Engineering-Based Problem Solving Strategies In AP Calculus:

An Investigation Into High School Student Performance On

Related Rate Free-Response Problems

by

John Thieken

A Dissertation Presented in Partial Fulfillment
Of the Requirements for the Degree
Doctor of Philosophy

Approved June 2012 by the
Graduate Supervisory Committee:

Tirupalavanam Ganesh, Chair
Finbarr Sloane
James Middleton

ARIZONA STATE UNIVERSITY

August 2012

ABSTRACT

A sample of 127 high school Advanced Placement (AP) Calculus students from two schools was utilized to study the effects of an engineering design-based problem solving strategy on student performance with AP style Related Rate questions and changes in conceptions, beliefs, and influences. The research design followed a treatment-control multiple post-assessment model with three periods of data collection. Four high school calculus classes were selected for the study, with one class designated as the treatment and three as the controls. Measures for this study include a skills assessment, Related Rate word problem assessments, and a motivation problem solving survey. Data analysis utilized a mixed methods approach. Quantitative analysis consisted of descriptive and inferential methods utilizing nonparametric statistics for performance comparisons and structural equation modeling to determine the underlying structure of the problem solving motivation survey. Statistical results indicate that time on task was a major factor in enhanced performance between measurement time points 1 and 2. In the experimental classroom, the engineering design process as a problem solving strategy emerged as an important factor in demonstrating sustained achievement across the measurement time series when solving volumetric rates of change as compared to traditional problem solving strategies. In the control classrooms, where traditional problem solving strategies were emphasized, a greater percentage of students than in the experimental classroom demonstrated enhanced achievement from point 1 to 2, but showed decrease in achievement from point 2 to 3 in the measurement time series. Results from the problem solving motivation survey demonstrated that neither time on task nor instruction strategy produced any effect on student beliefs about and perceptions of problem solving. Qualitative error analysis showed that type of instruction had little effect on the type and number of errors committed, with the exception of procedural errors from performing a derivative and errors decoding the problem statement. Results demonstrated that students who engaged in the engineering design-based committed a larger number of decoding errors specific to Pythagorean type Related Rate problems; while students who engaged in routine problem solving did not sustain their ability to correctly differentiate a volume equation over time. As a whole, students committed a larger number of *misused data* errors than other types of errors. Where, misused data errors are the discrepancy between the data as given in a problem and how the student used the data in problem solving.

ACKNOWLEDGEMENTS

Thank you to my committee for your continued support and patience throughout my studies.

Dr. Ganesh, I cannot express how much I appreciate all your support over my graduate career. You have given me so much by including me in your research and your life. I will never forget the lessons and opportunities you have shared with me.

Dr. Sloane, without you I would never have made it this far. I am very fortunate that you were in your office that summer day. You have taught me the subtleties and integrity of good research, while grounding me in the reality of the process. I am forever in your debt for always being there for me.

Dr. Middleton, your passion and enthusiasm for teaching are addictive. Thanks for all your help and direction on both my comprehensive exams and my dissertation. Your ability to conceptualize the larger picture has shown me that the "big" idea is always within reach.

TABLE OF CONTENTS

Page

LIST OF TABLES

Table Page

Table Page

LIST OF FIGURES

CHAPTER 1

INTRODUCTION

Overview

This dissertation study investigates the effects of an engineering-based problem solving strategy in high school AP Calculus. Problem solving has a long history, from the simple laboratory experiments of the Gestaltists in Germany to the investigations of cognition and its role in problem solving (Lester & Kehle, 2003). Problem solving can be described as the cognitive process applied to achieve a goal when no solution strategy is initially obvious to the problem solver (Mayer, 1992). With Mayer's description of problem solving two distinctions become clear. First, problem solving is a cognitive process. Second, the existence of a problem does not imply a lack of knowledge on the problem solver's part, only that the immediate solution method is not obvious. When students engage in non-routine problems, i.e., the problem solver does not have a learned solution method ready to apply (Mayer & Wittrock, 1996), the probability of success becomes a function of the problem solver's conceptions, beliefs, influences, and knowledge. To be good problem solvers we must possess the ability to explore and engage the unfamiliar. Without the ability to explore and engage the unfamiliar there could be no innovation or progress. As such, the importance of problem solving is considered a critically important cognitive attribute, and has been widely reviewed, across many domains. The many areas investigated include Labor Market (Binkley et al., 1999), Education (Didi et al., 1993), Schooling (OECD, 1997), Goal of School Curriculum (Svecnik, 1999), Problem Solving in

Mathematics (De Corte et al., 1996), and Roles in Society (Trier & Peschar, 1995) (as cited in Klime, 2004, pg. 8). The perceived critical importance begs the question of how do we get students' to make connections between their beliefs, influences and knowledge as a means of successfully engaging with the unfamiliar? The National Research Council (NRC, 2009) emphasized the act of design as a problem-solving process, capable of drawing together the abstract and the concrete. Research has shown that design-based curricula can lead to significant learning gains, improved retention, and enhanced engagement when contrasted with scripted inquiry (Mehalik et al., 2007). If students gain success by engaging in design-based curriculum, then why not approach problem solving as a design challenge? The field of engineering, by definition, is the science of engaging in the unfamiliar. At its core, all engineered creations result from the application of a design process. Consequently the Engineering Design Process allows for the integration of skills, knowledge, planning, and implementation by solving real world problems.

Problem Statement

Traditional problem solving strategies, where students are taught procedures to solve specific types of problems, provide a disjointed view of problem solving. As a result, students learn procedural processes, rather than methods to connect their prior knowledge and abilities to engage in all types of problems. Problem solving instruction needs to encapsulate procedures and strategies that transcend content domain, providing a means to successfully engage in any situation.

Research Questions

The purpose of this study was to investigate how students who were taught an engineering-based problem solving strategy compare to students taught traditional problem solving strategies. Groups were compared across the content domain Related Rates, concepts of motivation, and error structures to answer the following questions.

1. What impact does type of instruction have on high school student performance on AP calculus Related Rate problems (RRP)?
2. What impact does the type of instruction have on high school students' longitudinal achievement on AP calculus RRP?
3. What impact does the type of instruction have on the types of errors high school students make on AP calculus RRP?
4. Do the following five factors underlie the twenty problem solving motivation survey items in the following way?

Mastery:	Q3	Q4	Q16	Q20
Ability/Effort:	Q8	Q11	Q17	Q19
Expectations:	Q2	Q7	Q14	Q18
Performance:	Q1	Q5	Q9	Q15
Value:	Q6	Q10	Q12	Q13

5. Does the type of instruction have an impact on high school AP calculus students' beliefs and perceptions within the five motivation factors listed in research question 4 over time?

Broader Impact

This study served to explore and simplify the complexities of problem solving instruction as it relates to mathematics achievement. Even though the ability to be a good problem solver is a life necessity, we often overlook the connection between solving math problems and solving everyday problems. It is easy to see the connections between mathematics and professional fields like engineering, physics, accounting, etc. However, is problem solving in mathematics only relevant to math intensive professions? When students are taught to be good problem solvers through a structured process, and when that process and those skills are utilized within multiple domains, they will gain insight into problem solving as a holistic ability. I have an engineering background and therefore it was natural for me to link problem solving in mathematics to problem solving in engineering. By engaging students in the act of problem solving through the engineering design process I believe that students are afforded access to the skills necessary in transforming problem solving from the abstract to the concrete. I envision three broader impacts from this study. First, by combining problem solving and engineering, hands-on design activities can be used to involve students in the concrete and physical aspects of problem solving. When students start to engage in problem solving through design and innovation, they will own their learning and become intrinsically motivated. Second, by using the engineering design process as a model of problem solving students will be exposed to the core of exploration and innovation? As a by-product, we can demystify Science, Technology, Engineering, and Mathematics (STEM) influenced careers for middle class students who are likely capable, but may not have the confidence to purse those areas. Third, by using the instructional approaches described in this study the door is open for a true integration of STEM in the classroom.

CHAPTER 2

LITERATURE REVIEW

Problem Solving: A Brief Overview

Problem solving has a long history, from the simple laboratory experiments of the Gestaltists in Germany to the investigations of the social influences and the development of situated problem solving. Some of the earliest forms of problem solving research come from the work performed in Germany. The Gestaltists performed simple laboratory experiments placing as their primary assumption that the cognitive process engaged with simple problems were representative of the process engaged in when solving realistic problems (Lester & Kehle, 2003). Following the Gestaltists, cognitive scientists began to research the cognitive processes individuals engage as they problem solve. A key assumption for this work was that a person's ability to solve a problem was guided by internal goals and that the general cognitive process used was the same for all types of problems (Lester & Kehle, 2003). During this cognitive exploration period of research Newell and Simon (1972) sought to discover a global theory of problem solving and approached their research through the framework of information systems. The information systems model served to expand on the 3-stage model promoted by Poyla (1945) through investigations of cognitive structures as a function of content and knowledge domains. Following the work of Newell and Simon, researchers began to understand that problem solving varied across content and knowledge domains and could not be simplified into a global theory. As such researchers began to investigate problem solving as a function of knowledge domain. By concentrating or separating problem solving observations into a domain area, researchers were able to study aspects of problem difficulty, characteristics of good problem solvers, differences between novice and expert problems solvers, and domain specific strategies/heuristics. Some of the most recognized research in this era comes from Shoenfeld (1982, 1985) and his work on comparing the processes of good and poor problems solvers (Schoenfeld & Hermann, 1982). Along with expert/novice comparisons, Shoenfeld (1987) led the charge on the importance of metacognition within problem solving activities. He studied, how the knowledge of one's own beliefs, thought processes, and ability to regulate and monitor affected one's ability to problem solve. Finally, recent research has highlighted social influences and their affect on problem solving success.

The short history on problem solving is meant to give the reader a temporal development of problem solving, and to intellectually undergird the next section, "What is Problem Solving?" Although the term problem solving has taken on many forms over its history and in fact can still be referenced in many different texts containing similar and yet subtly different meanings. Consequently, it is important that a formal definition of problem solving is presented before moving forward. As stated above, there are many interpretations and definitions of problem solving. Perspectives and references to problem solving for this dissertation will follow Mayer's (1992) conceptions of problems solving, "Problem solving is cognitive processing directed at achieving a goal when no solution method is obvious to the problem solver (pg. 47)" (as cited in Mayer and

Wittrock, 1996). There are two items of note in this definition, (1) problem solving is a cognitive process and (2) existence of a problem does not imply that the problem solver has no have knowledge of the problem or the solution, only that the immediate solution method is not obvious. The second distinction allows the reader to differentiate between routine problems and non-routine problems. A routine problem exists when the problem solver already posses a solution method and a non-routine problem is when the problem solver does not have a previously learned solution method ready to apply (Mayer & Wittrock, 1996). This second point enables us to investigate problem solving as a function of the problem solver's conceptions, beliefs, influences, and knowledge. As such an overall framework of problem solving consists of a Problem Solving Process, Problem Solving Competencies, and Problem Solving Influences.

The Problem Solving Process

Similar to the definition of problem solving there are many different conceptions of what the problem solving processes entails. In a review of literature, spanning many different domains and disciplines, (Peschar, 2004) found a total of 151 different descriptions of problem solving. The problem solving process has been described by researchers in different ways and range from Polya's (1945) rather simplistic description of *Define the Problem*, *Form a Solution*, and *Evaluate the Solution*, to Peschar's (2004) more complex descriptions of *Search for Information*, *Structure and Integrate Into a Mental Representation*, *Reason Based on the Model*, *Plan Actions*, *Execute and Evaluate*, *Continuously Process External Information and Feedback*. Whether complex or simple we can see the basic underlying constructs in both representations: understanding the problem, devising a solution method, applying the solution method, and evaluating the solution. The overall structure of problem solving is overtly simple. The complexities come into to play when we investigate and break down each step for the purposes of instruction.

Problem Solving Competencies

Problem solving competencies include the problem solver's abilities, knowledge, and literacy domains. Problem solvers abilities are comprised of what they can do analytically, creatively, and practically (Peschar, 2004). Analytic abilities include identifying and understanding the problem (a well defined problem presents the given state, the goal state, and the allowable operators in a specific and clear manner (Mayer & Wittrock, 1996), setting up a strategy for solving the problem, and monitoring the solution process (also known as metacognition).

Creative abilities describe the problem solvers' ability to make connections and develop relationships between the problem and their own knowledge. A problem solvers' creative abilities allows him/her to purposefully engage appropriate options when problem solving. Practical abilities go beyond the process of solving problems to include one's ability to apply problem solving strategies to real-life problems. Note that applications to real-life do not necessarily include involvement with math or science, since the working definition for problem solving within this dissertation makes a point to describe a problem as any situation where a solution method does not immediately present itself. Problems can exist in all facets of life in various contexts, not all of which need to include mathematics and science, e.g. realizing that your car won't start and needing to get to work on time. Peschar's inclusion of Practical Ability to his working definition simply speaks to the problem solver's capacity to use problem solving strategies to engage with and successfully mitigate problems. For the purposes of this dissertation, problem solving will be confined to the realm of academic problems within the mathematical domain of AP Calculus, specifically in the area of Related Rates. Related Rate problems involve finding the rate at which some variable changes.

When considering one's ability to apply learned problem solving skills and strategies to different contexts and domains, we engage with the idea of transfer. Transfer as a topic of study in education has a long history (Mayer & Wittrock, 1996). Transfer can be thought of as an application of learning. Transfer differs from "mere learning" as to transfer implies applying learning to a task within the same domain or across domains as opposed to applying learning on the same task (Perkins and Salomon, 1994). In the previous sentence, there is mention of transfer occurring within a single domain or across different domains, Thorndike (1932) referred to this as near and far transfer respectively. Up to this point I have simply presented the notion of transfer, as the application of learning. The term "learning" is somewhat ambiguous and needs to be clarified. Learning constitutes any number of constructs from knowledge to skills. As such, transfer can be a label attached to any one of these constructs. We can discuss knowledge transfer, where prior knowledge affects new knowledge or we can discuss problem solving transfer, where prior problem solving experiences affects how one solves a new problem (Mayer & Wittrock, 1996). Irrespective of these specifics, Mayer and Wittrock list four types of transfer: General Transfer of General Skill, Specific Transfer of Specific Behavior, Specific Transfer of Specific Skills, and Metacognition Control of General and Specific Skill. Within any type of process there are varying degrees to which a learner can apply the process. Transfer of knowledge and learning is certainly no different. Salomon and Perkins (1989) distinguish between two levels of transfer, high-road and low-road transfer. Where high-road transfer is effortful and conscious and low-road transfer is automatic and does not require conscious attention.

Similar to the concepts of near and far transfer, we can think of knowledge within the space of problem solving as a form of near and far. Where near knowledge would constitute what the problem solver knows concerning the problem and far knowledge would constitute what the problem solver knows about the relevant domain. Since problem solving depends heavily on domain-specific knowledge (Chi et al., 1988; Weinert & Kluwe, 1987), care should be taken to ensure that the problem-solving tasks cannot be solved as a matter of routine. Furthermore, the problems cannot place excessive domain-specific demands on the students, triggering trial-and-error behavior (Peschar, 2004). To help resolve these problems, knowledge is parsed into a number of useable components.

Within any given domain, knowledge can be separated into declarative, procedural, conceptual, and strategic knowledge (Hiebert, & Lefevre, 1986;

Marzano, 2007; Phye, 1997). Declarative knowledge is considered “the what”; it consists of facts, vocabulary, concepts, etc. Declarative knowledge is strictly informational in nature. Examples of declarative knowledge in mathematics include knowing that a square has four sides or that a quadratic equation is a second-degree equation. Procedural knowledge accounts for the “the how”; it focuses on the ability to combine, restructure, or assimilate declarative knowledge.

Procedural knowledge is oriented more toward skills, strategies, and procedures (Phye, 1997). Procedural knowledge in mathematics comprises two systems. The first is the composition of formal language or symbolic representation of mathematics. The second consists of the algorithms or rules for engaging in mathematical tasks (Hiebert, & Lefevre, 1986). Examples of procedural knowledge would include finding the linear distance between two points or using the Pythagorean theorem to solve for the length of a hypotenuse.

Conceptual knowledge is rich in relationships and connections. Students with conceptual understanding know more than isolated facts and methods. They understand why a mathematical idea is important and the kinds of contexts in which it is useful (Greeno, Pearson, & Schonfeld, 1997; Heibert, 1986). A unit of conceptual knowledge cannot exist in isolation; by definition a piece of information is only identified as conceptual knowledge when the individual recognizes its connection with other pieces of information (Hiebert & Lefevre, 1986). The example of finding the linear distance between two points can be classified as conceptual knowledge if the student recognizes that this procedure is simply an application of the Pythagorean theorem where the linear distance is the hypotenuse of a right triangle intersecting the two points in a two-dimensional Cartesian coordinate system. Hiebert and Lefevre (1986) included a fourth type of knowledge, strategic knowledge, which constitutes one's ability to sift through the many possible possibilities and determine an appropriate solution path and the oversight to stay on that path.

However, Kilpatrick et al. (2001) presented a more in-depth description through their explanation of fluency, competence, and reasoning. Procedural and conceptual fluency refers to knowing when and how to use procedures, skills, relationships, and connections appropriately, accurately, and efficiently. Strategic competence represents the ability to formulate, represent, and solve problems. Along with these proficiencies, success in mathematics requires adaptive reasoning, and productive disposition (Kilpatrick, Swafford, & Findell, 2001). Kilpatrick, et al. (2001) defined adaptive reasoning as "the capacity for logical thought, reflection, explanation, and justification" (p. 129); the see productive disposition as "the habitual inclination to see mathematics as sensible, useful, and worthwhile, coupled with a belief in diligence and one's own efficiency" (p. 131).

Problem Solving Influences

Problem solving influences consist of all things within the task environment. Where the task environment can be thought of as the problem solver (internal) and the problem solving environment (external) (Newell & Simon, 1972). How the problem solver performs on any given problem is determined by the problem solvers competencies and motivation. As competencies and abilities were elaborated on in the previous paragraphs, they will not be described in this section.

Consequently, we now focus our attention on the role of motivation. The past fifty years of cognitive motivation research has served to broaden our understanding of the key roles that expectations and values play in motivation. Current notions of achievement motivation focus on the expectations of rewards (e.g., Ames, 1984; Dweck, 1986; Elliot, 1999; Maehr, 1984; Nicholls, 1984). Expectations exist in a variety of forms unique to the individual and the situation. In the locus of achievement, expectation can be conceptualized as the combination of personal perceptions and beliefs that shape how an individual expects to perform on a task on activity. As such, achievement motivation theory investigates how perceptions of cause, ability, and control influence achievement motivation (Stipek, 1996). Perceptions of cause include all aspects that an individual credits to the success or failure on a task. At its center, attribution theory assumes that individuals want to know the cause of certain outcomes (Wiener, 1992), in an effort to attain a stable environment and gain control (Heider, 1958). Studies have shown that improvement in effort and performance can be achieved by focusing on effort as the cause of performance (Forsterling, 1985). Additionally, attributing success to ability produces success, while attributing failure to lack of ability produces lack of achievement (Meyer & Fennema, 1985). As students attribute a lack of success to a lack of ability, they begin to view success as unobtainable (Dweck, 1986). Similar to expectations, the value an individual assigns to an activity or task influences their motivation and behavior. Atkinson's (1964) theory of achievement motivation proposed that achievement behavior is determined by the "incentive value" of the achievement goal (Stipek, 1996). Consequently, individuals are unlikely to engage with a task when they perceive no value in the outcome, regardless of their expectations for success (Stipek, 1996). The main characteristics of intrinsic motivation include: expectation of failure, perceived enjoyment, and outcome value. Achievement goal theory expands motivation theory to investigate *why* individuals are trying to achieve, rather than on *what* individuals are trying to achieve (Urban & Maehr, 1995). As a social-cognitive approach to motivation, achievement goal theory recognizes the personal and environmental factors of goal endorsement (Dweck & Leggett, 1988). The literature defines two distinct types of goals: Mastery and Performance (Ames & Archer, 1987). According to Elliot (1999), "Performance goals focus on the demonstration of competence relative to others, whereas mastery goals focus on the development of competence or task mastery" (p. 169). Adopting a mastery-goal orientation has been shown to promote positive processes and outcomes, including: persistence (Elliot & Dweck, 1988), associating effort to success (Ames & Archer, 1987; Nicholls, et al, 1985), preference for challenging work (Ames & Archer, 1988; Elliot & Dweck, 1988), creativity, and intrinsic motivation (Stipek & Kowalski, 1989). However, performance-goal orientations have also been associated with avoidance of challenging tasks (Dweck, 1986), negative affect after failure (Jagascinski & Nicholls, 1987), and use of procedural and short-term learning strategies (Ryan & Grolnik, 1986) (cited in Ames, 1992). Research suggests that a mastery-goal orientation encourages motivation and behavior likely to promote long-term and high-quality learning (Ames, 1992). Characteristics of the task can also play a key role in the degree of motivation students engage in a task. Decisions about the nature and structure of tasks, how tasks are evaluated, contingencies of rewards, level of autonomy, and level of control all influence student motivation (Stipek, 1996). "Even when students have a perception of personal control … they will not choose or be intrinsically motivated to complete tasks that are very easy, very hard, boring, repetitive, or meaningless" (Campbell & Stanley, 1963, p. 100). Students' perceptions about the difficulty level of a task can affect achievement-related cognition. Difficult tasks, as perceived by students in relation to their own ability or skill, can lower expectations for success, perceptions of control, and perceptions of self-efficacy as compared to easy tasks. Such perceptions have a direct influence over the student's view of self-worth. Self-worth theory focuses on the students need to protect their sense of worth or personal value. Similar to attribution theory, the theory of self-worth postulates that achievement behavior can best be conceptualized through self-perceptions of causality (Covington, 1984). However, self-worth theory focuses on the need to approach success and avoid failures that cause a sense of worthlessness and social disapproval. Factors that can influence an individual's self-worth include: perceived performance level, estimates of ability, and degree of effort expended on a task (Covington, 1984). Each of these theoretical and empirical insights has implications for the measurement of motivation and its critical dimensions. These implications are explored in the methodology section.

Conceptual Understanding and Inquiry

In the last twenty years, authorities, educators, and stakeholder within the Science and Mathematics communities have put forth a charge to improve the quality of K-12 education through engagement, inquiry, and enriched curriculum, e.g., NCTM Standards (1989, 2000). During the late 1990s the science education community evoked change by way of the development and implementation of a national set of education standards. Through their review of research, the developers of the *National Science Education Standards* (National Research Council, 1996) identified two distinct issues in education. First, most teachers were still using traditional, didactic methods in their classrooms (Stake and Easley, 1978; Harms and Yager, 1981; Weiss, 1987- cited in Olson & Loucks-Horsley, 2000); and second, students were mastering disconnected facts in lieu of broader understandings, critical reasoning, and problem-solving skills.

A short time after the science education community investigated their content and practices the math community reconsidered theirs. In 2008, the National Math Advisory Panel reported on a set of findings and recommendations, set in motion by a presidential charge, to evaluate the state of mathematics education in the US. Although a large amount of their efforts were focused on identifying key concepts and benchmarks within Algebra, many of their recommendations can extend beyond Algebra to encapsulate the general nature of mathematics learning. The report illustrates a need to enhance reasoning and conceptual understanding, while focusing on effort as benchmark for mathematical achievement (National Mathematics Advisory Panel, 2008). Such a recommendation makes explicit reference to current notions of learning and motivation; shifting from the behavioral notions of stimulus-response, to the cognitive notions of learning as an active process of mental constructions with connections between existing knowledge and beliefs, and the importance of self-monitoring (Shepard, 2009). Such a shift is also apparent in the most recent efforts to develop the Common Core Math Standards (CCMS, 2010). The underlying structure and framework of the CCMS standards speaks to the *processes* and *proficiencies* in mathematics. The first of these recommendations encapsulates the NCTM process standards of problem solving, reasoning and proof, communication, representation, and connections. The second embodies the National Research Council's proficiency standards of adaptive reasoning, strategic competence, conceptual understanding, procedural fluency, and productive disposition. These processes and proficiencies are similar in nature to Kilpatrick et al.'s (2001) description of knowledge through fluency, competence, and reasoning. Where procedural and conceptual fluency refers to knowing when and how to use procedures, skills, relationships, and connections appropriately, accurately, and efficiently; and strategic competence represents the ability to formulate, represent, and solve problems (Kilpatrick, Swafford, & Findell, 2001). Although not cited as an influence to the framework of the Common Core Standards, success in mathematics requires, and should include, adaptive reasoning and a productive disposition (Kilpatrick, Swafford, & Findell, 2001).

Where Kilpatrick, et al. (2001) defined adaptive reasoning as “the capacity for logical thought, reflection, explanation, and justification” (p. 129); and productive disposition as “the habitual inclination to see mathematics as sensible, useful, and worthwhile, coupled with a belief in diligence and one’s own efficiency” (p. 131).

Mathematics is similar to most subjects, where mastery is highly dependent on both the type of knowledge and the proficiencies for which the knowledge is applied. The study of mathematics is sequential in nature, where each level builds upon the ideas, concepts, and procedures that came before (Bloom et al., 1971). Beyond the procedures and concepts, mathematics involves a process, a way of being, an intuitive sense of seeing relationships, and a balance of knowing what is important and what is not (Wiliam, 2007). The current view of mathematics demonstrates a shift from a subject of “immutable truths and unquestionable certainty” to a subject derived, learned, and debated within a social system (Nickson, 1992, p. 103). Emphasis is placed on the importance of judgment and choice in choosing strategies and drawing conclusions. Therefore, students should be provided with opportunities to construct knowledge, grapple with ideas, and formulate conclusions in both private and public settings. Such an emphasis aligns well with problem solving and design approaches.

