

Essays in Education and Macroeconomics

by

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ABSTRACT

This dissertation consists of three essays on education and macroeconomics. The first chapter analyzes whether public education financing systems can account for large differences among developed countries in earnings inequality and intergenerational earnings persistence. I first document facts about public education in the U.S. and Norway, which provide an interesting case study because they have very different earnings distributions and public education systems. An overlapping generations model is calibrated to match U.S. data, and tax and public education spending functions are estimated for each country. The benchmark exercise finds that taxes and public education spending account for about 15% of differences in earnings inequality and 10% of differences in intergenerational earnings persistence between the U.S. and Norway. Differences in private education spending and early childhood education investments are also shown to be quantitatively important.

The second chapter develops a life-cycle model to study increases in college completion and average ability of college students born from 1900 to 1972. The model is disciplined with new historical data on real college costs from printed government surveys. I find that increases in college completion for 1900 to 1950 cohorts are due primarily to changes in college costs, which generate large endogenous increases in college enrollment. Additionally, I find strong evidence that post-1950 cohorts under-predicted large increases in the college earnings premium. Modifying the model to restrict perfect foresight of the education premia generates a slowdown in college completion consistent with empirical evidence for post-1950 cohorts. Lastly, I find that increased sorting of students by ability can be accounted for by increasingly precise ability signals over time.

The third chapter assesses how structural transformation is affected by sectoral differences in labor-augmenting technological progress, capital intensity, and capital-labor substitutability. CES production functions are estimated for agriculture, manufacturing, and services on post-war U.S. data. I find that sectoral differences in labor-augmenting

technological progress are the dominant force behind changes in sectoral labor and relative prices. Therefore, Cobb-Douglas production functions with labor-augmenting technological change capture the main technological forces behind post-war U.S. structural transformation.

DEDICATION

For Megan, whose love, support, and encouragement made everything possible.
And for my parents, who set high expectations and provided the foundation to help me
achieve my goals.

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Finally, the second chapter of this dissertation is coauthored with Kevin Donovan, and the third chapter is coauthored with Berthold Herrendorf and Ákos Valentinyi. I am grateful to have such talented and dedicated colleagues.

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Chapter 1

PUBLIC EDUCATION FINANCING SYSTEMS, EARNINGS INEQUALITY, AND INTERGENERATIONAL MOBILITY

1.1 Introduction

Among developed countries there are large and well-known differences in both earnings inequality and the persistence of earnings across family generations (commonly referred to as intergenerational earnings mobility).¹ There are also notable differences in how public education expenditures are financed and allocated across individuals. In the United States, for example, public primary and secondary schools receive a significant share of funding from local tax revenue. As a result, public education spending per student is positively correlated with local incomes and varies widely across school districts. By contrast, many European countries finance public primary and secondary schools with federal tax revenue and provide a more uniform distribution of expenditures per student across schools. The goals of this paper are: (i) to document empirical evidence on the distributions of public education expenditures that result from different public education financing systems; and (ii) to ask whether or not differences in taxation and public education spending can account for the large differences in earnings inequality and intergenerational earnings persistence across countries.

The empirical and quantitative exercises in this paper focus on the U.S. and Norway for two reasons. First, disaggregated education financing data are available for both countries, allowing for examination of public primary and secondary education spending at the school district level. Second, the U.S. and Norway provide an interesting case study because they are polar opposites in several important aspects relating to earnings distributions. For countries in the Organisation for Economic Co-operation and Development

¹See, e.g., evidence presented in Aaberge et al. (2002), Bratsberg et al. (2007), Andrews and Leigh (2009) and Corak (Forthcoming).

(OECD), the U.S. generally ranks among those with the highest earnings inequality and intergenerational earnings persistence, while Norway ranks among those with the lowest earnings inequality and intergenerational earnings persistence.

I begin by documenting several facts about the public education systems in the U.S. and Norway. First, the relative contributions from local, state, and federal funding sources varies widely across U.S. school districts, but Norwegian public schools are almost exclusively funded by the federal government. Second, there is a positive correlation between income and public expenditures per student across school districts in the U.S., whereas in Norway this correlation is strongly negative. Third, the variance of public spending per student across school districts in the U.S. is double that of Norway. Fourth, private sources account for nearly one-third of total education spending in the U.S., but only about 5% of total education spending in Norway.² Finally, tertiary education is essentially free in Norway, while publicly subsidized grants and loans in the U.S. are generally dependent on a student's other financial resources.

Motivated by these empirical observations, I depart from the traditional concept of "public education" in which all individuals receive the same amount of public resources. Instead I estimate functions for public education spending so that children of parents with different earnings receive different amounts of public spending on their education. I then incorporate these estimated public education spending functions into a calibrated overlapping generations model in order to assess quantitatively the impact of public education expenditures on earnings inequality and intergenerational earnings persistence. In the model, families consisting of parent-child pairs are heterogeneous with respect to the parent's acquired human capital (which determines labor earnings), the child's endowed learning ability, and the child's tastes for schooling. Public spending on compulsory (primary and secondary level) education for each child is determined by a function of parent earnings. Parents may

²The first three facts regarding education are documented in Section 1.3. The fourth fact is from OECD *Education at a Glance 2010*, Table B3.1.

supplement public education expenditures with their own private spending, but they may not borrow against the child's future earnings to do so. Upon completing compulsory education, children may obtain non-compulsory (tertiary level) education which is subsidized by the government. After completing all education children enter the labor force, become adults, and have children of their own. I examine the stationary recursive competitive equilibrium in this economy and conduct counterfactual experiments with respect to the public education and taxation systems.

There are several channels through which the model can generate cross-country differences in earnings inequality and intergenerational persistence. First, public education spending may ease credit constraints for low income parents who would otherwise invest less in high ability children. All else equal, this increases average human capital and income levels in the economy and decreases intergenerational earnings persistence. Second, a more equal distribution of resources across schools reduces variance in the resulting human capital distribution of the population, thus reducing earnings variance. Third, the level and progressivity of taxes on labor earnings affect net returns to human capital, and thus the incentives to make additional private investments beyond the publicly provided allotment.³

Ultimately, the magnitude of these effects must be determined quantitatively. In the main quantitative exercise, I estimate public education spending functions from data and calibrate remaining parameters of the model to match features of the U.S. education and earnings distributions. I then compute a counterfactual economy in which the U.S. public education and taxation systems are replaced by the Norwegian counterparts. I find that these features account for about 15% of the cross-country differences in earnings inequality and 10% of differences in intergenerational earnings persistence. Importantly, these differences are largely due to changes in the *distribution* of public education spending rather

³See, e.g., Trostel (1993), which finds a negative effect of proportional income taxation on human capital, Erosa and Koreshkova (2007), which find a negative effect of progressive taxation on human capital, and Guvenen, Kuruscu, and Ozkan (2011) which examine both average tax levels and progressivity in a cross-country study.

than average level differences. Furthermore, I find that the public education spending is responsible for most of predicted model differences in intergenerational earnings persistence, whereas tax system differences are responsible for most of the differences in earnings inequality. This result suggests that while earnings inequality and earnings persistence are highly correlated across countries, they are not necessarily driven by the same factors, and they may respond independently to tax and education spending policies.

The remainder of this paper proceeds as follows. Section 1.2 discusses the related literature. Section 1.3 presents motivating evidence on public education systems in the U.S. and Norway. Section 1.4 outlines the model. Section 1.5 covers estimation of the tax and public education spending functions, as well as calibration of remaining model parameters. Section 1.6 discusses the benchmark model fit and results from the main quantitative exercise. Section 1.7 presents additional experiments and sensitivity analysis, and Section 1.8 concludes.

1.2 Related Literature

This paper builds on an extensive literature dating back at least to early theoretical work by Becker and Tomes (1979), Loury (1981), and Becker and Tomes (1986), which examined the role of credit constraints and the transmission of ability from parents to children in generating income persistence over time within families. Solon (2004) contributed to this literature by expanding the model to explicitly account for cross-country differences in intergenerational persistence. On the empirical side, many papers have measured cross-country differences in both intergenerational earnings persistence and earnings inequality. A recent summary of these can be found in Corak (2006).

Other papers including Restuccia and Urrutia (2004), Seshadri and Yuki (2004), and Taska (2011) have also examined quantitatively the impact of taxation and public education spending on income inequality and intergenerational persistence. Several papers have also

studied these issues in a cross-country setting. For example, Björklund and Jäntti (1997) study the case of the U.S. and Sweden, Checchi, Ichino, and Rustichini (1999) examine the case of the U.S. and Italy, and Holter (2012) analyzes the U.S. versus 10 other OECD countries. The main value added of this paper relative to existing quantitative analyses is that I explicitly model the heterogeneity in public education spending both within and across countries. This allows me to conduct policy experiments that incorporate differences in the distribution of public education expenditures rather than only differences in aggregate measures of public spending, such as average public expenditure per student.

This paper is also related to another influential strand of literature including Glomm and Ravikumar (1992), Durlauf (1996), Bénabou (1996), and Fernandez and Rogerson (1998). These papers model households that are organized (either exogenously or endogenously) into separate local communities. They study the differences in income inequality, growth, and intergenerational income persistence when locally provided public education is replaced with a system in which education spending is equalized across communities through state redistribution. As I show in Section 1.3, public education spending is neither purely local nor equalized across communities in either the U.S. or Norway. By estimating public education spending as a function of parent income, I am able to account more precisely for the actual differences between the education financing systems in place in these two countries.

Of course, it is well-known that public education spending is not uniform, and some papers have studied how various education financing systems can affect the distribution of public education spending. One important contribution is by Fernandez and Rogerson (2003) who examine five different education financing systems in a general equilibrium political economy model, and compare the effects on welfare and the distribution of education resources. Also, Murray, Evans, and Schwab (1998) study changes in the distribution of public school resources following legal challenges from the 1970s through 1990s. They

find that inequality in education spending declined significantly during these decades in states where public finance reform was ordered by the courts. To my knowledge, though, this is the first paper to compare distributions of education spending across countries and examine implications for earnings distributions in a general equilibrium framework.

Perhaps the most closely related papers are Bénabou (2002) and Seshadri and Yuki (2004). Bénabou (2002) develops a dynastic heterogeneous-agent economy and calibrates to match U.S. tax and education finance policies. He then conducts separate exercises to assess the impact of progressivity in taxes and education spending for economic growth, aggregate welfare, inequality, and intergenerational mobility. In this paper, however, I integrate both the fiscal and education finance policies for a joint quantitative analysis, yet I am still able to quantify marginal effects of each, as well. Seshadri and Yuki (2004) also have a dynamic general equilibrium setting with heterogeneity in which they quantify the relative effects of monetary versus educational transfers. This paper extends their work along several important dimensions by modeling multiple stages of education in which both schooling time and expenditures matter for human capital production, and disciplining the magnitude of educational transfers using rich disaggregated data.

1.3 Empirical Evidence

This section first examines data on revenue sources and the distribution of public education expenditures in the U.S. and Norway. I then briefly discuss the higher education subsidies available in each country. The data discussed here are incorporated later in the quantitative analysis in Sections 1.5 and 1.6. Further details on the data construction and sources are found in Appendix A.

Public School District Revenues

In the United States, public primary and secondary school districts receive funding from local, state, and federal sources. Yet the share of revenue accruing from each of these sources varies widely across districts. By contrast, local governments in Norway, which are responsible for funding local public schools, are largely financed through federal government grants and federally regulated income tax sharing, as described in Fiva and Rønning (2008). Accordingly, federal sources account for the vast majority of total revenue in almost all Norwegian school districts.

Figure 1.1: Histogram of School Districts by Local Revenue Share

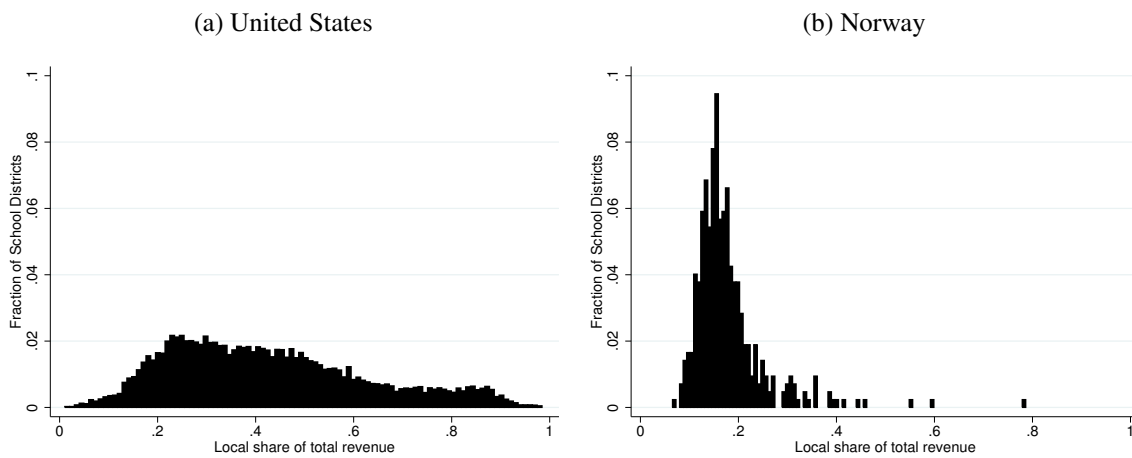


Figure 1.1 illustrates the distinction in financing systems between the two countries. Panel (a) shows the distribution of public school districts in the U.S. by the share of total revenue that is generated from local sources. Panel (b) shows the same data for Norway. Notably, school districts in the U.S. range from one extreme of having zero local funding to the other extreme of being completely reliant on local revenue. The system of strong federal control in Norway, however, results in the much more concentrated distribution seen in Panel (b). Federal government block grants are the primary source of revenues for

Table 1.1: Correlations Across School Districts Between Median Income and Public Education Revenues

Variable	United States	Norway
Local revenue	0.58	0.02
Non-local revenue	-0.36	0.03
Total revenue	0.31	0.03
Local share of total revenue	0.57	0.03

Notes: All correlations are significant at the 1% level.

municipalities, so nearly nine out of ten Norwegian school districts raise less than 25% of public education revenues from local sources.

Of course, a public school system funded strictly at the local level would exhibit strong positive correlations between local household incomes and local education expenditures, as demonstrated in Glomm and Ravikumar (1992) and Fernandez and Rogerson (1998). To offset the disparities that would arise in school systems funded strictly by local sources, state and federal governments redistribute money across districts. Figure 1.1 suggests that the Norwegian system likely results in greater redistribution than the U.S. system, and Table 1.1 confirms this fact. For both the United States and Norway, Table 1.1 reports correlations between school district median income and school district revenue variables, including the public education revenue from local sources, revenue from non-local sources, total revenue, and the share of total revenue from local sources.

Several remarkable differences between the U.S. and Norwegian public education financing systems are apparent from the table. As expected, the correlation between median income and local revenue for the U.S. is indeed strongly positive at 0.58. In other words, school districts with higher median income tend to raise greater amounts of local revenue. In addition, those school districts with higher median income tend also to raise a greater share of revenue locally, as indicated by the correlation of 0.57 between median

income and the local share of total revenue. Also, the correlation between median income and non-local revenue is strongly negative at -0.36 indicating that that state and federal governments do redistribute funds in an attempt to offset revenue disparities due to local funding. Nevertheless, the correlation between median income and total revenue is still positive at 0.31. In contrast, the correlations between public education revenue variables and median income in Norway are quite different. Unlike the U.S., median income in Norway is essentially uncorrelated across school districts with local, federal, or total revenue, as seen in the far right column of Table 1.1.

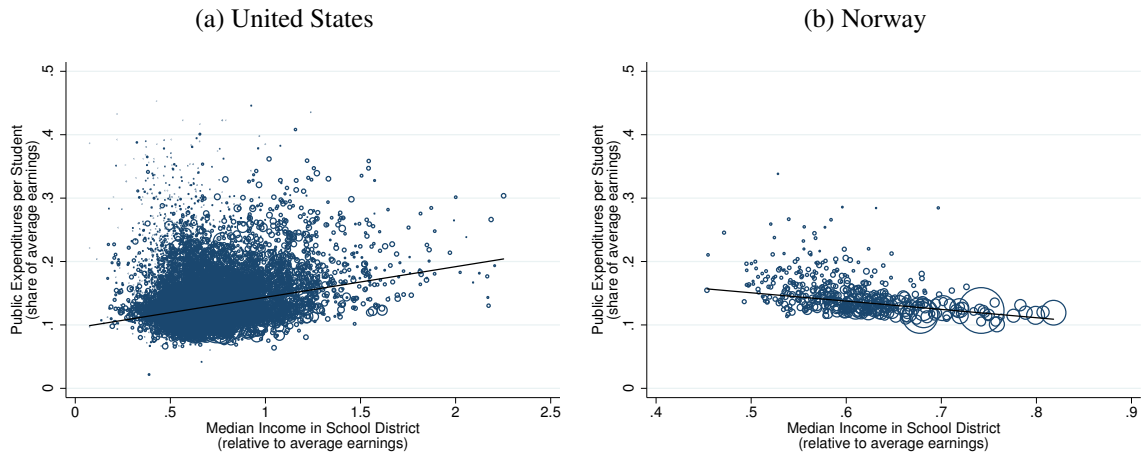
Public School District Spending

Having provided evidence for the sources of public primary and secondary education revenue in the U.S. and Norway, I now examine how public education spending is distributed among individuals within the two countries. Panel (a) of Figure 1.2 is a scatter plot of the “total current expenditures on instruction per student” for school districts in the U.S. against the median income in those districts. Likewise, Panel (b) of Figure 1.2 plots the “net operating expenditures on instruction per student” for school districts in Norway against the median income in those districts. While these data are from different sources and thus have different names, they are comparable in that both series include only public education expenditures directly related to student instruction. Both income and expenditure data in Figure 1.2 are divided by average annual wage earnings of full-time equivalent workers in the respective country in order to make the units comparable.⁴ Circles in the scatter plots vary in size proportional to the number of students in each school district, and regression lines overlaying the scatter plots are weighted by the number of students each district.⁵

⁴Annual wage earnings for the U.S. and Norway are from the OECD Taxing Wages database. Alternatively, one could convert the data from local currencies using PPPs to make the units comparable. However, it turns out that different PPP indexes (such as those by Gheary-Khamis and Èltetö-Köves-Szulc-Sergeev) provide very different answers for the U.S. and Norway, so I use this normalization in order to avoid PPP conversions altogether.

⁵The astute reader may be curious as to why the horizontal axis of the Norwegian scatter plot does not have data both above and below one, as in the U.S. version. The measure of income in each school district is

Figure 1.2: Distributions of Public Education Expenditures



Perhaps the most obvious difference between the distribution of public education expenditures in the U.S. and Norway is that there is a positive correlation of 0.27 between median income and expenditures on instruction per student in the U.S., whereas in Norway this correlation is strongly negative at -0.51. Thus, school districts in the U.S. with higher median income tend to have higher public education expenditures per student. In Norway, however, districts with higher median income tend to have lower public education expenditures per student.

Another notable difference in the panels of Figure 1.2 is that the variance of public expenditures across districts is much greater in the U.S. than Norway. One simple summary statistic which captures this difference is the coefficient of variation in public expenditures per student. For the U.S. this is 0.273, but for Norway it is only 0.136. This means that there is twice as much dispersion in public education expenditures per student (relative to the mean expenditures per student) across school districts in the U.S. as in Norway.⁶

median personal wage earnings for all persons over 17. These data are normalized relative to the mean annual wage earnings of full-time equivalent workers. Due to skewness in the income distribution, the median will tend to be smaller than the mean. Also, because some individuals work only part-time, or not at all, the numerator will generally be smaller than the denominator. In the U.S., there are some particularly high income districts where the numerator exceeds the denominator, but this is not the case for Norway.

⁶It is interesting to note that such large dispersion still existed in the U.S. as of 2000 despite reduced

Despite the distributional differences between the U.S. and Norway, it turns out that the differences for the “average” student are actually quite small. Using the same data as in Figure 1.2, the mean annual public expenditure on instruction per student is 13.3% of average earnings in the U.S., and 13.0% of average earnings in Norway. In essence, this is why modeling the distribution of public expenditures is potentially important. If one only examines differences in average public spending per student, then the U.S. and Norway appear quite similar, but there are significant differences once the full distribution of public spending is taken into account.

Public Subsidies for Higher Education

Public funding for higher education in the U.S. is a complicated web of subsidies, grants, and loans. Some of these public expenditures directly lower the prices paid by students, while others provide a low-cost source of borrowing for students who might otherwise be credit constrained. By contrast, the Norwegian higher education system is essentially free for all admitted students, with the exception of some small fees.

A couple of aggregate statistics illustrate well the fiscal implications of these different systems. According to OECD data, public spending on tertiary education in the year 2000 amounted to 1.1% of GDP in the U.S. and 1.7% of GDP in Norway.⁷ In other words, public spending on tertiary education relative to GDP was about 50% higher in Norway than the United States. The lower levels of public spending in the U.S. are compensated by higher levels of private spending. Public sources in the U.S. accounted for 31.1% of total tertiary education spending in 2000, and the remaining 68.9% was privately funded. By contrast, 96.3% of total tertiary education spending in Norway in the year 2000 was public.⁸

inequality across school districts in many states from the 1970s through 1990s, as described in Murray, Evans, and Schwab (1998).

⁷See OECD *Education at a Glance 2003*, Table B4.1.

⁸See OECD *Education at a Glance 2012*, Table B3.2b.

Overall the above data paints a picture of two countries in which systems for both funding and distributing public education expenditures are very different. Based on this evidence, modeling public education as a system in which every individual receives an identical allocation may result in misleading or erroneous results, especially when making cross-country comparisons. This paper provides a first step toward modeling a more realistic public education financing system. In the model and quantitative exercises to follow, I allow for heterogeneous individuals to receive different amounts of public education expenditures during compulsory education. I also model higher education subsidies that are dependent on parent income. This environment allows for more accurate accounting of the role that public education financing plays in generating cross-country differences in earnings inequality and intergenerational earnings persistence.

1.4 Model Economy

Timing, Demographics, and Preferences

Time in the model economy is discrete with an infinite horizon, and the economy is populated by two-period lived individuals. A model period corresponds to 30 years, where model ages 1 and 2 correspond to actual ages 5-34 and 35-64, respectively. The focus here is on public education and lifetime labor earnings, so I do not model the period of early childhood prior to formal schooling, nor the retirement period of life.

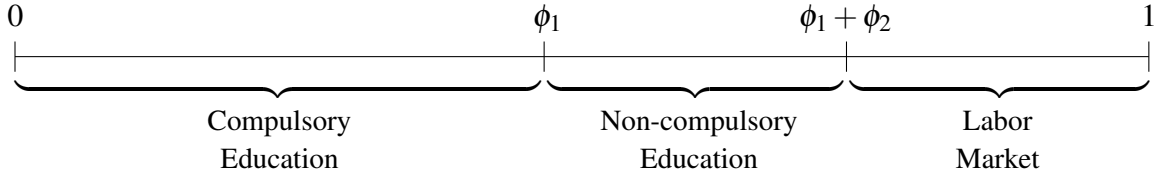
During the first period of life each individual is referred to as a “child,” and during the second period of life the individual is a “parent.” Each parent has a child at the beginning of the second period of life. A family at a given point in time consists of one parent and one child, and an infinite sequence of overlapping generations of parent-child pairs is referred to as a family dynasty. The parent in each family is assumed to make all decisions for the family in that period.

At the beginning of each period, each family is characterized by a state vector $\mathbf{x} = (h_p, \alpha, \zeta)$, where h_p is the human capital of the parent acquired through education in the previous period, α is the learning ability of the child, and ζ is the child's taste for schooling. The child's learning ability and tastes for school are both random endowments. Learning ability may be correlated across generations, but tastes for schooling are independently and identically distributed across individuals and time. The aggregate state of the economy is the distribution over individual state vectors, defined by $\mu(\mathbf{x})$.

Preferences are similar to those in Barro and Becker (1989). Parents value the family's consumption in the current period, and they are altruistic in that they also value the child's utility from schooling and the consumption of all future generations in their family dynasty. As in Restuccia and Vandenbroucke (2012), ζ is allowed to be either positive or negative so that schooling may provide either a utility benefit or a cost to the child.

All individuals are endowed with one unit of time each period. Individuals do not value leisure. Parents devote their full time endowment inelastically to the labor market. Children divide their time endowment into the following three fractions. First, they devote an exogenous fraction $\phi_1 \in (0, 1)$ of their time endowment to compulsory education. This assumption is consistent with the fact that all OECD countries require children to complete some minimum amount of education, generally corresponding to the primary and secondary levels. Next, children spend a fraction $\phi_2 \in [0, 1 - \phi_1]$ in non-compulsory education, where ϕ_2 is chosen by each parent for the child. The non-compulsory stage nests all forms of post-secondary education, including two and four-year colleges and universities, trade schools, professional schools, and graduate programs. Finally, the remaining fraction $1 - \phi_1 - \phi_2$ is supplied as market labor after all education is complete. Figure 1.3 provides a graphical example of the division of the child's time endowment.

