offset from the mid-sized profile, and the distance between them is  $\Delta F$ .

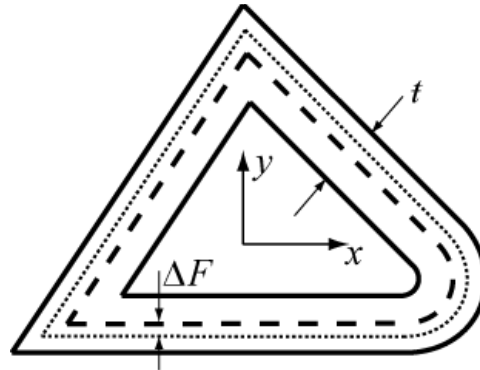


Figure 5.28: Exaggerated tolerance-zone (between two solid-lined triangles) for a triangular profile (dashed-lined triangle) with a rounded corner in figure 5.23. The dot-line is the offset profile for mid-sized profile.

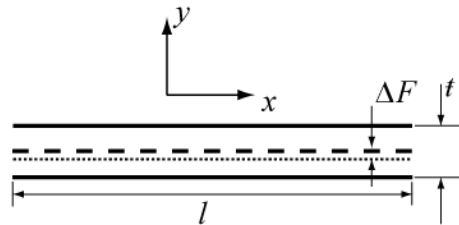


Figure 5.29: Exaggerated tolerance-zone (between the two solid-lines) for a line-segment (dashed). The dot-line is the location of the profile with zero translations and rotation.

For the line-segment element, the size change can be considered as a translation. In Figure 5.29, the dotted line can be gotten by either offsetting or translating from the true position. The extension of length of the line-segment caused by increased size can be ignored, because only keep first order terms. Figure 5.11(a) gives the tolerance-zone and the location of the profile with zero translations and rotation. It has the same tolerance-zone as the one in Figure 5.9, and the only difference is where we define the location of

the zero translations and rotation. Similar to Figure 5.9, the 2-D T-Map section is created in Figure 5.30(a). Compare to the 2-D section in Figure 5.11(a), they have the same geometry size and shape, but different locations in their reference systems. In other words, the T-Map model is translated by  $\Delta F$ , due to the size changed in same value. The 3-D model build process is same as the mid-sized profile. The process is extruding the 2-D T-Map along its normal direction, as shown in Figure 5.30(b). Then rotate and shear the model based on the differences between the WCS's, and get the primitive for intersection as shown in Figure 5.30(c).

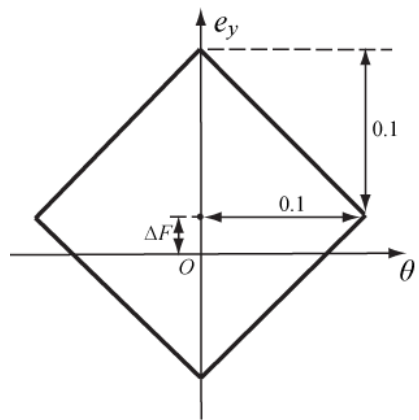


Figure 5.30(a): 2-D T-Map section (model is rescaled in direction  $\theta$ ) for the tolerance-zone in Figure 5.29.

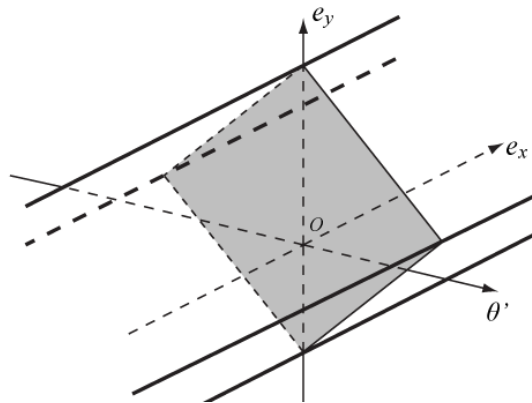


Figure 5.30(b): 3-D T-Map from extruding the 2-D T-Map in Figure 5.30(a) along its normal direction

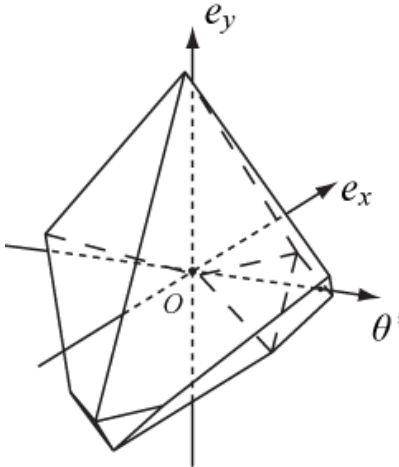


Figure 5.30(c): An example of 3-D primitive T-Map for the triangle profile with a size change as shown in Figure 5.30(a).

### 5.8.2 Size change for circular arc

The size-changed arc in its tolerance zone is shown in Figure 5.31(a).  $\Delta F$  is the value changed from the true profile. Section 5.2.3 gives a way to create the 2-D model for the mid-sized profile, and the 2-D T-Map model for the example is given in the Figure 5.6(c). The boundary of this 2-D T-Map model can be separated into two parts. One is from the outside boundary of the tolerance zone and the other is from the inside boundary of the tolerance zone. The arc BC in the 2-D T-Map model is mapping the limit locations of the profile that is moved to the outside boundary of the tolerance zone. Arc BC is on the tolerance circle which radius is half tolerance value. The tolerance value is equal to the distance between the true profile and the outside boundary of the tolerance zone. However, for the profile that has size change, the distances between the profile without translation or rotation and the boundaries of the tolerance zone is unequal. In Figure 5.31(a), the distances are  $t/2 - \Delta F$  to the outside boundary and  $t/2 + \Delta F$  to the inside boundary. Figure 5.31(b) is the 2-D model for the size-changed profile in Figure 5.31(a).

In Figure 5.31(a), arc BC and arc EF are on two tolerance circles with different radii. The enveloped area of these two arcs and their extension lines is the 2-D T-Map for the size-changed profile.

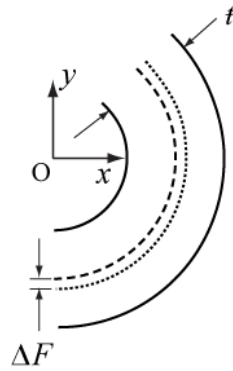


Figure 5.31(a): Exaggerated tolerance-zone for circular arc. Dash line is true profile and dot line is the profile with size change.

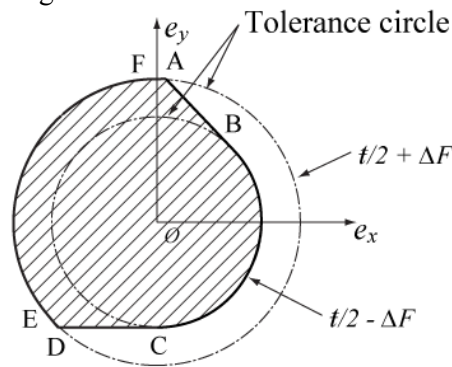


Figure 5.31(b): 2-D T-Map model (shaded area) limited by two arcs have radii  $t/2 \pm \Delta F$  for size-changed profile in 5.31(a).

Once the 2-D T-Map model is generated, the 3-D primitive is created follow the same steps that were used in creating the 3-D primitive for the mid-sized circular arc shaped profile. Because the size changed arcs have the same center, the 3-D model is sheared just as the 3-D model for the mid-sized profile. The result is the primitive T-Map of the arc ready for intersection.

### 5.8.3 Size change for free form curve

In the process to build sub-model for mid-sized free form curve, the sampling points on the curve is considered as arcs with zero arc length. The sub-model for a sampling point on the curve is shown in Figure 5.32(a). In this 2-D model, two lines are tangent to two tolerance circles. That means this sampling point has the distant  $t/2 - \Delta F$  to the outside boundary and  $t/2 + \Delta F$  to the inside boundary. Similar to circular arc, the 3-D sub-model is shown in 5.32(b). The 3-D primitive model is generated by intersecting all the sub-models.

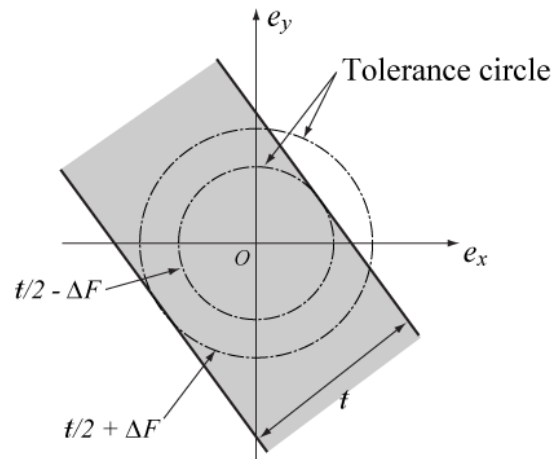


Figure 5.32(a): The 2-D T-Map model of translation in x and y direction.