Assessing Problem Solving

The assessment of problem solving can be analyzed through five sub-categories, which include the students ability to *Represent* the problem, *Plan* a solution, *Execute* the plan, *Self -Monitor* throughout the problem, and *Explain* the solution (Mayer & Wittrock, 1996; Peschar, 2004). These categories makeup the framework for quality problem solving, as they investigate the comprehension of the problem situation, the cognitive processes used to approach the problem and execute a solution plan, and the appropriateness of the solution (Peschar, 2004). Peschar suggests that problem solving assessments should require the student to continually process information within an authentic setting, allowing for the assessment strategies as well as outcome behaviors to be observed. Since problem solving depends heavily on domain-specific knowledge (Chi et al. 1988; Weinert & Kluwe 1987), care needs be taken to ensure that the problem-solving tasks cannot be solved as a matter of routine nor that they place excessive domain-specific demands on the students, thus triggering trial-and-error behavior (Peschar, 2004).

Bridging the Gap

Discussions to this point have served to inform the readers about the need for quality inquiry, to emphasize process and proficiencies, to focus on effort as a benchmark for achievement, to promote mastery-goal orientations, to mediate shifts of control from teacher to student, to encourage positive self-worth, and to package it all in a nice box of activities and assessments. The National Research Council (NRC, 2009) emphasized design as a problem-solving process, capable of drawing together the abstract and concrete. Research has shown that design-based curricula lead to significant learning gains, improved retention, and enhanced engagement when contrasted with scripted inquiry (Mehalik et al., 2007 - cited in Svihla & Petrosino, 2008). Design activities provide a unique ability to educators to target conceptual learning, logic and problem solving constructs, and metacognition through their curricular efforts. A fundamental component of any design-problem is the application of a design process (Ertas & Jones, 1993). The Engineering Design Process allows for the integration of skills, knowledge, understanding, planning, and implementing. As such, the engineering design process has the potential to enhance math and science learning (NRC, 2009) through inquiry, conceptualization, and applications. Problem solving provides a unique process through which inquiry, conceptualization, and application can be combined into practice and assessment. The 2003 framework developed by PISA's (Program for International Student Assessment) defined problem solving as "an individual's capacity to use cognitive processes to confront and resolve real, cross-disciplinary situations where the solution path is not immediately obvious and where the literacy domains or curricular areas that might be applicable are not within a single domain of mathematics, science, or reading" (OECD 2003, p. 156). As such, we can think of problem solving as devising and implementing a plan to solve problems for which no routine solution methods is available. Such a skill is profoundly important for the continuation of innovation in our society. Without the ability to explore and engage in the unfamiliar there will be no innovation or progress. The importance of problem solving is a given. When design-based curriculum is introduced with the intention of educating students in design, problems solving, and related STEM content, it is vital that activities and assessments cultivate the process of problem solving along with the generation of high quality products. However, assessments seem to be an afterthought in engineering education programs (Svihla & Petrosino, 2008).

Therefore, the goals of this dissertation are to examine problem solving through an engineering design-based framework with explicit attention to the assessment issues and the measurement and interaction of the many components of problem solving both quantitatively and qualitatively. As a means to explore these critical components, I devised an intervention of short duration in AP calculus.

CHAPTER 3

METHODS

Participants

The participants in this study included students in calculus AB from two different schools within the same school district. The academic structure of these schools locates calculus AB as a junior level course. However, it is common for calculus AB classes to contain both juniors and seniors, i.e., grade 11 and 12 students. Measures and protocols were pilot tested with a small sample of students from School A; two students who were enrolled in calculus BC and two students enrolled in calculus AB. The two calculus BC students were chosen based on an informal assessment of their performance in pre-calculus (each student was a former student of mine), their ability to communicate well, and their willingness to be interviewed. The sampling pool for the overall study was broader, depicting the full breadth of knowledge in the two settings. However, the students in the overall study were already allocated to their classes with no consideration of randomization. Therefore, sampling for the study followed a convenience sampling strategy. I was able to include three sections of calculus at my school (school A) and one section of calculus at the second school (school B), totaling four sections. School B was selected based on a relationship I had with the instructor and her willingness to participate in the study. The research design followed a treatment-control structure with one treatment group and three control groups. The treatment group consisted of one section from school A (n = 34 with 20 female and 14 male students), while the control groups consisted of two sections from school A (n = 28 with 17 female and 11 male; n = 27 with 15 female and 12 male) and one section from school B (n = 39 with 21 female and 18 male). I served as the instructor of the treatment group. Although I have been an instructor at school A for ten years this was my first year teaching calculus. The control group instructors included a twenty-five year veteran teacher (school A), teaching calculus AB for the fifth-year and a 10-year veteran teacher (school B), teaching calculus AB for the first time. Table 1 shows the number of students by race/ethnicity and sex for each school site.

Table 1
Student Demographics at Participant School Sites by Race/Ethnicity and Sex

	School A	School B
Asian/Not Hispanic	104	84
Native American/Not Hispanic	10	0
African American/Not Hispanic	47	47
Hispanic	233	133
White/Not Hispanic	2035	1878
Male	1224	1094
Female	1212	1061

Measures

Measures for this study consisted of free-response mathematical knowledge assessments and a problem solving motivation survey (Appendix C). The mathematical knowledge assessments included a skills assessment and content knowledge assessments within the domain of Related Rate. The skills assessment was given prior to the study and investigated students' ability to:

- Relate variables through rational equations,

- Take the derivative of an equation in one variable with respect to time using the constant derivative rule,
- Take the derivative of an equation in two variables with respect to time using the product rule,
- Rewrite a differential equation.
- Substitute given values into a differential equation and solve for a rate of change,
- Identify which given in a problem statement represents a rate of change,
- Determine whether the identified rate of change signifies an increasing or decreasing function, and
- Solve a simple rate of change problem through direct substitution.

I identified these eight skills as key elements necessary in solving Related Rate questions while designing and modifying the Related Rate assessment questions. Like all the measures given in this study, the skills assessment was field tested and modified during the pilot study. Unlike the content assessment and motivation survey, the skills assessment was only administered as a pre-assessment and was not given at the end of the study. The skills assessment was included to investigate and compare prior knowledge (Appendix C).

The Related Rate assessment was given prior to the treatment (pre), immediately following the treatment (post1), and six weeks after the treatment (post2). The Related Rate questions were designed to mimic the format of AP free-response questions. Released AP questions were referenced and used when possible as a means of assessing the students' Practical Abilities (Peschar, 2004).

Since the purpose of the AP test is to assess student understandings from an entire year of calculus and my study was conducted in the middle of the school year, I found that portions of the released items prompted students to engage in material they had yet to learn. As such I eliminated sub-parts of these questions and supplemented where necessary. The pre- and post2- assessments consisted of two free-response questions, while the post1-assessment contained three questions. The questions were selected to assess the three main types of Related Rate problems: Volumetric, Pythagorean, and Trigonometric. Volumetric Related Rate problems contain a type of geometric solid and prompts linked to changes in the volume, the height, and/or the base. Pythagorean Related Rate problems contain an object or objects in motions, where the directions of motion are orthogonal. Trigonometric Related Rate problems contain objects whose relationship can be determined through a right triangle. For the purposes of having different questions on the assessments, but still maintaining the ability to compare responses, questions across each assessment were modified by changing volumetric shapes, numerical values, units of measure, thematic content, and the order of sub-parts. Care was taken to select similar contexts across the assessments. For example, Question 1 (Volumetric) investigates the change in height of water as it leaves (pre) and enters (post and post2) similar containers. Question 2 (Pythagorean) prompts students to address the motion of orthogonally situated vehicles, cars (pre and post2) and trains (post1). For the purposes of this study, all students were allowed to use a graphing calculator on all Related Rate assessments.

The problem solving motivation survey was administered at two measurement points: pre and post1. This survey was developed prior to the study as a means to investigate the constructs of Ability, Effort, Performance, Mastery, Expectation, and Value. I identified these constructs and developed the survey from a comprehensive literature review in the areas of motivation and problem solving. The constructs, Ability and Effort, refer to student perceptions of how ability and effort affect their problem solving success (Dweck, 1986; Forsterling, 1985; Meyer & Fennema, 1985; Schunk, 1983; Stipek, 1996; Weiner, 1992). Performance and Mastery refer to student orientation, where a performance orientation can be described as a disposition where the student is interested in demonstrating competence and a mastery orientation describes a student interested in developing competence (Ames & Archer, 1987; Dweck, 1986; Dweck & Leggett, 1988; Elliot, 1999). Expectation refers to the expectations of success students possess before they engage in a difficult problem (Ames, 1984; Dweck, 1986; Elliot, 1999; Maehr, 1984; Nicholls, 1984). Lastly, Value refers to the amount of importance the student places on being a good problem solver and how success in problem solving can affect other academic areas (Atkinson, 1964; Stipek, 1996). Four questions were written for each construct. Through administration of the survey in the pilot study the five constructs were condensed into four (combining Ability and Effort into one construct) and the questions were modified as needed. The final version of the survey included twenty questions, which are given below, and are listed by category. The Q-notation preceding each question represents where the question was situated within the survey, i.e., Q1 indicates Question 1. The question order was randomly generated using a random number generator in Microsoft Excel.

Ability/Effort

Q19 - Being a good problem solver in math is an ability you either have or don't.

Q8 - The only way to be a good problem solver in math is to be born with the ability.

Q11 - Being a good problem solver in math is a skill that can be learned.

Q17 - Students who are poor problem solvers in math cannot learn to be good problem solvers in math.

Performance

Q5 - I only want to be a good problem solver so I can do well on exams.

Q15 - I will not attempt a math problem if I think I cannot solve it correctly.

Q9 - Attempting a math problem and not solving it correctly is a waste of time.

Q1 - When I solve math problems I am most interested in getting the correct answer.

Mastery

Q3 - When attempting math problems my goal is to learn the methods and strategies for solving that type of problem.

Q20 - When I engage in solving math problems, my goal is to learn how to be a better problem solver.

Q16 - While solving math problems I think about how I can be a better problem solver.

Q4 - When given a math problem, I do my best because I want to learn new concepts.

Value

Q12 - Taking the time now, and learning how to be a good problem solver in math will help me in future math courses.

Q13 - Learning how to be a good problem solver in math will help me outside of school.

Q6 - Learning how to be a good problem solver in math will help me in other courses, i.e. English, History, Social Studies, …

Q10 - I see no value in spending time to become a good problem solver in math.

Expectations

Q2 - I am surprised when I solve math problems correctly.

Q7 - When I attempt a math problem, I am confident that I will generate the correct solution.

Q18 - I am surprised when I have difficulties solving math problems.

Q14 - When I engage in a math problem I expect to be successful.

Response choices for each question were in the form of Likert type scales with anchors, Agree (1), Agree a little (2), Neutral (3), Disagree a Little (4), and Disagree (5). Where a Neutral response indicates a position of neither agreeing nor disagreeing. A five-point Likert scale was derived from the pilot studies. The initial survey contained a three-point scale and was later modified as a result of student feedback.

Procedures

The research deign for this study consisted of two phases, each containing multiple steps. The first phase consisted of pilot testing the skills assessment, the Related Rate pre-assessment, the problem solving motivation survey, and the problem solving strategy based on the engineering design process. Participants for the pilot study consisted of four students, two students in calculus AB and two students in calculus BC. Pilot sessions concentrated on gaining insight and feedback into the language used in the measures, the appropriate nature of the response categories used on the survey, and the practicality of delivering the designed problem solving strategy, i.e., the treatment. The pilot sessions spanned 30 minutes with each student engaging in two sessions (students were asked to come in at lunch, which is a 30 minute period) and followed a predetermined format. To begin the first session, the student was given the skills assessment and asked to read each question, state what he/she thought the question was asking, and then answer the question. Student interpretations of each question were used to inform modifications and gauge the content validity of the questions. Engagement with the skills assessment took about 10 minutes. Next, the student was given the problem solving motivation survey and asked to read each question, state what he/she thought the question was asking, identify any confusing words or phrases, and answer the question with the indicated Likert scale. While answering the question the student was asked to determine if the scale provided enough choices and if the response descriptions fit how he wanted to answer the question. Engagement with the problem solving motivation survey took about 15 minutes and ended the first session. Changes in the skills assessment were limited to langue use, while changes in the survey focused on both language use and the number of response choices from three to five. Following the first session the student was asked to come in the next day to go over the Related Rate assessment and the problem solving strategy. To start the second session the student was given a copy of the Related Rate pre-assessment and asked to read the questions, communicate his interpretations of key elements in the problem statement and state what he thought each prompt was "asking" him to do. Once the student had communicated this information he was given a copy of the engineering design-based problem solving strategy. I then explained each stage of the problem solving strategy, informally assessing his understanding of the process through observation and direct questioning. Once I was satisfied that the student understood the problem solving strategy, the student was asked to engage with the problem, generating a solution using the problem solving strategy. The second session was used to evaluate how the student interpreted and utilized the problem solving strategy in the context of solving an actual problem. The problem solving strategy was design based on the engineering design process. The strategy contains six steps consisting of Identify the Problem, Gather Information, Imagine a Solution, Plan Your Solution, Implement Your Solution, and Evaluate the Solution (Appendix D). These six steps represent Peschar's (2004) Analytic Ability within the domain of problem solving competencies. In the first step, *identify the problem*, students are asked to read the problem, list the givens, identify assumptions, and summarize key ideas, while making no attempt to solve the problem. The second step, *gather information*, has students begin to generate connections at a macro level, identifying prior knowledge relevant to the problem and clarifying parts of the problem they don't understand. The third step, *imagine a solution*, is where students brainstorm possible methods to solve the unknowns. Brainstorming is where students develop their Creative Ability (Peschar, 2004). This is the most important step, as is serves to connect knowledge and skills in an informal, stress free manner. The fourth step, *plan your solution*, is where the students review their brainstorming ideas and select the option they feel has the best chance of solving the problem or part of the problem. The fifth step, *Implement your solution*, allows students to use the process they selected in Step 4 on the problem. The final step, *evaluate the solution*, explores the solution generated from Step 5 through evaluations based on the solutions ability to satisfy the givens, assumptions, and the unknowns. Following the first set of pilot interviews, changes were made to the assessments, survey, and the problem solving strategy where appropriate. The modified measures and strategies were given to the second student and the process described above was repeated. Following each session with the second student, changes were made to the assessments, survey, and problem solving strategy where applicable. After meeting with the two calculus BC students and gathering their insights I was comfortable with the content validity of the measures, from the perspective of a calculus BC student sample. Since my intended sample consisted of calculus AB students I wanted to get the interpretations and understanding of each measure from the calculus AB student point of view. Therefore, I utilized my two student aides, both of whom were current calculus AB students (one enrolled in my class and one enrolled in my colleague's class) and asked them to read over the skills assessment and the problem solving motivation survey. I did not give them the Related Rate assessments, as they would be a part of the study in their respective classes and their review of the assessment might compromise their pre-assessment scores. I felt comfortable that any prior experience with the skills assessment and the survey would not skew their scores on these two measures. Each student was instructed to read over the questions and write a short sentence on what they thought each question was "asking". Using their written interpretations of the questions I asked follow-up questions based on perceptions that differed from my intent for a given question. Following my interactions with these two students I finalized the measures into the version shown in Appendix C.

The second phase of the study included administration of the measures and delivery of the curricula to the treatment and control groups. No attempt was made to place restrictions or conformities on what content the control group instructors delivered or how they delivered that content. The following is a description of the procedures each instructor used to teach problem solving of Related Rate problems. Instructor 1 and Instructor 2's descriptions are self-reported, as I was teaching during their instruction time and could not conduct any observations. Any handouts, assigned homework, and/or class work given with respect to each class and instructor referenced in this excerpt are presented in

Appendix D. Each class period for all instructors lasted the same amount of time, 50 minutes.

Instructor 1 (Treatment group)

Day 1: I administered the skills assessment, the problem solving motivation survey, and the Related Rate pre-assessment. For homework, I assigned the derivatives of areas and volumes worksheet.

Day 2: I introduced the engineering design process as a problem solving strategy. We discussed *Understanding the Problem* and *Gathering Information*. In preparation for the class, I selected nine problems I wanted students to engage with during class and for homework. I put-together a packet of materials that included the nine problems with prompts to document their work structured through the engineering design process as a problem solving strategy (Appendix D). These problems were different from the solo homework problems I assigned on the second to last day. I selected the first problem (melting snowball) and modeled how to set-up the *Understanding the Problem* stage of the strategy. We talked about reading the entire problem, generating a diagram, labeling the diagram, and generating relationship equations for the variables in the diagram. Then I had the students work on the second problem (streetlight-walker) in their small groups. Students were seated in groups of four. When they set-up the problem I asked each group (9 in total) to write their understanding of the problem on the board. As a class, we were only able to get through my example and one practice problem. For homework I asked students to set-up the *Understanding the Problem* and *Gathering Information* stages on problems 3 and 4 in the packet.

Day 3: I started the day by having groups display their understanding of the assigned homework problems on the board. Then I went through and commented on certain shortcomings. We returned to the second problem students worked-out in class and I introduced the concepts behind *Imagine a Solution, Plan, Implement, and Evaluate Your Solution*. As a class we worked through the process. We began by deciphering the problem statement to generate $d(x_1 + x_2)/dt$ =? Brainstorming produced 10 possible solution methods (ratios, trigonometric functions, areas, distance over time, Pythagorean theorem, completing the square, derivatives, find x_2, slope, and distance formula). I proceeded to ask which method they would like to try first. The class suggested that we should try ratios first. As a class we worked through finding x_2 and calculating (x_1+x_2)/time. At this point, I ruined the moment by suggesting that we couldn't find the time because we didn't know where the person started. This was not the case and it turns out that this method was a very interesting way to solve the problem. I used the experience to suggest that not all methods work and when a method doesn't work we need to go back to the brainstorming stage and try another method. The "failed" attempt of the problem was not received well. This was not an unexpected result. Students expressed that they should be able to solve problems on their first try and they were a little more than annoyed with doing work that doesn't pan-out. For homework I asked students to complete the problem solving strategy on the two homework problems I assigned the previous night.

Day 4: I began the class by revisiting the misunderstanding that occurred the previous day (streetlight-walker problem). Following this review, I selected groups to display their homework solutions on the board. Again, I went through each solution and commented on certain shortcomings. After discussing the homework I had the students work in groups on the "baseball problem." Groups were asked to present their solutions on the board. We didn't have time to finish the baseball problem in class so I assigned the problem and the "leaking cone problem" for homework.

Day 5: I began the class by giving students 5-10 minutes to compare their homework solutions. Then groups were asked to present their solutions on the board. We discussed the problem areas again. Following the homework review I had students work on the "changing triangle problem" in their groups. Students worked on the problem in small groups. There wasn't enough time to present solutions on the board. As I walked around I noticed that students were not constructing the correct relationship between the given variables and they were not labeling the rate of change correctly as dx/dt. I assigned the “boat problem” and the “kite problem” as homework.

Day 6: I began the class by walking around to each group and glancing at their homework. It was very clear that the students had trouble with these problems. I began to realize that the students had difficulty understanding the problem statement. Their initial drawings did not accurately depict how the system changed over time. We discussed this shortcoming and I gave the class 20-25 minutes to rework these problems in their groups. Once the problems were complete I had each group write their method and solution on the board. I assigned the rest of the packet and textbook problems for homework.

Day 7: Students began class by completing the problem solving motivation survey. We reviewed the homework problems in a lecture format where students asked questions and I answered by showing solution steps on the board.

Day 8: I administered the Related Rate post-assessment.

Instructor 2 (Control groups 1 and 2)

Day 1: The instructor administered the skills assessment, the problem solving motivation survey, and the pre-Related Rate assessment.

Day 2: The instructor began class by showing various animations of Related Rate problems using Calculus in Motion (Weeks, 2011), through Geometer’s Sketchpad (Version 4.06). Examples include a cone being filled with water, a ladder sliding down a wall, etc. While the students were watching the videos the instructor questioned the class about what they saw changing. Students were not asked to provide mathematical descriptions by referencing variables. Rather, they were prompted to give literal descriptions of what was changing. Following the animations, the instructor led a class discussion about rates of change and the relationship to derivatives. Once the class had formed a consensus and connected the animations to derivatives the instructor worked out a Pythagorean theorem type problem on the board. While working through this problem the instructor talked about general problem solving strategies of drawing a picture, identifying the variables, and finding the relationship between variables. During this problem presentation the instructor continuously connected parts of the problem to the animation videos shown at the beginning of class. Students were not assigned homework.

Day 3: The instructor started the class by giving the students a list of problems the class would be working through during the period and for homework. Assigned problems were references to problems in the class textbook and consisted of three types: (1) Pythagorean theorem problems, (2) Volume problems, and (3) Trigonometric problems. Similar to Day 2, the instructor worked out assigned problems in a lecture style, fielding student questions during the process. Students were assigned one of the given problems for homework.

Day 4: The instructor began class by fielding any questions the students had on the previous night's homework. Questions were answered in a verbal and written manner when applicable. Once all the questions were answered the students were instructed to complete the next problem from the previously assigned list. The physical environment of the classroom is structured so that the students are in groups of four. Therefore, each reference to students working on assigned problems is a reference to students collaboratively working on problems in their assigned groups. While the students worked on the problems, the instructor walked around the room observing their performance. When questions come up, the instructor would stop the class's activity and present the question. At this time the instructor would either lead a class discussion or he would work out a portion of the problem on the board. This structure was followed for the entire class period, continuously working through the assigned problems. For homework, students were asked to finish the problems they worked in class and to complete one additional problem from the list of assigned problems.

Days 5 and 6: Class continued with the same structure as in Day 4.

Day 7: Students began class by completing the problem solving motivation survey. The rest of the period continued with the same structure as discussed in Day 4. Students were not given homework.

Day 8: The instructor administered the Related Rate post-assessment.

Instructor 3 (Control group 3)

Day 1: The instructor administered the skills assessment, the problem solving motivation survey, and the Related Rate pre-assessment. The instructor introduced Related Rate and gave students the first worksheet. The instructor worked out the example problems from the worksheet on the board and assigned a second worksheet as homework.

Day 2: The instructor began the class by answering homework questions. Questions were answered in a verbal and written manner. After discussing the homework, students were given the third worksheet. Homework problems were assigned from the worksheet.

Day 3: Students reviewed for the chapter test.

Day 4: Students took the chapter test. It is important note that the post-assessment Related Rate problems of this study were not included in the chapter test.

Day 5: The instructor administered the problem solving motivation survey, and the Related Rate post-assessment.

Differences and Similarities Among the Three Instructors

The main difference between Instructors 1 and 2 and Instructor 3 was the focus and the amount of time spent on Related Rate instruction and problems. In the calculus curriculum Related Rate is housed within a chapter designated as derivatives. Instructors 1 and 2 completed the derivative aspects of this chapter, took a chapter test, and then focused on Related Rate, utilizing the post-assessment as a Related Rate unit test. As a by-product, Instructors 1 and 2 spent twice the amount of time on Related Rates. By separating Related Rates from the rest of the chapter, the students could focus solely on Related Rates and did not have to worry about a comprehensive chapter exam. Instructor 3 situated Related Rates within the chapter, spending two days on the Related Rate curriculum delivery, then returned focus to the chapter and the chapter exam. Once the chapter exam was given, Instructor 3 gave the post-assessment as a stand-alone activity.

The main difference between instructors 1 and 2 existed in the focus of instruction and the interactions between instructor and student. I focused on the concept of problem solving in the context of Related Rates, but not specific to Related Rates, while utilizing public displays of student work where possible. Instructor 2 focused on teaching the types of Related Rate problems and utilized a lecture style of questioning and answering. In contrast, Instructor 3 focused on worksheets and direct lecture in a non-cooperative learning style environment.

CHAPTER 4

DATA ANALYSIS AND RESULTS

Data analysis consisted of both descriptive and inferential statistics, parametric, nonparametric, and structural equation models when appropriate. For the purposes of order and clarity, this section will be structured around the measures. Descriptions of statistical techniques will be confined to the individual measures (where applicable). As I described in the methods section, measures for this study included a skills assessment, Related Rate assessments, and a problem solving motivation survey. Since similar considerations occurred within each analysis the following accommodations for data type, independence, normality, and elevated type-I error considerations will precede individual explanations of each data analysis procedure. Unless stated otherwise, all missing data were deleted pairwise.

Data Type

Data can be categorized into four basic components: Nominal, Ordinal, Interval, and Ratio (Ray & Ravizza, 1981; Singleton, Straits, & Straits, 1993; Thorndike, 1978). Nominal numbers, also called categorical numbers, are used to indicate group membership. In order for measurements to be nominal, cases assigned to the same category must be equivalent; each category must be exhaustive, and mutually exclusive. Ordinal numbers also convey group membership with the additional property of expressing order on the trait. However, equal differences between numbers on the scale does not equate to equal differences between values of the trait. Interval numbers are considered to be ordinal numbers with the addition that equal differences between scale values does represent equal differences between values of the trait. An interval scale contains an arbitrary zero point and as such can include negative values. Ratio numbers have the same properties as interval numbers, with the exception of containing a non-arbitrary zero point. The ratio scale of numbers cannot contain negative values, as the zero represents an absence of the trait.

Independence

Independence within a measure is evaluated by considering how one data point influences another (Field, 2009). Some statistical techniques require independence of observers, some of subjects, and some of residuals. In each case the underlying idea of independence is the same, "How does one data point influence another?" This study utilized convenience samples and therefore was afforded no control on how students were clustered within classrooms. Independence issues at the student level were minimized during the administration of the measures. A pre/post design prevents independence within individual students. However, care was taken during the administration of each measure to minimize the influences between students. Students were not allowed to confer while engaging in each measure and instructor observation served to deter students from cheating.

