Mediation as a Novel Method for Increasing Statistical Power

by

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ABSTRACT

Including a covariate can increase power to detect an effect between two variables. Although previous research has studied power in mediation models, the extent to which the inclusion of a mediator will increase the power to detect a relation between two variables has not been investigated. The first study identified situations where empirical and analytical power of two tests of significance for a single mediator model was greater than power of a bivariate significance test. Results from the first study indicated that including a mediator increased statistical power in small samples with large effects and in large samples with small effects. Next, a study was conducted to assess when power was greater for a significance test for a two mediator model as compared with power of a bivariate significance test. Results indicated that including two mediators increased power in small samples when both specific mediated effects were large and in large samples when both specific mediated effects were small. Implications of the results and directions for future research are then discussed.

This thesis is dedicated to my father Paul and my mother Susan, for believing in me.

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Introduction

Adequate statistical power is essential in hypothesis testing. Many studies are underpowered due to limited sample sizes or difficulty detecting effects. This issue extends from the test of a simple bivariate relationship to tests of models containing multiple independent variables, including mediation. Fritz and MacKinnon (2007) found in a literature survey that 75% of 166 articles on mediation did not have adequate power to detect effects. Research has indicated that including a mediator increases statistical power, but this finding has not been studied in detail. Furthermore, increasing power through use of multiple mediators is also likely to increase power, but models with multiple mediators have not been studied in this regard. The current studies demonstrate the situations in which the inclusion of a mediator in a bivariate model can increase power to detect effects, and extend findings to the inclusion of a second mediator.

Statistical Power in Hypothesis Significance Testing

The power of a statistical test is traditionally defined as that test's ability to detect an effect when an effect is truly present in the population (Neyman & Pearson, 1933). Power depends on several key parameters involved in hypothesis testing, namely Type I error rate, sample size, and effect size (Cohen, 1988). Power can be understood in terms of the types of errors that can occur in hypothesis testing. Statistically, α (a coefficient which ranges from zero to one) represents the rate at which a test incorrectly identifies the presence of a significant effect when no effect is actually present in the population, a Type I error. Power is defined as $(1 - \beta)$, where β represents the rate at which a test fails to find an effect that is truly present in the population (also known as a Type II error). Different levels of α determine the stringency of a test; keeping α closer to zero reduces the likelihood that a Type I error will be made but also decreases the chances of finding an effect that is truly present, thereby decreasing power. All else being equal, larger sample sizes increase the probability of detecting an effect if one exists. If an effect does exist, the effect size gives a measure of the magnitude of the effect. A very small effect may be difficult to detect, which would decrease power, and conversely a very large effect would increase power. The three parameters Type I error, sample size, and effect size are interdependent with power, and given any three, the fourth can be calculated (Cohen, 1988). Cohen (1988) named a power value of 0.80 as a general guideline for adequate power of a hypothesis test in the social sciences. Cohen (1988) defines a small effect size as a correlation of 0.10, a medium effect size as a correlation of 0.30, and a large effect size as a correlation of 0.50.

To calculate the analytical power of a bivariate correlation coefficient ρ_{xy}, the correlation must first be transformed using the Fisher transformation. The transformation is as follows:

$$\rho'_{\mathrm{XY}} = \frac{1}{2}\ln\frac{1+\rho_{\mathrm{XY}}}{1-\rho_{\mathrm{XY}}} \tag{1}$$

The transformed ρ'_{XY} is then divided by its standard error to create a *z* score, which is the noncentrality parameter for the alternative hypothesis distribution:

$$z_{\rho'} = \frac{\rho'_{\mathrm{XY}}}{\frac{1}{\sqrt{N - 3}}} \tag{2}$$

The difference between $z_{1-(\alpha/2)}$ on the central distribution and the noncentrality parameter on the alternative hypothesis distribution ($z_{1-(\alpha/2)} - z_{\rho'}$) is the *z* value for the alternative hypothesis distribution. Statistical power is one minus the probability of the *z* value for the alternative hypothesis distribution occurring under a normal distribution, or:

$$\pi = 1 - \Phi(z_{1-(\alpha/2)} - z_{\rho'}) \tag{3}$$

These formulas can also be used in conjunction with first- and second-order partial correlation coefficients to calculate power for bivariate relationships in models with multiple independent variables, such as single and multiple mediator models.

To calculate the analytical power of a regression coefficient *d*, the coefficient must first be divided by its standard error to produce the noncentrality parameter for the alternative hypothesis distribution:

$$t = \frac{d}{s_d} \tag{4}$$

Next, it is necessary to find the 97.5th percentile (for a two-tailed test) from the Student's *t* distribution with $N - k - 1$ degrees of freedom, where *k* is the number of predictors in the equation containing the regression coefficient. Statistical power is one minus the probability of the *t* value for the alternative hypothesis distribution occurring under the Student's *t* distribution at the 97.5th percentile with $N - k - 1$ degrees of freedom.

Third Variable Effects

The addition of a third variable to the association between two variables results in several different types of relationships (MacKinnon, 2008). A third variable (Z) can be related to either the independent variable (X), the dependent variable (Y), or both. When Z is related to Y such that both X and Z have an effect on Y, Z is called a covariate. When Z is related to Y, including it in a model will result in better prediction of Y, as more variability in Y is explained by both Z and X than by X alone. However, when Z is also related to X such that the inclusion of Z in a model alters the relationship between X and Y, Z is a confounding variable. The independent variables X and Z may be somewhat related, but as long as Z does not affect X's relationship with Y, it is a covariate and not a confounder. In some situations, including a third variable Z will affect the relationship between X and Y such that the X to Y relationship differs at different levels or extents of Z; such a variable is a moderator. In ANOVA, an interaction between two independent variables indicates that one of the variables moderates the effect of the other variable (Baron & Kenny, 1986). In other words, the moderator Z affects the relation between X and Y.

Mediators are defined by their intercession in the causal chain between X and Y, referred to as the mediational chain (Cook & Campbell, 1979). For a variable to be considered a mediator (M), X must cause M, and in turn M must cause Y (MacKinnon, 2008). A classic example of mediation in the social sciences is the hypothesis that attitudes affect intentions, which in turn affects behavior (Fishbein & Ajzen, 1975). A mediator differs from a covariate or a confounder in that causality does not play a role in defining either a covariate or confounder. Both mediators and confounders affect the relationship between X and Y, but while confounders are related to both X and Y, there are no causal assumptions. Baron and Kenny (1986) distinguish between moderators and mediators by specifying that moderators are always independent variables while mediators shift between effects and causes, and that moderators identify the circumstances under which a given effect will occur while mediators identify how or why a given effect occurs.

Mediation: An Overview

Single mediator model. Figure 1 shows how the relationship between an independent variable (X) and a dependent variable (Y) changes when a mediator (M) is added to a model.

- Insert Figure 1 about here -

The top diagram depicts a conventional bivariate regression model, which can also be represented by the equation following notation in Mackinnon (2008):

The top diagram depicts a conventional bivariate regression model, which can also be represented by the equation following notation in Mackinnon (2008):

$$Y = i_1 + cX + \varepsilon_1 \tag{5}$$

Where i1 is the intercept for the equation, c is the population relationship between X and Y (also known in reference to mediation as the total effect), and ε1 is the variability in Y that is not explained by X. Adding a mediator to the traditional regression model generates the bottom diagram, which can also be represented by two new equations:

$$Y = i_2 + c'X + bM + \varepsilon_2 \tag{6}$$

$$M = i_3 + aX + \varepsilon_3 \tag{7}$$

From equation 6, b is the population relationship between M and Y, c’ is the population relationship between X and Y controlling for the mediator M (also known at the direct effect), and ε2 is the variability in Y that is not explained by its relationships with X and M. From equation 7, a is the population relationship between X and M, and ε3 is the variability in M that is not explained by X. The constant coefficients i2 and i3 are the intercepts for the equations.

These parameters can be used to define the mediated effect in two ways. The mediated effect *ab* is the product of the *a* and *b* paths; the total effect *c* is equal to the mediated effect *ab* plus the direct effect *c'*. This results in the following equation:

$$ab = c - c' \tag{8}$$

Therefore, the mediated effect can be quantified as *ab* or as *c – c'*; as discussed in MacKinnon (2008), the two sides of the equation may not be equal for special cases of regression, or for unequal sample sizes across equations. When *c'* is zero and therefore *ab* is equal to *c*, this is known as full mediation; partial mediation occurs when *c'* is nonzero.

For the above mediation equations to yield correct results, the same assumptions that are required for regression analysis must hold (Cohen, Cohen, West, & Aiken, 2003). The assumptions are as follows:

I. Relationships among variables are assumed to be linear, and variables do not interact.
II. No theoretically important variables have been omitted from the model represented by the equations.
III. The measures of X, M, and Y have acceptable reliability and validity without significant measurement error.
IV. Residual errors of the equations are uncorrelated across equations, are uncorrelated with the predictor variables in each equation, and have equal variances across values of the predictor.

For a more in depth discussion of these assumptions with respect to the single mediator model, see MacKinnon (2008).

Parallel two mediator model. Adding a second mediator to the single mediator model results in two different multiple mediator models: the parallel two mediator model and the sequential two mediator model. The parallel two mediator model, which is depicted in Figure 2, is the focus of this paper; more information on the sequential two mediator model can be found in Taylor, MacKinnon, and Tein (2008). The parallel two mediator model is a simple extension of the single mediator model, where the independent variable (X) is now related to the dependent variable (Y) through a mediator (M_1) and also through an additional mediator (M_2). The mediators each have their own separate effects within the model (as opposed to transmitting the effect of X to M_1 to M_2 to Y), hence the use of the term parallel.

- Insert Figure 2 about here -

This diagram can be represented by the following equations following notation in MacKinnon (2008):

$$Y = i_1 + cX + \varepsilon_1 \tag{9}$$

$$Y = i_2 + c'X + b_1M_1 + b_2M_2 + \varepsilon_2 \tag{10}$$

$$M_1 = i_3 + a_1X + \varepsilon_3 \tag{11}$$

$$M_2 = i_4 + a_2X + \varepsilon_4 \tag{12}$$

Equation 9 is identical to equation 5, only containing the independent variable X and dependent variable Y. From equation 10, c' is the population relationship between X and Y controlling for the mediators M_1 and M_2, b_1 is the population relationship between M_1 and Y controlling for M_2 and X, b_2 is the population relationship between M_2 and Y controlling for M_1 and X, and ε_2 is the variability in Y not explained by its relationships with X, M_1, and M_2. From equation 11, a_1 is the population relationship between M_1 and

X, and ε_3 is the variability in M_1 that is not explained by X. From equation 12, a_2 is the population relationship between M_2 and X, and ε_4 is the variability in M_2 that is not explained by X. The constant coefficients i_1, i_2, i_3, and i_4 are the intercepts for the equations.

Because the two mediators each mediate the relationship between X and Y separately, there are two mediated effects in this model. The mediated effect of M_1 is the product of the a_1 and b_1 paths, and the mediated effect of M_2 is the product of the a_2 and b_2 paths. Individually, these are the specific mediated effects (a_1b_1 and a_2b_2); together, they are the total mediated effect ($a_1b_1 + a_2b_2$). In this model, since c is still the total effect and c' is still the direct effect of X on Y controlling for the mediators, the total effect c is equal to the direct effect c' plus the total mediated effect, resulting in the following equation:

$$a_1b_1 + a_2b_2 = c - c' \tag{13}$$

Again, there are certain situations in which this equality does not hold, but it is true for the situations discussed here.

As the parallel two mediator model is a simple extension of the single mediator model, the same assumptions of the single mediator model regression equations apply to the parallel two mediator model regression equations. The parallel two mediator model is advantageous in that in can ameliorate the single mediator model by addressing the assumption of no omitted influences. However, the assumption of no interactions among variables can become problematic in models with multiple mediators, as the number of possible interactions among variables increases exponentially as the number of mediators included in a model increases.

Mediation: Significance Tests

MacKinnon, Lockwood, Hoffman, West, and Sheets (2002) identified three main approaches to significance testing in mediation: causal steps tests, product of coefficients tests, and difference in coefficients tests. These tests have been developed in depth for significance testing of the single mediator model, and with varying modifications they can also be used to test the significance of the parallel two mediator model (MacKinnon, 2008).

Causal steps. The most common approach to mediation in the social sciences is the causal steps approach, which stems from Judd and Kenny's (1981) pivotal work discussing the circumstances necessary to determine the mediated effect; these circumstances are also discussed in detail in Baron and Kenny's classic mediation article (1986). Judd and Kenny's requirements for determining mediation in the single mediator model stipulate that:

I. The effect of X on Y (*c* path) is significant.
II. The effect of X on M (*a* path) is significant.
III. The effect of M on Y controlling for X (*b* path) is significant.
IV. The effect of X on Y when adjusted for M (*c'* path) is not significant.

Baron and Kenny relax the requirements of the fourth condition, and only require that the first three conditions hold. MacKinnon et al. (2002) found that in a literature review of 200 articles involving mediation, the majority of them that used a formal test of mediation used the causal steps approach following Baron and Kenny. Additionally, *Social Sciences Citation Index* indicates that the Baron and Kenny article has been cited over 16000 times. MacKinnon et al. (2002) have also developed a joint significance test based on the causal steps approach which tests the significance of *a* and *b* coefficients separately.

The Baron and Kenny causal steps approach can also be used to test the parallel two mediator model, with some adjustment of the requirements. As there are two specific mediated effects, the second condition must hold for the effects of X on both M_1 and M_2 such that the a_1 and a_2 paths are significant. The third condition must also hold for the effects of M_1 on Y and M_2 on Y controlling for X and the other mediator, such that the b_1 and b_2 paths are significant. While it is possible to use this method of significance testing for the parallel two mediator model, there are several important limitations to this method when additional mediators are involved (MacKinnon, 2008, pp. 110-111). A test of joint significance for the parallel two mediator model has not yet been developed.

Product of coefficients. The second category of mediation tests look at the significance of the mediated effect *ab*, known as product of coefficients tests. The significance for the product of the coefficients is most often tested using a *z* test with a standard error derived by Sobel (1982), who used the multivariate delta method based on first derivatives.

$$s_{ab} = \sqrt{a^2 s_b^2 + b^2 s_a^2} \tag{14}$$

The product of coefficients can also be tested using confidence limits based on the product of two random normally distributed variables (MacKinnon, Fritz, Williams, & Lockwood, 2007; MacKinnon, Lockwood, & Williams, 2004).

The multivariate delta method can also be extended to derive the solution for the standard error of the total mediated effect for the parallel two mediator model (MacKinnon, 2008). From equation 13, the total mediated effect is $a_1b_1 + a_2b_2$, so the standard error for the total mediated effect is as follows:

$$s_{a_1b_1+a_2b_2} = \sqrt{s_{a_1}^2 b_1^2 + s_{b_1}^2 a_1^2 + s_{a_2}^2 b_2^2 + s_{b_2}^2 a_2^2 + 2a_1a_2s_{b_1b_2}} \quad (15)$$

The covariance between b_1 and b_2 is $s_{b_1b_2}$, and this value is necessary for computing the standard error.

Difference in coefficients. The third set of significance tests for mediation, the difference in coefficients tests, involve testing the mediated effect $c – c'$. These tests for mediation are more commonly used in the fields of medicine and epidemiology. Several standard errors for these tests have been derived for the single mediator model by McGuigan and Langholtz (1988), Freedman and Schatzkin (1992), and Clogg, Petkova, and Shihadeh (1992). MacKinnon (2008) provides a formula for the standard error of $c – c'$ for the parallel two mediator model as well.

Increasing Power Using Additional Variables

There has been extensive literature published on the inclusion of a third variable in an experimental design to increase the power of a study. The use of covariates to increase power has been particularly well-documented. One of the primary uses of a covariate in analysis of covariance (ANCOVA) is to increase the precision of a randomized experiment by reducing error variability (Cochran, 1957; Huck, 1972). Miller and Chapman (2001) agree that the main goal of using ANCOVA should be to increase power instead of using it to control for group differences, as is often done in practice.

Sometimes researchers wish to study the effect of X on an outcome Y, but Y may be difficult or costly to measure. Instead of looking directly to Y, they will find an intermediate endpoint Z that is a surrogate for Y such that the dependent variable X affects the intermediate endpoint Z as it would Y, and Z is a predictor of Y (Prentice, 1989, p. 432). Surrogate endpoints are often used in medical research, as the true endpoints of interest are more expensive or harder to measure (such as death rates of participants as a measure of mortality). For example, the presence of polyps has been used as a surrogate endpoint for the outcome of colon cancer (Freedman & Schatzkin, 1992).

Surrogate endpoints are also a way to increase the power of a study. As surrogate endpoints are used in place of outcomes that are difficult or costly to measure, measuring those true outcomes could reduce the sample size or effect size of the study (Prentice, 1989). Therefore, sample size or effect size can be increased through the use of surrogate endpoints (thereby increasing power).

Related to these concepts is the idea of an intensive design: including intermediate points of measurement and using the weighted average of those responses for each subject as an outcome instead of just one response (Kraemer & Thiemann, 1989). The intensive design increases power without requiring an increased sample size over a posttest-only randomized experimental design (Kraemer & Thiemann, 1989), and can also have increased power over a pretest-posttest design given certain conditions, although this design does require more measurement points (Maxwell, 1998; Venter, Maxwell, & Bolig, 2002).

Extension: Increasing Power Using a Mediator

In light of this research, it follows that the inclusion of a mediating variable in a model would lead to increased power. MacKinnon et al. (2002) compared all known methods of testing mediation and identified the tests with the best power and Type I error rates, and found that due to the requirement of a significant X to Y relationship, the Baron and Kenny and Judd and Kenny causal steps tests for mediation are underpowered. Fritz and MacKinnon (2007) replicated this result, finding that when *a* and *b* paths were small and *c'* was zero, the sample size required to detect the mediated effect at 0.8 power for the Baron and Kenny test was 20886. Given that some tests of mediation have the ability to detect effects when the relationship between X and Y is nonsignificant, it follows that including a mediator can increase power over the bivariate model in some situations. MacKinnon (2008, pp. 394-395) and Fritz, Cox, and MacKinnon (2012) discuss this as well. With these findings in mind, the next logical step will be to determine the details of when this effect occurs for the inclusion of a single mediator, and to extend this idea through the inclusion of multiple mediators. The intention of this paper is to determine when a mediation model that includes one mediator or two parallel mediators will be more powerful than a model which only examines the relationship between X and Y.

Hypotheses

1) There are systematic differences between combinations of parameters and sample size where the test of X to M to Y is more powerful than the test of a bivariate relationship X to Y.
2) The increased power when a mediator is added will extend to the parallel two mediator model.

Method

Single Mediator Model Empirical Power

A Monte Carlo simulation was used to compute empirical power for the test of the total effect, the product of coefficients test of mediation for the single mediator model, and the joint significance test of mediation for the single mediator model. SAS syntax (Version 9.2 of the SAS System for Windows) for the simulation can be found in Appendix A. In addition, a summary of the following steps is included in Appendix B. A macro was designed to loop through 384 different combinations of population sample sizes (50, 100, 200, 500, 1000, and 5000) and population path parameters (0, 0.14, 0.39, and 0.59) for each of the *a*, *b*, and *c'* paths. Population path parameters were chosen in accordance with those used in prior research on mediation models (Fritz & MacKinnon, 2007; MacKinnon et al., 2002). PROC IML was then used with the RANNOR function within the macro to generate random data with normally distributed residuals for the independent variable X. Data for X can be represented with the following equation:

$$\mathrm{X}_{\mathrm{i}} = e_1 \tag{16}$$

The program then used the mediation regression equations to create M and Y by generating normally distributed residuals for each variable using RANNOR, and using those residuals along with the predetermined parameters and generated X variable in the mediation equations.

The data vectors for X, M, and Y were combined into a matrix and a data set was formed from that matrix. All three variables generated were continuous. The parameter estimates, their standard errors, and *p* values were then estimated from the simulated data in a series of regression analyses using the following regression equations:

$$\mathrm{Y_i} = \hat{i}_1 + \hat{c}\mathrm{X_i} + \hat{e}_1 \tag{17}$$

$$\mathrm{Y_i} = \hat{i}_2 + \hat{c}'\mathrm{X_i} + \hat{b}\mathrm{M_i} + \hat{e}_2 \tag{18}$$

$$\mathrm{M_i} = \hat{i}_3 + \hat{a}\mathrm{X_i} + \hat{e}_3 \tag{19}$$

Where $\hat{c}$ is the sample estimate for the relationship between X and Y, $\hat{a}$ is the sample estimate for the relationship between X and M, $\hat{b}$ is the sample estimate for the relationship between M and Y, and $\hat{c}'$ is the sample estimate for the relationship between X and Y controlling for M. The sample intercepts are represented here as $\hat{i}_1$, $\hat{i}_2$, and $\hat{i}_3$, and the sample error variability for the equations are $\hat{e}_1$, $\hat{e}_2$, and $\hat{e}_3$.

Each set of results was saved into individual data sets and then compiled into a single new data set. Within the new data set, several variables were created to conduct significance tests and generate power values. Two tests of mediation were included: The product of coefficients test using the multivariate delta standard error of the indirect effect *ab*, and the joint significance test testing *a* and *b* separately. These were used based on findings from MacKinnon et al. (2002), which listed the multivariate delta method as the most commonly used product of coefficients test and found the joint significance test to have a good balance of power and Type I error. Both tests included were two-tailed tests. The product of coefficients test was conducted by dividing the product of generated estimates $\hat{a}\hat{b}$ by its multivariate delta standard error:

$$z_{\hat{a}\hat{b}} = \frac{\hat{a}\hat{b}}{s_{\hat{a}\hat{b}}} \qquad (20)$$

A z test was then conducted by comparing the absolute value of $z_{\hat{a}\hat{b}}$ to 1.96 (the 97.5 percentile under the normal distribution). The hypotheses for the product of coefficients test are as follows:

H_0: $ab = 0$

H_1: $ab \neq 0$

The joint significance test was conducted by testing the significance of the $\hat{a}$ and $\hat{b}$ coefficients separately. For each coefficient, the saved p value from the regression analysis was compared to 0.05. The hypotheses for the joint significance test are as follows:

H_0: $a = b = 0$

H_1: $a \neq 0$ and $b \neq 0$ or $a = 0$ and $b \neq 0$ or $a \neq 0$ and $b = 0$

Three binary variables were then created to generate empirical power values for the test of the total effect, the product of coefficients test, and the joint significance test. For each of the three tests, a binary variable equaled zero for a nonsignificant test and one for a significant test.

The process of generating data, performing regression analyses, and testing for mediation was repeated for a total of 1000 replications for each combination of population parameters and sample size. The mean for each binary significance variable is the proportion of times out of 1000 that the test was significant (the power value for combinations with non-zero population path parameters, and the Type I error value for combinations with population path parameters equal to zero).

Single Mediator Model Analytical Power

In addition to estimating empirical power of the single mediator model, it is possible to compute analytical power for the single mediator model. The SAS program to compute analytical power for the single mediator model can be found in Appendix C. For the single mediator model, the program calculated population variances and covariances for X, M, and Y based on different combinations of population path parameters and sample sizes (the same ones used in the empirical simulation). The covariance matrix for the single mediator model can be found in MacKinnon (2008). Those variances and covariances were then used to calculate zero-order and first-order partial correlation effect sizes corresponding to the population *a* and *b* paths as found in MacKinnon (2008):

$$\rho_{\mathrm{XM}} = \frac{\sigma_{\mathrm{XM}}}{\sigma_{\mathrm{X}}\sigma_{\mathrm{M}}} \tag{21}$$

$$\rho_{\mathrm{YM.X}} = \frac{\rho_{\mathrm{MY}} - \rho_{\mathrm{XY}}\rho_{\mathrm{XM}}}{\sqrt{(1-\rho_{\mathrm{XY}}^2)(1-\rho_{\mathrm{XM}}^2)}} \tag{22}$$

Where ρ_{XM} is the correlation effect size corresponding to the *a* coefficient and $\rho_{\mathrm{YM.X}}$ is the first-order partial correlation effect size corresponding to the *b* coefficient. The first-order partial correlation effect size corresponding to the *c’* coefficient would be $\rho_{\mathrm{YX.M}}$.

The variances and covariances for the single mediator model were also used to calculate the true *a* and *b* coefficients and their true standard errors:

$$a = \frac{\sigma_{\mathrm{XM}}}{\sigma_{\mathrm{X}}^2} \tag{23}$$

$$\sigma_a = \sqrt{\frac{\sigma_{\varepsilon_3}^2}{(N-1)\sigma_{\mathrm{X}}^2}} \tag{24}$$

$$b = \frac{\sigma_{\mathrm{X}}^2\sigma_{\mathrm{MY}} - \sigma_{\mathrm{XM}}\sigma_{\mathrm{XY}}}{\sigma_{\mathrm{X}}^2\sigma_{\mathrm{M}}^2 - \sigma_{\mathrm{XM}}^2} \tag{25}$$

$$\sigma_b = \sqrt{\frac{\sigma_{\varepsilon_2}^2}{(N-1)\sigma_{\mathrm{M}}^2(1-\rho_{\mathrm{XM}}^2)}} \tag{26}$$

From the above standard error equations, $\sigma_{\varepsilon_3}^2$ is the population error variance from the equation with X predicting M and $\sigma_{\varepsilon_2}^2$ is the population error variance from the equation with X and M predicting Y. The values chosen for the *a*, *b*, and *c'* population parameters in the single mediator model are path coefficients corresponding approximately to Cohen's small, medium, and large effect sizes (MacKinnon et al., 2002).

Power was then calculated in two ways. First the correlations above corresponding to *a* and *b* were used to calculate a *z* test, which was then used to compute analytical power as described in the introduction of this paper. Then the path coefficients *a* and *b* and their true standard errors were each used to calculate a *t* test which was used to compute analytical power, also described above. Analytical formulas for power of the joint significance test of mediation for the single mediator model can be found in Wang and Xue (2012). For both methods of computing analytical power, the individual power values for *a* and *b* were then multiplied to calculate analytical power, analogous to the joint significance test. Analytical power of the total mediated effect *ab* cannot be computed using the methods just described, as the distribution of the product of two random normally distributed variables is not normal (Aroian, 1944; Craig, 1936). However, it can be approximated by forming a *z* score using the values of *ab* and s_{ab}. The formula used is as follows (Wang & Xue, 2012):

$$\pi_{ab} = 1 - \Phi(z_{1-(\alpha/2)} - \frac{ab}{s_{ab}}) \quad (27)$$

Covariance Matrix for the Parallel Two Mediator Model

In order to derive true formulas for the regression coefficients, standard errors of regression coefficients, and effect sizes based on correlations for the parallel two mediator model, it was necessary to first derive the parallel two mediator model covariance matrix, which is shown in Table 1.

Insert Table 1 about here

Covariance algebra was used to derive formulas for the variance of each variable in the parallel two mediator model, and the covariances between each pair of variables. For the full derivation of the covariance matrix, see Appendix D. Syntax to compute the true variances and covariances of the variables in SAS can be found in Appendix I.

True Path Coefficients and Standard Errors for the Parallel Two Mediator Model

After using covariance algebra to derive the covariance matrix for the parallel two mediator model, the variances and covariances were used to derive formulas for the true path coefficients and standard errors of the parallel two mediator model in terms of variable variances and covariances. The formulas for the true path coefficients are as follows:

$$a_1 = \frac{\sigma_{\mathrm{XM_1}}}{\sigma_{\mathrm{X}}^2} \tag{27}$$

$$b_1 = \frac{\sigma_{\mathrm{M_1Y}} - c'\sigma_{\mathrm{XM_1}} - b_2\sigma_{\mathrm{M_1M_2}}}{\sigma_{\mathrm{M_1}}^2} \tag{28}$$

$$a_2 = \frac{\sigma_{\mathrm{XM_2}}}{\sigma_{\mathrm{X}}^2} \tag{29}$$

$$b_2 = \frac{\sigma_{\mathrm{M_2Y}} - c'\sigma_{\mathrm{XM_2}} - b_1\sigma_{\mathrm{M_1M_2}}}{\sigma_{\mathrm{M_2}}^2} \tag{30}$$

$$c' = \frac{\sigma_{\mathrm{XY}} - b_1\sigma_{\mathrm{XM_1}} - b_2\sigma_{\mathrm{XM_2}}}{\sigma_{\mathrm{X}}^2} \tag{31}$$

For the full derivation of the regression coefficients, see Appendix E. The formulas for the standard errors of the regression coefficients required some additional work, as the mean squared errors of the regression equations for X predicting M_1 and X predicting M_2 are in the numerator of the standard errors of a_1 and a_2. In addition, several squared multiple correlations were part of the formulas for the standard errors of b_1, b_2, and c'. The formulas for the three error variances from the parallel two mediator model regression equations were found using the covariances between variables:

$$\sigma_{\varepsilon_2}^2 = \sigma_{\mathrm{Y}}^2 - c'\sigma_{\mathrm{X}}^2 - 2b_1c'\sigma_{\mathrm{XM_1}} - 2b_2c'\sigma_{\mathrm{XM_2}} - 2b_1b_2\sigma_{\mathrm{M_1M_2}} - b_1^2\sigma_{\mathrm{M_1}}^2 - b_2^2\sigma_{\mathrm{M_2}}^2 \tag{32}$$

$$\sigma_{\varepsilon_3}^2 = \sigma_{\mathrm{M_1}}^2 - a_1^2\sigma_{\mathrm{X}}^2 \tag{33}$$

$$\sigma_{\varepsilon_4}^2 = \sigma_{\mathrm{M_2}}^2 - a_2^2\sigma_{\mathrm{X}}^2 \tag{34}$$

In the above equations, $\sigma^2_{\varepsilon_2}$ is the population error variance of the regression equation where Y is predicted by X, M_1, and M_2, $\sigma^2_{\varepsilon_3}$ is the population error variance of the regression equation where M_1 is predicted by X, and $\sigma^2_{\varepsilon_4}$ is the population error variance of the regression equation where M_2 is predicted by X.

The formula for the squared multiple correlation uses standardized regression coefficients and bivariate correlations, and the formulas for multiple correlations between predictors use bivariate correlations between predictors only:

$$R^2_{\mathrm{Y.X,M_1,M_2}} = c'^{*}(\rho_{\mathrm{YX}}) + b^*_1(\rho_{\mathrm{M_1Y}}) + b^*_2(\rho_{\mathrm{M_2Y}}) \tag{35}$$

$$R^2_{\mathrm{X.M_1,M_2}} = \frac{\rho^2_{\mathrm{M_1X}} + \rho^2_{\mathrm{M_2X}} - 2\rho_{\mathrm{M_1M_2}}\rho_{\mathrm{M_1X}}\rho_{\mathrm{M_2X}}}{1-\rho^2_{\mathrm{M_1M_2}}} \tag{36}$$

$$R^2_{\mathrm{M_1.X,M_2}} = \frac{\rho^2_{\mathrm{M_1X}} + \rho^2_{\mathrm{M_1M_2}} - 2\rho_{\mathrm{M_1M_2}}\rho_{\mathrm{M_1X}}\rho_{\mathrm{M_2X}}}{1-\rho^2_{M_2X}} \tag{37}$$

$$R^2_{\mathrm{M_2.X,M_1}} = \frac{\rho^2_{\mathrm{M_2X}} + \rho^2_{\mathrm{M_1M_2}} - 2\rho_{\mathrm{M_1M_2}}\rho_{\mathrm{M_1X}}\rho_{\mathrm{M_2X}}}{1-\rho^2_{M_1X}} \tag{38}$$

Where *c'**, *b₁**, and *b₂** are true standardized regression coefficients. Using the above formulas, the standard errors of the regression coefficients can be computed. The standard errors for the regression coefficients are as follows:

$$\sigma_{a_1} = \sqrt{\frac{\sigma^2_{\varepsilon_3}}{(N-1)\sigma^2_{\mathrm{X}}}} \tag{39}$$

$$\sigma_{b_1} = \frac{\sigma_{\mathrm{Y}}}{\sigma_{\mathrm{M_1}}}\sqrt{\frac{1}{1-R^2_{\mathrm{M_1.X,M_2}}}}\sqrt{\frac{1-R^2_{\mathrm{Y.X,M_1,M_2}}}{N - 3 - 1}} \tag{40}$$

$$\sigma_{a_2} = \sqrt{\frac{\sigma_{\varepsilon_4}^2}{(N-1)\sigma_{\mathrm{X}}^2}} \tag{41}$$

$$\sigma_{b_2} = \frac{\sigma_{\mathrm{Y}}}{\sigma_{\mathrm{M_2}}}\sqrt{\frac{1}{1-R_{\mathrm{M_2.X,M_1}}^2}}\sqrt{\frac{1-R_{\mathrm{Y.X,M_1,M_2}}^2}{N\text{ - }3\text{ - }1}} \tag{42}$$

$$\sigma_{c'} = \frac{\sigma_{\mathrm{Y}}}{\sigma_{\mathrm{X}}}\sqrt{\frac{1}{1-R_{\mathrm{X.M_1,M_2}}^2}}\sqrt{\frac{1-R_{\mathrm{Y.X,M_1,M_2}}^2}{N\text{ - }3\text{ - }1}} \tag{43}$$

Effect Sizes for the Parallel Two Mediator Model

The variances and covariances were then used to derive full and partial correlation effect sizes for the parallel two mediator model. Formulas for the effect sizes are as follows:

$$\rho_{\mathrm{XM_1}} = \frac{\sigma_{\mathrm{XM_1}}}{\sqrt{\sigma_{\mathrm{X}}^2}\sqrt{\sigma_{\mathrm{M_1}}^2}} \tag{44}$$

$$\rho_{\mathrm{M_1Y.XM_2}} = \frac{\rho_{\mathrm{M_1Y.X}} - \rho_{\mathrm{M_1M_2.X}}\rho_{\mathrm{M_2Y.X}}}{\sqrt{1-\rho_{\mathrm{M_1M_2.X}}^2}\sqrt{1-\rho_{\mathrm{M_2Y.X}}^2}} \tag{45}$$

$$\rho_{\mathrm{XM_2}} = \frac{\sigma_{\mathrm{XM_2}}}{\sqrt{\sigma_{\mathrm{X}}^2}\sqrt{\sigma_{\mathrm{M_2}}^2}} \tag{46}$$

$$\rho_{\mathrm{M_2Y.XM_1}} = \frac{\rho_{\mathrm{M_2Y.X}} - \rho_{\mathrm{M_1M_2.X}}\rho_{\mathrm{M_1Y.X}}}{\sqrt{1-\rho_{\mathrm{M_1M_2.X}}^2}\sqrt{1-\rho_{\mathrm{M_1Y.X}}^2}} \tag{47}$$

$$\rho_{\mathrm{XY.M_1M_2}} = \frac{\rho_{\mathrm{XY.M_1}} - \rho_{\mathrm{XM_2.M_1}}\rho_{\mathrm{YM_2.M_1}}}{\sqrt{1-\rho_{\mathrm{XM_2.M_1}}^2}\sqrt{1-\rho_{\mathrm{YM_2.M_1}}^2}} \tag{48}$$

Where equation 44 is the zero-order correlation effect size for a_1, equation 45 is the second-order partial correlation effect size for b_1, equation 46 is the zero-order correlation effect size for a_2, equation 47 is the second-order partial correlation effect size for b_2, and equation 48 is the second-order partial correlation effect size for *c'*. Appendix F shows how the formulas can be produced from the variances and covariances between variables.

Using the above formulas for correlation effect sizes as well as the formulas for regression coefficients derived from the covariance matrix, a program was written to determine the size of the regression coefficients for this model that would correspond approximately to Cohen's small, medium, and large correlation effect sizes. The program first calculated variances and covariances and then correlation and second-order partial correlation effect sizes for the parallel two mediator model using path parameters. After iterating through, the parameter values were manually changed to adjust the correlation results to be as close to Cohen's guidelines for small, medium, and large effect sizes as possible. The program determined that for a_1, b_1, a_2, and b_2, a path coefficient of 0.101 corresponded to a small effect, a path coefficient of 0.314 corresponded to a medium effect, and a path coefficient of 0.577 corresponded to a large effect. For *c'*, path coefficients of 0.131, 0.400, and 0.740 corresponded to small, medium, and large effects. A SAS program that computes these values is shown in Appendix G. These path coefficients are necessary to set population path parameters for analytical calculation of power, and for generation of data to calculate empirical power for the parallel two mediator model.