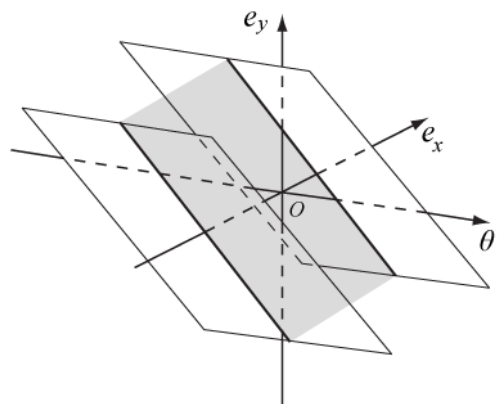


Figure 5.32(b): 3-D primitive sub-model for a sampling point, the shaded section is same with Figure 5.32(a).



### 5.8.4 Unequally disposed profile tolerance

A unilateral and unequally disposed profile tolerance is indicated with an unequally disposed profile symbol placed in the feature control frame. In Figure 5.33, a second value 0.15 is added to indicate the tolerance in the inward direction that would allow additional material to be removed from the true profile.

For the unequally disposed and unilateral profile tolerance, the true profile location has different distances to two boundaries of the tolerance zone. The distance to the inside boundary is 0.15, and the distance to the outside boundary is 0.05. Compare to the size-changed profile and its tolerance zone in Figure 5.28, the unequally disposed profile in the true profile location can be expressed as an equally disposed profile tolerance in the same tolerance zone, but added an initial size change.

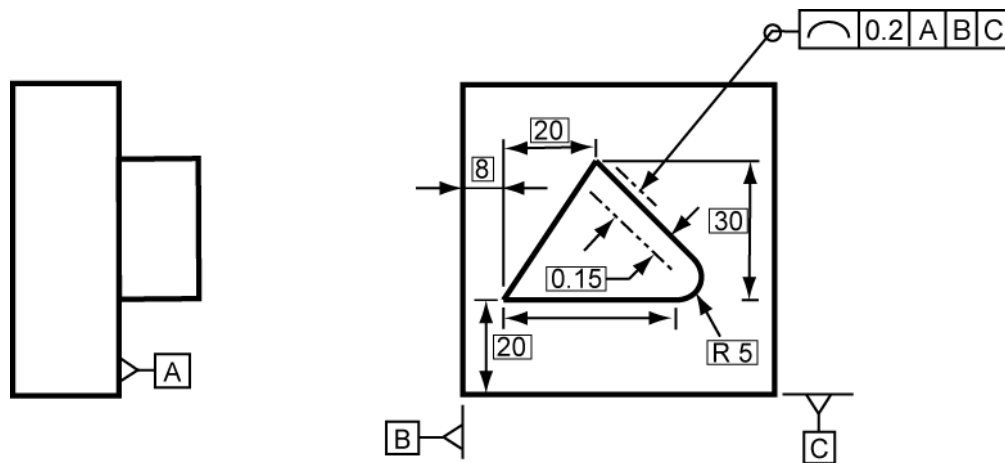


Figure 5.33: Boss with its shape controlled by the profile tolerance  $t = 0.2$  mm relative to Datums A, B, and C. A second value 0.15 is added. A rounded corner is added to the right corner.

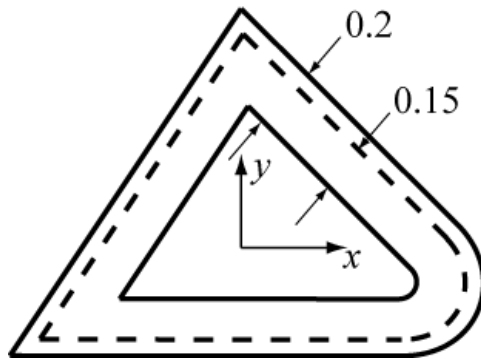


Figure 5.34: Exaggerated tolerance-zone (between two solid-lined triangles) for a triangular profile (dashed-lined triangle) with a rounded corner in figure 5.33. The dash-line is the true profile. The distance from true profile to inside boundary is 0.15.

## 5.9 4-D model

A group of 3-D T-Maps are generated under the different size changes. These 3-D T-Maps are used to depict the full 4-D T-Map for the whole profile. Figure 5.35 is a 4-D T-Map for the triangular profile.

## 5.10 Open profiles

ASME standard has not given the clear description for the ends of an open profile. However, profile tolerances apply normal (perpendicular) to the true profile at all points along the profile. Also, in ASME standard, it permits extended boundary lines for the sharp corners. That means two boundaries of tolerance zone for a segment of profile can be extended. In other word, the ends of profile cannot make any contribution to constrain the displacements of profile. For special profile shape, like the straight line, the line segment profile has two straight lines as its boundaries. The line segment can move along its direction to any position. The profile tolerance of this line segment cannot constrain the displacement of this line segment in its direction.

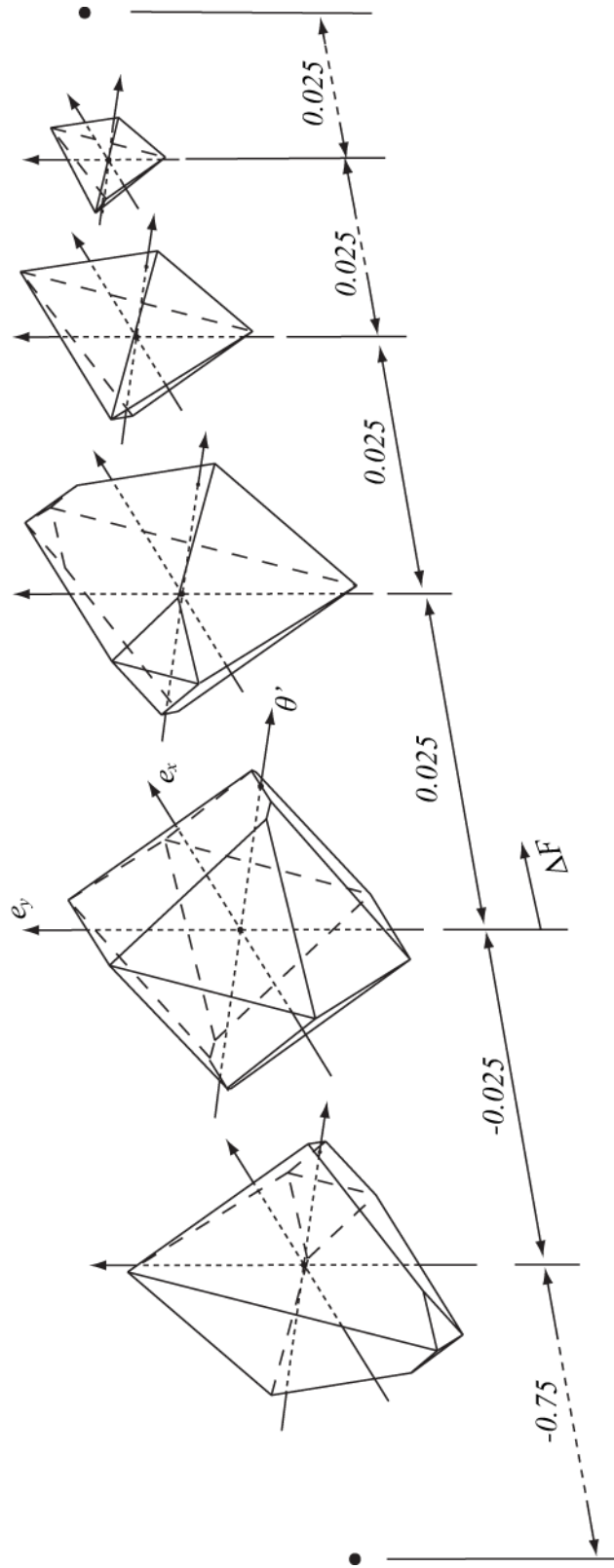


Figure 5.35: The morphology of the 4-D T-Map as a function of size for the triangular line-profile specified in Figure 5.1. The scale in the direction of size change is not linear. One shape is shown for size decrease to illustrate the through-the-origin symmetry in the direction of size.

