The Influence of Class Nonlinear Dynamics and Education on Socio-Economic Mobility

by

Cesar Paul Montalvo Clavijo

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Graduate Supervisory Committee:

Carlos Castillo-Chavez, Co-Chair
Anuj Mubayi, Co-Chair
Charles Perrings
Yun Kang

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ABSTRACT

The dissertation addresses questions tied in to the challenges posed by the impact of environmental factors on the nonlinear dynamics of social upward mobility. The proportion of educated individuals from various socio-economic backgrounds is used as a proxy for the environmental impact on the *status quo* state.

Chapter 1 carries out a review of the mobility models found in the literature and sets the economic context of this dissertation. Chapter 2 explores a simple model that considers poor and rich classes and the impact that educational success may have on altering mobility patterns. The role of the environment is modeled through the use of a modified version of the invasion/extinction model of Richard Levins. Chapter 3 expands the socio-economic classes to include a large middle class to study the role of social mobility in the presence of higher heterogeneity. Chapter 4 includes demographic growth and explores what would be the time scales needed to accelerate mobility. The dissertation asked how long it will take to increase by 22% the proportion of educated from the poor classes under demographic versus non-demographic growth conditions. Chapter 5 summarizes results and includes a discussion of results. It also explores ways of modeling the influence of nonlinear dynamics of mobility, via exogenous factors. Finally, Chapter 6 presents economic perspectives about the role of environmental influence on college success. The framework can be used to incorporate the impact of economic factors and social changes, such as unemployment, or gap between the haves and have nots. The dissertation shows that peer influence (poor influencing the poor) has a larger effect than class influence (rich influencing the poor). Additionally, more heterogeneity may ease mobility of groups but results depend on initial conditions. Finally, average well-being of the community and income disparities may improve over time. Finally, population growth may extend time scales needed to achieve a specific goal of educated poor.

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Chapter 1

INTRODUCTION

The ability of individuals to live in societies where upward mobility is possible during their lifetime or across generations is central to the progress, stability, and well-being of most modern societies. Hence, understanding the dynamics of upward mobility are of deep interest. Consequently, characterizing the socio-economic landscapes that facilitate the role of mechanisms that may be responsible for social mobility is essential. The statistics provided by international organizations like the World Bank [7] or the United Nations [8, 9] on changes in levels of poverty, education attainment, Gross Domestic Product (GDP) growth, and real value income over time are used to identify factors that may affect mobility. Over the past decade, we have seen dramatic decreases in the proportion of people living in extreme poverty, a marker of upward social mobility, particularly in some societies. In fact, the number of people surviving with less than 1.90 dollars a day dropped from 1,895 in 1990 to 736 millions in 2015, with China and India leading the way in the reduction of extreme poverty [7]. The World Bank reports that in the early 1990s, China and India housed 57% and 46% of the population living on less than 1.90 dollars per day, respectively, percentages that have decreased to 7.9% and 21.2% by 2011 [10].

Poverty reduction depends on factors that include investment in human talent. This perspective suggests that upward mobility may be difficult to achieve when access to education, particularly higher education, is limited. Historically, individuals or groups with deleterious initial conditions, such as those that are the result of racism, segregation, neglect of rural areas, and the abandonment of “barrios” or “neighborhoods”, have had reduced access to higher education. The role of these factors in maintaining poverty in the USA, has been documented extensively [11]. A glance at the origin of historical black colleges and universities (HBCUs) highlights some of the efforts carried out by minority communities to increase access to higher education [12]. In Latin America, for example, in the case of the nation of El Salvador, reduced access to higher education is tied in, among other factors, to the pervasive prevalence of gangs and the violence that comes with them [13].

In order to frame the central aspects of this dissertation, it is important to introduce some definitions and concepts associated with mobility. Inter-generational income mobility is often defined as the chance for a child to move up in the income distribution scale relative to her/his parents' economic status [14]. The World Bank identifies two modes of inter-generational mobility (IGM) [2]. Absolute IGM, which means that living standards of individuals within a generation are higher than those of the parents; on the other hand, relative IGM is a measure of the economic position of an individual, that is, it is independent of the socio-economic status of the parents. Relative mobility means that the achievements of individuals are less likely to be affected by the circumstances that they inherit, including parental education and income, race, gender, and birthplace [15].

The data from the World Bank demonstrates that relative and absolute mobility are higher in developed countries. The report "Economic Mobility Across Generations Around the World" studied cohorts from 1940s through 1980s with information from 148 economies, including 111 developing countries [2]. The data show that developing nations experience lower mobility and, in particularly, that South Asia and Africa ranked lower on mobility than the average within developing nations. The data on the Middle East, Northern Africa and Latin America show lower rates of income mobility, in general, possibly "due to a lack of an educated workforce and labor market deficiencies." Low rates of mobility provide further evidence that it is more difficult to move out of poverty in a poor country [2].

The focus on this dissertation is on the role of higher education, as defined by college attainment, on mobility, since it has been documented that college education generates higher increases in individuals' earnings. In the United States, according to the US Bureau of Labor Statistics, those having a college degree have median weekly earnings that are 65% higher than those with only a high school diploma. Further, vulnerability decreases for college graduates. For example, the rates of unemployment are considerably lower for those with a bachelor's degree (2.5%) than those with only a high school degree (4.6%). Figure 1.1 shows the change in earnings and employment among those with college degrees against those who have less years of education in 2017.

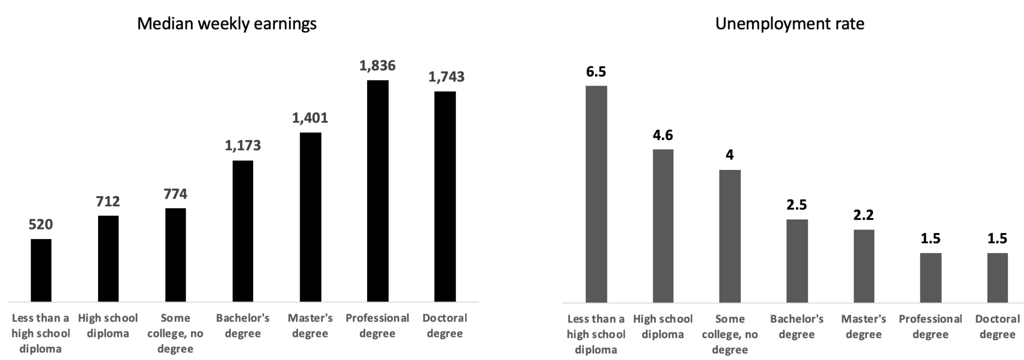


Figure 1.1: Median Weekly Earnings and Unemployment Rates by Educational Attainment in U.S., 2017. Source: Bureau of Labor Statistics

Figure 1.2 shows that offspring generally achieve higher levels of education than their parents. The left panel of Figure 1.2 highlights the fact that developing countries had reached by 1980s almost the same levels of college education than high-income nations had 40 years earlier. Further, we see that developed nations doubled the percentage of population with a college degree at faster rates over the 1940-1980 period. Also, in developed countries, the percentage of females with a college degree has surpassed the percentage of males since 1950 [2]. In short, there is still a large breach regarding access to higher education between developing and developed countries.

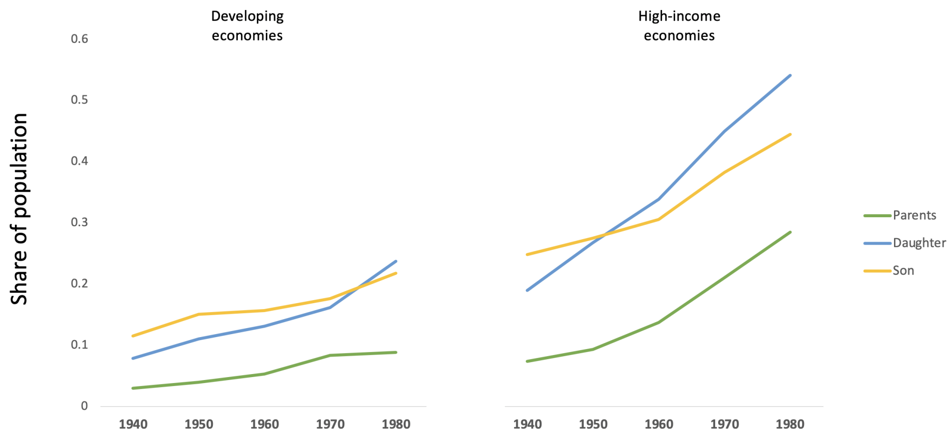


Figure 1.2: Share of Population with Higher Education (Tertiary) in Different Cohorts, using the Global Database on Intergenerational Mobility. Source: World Bank [2]

Inter-generational mobility can be assessed through changes in income category, that is, estimating the probability of change of income over time. These estimates differ considerably if ethnicity is taken into account. In the United States, for example, according to a study conducted by Chetty et al. using data from 1989 to 2015 [16] , 15% of White men who grew up poor may become rich while only 7% of poor Black men may do so. Further, the odds of falling into poverty double for Blacks who were born rich in contrast to Whites who also were born rich. Poverty is strongly tied in to ethnic groups, as well. For example, 2015 data show that 38% of the Black poor remains in poverty with only 26% of the White poor remaining poor. Table 1.1 presents matrices that summarize income mobility by socio economic status (row) for both White and Black men in the United States [17].

Population transition matrices, like the above, provide an initial simple way to

	rich	middle	poor
rich	39	51	10
middle	24	62	14
poor	15	59	26

White men

	rich	middle	poor
rich	17	62	21
middle	14	61	25
poor	7	55	38

Black men

Table 1.1: Transition Probabilities Representing Income Mobility for White and Black Men in the United States, Showing Socio-Economic Status at Birth and Adulthood.

roughly analyze the likelihood of mobility for future generations under unchanging conditions [18]. The above matrices, when re-scaled, can be considered as transitional matrices A for Markov processes where, the entry (i, j) for every row and column, respectively, provides the probability that an individual moves from the ith state to the jth state over the next period. Given an initial state of the population distribution P_0, the probability distribution of the groups after t periods, without any change in the transition probabilities, is given by $P_t = A^t P_0$. Hence, a rough estimate of proportions of classes can be estimated, regardless of where they were born. In the long term, 26.5%, 58.5% and 14,6% of White men; and 12.4%, 59.4% and 28,2% of Black men will be rich, middle or poor, respectively, in the United States.

The study "Creating Moves to Opportunity: Experimental Evidence on Barriers to Neighborhood Choice" conducted by Bergman et al. [19] addresses upward and downward social mobility based on the likelihood that someone can achieve higher or lower socio-economic status. They observed that moving to an area with low poverty figures at young age, has a sizable impact on the long-term success of an individual; his/her earnings may increase by 30 % while the odds that he/she may go to college also increases [20].

The majority of research and reports about mobility has focused particularly on the trends of income through time and how family and stochasticity (i.e. luck) have shaped mobility in various communities. However, there seems to be no modeling research that addresses neighborhood effects on the nonlinear dynamics of social mobility.

1.1 Social Mobility in the United States: the American Dream Revised

Mobility in the United States has slowed down according to the mobility report generated by the Federal Reserve [15]. The American Dream has been called by some authors that includes Nobel Laurate Stiglitz "at a larger extent, as a myth" [21]. The evidence suggests that economic mobility might have reached maximum levels, that is, a potential period of stagnation may now be in place. Chetty et al. [22] show that the probability that young generations may be earning more than their parents has declined over time. In 1940, 90% of children eventually earned more than their parents but by 1985, this success percentage had dropped to 50% for all children. Social mobility closely relates to their surroundings, which include the neighborhood that individuals live in, the schools that they attend, the type of employers that they have access to, the churches that they attend, the presence of unions, etc. Including these aspects in a full analysis of mobility becomes a tremendous challenge [15]. Chetty et al. [22] argue that mobility has slowed down in the United States possibly for two reasons: decaying rates of growth in the economy and inequity in wealth distributions, the latter, being the leading cause. Figure 1.3 gathers estimates obtained by Chetty et al. for 5 cohorts. The data captures income comparisons at age 30 for children and their parents, adjusted for inflation. Higher rates of GDP growth are required for the last cohorts in the study to catch up with the figures of mobility from 1940's. It is further observed in this study, that growth alone would be insufficient to maintain upward mobility but, deliberate measures in education and health are needed to maintain the flow of people reaching higher income classes.