Parallel Two Mediator Model Empirical Power

For the parallel two mediator model, a simulation was written to generate empirical power for the test of the total effect *c* and the test of the total mediated effect $a_1b_1 + a_2b_2$ over 500 replications. SAS syntax for the parallel two mediator model simulation can be found in Appendix H. A macro was designed to loop through 5120 different combinations of population sample sizes (50, 100, 200, 500, and 1000) and population path parameters (for a_1, b_1, a_2, and b_2: 0, 0.101, 0.314, and 0.577; for c': 0, 0.131, 0.400, and 0.740). PROC IML was then used with the RANNOR function within the macro to generate random data with normally distributed residuals for X. Data for M_1, M_2, and Y were generated by creating normally distributed residuals for each using RANNOR. The data vectors for X, M_1, M_2, and Y were then concatenated into a single matrix to form a data set. All four variables generated were continuous.

The parameter estimates, their standard errors, the covariance between the $\hat{b}_1$ and $\hat{b}_2$ estimates, and *p* values were then estimated from the simulated data in a series of regression analyses using the following regression equations:

$$\mathrm{Y}_\mathrm{i} = \hat{i}_1 + \hat{c}\mathrm{X}_\mathrm{i} + \hat{e}_1 \qquad (49)$$

$$\mathrm{Y}_\mathrm{i} = \hat{i}_2 + \hat{c}'\mathrm{X}_\mathrm{i} + \hat{b}_1\mathrm{M}_{1\mathrm{i}} + \hat{b}_2\mathrm{M}_{2\mathrm{i}} + \hat{e}_2 \qquad (50)$$

$$\mathrm{M}_{1\mathrm{i}} = \hat{i}_3 + \hat{a}_1\mathrm{X}_\mathrm{i} + \hat{e}_3 \qquad (51)$$

$$\mathrm{M}_{2\mathrm{i}} = \hat{i}_4 + \hat{a}_2\mathrm{X}_\mathrm{i} + \hat{e}_4 \qquad (52)$$

Where $\hat{c}$ is the sample estimate for the relationship between X and Y, $\hat{a}_1$ is the sample estimate for the relationship between X and M_1, $\hat{a}_2$ is the sample estimate for the relationship between X and M_2, $\hat{b}_1$ is the sample estimate for the relationship between M_1 and Y, $\hat{b}_2$ is the sample estimate for the relationship between M_2 and Y, and $\hat{c}'$ is the sample estimate for the relationship between X and Y controlling for the mediators. The sample intercepts are represented here as $\hat{i}_1$, $\hat{i}_2$, $\hat{i}_3$, and $\hat{i}_4$, and the sample error variability for the equations are $\hat{e}_1$, $\hat{e}_2$, $\hat{e}_3$, and $\hat{e}_4$.

Each set of results was saved into individual data sets and then compiled into a single new data set. Within the new data set, several variables were created to conduct significance tests and generate power values. The two-tailed test of the total mediated effect was conducted by dividing the sum of the specific mediated effect estimates $\hat{a}_1\hat{b}_1 + \hat{a}_2\hat{b}_2$ by its multivariate delta standard error, as it was with the single mediator model:

$$z_{\hat{a}_1\hat{b}_1+\hat{a}_2\hat{b}_2} = \frac{\hat{a}_1\hat{b}_1 + \hat{a}_2\hat{b}_2}{s_{\hat{a}_1\hat{b}_1+\hat{a}_2\hat{b}_2}} \tag{53}$$

The absolute value of the *z* statistic was then compared to the 97.5 percentile cutoff on the normal distribution. The hypotheses for the test of the total mediated effect are as follows:

H_0: $a_1b_1 + a_2b_2 = 0$

H_1: $a_1b_1 + a_2b_2 \neq 0$

Two binary variables were then created to generate empirical power values for the test of the total effect and the test of the total mediated effect. Each time a test was performed, a binary variable would be equal to zero for a nonsignificant test and equal to one for a significant test.

In addition to the test of the total mediated effect, the two specific mediated effects a_1b_1 and a_2b_2 were tested for significance using both the product of coefficients and joint significance tests. The hypotheses for the product of coefficients tests of the specific mediated effects are as follows:

H_0: $a_1 b_1 = 0$

H_1: $a_1 b_1 \neq 0$

H_0: $a_2 b_2 = 0$

H_1: $a_2 b_2 \neq 0$

The hypotheses for the joint significance test of the specific mediated effects are as follows:

H_0: $a_1 = 0$ and $b_1 = 0$

H_1: $a_1 \neq 0$ and $b_1 \neq 0$, or $a_1 = 0$ and $b_1 \neq 0$, or $a_1 \neq 0$ and $b_1 = 0$

H_0: $a_2 = 0$ and $b_2 = 0$

H_1: $a_2 \neq 0$ and $b_2 \neq 0$, or $a_2 = 0$ and $b_2 \neq 0$, or $a_2 \neq 0$ and $b_2 = 0$

Binary variables were created for each test for the specific mediated effects in the same way they were for the single mediator model.

The process of generating data, performing regression analyses, and testing for significance was repeated a total of 500 times for each combination of population parameters and sample size. The mean for each created binary variable is the proportion of times out of 500 that the test was significant (the power value for combinations with non-zero population path parameters and the Type I error value for combinations with population path parameters equal to zero).

Parallel Two Mediator Model Analytical Power

A program was also written to calculate analytical power of the parallel two mediator model. Syntax for the program is shown in Appendix I. For the parallel two mediator model, the program calculated population variances and covariances of X, M_1, M_2, and Y based on the combinations of parameters and sample sizes used in the empirical simulation. Those variances and covariances were then used to calculate zero-order, first-order, and second-order partial correlations, and the true formulas for each path coefficient and the standard errors of the coefficients. Power was then calculated for the specific mediated effects using both of the methods for calculating analytical power that were used in the single mediator model. Power for the total mediated effect was computed by dividing the sum of the specific mediated effects by the two mediator model multivariate delta standard error to calculate a *t* value. The *t* value was then used as the noncentrality parameter to calculate power as described in the introduction of this paper.

Results

Empirical Single Mediator Model Results

Empirical power values for combinations of parameters and sample size where power of the joint significance test exceeds power of the test of the total effect are shown in Appendix J. Empirical power values for combinations of parameters and sample size where power of the product of coefficients test exceeds power of the test of the total effect are shown in Appendix K. Of the 384 combinations of parameters and sample sizes, the joint significance test had greater power than the test of the total effect 64 times, and the product of coefficients test had greater power than the test of the total effect 53 times. Results indicate that the test of joint significance had more power than the product of coefficients test, as found earlier (MacKinnon et al., 2002; Fritz & MacKinnon, 2007). For all cases where power for the joint significance and product of coefficients tests was greater than power for the test of the total effect, the direct effect (*c'*) was always zero or small; when *c'* is equal to zero this indicates full mediation.

When *a* and *b* were small (*a* = *b* = .14), the joint significance and product of coefficients tests had more power than the test of the total effect at larger sample sizes. The test of the total effect had more power than the joint significance and product of coefficients tests at smaller sample sizes when *a* and *b* were small. For example, in Appendix J for the case where *a* and *b* were small (*a* = *b* = .14), the test of the total effect had more power than the joint significance test for a sample size of 50. The joint significance test had more power than the test of the total effect in every other sample size. In Appendix K for the case where *a* = *b* = .14, the test of the total effect had more power than the product of coefficients test until sample size reached 200. However, when *a* and *b* were large (*a* = *b* = .59), the joint significance and product of coefficients tests had more power than the test of the total effect at smaller sample sizes. In both Appendices J and K for *a* = *b* = .59, the joint significance and product of coefficients tests had more power than the test of the total effect in sample sizes up to 200. At sample sizes larger than 200, the test of the total effect and the joint significance and product of coefficients tests all had power of approximately one for large *a* and *b*.

In summary, two patterns of results emerged for the single mediator model. The joint significance and product of coefficients tests had more power than the test of the total effect when sample size was large and effects were small, and when sample size was small and effects were large. Results from the first empirical simulation provide support for the first hypothesis that the significance tests for the single mediator model systematically have more power than the test of the total effect.

Empirical Parallel Two Mediator Model Results

Logistic regression was used to summarize the results of the two mediator model simulation for all 2560000 datasets generated from the 500 replications of 5120 combinations of parameters and sample size. For each dataset, a variable was created which was equal to one if the test of the total mediated effect was significant and the test of the total effect was not, and was otherwise equal to zero. Frequencies were obtained for both values of this variable for each combination of population parameters and sample size, such that there was a value for the frequency of zeros and a value for the frequency of ones for each of the 5120 combinations. These were used to create a new dataset with 10240 observations. A weighted logistic regression with a binary dependent variable was then conducted. The predictors were centered population values of a_1, b_1, a_2, b_2, c', and N, with all interactions between predictors included. The frequency of the dependent variable for each combination of parameters and sample size was included as a weight. Syntax for the weighted logistic regression is shown in Appendix L.

Results for the main effects and interactions are shown in Appendix M. Because of the large number of main effects and interactions analyzed and the very large sample size, effects are considered important if $p < 0.0001$. The highest order important interactions were four-way interactions, so those are interpreted first.

Of the important four-way interactions, the interactions that most clearly describe the results are the interaction between a_1, b_1, c', and N and the interaction between a_2, b_2, *c'*, and *N*. A series of graphs demonstrating this interaction is shown in Figure 3, panels A to E.

- Insert Figure 3 about here -

The $a_1*b_1*c'*N$ interaction shows that as a_1 and b_1 increased and *N* decreased, the proportion of cases where the test of the total mediated effect was significant and the test of the total effect was not significant increased, but only when *c'* was zero or small, $\chi^2(1, N = 2559999) = 112.3053$, $p < 0.0001$. When *c'* was medium or large, the proportion of cases where the test of the total mediated effect was significant and the test of the total effect was not significant dropped to zero for all combinations of a_1 and b_1 parameters and sample size. The same pattern of results also held true for the $a_2*b_2*c'*N$ interaction, but for coefficients a_2 and b_2, $\chi^2(1, N = 2559999) = 86.5141$, $p < 0.0001$. A series of graphs demonstrating this interaction is shown in Figure 4, panels A to E.

- Insert Figure 4 about here -

Another way to interpret the interactions would be to look at the behavior of the individual coefficients across sample sizes. It appears that when *c'* is zero or small, there were a larger proportion of cases where the test of the total mediated effect was significant and the test of the total effect was not significant when the a_1, a_2, b_1, and b_2 coefficients were large and sample size was small. In addition, there were a larger proportion of cases where the test of the total mediated effect was significant and the test of the total effect was not significant when the coefficients were small and sample size was large. For example, in Figure 4a. where $N = 50$, $a_2 = b_2 = 0.577$, $c' = 0$, the proportion of cases where the test of the total mediated effect was significant and the test of the total effect was not significant was 0.29, the largest proportion at that combination of parameters and sample size. However, for the same combination of parameters, the proportion dropped across sample sizes to 0.14 at $N = 100$, 0.01 at $N = 200$, and zero at larger sample sizes. Conversely, in Figure 4a. where $N = 50$, $a_2 = b_2 = 0.101$, $c' = 0$, the proportion of cases where the test of the total mediated effect was significant and the test of the total effect was not significant was 0.07, and the proportion for that combination of parameters increased across sample sizes to 0.13 at $N = 100$, 0.16 at $N = 200$, 0.25 at $N = 500$, and 0.31 at $N = 1000$.

Tables 2 to 6 contain empirical power values for both the test of the total mediated effect for the parallel two mediator model and the test of the total effect where $c' = 0$ or 0.131 (corresponding to zero or small effect sizes for c', which were the only values of c' at which the test of the total mediated effect was found to have more power than the test of the total effect).

Insert Tables 2-6 about here

Combinations of parameters and sample size where the test of the total mediated effect was more powerful than the test of the total effect are highlighted in bold red. It appears that the test of the total mediated effect was more powerful than the test of the total effect when effect sizes were large at smaller sample sizes, and when effect sizes were small at larger sample sizes, as was true for the single mediator model. For example, the test of the total mediated effect was more powerful than the test of the total effect when the a_1, a_2, b_1, b_2 coefficients were all large for $N = 50$, but both power values were equal to one for all larger sample sizes. However, the test of the total mediated effect was more powerful than the test of the total effect when the a_1, a_2, b_1, b_2 coefficients were small for N = 200 and above. In addition, there were more combinations of parameters and sample size where the test of the total mediated effect was more powerful than the test of the total effect at lower sample sizes because as sample size increased, the power of both tests approached one for the larger effect sizes.

Beyond looking at combinations of parameters and sample size where the test of the total mediated effect was more powerful than the test of the total effect, it is of interest to look at combinations where the test of the total mediated effect exceeded adequate power of 0.80 (Cohen, 1988) and the test of the total effect did not. In Tables 2 to 6, those cases are highlighted in bold blue. There are 204 combinations where this occurs, and for each sample size it is relatively systematic. The tables show that the general trend described above, which applies to all combinations of parameters and sample size where power of the test of the total mediated effect exceeds power of the test of the total effect, is more pronounced for combinations where the test of the total mediated effect exceeded adequate power of 0.80 and the test of the total effect did not.

The logistic regression results showed that as effect size increased and sample size decreased, there was a larger proportion of cases where the test of the total mediated effect was significant and the test of the total effect was not significant (but only when *c'* was zero or small). When *c'* was zero or small, empirical power was greater for the test of the total mediated effect than for the test of the total effect in two cases: (1) when effect sizes were small and sample size was large, and (2) when effect sizes were large and sample size was small. Looking at the combinations where empirical power exceeded 0.80 for the test of the total mediated effect, and not for the test of the total effect, revealed that these combinations followed the same (albeit more pronounced) general trend that all combinations did. Results from the second empirical simulation provide support for the second hypothesis that the test of the parallel two mediator model systematically has more power than the test of the total effect.

Comparison of Analytical and Empirical Power: Single Mediator Model

Table 7 shows a comparison of results for analytical and empirical power of the joint significance test for the single mediator model, collapsed across levels of *c'*.

Insert Table 7 about here

Only power of the models is considered in this table (combinations where *a* or *b* is zero are measures of Type I error rate). Analytical power does not vary across levels of *c'*, so empirical power has been averaged across levels of *c'* for comparison. Empirical power is compared to analytical power calculated both using correlation coefficients and regression coefficients.

All three methods of calculating power yielded similar results. The largest discrepancy between the two methods of calculating analytical power was never greater than an absolute value of 0.02. The largest discrepancy between empirical power and analytical power calculated using correlation coefficients was 0.032 at N = 200, $a = b =$ 0.14. The largest discrepancy between empirical power and analytical power calculated using regression coefficients was 0.041 at N = 50, $a = b =$ 0.39. The discrepancies between empirical and analytical power decreased as sample size increased. Results from the program to compute analytical power of the single mediator model are very close to results from the empirical simulation, indicating that both programs produced relatively accurate power values for the joint significance test of the single mediator model.

Table 8 shows a comparison of results for analytical and empirical power of the product of coefficients test for the single mediator model, collapsed across levels of *c'*.

Insert Table 8 about here

Only combinations where all coefficients are greater than zero are shown (*a* and *b* at 0.14, 0.39, and 0.59). There were discrepancies between empirical and analytical power of the product of coefficients test for the single mediator model due to the way analytical power was calculated. Analytical power was smaller than empirical power by at least 0.03 for 6 combinations of parameters and sample size, and empirical power was smaller than analytical power by at least 0.03 for 11 combinations of parameters and sample size. When analytical power was smaller than empirical power, the greatest difference in power values was 0.097 at $N = 100$, $a = b = 0.39$. When empirical power was smaller than analytical power, the greatest difference in power values was 0.202 at $N = 200$, $a = b = 0.14$.

Differences between analytical and empirical power of the product of coefficients test for the single mediator model occurred because the method of computing analytical power that was used does not necessarily extend to functions of variables, as it assumes a normal distribution underlying variables. The distribution underlying the product of two variables is non-normal, but it approximates normal as effect size and sample size increase (Springer, 1979). When sample size and effect sizes are large, analytical power and empirical power should become very similar, and Table 8 indicates that they do.

Comparison of Analytical and Empirical Power: Parallel Two Mediator Model

Total mediated effect. Table 9 shows a comparison of results for analytical and empirical power of the test of the total mediated effect for the parallel two mediator model, collapsed across levels of *c'*.

Insert Table 9 about here

Only combinations where all coefficients from the specific mediated effects are greater than zero are shown (a_1, b_1, a_2, and b_2 at 0.101, 0.314, and 0.577). There were large discrepancies between empirical and analytical power of the test of the total mediated effect for the parallel two mediator model due to the way analytical power was calculated. Analytical power was smaller than empirical power by at least 0.03 for 163 combinations of parameters and sample size, and empirical power was smaller than analytical power by at least 0.03 for 36 combinations of parameters and sample size. When analytical power was smaller than empirical power, the greatest difference in power values was 0.306 at $N = 100$, $a_1 = 0.577$, $b_1 = 0.314$, $a_2 = 0.577$, and $b_2 = 0.101$. When empirical power was smaller than analytical power, the greatest difference in power values was 0.146 at $N = 50$, $a_1 = b_1 = 0.314$, $a_2 = b_2 = 0.101$.

As with the single mediator model, differences between analytical and empirical power of the test of the total mediated effect for the parallel two mediator model occurred because the method of computing analytical power that was used does not necessarily extend to functions of variables, as it assumes a normal distribution underlying variables. However, Table 9 indicates that when sample size and effect sizes are large, analytical power and empirical power become very similar, as they are expected to.

Specific mediated effects. Table 10 shows a comparison of results for analytical and empirical power of the joint significance test of the specific mediated effect a_1b_1 for the parallel two mediator model, collapsed across levels of a_2, b_2, and c'.

Insert Table 10 about here

Analytical power for a_1b_1 does not vary across levels of a_2, b_2, or c', so empirical power has been averaged across levels of a_2, b_2, and c' for comparison. Results show that the comparison of analytical and empirical power for the specific mediated effect a_1b_1 is similar to the comparison of power for the single mediator model, in that the three methods of calculating power yield very similar results. For a_1b_1, the largest discrepancy between the two methods of calculating analytical power was never greater than an absolute value of 0.018. The largest discrepancy between empirical power and analytical power calculated using correlation coefficients was 0.007 at $N = 200$, $a_1 = 0.101$, $b_1 = 0.14$. The largest discrepancy between empirical power and analytical power calculated using regression coefficients was -0.023 at $N = 50$, $a_1 = b_1 = 0.577$.

Table 11 shows a comparison of results for analytical and empirical power of the joint significance test of the specific mediated effect a_2b_2 for the parallel two mediator model, collapsed across levels of a_1, b_1, and c'.

Insert Table 11 about here

For a_2b_2, the largest discrepancy between the two methods of calculating analytical power was never greater than an absolute value of 0.018. The largest discrepancy between empirical power and analytical power calculated using correlation coefficients was 0.011 at $N = 50$, $a_2 = 0.314$, $b_2 = 0.577$. The largest discrepancy between empirical power and analytical power calculated using regression coefficients was -0.017 at $N = 50$, $a_2 = b_2 = 0.577$. Results from the program to compute analytical power of the specific mediated effects for the parallel two mediator model are very close to results for the specific mediated effects from the empirical simulation, indicating that both methods produced relatively accurate power values for the joint significance tests of the specific mediated effects for the parallel two mediator model.

Type I Error Rates

For the single mediator model, results from the empirical simulation with combinations where one or both of the population parameters a or b is equal to zero provided empirical Type I error rates. Empirical Type I error rates for both tests where $a = b = 0$ for the single mediator model can be found in Table 12.

Insert Table 12 about here

For all sample sizes, when $a = b = 0$ the product of coefficients test had Type I error rates around zero, and the test of joint significance had Type I error rates around 0.001. Both tests underestimated Type I error rates, as found in earlier research (MacKinnon et al., 2002). However, when only one coefficient was zero, Type I error rates increased to values closer to 0.05.

For the parallel two mediator model, when either a_1 or b_1 was zero the a_1b_1 term was zero, and when either a_2 or b_2 was zero the a_2b_2 term was zero. Therefore, results from the simulation with combinations where both the a_1b_1 and a_2b_2 terms are zero provide empirical Type I error rates. To reduce the amount of information presented, combinations where $a_1 = b_1 = a_2 = b_2 = 0$ are provided here as measures of empirical Type I error for the product of coefficients test in Table 13.

Insert Table 13 about here

As for the single mediator model, the product of coefficients test of the parallel two mediator model underestimated Type I error rates (Type I error rates were also around zero for this model). These Type I error rates are in accordance with previous research on Type I error rates of tests for the mediated effect (MacKinnon et al., 2002).

Power Comparison for Parameters Greater than One

The power differences for the total and indirect effects above were consistent for the single and parallel two mediator models. However, for both models, the parameters used to generate power values were less than one. It is of interest to know if these findings hold for parameters greater than one. To investigate this possibility, an empirical search was made using the program in Appendix H for $N = 100$ using coefficients of 1.01, 3.14, and 5.77 for a_1, b_1, a_2, and b_2, and 0 for c', for a total of 81 combinations of parameters. In addition, the variance of X and the error variances of M_1, M_2, and Y were changed from a value of one to a value of 10.

Power values for the test of the total effect and the test of the total mediated effect using coefficients greater than one at $N = 100$ can be found in Table 14.

Insert Table 14 about here

For many of the combinations of parameters at $N = 100$, power was equal to one for tests of both the total and total mediated effects. All combinations of parameters had power above 0.90 for tests of both effects. When power values were not equal, power of the test of the total mediated effect was always greater than power of the test of the total effect. For example, at $a_1 = b_1 = 1.01$, $a_2 = b_2 = 1.01$, power of the test of the total mediated effect was 0.994 and power of the test of the total effect was 0.946. Power for the test of the total mediated effect was greater than power of the test of the total effect for the combinations of small and medium coefficients, and when a_1 or a_2 was small and b_1 or b_2 was large.

The increase in coefficient size for a_1b_1, a_2b_2, and c led to more powerful tests for both the total and total mediated effects as compared with results using combinations of parameters less than one because the parameters are larger. All of the coefficients greater than one corresponded to large effect sizes, resulting in adequate power for all combinations of parameters tested at $N = 100$. Adding a mediator to increase power is of less interest when power values for all combinations are above Cohen’s (1988) guideline for adequate power, because it is not necessary to increase power when power is adequate.

Bootstrap Comparison to Empirical Power for Parallel Two Mediator Model

Analytical power for the total indirect effect for the parallel two mediator model was inaccurate because the assumed normal distribution for the test is only true for very large sample and effect sizes. That is, the ratio of the total mediated effect to the standard error of the total mediated effect does not have a normal distribution unless sample and effect size are very large. Because it was not possible to accurately compute analytical power for the parallel two mediator model, a SAS program was written to compute bootstrapped power values for the total mediated effect $a_1b_1+a_2b_2$ and for the total effect c in order to check the accuracy of the empirical power value comparisons found for the total and total mediated effects. The bootstrap method is an appropriate method for comparison because it generates asymmetric confidence intervals based on the distribution of the indirect effect instead of assuming a normal distribution. The program can be found in Appendix N. A flowchart of the bootstrap process can be found in Appendix O. The bootstrap program was used to obtain power for 10 combinations of parameters and sample size for comparison to the empirical results. The combinations to be bootstrapped were randomly selected from the combinations where the total mediated effect is more powerful than the total effect. The program simulated one sample of data for X, M_1, M_2, and Y based on the mediation equations, and then conducted bootstrap for that single dataset by sampling with replacement from that sample of data to get N observations, forming the first bootstrap sample. In the bootstrap sample, the values of a_1, b_1, a_2, b_2, and c were generated and saved. This process of randomly sampling with replacement and saving values of the coefficients was repeated 1000 times to create 1000 bootstrap samples for the first simulated data set, and the saved bootstrapped values of a_1, b_1, a_2, b_2, and c were used to create confidence intervals for the total mediated effect $a_1b_1 + a_2b_2$ and the total effect c. Binary variables were then generated for the total and total mediated effects that were equal to one when the confidence interval did not include zero and equal to zero when the confidence interval included zero.

This process of simulating a dataset, generating a bootstrap confidence interval, and creating binary variables for significance based on the confidence interval was repeated 1000 times, simulating 1000 datasets. The means of the binary variables from the 1000 simulated datasets were the bootstrapped power values of the total and total mediated effects.

A comparison of bootstrapped and empirical power values for the total and total mediated effects for the parallel two mediator model can be found in Table 15.

Insert Table 15 about here

Results show that the power values are very similar for the bootstrap and empirical methods. The largest discrepancy between empirical and bootstrapped power values for the total mediated effect was a discrepancy of 0.038, which occurred at $N = 50$, $a_1 = 0.101$, $b_1 = 0.314$, $a_2 = 0.577$, and $b_2 = 0.577$. The largest discrepancy between empirical and bootstrapped power values for the total effect was a discrepancy of 0.045, which occurred at $N = 50$, $a_1 = 0.577$, $b_1 = 0.577$, $a_2 = 0.314$, and $b_2 = 0.101$. These bootstrap results confirm that the empirical power value comparisons for the total and total mediated effects for the parallel two mediator model are accurate. Most importantly, the bootstrap results confirm that the cases where power to detect the total mediated effect is greater than power to detect the total effect.

Comparison of Standard Errors of Total and Indirect Effects

For the majority of the combinations of parameters and sample sizes in this study, the test of mediation is more powerful than the test of the total effect when c' is equal to zero, and therefore when the total and indirect effects are equal (that is, $a_1b_1 + a_2b_2 = c$ or $ab = c$). This means that for the test of the indirect effect to be more powerful, the test of significance for $a_1b_1 + a_2b_2$ or ab must be larger than the test of significance for c. Because the tests of significance are computed by dividing the effects by their standard errors, it follows that if the test of the indirect effect is more powerful than the test of the total effect, the standard error of the total effect must be larger than the standard error of the indirect effect.

Table 16 shows a comparison of the analytical standard errors of c and ab for the single mediator model next to power of the tests of c and ab for combinations where a and b are greater than zero and c' is equal to zero at $N = 100$, a sample of the combinations used in the full empirical simulation. The analytical program in Appendix C was used to compute these values.

Insert Table 16 about here

Results from the analytical program show that when ab is equal to c and c' is equal to zero, the standard error of c is larger than the standard error of ab. Furthermore, when the standard error of *c* is larger than the standard error of *ab*, power of the test of *ab* is larger than power of the test of *c*.

A further study of this effect at smaller sample sizes and smaller effect sizes showed that while the standard error of *c* is always larger than the standard error of *ab* when *ab* and *c* are equal, when *a* or *b* approaches zero the power of the test of *ab* becomes smaller than the test of *c*. This effect is true for cases where *a* or *b* is very small but not zero. For example at $N = 100$, when *a* is equal to 0.14 and *b* is equal to 0.001, *c* is equal to 0.0014. The standard error of *ab* is 0.01422 and the standard error of *c* is 0.10102 so the standard error of *ab* is smaller than the standard error of *c*, but power of the test of *ab* is equal to 0.00714 while power of the test of *c* is equal to 0.02508. Although power of the test of *c* approaches 0.025 for very small effects, power of the test of *ab* decreases to below 0.025.

Ranges of Correlations Between X and Y for Which Including a Mediator Will Increase Power

While analytical formulas are useful for determining the conditions under which power will be greater for the test of mediation than for the test of *c*, in practice researchers performing power analyses may only have an idea of the expected correlations between variables and an expected sample size. Therefore, it is also useful to find ranges of correlations between variables for which adding a mediator would increase power. Table 17 shows the ranges of correlations between X and Y for which adding one mediator would increase power for $N = 50$, 100, 200, 500, 1000, and 5000. The range of correlations only includes values of correlations that would result in inadequate power for the test of *c* (power of less than 0.80). That is, any correlation larger than the maximum correlation given would result in adequate power for the test of *c* (power of greater than 0.80).

Insert Table 17 about here

As sample size increases, the range of correlations between X and Y where the inclusion of a mediator would be beneficial to power decreases. The minimum and maximum correlations between X and Y also decrease as sample size increases, meaning the correlations between X and Y that would result in more power when a mediator is added are smaller for larger sample sizes, and larger for smaller sample sizes. In addition, while the table includes the range of correlations between X and Y for which adding a mediator would increase power when power of the test of *c* is less than 0.80, the inclusion of a mediator will not always increase power to be 0.80 until $N = 1000$. Even at $N = 1000$, the minimum power of *ab* is 0.781488, which does not quite reach adequate power of 0.80. At smaller sample sizes, the inclusion of a mediator may only marginally increase the power of a study. For example, the minimum power of *ab* at $N = 50$ is 0.0522368. In this case, power would still be much too low to detect effects.

However, the smallest correlation between X and Y that would result in increased power with the addition of a mediator is 0.0099499, and for $N = 1000$ or greater, the inclusion of a mediator will increase power to a minimum of 0.781488. A correlation between X and Y of 0.0099499 corresponds to a very small effect size. This is of interest for researchers with large sample sizes but very small effect sizes who wish to increase the power of their studies.

In addition, the largest correlation between X and Y that would result in increased power with the addition of a mediator is 0.3806169, which corresponds to a medium effect size. Any correlation larger than 0.3806169 would result in adequate power for the test of *c*. This indicates that researchers with limited sample sizes who expect to obtain a medium effect size can benefit from including a mediator in their model.

Discussion

Summary of Results

The purpose of this paper was to identify situations where the test of the indirect effect is more powerful than the test of the total effect for both single and two mediator models, to show that it is possible under certain circumstances to use a mediator or mediators to increase power. The results showed that the inclusion of mediators increases power when effects are small and sample size is large, and when effects are large and sample size is small. These results extended from the indirect effect *ab* for the single mediator model to the specific indirect effects a_1b_1 and a_2b_2 for the parallel two mediator model. These are the most important findings from this study, as they can inform a researcher with a fixed sample size and fixed effect sizes of how much power can be increased by including one or two mediators.

The conditions for when power of the test of the indirect effect will be greater than power of the test of the total effect were also found in the comparison of standard errors of *c* and *ab*, and therefore in terms of variances and covariances among variables. This means that if a researcher has an expected covariance matrix among variables based on previous literature it would be possible to use those variances and covariances to determine if the standard error of *c* will be larger than the standard error of *ab*. If it is, the researcher would benefit from adding a mediator to the existing model. In addition, if a researcher has only a fixed sample size and an expected correlation between two variables based on previous literature, the ranges of correlations between two variables that will result in increased power when a mediator is added are provided here for a subset of sample sizes that are common in the social sciences.

Fit with Earlier Literature

Previous research on significance tests of mediation has shown that some tests are more powerful because they do not require the total effect to be significant. The results of this study confirm that under certain conditions when the indirect effect and the total effect are equal, the test of the indirect effect will be significant while the test of the total effect will not. This supports findings in existing literature, and extends this concept to a model with multiple mediators as well. The results here also confirm that the joint significance test of mediation for the single mediator model is more powerful than the product of coefficients test using the multivariate delta standard error, which supports findings in MacKinnon et al. (2002). In addition, MacKinnon et al. (2002) found that the Type I error rates of the product of coefficients and joint significance tests are too low at less than 0.05. The Type I error rates examined in this study correspond to the error rates found in the aforementioned publication.

Proximal vs. Distal Mediators: Effects on Power

For the single mediator model, when b is larger than a (that is, the mediator is closer in time or more highly related to the outcome Y than to X), the mediator M is considered a distal mediator. When a is larger than b (that is, the mediator is closer in time or more highly related to X than to Y), M is considered a proximal mediator.

According the previous research, the test of *ab* will be more powerful for models with distal mediators than for models with proximal mediators due to collinearity between X and M (Hoyle & Kenny, 1999). When collinearity between X and M is high, the standard error of the *b* path is increased, leading to a less powerful test of significance. Results from Fritz and Mackinnon (2007) support this point, demonstrating that required sample size is larger for conditions where *a* is larger than *b*. This effect is discussed as well in Kenny and Judd (2013).

As the test of *ab* is known to be more powerful when *b* is larger than *a*, it follows that when comparing power of the test of *ab* to power of the test of *c*, the gain in power over the test of *c* would be greater in conditions where *b* is larger than *a*. This effect is seen in the results from this study. For example, in Appendix J at $N = 200$, $a = 0.39$, $b = 0.59$, $c' = 0$, adding a mediator increased power by 0.212, while at $N = 200$, $a = 0.59$, $b = 0.39$, $c' = 0$, adding a mediator increased power by 0.132. The increase in power was larger for the condition where *b* was larger than *a*.

The effect was also shown for the parallel two mediator model results, where when b_1 or b_2 was larger than a_1 or a_2, the increase in power would be greater. For example, in Table 9 at $N = 200$, $a_1 = a_2 = 0.101$, $b_1 = b_2 = 0.577$, adding two mediators increased power by 0.246, while at $N = 200$, $a_1 = a_2 = 0.577$, $b_1 = b_2 = 0.101$, adding two mediators increased power by 0.166. The increase in power was larger for the condition where the b_1 and b_2 coefficients were larger than the a_1 and a_2 coefficients.

Limitations

This study shows that mediators can be used to increase statistical power given certain circumstances. However, it is important to realize that including a mediator will not always increase power. In some situations, including a mediator may fail to change the relationship between X and Y or decrease the power to detect the relationship between X and Y. As the current study shows that including a mediator will increase power when the standard error of *c* is greater than the standard error of *ab*, it follows that the standard error of *c* will not always be larger than the standard error of *ab*. When *c* is equal to *ab* and the standard errors are equal there will be no difference in power to detect effects, and when *c* is equal to *ab* and the standard error of *ab* is larger than the standard error of *c* then the test of *c* will be more powerful than the test of *ab*.

Future Directions

Although the current study examines a single mediator model and a two mediator model, the gain in power achieved from adding mediators should be assessed for more complex models with multiple mediators. The two mediator model examined here is a parallel model, meaning the effect of X on Y is simultaneously transmitted through two mediators, as opposed to transmitting the effect of X to M_1 to M_2 to Y. Such a model would be considered a sequential two mediator model, as the effects of X on Y are transmitted sequentially first through M_1 and then through M_2. The increase in power from using a sequential two mediator model over a bivariate model should be studied in future research. The power gained from including more than two mediators in a model should be studied as well.

Future research should also address issues with causal inference for the two mediator model. Identification and sensitivity of causal mediation analysis has been studied for the parallel two mediator model (Imai & Yamamoto, 2013) and for the sequential two mediator model (Albert & Nelson, 2011; Avin, Shpitser, & Pearl, 2005;

Robins, 2003), but the decomposition of causal effects in a counterfactual framework becomes more complex as multiple mediators are added to a model (Daniel, De Stavola, & Cousens, 2013). Causal inference is a key component of models with mediators, as a mediator is hypothesized to be intermediate in the causal relationship between X and Y.

Sufficient power is a key component in the design and implementation of any study in the social sciences. This paper demonstrates that including one or more mediators can increase power to detect effects for researchers with fixed effect sizes or sample sizes. The most important result in this paper is the finding that including multiple mediators in a model will increase power over and above a bivariate model. Another finding is the specific conditions under which including one or more mediators will increase power. This is important for planning in research design, as it provides both an analytical formula for knowing when the inclusion of a mediator is beneficial as well as guidelines for researchers with an expected effect size and sample size who wish to use mediators to increase power. Indeed, these findings will be of use for all researchers who are interested in mediation as a method for increasing statistical power.