In the process of building T-Maps for closed profiles, the profiles are decomposed into some segments. They can be considered several independent profiles. The tolerance zones for these segments are open at their ends. In previous study of primitive T-Maps for three types of profile elements, there is no constraint for the profile at their ends. In Figure 5.9, Figure 5.22(a) and Figure 5.25(a), only the boundaries and their extension lines are used for primitive T-Map building. Because the tolerance zone for the open profile in ASME standard is same with the tolerance zone used for primitive T-Map, the process for building primitives meets the ASME standard. These segments can be considered open profiles. The T-Map of the whole open profile can be directly obtained by intersect the T-Map primitives of these segments. The steps for constructing T-Maps for the open profile are same as for the T-Maps for the closed profile.

### **5.11 T-Map under ISO Standard**

By the definition of the ISO standard, the tolerance zone is limited by two lines enveloping circles whose diameter is tolerance value. The centers of these circles are situated on the true profile. Take the triangular profile as an example, the tolerance zone for the profile under ISO standard is shown in Figure 5.36. The corner of outsider boundary is rounded, because the boundary envelop circles are situated on the true profile.

Comparing the tolerance zones under two standards, the tolerance zone under ISO standard is smaller than the tolerance zone under ASME standard near the corner. In Figure 5.36, the different areas are shown in shaded. The reduction of tolerance zone means more constraint for the displacements of the profile. Under ISO standard, the T-

Map model will be removed some volume from the model under the freedom permitted with the ASME standard.

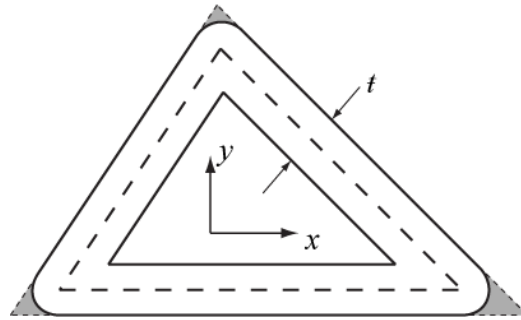


Figure 5.36: Exaggerated tolerance-zone (between two solid-lined triangles) for a triangular profile (dashed-lined triangle). The corner of outside boundary is rounded. Shaded area is the area that reduced from the tolerance zone under ASME standard.

Trim faces are created to modify the T-Map models for ISO standard. The trim face is generated from the circular arc of the boundary at each sharp corner. Take the right corner of the triangular profile as an example. The vertex at the corner is considered as a circular arc profile with zero radius. The circular arc profile has a tangent range that is between the tangents of the arc at its ends. This “point” arc profile keeps the tangent range that is between the directions of the two edges of the corner. The boundary of this “point” arc profile is shown in Figure 5.37(a). The boundary is offset from the “point” arc profile by half tolerance value. Based on the boundary, in the 2-D T-Map the plane is separated into two areas. The shaded area is the acceptable region of the mapping point space. Because the circular arc has infinite rotational value in the T-Map, the unsheared trim face can be got by extruding the 2-D T-Map in the direction of  $\theta'$ . The unsheared trim face separates the 3-D space into two parts, acceptable mapping points and unacceptable mapping points. Applying the shear deformation to the trim face by the

displacement between the “point” arc and the pole, the sheared trim face is ready to trim the T-Map model.

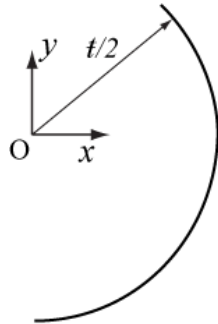


Figure 5.37(a): The boundary of the “point” arc profile. “point” arc profile is located at  $O$ .

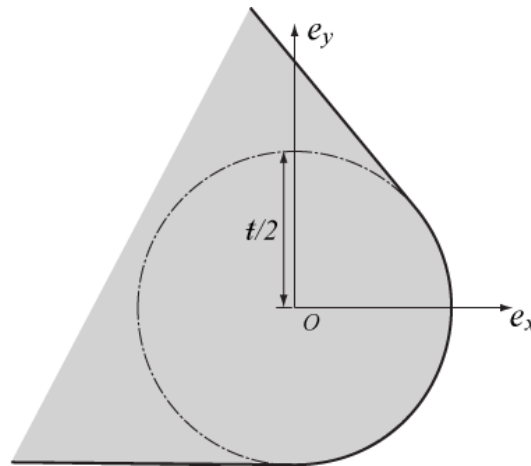


Figure 5.37(b): The 2-D T-Map of the “point” arc profile. The shaded area is the accept region of the mapping point space.

Three trim faces are created for the triangular profile. Trim the 3-D T-Map model for mid-sized profile in Figure 5.21, and get the T-Map model for mid-sized profile under the ISO standard. This T-Map model is shown in Figure 5.38.

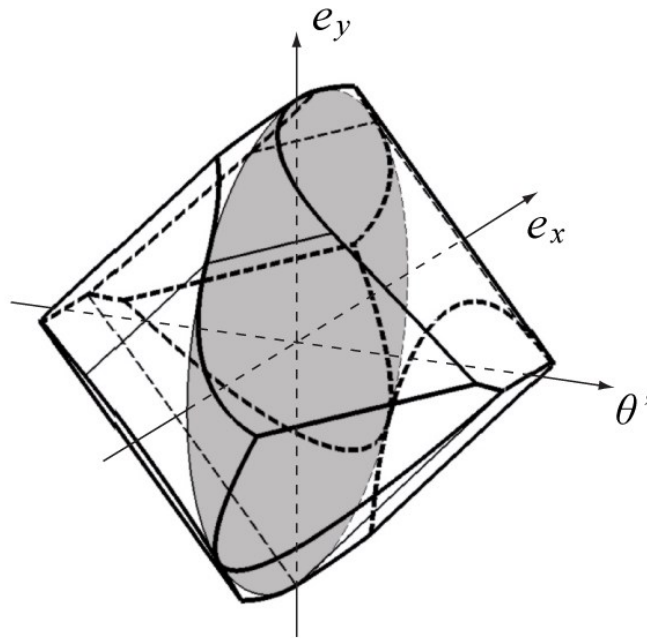


Figure 5.38: 3-D T-Map model for mid-sized profile under the ISO standard. Compare to the T-Map in Figure 5.21 for the triangular profile under the ASME standard. The heavy dashed lines are hidden, and the light dotted line identifying points having  $C^2$  discontinuity.

## 5.12 Hypersections of the 4-D T-Maps

The 3-D modeling discussed in sections from 5.5 to 5.8 is to create a 3-D hypersection of the 4-D T-Map in the dimensions of  $e_x$ ,  $e_y$ , and  $\theta'$ . In this section other hypersections of the 4-D T-Map will be discussed. The dimensions of these 3-D hypersections would be  $e_x\theta'\Delta F$ ,  $e_y\theta'\Delta F$ , and  $e_x e_y \Delta F$ . The procedure of creating these 3-D hypersections is similar to the method for constructing the 3-D T-Map for the mid-sized profile. The first step is to build a primitive model for each decomposed segment in its WCS, and then transfer the primitive model to the global reference system. Finally, do the Boolean operation to the primitive.

The primitives of each element may have different shapes in the different dimensions. For the straight line segment in Figure 5.9, the primitives of the straight line

segment in the space with the dimensions  $e_x\theta'\Delta F$  are shown in the Figure 5.39(a). The primitive model with the change in the fourth dimension  $e_y$  is shown in Figure 5.39(b). Because the size change of the profile can be seen as an additional translation of the profile in y direction, the primitive will translate in the  $\Delta F$  direction when there is a non-zero  $e_y$ . After transfer these primitives to the global reference system, the primitive will be sheared. In the space  $e_x\theta'\Delta F$ , the 3-D primitive need sheared only in the  $e_x$  direction.

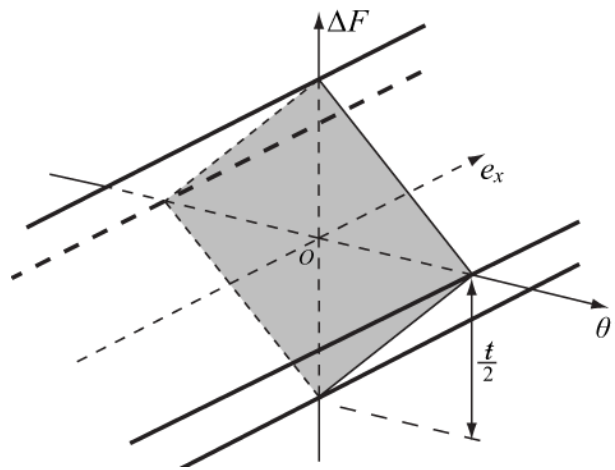


Figure 5.39(a): The 3-D primitives of the T-Map in the space with dimensions  $e_x\theta'\Delta F$  at zero  $e_y$ . The shaded area is the 2-D section at the plane  $O\theta'\Delta F$ .