Normality

Normality is a fundamental issue in data analysis. Assumptions of normality refer to the population being normally distributed, i.e. following a normal distribution, perfectly symmetric with skewness and kurtosis equal to zero

(Field and Miles, 2010). Investigations into the distribution of the sample data are necessary for the integrity of the mathematics behind the statistics and the inferences made from the results. Investigations into normality consisted of constructing histograms and Q-Q plots, inspecting the skewness and kurtosis, and performing Kolmogorov-Smirnov Tests of Normality.

Type-I Error

Elevated Type-I error can occur when multiple hypothesis tests are conducted on the same set of data to investigate the same empirical question. This is commonly referred to as familywise error and can be described as the probability of rejecting the null hypothesis (p-value) in multiple tests when the null hypothesis is true in each case (Field, 2009). Hypothesis testing for this study was utilized for exploration and testing theory. When hypothesis testing was used to explore the data no attempts were made to control for familywise error, e.g., identifying differences between-classes as a means of selecting student work that would be interesting to compare (searching for similarities an differences). When hypothesis testing was used to test theory, i.e., do students in the treatment group outperform students in the control groups, efforts were made through Bonferroni corrections to control for familywise error. A Bonferroni correction controls Type-I error by scaling the p-value for each hypothesis test by dividing the probability by the total number of hypothesis tests performed.

Skills assessment

The skills assessment was a multiple part, five-question assessment. Each question was evaluated based on a predetermined point scale. The allocation of points assigned to each question was derived from the number of sub-parts and the complexity of the required answer. The rubric for this assessment (Appendix E) includes the number and allocation of points awarded, and was developed through an iterative process. The rubric was initially created from my expectations and previous experiences with teaching and assessment. This first version was used to score the entire set of assessments (classes A, B, C, and D).

Table 2

Skills assessment Questions 4

4. Given $\frac{dA}{dt} = 4\pi s^2 \frac{ds}{dt}$. If side s is 4 inches and $\frac{dA}{dt} = 23$ in^2/sec …

   a. What is the equation for *ds/dt*?
   b. At what rate is the side changing? What are the units?

During the scoring process I identified and recorded discrepancies and complications. After completing the first iteration of scoring I found two areas in need of change. First, the responses to Question 4 (Table 2) contained variation in how students responded to each prompt. Students were consistently giving the differential equation, but some were in response to prompt (a), while others to prompt (b). After reexamining the wording of Question 4, I concluded that it was in fact poorly worded and decided to change the number of points awarded from two points to one point. The single point was awarded to the student if he/she correctly represented the differential equation as a response to either prompt. The second area of change was concerned with the first part of Question 5 (Table 3). My intentions for Question 5 were to assess the students' ability to identify and interpret key pieces of information within a problem and to solve a simple rate problem through the use of substitution.

Table 3

Skills assessment Questions 5

5. A right circular cone is filled with water such that the height *h* of the water in the cone changes at a constant rate of 0.4 meters per minute. If the radius of the cone is 3 meters and the volume of the cone is given by $V = \pi r^2 h$ …

    a. Write an equation for the change in volume as a function of time.
    b. What is the value of *dh/dt*?
    c. Is the height increasing or decreasing when the radius of the cone is 3?
    d. If the right circular cone is empty at time $t = 0$ and a height of 2 meters corresponds to a volume of 56.55 cubic meters. How long will it take to fill the cone to a volume of 56.55 cubic meters?

Part (a) of Question 5 prompted students to write an equation representing the volumetric rate of change for a right circular cone. Since Question 4 prompted students to take the derivative of the volume equation for a right circular cylinder, even though the derivative in Question 4 was removed from context, I decided to eliminate the initial point awarded for part (a) of Question 5.

Table 4

Skills assessment: Percent Correct Response by Question

	Class A		Class B		Class C		Class D	
	Percent	SD	Percent	SD	Percent	SD	Percent	SD
Q1: Solve a Ratio								
	90.32	30.05	100.00	0.00	100.00	0.00	94.73	22.62
Q2: Derivatives in a Single Variable								
	69.35	35.77	53.70	33.75	53.70	36.49	32.89	29.12
Q3: Derivatives in Two Variables								
	58.70	39.97	37.77	32.02	40.74	35.72	23.68	27.05
Q4: Solving Rate of Change								
	90.32	30.05	66.66	48.03	88.88	32.02	71.05	45.96
Q5: Simple Rate of Change Using Substitution								
	27.41	21.75	21.29	23.72	19.44	27.15	21.05	18.85
Total	55.58	21.45	42.16	19.36	44.44	23.20	33.40	14.99

Once the rubric was amended the entire set of assessments were rescored. The second iteration of scoring yielded no further need for rubric modifications. Descriptive statistics by question totals are provided in Table 4 and by question sub-part in Appendix F, Table 24. For a graphical representation see Appendix G, Figure 2.

The results of the skills assessment reveal a few interesting phenomena. First, the treatment group (class A) consistently outscores classes B, C, and D on all but Question 1. Second the standard deviations associated with these scores are relatively large, between 19% and 48% relatively. These high values suggest a large dispersion between the students who know the material and the students who do not. It has been my experience that such disparities are common in AP calculus due to the fact that a non-AP version of calculus is not offered at my school. Therefore, students who may not be ready for the pace and structure of an AP class have no choice but to take AP calculus. As such, the make up of an AP calculus class contains a wide range of student abilities, beliefs, and work ethics. Third, there are clearly issues with Question 5 across each class as the reported scores are between 19% and 27% correct. Although knowledge of calculus and Related Rates was not a necessity to correctly solve this problem, the results are not surprising and suggest a common starting level across the four classes.

Table 5

Related Rate Pre-Assessment: Percent Correct by Total

	Class A			Class B			Class C			Class D		
Related Rate Problem	N	Percent Correct	SD	N	Percent Correct	SD	N	Percent Correct	SD	N	Percent Correct	SD
Volumetric Score (Q1)	34	32.97	33.68	28	32.94	35.22	27	29.63	33.66	37	15.92	21.75
Rate of Change (Q1b)	34	34.84	40.61	28	31.43	34.36	27	28.15	33.79	37	11.35	25.27
Pythagorean Score (Q2)	34	19.71	20.32	28	12.30	10.64	27	13.17	11.95	38	19.30	34.08
Rate of Change (Q2b)	34	3.23	16.40	28	0.00	0.00	27	1.06	3.81	38	13.16	30.24
Total Score (Q1 + Q2)	34	26.34	27.00	28	22.62	22.93	27	21.40	22.81	37	17.61	27.91

Table 6

Related Rate Post1-Assessment: Percent Correct by Total

	Class A			Class B			Class C			Class D		
Related Rate Problem	N	Percent Correct	SD	N	Percent Correct	SD	N	Percent Correct	SD	N	Percent Correct	SD
Volumetric Score (Q1)	33	68.01	41.76	28	67.86	35.69	27	68.72	35.24	38	37.89	39.95
Rate of Change (Q1b)	33	67.27	40.68	28	71.43	28.41	27	77.04	29.66	38	35.38	39.44
Pythagorean Score (Q2)	34	67.97	38.01	28	86.11	22.92	27	83.13	19.72	38	47.01	42.58
Rate of Change (Q2b)	34	61.34	40.64	28	82.14	29.46	27	78.31	25.35	39	38.10	42.87
Total Score (Q1 + Q2)	34	67.99	39.88	28	76.98	29.30	27	75.93	27.48	38	42.45	41.26
Trigonometric Score (Q3)	34	32.35	47.05	28	41.07	49.79	27	50.00	50.64	38	21.79	41.56
Rate of Change (Q3a)	34	35.71	45.06	28	48.98	46.93	27	56.61	45.23	38	2.93	12.16

Table 7

Related Rate Post2-Assessment: Percent Correct by Total

	Class A			Class B			Class C		
Related Rate Problem	N	Percent Correct	SD	N	Percent Correct	SD	N	Percent Correct	SD
Volumetric Score (Q1)	34	62.87	39.73	28	55.81	44.62	27	53.31	42.62
Rate of Change (Q1b)	34	54.38	43.56	28	49.63	45.30	27	46.15	44.24
Pythagorean Score (Q2)	34	63.54	42.86	28	69.14	43.23	27	65.81	44.01
Rate of Change (Q2b)	34	58.93	43.78	28	64.55	45.72	27	60.99	45.59
Total Score (Q1 + Q2)	34	63.20	41.29	28	62.47	43.93	27	59.56	43.32

Related Rate Assessments

Analyses for this assessment served to answer the first research question, "What impact does type of instruction have on high school student performance on AP calculus Related Rate problems (RRP)?" Similar to analysis procedures conducted on the skills assessment, the Related Rate assessments were also evaluated utilizing a rubric (Appendix E) developed through an iterative process. The style of scoring and the points assigned to each question mimicked the structure used by the College Board's scoring of the free-response questions on the AP calculus exams (College Board, 2011). Each question is assigned a total of nine points, separated into partial points based on skills, concepts, and knowledge needed to solve the problem. The Related Rate assessment rubric was initially created based on my review of previously scored AP calculus exams and concepts that I felt were important to the success of this study. The first version of the rubric was used to score the entire set of assessments (classes A, B, C, and D). After completing the first iteration I found three areas in need of change. The first was the initial awarding of points for identifying the units of an answer or set of answers. Although I feel this is an area that needs to be addressed on the AP Calculus exam, I chose to free up that point in the hopes that I could use it elsewhere to help study my research question. Since these problems are situated in the context of Related Rates, a necessary component for success is knowing when and how to take the derivative. I used the point previously assigned to *units* and decided to award a point for taking the derivative in the context of the problem. This allocation helped to distinguish between students who knew when and where to take a derivative and those who did not. I awarded this point for two reasons. First, it provided a way of quantifying student understanding of the connection between Related Rate and derivatives. Second, it allowed me to distinguish between students who left the question blank, and students who used derivatives incorrectly. The final area of change involved justifying answers when prompted. Questions that prompted students to justify their answers were in a two-part structure: (1) prompt for an answer, and (2) justify your answer; awarding a single point for the answer and a single point for the justification. Initially, I credited students with a point for the correct answer even if the justification was incorrect. Then I decided that the questions were structured in such a way that a student could supply a correct answer without truly knowing the concept. Therefore, I decided to award zero points for both parts of the question if there was no justification or the justification was incorrect. With each correction to the rubric the entire set of assessments was rescored. The second round of scoring produced no new discrepancies or complications resulting in the final version given in Appendix E.

Since the Related Rate assessments were given in three iterations, for clarity, discussion of the results will follow the same sequence. The Related Rate pre-, post1-, and post2-assessments had three major ideas: Volumetric Related Rate (Q1), Pythagorean Related Rate (Q2), and Trigonometric Related Rate (Q3). Questions 1 and 2 are located on all three assessments, while Question 3 is only present on post1-assessment. Reasoning for the addition of Question 3 to the post1-assessment was given in the methods section. The three assessment questions had the following common elements: Calculation of Rate of Change (Q1b, Q2b, Q3a) and Interpretation of Rate of Change (Q1c, Q2a, Q3b).

As stated in the method section, the order of sub-parts within Question 2 (Pythagorean) reversed from pre-assessment to post1- and post2-assessments, where post1 and post2 have the same order. Question 2 on the pre-assessment prompted students to *Interpret the Rate of Change* in part (a) and *Calculate the Rate of Change* in part (b), while the post1-assessment reversed the order prompting for students to *Calculate the Rate of Change* in part (a) and *Interpret the Rate of Change* in part (b). For purposes of clarity in the analysis the question order was recoded for all the assessments to follow the order specified in the pre-assessment, *Interpret the Rate of Change* in part (a) and *Calculate the Rate of Change* in part (b).

Pre-Assessment

Before discussion of the results it is important to elaborate on a few points of interest. Volumetric Score (Q1) and Pythagorean Score (Q2) are the average student scores on the entire question and were both worth 9-points. Calculating the rate of change is an integral part of both Volumetric and Pythagorean questions. These parts are labeled Q1b and Q2b respectively and represent the average student score on Question 1, part b (worth 5-points) and Question 2 part b (worth 7-points).

Class D differences in the sample size from Question 1 to Question 2 are a consequence of one student who had a missing assessment page. After I had received the assessments from the cooperating teacher and began to input the data

I noticed that one student was missing Question 1. Investigations into the missing question were uneventful and the response never materialized

Pre-assessment descriptive statistics of percent correct for each question are shown in Table 5. As demonstrated in Table 5, students performed poorly on the pre-assessment with scores ranging from 0% to 40%. Comparing Volumetric (Q1) and Pythagorean (Q2) questions, it is evident that average student scores on Volumetric (Q1) are higher than average scores on Pythagorean (Q2). The results of the pre-assessment are not unexpected as students had yet to engage in material and problem solving of this kind.

Post1-Assessment

Point totals for the question and sub-parts remained the same for Questions 1 and 2 as compared to the pre-assessment. In addition to Questions 1 and 2, post1-assessment contains a third question, Question 3. Differences in the sample sizes for the treatment group (class A) are a result of administrator error. A student in class A was absent during the administration of the post-assessment and I inadvertently gave the student an incorrect version of Question 1. The specifics in the question were different enough that I felt any comparisons between that score and the rest of the scores would be inappropriate. Post1-assessment descriptive statistics of percent correct for each question are shown in Table 6. As indicated by Table 6, average students scores across all four classes are higher for post1-assessment as compare to the pre-assessment. Average scores for the treatment (class A) and the control groups within the same school (classes B and C) show similar gains on Question 1 (Volumetric) and higher gains for classes B and C on Question 2 (Pythagorean) and Question 3 (Trigonometric). Standard deviations for Question 3 across all four classes are extremely high, suggesting a large discrepancy between scores. The control group from School 2 (class D) has lower average scores on all three questions as compared to the other three classes.

Post2-Assessment

Point totals for the question and sub-parts remained the same for Questions 1 and 2 as compared to the pre-assessment. Post2-assessment descriptive statistics of percent correct for each question are shown in Table 7. For a graphical representation see Appendix G, Figure 3. The control group from School 2 (class D) was not administered post2-assessment. I felt that class D's scores on the post1-assessment demonstrated a lack of understanding and further comparisons to demonstrate this point were unnecessary.

Average scores for both the treatment group (class A) and the control groups (classes B and C) have decreased from post1 to post2. The treatment group shows higher average scores on Question 1 (Volumetric) and lower average scores on Question 2 (Pythagorean) as compared to the control groups. Although, differences between the treatment and control groups, on Question 2, decrease from post1 to post2. Standard deviations for the treatment group remain consistent across post1 and post2, while the control group's standard deviations have increased from post1 to post2.

Within-Class Differences

Following investigations of the descriptive statistics, hypothesis tests were conducted within- and between-classes. Investigations continued to focus around question totals, sub-part totals, and overall assessment totals. Since Question 3 (Trigonometric) was only given at post1-assessment, dependent-within-class tests could not be conducted. Similar reasoning precluded any within-class analysis for Class D in regards to comparisons to the post2-assessments. Data for all assessments were considered to be interval in nature, dependent within classrooms, independent across classrooms, and non-normal. Since the data deviated from normality, analysis was conducted using nonparametric significance testing. Investigations into the differences within each class and between the three classes utilized SPSS (Version 20). Analysis of within-class differences was conducted using a Wilcoxon-Sign Rank test for dependent samples, at a p-value of .05. Assumptions of the Wilcoxon-Sign Rank test specify that the each pair of measurements is taken on the same subjects, population differences are symmetric about their median, the differences are independent, and the differences are measured on at least an interval scale. All assumptions were met and the analysis was conducted. Results for these tests show statistically significant increases across all four classes (Table 28, Appendix F). Table 28 includes the test statistic, standard error, standardized test statistic, p-value, and effect size. Statistical testing revealed significant differences between pre and post for all four classes, $p < .001$, on all questions and question sub-parts. Again, these findings are not surprising since students had no familiarity with the material before the pre-assessment. As part of the output, SPSS returns a test statistic value, a standardized-value, and a p-value for all significant pairwise comparisons. To determine which results were meaningful, effect sizes were calculated by dividing the standardized test statistic by the square root of the sample size. Effect sizes are large ranging from .67 to .90. Cohen (1988) considers effect sizes of 0.1 to be small, 0.3 medium, and 0.5 large. Within-class differences between post1 and post2 show a different set of results. Even though the average scores have decreased from post1 to post2 for all three classes (A, B, and C), statistically significant differences vary across the three classes. Analysis for the treatment group (class A) yielded significant differences for the total scores on Question 1 (Volumetric) and Question 2 (Pythagorean), but no significant differences on *Calculating the Rate of Change* (Q1b, Q2b) or the total assessment score (Q1 + Q2). Effect sizes for the treatment group consisted of -.35 and -.48. In contrast, the first control group (class B) contained statistically significant differences for all question totals and subparts, except for *Calculating the Rate of Change* on the Volumetric problem (Q1b). While the second control group (class C) showed statistically significant differences for all question totals and subparts. Effect sizes for both control groups ranged from -.48 to -.78.

Between-class Differences

The second part of the analysis investigated between-class differences using a Kruskal-Wallis, at a p-value of .05, with follow-up pairwise comparisons tests using Dunn's method for unequal sample sizes at a p-value of .05. Family-wise error for these tests was controlled using a Bonferroni correction ($\alpha/3$ =

.017). The Bonferroni correction utilized three comparisons, because follow-up comparison tests were only conducted between the treatment group (class A) and the control groups (classes B, C, and D). Assumptions of the Kruskal-Wallis state that the data are random, the observations are independent, the measurement scale is at least ordinal, and the populations are identical except for a possible difference in location for at least one population. All assumptions were not met, as the data were not randomly selected. Since deviations from this assumption have the potential to influence generalizations and inferences, care was taken in drawing conclusions.

Table 8
Related Rate Assessment Significant Follow-Up Comparisons

Groups		Question	Test Statistic	Std Error	Adj. p-value	Effect Size
A - B	Post1	Pythagorean Score (Q2)	-32.25	9.35	0.001	0.44
	Post1	Rate of Change (Q2b)	-31.71	9.06	0.001	0.44
A - C	Post1	Volumetric Score (Q1)	-29.14	9.15	0.004	0.41
	Post1	Rate of Change (Q2b)	-26.23	9.44	0.015	0.36
A – D	Pre	Rate of Change (Q1b)	28.40	8.12	0.001	0.42
	Pre	Volumetric Score (Q1)	32.67	8.45	<.001	0.46
	Pre	Total Score (Q1 + Q2)	28.59	8.60	0.003	0.40
	Post1	Rate of Change (Q1b)	38.09	8.41	<.001	0.54
	Post1	Volumetric Score (Q1)	43.05	8.62	<.001	0.59
	Post1	Pythagorean Score (Q2)	22.92	8.60	0.024	0.31
	Post1	Total Score (Q1 + Q2)	36.93	8.66	<.001	0.51
	Post1	Rate of Change (Q3a)	36.88	8.34	<.001	0.52
	Post1	Trigonometric Score (Q3)	38.87	8.58	<.001	0.53

Table 8 shows significant follow-up tests as determined by the adjusted p-value. For a complete list of significant results see Appendix F, Table 27. Effect sizes were calculated by dividing the standardized test statistic by the square root of the sum of the two samples being compared. Corrections were made for missing values and ties within the original Kruskal-Wallis ranks. Statistical testing revealed many significant results.

The first item to note in Table 8 is the absence of any statistically significant differences between the treatment (class A) and the control groups (classes B and C), for Question 3 and on all post2 questions. However, there are significant between these three classes on Question 1 (Volumetric) and Question 2 (Pythagorean), to include *Calculating the Rate of Change* for the Pythagorean question (Q2b). Conversely, the treatment (class A) significantly outscored the control group from School 2 (class D) on all three questions and the total scores for both the pre- and post1-assessments.

Longitudinal Achievement

Analyses into longitudinal achievement served to answer the second research question, “What impact does type of instruction have on high school students’ longitudinal achievement on AP calculus RRP?” To investigate longitudinal achievement, student scores were calculated and compared by category. The categories were generated from the literature review on problem solving and what I consider to be the key features for success on RRP. Through this process I identified three specific categories: Determining Relationships Between Variables, Correctly Finding the Derivative, and Indentifying Key Information within the question and prompts. Each of these categories is directly linked to a particular question and rubric point. Data for this analysis was generated and compared by combining student scores across the three assessments. If the student correctly answered a particular question-part they were awarded one point. Question-parts answered incorrectly were awarded zero points. Scores for each assessment were then grouped together to form a three-digit code representing a student's composite performance over the three assessments. For example, a score of 000 represents an incorrect answer for a particular question on all three assessments: pre, post1, and post2. A score of 011 signifies an incorrect answer on the pre, followed by a correct answer on both post1 and post2. The following sections describe the data and present summarized results as a function of the percent of students within each code. The analysis only focused on students who had an incorrect answer on the pre-assessment, that is, 000, 010, 001, and 011. Since each table represents the percent of students based on a portion of the sample, percents were calculated by dividing the number of students within each coded group by the total number of students with incorrect answers on the pre-assessment. In the following sections, the code of 011 will be referred to as "sustained achievement" from measurement point 1 to point 3, as the student progressed from an incorrect answer on the pre-assessment to a correct answer on the post1assessment and retained this achievement on the post2-assessment. The code of 010 will be referred to as "non-sustained achievement" as the student did not sustain the achievement score from post1 to post2. The code of 000 signified "non-achievement".

Header coding for the following tables follow the same system as described in previous sections, with the addition of a number to indicate the specific rubric point. The common elements for each question, Calculation of rate of Change (Q1b, Q2b, Q3a) and Interpretation of Rate of Change (Q1c, Q2a, Q3b), were separated further within the rubric. The common rubric categories used in the longitudinal analysis include: Relating Two Variables (Q1a2, Q1b3), Differentiating an Equation (Q1b2, Q2b3), and Identifying Key Information (Q1b4, Q2b5).

Table 9
Percent of Students Correctly Relating Two Variables

		Question 1a2 (Solve a Rational Eq.)				Question 1b3 (Differentiate a Rational Eq.)				
Class	N	000	010	011	001	N	000	010	011	001
A	28	36%	14%	46%	4%	28	64%	21%	7%	7%
B	23	48%	17%	13%	22%	27	63%	11%	0%	26%
C	25	52%	20%	12%	16%	26	69%	19%	8%	4%

Relating Two Variables

To investigate students' ability to relate two variables, percents were calculated for Q1a2 and Q1b3 across the three assessments (Table 9). For a graphical representation see Appendix G, Figure 4. Question 1a2 (Q1a2) represents a student's ability to setup and solve for a single variable in a rational equation and Q1b3 represents their ability to differentiate a rational equation containing two-variables. When comparing the percentage of students who setup and solved the rational equation correctly (Q1a2), the treatment group (class A) contained a larger percentage of students with sustained achievement (011) over time as compared to the two control groups (classes B and C). In contrast, the percentages of student who could setup and differentiate a rational equation (Q1b3) are relatively consistent across the three classes, with large percentages of non-achievement (000) and small percentages of sustained achievement (011).

Table 10
Percent of Students Differentiating an Equation

	Question 1b2 (Volumetric Differentiation)					Question 2b3 (Pythagorean Differentiation)				
Class	N	000	010	011	001	N	000	010	011	001
A	20	40%	15%	40%	5%	29	14%	24%	52%	10%
B	24	25%	50%	25%	0%	27	4%	30%	67%	0%
C	22	9%	77%	14%	0%	26	0%	31%	69%	0%

Differentiating an Equation

To investigate students' ability to correctly differentiate an equation, percents were calculated for Q1b2 and Q2b3 (Table 10). For a graphical representation see Appendix G, Figure 5. Question 1b2 (Q1b2) represents a student's ability to differentiate a volumetric equation in two-variables and Q2b3 represents their ability to differentiate a Pythagorean equation in two-variables. Patterns across Table 10 are similar to the scoring patterns in Table 9. The treatment group (class A) has the largest percentage of students who demonstrated a sustained ability (011) and the smallest percentage of students who demonstrated a non-sustained ability (010) to take the derivative of a volumetric equation in two-variables (Q1b2) over time.

Table 11
Percent of Students Correctly Identifying Key Information

Q1b4	Question 1b4 (Volume Rate)					Question 2b5 (Direction Rates)				
Class	N	000	010	011	001	N	000	010	011	001
A	23	17%	26%	43%	13%	29	7%	31%	59%	3%
B	22	14%	36%	50%	0%	27	0%	22%	78%	0%
C	23	13%	39%	48%	0%	24	0%	17%	83%	0%

In contrast, both control groups have a larger percentage of students who sustained the ability (011) to differentiate a

Pythagorean equation in two-variables (Q2b3) over time, as compared to the treatment group. For both questions (Q1b2, Q2b3), class A has the largest percentage of students with non-achievement (000) across the three assessments.

Indentifying key information

To investigate students' ability to identify key information within the text of a given problem, percents were calculated for Q1b4 and Q2b5 (Table 11). For a graphical representation see Appendix G, Figure 6. Question 1b4 (Q1b4) represents a student's ability to identify the given rate of change as the volumetric rate of change (dV/dt) and Q2b5 represents their ability to indentify two directional rates of change. The percentages are similar across all three classes when identifying the volumetric rate of change (Q1b4), with the exception that the treatment group (class A) has the smallest percentage of students who demonstrated non-sustained achievement (010). When comparing the percentages of students who identified the directional rates of change (Q2b5), the treatment group has the largest percentage of students with non-sustained achievement (010) and the smallest percentage of students with sustained achievement (011) as compared to the control groups (classes B and C). In contrast, classes B and C have consistent percentages across both questions in terms of non-achievement (000), non-sustained achievement (010) and sustained achievement (011).