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Tables

Table 1

Covariance Matrix for the Parallel Two Mediator Model

$$\begin{array}{c|cccc}
 & X & M_1 & M_2 & Y \\
\hline
X & \sigma_X^2 & & & \\
M_1 & a_1\sigma_X^2 & a_1^2\sigma_X^2+\sigma_{e3}^2 & & \\
M_2 & a_2\sigma_X^2 & a_1a_2\sigma_X^2 & a_2^2\sigma_X^2+\sigma_{e4}^2 & \\
Y & c'\sigma_X^2+a_1b_1\sigma_X^2+a_2b_2\sigma_X^2 & a_1c'\sigma_X^2+a_1^2b_1\sigma_X^2+a_1a_2b_2\sigma_X^2+b_1\sigma_{e3}^2 & a_2c'\sigma_X^2+a_1a_2b_1\sigma_X^2+a_2^2b_2\sigma_X^2+b_2\sigma_{e4}^2 & c'^2\sigma_X^2+2a_1b_1c'\sigma_X^2+2a_2b_2c'\sigma_X^2+2a_1a_2b_1b_2\sigma_X^2+b_1^2(a_1^2\sigma_X^2+\sigma_{e3}^2)+b_2^2(a_2^2\sigma_X^2+\sigma_{e4}^2)+\sigma_{e2}^2
\end{array}$$

Table 2

Empirical Power Values for the Test of the Total Mediated Effect and the Test of the Total Effect for the Parallel Two Mediator Model at c' = 0 and 0.131, N = 50

		a_1b_1																															
		ZZ		ZS		SZ		ZM		MZ		ZL		LZ		SS		SM		MS		SL		LS		MM		ML		LM		LL	
a_2b_2		$a_1b_1+a_2b_2$	c	$a_1b_1+a_2b_2$	c	$a_1b_1+a_2b_2$	c	$a_1b_1+a_2b_2$	c	$a_1b_1+a_2b_2$	c	$a_1b_1+a_2b_2$	c	$a_1b_1+a_2b_2$	c	$a_1b_1+a_2b_2$	c	$a_1b_1+a_2b_2$	c	$a_1b_1+a_2b_2$	c	$a_1b_1+a_2b_2$	c	$a_1b_1+a_2b_2$	c	$a_1b_1+a_2b_2$	c	$a_1b_1+a_2b_2$	c	$a_1b_1+a_2b_2$	c	$a_1b_1+a_2b_2$	c
ZZ	c'=0	0	0.046	0.002	0.044	0	0.058	0.004	0.068	0.006	0.05	0.02	0.038	0.02	0.06	0	0.052	0.016	0.058	0.024	0.048	0.064	0.07	0.036	0.076	***0.118***	***0.09***	***0.364***	***0.164***	***0.322***	***0.234***	***0.838***	***0.48***
	c'=0.131	0	0.108	0.002	0.15	0	0.148	0.002	0.092	0.006	0.158	0.024	0.124	0.034	0.178	0.004	0.168	0.004	0.162	0.026	0.164	0.068	0.19	0.04	0.236	0.128	0.318	0.39	0.46	0.41	0.516	***0.842***	***0.804***
ZS	c'=0	0	0.054	0	0.052	0	0.052	0.004	0.058	0.002	0.036	0.022	0.056	0.02	0.042	0	0.072	0.012	0.056	0.01	0.054	0.038	0.044	***0.068***	***0.06***	***0.12***	***0.08***	***0.322***	***0.192***	***0.374***	***0.226***	***0.834***	***0.452***
	c'=0.131	0	0.11	0.006	0.138	0	0.13	0.006	0.126	0.004	0.158	0.014	0.134	0.03	0.134	0.002	0.16	0.018	0.184	0.026	0.186	0.05	0.186	0.054	0.222	0.126	0.34	0.376	0.452	0.37	0.568	***0.848***	***0.788***
SZ	c'=0	0	0.054	0.002	0.038	0	0.054	0.012	0.056	0.01	0.068	0.034	0.06	0.032	0.066	0	0.06	0.016	0.032	0.012	0.068	0.052	0.064	0.056	0.058	***0.112***	***0.098***	***0.35***	***0.196***	***0.356***	***0.216***	***0.806***	***0.522***
	c'=0.131	0.002	0.156	0	0.148	0	0.142	0.01	0.158	0.018	0.134	0.024	0.126	0.02	0.134	0	0.15	0.018	0.218	0.01	0.206	0.054	0.206	0.074	0.254	0.126	0.308	0.396	0.496	0.37	0.522	***0.834***	***0.762***
ZM	c'=0	0.002	0.042	0.008	0.046	0.008	0.044	0.016	0.07	0.022	0.044	0.036	0.056	0.038	0.04	0.008	0.076	0.018	0.052	0.014	0.062	0.06	0.076	0.072	0.084	***0.136***	***0.08***	***0.36***	***0.176***	***0.322***	***0.196***	***0.746***	***0.484***
	c'=0.131	0.006	0.114	0.002	0.122	0.004	0.154	0.024	0.124	0.014	0.118	0.026	0.096	0.02	0.126	0.008	0.136	0.022	0.178	0.038	0.198	0.06	0.188	0.05	0.25	0.11	0.3	0.342	0.434	0.326	0.502	***0.772***	***0.768***
MZ	c'=0	0.004	0.054	0	0.068	0.012	0.048	0.018	0.056	0.012	0.056	0.032	0.056	0.024	0.072	0.02	0.044	0.024	0.072	0.03	0.05	0.044	0.05	0.048	0.07	***0.122***	***0.102***	***0.354***	***0.21***	***0.318***	***0.242***	***0.76***	***0.506***
	c'=0.131	0.014	0.13	0.016	0.152	0.004	0.118	0.02	0.136	0.01	0.132	0.032	0.11	0.042	0.16	0.008	0.14	0.022	0.178	0.02	0.206	0.05	0.184	0.042	0.232	0.138	0.33	0.346	0.476	0.286	0.536	***0.8***	***0.766***
ZL	c'=0	0.01	0.056	0.018	0.04	0.014	0.052	0.02	0.046	0.026	0.046	0.026	0.052	0.04	0.046	0.026	0.05	0.032	0.074	0.024	0.058	***0.054***	***0.052***	***0.058***	***0.054***	***0.128***	***0.106***	***0.236***	***0.156***	***0.24***	***0.176***	***0.63***	***0.404***
	c'=0.131	0.03	0.144	0.026	0.092	0.032	0.118	0.026	0.114	0.028	0.11	0.036	0.126	0.042	0.116	0.028	0.14	0.038	0.15	0.036	0.158	0.07	0.186	0.07	0.224	0.106	0.256	0.274	0.398	0.288	0.442	0.62	0.704
LZ	c'=0	0.03	0.056	0.026	0.046	0.03	0.032	0.034	0.05	0.028	0.046	0.024	0.036	0.032	0.046	0.026	0.052	0.02	0.052	0.04	0.046	0.058	0.068	0.054	0.058	***0.104***	***0.092***	***0.234***	***0.184***	***0.268***	***0.23***	***0.652***	***0.532***
	c'=0.131	0.028	0.16	0.022	0.132	0.028	0.142	0.026	0.12	0.04	0.14	0.038	0.084	0.036	0.148	0.016	0.162	0.034	0.202	0.072	0.224	0.06	0.21	0.052	0.242	0.128	0.322	0.256	0.444	0.27	0.496	0.632	0.794
SS	c'=0	0	0.054	0.002	0.048	0.002	0.05	0.006	0.044	0.006	0.05	0.022	0.04	0.02	0.052	0.006	0.064	0.018	0.042	0.014	0.054	0.068	0.078	***0.072***	***0.054***	***0.146***	***0.102***	***0.408***	***0.208***	***0.396***	***0.24***	***0.83***	***0.546***
	c'=0.131	0	0.178	0.004	0.134	0.004	0.178	0.012	0.126	0.016	0.16	0.02	0.114	0.044	0.168	0.012	0.174	0.016	0.218	0.038	0.218	0.066	0.214	0.066	0.286	0.14	0.32	0.428	0.5	0.416	0.548	***0.852***	***0.768***
SM	c'=0	0.008	0.046	0.014	0.052	0.014	0.052	0.034	0.058	0.018	0.054	0.036	0.054	0.034	0.06	0.022	0.05	0.056	0.076	0.054	0.074	***0.106***	***0.08***	***0.104***	***0.09***	***0.212***	***0.128***	***0.468***	***0.244***	***0.446***	***0.282***	***0.84***	***0.574***
	c'=0.131	0.02	0.192	0.014	0.214	0.004	0.162	0.024	0.168	0.03	0.202	0.034	0.148	0.028	0.212	0.022	0.2	0.066	0.23	0.056	0.268	0.084	0.244	0.1	0.286	0.24	0.39	0.464	0.488	0.472	0.536	***0.854***	***0.796***
MS	c'=0	0.02	0.048	0.028	0.044	0.012	0.048	0.028	0.064	0.016	0.068	0.04	0.06	0.038	0.062	0.036	0.076	0.054	0.07	***0.068***	***0.056***	***0.086***	***0.078***	***0.102***	***0.084***	***0.2***	***0.124***	***0.46***	***0.212***	***0.466***	***0.296***	***0.864***	***0.554***
	c'=0.131	0.01	0.172	0.014	0.2	0.02	0.194	0.024	0.2	0.02	0.176	0.034	0.148	0.034	0.176	0.016	0.186	0.034	0.22	0.07	0.254	0.1	0.204	0.084	0.308	0.224	0.39	0.43	0.52	0.434	0.588	***0.85***	***0.816***
SL	c'=0	0.058	0.08	***0.064***	***0.062***	***0.058***	***0.04***	0.058	0.068	***0.054***	***0.052***	***0.062***	***0.054***	0.054	0.068	0.05	0.052	***0.1***	***0.074***	***0.108***	***0.098***	***0.106***	***0.09***	***0.118***	***0.1***	***0.254***	***0.152***	***0.432***	***0.246***	***0.44***	***0.3***	***0.788***	***0.572***
	c'=0.131	0.078	0.208	0.038	0.188	0.06	0.186	0.046	0.188	0.056	0.184	0.052	0.188	0.044	0.188	0.072	0.232	0.126	0.242	0.106	0.29	0.138	0.276	0.148	0.338	0.234	0.352	0.434	0.542	0.45	0.556	0.772	0.796
LS	c'=0	0.05	0.074	0.042	0.064	0.058	0.064	0.05	0.074	***0.08***	***0.066***	***0.064***	***0.062***	0.054	0.062	***0.082***	***0.068***	***0.094***	***0.086***	***0.126***	***0.08***	***0.146***	***0.124***	***0.132***	***0.126***	***0.282***	***0.204***	***0.42***	***0.292***	***0.412***	***0.368***	***0.796***	***0.604***
	c'=0.131	0.05	0.29	0.07	0.246	0.052	0.272	0.044	0.218	0.054	0.244	0.06	0.206	0.072	0.23	0.084	0.258	0.09	0.32	0.09	0.358	0.134	0.268	0.15	0.414	0.226	0.436	0.408	0.604	0.442	0.668	0.77	0.844
MM	c'=0	0.1	0.112	***0.118***	***0.094***	0.096	0.106	***0.12***	***0.092***	***0.152***	***0.09***	0.09	0.09	***0.11***	***0.102***	***0.168***	***0.096***	***0.212***	***0.106***	***0.244***	***0.14***	***0.23***	***0.146***	***0.242***	***0.142***	***0.51***	***0.24***	***0.678***	***0.362***	***0.668***	***0.418***	***0.95***	***0.686***
	c'=0.131	0.122	0.3	0.144	0.362	0.112	0.322	0.124	0.32	0.146	0.342	0.122	0.246	0.104	0.326	0.162	0.368	0.234	0.41	0.198	0.374	0.26	0.386	0.23	0.49	0.484	0.504	0.676	0.676	0.674	0.726	***0.942***	***0.882***
ML	c'=0	***0.39***	***0.21***	***0.374***	***0.202***	***0.35***	***0.18***	***0.318***	***0.21***	***0.344***	***0.214***	***0.272***	***0.142***	***0.262***	***0.19***	***0.432***	***0.228***	***0.474***	***0.244***	***0.462***	***0.226***	***0.396***	***0.254***	***0.456***	***0.304***	***0.664***	***0.394***	***0.81***	***0.452***	***0.774***	***0.56***	***0.96***	***0.758***
	c'=0.131	0.362	0.466	0.356	0.428	0.366	0.462	0.344	0.408	0.332	0.402	0.258	0.366	0.236	0.454	0.446	0.522	0.464	0.492	0.426	0.522	0.428	0.526	0.45	0.588	0.664	0.668	***0.79***	***0.69***	0.784	0.796	***0.964***	***0.928***
LM	c'=0	***0.384***	***0.192***	***0.416***	***0.222***	***0.378***	***0.222***	***0.358***	***0.204***	***0.288***	***0.236***	***0.258***	***0.188***	***0.244***	***0.216***	***0.41***	***0.236***	***0.436***	***0.238***	***0.478***	***0.272***	***0.444***	***0.29***	***0.412***	***0.36***	***0.69***	***0.402***	***0.8***	***0.53***	***0.788***	***0.646***	***0.954***	***0.832***
	c'=0.131	0.396	0.52	0.366	0.52	0.35	0.5	0.286	0.492	0.322	0.56	0.25	0.398	0.234	0.55	0.378	0.558	0.44	0.582	0.414	0.582	0.438	0.558	0.412	0.648	0.7	0.71	0.772	0.774	0.794	0.862	***0.956***	***0.944***
LL	c'=0	***0.856***	***0.516***	***0.802***	***0.536***	***0.824***	***0.486***	***0.756***	***0.498***	***0.72***	***0.486***	***0.602***	***0.408***	***0.642***	***0.486***	***0.86***	***0.542***	***0.89***	***0.576***	***0.868***	***0.606***	***0.776***	***0.57***	***0.784***	***0.556***	***0.95***	***0.692***	***0.956***	***0.754***	***0.954***	***0.828***	***0.992***	***0.942***
	c'=0.131	***0.862***	***0.796***	***0.83***	***0.766***	***0.824***	***0.772***	0.77	0.782	***0.758***	***0.738***	0.608	0.684	0.646	0.768	***0.868***	***0.8***	***0.856***	***0.822***	***0.852***	***0.822***	0.774	0.774	0.766	0.87	***0.95***	***0.89***	***0.96***	***0.898***	***0.96***	***0.934***	***0.994***	***0.98***

Note: For effect sizes, Z = zero, S = 0.101, M = 0.314, and L = 0.577, as per the path parameters used in the simulation. Different values of c' are listed down the lefthand column and the different tests are listed across the top.

Table 3

Empirical Power Values for the Test of the Total Mediated Effect and the Test of the Total Effect for the Parallel Two Mediator Model at c' = 0 and 0.131, N = 100

		a_1b_1																															
		ZZ		ZS		SZ		ZM		MZ		ZL		LZ		SS		SM		MS		SL		LS		MM		ML		LM		LL	
a_2b_2		$a_1b_1+a_2b_2$	c	$a_1b_1+a_2b_2$	c	$a_1b_1+a_2b_2$	c	$a_1b_1+a_2b_2$	c	$a_1b_1+a_2b_2$	c	$a_1b_1+a_2b_2$	c	$a_1b_1+a_2b_2$	c	$a_1b_1+a_2b_2$	c	$a_1b_1+a_2b_2$	c	$a_1b_1+a_2b_2$	c	$a_1b_1+a_2b_2$	c	$a_1b_1+a_2b_2$	c	$a_1b_1+a_2b_2$	c	$a_1b_1+a_2b_2$	c	$a_1b_1+a_2b_2$	c	$a_1b_1+a_2b_2$	c
ZZ	c'=0	0	0.038	0.002	0.058	0.002	0.042	0.01	0.054	0.006	0.038	0.03	0.042	0.028	0.048	0.008	0.052	***0.072***	***0.054***	***0.058***	***0.054***	***0.14***	***0.058***	0.098	0.098	***0.482***	***0.154***	***0.798***	***0.348***	***0.77***	***0.39***	***1***	***0.81***
	c'=0.131	0	0.238	0	0.224	0.002	0.262	0.012	0.224	0.004	0.25	0.022	0.216	0.028	0.258	0.004	0.28	0.054	0.306	0.038	0.378	0.136	0.38	0.146	0.416	0.506	0.53	***0.814***	***0.728***	0.814	0.824	***0.996***	***0.98***
ZS	c'=0	0.004	0.036	0.002	0.05	0.002	0.054	0.012	0.048	0.018	0.034	0.028	0.036	0.034	0.054	0.008	0.048	0.054	0.086	0.054	0.06	***0.136***	***0.08***	***0.124***	***0.09***	***0.478***	***0.142***	***0.784***	***0.338***	***0.766***	***0.392***	***0.996***	***0.81***
	c'=0.131	0	0.24	0.006	0.238	0	0.264	0.01	0.252	0.02	0.268	0.03	0.22	0.046	0.272	0.004	0.26	0.056	0.334	0.054	0.346	0.128	0.344	0.114	0.486	0.498	0.586	***0.81***	***0.784***	***0.832***	***0.796***	***0.998***	***0.968***
SZ	c'=0	0	0.04	0.004	0.052	0.002	0.064	0.008	0.052	0.014	0.06	0.046	0.054	0.042	0.054	0.006	0.048	***0.07***	***0.064***	0.048	0.048	***0.136***	***0.07***	***0.15***	***0.07***	***0.532***	***0.168***	***0.818***	***0.318***	***0.804***	***0.42***	***0.996***	***0.824***
	c'=0.131	0.006	0.318	0	0.214	0.004	0.234	0.014	0.198	0.01	0.232	0.032	0.184	0.038	0.232	0.002	0.3	0.044	0.362	0.062	0.332	0.13	0.342	0.128	0.49	0.552	0.56	***0.804***	***0.758***	0.808	0.816	***0.994***	***0.962***
ZM	c'=0	0.022	0.042	0.014	0.04	0.024	0.054	0.014	0.046	0.018	0.04	0.032	0.052	0.032	0.048	0.02	0.056	0.052	0.056	0.046	0.082	***0.096***	***0.076***	***0.114***	***0.094***	***0.374***	***0.124***	***0.7***	***0.322***	***0.658***	***0.36***	***0.988***	***0.772***
	c'=0.131	0.02	0.264	0.006	0.228	0.006	0.216	0.016	0.224	0.028	0.212	0.04	0.18	0.032	0.232	0.018	0.238	0.056	0.264	0.056	0.342	0.122	0.368	0.128	0.412	0.368	0.574	***0.716***	***0.71***	0.704	0.796	***0.988***	***0.958***
MZ	c'=0	0.02	0.064	0.012	0.044	0.016	0.042	0.02	0.068	0.01	0.05	0.04	0.062	0.042	0.054	0.006	0.048	0.056	0.07	***0.076***	***0.064***	***0.12***	***0.074***	***0.144***	***0.09***	***0.372***	***0.138***	***0.678***	***0.296***	***0.66***	***0.424***	***0.988***	***0.83***
	c'=0.131	0.016	0.264	0.012	0.25	0.012	0.252	0.02	0.224	0.032	0.248	0.054	0.206	0.046	0.25	0.016	0.276	0.062	0.354	0.076	0.38	0.124	0.366	0.108	0.484	0.34	0.536	0.702	0.756	0.682	0.824	0.98	0.982
ZL	c'=0	0.03	0.034	0.05	0.052	0.044	0.048	0.042	0.058	***0.05***	***0.048***	0.028	0.056	0.036	0.056	0.04	0.054	0.054	0.058	0.07	0.072	***0.096***	***0.086***	***0.094***	***0.086***	***0.238***	***0.126***	***0.484***	***0.266***	***0.5***	***0.33***	***0.922***	***0.716***
	c'=0.131	0.036	0.212	0.036	0.192	0.062	0.21042	0.04	0.174	0.034	0.18	0.034	0.176	0.052	0.186	0.046	0.204	0.056	0.28	0.058	0.262	0.106	0.352	0.078	0.366	0.24	0.494	0.516	0.65	0.526	0.738	0.936	0.948
LZ	c'=0	0.028	0.034	0.034	0.068	0.046	0.066	0.046	0.048	0.054	0.054	0.05	0.052	0.034	0.058	0.034	0.052	0.048	0.06	***0.064***	***0.048***	***0.068***	***0.062***	0.08	0.09	***0.234***	***0.164***	***0.51***	***0.346***	***0.506***	***0.418***	***0.932***	***0.816***
	c'=0.131	0.024	0.238	0.032	0.24	0.034	0.278	0.038	0.246	0.032	0.24	0.036	0.184	0.052	0.276	0.04	0.276	0.062	0.376	0.052	0.39	0.07	0.34	0.098	0.46	0.238	0.59	0.526	0.746	0.514	0.824	0.938	0.97
SS	c'=0	0.002	0.042	0.004	0.056	0.012	0.042	0.026	0.05	0.022	0.056	0.038	0.042	***0.056***	***0.044***	0.026	0.066	0.082	0.084	***0.104***	***0.064***	***0.152***	***0.082***	***0.14***	***0.092***	***0.576***	***0.148***	***0.85***	***0.36***	***0.868***	***0.412***	***0.998***	***0.832***
	c'=0.131	0.004	0.262	0.004	0.258	0.01	0.3	0.026	0.254	0.018	0.3	0.034	0.214	0.034	0.304	0.03	0.354	0.098	0.392	0.09	0.388	0.14	0.392	0.154	0.464	0.614	0.63	***0.826***	***0.786***	0.816	0.844	***1***	***0.982***
SM	c'=0	***0.056***	***0.052***	0.05	0.066	0.052	0.06	0.044	0.06	***0.074***	***0.06***	***0.07***	***0.064***	***0.068***	***0.048***	***0.096***	***0.068***	***0.172***	***0.098***	***0.194***	***0.102***	***0.236***	***0.132***	***0.246***	***0.146***	***0.616***	***0.222***	***0.848***	***0.408***	***0.858***	***0.474***	***0.998***	***0.836***
	c'=0.131	0.066	0.332	0.048	0.35	0.056	0.334	0.068	0.302	0.062	0.336	0.054	0.252	0.068	0.326	0.088	0.332	0.192	0.396	0.154	0.444	0.22	0.452	0.23	0.512	***0.644***	***0.636***	***0.868***	***0.82***	0.856	0.878	***0.996***	***0.976***
MS	c'=0	0.042	0.06	0.054	0.058	0.05	0.072	0.06	0.08	0.048	0.07	***0.066***	***0.05***	0.042	0.05	***0.098***	***0.078***	***0.174***	***0.088***	***0.176***	***0.1***	***0.224***	***0.136***	***0.242***	***0.142***	***0.624***	***0.246***	***0.856***	***0.446***	***0.832***	***0.502***	***0.994***	***0.886***
	c'=0.131	0.052	0.32	0.074	0.322	0.064	0.362	0.09	0.304	0.054	0.354	0.058	0.294	0.076	0.354	0.086	0.404	0.202	0.466	0.172	0.492	0.244	0.444	0.21	0.63	0.642	0.692	***0.846***	***0.82***	0.836	0.918	***0.996***	***0.986***
SL	c'=0	***0.1***	***0.058***	***0.118***	***0.078***	***0.126***	***0.068***	***0.11***	***0.07***	***0.126***	***0.082***	***0.094***	***0.08***	***0.108***	***0.082***	***0.182***	***0.088***	***0.272***	***0.128***	***0.242***	***0.134***	***0.246***	***0.158***	***0.288***	***0.18***	***0.568***	***0.276***	***0.808***	***0.462***	***0.754***	***0.512***	***0.986***	***0.834***
	c'=0.131	0.134	0.374	0.106	0.332	0.104	0.326	0.126	0.376	0.132	0.38	0.112	0.314	0.092	0.348	0.172	0.41	0.23	0.458	0.218	0.48	0.264	0.446	0.272	0.506	0.514	0.664	0.77	0.816	0.784	0.866	***0.986***	***0.976***
LS	c'=0	***0.118***	***0.08***	***0.124***	***0.092***	***0.12***	***0.09***	***0.126***	***0.062***	***0.1***	***0.088***	0.076	0.102	***0.086***	***0.07***	***0.184***	***0.088***	***0.222***	***0.132***	***0.234***	***0.144***	***0.256***	***0.202***	***0.252***	***0.216***	***0.556***	***0.304***	***0.8***	***0.518***	***0.808***	***0.598***	***0.98***	***0.882***
	c'=0.131	0.132	0.458	0.122	0.456	0.132	0.458	0.106	0.406	0.108	0.456	0.108	0.352	0.094	0.462	0.18	0.422	0.236	0.55	0.22	0.602	0.216	0.532	0.256	0.67	0.582	0.772	0.782	0.836	0.78	0.938	0.982	0.992
MM	c'=0	***0.506***	***0.16***	***0.536***	***0.138***	***0.48***	***0.156***	***0.366***	***0.146***	***0.354***	***0.148***	***0.228***	***0.142***	***0.244***	***0.146***	***0.56***	***0.17***	***0.646***	***0.226***	***0.62***	***0.234***	***0.576***	***0.286***	***0.544***	***0.316***	***0.936***	***0.438***	***0.968***	***0.646***	***0.978***	***0.704***	***1***	***0.914***
	c'=0.131	0.502	0.54	0.472	0.578	0.474	0.582	0.362	0.502	0.35	0.572	0.276	0.49	0.234	0.55	0.54	0.586	0.59	0.624	0.638	0.664	0.536	0.68	0.516	0.772	***0.948***	***0.846***	***0.97***	***0.924***	***0.964***	***0.952***	***1***	***0.996***
ML	c'=0	***0.806***	***0.324***	***0.764***	***0.344***	***0.77***	***0.356***	***0.72***	***0.308***	***0.684***	***0.336***	***0.538***	***0.296***	***0.524***	***0.324***	***0.82***	***0.342***	***0.83***	***0.444***	***0.846***	***0.45***	***0.772***	***0.462***	***0.746***	***0.528***	***0.964***	***0.626***	***0.992***	***0.802***	***0.992***	***0.824***	***1***	***0.974***
	c'=0.131	***0.792***	***0.774***	***0.798***	***0.784***	***0.796***	***0.736***	0.688	0.69	0.712	0.758	0.516	0.634	0.498	0.734	***0.85***	***0.776***	***0.854***	***0.802***	***0.852***	***0.828***	0.79	0.81	0.772	0.892	***0.982***	***0.92***	***0.984***	***0.96***	0.982	0.984	1	1
LM	c'=0	***0.776***	***0.384***	***0.752***	***0.404***	***0.764***	***0.374***	***0.714***	***0.38***	***0.72***	***0.352***	***0.5***	***0.324***	***0.512***	***0.424***	***0.836***	***0.424***	***0.842***	***0.514***	***0.864***	***0.522***	***0.772***	***0.49***	***0.782***	***0.606***	***0.972***	***0.68***	***0.98***	***0.834***	***0.988***	***0.896***	***1***	***0.992***
	c'=0.131	***0.806***	***0.804***	0.814	0.842	0.768	0.844	0.688	0.824	0.698	0.8	0.538	0.708	0.496	0.824	***0.848***	***0.846***	0.818	0.842	0.846	0.89	0.758	0.844	0.752	0.924	***0.96***	***0.958***	***0.982***	***0.976***	0.986	0.99	***1***	***0.998***
LL	c'=0	***0.992***	***0.77***	***0.998***	***0.81***	***0.998***	***0.784***	***0.986***	***0.762***	***0.984***	***0.796***	***0.936***	***0.732***	***0.918***	***0.83***	***1***	***0.828***	***0.998***	***0.842***	***0.998***	***0.828***	***0.988***	***0.834***	***0.986***	***0.894***	***1***	***0.938***	***1***	***0.968***	***1***	***0.988***	1	1
	c'=0.131	***1***	***0.964***	***0.994***	***0.976***	***1***	***0.96***	***0.988***	***0.956***	***0.984***	***0.972***	0.916	0.918	0.906	0.96	***1***	***0.974***	***0.992***	***0.978***	***0.996***	***0.982***	0.972	0.976	0.976	0.988	***1***	***0.996***	1	1	1	1	1	1

Note: For effect sizes, Z = zero, S = 0.101, M = 0.314, and L = 0.577, as per the path parameters used in the simulation. Different values of c' are listed down the lefthand column and the different tests are listed across the top.

Table 4

Empirical Power Values for the Test of the Total Mediated Effect and the Test of the Total Effect for the Parallel Two Mediator Model at c' = 0 and 0.131, N = 200

		a_1b_1																															
		ZZ		ZS		SZ		ZM		MZ		ZL		LZ		SS		SM		MS		SL		LS		MM		ML		LM		LL	
a_2b_2		$a_1b_1+a_2b_2$	c	$a_1b_1+a_2b_2$	c	$a_1b_1+a_2b_2$	c	$a_1b_1+a_2b_2$	c	$a_1b_1+a_2b_2$	c	$a_1b_1+a_2b_2$	c	$a_1b_1+a_2b_2$	c	$a_1b_1+a_2b_2$	c	$a_1b_1+a_2b_2$	c	$a_1b_1+a_2b_2$	c	$a_1b_1+a_2b_2$	c	$a_1b_1+a_2b_2$	c	$a_1b_1+a_2b_2$	c	$a_1b_1+a_2b_2$	c	$a_1b_1+a_2b_2$	c	$a_1b_1+a_2b_2$	c
ZZ	c'=0	0	0.042	0.004	0.056	0.002	0.04	0.022	0.066	0.018	0.046	0.03	0.05	0.046	0.046	0.018	0.068	***0.18***	***0.084***	***0.18***	***0.086***	***0.286***	***0.114***	***0.218***	***0.124***	***0.928***	***0.26***	***0.988***	***0.54***	***0.992***	***0.68***	***1***	***0.984***
	c'=0.131	0.002	0.43	0.002	0.44	0.004	0.46	0.03	0.416	0.03	0.42	0.058	0.346	0.046	0.476	0.022	0.536	0.2	0.602	0.196	0.644	0.288	0.666	0.25	0.726	***0.95***	***0.862***	***0.986***	***0.97***	0.984	0.994	1	1
ZS	c'=0	0	0.04	0.002	0.03	0.002	0.04	0.032	0.06	0.034	0.06	0.032	0.044	0.046	0.05	0.016	0.048	***0.174***	***0.064***	***0.21***	***0.054***	***0.244***	***0.092***	***0.238***	***0.106***	***0.898***	***0.252***	***0.982***	***0.596***	***0.992***	***0.716***	***1***	***0.984***
	c'=0.131	0.004	0.42	0.002	0.466	0.002	0.436	0.018	0.45	0.028	0.46	0.038	0.36	0.036	0.436	0.03	0.5	0.148	0.556	0.176	0.634	0.254	0.602	0.25	0.758	***0.936***	***0.858***	***0.994***	***0.964***	***0.984***	***0.98***	1	1
SZ	c'=0	0.002	0.048	0.006	0.054	0.004	0.056	0.032	0.05	0.016	0.04	0.044	0.046	0.038	0.058	0.016	0.036	***0.184***	***0.062***	***0.18***	***0.084***	***0.264***	***0.112***	***0.276***	***0.12***	***0.934***	***0.254***	***0.978***	***0.584***	***0.988***	***0.674***	***1***	***0.98***
	c'=0.131	0	0.442	0.004	0.478	0.004	0.456	0.024	0.438	0.026	0.452	0.03	0.376	0.028	0.454	0.016	0.5	0.154	0.592	0.16	0.636	0.246	0.612	0.25	0.72	***0.914***	***0.878***	***0.986***	***0.978***	***0.99***	***0.986***	***1***	***0.998***
ZM	c'=0	0.044	0.06	0.032	0.052	0.02	0.056	0.032	0.048	0.038	0.056	0.034	0.048	0.04	0.04	0.034	0.048	***0.146***	***0.066***	***0.114***	***0.064***	***0.224***	***0.102***	***0.202***	***0.108***	***0.706***	***0.244***	***0.956***	***0.548***	***0.94***	***0.64***	***1***	***0.968***
	c'=0.131	0.03	0.436	0.026	0.444	0.028	0.422	0.044	0.362	0.044	0.408	0.056	0.356	0.042	0.426	0.05	0.478	0.128	0.536	0.156	0.592	0.212	0.62	0.202	0.72	0.744	0.852	***0.938***	***0.934***	0.946	0.982	1	1
MZ	c'=0	0.026	0.064	0.03	0.052	0.026	0.052	0.042	0.06	0.026	0.05	***0.054***	***0.036***	0.044	0.044	0.032	0.054	***0.15***	***0.076***	***0.134***	***0.074***	***0.19***	***0.108***	***0.2***	***0.108***	***0.724***	***0.266***	***0.956***	***0.59***	***0.968***	***0.682***	***1***	***0.982***
	c'=0.131	0.018	0.466	0.022	0.434	0.02	0.478	0.044	0.442	0.046	0.454	0.04	0.334	0.052	0.444	0.038	0.54	0.116	0.552	0.142	0.608	0.202	0.658	0.2	0.754	0.704	0.86	0.954	0.956	0.96	0.984	1	1
ZL	c'=0	0.04	0.044	***0.04***	***0.036***	0.046	0.054	0.054	0.064	0.048	0.062	0.042	0.044	0.05	0.062	***0.052***	***0.038***	***0.084***	***0.064***	***0.104***	***0.068***	***0.168***	***0.098***	***0.166***	***0.108***	***0.444***	***0.206***	***0.848***	***0.5***	***0.816***	***0.598***	***0.998***	***0.93***
	c'=0.131	0.04	0.38	0.04	0.366	0.05	0.38	0.034	0.296	0.026	0.412	0.048	0.278	0.04	0.35	0.052	0.384	0.092	0.484	0.096	0.518	0.162	0.554	0.16	0.638	0.494	0.78	0.808	0.92	0.852	0.958	1	1
LZ	c'=0	0.026	0.058	0.048	0.052	***0.054***	***0.04***	0.048	0.052	0.026	0.058	0.05	0.058	***0.066***	***0.052***	0.03	0.084	***0.078***	***0.062***	***0.092***	***0.054***	***0.138***	***0.122***	***0.142***	***0.116***	***0.498***	***0.272***	***0.838***	***0.6***	***0.846***	***0.664***	***1***	***0.966***
	c'=0.131	0.048	0.454	0.062	0.434	0.052	0.47	0.04	0.386	0.064	0.48	0.044	0.354	0.05	0.472	0.058	0.53	0.106	0.588	0.09	0.618	0.132	0.604	0.166	0.756	0.488	0.856	0.83	0.974	0.854	0.99	0.994	1
SS	c'=0	0.012	0.054	0.022	0.068	0.012	0.056	0.044	0.062	***0.056***	***0.034***	0.042	0.062	0.038	0.038	***0.088***	***0.058***	***0.278***	***0.08***	***0.266***	***0.084***	***0.366***	***0.142***	***0.356***	***0.17***	***0.962***	***0.31***	***0.988***	***0.666***	***1***	***0.718***	***1***	***0.982***
	c'=0.131	0.014	0.514	0.018	0.496	0.038	0.528	0.026	0.486	0.056	0.51	0.05	0.434	0.042	0.538	0.106	0.536	0.322	0.672	0.28	0.7	0.3	0.674	0.35	0.79	***0.962***	***0.896***	***0.996***	***0.966***	***0.99***	***0.984***	1	1
SM	c'=0	***0.176***	***0.062***	***0.192***	***0.082***	***0.186***	***0.078***	***0.162***	***0.092***	***0.122***	***0.064***	***0.108***	***0.092***	***0.09***	***0.052***	***0.314***	***0.072***	***0.456***	***0.126***	***0.468***	***0.13***	***0.454***	***0.174***	***0.462***	***0.256***	***0.952***	***0.39***	***0.996***	***0.706***	***0.994***	***0.786***	***1***	***0.978***
	c'=0.131	0.186	0.592	0.158	0.564	0.162	0.56	0.142	0.556	0.134	0.558	0.078	0.442	0.102	0.598	0.302	0.616	0.434	0.742	0.422	0.732	0.474	0.702	0.43	0.832	***0.918***	***0.906***	***0.996***	***0.97***	0.992	0.994	1	1
MS	c'=0	***0.176***	***0.068***	***0.214***	***0.076***	***0.188***	***0.078***	***0.13***	***0.054***	***0.116***	***0.06***	***0.086***	***0.048***	***0.104***	***0.062***	***0.29***	***0.086***	***0.434***	***0.108***	***0.428***	***0.148***	***0.46***	***0.194***	***0.438***	***0.224***	***0.944***	***0.422***	***1***	***0.752***	***0.986***	***0.796***	***1***	***0.994***
	c'=0.131	0.18	0.57	0.194	0.626	0.19	0.58	0.112	0.604	0.104	0.616	0.098	0.504	0.086	0.63	0.31	0.66	0.454	0.724	0.454	0.748	0.448	0.74	0.502	0.9	***0.952***	***0.93***	***0.992***	***0.98***	***0.996***	***0.99***	1	1
SL	c'=0	***0.282***	***0.126***	***0.262***	***0.11***	***0.256***	***0.122***	***0.198***	***0.096***	***0.236***	***0.108***	***0.136***	***0.084***	***0.168***	***0.106***	***0.354***	***0.136***	***0.46***	***0.21***	***0.448***	***0.198***	***0.488***	***0.242***	***0.532***	***0.282***	***0.854***	***0.46***	***0.982***	***0.758***	***0.98***	***0.796***	***1***	***0.982***
	c'=0.131	0.292	0.652	0.238	0.618	0.278	0.622	0.25	0.6	0.22	0.656	0.18	0.538	0.158	0.672	0.352	0.678	0.462	0.728	0.454	0.75	0.532	0.802	0.466	0.818	0.862	0.9	0.974	0.974	0.974	0.986	1	1
LS	c'=0	***0.264***	***0.106***	***0.278***	***0.146***	***0.266***	***0.124***	***0.204***	***0.106***	***0.214***	***0.122***	***0.142***	***0.096***	***0.156***	***0.134***	***0.294***	***0.174***	***0.492***	***0.224***	***0.436***	***0.23***	***0.502***	***0.314***	***0.532***	***0.366***	***0.858***	***0.562***	***0.968***	***0.83***	***0.984***	***0.868***	***1***	***0.998***
	c'=0.131	0.262	0.752	0.252	0.732	0.262	0.746	0.21	0.708	0.24	0.768	0.144	0.632	0.154	0.732	0.346	0.794	0.442	0.866	0.436	0.874	0.496	0.856	0.48	0.928	0.876	0.978	0.972	0.99	0.974	0.998	1	1
MM	c'=0	***0.934***	***0.272***	***0.93***	***0.224***	***0.922***	***0.26***	***0.702***	***0.248***	***0.704***	***0.23***	***0.488***	***0.218***	***0.47***	***0.266***	***0.974***	***0.302***	***0.948***	***0.398***	***0.96***	***0.402***	***0.864***	***0.458***	***0.85***	***0.55***	***1***	***0.716***	***1***	***0.908***	***1***	***0.928***	***1***	***0.998***
	c'=0.131	***0.928***	***0.864***	***0.9***	***0.85***	***0.92***	***0.844***	0.698	0.812	0.73	0.852	0.47	0.736	0.484	0.868	***0.974***	***0.886***	0.934	0.946	0.936	0.948	0.838	0.912	0.886	0.956	***1***	***0.98***	***1***	***0.996***	***1***	***0.998***	1	1
ML	c'=0	***0.988***	***0.614***	***0.992***	***0.584***	***0.988***	***0.594***	***0.958***	***0.572***	***0.95***	***0.592***	***0.868***	***0.522***	***0.83***	***0.604***	***0.994***	***0.646***	***1***	***0.742***	***0.99***	***0.712***	***0.972***	***0.712***	***0.988***	***0.808***	***1***	***0.91***	***1***	***0.974***	***1***	***0.996***	1	1
	c'=0.131	***0.998***	***0.958***	***0.992***	***0.962***	***0.98***	***0.966***	***0.958***	***0.944***	0.96	0.966	0.84	0.936	0.842	0.966	***0.99***	***0.966***	***0.988***	***0.97***	***0.988***	***0.986***	0.98	0.984	0.97	0.988	***1***	***0.998***	1	1	1	1	1	1
LM	c'=0	***0.984***	***0.642***	***0.988***	***0.666***	***0.992***	***0.694***	***0.944***	***0.6***	***0.942***	***0.676***	***0.852***	***0.532***	***0.824***	***0.662***	***0.996***	***0.744***	***0.994***	***0.76***	***0.986***	***0.824***	***0.98***	***0.796***	***0.976***	***0.89***	***0.998***	***0.952***	***1***	***0.99***	***1***	***0.994***	1	1
	c'=0.131	***0.99***	***0.978***	0.986	0.988	0.982	0.988	0.948	0.972	0.96	0.988	0.84	0.95	0.832	0.978	***0.994***	***0.976***	***0.988***	***0.986***	0.982	0.994	0.968	0.992	0.978	1	***1***	***0.998***	1	1	1	1	1	1
LL	c'=0	***1***	***0.984***	***1***	***0.984***	***1***	***0.98***	***1***	***0.954***	***1***	***0.984***	***0.996***	***0.944***	***1***	***0.976***	***1***	***0.982***	***1***	***0.988***	***1***	***0.992***	***1***	***0.994***	***1***	***0.998***	1	1	1	1	1	1	1	1
	c'=0.131	1	1	1	1	1	1	1	1	1	1	0.992	0.994	1	1	1	1	***1***	***0.998***	1	1	1	1	1	1	1	1	1	1	1	1	1	1

Note: For effect sizes, Z = zero, S = 0.101, M = 0.314, and L = 0.577, as per the path parameters used in the simulation. Different values of c' are listed down the lefthand column and the different tests are listed across the top.