Figure 1.3: Division of a child's time endowment



Human Capital and Ability

As previously mentioned, each child is endowed with learning ability α . I use the term *ability* to describe an individual's efficiency in producing human capital, while *human capital* determines an individual's labor efficiency in the production of final output. In practice, an individual's learning ability is likely affected by genetic endowment, early childhood environment, parental education, peer influence, and many other factors. For simplicity, however, I assume that learning ability is transmitted stochastically from parent to child via a transition function $Q(\alpha, \alpha')$. This modeling assumption has been widely employed in similar contexts, including Becker and Tomes (1979) and (1986), as well as Restuccia and Urrutia (2004).

All individuals are assumed to begin life with initial human capital h_1 , which is normalized to one. New human capital is created by a human capital production function which takes the individual's learning ability, current human capital stock, time, and education spending as inputs. Production in each of the two education stages is of the form in Ben-Porath (1967). The acquired human capital stock after compulsory education is denoted h_2 , and human capital after non-compulsory education denoted h_3 . Human capital evolves as follows:

$$h_2 = h_1 + \alpha[(h_1 \phi_1)^v s_1^{1-v}]^{\gamma_1} \quad (1.1)$$

$$h_3 = h_2 + \alpha[(h_2 \phi_2)^v s_2^{1-v}]^{\gamma_2} \quad (1.2)$$

where α is ability, ϕ_j is the fraction of time devoted to schooling in stage $j = \{1, 2\}$, s_j is education spending in stage $j = \{1, 2\}$, and $v \in (0, 1)$, $\gamma_1 \in (0, 1)$, and $\gamma_2 \in (0, 1)$ are exogenous parameters. I assume that public and private expenditures on education, denoted g_j and e_j respectively, are perfect substitutes, so that total spending in each stage is $s_j = g_j + e_j$ for $j = \{1, 2\}$.

To simplify notation in the recursive formulation to follow, denote the human capital production function obtained after substituting equation (1.1) into (1.2) by $f(s_1, s_2, \phi_2; \mathbf{x})$. In addition, to distinguish between the human capital of parent and child within a family, denote the acquired human capital of the child after completing all education by h_c . Thus,

$$h_c \equiv h_3 = f(s_1, s_2, \phi_2; \mathbf{x}) \quad (1.3)$$

Some additional properties of the human capital production function should be noted. First, human capital does not depreciate, so a child who obtains no additional education beyond the compulsory stage will enter the labor market with human capital they acquired through compulsory education. Also, the human capital acquired in compulsory education is an input to the non-compulsory stage human capital production function. In other words, individuals with more human capital after high school would gain more from additional time spent in college.

While all human capital investment occurs in the first period of life, i.e., prior to age 35, I follow Erosa, Koreshkova, and Restuccia (2010) in assuming that each individual receives a shock to their human capital stock at the beginning of the second period of life. This type of shock is commonly referred to as “market luck,” but it may also simply represent unobserved heterogeneity resulting in different earnings among individuals with similar levels of observable human capital. More specifically, a child with human capital h_c today will have human capital h'_p when they become a parent tomorrow according

to:

$$h'_p = \eta h_c \quad (1.4)$$

where $\ln(\eta) \sim N(0, \sigma_\eta^2)$. The inclusion of market luck shocks is made for consistency with the following facts: (i) earnings variance within cohorts grows over the life cycle, as documented by Huggett, Ventura, and Yaron (2006); and (ii) more than one-third of the variance in lifetime earnings is attributable to post-education factors (after age 23), as documented by Huggett, Ventura, and Yaron (2011).

Final Output Technology

A representative firm produces the single final output good according to a linear production function $Y = L$, where Y is aggregate output and L is aggregate effective labor supply. Since labor efficiency units are equal to human capital, aggregate effective labor supply in a given period is:

$$L = \int [h_p(\mathbf{x}) + (1 - \phi_1 - \phi_2(\mathbf{x}))h_c(\mathbf{x})] d\mu(\mathbf{x}). \quad (1.5)$$

Additionally, the wage per efficiency unit of labor with this technology is normalized to one, so the labor earnings of parent and child, denoted y_p and y_c , are equivalent to the amount parent and child human capital supplied to the labor market, h_p and $(1 - \phi_1 - \phi_2(\mathbf{x}))h_c(\mathbf{x})$.

The final output good is used for family consumption c , government consumption G_c (discussed below), and as the expenditure input to the human capital production function via e_1 , e_2 , g_1 , and g_2 . Denoting aggregate quantities by capital letters, market clearing in final output each period requires:

$$Y = C + E_1 + E_2 + G_1 + G_2 + G_c. \quad (1.6)$$

Government

A government imposes taxes on labor earnings according to the average tax rate function $\tau(y)$, where y is labor earnings and $\tau'(y) > 0$. Taxes are levied at the individual rather than family level, so that a parent and child within a family may face different average and marginal tax rates. The total tax obligation of a family is denoted by $T(y_p, y_c) = \tau(y_p) \cdot y_p + \tau(y_c) \cdot y_c$, so a family's period net earnings are: $y_p + y_c - T(y_p, y_c)$. Tax revenues fund public spending on compulsory education, subsidies for non-compulsory education, and government consumption, which provides no utility to individuals. The government budget balances each period.

In the computational work to follow, the key distinctions between the U.S. and Norway will be the tax functions and the public education spending functions. These are estimated for each country in the next section. For now, however, general public education spending functions are defined as follows. First, compulsory education spending is potentially a function of the entire family state vector $\mathbf{x} = (h_p, \alpha, \zeta)$, and parents observe $g_1(\mathbf{x})$ when making decisions for the family. Recall that total spending on compulsory education is $s_1 = g_1 + e_1$, so parents may choose to supplement the public spending on their child with private spending. Second, government subsidies for non-compulsory education are modeled as a fraction of the total cost of non-compulsory education, where that fraction may depend on the family state vector \mathbf{x} . Specifically, if a family described by state vector \mathbf{x} chooses a college or university education with total cost s_2 , then the government subsidy will be $g_2 = \theta(\mathbf{x})s_2$ and the share paid out-of-pocket by the family will be $e_2 = (1 - \theta(\mathbf{x}))s_2$, where $\theta(\mathbf{x}) \in [0, 1]$ for all \mathbf{x} .

Decision Problems

A parent who enters a period with state vector $\mathbf{x} = (h_p, \alpha, \zeta)$ chooses consumption, private education spending on the child's compulsory education, total spending on non-compulsory education (which, given the subsidy $g_2(\mathbf{x})$, also yields a choice for private spending), and the fraction of the child's time spent in non-compulsory education. The parent's objective is to maximize utility from current consumption and the child's taste for school, as well as the expected discounted utility of future generations in the family dynasty. The full decision problem is specified recursively as follows:

$$V(h_p, \alpha, \zeta) = \max_{c, e_1, s_2, \phi_2} \{u(c) + \zeta \phi_2 + \beta \mathbb{E}_{\alpha', \eta', \zeta'} [V(h'_p, \alpha', \zeta') | \alpha]\} \quad (1.7)$$

subject to

$$c + e_1 + (1 - \theta(\mathbf{x}))s_2 = y_p + y_c - T(y_p, y_c)$$

$$y_p = h_p; \quad y_c = (1 - \phi_1 - \phi_2)h_c$$

$$h_c = f(s_1, s_2, \phi_2; \mathbf{x})$$

$$s_1(\mathbf{x}) = g_1(\mathbf{x}) + e_1$$

$$h'_p = \eta h_c$$

$$\phi_2 \in [0, 1 - \phi_1]$$

where $V(h_p, \alpha, \zeta)$ is the value function of a family with state $\mathbf{x} = (h_p, \alpha, \zeta)$. Substituting the budget constraint into the objective function, the problem above can be written as a decision problem for three choice variables: private education spending on compulsory education e_1 , total spending on non-compulsory education s_2 (which, given $\theta(\mathbf{x})$, implies a choice for private spending e_2), and time spent in non-compulsory education ϕ_2 . A solution to this problem consists of optimal decision rules $e_1^*(\mathbf{x})$, $s_2^*(\mathbf{x})$, and $\phi_2^*(\mathbf{x})$. I will examine the stationary recursive competitive equilibrium in this economy, defined below.

Equilibrium

A stationary recursive competitive equilibrium in this economy consists of optimal decision rules $e_1^*(\mathbf{x})$, $s_2^*(\mathbf{x})$, and $\phi_2^*(\mathbf{x})$, labor demand L^* , and stationary distribution $\mu(\mathbf{x})$ such that in every period:

1. Parents choose $e_1^*(\mathbf{x})$, $s_2^*(\mathbf{x})$, and $\phi_2^*(\mathbf{x})$ to solve their decision problem;
2. The representative firm chooses L^* to maximize profits;
3. The government budget balances each period;
4. The stationary distribution $\mu(\mathbf{x})$ is consistent with the decision rules and exogenous stochastic processes for α , η , and ζ ;
5. Output and labor markets clear.

1.5 Model Parameterization

The main quantitative exercise consists of parameterizing the model to match important features of the U.S. data, and then computing a counterfactual economy in which the U.S. education financing system (including the progressive tax functions, compulsory public education expenditures, and subsidies for non-compulsory education) are replaced by the Norwegian counterparts, holding all else fixed. Comparing earnings inequality and inter-generational earnings persistence between the benchmark and counterfactual economies then identifies the share of cross-country differences accounted for *only* by the public education financing systems. Toward this end, this section discusses the benchmark parameterization of the model. I first estimate the labor income tax functions $\tau(y)$ and public education spending functions $g_1(\mathbf{x})$ and $g_2(\mathbf{x})$ for both the U.S. and Norway. Then I calibrate the remaining parameters for preferences and human capital production, as well as the stochastic processes for ability, market luck, and tastes for schooling.

Table 1.2: Tax Function Parameter Estimates

Parameter	United States	Norway
β_0	0.434 (0.007)	1.106 (0.060)
β_1	0.003 (0.0004)	-0.002 (0.0005)
β_2	-0.321 (0.008)	-0.921 (0.060)
β_3	-0.719 (0.018)	-0.190 (0.013)
R^2	0.993	0.998

Tax Functions

Tax systems in OECD countries vary along many dimensions, including average and marginal labor income tax rates, social security taxes, and the credits and benefits available for families with children. Variations in average and marginal tax rates affect incentives to invest in human capital by altering the after-tax return on investment. In addition, implicit transfers embedded in the tax code - such as credits for families with children - should be taken into account when making cross-country comparisons of pre- and post-tax earnings inequality. For these reasons, I utilize OECD data which are comparable across countries and include central and local government taxes, family tax benefits, and social security tax contributions.

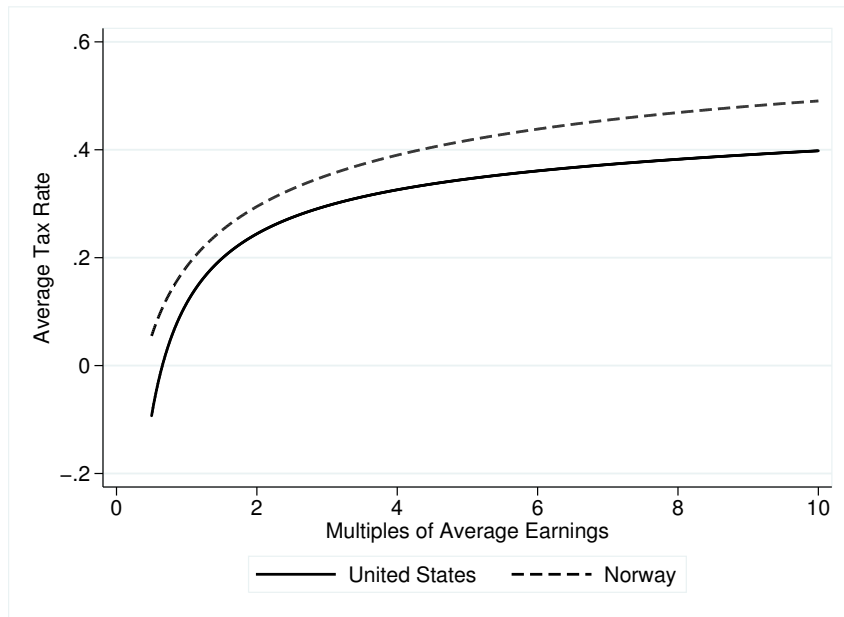
For both the U.S. and Norway, I estimate a tax function of the following form:

$$\tau(\hat{y}) = \beta_0 + \beta_1\hat{y} + \beta_2\hat{y}^{\beta_3} \quad (1.8)$$

where \hat{y} denotes annualized individual earnings relative to the average annual earnings in

that country, and $\tau(\hat{y})$ denotes the *net average tax rate* paid by an individual with relative earnings \hat{y} .⁹ This form has recently been employed for similar use in cross-country quantitative analysis by Guvenen, Kuruscu, and Ozkan (2011), and builds on earlier use of the isoelastic form by Bénabou (2000). The nonlinear least squares regression results are reported in Table 1.2 and the estimated tax functions are plotted in Figure 1.4. For full details of the estimation procedure, see Appendix B. It is worth noting that net average tax rates (both in the raw OECD data and resulting from the estimated tax functions) are actually negative for some individuals with very low earnings. This is due to features of the tax code (such as the earned income tax credit in the U.S.) which result in some individuals receiving transfers from the government that are larger than the taxes they pay.

Figure 1.4: Estimated Average Tax Functions



⁹The OECD reports average annual wage earnings in each country and normalizes all tax calculations relative to these numbers. Hence, I do the same. For the year 2000, these amounts were 33,129 (in U.S. dollars) for the U.S. and 298,385 (in Norwegian Kroner) for Norway.

Public Education Expenditures

Compulsory Stage

In defining the model, I allowed for government spending to be a function of the family state vector (h_p, α, ζ) . However, public education expenditures are generally not conditioned on child-specific characteristics such as ability and tastes for schooling.¹⁰ For this reason, and in order to utilize the data discussed in Section 1.3, I assume that government spending on compulsory education depends only on parent human capital, not child ability or tastes for school. Since parent income is proportional to parent human capital in the model, I can use the data shown in Figure 1.2 to estimate children's public education expenditure as a function of their parent's human capital via the following form:

$$\hat{g}_1(\hat{y}_p) = a_1 + b_1\hat{y}_p. \quad (1.9)$$

where, as before, \hat{g}_1 and \hat{y}_p indicate that those variables are normalized with respect to average wage earnings in the respective economy. This ensures that $\hat{g}_1(\hat{y}_p)$ in the model does not depend on the units (e.g., U.S. dollars or Norwegian kroner) in which income and education spending are measured in the data.

Equation (1.9) is estimated by ordinary least squares regression (weighted by the number of students in each school district) for both the U.S. and Norway using the data from the year 2000.¹¹ Table 1.3 provides the estimated parameters for each country. Consistent with the observations made regarding Figure 1.2 earlier, two points should be noted. First, the intercept term a_1 is more than twice as large for Norway as for the U.S., indicating that individuals at the bottom of the earnings distribution in Norway receive much

¹⁰Notable exceptions where public education spending might depend on a child's ability or tastes for schooling include public charter or magnet schools, special education and gifted education programs, etc. However, these are a small fraction of overall public schooling; therefore this paper abstracts from these special schools and programs.

¹¹As noted in Appendix A, the Norwegian data for 2000 are missing many observations; nevertheless, the estimated parameters for Norway in 2000 and 2002 are remarkably similar. Therefore, I use the estimated parameters for 2000 in computation in order to be consistent with the U.S. year 2000 estimation.

Table 1.3: Parameters for $\hat{g}_1(\hat{y}_p)$

Parameter	United States	Norway
a_1	0.096 (0.00002)	0.210 (0.0003)
b_1	0.051 (0.00003)	-0.121 (0.0004)
R^2	0.08	0.22

greater public education funding (as a share of that country's average wage earnings) than would individuals at the bottom of the U.S. earnings distribution. Additionally, the fact that b_1 is positive for the U.S. and negative for Norway shows that public expenditures will be increasing with respect to parent earnings (i.e., with respect to parent human capital) in the U.S. but decreasing in Norway.¹²

Non-compulsory Stage

Now consider public investment in non-compulsory education. As discussed in Section 1.3 public colleges and universities in the U.S. are subsidized by tax dollars, so the list price of tuition is lower than would prevail without such subsidies. In addition, students are eligible for federal and state grants and loans based on their financial need. Students who apply for public financial aid submit detailed financial information including their parents' income and assets. Based on that information an amount called Expected Family Contribution (EFC) is computed. EFC is the amount that the government expects a student's family to

¹²Due to the linear form of $\hat{g}_1(\hat{y}_p)$, it is possible that households at the extreme ends of the earnings distribution may receive unreasonably high or low (or possibly even negative, in the case of Norway) amounts of public education funding. To avoid this problem in computation, I bound $\hat{g}_1(\hat{y}_p)$ below and above by the 1st and 99th percentiles of public education expenditures in the data. As a share of average earnings, these limits are approximately 0.08 and 0.31 for the U.S., and 0.11 and 0.25 for Norway. As noted in Section 1.3, these limits confirm that the variance of public education spending is much larger in the U.S. than Norway.

Table 1.4: Parameters for $\text{EFC}(\hat{y}_p)$

Parameter	United States
a_2	-0.093 (0.010)
b_2	0.226 (0.007)
R^2	0.986

provide out-of-pocket for the student's post-secondary education expenses.¹³ I use the EFC concept to discipline the public subsidy for higher education, $\theta(\mathbf{x})$, by assuming that EFC is the fraction of higher education expenses remaining after the government subsidy.

Because this model does not include assets, the actual EFC formula can not be included directly. However, as pointed out by Belley, Frenette, and Lochner (2011), assets only play a minor role in the calculation of EFC, so abstracting from asset holdings should not seriously affect the estimation. Fortunately, the U.S. National Center for Education Statistics (NCES) publishes data on the average EFC for families of various income levels, which I use to estimate the following relationship between average EFC and parent earnings:

$$\text{EFC}(\hat{y}_p) = a_2 + b_2(\hat{y}_p) \quad (1.10)$$

where, as before, \hat{y}_p is parent income relative to average wage earnings. Average EFC is normalized by the average 4-year public university cost, so the left hand side of equation (1.10) should be interpreted as the average *share* of total public university costs that a parent with relative earnings \hat{y}_p would be expected to pay for her child's college education. The estimated coefficients for equation (1.10) are shown in Table 1.4. The key point to take

¹³The EFC concept has also been utilized recently by Brown, Scholz, and Seshadri (2012). Whereas they use it to determine which families are potentially borrowing constrained in financing college education for their children, I take EFC as a proxy for the average private share of total higher education expenses.

from those estimates is that the coefficient on parent income is positive, so parents with higher income must pay a larger share of their child's college education expenses.

With this linear formulation, I need to bound the government subsidy above and below, so I utilize the following in computation:

$$g_2(\hat{y}_p, s_2) = \begin{cases} 0, & \text{if } \text{EFC}(\hat{y}_p) \geq 1 \\ (1 - \text{EFC}(\hat{y}_p))s_2, & \text{if } \text{EFC}(\hat{y}_p) \in (0, 1) \\ \bar{g}_2, & \text{if } \text{EFC}(\hat{y}_p) \leq 0 \end{cases}$$

where \bar{g}_2 is the maximum government subsidy, which prevents individuals with low earnings in the model from getting infinitely large government subsidies. To summarize this formulation in words, parents with relative earnings \hat{y}_p choose total education spending s_2 for their child, receive public subsidy $g_2(\hat{y}_p, s_2)$, and pay the remaining share of the total cost $\text{EFC}(\hat{y}_p)s_2$. The maximum public subsidy \bar{g}_2 is set equal to the average public university tuition in the United States. According to the NCES, that amount was \$7,586 for the 2000-2001 school year, or about 23% of average annual wage earnings. A four year degree, therefore, costs about 92% of average annual earnings.¹⁴

Norwegian public subsidies for non-compulsory are much simpler to specify because higher education in Norway is essentially free.¹⁵ As mentioned earlier, 96.3% of tertiary education expenditures in Norway for the year 2000 came from public sources and only 3.7% from private sources. Therefore, I assume that the share of total higher education costs paid by the government in the case of Norway is constant at 0.963 for all individuals independent of parental income. As in the case of the U.S., I constrain the government subsidy function for Norway to the interval $[0, \bar{g}_2]$, where \bar{g}_2 is the same as in the U.S. case described above, so that parents can not extract arbitrarily large public subsidies.

¹⁴Since a model period is 30 years, "annual earnings" in the model are 1/30 of period earnings. Annualizing \bar{g}_2 , therefore, implies $\bar{g}_2 = 0.92(\frac{1}{30})\bar{y} \approx 0.03\bar{y}$, where \bar{y} is average earnings.

¹⁵A complete description of the organization and funding of the education system in Norway is available at: http://eacea.ec.europa.eu/education/eurydice/eurybase_en.php.

Selection of Remaining Parameters

Having estimated functions for taxes and public education spending, I now discuss calibration of the utility function, the human capital production function, and the stochastic processes for initial ability, tastes for schooling, and market luck shocks. The utility function is assumed to exhibit constant relative risk aversion and is given by $u(c) = \frac{c^{(1-\sigma)} - 1}{1 - \sigma}$ with discount factor β . I also assume that ability is transmitted across generations according to a first-order autoregressive process:

$$\ln \alpha_{it} = \rho_{\alpha} \ln \alpha_{i,t-1} + \varepsilon_{it}, \quad (1.11)$$

where $\varepsilon_{it} \sim N(0, \sigma_{\alpha}^2)$, α_{it} and $\alpha_{i,t-1}$ denote the ability of the individual in family i born in periods t and $t - 1$, and ρ_{α} determines the persistence of ability across generations. And as stated previously, the shocks to schooling tastes ζ and market luck η are distributed according to $\zeta \sim N(0, \sigma_{\zeta}^2)$ and $\ln(\eta) \sim N(0, \sigma_{\eta}^2)$. Hence, the remaining parameters to be chosen are σ , β , ρ_{α} , σ_{α} , ν , γ_1 , γ_2 , ϕ_1 , σ_{η} , and σ_{ζ} . Table 1.5 summarizes the benchmark parameter values and the remainder of the section discusses their selection.

First, σ and ϕ_1 are chosen prior to solving the model. I set $\sigma = 1$, which implies $u(c) = \log(c)$. The fraction of time spent in compulsory schooling is $\phi_1 = 0.345$. This corresponds to 10.4 actual years of compulsory education, which is the average number of years of compulsory schooling across U.S. states in 2000.¹⁶ The remaining eight parameters β , ρ_{α} , σ_{α} , ν , γ_1 , γ_2 , σ_{η} , and σ_{ζ} are jointly calibrated to minimize a quadratic loss function so that the model replicates relevant statistics from U.S. data.

The targeted statistics are chosen so that the model captures salient features of both the earnings and education distributions. Data targets related to the earnings distribution are the intergenerational earnings elasticity, the pre-tax Gini coefficient, and the share of

¹⁶Calculated from data in Table 165 in the 2008 Digest of Education Statistics.

Table 1.5: Summary of Benchmark Model Parameters

Independently Chosen Parameters			
Description	Parameter	Value	Target
Coefficient of relative risk aversion	σ	1	$u(c) = \log(c)$
Fraction of time in compulsory schooling	ϕ_1	0.35	Calculated from U.S. Digest of Education Statistics
Jointly Calibrated Parameters			
Description	Parameter	Value	Target
Generational time discounting (altruism)	β	0.94 ³⁰	Private share of total education spending = 0.327
Persistence in AR(1) process for α	ρ_α	0.71	Intergenerational earnings elasticity = 0.47
Std dev of noise in AR(1) process for α	σ_α	0.36	Pre-tax Gini coefficient = 0.44
Elasticity of HC output w.r.t. inputs	ν	0.60	Average years of schooling = 13.1
Exponent on compulsory HC production	γ_1	0.51	High school completion rate = 0.841
Exponent on non-compulsory HC production	γ_2	0.76	College completion rate = 0.256
Std dev of market luck shocks	σ_η	0.53	Share of earnings variance post-schooling = 0.385
Std dev of taste for schooling shocks	σ_ζ	0.25	Mincer returns to schooling = 0.10

Notes: Data for the private share of total U.S. education spending is from Table 3B.1 in OECD *Education at a Glance 2012*. The intergenerational earnings elasticity is taken from Corak (2006). The Gini coefficient is an OECD estimate before taxes and transfers, for the working age population (18-65) around the year 2000, and is taken from <http://stats.oecd.org/>. Average years of schooling is the author's calculation from 2000 U.S. Census data. High school and college completion rates are from National Center for Education Statistics *Digest of Education Statistics*. The share of earnings variance due to post-schooling factors is based on the finding of Huggett, Ventura, and Yaron (2011) that the fraction of variance in lifetime earnings due to conditions before age 23 is 0.615. Finally, the target of 10% Mincer returns per year of schooling is approximately the average return as estimated by many studies of the U.S. since 1950, as surveyed, for example, in Heckman, Lochner, and Todd (2006).

lifetime earnings variance due to post-schooling factors.¹⁷ Education related statistics are the private share of total education spending, average years of schooling, high school and college completion rates, and the Mincer returns for an additional year of schooling.