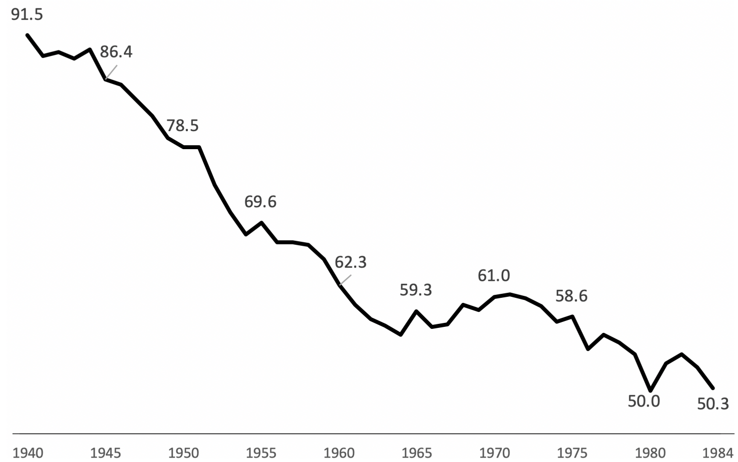


Figure 1.3: Children Earning More than Parents (Percentage) by Year of Birth

The United States has been criticized for having lower rates of mobility when compared to Scandinavian countries [23]. However, a careful comparison of income and earnings conducted by Corak et al. [24] shows that the levels of mobility in the American society are not that different than those in Scandinavian countries with only Denmark exceeding American figures. However, Corak's study observes that there still exists a racial penalty, highlighting "upward mobility that are intolerably low and rates of downward mobility that are unacceptably high" for African Americans.

According to Chetty et al. [22, 20], several geographical aspects have a strong association with upward mobility in the United States. They find robust evidence that long exposure of children to neighborhoods characterized by low poverty is a factor that affects earnings and college attendance. Also, living in areas with stable family structures, measured by fewer single parental homes, seem to support higher rates of upward mobility. Finally, communities with a considerable proportion of better public schools, also seem to support higher rates of mobility. The rate of innovation and entrepreneurship boost growth of local economies and, therefore, become factors that improve opportunities for social mobility. Among other variables that may not be obvious, it is seen that patent applications are a factor that explains innovation capability of countries in the Global Competitiveness Index [25]. The growth of patent records are "attributed to differences in childhood environment and exposure between low-and-high-income families". [26].

Alan Krueger, Chairman of the Council of Economic Advisors, introduced in 2012 a relationship connecting inequality and the degree of lack of mobility that was used to explain the strong positive correlation between them. Inequality has been most often measured by the Gini coefficient with inter-generational income elasticity being captured by the β coefficient from the ordinary least squares regression $lnY_i = \alpha + \beta lnX_i + \epsilon$, where Y_i is the son's income, X_i is the father's income, and ϵ represents the error term. Krueger concluded that "as inequality increases, we see that year-to-year or generation-to-generation economic mobility decreases," an empirical result known as the Great Gatsby Curve. Figure 1.4 shows this correlation for a group of selected countries based on the work of Corak [3]. Brazil is a country with high levels of income inequality and the relationship suggests that the success of Brazilian children depend largely on the success of the parent. Sweden, on the other end, is a country with low inequality where the success of Swedish children seems not to depend to a large extent, on the parental income.

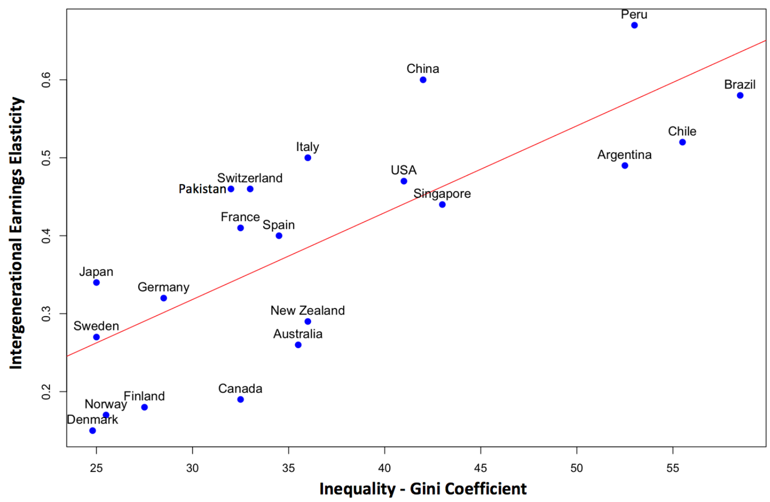
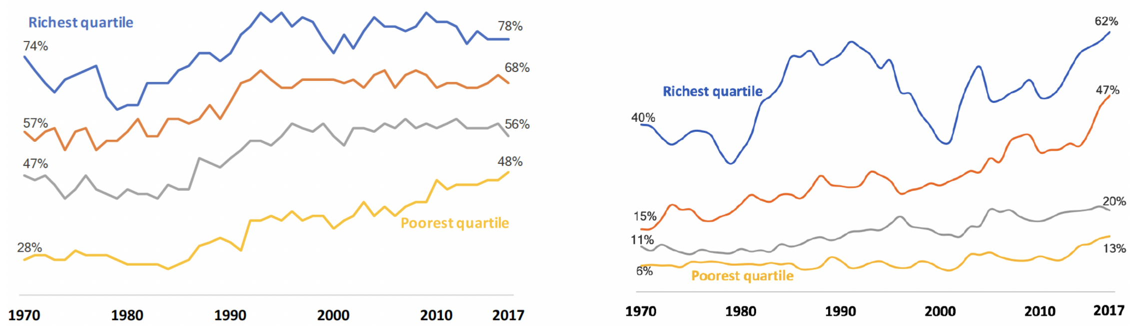


Figure 1.4: Great Gatsby Curve of Inequality and Economic Immobility. Source: Corak [3].

Higher education faces formidable challenges with respect to equality of opportunity and efficiency in the United States. Figure 1.5.a presents the levels of college participation of the population according to income. The time series represent the richest 25%, the 3rd quartile, 2nd quartile and the poorest 25% of the population, respectively. Rich young have 78% probability to enter college in contrast with 48% from poor young. Rich have mostly maintained the proportion of college participation above 70 % while the poorest have increased their percentages considerably from the 28% estimate of 1940. The gap has been narrowed but it is still large: rich go to college almost two times more than the poor. Further, the participation of the poor has gone up dramatically but their success has not. The odds of the rich to complete a college degree are 4.8 times to that of the poor in 2017, as it is seen in Figure 1.5.b. From 1940, the proportion of successful graduation among the poorest started as low as 6% and it has doubled over decades but it is still at low levels (13%) in 2017.



(a)

(b)

Figure 1.5: (a) College Participation and (b) College Degree Attainment by Family Income (Percentage).

1.2 Brief Overview of Dynamic Mathematical Models Tied to Mobility

In 1886, Galton [27] presented a model that became the basis for the analysis of mobility. His research focused on identifying determinants of height for individuals. He stated that heights of individuals can be seen as the results of parental influence and the average heights of ancestors. Galton estimated differences in mean and variances within generations to explain "inequalities" of height and to highlight auto-correlations between the heights of parents and offspring in order to capture inter-generational differences, a framework that can be used to study the determinants of inter-generational mobility. Galton used his model to address the association of the height of the child to that of the parents' height and the "inherited" height from ancestors.

Prais et al. [28] introduced a simple social mobility model; a discrete Markov chain model. These researchers divided the population into proportions defined by socio economic classes. Specifically, the entries of S_t collect the estimates of proportions in each class at time or generation t. The next generation is obtained from $S_{t+1} = PS_t$ where P collects the estimated, assumed invariant, transition proportions from class to class. Under the assumption that P remains unchanged over time, the model concludes that after n units of time (or n generations), the composition of the population is modeled by $S_{n+1} = P^n S_n$. If P is an irreducible matrix then, the distribution of classes approaches the vector $\bar{s}$, as $n \to \infty$; $\bar{s}$ is identified as the equilibrium distribution. As it is known from Markov chain models, the total time spent in a specific class j is given by $t_j = \frac{1}{1-p_{jj}}$, where p_{jj} is the proportion of fathers in the jth social class whose sons move into the jth social class [29]. Prais et al. estimated differences in distributions of classes on a long-time scale using the equilibrium distribution. The matrix contained seven categories defined by occupational level. These researchers also estimated the average number of generations spent in each social class in England. They found, via this model, that skilled manual workers tend to spend almost 2 generations in this category (1.90), professional workers 1.63, and lower grade non manual worker 1.23 generations. Finally, they also estimated immobility ratios and found that the professional class had the highest immobility records, meaning difficult to enter and low probability to leave this occupational category.

Breen et al. [30] constructed a discrete time model of inter-generational mobility to study relationships between economic growth, inequality, and social mobility. They identified factors that reduce economic efficiency in the transmission of wealth and privilege, and that enhance ethnic and gender disadvantage. Their model used the following discrete time system of equations:

$$
\begin{aligned}
S_{it} &= \alpha A_{it} + (1-\alpha) R_{it-1} \\
P_{it} &= \frac{S_{it} - \bar{s}_t}{\sigma_s} + \frac{\epsilon_{it}}{\beta} \\
R_{it} &= \beta P_{it}, \\
& i = 1, 2, ..., I \\
& t = 1, 2, ..., T
\end{aligned}
$$

where S_{it} represents the resources for family i at the generation t. S_{it} dynamics include the role of ability (A_{it}) and parents' resources (R_{it-1}). Ability refers, in this model, as the level of education, measured commonly in number of years in school. P_{it} accounts for the position occupied by the family/individual as a function of the resources. The researchers also included stochastic effects ($\epsilon_{it} \sim N(0, \sigma)$). R_{it} corresponds to rewards or returns obtained as determined by a factor β, that is associated with a degree of reward inequality. Ability is considered to be random and normally distributed, that is, $A_{it} \sim N(0, \sigma)$; α captures the relative importance of parental rewards and it varies between zero and one. Average amount of resources at period time t is given by $\bar{s_t}$ with standard deviation given by σ_s. Inequality of reward recognizes the importance of positive signs for the creation of incentives in this model. These researchers stressed that too much inequality, that is, inequalities beyond a certain threshold, may be detrimental or counterproductive for growth. Breen et al. theoretical results demonstrate, among other things, that inequality and economic efficiency do not follow a linear relationship. They also noted that different rates of social mobility can be compatible with economic efficiency.

One of the most influential works on the distribution of income, generation of human capital and inter-generational mobility in the economic literature is that of Becker and Tomes [31]. Their work is based on family model of transmission of wealth that includes the role of abilities and "luck". Their model reflects the perspective between present and future value in which, a generation can increase their consumption at the expense of future generations albeit such levels of consumption may be discouraged by concerns over their children' s future. The mediating element comes from the introduction of endowments that are transferred from parents to children, such as financial or physical inheritances, as well as, non-monetary aspects, such as, caste, race, culture, reputation, connections, etc. Thus, parents try to maximize a utility by choosing optimal investments for children. The utility function of the parent is given by a function $U_t = U_t(Z_t, I_{t+1})$, where Z_t corresponds to consumption of parents and I_{t+1} is children' s wealth. Parents wealth is represented by I_t and it can be used to purchase consumption goods or physical units for children.