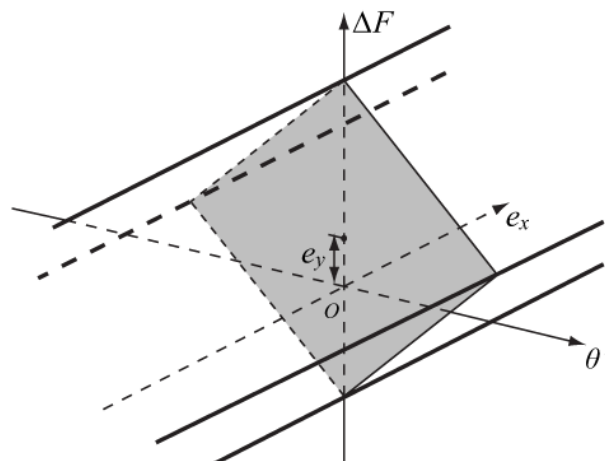


Figure 5.39(b): The 3-D primitives of the T-Map in the space with dimensions  $e_x\theta'\Delta F$  at non-zero  $e_y$ . The shaded area is the 2-D section at the plane  $O\theta'\Delta F$ .



The primitive of the straight line segment (Figure 5.9) in the space with the dimensions  $e_y\theta'\Delta F$  is shown in the Figure 5.40. The primitive model will not change with the change in the fourth dimension  $e_x$ , because there is no influence between  $e_x$  and other dimensions. In the space  $e_y\theta'\Delta F$ , the 3-D primitive need sheared in direction only in the  $e_y$  direction.

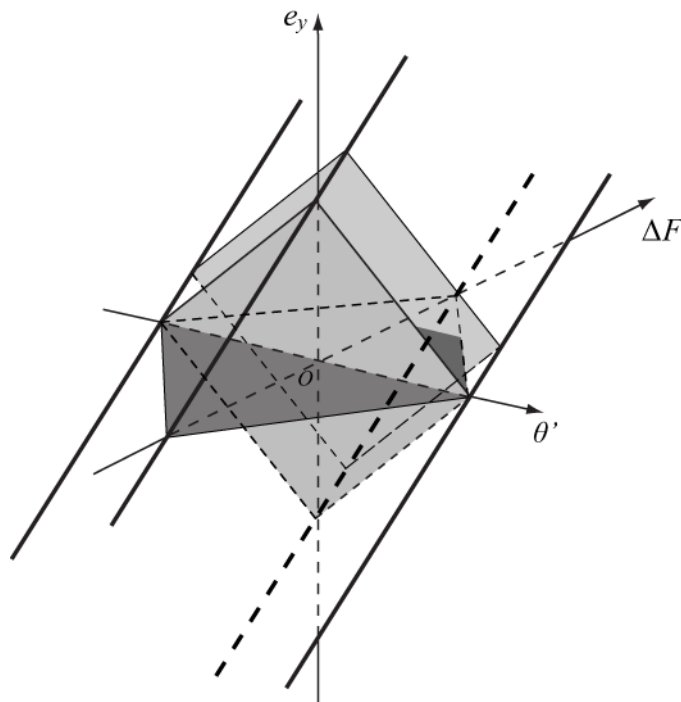


Figure 5.40: The 3-D primitives of the T-Map in the space with dimensions  $e_y\theta'\Delta F$ . The light shaded areas are the 2-D sections with the dimensions  $e_y$  and  $\theta'$ . The dark one is the 2-D section at the plane  $O\theta'\Delta F$ . The extrude direction is 45 degree to axis  $\Delta F$  and axis  $e_y$ .

The primitives of the straight line segment (Figure 5.9) in the space with the dimensions  $e_x e_y \Delta F$  are shown in the Figure 5.41(a). When the rotational angle changed, the allowable range of the translation in  $e_y$  and size change  $\Delta F$  will decrease, and the

primitive model is shown in the Figure 5.41(b). Because the space  $e_x e_y \Delta F$  does not have dimension  $\theta'$ , the sharing deformation does not apply to the primitive.

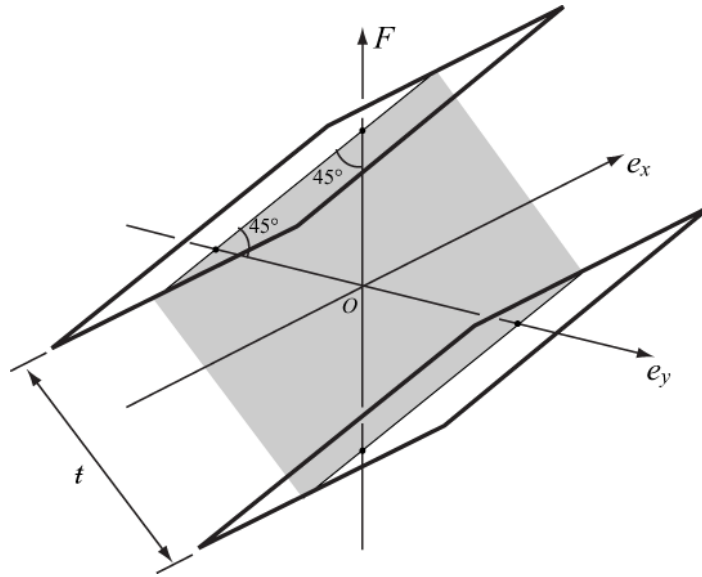


Figure 5.41(a): The 3-D primitives of the T-Map in the space with dimensions  $e_x e_y \Delta F$  at zero  $e_x$ . The model is a volume between two planes. The shaded area is the 2-D section at the plane  $Oe_y \Delta F$ .

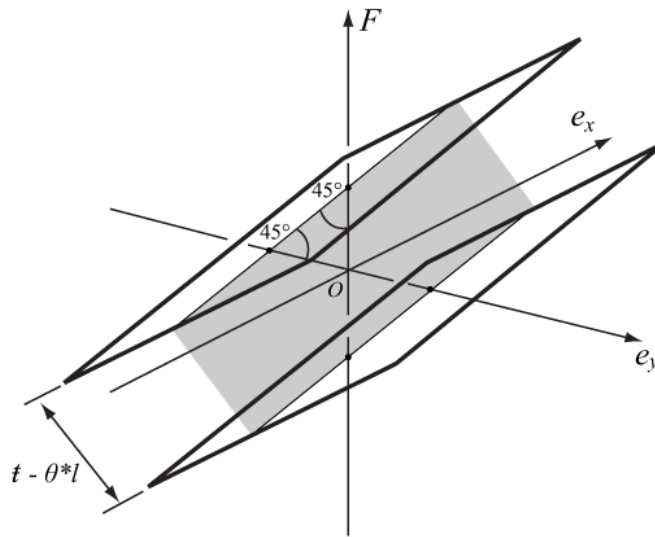


Figure 5.41(b): The 3-D primitives of the T-Map in the space with dimensions  $e_x e_y \Delta F$  non-zero  $e_x$ . The model is a volume between two planes. The shaded area is the 2-D section at the plane  $Oe_y \Delta F$ .

For the circular arc in Figure 5.6(a), the primitive of circular arc in the space with the dimensions  $e_x e_y \Delta F$  is shown in the Figure 5.42. The section at  $\Delta F = 0$  is same as the 2-D T-Map in Figure 5.6(c), and the section at a non-zero  $\Delta F$  value is similar to Figure 5.31(b). The shear deformation is the same to straight line segment. Because there is no constraint for the profile in the tolerance zone, the rotational angle is not influence other dimensions. The primitive model will not change with the change in the fourth dimension  $\theta'$ .