Errors Analysis

Error analyses served to answer the third research question, "What impact does the type of instruction have on types of errors high school students make on AP calculus RRP?" Errors were analyzed using an iterative coding process to identify and categorize error structures. These Initial error structures were then mapped to a modified version of Movshovitz-Hadar and Zaslavsky's (1987) descriptive error model. The data for this analysis consisted of Volumetric Rates of Change (Question 1) and Pythagorean theorem (Question 2) inclusive of the sub-parts on all three assessments for classes A, B, and C. I decided to eliminate Question 3 because it was not a common question across the three assessments. I also eliminated class D from this analysis as I felt that results from any comparisons would not help answer the research question. To begin the analysis, each student's pre-assessment in class A was reviewed and errors were identified and coded. Description and codes were based on my experiences as an educator and were meant to answer the question, "How did the student's work deviate from the assessment rubric?" Once the review of class A's pre-assessments was completed, the pre-assessments for class B were reviewed and coded based on the initial codes generated from the errors of class A. New codes were added when needed and the entire set of class A pre-assessments were reviewed again and corrected where appropriate. Upon completion of class B's pre-assessments, the pre-assessments for class C were reviewed and coded based on the initial coding generated from classes A and B. New codes were added when needed and the entire set of class A and B pre-assessments was reviewed again and corrected where appropriate. This process was continued for both the post1- and the post2-assessments. For each instance where new codes were generated the entire set of data were reviewed and modified where appropriate. This exhaustive and comprehensive coding process generated seventy-five errors for Question 1

(Volumetric) and fifty-four errors for Question 2 (Pythagorean). I then mapped the individual errors to a modified version of Movshovitz-Hadar and Zaslavsky's (1987) descriptive error model. Movshovitz-Hadar and Zaslavsky (1987) classified six error types in their descriptive model: Misused Data, Misinterpreted Language, Logically Invalid Inference, Distorted Theorem or Definition, Unverified Solution, and Technical Errors. I decided to eliminate the category Unverified Solution and added Unclear Solution and Blank to represent solutions that I could not follow and identify when a student left the question blank. In the following sections I elaborate on the characteristics of each category, give specific examples of student errors, and provided comparisons between-classes A, B, and C for the specific types of errors. Examples of student work within each error category were selected within the context of a single problem. This allowed the problem statement and prompts to be included in the tables without over complication. It should be noted that this type of selection is purposeful and not representative of error patterns. In order to compare errors between-classes, total errors were computed by class for each question sub-part according to the categories indicated above. Take note that the pre-assessment counts can be misleading due to student ignorance of the pre-assessment material. Low error counts could represent fewer students attempting the problem or getting to a point in the problem with their work where they could answer the question. While large error counts could represent a misunderstanding of the initial problem carried through parts of the problem. Such misunderstandings would only inflate the *misinterpreted language* errors counts. As these types of situations exist in not fully understanding the problem and the necessary solution process. It is also important to note that questions are cumulative in nature, meaning that some parts build on the previous work, and “redundant errors” were not counted in the total. Where a redundant error would include missing a point for not taking the derivative correctly, when in fact the student didn’t know to take a derivative. In this example the student would loose both points in the scoring, but only be counted for one error in the initial point of not knowing that a derivative was needed.

Question coding for the following tables follow the same system as described in previous sections, question number, question part, and rubric sub-part. For example AQ1b2 would indicate an error for class A on Question 1, part (b), rubric point number 2. The common elements for each question, Calculation of rate of Change (Q1b, Q2b, Q3a) and Interpretation of Rate of Change (Q1c, Q2a, Q3b), were separated further within the rubric. Rubric categories used in the error analysis include: Solve a Rational Equation (Q1a2), Differentiated a Rational Equation (Q1b3), Identify Correct Starting Equation (Q2b1), Know to Differentiate (Q1b1, Q2b2), Correctly Differentiate an Equation (Q1b2, Q2b3), Identify Rate of Change as Negative (Q2b7), Interpret Rate of Change (Q1c1, Q2a1), and Justify an Answer (Q1c2, Q2a2). For reasons of parsimony, only error totals of four and above are reported in this section. As a result, sections including Technical Errors and Unclear Errors were excluded, as their error counts were less than 4.

Misused Data

This category includes errors where students used given data in an inappropriate place, used a value of one variable for another, or assigned a value to a variable that was inconsistent with the problem.

Table 12
Number of Misused Data Errors

Type	Code	Error	Pre	Post1	Post2
Solve a Rational Eq.	AQ1a2	MD	26	12	14
	BQ1a2	MD	21	14	13
	CQ1a2	MD	20	17	16
Differentiate a Rational Eq.	AQ1b3	MD	3	9	9
	BQ1b3	MD	2	11	7
	CQ1b3	MD	6	12	9
Justify an Answer	AQ1c2	MD	10	3	3
	BQ1c2	MD	4	5	4
	CQ1c2	MD	5	3	1

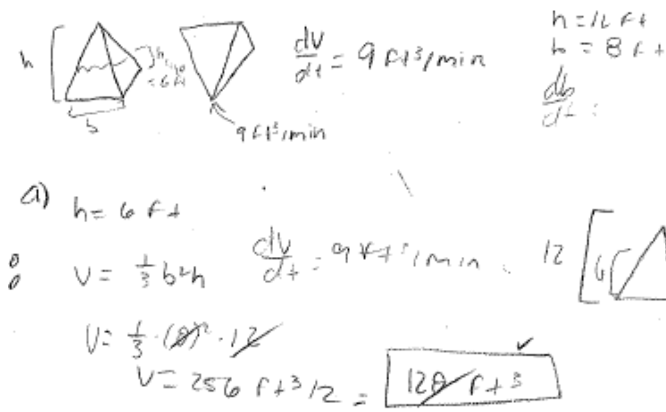
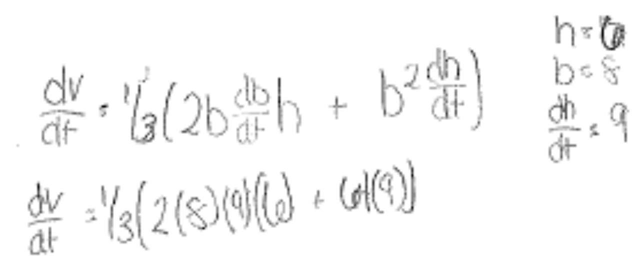
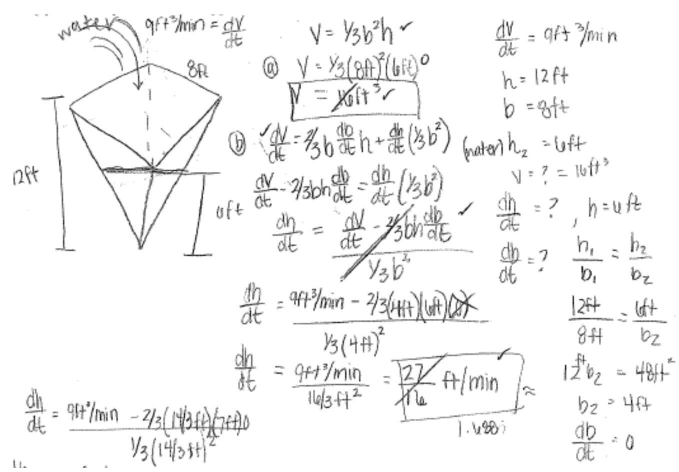
Error totals by question and class for pre, post1, and post2 are given in Table 12. Note in Table 12 that students committed *misused data* errors on Question 1 (Volumetric) and not Question 2 (Pythagorean). Further, results indicate that students within all three classes are committing large numbers of *misused data* errors when solving a rational equation (Q1a2). Student totals on this question decrease across the post-assessments, but remain relatively high. Differentiate a rational equation errors (Q1b3) are relatively consistent across the three classes, increasing from pre to post1, then decreases slightly from post1 to post2. The treatment group (class A) contains the largest number of errors on the pre-assessment when justifying an answer (Q1c2).

Table 13

Examples of Students' Misused Data Errors

Water runs into a square pyramid shaped tank at a rate of 9 ft^3/min. The tank stands point down and has a height of 12 ft and a base of 8 ft. (Note: The volume *V* of a square pyramid with base b and height h is $V=1/3b^2h$)

a) Find the volume *V* of water in the container when h = 6 ft. Indicate units of measure.
b) Find the rate of change of the height of water in the container, with respect to time, when h = 6 ft. Indicate units of measure.
c) As time passes, what happens to the rate at which the water level rises? Justify.

Inappropriate Place	
Transposed Values	
Assigned Incorrect Value	

Examples of these errors are given in Table 13. The most common *inappropriate placement* error occurred when students substituted an accurate given value into a correct equation, but that substituted value was not appropriate in the context of the problem. We can see from the first example in Table 13 that the student substituted a base value of 8 ft into the specified volume equation. In this particular case the base of the pyramid is correctly specified as 8 ft (with a height of 12 ft), but this is inappropriate within the context of the problem, as the prompt specified the volume of the pyramid when the height of the water inside the pyramid was 6ft. The second misused data error occurred when students transposed values by switching the values of two variables. We can see from the second example that with the exception of leaving out the π, the student has correctly generated an equation that represents the derivative of volume. It is also clear that the student has equated dh/dt to 9. This is the given value of the rate at which the volume changes. Therefore the student had transposed the value given for the volumetric rate of change with the rate of change in height. The final type of misused data error consisted of students assigning incorrect values to variables. Such an error occurred when students assigned values to variables that were neither given in the problem nor computed by the student. As we can see in the third example, part (b), the student generated a derivative equation, albeit incorrectly, then manipulated the equation to solve for *dh/dt*. The line after this equation has a zero in the parenthesis. In this particular case the rate at which the base changes is not zero as indicated by the substitution and the *db/dt* = 0 on the right-hand side. The student has incorrectly assigned a value to *db/dt*.

Misinterpreted Language

This category includes errors where students generated an equation different from the situation described verbally, incorrectly interpreted information given in the problem, or solved for a variable different from the variable specified by the prompt.

Table 14
Number of Misinterpreted Language Errors

Type	Question	Error	Pre	Post1	Post2
Know to Differentiate	AQ1b1	ML	7	0	2
	BQ1b1	ML	5	0	2
Correctly Differentiate an Eq.	AQ1b2	ML	14	2	5
	BQ1b2	ML	11	0	2
	CQ1b2	ML	6	0	5
Solve for the Correct Variable	AQ1b5	ML	8	3	7
	BQ1b5	ML	6	3	9
	CQ1b5	ML	4	1	4
Justify an Answer	AQ1c2	ML	5	4	4
	BQ1c2	ML	5	3	5
	CQ1c2	ML	4	5	3
Identify Correct Starting Equation	AQ2b1	ML	18	0	2
	BQ2b1	ML	10	0	0
	CQ2b1	ML	13	0	2
Know to Differentiate	AQ2b2	ML	29	3	5
	BQ2b2	ML	21	0	1
	CQ2b2	ML	21	0	3
Identify Rate as Negative	AQ2b7	ML	0	24	17
	BQ2b7	ML	0	8	10
	CQ2b7	ML	0	9	12

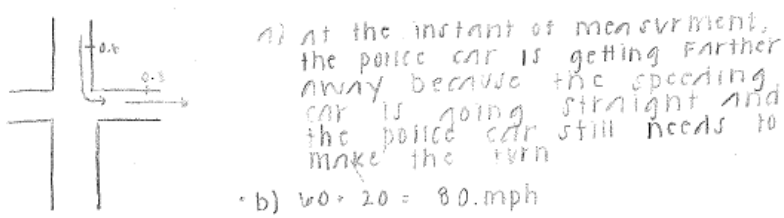
Error totals by question and class for pre, post1, and post2 are given in Table 14. Overall, *misinterpreted language* error totals are low for post1 and remain constant or increase slightly from post1 to post2. Students across all three classes committed a large number of pre-assessment errors for differentiating the equation (Q1b2), identifying the correct starting equation (Q2b1), and knowing to differentiate (Q2b2). Note that the absences of errors on the pre-assessment for Q2b7 is misleading, as students did not take the derivative of their initial equation (errors for Q2b2) and therefore did not generate an equation containing rates of change.

Table 15
Examples of Students' Misinterpreted Language Errors

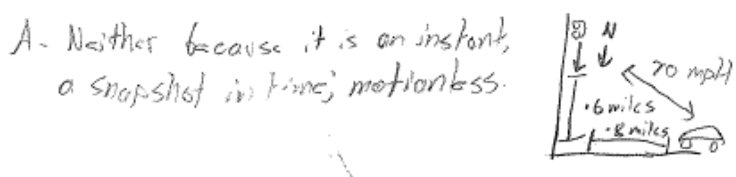
A police cruiser, approaching a right-angle intersection from the north, is chasing a speeding car that has turned the corner and is now moving straight east. When the cruiser is 0.6 miles north of the intersection and the car is 0.8 miles to the east, the police determine with radar that the distance between them and the car is increasing at 20 mph.

a) At the instant of measurement, is the police car getting closer to the speeding car or farther away? Justify your answer.
b) If the police cruiser is moving at 60 mph at the instant of measurement, what is the speed of the car? Indicate units.

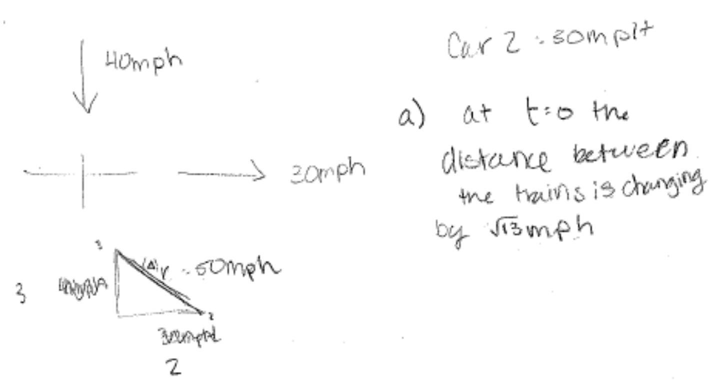
Generated Incorrect Equation



Incorrectly Interpreted Information



Solved for Incorrect Variable



However, error counts on post1 and post2 for Q2b7 are telling and show a large number of students in the treatment (class A) unable to identify a given rate of change as negative on post1 and post2, with a decrease in the number of errors from post1 to post2. The control groups (classes B and C) consistently show a smaller number of this type of identification error on post1 and post2, but are consistently high overall. Examples of these errors are given in Table 15. As we can see from the first example, the student's response to part (b) added the given rates to determine the rate of change between the two vehicles. In this case an additive procedure misrepresents the relationship between the two objects. A correct approach to this part of the question is to differentiate an equation representing the distance between the two objects, i.e., the Pythagorean theorem. The second example demonstrates how the student misinterpreted the given information to formulate an answer to part (a). In this case the given information describes a continuous relationship between the two cars. The prompt states that the student needs to investigate the relationships at a single instant. This particular student interpreted this to mean that the cars were frozen in time and therefore not in motion. The final example demonstrates how students solve for an incorrect variable. This example also involves part (a). As we can see from the student's work the answers is stated to be the square root of thirteen. This number is generated by taking the square root of the sum of squares of the distances between the two cars. The resulting answer is in fact a distance and not a rate of change.

Logically Invalid Inference

This category includes errors where students used incorrect logic to answer a question. Errors of this type include instances when students use invalid logic or when students incorrectly make conclusions based on prior numerical calculations.

Table 16
Number of Logically Invalid Inference Errors

Type	Question	Error	Pre	Post1	Post2
Interpret Rate of Change	AQ1c1	IL	15	9	6
	BQ1c1	IL	11	5	2
	CQ1c1	IL	8	6	4
Justify an Answer	AQ1c2	IL	4	4	1
	BQ1c2	IL	7	2	1
	CQ1c2	IL	5	1	2
Interpret Rate of Change	AQ2a1	IL	0	6	3
	BQ2a1	IL	0	7	0
Justify an Answer	AQ2a2	IL	11	14	6
	BQ2a2	IL	9	9	2
	CQ2a2	IL	15	9	7

It is important to note that the prompts in these questions could be answered through mathematics and numerical interpretations and at no time was it necessary to reason out an answer in a logical manner. Error totals by question and class for pre, post1, and post2 are given in Table 16. For the most part, *logically invalid inference* errors decrease across the three assessments for each class, with a few exceptions. Increases in interpreting the rate of change (Q2a1) occur from pre to post1 for classes A and B, while increases for justifying an answer in Question 2 (Pythagorean) only occur for class A from pre to post1.

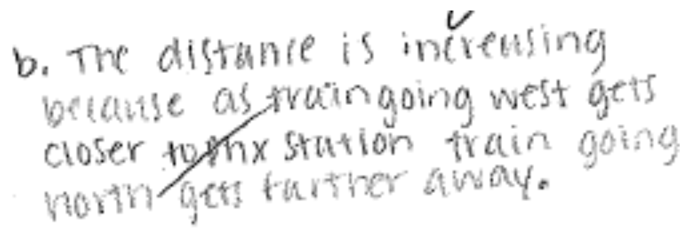
Table 17

Examples of Students' *Logically Invalid Inference Errors*

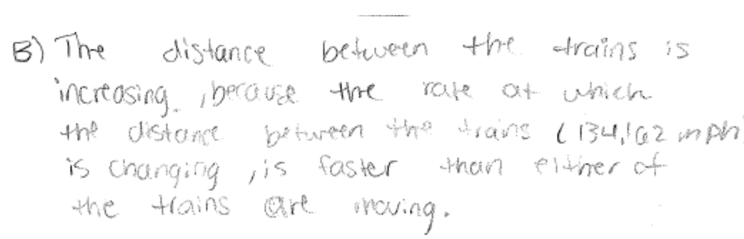
One train travels west towards Phoenix at 120 mph, while a second train travels north away from Phoenix at 90 mph. At time $t = 0$, the first train is 10 miles east and the second train is 20 miles north of Phoenix station.

a) Calculate the rate at which the distance between the trains is changing at $t = 0$.
b) Is the distance between the trains increasing or decreasing at $t = 0$? Justify your answer.

Invalid Logic



Incorrect Numerical Inference



Examples of logically invalid inferences are given in Table 17. We can see in the first example that the student has come to the conclusion that since the two trains are traveling in different directions the trains are getting farther apart, regardless of their speeds. The second example demonstrates how students improperly use numerical values to make inferences. In this example the student states that the rate of change between the trains is 134 mph, but doesn't understand the literal meaning of this number and therefore compares it to irrelevant values to formulate a conclusion.

Distorted Theorem or Definition

This category includes errors where students used a theorem, definition, or procedure incorrectly. Errors of this type include using the wrong differentiation rule or using an equation incorrectly within the context of the problem.

Table 18
Number of Distorted Theorem or Definition Errors

Type	Question	Error	Pre	Post1	Post2
Correctly Differentiate an Eq.	AQ1b2	DT	7	7	9
	BQ1b2	DT	11	7	16
	CQ1b2	DT	12	2	12
Identify Correct Starting Equation	AQ2b1	DT	8	3	1
	BQ2b1	DT	7	0	0
	CQ2b1	DT	7	0	1

Error totals by question and class for pre, post1, and post2 are given in Table 18. Comparatively, Question 1 (Volumetric) seems to produce more *distorted theorem or definition* errors than Question 2 (Pythagorean). Each class increases in the number of errors committed across post1 and post2 for Question 1. While error counts decrease across post1 and post2 for Question 2. Note that the types of errors for Questions 1 and 2 are different. In Question 1, errors are committed when the students perform procedural actions (taking the derivative of an equation) and Question 2, errors are committed when perform conceptual actions (generating an appropriate equation).

Examples of these errors are given in Table 19. The correct differentiation process for this question is to use the product rule to separate the independent variables and then take the derivative of each piece utilizing the chain rule with respect to time *t*. As we can see from the first example the student has separated the original volume equation using the product rule, but has not performed the individual chain rules correctly. In the second example we can see that the student did not separate the original equation using the product rule, but did in fact perform the individual chain rules correctly if the equation had been separated properly.

Table 19

Examples of Students' Distorted Theorems or Definitions Errors

A container has the shape of an open right circular cone. The height of the container is 10 centimeters and the diameter of the opening is 10 centimeters. Water in the container is evaporating so that its depth *h* is changing at the constant rate of -3/10 centimeters per hour. (Note: The volume *V* of a right circular cone with radius *r* and height *h* is given by $V=\pi r^2 h$).

a) Find the volume *V* of water in the container when *h* = 5 centimeters. Indicate units of measure.
b) Find the rate of change of the volume of water in the container, with respect to time, when *h* = 5 centimeters. Indicate units or measure.
c) As time passes, what happens to the rate at which the water volume changes? Justify your answer.

Incorrect Differentiation	b) $V=\pi r^2 h$ $V' = \pi\left(2rh + r^2 \frac{dh}{dt}\right)$ $V' = 2\pi rh + \pi r^2 \frac{dh}{dt}$
Incorrect Differentiation	B) $v' = \frac{1}{3}\pi r^2 \frac{dr}{dt} \cdot \frac{dh}{dt} h$ $v' = \frac{1}{3}\pi (5)^2 \frac{dr}{dt}) \cdot \frac{dh}{dt}(5)$

Problem Solving Motivation Survey

Analysis of the problem solving motivation survey consisted of exploring the factor structure of the survey items and comparing latent mean differences generated from the factor structure. Investigations into the factor structure served to answer the fourth research question, "Do the following five factors underlie the twenty problem solving motivation survey items in the following way: *Mastery*

(Q3, Q4, Q16, Q20), *Ability/Effort* (Q8, Q11, Q17, Q19), *Expectations* (Q2, Q7, Q14, Q18), *Performance* (Q1, Q5, Q9, Q15), and *Value* (Q6, Q10, Q12, Q13)." Data for the problem solving motivation survey were considered to be ordinal in nature, independent between-classes and dependent within-classes, with deviations from normality.

Exploratory Factor Analysis

Normally a Confirmatory Factor Analysis would be used to investigate the fit of the five factors to the measures. Unfortunately the sample size (n = 128) is too small to be sure of model stability. A general rule of thumb for a CFA is 10 samples for each estimated parameter (Field, 2009). A CFA for this model would include 20 error, 15 factor loadings (scaling will set 5 of the twenty loadings to 1), 5 factor variances, and 10 factor covariances equally 50 estimated parameters. Which translates into a minimum sample size of 500. Therefore, an Exploratory Factor Analysis (EFA) was employed. A five-factor EFA at two time points with oblique rotation (Geomin) was performed using the SEM software package Mplus 6.0, on the 128 students participating in this study. A WLSMV estimator was used because the data were ordinal (5-point Likert) and deviated from normality.

Table 20

Two Time Points Exploratory Factor Analysis, Factor Loadings

	Pre-Survey					Post-Survey				
	F1	F2	F3	F4	F5	F1	F2	F3	F4	F5
Q3	0.691					0.702				
Q4	0.673					0.683				
Q16	0.627					0.637				
Q20	0.789					0.801				
Q8		0.827					0.818			
Q11		0.750					0.743			
Q17		0.626					0.619			
Q19		0.822					0.814			
Q2			0.643					0.65		
Q7			0.899					0.908		
Q14			0.833					0.841		
Q18			0.743					0.750		
Q5				0.416					0.440	
Q9				0.543					0.575	
Q10				-0.623					-0.659	
Q15				0.551					0.584	
Q6					0.554					0.543
Q12					0.460					0.450
Q13					0.929					0.910

Missing data included 110 data points and were eliminated pairwise. One participant had missing data for all entries and was therefore eliminated from the data set, making the final sample size 127. Correlation tables with means and standard deviations for pre and post1 are given in tables 28 and 29, Appendix F. Model fit was determined utilizing Hu and Bentler's (1999) cutoff values of comparative fit index (CFI) > .95, Tucker-Lewis fit index (TLI) > .95, and root mean square error of approximation (RMSEA) < .06. The EFA produced a CFI = .965, TLI = .957, and RMSEA = .037 (.024 .048) indicating a good fit between the model and the observed data. Final factor loadings were determined by deleting non-significant parameter estimates and factor loading less than 0.4 (Field, 2009). Factor loading are given in Table 20. Results from the EFA produced an interesting finding. We can see from Table 20 that Question 1 did not load on any factor. Recall that Question 1 was the following, "When I solve math problems I am most interested in getting the correct answer." Upon further reflection I concluded that this question was poorly worded and decided to eliminate it from the analysis.

Table 21

Second Exploratory Factor Analysis, Factor Loadings

	Pre-Survey					Post-Survey				
	F1	F2	F3	F4	F5	F1	F2	F3	F4	F5
Q3	0.712					0.712				
Q4	0.703					0.703				
Q16	0.640					0.640				
Q20	0.780					0.780				
Q8		0.823					0.823			
Q11		0.759					0.759			
Q17		0.628					0.628			
Q19		0.818					0.818			
Q2			0.657					0.657		
Q7			0.907					0.907		
Q14			0.821					0.821		
Q18			0.741					0.741		
Q5				0.431					0.431	
Q9				0.512					0.512	
Q10				-0.589					-0.589	
Q15				0.563					0.563	
Q6					0.570					0.570
Q12					0.482					0.482
Q13					0.945					0.945
α	.768	.780	.807	.606	.669	.767	.776	.855	.657	.642

As such a second EFA was run utilizing Questions 2-20. This second analysis yielded a CFI = .964, TLI = .955, and RMSEA = .040 (.027 .051) indicating a good fit between the model and the observed data.