Wealth can be distributed via a combination of these two goods, represented by $I_t = Z_t + \Pi_t y_t$, where Π_t is the cost of goods for children and y_t are physical units of investment in children. Physical units for children can be better understood as clothing, education, food, etc. Families face this disjunctive: consume or invest in children. Economists refer to the opportunity cost of a decision to refer to the loss of opportunity to consume or do something else [32]. The cost of opportunity of investing in children per generation is given by w_{t+1} and it is the product of the cost of goods for children times the rate of return per generation $1 + r_t$, therefore, $w_{t+1} = (1 + r_t)\Pi_t$

According to Becker and Tomes, income inequality and inter-generational mobility provide an equilibrium that depend on luck and family-related parameters, such as, "inheritability" of endowments and the propensity to invest in children are addressed. Hence, children's wealth is defined as the capital invested plus the endowment received from parents and market capital gains. This relationship is given by the equation $I_{t+1} = \Pi_{t+1} y_t + w_{t+1} e_{t+1} + w_{t+1} u_{t+1}$. The inter-temporal budget constraint expresses that the present value of rewards equals the present value of investments and consumption, which then takes the following form:

$$I_t + \frac{w_{t+1}}{1 + r_t} e_{t+1} + \frac{w_{t+1}}{1 + r_t} u_{t+1} = Z_t + \frac{I_{t+1}}{1 + r_t} \tag{1.1}$$

Expression 1.1 connects the present value of parental assets (wealth, endowments and gains) that can be used for the consumption and investment of their children. The decisions of consumption today can be based on what families have today and on what they may have in the future. Families do not make decision for just one year. The maximization of the utility subject to inter-temporal constraints attempts to capture how families decide; trade-offs between consumption or investment in children; a result known as the marginal rate of substitution between investment in children and consumption, which is equal to the rate of return per generation given by $1 + r_t$.

One of the results is that parents will be indifferent to the urge to consume more or invest more in their children as long as the ratio $\frac{w_{t+1}}{\Pi_t}$ matches the rate of return $(1 + r_t)$, which represents the cost of opportunity for parents between their own consumption and their investment on their children.

Becker and Tomes conclude that the long-term income of children is a function of individual's income, endowment of parents, inheritability and parents' investment in children, and luck. These researchers highlighted the possibility that progressive taxes and subsidies may increase income inequality since parents may be discouraged to invest in their children if decrease in returns are foreseen. With respect to mobility, family is identified to be more important when the degree of inheritability and the propensity to invest are larger.

Goldberger [33] provides a comprehensive summary of the extensive work done by Becker and Tomes. Wealth and inheritance are shown to be a central determinant of mobility. Fisher [34] highlights the fact that wealth acts as an absorption buffer against income changes that are in part responsible for the reduction of inter-generational mobility.

Econometric models

The initial and influential work by Kuznets [35] is the basis of one of the first attempts to explain economic growth and inequality tied in to structural transformation as a country develops. Kuznets hypothesized an inverted U-shaped curve to explain why inequality rises at early stages of development (a rural agricultural society) further decreasing after reaching a threshold of economic growth in per capita income under a modern industrialized environment. The original hypothesis made by Kuznets has been challenged both on theoretical and empirical grounds [36].

There exists an extensive literature that make use of econometric models to explain the influence of parental socio-economic status over children' s future standard of living. For instance, a classic model representing the long term economic status captured by annual earnings takes the following form: $Y_{1i} = \rho Y_{0i} + \varepsilon_i$, where Y_{1i} and Y_{0i} represent income of children and parents, respectively. The coefficient ρ corresponds to the correlation between Y_{1i} and Y_{0i} and it may be estimated by applying a least squares regression. This has become the standard approach for estimating the intergenerational mobility coefficient. Variations of the model have been used to capture separate effects related to income growth such as ability, education, age, experience, etc.

1.2.1 Ecological Models

Ecology is a term refering to the "interrelationships between organisms and their environments" with ecological models providing a framework capable of integrating social levels of influence [37]. Ecological models offer a useful comprehensive perspective for addressing, for example, population level dynamics as a function of the environment. Several aspects of ecological models have helped answer questions related to the importance of surrounding environments on behavioral changes experienced by individuals. Ecological models have been used in social sciences and public health to study "the nature of transactions of individuals with their physical and socio-cultural environments" [38]. Sallis et al. [37] provide a detailed history on the progression of ecological models in public health where they have been used to address perceptions of influence and their role on policy.

Public health models provide a useful example on the use of ecological perspectives applied in the evaluation of intervention measures at the population. Ecological perspectives have been used to assess the impact of tobacco control policies, the role of sedentary behavior on obesity, needle exchange policies on HIV, or the impact of environment (fast food) on obesity [37]. There exist several models that are used to capture the role of the environment in health. Sanchez et al. [39] constructed a model to explain the relationship between alcohol abuse, prevention and relapse issues. Sallis et al. [37] explain the use of ecological frameworks in addressing health behaviors that may well extend to social behaviors or life decisions that include educational trade-offs like whether or not to enroll in college. There are multiple levels of influence, including intrapersonal, organizational, and at the community level. The environment often is a significant determinant of health behavior since situations and contexts may shape or constrain individuals' decisions.

Environmental factors, namely, residential density, access to destinations, aesthetics, all play an important role on physical activity. Ishii et al. [40] point out that people become engaged into physical activity under the influence of environmental factors and interaction of demographic, psychological, and social variables. The environment, they argue, may have long term consequences on behaviors affecting health decisions. Social support, for instance, may be seen as a "mediation mechanism" between the environment and physical activity. Owen [41] also highlights that "convenience of facilities and accessibility of destinations for walking such as sidewalks, or even perceptions about traffic", affect decisions on whether or not to engage in healthy activities including walking regularly.

Based on a community sampled study among adults, McNeill et al. [42] recognized that social support indirectly influences physical activity through peer motivation. Other factors associated with physical activity include neighborhood quality (i.e. criminal activity, traffic) and access to facilities. Mediating structures are an important component of the community as Berger and Neuhaus have stated [43]. Some examples of such structures include family, informal social networks, churches, neighborhoods that develop a social identity for the individual. These social structures can contribute to the building of strong ties within a community and, as a result, become a force of behavioral change, at the population level, that are difficult to achieve without them.

1.2.2 Ecological Models for Human Development

Lewin [44] postulates the existence of a behavioral principle, namely behavior (B) that evolves as a function of the reciprocal interaction between the individual and the environment PE, namely, that $B = f(PE)$. Bronfenbrenner's critique of Lewin's postulate [45] is that much of the attention has centered on individuals' properties that drives an asymmetry in characterizing the environment in which individuals develop. The extent of the research that has focused on interpersonal influences, that is, "face to face situations as part of one's environment ... that creates reinforcement, modeling, identification and social learning". They identified a delimiting micro system for the person. Bronfenbrenner extends the definition of ecology of human development which "involves the scientific study of the progressive, mutual accommodation between an active, growing human being and the changing properties of the immediate settings in which the developing person lives, as a process affected by relations between these settings, as well as by the larger context in which the setting are embedded." Bronfenbrenner proceeds to categorize three levels of environmental influences. Micro system interactions, explained by " family members and work groups interactions", a meso-system "consisting of physical family, school, and work surroundings and finally," an exosystem consisting of the "larger social system of economics, culture, and politics ".

Neighborhood Matters

The role of the environment linked to factors that may affect mobility include but are not limited to urban-rural status, the average income of the area, economic segregation, poverty status and, resources for education, including availability and quality [15]. Concentration of poor families in areas with little job opportunities, lack of trusting neighbors, and fewer community institutions tend to increase economic segregation possible responsible for creating an important "racial penalty for Blacks" [46]. For example, African Americans living in poor neighborhoods in Chicago had lower median income, approximately $19,000 less when compared to that of Whites and, $8,000 less in the case of Los Angeles [46]. "Black neighborhoods are more exposed to higher levels of poverty, unemployment, crime, disorder, etc. than White's neighborhoods." Education or the lack of education, may magnify, or decrease, respectively, the extent of inequalities. Characteristics of the social environment, such as, "violence, lead exposure, and incarceration, directly predict lower inter-generational income mobility, adult incarceration, and teenage birth among children who grow up poor" [47].

Changes in the ecology of neighborhoods can lead to changes in the prosperity outcomes or economic hardship that afflict the people who inhabit them. Case et al. [48] examines the effects of family background and neighborhood characteristics on the future social and economic outcomes of urban youths in Boston. This connection between families and their immediate social environment is used to explain criminal activity, drug and alcohol use, childbearing, school outcomes, and church and work attendance. According to Case and Katz in the study "The Company You Keep: The Effects of Family and Neighborhood on Disadvantaged Youths" [48], neighborhood peers substantially affect the behavior of the youth behaviors as a contagious process. For instance, having a large proportion neighbors involved in crime is associated with a significant increase the probability of individuals being involved in crime.

Other types of models capturing neighborhood effects have been proposed by Wilson (1987) [49] under the "Collective socialization" scheme. In this model, adults have an influence on youths who are not their children, specifically, affluent neighbors may act as role models for the youth, signaling success that can be obtained by hard work. Gladwell [50] makes an elegant elaboration on the effect of environment on individuals and their responses to different situations. The power of context, he claimed, "makes human beings more sensitive to their environment than they may seem".

Also, peer influence is highlighted using "epidemic" or "contagion models". Crane [51] explored the "peer effect" in ghettos or poor neighborhoods on school dropout rates and teenage pregnancy. Crane suggests that "ghettos are communities that experience epidemics [contagion effect] of social problems". He made use of the term "neighborhood quality" to refer to attributes that increase the probability of having problems when a threshold is passed, that is, a critical mass of "problem" individuals, in the lowest sector of quality of the neighborhood. Crane used records of the Public Use Microdata Samples (PUMS) from the Census in the US in 1970 and a piecewise linear logit model to "estimate the pattern of neighborhood effects across the distribution of neighborhood quality". The collected evidence suggested that neighborhood affects educational attainment minimally, albeit their impact is higher on pregnancies. According to logit estimates by Crane [51], living in the worst neighborhoods is associated with the highest dropout rates for Blacks (0.192), Hispanics (0.166) and, Whites (0.146). One percentage point decrease in the high status of a neighborhood increases the probability of school dropout by 0.0023 in Hispanics, 0.0041 for Blacks and 0.00095 for Whites. Gladwell [50] also makes reference to the work conducted by Crane on the "effect of role models in a community", particularly, the influence of professionals, managers, teacher on the lives of teenagers in the same neighborhood. There exist a "tipping point", below which, problems explode in a community, for instance, if the percentage of professionals goes below 5 percent, drop out rates more than double for black schoolchildren with childbearing for teenage girls also doubling.

In 1978, Schelling [52] used dynamical approaches to recognized segregation through neighborhoods. This author identified a neighborhood tipping point. He based his ideas on the relationship between neighborhood quality and the incidence of social problems. His research highlighted nonlinear increase in social problem as neighborhood quality declines with a jump of problems increasing at the bottom of Schelling's distribution of quality. "Tipping" occurs, according to this author, when an identifiable new minority enters a neighborhood in numbers that make earlier residents to begin evacuating the neighborhood. Grodzins [53] estimated that White Americans neighborhoods have a tipping point of about 20% with respect to the presence of Blacks in their neighborhoods.

Another critical aspect is the exposure of young children and their families to the influence of role models for the formation of aspirations. For instance, Jensen [54] documented a significant improvement in the perception of returns to schooling after providing information of these returns to students in the Dominican Republic. Also, the presence of role models raised aspirations as it is documented in Beaman [55], where female leadership had the potential to influence the educational attainment and career aspirations of teenage girls in India. Using surveys in almost 500 villages, Beaman found that girls spend less time on household chores, suggesting an important impact of women leaders through a role model effect. The evidence, however, is not conclusive on whether the improvement happens when these influences are exerted on aspirations of parents or children.