While there are no direct one-to-one mappings between the parameters and moments above, the target statistics are justified as follows. First, β controls time discounting (altruism) across generations, i.e., how much the current generation values the utility of future generations relative to own utility. Parents affect the income and consumption of future generations by investing in education in the current period, so β primarily impacts the amount of private education spending in equilibrium. Parameter ρ_α determines how persistent the transmission of ability is across generations, which in turn affects how persistent human capital, and thus earnings, is across generations. Parameter σ_α determines the variance of the ability distribution in the population, which affects earnings dispersion. Parameter ν affects the relative importance of time versus expenditures as inputs to human capital production, and so is intended to target the average time spent in school. Parameters γ_1 and γ_2 determine the returns to compulsory and non-compulsory education, and so affect the share of individuals in the population completing high school and college education. Parameter σ_η determines the variance of market luck shocks, which transforms human capital from childhood to parenthood according to $h'_p = \eta h_c$. Thus, σ_η effectively controls how much of the variance in lifetime earnings is due to post-schooling shocks relative to the differences already present at the time of labor market entry. Finally, σ_ζ controls the variance of schooling taste shocks in the population, but indirectly it also affects the average return to an additional year of schooling by ensuring that time spent in school is not perfectly correlated with learning ability. Idiosyncratic tastes for schooling result in some higher ability children spending less time in school, while some lower ability chil-

¹⁷I focus on *pre-tax* earnings inequality rather than *post-tax* because I want to uncover differences due only to public education financing systems, not due to other social programs which re-distribute income among the population. Such programs are likely responsible for large cross-country differences in post-tax income inequality and consumption inequality, but they are not the focus of this paper.

dren spending more time in school. Hence, increasing the variance of taste shocks lowers the average return to an additional year of schooling.

1.6 Results

Benchmark Model Fit

Before using the model to conduct experiments and examine the implications of alternative policies, it must be verified that the model replicates relevant features of the U.S. economy. Table 1.6 shows that the model matches well the features of the earnings and education distributions that were targeted in calibration. Regarding the earnings distribution, the two main statistics of interest are the intergenerational earnings elasticity and earnings inequality. The model predicts an elasticity of parent-child earnings of 0.475 and pre-tax Gini coefficient of 0.440, both of which are very close to their targeted values. Moving down the table, one sees that 34% of total education spending in the model is private, which is also quite close to the data value of 32.7%. A statistic that the model does not match quite as well is the average share of children's time endowment spent in school, which is somewhat higher in the model than the data. The model value of 0.493 corresponds to 14.8 years of schooling, whereas the data value of 0.437 corresponds to only 13.1 years of schooling. However, the next two rows indicate that despite overestimating the average years of schooling, the model nonetheless matches the completion rates for high school and college very well. In the model, 86.1% of individuals complete high school, compared to 84.1% in the data. Likewise, the college completion rate of 25.8% is only slightly above the data value of 25.6%.¹⁸ The share of earnings variance due to factors after schooling completion in the data is 0.385, and the model value is 0.394. Finally, I target Mincer returns of 10% per year of schooling, and the model predicts annual returns of 9.7%.

¹⁸The reason that the model matches the high school and college completion rates while overestimating the mean years of school is that data for years of schooling is lumpy, i.e., there are spikes in the distribution around high school and college graduation points. The model has no feature to generate such a lumpy distribution.

Table 1.6: Benchmark Model Fit

Target	Data	Model
Intergenerational earnings elasticity	0.470	0.475
Gini coefficient (pre-tax)	0.440	0.440
Private share of total education spending	0.327	0.340
Average share of time in school	0.437	0.493
High school completion rate	0.841	0.861
College completion rate	0.256	0.258
Share of earnings variance post-schooling	0.385	0.394
Mincer returns to additional year of schooling	0.100	0.097

Overall, the model matches nicely the data moments targeted in calibration. But in fact the model also makes predictions for other important, non-targeted statistics. For instance, the model is not constructed to match the division of private education spending between compulsory and non-compulsory levels. Despite this, it actually matches well the fact that much of the private education spending in the U.S. occurs at the college and university level. According to the OECD, only 8.6% of primary and secondary education spending in the U.S. is private. This is quite comparable to the model, where in equilibrium 12.7% of compulsory stage expenditures are privately funded. OECD data also indicates that 68.9% of all tertiary level expenditures in the U.S. are private, and in the model 53.2% of non-compulsory stage expenditures are private. Thus, while the model targeted only the aggregate level of private spending, it nonetheless replicates an important pattern in the U.S. data in that most private spending is for higher education. I conclude that the private education spending decisions of model households accurately represent the actual decisions made by U.S. households. It is important that the model is correct along this dimension because I show later that cross-country differences in private spending are critical.

Quantitative Effects of Taxes and Public Education Expenditures

The goal of the main computational exercise is to determine how much of the observed differences in earnings inequality and intergenerational persistence between the U.S. and Norway in the data is due to the previously discussed differences in taxes and public education systems. Toward this end, I first compute the stationary recursive equilibrium of the benchmark U.S. economy calibrated above. I then compute the stationary equilibrium in three counterfactual economies. The first counterfactual assumes that the U.S. adopts only the Norwegian public education spending, i.e., I replace the functions $g_1(\cdot)$ and $g_2(\cdot)$ for the U.S. with those for Norway. Any differences in equilibrium earnings inequality and intergenerational persistence generated by this experiment will be attributable to the different public education expenditures. The second counterfactual assumes that the U.S. keeps its own public education system while adopting the Norwegian tax function, i.e., I replace $\tau(\cdot)$ for the U.S. with that for Norway. This experiment reveals the marginal effects of the tax system on earnings inequality and intergenerational earnings persistence. The final counterfactual assumes the U.S. adopts both the Norwegian public education and tax functions.

Table 1.7 shows results for these exercises. The first row shows again that the benchmark calibration replicates the U.S. intergenerational earnings elasticity and earnings inequality (as measured by the pre-tax Gini coefficient). The second row shows that the model generates a decrease in the intergenerational earnings elasticity from 0.475 to 0.441 when the U.S. public education functions are replaced by those for Norway. Additionally, earnings inequality as measured by the pre-tax Gini coefficient declines from 0.440 to 0.434. The marginal effects of the tax system are shown in the third row, where intergenerational earnings mobility falls from 0.475 to 0.471, and the Gini coefficient declines from 0.440 to 0.430. Finally, the fourth row shows the combined effects if the U.S. were to

Table 1.7: Earnings Distribution Statistics for Various Education and Tax Systems

Education System	Tax System	Intergenerational earnings elasticity		Pre-tax Gini coefficient	
		Data	Model	Data	Model
U.S.	U.S.	0.470	0.475	0.440	0.440
Norway	U.S.	-	0.441	-	0.434
U.S.	Norway	-	0.471	-	0.430
Norway	Norway	0.170	0.444	0.377	0.431

Notes: Data estimates of earnings elasticities are from Corak (2006) and pre-tax Gini coefficients are from the OECD.

adopt both Norwegian taxes and public education functions. The intergenerational earnings elasticity declines to 0.444, and the Gini coefficient declines to 0.431.

The change in intergenerational earnings elasticity between the benchmark U.S. economy and counterfactual economy with both Norwegian taxes and public education spending represents 10.6% of the observed difference between the U.S. and Norway in the data, while the decline in the Gini coefficient is 14.3% of the data difference.¹⁹ Importantly, most of the decrease in the intergenerational earnings elasticity is due to changing the public education spending functions, whereas most of the decrease in the Gini coefficient is due to changing the tax system. From these experiments I conclude that the redistribution of public education spending has a larger impact on intergenerational earnings mobility, while progressivity of the tax system has a larger impact on earnings inequality. It is also worth emphasizing that, as in Guvenen, Kuruscu, and Ozkan (2011), general equilibrium effects of the tax and transfer system are borne out even in pre-tax earnings. This result stems from the fact that progressivity in the tax function provides a disincentive to invest

¹⁹The share of intergenerational earnings elasticity accounted for by the model is calculated as $\frac{\Delta_{\text{model}}}{\Delta_{\text{data}}} = \frac{0.475-0.444}{0.47-0.17} = 0.106$. Similarly, the share of earnings inequality accounted for by the model is calculated as $\frac{\Delta_{\text{model}}}{\Delta_{\text{data}}} = \frac{0.440-0.431}{0.44-0.377} = 0.143$

in human capital. This is especially true for the highest ability individuals who would experience the largest earnings gains from investing in education, and therefore also face the largest tax penalties.

Recall from Section 1.3 that average public spending per student is nearly identical in the U.S. and Norwegian data. Because there are not large differences in the average level of public spending between the two countries, it is actually the *distribution* of public education spending across individuals driving these results. To confirm this, I conduct an additional experiment by taking the benchmark U.S. model and changing only the compulsory public education spending function g_1 to the Norwegian version, leaving the U.S. tax system and subsidies for non-compulsory education the same. With this change, the average amount of public education spending (which is endogenous because it depends on average model income) decreases by only 0.2%.²⁰ Thus, the average level of public spending is essentially unchanged, but the distribution of spending across individuals shifts substantially. Low income children now receive much more public spending, while high income children receive much less. In this experiment the intergenerational earnings elasticity falls from 0.475 to 0.448. Compare this to the last row of Table 1.7 where Norwegian tax and non-compulsory education subsidies were also changed, and the intergenerational earnings elasticity fell to 0.444. Similarly, in this experiment the pre-tax Gini coefficient decreases to 0.436, compared to 0.431 when the tax and non-compulsory education subsidies also change to the Norwegian versions. To summarize, nearly 90% of the model difference in intergenerational earnings persistence, and nearly half of the model difference in pre-tax Gini coefficients is due only to changing the distribution of compulsory level public education spending.

This confirms an important result from existing research including Restuccia and Urrutia (2004) and Holter (2012). These papers examine differences in average public

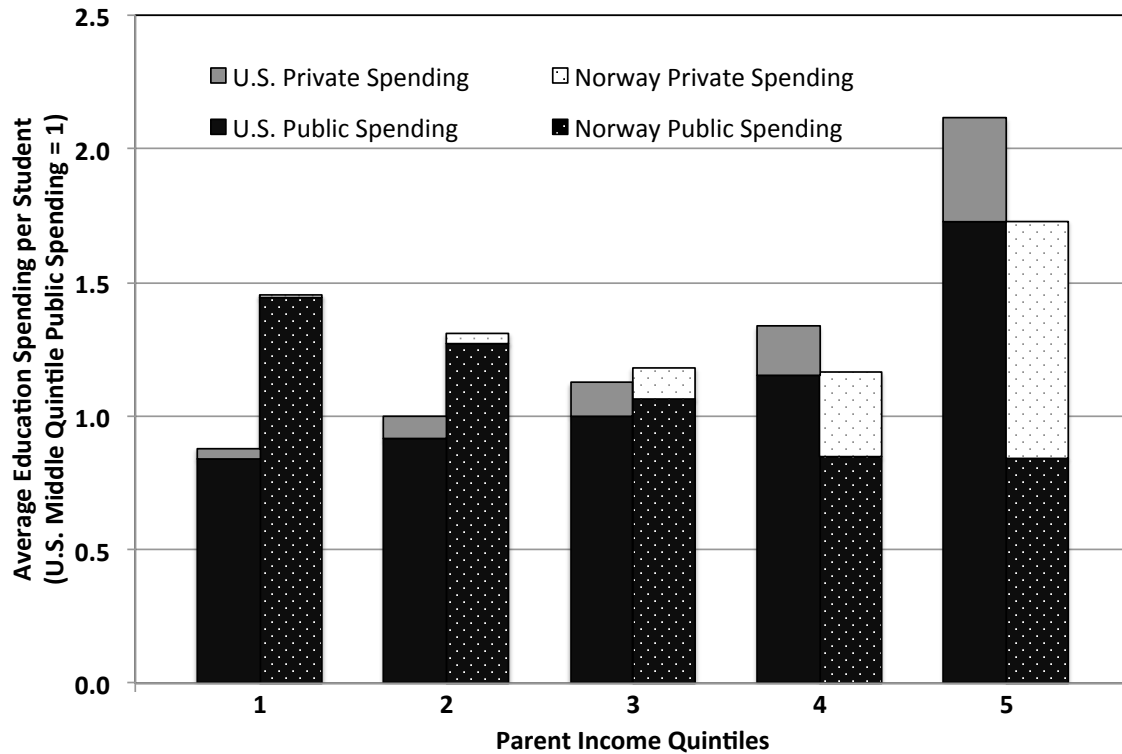
²⁰Again, this model prediction is consistent with the data from Section 1.3 showing that average public spending per student in compulsory education is nearly identical for the U.S. and Norway.

spending per student and find that additional public spending on early education has a larger impact on intergenerational earnings persistence than additional public subsidies for higher education. While the change in average public spending per student in my experiment is essentially zero, I still find that the distribution of public spending on earlier (compulsory) education is quantitatively much more important than changes in public subsidies for non-compulsory education. As in their models, this result is due to the dynamic complementarity between stages of education. That is, human capital production in non-compulsory education depends on the human capital produced during compulsory education, so redistribution of public expenditures has greater impact on earnings mobility when it is targeted toward earlier stages of education.

The Private Response to Public Policy Change

Finally, I want to emphasize the importance of the private education spending for these results. Figure 1.5 depicts the average public and private spending on compulsory level education by parent income quintiles in the benchmark U.S. version of the model and the counterfactual model with Norwegian taxes and education systems (the results for which were shown in the first and last rows of Table 1.7). By construction, public spending is increasing with respect to parent income in the U.S. version, but decreasing in the Norwegian version. As discussed before, because average public expenditures per student are nearly identical in the two economies, children of parents near the mean income level receive public education spending that is very similar in the two countries. However, in moving from the U.S. to Norwegian systems, children of low income parents see large increases in their public education spending, while children of high income parents see large decreases. The model predicts that high income parents, therefore, increase their private spending to offset the decrease in public funds. This behavior serves to dampen the impact that changes in public policy may have on earnings inequality and intergenerational persistence. Additionally, a second factor dampening the impact of such a policy change is that spending

Figure 1.5: Public and Private Education Spending in Benchmark Model Exercise



redistributed from children of high income parent to children of low income parents will be less effective in producing human capital, on average. The reason is that low income parents will have lower average learning ability, α , and because learning ability is correlated across generations, then children of low income parents will also tend to have lower average learning ability.

While the benchmark model matches the private share of education spending in the U.S. education system by construction, the private education spending in the counterfactual exercise with Norwegian public education and tax systems is higher than in the data. According to the OECD, fully 99% of spending on primary and secondary education in Norway is publicly provided. One reason for this is that private schools in Norway are subject to government approval, and those approved have 85% of their expenses covered

by the government.²¹ Hence, a simple and reasonable way to make the model more accurately reflect the Norwegian education system is to limit private education spending. By exogenously restricting private spending on compulsory education to zero and limiting private spending on non-compulsory education to levels matching the Norwegian data (5%, as compared to nearly 33% in the U.S.), the intergenerational earnings elasticity decreases to 0.34, and the pre-tax Gini coefficient decreases to 0.35. In other words, this model accounts for nearly half of the cross-country difference in intergenerational earnings persistence and 150% of the difference in earnings inequality when private education spending is restricted to reflect the laws in Norway. The reason this policy change has such a large impact is that the restriction primarily impacts the education spending for children of high income parents who, as discussed above, have higher average learning ability and therefore turn dollars into human capital more efficiently. This suggests that cross-country differences in private education spending are perhaps even more important than public education spending for understanding the cross-country differences in earnings distributions.

1.7 Discussion

Having presented the benchmark model results, this section discusses other potentially important differences between the U.S. and Norwegian economies that may also help explain the residual differences in earnings inequality and intergenerational earnings persistence which are unaccounted for by public education spending and taxation policies.

The Role of Time and Goods in Human Capital Production

Human capital production functions take many forms in the economics literature. Some authors, such as Guvenen, Kuruscu, and Ozkan (2011), model human capital production requiring only the individual's effective time, i.e., the product of time and the current human

²¹According to the European Commission's *National System Overview of Education Systems in Europe*, 2011 Edition.

Table 1.8: Results from Varying the Relative Weight of Time vs. Goods in Education

ν	Intergenerational earnings elasticity			Pre-tax Gini coefficient		
	U.S.	Norway	$\frac{\Delta_{model}}{\Delta_{data}}$	U.S.	Norway	$\frac{\Delta_{model}}{\Delta_{data}}$
0.2	0.573	0.544	0.097	0.531	0.508	0.375
0.3	0.553	0.520	0.110	0.522	0.494	0.444
0.4	0.522	0.488	0.112	0.488	0.465	0.370
0.5	0.495	0.467	0.095	0.461	0.448	0.218
0.6	0.475	0.444	0.106	0.440	0.431	0.143
0.7	0.456	0.423	0.110	0.421	0.407	0.208
0.8	0.445	0.413	0.106	0.410	0.399	0.160

capital stock. Other authors, such as Restuccia and Urrutia (2004), model human capital production with goods as the only input. In this paper, I have chosen a human capital production function requiring the both individual's time and goods. I now ask whether the relative weight of time versus goods is important for the cross-country accounting exercise. To answer this, I vary the parameter ν in both the U.S. and Norwegian versions of the economy from the benchmark value of 0.6 to a low of 0.2 and a high of 0.8. At lower values for ν , goods are more important for human capital production, and at higher values of ν goods become less important. Table 1.8 reports the results of these experiments.

Consider first the effect on intergenerational earnings elasticity. As seen in Table 1.8, the model accounts for about 10 – 11% of the data difference in intergenerational earnings elasticity, regardless of the value of ν . To understand why this is true, notice that when ν is lower the child's time is relatively less important and goods are relatively more important for human capital production. While this implies that public investments are more effective for improving the earnings prospects of low income, high ability children, it also means that parent's private education investments are also more effective. As a

result, I find that as ν decreases, the private share of total education spending increases, particularly among higher income families. Thus, for all values of ν , moving from the U.S. to the Norwegian public education financing system has essentially the same effect – low income, high ability children are somewhat more likely to rise within the earnings distribution, but increased private education spending by high income parents means that high income, low ability children are less likely to move down the income distribution. Thus, cross-country exercise of accounting for intergenerational income mobility is largely unaffected by ν .

Turning to the effects of ν on earnings inequality, there is a different pattern. As ν decreases the model is able to generate more of the difference in pre-tax Gini coefficients seen in the data. Why is this the case? As discussed above, when ν decreases goods become relatively more important in human capital production, so private education spending increases. More importantly for understanding inequality, though, is that the *variance* of private education spending increases, which means the variance of total education spending also increases. This leads to greater earnings inequality in both countries, but the effect is smaller in the Norwegian version of the economy because average and marginal tax rates are higher, which serves as a disincentive for private investment in human capital. Thus, the pre-tax Gini coefficient increases by a smaller amount with Norwegian taxes.

I draw two main conclusions from these experiments. First, the model results do not seem particularly sensitive to ν , and if anything the benchmark results may be a lower bound for the share of earnings inequality accounted for by the model. Second, the magnitude of private education spending is a major difference between the U.S. and Norway, and once again the option of private spending is shown to dampen what might otherwise be large cross-country differences in the model. This underscores the earlier point that understanding private education spending differences is a promising avenue for future research in accounting for cross-country differences in earnings inequality and persistence.

Cross-Country Differences in Variance of Idiosyncratic Earnings Shocks

In this model there are three sources of earnings variance: variance in learning ability, variance in education investments, and the idiosyncratic earnings shocks denoted by η . The main exercise assumes equal variance of the idiosyncratic shocks in both the U.S. and Norway; however, this may be an invalid assumption due to cross-country differences in labor market factors such as employment protection policies or unionization rates. For example, the OECD reports an employment protection index that ranges from zero to six, where higher numbers indicate that costs and procedures for dismissing workers are higher in that country. In 2008, the employment protection index for Norway was 2.7, whereas for the U.S. it was 0.7. This placed Norway among those OECD countries with the strictest employment protection laws, while the U.S. was among the least strict. Similarly, the OECD reports that in 2008, 53.3% of employees in Norway were unionized, but only 11.9% of U.S. employees were unionized. Based on statistics like these, it is reasonable to think that employees in Norway experience less uncertainty about their lifetime earnings once they complete school, which would show up in the model as lower variance of the market luck shocks. To investigate any effect this may have on the results, I incrementally decrease the variance of the market luck shock from its benchmark level in the economy with Norwegian public education spending and taxes. Table 1.9 reports the results for this experiment. As before, the first row repeats the benchmark intergenerational earnings elasticity and pre-tax Gini coefficients obtained in the Norwegian version of the economy. Subsequent rows show the effects from decreasing σ_η in 10% increments.

From Table 1.9 it is apparent that cross-country differences in post-schooling earnings shocks can not account for the remaining differences in earnings inequality and persistence. As σ_η decreases, the intergenerational earnings elasticity increases, but earnings inequality decreases. Why is this the case? First, the effect on inequality is simply me-

Table 1.9: Results from Decreasing the Variance of Market Luck Shocks

Experiment	Value of σ_η	Intergenerational earnings elasticity	Pre-tax Gini coefficient
Norway Benchmark	0.530	0.444	0.431
10% decrease in σ_η	0.477	0.480	0.422
20% decrease in σ_η	0.424	0.507	0.412
30% decrease in σ_η	0.371	0.545	0.405
40% decrease in σ_η	0.318	0.580	0.400
50% decrease in σ_η	0.265	0.616	0.396

chanical, i.e., σ_η is one source of earnings variance, so when it decreases, earnings variance decreases. The effect on intergenerational persistence is slightly more subtle. As σ_η decreases, the other two sources of earnings variance – learning ability and investments in education – become relatively more important. Since both of these are positively correlated with parent earnings, the persistence of earnings across generations increases.

This exercise is particularly notable because it runs counter to perceived wisdom that labor market institutions should help explain the strong positive correlation of earnings inequality and intergenerational persistence across countries. In summary, this result suggests that lower variance of idiosyncratic earnings shocks may help account for cross-country differences in inequality, but will not help explain why countries with lower earnings inequality also tend to have lower intergenerational earnings persistence.

Cross-Country Differences in Variance of Learning Ability

Recall that the model begins with children at age five because that is approximately the age when children in both the U.S. and Norway enter the compulsory public education system. Furthermore, the endowment of learning ability, α , is broadly interpreted to include not only innate factors, but also external influences prior to age five that affect the child's effi-

ciency in producing human capital. The benchmark quantitative exercise held the variance of α constant, but in this section I ask if cross-country differences in this variance can help account for differences in earnings inequality and intergenerational persistence.

First, though, what might cause cross-country differences in the variance of learning ability at early ages? Contrasting the U.S. and Norway, there are many reasons why the variance of learning ability may differ. For example, the variance of learning ability may be positively related to the ethnic and cultural diversity in a population. As noted by Alesina et al. (2003) and Fearon (2003), the U.S. is much more heterogeneous in this respect than Norway. In addition, differences in early childhood education may affect the distribution of learning ability upon entrance to public compulsory schooling. Again, there is evidence that differences between the U.S. and Norway are substantial along this dimension. New mothers in Norway receive paid maternal leave for nearly a year, after which time most enroll their children in publicly subsidized pre-primary educational institutions. The OECD reports that in 2010, 95% of three year olds in Norway were enrolled in early childhood education, whereas only 51% of three year olds in the U.S. were enrolled. This evidence suggests that the variance of learning ability in Norway is likely smaller than in the U.S.

How are the results affected if this is the case? To answer this question, I take the model as previously calibrated and incrementally decrease the parameter σ_α in the version with Norwegian public education spending and tax functions. Table 1.10 reports the results from these experiments. The first row repeats, for purpose of comparison, the intergenerational earnings elasticity and pre-tax Gini coefficients obtained in the benchmark Norwegian model. Subsequent rows show the effects from decreasing σ_α in 10% increments. The results indicate that even small changes in the variance of learning ability can lead to large changes in the earnings distribution.