Table 5

Empirical Power Values for the Test of the Total Mediated Effect and the Test of the Total Effect for the Parallel Two Mediator Model at c' = 0 and 0.131, N = 500

		a_1b_1																															
		ZZ		ZS		SZ		ZM		MZ		ZL		LZ		SS		SM		MS		SL		LS		MM		ML		LM		LL	
a_2b_2		$a_1b_1+a_2b_2$	c	$a_1b_1+a_2b_2$	c	$a_1b_1+a_2b_2$	c	$a_1b_1+a_2b_2$	c	$a_1b_1+a_2b_2$	c	$a_1b_1+a_2b_2$	c	$a_1b_1+a_2b_2$	c	$a_1b_1+a_2b_2$	c	$a_1b_1+a_2b_2$	c	$a_1b_1+a_2b_2$	c	$a_1b_1+a_2b_2$	c	$a_1b_1+a_2b_2$	c	$a_1b_1+a_2b_2$	c	$a_1b_1+a_2b_2$	c	$a_1b_1+a_2b_2$	c	$a_1b_1+a_2b_2$	c
ZZ	c'=0	0	0.068	0.012	0.042	0	0.072	0.032	0.034	0.024	0.066	***0.06***	***0.056***	0.04	0.062	***0.162***	***0.066***	***0.516***	***0.066***	***0.55***	***0.122***	***0.59***	***0.21***	***0.586***	***0.268***	***1***	***0.578***	***1***	***0.928***	***1***	***0.976***	1	1
	c'=0.131	0	0.834	0.008	0.86	0.01	0.832	0.032	0.802	0.048	0.834	0.058	0.704	0.036	0.832	0.158	0.886	0.544	0.936	0.56	0.926	0.604	0.96	0.588	0.982	***1***	***0.998***	1	1	1	1	1	1
ZS	c'=0	0.004	0.04	0.018	0.046	0.018	0.044	***0.04***	***0.038***	***0.046***	***0.034***	0.058	0.074	0.046	0.048	***0.158***	***0.052***	***0.532***	***0.094***	***0.53***	***0.11***	***0.578***	***0.188***	***0.624***	***0.256***	***1***	***0.542***	***1***	***0.948***	***1***	***0.972***	1	1
	c'=0.131	0.002	0.858	0.006	0.842	0.014	0.832	0.044	0.824	0.036	0.824	0.058	0.736	0.052	0.852	0.138	0.89	0.492	0.934	0.566	0.958	0.586	0.932	0.592	0.992	0.998	0.998	1	1	1	1	1	1
SZ	c'=0	0.004	0.038	0.008	0.048	0.014	0.034	0.04	0.052	0.042	0.052	0.06	0.064	***0.05***	***0.04***	***0.132***	***0.046***	***0.53***	***0.098***	***0.496***	***0.104***	***0.562***	***0.194***	***0.582***	***0.254***	***1***	***0.554***	***1***	***0.948***	***1***	***0.972***	1	1
	c'=0.131	0.004	0.856	0.026	0.822	0.016	0.804	0.032	0.788	0.038	0.826	0.046	0.692	0.066	0.838	0.152	0.89	0.526	0.92	0.49	0.936	0.546	0.952	0.58	0.986	1	1	1	1	1	1	1	1
ZM	c'=0	***0.046***	***0.042***	0.032	0.042	0.04	0.046	***0.05***	***0.042***	0.026	0.038	***0.072***	***0.052***	0.05	0.058	***0.094***	***0.056***	***0.33***	***0.106***	***0.344***	***0.114***	***0.518***	***0.166***	***0.466***	***0.208***	***0.986***	***0.568***	***1***	***0.914***	***1***	***0.944***	1	1
	c'=0.131	0.034	0.796	0.048	0.77	0.06	0.808	0.044	0.758	0.052	0.81	0.052	0.702	0.054	0.818	0.102	0.834	0.35	0.926	0.32	0.948	0.47	0.94	0.482	0.974	0.992	0.996	***1***	***0.998***	1	1	1	1
MZ	c'=0	0.03	0.058	0.038	0.044	0.046	0.048	0.044	0.064	0.046	0.052	0.05	0.058	0.02	0.06	***0.092***	***0.056***	***0.334***	***0.092***	***0.344***	***0.096***	***0.48***	***0.202***	***0.502***	***0.246***	***0.988***	***0.534***	***1***	***0.922***	***1***	***0.97***	1	1
	c'=0.131	0.032	0.822	0.058	0.826	0.04	0.82	0.03	0.794	0.046	0.82	0.048	0.766	0.044	0.836	0.076	0.896	0.334	0.922	0.3	0.952	0.518	0.964	0.48	0.982	0.986	0.998	1	1	1	1	1	1
ZL	c'=0	***0.06***	***0.042***	***0.07***	***0.05***	0.042	0.052	***0.046***	***0.044***	0.04	0.058	0.046	0.048	***0.046***	***0.042***	***0.054***	***0.04***	***0.194***	***0.098***	***0.182***	***0.108***	***0.358***	***0.19***	***0.36***	***0.188***	***0.856***	***0.45***	***0.994***	***0.872***	***0.998***	***0.914***	1	1
	c'=0.131	0.042	0.724	0.042	0.708	0.042	0.726	0.044	0.66	0.05	0.734	0.05	0.624	0.042	0.748	0.06	0.756	0.202	0.884	0.17	0.878	0.352	0.906	0.358	0.938	0.886	0.988	1	1	1	1	1	1
LZ	c'=0	0.048	0.058	0.05	0.056	0.05	0.056	0.054	0.066	0.048	0.048	0.05	0.05	0.052	0.056	0.06	0.062	***0.154***	***0.11***	***0.186***	***0.094***	***0.344***	***0.176***	***0.332***	***0.248***	***0.846***	***0.476***	***0.998***	***0.932***	***0.996***	***0.968***	1	1
	c'=0.131	0.04	0.826	0.058	0.824	0.044	0.838	0.046	0.8	0.052	0.812	0.056	0.704	0.04	0.8	0.06	0.884	0.184	0.93	0.17	0.946	0.35	0.952	0.392	0.988	0.862	0.996	0.996	1	1	1	1	1
SS	c'=0	***0.142***	***0.058***	***0.128***	***0.06***	***0.098***	***0.058***	***0.092***	***0.06***	***0.084***	***0.05***	0.066	0.07	***0.086***	***0.036***	***0.546***	***0.07***	***0.794***	***0.14***	***0.746***	***0.15***	***0.746***	***0.266***	***0.714***	***0.344***	***1***	***0.602***	***1***	***0.94***	***1***	***0.982***	1	1
	c'=0.131	0.16	0.854	0.134	0.85	0.132	0.874	0.102	0.85	0.072	0.874	0.062	0.794	0.084	0.866	0.546	0.918	0.734	0.966	0.752	0.972	0.736	0.97	0.686	0.986	***1***	***0.996***	1	1	1	1	1	1
SM	c'=0	***0.536***	***0.104***	***0.524***	***0.088***	***0.506***	***0.084***	***0.308***	***0.116***	***0.32***	***0.076***	***0.162***	***0.11***	***0.198***	***0.112***	***0.764***	***0.128***	***0.856***	***0.246***	***0.868***	***0.262***	***0.832***	***0.364***	***0.856***	***0.466***	***1***	***0.728***	***1***	***0.98***	***1***	***0.994***	1	1
	c'=0.131	0.596	0.938	0.522	0.924	0.54	0.944	0.322	0.912	0.314	0.928	0.19	0.85	0.144	0.938	0.774	0.938	0.906	0.984	0.892	0.98	0.828	0.982	0.838	0.996	1	1	1	1	1	1	1	1
MS	c'=0	***0.558***	***0.092***	***0.496***	***0.092***	***0.554***	***0.118***	***0.334***	***0.112***	***0.362***	***0.114***	***0.204***	***0.094***	***0.182***	***0.13***	***0.778***	***0.146***	***0.892***	***0.27***	***0.87***	***0.298***	***0.85***	***0.462***	***0.854***	***0.502***	***1***	***0.786***	***1***	***0.982***	***1***	***0.988***	1	1
	c'=0.131	0.538	0.958	0.49	0.932	0.502	0.934	0.324	0.92	0.336	0.944	0.184	0.902	0.154	0.946	0.786	0.952	0.862	0.98	0.868	0.986	0.848	0.98	0.836	0.998	1	1	1	1	1	1	1	1
SL	c'=0	***0.628***	***0.16***	***0.55***	***0.2***	***0.582***	***0.21***	***0.51***	***0.18***	***0.476***	***0.192***	***0.33***	***0.186***	***0.356***	***0.2***	***0.716***	***0.25***	***0.884***	***0.414***	***0.856***	***0.396***	***0.88***	***0.512***	***0.884***	***0.588***	***0.998***	***0.804***	***1***	***0.982***	***1***	***0.994***	1	1
	c'=0.131	0.582	0.954	0.578	0.932	0.56	0.96	0.476	0.94	0.52	0.946	0.368	0.914	0.35	0.96	0.742	0.966	0.854	0.982	0.872	0.98	0.888	0.986	0.856	0.998	1	1	1	1	1	1	1	1
LS	c'=0	***0.636***	***0.254***	***0.564***	***0.202***	***0.612***	***0.234***	***0.49***	***0.218***	***0.502***	***0.242***	***0.302***	***0.224***	***0.352***	***0.264***	***0.728***	***0.324***	***0.874***	***0.488***	***0.85***	***0.506***	***0.908***	***0.626***	***0.856***	***0.724***	***1***	***0.916***	***1***	***0.998***	1	1	1	1
	c'=0.131	0.626	0.988	0.578	0.986	0.578	0.984	0.51	0.984	0.498	0.982	0.324	0.964	0.334	0.992	0.712	0.992	0.846	0.998	0.842	1	0.872	1	0.898	1	1	1	1	1	1	1	1	1
MM	c'=0	***1***	***0.534***	***1***	***0.534***	***1***	***0.59***	***0.982***	***0.516***	***0.994***	***0.57***	***0.86***	***0.472***	***0.856***	***0.558***	***1***	***0.618***	***1***	***0.762***	***1***	***0.786***	***1***	***0.846***	***0.998***	***0.91***	***1***	***0.982***	1	1	1	1	1	1
	c'=0.131	***1***	***0.998***	***1***	***0.996***	1	1	0.984	0.998	0.986	0.998	0.862	0.984	0.868	1	1	1	1	1	1	1	1	1	0.996	1	1	1	1	1	1	1	1	1
ML	c'=0	***1***	***0.922***	***1***	***0.934***	***1***	***0.946***	***1***	***0.924***	***1***	***0.934***	***0.998***	***0.874***	***0.996***	***0.942***	***1***	***0.936***	***1***	***0.984***	***1***	***0.99***	***1***	***0.98***	1	1	1	1	1	1	0.998	1	1	1
	c'=0.131	1	1	1	1	1	1	1	1	1	1	0.998	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
LM	c'=0	***1***	***0.96***	***1***	***0.958***	***1***	***0.978***	***1***	***0.974***	***1***	***0.968***	***0.998***	***0.896***	***0.998***	***0.962***	***1***	***0.988***	***1***	***0.998***	***1***	***0.996***	***1***	***0.99***	1	1	1	1	1	1	1	1	1	1
	c'=0.131	1	1	1	1	1	1	1	1	1	1	0.998	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
LL	c'=0	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
	c'=0.131	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1

Note: For effect sizes, Z = zero, S = 0.101, M = 0.314, and L = 0.577, as per the path parameters used in the simulation. Different values of c' are listed down the lefthand column and the different tests are listed across the top.

Table 6

Empirical Power Values for the Test of the Total Mediated Effect and the Test of the Total Effect for the Parallel Two Mediator Model at c' = 0 and 0.131, N = 1000

		a_1b_1																															
		ZZ		ZS		SZ		ZM		MZ		ZL		LZ		SS		SM		MS		SL		LS		MM		ML		LM		LL	
a_2b_2		$a_1b_1+a_2b_2$	c	$a_1b_1+a_2b_2$	c	$a_1b_1+a_2b_2$	c	$a_1b_1+a_2b_2$	c	$a_1b_1+a_2b_2$	c	$a_1b_1+a_2b_2$	c	$a_1b_1+a_2b_2$	c	$a_1b_1+a_2b_2$	c	$a_1b_1+a_2b_2$	c	$a_1b_1+a_2b_2$	c	$a_1b_1+a_2b_2$	c	$a_1b_1+a_2b_2$	c	$a_1b_1+a_2b_2$	c	$a_1b_1+a_2b_2$	c	$a_1b_1+a_2b_2$	c	$a_1b_1+a_2b_2$	c
ZZ	c'=0	0	0.06	0.014	0.062	0.012	0.046	0.046	0.066	***0.05***	***0.036***	0.044	0.054	0.048	0.052	***0.584***	***0.042***	***0.894***	***0.166***	***0.834***	***0.172***	***0.9***	***0.328***	***0.864***	***0.424***	***1***	***0.858***	***1***	***0.996***	1	1	1	1
	c'=0.131	0	0.98	0.016	0.986	0.016	0.986	0.044	0.962	0.042	0.982	0.062	0.938	0.042	0.99	0.562	0.99	0.898	0.996	0.884	0.998	0.902	1	0.886	1	1	1	1	1	1	1	1	1
ZS	c'=0	0.022	0.058	0.028	0.046	0.032	0.074	0.04	0.042	0.04	0.066	***0.044***	***0.038***	0.054	0.058	***0.386***	***0.064***	***0.832***	***0.156***	***0.868***	***0.154***	***0.892***	***0.412***	***0.882***	***0.43***	***1***	***0.844***	1	1	1	1	1	1
	c'=0.131	0.014	0.988	0.018	0.984	0.024	0.988	0.038	0.97	0.054	0.986	0.03	0.962	0.04	0.99	0.406	0.996	0.842	1	0.838	1	0.864	1	0.896	1	1	1	1	1	1	1	1	1
SZ	c'=0	0.008	0.062	0.036	0.05	0.026	0.048	0.044	0.072	0.036	0.044	***0.058***	***0.052***	0.046	0.06	***0.418***	***0.054***	***0.848***	***0.172***	***0.826***	***0.154***	***0.876***	***0.354***	***0.854***	***0.45***	***1***	***0.86***	1	1	1	1	1	1
	c'=0.131	0.012	0.984	0.024	0.988	0.026	0.984	0.04	0.982	0.058	0.994	0.044	0.964	0.04	0.988	0.382	0.996	0.846	0.988	0.854	0.998	0.866	0.998	0.902	1	1	1	1	1	1	1	1	1
ZM	c'=0	0.046	0.062	0.046	0.07	0.04	0.044	0.044	0.056	0.042	0.048	0.042	0.048	0.046	0.07	***0.15***	***0.068***	***0.586***	***0.164***	***0.588***	***0.14***	***0.764***	***0.32***	***0.82***	***0.406***	***1***	***0.822***	1	1	1	1	1	1
	c'=0.131	0.038	0.98	0.042	0.978	0.028	0.988	0.032	0.95	0.064	0.978	0.042	0.938	0.05	0.98	0.138	0.98	0.586	0.996	0.54	0.998	0.794	1	0.814	1	1	1	1	1	1	1	1	1
MZ	c'=0	0.048	0.052	***0.06***	***0.05***	0.034	0.042	0.044	0.048	0.04	0.042	***0.056***	***0.044***	0.062	0.064	***0.152***	***0.074***	***0.6***	***0.158***	***0.552***	***0.152***	***0.776***	***0.32***	***0.76***	***0.476***	***1***	***0.854***	1	1	1	1	1	1
	c'=0.131	0.032	0.992	0.04	0.986	0.04	0.986	0.046	0.982	0.066	0.98	0.054	0.94	0.044	0.982	0.138	0.99	0.604	0.998	0.574	0.994	0.78	1	0.794	1	1	1	1	1	1	1	1	1
ZL	c'=0	0.032	0.05	0.046	0.068	0.056	0.064	0.046	0.054	0.052	0.066	0.032	0.052	0.044	0.044	***0.084***	***0.066***	***0.322***	***0.148***	***0.302***	***0.128***	***0.604***	***0.31***	***0.638***	***0.356***	***0.994***	***0.762***	***1***	***0.996***	1	1	1	1
	c'=0.131	0.05	0.942	0.054	0.942	0.03	0.948	0.04	0.938	0.058	0.942	0.052	0.882	0.052	0.936	0.094	0.96	0.35	0.986	0.344	0.998	0.644	0.998	0.608	0.998	0.996	1	1	1	1	1	1	1
LZ	c'=0	0.048	0.06	***0.06***	***0.048***	***0.066***	***0.064***	0.054	0.06	***0.058***	***0.046***	0.046	0.058	0.054	0.058	0.066	0.078	***0.294***	***0.158***	***0.322***	***0.162***	***0.58***	***0.338***	***0.61***	***0.446***	***0.988***	***0.872***	1	1	***1***	***0.998***	1	1
	c'=0.131	0.058	0.974	0.056	0.978	0.054	0.988	0.05	0.98	0.05	0.996	0.062	0.954	0.062	0.982	0.076	0.99	0.34	0.998	0.326	1	0.572	1	0.598	1	0.992	1	1	1	1	1	1	1
SS	c'=0	***0.534***	***0.07***	***0.384***	***0.05***	***0.406***	***0.062***	***0.144***	***0.064***	***0.14***	***0.076***	***0.064***	***0.054***	***0.096***	***0.06***	***0.962***	***0.11***	***0.962***	***0.216***	***0.968***	***0.254***	***0.936***	***0.456***	***0.96***	***0.572***	***1***	***0.884***	1	1	1	1	1	1
	c'=0.131	0.548	0.996	0.428	0.994	0.39	0.998	0.12	0.988	0.158	0.992	0.098	0.948	0.076	0.994	0.972	0.998	0.972	0.998	0.972	0.998	0.948	1	0.956	1	1	1	1	1	1	1	1	1
SM	c'=0	***0.88***	***0.156***	***0.814***	***0.146***	***0.818***	***0.136***	***0.57***	***0.14***	***0.602***	***0.126***	***0.324***	***0.132***	***0.344***	***0.19***	***0.966***	***0.236***	***0.998***	***0.45***	***1***	***0.492***	***0.986***	***0.616***	***0.982***	***0.754***	***1***	***0.974***	1	1	1	1	1	1
	c'=0.131	0.868	0.998	0.854	0.998	0.848	1	0.566	0.996	0.596	0.996	0.302	0.994	0.326	0.992	0.968	1	0.996	1	0.998	1	0.994	1	0.988	1	1	1	1	1	1	1	1	1
MS	c'=0	***0.866***	***0.158***	***0.842***	***0.144***	***0.842***	***0.154***	***0.6***	***0.168***	***0.616***	***0.172***	***0.332***	***0.134***	***0.342***	***0.17***	***0.96***	***0.266***	***0.99***	***0.432***	***0.99***	***0.516***	***0.994***	***0.68***	***0.986***	***0.778***	***1***	***0.988***	1	1	1	1	1	1
	c'=0.131	0.884	1	0.862	1	0.866	1	0.552	1	0.63	0.998	0.282	0.994	0.352	1	0.98	0.998	1	1	0.99	1	0.986	1	0.996	1	1	1	1	1	1	1	1	1
SL	c'=0	***0.89***	***0.352***	***0.892***	***0.366***	***0.89***	***0.394***	***0.788***	***0.358***	***0.806***	***0.356***	***0.614***	***0.302***	***0.582***	***0.352***	***0.958***	***0.472***	***0.998***	***0.628***	***0.99***	***0.708***	***0.998***	***0.826***	***0.998***	***0.896***	***1***	***0.99***	1	1	1	1	1	1
	c'=0.131	0.89	1	0.898	1	0.868	1	0.812	1	0.814	1	0.57	0.998	0.63	1	0.964	1	0.986	1	0.984	1	0.994	1	1	1	1	1	1	1	1	1	1	1
LS	c'=0	***0.904***	***0.432***	***0.872***	***0.478***	***0.868***	***0.448***	***0.792***	***0.404***	***0.78***	***0.464***	***0.612***	***0.35***	***0.614***	***0.46***	***0.954***	***0.582***	***0.98***	***0.784***	***0.992***	***0.814***	***0.996***	***0.882***	***0.994***	***0.958***	***1***	***0.998***	1	1	1	1	1	1
	c'=0.131	0.894	1	0.862	1	0.876	1	0.82	1	0.768	1	0.574	1	0.594	1	0.952	1	0.998	1	0.992	1	0.994	1	0.992	1	1	1	1	1	1	1	1	1
MM	c'=0	***1***	***0.842***	***1***	***0.856***	***1***	***0.806***	***1***	***0.81***	***1***	***0.858***	***0.986***	***0.708***	***0.996***	***0.842***	***1***	***0.898***	***1***	***0.97***	***1***	***0.966***	***1***	***0.982***	***1***	***0.996***	***1***	***0.998***	1	1	1	1	1	1
	c'=0.131	1	1	1	1	1	1	1	1	1	1	0.992	1	0.994	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
ML	c'=0	***1***	***0.996***	***1***	***0.998***	***1***	***0.998***	***1***	***0.996***	***1***	***0.998***	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
	c'=0.131	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
LM	c'=0	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
	c'=0.131	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
LL	c'=0	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
	c'=0.131	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1

Note: For effect sizes, Z = zero, S = 0.101, M = 0.314, and L = 0.577, as per the path parameters used in the simulation. Different values of c' are listed down the lefthand column and the different tests are listed across the top.

Table 7

Comparison of Empirical and Analytical Power for the Joint Significance Test for the Single Mediator Model Across Levels of c'

	N = 50			N = 100			N = 200			N = 500			N = 1000			N = 5000		
	Emp	*z*	*t*	Emp	*z*	*t*	Emp	*z*	*t*	Emp	*z*	*t*	Emp	*z*	*t*	Emp	*z*	*t*
a = 0.14, *b* = 0.14	0.018	0.025	0.024	0.057	0.078	0.077	0.218	0.249	0.249	0.745	0.766	0.768	0.987	0.986	0.986	1	1	1
a = 0.14, *b* = 0.39	0.094	0.117	0.117	0.254	0.269	0.269	0.504	0.499	0.500	0.875	0.875	0.877	0.994	0.993	0.993	1	1	1
a = 0.14, *b* = 0.59	0.148	0.153	0.153	0.282	0.279	0.278	0.517	0.499	0.500	0.883	0.875	0.877	0.995	0.993	0.993	1	1	1
a = 0.39, *b* = 0.14	0.106	0.117	0.116	0.272	0.269	0.267	0.507	0.499	0.498	0.881	0.875	0.876	0.994	0.993	0.993	1	1	1
a = 0.39, *b* = 0.39	0.521	0.551	0.562	0.926	0.928	0.937	0.999	0.9993	0.9995	1	1	1	1	1	1	1	1	1
a = 0.39, *b* = 0.59	0.717	0.720	0.737	0.965	0.963	0.969	1	0.9996	0.9998	1	1	1	1	1	1	1	1	1
a = 0.59, *b* = 0.14	0.146	0.153	0.151	0.273	0.279	0.276	0.503	0.499	0.498	0.867	0.875	0.876	0.993	0.993	0.993	1	1	1
a = 0.59, *b* = 0.39	0.713	0.720	0.730	0.963	0.963	0.967	1	0.9996	0.9998	1	1	1	1	1	1	1	1	1
a = 0.59, *b* = 0.59	0.943	0.941	0.957	1	0.9996	0.9999	1	1	1	1	1	1	1	1	1	1	1	1

Note: Empirical power is represented here as ‘Emp’, and analytical power calculated using regression coefficients and their standard errors is represented here as ‘*t*’.

Table 8

Comparison of Empirical and Analytical Power for the Product of Coefficients Test for the Single Mediator Model Across Levels of c'

	N = 50		N = 100		N = 200		N = 500		N = 1000		N = 5000	
	Emp	Ana	Emp	Ana	Emp	Ana	Emp	Ana	Emp	Ana	Emp	Ana
a = 0.14, *b* = 0.14	0.005	0.101	0.015	0.163	0.083	0.285	0.577	0.598	0.968	0.878	1	1
a = 0.14, *b* = 0.39	0.037	0.147	0.163	0.256	0.444	0.458	0.865	0.837	0.993	0.986	1	1
a = 0.14, *b* = 0.59	0.097	0.155	0.240	0.270	0.503	0.483	0.878	0.860	0.995	0.990	1	1
a = 0.39, *b* = 0.14	0.051	0.146	0.172	0.254	0.447	0.456	0.872	0.836	0.993	0.986	1	1
a = 0.39, *b* = 0.39	0.357	0.476	0.874	0.777	0.999	0.972	1	1	1	1	1	1
a = 0.39, *b* = 0.59	0.607	0.613	0.957	0.895	1.000	0.996	1	1	1	1	1	1
a = 0.59, *b* = 0.14	0.103	0.153	0.240	0.268	0.483	0.481	0.864	0.859	0.993	0.990	1	1
a = 0.59, *b* = 0.39	0.615	0.609	0.951	0.894	1.000	0.995	1	1	1	1	1	1
a = 0.59, *b* = 0.59	0.904	0.820	1	0.985	1	1	1	1	1	1	1	1

Note: Empirical power is represented here as ‘Emp’, and analytical power calculated using regression coefficients and their standard errors is represented here as ‘*t*’.

Table 9

Comparison of Empirical and Analytical Power of the Test of the Total Mediated Effect for the Parallel Two Mediator Model Across Levels of c'