Higher levels of income inequality are associated with lower rates of social mobility that could make low-income youth develop a perception that investment in education is tied in to a lower rate of return, in contrast to, getting a job for example. This counter effect will diminish any potential "aspirational" effect resulting from higher educational wage benefits. The evidence shows greater school dropout rate in areas where inequality prevails which, perpetuates low levels of mobility [56]. With respect to mobility, Chetty et al 2018 [16] analyzed the sources of racial disparities, focusing on income across generations using longitudinal data from the U.S. Census Bureau from 1989 to 2015. Both Blacks and American Indians have modest mobility rates, relatively lower than Whites'. However, Hispanics are moving up significantly in income distribution across generations. Also, they found that inter-generational gaps might be narrowed by at most 25% in the case where Black and White boys were to grow up in the same neighborhoods. Other environmental factors affecting outcomes of Blacks positively are low levels of racial bias and high rates of father presence among Blacks. In a different study, Chetty et al. [20] found that when poor families move to "better" neighborhoods (higher income), their children were more likely to attend college by 16%, attended more selective colleges, and increased annual earnings by more than $1,600. Additionally, in connection to these previous works, Chetty et al. [57] found additional factors influencing upward mobility, namely less residential segregation, less income inequality, better primary schools, greater social capital, and greater stability in families.

This dissertation attempts to address as a general research question the impact of education, particularly higher education, as a force that enhances social mobility. Specifically, we would like to explore to what extent, group-specific mobility alters the *status quo* of communities with prevalence and levels of education serving as a proxy for environmental quality. There exist specific sub-questions that address the impact that environmental-driven changes may have in altering the *status quo*. Do the increases in the level of education among groups with a history of limited access, have an impact in accelerating the desire of unprivileged members of the population to overcome the barriers that limit their access to and success in higher education? What are the possible time scales needed to reach a critical mass of individuals needed to maintain a sustainable positive group influence over time? What is the result occurring in mobility in a community with more heterogeneity of the population? Finally, can deliberate measures to achieve sustainable change within reasonable time scales be identified and implemented?

We address the impact of education first via the data collected in the USA on the access and success of underrepresented minorities, particularly African Americans, cited in this chapter. We address the potential role of access to college education via artificial constructs where only access and success in higher education can generate sustainable change. We build a series of models starting with a caricature of the world stratified by economics, rich and poor, relative terms that do not account for within subgroups differences and, by the prevalence of college educated in each group.

The basic model, rich and poor, ignores demographic growth and the population is assumed to be at a steady state. As individuals in each category become more educated, we proceed to explore whether or not increasing the levels of groups' access or within group success (becoming educated) are likely to impact the desire of other members to become educated? In other words, if 1% of the African American population is educated, what would the impact of this 1% may have on the other 99%. And if the proportion of educated people grows, does its growth influence the desire (and the likelihood of success) of individuals to become more educated. In general, the model assumes that if a community improves its level of education, then it experiences increases in average income, which in turn, may generate changes in the environment that alter the desire of the rest of the members of the community to find ways of becoming educated. The studies cited in this chapter support this assumption.

These are questions that are embedded in high levels of social complexity that are not addressed in this dissertation. We address our research questions exclusively with the aid of simplified nonlinear dynamic models, that is, models that consider communities comprised of simplified socio-economic structures, rich and poor, divided by their level of college education. The study of these questions is carried out through the formulation of compartmental models under various assumptions on the nonlinear rates of transition often, a function of the state of the system, that model the movement from one compartment to another.

The rest of this PhD dissertation is organized as follows. In Chapter 2, three compartmental models are introduced and used to explore the environmental influence of specific educated groups on the less privileged subgroups. Chapter 3 extends the analysis of Chapter 2 to models that include a middle class in order to gauge the impact of non-dichotomous community structure on mobility. Chapter 4 addresses the questions of Chapter 2 under the assumption that population growth is ongoing. Chapter 5 highlights conclusions, discussion, and it incorporates a model with an altered assumption to include, a data driven approach, via the use of non-autonomous differential equations. Finally, Chapter 6 presents the views from an economic perspective about of higher education, and the role of factors affecting environmental effects.

Chapter 2

MODELS FOR SOCIO-ECONOMIC MOBILITY

Studying the role of environmental influence on social mobility offers a multitude of challenges due to the high levels of heterogeneity involved in existing communities, as well as, external factors that depend in fundamental ways on economic, social and political structures that have been often built over centuries. This is the type of challenges that theoreticians face constantly when the focus is on understanding the impact of the nonlinear dynamics, typical of complex systems, that have emerged under different cultural, social and political norms. We follow the approach that is often used in population biology, particularly in epidemiology, that is, we use simple models to tease out the role of specific assumptions under simplistic controled scenarios [58, 59].

In this chapter, we introduce a mobility model that divides the population of interest according to their socio-economic and educational status. Socio-economic status divides the population, in our simplified setting, into born rich or born poor individuals. We add the educational status, that is, college Uneducated or Educated, plus the underlying hypothesis that social mobility may be a function of "contagion". That is, influence from the environment is assumed to be generated from the ability of subgroups of educated people to impact (copycat effect) those individuals lacking higher education, a hypothesis that comes from research on the effect of surrounding environments on the propensity of uneducated individuals to decide to increase their level of education. The assumption is that an environment populated by direct or indirect role models is likely to influence the interest of some individuals in higher education [60, 24, 33, 48, 55, 61, 19, 43, 28, 29, 22, 51, 62, 34].

We study the nonlinear dynamics of upward socio-economic mobility via educational mobility. The goal is to assess the impact that changing distributions or frequencies of educated individuals, modeled on some appropriate function of the state variables, may possibly have on upward educational mobility. It is assumed that the average income of the poor college educated is higher or significantly higher than the uneducated poor; that the income of the college educated rich is higher than that of the uneducated rich; that the income of the uneducated rich is significantly higher than that of the uneducated poor and possibly higher or slightly less than that of the average income of the educated (born) poor. These assumptions are introduced in order to avoid a detailed economic stratification by income level since the goal of this chapter is to look at those questions in oversimplified scenarios. According to the U.S. Census Bureau, 51% of poor people are either African Americans or Hispanics and, these figures may be tied in to low participation and success rates of educated college groups. We may offer an initial scenario where parameters for the born poor can be approximated from these two groups. This view is tied in to the growth of the educated groups which is often tied in to race in the U.S. We are aware that being born poor regardless of race or ethnic origin is by itself a significant obstacle when it comes to access and success in higher (college) education. Our social framework is coupled with a model that attempts to capture social environmental effects generated by increases in the likelihood that the born poor uneducated may become educated. We use Levins' model [63] to capture this environmental effect, a phenomenological approach, that we believe proves to be useful.

The Census Bureau defines an individual to be born poor if its income is less than a particular threshold. For the year 2019, a poor family is defined as such, if it has an annual income of less than $24,858 for a family of 4 persons (2 children included) or less than $12,752 for a single individual under the age of 65. There is not an official definition of middle or rich classes in the US. The Pew Research Center promotes the view that a middle class family is the one that has an income between 67% to 200 % of the national median income. Also, as a reference, a family is categorized as rich, if its income is beyond 200% of the national median income.

An educated person refers explicity to an individual with a college degree. According to National Census Bureau, having a college degree produces a considerable increase in earnings over a lifetime. The model integrates the gross enrollment ratio, defined by UNESCO as the number of students enrolled in a given level of education regardless of age as the percentage of the official school-age population corresponding to the same level of education [5]. The model accounts for the fact that enrollment rates vary according to their socio economic status. For instance, the National Center for Education Statistics (NCES) indicates that gross college enrollment rates are 28% for individuals considered to be poor (lowest fifth of income distribution), 44% on average for middle class (second lowest to second highest fifth of income distribution) and 78% of rich (highest fifth) [64].

Social mobility is the result of mechanisms that may include education, habitat or neighborhood influence, political and social connections, economic growth, institutional strengthening, luck, etc. The model in this dissertation assumes that mobility is the exclusive result of access and success in higher education or, in other words, for Chapters 2, 3 and 4, education is used as **a proxy** for all the social environmental factors that impact mobility, an approach that we believe it is not too restrictive, as the framework can allow for the inclusion of alternative “proxy” measures. The dynamical process that represents the influence of education is modeled via nonlinear dynamical coupled systems that includes various levels of organization. The overall goal is to study the impact of nonlinear effects (environmental) on mobility via a series of models that involve increases levels of complexity.

Research conducted by Phinney [1] highlights the reasons for attending college, such as, the aspiration of higher wages, “help one’s family, to prove one’s self-worth, and from encouragement”. The identification of the reasons why individuals choose college life over a list of possible substitutes, helps understand the main drivers of motivation that impact such a decision among young people. Economic constraints of individuals are taken into consideration through the rates of progression of education, for instance, a poor individual experiences a lower rate of college success, namely, enrollment and completion. How do we capture all these factors in an equation that models environmental influence? We address this challenge by using a modified version of Levins’ model [63].

Cote et al. [62] identify five goals as the drivers of college attendance among populations of high school students. These drivers include obtaining a good job, a feeling of being successful, intellectual development, helping to improve the community, and to avoid employment in activities categorized as less desirable. Further, these researchers observe that college students set their goals related to higher education, in part, based on the family’s experience. They concluded that, for ethnic minorities and immigrants, college attendance is driven, in general, by social class and cultural backgrounds. Astin [65] showed that family, peers, and faculty can have a strong impact on the attitudes and achievements of college students. Here, there is not enough space to document and explore the variety of specific reasons that may influence decisions to attend college and the factors affecting such decisions. Further, the fact that a group of established researchers recognize that there is almost “no research on this topic among students from ethnically diverse backgrounds”, has inspired us to find simple ways of capturing environmental effects on mobility. We have selected to use a modified version of Levins’ equation [63] to address this modeling challenge.

Our structure choice may be supported by studies that show that the differentiation between the family's status of students from low income and those of other segments of society, "may offer young people with strong motivation to attend college in order to better themselves" [1]. Lopez [66] shows that Latino parents tell their children the difficulties they faced due to the their "lack of education and encourage their children to do better". Young generations that experience the family struggle with poverty may wish to obtain an education that allows them to get a good job and contribute back monetarily to their families [67, 68]. Family interdependence plays an enormous role in the motivation to go to college when ethnic backgrounds are considered.

The models seek to compare the dynamics on a pre-defined *statu quo*, a distribution of college educated and college uneducated classes as the current state, with the distribution obtained under environmental influence, the influence driven equilibrium. The *status quo* corresponds to an equilibrium where there is no environmental influence of other members in the community on the desire to pursue higher education while the influence equilibrium accounts for the effects on the determination and success of vulnerable groups to pursue higher education with the success of within a group or via external members of the community, to do so.

In the simple nonlinear model, the impact of influence is modeled as a function of the number, frequency, or a combination of poor or non educated members of a population. The effect created by the presence of such population is modeled by its ability and speed to "invade" the influence-free or *status quo* state. With these models, we hope to answer questions like what are the time scale needed to observe significant changes of proportion of individuals who are college educated from poor origin? What are the initial conditions needed to accelerate change? If the influence is weak but "enough", how long does it take to generate significant change? Under what conditions, is there a dominant, that is, larger positive effect from a particular subgroup?

This chapter is organized as follows: Section 2.1 introduces the basic mathematical characterization of an artificial community divided by socio-economic and educational status. The model considers the influence of the environment, first using as a proxy the result of the successes by the educated rich. Section 2.2 models influence as entirely dependent on the success of the educated poor, which are assumed to be no longer poor, as the result of the education acquired. Section 2.3 collects the simulations of a model that accounts for the impact of both groups. Clearly, modeling environmental influence using education as proxy is simplistic but as we will see, such an approach let us explore the impact of the nonlinear dynamics that emerge from the coupling of the environment and mobility. It will no be a huge leap to include a coupling with an environment, modeled in less restrictive ways. This will be briefly discuss in Chapter 5.