Consider first the changes in the intergenerational earnings elasticity. As σ_α decreases, the ability distribution becomes more compressed. Then under the Norwegian

Table 1.10: Results from Decreasing the Variance of Ability

Experiment	Value of σ_α	Intergenerational earnings elasticity	Pre-tax Gini coefficient
Norway Benchmark	0.360	0.444	0.431
10% decrease in σ_α	0.324	0.387	0.378
20% decrease in σ_α	0.288	0.318	0.327
30% decrease in σ_α	0.252	0.242	0.285
40% decrease in σ_α	0.216	0.166	0.245
50% decrease in σ_α	0.180	0.093	0.222

public education system in which more funds are targeted to children in lower income households, it becomes more likely that low income, high ability children will surpass high income, low ability children in human capital production, and thus earnings. This increased “churning” decreases earnings persistence across generations.

A similar intuition can be applied toward understanding changes in the Gini coefficient. As σ_α decreases children are more similar in learning ability, so the resulting distribution of human capital and earnings exhibits less inequality. Recall that the Norwegian pre-tax Gini coefficient in the data is 0.377. This experiment indicates that just a 10% decrease in the variance of ability at age five will decrease the model’s predicted pre-tax Gini coefficient to 0.378, which is sufficient to account for essentially all of the differences in pre-tax inequality between the U.S. and Norway. Based on these exercises, it seems promising that cross-country differences in early childhood education policies may explain a large share of the remaining differences unaccounted for by public education spending and tax progressivity.

1.8 Conclusion

This paper first provided evidence for cross-country differences in public education funding at both the compulsory level and for higher education. I argued that aggregate measures of public education spending, such as average expenditures per student, do not capture all of the relevant differences in the allocation of public education spending either within or across countries. As a step toward more accurately modeling the many differences in public education financing systems, I estimated public education spending functions from financial data at the school district level and incorporated these into a dynamic general equilibrium model. In addition, I estimated functions for higher education subsidies and taxes on labor earnings. The fundamental question was whether differences in public education spending and taxes can help account for large differences across countries in labor earnings inequality and intergenerational earnings persistence.

In the benchmark exercise, I calibrated the model to match U.S. earnings and education statistics, then computed the change in earnings inequality and intergenerational persistence when U.S. taxes and public education financing systems were replaced with those for Norway. I found that about 15% of differences in earnings inequality and about 10% of differences in intergenerational earnings persistence are due to differences in public education spending and taxes. In contrast to an existing literature which has focused on cross-country differences in average public spending per student, the earnings distribution changes in this model are primarily due to changing the distribution of public education expenditures across students, rather than changing the average level of spending per student. Furthermore, I showed that the ability of high income parents to spend privately on education for their children counteracts the redistribution of public funds from higher to lower income families, thus reducing the impact of such a policy change.

Note that the model employed in this paper was simplified along some dimensions to maintain tractability. I close by discussing some caveats related to these simplifying assumptions, and discuss implications for future work. First, the parameter α has been referred to as “learning ability,” but in a reduced form way it captures all factors which affect a child’s productivity in turning educational inputs (time and money) into educational outcomes (human capital). The factors affecting α may include internal elements such as the child’s genetic, biological, or psychological make-up, but also may include external influences such as parental and peer effects. To the extent that there may exist cross-country differences in these and other factors, the mean and variance of α may also differ. As demonstrated in Section 1.7, these differences can have important implications for labor market outcomes, so future work should consider more carefully how “learning ability” is produced during early childhood and subsequently may evolve over the life-cycle.

Second, the two period life-cycle structure imposes some limitations. For one, the single period budget constraint for the family implies that income earned by a child after the completion of schooling can be used to contribute to the family’s current consumption and the child’s own schooling costs. This also implies that parents have an additional incentive to invest in their children’s education beyond the pure altruism built into the preferences. Additionally, the two-period structure limits the realism of the idiosyncratic shocks to earnings. In this setup, there is a single shock to earnings, but a life-cycle structure with more refined periods would allow for a richer wage process, potentially including both permanent and transitory shock components. As discussed in Section 1.7, idiosyncratic earnings shocks likely differ across countries due to public policies and labor market structures. An interesting extension of this work would allow for shorter periods and examine whether the conclusions of Section 1.7 are confirmed under a more refined wage process.

Chapter 2

FACTORS AFFECTING COLLEGE COMPLETION AND STUDENT ABILITY IN THE U.S. SINCE 1900

2.1 Introduction

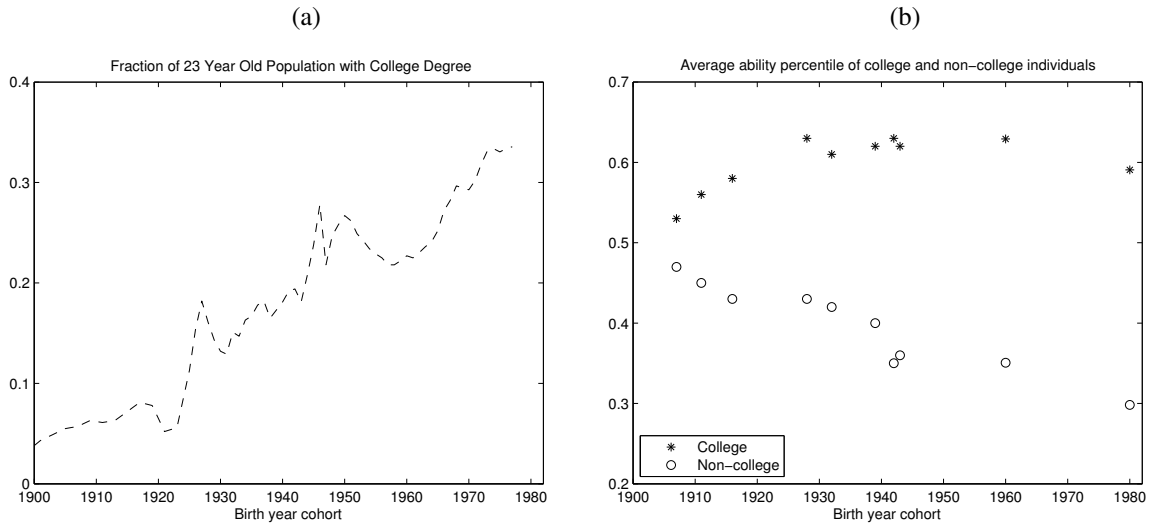
The twentieth century saw a dramatic expansion of higher education in the United States. Among those in the 1900 birth cohort, less than 4% held a bachelor's or first professional degree at age 23, but by the 1970 birth cohort this share had risen to more than 30%. Panel (a) of Figure 2.1 plots this series for all cohorts from 1900 through 1977.¹ Concurrent with the increase in college attendance, the ability gap widened substantially between college students and those individuals with a high school degree and no college experience, i.e., “non-college” individuals. This pattern is seen in Panel (b) of Figure 2.1, which plots the average IQ percentile (a proxy for “ability”) of college and non-college individuals.² For example the average college student born in 1907 had an IQ in the 53rd percentile, very close to the average non-college individual whose IQ was in the 47th percentile. Yet over the next several decades, the average IQ percentile increased among college enrollees and decreased among those with only a high school degree. Most intriguing is that this trend of increased ability sorting occurred even as the share of students attempting college grew steadily larger.

The goal of this paper is to understand the causes of these two empirical trends. However, this task is complicated by the vast number of changes in both the aggregate economy and education sector over this time period. I combat this by developing an overlapping generations lifecycle model populated by high school graduates who are hetero-

¹The 1977 cohort was 23 years old in 2000 when this data series ends. Data for cohorts born up to 1967 are taken from Snyder (1993), and from 1968 through 1977 are the authors' calculation.

²These two data trends have also been documented by other authors, including Hendricks and Schoellman (2012). In panel (b), data points for cohorts prior to 1950 are from Taubman and Wales (1972). The 1960 data point is from the NLSY79, as calculated by Hendricks and Schoellman (2012). The 1980 data point is the author's calculation based on data from the NLSY97.

Figure 2.1: College Completion and Average Student Ability in the U.S. since 1900



generous in both ability and financial assets. An important feature of the model is that individuals only see a noisy signal of their true ability when making risky decisions about college enrollment. I incorporate newly constructed data on college costs obtained from historical printed government sources. Additionally, I estimate life-cycle wage profiles for men and women in each birth-year cohort in order to accurately model the opportunity costs of wages foregone by college attendees and the education earnings premia realized by those who either complete some college or successfully graduate from college.

I calibrate parameters of the model to match the U.S. data and then conduct a series of experiments in order to understand changes in college completion and ability sorting over time. First, I find that the secular increase in high school completion is responsible for less than half of the increase in college completion over the entire time period. The remainder is due to changes in college enrollment and completion rates *conditional on high school graduation*. Interestingly, however, the key features of the model allowing us to match the data depend critically on the time period considered. For cohorts born from 1930 to 1950, I find that changes in college costs are key for generating the increase in college completion,

as they generate a large endogenous increase in college enrollment. Endogenous changes in the average ability of college students also affects college completion rates, but the impact is quantitatively much smaller.

For cohorts born after 1950, the benchmark model significantly overpredicts college completion rates in the data. I show that this is likely due to a sharp increase in the growth rate of the college earnings premium. While the college earnings premium was roughly flat for cohorts born between 1900 and 1950, the growth rate increased sharply for cohorts born after 1950. I find that modifying the model to allow for imperfect forecasting of the college wage premium improves substantially the predictions for college completion for cohorts born after 1950, while leaving the results for cohorts born before 1950 largely unaffected.

In terms of capturing increased ability sorting over time, I consistently find that changes in economic factors (i.e., earnings premia, college costs, opportunity costs, and asset endowments) have little impact. Instead, the key feature in the model that accounts for this is uncertainty about ability. I show that a decrease in the variance of ability signals can generate an increase in ability sorting similar to that in the data. I attribute this change to the increases in standardized testing which improved students knowledge of their own ability relative to other students in their cohort, as discussed in Hoxby (2009).

This paper is related to a large literature on the joint determination of enrollment changes and ability sorting, but previous work focuses almost exclusively on the post-World War II period. Lochner and Monge-Naranjo (2011) look at the role of student loan policies with limited commitment, and shows that this can generate ability sorting. My focus on an earlier time period excludes the student loan innovations they consider, so I instead investigate other factors that may be relevant in understanding ability sorting. Garriga and Keightley (2007) consider the impact of different education subsidies for enrollment and time-to-degree decisions, in a model with borrowing constraints and risky education investment. Hendricks and Leukhina (2011) consider the role of borrowing constraints and

learning in understanding the evolution of educational earnings premia. Like this paper, Altonji (1993) and Manski (1989) assume that high school students do not perfectly know their own ability, and they use this feature to investigate the role of preferences, ability, and earnings premia for enrollment and dropout. Cunha, Heckman, and Navarro (2005) extend the model developed in Willis and Rosen (1979) to include uncertain ability, and find that roughly sixty percent of the variability in returns to schooling is forecastable.

Hendricks and Schoellman (2012) study the same time period as I do, but they take data on college completion and student ability as given in order to understand changes in the college earnings premium in a complete markets model. By contrast, I seek to understand the economic factors that affected college completion and average student ability for cohorts since 1900. Perhaps most related to this paper is Castro and Coen-Pirani (2012), who ask whether educational attainment over time can be explained by earnings premia in a complete markets model. They find that it cannot. My model, with limited borrowing and uncertainty about ability, matches college attainment well for early cohorts, but shares the problem that the model overpredicts attainment after 1950 due to the increase in the earnings premia for these cohorts. In both, disregarding individuals' ability to perfectly forecast future earnings premia helps the model fit, but not entirely.

This work also relates to a number of empirical papers on the impact of different economic forces on historical post-secondary completion, including college costs and income (Campbell and Siegel, 1967), student ability (Taubman and Wales, 1972), academic quality (Kohn, Manski, and Mundel, 1976) and borrowing constraints (Hansen and Weisbrod, 1969).

2.2 Model

In this section, I develop an overlapping generations model to investigate the simultaneous trends of increasing college completion rates and increasing ability sorting between college

and non-college individuals.³ The key features include borrowing limits, uncertainty about own ability, and risky completion of college education.

Demographics and Preferences

Time in the model is discrete, and a model period is one year. Each period, N_{mt} males and N_{ft} females are born, each of whom lives for a total of T periods. Let $a = 1, 2, \dots, T$ denote age. Each individual maximizes expected lifetime consumption

$$\mathbb{E}_0 \sum_{a=1}^T \beta^{a-1} \left(\frac{c_a^{1-\sigma} - 1}{1-\sigma} \right)$$

Endowments and Signals

Individuals are ex-ante heterogeneous along three dimensions: their sex, m or f , initial asset endowment k_0 , and ability to complete college, denoted α . The probability that any individual completes his or her current year of college is given by $\pi(\alpha)$, where $\pi' > 0$. Log initial assets, $\log(k_0)$, and ability α are drawn from a joint normal distribution with correlation ρ_t , means $\mu_{\alpha,t}$ and $\mu_{k,t}$, and standard deviations $\sigma_{\alpha,t}$ and $\sigma_{k,t}$. Note that the parameters on the joint distribution for $\{\alpha, k_0\}$ are potentially time-varying.

While sex and asset endowments are perfectly observable, ability α is not. Instead, each individual receives a signal $\theta = \alpha + \varepsilon$ at the beginning of life. The error term is $\varepsilon \sim N(0, \sigma_\varepsilon^2)$. Note that because assets and ability are jointly distributed, individuals actually receive two pieces of information about ability – the signal θ and asset endowment k_0 . Let $v = (k_0, \theta)$ be the information an individual has about his true ability. After the initial college enrollment decision, ability α becomes publicly observable.

³The counterpart to ability in the data is IQ.

Education Decisions

The population I am considering consists of high school graduates, so that birth in this model translates to a high school graduation in the real world. At birth, every individual decides whether or not to enroll in college, given sex, asset endowment k_0 , and signal θ . This is the only time this decision can be made. Once enrolled in college, individuals can only exit college by graduating or failing out with annual probability $\pi(\alpha)$. After failure, individuals enter the labor force and may not re-enroll, consistent with the finality of dropout decisions discussed in Card and Lemieux (2001). Graduating college requires C years of full-time education at a cost of λ_t per year. If an individual decides to not enter college, he or she immediately enters the labor market and begins to work.

Labor Market

I adopt the common assumption that individuals of different ages, a , sex s , and education, e , are different inputs into a constant returns to scale production function that requires only labor. Therefore, wages depend on age, sex, education level, and the year. I write wages as $w_{a,t}(e, s)$ for $s \in \{f, m\}$ and $e \in \{0, 1, \dots, C\}$. While ability α has no direct effect on realized wages, it does affect expected wages because higher ability students are more likely to graduate college and earn higher wages.

Savings Market

Each individual can borrow and save at an exogenous interest rate r_t . I assume individuals must die with zeros assets, so $k_{T+1} = 0$. Borrowing is constrained to be a fraction $\gamma \in [0, 1]$ of expected discounted future earnings. Therefore, individuals must keep assets k_t each period above some threshold \bar{k} , where

$$\bar{k} = -\gamma \cdot \mathbb{E} \sum_{n=a}^{n=T} \frac{w_{n,t}}{1+r_t}$$

Note that both the expectations operator and wage can depend on a number of factors, including ability α , age a , year t , education e , and sex s . Therefore, the borrowing constraint will be written as the function $\bar{k}(\alpha, a, t, e, s)$. In a slight abuse of notation, I will write $\bar{k}(a, t, e, s)$ when the borrowing constraint does not depend on ability α , as is the case once an individual finishes college.

Timing and Recursive Problem

At the beginning of year t , N_{mt} men and N_{ft} women are born at age $a = 1$. Again, each individual is initially endowed with assets k_0 , sex s , ability α , and a signal θ of true ability. Immediately, each individual decides whether or not to enroll in college. If he or she enrolls in college, true ability is immediately realized, and the individual proceeds through college. In the case of failure (due to $\pi(\alpha)$) or graduation, he or she proceeds to the labor market and works for the remainder of his or her life. Individuals who do not enroll in college proceed directly to the labor market, where they receive the wage associated with age a , education $e = 0$, and sex s .

Recursive Problem for Worker

For individuals currently not enrolled in college, their ability is irrelevant for their decision problem. Therefore, the value of entering year t at age a with assets k , years of college education e , and sex $s \in \{f, m\}$ is:

$$\begin{aligned}
 V_{a,t}^w(k, e, s) &= u(c) + \beta V_{a+1,t+1}^w(k', e, s) \\
 s.t. \quad c + k' &= (1+r)k + w_{a,t}(e, s) \\
 k' &\geq \bar{k}(a, t, e, s) \\
 k_{T+1} &= 0
 \end{aligned}$$

Recursive Problem for College Student

If instead an individual is currently enrolled in college, he has already completed e years of his education and must pay λ_t in college costs for the current year. The probability that he passes and remains enrolled the next year, however, depends on his ability α . Recall that α is known with certainty as soon as the education decision is made, so there is no uncertainty about ability.

The value of being enrolled in college at year t at age a , with assets k , ability α , e years of education completed, and sex $s \in \{f, m\}$ is:

$$V_{a,t}^c(k, \alpha, e, s) = u(c) + \beta \left[\pi(\alpha) V_{a+1,t+1}^c(k', \alpha, e+1, s) + (1 - \pi(\alpha)) V_{a+1,t+1}^w(k', \alpha, e, s) \right]$$

subject to

$$\begin{aligned} c + k' - \lambda_t &= (1+r)k \\ k' &\geq \bar{k}(\alpha, a, t, e, s) \\ \pi(\alpha) &= 0 \quad \text{if } a = C \quad \forall \alpha \end{aligned}$$

The last restriction simply states that if $a = C$, that individual is graduating college and cannot acquire any more years of college education.

The College Enrollment Decision

Given the value of being enrolled in college and working, it is possible then to define the educational decision rule at the beginning of life. Recall that at this point, α is unknown, but each individual receives a signal $v = (k_0, \theta)$. Each individual then constructs beliefs over possible ability levels by using Bayes' Rule.

Let $F(\alpha; k_0, \theta)$ be the cumulative distribution function of beliefs (as defined by Bayes' Rule) over ability levels. Given all this, an individual born in year t of sex s with

assets k_0 and signal θ enters college if and only if the expected value of entering college is higher than the (certain) value of entering the workforce. This is given by the inequality

$$\int_{\alpha} V_{1,t}^c(k_0, \alpha, 1, s) F(d\alpha; k_0, \theta) \geq V_{1,t}^w(k_0, 0, s) \quad (2.1)$$

2.3 Calibration

The goal of this paper is to assess the role played by a number of features of the economy in understanding ability sorting and college enrollment over time. I therefore take a multi-faceted approach to parameterizing the model. First, I construct historical data series for N_{mt} , N_{ft} , and λ_t , which are incorporated directly into the model. Second, I estimate life-cycle wage profiles $w_{a,t}(e, s)$, which are taken as given by model individuals solving their dynamic problem. Third, I exogenously choose values for T , C , r_t , β , ρ_t , $\mu_{\alpha,t}$, $\mu_{k,t}$, $\sigma_{\alpha,t}$, $\sigma_{k,t}$, and $\pi(\alpha)$. Finally, I calibrate $\sigma_{\varepsilon,t}$, and γ in order to match important features of the time series data. Each of these are discussed in more detail below.

Historical Time Series Data

As previously mentioned, N_{mt} males and N_{ft} females are “born” into the model each year, meaning they graduate high school and enter the model eligible to make college enrollment decisions. I take high school completion, and thus the population of potential college enrollees, as exogenous. The series for N_{mt} and N_{ft} are taken directly from the U.S. Statistical Abstract Historical Statistics, and I use linear interpolation to supply missing values.

Annual college costs per student, λ_t , are calculated as the average tuition and fee expenses paid out-of-pocket by students each year.⁴ Note that because I measure average *out-of-pocket* costs in the data, λ_t accounts for changes over time in the average amount of financial aid received by students in the form of public and private scholarships and

⁴Additional student expenses, such as room and board, could also be included, and in fact I do consider these costs as a robustness exercise in Section 2.5. I choose to leave these out of the benchmark specification because such costs are usually more accurately classified as consumption rather than education expenses, and must be paid regardless of college enrollment status.

grants. Full details of the data construction are relegated to Appendix C. Briefly, however, I compute λ_t each period as the total revenues from student tuition and fees received by all institutions of higher education divided by the total number of students enrolled in those institutions. The complete time series is constructed by splicing together data from historical print sources including the *Biennial Surveys of Education* (1900 to 1958) and the *Digests of Education Statistics* (since 1962).

Life-Cycle Wage Profiles

Life-cycle wage profiles $w_{a,t}(e,s)$ are estimated using decennial U.S. Census data from 1940 through 2000, along with American Community Survey (ACS) data from 2006-2010. Each ACS data set is a 1% sample of the U.S. population, so that when combined they constitute a 5% of the U.S. population, similar to a decennial census. The data are collected from the Integrated Public-Use Microdata Series (IPUMS) (Ruggles et al., 2010), and include wage and salary income, educational attainment, age, and sex. From age and education data I compute potential labor market experience, x , as age minus years of education minus six. I assume that wages can be drawn from one of three education categories - high school, some college, or college. These correspond to $e = 0$, $e \in [1, C - 1]$ and $e = C$ in the model. For each education category, I estimate wage profiles for the non-institutionalized population between ages 17 and 65 who report being in the labor force using the following regression:

$$\log(w_{i,t}) = \delta_{i,t}^b + \sum_{j=1}^4 \beta_j^s x_{i,t}^j \quad (2.2)$$

where i denotes individuals, b is birth-year cohort, s is sex, and x is potential labor market experience. In words, I regress log wages on a full set of birth year dummies plus sex specific quartics in experience.

Exogenous Parameters

Parameters set exogenously prior to solving the model are: T , C , r_t , β , ρ_t , $\mu_{\alpha,t}$, $\mu_{k,t}$, $\sigma_{\alpha,t}$, $\sigma_{k,t}$, and $\pi(\alpha)$. I set the length of working life at $T = 48$, implying that individuals born into the model at age 18 would retire at age 65. The number of periods required to complete college is $C = 4$, so that all individuals in the model have post-secondary education $e \in \{0, 1, 2, 3, 4\}$.⁵ The real interest rate is set to $r_t = 0.04$ in all periods, and the discount rate is $\beta = 0.96$, which is a standard value in macroeconomic models with annual periods.

I now turn to the parameters for the joint normal distribution over $\{\alpha, k\}$. Recall from Section 2.2 that α only affects an individual's probability of passing college. Furthermore, my interest in "ability" is limited to understanding changes over time in the average ability of college versus non-college students *within cohorts*. In other words, I only care here about the relative ability of students within the same birth year, as in the data from Figure 2.1b, not across birth years. As this is the objective, I do not have to worry about trends in average student ability (such as the so-called "Flynn effect") and can normalize the ability distribution for each birth cohort. For this reason, I set $\mu_{\alpha,t} = 0$ and $\sigma_{\alpha,t} = 1$, for all t , so the distribution for α is a standard normal, conditional on k_0 .

Unlike with ability, I am certainly concerned about changes over time in the mean and variance of the initial asset distribution. I interpret k_0 as a reduced-form way of capturing all of the personal financial resources available to a new high school graduate, including but not limited to parental gifts and bequests, and the individual's own income and savings. Additionally, since the model does not allow for individuals to work while in college, I interpret initial assets to also include the present value of income earned while enrolled.

⁵I am not presently concerned with educational attainment beyond the bachelor's degree level, so I do not model post-graduate education in this paper.

With this in mind, I require that the mean and standard deviation of initial assets in the model to track the mean and standard deviation of income in U.S. data. To this end, I start with $\mu_{k,t}$ equal to the annual mean real income per person, as in Piketty and Saez (2006) so that the average real asset endowment in the model equals the actual real mean income in the U.S. each year. Then, in order to account for the fact that $\mu_{k,t}$ includes the individuals' own earnings while in college, I adjust it upward for men and downward for women so that the difference between mean asset endowments for men and women matches the gender earnings gap in the estimated wage profiles during college years.

Piketty and Saez (2006) also provide historical data on the share of income received by the top ten percent of individuals, as well as the cut-off income level for the 90th percentile. Assuming that the U.S. income distribution is log-normal as predicted by Gibrat's law, I can use these data to back out the implied standard deviation of the U.S. income distribution each year. The procedure is as follows. Let real income in year t , denoted Y_t , be a random variable with realization y_t such that $Y_t \sim \ln \mathcal{N}(\mu_t, \sigma_t^2)$ and the associated cumulative distribution function is $F_Y(y_t; \mu_t, \sigma_t^2)$. Observed data are the real mean income in the U.S. in year t , denoted \bar{y}_t , and the 90th percentile of real income in year t , denoted $y_{90,t}$. A standard property of the log-normal distribution is that $\mathbb{E}[Y_t] = \exp(\mu_t + \frac{\sigma_t^2}{2})$. Since $\mathbb{E}[Y_t] = \bar{y}_t$ is observed, I can guess a value $\tilde{\sigma}_t^2$ and solve for the associated mean of the distribution:

$$\tilde{\mu} = \ln(\bar{y}_t) - \frac{\tilde{\sigma}_t^2}{2}$$

Next, I compute $1 - F_Y(y_{90,t}; \tilde{\mu}, \tilde{\sigma}_t^2)$, which would be the fraction of total income received by those with income above the threshold value $y_{90,t}$ if the mean and variance of the income distribution were actually $\tilde{\mu}$ and $\tilde{\sigma}_t^2$. This process continues iteratively until I find a value σ_t^2 , and associated μ_t such that the fraction of income received by the top ten percent equals that observed in the data. I then set $\sigma_{k,t} = \sigma_t$.