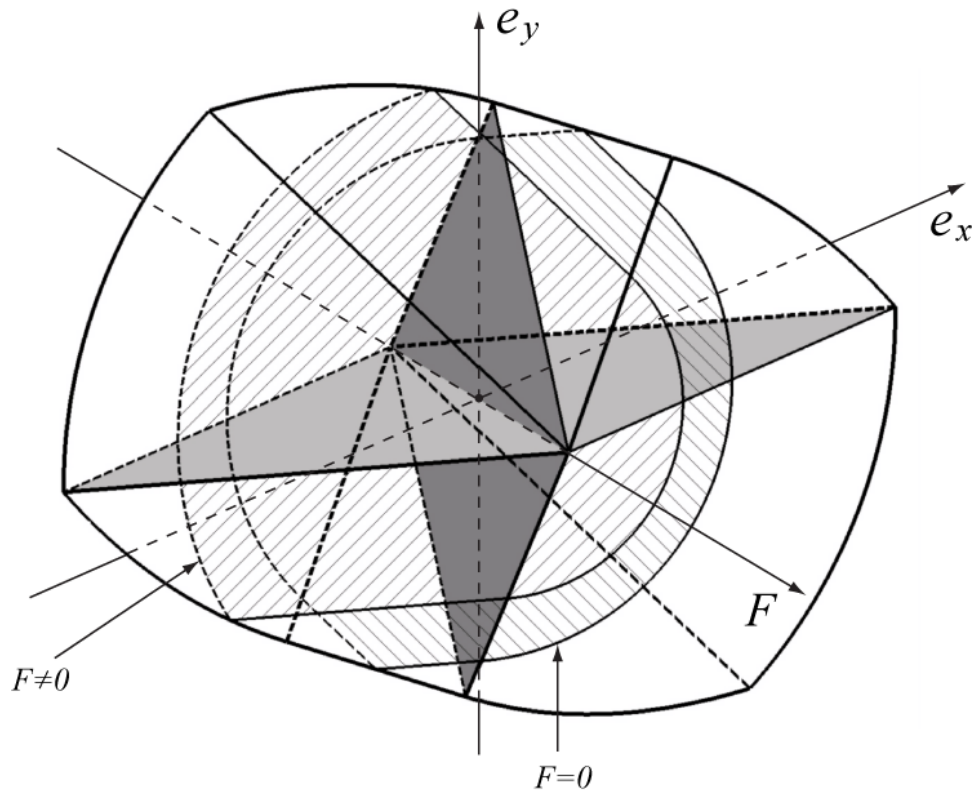


Figure 5.42: The 3-D primitives of the T-Map in the space with dimensions  $e_x e_y \Delta F$  for the profile segment in Figure 5.31. The light shaded area is the 2-D section at the plane  $Oe_x \Delta F$ . The dark one is the 2-D section at the plane  $Oe_y \Delta F$ .

Since, for the circular arc, there is no limit in the rotation, the primitives of the circular arc (Figure 5.6(a)) in the space with the dimensions  $e_x\theta'\Delta F$  or  $e_y\theta'\Delta F$  can be obtained by extruding a 2-D T-Map to infinity along the  $\theta'$  axis. Extruding the light shaded area (section in  $e_y=0$ ) in Figure 5.42 along the  $\theta'$  axis will create the primitive in the space  $e_x\theta'\Delta F$ , and extruding the dark shaded area (section in  $e_x=0$ ) along the  $\theta'$  axis will create the primitive in the space  $e_y\theta'\Delta F$ . When the fourth dimension changes, for example,  $e_y$  is a non-zero value for the space with the dimensions  $e_x\theta'\Delta F$ , the 2-D T-Map will be created by intersecting the 3-D model in Figure 5.42 with a plane parallel to the  $Oe_x\Delta F$ . The distance between these two planes is equal to the change of  $e_y$ .

The primitive for a free form curve is the combination of the sub-models of the profile at the sampling points. In the study in section 3.7, the sub-model is considered as a special case of the circular. For example, there is a sampling point at which the tangent of the profile is along the x axis. The 3-D sub-model in the space  $e_xe_y\Delta F$  is shown in Figure 5.43. Same as the construction of primitive for circular arc, the sub-model will not change, when the fourth dimension  $\theta'$  changed. The sub-model in the space  $e_x\theta'\Delta F$  or  $e_y\theta'\Delta F$  can be built using the same constructing method for the primitive for circular arc.

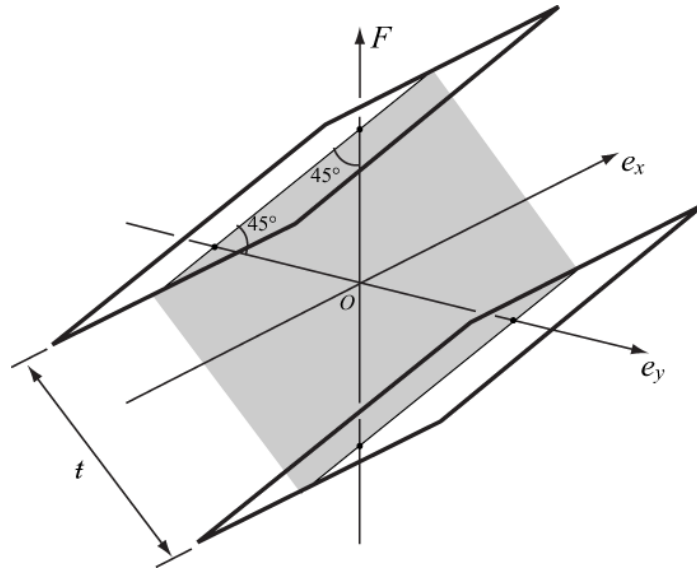


Figure 5.43: The 3-D primitives of the T-Map in the space with dimensions  $e_x, e_y, \Delta F$ . The shaded area is the 2-D section at the plane  $Oe_x\Delta F$ .

## Chapter 6

### COMPUTER PROGRAMMING OF T-MAP CONSTRUCTION

#### 6.1 Requirements of programming

The object of programming is to use computer software to create 3-D T-Maps for tolerances applied to line profiles. Because the 4-D model cannot be directly shown by the computer system, the 4-D T-Map model is depicted in this thesis with a set of 3-D models at different allowable sizes within the tolerance-zone. Consequently, the software creates a 3-D T-Map (hypersection of the 4-D T-Map for a specific size  $\Delta F$ ). The program supports the profiles formed from combinations of straight line-segments, circular arcs, and free form curves. The program supports both closed profiles and open profiles, and it supports convex profiles and concave profiles. The program is under the ASME Standard. The codes are packed and can be used in other programs.

The input information includes the profile geometric data in the form of computer math data. Another input is the tolerance value. The tolerance value has two parts to define the maximum allowable displacements of the profile from the true profile position in the two opposite direction. Because the tolerance may be disposed unilaterally or unequally, the tolerance value is entered with two values. And a change in size can be presented by adjusting these two tolerance values. Amplification factors for final presentation can be set in the input information. With the appropriate amplification factor, the model can represent the T-Map more clearly. Because the free form curves are approximated with short line segments, a value is needed to control the accuracy of the construction for the free form curves.

The output of the program is a 3-D T-Map with the given tolerance values. The model is in the form of computer math data. And the program also gives the coordinates of all points in the fixed reference system having its origin at the invariant point (pole). The coordinates of the pole may be used in the operation of the T-Map in the future works.

## **6.2 Programming environment and tools**

The program is created in Visual Studio 2008, and the programming language is C++. It is because in the previous researches of T-Map, C++ is used for most programming works. Considering the factors of heritability and compatibility, this new program should be in the same programming environment.

The 3D ACIS Modeler (ACIS) is the tool used to create and operate the models in the C++. ACIS is an open, object-oriented C++ architecture that enables robust, 3D modeling capabilities. It gives a sound base of 3D modeling functionality, plus the flexibility to meet our programming requirements. The T-Map building process for line profiles needs the functions to extract the geometric properties of the profiles, to create the solid geometric models and to operate these models. All these requirements can be realized with ACIS. Also ACIS is widely used in previous works by the researchers in ASU Design Automation Lab. The math data of T-Map model is outputted in the form of Standard ACIS Text (file extension .sat). SolidWorks is used to visualize and read the .sat file.

### 6.3 Class and functions

A class is created for the generation of 3-D T-Map for the line profile. Its name is LineProfileToTMAP. This class saves all the data of the profile and T-Map models, and also provides all the functions for the 3-D T-Map for the line profile. This class can be used in other programs, and it is created and tested in Visual C++ Win32 Console Application.

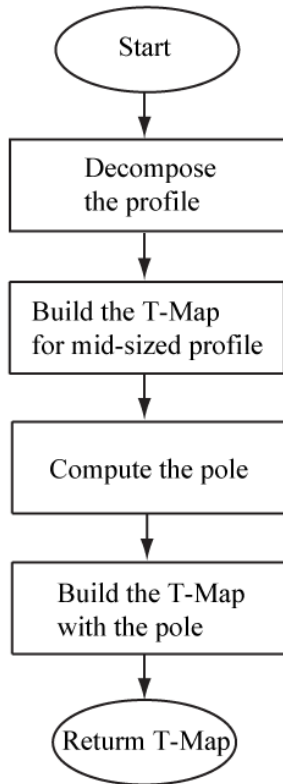
The variables of the class are shown in the Table 6.1. The amplification factors are used to change the scale reference frame, in order to get a better view of the T-Map models. Inside tolerance value and outside tolerance value are calculated from the size change and the unilaterally dispose profile tolerance.