2.1 Model 1: Rich-Poor and Influence from Rich Educated

2.1.1 Model Derivation

The artificial community to explore the dynamics of mobility is stratified by socio-economic status, namely (born) rich (R) and (born) poor (P). It is further divided by college education status: Uneducated (U) and Educated (E). Total population is given by $N = U_R + U_P + E_R + E_P$. We let $\widehat{U}_P$ denote the average income of the poor uneducated; $\widehat{E}_P$ of the poor educated; $\widehat{U}_R$ of the rich uneducated; and $\widehat{E}_R$ of the rich educated. It is assumed that $\widehat{U}_P < \widehat{E}_P \leq \widehat{U}_R << \widehat{E}_R$ or that $\widehat{U}_P < \widehat{U}_R \leq \widehat{E}_P << \widehat{E}_R$. Under this conditions, we could estimate average changes in income. For example, we will have that $\widehat{P}(t) = \widehat{U}_P \frac{U_P(t)}{P(t)} + \widehat{E}_P \frac{E_P(t)}{P(t)}$. A positive change would be defined, for example, whenever $\frac{E_P}{N}$ increases, with the degree of success being given by $\frac{\widehat{P}(t)}{\widehat{P}(0)}$, that is, when $\frac{\widehat{P}(t)}{\widehat{P}(0)} > 1$ implying that upward mobility, among the poor, is on the rise. Furthermore, it is assumed that $U_P(0) >> E_P(0)$, $U_R(0) > E_R(0)$, and that $P(0) >> R(0)$. The recruitment rate into each class is assumed to be a fraction q_R and q_P of total recruitment rate Λ, with $q_R + q_P = 1$. It is further assumed that there is no recruitment of individuals into the educated classes. Moreover, it is assumed that $0 < \frac{q_R}{q_P} << 1$, in general.

The model assumes that Uneducated individuals become Educated at the per capita rate α_i, $i \in \{R, P\}$. The rates of transition to become educated vary according to groups. We focus on the case $\alpha_R >> \alpha_P$, that is, it is assumed that rich individuals have larger college enrollment and success rates due to, among other things, affordability. According to the report "Indicators of Higher Education Equity in the United States" by the Pell Insitute [69], the average college costs that include undergraduate tuition, fees, and room and board for a full-time student is approximately \$ 43,000 in a 4 year private college or \$20,000 in a 4 year public institution.

This expense is greater than 80% of the average annual income of a poor household if student attends a public university, and more than double in the case that the student goes to a private university. For a rich household (mean annual income of $ 186,000 or more), the percentage of college education expenditure is 11% and 23 % for a public or private university, respectively. These amounts do not account for financial aid, including, for example, loans, or in the case of the very poor, Pell grants.

Education as an environmental influence

Exploring various versions of the nonlinear effects of the environment on social mobility is central towards the further understanding of the forces that impact mobility, in the absence of specific state-driven policies that directly address mobility via higher education. The uneducated poor may be encouraged or discouraged by their environment. The environmental influence for the poor to enroll and succeed in college, is modeled by an "effectiveness" parameter β_P. The role of the environment as an inspiring or negative force that increases or decreases the likelihood of enrollment and success (graduation) from college is rather difficult to capture; and so, we proceed in a phenomenological way. Specifically, we build a highly simplified environmental influence model, a caricature model of social mobility. In our simplified community of individuals belonging to U_P, U_R, E_P, and E_R, we model influence as a force of change (mobility) operating within an established population, the *status quo* state. Further it is assumed that going from $U \to E$, increases average income regardless of whether people are born poor or born rich, that is, "upward" mobility takes place. Subscript P defines origin at birth and it is used only to track the origin of the socio-economic status. Educated poor E_P does not mean that this individuals are economically poor, it means "born poor becoming educated".

We model influence as the state-dependent index of change (if it is in $(0, 1)$, we call it probability); that is, as a "probability" function that depends on the number of individuals in E_R, or the fraction $\frac{E_R}{N}$, or the number of E_P, or the fraction $\frac{E_P}{N}$, or the weighted fraction $\frac{lE_R+(1-l)E_P}{N}$ with $0 < l << 1$, or some other variation. This probability or influence function operates in the world defined by U_P, U_R, E_P and E_R. Hence, this "probability" or function $Q(\cdot)$ is being explicitly modeled as a function of some of the state variables. First, $Q \equiv Q(E_R)$, then $Q \equiv Q(E_P)$ and finally, $Q \equiv Q(E_R, E_P)$. This may be analogous to the approach followed, for example, by Song et al. [70], in modeling peer pressure rate in their ecstasy model of the dynamics of a population of young individuals that regularly attend raves.

Richard Levins modeled the probability Q of patch colonization via $\frac{d}{dt}Q = rQ(1-Q) - eQ$, where Q represents the proportion of colonized sites and $\frac{1}{e}$ the life-span of a site as colonized [63]. This characterization provides a useful framework to model environmental influence. We observed that $Q > 0$ if and only if $\frac{r}{e} > 1$ and $Q(0) > 0$; in this case, the limit of $Q(t)$ as $t \to \infty$ is $1 - \frac{e}{r}$, that is, $Q(\infty) \equiv 1 - \frac{e}{r}$. Further, we have that whenever $\frac{r_0}{e} \leq 1$, $Q(t) \to 0$. In short, Q^* can be interpreted as the probability of "influence" with $\frac{1}{e}$ denoting the average impact of the "influence" effect. We proceed to use a modified version of Levins framework to model environmental influence. For the three cases under consideration in this chapter, we will have that either $r = r(E_R)$, $r = r(E_P)$ or $r = r(E_R, E_P)$. We will also assume that e is constant albeit, we may have for example the situation when $e_P < e_{PR} < e_R$, that is, $\frac{1}{e_P} > \frac{1}{e_{PR}} > \frac{1}{e_R}$, with other options possible. To illustrate some results, we have chosen $\frac{dQ}{dt} = r_0 \frac{E_R}{\theta_R + E_R} Q(1-Q) - e_R Q$, so that, $Q \in [0, 1)$ whenever $\frac{r_0}{e_P} \frac{E_R}{\theta_R E_R} > 1$ with $Q_{max} \equiv 1 - \frac{e_R}{r_0}$, that is, the case when θ_R, the handling time, is equal to zero. In general, for a fixed θ_R, we have that $Q(E_R)$ increases as E_R increases. Influence is introduced by Levins' equation to support the argument that increases in education may generate increases in social mobility within specific groups.

Hence, the use of $Q \equiv Q(E_R)$ leads to a simplified compartmental model that accounts for the role of environmental influence in accelerating educational achievement in the poor class, namely,

$$\frac{dU_R}{dt} = q_R\Lambda - \alpha_R U_R - \mu U_R, \tag{2.1}$$
$$\frac{dU_P}{dt} = q_P\Lambda - \alpha_P U_P - \beta_P U_P Q - \mu U_P, \tag{2.2}$$
$$\frac{dE_R}{dt} = \alpha_R U_R - \mu E_R, \tag{2.3}$$
$$\frac{dE_P}{dt} = \alpha_P U_P + \beta_P U_P Q - \mu E_P, \tag{2.4}$$
$$\frac{dQ}{dt} = r_0 \frac{E_R}{\theta_R + E_R} Q(1-Q) - e_R Q, \tag{2.5}$$
$$\tag{2.6}$$

with $q_R + q_P = 1$. The function $r(E_R) = r_0 \frac{E_R}{\theta_R + E_R}$ corresponds to the intrinsic rate of growth of influence, that is, the positive impact that the growth of E_R may have in the growth of Q, and hence, of E_P. In short, $r(\cdot)$ is an increasing concave down function of the state variable E_R

Parameter r_0 denotes the maximal growth rate; $\frac{1}{e_R}$ the average time of the influence of the class E_R; and θ_R the reduction on the effect of Q due to "handling" time, that is, the strength of the influence, with $\theta_R = 0$ corresponding to the case when the impact is instantaneous. θ_R captures the accelerating or de-accelerating impact of the influence. The transitions of the different groups in the population are depicted in the flow diagram derived from the Figure 2.1, below:

2.1.2 Mathematical Analysis

Adding Equations 2.1 - 2.4 shows that the population is asymptotically constant, that is, $N \to \frac{\Lambda}{\mu}$ as $t \to \infty$ [71] and so we assume, without loss of generality, that

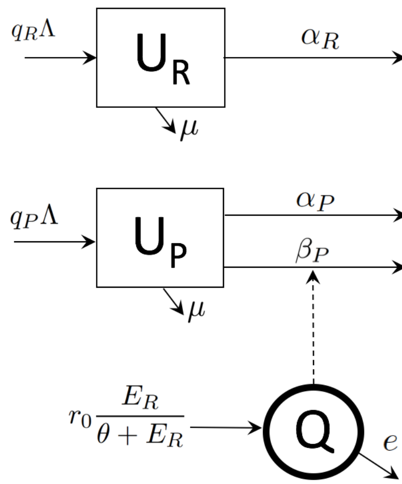


Figure 2.1: Flow Diagram of Compartmental Model 1

$N(\infty) = \frac{\Lambda}{\mu}$. System 2.1 - 2.4 supports at least two equilibria: the influence-free equilibrium when $Q^* = 0$, and a positive influence equilibrium when $Q^* > 0$. The influence-free equilibrium will be referred to as the *status quo* equilibrium and it is given by the following relationships:

$$U_R^0 = \frac{q_R\Lambda}{\alpha_R + \mu},$$
$$U_P^0 = \frac{q_P\Lambda}{\alpha_P + \mu},$$
$$E_R^0 = q_R\frac{\Lambda}{\mu}\frac{\alpha_R}{\alpha_R + \mu},$$
$$E_P^0 = q_P\frac{\Lambda}{\mu}\frac{\alpha_P}{\alpha_P + \mu},$$
$$Q^0 = 0.$$

Under the influence-free or *status quo* equilibrium, the educated rich or E_R^0 is directly proportional to the total rich population, namely $q_R\frac{\Lambda}{\mu}$, with the constant of proportionality given by the successfully educated rich proportion, $\frac{\alpha_R}{\alpha_R+\mu}$. Similarly, the E_P^0 equilibrium is directly proportional to the total poor population $q_P\frac{\Lambda}{\mu} = (1-q_R)\frac{\Lambda}{\mu}$ with the constant of proportionality given by the successfully educated poor proportion,

variable	Description
U_R	number of rich uneducated individuals
U_P	number of poor uneducated individuals
E_R	number of rich individuals with college degree
E_P	number of poor individuals with college degree
Q	influence probability function

parameter	Description
q_R	proportion of rich recruited
q_P	proportion of poor recruited
α_R	per capita rate to become educated for the rich
α_P	per capita rate to become educated for the poor
β_P	effectiveness of environmental transmission rate for the poor
r_0	intrinsic rate of growth of Q
θ_R	influence impact delay
e_R	rate of loss of influence
μ	per capita system exit, $\frac{1}{\mu}$: average educational-time window
Λ	constant recruitment rate

Table 2.1: Variables and Parameters of the Model

$\frac{\alpha_P}{\alpha_P+\mu}$. In general, one may expect that a community with high degree of inequities would satisfy the relationship $0 < q_R << q_P < 1$, $q_R + q_P = 1$. We observe that the *status quo* equilibrium is always viable.

A second equilibrium can be supported when $Q^* > 0$, that is, when the environment has a positive effect on the mobility from U_P to E_P. This occurs when E_R^* is such that $Q^*(E_R) > 0$. Here, the impact of $E_R^* > 0$ on the transition from U_R to E_R

is considered negligible and hence ignored. System 2.1 - 2.5 can therefore support the influence equilibrium given by the expressions:

$$
\begin{aligned}
U_R^* &= \frac{q_R \Lambda}{\alpha_R + \mu}, \\
U_P^* &= \frac{q_P \Lambda}{\alpha_p + \beta_p Q^* + \mu}, \\
E_R^* &= q_R \frac{\Lambda}{\mu} \frac{\alpha_R}{\alpha_R + \mu}, \\
E_P^* &= q_P \frac{\Lambda}{\mu} \frac{\alpha_P + \beta_P Q^*}{\alpha_P + \beta_P Q^* + \mu}, \\
Q^* &= 1 - \frac{e_R}{r_0} \frac{\theta_R + E_R^*}{E_R^*}.
\end{aligned}
$$

A positive influence equilibrium may occur when $Q^* > 0$, that is, whenever $\frac{E_R^*}{\theta_R + E_R^*} \frac{r_0}{e_R} > 1$, otherwise $Q^* \leq 0$.