The last parameter related to the stochastic endowment process that I need to determine is ρ_t , the correlation between ability and initial asset endowments. Lacking the rich historical data that would be required to properly identify this parameter, I will assume for the benchmark parameterization that $\rho_t = 0$ for all t , so that ability and assets are independent random variables. Intuitively, though, one would expect some positive correlation between a student's financial resources and his or her probability of completing college. It is well known, for example, that parental income is positively related with student test scores and performance (Black, Devereux, and Salvanes, 2005; Cameron and Heckman, 1998). Moreover, as the absolute value of the correlation increases, this also implies a more precise signal of ability. Thus, I later examine in Section 2.5 how the results may change as I allow ρ to increase.

Finally, I need to set the annual probability of passing college, $\pi(\alpha)$. Note that $\pi(\alpha)$ is a reduced form way to capture college non-completion for any reason, including failure and voluntary drop-out. I employ the simple assumption that an individual's *cumulative* probability of completing college equals her percentile rank in the ability distribution. For example, an individual whose ability is higher than 75% of the peers in her birth-year cohort will complete college with probability 0.75, conditional on enrollment. With the length of college set to $C = 4$, there are 3 independent opportunities for failure - after the first, second, and third years of school. Thus, the *annual* probability $\pi(\alpha)$ is simply the cumulative probability raised to the power one-third.

Calibrated Parameters

Finally, I choose the borrowing constraint, γ , and the variance of the noise on the ability signal, $\sigma_{\varepsilon,t}$, to replicate the two main data series of interest – college completion and the average ability of college relative to non-college individuals. The borrowing constraint is set to $\gamma = 0.025$ in order to match the time series of college completion. Intuitively, this

means that in any given period an individual can borrow up to 2.5% of his expected lifetime income. Post-schooling, this amount is known with certainty because the wage profiles are given, but during college the expected lifetime income is conditional on the probability of passing college.

Unfortunately, I do not have direct evidence on the precision with which individuals in a given cohort know their own ability relative to their peers. At a qualitative level, it is likely that this precision has increased – i.e., $\sigma_{\epsilon,t}$ has likely decreased – over time. In the early part of the 20th century, no standardized exams existed to compare students within cohorts across schools. Those college admissions exams that did exist were generally school-specific, so there was little scope for comparison of students across schools. During World War I, the U.S. military began testing recruits using the Army Alpha and Army Beta aptitude tests. By World War II, these tests were replaced by the Army General Classification Test (AGCT), a precursor to the Armed Forces Qualification Test (AFQT). On the civilian side, the introduction of the Scholastic Aptitude Test (SAT) in 1926 started a trend toward more widespread use of standardized exams as a college admissions criteria. As standardized testing became more common, students obtained more and more precise signals of their own ability relative to peers. In the modern era, virtually every student contemplating college takes either (or both) of the SAT or the ACT (American College Testing) exams. Even those who do not take these college admissions exams still have quite precise information about their relative ability because other standardized exams are mandated at public schools.

With this historical background in mind, I make the following assumptions on the time series structure of $\sigma_{\epsilon,t}$. For cohorts making college decisions prior to World War II, i.e., those born 1900 through 1923 and graduating high school from 1918 through 1941, I assume that $\sigma_{\epsilon,t}$ decreases linearly from $\sigma_{\epsilon,1900} = 2$ to $\sigma_{\epsilon,1923} = 0.2$. For cohorts born after 1923, $\sigma_{\epsilon,t}$ remains constant at 0.2. This is an admittedly ad hoc construction, but in a simple

way it captures the trend of each subsequent cohort getting slightly better information than the previous cohort as aptitude and ability tests became more common in the time between the world wars. By the completion of World War II, such tests were in widespread use and students likely had quite precise signals about their own ability relative to peers.

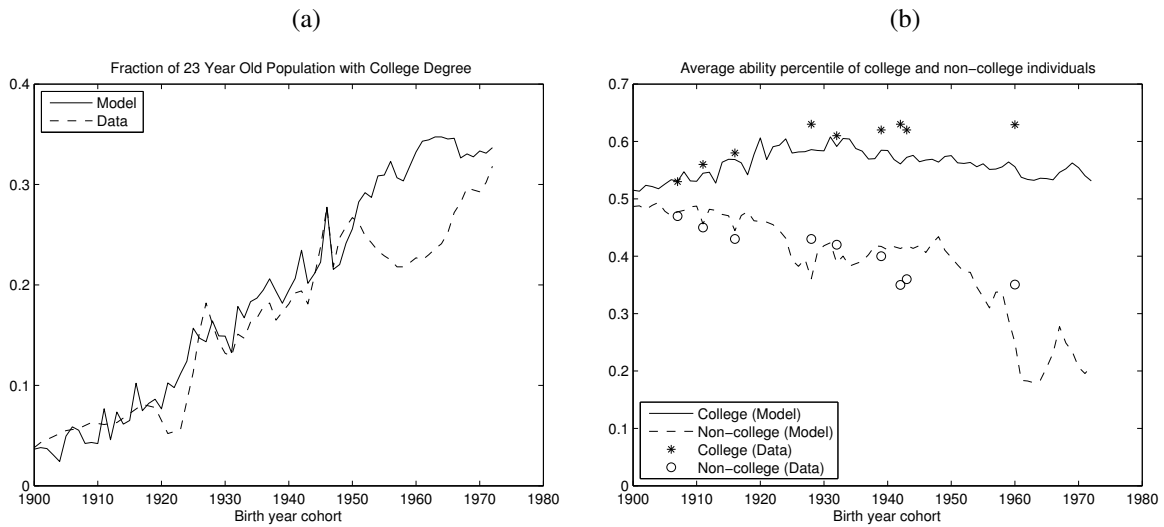
2.4 Results

The main computational exercise consists of first simulating the model for U.S. birth cohorts from 1900 through 1972 (i.e., students who graduated high school from 1918 through 1990), verifying that the model replicates important features of the historical data, and then running counterfactual simulations to quantify the impact of changes in direct college costs, education earnings premia, and opportunity costs of college (foregone wages) on college completion and average student ability. Having discussed the benchmark model parameterization, I now examine how well the simulated model matches U.S. data.

Benchmark Model Fit

Figure 2.2 depicts the model predictions along with historical U.S. data for college completion and average student ability. The measure of college completion that I choose to match is the share of 23-year-olds with a college degree. While educational attainment is often measured later in life to capture those who complete college at older ages, I prefer this series for a couple of reasons. First, to my knowledge it is the only measure of college completion with consistent time series data for birth cohorts back to 1900. Second, the model is not constructed to evaluate college enrollment decisions of older students who: (i) are generally less financially-dependent upon parents when paying for education; (ii) face different opportunity costs of school after having been in the workforce for some time; and (iii) may anticipate different return on investment in education due to later-life completion.

Figure 2.2: Benchmark Model Results



Panel (a) of Figure 2.2 shows that, overall, the model replicates well the trends in U.S. college completion over much of the 20th century, with one notable exception. The model does not capture the initial decline and subsequent increase in college completion for cohorts born in the 1950s and 1960s. This deviation is due primarily to the modeling assumption that individuals know their lifetime wage profile with certainty, implying that they can perfectly forecast changes in the education earnings premium. Later I consider alternative assumptions, and find that the model can generate more accurate predictions over this time period.

Panel (b) of Figure 2.2 plots the average ability percentile of students who attempt college (even if they do not complete), and those who have only high school education. While I only have a few reliable data points to match, those I do have show a clear pattern of increased sorting by ability over time. For cohorts born at the beginning of the 20th century, college and non-college students had similar ability on average, but the ability gap widened throughout the century. This general pattern is also predicted by the model.

Table 2.1: Measures of Fit for Various Model Specifications

Model \ Cohorts	Fraction of 23-year-olds with College Degree				Average Ability Difference
	1900-1972	1900-1925	1926-1950	1951-1972	1900-1972
Benchmark	0.158	0.015	0.009	0.133	0.034
Imperfect foresight	0.055	0.020	0.013	0.022	0.028
Constant costs rel. to income	0.134	0.024	0.011	0.099	0.034
Corr(α, k_0) = 0.30	0.183	0.023	0.017	0.143	0.038
Include room and board	0.159	0.019	0.017	0.123	0.035

In order to facilitate quantitative comparison with alternative specifications, I also provide measures of model fit over various time periods in Table 2.1. The measure of fit I report is the sum of squared deviations between model and data. The columns labeled “Fraction of 23-year-olds with college degree” refer to the series in Panel (a) of Figure 2.2. For this series, I compute the fit over all cohorts 1900-1972, and three subsamples: 1900-1925, 1926-1950, and 1951-1972. As seen in the “Benchmark” model specification in Panel (a) of Figure 2.2, the model matches the data very closely for cohorts born pre-1950, but does less well for cohorts born after 1950. The column labeled “average ability difference” measures how well the model matches the difference between the average ability percentile of college and non-college individuals. I only report the full sample for this statistic because there are so few data points to match within the sub-sample periods.

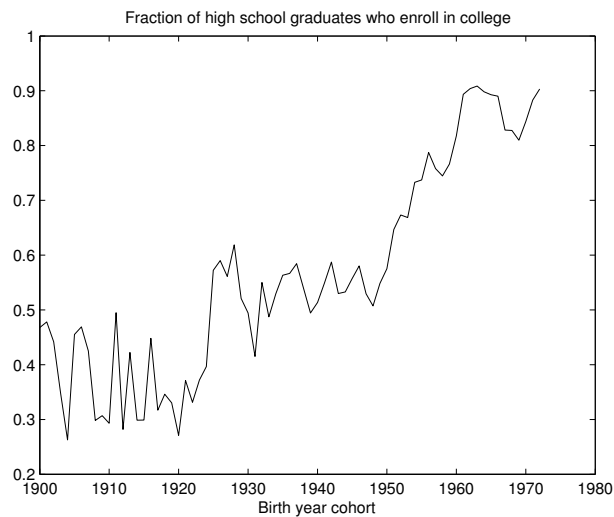
Discussion of Benchmark Results

The measure of college completion – the fraction of twenty-three year olds with a college degree – can be decomposed as

$$\frac{p^{grad}}{p^{23}} = \left(\frac{p^{HS}}{p^{23}} \right) \left(\frac{p^{enroll}}{p^{HS}} \right) \left(\frac{p^{grad}}{p^{enroll}} \right) \quad (2.3)$$

where P^{HS} , P^{enroll} , and P^{grad} are the number of people that complete high school, enroll in college, and graduate college. The model's predictions for college completion can be decomposed into the three terms on the right hand side of equation (2.3). While the first is exogenous, the second and third terms are endogenous to the model. In this section, I use this decomposition to understand what drives the change in college completion predicted by the model.

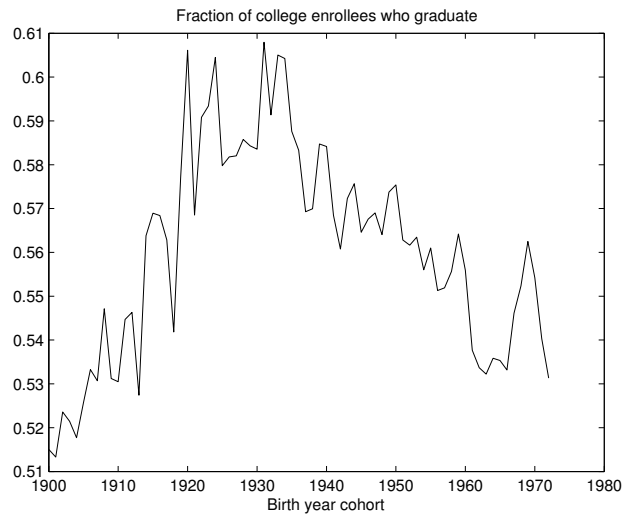
Figure 2.3: College Enrollment Conditional on High School Graduation



First, Figure 2.3 plots the share of high school graduates that enroll in college, as predicted by the model. In the language of equation (2.3), this is P^{enroll}/P^{HS} . Figure 2.3 shows that for cohorts born between 1900 and 1920, college enrollment rates conditional on high school graduation were between 30 and 50 percent, albeit with a lot of noise. This rate increased for cohorts born in the 1920s and generally remained between 50 and 60 percent for cohorts through 1950, after which the rate again increased substantially.

The third term in equation (2.3) is the share of college enrollees that graduate by age twenty-three. This is given by the ratio P^{grad}/P^{enroll} and is plotted in Figure 2.4. While Figure 2.4 shows that the college pass rate has a fair amount of year-to-year noise, the

Figure 2.4: College Pass Rate



hump-shaped trend is still evident. From the 1900 through 1930 birth cohorts, the college pass rate increased from about 51% to nearly 61%. After the 1930 cohort, however, this trend reverses, and the pass rate steadily declines back down to around 53%. This result is consistent with evidence from Bound, Lovenheim, and Turner (2010), who compare the high school class of 1972 (roughly birth cohort 1954) to that of 1992 (birth cohort 1974) and find a significant decrease in college completion conditional on enrollment. In my model, this pattern is due entirely to the ability composition of college students. Recall from Panel (b) of Figure 2.2 that the average ability of college enrollees was generally increasing through the 1930 cohort, then decreasing in the following cohorts. Unfortunately, I have found no reliable historical data to compare with the model's predicted pass rates. However, the National Center for Education Statistics (NCES) does provide more recent data that provides a rough comparison. For the cohort beginning college in 1996 (assuming they are around 18 years old, this is approximately the 1976 birth cohort), the share completing college within five years was 50.2%.⁶ The last birth cohort in this model is 1972, so the comparison is not perfect, but the model pass rate of 53.1% for that cohort is quite close.

⁶See Table 341 in the 2010 *Digest of Education Statistics*.

I now isolate the effects of the college enrollment and college pass rates through two counterfactual experiments. I ask two questions. First, how does college completion change relative to the benchmark if there were no endogenous increase in the college enrollment rate, as in Figure 2.3? Second, how does college completion change if there were no endogenous changes in the college pass rate, as in Figure 2.4? Results from these two experiments are plotted in Figure 2.5, along with the benchmark prediction for college completion.

Figure 2.5: College Completion if Enrollment Rates and Pass Rates were Constant

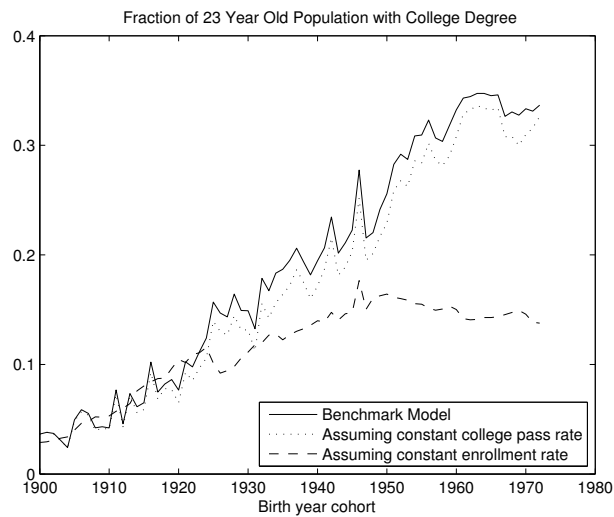


Figure 2.5 shows that if the college enrollment rate had remained constant instead of rising after the 1920 cohort, the model would have under-predicted college completion rates by more than half by the end of the time series.⁷ Similarly, if the college pass rate had instead remained constant at the 1900 value of 51.5%, then college completion would have been several percentage points lower than in the benchmark model. It is clear, however, that the quantitative effects of changes in college enrollment are much larger than those due to changing college completion rates.

⁷In Figure 2.5, I assume that the college enrollment rate conditional on high school graduation is constant at 36.9%, which is the average enrollment rate for cohorts 1900 through 1920.

Counterfactual Experiments

What if individuals do not have perfect foresight of education earnings premia?

Figure 2.6 shows that for cohorts born in the U.S. prior to 1950, the education premia implied by the estimated life-cycle wage profiles exhibit some year to year variation, but essentially no trend. Beginning around the 1950 cohort, however, the college earnings premia began increasing steadily. I now examine how the model predictions for college completion and average student ability would differ if, instead of predicting changes in the education premium exactly, model individuals expected an historical average education earnings premia to prevail in the future as well. For this exercise, I assume that the high school wage for each cohort is observable, but the earnings premia for individuals who complete college or some college are not observable. Rather, individuals observe a moving average of the earnings premia earned by previous cohorts and assume their own cohort's earnings premia will be the same. Thus, as the true college earnings premium begins rising, newly born cohorts will predict the increase imperfectly and with several years lag.

Figure 2.6: Education Premia Implied by Estimated Life-Cycle Wage Profiles

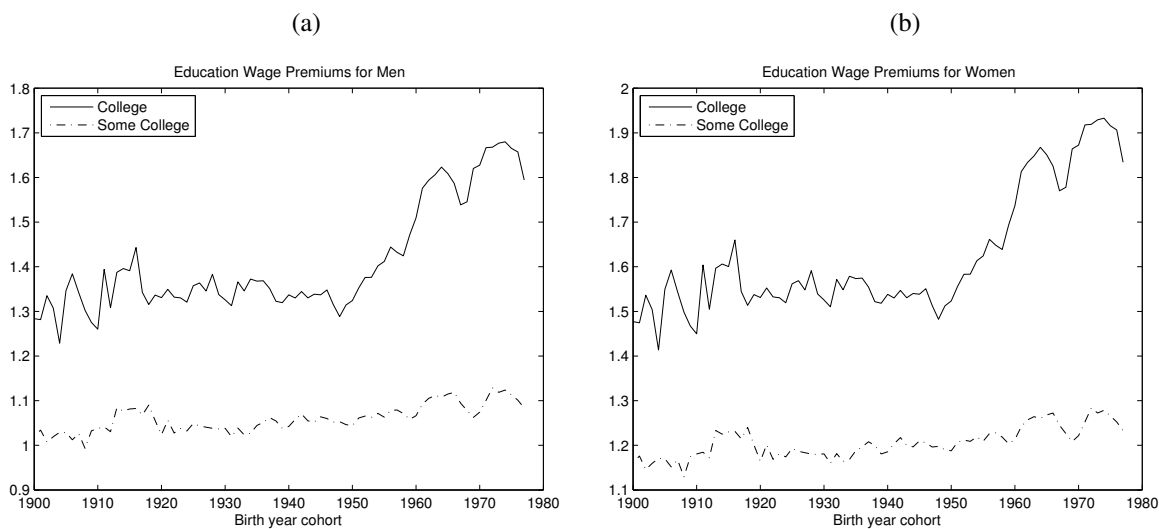


Figure 2.7: Results with Imperfect Foresight of Education Earnings Premia

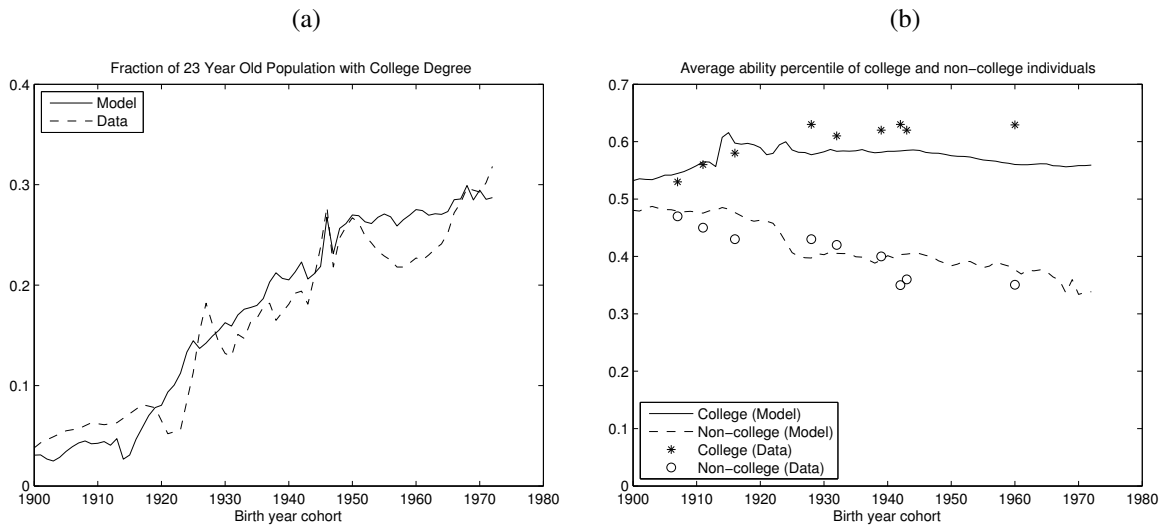


Figure 2.7 shows the model predictions under this counterfactual experiment, assuming a 25-year moving average. Relative to the benchmark model results, notice that the model now comes much closer to the actual college completion rate in the data for cohorts born after 1950. The model still does not capture all of the decline for the cohorts in the 1950s, but as Table 2.1 clearly shows, this specification fits the data much better than the benchmark assumption that individuals perfectly forecast changes in the education premia. Over the entire time period, the sum of squared deviations declines by almost two-thirds from the benchmark value of 0.158 to 0.055. All of this gain is due to the 1951-1972 cohorts, where the sum of squared deviations changes from 0.133 to 0.022, a decrease of more than 83%. Additionally, the model's ability to match changes in average ability of college and non-college students also improves under this specification. According to the last column of Table 2.1, the sum of squared deviations declines from 0.034 to 0.028. These improvements strongly suggest that perfect foresight of education earnings premia is a problematic assumption. Accurately modeling students' expectations about the returns to education is crucial for understanding college enrollment decisions, particularly during periods of time when education premia are changing rapidly.

What if real college costs increased proportional to real disposable incomes?

I now ask how college completion rates and average student ability would have differed over the time period in question if real college costs were constant with respect to real average income. Figure 2.8 depicts the actual time series data for real college costs that I use in the benchmark model (solid line), along with a hypothetical series for college costs which are a constant fraction of annual real average income (dashed line). From 1920 to around 1940, the actual series exceeds the hypothetical series due to the fact that per student tuition and fees spiked relative to income during the Great Depression. Then from the early 1940s until about 1990, the hypothetical series is above the actual series. Holding all else constant, I would expect that individuals in the counterfactual model facing the hypothetical college costs should attend college in greater numbers for the cohorts born from about 1900 to 1920 (those in school from around 1920 to 1940), and fewer of those born after 1920 would attend college.