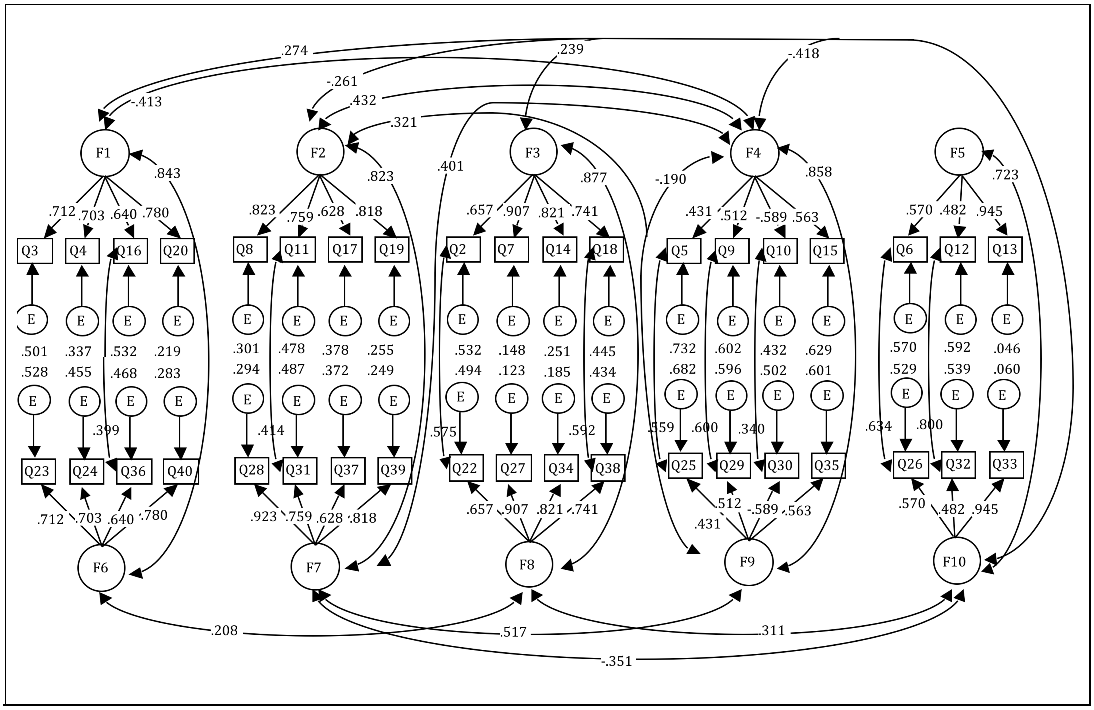


Figure 1. Exploratory Factor Analysis Parameter Estimates

Factor loadings are shown in Table 21 and standardized parameter estimates in Figure 1. Master (F1), Ability/Effort (F2), and Expectation (F3) subscales for pre and post of the Problem Solving Motivation Survey all had high reliabilities, Cronbach's α between .767 and .855. Reliability of .7 to .8 are considered to be acceptable values for Cronbach's α (Field, 2009). Performance (F4) and Value (F5) subscales had lower reliabilities, Cronbach's α between .606 and .669.

With the results from the second EFA tallied and the factor loadings categorized, I compare the hypothesized model with the EFA model.

Hypothesized Model:

Mastery:	Q3	Q4	Q16	Q20
Ability/Effort:	Q8	Q11	Q17	Q19
Expectations:	Q2	Q7	Q14	Q18
Performance:	Q1	Q5	Q9	Q15
Value:	Q6	Q10	Q12	Q13

EFA Model:

Mastery (F1):	Q3	Q4	Q16	Q20
Ability/Effort (F2):	Q8	Q11	Q17	Q19
Expectations (F3):	Q2	Q7	Q14	Q18
Performance (F4):	Q5	Q9	<u>Q10</u>	Q15
Value (F5):	Q6	Q12	Q13	

Comparison of the two models yielded a discrepancy between the hypothesized and EFA models is the elimination of Question 1 (as stated in the preceding analysis) and the placement of Question 10. In the hypothesized model Question 10 was assigned to Value, whereas the EFA model has Question 10 loading negatively on Performance. Theoretically this makes sense as Question 10, "I see no value in spending time to become a good problem solver in math", should decrease as a student increases their orientation towards performance. A student whose primary motivation is the display of competence would not be interested in developing the strategies and competences needed to be a good problem solver.

Latent Mean Differences

Following the EFA, analysis was conducted on latent mean differences. Explorations into latent mean differences served to answer the fifth research question, “Does the type of instruction have an impact on high school AP calculus students’ beliefs and perceptions within the five motivation factors listed in Research Question 4 over time?” Factor scores were computed using the standardized factor loading shown above. The survey data were considered to be interval in nature, dependent within classrooms, independent across classrooms, and non-normal. Since the data deviated from normality, analysis was conducted using nonparametric significance testing. Investigations into the differences within each class and between the three classes utilized SPSS (Version 20).

Analysis of within-class differences was conducted using a Wilcoxon-Sign Rank test for dependent samples, at a p-value of .05. Assumptions of the Wilcoxon-Sign Rank test specify that the each pair of measurements is taken on the same subjects, population differences are symmetric about their median, the differences are independent, and the differences are measured on at least an interval scale.

Table 22
Problem Solving Motivation Survey Within-Class Wilcoxon Sign-Rank Results

Class		Factor	Test Statistic	Std Error	p-value	Effect Size
A	Post-Pre	Mastery	388	54.47	.020	0.41
B	Post-Pre	Ability	74.0	43.91	.003	-0.18

All assumptions were met and the analysis was conducted. Significant results of the within-class analysis are given in Table 22 (for all analysis results see Table

28, Appendix F). Effect sizes were calculated by dividing the standardized test statistic by the square root of the sum of the two samples being compared. Results show that the treatment group (class A) increased from pre to post within the *Mastery* construct. Such an increases represents an average change in students' orientation towards mastery goals, where students with mastery goal orientations focus on the development of competence or task mastery (Elliot, 1999). The first control group (class B) demonstrated a decrease from pre to post within the construct of *Ability*. Such a decrease represents an average change in student perceptions of their mathematical problem solving ability. Although class B does show a statistically significant difference, the effect size is low at -0.18.

Analysis of between-class differences was conducted using a Kruskal-Wallis test, at a p-value of .05, with follow-up pairwise comparisons tests using Dunn's method for unequal sample sizes at a p-value of .05. Family-wise error for these tests was controlled using a Bonferroni correction ($\alpha/3 = .017$). The Bonferroni correction utilized three comparisons, because follow-up comparison tests were only conducted between the treatment group (class A) and the control groups (classes B, C, and D). Assumptions of the Kruskal-Wallis state that the data are random, the observations are independent, the measurement scale is at least ordinal, and the populations are identical except for a possible difference in location for at least one population. All assumptions were not met, as the data were not randomly selected. Since deviations from this assumption have the potential to influence generalizations and inferences, care was taken in drawing conclusions. The results for all comparisons are given in Table 29 (Appendix F).

Table 23
Problem Solving Motivation Survey Follow-Up Comparisons

Groups		Factor	Test Statistic	Std Error	Adj. p-value	Effect Size
A - B	Pre	Expectation	20.82	9.37	0.078	0.29
A - D	Pre	Performance	-23.91	8.66	0.018	-0.33

Table 23 shows the significant pairwise comparisons and includes the test statistic, standard error, adjusted p-value, and the effect size for each comparison. Effect sizes were calculated by dividing the standardized test statistic by the square root of the sum of the two samples being compared. Corrections were made for missing values and ties within the original Kruskal-Wallis ranks. Overall results show two statistically significant follow-up comparisons. Differences between the treatment group (class A) and a control group at the same school (class B) suggest that class A initially possessed a higher expectation for success for mathematical problem solving. Differences between the treatment group (class A) and a control group at the alternate school (class D) suggest that class D initially possessed a higher disposition towards performance goals, focusing on the demonstration of competence relative to others (Elliot, 1999).

CHAPTER 5

CONCLUSION AND RECOMMENDATIONS

Each portion of the data analysis was performed to answer a specific research question. Therefore, the following section will be structured by these research questions. Since Research Question 4 was generated to investigate a dichotomous relationship, the data analysis and results described in Chapter 4 are enough to provide an informed answer. Therefore, the results of Research Question 4 will not be repeated in this section.

1. What impact does type of instruction have on high school student performance on AP calculus Related Rate problems (RRP)?

The skills assessment demonstrated that even though there were variations in the scores, no extreme differences occurred. Therefore, it is realistic to assume that each group was relatively similar in their starting skills and abilities. The results between the Related Rate pre- and post1-assessments showed statistically significant increases from pre to post1 for each class, with moderate to large effect sizes across the four classes. This was expected, as the students had yet to engage with problem solving in the context of Related Rates. The between-class analysis and follow-up pairwise comparisons show statistically significant differences between the two schools A and B. Since the main differences between the schools were time on task and content focus, the results suggest that these two differences are key factors in problem solving achievement. Note that school A and school B instruction also differed on the style of instruction. Instructors from school A emphasized cooperative learning and classroom discussion, while the instructor from school B utilized direct teaching without cooperative groups. Due to the extreme differences in the amount of time spent on task and the content focus, it would be irresponsible to make any claims or assumptions about the affect "teaching style" has on problem solving. Not to mention that the study was not designed to capture these differences. Moreover, investigations into teaching style would change the unit of analysis.

As a means of studying general transfer of general skill (Mayer & Wittrock, 1996), the post1-assessment contained a third question. Recall that Questions 1 and 2 on the three assessments were classified as Volumetric and Pythagorean type problems. Question 3 differed from these two questions, engaging students in Trigonometric relationships. Results of the between-class comparisons showed no significant differences between the treatment group (class A) and the control groups (classes B and C) within school A. Suggesting that instructional differences between these three classes had little effect on the students' general transfer of general skill. However, results of the between-class comparisons showed significant differences between the treatment group (class A) and the control group (classes D) at school B. Again, this difference can be attributed to the discrepancies in the amount of time spent on task and the curricular focus within each classroom, rather than variations in problem solving instruction.

2. What impact does type of instruction have on high school students' longitudinal achievement on AP calculus RRP?

Longitudinal analysis generated from the Related Rate assessments served to answer this research question. There are two aspects of the analysis and results that deserve further attention. First, class A consistently contains a larger percentage of students with sustained achievement and a lower percentage of students with non-sustained achievement across Question 1 (Volumetric), but not Question 2 (Pythagorean). This result can be explained by categorizing the differences between the two types of problems. Both questions are similar in the steps needed to successfully produce a solution: generate an equation, differentiate the equation, identify key information in the problem statement, determine needed relationships between variables, substitute appropriate values, solve for the indicated variable. The differences between the two problems exist in the complexities of each step. In the Volumetric problem (Q1), the system is more complex in the fact that multiple states can exist within the system. Students are given a geometric shape with constant dimensions and prompted to determine a rate of change utilizing a proportion of the geometric shape. In these problems a large number of variations between the conditions can exist, allowing individual problems to be unique in their own right. Furthermore, Volumetric problems require the use and combination of two equations, one given in the problem statement and one needing to be generated by the student. In contrast, Pythagorean (Q2) problems are simplistic in the lack of variation the system can support and the number of equations needed to successfully generate a solution. Pythagorean problems form a system representing a right triangle, with all relationships between the variables existing through the Pythagorean theorem. To successfully solve Pythagorean problems students only need to recognize the system as a right triangle, generate the Pythagorean equation, differentiate correctly, and substitute in the correct values. Since Pythagorean problems allow for little variation, difficulties exist in identifying the information given in the problem statement. It is my contention that the instructional style of Instructor 2 emphasized solving types of problems and not general problem solving. When instruction focuses on problem type, mastery of a specific type of problem converts non-routine problems, where a problem solving approach in required, to routine problems, where learned heuristics are sufficient to solve the problem. This approach works well when a given problem is similar enough to the problems used in practice. But, if the problem differs in ways that are unfamiliar to the student, their ability to perform successfully decreases. As such, complexities of the Volumetric problem (Q1) favor students with a general understanding of problem solving.

Second, the treatment group (class A) contained a larger percentage of students who showed non-achievement across the three assessments. Such a result can be explained by the complexity and duration of the treatment. Since the study engaged students in general problem solving, the concepts were more complex than teaching students how to complete a type of problem. As such, to gain a working understanding of the process, the students, especially those students who started out at a disadvantage, needed more instruction time. A 5-day period wasn't enough time for all students to successfully engage with the complexities of the given problem solving strategy.

3. What impact does type of instruction have on the types of errors high school students make on AP calculus RRP?

The first error code was *misused data* and the analysis showed that all three classes (A, B, and C) were similar in the patterns of errors committed across the pre-, post1-, and post2-assessments. These results suggest that the type of problem solving instruction had comparatively little effect on misusing data. Students from all three classes had a larger number of errors for Question 1 (Volumetric) and fewer errors for Question 2 (Pythagorean). Such results indicate that Question type plays a role in problem solving success.

The second error code was *misinterpreting language* and the analysis also showed that all three classes were similar in the patterns structure across the three assessments, with the exception of Question 2b7 (assigning a negative direction to the speed, i.e., negative rate of change). The general pattern across the pre-, post1-, and post2-assessments consists of moderate to large decreases across pre and post1, with slight to no increase across post1 and post2. In contrast to the *misused data* patters, language errors are larger for Question 2 (Pythagorean) than Question 1 (Volumetric). The treatment group (class A) committed a significantly larger number of errors on Question 2b7 as compared to the two control groups

(Classes B and C). Although class A decreased in the number of errors across post1 and post2, while classes B and C increased, class A retained the largest number of errors on post2. This result suggests that differences in the type of problem solving instruction effected students' ability to detect subtleties in the problem statement of a less complex problem (see the response to Research Question 2 for elaboration on the complexities between the two assessment problems). Such differences are a result of the treatment class engaging with a general problem solving strategy and the control groups investigating how to solve specific types of problem, where problem solving was reduced to engaging in routine problems.

The third error was *logically invalid inferences* and the analysis showed that all three classes were similar in the patterns of errors committed across the three assessments. All three classes decreased from pre to post1 and either increased slightly or remained the constant from post1 to post2, with the exception of justifying an answer on Question 2 (Q2a2). For Question 2a2, the treatment group (class A) increased from pre to post1 then produced a large decrease from post1 to post2. This result suggests that differences in the type of problem solving instruction had little effect on students' ability to justify an answer.

The last error was *distorted theorem or definition* and the analysis showed similar patterns across the three assessments for each class. However, the control groups (classes B and C) showed a larger increase in the number of errors committed while taking the derivative of volumetric equation in two-variables (Q1b2) from post1 to post2. These results would seem to suggest that engagement with the treatment had an effect on the procedural skills of taking derivatives. However, the treatment group's (class A) lower error counts and consistency across the three assessments suggests that the treatment group was initially better at taking derivatives than the control groups and the treatment groups engagement in the engineering design-based problem solving strategy neither helped nor hindered their ability to take a derivative. However, the control groups' (classes B and C) decrease from pre to post1 and increase from post1 to post2 in the number of errors committed suggests an instructional effect. Students within the control group learned how to successfully take the derivative within Volumetric problem during instruction, but could not produce the same success at a later time. These results suggest that problem solving instruction focused on making problem routine has a negative effect on procedural skills in complex setting over time.

Overall results of the error analysis yielded three interesting points. First, traditional problem solving instruction vs. an engineering design-based problem solving instruction had little effect on the type and number of errors committed. Except for class A's ability to correctly determine the sign of a given rate of change within the context of the problem statement and classes B and C's inability to correctly take the derivative of a two-variable equation in the post2-assessment. Second, the type of question and sub-part are key factors in problem solving success. Students have a harder time finding relationships and justifying answers within Volumetric type Related Rate problems. Third, students committed a larger number of *misused data* error than the other types of errors.

5. Does the type of instruction have an impact on high school AP calculus students' beliefs and perceptions within the five motivation factors listed in research question 4 over time?

Analysis conducted comparing latent mean differences resulted in only two statistically significant differences between the four classes. Either the measure wasn't sensitive enough to capture differences or none of the instruction methods had an effect on problem solving motivation.

Recommendations

The results should in no means be taken as exhaustive. Statistical results showed that time and focus are large factors in the development of problem solving success in the context of Related Rates. Error analysis demonstrated that problem type plays a role in the students' ability to reason and find mathematical relationships. Longitudinal analysis results showed that teaching an engineering design-based problem solving strategy, rather than focusing on types of problems, had a positive effect on sustained achievement. Although there were some interesting findings in this study the treatment length was too short and the content choice too difficult for critical substantive power in the study. In further studies the treatment needs to be longer and the content should be scaffolded to introduce problem solving in a simpler context, working up to more complex domains such as Related Rates.

Implications for design exist across two areas: Measurement and Treatment. The content measures for this study served to answer the research questions as a function of Related Rate achievement. Further investigations into problem solving at the AP calculus level need to incorporate measures that are sensitive to both content achievement and problem solving ability. As such, the construct of problem solving ability, at the student level, needs to be defined and evaluated within AP calculus. Before further studies are conducted, it is important to have concrete answers for the following questions.

What does it mean to be a good problem solver in AP calculus?

What abilities and skills, both procedural and cognitive, are necessary to be a good problem solver in AP calculus?

How does problem solving ability affect instruction, time, and motivation needed to be a good problem solver?

Following the answers to these questions new treatment designs need to focus on transitioning from the individual problem solver to a classroom of problem solvers. When instruction transitions from a singular focus to a group focus two obstacles become apparent.

First, how can the diversity in problem solving ability within a classroom be accounted for by instruction? Follow-up studies need to include a measurement variable that can be used to classify students by problem solving ability. Such an indicator can then be used to control for variation across students.

Second, when comparing instruction across classrooms, research designs need to account for instruction overlap. When working in the problem solving domain, instruction in the control will have similarities with instruction in the treatment regardless of treatment innovations. Therefore, instruction of problem solving needs to be defined and evaluated at the classroom level. What is takes to be a good problem solver, at the student level, may be very different from what is necessary for instruction.

REFERENCES

Ames, C. (1984). Competitive, cooperative, and individualistic goal structures: A cognitive-motivational analysis. In C. Ames & R. Ames (Eds.), Research on motivation in education (Vol. 3, pp. 177-207). New York: Academic.

Ames, C. (1992). Classrooms: Goals, Structures, and Student Motivation. Journal of Educational Psychology, 84(3), 261-271.

Ames, C, & Archer, J. (1987). Mothers' beliefs about the role of ability and effort in school learning. Journal of Educational Psychology, 79, 409-414.

Atkinson, J. (1964). An introduction to motivation. Princeton, NJ: Van Nostrand.

Bloom, B. S., Engeihart, M. D., Furst, E. J., Hill, W. H., & Krathwohl, D. (1956). *Taxonomy of educational objectives: The cognitive domain*. New York: McKay.

Bloom, B. S., Hastings, J. T. & Madaus, G. F. (Eds.). (1971). Handbook on the formative and summative evaluation of student learning. New York, NY: McGraw-Hill.

Campbell, D. T, & Stanley, J. C. (1963). Experimental and quasi-experimental designs for research. In N. L. Gage (Ed.), Handbook of research on teaching. Chicago: Rand McNally.

Cohen, J. (1988). *Statistical power analysis for the behavioral sciences* (2nd ed.). Hillsdale, NJ: Lawrence Earlbaum Associates.

College Board (September, 2011). Retrieved from http://apcentral.collegeboard.com/apc/public/courses/teachers_corner/2178.html

Common Core Standards Initiative. (2010). Common Core Standards Initiative. Retrieved from http://www.corestandards.org/

Covington, M. V. (1984). The Self-Worth Theory of Achievement Motivation: Findings and Implications. The Elementary School Journal, 85(1), pp. 5-20.

De Corte, E., Greer, B., & Verschaffel, L. (1996). Mathematics learning and teaching. In D. Berliner and R. Calfee (Eds.), *Handbook of educational psychology* (pp. 491-549). New York: Macmillan.

Dossey, J. A., McCrone, S.A., & O'Sullivan, C. (2006). Problem Solving in the PISA and TIMSS 2003 Assessments (NCES 2007-049). U.S. Department of Education.

Dweck, C. S. (1986). Motivational Processes Affecting Learning. *American Psychologist*. 41(10), pp. 1040-1048. 16

Dweck, C., & Leggett, E. (1988). A social-cognitive approach to motivation and personality. Psychological Review, 95, 256-273.

Elliot, A. J. (1999). Approach and avoidance motivation and achievement goals. Educational Psychologist, 34, 169–189.

Elliot, A., & Dweck, C., (1988). *Goals: An approach to achievement and motivation.* Journal of Personality and Social Psychology, **54**(1): p. 5-12.

Field, A. P. (2009). *Discovering statistics with SPSS* (6th ed.). London: Sage.

Field, A. P., & Miles, J. (2010). *Discovering statistics using SAS*. London: Sage.

Forsterling, F. (1985). Attributional retraining: A review. Psychological Bulletin, 98, 495-512.

Greeno, J. G., Pearson, P. D., & Shoenfeld, A. H. (1997). Implications for the NAEP of research on learning and cognition. In R. Linn, R. Glaser, & G. Bohrnstedt (Eds.), Assessment in transition: Monitoring the nation's educational progress (pp. 151-215). Stanford, CA: National Academy of Education.

Heider, F. (1958). *The psychology of interpersonal relations.* New York: Wiley.

Hiebert, J. & Lefevre, P. (1986). Procedural and Conceptual Knowledge. In J. Hiebert, (Ed.), *Conceptual and Procedural Knowledge: The Case of Mathematics* (pp. 1-27). Hillsdale, NJ: Erlbaum.

Urdan, T., & Maehr, M. (1995). Beyond a two-goal theory of motivation and achievement: A case for social goals. Review of Educational Research, 65,213-243.

Jagacinski, C. M., & Nicholls, J. G. (1987). Competence and affect in task involvement and ego involvement: The impact of social comparison information. Journal of Educational Psychology, 79, 107-114.

Kilpatrick, J., Swafford, J., & Findell, B. (Eds.). (2001). Adding it up: Helping children learn mathematics. Washington, DC: National Academies Press.

Klime, E. (2004). Assessment of cross-curricular problem-solving competencies. In Moskowitz, J. & Stephens, M. (Eds.), Comparing Learning Outcomes: International assessment and education policy (pp. 81-107). New York, New York: Routledge Falmer.

Lester, F. K. & Kehle, P. E. (2003). From Problem Solving to Modeling: The Evolution of Thinking About Research on Complex Mathematical Activity. In Lesh, R. & Doerr, H. (Eds.), Beyond Contructivism (pp. 501-518). Mahwah, New Jersey: Lawrence Erlbaum Associates, Publishers.

Marzano, R. (2007). The Art and Science of Teaching: A Comprehensive Framework For Effective Instruction. Alexandria, VA: Association for Supervision and Curriculum Development.

Maehr, M. (1984). Meaning and motivation. In R. Ames & c. Ames (Eds.). Research in motivation in education: Student motivation (Vol. I. pp. 115-144). New York: Academic.

Mayer, R.E. (1992). *Thinking, Problem Solving, Cognition* (2nd ed.). New York: Freeman.

Mayer, R. E., & Wittrock, M. C. (1996). Problem-solving transfer. In D. Berliner & R. Calfee (Eds.), Handbook of educational psychology (pp. 45–61). New York: Macmillan.

Mehalik, M. , Doppelt, Y., & Schunn, C. D. (2007). Middle-School Science Through Design-Based Learning versus Scripted Inquiry: Better Overall Science Concept Learning and Equity Gap Reduction, Journal of Engineering Education.

Meyer, M. R., & Fennema, E. (1985). Predicting mathematics achievement for females and males from causal attributions. In S. K. Damarin & M. Shelton (Eds.), *Proceedings of the seventh annual meeting of the North American Chapter of the International Group for the Psychology of Mathematics Education* (pp. 201–206). Columbus, OH: Authors.

National Council of Teachers of Mathematics. (1989). *Professional standards for teaching mathematics.* Reston, VA: Author.

National Council of Teachers of Mathematics. (2000). *Professional standards for teaching mathematics.* Reston, VA: Author.

National Mathematics Advisory Panel. (2008a). *Foundations for success: The final report of the National Mathematics Advisory Panel.* Washington, DC: U.S. Department of Education.

National Research Council. (1996). *National science education standards.* Washington, DC: National Academy Press.

National Research Council. (2009). *Learning science in informal environments: People, places, and pursuits.* Committee on Learning Science in Informal Environments. Philip Bell, Bruce Lewenstein, Andrew W. Shouse, and Michael A. Feder, Editors. Board on Science Education, Center for Education, Division of Behavioral and Social Sciences and Education. Washington, DC: The National Academies Press.

Newell, A., & Simon, H.A. (1972). *Human Problem Solving.* Englewood Cliffs NJ: Prentice Hall.

Nicholls, J. (1984). Achievement motivation: Conceptions of ability, subjective experience, task choice, and performance. Psychological Review, 91, 328-346.

Nickson, M. (1992). The Culture of the Mathematics Classroom: An Unknown Quantity?. In D.A. Grouws (ed.), Handbook of Research on Mathematics Teaching and Learning (pp. 101-114), Macmillan, New York.

OECD (1997). (Eds.). *Prepared for life?* (Paris: OECD).

OECD (2033). (Eds.). *The PISA 2003 Assessment Framework. Mathematics, Reading, Science and Problem Solving Knowledge and Skills* (Paris: OECD)

Olson, S., & Loucks-Horsley, S. (Eds.). (2000). Inquiry and the National Science Education Standards: A guide for teaching and learning. Washington, DC: National Research Council

Perkins, D. and Saloman, G. (1994) Transfer of learning. In T. Husen and T. Postlethwaite (eds) *The international encyclopedia of education* 2nd Edition*, Vol. 11* (Oxford: Elsevier Science Ltd)

Peschar, J. (2004). Assessment of Cross-Curricular Problem Solving Competencies in Comparing Learning Outcomes. In J. Moskowitz & M. Stephens (eds.), Comparing Learning Outcomes. New York: RoutledgeFalmer

Phye, G. D. (1997). Classroom Assessment: A Multidimensional Perspective. In G. D. Phye (Ed.) Handbook of classroom assessment: Learning, adjustment, and achievement. NY: Academic Press.

Polya, G. (1945). How to solve it. Princeton, NJ: Princeton University Press.