Table 6.1 Variables of class LineProfileToTMAP

Variable name	Description
amp_angle	The amplification factor of the axis of rotational angle.
amp_tmap	The amplification factor of the whole T-Map model.
input_profile	Save the input math data of profile as an ENTITY.
inside_tol	The tolerance value to the inside direction.
outside_tol	The tolerance value to the outside direction.
pole	The coordinates of the pole position.
primitive_list	Save the primitive T-Map models in the ENTITY_LIST.
ptmap	The pointer of output 3-D T-Map model.
segments_list	Save the segments of decomposed profile in the ENTITY_LIST.



get\_tmap:



build\_tmap:

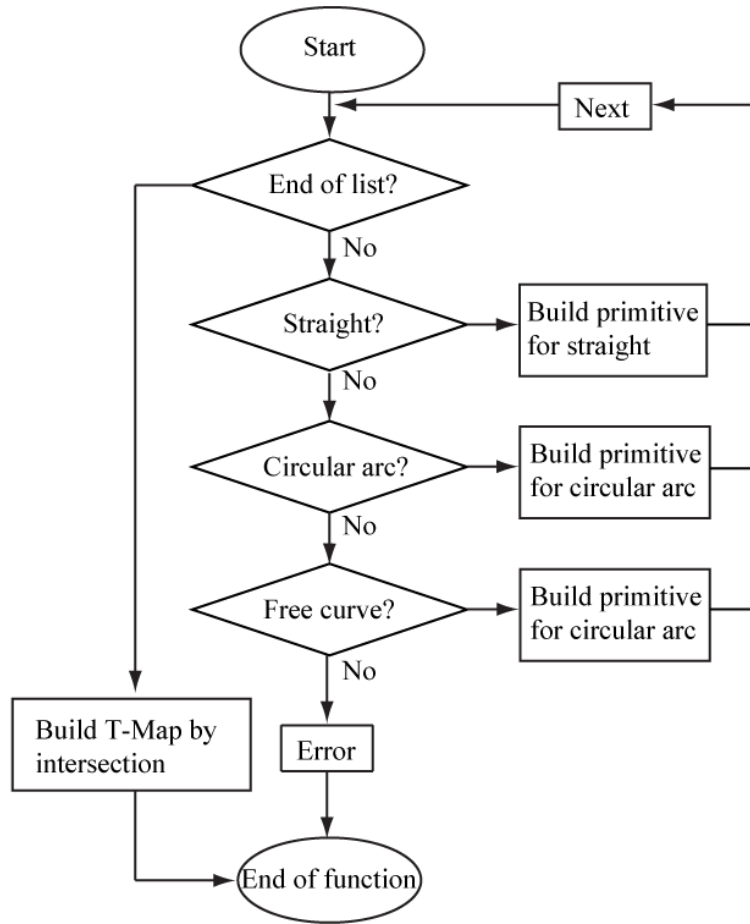


Figure 6.1: Flowchart of function get\_tmap and function build\_tmap.

The functions of the class are used to compute the model of T-Map, input and output the data. The function get\_tmap is the main entrance of the program. This function will return the final T-Map with the given tolerance and size change, also will compute and save the pole coordinates. Flowchart of the function get\_tmap is shown in Figure 6.1. Decompose the profile is realized by the function decompose\_to\_segments. Here the profile is broken at its vertexes. The segments of the profile are saved in the segments\_list. The location of the pole needs to be got first. From the maximum rotational value of the mid-sized T-Map model, the true location of the pole can be

calculated. With the true location of the pole, the required T-Map model can be generated.

The function `build_tmap` is used twice in the function `get_tmap`. One time is building the T-Map for the mid-sized profile which is used for calculating the pole. The other time is building the T-Map with the true location of the pole. In function `build_tmap`, the primitives are created for each member in the `segments_list` and the models of primitives are saved in the `primitive_list`. After all of the primitives are created, intersect the primitives to get a T-Map model and save the pointer of the model to `ptmap`.

The primitive of straight line is built by the function `build_primitive_straight`. In `build_primitive_straight`, four vertexes of the 2-D T-Map are created, as the four vertexes in Figure 5.11(a). Then the four vertexes are translated based on position relation between the WCS and the pole. A face is created by these vertexes, and the primitive solid is created by extruding the face in its normal direction.

The function `build_primitive_arc` builds the primitive for a circular arc. In this function the circular arc will be separated into two pieces, if the angle of the arc is larger than 180 degrees. The function `build_primitive_sub_arc` is used to build the sub-models for the separated arcs, and the sub-models are intersected in function `build_primitive_arc` to get the final primitive model for the arc. In the function `build_primitive_sub_arc`, a face with six vertexes is created. The 2-D model shown in the Figure 5.6(c) is one example of this face. The 3-D model of the primitive solid is created by extruding the face in a direction. The direction is calculated by the relation between the center of the circular arc and the location of the pole. The function `build_primitive_sub_arc` will return this 3-D model to the function `build_primitive_arc`.

The primitive of free form curve is built by the function `build_primitive_intcur`. In ACIS all curves can be classified into four types, ellipse, helix, intcurve, and straight. Ellipse, intcurve and straight can be planar curves. The intcurve is used to restore and operate the free form curves in ACIS. In the function `build_primitive_intcur`, the free curve is broken into segments at the zero curvature, and `build_primitive_sub_intcur` creates the models for these free curve segments. In the function `build_primitive_sub_intcur`, a group sampling points are picked from the curve. The tangents and curvatures of the curve at those sampling points are used to build the sub-models. The 3-D model is built same as introduced in section 5.7.

#### **6.4 Test of the class**

The class `LineProfileToTMAP` is tested in the Win32 console application. The steps to use this class as follow:

1. Include the class `LineProfileToTMAP` in the header file.
2. Create an object of this class.
3. Set the parameters of this object, including inside and outside tolerances, maximum distance of sampling point, and amplify factor of angle and whole T-Map.
4. Import the profile into the object.
5. Run the function `get_tmap`, and get the 3-D T-Map model.

If there are some errors in the program, error information will be output. The information includes the notes to locate the error, and also includes the ACIS error codes.

CONCLUSION AND FUTURE WORK

**7.1 Conclusion**

In this thesis new methods were developed to generate the Tolerance Map (T-Map) for a line profile. Two methods were explored: the parametric method and the method of the decomposition and intersection, with the objective of identifying and implementing the most promising one for use in computer aided tolerancing. The T-Map is a new math model to represent the variations of a tolerated feature as points in a hypothetical, Euclidian space. The points in the 4-D T-Map model for line profile represented the deviations of a manufactured profile from the true profile. The deviations are represented with four variables: translations in two directions of the plane, one rotation in the plane, and size change. With the examples of profiles, the decomposition method is introduced. In this method, the profile is decomposed at its vertexes into three types: straight line, circular arc and free form curve. Each of these types has a procedure to build the corresponding primitive model. The primitive models are deformed to be represented in the same reference system. Apply the intersection of the Boolean operation to all primitives, and get the T-Map for the whole profile. A parametric method for T-Map generation is also developed sufficient to verify the decomposition method.

The decomposition method is the only method that has been developed for generating the T-Map for line profile having any geometric shape. It can be used to generate open and closed profiles, convex profiles and ones having concave portions. And this method is compatible with the ASME standard Y 14.5 and ISO standards.

The decomposition method has been implemented in computer software to generate the 3-D hypersections of a T-Map for a given tolerance and size change on a line-profile. T-Maps for some example profiles are included in the appendix of the thesis.

## **7.2 Future work**

The T-Map created in this thesis is for a single class of geometric tolerance defined from designers, the line-profile tolerance. Line profiles are usually linked to other features, such as one or more planes or lines, so designers need a way to integrate the T-Map for a line-profile with those of other features and tolerance classes. So, one extension of this thesis would be to apply T-Maps for line-profiles to parts and assemblies that require such integration.

Another line of investigation would be to use the decomposition idea of this thesis to develop T-Maps for surface profiles. It should be possible to again use T-Maps for primitive surface elements and use Boolean intersection of these to get the T-Map for the whole surface profile.

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## APPENDIX A

### OUTPUTS OF THE COMPUTER PROGRAM TESTING

Examples of profile tolerances are used to test the program. Figure A.1 is the profile tolerance used in Chapter 4. 3-D T-Maps outputted from the program with different size changes are shown in Table A.1. The models get from the computer are same as those in Chapter 4.