Figure 2.8: College Costs

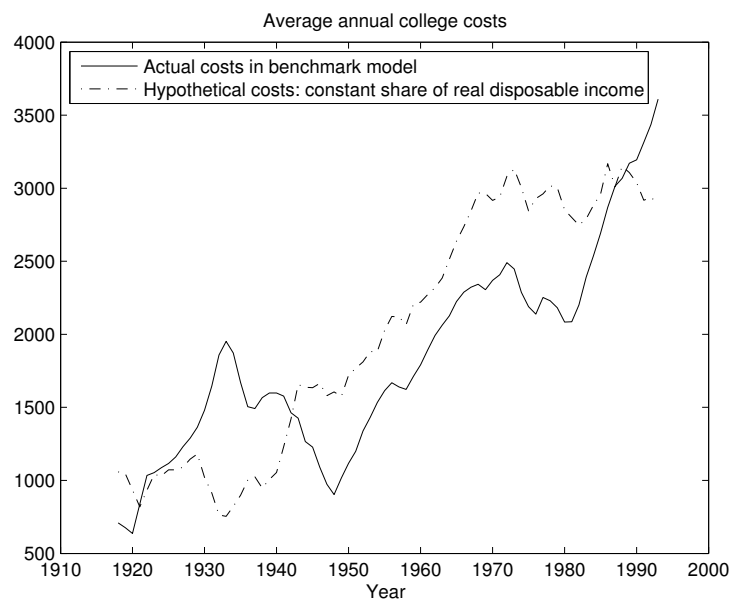
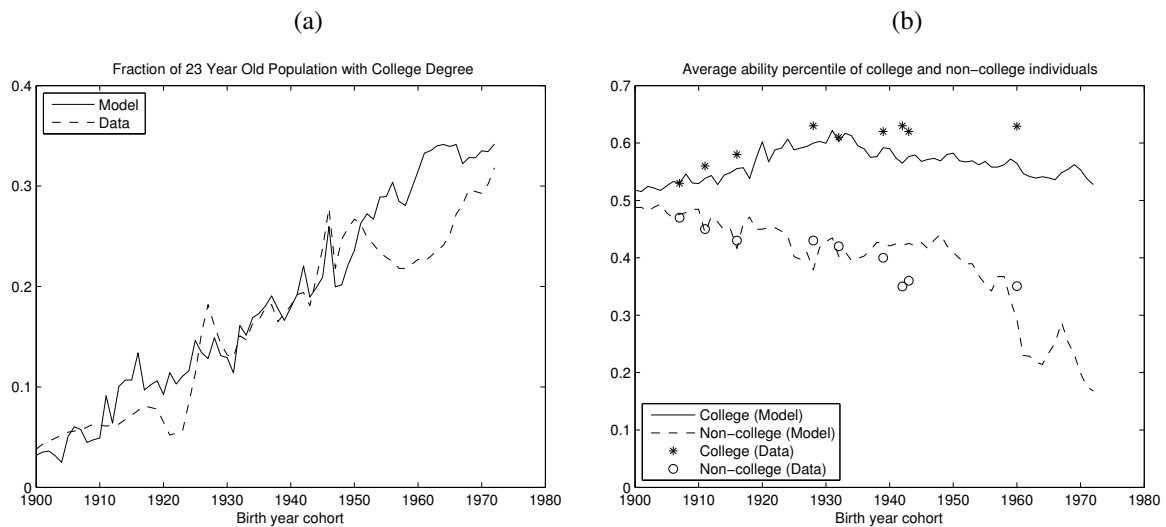


Figure 2.9 largely confirms these predictions. Relative to the data, the model predicts too many people attending college for those cohorts born between about 1910 and 1925. For the cohorts from 1925 through 1950, the model does predict slightly fewer college graduates, but still matches the data quite closely. And finally, for the cohorts born after 1950, the model still predicts more college graduates than in the data. However, as can be seen in Table 2.1, the model fit improves over this period since the sum of squared deviations fall from 0.133 to 0.099, a decrease of more than 25%. Turning to Panel (b) of Figure 2.9, there are hardly any discernible differences in average ability of college and non-college students relative to the benchmark model. This can also be confirmed by noting that sum of squared deviations for the average ability difference in Table 2.1 is unchanged from the benchmark value of 0.034. I conclude that the fluctuations in real college costs relative to real income are not a major factor in accounting for the increased ability sorting over time.

Figure 2.9: Results with Alternative College Costs



2.5 Robustness

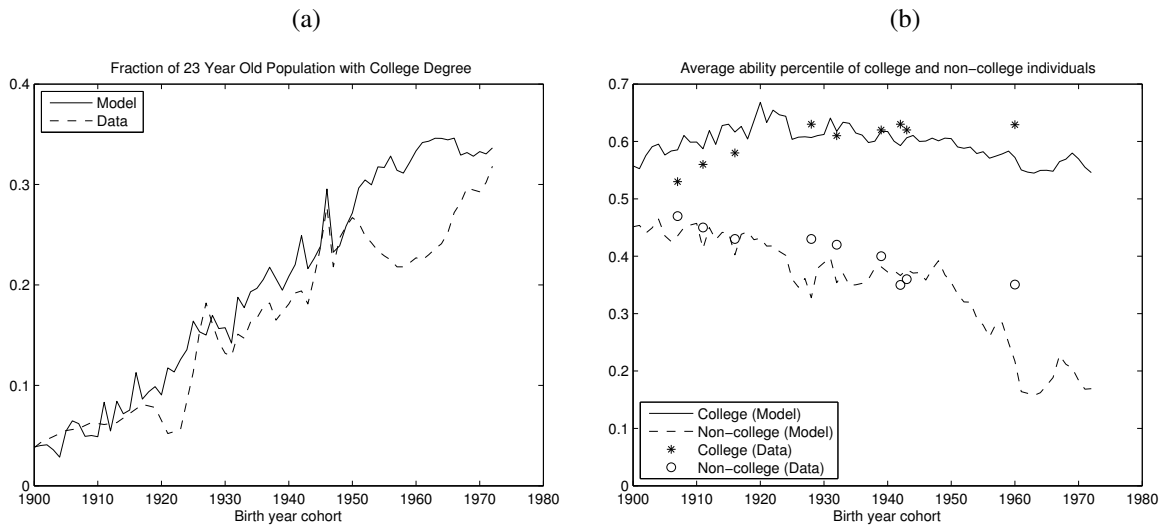
Having discussed the benchmark model results and counterfactual experiments, I now make a few remarks about the robustness of some modeling assumptions. In particular, I made the strong assumption that ability and initial assets were uncorrelated. I also assumed that room and board were excluded from college costs. I now relax these assumptions and see how they affect the results.

Correlation of Ability and Initial Assets

In the benchmark specification, I assumed that the random endowments for ability and assets were uncorrelated. However, there is evidence to suggest that these may be positively correlated, and I want to understand how this affects the results. I maintain the assumption that α and $\log(k_0)$ share a bivariate normal distribution, only now I set $\rho = 0.3$. All other parameters are maintained as in the benchmark specification. Figure 2.10 shows the model predictions for college completion and ability sorting between college and non-college individuals.

Relative to the benchmark model results, two things are notable. The positive correlation between ability and assets increases college completion minimally throughout the time period, and it increases the difference in ability between college and non-college students during earliest birth cohorts. Both of these effects reduce the model fit slightly, as seen in Table 2.1. The increase in completion is simply due to the fact that higher ability students are now more likely to have greater financial resources as well, thus making them more likely to attend college. The effect on average ability is also quite intuitive. Recall that individuals receive information $v = (k_0, \theta)$, where $\theta = \alpha + \varepsilon$ is the noisy signal of true ability α . As ρ increases k_0 becomes more informative about α , so individuals with high initial assets will infer that they have higher ability, and thus be more likely to enroll in

Figure 2.10: Results with Positive Correlation between Ability and Initial Assets



college. This increases the average ability of individuals who attempt college, while simultaneously decreasing the average ability of non-college individuals. The effect is largest for earlier birth cohorts because later birth cohorts received more accurate signals about their true ability.

College Costs Including Room and Board

College costs in the benchmark model were restricted only to tuition and fees. Now, I take a broader view of college costs and examine whether or not the results are sensitive to the inclusion of room and board expenses. Like the earlier time series data on college tuition and fees, I construct this data from printed historical government documents. Again, the details are found in Appendix C. For this experiment, all calibrated values are maintained just as in the benchmark economy, with the exception of the borrowing constraint, γ . I need to adjust γ because students now face additional college expenses, so college completion rates would be too low if I held γ constant at the benchmark value. The new borrowing constraint which allows us to match the time series of college completion is $\gamma = 0.04$.

Figure 2.11: Results for College Costs including Tuition, Fees, Room, and Board

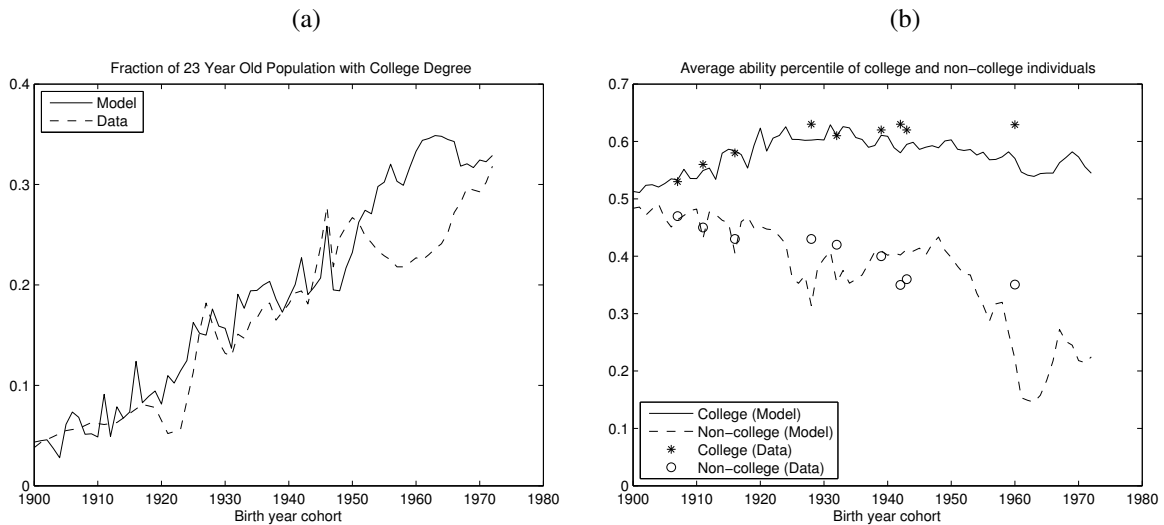


Figure 2.11 shows the model predictions for college completion and average student ability when room and board costs are included. Relative to the benchmark results in Figure 2.2, very little has changed. The model still predicts college completion rates in line with the data up until the 1950s and 1960s cohorts, when model and data diverge. Additionally, average ability of college and non-college students diverges over time just as in the benchmark model. Referring to Table 2.1, it is clear that while the model fits college completion slightly worse than the benchmark model pre-1950, it does slightly better post-1950. On the whole, this model fits almost exactly as well as the benchmark model for both college completion and average ability difference.

2.6 Conclusion

I develop an overlapping generations model with unobservable ability and borrowing constraints to investigate post-secondary completion and ability sorting in the birth cohorts of 1900–1972. To discipline the model, I digitize and utilize historical data series including statistics on college costs and high school graduation rates. I find that the share of high

school graduates enrolling in college and the subsequent college pass rate are both key for understanding increased college graduation rates. However, I find no evidence that economic factors – including real college costs, opportunity costs, education wage premia, or asset endowments – have a major impact on increasing ability sorting over time. I do find, however, that a decrease in the variance of ability signals can properly match this fact, a trend which I attribute to increases over time in standardized testing.

An important deviation between the benchmark model and historical data is that the model does not properly match college completion after the 1950 birth cohort. I show that this could be due to individuals having imperfect foresight about the college earnings premium. If individuals observe a moving average of the earnings premia from previous cohorts and use this to estimate the future earnings premium, then changes in the earnings premium are taken into account only with a lag. I build this into the model and find that it significantly improves the model's fit. I therefore view this as evidence of backward looking wage estimation when making college enrollment decisions.

An interesting use of this framework would be an extension to multiple countries. Evidence suggests that ability is strongly related to growth (Hanushek and Kimko, 2000), but the causality from formal schooling to economic growth is somewhat tenuous (Bils and Klenow, 2000). If developing countries have very little ability sorting between education levels, as was the case in the early U.S., there may be a weak correlation between education level and labor efficiency. In a cross-country context, this could arise due to tighter borrowing constraints or less precise signals about true ability. I will explore this link in future research.

Chapter 3

SECTORAL TECHNOLOGY AND STRUCTURAL TRANSFORMATION

3.1 Introduction

The reallocation of production factors across the broad sectors agriculture, manufacturing, and services is one of the important stylized facts of growth and development: as economies develop agriculture shrinks, manufacturing first grows and then shrinks, and services grow. A growing recent literature has studied this so called structural transformation and has shown that it has important implications for the behavior of aggregate variables such as output per worker, hours worked, and human capital.¹ This paper is part of a broader research program that asks what economic forces are behind structural transformation. Herrendorf, Rogerson, and Valentinyi (2013b) addressed the preference aspect of this question and quantified the importance of the effects of changes in income and relative prices for changes in the composition of households consumption bundles. In this paper, I focus on the technology aspect of this question. In particular, I ask how important are sectoral differences in technological progress, capital intensity, and the substitutability between capital and labor for structural transformation.

There are two different views in the literature about this question. Most papers on structural transformation use sectoral production functions of the Cobb-Douglas form with capital shares that are equal to the aggregate capital share. The advantage of this way of proceeding is that it is convenient, as sectoral Cobb-Douglas production functions with equal capital shares can be aggregated to an economy-wide Cobb-Douglas production function with the same capital share. However, this way of proceeding assumes away differences in sectoral capital intensity and the substitutability between capital and labor that may lead to potentially important economic forces behind structural transformation.

¹The recent literature started with the papers by Kongsamut, Rebelo, and Xie (2001) and Ngai and Pissarides (2007). Herrendorf, Rogerson, and Valentinyi (2013a) provide a review of this literature.

To see how these forces operate, suppose first that technological progress is even (i.e., is the same in all sectors) and compare two sectoral production functions that only differ in the relative capital intensity. When the economy is poor and capital is relatively scarce compared to labor, then the price of the output of the capital-intensive sector relative to that of the labor-intensive sector will be high. As even technological progress takes place, the economy develops and capital becomes less scarce compared to labor and the relative price of the output of the capital-intensive sector will fall. Acemoglu and Guerrieri (2008) emphasized this economic force behind structural transformation. Now suppose that technological progress is still even and compare two sectoral production functions that only differ in the elasticity of substitution between capital and labor. As before, when the economy is poor, the relative price of the output of the sector with low substitutability will be high. As even technological progress takes place, the relative price of the output of the sector with low substitutability will fall. Alvarez-Cuadrado, Long, and Poschke (2012) emphasized this economic force behind structural transformation.

In order to assess how quantitatively important the different features of sectoral technology are for structural transformation, I estimate constant elasticity of substitution (CES) production functions for agriculture, manufacturing, and services on postwar U.S. data. I also estimate Cobb-Douglas production functions with sector-specific capital shares and Cobb-Douglas production functions with a common capital share equal to the aggregate capital share. I then endow competitive stand-in firms in each sector with the estimated technologies and ask how well their optimal choices replicate the observed sectoral allocation of labor and the sectoral relative prices. The reason for focusing on sectoral labor is that it is the most widely available measure of sectoral activity, which is most commonly used in the context of structural transformation.

The estimation of the sectoral CES production functions yields the following results. First, labor-augmenting technological progress is fastest in agriculture and slowest in

services and the differences in the growth rates are sizeable. Second, agriculture is the most capital-intensive sector and manufacturing is the least capital-intensive sector; services are more capital intensive than manufacturing because they include the capital-intensive industry owner-occupied housing. Third, capital and labor are most easily substitutable in agriculture and least easily substitutable in services; moreover, in agriculture capital and labor are more substitutable than in the Cobb-Douglas case and in manufacturing and services they are less substitutable than in the Cobb-Douglas case.²

In order to assess how quantitatively important the different features of the estimated sectoral production functions are for structural transformation, I compare the predicted trends of sectoral labor with those of Cobb-Douglas production functions with equal and with different capital shares. It turns out that uneven labor-augmenting technological progress is the dominant force behind these trends. As a result, sectoral Cobb-Douglas production functions with equal capital shares (which by construction abstract from differences in the elasticity of substitution and in capital shares) do a good job at capturing the reallocation of labor during the process of U.S. structural transformation. The reason for this finding is that the CES production function of agriculture has both by far the largest relative weight on capital and the largest elasticity of substitution whereas the other two CES production functions have fairly similar relative weights and elasticities of substitution. Hence, the effects on structural transformation out of agriculture of the relatively large weight on capital and the relatively large elasticities of substitution go in opposite directions and largely cancel each other, leaving the effects of uneven labor-augmenting technological progress as the dominating force. I also show that similar conclusions hold for relative prices, that is, Cobb-Douglas production functions with equal shares do a good job at predicting the relative prices of sectoral outputs.

²The finding that in agriculture capital and labor are more substitutable than in the Cobb-Douglas case is consistent with the view that a mechanization wave led to massive substitution of capital for labor in U.S. agriculture during the 1950s and 1960s; see for example Schultz (1964).

This paper falls into a large literature that estimates production functions at the aggregate level, the industry level, or the firm level. Antràs (2004), Klump, McAdam, and Willman (2007) and León-Ledesma, McAdam, and Willman (2010) are the contributions to this literature which are most closely related to this work. They revisited the question how substitutable capital and labor are at the level of the aggregate U.S. economy. I adopt the methodology of León-Ledesma, McAdam, and Willman (2010) and apply it at the more disaggregate level of the three broad sectors that are relevant in the context of structural transformation.

The remainder of the paper is organized as follows. In Section 3.2 I introduce the concept of value-added production functions. Section 3.3 discusses the estimation issues that arise and the data that I use. In Section 3.4, I present the estimation results and in Section 3.5 I compare the CES production function with the Cobb-Douglas production functions. Section 3.6 discusses the implications of these results for building multi-sector models and section 3.7 concludes.

3.2 Value-added Production Functions

I start with the question of whether to write production functions in gross-output form or in value-added form. Since value added equals the difference between gross output and intermediate inputs, the difference between the two possibilities is whether one counts everything that the sector produces (“gross output”) or whether one counts only what the sector produces beyond the intermediate inputs that it uses (“value added”). To see the issues involved in this question, it is useful to start with the aggregate production function. In a closed economy, GDP equals value added by definition. Therefore, GDP G is ultimately produced by combining domestic capital K and labor L , and I can write the aggregate production function as a value-added production function:

$$G = H(K, L)$$

where H has the usual regularity conditions including homogeneity of degree one. In an open economy, GDP is in general not equal to domestic value added anymore because of imported intermediate inputs. Therefore, GDP is ultimately produced with domestic capital, labor, and imported intermediate inputs Z :

$$G = H(K, L, Z)$$

While imported intermediate inputs are often abstracted from, they can be quantitatively important, in particular in small open economies that have few natural resources.

Turning now to sectoral production functions, the question which production function to use arises even in a closed economy. The reason for this is that all sectors use intermediate inputs from other sectors, and so sectoral output practically never equals sectoral value added. Therefore, it is natural to start with a production function for gross output and to ask under what conditions a production function for value added exists.

Denoting the sector index by $i \in \{a, m, s\}$, the production function for sectoral gross output can be written as:

$$G_i = H_i(K_i, L_i, Z_i)$$

The question I ask here is under which conditions a value-added production functions $F_i(\cdot, \cdot)$ exist such that sectoral value added is given by:

$$Y_i \equiv \frac{P_{gi}G_i - P_{zi}Z_i}{P_{yi}} = F_i(K_i, L_i) \quad (3.1)$$

where P_{gi} , P_{zi} , and P_{yi} denote the prices of gross output, intermediate inputs, and value added, all expressed in current dollars.

In the 1970s Sato (1976) essentially showed that a value added production function exists if there is perfect competition and if the other input factors are separable from intermediate inputs, that is, the gross-output production function is of the form

$$G_i = H_i(F_i(K_i, L_i), Z_i) \quad (3.2)$$

where H_i and F_i have the usual regularity conditions (i.e., they are continuously differentiable and concave in each input factor and the Inada conditions hold). To see Sato's argument, consider the problem of a stand-in firm that takes prices and gross output as given and chooses capital, labor, and intermediate inputs to minimize its costs subject to the constraint that it produces the given output:

$$\min_{K_i, L_i, Z_i} R_i K_i + W_i L_i + P_{z_i} Z_i \quad \text{s.t.} \quad H_i(F_i(K_i, L_i), Z_i) \geq G_i \quad (3.3)$$

where R_i and W_i denote the rental rates for capital and labor, both expressed in current dollars. The first order conditions to this problem imply:³

$$P_{y_i} = \lambda_i \frac{\partial H_i(F_i(K_i, L_i), Z_i)}{\partial Y_i} \quad (3.4)$$

$$R_i = \lambda_i \frac{\partial H_i(F_i(K_i, L_i), Z_i)}{\partial Y_i} \frac{\partial F_i(K_i, L_i)}{\partial K_i} \quad (3.5)$$

$$W_i = \lambda_i \frac{\partial H_i(F_i(K_i, L_i), Z_i)}{\partial Y_i} \frac{\partial F_i(K_i, L_i)}{\partial L_i} \quad (3.6)$$

where λ_i is the multiplier on the constraint. Substituting the first equation into the second and third equation gives:

$$R_i = P_{y_i} \frac{\partial F_i(K_i, L_i)}{\partial K_i} \quad (3.7)$$

$$W_i = P_{y_i} \frac{\partial F_i(K_i, L_i)}{\partial L_i} \quad (3.8)$$

Using that the envelope theorem implies that the multiplier on the constraint equals the price of value added P_{y_i} , it is straightforward to show that these are the first order conditions to the problem of a stand-in firm that takes prices and value added as given and chooses capital and labor to minimize its costs subject to the constraint that it produces the given value added:

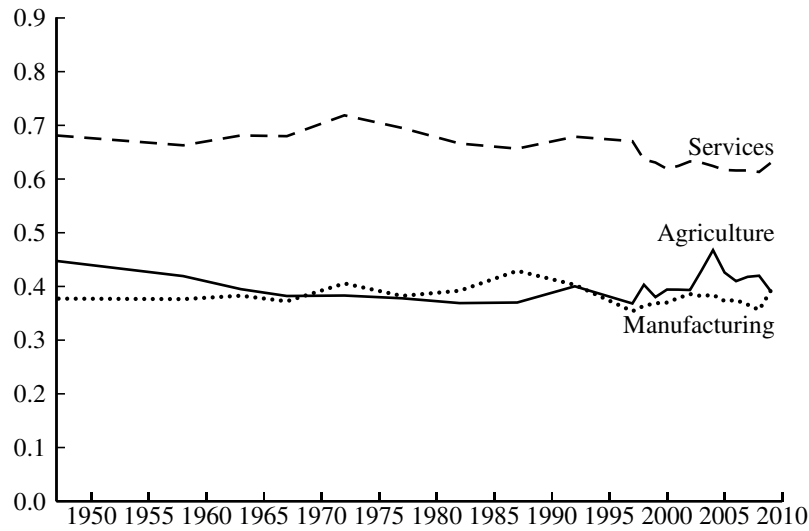
$$\min_{K_i, L_i} R_i K_i + W_i L_i \quad \text{s.t.} \quad F_i(K_i, L_i) \geq Y_i \quad (3.9)$$

³To obtain (3.4), note that I assumed that H is linear homogeneous, implying that

$$G_i = H_i \partial H_i / \partial Y_i + Z_i \partial H_i / \partial Z_i$$

Then, the first equation follows by combining this equation with the first order condition for the optimal choice of Z and compare the result with equation (3.1).

Figure 3.1: Intermediate Inputs Shares in the U.S.



Source: Input-Output Tables for the United States, Bureau of Economic Analysis

The question remains if condition (3.2) holds in the data. A sufficient condition is that the sectoral production function is Cobb-Douglas between value added and intermediate inputs:

$$G_i = [F_i(K_i, L_i)]^{\eta_i} Z_i^{1-\eta_i} \quad (3.10)$$

In this case, perfect competition implies that the share of intermediate inputs is constant over time. Figure 3.1 plots the intermediate good shares for the post-war U.S., and none of them has a pronounced trend. I take that to mean that the functional form (3.10) is a reasonable approximation when one is interested in secular trends in the U.S., which is what the literature on structural transformation focuses on. I will therefore proceed under the assumption that sectoral value-added production functions exist and estimate them.

3.3 Estimating Sectoral Production Functions

Deriving the system to estimate

I restrict attention to the class of CES production functions:

$$Y_{it} = A_i \left[\theta_i (K_{it})^{\frac{\sigma_i-1}{\sigma_i}} + (1 - \theta_i) (\exp(\gamma_{it}) L_{it})^{\frac{\sigma_i-1}{\sigma_i}} \right]^{\frac{\sigma_i}{\sigma_i-1}} \quad (3.11)$$

where $i \in \{a, m, s\}$ denotes the sector, A_i is TFP, θ_i is the relative weight of capital, σ_i is the elasticity of substitution, and γ_i is the growth rate of labor-augmenting technological progress.⁴

León-Ledesma, McAdam, and Willman (2010) show that for estimation purposes it is advantageous to reparameterize this production function in the following “normalized” form:

$$Y_{it} = F_i(K_{it}, L_{it}) = \xi_i \bar{Y}_i \left[\bar{\theta}_i \left(\frac{K_{it}}{\bar{K}_i} \right)^{\frac{\sigma_i-1}{\sigma_i}} + (1 - \bar{\theta}_i) \left(\frac{\exp(\gamma_i(t - \bar{t})) L_{it}}{\bar{L}_i} \right)^{\frac{\sigma_i-1}{\sigma_i}} \right]^{\frac{\sigma_i}{\sigma_i-1}} \quad (3.12)$$

where \bar{Y}_i , \bar{K}_i and \bar{L}_i are the geometric averages of output, capital and labor over the sample period; \bar{t} is the arithmetic average of the time index; and ξ_i is an auxiliary parameter close to unity. The advantage of working with the normalized form (3.12) instead of the original form (3.11) is that $\bar{\theta}_i$ equals the average capital share in sector i . This means that the value of $\bar{\theta}_i$ can be obtained from the data directly and independently of the estimated value of σ_i . In contrast, θ_i depends on σ_i , and so identification often is challenging when one estimates the two parameters together.