		N = 50		N = 100		N = 200		N = 500		N = 1000	
		Emp	*t*	Emp	*t*	Emp	*t*	Emp	*t*	Emp	*t*
$a_1 = 0.101, b_1 = 0.101$	$a_2 = 0.101, b_2 = 0.101$	0.006	0.082	0.023	0.126	0.098	0.211	0.534	0.452	0.965	0.740
$a_1 = 0.101, b_1 = 0.101$	$a_2 = 0.101, b_2 = 0.314$	0.023	0.113	0.089	0.187	0.305	0.331	0.771	0.676	0.970	0.928
$a_1 = 0.101, b_1 = 0.101$	$a_2 = 0.101, b_2 = 0.577$	0.064	0.116	0.174	0.193	0.358	0.343	0.733	0.694	0.959	0.937
$a_1 = 0.101, b_1 = 0.101$	$a_2 = 0.314, b_2 = 0.101$	0.025	0.096	0.089	0.154	0.289	0.268	0.767	0.567	0.969	0.854
$a_1 = 0.101, b_1 = 0.101$	$a_2 = 0.314, b_2 = 0.314$	0.150	0.292	0.577	0.526	0.968	0.822	1.000	0.996	1	1
$a_1 = 0.101, b_1 = 0.101$	$a_2 = 0.314, b_2 = 0.577$	0.443	0.452	0.838	0.750	0.994	0.963	1	1.000	1	1
$a_1 = 0.101, b_1 = 0.101$	$a_2 = 0.577, b_2 = 0.101$	0.072	0.100	0.171	0.162	0.337	0.283	0.711	0.596	0.947	0.877
$a_1 = 0.101, b_1 = 0.101$	$a_2 = 0.577, b_2 = 0.314$	0.413	0.414	0.833	0.706	0.992	0.946	1	1.000	1	1
$a_1 = 0.101, b_1 = 0.101$	$a_2 = 0.577, b_2 = 0.577$	0.860	0.751	1.000	0.966	1	1.000	1	1	1	1
$a_1 = 0.101, b_1 = 0.314$	$a_2 = 0.101, b_2 = 0.101$	0.019	0.113	0.085	0.187	0.314	0.331	0.754	0.676	0.969	0.928
$a_1 = 0.101, b_1 = 0.314$	$a_2 = 0.101, b_2 = 0.314$	0.053	0.144	0.191	0.250	0.472	0.448	0.881	0.827	0.997	0.984
$a_1 = 0.101, b_1 = 0.314$	$a_2 = 0.101, b_2 = 0.577$	0.109	0.146	0.242	0.253	0.477	0.453	0.857	0.832	0.991	0.985
$a_1 = 0.101, b_1 = 0.314$	$a_2 = 0.314, b_2 = 0.101$	0.050	0.128	0.189	0.218	0.445	0.390	0.881	0.761	0.993	0.966
$a_1 = 0.101, b_1 = 0.314$	$a_2 = 0.314, b_2 = 0.314$	0.220	0.315	0.613	0.563	0.951	0.854	1	0.998	1	1
$a_1 = 0.101, b_1 = 0.314$	$a_2 = 0.314, b_2 = 0.577$	0.465	0.473	0.842	0.773	0.993	0.971	1	1.000	1	1
$a_1 = 0.101, b_1 = 0.314$	$a_2 = 0.577, b_2 = 0.101$	0.091	0.127	0.217	0.217	0.459	0.388	0.855	0.758	0.990	0.965
$a_1 = 0.101, b_1 = 0.314$	$a_2 = 0.577, b_2 = 0.314$	0.437	0.439	0.839	0.736	0.992	0.958	1	1.000	1	1
$a_1 = 0.101, b_1 = 0.314$	$a_2 = 0.577, b_2 = 0.577$	0.864	0.759	0.997	0.968	1	1.000	1	1	1	1
$a_1 = 0.101, b_1 = 0.577$	$a_2 = 0.101, b_2 = 0.101$	0.063	0.116	0.155	0.193	0.328	0.343	0.740	0.694	0.950	0.937
$a_1 = 0.101, b_1 = 0.577$	$a_2 = 0.101, b_2 = 0.314$	0.092	0.146	0.244	0.253	0.456	0.453	0.852	0.832	0.991	0.985
$a_1 = 0.101, b_1 = 0.577$	$a_2 = 0.101, b_2 = 0.577$	0.115	0.159	0.258	0.278	0.517	0.496	0.880	0.872	0.995	0.992
$a_1 = 0.101, b_1 = 0.577$	$a_2 = 0.314, b_2 = 0.101$	0.098	0.137	0.229	0.235	0.452	0.422	0.853	0.799	0.992	0.977
$a_1 = 0.101, b_1 = 0.577$	$a_2 = 0.314, b_2 = 0.314$	0.237	0.286	0.547	0.514	0.853	0.810	0.999	0.995	1	1.000
$a_1 = 0.101, b_1 = 0.577$	$a_2 = 0.314, b_2 = 0.577$	0.419	0.440	0.780	0.735	0.975	0.957	1	1.000	1	1
$a_1 = 0.101, b_1 = 0.577$	$a_2 = 0.577, b_2 = 0.101$	0.129	0.144	0.245	0.250	0.502	0.447	0.884	0.827	0.994	0.984
$a_1 = 0.101, b_1 = 0.577$	$a_2 = 0.577, b_2 = 0.314$	0.435	0.416	0.776	0.707	0.972	0.946	1	1.000	1	1
$a_1 = 0.101, b_1 = 0.577$	$a_2 = 0.577, b_2 = 0.577$	0.774	0.720	0.983	0.954	1	0.999	1	1	1	1
$a_1 = 0.314, b_1 = 0.101$	$a_2 = 0.101, b_2 = 0.101$	0.024	0.096	0.091	0.154	0.293	0.268	0.750	0.567	0.970	0.854
$a_1 = 0.314, b_1 = 0.101$	$a_2 = 0.101, b_2 = 0.314$	0.056	0.128	0.173	0.218	0.466	0.390	0.881	0.761	0.997	0.966
$a_1 = 0.314, b_1 = 0.101$	$a_2 = 0.101, b_2 = 0.577$	0.105	0.137	0.214	0.235	0.429	0.422	0.861	0.799	0.988	0.977
$a_1 = 0.314, b_1 = 0.101$	$a_2 = 0.314, b_2 = 0.101$	0.059	0.099	0.176	0.161	0.461	0.282	0.862	0.593	0.991	0.875
$a_1 = 0.314, b_1 = 0.101$	$a_2 = 0.314, b_2 = 0.314$	0.215	0.243	0.612	0.441	0.946	0.732	1	0.984	1	1.000
$a_1 = 0.314, b_1 = 0.101$	$a_2 = 0.314, b_2 = 0.577$	0.448	0.400	0.846	0.687	0.989	0.936	1	1.000	1	1
$a_1 = 0.314, b_1 = 0.101$	$a_2 = 0.577, b_2 = 0.101$	0.106	0.101	0.231	0.164	0.439	0.288	0.862	0.604	0.991	0.883
$a_1 = 0.314, b_1 = 0.101$	$a_2 = 0.577, b_2 = 0.314$	0.461	0.337	0.854	0.598	0.987	0.882	1	0.999	1	1
$a_1 = 0.314, b_1 = 0.101$	$a_2 = 0.577, b_2 = 0.577$	0.846	0.652	0.997	0.921	1	0.998	1	1	1	1
$a_1 = 0.314, b_1 = 0.314$	$a_2 = 0.101, b_2 = 0.101$	0.144	0.292	0.593	0.526	0.970	0.822	1	0.996	1	1
$a_1 = 0.314, b_1 = 0.314$	$a_2 = 0.101, b_2 = 0.314$	0.232	0.315	0.635	0.563	0.943	0.854	1	0.998	1	1
$a_1 = 0.314, b_1 = 0.314$	$a_2 = 0.101, b_2 = 0.577$	0.249	0.286	0.530	0.514	0.864	0.810	0.998	0.995	1	1.000
$a_1 = 0.314, b_1 = 0.314$	$a_2 = 0.314, b_2 = 0.101$	0.210	0.243	0.631	0.441	0.951	0.732	1	0.984	1	1.000
$a_1 = 0.314, b_1 = 0.314$	$a_2 = 0.314, b_2 = 0.314$	0.488	0.422	0.944	0.715	1.000	0.950	1	1.000	1	1
$a_1 = 0.314, b_1 = 0.314$	$a_2 = 0.314, b_2 = 0.577$	0.675	0.564	0.969	0.860	1	0.991	1	1	1	1
$a_1 = 0.314, b_1 = 0.314$	$a_2 = 0.577, b_2 = 0.101$	0.254	0.204	0.553	0.370	0.857	0.640	0.999	0.957	1	0.999
$a_1 = 0.314, b_1 = 0.314$	$a_2 = 0.577, b_2 = 0.314$	0.666	0.488	0.967	0.791	1.000	0.977	1	1.000	1	1
$a_1 = 0.314, b_1 = 0.314$	$a_2 = 0.577, b_2 = 0.577$	0.949	0.765	1	0.970	1	1.000	1	1	1	1
$a_1 = 0.314, b_1 = 0.577$	$a_2 = 0.101, b_2 = 0.101$	0.414	0.452	0.835	0.750	0.991	0.963	1	1.000	1	1
$a_1 = 0.314, b_1 = 0.577$	$a_2 = 0.101, b_2 = 0.314$	0.454	0.473	0.851	0.773	0.994	0.971	1	1.000	1	1
$a_1 = 0.314, b_1 = 0.577$	$a_2 = 0.101, b_2 = 0.577$	0.428	0.440	0.772	0.735	0.975	0.957	1	1.000	1	1
$a_1 = 0.314, b_1 = 0.577$	$a_2 = 0.314, b_2 = 0.101$	0.451	0.400	0.848	0.687	0.995	0.936	1	1.000	1	1
$a_1 = 0.314, b_1 = 0.577$	$a_2 = 0.314, b_2 = 0.314$	0.679	0.564	0.969	0.860	1	0.991	1	1	1	1
$a_1 = 0.314, b_1 = 0.577$	$a_2 = 0.314, b_2 = 0.577$	0.799	0.680	0.985	0.935	1	0.999	1	1	1	1
$a_1 = 0.314, b_1 = 0.577$	$a_2 = 0.577, b_2 = 0.101$	0.421	0.339	0.781	0.601	0.976	0.884	1	0.999	1	1
$a_1 = 0.314, b_1 = 0.577$	$a_2 = 0.577, b_2 = 0.314$	0.776	0.615	0.986	0.898	1	0.996	1	1	1	1
$a_1 = 0.314, b_1 = 0.577$	$a_2 = 0.577, b_2 = 0.577$	0.957	0.834	1	0.987	1	1.000	1	1	1	1
$a_1 = 0.577, b_1 = 0.101$	$a_2 = 0.101, b_2 = 0.101$	0.076	0.100	0.145	0.162	0.337	0.283	0.716	0.596	0.959	0.877
$a_1 = 0.577, b_1 = 0.101$	$a_2 = 0.101, b_2 = 0.314$	0.099	0.127	0.227	0.217	0.445	0.388	0.852	0.758	0.989	0.965
$a_1 = 0.577, b_1 = 0.101$	$a_2 = 0.101, b_2 = 0.577$	0.133	0.144	0.269	0.250	0.497	0.447	0.874	0.827	0.997	0.984
$a_1 = 0.577, b_1 = 0.101$	$a_2 = 0.314, b_2 = 0.101$	0.104	0.101	0.230	0.164	0.464	0.288	0.843	0.604	0.992	0.883
$a_1 = 0.577, b_1 = 0.101$	$a_2 = 0.314, b_2 = 0.314$	0.245	0.204	0.539	0.370	0.869	0.640	0.998	0.957	1	0.999
$a_1 = 0.577, b_1 = 0.101$	$a_2 = 0.314, b_2 = 0.577$	0.428	0.339	0.781	0.601	0.979	0.884	1	0.999	1	1
$a_1 = 0.577, b_1 = 0.101$	$a_2 = 0.577, b_2 = 0.101$	0.134	0.102	0.264	0.165	0.489	0.290	0.890	0.608	0.992	0.886
$a_1 = 0.577, b_1 = 0.101$	$a_2 = 0.577, b_2 = 0.314$	0.432	0.277	0.779	0.502	0.977	0.800	1	0.994	1	1
$a_1 = 0.577, b_1 = 0.101$	$a_2 = 0.577, b_2 = 0.577$	0.775	0.546	0.980	0.846	1	0.989	1	1	1	1
$a_1 = 0.577, b_1 = 0.314$	$a_2 = 0.101, b_2 = 0.101$	0.401	0.414	0.821	0.706	0.993	0.946	1	1	1	1
$a_1 = 0.577, b_1 = 0.314$	$a_2 = 0.101, b_2 = 0.314$	0.447	0.439	0.853	0.736	0.991	0.958	1	1	1	1
$a_1 = 0.577, b_1 = 0.314$	$a_2 = 0.101, b_2 = 0.577$	0.423	0.416	0.769	0.707	0.974	0.946	1	1	1	1
$a_1 = 0.577, b_1 = 0.314$	$a_2 = 0.314, b_2 = 0.101$	0.438	0.337	0.849	0.598	0.992	0.882	1	0.999	1	1
$a_1 = 0.577, b_1 = 0.314$	$a_2 = 0.314, b_2 = 0.314$	0.677	0.488	0.971	0.791	1	0.977	1	1	1	1
$a_1 = 0.577, b_1 = 0.314$	$a_2 = 0.314, b_2 = 0.577$	0.773	0.615	0.989	0.898	1	0.996	1	1	1	1
$a_1 = 0.577, b_1 = 0.314$	$a_2 = 0.577, b_2 = 0.101$	0.426	0.277	0.778	0.502	0.972	0.800	1	0.994	1	1
$a_1 = 0.577, b_1 = 0.314$	$a_2 = 0.577, b_2 = 0.314$	0.791	0.521	0.984	0.823	1	0.984	1	1	1	1
$a_1 = 0.577, b_1 = 0.314$	$a_2 = 0.577, b_2 = 0.577$	0.955	0.758	1	0.968	1	1	1	1	1	1
$a_1 = 0.577, b_1 = 0.577$	$a_2 = 0.101, b_2 = 0.101$	0.851	0.751	0.999	0.966	1	1	1	1	1	1
$a_1 = 0.577, b_1 = 0.577$	$a_2 = 0.101, b_2 = 0.314$	0.853	0.759	0.998	0.968	1	1	1	1	1	1
$a_1 = 0.577, b_1 = 0.577$	$a_2 = 0.101, b_2 = 0.577$	0.795	0.720	0.985	0.954	1	0.999	1	1	1	1
$a_1 = 0.577, b_1 = 0.577$	$a_2 = 0.314, b_2 = 0.101$	0.856	0.652	0.998	0.921	1	0.998	1	1	1	1
$a_1 = 0.577, b_1 = 0.577$	$a_2 = 0.314, b_2 = 0.314$	0.941	0.765	1	0.970	1	1	1	1	1	1
$a_1 = 0.577, b_1 = 0.577$	$a_2 = 0.314, b_2 = 0.577$	0.958	0.834	1	0.987	1	1	1	1	1	1
$a_1 = 0.577, b_1 = 0.577$	$a_2 = 0.577, b_2 = 0.101$	0.775	0.546	0.986	0.846	1	0.989	1	1	1	1
$a_1 = 0.577, b_1 = 0.577$	$a_2 = 0.577, b_2 = 0.314$	0.955	0.758	1	0.968	1	1	1	1	1	1
$a_1 = 0.577, b_1 = 0.577$	$a_2 = 0.577, b_2 = 0.577$	0.994	0.900	1	0.996	1	1	1	1	1	1

Note: Empirical power is represented here as ‘Emp’, and analytical power calculated using regression coefficients and their standard errors is represented here as ‘*t*’.

Table 10

Comparison of Empirical and Analytical Power of the tests of the Specific Mediated Effect a_1b_1 for the Parallel Two Mediator Model Across Levels of c'

	N = 50			N = 100			N = 200			N = 500			N = 1000		
	Emp	*z*	*t*	Emp	*z*	*t*	Emp	*z*	*t*	Emp	*z*	*t*	Emp	*z*	*t*
a_1 = 0.101, b_1 = 0.101	0.011	0.010	0.010	0.028	0.028	0.027	0.088	0.086	0.085	0.373	0.376	0.376	0.788	0.791	0.792
a_1 = 0.101, b_1 = 0.314	0.056	0.058	0.057	0.143	0.144	0.144	0.297	0.290	0.290	0.617	0.613	0.614	0.893	0.889	0.890
a_1 = 0.101, b_1 = 0.577	0.101	0.099	0.099	0.166	0.167	0.166	0.295	0.293	0.293	0.614	0.613	0.614	0.889	0.889	0.890
a_1 = 0.314, b_1 = 0.101	0.059	0.058	0.057	0.140	0.144	0.143	0.291	0.290	0.289	0.611	0.613	0.613	0.889	0.889	0.890
a_1 = 0.314, b_1 = 0.314	0.311	0.317	0.318	0.737	0.741	0.751	0.983	0.983	0.985	1	1	1	1	1	1
a_1 = 0.314, b_1 = 0.577	0.540	0.543	0.552	0.867	0.861	0.868	0.991	0.991	0.993	1	1	1	1	1	1
a_1 = 0.577, b_1 = 0.101	0.100	0.099	0.098	0.159	0.167	0.165	0.294	0.293	0.291	0.609	0.613	0.613	0.887	0.889	0.890
a_1 = 0.577, b_1 = 0.314	0.538	0.543	0.545	0.859	0.861	0.865	0.990	0.991	0.992	1	1	1	1	1	1
a_1 = 0.577, b_1 = 0.577	0.925	0.930	0.948	1	0.999	1	1	1	1	1	1	1	1	1	1

Table 11

Comparison of Empirical and Analytical Power of the tests of the Specific Mediated Effect a_2b_2 for the Parallel Two Mediator Model Across Levels of c'

	N = 50			N = 100			N = 200			N = 500			N = 1000		
	Emp	*z*	*t*	Emp	*z*	*t*	Emp	*z*	*t*	Emp	*z*	*t*	Emp	*z*	*t*
$a_2 = 0.101$, $b_2 = 0.101$	0.011	0.010	0.010	0.028	0.028	0.027	0.085	0.086	0.085	0.372	0.376	0.376	0.790	0.791	0.792
$a_2 = 0.101$, $b_2 = 0.314$	0.055	0.058	0.057	0.145	0.144	0.144	0.291	0.290	0.290	0.616	0.613	0.614	0.889	0.889	0.890
$a_2 = 0.101$, $b_2 = 0.577$	0.103	0.099	0.099	0.169	0.167	0.166	0.294	0.293	0.293	0.615	0.613	0.614	0.892	0.889	0.890
$a_2 = 0.314$, $b_2 = 0.101$	0.059	0.058	0.057	0.142	0.144	0.143	0.281	0.290	0.289	0.613	0.613	0.613	0.889	0.889	0.890
$a_2 = 0.314$, $b_2 = 0.314$	0.312	0.317	0.318	0.739	0.741	0.751	0.981	0.983	0.985	1	1	1	1	1	1
$a_2 = 0.314$, $b_2 = 0.577$	0.554	0.543	0.552	0.861	0.861	0.868	0.990	0.991	0.993	1	1	1	1	1	1
$a_2 = 0.577$, $b_2 = 0.101$	0.100	0.099	0.098	0.166	0.167	0.165	0.286	0.293	0.291	0.612	0.613	0.613	0.888	0.889	0.890
$a_2 = 0.577$, $b_2 = 0.314$	0.537	0.543	0.545	0.857	0.861	0.865	0.990	0.991	0.992	1	1	1	1	1	1
$a_2 = 0.577$, $b_2 = 0.577$	0.931	0.930	0.948	0.999389	0.999	1	1	1	1	1	1	1	1	1	1

Table 12

Type I Error Rates for the Single Mediator Model, a = b = 0

					Type I Error Rates (α)	
					Product of Coefficients	Joint Significance
		a	*b*	*c'*		
N	50	0	0	0	0	0
		0	0	0.14	0	0.001
		0	0	0.39	0	0.001
		0	0	0.59	0	0.001
	100	0	0	0	0	0.001
		0	0	0.14	0	0.001
		0	0	0.39	0	0.001
		0	0	0.59	0	0.001
	200	0	0	0	0	0
		0	0	0.14	0	0.001
		0	0	0.39	0	0
		0	0	0.59	0	0.001
	500	0	0	0	0	0.001
		0	0	0.14	0	0
		0	0	0.39	0	0
		0	0	0.59	0.001	0.001
	1000	0	0	0	0	0.002
		0	0	0.14	0	0.004
		0	0	0.39	0	0.001
		0	0	0.59	0	0.001
	5000	0	0	0	0	0.004
		0	0	0.14	0	0.001
		0	0	0.39	0	0.001
		0	0	0.59	0	0.003

Table 13

Type I Error Rates for the Parallel Two Mediator Model, $a_1b_1 = a_2b_2 = 0$

							Type I Error Rates (α)
		a_1	b_1	a_2	b_2	c'	Product of Coefficients
N	50	0	0	0	0	0	0
		0	0	0	0	0.14	0
		0	0	0	0	0.39	0
		0	0	0	0	0.59	0
	100	0	0	0	0	0	0
		0	0	0	0	0.14	0
		0	0	0	0	0.39	0
		0	0	0	0	0.59	0
	200	0	0	0	0	0	0
		0	0	0	0	0.14	0.002
		0	0	0	0	0.39	0
		0	0	0	0	0.59	0.002
	500	0	0	0	0	0	0
		0	0	0	0	0.14	0
		0	0	0	0	0.39	0
		0	0	0	0	0.59	0
	1000	0	0	0	0	0	0
		0	0	0	0	0.14	0
		0	0	0	0	0.39	0
		0	0	0	0	0.59	0

Table 14

Empirical Power Values for the Test of the Total Mediated Effect and the Test of the Total Effect for the Parallel Two Mediator Model at $c' = 0$, $N = 100$ for Coefficients Greater than One

	a_1b_1																	
	1.01, 1.01		1.01, 3.14		3.14, 1.01		1.01, 5.77		5.77, 1.01		3.14, 3.14		3.14, 5.77		5.77, 3.14		5.77, 5.77	
a_2b_2	$a_1b_1+a_2b_2$	c	$a_1b_1+a_2b_2$	c	$a_1b_1+a_2b_2$	c	$a_1b_1+a_2b_2$	c	$a_1b_1+a_2b_2$	c	$a_1b_1+a_2b_2$	c	$a_1b_1+a_2b_2$	c	$a_1b_1+a_2b_2$	c	$a_1b_1+a_2b_2$	c
1.01, 1.01	0.994	0.946	0.97	0.97	1	1	0.954	0.948	1	1	1	1	1	1	1	1	1	1
1.01, 3.14	0.972	0.948	0.984	0.976	1	1	0.984	0.984	1	1	1	1	1	1	1	1	1	1
3.14, 1.01	1	1	1	1	1	1	0.99	0.986	1	1	1	1	1	1	1	1	1	1
1.01, 5.77	0.94	0.93	0.992	0.992	0.996	0.996	0.988	0.986	1	1	1	1	1	1	1	1	1	1
5.77, 1.01	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
3.14, 3.14	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
3.14, 5.77	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
5.77, 3.14	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
5.77, 5.77	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1

Table 15

Comparison of Bootstrapped and Empirical Power Values of the Total and Total Mediated Effects for the Parallel Two Mediator Model

						Power of $a_1b_1 + a_2b_2$			Power of c		
N	a_1	b_1	a_2	b_2	c'	Empirical	Bootstrap	Difference	Empirical	Bootstrap	Difference
50	0.101	0.314	0.577	0.577	0	0.89	0.876	0.038	0.576	0.535	0.041
50	0.577	0.577	0.314	0.101	0	0.864	0.864	0	0.554	0.599	-0.045
100	0.101	0.101	0.314	0.577	0	0.82	0.852	-0.032	0.342	0.371	-0.029
100	0.101	0.314	0.577	0.314	0	0.842	0.859	-0.017	0.514	0.518	-0.004
200	0	0	0.314	0.314	0	0.934	0.962	-0.028	0.272	0.26	0.012
200	0.314	0.314	0.101	0	0	0.934	0.925	0.009	0.254	0.281	-0.027
500	0.577	0.101	0.577	0.101	0	0.856	0.881	-0.025	0.724	0.713	0.011
500	0.101	0.314	0.314	0.101	0	0.892	0.863	0.029	0.27	0.237	0.033
1000	0.101	0.101	0.101	0.101	0	0.962	0.968	-0.006	0.11	0.095	0.015
1000	0.101	0.577	0.314	0.101	0	0.994	0.998	-0.004	0.68	0.698	-0.018

Table 16

Comparison of Power and Standard Errors of ab and c for the Single Mediator Model Where c’ = 0 and N = 100

a	b	ab	c	s_c	s_{ab}	π_c	π_{ab}
0.14	0.14	0.0196	0.0196	0.1020	0.0201	0.0385	0.0779
0.14	0.39	0.0546	0.0546	0.1084	0.0419	0.0723	0.2688
0.14	0.59	0.0826	0.0826	0.1173	0.0613	0.1038	0.2790
0.39	0.14	0.0546	0.0546	0.1020	0.0420	0.0767	0.2688
0.39	0.39	0.1521	0.1521	0.1084	0.0559	0.2847	0.9279
0.39	0.59	0.2301	0.2301	0.1173	0.0716	0.4917	0.9631
0.59	0.14	0.0826	0.0826	0.1020	0.0616	0.1240	0.2790
0.59	0.39	0.2301	0.2301	0.1084	0.0717	0.5539	0.9631
0.59	0.59	0.3481	0.3481	0.1173	0.0845	0.8290	0.9996

Table 17

Ranges of Correlations Between X and Y for Which Including a Mediator Will Increase Power

N	Minimum r_{xy}	Maximum r_{xy}
50	0.0287229	0.3806169
100	0.0196078	0.2721655
200	0.0099499	0.1929221
500	0.0099499	0.1239751
1000	0.0099499	0.0881474
5000	0.0099499	0.0391931

Figure Captions

Figure 1. Path diagrams for the regression and one mediator models. Adapted from MacKinnon, 2008.

Figure 2. Path diagram for the parallel multiple mediator model. Adapted from MacKinnon, 2008.

Figure 3. Plots for the Four-Way Interaction between a_1, b_1, c', and N Across N.

Figure 4. Plots for the Four-Way Interaction between a_2, b_2, c', and N Across N.

Figures

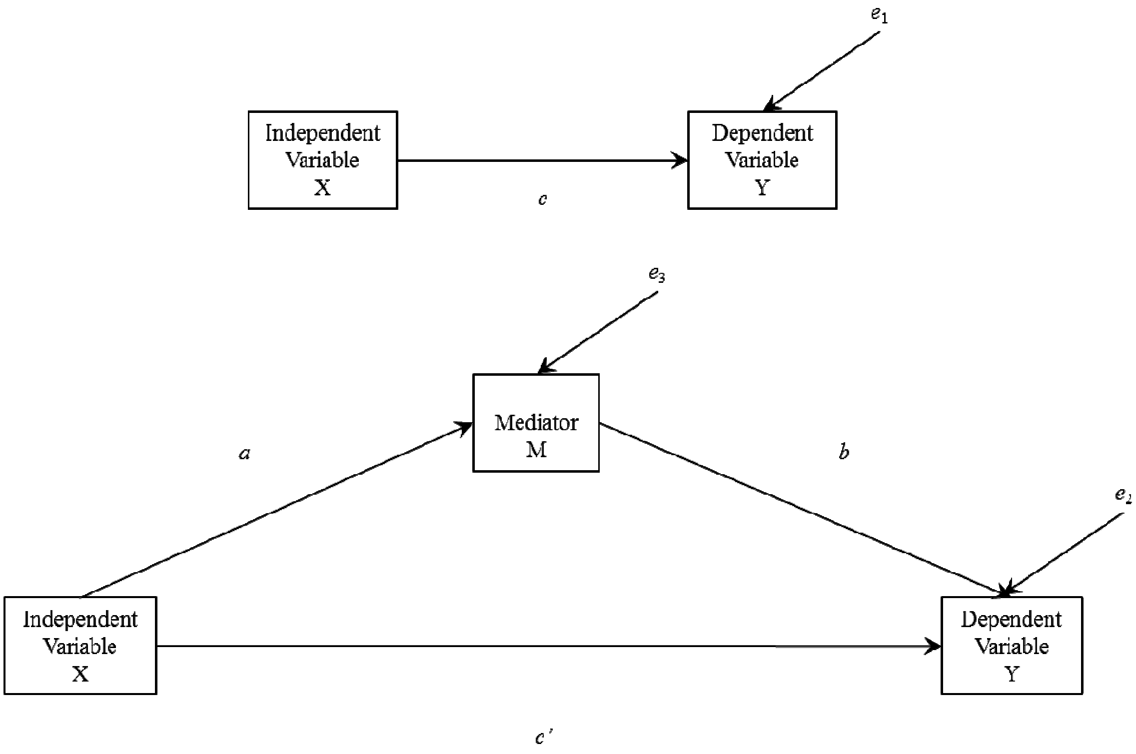


Figure 1.

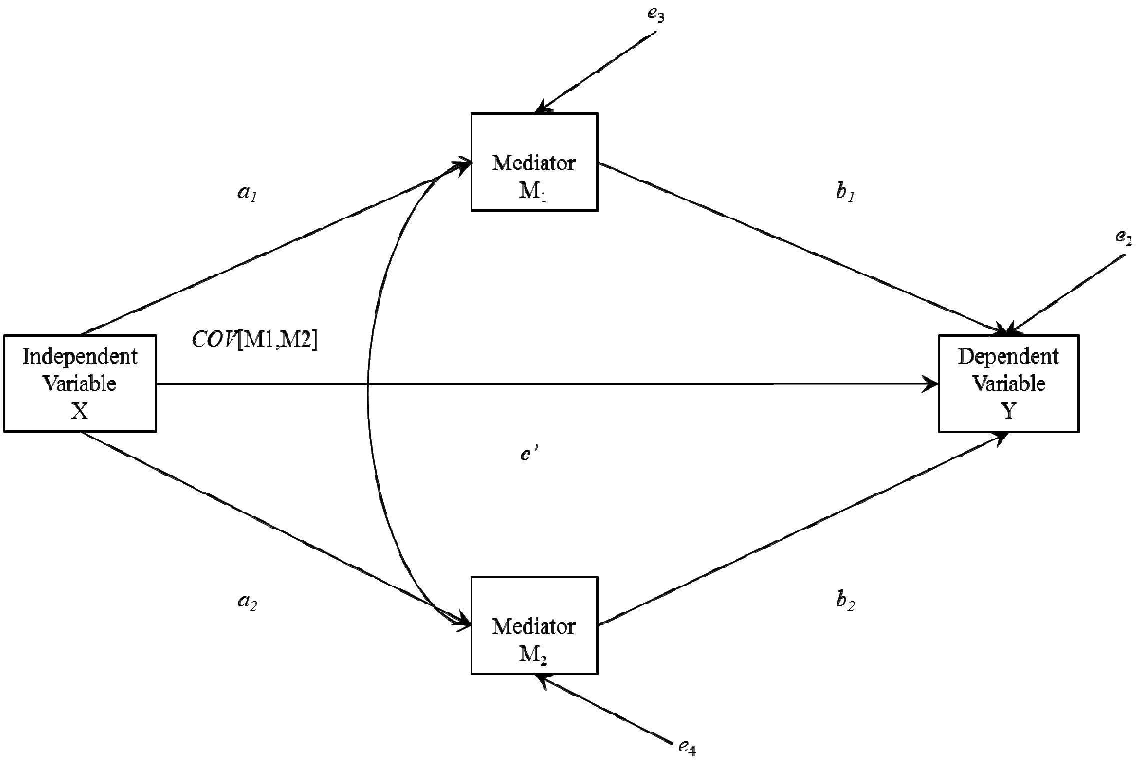

e_3
Mediator
M_1
a_1
b_1
e_2
Independent
Variable
X
COV[M1,M2]
Dependent
Variable
Y
c'
a_2
b_2
Mediator
M_2
e_4

Figure 2.

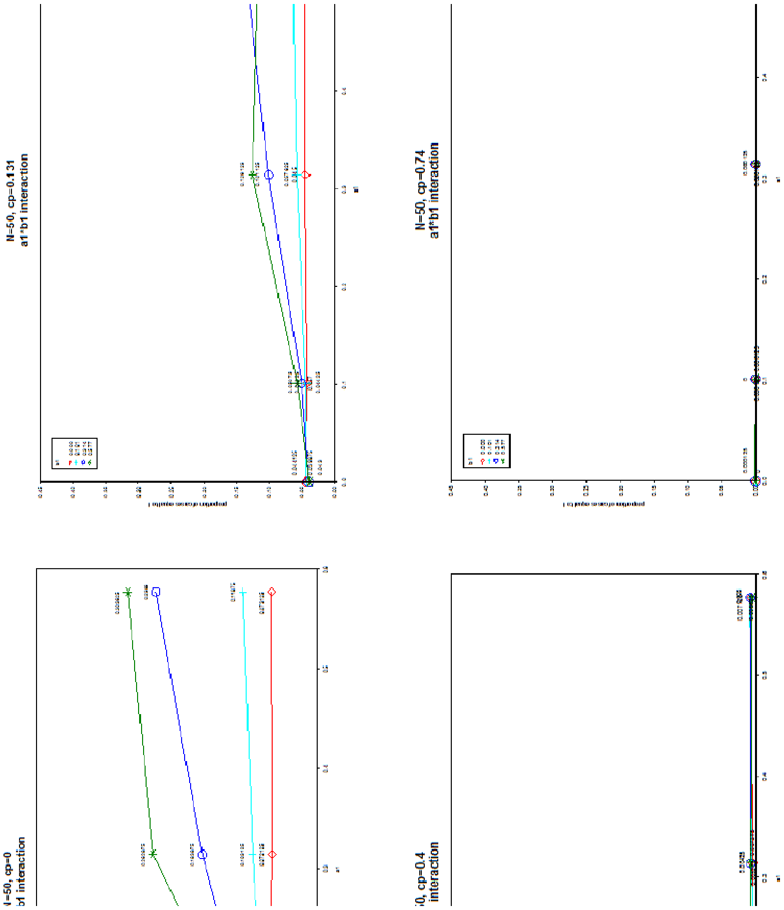


Figure 3, Panel A.

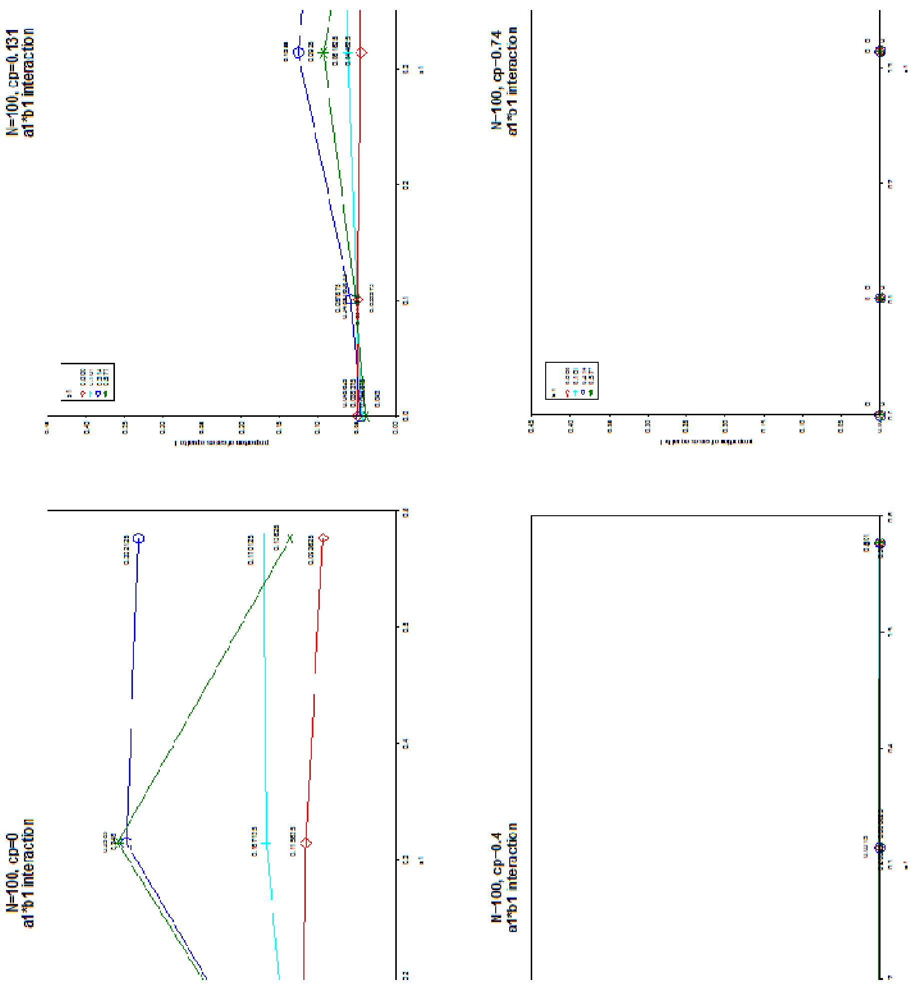


Figure 3, Panel B.

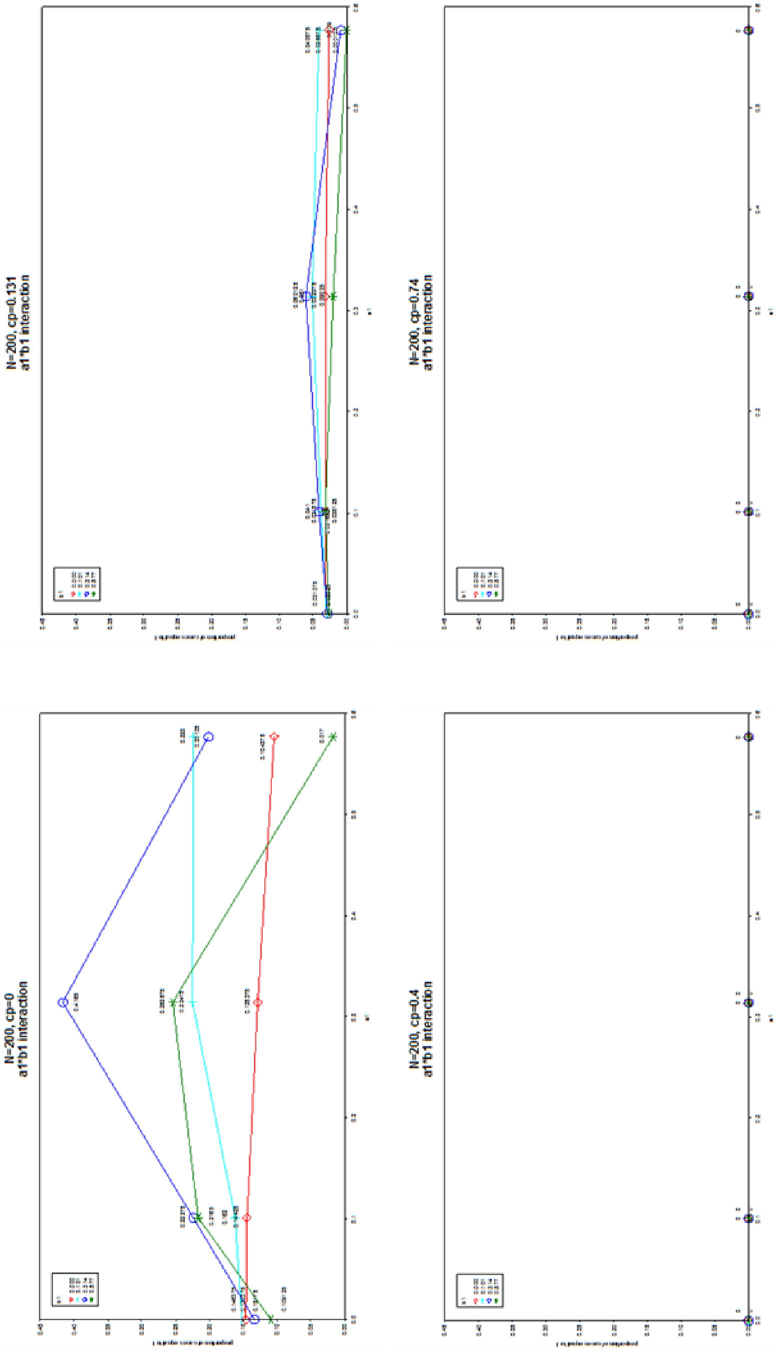


Figure 3, Panel C.

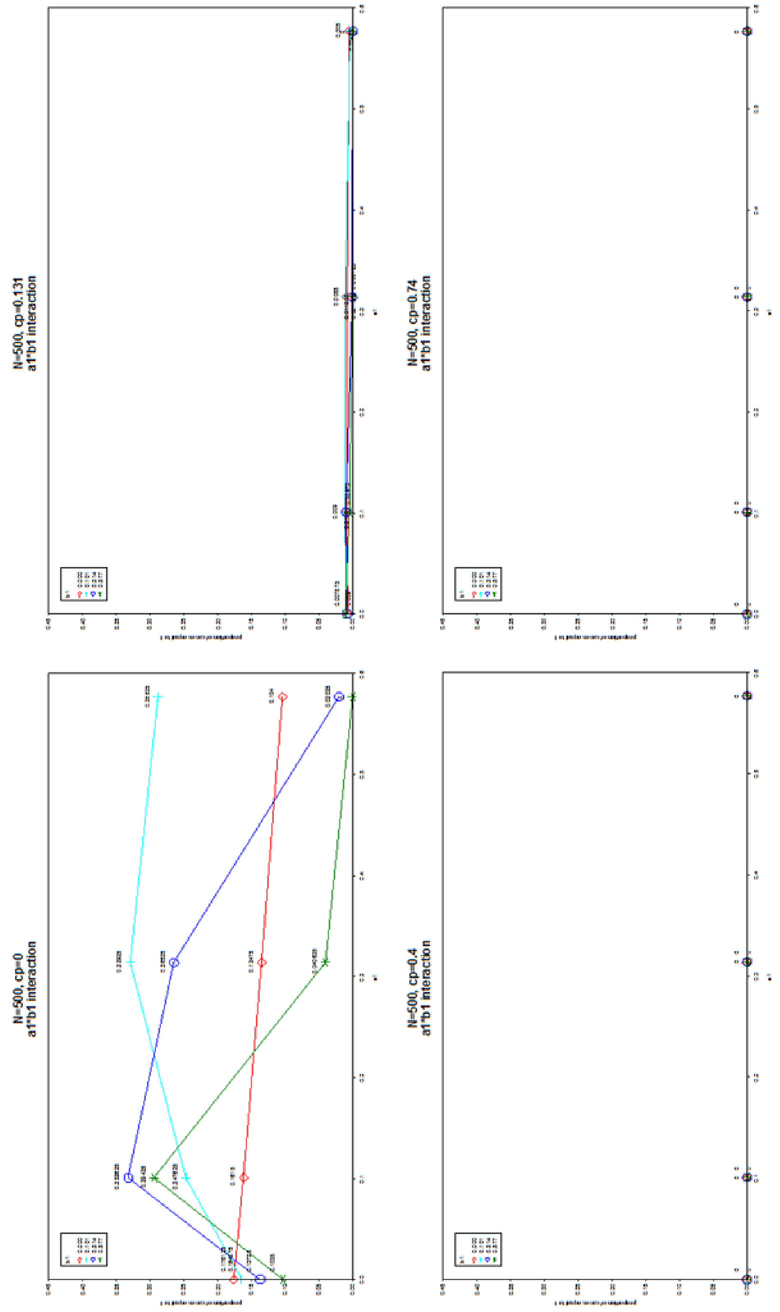


Figure 3, Panel D.