Ryan, R. M., & Grolnick, W. S. (1986). Origins and pawns in the classroom: Self-report and projective assessments of individual differences in children's perceptions. Journal of Personality and Social Psychology, 50, 550-558.

Salomon, G. & Perkins, D. (1998) Individual and Social Aspects of Learning. Review of Research in Education, 23, 1-24.

Schoenfeld, A. H. (1982). Measures of problem-solving performance and of problem-solving instruction. Journal for Research in Mathematics Education, 13(1), 31-49.

Schoenfeld, A. H. (1985). Mathematical problem solving. New York: Academic Press.

Schoenfeld, A. H. (1987). What's all the fuss about metacognition? In A. Schoenfeld (Ed.), *Cognitive science and mathematics education* (pp. 189-215). Hillsdale, NJ: Erlbaum.

Schoenfeld, A. H., & Herrmann, D. J. (1982). Problem perception and knowledge structure in expert and novice mathematical problem solvers. Journal of Experimental Psychology: Learning, Memory, and Cognition, 8, 484-494.

Schunk, D. (1983). Ability versus effort attributional feedback: Differential effects on self-efficacy and achievement. Journal of Educational Psychology, 75, 848-856.

Shepard, L. (2009). The Role of Assessment in a Learning Culture. Educational Researcher, 29 (4), pp. 4-14.

Stipek, D. (1996). Motivation and instruction. In D. Berliner & R. Calfee (Eds.), Handbook of educational psychology (pp. 85–113). New York: Macmillan.

Ertas, A., & Jones, J. (1993). *The Engineering Design Process*. New York: John Wiley & Sons, Inc.

Thorndike, Edward (1932), *The Fundamentals of Learning*, AMS Press Inc.

Thorndike, R. M. (1978). *Correlational Procedures for Research.* New York: Gardner Press, Inc.

Trier, U. P., & Peschar, J. L. (1995). Cross curricular competencies: Rationale and strategy for a new educational indicator. In G. Phillips, A. Grisay, J. L. Peschar, & M. Granheim (Eds.), *Measuring what students learn* (pp. 97–108). Paris: Organization for Economic Co-operation and Development.

Urdan, T., & Maehr, M. (1995). Beyond a two-goal theory of motivation and achievement: A case for social goals. Review of Educational Research, 65,213-243.

Weeks, A. (2011). Calculus in Motion (Software), Available from http://www.calculusinmotion.com/.

Weiner, B. (1992). Human motivation: Metaphors, theories and research. London: Sage Publications.

Weinert, EE., & Kluwe, R.H. (1987). *Metacognition, Motivation and Understanding:* Hillsdale NJ: Edbaum.

William, D. (2007). Keeping learning on track: classroom assessment and the regulation of learning. In F. K. Lester Jr (Ed.), Second handbook of mathematics teaching and learning (pp. 1053-1098). Greenwich, CT: Information Age Publishing.

APPENDIX A

IRB APPROVAL LETTER

ASU Knowledge Enterprise Development

Office of Research Integrity and Assurance

To:	Tirupalavanam Ganesh EDUC - I.
From:	Mark Roosa, Chair Soc Beh IRB
Date:	11/09/2011
Committee Action:	**Exemption Granted**
IRB Action Date:	11/09/2011
IRB Protocol #:	1111007074
Study Title:	A Study of Problem Solving Strategies in High School Mathematics

The above-referenced protocol is considered exempt after review by the Institutional Review Board pursuant to Federal regulations, 45 CFR Part 46.101(b)(1) .

This part of the federal regulations requires that the information be recorded by investigators in such a manner that subjects cannot be identified, directly or through identifiers linked to the subjects. It is necessary that the information obtained not be such that if disclosed outside the research, it could reasonably place the subjects at risk of criminal or civil liability, or be damaging to the subjects' financial standing, employability, or reputation.

You should retain a copy of this letter for your records.

APPENDIX B

CONSENT AND ASSENT LETTERS

STUDENT ASSENT LETTER

Written Student Assent Form

I have been informed that my parent(s) have given permission for me to participate in a research study on the best ways to implement problem-solving strategies necessary for success on the upcoming AP exam.

I understand that information will be collected investigating the impact the problem solving strategy has on my performance, motivation, and learning. Data collection will include surveys of beliefs on aspects of problem-solving, motivations about problem-solving, and calculus knowledge assessments. As this study coincides with the Related Rate section, I will complete relevant related rate questions in class, for homework, and as a section quiz. These assignments are part of the calculus curriculum and will be graded as normal class assignments, regardless of participation in the study.

I will be asked to participate in group-discussions and these discussions may be audio-taped.

My participation in this project is voluntary and I have been told that I may stop my participation at any time. If I choose not to participate, it will not affect my grade in the class in any way.

Permission to participate in audio-taped focus group discussions:

_____ (initials) **Yes**; I grant the researchers' permission to audio-tape the discussions I take part in.

Permission to participate in the study:

_____ (initials) **Yes**; I will participate in the study.

Signature	Printed Name	Date

PARENT CONSENT LETTER

Parental Letter of Consent for Minors

Page 1

November 9, 2011

Dear Parent,

I will be conducting a research study on the best ways to implement problem-solving strategies necessary for success on the upcoming AP exam. I have developed a problem-solving strategy that integrates current research about problem-solving, motivation, and how students learn. My study will coincide with the calculus curriculum and will serve to add positive strategies related to course content.

As a part of your child's participation, information will be collected concerning the impact the problem solving strategy has on performance, motivation, and learning. Data collection will include surveys of beliefs on aspects of problem-solving, motivations about problem-solving, and calculus knowledge assessments. As this study coincides with the *Related Rate* section, students will complete relevant related rate questions in class, for homework, and as a section quiz. These assignments are part of the calculus curriculum and will be graded as normal class assignments, regardless of participation in the study. Normal classroom seating-assignments locate students in groups of four and will continue throughout this study. At some point during the class a digital audio recorder may be placed at your child's table to record the group discussions.

Your child's participation in this study is voluntary. Precautions will be taken to ensure the anonymity of your child in regards to all collected work (paper and audio) pertaining to any report findings of this study. If the research study is published, a pseudonym will be used in place of your child's name.

Any decision concerning your child not participating in the study will have no negative affects on your child's grade or treatment in class. Participation in the study can be terminated at any time during or after the study.

Professor Dr. Tirupalavanam Ganesh, ASU School of Engineering, will supervise all aspects of this study. If you have any questions concerning the research study or your child's participation in this study, please contact me at jthieken@pvlearners.net or Dr. Ganesh at tganesh@asu.edu.

Sincerely,

Mr. Thieken
Pinnacle High School
jthieken@pvlearners.net

APPENDIX C

MEASURES

SKILLS ASSESSMENT

Answer each of the following questions.

1. A right triangle has a base of 4 and a height of 10. If a similar right triangle has a height of 5, what would be the length of its base?

2. Given $A = \pi r^2$, find $\frac{dA}{dt}$.

3. Given $V = \frac{1}{3}\pi r^2 h$, find $\frac{dV}{dt}$.

4. Given $\frac{dA}{dt} = 4\pi s \frac{ds}{dt}$.

   a. Write an equation for *ds/dt*?

   b. At what rate is side *s* changing the instant side *s* is 4 inches and $\frac{dA}{dt} = 23$ inches squared per second?

5. A right circular cone is filled with water such that the height *h* of the water in the cone changes at a rate of 0.4 meters per minute. If the radius of the cone is 3 meters and the volume of the cone is given by $V = \pi r^2 h$ …

   a. Write an equation for the change in volume as a function of time.
   b. What is the value of *dh/dt*?
   c. Is the height increasing or decreasing when the radius of the cone is 3?
   d. If the right circular cone is empty at time $t = 0$ and a height of 2 meters corresponds to a volume of 56.55 cubic meters. How long will it take to fill the cone to a volume of 56.55 cubic meters?

RELATED RATE PRE-ASSESSMENT

Problem 1

A container has the shape of an open right circular cone. The height of the container is 10 centimeters and the diameter of the opening is 10 centimeters. Water in the container is evaporating so that its depth *h* is changing at the constant rate of $-\frac{3}{10}$ centimeters per hour. (Note: The volume *V* of a right circular cone with radius *r* and height *h* is given by $V = \frac{1}{3}\pi r^2 h$)

(a) Find the volume *V* of water in the container when *h* = 5 centimeters. Indicate units of measure.

(b) Find the rate of change of the volume of water in the container, with respect to time, when *h* = 5 centimeters. Indicate units or measure.

(c) As time passes, what happens to the rate at which the water volume changes? Justify your answer.

Problem 2

A police cruiser, approaching a right-angle intersection from the north, is chasing a speeding car that has turned the corner and is now moving straight east. When the cruiser is 0.6 miles north of the intersection and the car is 0.8 miles to the east, the police determine with radar that the distance between them and the car is increasing at 20 mph.

(a) At the instant of measurement, is the police car getting closer to the speeding car or farther away? Justify your answer.

(b) If the police cruiser is moving at 60 mph at the instant of measurement, what is the speed of the car? Indicate units.

RELATED RATE POST1-ASSESSMENT

Problem 1

Water runs into a square pyramid shaped tank at a rate of 9 ft^3/min. The tank stands point down and has a height of 12 ft and a base of 8 ft. (Note: The volume V of a square pyramid with base b and height h is $V=1/3b^2h$)

(a) Find the volume V of water in the container when h = 6 ft. Indicate units of measure.

(b) Find the rate of change of the height of water in the container, with respect to time, when h = 6 ft. Indicate units of measure.

(c) As time passes, what happens to the rate at which the water level rises? Justify.

Problem 2

One train travels west towards Phoenix at 120 mph, while a second train travels north away from Phoenix at 90 mph. At time t = 0, the first train is 10 miles east and the second train is 20 miles north of Phoenix station.

(a) Calculate the rate at which the distance between the trains is changing at t = 0.

(b) Is the distance between the trains increasing or decreasing at t = 0? Justify your answer.

Problem 3

A searchlight rotates at a rate of 3 revolutions per minute. The beam hits a wall located 10 miles away and produces a dot of light that moves horizontally along the wall. (Note: there are 2π radians in 1 revolution)

(a) How fast is this dot moving when the angle θ between the beam and the line through the searchlight perpendicular to the wall is $\pi/6$?

(b) If the distance to the wall increased to 15 miles, would the dot's speed across the wall increase or decrease? Justify your answer.

RELATED RATE POST2-ASSESSMENT

Problem 1

Water runs into a right circular cone at a rate of 5 ft^3/min. The cone stands point down and has a height of 15 ft and a diameter at the opening of 10 ft. (Note: The volume *V* of a right circular cone with radius *r* and height *h* is given by $V=1/3\pi r^2 h$)

(a) Find the volume *V* of water in the container when $h = 9$ ft. Indicate units of measure.

(d) Find the rate of change of the height of water in the container, with respect to time, when $h = 9$ ft. Indicate units of measure.

(e) As time passes, what happens to the rate at which the water level rises? Justify.

Problem 2

A car travels south towards and intersection at 40 mph, while a second car travels east away from the same intersection at 30 mph. At time $t = 0$, the first car is 3 miles north and the second car is 2 miles east of the intersection.

(a) Calculate the rate at which the distance between the cars is changing at $t = 0$.

(b) Is the distance between the cars increasing or decreasing at $t = 0$? Justify your answer.

PROBLEM SOLVING MOTIVATION SURVEY

Mathematical Problem: A problem in math where the initial solution or method to solve the problem is not obvious.

Ex) If you have 3 quarters, 8 dimes, 5 nickels, and 10 pennies how many different ways can you create $0.55?

Mathematical Problem Solving: Creating and implementing a plan to solve mathematical problems for which the initial solution or method to solve the problem is not obvious.

For questions 1 - 20, bubble in the letter that best describes how you feel about the given statements.

	Agree	**Agree a little**	**Neutral**	**Disagree a little**	**Disagree**
1. When I solve math problems I am most interested in getting the correct answer.	**A**	**B**	**C**	**D**	**E**
2. I am surprised when I solve math problems correctly.	**A**	**B**	**C**	**D**	**E**
3. When attempting math problems my goal is to learn the methods and strategies for solving that type of	**A**	**B**	**C**	**D**	**E**
4. When given a math problem, I do my best because I want to learn new concepts.	**A**	**B**	**C**	**D**	**E**
5. I only want to be a good problem solver so I can do well on exams.	**A**	**B**	**C**	**D**	**E**
6. Learning how to be a good problem solver in math will help me in other courses, i.e. English, History, Social	**A**	**B**	**C**	**D**	**E**
7. When I attempt a math problem, I am confident that I will generate the correct solution.	**A**	**B**	**C**	**D**	**E**
8. The only way to be a good problem solver in math is to be born with the ability.	**A**	**B**	**C**	**D**	**E**
9. Attempting a math problem and not solving it correctly is a waste of time.	**A**	**B**	**C**	**D**	**E**
10. I see no value in spending time to become a good problem solver in math.	**A**	**B**	**C**	**D**	**E**
11. Being a good problem solver in math is a skill that can be learned.	**A**	**B**	**C**	**D**	**E**

	Agree	Agree a little	Neutral	Disagree a little	Disagree
12. Taking the time now, and learning how to be a good problem solver in math will help me in future math	**A**	**B**	**C**	**D**	**E**
13. Learning how to be a good problem solver in math will help me outside of school.	**A**	**B**	**C**	**D**	**E**
14. When I engage in a math problem I expect to be successful.	**A**	**B**	**C**	**D**	**E**
15. I will not attempt a math problem if I think I cannot solve it correctly.	**A**	**B**	**C**	**D**	**E**
16. While solving math problems I think about how I can be a better problem solver.	**A**	**B**	**C**	**D**	**E**
17. Students who are poor problem solvers in math cannot learn to be good problem solvers in math.	**A**	**B**	**C**	**D**	**E**
18. I am surprised when I have difficulties solving math problems.	**A**	**B**	**C**	**D**	**E**
19. Being a good problem solver in math is an ability you either have or don't.	**A**	**B**	**C**	**D**	**E**
20. When I engage in solving math problems, my goal is to learn how to be a better problem solver.	**A**	**B**	**C**	**D**	**E**

APPENDIX D

INSTRUCTOR CURRICULAR MATERIALS

INSTRUCTOR 1 MATERIALS

Problem Solving

Understand the Problem: Generate an understanding of the problem.

- Read the problem statement.
- List the givens and identify their place in the problem, i.e. label using correct notations.
- Summarize your understanding of the problem.
  - Identify key components.
  - Draw a picture if necessary
- Not Try to Solve the Problem

Gather Information: Gather and organize information applicable to the problem.

- Begin to generate connections at a macro level. What procedural knowledge/skills do you have on the subject?
- Use reference materials to clarify parts of the problem you don't understand.
- Have you solved a similar problem
- Do Not Consider the Unknowns at this stage.

Imagine a Solution: Brainstorm possible methods to solve the unknowns.

- Consider each prompt separately
- List any additional givens
- List the unknown(s).

Brainstorm ideas to determine the unknown based on the givens and the information in the GI stage.

- List any idea that comes to mind.
- Do not focus on whether or not the idea will work.
- Ideas should be simple.
- There are no bad ideas … you are hoping to make connections.

Plan your solution: Choose the most appropriate idea from the IS stage.

- Review your list of ideas generated from the brainstorming.
- Determine which idea has the best chance of solving the problem.

Implement your solution: Use your plan and calculate the solution.

- If you find that when you implement your plan another unknown develops, start the process over at the *Imagine a Solution* stage.
- If your plan doesn't work return to brainstorming and select another idea or try brainstorming again.

Evaluate the solution: Determine the appropriateness of your solution.

- Compare givens, assumptions, and unknowns to evaluate the validity of your solution.

INSTRUCTOR 1 MATERIALS

Derivative of Area and Volume Worksheet

	Area	Diagram	*dA/dt*
Triangle	$A = \frac{1}{2}bh$		
Rectangle	$A = lw$		
Trapezoid	$A = \frac{1}{2}h(b_1 + b_2)$		
Parallelogram	$A = bh$		
Circle	$A = \pi r^2$		

	Volume	Surface Area	Diagram	*dV/dt* & *dT/dt*
Right Circular Cone	$V = \frac{1}{3}\pi r^2 h$	$T = \pi rl + \pi r^2$		
Pyramid	$V = \frac{1}{3}Bh$	$T = B + \frac{1}{2}Pl$		
Sphere	$V = \frac{4}{3}\pi r^3$	$T = 4\pi r^2$		
Right Circular Cylinder	$V = \pi r^2 h$	$T = 2\pi rh + 2\pi r^2$		
Rectangular solid	$V = Bh$	$T = 2B + Ph$		

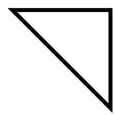
Practice Problems

1. (8) If a snowball melts so that its surface area decreases at a rate of 1 cm^2/min, find the rate at which the diameter decreases when the diameter is 10 cm.
2. (Problem 9) A streetlight is mounted at the top of a 15-ft tall pole. A man 6 ft tall walks away from the pole at a speed of 5 ft/s along a straight path. How fast is the tip of his shadow moving when he is 40 ft from the pole?
3. (Problem 14) A baseball diamond is a square with side 90 ft. A batter hits the ball and runs toward first base with a speed of 24 ft/s.
    a. At what rate is his distance from second base decreasing when he is halfway to first base?
    b. At what rate is his distance from third base increasing at the same point?
4. (Problem 10) At noon, ship A is 150 km west of ship B. Ship A is sailing east at 35 km/hr and ship B is sailing north at 25 km/h. How fast is the distance between the ships changing at 4:00 p.m.?
5. (Problem 18) A particle is moving along the curve $y = \sqrt{x}$. As the particle passes through the point (4,2), its x-coordinate increases at a rate of 3 cm/s. How fast is the distance from the particle to the origin changing at this point?
6. (Problem 19) Water is leaking out of an inverted conical tank at a rate of 10,000 cm^3/min at the same time that water is being pumped into the tank at a constant rate. The tank has height 6 m and the diameter at the top is 4 m. If the water level is rising at a rate of 20 cm/min when the height of the water is 2 m, find the rate at which water is being Problem pumped into the tank.
7. (Problem 20) A trough is 10 ft long and its ends have the shape of isosceles triangles that are 3 ft across at the top and have a height of 1 ft. If the trough is being filled with water at a rate of 12 ft^3/min, how fast is the water rising when the water is 6 inches deep?
8. (Problem 24) A kite 100 ft above the ground moves horizontally at a speed of 8 ft/s. At what rate is the angle between the string and the horizontal decreasing when 200 ft of string have been let out?
9. (Problem 25) Two sides of a triangle are 4 m and 5 m in length and the angle between them is increasing at a rate of 0.06 rad/s. Find the rate at which the area of the triangle is increasing when the angle between the sides of fixed length is $\pi/3$.

INSTRUCTOR 3 MATERIALS

Related Rate Worksheet 1

Example 1: Carry leaves Horizon at the same time as Tom. Carry travels south on 56th St. at 4 m/h. Bob is traveling West on Greenway at 7.5 m/h. At what rate are they moving apart after 2 hours?

1. Draw a picture: 

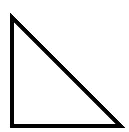
2. Label and put in values
   - Rate that Carry is walking: $dy/dt = 4$ m/h.
   - Rate that Tom is walking: $dx/dt = 7.5$ m/h.

3. Figure out what formula to use: $x^2 + y^2 = s^2$

4. Take the derivative with respect to t: $?/dt$

5. Plug in and solve: $ds/dt = 8.5$ m/h

Example 2: The foot of a 26 ft ladder is 10 feet away from the wall as it leans against the house. The base is moving at the rate of 3 ft/sec. As it slides away, how fast is the top moving? How fast is the area formed by the house, ladder, and ground changing?

1. Draw a picture: 

2. Label and put in values
   - Rate the base is moving away from the house: $dx/dt = 3$ ft/sec.
   - Ladder is 26 ft long: $s = 26$ ft.
   - The ladder doesn't change length: $ds/dt = 0$ ft/sec

3. Figure out what formula to use: $x^2 + y^2 = s^2$

4. Take the derivative with respect to t: $?/dt$

5. Plug in and solve: $dy/dt = -5/4$ ft/sec

6. Figure out what formula to use: $A = 1/2bh$

7. Take the derivative with respect to t: $?/dt$

8. Plug in and solve: $dA/dt = 29.75$ ft/sec

INSTRUCTOR 3 MATERIALS

Related Rate Worksheet 2

1. *Heating a plate.* As a circular plate of metal is heated in an oven, its radius increases at a rate of 0.01 cm/min. At what rate is the plate's area increasing when the radius is 50 cm?

2. *Changing dimensions in a rectangle.* The length l of a rectangle is decreasing at the rate of 2 cm/sec while the width, w is increasing at the rate 2 cm/sec. When $l = 12$ cm and w = 5 cm, find the rates of change of (a) the area, (b) the perimeter, (c) the lengths of the diagonals of the rectangle. Indicate which of these quantities are increasing or decreasing.

3. *Commercial air traffic.* Two commercial jets at 40,000 feet are flying at 520 mph along straight line courses that cross at right angles. How fast is the distance between the planes closing when plane A is 5 miles from the intersection point and plane B is 12 miles from the intersection point? How fast is the distance closing at any time?

4. *Sliding ladder.* A 13-foot ladder is leaning against a house when its base starts to slide away. By the time the base is 12 feet from the house the base is moving at the rate of 5 feet/sec. How fast is the top of the ladder sliding down the wall then? How fast is the area of the triangle, formed by the ladder, wall and the ground changing?

5. *The radius of an inflating balloon.* A spherical balloon is inflated with helium at the rate of $100\pi\, ft^3\ /min$. How fast is the balloon's radius increasing at the instant the radius is 5 feet? How fast is the surface area increasing?

6. *Hauling in a boat:* A boat is pulled towards a dock by a rope from the bow through a ring on the dock 6 feet above the bow. If the rope is hauled in at the rate of 2 ft/sec, how fast is the boat approaching the dock when 10 feet of rope are out?

Challenge

7. *A growing sand pile.* Sand falls from a conveyor belt at the rate of 10 m^3/min onto the top of a conical pile. The height of the pile is always three-eighths of the base of the diameter. Haw fast are the (a) height and (b) radius changing when the pile is 4m high? *Answer in centimeters per minute.*

TEACHER 3 MATERIALS

Related Rate Worksheet 3

1. Gas is escaping from a spherical balloon at a rate of 10 ft^3/hr. At what rate is the radius changing when the volume is 400 ft^3? 0.038 ft/hr

2. As a circular griddle is being heated, its *diameter* changes at a rate 0.01 cm/min. When the diameter is 30 cm at what rate is the area of one side changing? 0.471 cm^2/min

3. A ladder 20 feet long leans against a vertical building. If the bottom of the ladder slides away from the building horizontally at a rate of 2 ft/sec, how fast is the ladder sliding down the building when the top of the ladder is 12 feet above the ground? -8/3 ft/sec

4. A man on the dock is pulling in a boat by means of a rope attached to the bow of the boat 1 ft above the water level and passing through a simple pulley located on a dock 8 feet above the water level. If he pulls in the rope at a rate of 2 ft/sec, how fast is the boat approaching the dock when the bow of the boat is 24 feet from the point that is directly below the pulley? -25/12 ft/sec

5. A girl starts at point A and runs east at a rate of 10 ft/sec. One minute later, another girl starts at point A and runs north at a rate of 8 ft/sec. At what rate is the distance between them changing one minute after the second girl starts? 12.256 ft/sec

6. A water tank has the shape of an inverted right circular cone of altitude 12 feet and base radius of 6 feet. If water is pumped into the tank at the rate of 10 gal/min, approximate the rate at which the water level is rising-when it is 3 feet deep. 0.189 ft/min
   Note: One gallon is approximately 0.1337 cubic feet.

APPENDIX E

ASSESSMENT RUBRICS

SKILLS ASSESSMENT RUBRIC

Answer each of the following questions.

1. A right triangle has a base of 4 and a height of 10. If a similar right triangle has a height of 5, what would be the length of its base?

$$\boxed{1:\ x=2}$$

2. Given $A=\pi r^2$, find $\frac{dA}{dt}$.

$$\frac{dA}{dt}=2\pi r\frac{dr}{dt}$$

$$2:\begin{cases}1:\ 2\pi r\\ 1:\ \dfrac{dr}{dt}\end{cases}$$

zero points if an incorrect rule is initially used

3. Given $V=\frac{1}{3}\pi r^2 h$, find $\frac{dV}{dt}$.

$$\frac{dV}{dt}=\frac{1}{3}\pi\left(2rh\frac{dr}{dt}+r^2\frac{dh}{dt}\right)$$

$$5:\begin{cases}1:\ \text{product rule}\\ 1:2rh\\ {}^{*}1:\dfrac{dr}{dt}\\ 1:r^2\\ {}^{*}1:\dfrac{dh}{dt}\end{cases}$$

*No second dh/dt or dr/dt point if they didn't use the product rule

4. Given $\frac{dA}{dt} = 4\pi s^2 \frac{ds}{dt}$. If side s is 4 inches and $\frac{dA}{dt} = 23$ in^2/sec …

   a. What is the equation for *ds/dt*?
   b. At what rate is the side changing? What are the units?

   a. $\boxed{1:\ \frac{ds}{dt} = \frac{1}{4\pi s^2}\frac{dA}{dt}}$

   *The question was poorly worded, therefore only (a) is counted.

   b. $\boxed{1:\ \frac{ds}{dt} = \frac{23}{64\pi} = 0.114}$

5. A right circular cone is filled with water such that the height h of the water in the cone changes at a constant rate of 0.4 meters per minute. If the radius of the cone is 3 meters and the volume of the cone is given by $V = \pi r^2 h$ …

   a. Write an equation for the change in volume as a function of time.
   b. What is the value of *dh/dt*?
   c. Is the height increasing or decreasing when the radius of the cone is 3?
   d. If the right circular cone is empty at time $t = 0$ and a height of 2 meters corresponds to a volume of 56.55 cubic meters. How long will it take to fill the cone to a volume of 56.55 cubic meters?