I assume that each sector has a stand-in firm, which behaves competitively and takes as given sectoral value added, the sectoral interest rate and wage when it chooses sectoral

⁴In contrast, Jorgenson, Gollop, and Fraumeni (1987) estimated translog production functions for 45 disaggregate U.S. industries during 1948–79. Although I recognize that translog production functions have many advantages in empirical work, I focus on CES production functions here because they are more common in multi-sector growth models.

capital and labor to minimize its costs subject to the constraint that it produces at least the given sectoral output. Denoting the price of value added in sector i by P_{yit} and the real interest rate and real wage in sector i by

$$r_{it} \equiv \frac{R_{it}}{P_{yit}}, \quad w_{it} \equiv \frac{W_{it}}{P_{yit}} \quad (3.13)$$

I can write the problem of the stand-in firm as:

$$\min_{K_{it}, L_{it}} r_{it}K_{it} + w_{it}L_{it} \quad \text{s.t.} \quad F_i(K_{it}, L_{it}) \geq Y_{it} \quad (3.14)$$

The first order conditions to this problem imply

$$r_{it} = \frac{\bar{\theta}_i \bar{Y}_i}{\bar{K}_i} \xi_i^{\frac{\sigma_i-1}{\sigma_i}} \left(\frac{Y_{it} \bar{K}_i}{\bar{Y}_i K_{it}} \right)^{\frac{1}{\sigma_i}} \quad (3.15)$$

$$w_{it} = \frac{(1 - \bar{\theta}_i) \bar{Y}_i}{\bar{L}_i} \xi_i^{\frac{\sigma_i-1}{\sigma_i}} \exp\left(\frac{\sigma_i}{\sigma_i - 1} \gamma_i(t - \bar{t})\right) \left(\frac{Y_{it} \bar{L}_i}{\bar{Y}_i L_{it}} \right)^{\frac{1}{\sigma_i}} \quad (3.16)$$

Taking logs of (3.12) and (3.15)–(3.16) and rearranging, I arrive at a system of three equations for each sector:

$$\log\left(\frac{Y_{it}}{\bar{Y}_i}\right) = \frac{\sigma_i}{\sigma_i - 1} \log\left[\bar{\theta}_i \left(\frac{K_{it}}{\bar{K}_i}\right)^{\frac{\sigma_i-1}{\sigma_i}} + (1 - \bar{\theta}_i) \left(\frac{\exp(\gamma_i(t - \bar{t})) L_{it}}{\bar{L}_i}\right)^{\frac{\sigma_i-1}{\sigma_i}}\right] + \log(\xi_i) \quad (3.17)$$

$$\log(r_{it}) = \log\left(\frac{\bar{\theta}_i \bar{Y}_i}{\bar{K}_i}\right) + \frac{1}{\sigma_i} \log\left(\frac{Y_{it} \bar{K}_i}{\bar{Y}_i K_{it}}\right) + \frac{\sigma_i - 1}{\sigma_i} \log(\xi_i) \quad (3.18)$$

$$\log(w_{it}) = \log\left(\frac{(1 - \bar{\theta}_i) \bar{Y}_i}{\bar{L}_i}\right) + \frac{1}{\sigma_i} \log\left(\frac{Y_{it} \bar{L}_i}{\bar{Y}_i L_{it}}\right) + \frac{\sigma_i - 1}{\sigma_i} \left[\gamma_i(t - \bar{t}) + \log(\xi_i)\right] \quad (3.19)$$

I observe Y_{it}/\bar{Y}_i , K_{it}/\bar{K}_i , L_{it}/\bar{L}_i , r_{it} , w_{it} , and $\bar{\theta}_i$. Specifically, w_{it} is the part of value added that goes to labor divided by the product of sectoral labor and the sectoral price level and r_{it} is the part of value added that does not go to labor divided by the product of sectoral capital (which includes sectoral land) and the sectoral price level. $\bar{\theta}_i$ is the share of capital income in sector i 's value added, which I calculate according to the method of Gollin (2002).

I estimate σ_i , γ_i , and ξ_i from the equations (3.17)–(3.19) for the aggregate economy and the three sectors using three stage least squares with an AR(1) error structure. For the aggregate economy this results in a three-equation system, and for the sectoral estimation in a nine-equation system with three equations for each of the three sectors. By estimating the three sectors together, I allow for the possibility that error terms across equations and sectors are correlated. Several right-hand side variables are endogenous. To deal with that, I follow León-Ledesma, McAdam, and Willman (2010) and use as instrumental variables the one-period lagged values (appropriate to each sector or the aggregate economy) of the log rental rate on capital, log real wage, log normalized output, log normalized capital, and log normalized labor. I also add a second lag of each of these instrumental variables for equation (3.19) in manufacturing to ensure stationarity of the AR(1) term coefficient in that equation. Additionally, I include a time trend with the instruments for equations (3.17) and (3.19) because it is an exogenous right-hand side variable in both equations.

Data

For output, I use the BEA’s “GDP-by-Industry” tables 1947–2010, which contain value added at current prices and quantity indexes of value added by industries according to the North American Industrial Classification (NAICS). I define sectors in the obvious way: agriculture comprises farms, fishing, forestry; manufacturing comprises construction, manufacturing, and mining;⁵ and services comprise all other industries (i.e. government, education, real estate, trade, transportation, etc.). Real output for sector i , Y_{it} , is defined by the sector’s value added expressed in 2005 prices.

An additional issue arises with measuring value added in agriculture. As is standard in national income accounting, NIPA reports “Rent paid to nonoperator landlords” as part of value added of the real estate industry. Since both the labor and capital that generate this

⁵A better name for this sector might be industry. I don’t use industry here because it typically refers to a generic production category.

rent are reported as input factors in agriculture, consistency requires us to treat this rent as part of value added of agriculture. Therefore, I add “Rent paid to nonoperator landlords” (as reported by the BEA in NIPA Table 7.3.5 “Farm Sector Output, Gross Value Added, and Net Value Added”) to the value added of agriculture and subtract it from the value added of services.

Turning to inputs, I calculate the capital stocks by sector from the BEA’s “Fixed Asset” tables 1947–2010, which contain the year-end current cost and quantity index in 2005 prices of the net stock of fixed assets. Fixed assets are constructed according to NAICS. Since the BEA fixed assets only includes reproducible capital, I add the value of farm land from the USDA to the fixed assets in agriculture.⁶ Given that the data report year-end capital stocks, I calculate the capital stocks during period t as the geometric averages of the year-end capital stocks in $t - 1$ and t .⁷

I calculate labor input by sector from two data sources. The BEA’s “GDP-by-Industry” tables 1947–2010 follow the NAICS classification consistently, but report only full- and part-time employees by industry. The “Income-and-Employment-by-Industry” tables 1948–2010 again change the industry classification: they use SIC72 during 1947–1987, SIC87 during 1987–1997, and NAICS during 1998–2010, but they contain much more detailed information about hours worked by full-time and part-time employees by industry; full-time equivalent employees by industry; self-employed persons by industry; and persons engaged in production by industry. To construct sectoral hours worked, I use the GDP-by-Industry Tables to the maximum extent possible and the Income-and-Employment-by-Industry Tables to the minimum extent possible. In particular, I merge

⁶The data are from “Land in farms” and “Farm real estate values” tables of the “U.S. and State Farm Income and Wealth Statistics” tables from the U.S. Department of Agriculture (USDA). The data include information on the quantity of land in acres and the value of land per acre.

⁷Since the BEA publishes neither the value added nor the capital stock data for the sectors as I define them, I have to construct these aggregates from the underlying BEA data. Since the BEA calculates real quantities with the chain-weighted method, they are not additive. I use the so called cyclical expansion procedure to aggregate real quantities; see Appendix E for a description of this method.

the two data sources as follows:

$$\begin{aligned} \text{Self-emp}_{NAICS} &= \frac{\text{Self-emp}_{SIC}}{\text{Part- and full-time emp}_{SIC}} \text{Part \& full-time emp}_{NAICS} \\ \text{Full-time eq emp}_{NAICS} &= \frac{\text{Full-time eq emp}_{SIC}}{\text{Part \& full-time emp}_{SIC}} \text{Part \& full-time emp}_{NAICS} \\ \text{Hours full-time eq emp}_{NAICS} &= \frac{\text{Hours full-time eq emp}_{SIC}}{\text{Full-time eq emp}_{SIC}} \text{Full-time eq emp}_{NAICS} \\ \text{Hours persons engaged}_{NAICS} &= \text{Hours full-time eq emp}_{NAICS} \\ &\quad + \frac{\text{Hours full-time eq emp}_{NAICS}}{\text{Full-time eq emp}_{NAICS}} \text{Self-emp}_{NAICS} \end{aligned}$$

Labor input for sector i , L_{it} , is defined as hours worked in sector i constructed above.

I also need the rental prices of the production factors, which for sector i are defined as:

$$r_{it} = \frac{\theta_{it} Y_{it}}{K_{it}} \quad w_{it} = \frac{(1 - \theta_{it}) Y_{it}}{L_{it}}$$

where θ_{it} is the share of capital income in sector's i value added in period t . I already described the construction of Y_{it} , K_{it} and L_{it} from the data, so I only need to describe the construction of the capital share in value added. I use the BEA's "Components-of-Value-Added-by-Industry" Tables 1947–2010 as follows: "compensation of employees" is labor income; "gross operating surplus minus proprietors' income" is capital income; proprietors' income is split into capital and labor income according to above shares. In the case of agriculture, I add "Rent paid to nonoperator landlords" to "gross operating surplus minus proprietors income" since rent is capital income. An issue arises because the industry classification changes over time in these tables. In particular, SIC72 applies to 1947–1987, SIC87 applies to 1987–1997, and NAICS applies to 1998–2010. I calculate the sectoral capital shares for each of the three sub-periods and assume that the same capital share also applies to the corresponding NAICS classifications. Since the three sectors I use are fairly aggregated, this should not be a big issue.

Table 3.1: Estimation Results

	Aggregate	Agriculture	Manufacturing	Services
σ	0.69*	1.09*	0.78*	0.64*
	(0.022)	(0.032)	(0.021)	(0.015)
γ	0.017*	0.092*	0.019*	0.012*
	(0.001)	(0.001)	(0.003)	(0.003)
ξ	1.02*	0.98*	0.98*	1.05*
	(0.018)	(0.010)	(0.040)	(0.064)
$\bar{\theta}$	0.33	0.61	0.29	0.34

Standard errors in parentheses; * ($p < 0.01$)

3.4 Estimation Results

The estimation results are summarized in Table 3.1.⁸ I find that capital and labor are most substitutable in agriculture and least substitutable in services. In agriculture capital and labor are more substitutable than in the Cobb-Douglas case, which is consistent with the view that a mechanization wave led to massive substitution of capital for labor in agriculture after World War II. In manufacturing and services capital and labor are less substitutable than Cobb-Douglas. On the aggregate, I find that capital and labor are less substitutable than Cobb-Douglas, which is consistent with the previous results of Antràs (2004), Klump, McAdam, and Willman (2007) and León-Ledesma, McAdam, and Willman (2010).

⁸Appendix D contains further information that shows that the fit (as measured by mean squared error) is good. Moreover, it reports multivariate Ljung-Box Adjusted Q-statistics, which test for autocorrelation in the residuals, and do not reject the null hypothesis of no residual autocorrelations. To conserve space I only report the test statistics for the second lag, but the existence of higher order autocorrelation is also strongly rejected.

Labor-augmenting technological progress is fastest in agriculture and slowest in services and the differences in the growth rates of technological progress are sizeable: in agriculture technological progress grew by 9.2% per year, whereas in manufacturing it grew by 2.0% and in services it grew by just 1.2%; these growth rates result in an average of 1.7% annual growth of aggregate labor-augmenting technological progress. Since these numbers appear rather large, it is useful to remember two qualifications. First, the growth in TFP implied by these numbers is smaller than what I find for labor-augmenting technological progress because the labor share is smaller than one. Second, I have used measures of raw sectoral labor that do not take into account sectoral human capital. Increases in sectoral human capital then show up as an increases in labor-augmenting technological progress.

The fact that technological progress is slowest in services while the share of value added produced in services is growing is sometimes referred to as Baumol “disease”, reflecting that Baumol (1967) was the first to point out that these two facts imply decreasing growth rates of real GDP. Moreover, if the current trends of structural transformation continue, then services will dominate the economy in the limit, and so assuming that current trends for sectoral technological progress continue, aggregate labor-augmenting technological progress will fall from its 1.7% post-war average to the lower 1.2% post-war average for services.

The last row of Table 3.1 reports $\bar{\theta}$, that is, the average capital share in the post-war period. The aggregate capital share comes out as the standard value of 1/3, and sectoral capital shares differ from that standard value. However, while the agricultural capital share is considerably larger than the aggregate capital share, the capital shares in manufacturing and services are fairly close to the aggregate capital share. The capital share in agriculture is much larger than the other two capital shares because capital includes land and agriculture is land intensive. The capital share in services is larger than in manufacturing because the capital-intensive industry owner-occupied housing is part of services.

3.5 Sectoral Technology and Structural Transformation

Cobb-Douglas sectoral production functions

In this section, I evaluate the implications of the different features of sectoral technology for structural transformation. To this end, I compare the unrestricted CES production functions that I have estimated above with two restricted CES production functions: (i) I impose $\sigma_i = 1$ which results in a Cobb-Douglas production function with possibly different capital shares; (ii) I impose $\sigma_i = 1$ and $\bar{\theta}_i = \bar{\theta}$, which results in Cobb-Douglas production functions with a common capital share equal to the aggregate capital share. I write these three functional forms as follows:

$$Y_{it} = \left[\bar{\theta}_i \left(\frac{\xi_i \bar{Y}_i}{\bar{K}_i} K_{it} \right)^{\frac{\sigma_i-1}{\sigma_i}} + (1 - \bar{\theta}_i) \left(\frac{\xi_i \bar{Y}_i \exp(\gamma_i(t - \bar{t}))}{\bar{L}_i} L_{it} \right)^{\frac{\sigma_i-1}{\sigma_i}} \right]^{\frac{\sigma_i}{\sigma_i-1}}$$

$$Y_{it} = \left(\frac{\bar{Y}_i}{\bar{K}_i} K_{it} \right)^{\bar{\theta}_i} \left(\frac{\bar{Y}_i \exp(\gamma_i(t - \bar{t}))}{\bar{L}_i} L_{it} \right)^{1-\bar{\theta}_i}$$

$$Y_{it} = \left(\frac{\bar{Y}_i}{\bar{K}_i} K_{it} \right)^{\bar{\theta}} \left(\frac{\bar{Y}_i \exp(\gamma_i(t - \bar{t}))}{\bar{L}_i} L_{it} \right)^{1-\bar{\theta}}$$

To simplify the notation, I define (where $\xi_i = 1$ in the Cobb-Douglas cases):

$$A_{ik} \equiv \frac{\xi_i \bar{Y}_i}{\bar{K}_i}, \quad A_{ilt} \equiv \frac{\xi_i \bar{Y}_i \exp(\gamma_i(t - \bar{t}))}{\bar{L}_i}$$

and write:

$$Y_{it} = \left[\bar{\theta}_i (A_{ik} K_{it})^{\frac{\sigma_i-1}{\sigma_i}} + (1 - \bar{\theta}_i) (A_{ilt} L_{it})^{\frac{\sigma_i-1}{\sigma_i}} \right]^{\frac{\sigma_i}{\sigma_i-1}} \quad (3.20)$$

$$Y_{it} = (A_{ik} K_{it})^{\bar{\theta}_i} (A_{ilt} L_{it})^{1-\bar{\theta}_i} \quad (3.21)$$

$$Y_{it} = (A_{ik} K_{it})^{\bar{\theta}} (A_{ilt} L_{it})^{1-\bar{\theta}} \quad (3.22)$$

To obtain the parameters the Cobb-Douglas production functions, I set $\bar{\theta} = 1/3$, $\bar{\theta}_a = 0.54$, $\bar{\theta}_m = 0.29$, and $\bar{\theta}_s = 0.34$. This leaves γ_i to estimate. I drop equations (3.18)–(3.19) and estimate the output equations (3.17) jointly for the three sectors where I parameterize A_k and A_l in the same way as in the case in the CES and again assume AR(1) error

Table 3.2: Average Annual Growth Rates of Labor-augmenting Technological Progress (in %)

	Aggregate	Agriculture	Manufacturing	Services
CES	1.7	9.2	2.0	1.3
C-D with $\bar{\theta}_i$	1.8	9.3	2.1	1.5
C-D with $\bar{\theta}$	1.8	6.2	2.1	1.5

Table 3.3: Average Annual Growth Rates of TFP (in %)

	Aggregate	Agriculture	Manufacturing	Services
C-D with $\bar{\theta}_i$	1.2	3.5	1.5	1.0
C-D with $\bar{\theta}$	1.2	4.1	1.4	1.0

terms. Table 3.2 reports the resulting average annual growth rates of labor-augmenting technological progress. To put them into perspective, it is useful to calculate the implied growth rates of TFP. For the two Cobb-Douglas production functions, they are obtained as $\exp(\gamma_i \bar{\theta}_i)$. It is not clear how to calculate TFP for the CES production function, so I don't attempt to do this here. Table 3.3 shows the growth rates for TFP. They are sizeable compared to what other studies find; see for example Jorgenson, Gollop, and Fraumeni (1987). The reason for this is that I have not taken into account improvements in the quality of sectoral labor (e.g., through increases in years of schooling and experience). In the estimation, such improvements show up as labor-augmenting technological progress.

Sectoral labor allocations

I now turn to the sectoral labor allocations that result from the optimal choices of stand-in firms with are endowed with these production functions. Solving the first order conditions

to the firm problem, (3.15)–(3.16), for sectoral labor, I obtain for each functional form:

$$L_{it} = \left(\bar{\theta}_i \left(\frac{1 - \bar{\theta}_i}{\bar{\theta}_i} \frac{A_{ilt} r_{it}}{A_{ik} w_{it}} \right)^{1 - \sigma_i} + (1 - \bar{\theta}_i) \right)^{\frac{\sigma_i}{1 - \sigma_i}} \frac{Y_{it}}{A_{ilt}} \quad (3.23)$$

$$L_{it} = \left(\frac{1 - \bar{\theta}_i}{\bar{\theta}_i} \frac{A_{ilt} r_{it}}{A_{ik} w_{it}} \right)^{\bar{\theta}_i} \frac{Y_{it}}{A_{ilt}} \quad (3.24)$$

$$L_{it} = \left(\frac{1 - \bar{\theta}}{\bar{\theta}} \frac{A_{ilt} r_{it}}{A_{ik} w_{it}} \right)^{\bar{\theta}} \frac{Y_{it}}{A_{ilt}} \quad (3.25)$$

It is worth to take a moment and build intuition for how the different features of technology affect the allocation of labor across the three broad sectors. The term Y_{it}/A_{ilt} is common to the right-hand sides because more labor-augmenting technological progress implies that less labor is needed to produce the given quantity Y_{it} of sectoral value added. The other right-hand side terms differ among the different functional forms. It is easiest to start with the Cobb-Douglas cases. The term $[(1 - \bar{\theta}_i)/\bar{\theta}_i]^{\bar{\theta}_i}$ is decreasing in $\bar{\theta}_i$ and captures that a sector with a larger capital share receives less labor than a sector with a smaller capital share. The term $[(A_{ilt} r_{it})/(A_{ik} w_{it})]^{\bar{\theta}_i}$ captures that an increase in the relative rental rate of capital to labor (where both rental rates are expressed relative to the relevant A 's) leads to a decrease in the sectoral capital-labor ratio and an increase in sectoral labor, and that this increase is larger when the sectoral capital share is larger.

When the economy is poor, the economy-wide capital-labor ratio is low and the relative rental rate of capital to labor is high, implying that a sector with a larger capital share receives relatively less labor. As the economy develops, the capital-labor ratio increases and the relative rental rate of capital to labor decreases, implying an increase in the relative labor of this sector. This is the mechanism that Acemoglu and Guerrieri (2008) emphasized.

For the case of the CES production functions, there is an additional substitution effect: if the elasticity of substitution is larger than one, a higher rental rate of capital relative to labor leads to larger reduction of the capital-labor ratio than in the Cobb-Douglas case;

if the elasticity of substitution is smaller than one, a higher rental rate of capital relative to labor leads to smaller reduction of the capital-labor ratio than in the Cobb-Douglas case. Hence, when the economy is poor and the relative rental rate of capital to labor is high, a sector with a smaller elasticity of substitution receives relatively less labor. As the economy develops, the relative rental rate of capital to labor decreases, implying an increase in the relative labor in this sector. This is the mechanism that Alvarez-Cuadrado, Long, and Poschke (2012) emphasized.

Figure 3.2 plots the labor allocations that are implied by equations (3.23)–(3.25) when I plug in the estimated parameter values for σ_i and $\bar{\theta}_i$ and the data values of the exogenous variables A_{ik} , A_{ilt} , Y_{it} , r_{it} , and w_{it} . Note that I have normalized hours worked in 1948 to one. All three functional forms do a reasonable job at capturing the long-run secular changes in sectoral hours worked. In particular, the CES and the Cobb-Douglas with different capital shares perform very similarly. The Cobb-Douglas with equal capital shares does somewhat worse, in particular in manufacturing and in agriculture. The reason for this is that it misses that manufacturing has a larger labor share and agriculture has a smaller labor share than the aggregate. As a result, the Cobb-Douglas with equal labor shares systematically allocates too little labor to manufacturing and too much labor to agriculture. Compared to the other two functional forms, manufacturing hours predicted by the Cobb-Douglas with equal shares are therefore lower and agricultural hours are higher. Nonetheless, even the Cobb-Douglas with equal shares gets the main secular trends of hours mostly right.

The reason why the Cobb-Douglas production function with equal shares gets the main secular trends of hours mostly right is that the CES production function of agriculture has both by far the largest relative weight on capital and the largest elasticity of substitution whereas the other two CES production functions have fairly similar relative weights and elasticities of substitution. Hence, the effects on structural transformation out of agriculture

of the relatively large weight on capital and the relatively large elasticities of substitution go in opposite directions and largely cancel each other, leaving the effects of uneven labor-augmenting technological progress as the dominating force.

Relative prices

I continue with the relative prices of sectoral value added that each of the three functional forms implies under the maintained assumption that the sectoral stand-in firm behave competitively. The first order conditions to the firm problem (3.14) imply that the real wage w_{it} equals the marginal product of labor. Hence, cost minimization implies that the price of sector i 's value added relative to services is given by:

$$P_{it} = \frac{P_{it}}{P_{st}} = \frac{W_{it}}{W_{st}} \frac{MPL_{st}}{MPL_{it}}$$

While I observe the nominal wages W_{it} and W_{st} in the data, the model implies the values for the marginal products MPL_{it} and MPL_{st} .

Figure 3.3 reports the results that the three functional forms imply for the relative prices, all of which do reasonably well with respect to the relative price of agriculture. In contrast, the CES does worst with respect to the relative price of manufacturing and the two Cobb-Douglas perform nearly identically well.