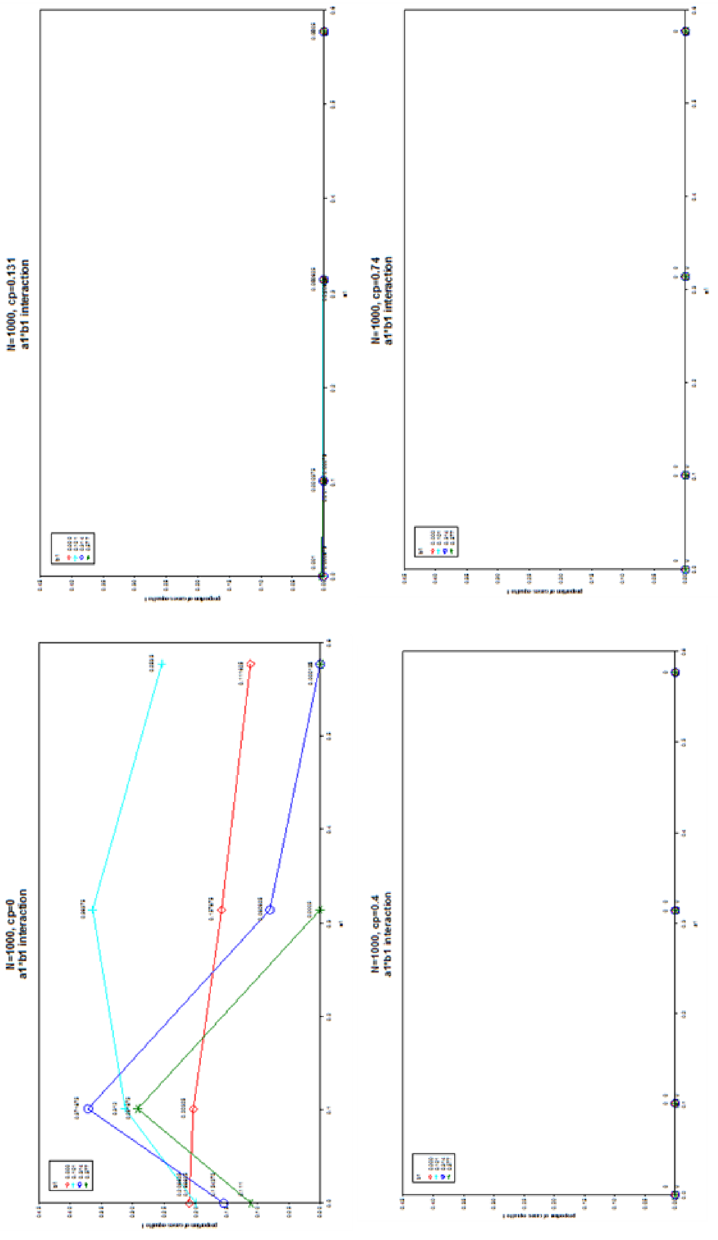


Figure 3, Panel E.

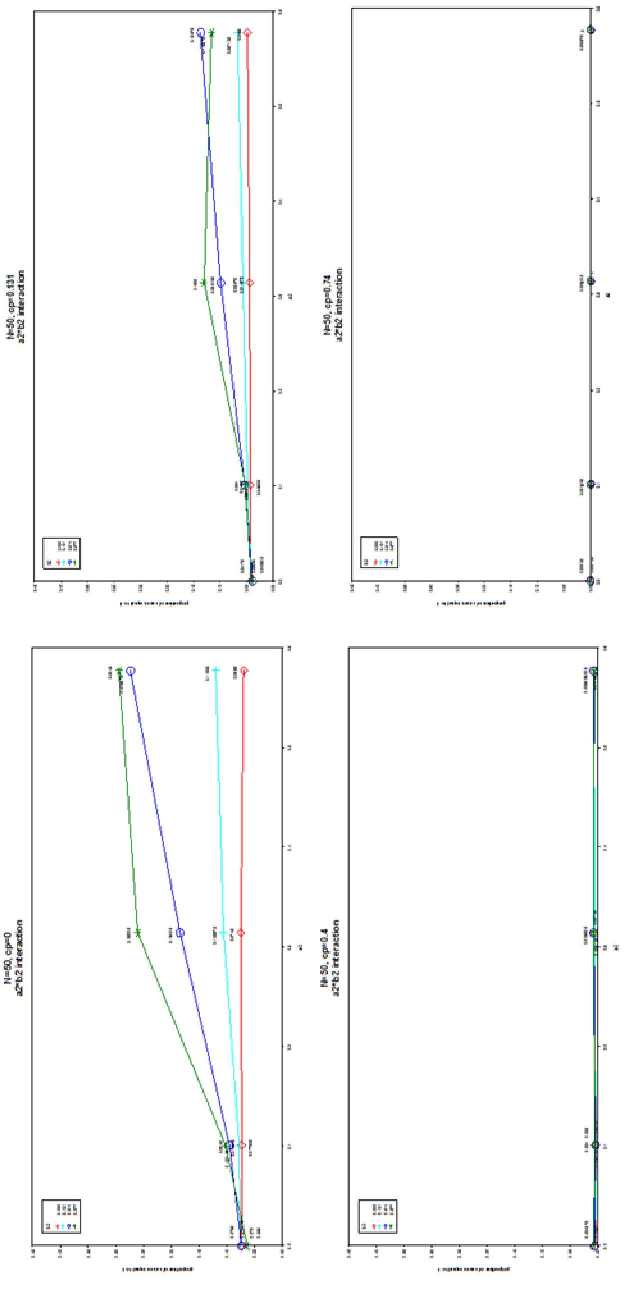


Figure 4, Panel A.

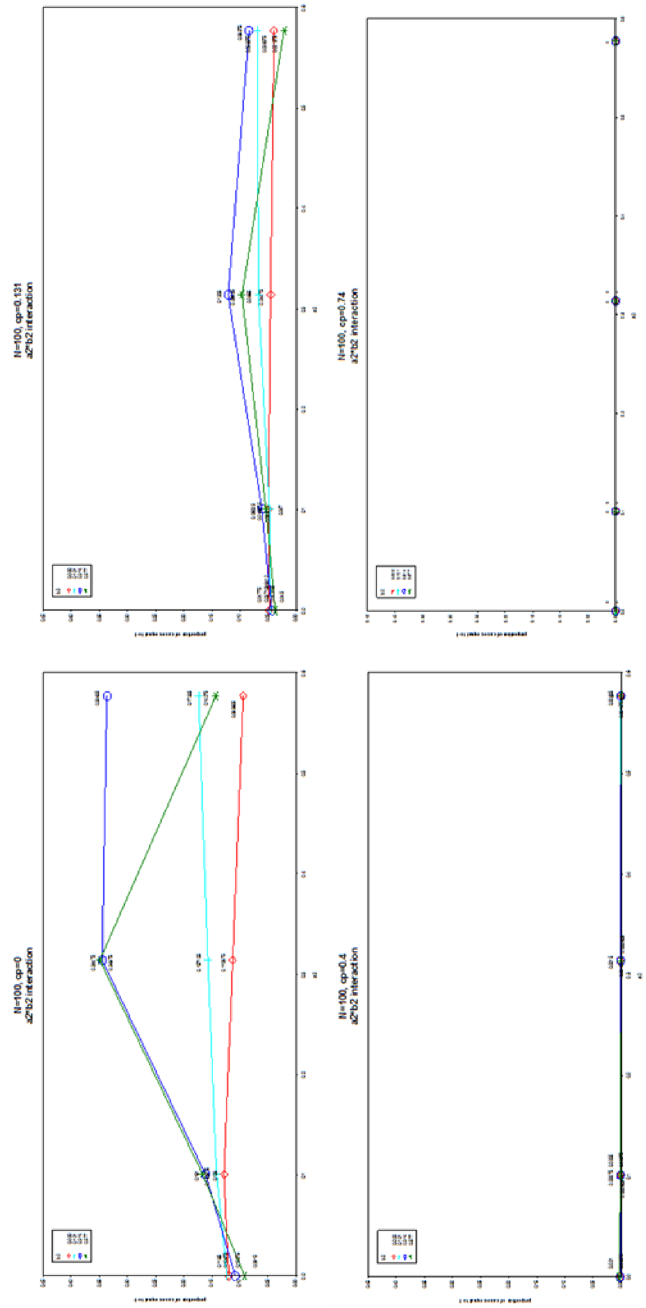


Figure 4, Panel B.

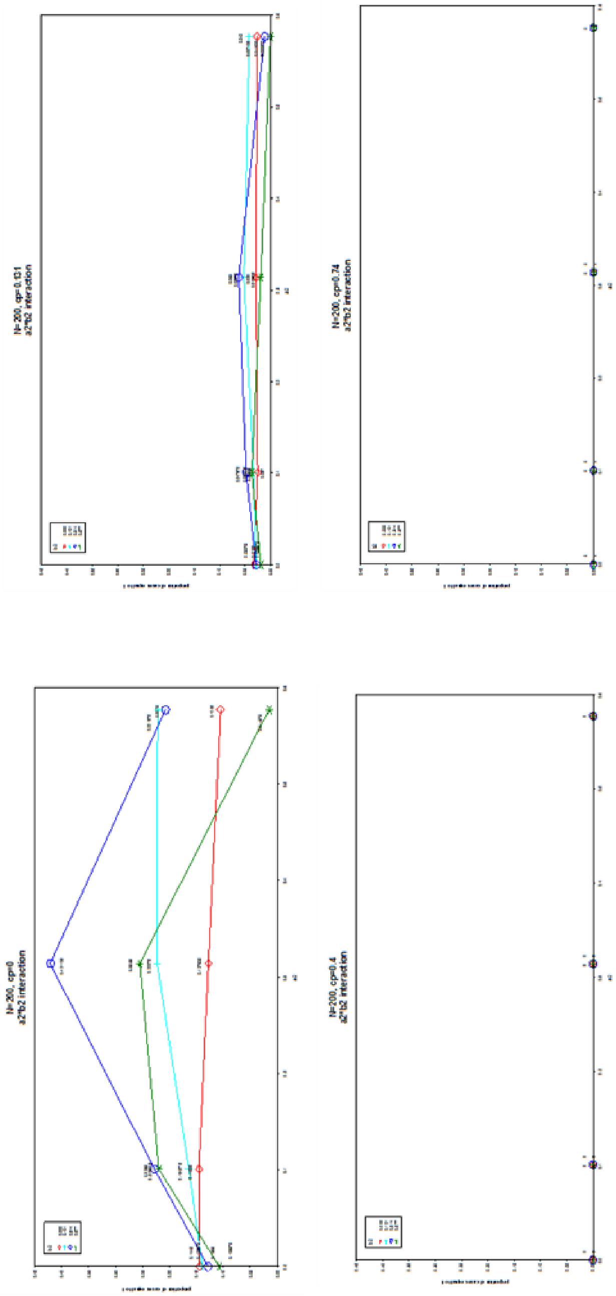


Figure 4, Panel C.

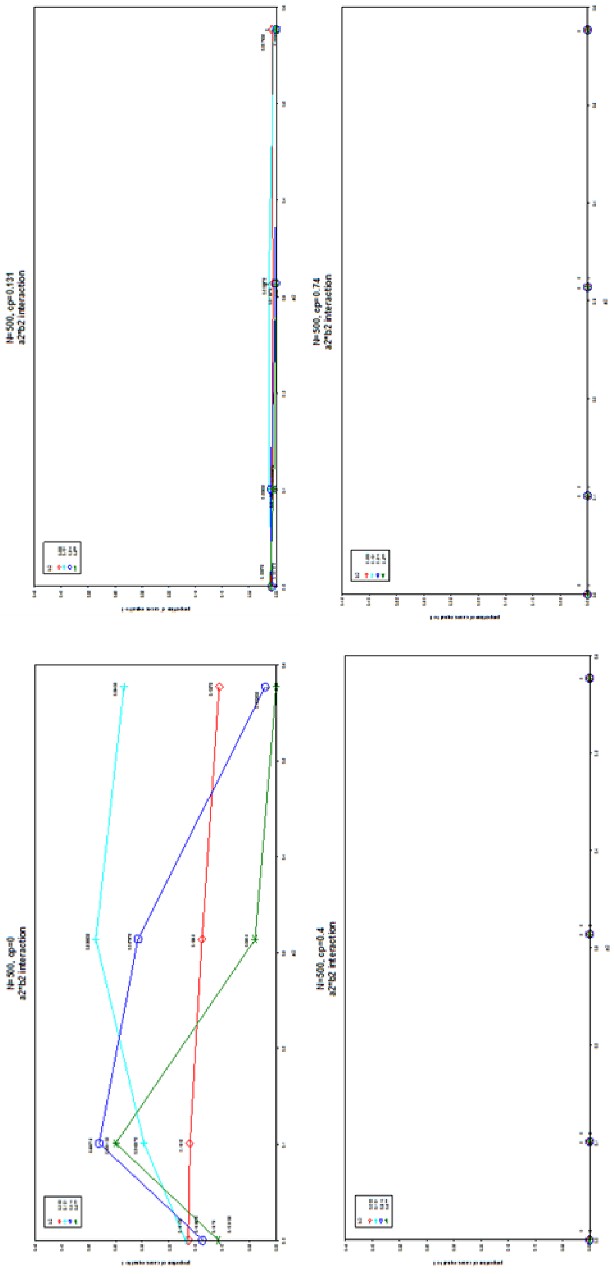


Figure 4, Panel D.

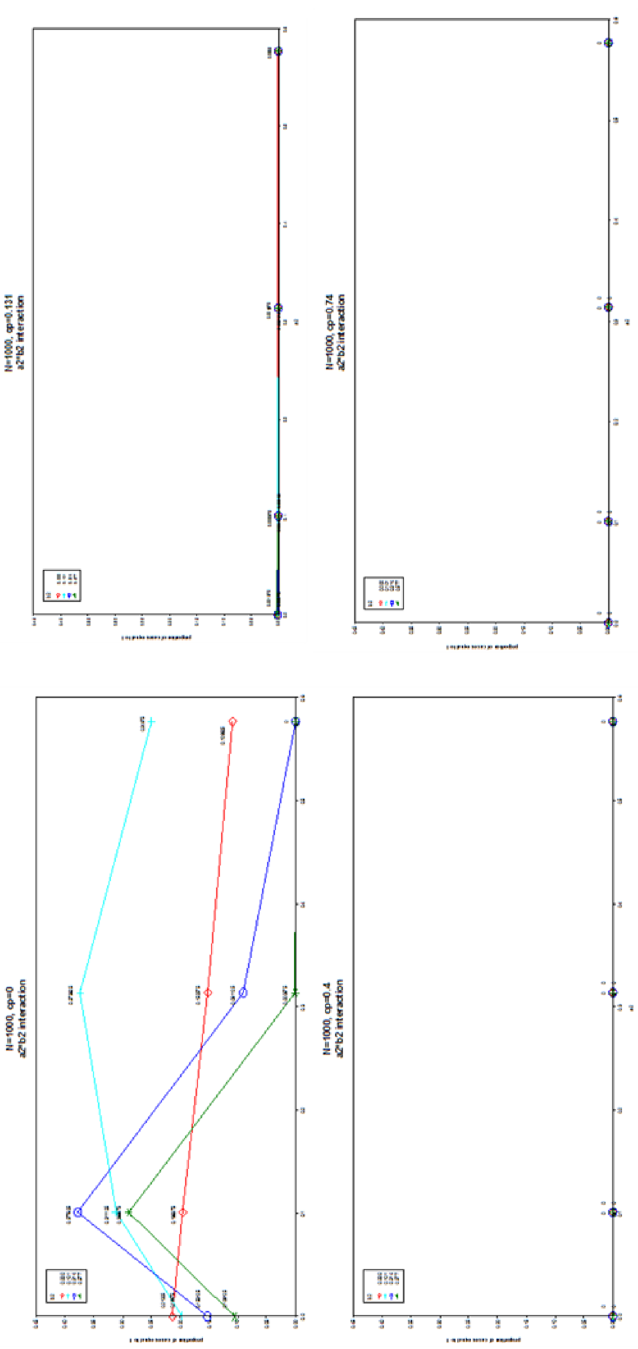


Figure 4, Panel E.

APPENDIX A

PROGRAM TO COMPUTE EMPIRICAL POWER FOR THE SINGLE MEDIATOR MODEL

```
*/ Automatically deletes log so it doesn't have to be cleared */
FILENAME NULLOG DUMMY 'C:\NULL';
PROC PRINTTO LOG=NULLOG;

%macro medsim;

%do nsize = 1 %to 1;
%do aparm = 1 %to 4;
%do bparm = 1 %to 4;
%do cparm = 1 %to 4;
%do numsamps = 1 %to 1000;

proc iml;

a = {0,.14,.39,.59};
b = {0,.14,.39,.59};
cp = {0,.14,.39,.59};
n = {50,100,200,500,1000,5000};
varx = 1;
resvarm = 1;
resvary = 1;

/* calculate variances based on paths */
covxm = a[&aparm,1]*varx;
varm = (a[&aparm,1]**2)*varx + resvarm;
*resvarm = 1 - ((a[aparm,1]**2)*varx);
*resvary = 1 - ((b[bparm,1]**2)*varm + (cp[cparm,1]**2)*varx + 2*b[bparm,1]*cp[cparm,1]*covxm);

/* create x scores */
x = rannor(j(n[&nsize,1],1,0));

/* generate residuals for m */
/* sqrt(resvarm) = the std for the residual distribution */
resm = sqrt(resvarm)*rannor(j(n[&nsize,1],1,0));

/* generate m via regression equation */
m = a[&aparm,1]*x + resm;

/* generate residuals for y */
/* sqrt(resvary) = the std for the residual distribution */
resy = sqrt(resvary)*rannor(j(n[&nsize,1],1,0));

/* generate y via regression equation */
y = b[&bparm,1]*m + cp[&cparm,1]*x + resy;


/* concatenate vectors into single matrix */
medvars = x || m || y ;

/* create sas data set dat from iml matrix medvars */
create dat from medvars[colname = {x m y}];
append from medvars;

*proc means data = dat;
*var x m y resm resy;
```

```
ods listing close;
proc reg data = dat;
model m = x;
ods output ParameterEstimates = apath;
run;

ods listing close;
proc reg data = dat;
model y = x m;
ods output ParameterEstimates = bpath;
run;

ods listing close;
proc reg data = dat;
model y = x;
ods output ParameterEstimates = total;
run;

data apath;
set apath;
where variable = 'X';
keep estimate stderr probt;
run;

data apath;
set apath;
rename estimate = a;
rename stderr = ase;
rename probt = pa;
run;

data cprime;
set bpath;
where variable = 'X';
keep estimate stderr;
run;

data cprime;
set cprime;
rename estimate = cprime;
rename stderr = cprimese;
run;

data bpath;
set bpath;
where variable = 'M';
keep estimate stderr probt;
run;

data bpath;
set bpath;
rename estimate = b;
rename stderr = bse;
rename probt = pb;
```

```
run;

data total;
set total;
where variable = 'X';
keep estimate stderr probt;
run;

data total;
set total;
rename estimate = c;
rename stderr = cse;
rename probt = cpvalue;
run;

data medparms;
merge apath bpath cprime total;
run;

data medparms;
set medparms;

csig = 0;
if cpvalue < .05 then csig = 1;

/*Product of Coefficients Test*/
ab = a*b;
sobelse = sqrt(a*a*bse*bse + b*b*ase*ase);
sobelz = ab / sobelse;
sobelsig = 0;
if abs(sobelz) > 1.96 then sobelsig = 1;

/*Joint Significance Test*/
jointsig = 0;
if pa < .05 and pb < .05 then jointsig = 1;

if &nsize = 1 then sampsize = 50;
if &nsize = 2 then sampsize = 100;
if &nsize = 3 then sampsize = 200;
if &nsize = 4 then sampsize = 500;
if &nsize = 5 then sampsize = 1000;
if &nsize = 6 then sampsize = 5000;

if &aparm = 1 then apath = 0;
if &aparm = 2 then apath = .14;
if &aparm = 3 then apath = .39;
if &aparm = 4 then apath = .59;

if &bparm = 1 then bpath = 0;
if &bparm = 2 then bpath = .14;
if &bparm = 3 then bpath = .39;
if &bparm = 4 then bpath = .59;

if &cparm = 1 then cpath = 0;
if &cparm = 2 then cpath = .14;
```

```
if &cparm = 3 then cpath = .39;
if &cparm = 4 then cpath = .59;

file "c:\HPO\medsimple50.dat" mod;
put @1 (a) (8.6)
@10 (ase) (8.6)
@20 (pa) (8.6)
@30 (b) (8.6)
@40 (bse) (8.6)
@50 (pb) (8.6)
@60 (cprime) (8.6)
@70 (cprimese) (8.6)
@80 (c) (8.6)
@90 (cse) (8.6)
@100 (cpvalue) (8.6)
@110 (csig) (8.6)
@120 (ab) (8.6)
@130 (sobelse) (8.6)
@140 (sobelz) (8.6)
@150 (sobelsig) (8.6)
@160 (jointsig) (8.6)
@170 (sampsize) (8.6)
@180 (apath) (8.6)
@190 (bpath) (8.6)
@200 (cpath) (8.6);

run;

%end;
%end;
%end;
%end;
%end;

%mend;
%medsim;
run;

data medparms;
infile "c:\HPO\medsimple50.dat";
input a ase pa b bse pb cprime cprimese c cse cpvalue csig
ab sobelse sobelz sobelsig jointsig sampsize apath bpath cpath;
run;

proc sort data = medparms;
by sampsize apath bpath cpath;
run;

proc means data = medparms noprint;
var a b c cprime ab pa pb cpvalue sobelsig csig jointsig;
by sampsize apath bpath cpath;
output out = powertable mean = a b c cprime ab pa pb cpvalue sobelsig csig jointsig;
run;

proc print data = powertable;
```

```
run;
```

APPENDIX B

STEP-BY-STEP DESCRIPTION OF SAS EMPIRICAL SIMULATION FOR SINGLE MEDIATOR MODEL

1. Specify sample sizes (50, 100, 200, 500, 1000, 5000), *a* paths (0, 0.14, 0.39, 0.59), *b* paths (0, 0.14, 0.39, 0.59), and *c'* paths (0, 0.14, 0.39, 0.59)

   Within PROC IML, generating data of specified sample size:
2. specify variance of X as 1, residual variances of M and Y as 1
3. Calculate the covariance between X and M and the variance of M based on specified variances
4. Using RANNOR, generate random data for X (independent variable) with normally distributed residuals
5. Generate data for M (mediator), using RANNOR to generate normally distributed residuals then using those residuals in a regression equation with the specified parameter
6. Generate data for Y (dependent variable), using RANNOR to generate normally distributed residuals then using those residuals in a regression equation with the specified parameters
7. Concatenate vectors of variables X, M, and Y into a single matrix, then create a SAS data set from the matrix
8. Run series of regression equations and rename and save the resulting parameter estimates, standard errors, and p values for the *a*, *b*, *c'*, and *c* coefficients in a series of data sets
9. Create a new data set by combining the *a*, *b*, *c'*, and *c* data sets
10. In the new data set, create a variable that is equal to 0 when the *p* value for *c* is greater than .05 (nonsignificant) and equal to 1 when the *p* value for *c* is less than .05 (significant)

11. In the new data set, calculate the product of coefficients *z* test using the mediated effect *ab* and the multivariate delta standard error, and create a variable that is equal to 0 when the product of coefficient *z* is less than 1.96 (nonsignificant) and equal to 1 when the product of coefficients *z* is greater than 1.96 (significant)
12. In the new data set, create another variable that is equal to 0 when the *p* values for *a* and *b* are greater than .05 (nonsignificant) and equal to 1 when the *p* values for *a* and *b* are less than .05 (significant)
13. Export the variables from the new data set into a text file to be saved to a specified location (generates 1000 replications of each specified combination of parameters)
14. End macro
15. Import saved data set into SAS and use PROC MEANS to get the means of the variables (mean over 1000 replications); ex. the variable that is 0 when *c* is not significant and 1 when significant displays as a proportion of times over 1000 replications that *c* is significant (hence, the power value when *c* is not zero and the Type I error when *c* is zero)

APPENDIX C

PROGRAM TO COMPUTE ANALYTICAL POWER OF THE SINGLE MEDIATOR MODEL

```
*Last edited September 16, 2012;
*This program computes the power to detect the mediated effect;

ods html close;
ods listing;
data a;
input a b cp N;
 do a =0, 0.14, 0.39, 0.59;
 do b =0, 0.14, 0.39, 0.59;
 do cp =0, 0.14, 0.39, 0.59;
 do n =50, 100, 200, 500, 1000, 5000;
c=a*b+cp; ab=a*b;
sa=sqrt(1/(N-2)); sb=sqrt(1/(N-3)); sc=sqrt(1/(n-2));

*This section computes true variances and covariances as in Section 4.10 based
on residual error variance equal to 1. Note that VX1X1, VX2X2, and VX3X3 are
the residual variance in equations 3.1, 3.2, and 3.3, respectively;
ERROR=1;ERRORM=1;ERRORY=1;
BMX=A;BYM=B; BYX=CP;NOBS=N;
EMOD1=(ERROR)**2;
EMOD2=(ERRORm)**2;
EMOD3=(ERRORy)**2;
VX1X1=EMOD1;
CY1X1=BMX*EMOD1;
CY2X1=BYM*BMX*VX1X1+BYX*EMOD1;
CY1Y1=BMX*BMX*VX1X1+EMOD2;
CY2Y1=BMX*BMX*BYM*VX1X1+BMX*BYX*EMOD1+BYM*EMOD2;
CY2Y2=BYM*BYM*(BMX*BMX*EMOD1+EMOD2)+2*BMX*BYM*BYX*VX1X1+BYX*BYX*EM
OD1
+EMOD3;

*This section computes population correlations;
RY1X1=CY1X1/SQRT(VX1X1*CY1Y1);
RY2X1=CY2X1/SQRT(VX1X1*CY2Y2);
RY2Y1=CY2Y1/SQRT(CY1Y1*CY2Y2);
partryxm=(ry2x1-ry2y1*ry1x1)/sqrt((1-ry2y1*ry2y1)*(1-ry1x1*ry1x1));
partrymx=(ry2y1-ry2x1*ry1x1)/sqrt((1-ry2x1*ry2x1)*(1-ry1x1*ry1x1));

TRUEA=emod2/((NOBS-2)*(VX1X1));
TRUESEA=sqrt(TRUEA);
TRUEB=(emod3/(NOBS-3))*(1/CY1Y1/(1-RY1X1*RY1X1));
TRUESEB=sqrt(TRUEB);
TRUERAT=(BMX*BYM)/BYX;
TRUEPROP=(BMX*BYM)/((BMX*BYM)+BYX);

*Calculation of true mean squared error;
dermse1=cy2y2-(byx+bmx*bym)*(byx+bmx*bym)*vx1x1;
dermse2=cy2y2-bym*bym*cy1y1-byx*byx*vx1x1-2*byx*bym*cy1x1;
dermse3=cy1y1-bmx*bmx*vx1x1;

*Calculation of true standard errors pow refers to power;
powsea=sqrt(dermse3/((nobs-2)*Vx1x1));powsec=sqrt(dermse1/((nobs-2)*Vx1x1));
rmod2=(ry2y1*ry2y1+ry2x1*ry2x1-2*ry2y1*ry2x1*ry1x1)/(1-ry1x1*ry1x1);
powseb=sqrt(dermse2/((nobs-3)*cy1y1*(1-ry1x1*ry1x1)));
powsecp=sqrt(dermse2/((nobs-3)*Vx1x1*(1-ry1x1*ry1x1)));
```

```
/**Calculation of power of RY2X1 (correlation for c coeff) using z test;*/
/*CPRIME=((1/2)*LOG((1+RY2X1)/(1-RY2X1)));*/
/*SDCPRIME=1/SQRT(NOBS-3);*/
/*ZCPRIME=CPRIME/SDCPRIME;*/
/*ZC=1.96-ZCPRIME;*/
/*ZPOWERC=1-PROBNORM(ZC);*/

/**Calculation of power of c using t test;*/
/*TC=C/POWSEC;*/
/*TZC=1.96-TC;*/
/*DFC=NOBS-2;*/
/*TPOWERC=1-PROBT(TZC,DFC);*/

*Calculation of power of RY1X1 (correlation for a coeff) using z test;
APRIME=((1/2)*LOG((1+RY1X1)/(1-RY1X1)));
SDAPRIME=1/SQRT(NOBS-3);
ZAPRIME=APRIME/SDAPRIME;
ZA=1.96-ZAPRIME;
ZPOWERA=1-PROBNORM(ZA);

*Calculation of power of a using t test;
TA=ABS(A/POWSEA);
IF TA GT 20 THEN TA = 20;
TCRITA=TINV(.975,NOBS-2);
TPOWERA=1-PROBT(TCRITA,N-2,TA);

*Calculation of power of partrymx (correlation for b coeff) using z test;
BPRIME=((1/2)*LOG((1+partrymx)/(1-partrymx)));
SDBPRIME=1/SQRT(NOBS-3);
ZBPRIME=BPRIME/SDBPRIME;
ZB=1.96-ZBPRIME;
ZPOWERB=1-PROBNORM(ZB);

*Calculation of power of b using t test;
TB=B/POWSEB;
IF TB GT 20 THEN TB = 20;
TCRITB=TINV(.975,NOBS-3);
TPOWERB=1-PROBT(TCRITB,N-3,TB);

*Calculation of difference in power calculations for a and b;
APOWERDIFF=ZPOWERA-TPOWERA;
BPOWERDIFF=ZPOWERB-TPOWERB;

*Calculation of power of ab from t and z tests;
ZPOWERAB=ZPOWERA*ZPOWERB;
TPOWERAB=TPOWERA*TPOWERB;

*Calculation of differences between power values;
ABPOWERDIFF=ZPOWERAB-TPOWERAB;

 output;
 end;
 end;
 end;
```

```
 end;
 cards;
 0 0 0 50
 ;

 proc sort;
		BY NOBS A B CP;
RUN;

ods html body = 'C:\Users\Dropbox\Masters\Analytical work\Power Calculations\poweroutab.xls';
 proc print; var NOBS A B CP ta tcrita tpowera zpowera apowerdiff tpowerb zpowerb bpowerdiff
zpowerab tpowerab abpowerdiff;
 run;
ods html close;
```

APPENDIX D

DERIVATION OF THE COVARIANCE MATRIX FOR THE PARALLEL TWO MEDIATOR MODEL

$Y = i_1 + cX + \varepsilon_1$
$Y = i_2 + c'X + b_1M_1 + b_2M_2 + \varepsilon_2$
$M_1 = i_3 + a_1X + \varepsilon_3$
$M_2 = i_4 + a_2X + \varepsilon_4$

$\mathrm{Cov}[X, M_1] = \mathrm{Cov}(X, a_1X + \varepsilon_3)$
$= a_1\mathrm{Cov}(X, X) + \mathrm{Cov}(X, \varepsilon_3)$
$= \underline{a_1\sigma^2_X}$

$\mathrm{Cov}[X, M_2] = \mathrm{Cov}(X, a_2X + \varepsilon_4)$
$= a_2\mathrm{Cov}(X, X) + \mathrm{Cov}(X, \varepsilon_4)$
$= \underline{a_2\sigma^2_X}$

$\mathrm{Cov}[X, Y] = \mathrm{Cov}(X, c'X + b_1M_1 + b_2M_2 + \varepsilon_2)$
$= c'\mathrm{Cov}(X, X) + b_1\mathrm{Cov}(X, M_1) + b_2\mathrm{Cov}(X, M_2) + \mathrm{Cov}(X, \varepsilon_2)$
$= \underline{c'\sigma^2_X + a_1b_1\,\sigma^2_X + a_2b_2\,\sigma^2_X}$

$\mathrm{Cov}[M_1, Y] = \mathrm{Cov}(a_1X + \varepsilon_3, c'X + b_1M_1 + b_2M_2 + \varepsilon_2)$
$= a_1c'\mathrm{Cov}(X, X) + a_1b_1\mathrm{Cov}(X, M_1) + a_1b_2\mathrm{Cov}(X, M_2) + a_1\mathrm{Cov}(X, \varepsilon_2) +$
$c'\mathrm{Cov}(X, \varepsilon_3) + b_1\mathrm{Cov}(M_1, \varepsilon_3) + b_2\mathrm{Cov}(M_2, \varepsilon_3) + \mathrm{Cov}(\varepsilon_3, \varepsilon_2)$
$= a_1c'\,\sigma^2_X + a^2_1b_1\,\sigma^2_X + a_1a_2b_1\,\sigma^2_X + b_1\mathrm{Cov}(a_1X + \varepsilon_3, \varepsilon_3) + b_2\mathrm{Cov}(a_2X +$
$\varepsilon_4, \varepsilon_3)$
$= \underline{a_1c'\,\sigma^2_X + a^2_1b_1\,\sigma^2_X + a_1a_2b_1\,\sigma^2_X + b_1\,\sigma^2_{\varepsilon 3}}$

$\mathrm{Cov}[M_2, Y] = \mathrm{Cov}(a_2X + \varepsilon_4, c'X + b_1M_1 + b_2M_2 + \varepsilon_2)$
$= a_2c'\mathrm{Cov}(X, X) + a_2b_1\mathrm{Cov}(X, M_1) + a_2b_2\mathrm{Cov}(X, M_2) + a_2\mathrm{Cov}(X, \varepsilon_2) +$
$c'\mathrm{Cov}(X, \varepsilon_4) + b_1\mathrm{Cov}(M_1, \varepsilon_4) + b_2\mathrm{Cov}(M_2, \varepsilon_4) + \mathrm{Cov}(\varepsilon_2, \varepsilon_4)$
$= a_2c'\,\sigma^2_X + a_1a_2b_1\,\sigma^2_X + a^2_2b_2\,\sigma^2_X + b_1\mathrm{Cov}(a_1X + \varepsilon_3, \varepsilon_4) + b_2\mathrm{Cov}(a_2X +$
$\varepsilon_4, \varepsilon_4)$
$= \underline{a_2c'\,\sigma^2_X + a_1a_2b_1\,\sigma^2_X + a^2_2b_2\,\sigma^2_X + b_2\,\sigma^2_{\varepsilon 4}}$

$\mathrm{Cov}[M_1, M_2] = \mathrm{Cov}(a_1X + \varepsilon_3, a_2X + \varepsilon_4)$
$=\mathrm{Cov}(a_1X, a_2X) + \mathrm{Cov}(\varepsilon_3, \varepsilon_4)$
$= \underline{a_1a_2\sigma^2_X}$

$\mathrm{Cov}[X, X] = \underline{\sigma^2_X}$

$\mathrm{Cov}[M_1, M_1] = \underline{a^2_1\sigma^2_X + \sigma^2_{\varepsilon 3}}$

$\mathrm{Cov}[M_2, M_2] = \underline{a^2_2\sigma^2_X + \sigma^2_{\varepsilon 4}}$

$\mathrm{Cov}[Y, Y] = \mathrm{Cov}(c'X + b_1M_1 + b_2M_2 + \varepsilon_2, c'X + b_1M_1 + b_2M_2 + \varepsilon_2)$
$= \mathrm{Cov}(c'X, c'X) + \mathrm{Cov}(c'X, b_1M_1) + \mathrm{Cov}(c'X, b_2M_2) + \mathrm{Cov}(c'X, \varepsilon_2) +$
$\mathrm{Cov}(b_1M_1, c'X) + \mathrm{Cov}(b_1M_1, b_1M_1) + \mathrm{Cov}(b_1M_1, b_2M_2) + \mathrm{Cov}(b_1M_1, \varepsilon_2) +$