In short, when the maximal influence rate $\frac{r_0}{e_R}$ (assumed always to be greater than 1) times the effectiveness of the E_R^* class, given by $\frac{E_R^*}{\theta_R + E_R^*}$ is greater than 1. We let

$$
\mathcal{R}(\theta, E_R^*) \equiv \frac{r_0}{e_R} \frac{E_R^*}{\theta_R + E_R^*}, \tag{2.7}
$$

denote the influence reproduction number, with $\mathcal{R}(0, E_R^*)$, $E_R^* > 0$ and $Q^* > 0$, modeling the case when the impact of influence is instantaneous and hence maximal; $\mathcal{R}(\theta_R, 0)$ denotes the case when there is no environmental influence ($Q^* = 0$).

There is a possibility of $Q^* < 0$ with U_R^*, U_P^*, E_R^* and E_P^*, remaining still positive. When $0 < \mathcal{R}(\theta_R, E_R^*) < 1$, influence may become negative, that is, $Q^* < 0$. In fact, a simple calculation shows that $-1 < Q^* < 0$, with all state variables positive as long as $\alpha_P > \beta_P$ and $\frac{1}{2} < \mathcal{R}(\theta_R, E_R^*) < 1$. In such a case, Q could be interpreted as an influence coefficient $Q \in [-1, 1]$. Under the conditions $\frac{1}{2} < \mathcal{R}(\theta_R, E_R^*) < 1$ and $\alpha_P > \beta_P$. We will show that under this conditions, influence equilibrium falls below the *status quo*; an unstable equilibrium.

In order to illustrate the above results, we take parameter values $r_0 = 0.36, e_R = 0.15, \theta_R = 0.4$, and observe that we would need 28% of the Rich population in E_R to generate $Q^* > 0$. On the other hand, if r_0 increases to 0.45 then we would only need 20% in E_R to get $Q^* > 0$. If the social environment is propitious, the community requires less percentage of influential individuals to "produce" educated poor.

Note that the Max $\mathcal{R}(\theta_R, E_R^*) = \frac{r_0}{e_R}$ takes places when $\theta_R = 0$ provided that $E_R^* > 0$. Min $\mathcal{R}(\theta_R, E_R^*) = 0$ also whenever $E_R^* \geq 0$. Figure 2.2 presents the expression 2.7 as a function of θ_R. $\mathcal{R}(\theta_R, E_R^*)$ is a decreasing function on θ_R with everything else fixed. There is a unique intersection in Figure 2.2.a that identifies a threshold level that it is named as the critical influence θ_R^c. When $\frac{r_0}{e_R} < 1$, critical level of influence does not exist (Figure 2.2.b).

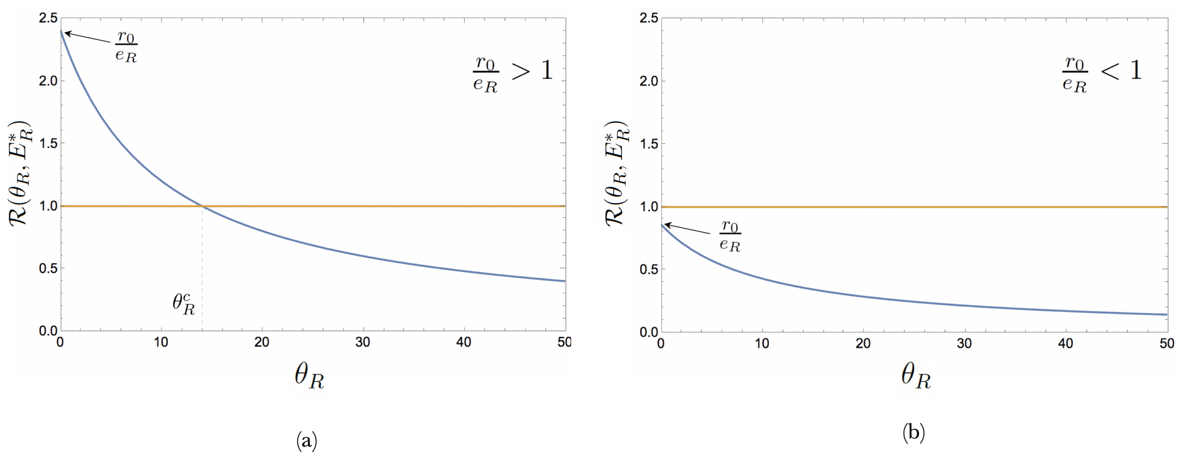


Figure 2.2: Influence Reproductive Number and Influence Delay with Interaction $\frac{r_0}{e_R} > 1$ (a) and No Interaction $\frac{r_0}{e_R} < 1$ (b).

A simple calculation shows that $\mathcal{R}(\theta_R, E_R^*) > 1 \iff \frac{r_0}{e_R} > \frac{\theta_R + E_R^*}{E_R^*}$ with $\frac{r_0}{e_R} > 1$, from where we conclude that $E_R^* \left(\frac{r_0}{e_R} - 1 \right) > \theta_R$, as long as, $\frac{r_0}{e_R} > 1$, or equivalently, that

$$0 \leq \theta_R < E_R^* \left(\frac{r_0}{e_R} - 1 \right)$$

.

Hence, if $0 \leq \theta_R < E_R^* \left(\frac{r_0}{e_R} - 1 \right)$ which requires that $\frac{r_0}{e_R} > 1$, we have treat $Q^* > 0$ while if $\theta_R > E_R^* \left(\frac{r_0}{e_R} - 1 \right)$ and $\frac{r_0}{e_R} > 1$, $Q^* \leq 0$. Hence, the delay in influence θ_R, its value depending on E_R^* (always positive), and the condition $\frac{r_0}{e_R}$, will determine whether or no $0 \leq \theta_R < E_R^* \left(\frac{r_0}{e_R} - 1 \right)$. Hence, we define the critical value for the delay influence as $\theta_R^c \equiv \left(\frac{r_0}{e_R} - 1 \right) E_R^*$.

Figure 2.3 shows the role of θ_R^c on $\mathcal{R}(\theta_R, E_R^*)$ and Q^*. Figure 2.3.a shows that if $0 \leq \theta_R < \theta_R^c$, the influence is sufficiently fast to have, $Q^* > 0$. Then, equilibrium for educated rich E_R^* is always positive and the net influence of the environment alone $\frac{r_0}{e_R} > 1$. When $\theta_R > \theta_R^c$, the "effectiveness" of influence is weak and $Q^* \leq 0$. Further, Figure 2.3.b shows that when $\frac{1}{2} < \mathcal{R}(\theta_R, E_R^*) < 1$ and $\alpha_P > \beta_P$, $-1 < Q^* < 0$.

(a)

(b)

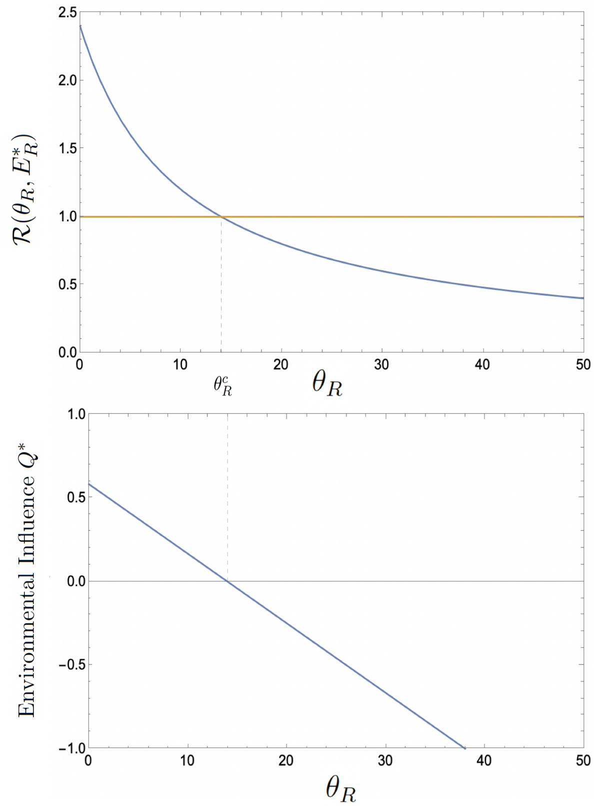


Figure 2.3: Influence Reproductive Number $\mathcal{R}(\theta_R, E_R^*)$ and Environmental Influence Q as a Function of Influence Delay.

If E_R^* increases, then θ_R^c grows and $Q^* > 0$, for a larger range of delays, under a fixed $\frac{r_0}{e_R} > 1$. So influence can be slower if E_R^* is bigger with still $Q^* > 0$. The larger $\frac{r_0}{e_R} > 1$ is, the stronger the effect on Q^*. In short, if $0 < \theta_R < \theta_R^c$, then $Q^* > 0$. When $\mathcal{R}(\theta_R, E_R^*) > 1$, the corresponding environment is $Q^* > 0$. When $\mathcal{R}(\theta_R, E_R^*) < 1$, the influence Q^* becomes negative, starting right after the critical delay θ_R^c; further we have that $Q \in [-1, 0)$, whenever $\frac{1}{2} < \mathcal{R}(\theta_R, E_R^*) < 1$ and $\alpha_P > \beta_P$.

The proportion of educated poor at equilibrium E_P^* is depicted as a function of the parameter θ_R in Figure 2.4. Whenever $0 < \theta_R < \theta_R^c$, the educated poor equilibrium is larger than the *status quo* equilibrium while for $\theta_R > \theta_R^c$, the equilibrium E_P^* falls below the levels of the *status quo*.

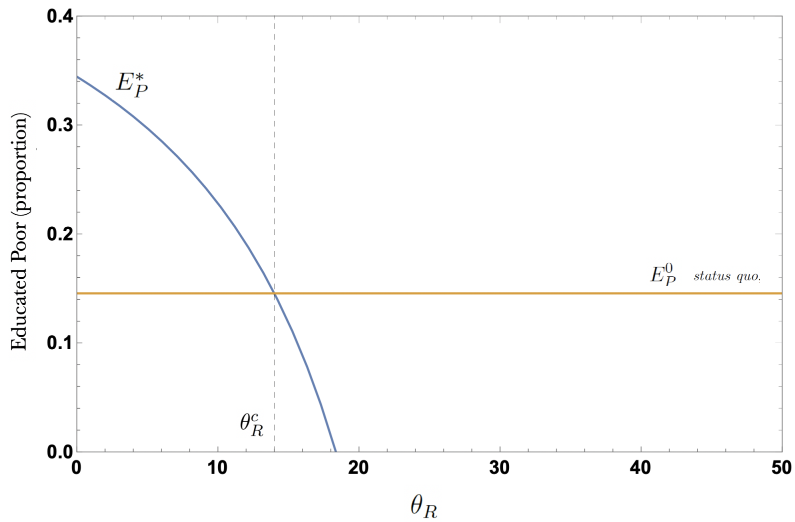


Figure 2.4: Educated Poor at Equilibrium and Influence Delay

Changes in the influence Q over time can also be written in terms of the logistic equation.

$$\begin{aligned}\frac{dQ}{dt} &= r_0 \frac{E_R}{\theta_R + E_R} Q(1-Q) - e_R Q \\ &= \left[r_0 \frac{E_R}{\theta_R + E_R} - e_R \right] Q - r_0 \frac{E_R}{\theta_R + E_R} Q^2 \\ &= \tilde{r} Q \left[1 - \frac{Q}{\frac{\tilde{r}}{r}} \right] \\ &= \tilde{r} Q \left[1 - \frac{Q}{K} \right]\end{aligned}$$

where, $\tilde{r} = \left[r_0 \frac{E_R}{\theta_R + E_R} - e_R \right]$, $r = r_0 \frac{E_R}{\theta_R + E_R}$ and $K = \frac{\tilde{r}}{r}$. If $\tilde{r} < 0$ then, the equilibrium tends to $K < 0$, and Q is a stable equilibrium as $Q \to 0$ as $t \to \infty$.

If $\tilde{r} > 0 \iff \mathcal{R}(\theta_R, E_R^*) > 1$, then $Q \to K$ and it is a stable equilibrium. If $\tilde{r} < 0 \iff \mathcal{R}(\theta_R, E_R^*) < 1$, then $Q \to 0$ and it is a stable equilibrium. If $\frac{1}{2} < \mathcal{R}(\theta_R, E_R^*) < 1$ and $\alpha_P > \beta_P$, there is a positive E_P^* equilibrium that is below the *status quo* equilibrium $E_P^* < E_P^0$ with E_P^* unstable.