3.6 Implications for Building Multi-sector Models

Equalizing marginal value products

Many builders of multi-sector models assume that the marginal value products of each primary factor of production (here capital and labor) are equalized across sectors. A set of assumptions that implies this is: (i) competitive firms rent each factor of production in a common factor market at a common nominal rental rate; (ii) each factor of production can be moved across sectors without any frictions or costs. Unfortunately, it turns out that in the U.S. the nominal rental rates are not equalized across sectors. Figure 3.4 shows that

Figure 3.2: Hours Worked (Data=1 in 1948)

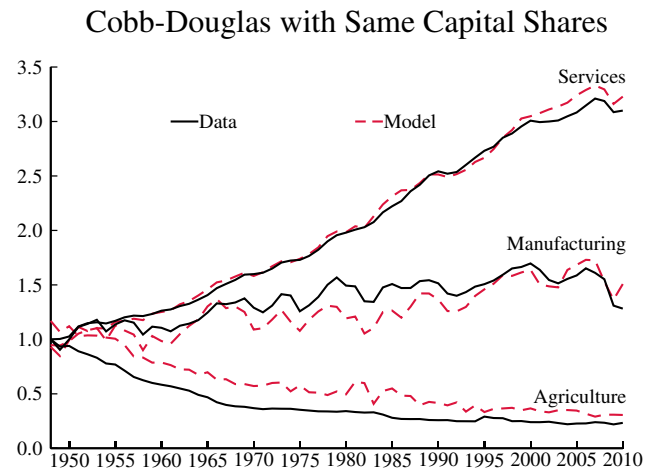
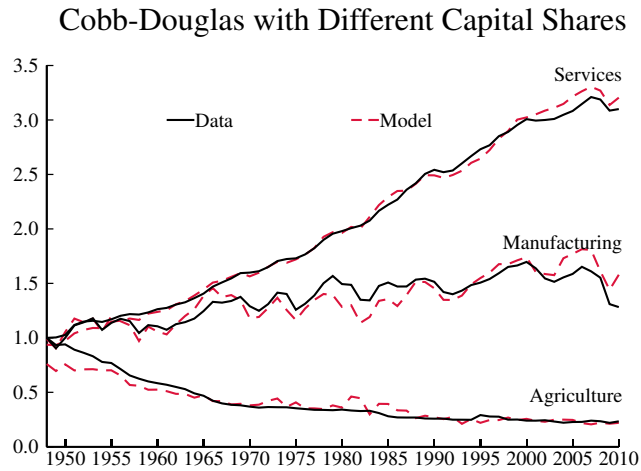
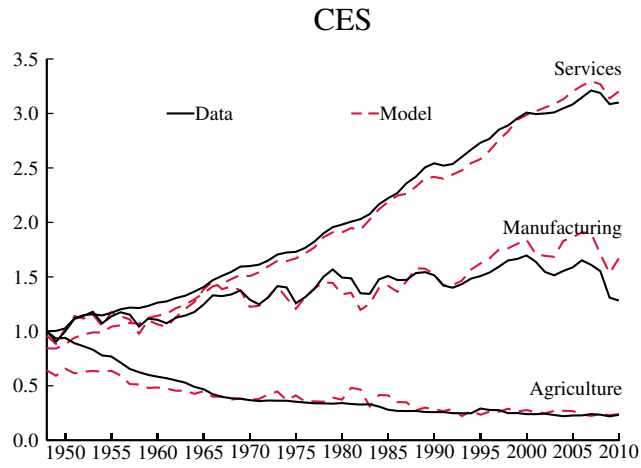
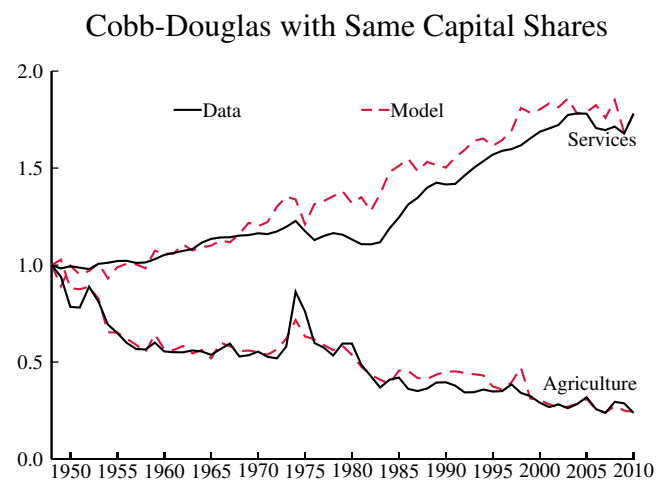
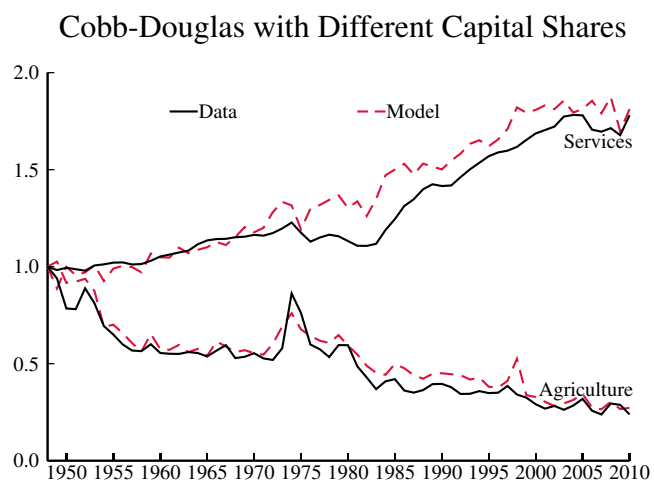
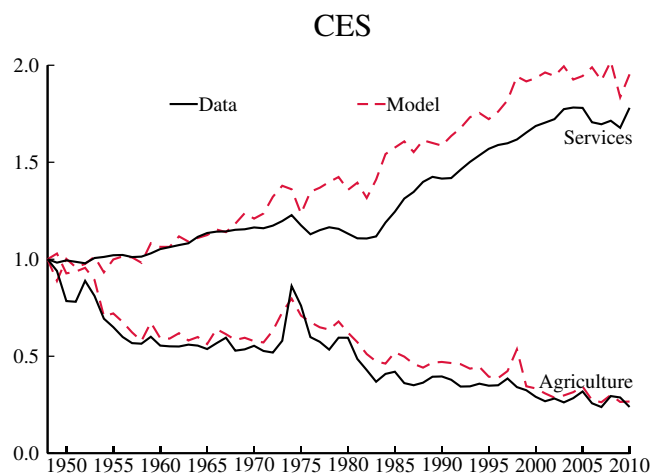


Figure 3.3: Sectoral Prices Relative to Manufacturing (Data=1 in 1948)



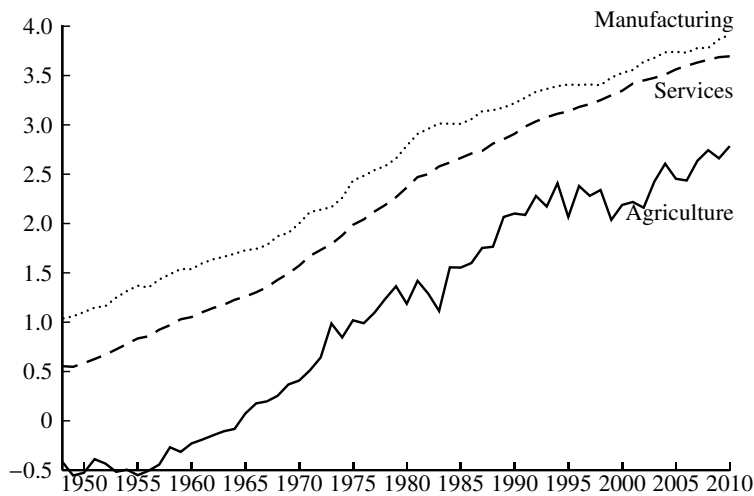
the marginal value product is somewhat higher in manufacturing than in services, and is much lower in agriculture than in the other two sectors. Given this evidence, my estimation strategy of system (3.17)–(3.19) has been to use the *observed* nominal rental rates and prices of sectoral value added instead of imposing that nominal rental rates are equalized across sectors.

The previous paragraph raises the question, in which way, if any, the estimated sectoral production functions may be used in multi-sector models that equalize marginal value products across sectors. The answer is that in order to incorporate the estimated production functions in a multi-sector model, one needs to add a reason for the difference in the marginal value products across sectors. In the case of labor, the most obvious reason is differences in sectoral human capital that reflect difference in innate ability, experience, and years of schooling like in Jorgenson, Gollop, and Fraumeni (1987) or Herrendorf and Schoellman (2012). The latter paper, for example, found that average sectoral human capital is lower in agriculture than in the rest of the U.S. economy, and that the difference accounts for almost all of the difference in nominal wages. This implies that per efficiency unit of labor the average nominal wages were roughly equal in agriculture and the rest of the U.S. economy during the last thirty years. In the case of capital, obvious reasons for the difference in the marginal value products across sectors are unmeasured quality differences in the measured stock of sectoral capital and unmeasured parts of the stock of capital; see Jorgenson, Gollop, and Fraumeni (1987) and McGrattan and Prescott (2005) for further discussion.

Value-added versus final expenditure production functions

So far, I have focused on value-added production functions. While this is a natural starting point when studying the technology side of structural transformation, Herrendorf, Rogerson, and Valentinyi (2013b) pointed out that one can also interpret the sectoral outputs as

Figure 3.4: Sectoral Marginal Value Products of Labor (in logs)



final goods that are consumed or invested. In this subsection I discuss the implications for models of structural transformation that interpret sectoral outputs as final goods.

Before I delve into the details, an example may be helpful. Consider a household which derives utility from the three consumption categories agriculture, manufacturing, and services. Herrendorf, Rogerson, and Valentinyi (2013b) pointed out that one can take two different perspectives on what these categories are: the value-added perspective and the final expenditure (or final goods) perspective. The value-added perspective breaks the household's consumption into the value-added components from the three sectors and assigns each value-added component to a sector. For example, if the household consumes a cotton shirt, then the value added of producing raw cotton goes to agriculture, the value added of processing to manufacturing, and the value added of distribution to services. This means that the consumption categories in the utility function of the household are the value added that is produced in the three sectors agriculture, manufacturing, and services. In contrast, the final expenditure perspective assigns each consumption good to one of the three consumption categories. The cotton shirt, for example, would typically be assigned

to manufacturing. This means that the consumption categories in the utility function of the household become final goods categories. This dramatically changes the meaning of the three sectors, as the manufacturing sector now produces the entire cotton shirt, implying that it combines the value added from the different industries that is required to produce the cotton shirt.

Although the sectoral production functions under the two perspectives are very different objects, I emphasize that they are two representations of the *same* underlying data, which are linked through intricate input-output relationships. To see the implications of this, it is useful to think that at a first approximation the sectoral output under the final goods perspective are some weighted average of the sectoral value added from the value-added perspective. This implies that the properties of the production function under the final goods perspective are a weighted average of the properties of the properties of the production functions under the value-added perspective. Valentinyi and Herrendorf (2008) showed that as a result the capital shares of industry gross output tend to be closer to the aggregate capital share than the capital shares of industry value added. This suggests that the sectoral capital shares under the final goods perspective should be closer to the aggregate capital share than the sectoral capital shares under the value-added perspective. I conjecture that a similar argument applies also to the elasticity of substitution, that is, for a given sector the elasticity of substitution is closer to one under the final goods perspective than under the value-added perspective.

These arguments suggest that under the final goods perspective the sectoral production functions are closer to the Cobb-Douglas production function with a common capital share than under the value-added perspective. Since I have shown above that the Cobb-Douglas production functions with a common capital share do a reasonable job at capturing sectoral employment and relative prices under the value-added perspective, this suggests that they will also do a reasonable job under the final goods perspective. Note that since

the aggregate capital share is the same under both perspectives, it is straightforward to parameterize the Cobb-Douglas production functions with a common capital share under the final goods perspective.

3.7 Conclusion

In this paper, I have assessed the technological forces behind structural transformation, i.e., the reallocation of production factors across agriculture, manufacturing, and services. In particular, I have asked how important for structural transformation are sectoral differences in labor-augmenting technological progress, the elasticity of substitution between capital and labor, and the intensities of capital. I have estimated CES production functions for agriculture, manufacturing, and services on postwar U.S. data. I have found that differences in labor-augmenting technological progress are the predominant force behind structural transformation. As a result, sectoral Cobb-Douglas production functions with equal capital shares (which by construction abstract from differences in the elasticity of substitution and in capital shares) do a reasonably good job of capturing the main trends of U.S. structural transformation.

Finally, this paper restricted attention to the postwar United States. It is also of interest to extend this analysis to a larger set of countries, in particular to situations which feature a larger range of real incomes. This will be useful in assessing the extent to which one can account for the process of structural transformation with stable sectoral technologies. This exercise is left for future work.

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APPENDIX A

DETAILS ON U.S. AND NORWAY DATA FOR CHAPTER 1

U.S. data in Section 1.3 are obtained from the National Center for Education Statistics Common Core of Data and School District Demographics System. Norwegian data are obtained from the StatBank data service through Statistics Norway. For all graphs and tables in Section 1.3, U.S. data are from the year 2000, while Norwegian data are from 2002. Data are available for Norway beginning in 2000 as well, but there are many missing values in 2000 and 2001. In the graphs I use year 2002 data for Norway because it was the first year with complete data for nearly all districts; however, the patterns described are broadly consistent across all years since 2000.

In the U.S. data, total revenues are categorized as coming from local, state, and federal sources. In the Norwegian school district data, however, the sources are not defined this way, so I categorize total revenue into local and federal sources as follows. Local revenue is defined as the amount coming from user payments, revenues from sales and hiring, property tax, and other direct and indirect taxes. All other revenue sources are classified as federal.

The school district level income measure for the U.S. is median earnings for the total population 16 and over, from Table P85 of the 2000 Census School District Tabulation (STP2) Data. The school district level income measure for districts in Norway is median gross income for residents 17 and over, from Table 05671 in Statistics Norway's StatBank.

APPENDIX B

DETAILS ON ESTIMATION OF TAX FUNCTIONS FOR CHAPTER 1

Tax functions for the U.S. and Norway are estimated using data from the 2010 edition of the OECD publication *Taxing Wages*. This edition contained a special feature section on tax reforms and changes in tax burdens from 2000–2009. Included with this special section are year 2000 net personal average tax rates for the following types of households: single individuals with either zero or two children; one earner married couples with either zero or two children; and two earner married couples with either zero or two children.

Net personal average tax rates include employee social security contributions plus central and local government income taxes, less cash benefits to families. The tax rates are computed for earnings levels varying from 50% of the average wage (AW) to 250% of the AW, in 1% increments, where the average wage is defined as the average gross wage earnings of a private sector adult male full-time (manual or non-manual) worker in that year and country.

For the two earner married couples, net personal average tax rates are computed assuming: (i) the husband’s earnings are variable, and the wife earns 67% of the average wage, or (ii) the husband’s earnings are variable, and the wife earns 100% of the average wage. Thus, in the first case tax rates are computed for households whose total earnings range from $50\% + 67\% = 117\%$ of AW up to $250\% + 67\% = 317\%$ of AW. Similarly, in the second case tax rates are computed for households whose total earnings range from $50\% + 100\% = 150\%$ of AW up to $250\% + 100\% = 350\%$ of AW.

Because childless families are not modeled, I only utilize tax data for households with children in the estimation. As in the text, denote the net average tax rate by $\tau(\hat{y})$, where $\hat{y} = \frac{y}{AW}$. Thus, for example, $\tau(2.5)$ is the net average tax rate paid by a parent whose labor earnings are 250% of the average wage. The estimation procedure is as follows:

1. For the four types of households with children, average the available OECD estimated tax rates at each 1% increment from 50% to 350% of AW.
2. Compute the marginal tax rate from 325% to 350% of AW as:

$$\frac{3.5 \cdot \tau(3.5) - 3.25 \cdot \tau(3.25)}{3.5 - 3.25}$$

3. Tax each additional 1% income increment up to the top income tax bracket (see next step) based on the marginal tax rate computed in step 2.
4. The OECD provides the top marginal personal income tax rate rates in each country, along with the level (as a % of AW) at which that rate becomes effective. For the U.S. in 2000, this rate was 48% at 8.9 times AW, and for Norway in 2000 it was 55.3% at 2.6 times AW. Assuming a two earner household where the wife makes 100% of AW, then this rate would become effective on the husband’s variable earnings when household earnings are 9.9 times AW for the U.S. and 3.6 times AW for Norway. I thus assume that each additional 1% income increment beyond these thresholds is taxed at their respective top marginal rates.

5. Having constructed this series from 0.5 times AW to 10 times AW, estimate the following functional form:

$$\tau(\hat{y}) = \beta_0 + \beta_1\hat{y} + \beta_2\hat{y}^{\beta_3} \quad (3.26)$$

6. The estimated parameters imply a tax liability that approaches negative infinity as income approaches zero. Therefore, I assume that $\tau(\hat{y}) \geq 0.10AW, \forall \hat{y}$. This lower bound was chosen because the lowest net personal average tax rate in the OECDU data is -11.8% for single parents with two children in the U.S. at 0.5 times AW, meaning these parents are net recipients of transfer payments from the government equal to 5.9% of AW. Setting the bound at 10% of AW allows for the possibility that individuals between 0% and 50% of AW may receive somewhat larger transfers, but the government will not write anyone a blank check. I have set the bound as high as 100% of AW, and results are not sensitive to this choice.

APPENDIX C

CONSTRUCTION OF HISTORICAL DATA FOR CHAPTER 2

I take several historical data series as exogenous to the model, and this section details the construction of those series. Data are taken from several sources in order to construct a consistent series since 1900. From 1900 to 1958, most data were collected every two years and published in the Biennial Survey of Education (BSE). Since 1962, the Digest of Education Statistics (DES) has been published annually. Other publications including the annual U.S. Statistical Abstract, the Bicentennial Edition “Historical Statistics of the United States: Colonial Times to 1970”, and “120 Years of American Education: A Statistical Portrait” help in bridging breaks between series, as well as verifying continuity of series that may have changed names from year to year. Also, many data were revised in later publications, so I take the most recent published estimates where available.

First, let c_t be the total annual cost of college per student. I assume that the total cost for educating *all* students in the U.S. in a given year equals the total revenues received in the current period by all institutions of higher education. Dividing this by the total enrollment each year yields the total annual cost per student. Alternatively, one could use the total current expenditures rather than revenues as the measure of total cost, but this makes little difference quantitatively because revenues and expenditures track each other quite closely. In addition, the revenue data is preferable because it allows me to determine how much of costs are paid out-of-pocket by students for tuition and fees, and how much comes from other sources such as state, local, and federal governments, private gifts, endowment earnings, auxiliary enterprises (athletics, dormitories, meal plans, etc.), and other sources. The numerator for c_t is constructed as follows:

- 1997-2000: total current revenue must be computed as the sum of current-fund revenue for public and private institutions, from the DES.
- 1976-1996: total current revenue equals “current-fund revenue of institutions of higher education” from the DES.
- 1932-1975: total current revenue equals “current-fund revenue of institutions of higher education” in “120 Years of American Education: A Statistical Portrait”.
- 1908-1930: total current revenue equals “total receipts exclusive of additions to endowment” for colleges, universities, and professional schools, from the BSE.
- 1900-1908: total current revenue equals “total receipts exclusive of additions to endowment” for colleges, universities, and professional schools, and is computed as (income per student)*(total students, excluding duplicates) from the BSE. Continuity with later years can be verified using the “income per student” series, which was published from 1890-1920.

The denominator for c_t is constructed as follows:

- 1946-2000: total fall enrollment for institutions of higher education, from the DES.

- 1938-1946: resident college enrollments, from the BSE. Continuity with the later series can be verified in that year 1946 data matches in both.
- 1900-1938: total students, excluding duplicates, in colleges, universities, and professional schools, from the BSE. Continuity with the later series can be verified in that year 1938 data matches in both.

Second, I construct two time series which estimate the share of annual college costs paid out-of-pocket by students. One measure, λ_t , includes only tuition and fees paid by students, and the other measure, ϕ_t includes tuition, fees, room, and board. In each year λ_t equals total tuition and fees paid by all students divided by total current revenue received by institutions of higher education. Similarly, ϕ_t equals total tuition, fees, room, and board aid by all students divided by total current revenue received by institutions of higher education. In each case, the measure of total current revenue is the same time series as was used above in constructing c_t . The time series for λ_t is constructed as follows:

- 1997-2000: current fund revenues from tuition and fees for all institutions of higher education is computed as the sum of the series for public and private institutions, from the DES.
- 1976-1996: current fund revenues from student tuition and fees, from the DES.
- 1930-1975: current fund revenues from student tuition and fees, from “120 Years of American Education: A Statistical Portrait”.
- 1918-1930: receipts of universities, colleges, and professional schools for student tuition and fees, from BSE.
- 1900-1918: we are unable to obtain proper data for these years.

The time series for ϕ_t is constructed as follows:

- 1976-2000: Average tuition, fees, room, and board paid by full-time equivalent (FTE) students is obtained from the DES. We multiply this by enrollment of FTE students, also from the DES, and divide by the current fund revenues to compute ϕ_t .
- 1960-1976: we are unable to obtain proper data for these years.
- 1932-1958: Data available biennially on total revenues from student tuition and fees, as well as revenue from auxiliary enterprises and activities (room and board), in the BSE. ϕ_t computed as the sum of these, divided by total current revenue.
- 1900-1930: ϕ_t computed as total revenue from student fees (included tuition, fees, room, and board) divided by total current revenue.

APPENDIX D
ESTIMATION DETAILS FOR CHAPTER 3

Table 3.4: Standard Errors of Regression Equations (3.17)–(3.19)

Specification	(16)			(17)			(18)		
	Agr	Man	Ser	Agr	Man	Ser	Agr	Man	Ser
C-D (equal)	0.080	0.026	0.010	-	-	-	-	-	-
C-D (unequal)	0.070	0.025	0.010	-	-	-	-	-	-
CES	0.072	0.026	0.010	0.036	0.050	0.020	0.054	0.027	0.011

Table 3.5: Multivariate Ljung-Box Q-Statistics

Specification	# of Lags	Degrees of freedom	Adj. Q-stat	p-value
C-D (equal)	2	18	11.157	0.888
C-D (unequal)	2	18	16.099	0.586
CES	2	162	191.927	0.054

Table 3.6: Mean Square Errors – Labor Allocation and Relative Prices

Specification	Labor Allocation			Relative Prices	
	Ag	Man	Ser	Ag	Ser
C-D (equal)	0.058	0.106	0.057	0.047	0.091
C-D (unequal)	0.074	0.103	0.054	0.046	0.088
CES	0.106	0.114	0.075	0.047	0.094

APPENDIX E

APPROXIMATE AGGREGATION OF CHAINED QUANTITY INDICES FOR
CHAPTER 3

Chain indices relate the value of an index number to its value in the previous period. In contrast, fixed–base indices relate the value of an index number to its value in a fixed base period. While chain indices are preferable to fixed–base indices when prices change considerably over time, using them may lead to problems because real quantities are not additive in general, that is, the real quantity of an aggregate does not equal the sum of the real quantities of its components. In practice, this becomes relevant when one is interested in the real quantity of an aggregate, but the statistical agencies only report the real quantities of the components of this aggregate. This appendix explains how to construct the real quantity of the aggregate according to the so called cyclical expansion procedure.

Let Y_{it} be the nominal value, y_{it} the real value, Q_{it} the chain–weighted quantity index, and P_{it} the chain–weighted price index for variable $i \in \{1, \dots, n\}$ in period t . Let $t = b$ be the base year for which we normalize $Q_{ib} = P_{ib} = 1$. The nominal and real values of variable i in period t are then given by:

$$Y_{it} = P_{it} \frac{Q_{it}}{Q_{ib}} Y_{ib} = P_{it} Q_{it} Y_{ib},$$

$$y_{it} = \frac{Y_{it}}{P_{it}} = Q_{it} Y_{ib}.$$

Let $Y_t = \sum_{i=1}^n Y_{it}$ and suppose that the statistical agency reports y_{it} , Q_{it} and P_{it} for all components i but not y_t , Q_t and P_t . Since in general $y_t \neq \sum_i y_{it}$, we need to find a way of calculating y_t .

We start by approximating Q_t using the “chain–summation” method:⁹

$$\frac{Q_t}{Q_{t-1}} = \sqrt{\frac{\sum_i P_{it-1} y_{it}}{\sum_i P_{it-1} y_{it-1}} \frac{\sum_i P_{it} y_{it}}{\sum_i P_{it} y_{it-1}}}.$$

Using this expression iteratively, we obtain Q_t as:

$$Q_t = \frac{Q_t}{Q_{t-1}} \frac{Q_{t-1}}{Q_{t-2}} \dots \frac{Q_{b+1}}{Q_b} Q_b = \frac{Q_t}{Q_{t-1}} \frac{Q_{t-1}}{Q_{t-2}} \dots \frac{Q_{b+1}}{Q_b},$$

where the last step used the normalization $Q_b = 1$. The real value and the price in period t then follow as:

$$y_t = Q_t Y_b,$$

$$P_t = \frac{Y_t}{Q_t Y_b}.$$

⁹This is only an approximation because sums like $\sum_i P_{it-1} y_{it}$ are not directly observable and the statistical agency typically uses more disaggregate categories than $i \in \{1, \dots, n\}$ to calculate them.

BIOGRAPHICAL SKETCH

Christopher M. Herrington is from Birmingham, Alabama. He earned a bachelor's degree, *summa cum laude*, in Business Administration from Birmingham-Southern College in 2005 and was also inducted to Phi Beta Kappa the same year. From 2005 through 2008, he worked as an Assistant Economist at the Federal Reserve Bank of Richmond.

During graduate school at Arizona State University, Christopher served as a Research Assistant to Professors Richard Rogerson, Berthold Herrendorf, and Edward Prescott. He also served as a Research Analyst at the Federal Reserve Bank of Minneapolis during the summers of 2011 and 2012. He earned a master's degree in economics from Arizona State University in 2010. He was honored to receive the Hardison Award for the best microeconomics comprehensive examination, the Prescott Award for summer research support, and the Rondthaler Award for outstanding dissertation research.

Starting in the Fall of 2013, Christopher will serve as an Assistant Professor of Economics and Finance in the Mitchell College of Business at the University of South Alabama.