$\mathrm{Cov}(b_2M_2, c'X) + \mathrm{Cov}(b_2M_2, b_1M_1) + \mathrm{Cov}(b_2M_2, b_2M_2) + \mathrm{Cov}(b_2M_2, \varepsilon_2) +$
$\mathrm{Cov}(\varepsilon_2, c'X) + \mathrm{Cov}(\varepsilon_2, b_1M_1) + \mathrm{Cov}(\varepsilon_2, b_2M_2) + \mathrm{Cov}(\varepsilon_2, \varepsilon_2)$
$= c'^{\,2}\sigma^2_X + c'b_1a_1\,\sigma^2_X + c'b_2a_2\,\sigma^2_X + b_1c'a_1\,\sigma^2_X + b^2_1(a^2_1\sigma^2_X + \sigma^2_{\varepsilon 3}) +$
$a_1a_2b_1b_2\,\sigma^2_X + c'b_2a_2\,\sigma^2_X + a_1a_2b_1b_2\,\sigma^2_X + b^2_2(a^2_2\sigma^2_X + \sigma^2_{\varepsilon 4}) + \sigma^2_{\varepsilon 2}$
$\underline{= c'^{\,2}\sigma^2_X + 2c'b_1a_1\,\sigma^2_X + 2c'b_2a_2\,\sigma^2_X + 2a_1a_2b_1b_2\sigma^2_X + b^2_1(a^2_1\sigma^2_X + \sigma^2_{\varepsilon 3})}$
$\underline{+ b^2_2(a^2_2\sigma^2_X + \sigma^2_{\varepsilon 4}) + \sigma^2_{\varepsilon 2}}$

APPENDIX E

DERIVATION OF TRUE REGRESSION COEFFICIENTS FOR THE PARALLEL TWO MEDIATOR MODEL

a_1:

$$\mathrm{Cov}[X, M_1] = a_1\mathrm{Cov}[X, X]$$

$$a_1 = \frac{\mathrm{Cov}[X, M_1]}{\mathrm{Cov}[X,X]}$$

a_2:

$$\mathrm{Cov}[X, M_2] = a_2\mathrm{Cov}[X, X]$$

$$a_2 = \frac{\mathrm{Cov}[X, M_2]}{\mathrm{Cov}[X,X]}$$

b_1:

$$\begin{aligned}\mathrm{Cov}[M_1, Y] &= a_1 c' \sigma^2_X + a^2_1 b_1 \sigma^2_X + a_1 a_2 b_1 \sigma^2_X + b_1 \sigma^2_{\varepsilon 3}\\ &= c'\mathrm{Cov}[X, M_1] + b_1\mathrm{Cov}[M_1, M_1] + b_2\mathrm{Cov}[M_1, M_2]\end{aligned}$$

$$b_1\mathrm{Cov}[M_1, M_1] = \mathrm{Cov}[M_1, Y] - c'\mathrm{Cov}[X, M_1] - b_2\mathrm{Cov}[M_1, M_2]$$

$$b_1 = \frac{\mathrm{Cov}[M_1, Y] - c'\mathrm{Cov}[X, M_1] - b_2\mathrm{Cov}[M_1, M_2]}{\mathrm{Cov}[M_1, M_1]}$$

b_2:

$$\begin{aligned}\mathrm{Cov}[M_2, Y] &= a_2 c' \sigma^2_X + a_1 a_2 b_1 \sigma^2_X + a^2_2 b_2 \sigma^2_X + b_2 \sigma^2_{\varepsilon 4}\\ &= c'\mathrm{Cov}[X, M_2] + b_1\mathrm{Cov}[M_1, M_2] + b_2\mathrm{Cov}[M_2, M_2]\end{aligned}$$

$$b_2\mathrm{Cov}[M_2, M_2] = \mathrm{Cov}[M_2, Y] - c'\mathrm{Cov}[X, M_2] - b_1\mathrm{Cov}[M_1, M_2]$$

$$b_2 = \frac{\mathrm{Cov}[M_2, Y] - c'\mathrm{Cov}[X, M_2] - b_1\mathrm{Cov}[M_1, M_2]}{\mathrm{Cov}[M_2, M_2]}$$

c':

$$\begin{aligned}\mathrm{Cov}[X, Y] &= c'\sigma^2_X + a_1 b_1 \sigma^2_X + a_2 b_2 \sigma^2_X\\ &= c'\mathrm{Cov}[X, X] + b_1\mathrm{Cov}[X, M_1] + b_2\mathrm{Cov}[X, M_2]\end{aligned}$$

$$c'\mathrm{Cov}[X, X] = \mathrm{Cov}[X, Y] - b_1\mathrm{Cov}[X, M_1] - b_2\mathrm{Cov}[X, M_2]$$

$$c' = \frac{\mathrm{Cov}[X, Y] - b_1\mathrm{Cov}[X, M_1] - b_2\mathrm{Cov}[X, M_2]}{\mathrm{Cov}[X, X]}$$

APPENDIX F

EFFECT SIZE FORMULAS FOR THE PARALLEL TWO MEDIATOR MODEL

$$\rho_{\mathrm{M_1M_2}} = \frac{\sigma_{\mathrm{M_1M_2}}}{\sqrt{\sigma^2_{\mathrm{M_1}}}\sqrt{\sigma^2_{\mathrm{M_2}}}}$$

$$\rho_{\mathrm{M_1Y}} = \frac{\sigma_{\mathrm{M_1Y}}}{\sqrt{\sigma^2_{\mathrm{M_1}}}\sqrt{\sigma^2_{\mathrm{Y}}}}$$

$$\rho_{\mathrm{M_2Y}} = \frac{\sigma_{\mathrm{M_2Y}}}{\sqrt{\sigma^2_{\mathrm{M_2}}}\sqrt{\sigma^2_{\mathrm{Y}}}}$$

$$\rho_{\mathrm{XY}} = \frac{\sigma_{\mathrm{XY}}}{\sqrt{\sigma^2_{\mathrm{X}}}\sqrt{\sigma^2_{\mathrm{Y}}}}$$

For *a₁*: $\rho_{\mathrm{XM_1}} = \dfrac{\sigma_{\mathrm{XM_1}}}{\sqrt{\sigma^2_{\mathrm{X}}}\sqrt{\sigma^2_{\mathrm{M_1}}}}$

For *a₂*: $\rho_{\mathrm{XM_2}} = \dfrac{\sigma_{\mathrm{XM_2}}}{\sqrt{\sigma^2_{\mathrm{X}}}\sqrt{\sigma^2_{\mathrm{M_2}}}}$

For *b*₁:

$$\rho_{\mathrm{M_1Y.XM_2}} = \frac{\rho_{\mathrm{M_1Y.X}} - \rho_{\mathrm{M_1M_2.X}}\rho_{\mathrm{M_2Y.X}}}{\sqrt{1-\rho^2_{\mathrm{M_1M_2.X}}}\sqrt{1-\rho^2_{\mathrm{M_2Y.X}}}}, \text{ where}$$

$$\rho_{\mathrm{M_1Y.X}} = \frac{\rho_{\mathrm{M_1Y}} - \rho_{\mathrm{XM_1}}\rho_{\mathrm{XY}}}{\sqrt{1-\rho^2_{\mathrm{XM_1}}}\sqrt{1-\rho^2_{\mathrm{XY}}}},$$

$$\rho_{\mathrm{M_1M_2.X}} = \frac{\rho_{\mathrm{M_1M_2}} - \rho_{\mathrm{XM_1}}\rho_{\mathrm{XM_2}}}{\sqrt{1-\rho^2_{\mathrm{XM_1}}}\sqrt{1-\rho^2_{\mathrm{XM_2}}}},$$

$$\rho_{\mathrm{M_2Y.X}} = \frac{\rho_{\mathrm{M_2Y}} - \rho_{\mathrm{XM_2}}\rho_{\mathrm{XY}}}{\sqrt{1-\rho^2_{\mathrm{XM_2}}}\sqrt{1-\rho^2_{\mathrm{XY}}}}$$

For *b*₂:

$$\rho_{\mathrm{M_2Y.XM_1}} = \frac{\rho_{\mathrm{M_2Y.X}} - \rho_{\mathrm{M_1M_2.X}}\rho_{\mathrm{M_1Y.X}}}{\sqrt{1-\rho^2_{\mathrm{M_1M_2.X}}}\sqrt{1-\rho^2_{\mathrm{M_1Y.X}}}}, \text{ where}$$

$$\rho_{\mathrm{M_2Y.X}} = \frac{\rho_{\mathrm{M_2Y}} - \rho_{\mathrm{XM_2}}\rho_{\mathrm{XY}}}{\sqrt{1-\rho^2_{\mathrm{XM_2}}}\sqrt{1-\rho^2_{\mathrm{XY}}}},$$

$$\rho_{\mathrm{M_1M_2.X}} = \frac{\rho_{\mathrm{M_1M_2}} - \rho_{\mathrm{XM_1}}\rho_{\mathrm{XM_2}}}{\sqrt{1-\rho^2_{\mathrm{XM_1}}}\sqrt{1-\rho^2_{\mathrm{XM_2}}}},$$

$$\rho_{\mathrm{M_1Y.X}} = \frac{\rho_{\mathrm{M_1Y}} - \rho_{\mathrm{XM_1}}\rho_{\mathrm{XY}}}{\sqrt{1-\rho^2_{\mathrm{XM_1}}}\sqrt{1-\rho^2_{\mathrm{XY}}}}$$

For *c'*:

$$\rho_{\mathrm{XY.M_1M_2}} = \frac{\rho_{\mathrm{XY.M_1}} - \rho_{\mathrm{XM_2.M_1}}\rho_{\mathrm{YM_2.M_1}}}{\sqrt{1-\rho^2_{\mathrm{XM_2.M_1}}}\sqrt{1-\rho^2_{\mathrm{YM_2.M_1}}}}, \text{ where}$$

$$\rho_{\mathrm{XY.M_1}} = \frac{\rho_{\mathrm{XY}} - \rho_{\mathrm{XM_1}}\rho_{\mathrm{M_1Y}}}{\sqrt{1-\rho^2_{\mathrm{XM_1}}}\sqrt{1-\rho^2_{\mathrm{M_1Y}}}},$$

$$\rho_{\mathrm{XM_2.M_1}} = \frac{\rho_{\mathrm{XM_2}} - \rho_{\mathrm{XM_1}}\rho_{\mathrm{M_1M_2}}}{\sqrt{1-\rho^2_{\mathrm{XM_1}}}\sqrt{1-\rho^2_{\mathrm{M_1M_2}}}},$$

$$\rho_{\mathrm{YM_2.M_1}} = \frac{\rho_{\mathrm{M_2Y}} - \rho_{\mathrm{M_1Y}}\rho_{\mathrm{M_1M_2}}}{\sqrt{1-\rho^2_{\mathrm{M_1Y}}}\sqrt{1-\rho^2_{\mathrm{M_1M_2}}}}$$

APPENDIX G

PROGRAM TO DETERMINE PATH COEFFICIENT VALUES BASED ON EFFECT SIZES FOR THE PARALLEL TWO MEDIATOR MODEL

```
*This program computes analytical effect sizes for the parallel two mediator model;
*The first four sets of "do" statements were used to iterate through values of parameters
that would produce correlations of 0.1, 0.3, and 0.5;
*The final set of "do" statements were the final values that resulted in the desired correlations;

data a;
input a1 a2 b1 b2 cp N;

/* do a1 = 0, 0.05, 0.1, 0.15, 0.20, 0.25, 0.30, 0.35, 0.40, 0.45, 0.50, 0.55, 0.60;*/
/* do a2 = 0, 0.05, 0.1, 0.15, 0.20, 0.25, 0.30, 0.35, 0.40, 0.45, 0.50, 0.55, 0.60;*/
/* do b1 = 0, 0.05, 0.1, 0.15, 0.20, 0.25, 0.30, 0.35, 0.40, 0.45, 0.50, 0.55, 0.60;*/
/* do b2 = 0, 0.05, 0.1, 0.15, 0.20, 0.25, 0.30, 0.35, 0.40, 0.45, 0.50, 0.55, 0.60;*/
/* do cp = 0, 0.05, 0.1, 0.15, 0.20, 0.25, 0.30, 0.35, 0.40, 0.45, 0.50, 0.55, 0.60;*/
/**/
/* do a1 = 0, 0.14, 0.39, 0.59;*/
/* do a2 = 0, 0.14, 0.39, 0.59;*/
/* do b1 = 0, 0.14, 0.39, 0.59;*/
/* do b2 = 0, 0.14, 0.39, 0.59;*/
/* do cp = 0, 0.14, 0.39, 0.59;*/
/**/
/* do a1 = 0.105, 0.110, 0.115, 0.320, 0.325, 0.330, 0.585, 0.590, 0.595;*/
/* do a2 = 0.105, 0.110, 0.115, 0.320, 0.325, 0.330, 0.585, 0.590, 0.595;*/
/* do b1 = 0.105, 0.110, 0.115, 0.320, 0.325, 0.330, 0.585, 0.590, 0.595;*/
/* do b2 = 0.105, 0.110, 0.115, 0.320, 0.325, 0.330, 0.585, 0.590, 0.595;*/
/* do cp = 0.120, 0.125, 0.130, 0.370, 0.375, 0.380, 0.7;*/
/**/
/* do a1 = 0.102, 0.103, 0.104, 0.317, 0.318, 0.319, 0.582, 0.583, 0.584;*/
/* do a2 = 0.102, 0.103, 0.104, 0.317, 0.318, 0.319, 0.582, 0.583, 0.584;*/
/* do b1 = 0.102, 0.103, 0.104, 0.317, 0.318, 0.319, 0.582, 0.583, 0.584;*/
/* do b2 = 0.102, 0.103, 0.104, 0.317, 0.318, 0.319, 0.582, 0.583, 0.584;*/
/* do cp = 0.131, 0.385, 0.715;*/

 do a1 = 0, 0.101, 0.314, 0.577;
 do a2 = 0, 0.101, 0.314, 0.577;
 do b1 = 0, 0.101, 0.314, 0.577;
 do b2 = 0, 0.101, 0.314, 0.577;
 do cp = 0, 0.131, 0.40, 0.74;

do n =50;

a1b1=a1*b1;
a2b2=a2*b2;
c=a1b1+a2b2+cp;
sa=sqrt(1/(N-2)); sb=sqrt(1/(N-3)); sc=sqrt(1/(n-2));

*This section computes true variances and covariances based
on residual error variance equal to 1;

ERROR=1;ERRORM1=1;ERRORM2=1;ERRORY=1;

EMOD1=(ERROR)**2;
EMOD2=(ERRORY)**2;
EMOD3=(ERRORM1)**2;
EMOD4=(ERRORM2)**2;
CXX=EMOD1;
```

```
CM1X=A1*EMOD1;
CM2X=A2*EMOD1;
CYX=CP*EMOD1+B1*CM1X+B2*CM2X;
CM1M1=A1*A1*EMOD1+EMOD3;
CM2M1=A1*CM2X;
CYM1=A1*CYX+B1*EMOD3;
CM2M2=A2*A2*EMOD1+EMOD4;
CYM2=A2*CYX+B2*EMOD4;
CYY=(CP*CP*EMOD1)+(2*A1*B1*CP*EMOD1)+(2*A2*B2*CP*EMOD1)+(2*A1*A2*B1*B2*EMO
D1)+(B1*B1*(A1*A1*EMOD1+EMOD2))+(B2*B2*(A2*A2*EMOD1+EMOD3))+EMOD4;

*This section computes population correlations;
RM1X=CM1X/SQRT(CXX*CM1M1);
RM2X=CM2X/SQRT(CXX*CM2M2);
RYX=CYX/SQRT(CXX*CYY);
RM1M2=CM2M1/SQRT(CM1M1*CM2M2);
RM1Y=CYM1/SQRT(CM1M1*CYY);
RM2Y=CYM2/SQRT(CM2M2*CYY);

XRM1Y=(RM1Y-RM1X*RYX)/SQRT((1-RM1X*RM1X)*(1-RYX*RYX));
XRM2Y=(RM2Y-RM2X*RYX)/SQRT((1-RM2X*RM2X)*(1-RYX*RYX));
XRM1M2=(RM1M2-RM1X*RM2X)/SQRT((1-RM1X*RM1X)*(1-RM2X*RM2X));
M1RXY=(RYX-RM1X*RM1Y)/SQRT((1-RM1X*RM1X)*(1-RM1Y*RM1Y));
M1RXM2=(RM2X-RM1X*RM1M2)/SQRT((1-RM1X*RM1X)*(1-RM1M2*RM1M2));
M1RM2Y=(RM2Y-RM1Y*RM1M2)/SQRT((1-RM1Y*RM1Y)*(1-RM1M2*RM1M2));

XM2RM1Y=(XRM1Y-XRM1M2*XRM2Y)/SQRT((1-XRM1M2*XRM1M2)*(1-XRM2Y*XRM2Y));
XM1RM2Y=(XRM2Y-XRM1M2*XRM1Y)/SQRT((1-XRM1M2*XRM1M2)*(1-XRM1Y*XRM1Y));
M1M2RXY=(M1RXY-M1RXM2*M1RM2Y)/SQRT((1-M1RXM2*M1RXM2)*(1-
M1RM2Y*M1RM2Y));

 output;
 end;
 end;
 end;
 end;
 end;
 end;
 cards;
 0 0 0 0 0 50
 ;

 proc print; var a1 a2 b1 b2 cp n rm1x rm2x xm2rm1y xm1rm2y m1m2rxy;
 run;

 title 'Plot of effect sizes for the parallel two mediator model';
 proc gplot;
             plot rm1x * a1;
             plot xm2rm1y * b1;
             plot m1m2rxy * cp;
             symbol1 value=dot;
run;
quit;
```

APPENDIX H

PROGRAM TO COMPUTE EMPIRICAL POWER FOR THE PARALLEL TWO MEDIATOR MODEL

```
/*Simulation for two mediator parallel model*/
/*Corresponding with MacKinnon (2008) notation,
a = a1
b = b1
cp = c'
d = a2
e = b2

x = X
m = M1
q = M2
y = Y
*/

FILENAME NULLOG DUMMY 'C:\NULL';
PROC PRINTTO LOG=NULLOG;

%macro medsimpara;

%do nsize = 5 %to 5;
%do aparm = 2 %to 2;
%do bparm = 1 %to 4;
%do cparm = 1 %to 4;
%do dparm = 1 %to 4;
%do eparm = 1 %to 4;
%do numsamps = 1 %to 500;

proc iml;

a = {0,0.101,0.314,0.577};
b = {0,0.101,0.314,0.577};
cp = {0,0.131,0.400,0.740};
d = {0,0.101,0.314,0.577};
e = {0,0.101,0.314,0.577};
n = {50,100,200,500,1000};
varx = 1;
resvarm = 1;
resvarq = 1;
resvary = 1;

/* calculate variances based on paths */
*covxm = a[&aparm,1]*varx;
*varm = (a[&aparm,1]**2)*varx + resvarm;
*resvarm = 1 - ((a[aparm,1]**2)*varx);
*resvary = 1 - ((b[bparm,1]**2)*varm + (cp[cparm,1]**2)*varx + 2*b[bparm,1]*cp[cparm,1]*covxm);

/* create x scores */
x = rannor(j(n[&nsize,1],1,0));

/* generate residuals for m */
/* sqrt(resvarm) = the std for the residual distribution */
resm = sqrt(resvarm)*rannor(j(n[&nsize,1],1,0));

/* generate m via regression equation */
m = a[&aparm,1]*x + resm;
```

```
/* generate residuals for q */
/* sqrt(resvarq) = the std for the residual distribution */
resq = sqrt(resvarq)*rannor(j(n[&nsize,1],1,0));

/* generate q via regression equation */
q = d[&dparm,1]*x + resq;

/* generate residuals for y */
/* sqrt(resvary) = the std for the residual distribution */
resy = sqrt(resvary)*rannor(j(n[&nsize,1],1,0));

/* generate y via regression equation */
y = b[&bparm,1]*m + e[&eparm,1]*q + cp[&cparm,1]*x + resy;

/* concatenate vectors into single matrix */
medvars = x || m || q || y ;

/* create sas data set dat from iml matrix medvars */
create dat from medvars[colname = {x m q y}];
append from medvars;

*proc means data = dat;
*var x m y resm resy;

ods listing close;
proc reg data = dat;
model m = x;
ods output ParameterEstimates = apath;
run;

ods listing close;
proc reg data = dat;
model q = x;
ods output ParameterEstimates = dpath;

ods listing close;
proc reg data = dat;
model y = x m q / covb;
ods output ParameterEstimates = cprimepath Covb = covest;
run;

/*ods listing close;*/
/*proc reg data = dat;*/
/*model y = x m q / covb;*/
/*ods output ParameterEstimates = bpath;*/
/*run;*/
/**/
/*ods listing close;*/
/*proc reg data = dat;*/
/*model y = x m q / covb;*/
/*ods output ParameterEstimates = epath;*/
/*run;*/

ods listing close;
```

```
proc reg data = dat;
model y = x;
ods output ParameterEstimates = total;
run;

data apath;
set apath;
where variable = 'X';
keep estimate stderr;
run;

data apath;
set apath;
rename estimate = a;
rename stderr = ase;
run;

data dpath;
set dpath;
where variable = 'X';
keep estimate stderr;
run;

data dpath;
set dpath;
rename estimate = d;
rename stderr = dse;
run;

data bpath;
set cprimepath;
where variable = 'M';
keep estimate stderr;
run;

data bpath;
set bpath;
rename estimate = b;
rename stderr = bse;
run;

data epath;
set cprimepath;
where variable = 'Q';
keep estimate stderr;
run;

data epath;
set epath;
rename estimate = e;
rename stderr = ese;
run;

data cprimepath;
set cprimepath;
```

```
where variable = 'X';
keep estimate stderr;
run;

data cprimepath;
set cprimepath;
rename estimate = cprime;
rename stderr = cprimese;
run;

data total;
set total;
where variable = 'X';
keep estimate stderr probt;
run;

data total;
set total;
rename estimate = c;
rename stderr = cse;
rename probt = cpvalue;
run;

data covest;
set covest;
where variable = 'Q';
keep m;
run;

data covest;
set covest;
rename m = covbe;
run;

data medparms;
merge apath dpath bpath epath cprimepath total covest;
run;

data medparms;
set medparms;

csig = 0;
if cpvalue < .05 then csig = 1;

ab = a*b;
de = d*e;
sobelse = sqrt(a*a*bse*bse + b*b*ase*ase + d*d*ese*ese + e*e*dse*dse + 2*a*d*covbe);
sobelz = (ab + de) / sobelse;
sobelsig = 0;
if abs(sobelz) > 1.96 then sobelsig = 1;

ta=a/ase;
pa=(1-probt(abs(ta),df))*2;

tb=b/bse;
```

```
pb=(1-probt(abs(tb),df))*2;

td=d/dse;
pd=(1-probt(abs(td),df))*2;

te=e/ese;
pe=(1-probt(abs(te),df))*2;

asig=0;
if pa < 0.05 then asig=1;

bsig=0;
if pb < 0.05 then bsig=1;

dsig=0;
if pd < 0.05 then dsig=1;

esig=0;
if pe < 0.05 then esig=1;

abse=sqrt(a*a*bse*bse + b*b*ase*ase);
zab=ab/abse;
absobelsig=0;
if abs(zab) > 1.96 then absobelsig=1;

abjointsig=0;
if pa < 0.05 and pb < 0.05 then abjointsig=1;

dese=sqrt(d*d*ese*ese + e*e*dse*dse);
zde=de/dese;
desobelsig=0;
if abs(zde) > 1.96 then desobelsig=1;

dejointsig=0;
if pd < 0.05 and pe < 0.05 then dejointsig=1;

if &nsize = 1 then sampsize = 50;
if &nsize = 2 then sampsize = 100;
if &nsize = 3 then sampsize = 200;
if &nsize = 4 then sampsize = 500;
if &nsize = 5 then sampsize = 1000;

if &aparm = 1 then apath = 0;
if &aparm = 2 then apath = 0.101;
if &aparm = 3 then apath = .314;
if &aparm = 4 then apath = .577;

if &bparm = 1 then bpath = 0;
if &bparm = 2 then bpath = .101;
if &bparm = 3 then bpath = .314;
if &bparm = 4 then bpath = .577;

if &dparm = 1 then dpath = 0;
if &dparm = 2 then dpath = .101;
if &dparm = 3 then dpath = .314;
```

```
if &dparm = 4 then dpath = .577;

if &eparm = 1 then epath = 0;
if &eparm = 2 then epath = .101;
if &eparm = 3 then epath = .314;
if &eparm = 4 then epath = .577;

if &cparm = 1 then cpath = 0;
if &cparm = 2 then cpath = .131;
if &cparm = 3 then cpath = .40;
if &cparm = 4 then cpath = .74;

file "c:\HPO\medsimpara.dat" mod;
put @1 (a) (8.6)
@10 (ase) (8.6)
@20 (b) (8.6)
@30 (bse) (8.6)
@40 (d) (8.6)
@50 (dse) (8.6)
@60 (e) (8.6)
@70 (ese) (8.6)
@80 (cprime) (8.6)
@90 (cprimese) (8.6)
@100 (c) (8.6)
@110 (cse) (8.6)
@120 (cpvalue) (8.6)
@130 (csig) (8.6)
@140 (ab) (8.6)
@150 (de) (8.6)
@160 (sobelse) (8.6)
@170 (sobelz) (8.6)
@180 (sobelsig) (8.6)
@190 (sampsize) (8.6)
@200 (apath) (8.6)
@210 (bpath) (8.6)
@220 (dpath) (8.6)
@230 (epath) (8.6)
@240 (cpath) (8.6)
@250 (covbe) (8.6);

run;

%end;
%end;
%end;
%end;
%end;
%end;
%end;

%mend;
%medsimpara;
run;

data medparms;
```

```
infile "c:\HPO\medsimpara.dat";
input a ase b bse d dse e ese cprime cprimese c cse cpvalue csig
ab de sobelse sobelz sobelsig sampsize apath bpath dpath epath cpath covbe;
run;

proc sort data = medparms;
by sampsize apath bpath dpath epath cpath;
run;

proc means data = medparms noprint;
var a b d e c cprime ab de sobelsig csig;
by sampsize apath bpath dpath epath cpath;
output out = powertablethree mean = a b d e c cprime ab de sobelsig csig;
run;

proc means data = medparms;
var covbe;
run;

proc print data = powertablethree;
run;
```

APPENDIX I

PROGRAM TO CALCULATE ANALYTICAL POWER OF THE PARALLEL TWO MEDIATOR MODEL

```
*Last edited September 16, 2012;
*This program computes the power to detect the mediated effect;

data a;
input a1 b1 a2 b2 cp N;
 do a1 = 0, 0.101, 0.314, 0.577;
 do b1 = 0, 0.101, 0.314, 0.577;
 do a2 = 0, 0.101, 0.314, 0.577;
 do b2 = 0, 0.101, 0.314, 0.577;
 do cp =0, 0.131, 0.4, 0.74;
 do n =50, 100, 200, 500, 1000;

c=a1*b1+a2*b2+cp;
a1b1=a1*b1;
a2b2=a2*b2;

sa1=sqrt(1/(N-2));
sa2=sqrt(1/(N-2));
sb1=sqrt(1/(N-4));
sb2=sqrt(1/(N-4));

*This formula does not match the formula below for the true SEcp;
scp=sqrt(1/(N-4));

ERROR=1;
ERRORM1=1;
ERRORM2=1;
ERRORY=1;

*This section computes true variances and covariances;
EMOD1=(ERROR)**2;
EMOD2=(ERRORY)**2;
EMOD3=(ERRORM1)**2;
EMOD4=(ERRORM2)**2;
NOBS=N;
CXX=EMOD1;
CM1X=A1*EMOD1;
CM2X=A2*EMOD1;
CYX=CP*EMOD1+A1*B1*EMOD1+A2*B2*EMOD1;
CM1M1=A1*A1*EMOD1+EMOD3;
CM1M2=A1*A2*EMOD1;
CM1Y=A1*CP*EMOD1+A1*A1*B1*EMOD1+A1*A2*B2*EMOD1+B1*EMOD3;
CM2M2=A2*A2*EMOD1+EMOD4;
CM2Y=A2*CP*EMOD1+A1*A2*B1*EMOD1+A2*A2*B2*EMOD1+B2*EMOD4;
CYY=(CP*CP*EMOD1)+(2*A1*B1*CP*EMOD1)+(2*A2*B2*CP*EMOD1)+(2*A1*A2*B1*B2*EMO
D1)+(B1*B1*(A1*A1*EMOD1+EMOD3))+(B2*B2*(A2*A2*EMOD1+EMOD4))+EMOD2;

*This section computes true standard errors;
SDX=SQRT(CXX);
SDM1=SQRT(CM1M1);
SDM2=SQRT(CM2M2);
SDY=SQRT(CYY);

*This section computes population correlations (RM1X and RM2X serve as effect sizes for a1 and a2);
RM1X=CM1X/SQRT(CXX*CM1M1);
```

```
RM2X=CM2X/SQRT(CXX*CM2M2);
RYX=CYX/SQRT(CXX*CYY);
RM1M2=CM1M2/SQRT(CM1M1*CM2M2);
RM1Y=CM1Y/SQRT(CM1M1*CYY);
RM2Y=CM2Y/SQRT(CM2M2*CYY);

*This section computes population first-order partial correlations ("XRM1Y" would be the corr btwn m1
and y with x partialled);
XRM1Y=(RM1Y-RM1X*RYX)/SQRT((1-RM1X*RM1X)*(1-RYX*RYX));
XRM2Y=(RM2Y-RM2X*RYX)/SQRT((1-RM2X*RM2X)*(1-RYX*RYX));
XRM1M2=(RM1M2-RM1X*RM2X)/SQRT((1-RM1X*RM1X)*(1-RM2X*RM2X));
M1RXY=(RYX-RM1X*RM1Y)/SQRT((1-RM1X*RM1X)*(1-RM1Y*RM1Y));
M1RXM2=(RM2X-RM1X*RM1M2)/SQRT((1-RM1X*RM1X)*(1-RM1M2*RM1M2));
M1RM2Y=(RM2Y-RM1Y*RM1M2)/SQRT((1-RM1Y*RM1Y)*(1-RM1M2*RM1M2));

*This section computes population second-order partial correlations (effect sizes for b1, b2, and c prime);
XM2RM1Y=(XRM1Y-XRM1M2*XRM2Y)/SQRT((1-XRM1M2*XRM1M2)*(1-XRM2Y*XRM2Y));
XM1RM2Y=(XRM2Y-XRM1M2*XRM1Y)/SQRT((1-XRM1M2*XRM1M2)*(1-XRM1Y*XRM1Y));
M1M2RXY=(M1RXY-M1RXM2*M1RM2Y)/SQRT((1-M1RXM2*M1RXM2)*(1-
M1RM2Y*M1RM2Y));

*This section computes true standardized coefficients for b1, b2, and c';
B1STAR=B1*SDM1/SDY;
B2STAR=B2*SDM2/SDY;
CPSTAR=CP*SDX/SDY;

*This section computes the squared multiple correlation and multiple correlations for standard errors;
RSQYXM1M2=(CPSTAR*RYX)+(B1STAR*RM1Y)+(B2STAR*RM2Y);
RSQM1=(RM1X*RM1X+RM1M2*RM1M2-2*RM2X*RM1X*RM1M2)/(1-RM2X*RM2X);
RSQM2=(RM2X*RM2X+RM1M2*RM1M2-2*RM1X*RM2X*RM1M2)/(1-RM1X*RM1X);
RSQX=(RM1X*RM1X+RM2X*RM2X-2*RM1M2*RM1X*RM2X)/(1-RM1M2*RM1M2);

*This section computes population coefficients based on variances and covariances;
TRUEA1=CM1X/CXX;
TRUEA2=CM2X/CXX;
TRUEB1=(CM1Y-CP*CM1X-B2*CM1M2)/CM1M1;
TRUEB2=(CM2Y-CP*CM2X-B1*CM1M2)/CM2M2;
TRUEB22=(((CXX*CM2Y-CM2X*CYX)*(CXX*CM1M1-CM1X*CM1X))-((CXX*CM1Y-
CM1X*CYX)*(CXX*CM1M2-CM1X*CM2X)))/(((CM1X*CM2X-CXX*CM1M2)*(CXX*CM1M2-
CM1X*CM2X))-((CM2X*CM2X-CXX*CM2M2)*(CXX*CM1M1-CM1X*CM1X)));
TRUECP=(CYX-B1*CM1X-B2*CM2X)/CXX;

*This code computes population mean squared errors for the three regression equations;
TRUEMSE2=CYY-(CP*CP*CXX)-(2*B1*CP*CM1X)-(2*B2*CP*CM2X)-(2*B1*B2*CM1M2)-
(B1*B1*CM1M1)-(B2*B2*CM2M2);
TRUEMSE3=CM1M1-A1*A1*CXX;
TRUEMSE4=CM2M2-A2*A2*CXX;

*This code computes population variances/standard errors of the coefficients;
TRUEVARA1=TRUEMSE3/((NOBS-2)*(CXX));
TRUESEA1=SQRT(TRUEVARA1);
TRUEVARA2=TRUEMSE4/((NOBS-2)*(CXX));
TRUESEA2=SQRT(TRUEVARA2);
TRUESEB1=(SDY/SDM1)*SQRT((1-(RSQYXM1M2))/(NOBS-3))*SQRT(1/(1-(RSQM1)));
TRUESEB2=(SDY/SDM2)*SQRT((1-(RSQYXM1M2))/(NOBS-3))*SQRT(1/(1-(RSQM2)));
```

```
*This formula does not match the formula above, scp;
TRUESECP=(SDY/SDX)*SQRT((1-(RSQYXM1M2))/(NOBS-3))*SQRT(1/(1-(RSQX)));

*This code computes both the population and sample product of coefficients standard errors of a1b1+a2b2;
TRUESEPOC=SQRT(TRUESEA1*TRUESEA1*B1*B1+TRUESEB1*TRUESEB1*A1*A1+TRUESEA2
*TRUESEA2*B2*B2+TRUESEB2*TRUESEB2*A2*A2+2*A1*A2*TRUESEB1*TRUESEB2);

*Calculation of power of RM1X (correlation for a1 coeff) using z test;
A1PRIME=((1/2)*LOG((1+RM1X)/(1-RM1X)));
SDA1PRIME=1/SQRT(NOBS-3);
ZA1PRIME=A1PRIME/SDA1PRIME;
ZA1=1.96-ZA1PRIME;
ZPOWERA1=1-PROBNORM(ZA1);

*Calculation of power of a1 using t test;
TA1=ABS(A1/TRUESEA1);
IF TA1 GT 20 THEN TA1 = 20;
TCRITA1=TINV(.975,NOBS-2);
TPOWERA1=1-PROBT(TCRITA1,NOBS-2,TA1);

*Calculation of power of RM2X (correlation for a2 coeff) using z test;
A2PRIME=((1/2)*LOG((1+RM2X)/(1-RM2X)));
SDA2PRIME=1/SQRT(NOBS-3);
ZA2PRIME=A2PRIME/SDA2PRIME;
ZA2=1.96-ZA2PRIME;
ZPOWERA2=1-PROBNORM(ZA2);

*Calculation of power of a2 using t test;
TA2=ABS(A2/TRUESEA2);
IF TA2 GT 20 THEN TA2 = 20;
TCRITA2=TINV(.975,NOBS-2);
TPOWERA2=1-PROBT(TCRITA2,NOBS-2,TA2);

*Calculation of power of XM2RM1Y (correlation for b1 coeff) using z test;
B1PRIME=((1/2)*LOG((1+XM2RM1Y)/(1-XM2RM1Y)));
SDB1PRIME=1/SQRT(NOBS-3);
ZB1PRIME=B1PRIME/SDB1PRIME;
ZB1=1.96-ZB1PRIME;
ZPOWERB1=1-PROBNORM(ZB1);

*Calculation of power of b1 using t test;
TB1=ABS(B1/TRUESEB1);
IF TB1 GT 20 THEN TB1 = 20;
TCRITB1=TINV(.975,NOBS-2);
TPOWERB1=1-PROBT(TCRITB1,NOBS-2,TB1);

*Calculation of power of XM1RM2Y (correlation for b2 coeff) using z test;
B2PRIME=((1/2)*LOG((1+XM1RM2Y)/(1-XM1RM2Y)));
SDB2PRIME=1/SQRT(NOBS-3);
ZB2PRIME=B2PRIME/SDB2PRIME;
ZB2=1.96-ZB2PRIME;
ZPOWERB2=1-PROBNORM(ZB2);

*Calculation of power of b2 using t test;
TB2=ABS(B2/TRUESEB2);
```

```
IF TB2 GT 20 THEN TB2 = 20;
TCRITB2=TINV(.975,NOBS-2);
TPOWERB2=1-PROBT(TCRITB2,NOBS-2,TB2);

*Calculation of difference in power calculations for a1, a2, b1, and b2;
A1POWERDIFF=ZPOWERA1-TPOWERA1;
A2POWERDIFF=ZPOWERA2-TPOWERA2;
B1POWERDIFF=ZPOWERB1-TPOWERB1;
B2POWERDIFF=ZPOWERB2-TPOWERB2;

*Calculation of power of a1b1 and a2b2 using z test;
ZPOWERA1B1=ZPOWERA1*ZPOWERB1;
ZPOWERA2B2=ZPOWERA2*ZPOWERB2;

*Calculation of power of a1b1 and a2b2 using t test;
TPOWERA1B1=TPOWERA1*TPOWERB1;
TPOWERA2B2=TPOWERA2*TPOWERB2;

*Calculation of difference in power calculations for specific mediated effects;
A1B1POWERDIFF=ZPOWERA1B1-TPOWERA1B1;
A2B2POWERDIFF=ZPOWERA2B2-TPOWERA2B2;

*Calculation of power of product of coefficients using t test;
A1B1=A1*B1;
A2B2=A2*B2;
A1B1A2B2=ABS((A1B1+A2B2)/TRUESEPOC);
Z2MED=1.96-A1B1A2B2;
ZPOWER2MED=1-PROBNORM(Z2MED);

 output;
 end;
 end;
 end;
 end;
 end;
 end;
 cards;
 0 0 0 0 0 50
 ;

/* PROC SORT;*/
/*          BY A2B2POWERDIFF;*/
/*RUN;*/
/**/
/*PROC PRINT;*/
/*          VAR A2B2POWERDIFF;*/
/*RUN;*/

 proc sort;
          by n a1 b1 a2 b2 cp;
run;

ods html body = 'C:\Users\Dropbox\Masters\Analytical work\Power Calculations\poweroutab.xls';
 proc print;
          VAR n a1 b1 a2 b2 cp zpowera1b1 tpowera1b1 zpowera2b2 tpowera2b2 ZPOWER2MED;
```

```
RUN;
 ods html close;
```