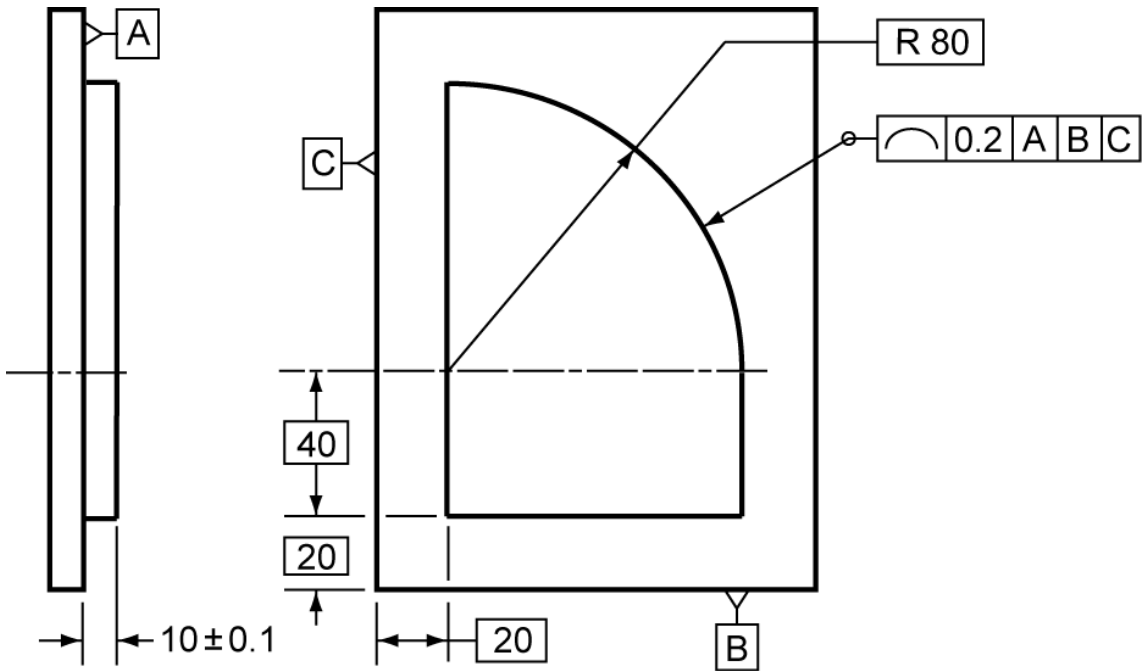
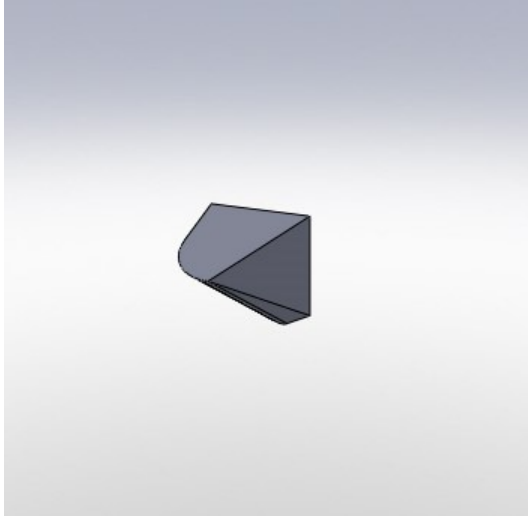
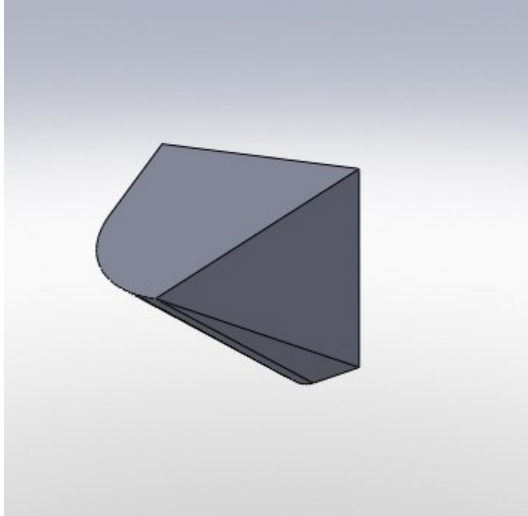
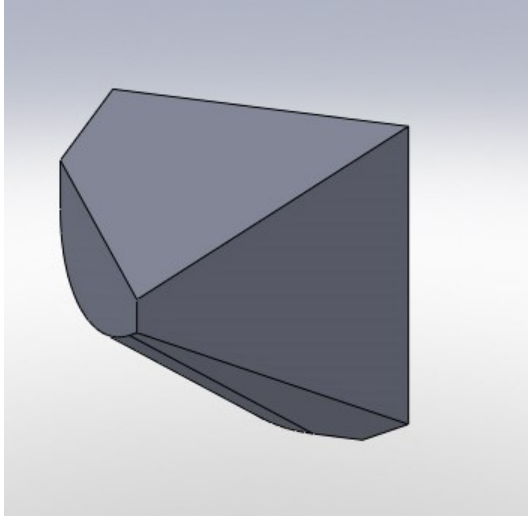
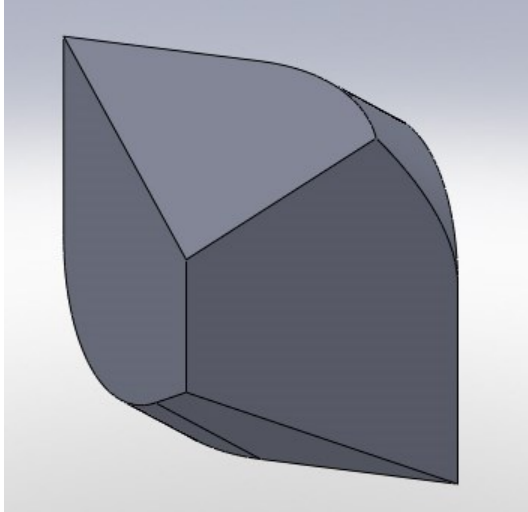
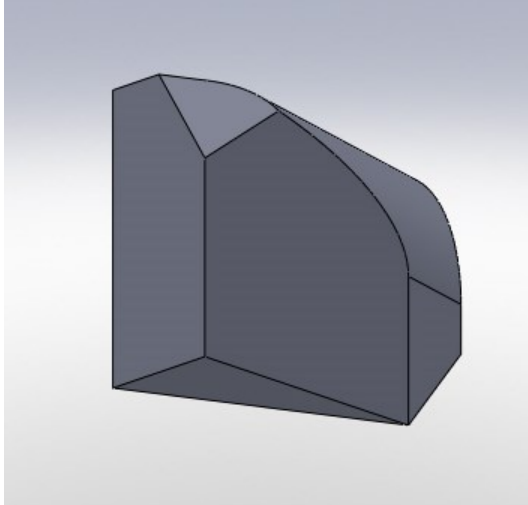
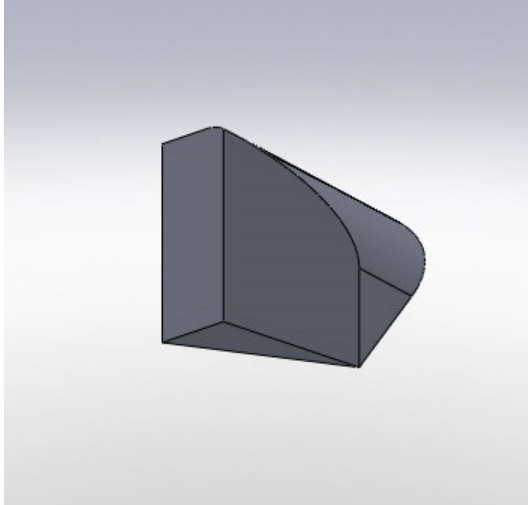


Figure A.1: Boss with its shape controlled by the profile tolerance  $t = 0.2$  mm relative to Datums A, B, and C.