*Original volume equation is not the volume of a circular cone. This is OK, the work given for the equation is the important aspect.

   a. $\boxed{1: \frac{dV}{dt} = 2\pi rh\frac{dr}{dt} + \pi r^2\frac{dh}{dt}}$

   b. $\boxed{1:\ \frac{dh}{dt} = 0.4 \text{ m/s}}$

   c. $\boxed{\text{1: Increasing; } dh/dt > 0}$

   d. $\boxed{1:\ t = (2 \text{ m})\frac{1 \text{ min}}{0.4 \text{ m}} = 5 \text{ min}}$

RELATED RATE PRE-ASSESSMENT RUBRIC

Question 1

A container has the shape of an open right circular cone. The height of the container is 10 centimeters and the diameter of the opening is 10 centimeters. Water in the container is evaporating so that its depth *h* is changing at the constant rate of $-\frac{3}{10}$ centimeters per hour. (Note: The volume *V* of a right circular cone with radius *r* and height *h* is given by $V = \frac{1}{3}\pi r^2 h$)

(a) Find the volume *V* of water in the container when *h* = 5 centimeters. Indicate units of measure.

(b) Find the rate of change of the volume of water in the container, with respect to time, when *h* = 5 centimeters. Indicate units or measure.

(c) As time passes, what happens to the rate at which the water volume changes? Justify your answer.

(a) $V = \frac{1}{3}\pi r^2 h$ $\frac{5}{10} = \frac{r}{5} \rightarrow r = 2.5$ $V = \frac{1}{3}\pi(2.5)^2(5) = 10.416\pi$ or 10.417π cm^3	$2: \begin{cases} 1: \text{ V} = \frac{5\pi}{3}r^2 \\ 1: \text{ r} = 2.5 \end{cases}$
(b) $\frac{1}{2}h = r \rightarrow \frac{dr}{dt} = \frac{1}{2}\frac{dh}{dt}$ $\frac{dV}{dt} = \frac{\pi}{4}h^2\frac{dh}{dt}$ or $\frac{dV}{dt} = \frac{1}{3}\pi\left(r^2\frac{dh}{dt} + 2hr\frac{dr}{dt}\right)$ $\frac{dV}{dt} = \frac{25\pi}{4}\left(\frac{-3}{10}\right) = -\frac{15\pi}{8}$ cm^3/hr	$5: \begin{cases} 1: \text{ took derivative of V} \\ 1: \text{ correct } \frac{dV}{dt} \\ 1: {}^{*}\frac{dr}{dt} = \frac{1}{2}\frac{dh}{dt} \text{ or r} = \frac{1}{2}h \\ 1: \frac{dh}{dt} = -\frac{3}{10} \\ 1: \frac{dV}{dt} = \text{ (correct for their work)} \end{cases}$ *correctly in V or dV No $\frac{dV}{dt}$ = point if $\frac{dr}{dt}$ is wrong
(c) Since dh/dt is negative, dV/dt is decreasing. As *h* gets smaller, dV/dt is decreasing at a decreasing rate, i.e. that rate at which the volume changes decreases.	$2: \begin{cases} 1: \text{ answer} \\ 1: \text{ justification} \end{cases}$ *2: used wrong equation in (b) and justified correctly.

Question 2

A police cruiser, approaching a right-angle intersection from the north, is chasing a speeding car that has turned the corner and is now moving straight east. When the cruiser is 0.6 miles north of the intersection and the car is 0.8 miles to the east, the police determine with radar that the distance between them and the car is increasing at 20 mph.

(a) At the instant of measurement, is the police car getting closer to the speeding car or farther away? Justify your answer.

(b) If the police cruiser is moving at 60 mph at the instant of measurement, what is the speed of the car? Indicate units.

(a) $dz/dt > 0$, therefore the speeding car is getting farther away.	$2:\begin{cases}1: \text{ answer}\\1: \text{ justification}\end{cases}$ zero points if no justification is given 2: used wrong equation in (b) and justified correctly.
(b) $x^2+y^2=z^2$ $x\frac{dx}{dt}+y\frac{dy}{dt}=z\frac{dz}{dt}$ $\frac{dx}{dt}=\left(z\frac{dz}{dt}-y\frac{dy}{dt}\right)\frac{1}{x}$ $\frac{dx}{dt}=\left((1)(20)-(0.6)(-60)\right)\frac{1}{0.8}=70\text{ mph}$	$7:\begin{cases}1: \ {*z^2=x^2+y^2}\\1: \text{ took derivative of z}\\1: \text{ correct } dz/dt\\1: \ z=1\\1: \ x=0.8\ \&\ y=0.6 \text{ in } dz/dt\\1: \ dy/dt=60\ \&\ dz/dt=20\\1: \ dy/dt \text{ (negative)}\end{cases}$ *Identified as the equation to use Values correctly substituted into the derivative equation

RELATED RATE POST1-ASSESSMENT RUBRIC

Question 1

Water runs into a square pyramid shaped tank at a rate of 9 ft^3/min. The tank stands point down and has a height of 12 ft and a base of 8 ft. (Note: The volume V of a square pyramid with base b and height h is $V=1/3b^2h$)

(a) Find the volume V of water in the container when h = 6 ft. Indicate units of measure.

(b) Find the rate of change of the height of water in the container, with respect to time, when h = 6 ft. Indicate units or measure.

(c) As time passes, what happens to the rate at which the water level rises? Justify.

(a) $V = \frac{1}{3}b^2(6)$ $\frac{b}{6} = \frac{8}{12} \rightarrow b = 6\left(\frac{8}{12}\right) = 4$ $V = \frac{1}{3}(4)^2(6) = 32 \text{ ft}^3$	$2: \begin{cases} 1: \ V = 2b^2 \\ 1: \ b = 4 \end{cases}$
(b) $\frac{2}{3}h = b \rightarrow \frac{db}{dt} = \frac{1}{3}\frac{dh}{dt}$ $\frac{dV}{dt} = \frac{4h^2}{9}\frac{dh}{dt}$ $\frac{dh}{dt} = \frac{9}{4h^2}\frac{dV}{dt}$ $\frac{dh}{dt} = \frac{9}{16} = .05625$ ft/min	$5: \begin{cases} 1: \text{ took derivative of V} \\ 1: \text{ correct } \frac{dV}{dt} \text{ or } \frac{dh}{dt} \\ 1: \ ^{*}\frac{db}{dt} = \frac{2}{3}\frac{dh}{dt} \text{ or } b = \frac{2}{3}h \\ 1: \ \frac{dV}{dt} = 9 \\ 1: \ \frac{dh}{dt} = \text{ (correct for their work)} \end{cases}$ *correctly in V or dV No $\frac{dh}{dt} =$ point if $\frac{db}{dt}$ is wrong
(c) The rate of change of h will decrease as time passes. As time passes b and h increase, since dV/dt is constant dh/dt will decrease.	$2: \begin{cases} 1: \text{ answer} \\ 1: \text{ justification} \end{cases}$ zero points if no justification is given or answer does not represent how the rate changes 2: used wrong equation in (b) and justified correctly.

Question 2

One train travels west towards Phoenix at 120 mph, while a second train travels north away from Phoenix at 90 mph. At time $t = 0$, the first train is 10 miles east and the second train is 20 miles north of Phoenix station.

(a) Calculate the rate at which the distance between the trains is changing at $t = 0$. Indicate units of measure.

(b) Is the distance between the trains increasing or decreasing at $t = 0$? Justify your answer.

(a) $z^2 = x^2 + y^2$ $\frac{dz}{dt} = \frac{1}{z}\left(x\frac{dx}{dt} + y\frac{dy}{dt}\right)$ $\frac{dz}{dt} = \frac{1}{\sqrt{500}}\left[10(-120) + 20(90)\right]$ $\frac{dz}{dt} = 12\sqrt{5} = 26.832$ mph *dz/dt= 134.164 when x = 120, not -120	$7: \begin{cases} 1: \ {}^{*}z^2 = x^2 + y^2 \\ 1: \text{ took derivative of z} \\ 1: \text{ correct derivative } dz/dt \\ 1: \ z = \sqrt{500} \\ 1: \ x = 10 \ \& \ y = 20 \text{ in } dz/dt \\ 1: \ dx/dt = 120 \ \& \ dy/dt = 90 \\ 1: \ dx/dt \text{ (negative)} \end{cases}$ *Identified as the equation to use
(b) The distance between trains in increasing, dz/dt > 0.	$2: \begin{cases} 1: \text{ answer} \\ 1: \text{ justification} \end{cases}$ zero points if no justification is given 2: used wrong answer in (a) and justified correctly.

Question 3

A searchlight rotates at a rate of 3 revolutions per minute. The beam hits a wall located 10 miles away and produces a dot of light that moves horizontally along the wall. (Note: there are 2π radians in 1 revolution).

(a) How fast is this dot moving when the angle θ between the beam and the line through the searchlight perpendicular to the wall is $\pi/6$? Indicate units of measure.

(b) If the distance to the wall increased to 15 miles, would the dot's speed across the wall increase or decrease? Justify your answer.

(a)

$$\tan\theta = \frac{x}{10}$$

$$\sec^2\theta \frac{d\theta}{dt} = \frac{1}{10}\frac{dx}{dt}$$

$$\frac{dx}{dt} = \frac{10}{\cos^2\theta}\frac{d\theta}{dt}$$

$$\frac{dx}{dt} = \frac{10}{\cos^2(\pi/6)}(6\pi)$$

$$\frac{dx}{dt} = 80\pi \text{ or } 251.327 \text{ miles/min}$$

$$7: \begin{cases} 1: \ y = 10 \text{ (constant - correct diagram place)} \\ 1: \ \frac{d\theta}{dt} = 3 \text{ rev/min or } 6\pi \text{ rad/min} \\ 1: \ dx/dt = ? \text{ (find correct unknown)} \\ 1: \ {}^*\tan\theta = x/10 \text{ (or equivalent relation)} \\ 1: \ {}^*\text{correct derivative} \\ 1: \ dx/dt = \text{ correct equation with answer} \\ 1: \ \text{units} \end{cases}$$

*Equation and derivative must end up incorporating θ and *dθ/dt* to earn both points.

(b) Increase. The distance to the wall is in the numerator or dx/dt. Therefore, if the distances increase, the speed across the wall increases.

$$2: \begin{cases} 1: \ \text{answer} \\ 1: \ \text{justification} \end{cases}$$

zero points if no justification is given

2: used wrong answer in (a) and justified correctly.

RELATED RATE POST2-ASSESSMENT RUBRIC

Question 1

Water runs into a right circular cone at a rate of 5 ft^3/min. The cone stands point down and has a height of 15 ft and a diameter at the opening of 10 ft. (Note: The volume *V* of a right circular cone with radius *r* and height *h* is given by $V=1/3\pi r^2 h$)

(a) Find the volume *V* of water in the container when $h = 9$ ft. Indicate units of measure.
(b) Find the rate of change of the height of water in the container, with respect to time, when $h = 9$ ft. Indicate units of measure.
(c) As time passes, what happens to the rate at which the water level rises? Justify.

(a) $V = \frac{1}{3}\pi r^2(9)$ $\frac{r}{9} = \frac{5}{15} \rightarrow r = 9\left(\frac{5}{3}\right) = 3$ $V = \frac{1}{3}\pi(3)^2(9) = 27\pi \text{ ft}^3$	$2:\begin{cases}1:\ V = 3r^2\\ 1:\ \text{r} = 3\end{cases}$
(b) $\frac{1}{3}h = r \rightarrow \frac{dr}{dt} = \frac{1}{3}\frac{dh}{dt}$ $\frac{dV}{dt} = \frac{\pi h^2}{9}\frac{dh}{dt}$ $\frac{dh}{dt} = \frac{9}{\pi h^2}\frac{dV}{dt} = \frac{9}{81\pi}(5)$ $\frac{dh}{dt} = \frac{5}{9\pi} = .1768$ ft/min	$5:\begin{cases}1:\ \text{took derivative of V}\\ 1:\ \text{correct } \frac{dV}{dt} \text{ or } \frac{dh}{dt}\\ 1:\ {}^{*}\frac{db}{dt} = \frac{1}{3}\frac{dh}{dt} \text{ or } b = \frac{1}{3}h\\ 1:\ \frac{dV}{dt} = 5\\ 1:\ \frac{dh}{dt} = \text{(correct for their work)}\end{cases}$ *correctly in V or dV No $\frac{dh}{dt} =$ point if $\frac{db}{dt}$ is wrong
(c) The rate of change of *h* will decrease as time passes. As time passes b and h increase, since dV/dt is constant dh/dt will decrease.	$2:\begin{cases}1:\ \text{answer}\\ 1:\ \text{justification}\end{cases}$ zero points if no justification is given or answer does not represent how the rate changes 2: referenced wrong equation in (b) and justified correctly.

Question 2

A car travels south towards and intersection at 40 mph, while a second car travels east away from the same intersection at 30 mph. At time $t = 0$, the first car is 3 miles north and the second car is 2 miles east of the intersection.

(a) Calculate the rate at which the distance between the cars is changing at $t = 0$.

(b) Is the distance between the cars increasing or decreasing at $t = 0$? Justify your answer.

$z^2 = x^2 + y^2$ $\frac{dz}{dt} = \frac{1}{z}\left(x\frac{dx}{dt} + y\frac{dy}{dt}\right)$ (a) $\frac{dz}{dt} = \frac{1}{\sqrt{13}}\left[2(30) + 3(-40)\right]$ $\frac{dz}{dt} = \frac{-60}{\sqrt{13}} = -\frac{60\sqrt{13}}{13} = -16.641$ mph *dz/dt= 134.164 when x = 120, not -120	$7: \begin{cases} 1: \ ^*z^2 = x^2 + y^2 \\ 1: \text{ took derivative of z} \\ 1: \text{ correct derivative } dz/dt \\ 1: \ z = \sqrt{13} \\ 1: \ x = 2 \ \& \ y = 3 \text{ in } dz/dt \\ 1: \ dx/dt = 30 \ \& \ dy/dt = 40 \\ 1: \ dy/dt \text{ (negative)} \end{cases}$ *Identified as the equation to use
(b) The distance between trains in decreasing, dz/dt < 0.	$2: \begin{cases} 1: \text{ answer} \\ 1: \text{ justification} \end{cases}$ zero points if no justification is given 2: referenced wrong answer in (a) and justified correctly.

APPENDIX F

STATISTICAL TABLES

Table 24
Skills assessment: Number Correct on Question Part

	Class A			Class B			Class C			Class D		
	N	Sum (%)		N	Sum (%)		N	Sum (%)		N	Sum (%)	
Q1	31	28	(90.3)	27	27	(100)	27	27	(100)	38	36	(94.7)
Q2a	31	27	(87.1)	27	22	(81.5)	27	21	(77.8)	38	22	(57.9)
Q2b	31	16	(51.6)	27	7	(25.9)	27	8	(29.6)	38	3	(7.9)
Q3a	31	23	(74.2)	27	12	(44.4)	27	11	(40.7)	38	14	(36.8)
Q3b	31	18	(58.1)	27	20	(74.1)	27	20	(74.1)	38	17	(44.7)
Q3c	31	16	(51.6)	27	5	(18.5)	27	8	(29.6)	38	3	(7.9)
Q3d	31	18	(58.1)	27	9	(33.3)	27	11	(40.7)	38	8	(21.1)
Q3e	31	16	(51.6)	27	5	(18.5)	27	5	(18.5)	38	3	(7.9)
Q4a	31	28	(90.3)	27	18	(66.7)	27	24	(88.9)	38	27	(71.1)
Q5a	31	12	(38.7)	27	3	(11.1)	27	4	(14.8)	38	2	(5.3)
Q5b	31	4	(12.9)	27	6	(22.2)	27	4	(14.8)	38	5	(13.2)
Q5c	31	12	(38.7)	27	11	(40.7)	27	10	(37.0)	38	21	(55.3)
Q5d	31	6	(19.4)	27	3	(11.1)	27	3	(11.1)	38	4	(10.0)

Table 25
Related Rate Assessment Within-Class Wilcoxon

	Class	N	Test Statistic	Std Error	Std Test Statistic	p-value	Effect Size
PostQ1a-PreQ1b	A	30	298.5	36.85	3.69	<.001	0.67
PostQ1-PreQ1	A	30	399.0	43.71	4.48	<.001	0.82
PostQ2b-PreQ2b	A	31	496.0	50.12	4.95	<.001	0.89
PostQ2-PreQ2	A	31	496.0	50.78	4.88	<.001	0.88
PostTotal-PreTotal*	A	30	465.0	48.53	4.79	<.001	0.87
Post2Q1-PostQ1	A	31	223.5	32.12	-2.66	0.008	-0.48
Post2Q1b-PostQ1b	A	31	267.0	43.32	-1.48	0.140	
Post2Q2-PostQ2	A	32	202.0	32.57	-1.97	0.049	-0.35
Post2Q2b-PostQ2b	A	32	225.0	39.11	-1.27	0.206	
Post2Total-PostTotal*	A	29	233.0	39.24	-1.47	0.143	
PostQ1a-PreQ1b	B	28	325.0	36.73	4.42	<.001	0.84
PostQ1-PreQ1	B	28	378.0	41.42	4.56	<.001	0.86
PostQ2b-PreQ2b	B	28	406.0	42.93	4.73	<.001	0.89
PostQ2-PreQ2	B	28	406.0	43.59	4.66	<.001	0.88
PostTotal-PreTotal*	B	28	406.0	43.75	4.64	<.001	0.88
Post2Q1-PostQ1	B	27	188.0	26.33	-3.15	0.002	-0.61
Post2Q1b-PostQ1b	B	27	156.5	26.50	-1.94	0.052	
Post2Q2-PostQ2	B	27	186.0	26.24	-3.09	0.002	-0.59
Post2Q2b-PostQ2b	B	27	186.0	28.53	-2.47	0.013	-0.48
Post2Total-PostTotal*	B	27	270.5	36.96	-2.92	0.003	-0.56

Table 25
Continued

	Class	N	Test Statistic	Std Error	Std Test Statistic	p-value	Effect Size
PostQ1a-PreQ1b		27	325.0	36.86	4.41	<.001	0.85
PostQ1-PreQ1		27	376.0	41.46	4.51	<.001	0.87
PostQ2b-PreQ2b		27	378.0	40.40	4.68	<.001	0.90
PostQ2-PreQ2		27	378.0	41.35	4.57	<.001	0.88
PostTotal-PreTotal*		27	378.0	41.50	4.55	<.001	0.88
Post2Q1-PostQ1	C	26	210.0	26.49	-3.96	<.001	-0.78
Post2Q1b-PostQ1b		26	192.0	26.54	-3.28	0.001	-0.64
Post2Q2-PostQ2		26	179.0	24.41	-3.44	0.001	-0.67
Post2Q2b-PostQ2b		26	195.5	28.48	-2.81	0.005	-0.55
Post2Total-PostTotal*		26	211.5	28.69	-3.35	0.001	-0.66
PostQ1a-PreQ1b		37	435.0	44.96	4.84	<.001	0.80
PostQ1-PreQ1		37	513.0	53.08	4.69	<.001	0.77
PostQ2b-PreQ2b		38	465.0	48.43	4.80	<.001	0.78
PostQ2-PreQ2		38	447.5	48.42	4.44	<.001	0.72
PostTotal-PreTotal*		37	627.0	60.93	5.12	<.001	0.84
Post2Q1-PostQ1	D						
Post2Q1b-PostQ1b							
Post2Q2-PostQ2		Class D was not given the post2-assessment					
Post2Q2b-PostQ2b							
Post2Total-PostTotal*							

*The total score is the sum of Question 1 and Question 2 scores.

Table 26

Related Rate Assessment Between-Class Kruskal-Wallis

	N	Test Statistic	dof	p-value	Average Rank Class A	Class B	Class C	Class D
PreQ1b	123	15.39	3	.002				
PreQ1	123	21.04	3	<.001	73.02	72.89	67.72	40.50
PreQ2b	124	11.44	3	.010				
PreQ2	124	7.17	3	.067				
PreTotal	123	11.72	3	.008				
PostQ1b	127	42.40	3	<.001	72.00	76.11	85.13	33.91
PostQ1	127	36.69	3	<.001	76.47	77.62	78.80	33.42
PostQ2b	128	46.43	3	<.001	57.24	88.95	86.37	38.14
PostQ2	128	47.82	3	<.001	58.90	91.14	85.13	35.97
PostQ3a	128	52.60	3	<.001	68.68	81.32	89.04	31.79
PostQ3	128	51.19	3	<.001	69.97	80.23	89.54	31.10
PostTotal*	124	52.23	3	<.001	66.11	85.05	81.85	29.18
Post2Q1b	85	1.08	2	.584	There were no significant differences between-classes on the post2-assessment			
Post2Q1	85	2.37	2	.305				
Post2Q2b	85	1.13	2	.570				
Post2Q2	86	1.75	2	.418				
Post2Total	85	1.22	2	.543				

*The total score is the sum of Question 1 and Question 2 scores.

Table 27. Related Rate Assessment Follow-Up Comparisons

Comparison	Question	Test Statistic	Std Error	Std Test Statistic	p-value	Adj. p-value	Effect Size
A - B	PostQ2b	-31.71	9.06	-3.50	<.001	.001	0.44
	PostQ2	-32.25	9.35	-3.45	<.001	.001	0.44
	Post Total	-18.94	9.33	-2.03	.042	.126	
A - C	PostQ1	-29.14	9.15	-3.18	.001	.004	0.41
	PostQ2b	-26.23	9.44	-2.78	.005	.015	0.36
	PostQ3a	-20.36	9.16	-2.22	.026	.078	
	PostQ3	-19.56	9.42	-2.07	.038	.114	
A - D	PreQ1b	28.40	8.12	3.50	<.001	.001	0.42
	PreQ1	32.67	8.45	3.87	<.001	<.001	0.46
	PreQ2b	-10.83	4.62	-2.34	.019	.057	
	PreQ2						
	Pre Total	28.59	8.60	3.33	.001	.003	0.40
	PostQ1b	38.09	8.41	4.53	<.001	<.001	0.54
	PostQ1	43.05	8.62	5.00	<.001	<.001	0.59
	PostQ2b	19.09	8.33	2.29	.022	.066	
	PostQ2	22.92	8.60	2.67	.008	.024	0.31
	Post Total	36.93	8.66	4.26	<.001	<.001	0.51
	PostQ3a	36.88	8.34	4.42	<.001	<.001	0.52
	PostQ3	38.87	8.58	4.53	<.001	<.001	0.53

Table 28
Problem Solving Motivation Survey Within-class Wilcoxon

Pre-Post	Class	N	Test Statistic	Std Error	Std Test Statistic	p-value	Effect Size
Mastery		32	388.0	54.47	2.32	.020	0.41
Ability		32	240.0	63.45	-0.45	.653	
Expectation	A	32	170.0	53.48	-1.76	.079	
Performance		31	212.0	51.03	-0.71	.481	
Value		32	315.0	53.45	0.95	.340	
Mastery		27	258.0	41.62	1.66	.097	
Ability		28	74.0	43.91	-0.94	.003	-0.18
Expectation	B	28	205.0	43.92	0.05	.964	
Performance		28	200.0	43.91	-0.07	.946	
Value		28	142.0	43.90	-1.39	.165	
Mastery		25	222.0	37.16	1.60	.109	
Ability		26	158.0	39.36	-0.45	.657	
Expectation	C	26	184.0	39.37	0.22	.829	
Performance		25	231.0	37.17	1.84	.065	
Value		26	148.0	39.23	-0.70	.483	
Mastery		37	475.0	66.29	1.86	.062	
Ability		38	358.0	6894.00	-0.18	.856	
Expectation	D	37	361.0	66.29	0.14	.886	
Performance		36	299.0	63.65	-0.53	.593	
Value		38	362.0	68.95	-0.12	.902	

Table 29
Problem Solving Motivation Survey Between-class Kruskal-Wallis Results

				Average Rank			
	N	Test Statistic	p-value	Class A	Class B	Class C	Class D
Pre Mastery	124	4.41	0.220				
Pre Ability	125	2.64	0.451				
Pre Expectation	125	8.70	0.034	70.42	49.61	55.43	72.00
Pre Performance	122	14.65	0.002	54.06	63.73	45.76	77.97
Pre Value	125	3.75	0.290				
Post Mastery	123	1.44	0.695				
Post Ability	126	2.43	0.489				
Post Expectation	125	6.68	0.083				
Post Performance	125	3.45	0.322				
Post Value	126	3.55	0.315				

Table 30

Pre-Survey Correlations With Means And Standard Deviations (Kendall's tau)