APPENDIX J

CONDITIONS WHERE POWER OF THE JOINT SIGNIFICANCE TEST EXCEEDS POWER OF THE TEST OF THE TOTAL EFFECT FOR THE SINGLE MEDIATOR MODEL

Conditions where Power of the Joint Significance Test Exceeds Power of the Test of the Total Effect for the Single Mediator Model

						1-β	
		a	*b*	*c'*	*c*	Total	Joint Significance
N	50	0.14	0.39	0	0.0546	0.063	0.097
		0.14	0.59	0	0.0826	0.083	0.153
		0.39	0.14	0	0.0546	0.073	0.104
		0.39	0.39	0	0.1521	0.173	0.518
		0.39	0.39	0.14	0.2921	0.455	0.498
		0.39	0.59	0	0.2301	0.267	0.688
		0.39	0.59	0.14	0.3701	0.559	0.704
		0.59	0.14	0	0.0826	0.075	0.143
		0.59	0.39	0	0.2301	0.311	0.707
		0.59	0.39	0.14	0.3701	0.646	0.709
		0.59	0.59	0	0.3481	0.503	0.93
		0.59	0.59	0.14	0.4881	0.823	0.947
	100	0	0.39	0	0	0.046	0.049
		0.14	0.14	0	0.0196	0.056	0.062
		0.14	0.39	0	0.0546	0.077	0.274
		0.14	0.59	0	0.0826	0.117	0.299
		0.39	0.14	0	0.0546	0.076	0.286
		0.39	0.39	0	0.1521	0.306	0.935
		0.39	0.39	0.14	0.2921	0.779	0.92
		0.39	0.59	0	0.2301	0.504	0.963
		0.39	0.59	0.14	0.3701	0.889	0.971
		0.59	0.14	0	0.0826	0.11	0.263
		0.59	0.39	0	0.2301	0.567	0.971
		0.59	0.39	0.14	0.3701	0.92	0.965
		0.59	0.59	0	0.3481	0.845	0.999
		0.59	0.59	0.14	0.4881	0.982	1
	200	0	0.59	0	0	0.04	0.047
		0.14	0.14	0	0.0196	0.068	0.22
		0.14	0.39	0	0.0546	0.119	0.489
		0.14	0.59	0	0.0826	0.17	0.497
		0.39	0	0	0	0.034	0.046
		0.39	0.14	0	0.0546	0.113	0.516
		0.39	0.39	0	0.1521	0.504	0.997
		0.39	0.39	0.14	0.2921	0.97	1

	0.39	0.59	0	0.2301	0.788	1
	0.39	0.59	0.14	0.3701	0.994	0.999
	0.59	0.14	0	0.0826	0.199	0.505
	0.59	0.39	0	0.2301	0.868	1
	0.59	0.39	0.14	0.3701	0.994	1
	0.59	0.59	0	0.3481	0.978	1
500	0	0.59	0	0	0.049	0.054
	0.14	0.14	0	0.0196	0.069	0.735
	0.14	0.39	0	0.0546	0.234	0.894
	0.14	0.59	0	0.0826	0.329	0.881
	0.39	0	0	0	0.05	0.056
	0.39	0.14	0	0.0546	0.236	0.878
	0.39	0.39	0	0.1521	0.883	1
	0.39	0.59	0	0.2301	0.99	1
	0.59	0.14	0	0.0826	0.442	0.872
	0.59	0.39	0	0.2301	0.995	1
1000	0	0.39	0	0	0.055	0.059
	0.14	0	0	0	0.045	0.065
	0.14	0.14	0	0.0196	0.082	0.99
	0.14	0.39	0	0.0546	0.375	0.993
	0.14	0.59	0	0.0826	0.612	0.994
	0.39	0.14	0	0.0546	0.388	0.996
	0.39	0.39	0	0.1521	0.996	1
	0.59	0.14	0	0.0826	0.728	0.989
5000	0	0.14	0	0	0.065	0.066
	0	0.39	0	0	0.044	0.045
	0.14	0.14	0	0.0196	0.28	1
	0.14	0.39	0	0.0546	0.94	1
	0.14	0.59	0	0.0826	0.999	1
	0.39	0.14	0	0.0546	0.967	1

APPENDIX K

CONDITIONS WHERE POWER OF THE PRODUCT OF COEFFICIENTS TEST EXCEEDS POWER OF THE TEST OF THE TOTAL EFFECT FOR THE SINGLE MEDIATOR MODEL

Conditions where Power of the Product of Coefficients Test Exceeds Power of the Test of the Total Effect for the Single Mediator Model

						1-β	
		a	*b*	*c'*	*c*	Total	Sobel
N	50	0.14	0.59	0	0.0826	0.083	0.089
		0.39	0.39	0	0.1521	0.173	0.364
		0.39	0.59	0	0.2301	0.267	0.579
		0.39	0.59	0.14	0.3701	0.559	0.598
		0.59	0.14	0	0.0826	0.075	0.098
		0.59	0.39	0	0.2301	0.311	0.61
		0.59	0.59	0	0.3481	0.503	0.89
		0.59	0.59	0.14	0.4881	0.823	0.908
	100	0.14	0.39	0	0.0546	0.077	0.182
		0.14	0.59	0	0.0826	0.117	0.248
		0.39	0.14	0	0.0546	0.076	0.185
		0.39	0.39	0	0.1521	0.306	0.885
		0.39	0.39	0.14	0.2921	0.779	0.868
		0.39	0.59	0	0.2301	0.504	0.952
		0.39	0.59	0.14	0.3701	0.889	0.963
		0.59	0.14	0	0.0826	0.11	0.227
		0.59	0.39	0	0.2301	0.567	0.956
		0.59	0.39	0.14	0.3701	0.92	0.955
		0.59	0.59	0	0.3481	0.845	0.999
		0.59	0.59	0.14	0.4881	0.982	1
	200	0	0.59	0	0	0.04	0.043
		0.14	0.14	0	0.0196	0.068	0.082
		0.14	0.39	0	0.0546	0.119	0.435
		0.14	0.59	0	0.0826	0.17	0.484
		0.39	0.14	0	0.0546	0.113	0.452
		0.39	0.39	0	0.1521	0.504	0.997
		0.39	0.39	0.14	0.2921	0.97	0.999
		0.39	0.59	0	0.2301	0.788	1
		0.39	0.59	0.14	0.3701	0.994	0.999
		0.59	0.14	0	0.0826	0.199	0.486
		0.59	0.39	0	0.2301	0.868	1
		0.59	0.39	0.14	0.3701	0.994	1
		0.59	0.59	0	0.3481	0.978	1
	500	0	0.59	0	0	0.049	0.052

	0.14	0.14	0	0.0196	0.069	0.582
	0.14	0.39	0	0.0546	0.234	0.884
	0.14	0.59	0	0.0826	0.329	0.876
	0.39	0.14	0	0.0546	0.236	0.873
	0.39	0.39	0	0.1521	0.883	1
	0.39	0.59	0	0.2301	0.99	1
	0.59	0.14	0	0.0826	0.442	0.87
	0.59	0.39	0	0.2301	0.995	1
1000	0.14	0.14	0	0.0196	0.082	0.971
	0.14	0.39	0	0.0546	0.375	0.992
	0.14	0.59	0	0.0826	0.612	0.992
	0.39	0.14	0	0.0546	0.388	0.995
	0.39	0.39	0	0.1521	0.996	1
	0.59	0.14	0	0.0826	0.728	0.989
5000	0	0.39	0	0	0.044	0.045
	0.14	0.14	0	0.0196	0.28	1
	0.14	0.39	0	0.0546	0.94	1
	0.14	0.59	0	0.0826	0.999	1
	0.39	0.14	0	0.0546	0.967	1

APPENDIX L

SYNTAX FOR LOGISTIC REGRESSION ANALYSIS OF SIMULATION DATA

```
data a;
input sampsize apath bpath dpath epath cpath case Frequency Percent;
cards;

…

;
run;

proc means data=a;
run;

data a2;
		set a;
sampsizecent = sampsize - 370;
run;

data a3;
		set a2;
apathcent = apath - 0.248;
run;

data a4;
		set a3;
bpathcent = bpath - 0.248;
run;

data a5;
		set a4;
dpathcent = dpath - 0.248;
run;

data a6;
		set a5;
epathcent = epath - 0.248;
run;

data a7;
		set a6;
cpathcent = cpath - 0.31775;
run;

*The following code produces odds ratio estimates, and does not include a class statement so produces
main effects and interactions;
title 'Logistic regression with weights';
proc logistic data=a7;
		weight frequency;
		model case = sampsizecent|apathcent|bpathcent|dpathcent|epathcent|cpathcent / expb;
		ods output ParameterEstimates = params;
run;
```

APPENDIX M

LOGISTIC REGRESSION RESULTS PREDICTING CASES WHERE POWER TO DETECT THE TOTAL MEDIATED EFFECT EXCEEDED POWER TO DETECT THE TOTAL EFFECT FOR THE PARALLEL TWO MEDIATOR MODEL

Logistic Regression Results Predicting Cases Where Power to Detect the Total Mediated Effect Exceeded Power to Detect the Total Effect for the Parallel Two Mediator Model

Predictor	β	SE_{β}	Wald's $\chi 2$	*df*	*p*	e^{β} (odds ratio)
Intercept	9.914	0.083	14175.674	1	<.0001***	20219.2
N	0.018	0.000	4152.833	1	<.0001***	1.018
a_1	7.444	0.379	385.188	1	<.0001***	1710.182
$N*a_1$	0.027	0.001	486.746	1	<.0001***	1.028
b_1	6.662	0.383	303.197	1	<.0001***	782.03
$N*b_1$	0.024	0.001	361.422	1	<.0001***	1.024
a_1*b_1	28.407	1.727	270.594	1	<.0001***	2.173E+12
$N*a_1*b_1$	0.085	0.006	230.500	1	<.0001***	1.089
a_2	7.203	0.381	357.823	1	<.0001***	1343.884
$N*a_2$	0.026	0.001	455.892	1	<.0001***	1.027
a_1*a_2	15.048	1.735	75.235	1	<.0001***	3428244
$N*a_1*a_2$	0.033	0.006	34.709	1	<.0001***	1.034
b_1*a_2	14.314	1.750	66.916	1	<.0001***	1645465
$N*b_1*a_2$	0.033	0.006	34.147	1	<.0001***	1.034
$a_1*b_1*a_2$	35.805	7.898	20.551	1	<.0001***	3.547E+15
$N*a_1*b_1*a_2$	0.120	0.026	21.916	1	<.0001***	1.127
b_2	6.420	0.385	278.228	1	<.0001***	613.873
$N*b_2$	0.023	0.001	335.073	1	<.0001***	1.023
a_1*b_2	12.793	1.753	53.268	1	<.0001***	359739.1
$N*a_1*b_2$	0.027	0.006	22.819	1	<.0001***	1.028
b_1*b_2	12.046	1.768	46.420	1	<.0001***	170409.6
$N*b_1*b_2$	0.026	0.006	20.618	1	<.0001***	1.026
$a_1*b_1*b_2$	29.453	7.978	13.629	1	0.0002	6.186E+12
$N*a_1*b_1*b_2$	0.100	0.026	15.069	1	0.0001	1.105
a_2*b_2	25.852	1.748	218.732	1	<.0001***	1.687E+11
$N*a_2*b_2$	0.078	0.006	189.519	1	<.0001***	1.081

$a_1*a_2*b_2$	27.301	7.959	11.766	1	0.0006	7.19E+11
$N*a_1*a_2*b_2$	0.091	0.026	12.574	1	0.0004	1.096
$b_1*a_2*b_2$	24.234	8.028	9.112	1	0.0025	33460000000
$N*b_1*a_2*b_2$	0.085	0.026	10.723	1	0.0011	1.089
$a_1*b_1*a_2*b_2$	104.500	36.208	8.328	1	0.0039	2.401E+45
$N*a_1*b_1*a_2*b_2$	0.241	0.117	4.237	1	0.0395	1.272
c'	24.966	0.265	8905.779	1	<.0001***	69610000000
$N*c'$	0.052	0.001	3688.429	1	<.0001***	1.054
a_1*c'	20.211	1.205	281.475	1	<.0001***	598930000
$N*a_1*c'$	0.061	0.004	245.922	1	<.0001***	1.063
b_1*c'	17.899	1.215	217.017	1	<.0001***	59336260
$N*b_1*c'$	0.052	0.004	171.139	1	<.0001***	1.053
a_1*b_1*c'	68.806	5.482	157.522	1	<.0001***	7.618E+29
$N*a_1*b_1*c'$	0.188	0.018	112.305	1	<.0001***	1.207
a_2*c'	19.513	1.209	260.339	1	<.0001***	298000000
$N*a_2*c'$	0.059	0.004	225.344	1	<.0001***	1.061
a_1*a_2*c'	21.761	5.507	15.612	1	<.0001***	2822900000
$N*a_1*a_2*c'$	0.069	0.018	14.896	1	0.0001	1.071
b_1*a_2*c'	21.177	5.555	14.535	1	0.0001	1574000000
$N*b_1*a_2*c'$	0.071	0.018	15.721	1	<.0001***	1.074
$a_1*b_1*a_2*c'$	58.033	25.064	5.361	1	0.0206	1.597E+25
$N*a_1*b_1*a_2*c'$	0.216	0.081	7.093	1	0.0077	1.241
b_2*c'	17.199	1.222	198.051	1	<.0001***	29482576
$N*b_2*c'$	0.049	0.004	154.292	1	<.0001***	1.051
a_1*b_2*c'	16.725	5.564	9.035	1	0.0026	18346367
$N*a_1*b_2*c'$	0.052	0.018	8.267	1	0.0040	1.053
b_1*b_2*c'	15.519	5.612	7.647	1	0.0057	5491654
$N*b_1*b_2*c'$	0.050	0.018	7.667	1	0.0056	1.052
$a_1*b_1*b_2*c'$	42.728	25.316	2.849	1	0.0915	3.602E+18
$N*a_1*b_1*b_2*c'$	0.164	0.082	4.039	1	0.0445	1.179

a_2*b_2*c'	60.599	5.548	119.299	1	<.0001***	2.078E+26
$N*a_2*b_2*c'$	0.167	0.018	86.514	1	<.0001***	1.182
$a_1*a_2*b_2*c'$	29.099	25.254	1.328	1	0.2492	4.341E+12
$N*a_1*a_2*b_2*c'$	0.127	0.082	2.407	1	0.1208	1.135
$b_1*a_2*b_2*c'$	27.901	25.474	1.200	1	0.2734	1.31E+12
$N*b_1*a_2*b_2*c'$	0.117	0.082	2.012	1	0.1560	1.124
$a_1*b_1*a_2*b_2*c'$	39.386	114.900	0.118	1	0.7317	1.273E+17
$N*a_1*b_1*a_2*b_2*c'$	0.225	0.371	0.368	1	0.5439	1.252

APPENDIX N

PROGRAM TO COMPUTE BOOTSTRAP POWER FOR THE PARALLEL TWO MEDIATOR MODEL

```
/*Program edited from MacKinnon (2008) Ch. 12;*/

FILENAME NULLOG DUMMY 'C:\NULL';
proc printto log=nullog;

%MACRO
SIMULATE(NSIM,NOBS,A1,A2,B1,B2,CP,FILE,TYPE,ERROR,ERRORM1,ERRORM2,ERRORY,NB
OOT);

DATA SUMMARY; SET _NULL_;
%DO I=1 %TO &NSIM;
DATA SIM;
DO I=1 TO &NOBS;
X=(&ERROR)*RANNOR(0);
M1=&A1*X+(&ERRORM1)*RANNOR(0);
M2=&A2*X+(&ERRORM2)*RANNOR(0);
Y=&CP*X+&B1*M1+&B2*M2+(&ERRORY)*RANNOR(0);
X2=X*X;
%LET CC=(&A1*&B1)+(&A2*&B2)+&CP;
OUTPUT;
END;

/*Bootstrap*/
*This is where the bootstrap samples are made.;
*sampsize should be equal to the number of observations in the dataset;
*rep is the number of bootstrap samples you want;

proc surveyselect data=sim noprint out=out2 method=urs sampsize=&NOBS rep=&nboot outhits;
run;
quit;

proc reg data=OUT2 outest=out3 tableout covout noprint;
by Replicate;
model y = m1 m2 x;
model m1 = x;
model m2 = x;
model y = x;

data parm; set out3;
if _TYPE_^='PARMS' then delete;

data se; set out3;
if _TYPE_^='STDERR' then delete;

data p; set out3;
if _TYPE_^='PVALUE' then delete;

data cov; set out3;
if _TYPE_^='COV' then delete;
if _NAME_^='M2' then delete;

data b1; set parm;
if _MODEL_^='MODEL1' then delete;
b1=m1;
keep Replicate b1;
```

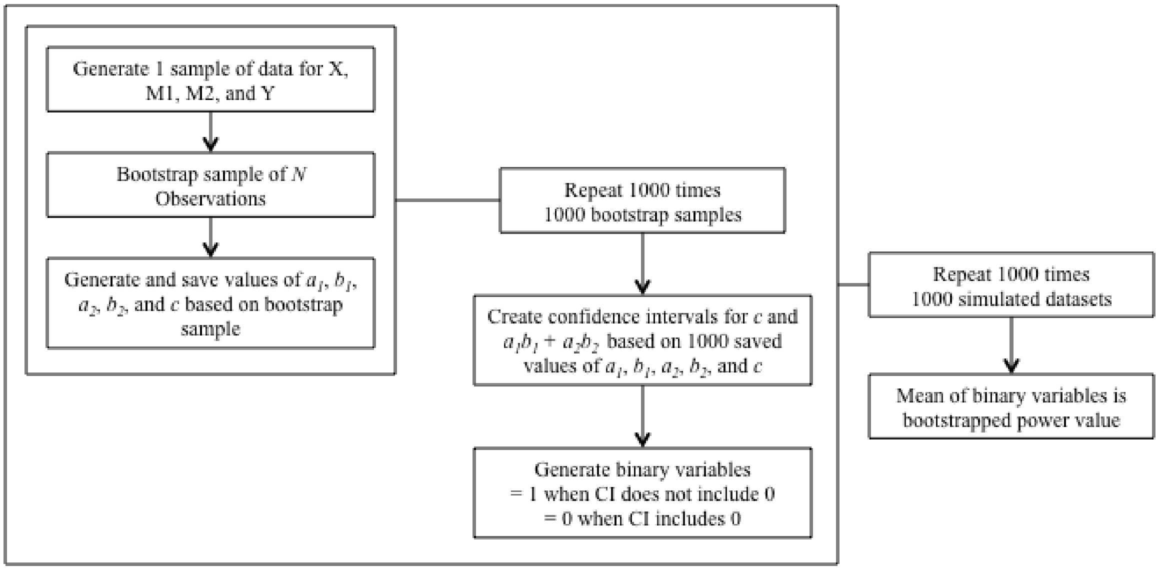
```
data b1se; set se;
if _MODEL_^='MODEL1' then delete;
b1se=m1;
keep Replicate b1se;

data b2; set parm;
if _MODEL_^='MODEL1' then delete;
b2=m2;
keep Replicate b2;

data b2se; set se;
if _MODEL_^='MODEL1' then delete;
b2se=m2;
keep Replicate b2se;

data covb1b2; set cov;
if _MODEL_^='MODEL1' then delete;
covb1b2=m1;
keep Replicate covb1b2;

data a1; set parm;
if _MODEL_^='MODEL2' then delete;
a1=x;
keep Replicate a1;

data a1se; set se;
if _MODEL_^='MODEL2' then delete;
a1se=x;
keep Replicate a1se;

data a2; set parm;
if _MODEL_^='MODEL3' then delete;
a2=x;
keep Replicate a2;

data a2se; set se;
if _MODEL_^='MODEL3' then delete;
a2se=x;
keep Replicate a2se;

data c; set parm;
if _MODEL_^='MODEL4' then delete;
c=x;
keep Replicate c;

data cse; set se;
if _MODEL_^='MODEL4' then delete;
cse=x;
keep Replicate cse;

data cpval; set p;
if _MODEL_^='MODEL4' then delete;
cpval=x;
keep Replicate cpval;
```

```
data d; merge b1 b1se b2 b2se covb1b2 a1 a1se a2 a2se c cse cpval; by Replicate;

a1b1=a1*b1;
a2b2=a2*b2;
med=a1b1+a2b2;
if med<=(&a1*&b1)+(&a2*&b2) then zmed=1; else zmed=0;
if c<=&CC then zc=1; else zc=0;

proc means data=d noprint;
var zmed zc;
output out=out4 mean(zmed)=meanzmed mean(zc)=meanzc;


data out4; set out4;
call symput("meanzmed", meanzmed);
call symput("meanzc", meanzc);

proc sort data=d;
by med;

*Percentile Bootstrap;
data e; set d;
z0=probit(&meanzmed);
if _N_=(ceil(.025*&nboot)) then call symput("LCL95med",med);
if _N_=(ceil(.975*&nboot)) then call symput("UCL95med", med);
if _N_=(ceil(&nboot*probnorm((2*z0)+probit(.025)))) then call symput("BCLCL95med", med);
if _N_=(ceil(&nboot*probnorm((2*z0)+probit(.975)))) then call symput("BCUCL95med", med);
run;
quit;

proc sort data=d;
by c;

data g; set d;
z0c=probit(&meanzc);
if _N_=(ceil(.025*&nboot)) then call symput("LCL95c",c);
if _N_=(ceil(.975*&nboot)) then call symput("UCL95c", c);
if _N_=(ceil(&nboot*probnorm((2*z0c)+probit(.025)))) then call symput("BCLCL95c", c);
if _N_=(ceil(&nboot*probnorm((2*z0c)+probit(.975)))) then call symput("BCUCL95c", c);
run;
quit;

data f; merge e g;
LCL95med=&LCL95med;
UCL95med=&UCL95med;
BCLCL95med=&BCLCL95med;
BCUCL95med=&BCUCL95med;
LCL95c=&LCL95c;
UCL95c=&UCL95c;
BCLCL95c=&BCLCL95c;
BCUCL95c=&BCUCL95c;
MEDSIGBC=0;
IF BCLCL95med GT 0 AND BCUCL95med GT 0 THEN MEDSIGBC=1;
IF BCLCL95med LT 0 AND BCUCL95med LT 0 THEN MEDSIGBC=1;
```

```
MEDSIG=0;
IF LCL95med GT 0 AND UCL95med GT 0 THEN MEDSIG=1;
IF LCL95med LT 0 AND UCL95med LT 0 THEN MEDSIG=1;
CSIGBC=0;
IF BCLCL95c GT 0 AND BCUCL95c GT 0 THEN CSIGBC=1;
IF BCLCL95c LT 0 AND BCUCL95c LT 0 THEN CSIGBC=1;
CSIG=0;
IF LCL95c GT 0 AND UCL95c GT 0 THEN CSIG=1;
IF LCL95c LT 0 AND UCL95c LT 0 THEN CSIG=1;

*appends results into summary dataset;
data test; set f;
data new; set summary;
data summary; set new test;

%END;
%MEND;
/**/
/*%SIMULATE(NSIM=1000,NOBS=100,A1=0.101,A2=0.314,B1=0.101,B2=0.577,CP=0,*/
/*FILE=TEMP,TYPE='CCC',ERROR=1,ERRORM1=1,ERRORM2=1,ERRORY=1,NBOOT=1000);*/
/**/
/*%SIMULATE(NSIM=1000,NOBS=200,A1=0,A2=0.314,B1=0,B2=0.314,CP=0,*/
/*FILE=TEMP,TYPE='CCC',ERROR=1,ERRORM1=1,ERRORM2=1,ERRORY=1,NBOOT=1000);*/

/*%SIMULATE(NSIM=1000,NOBS=1000,A1=0.101,A2=0.101,B1=0.101,B2=0.101,CP=0,*/
/*FILE=TEMP,TYPE='CCC',ERROR=1,ERRORM1=1,ERRORM2=1,ERRORY=1,NBOOT=1000);*/

/*%SIMULATE(NSIM=1000,NOBS=50,A1=0.101,A2=0.577,B1=0.314,B2=0.577,CP=0,*/
/*FILE=TEMP,TYPE='CCC',ERROR=1,ERRORM1=1,ERRORM2=1,ERRORY=1,NBOOT=1000);*/
/**/
/*%SIMULATE(NSIM=1000,NOBS=50,A1=0.577,A2=0.314,B1=0.577,B2=0.101,CP=0,*/
/*FILE=TEMP,TYPE='CCC',ERROR=1,ERRORM1=1,ERRORM2=1,ERRORY=1,NBOOT=1000);*/

/*%SIMULATE(NSIM=1000,NOBS=100,A1=0.101,A2=0.577,B1=0.314,B2=0.314,CP=0,*/
/*FILE=TEMP,TYPE='CCC',ERROR=1,ERRORM1=1,ERRORM2=1,ERRORY=1,NBOOT=1000);*/
/**/
/*%SIMULATE(NSIM=1000,NOBS=200,A1=0.314,A2=0.101,B1=0.314,B2=0,CP=0,*/
/*FILE=TEMP,TYPE='CCC',ERROR=1,ERRORM1=1,ERRORM2=1,ERRORY=1,NBOOT=1000);*/

/*%SIMULATE(NSIM=1000,NOBS=500,A1=0.577,A2=0.577,B1=0.101,B2=0.101,CP=0,*/
/*FILE=TEMP,TYPE='CCC',ERROR=1,ERRORM1=1,ERRORM2=1,ERRORY=1,NBOOT=1000);*/

%SIMULATE(NSIM=1000,NOBS=500,A1=0.101,A2=0.314,B1=0.314,B2=0.101,CP=0,
FILE=TEMP,TYPE='CCC',ERROR=1,ERRORM1=1,ERRORM2=1,ERRORY=1,NBOOT=1000);

/*%SIMULATE(NSIM=1000,NOBS=1000,A1=0.101,A2=0.314,B1=0.577,B2=0.101,CP=0,*/
/*FILE=TEMP,TYPE='CCC',ERROR=1,ERRORM1=1,ERRORM2=1,ERRORY=1,NBOOT=1000);*/

proc means data=SUMMARY;
run;
/*proc print data=f noobs;*/
/*var LCL95 UCL95 BCLCL95 BCUCL95;*/
/**/
/*run;*/
/*quit;*/
```

APPENDIX O

FLOWCHART OF BOOTSTRAP SIMULATION FOR POWER OF PARALLEL TWO MEDIATOR MODEL


Generate 1 sample of data for X, M1, M2, and Y
Bootstrap sample of *N* Observations
Generate and save values of a_1, b_1, a_2, b_2, and c based on bootstrap sample
Repeat 1000 times
1000 bootstrap samples
Create confidence intervals for c and $a_1b_1 + a_2b_2$ based on 1000 saved values of a_1, b_1, a_2, b_2, and c
Generate binary variables
= 1 when CI does not include 0
= 0 when CI includes 0
Repeat 1000 times
1000 simulated datasets
Mean of binary variables is bootstrapped power value

APPENDIX P

DOCUMENT NOTATION

α	Type I error, the rate at which a test incorrectly identifies the presence of a significant effect when no effect is present.
β	Type II error, the rate at which a test fails to find an effect that is truly present.
ε_j	True error variability in the mediation regression equations.
π	Power, an alternative representation equal to one minus the probability of a *z* value for the alternative hypothesis distribution occurring under a normal distribution.
$\rho_{\mathrm{M_1M_2.X}}$	Population first-order partial correlation between M_1 and M_2 partialling X.
$\rho_{\mathrm{M_1Y.X}}$	Population first-order partial correlation between M_1 and Y partialling X.
$\rho_{\mathrm{M_1Y.XM_2}}$	Population second-order partial correlation between M_1 and Y partialling X and M_2, and the effect size measure for the b_1 coefficient.
$\rho_{\mathrm{M_2Y.X}}$	Population first-order partial correlation between M_2 and Y partialling X.
$\rho_{\mathrm{M_2Y.XM_1}}$	Population second-order partial correlation between M_2 and Y partialling X and M_1, and the effect size measure for the b_2 coefficient.
ρ_{XM}	Population bivariate correlation between X and M, and the effect size measure for the *a* coefficient.
$\rho_{\mathrm{XM_1}}$	Population bivariate correlation between X and M_1, and the effect size measure for the a_1 coefficient.
$\rho_{\mathrm{XM_2}}$	Population bivariate correlation between X and M_2, and the effect size measure for the a_2 coefficient.
$\rho_{\mathrm{XM_2.M_1}}$	Population first-order correlation between X and M_2 partialling M_1.
ρ_{XY}	Population bivariate correlation between variables X and Y.
ρ'_{XY}	Population bivariate correlation between variables X and Y, transformed using a Fisher transformation for use in the calculation of analytical power.

$\rho_{\mathrm{XY.M_1}}$	Population first-order correlation between X and Y partialling M_1.
$\rho_{\mathrm{XY.M_1M_2}}$	Population second-order partial correlation between X and Y, and the effect size measure for the *c'* coefficient.
$\rho_{\mathrm{YM.X}}$	Population first-order correlation between M and Y partialling X, and the first-order partial correlation effect size measure for the *b* coefficient.
$\rho_{\mathrm{YM_2.M_1}}$	Population first-order correlation between Y and M_2 partialling M_1.
σ_a	True standard error of the *a* coefficient.
σ_{a_1}	True standard error of the a_1 coefficient.
σ_{a_2}	True standard error of the a_2 coefficient.
σ_b	True standard error of the *b* coefficient.
σ_{b_1}	True standard error of the b_1 coefficient.
σ_{b_2}	True standard error of the b_2 coefficient.
$\sigma_{c'}$	True standard error of the *c'* coefficient.
σ_{M}^2	True variance of M.
σ_{X}^2	True variance of X.
σ_{Y}^2	True variance of Y.
$\sigma_{\varepsilon_2}^2$	Population error variance from the equation with X and M predicting Y for the single mediator model, and from the equation with X, M_1, and M_2 predicting Y for the parallel two mediator model.
$\sigma_{\varepsilon_3}^2$	Population error variance from the equation with X predicting M for the single mediator model, and from the equation with X predicting M_1 for the parallel two mediator model.

$\sigma^2_{\varepsilon_4}$	Population error variance from the equation with X predicting M_2 for the parallel two mediator model.
$1 - \beta$	Power, a test’s ability to detect an effect when an effect is truly present.
a	Population path coefficient representing relationship between X and M.
$\hat{a}$	Sample path coefficient representing relationship between X and M.
a_1	Population path coefficient representing relationship between X and M_1.
$\hat{a}_1$	Sample path coefficient representing relationship between X and M_1.
a_2	Population path coefficient representing relationship between X and M_2.
$\hat{a}_2$	Sample path coefficient representing relationship between X and M_2.
b	Population path coefficient representing relationship between M and Y.
$\hat{b}$	Sample path coefficient representing relationship between M and Y.
b_1	Population path coefficient representing relationship between M_1 and Y.
b_1*	Population standardized path coefficient representing relationship between M_1 and Y.
$\hat{b}_1$	Sample path coefficient representing relationship between M_1 and Y.
b_2	Population path coefficient representing relationship between M_2 and Y.
b_2*	Population standardized path coefficient representing relationship between M_2 and Y.
$\hat{b}_2$	Sample path coefficient representing relationship between M_2 and Y.
c	Population path coefficient representing relationship between X and Y.
$\hat{c}$	Sample path coefficient representing relationship between X and Y.
c'	Population path coefficient representing relationship between X and Y controlling for M in the single mediator model, and for M_1 and M_2 in the parallel two mediator model.

$c'*$	Population standardized path coefficient representing relationship between X and Y controlling for M in the single mediator model, and for M_1 and M_2 in the parallel two mediator model.
$\hat{c}'$	Sample path coefficient representing relationship between X and Y controlling for M in the single mediator model, and for M_1 and M_2 in the parallel two mediator model.
d	General population regression coefficient, used here to describe calculation of power.
e_1	Sample error variability in X.
$\hat{e}_j$	Sample error variability in the mediation regression equations.
i_j	Population intercept in the mediation regression equations.
$\hat{i}_j$	Sample intercept in the mediation regression equations.
k	Number of predictors in a regression equation.
M	Mediator in the single mediator model.
M_1	Mediator in the parallel two mediator model.
M_2	Mediator in the parallel two mediator model.
N	Sample size.
p	Significance level of a statistical test.
$R^2_{Y.X,M_1,M_2}$	Population squared multiple correlation for the parallel two mediator model.
$R^2_{X.M_1,M_2}$	Population squared multiple correlation for X with M_1 and M_2.
$R^2_{M_1.X,M_2}$	Population squared multiple correlation for M_1 with X and M_2.
$R^2_{M_2.X,M_1}$	Population squared multiple correlation for M_2 with X and M_1.
s_{ab}	Multivariate delta standard error for the single mediator model.

$s_{\hat{a}\hat{b}}$	Sample multivariate delta standard error for the single mediator model.
$s_{a_1b_1+a_2b_2}$	Multivariate delta standard error for the parallel two mediator model.
s_d	Standard error of a general population regression coefficient.
t	Parametric test based on the *t* distribution.
X	Independent variable.
Y	Dependent variable.
Z	Third variable, either a covariate, confounder, or moderator; used in conceptual examples in this document.
$z_{1-\alpha/2}$	Value of *z* corresponding to the 97.5th percentile point of the standard normal distribution when α = 0.05, with a value of 1.96.
$z_{\hat{a}\hat{b}}$	Product of coefficients test of significance for the simulated sample mediated effect for the single mediator model.
$z_{\hat{a}_1\hat{b}_1+\hat{a}_2\hat{b}_2}$	Product of coefficients test of significance for the simulated sample mediated effect for the parallel two mediator model.
$z_{\rho'}$	Noncentrality parameter for the alternative hypothesis distribution, used in the calculation of analytical power of a bivariate relationship.