Table A.1 The 3-D models for the profile tolerance in Figure A.1 in different size change values

NO	Size change value	3-D T-Map model
1	-0.75	 <p>A 3-D T-Map model showing a small, dark gray, curved surface on a light gray background. The surface is a shallow, rounded wedge-like shape, representing a profile tolerance deviation of -0.75.</p>
2	-0.5	 <p>A 3-D T-Map model showing a medium-sized, dark gray, curved surface on a light gray background. The surface is a more pronounced rounded wedge-like shape, representing a profile tolerance deviation of -0.5.</p>
3	-0.25	 <p>A 3-D T-Map model showing a large, dark gray, curved surface on a light gray background. The surface is a deep, rounded wedge-like shape, representing a profile tolerance deviation of -0.25.</p>

4	0	
5	0.25	
6	0.5	

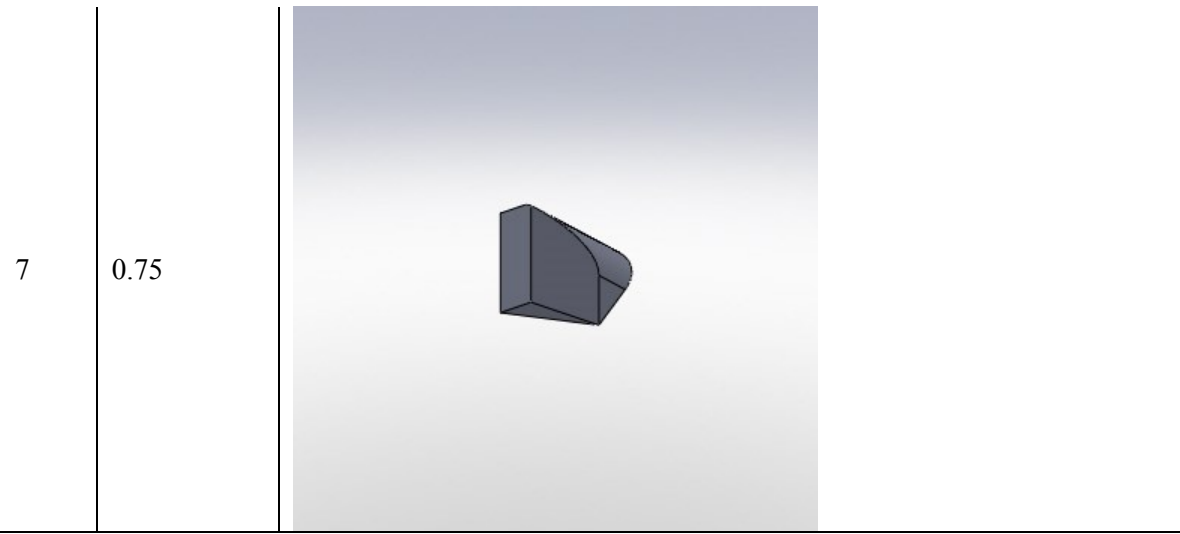


Figure A.2 is an example of concave shape of the profile tolerance. Table A.2 give the 3-D T-Maps outputted from the program with different size changes. All of the T-Map models are convex.

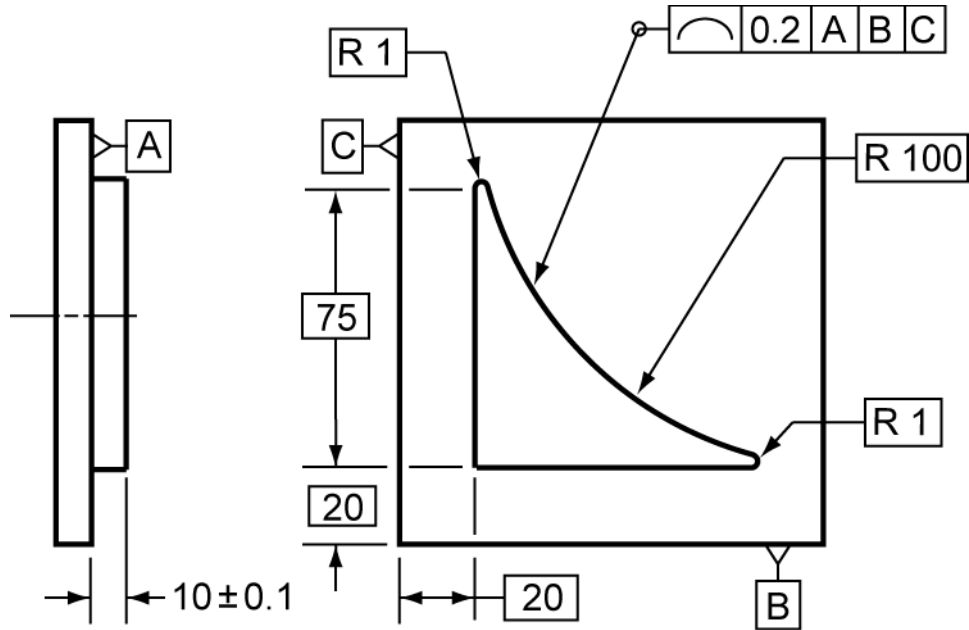
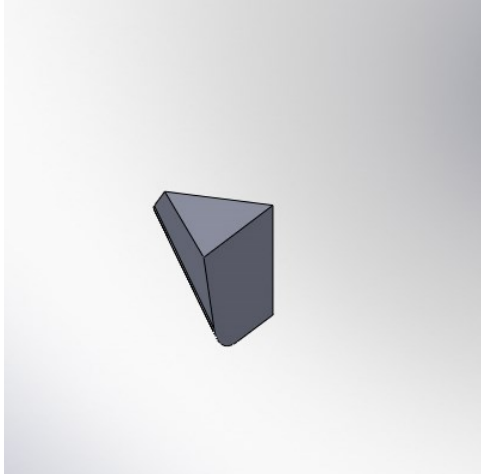
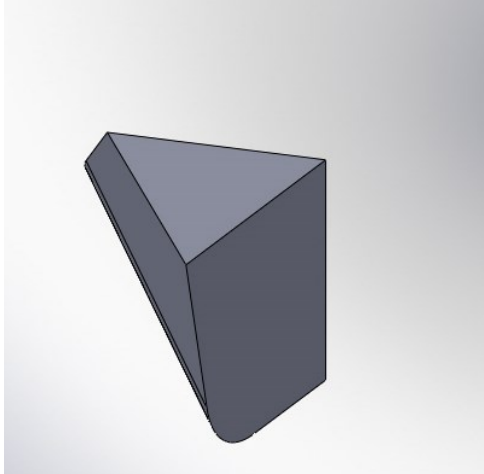
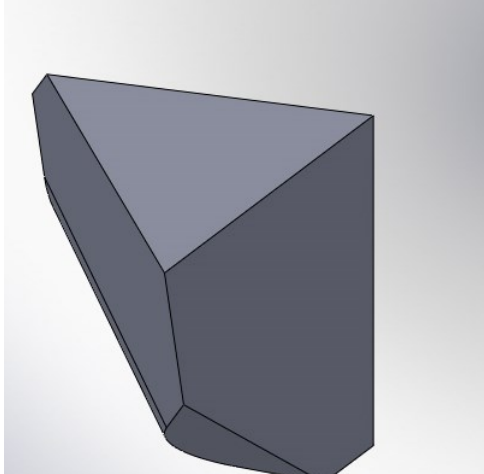
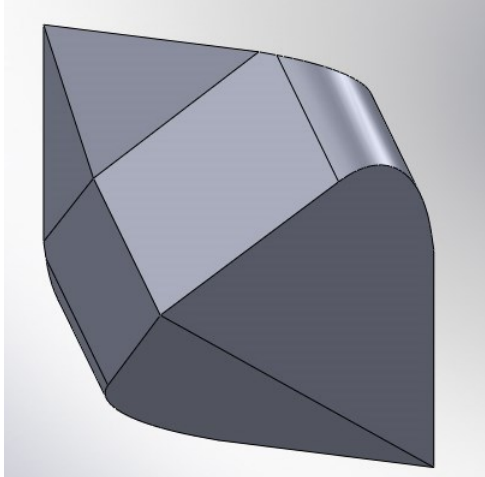
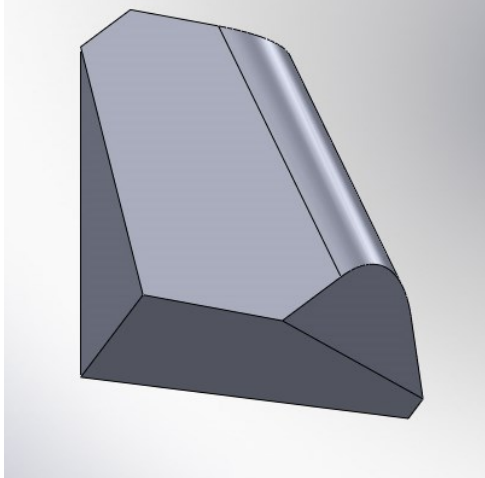


Figure A.2: Boss with concave shape controlled by the profile tolerance  $\pm 0.2$  mm relative to Datums A, B, and C.

Table A.2 The 3-D models for the profile tolerance in Figure A.2 in different size change values

NO	Size change value	3-D T-Map model
1	-0.75	 <p>A 3D model of a triangular prism with a small negative size change, appearing as a thin, slightly tapered triangular block.</p>
2	-0.5	 <p>A 3D model of a triangular prism with a moderate negative size change, appearing as a thicker triangular block with a more pronounced taper.</p>
3	-0.25	 <p>A 3D model of a triangular prism with a small positive size change, appearing as a thick triangular block with a slight outward flare on the top surface.</p>



4	0	
5	0.25	
6	0.5	