Computation of $\mathcal{R}(\theta, E_R^*)$

The influence reproductive number is estimated using the next generation method [72]. It represents the average number of uneducated poor individuals that become college-educated generated by the influence of educated rich individuals affecting the environment. The "infected" compartments correspond to E_P and Q. Then, the matrices $\mathcal{F}$ and $\mathcal{V}$ are as follows:

$$\mathcal{F} = \begin{pmatrix} \beta_P U_P Q \\ \left(r_0 \frac{E_R}{a+E_R}\right) Q(1-Q) \end{pmatrix} \mathcal{V} = \begin{pmatrix} -\alpha_P U_P + \mu E_P \\ e_R Q \end{pmatrix}$$

The linearization of the above matrices is obtained from the derivative with respect to the state variables E_P and Q, to finally obtain the matrices F and V, representing the rates of transmission and the average time length in every compartment, as follows:

$$\mathbf{F} = \begin{pmatrix} 0 & \beta_P \alpha_P U_P \\ 0 & \left(r_0 \frac{E_R}{a+E_R}\right)(1-2Q) \end{pmatrix} \mathbf{V} = \begin{pmatrix} \mu & 0 \\ 0 & e_R \end{pmatrix}$$

The influence reproductive number or $\mathcal{R}(\theta, E_R^*)$ is given by the spectral radius of the next generation matrix expression defined as $F \cdot V^{-1}$, and evaluated at the *status quo* equilibrium:

$$\mathcal{R}(\theta, E_R^*) = \frac{r_0}{e_R} \frac{E_R^0}{\theta + E_R^0} \tag{2.8}$$

where, $E_R^0 = E_R^* = \frac{\alpha_R q_R \Lambda}{\mu(\mu+\alpha_R)}$. This number suggests that the likelihood of influence of educated rich people on the uneducated poor population depends on the coupled effect between the environment and individuals, given by the dynamics $E_R \to Q \to E_P$: the educated rich generate an influence proportional to their presence $\frac{E_R^0}{\theta+E_R^0}$ and amplified by the net influence of the environment represented by $\frac{r_0}{e}$, where it is noticeable that the environment is able to increase the strength of the influence $\mathcal{R}(\theta, E_R^*)$ as r_0 is larger than the rate of loss of influence e_R. We observe that $\mathcal{R}(0, E_R^*) = \frac{r_0}{e_R}$ and $\mathcal{R}(\infty, E_R^*) = 0$, that is, $0 < \mathcal{R}(\theta, E_R^*) \leq \frac{r_0}{e_R}$.

Stability

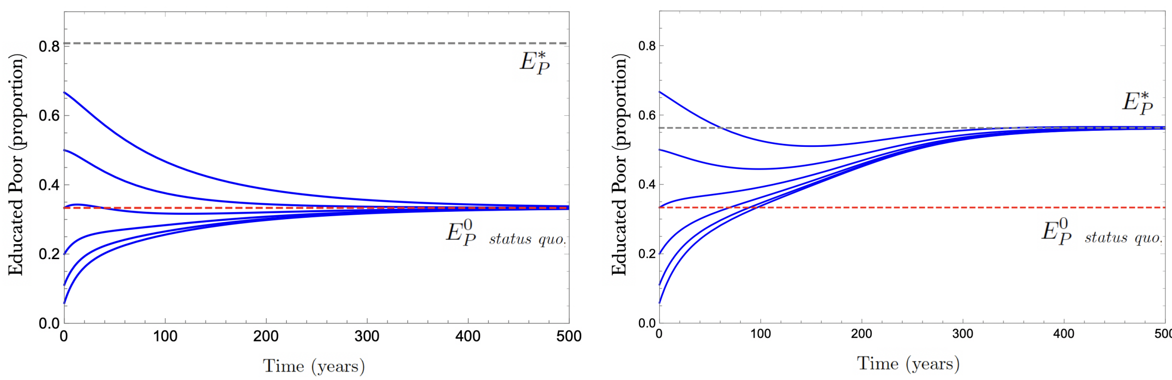
If we linearize the system 2.1 - 2.5 around the equilibria points, we can analyze the asymptotic behavior of the solutions. The linearization is given by the Jacobian (J) evaluated at each equilibrium. Hence, we have that:

$$\mathbf{J} = \begin{pmatrix} -\mu - \alpha_r & 0 & 0 & 0 & 0 \\ 0 & -\mu - \alpha_p - \beta_p Q & 0 & 0 & \beta_p U_P \\ \alpha_r & 0 & -\mu & 0 & 0 \\ 0 & \alpha_p + \beta_p Q & 0 & -\mu & \beta_p U_P \alpha_p \\ 0 & 0 & -\frac{E_R(1-Q)Qr_0}{(\theta+E_R)^2} + \frac{(1-Q)Qr_0}{\theta+E_R} & 0 & e_R + \frac{E_R(1-Q)r_0}{\theta+E_R} - \frac{E_R Q r_0}{\theta+E_R} \end{pmatrix}$$

For the influence-free equilibrium to be locally asymptotically stable (or l.a.s.), the eigenvalues of the Jacobian matrix evaluated at the equilibria have to have negative real parts. Substitution of the corresponding equilibrium gives the following eigenvalues:$\{-\mu, -\mu, -\mu, -(\mu + \alpha_P), e_R(\mathcal{R} - 1)\}$, so then, the influence-free equilibrium is locally asymptotically stable whenever $\mathcal{R}(\theta, E_R^*) < 1$. The *status quo* equilibrium is maintained as long as there is not enough "support" from the environment to make $\mathcal{R}(\theta, E_R^*)$ greater than one, that is, an intense enough level of net production of influence from the environment represented by $\frac{r_0}{e_R}$ that takes advantage of the presence of educated rich $\frac{E_R^0}{\theta + E_R^0}$ that would turn the current state of affairs.

In the case of the positive-influence equilibrium ($Q^* > 0$), the eigenvalues are given by the following expressions: $\left\{-\mu, -\mu, -(\mu + \alpha_R), e(1 - \mathcal{R}(\theta, E_R^*)), \frac{\beta_p}{\mathcal{R}(\theta, E_R^*)} - (\beta_p + \mu + \alpha_p)\right\}$. The value that makes all eigenvalues negative is $\mathcal{R}(\theta, E_R^*) > 1$. The necessary condition to obtain a stable positive-influence equilibrium, is that $\mathcal{R}(\theta, E_R^*) > 1$, that is, that the net effect of the environment favors an amplification effect of the presence of educated rich.

Figure 2.5.a illustrates how the stability is centered at the *status quo* equilibrium when $\mathcal{R}(\theta, E_R^*) < 1$ for various initial conditions. When the potential to influence the community is less than 1, the community tends to conserve the current state of affairs (the result is mathematically just a local result). If conditions in the community switch to a state where *status quo* became unstable, then the proportion of educated poor converge to the positive equilibrium E_P^*, and this is shown in Figure 2.5.b.



(a) $\mathcal{R}(\theta, E_R^*) < 1$ (b) $\mathcal{R}(\theta, E_R^*) > 1$

Figure 2.5: Educated Poor and Stability of Equilibrium Under Different Initial Conditions.

The positive-influence equilibrium for the poor educated is expressed as follows:

$$E_P^* = q_P \frac{\Lambda}{\mu} \left[\frac{\alpha_p \mathcal{R}(\theta, E_R^*)}{\mathcal{R}(\alpha_p + \beta_p + \mu) - \beta_p} + \frac{\beta_p(\mathcal{R}(\theta, E_R^*) - 1)}{\mathcal{R}(\theta, E_R^*)(\alpha_p + \beta_p + \mu) - \beta_p} \right] \tag{2.9}$$

Figure 2.6 depicts the proportion of educated poor at equilibrium, E_P^*, in terms of $\mathcal{R}(\theta_P, E_R^*)$. First, educated poor maintain a *status quo* equilibrium whenever $\mathcal{R}(\theta, E_R^*) < 1$, that is, even when influence $Q^* < 0$. We observe that the possibility of experiencing negative influence when $-1 \leq Q < 0$ is allowed with $\frac{1}{2} < \mathcal{R}(\theta_P, E_R^*) < 1$, and $\alpha_P > \beta_P$. As $\mathcal{R}(\theta_P, E_R^*) > 1$, the *status quo* equilibrium becomes unstable and the environment gives rise an equilibrium for poor educated, with $E_P^* > E_P^0$, that is, whenever $\mathcal{R}(\theta_P, E_R^*) > 1$ and equivalent to $Q^* > 0$.

The proportion of educated poor due to the influence of the environment alone can be estimated by the difference between equilibria $E_P^* - E^0$, an increasing function over $\mathcal{R}(\theta_P, E_R^*)$, whenever $\mathcal{R}(\theta_P, E_R^*) > 1$, which is equivalent to $Q^* > 0$.

We have seen that a positive equilibrium E_P^* decreases (from E_P^0) when $\frac{1}{2} < \mathcal{R}(\theta_P, E_R^*) < 1$ and $\alpha_P > \beta_P$, with E_P^* being an unstable equilibrium. That is all trajectories near E_P^* (not on E_P^*) approach the *status quo* equilibrium, which is locally asymptotically stable when $0 < \mathcal{R}(\theta_P, E_R^*) < 1$. In other words, the equilibria corresponding to $\frac{1}{2} < \mathcal{R}(\theta_P, E_R^*) < 1$ with $\alpha_P > \beta_P$ when $0 < E_P^* < E_P^0$ are all unstable. Further, since $Q^* \to 0$, as long as $\mathcal{R}(\theta, E_R^*) < 1$ then, we see that small perturbations from $0 < E_P^* (< E_P^0)$ will converge to E_P^0 as verified on simulations.

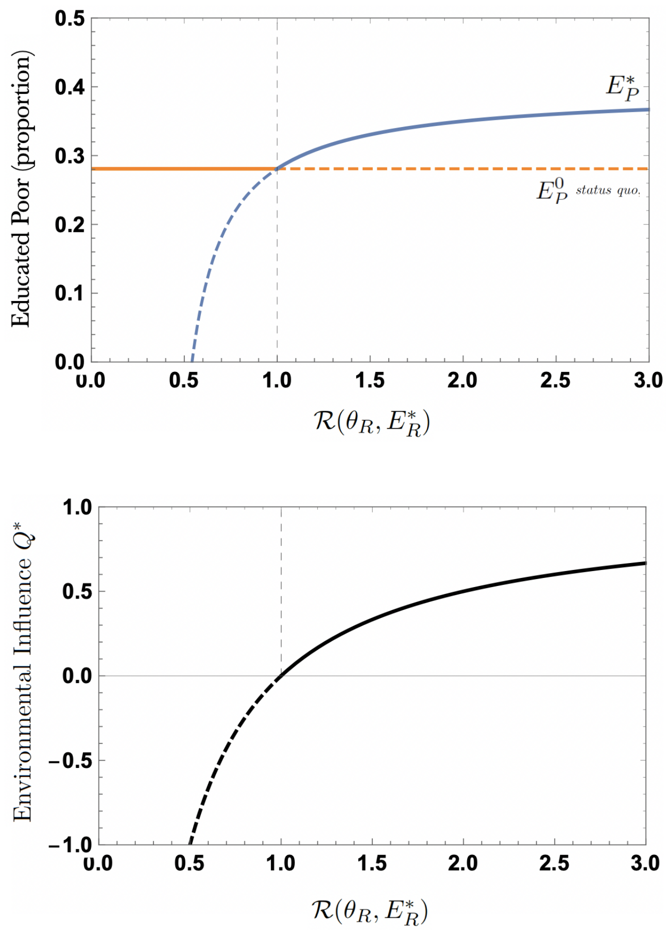


Figure 2.6: Bifurcation Diagram Showing Educated Poor and Environmental Influence as a Function of $\mathcal{R}(\theta_P, E_R^*)$. Solid: Stable Equilibrium. Dashed: Unstable Equilibrium.