	Q1	Q2	Q3	Q4	Q5	Q6	Q7	Q8	Q9	Q10	Q11
Q1	1.000	.061	-.139	.005	.060	.024	.041	-.036	.030	-.074	-.061
Q2	.061	1.000	-.078	.126	-.100	.083	.474	-.239	.017	.069	-.087
Q3	-.139	-.078	1.000	.432	.068	-.033	-.112	-.037	-.119	.179	-.133
Q4	.005	.126	.432	1.000	-.193	.080	.107	-.064	-.232	.429	-.170
Q5	.060	-.100	.068	-.193	1.000	-.073	-.207	.097	.217	-.243	.076
Q6	.024	.083	-.033	.080	-.073	1.000	-.044	-.211	-.041	.170	-.198
Q7	.041	.474	-.112	.107	-.207	-.044	1.000	-.061	-.037	.163	-.009
Q8	-.036	-.239	-.037	-.064	.097	-.211	-.061	1.000	.241	-.157	.394
Q9	.030	.017	-.119	-.232	.217	-.041	-.037	.241	1.000	-.384	.150
Q10	-.074	.069	.179	.429	-.243	.170	.163	-.157	-.384	1.000	-.211
Q11	-.061	-.087	-.133	-.170	.076	-.198	-.009	.394	.150	-.211	1.000
Q12	-.038	.048	.092	.223	-.031	.177	.019	-.101	-.157	.203	-.313
Q13	-.146	.101	-.026	.148	-.170	.408	.063	-.048	-.165	.242	-.198
Q14	.117	.406	-.059	.217	-.097	.021	.647	-.088	-.005	.169	-.077
Q15	.001	-.206	-.161	-.222	.143	.082	-.270	.121	.185	-.222	.058
Q16	-.076	.044	.207	.320	-.230	.062	.151	-.094	-.151	.167	-.044
Q17	-.047	-.041	-.249	-.267	.097	-.154	.022	.374	.255	-.303	.475
Q18	.067	.253	-.175	-.058	-.050	.040	.437	.005	.193	.106	.096
Q19	-.050	-.166	-.101	-.098	.217	-.248	-.024	.513	.180	-.179	.407
Q20	-.144	.092	.381	.504	-.155	.110	.107	-.033	-.135	.232	-.126

Table 30
Continued

	Q12	Q13	Q14	Q15	Q16	Q17	Q18	Q19	Q20	Mean	SD
Q1	-.038	-.146	.117	.001	-.076	-.047	.067	-.050	-.144	4.55	.979
Q2	.048	.101	.406	-.206	.044	-.041	.253	-.166	.092	3.14	1.278
Q3	.092	-.026	-.059	-.161	.207	-.249	-.175	-.101	.381	4.24	.974
Q4	.223	.148	.217	-.222	.320	-.267	-.058	-.098	.504	3.76	.954
Q5	-.031	-.170	-.097	.143	-.230	.097	-.050	.217	-.155	3.39	1.305
Q6	.177	.408	.021	.082	.062	-.154	.040	-.248	.110	3.62	1.169
Q7	.019	.063	.647	-.270	.151	.022	.437	-.024	.107	3.40	1.070
Q8	-.101	-.048	-.088	.121	-.094	.374	.005	.513	-.033	1.78	.989
Q9	-.157	-.165	-.005	.185	-.151	.255	.193	.180	-.135	2.07	1.113
Q10	.203	.242	.169	-.222	.167	-.303	.106	-.179	.232	4.40	.816
Q11	-.313	-.198	-.077	.058	-.044	.475	.096	.407	-.126	1.56	.756
Q12	1.000	.361	.083	-.086	.048	-.187	-.126	-.117	.211	4.82	.459
Q13	.361	1.000	.144	-.128	.141	-.136	-.036	-.188	.254	4.31	.945
Q14	.083	.144	1.000	-.199	.236	.028	.430	.046	.220	3.82	1.027
Q15	-.086	-.128	-.199	1.000	-.158	.176	-.125	.227	-.222	2.13	1.055
Q16	.048	.141	.236	-.158	1.000	-.057	.029	-.031	.513	2.76	1.174
Q17	-.187	-.136	.028	.176	-.057	1.000	.072	.531	-.160	1.70	.900
Q18	-.126	-.036	.430	-.125	.029	.072	1.000	.118	-.035	2.82	1.174
Q19	-.117	-.188	.046	.227	-.031	.531	.118	1.000	-.115	1.97	1.031
Q20	.211	.254	.220	-.222	.513	-.160	-.035	-.115	1.000	3.42	1.173

Table 31

Post-Survey Correlations With Means And Standard Deviations (Kendall's tau)

	Q1	Q2	Q3	Q4	Q5	Q6	Q7	Q8	Q9	Q10	Q11
Q1	1.000	-.002	-.097	-.113	.161	-.037	.136	.091	-.097	-.022	-.006
Q2	-.002	1.000	-.046	.140	-.242	.171	.491	-.192	-.063	.119	-.123
Q3	-.097	-.046	1.000	.451	-.048	.021	.074	-.098	-.182	.151	-.010
Q4	-.113	.140	.451	1.000	-.151	.047	.231	-.188	-.041	.222	-.144
Q5	.161	-.242	-.048	-.151	1.000	-.264	-.044	.265	.216	-.191	.069
Q6	-.037	.171	.021	.047	-.264	1.000	.199	-.193	-.131	.216	-.161
Q7	.136	.491	.074	.231	-.044	.199	1.000	-.065	-.071	.113	-.070
Q8	.091	-.192	-.098	-.188	.265	-.193	-.065	1.000	.252	-.217	.455
Q9	-.097	-.063	-.182	-.041	.216	-.131	-.071	.252	1.000	-.348	.069
Q10	-.022	.119	.151	.222	-.191	.216	.113	-.217	-.348	1.000	-.183
Q11	-.006	-.123	-.010	-.144	.069	-.161	-.070	.455	.069	-.183	1.000
Q12	-.028	.244	.095	.107	-.131	.211	.193	-.333	-.153	.198	-.341
Q13	-.115	.190	.100	.149	-.300	.501	.195	-.276	-.167	.224	-.166
Q14	.110	.440	.071	.242	-.055	.083	.666	-.050	.061	.113	-.114
Q15	.009	-.174	.005	-.091	.173	-.146	-.155	.276	.312	-.246	.179
Q16	-.067	.097	.280	.294	-.014	.181	.223	-.117	-.130	.051	-.191
Q17	.048	-.082	.019	.010	.210	-.161	-.042	.384	.233	-.198	.357
Q18	.061	.394	.041	.142	-.051	.078	.539	-.047	.031	.106	-.038
Q19	.092	-.172	-.034	-.092	.235	-.237	-.010	.597	.153	-.232	.482
Q20	-.264	.066	.311	.446	-.164	.121	.147	-.159	-.233	.190	-.226

Table 31

Continued

	Q12	Q13	Q14	Q15	Q16	Q17	Q18	Q19	Q20	Mean	SD
Q1	-.028	-.115	.110	.009	-.067	.048	.061	.092	-.264	4.43	.898
Q2	.244	.190	.440	-.174	.097	-.082	.394	-.172	.066	3.02	1.284
Q3	.095	.100	.071	.005	.280	.019	.041	-.034	.311	4.17	.821
Q4	.107	.149	.242	-.091	.294	.010	.142	-.092	.446	3.89	.981
Q5	-.131	-.300	-.055	.173	-.014	.210	-.051	.235	-.164	3.31	1.253
Q6	.211	.501	.083	-.146	.181	-.161	.078	-.237	.121	3.88	1.107
Q7	.193	.195	.666	-.155	.223	-.042	.539	-.010	.147	3.34	1.121
Q8	-.333	-.276	-.050	.276	-.117	.384	-.047	.597	-.159	1.77	.859
Q9	-.153	-.167	.061	.312	-.130	.233	.031	.153	-.233	2.24	1.194
Q10	.198	.224	.113	-.246	.051	-.198	.106	-.232	.190	4.45	.786
Q11	-.341	-.166	-.114	.179	-.191	.357	-.038	.482	-.226	1.56	.744
Q12	1.000	.389	.204	-.142	.038	-.325	.166	-.315	.059	4.79	.462
Q13	.389	1.000	.191	-.208	.181	-.176	.158	-.261	.251	4.29	.811
Q14	.204	.191	1.000	-.063	.234	.035	.522	-.014	.175	3.67	1.158
Q15	-.142	-.208	-.063	1.000	-.001	.184	-.082	.283	-.097	2.29	1.096
Q16	.038	.181	.234	-.001	1.000	.012	.059	-.157	.479	3.10	1.141
Q17	-.325	-.176	.035	.184	.012	1.000	.140	.553	-.123	1.70	.813
Q18	.166	.158	.522	-.082	.059	.140	1.000	.105	.102	2.76	1.148
Q19	-.315	-.261	-.014	.283	-.157	.553	.105	1.000	-.215	1.86	.961
Q20	.059	.251	.175	-.097	.479	-.123	.102	-.215	1.000	3.67	1.030

APPENDIX F

Graphs

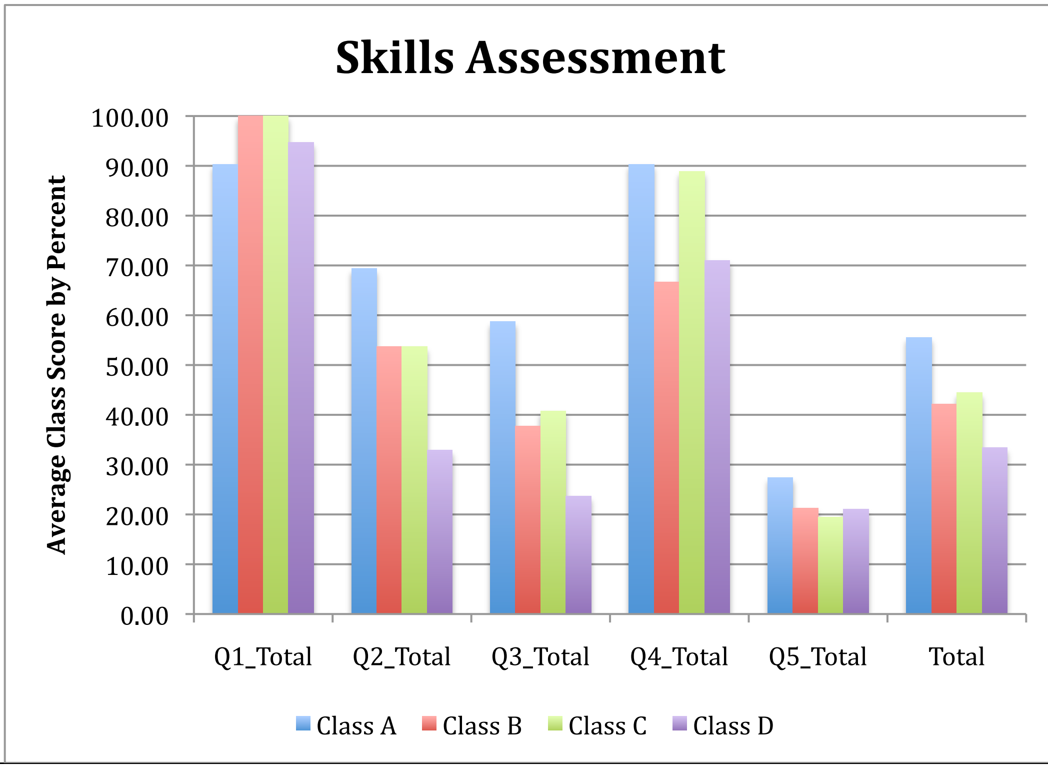


Figure 2. Average Class Score on the Skills Assessment

Figure 2 shows the average student score for each question. Question 1 (Q1) prompted students to generate and solve a rational equation in the context of similar triangles. Question 2 (Q2) prompted students to differentiate an equation in one-variable. Question 3 (Q3) prompted students to differentiate an equation in two-variables. Question 4 (Q4) prompted students to rewrite a differential equation. Question 5 (Q5) prompted students to identity the rate of change in the problem statement and solve a simple related rate problem through substitution. Referencing Figure 2, it is clear that the treatment group (class A) has a greater proficiency for differentiating one- and two-variable equations prior to the study, as compared to all three control groups (classes B, C, and D). Classes A and C scored higher than classes B and D on rewriting a differential equation (Q4). And, all four classes were similar in average scores for solving a rational equation (Q1) and solving a simple related rate problem (Q5).

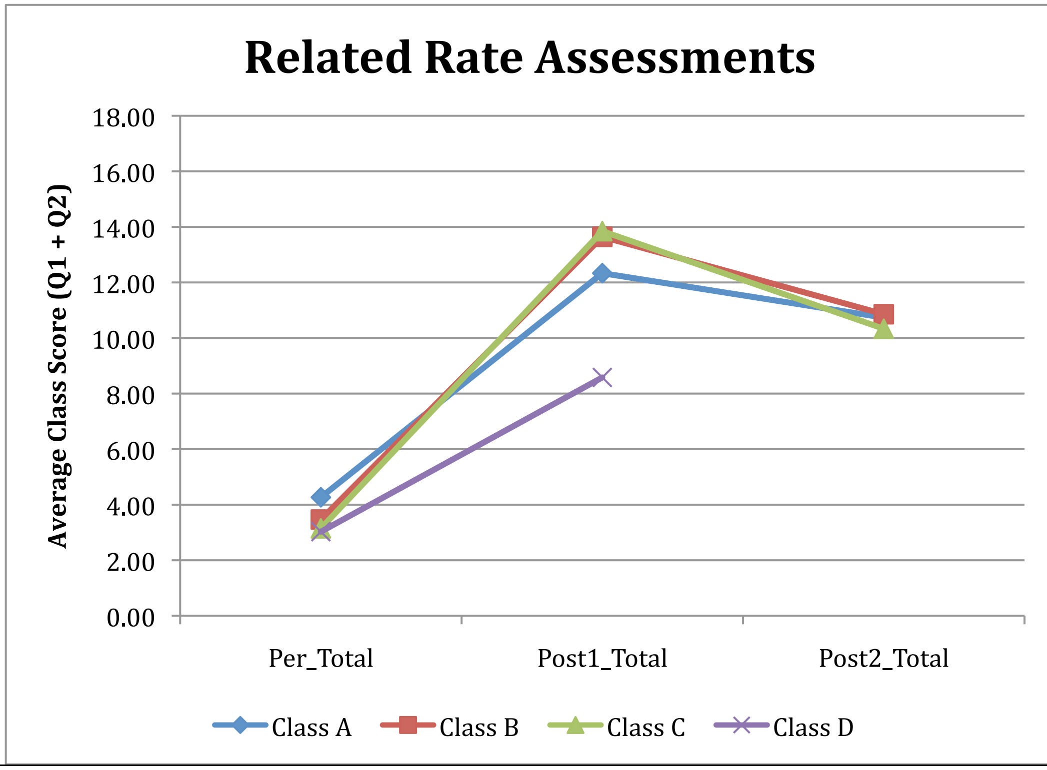


Figure 3. Average Class Score on the Related Rate Assessments

Figure 3 shows average class score for Questions 1 and 2 across the three assessed time points. Average scores within each assessment represent the sum of the average scores for Question 1 (Volumetric) and Question 2 (Pythagorean). Both Question 1 and Question 2 were worth nine points, for a combined maximum score of eighteen points. Results shown in Figure 3 demonstrate an increase from pre to post1 for all four classes and a decrease from post1 to post2 for classes A, B, and C (class D was not administered the post2-assessment).

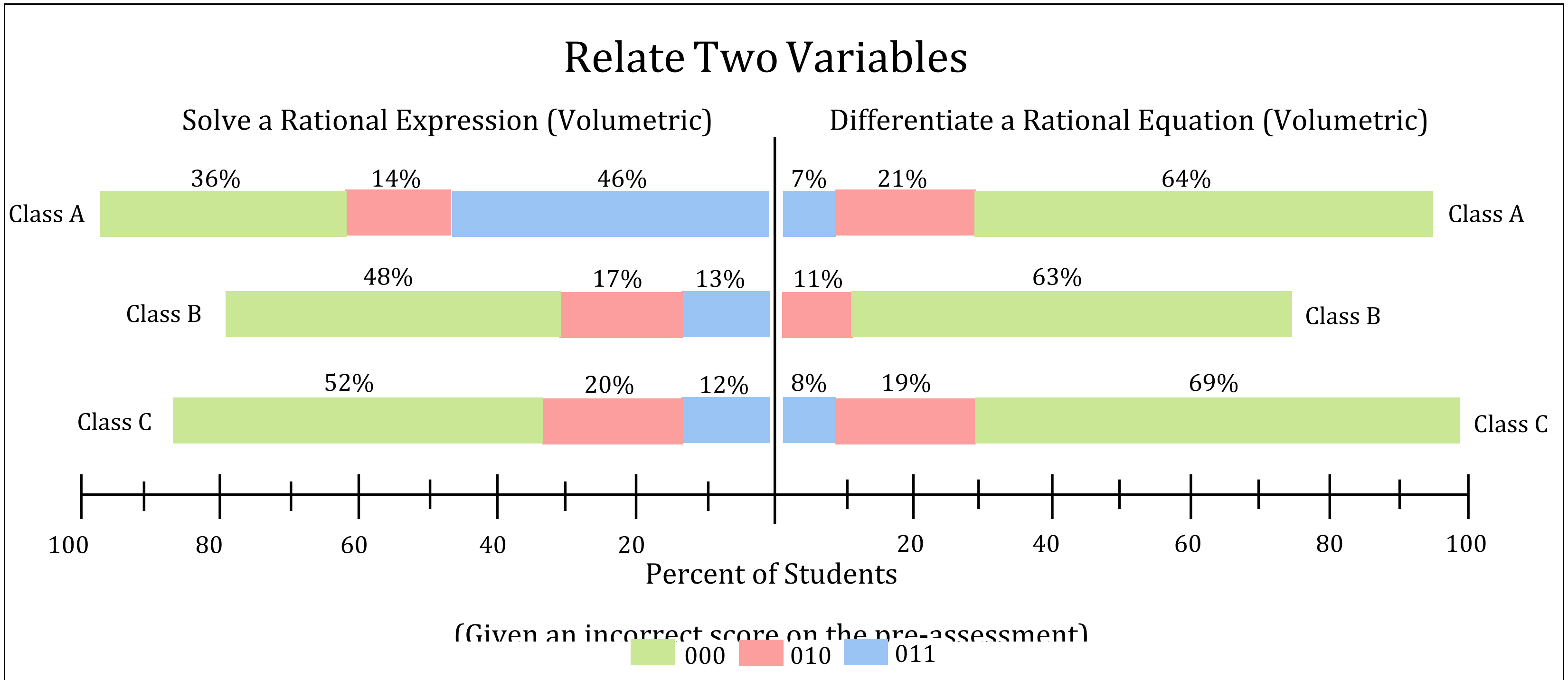


Figure 4. Longitudinal Analysis Results for the Category of Relating Two Variables

When investigating the percentage of students who setup and solved the rational equation correctly, the treatment group (class A) contained a larger percentage of students with sustained achievement (011) as compared to the two control groups (classes B and C). In contrast, the percentages of student who could setup and differentiate a rational equation are relatively consistent across the three classes, with large percentages of non-achievement (000) and small percentages of sustained achievement (011).

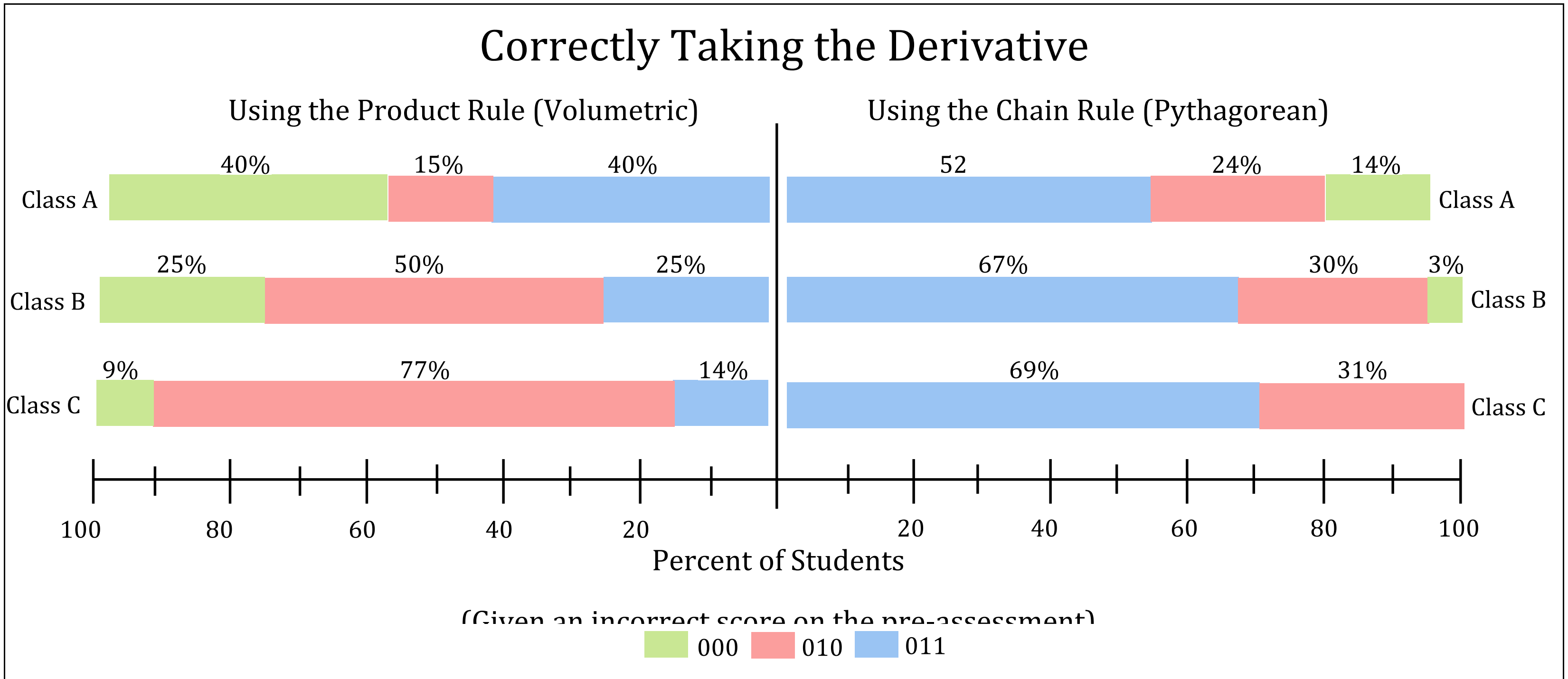


Figure 5. Longitudinal Analysis Results for the Category of Correctly Taking a Derivative

Referencing the results given in Figure 5, the treatment group (class A) has the largest percentage of students who demonstrated a sustained ability (011) and the smallest percentage of students who demonstrated a non-sustained ability (010) to take a derivative using the product rule. In contrast, both control groups have a larger percentage of students who sustained the ability (011) to differentiate a Pythagorean equation using the chain rule, as compared to the treatment group. For both categories, class A has the largest percentage of students with non-achievement (000) across the three assessments.

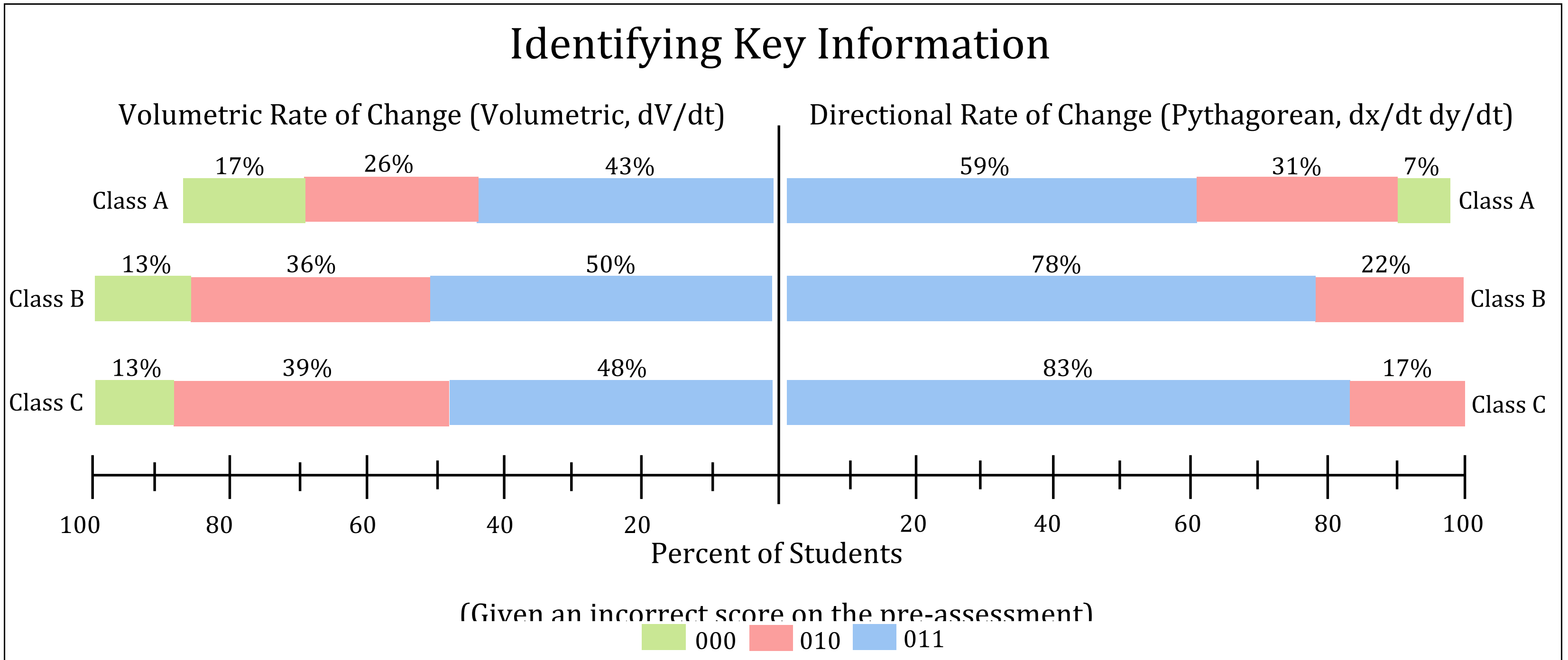


Figure 6. Longitudinal Analysis Results for the Category of Identifying Key Information

The percentages illustrated in Figure 6 are similar across all three classes when identifying the volumetric rate of change. When classes and students who identified the directional rates of change, the treatment group has the largest percentage of students with non-sustained achievement (010) and the smallest percentage of students with sustained achievement (011) as compared to the control groups (classes B and C).