Figure 2.6 summarizes the proportion of educated poor in connection to the activation of the environment when $Q^* > 0$. The equilibrium number of educated poor saturates after $\mathcal{R}(\theta_P, E_R^*) = 1$. Every additional effort of influence has a diminishing return, the result of $\mathcal{R}(\theta_P, E_R^*) \leq \frac{r_0}{e_R}$ and nonlinear dynamics. The principle of diminishing returns (non linear dynamics) states that "adding more of one factor [of production], yields lower incremental returns per-unit of factor used" [73]. Increases in the likelihood of influence $\mathcal{R}(\theta_P, E_R^*)$ generate more educated poor but possibilities of influence get eventually exhausted.

Sensitivity Analysis

Figure 2.7 presents the sensitivity analysis with respect to the parameters affecting E_P^*. The estimates correspond to the partial rank correlation coefficient which, measure the effect of every parameter on educated poor considering the rest of parameters as independent. This technique is used to help identifying principal input variables contributing to the changes of educated poor [74, 61]. The higher the coefficient, the larger the effect on educated poor after a variation in the parameter. According to this figure, the rate of loss of influence e_R has the largest depressing effect on the number of educated poor in the model, suggesting the key importance of factors in the environment that diminish the influence, such as, community cohesion, or lack of respect for authority or even corruption perception, etc. Rate of education of the poor α_P increases the number of educated, as well as, the rate of influence from the environment "effectiveness" β_P.

2.1.3 Simulations

Numerical simulations highlight the trends in proportions of educated individuals and the evolution of the environment over time. The parameters used to calibrate the model were obtained using information of the United States, recorded by official institutions such as the Office of the Census, the National Center for Education Statistics, the Federal Reserve, Unesco, the World Bank, as well as, private institutions and foundations such as the Pew Research Center, the Pell Institute, etc. The description, units and references of parameters are summarized in Table 2.3. Simulations highlight the proportion of educated poor associated with variations in different parameters of the model. We focus on the proportion of educated poor E_P as it is

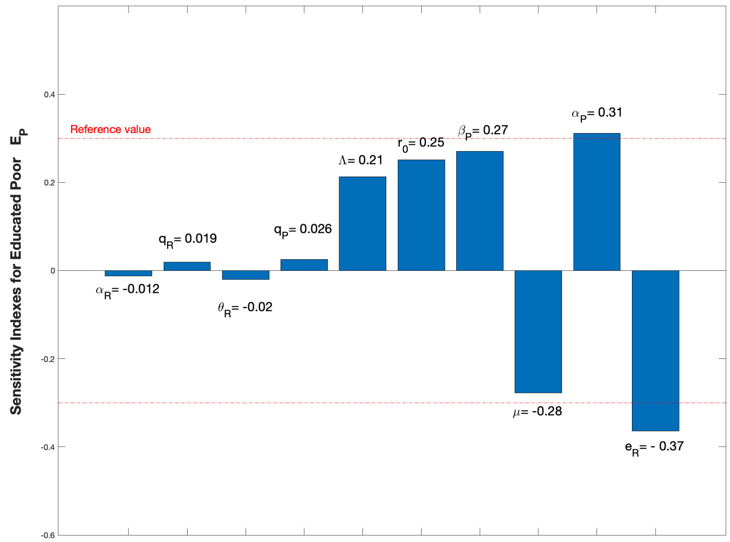


Figure 2.7: Sensitivity Analysis of Parameters Affecting E_P^*.

the group of main interest representing mobility.

Figure 2.8 presents the numerical simulations of the educated poor over time. At time $t = 0$, the proportion of educated poor is 10 %. The trajectories in red color in the figure represent different scenarios for the influence delay component, θ_R. The lower the value of the delay, the faster the influence exerted and greater the proportion of educated poor. θ_R may take values corresponding to $\theta_R = \{0, 17, 34, 51, 68\}$, with $\theta_R = 34$ as the average value of the influence delay throughout the manuscript.

For instance, when handling time is at $\theta_R = 34$, it takes almost 36 years to reach 22% of the poor population to be college educated. When the influence is $\theta_R = 17$, it takes 27 years to achieve the 22% goal and finally, when $\theta_R = 0$, that is, immediate influence, the target policy is achieved after 22 years. When influence is very weak ($\theta_R = 68$), the goal is not even achieved within 120 years. Environment influence is presented in Figure 2.9. Note how the influence of the environment is stronger as

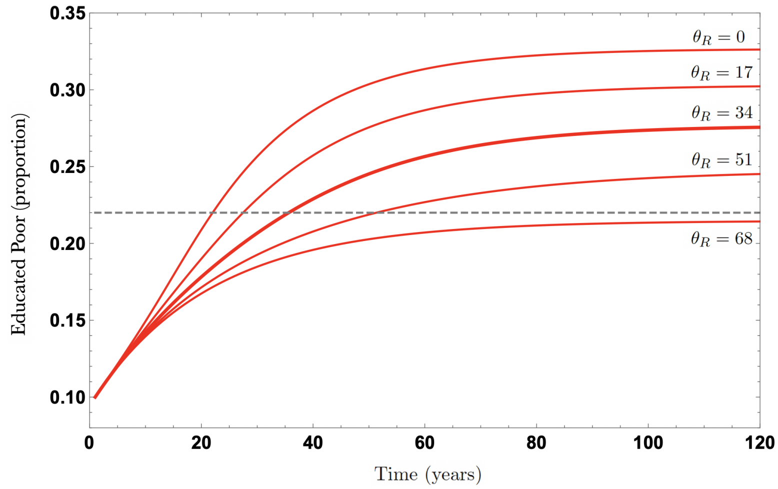


Figure 2.8: Time Series Simulations of Educated Poor and Rich, Under Different Scenarios of θ_R

long the handling time θ_R is lower.

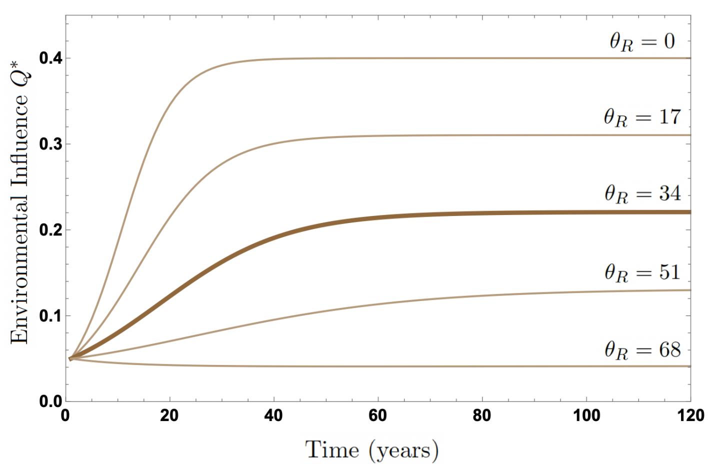


Figure 2.9: Environmental Influence of Education Simulations

2.2 Model 2: Rich-Poor Model with Influence from Poor Educated

2.2.1 Model Derivation

In this section, we focus on the situation when the influence to tackle the challenges of college education come from the educated poor socio-economic class. That is, it is assumed that $Q \equiv Q(E_P)$. It is implicitly assumed that the impact of E_R on U_R is negligible and hence ignored.

In this variation of the model of Section 2.1, the dynamic "probability" or function of environmental influence is given by Q. The per-capita rate of growth of Q is given by $r(\theta_P, E_P)(1-Q)$ with $r(\theta_P, E_P) = r_0 \frac{E_P}{\theta_P + E_P}$ and hence, $r(\theta_P, 0) \equiv 0$ and $r(0, E_P) \equiv r_0$, whenever $E_P > 0$. Here, r_0 denotes the maximum rate of growth of Q as noted in Section 2.1. These definitions and assumptions lead to the following nonlinear system of differential equations:

$$\frac{dU_R}{dt} = q_R \Lambda - \alpha_R U_R - \mu U_R, \tag{2.10}$$
$$\frac{dU_P}{dt} = q_P \Lambda - \alpha_P U - \beta_P U_P Q - \mu U_P, \tag{2.11}$$
$$\frac{dE_R}{dt} = \alpha_R U_R - \mu E_R \tag{2.12}$$
$$\frac{dE_P}{dt} = \alpha_P U + \beta_P U_P Q - \mu E_P, \tag{2.13}$$
$$\frac{dQ}{dt} = r_0 \frac{E_P}{\theta_P + E_P} Q(1-Q) - e_P Q, \tag{2.14}$$

where r_0 is the intrinsic growth rate of Q; $\frac{1}{e_P}$ is the average time impact of the influence Q on U_P; θ_P determines whether or not there is a delay on the impact of E_P on Q (handling time); α_R and α_P denote the rates of progression from U_R and U_P to E_R and E_P, respectively; $\frac{1}{\mu}$ is the average educational "life-span", that is, it is the window in time when individuals can still select the route of higher education, and β_P denotes the "effectiveness" of the transmission of influence.

The population reaches a limiting value given by $N \to \frac{\Lambda}{\mu}$ as $t \to \infty$, and hence, we assume that $N(0) \equiv \frac{\Lambda}{\mu}$. Figure 2.10 depicts the graphical flow of Model 2.

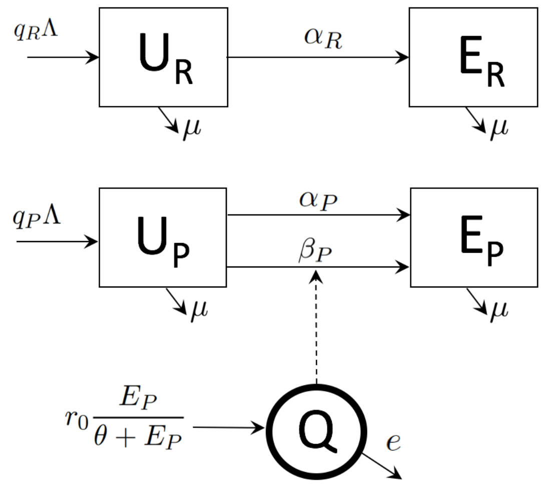


Figure 2.10: Flow Diagram of Compartmental Model 2

2.2.2 Mathematical Analysis

Equilibria

The equilibria of System 2.10 - 2.14 is obtained by setting every equation equal to zero. This system may support up to three equilibria: the influence-free equilibrium referred to as the *status quo* equilibrium, when $Q^* = 0$, is given by the following expressions:

$$
\begin{aligned}
U_R^0 &= \frac{q_R\Lambda}{\alpha_R + \mu}, \\
U_P^0 &= q_P\frac{\Lambda}{\alpha_P\mu}, \\
E_R^0 &= q_R\frac{\Lambda}{\mu}\frac{\alpha_R}{\alpha_R + \mu}, \\
E_P^0 &= q_P\frac{\Lambda}{\mu}\frac{\alpha_P}{\alpha_P + \mu}, \\
Q^0 &= 0.
\end{aligned}
$$

Influence equilibria corresponding to $Q^* \neq 0$ is obtained by the sub system representing the rich compartments U_R' and E_R', which is independent and can be solved separately, and hence, we have that,

$$U_R^* = \frac{q_R \Lambda}{\alpha_R + \mu}$$
$$E_R^* = q_R \frac{\Lambda}{\mu} \frac{\alpha_R}{\alpha_R + \mu}$$

From 2.11 and 2.13, we obtain expressions for uneducated and educated poor by solving $U_P' = 0$ and $E_P' = 0$. And so,

$$U_P^* = \frac{q_p \Lambda}{\alpha_p + \beta_p Q^* + \mu},$$
$$E_P^* = \frac{1}{\mu} \left(\alpha_P + \beta_P Q^*\right) U_P^*,$$

A positive influence equilibrium arises when $Q^* > 0$. The conditions are derived from the solutions of $Q' = 0$,

$$Q^* = 1 - \frac{e_P}{r_0} \frac{\theta_P + E_P^*}{E_P^*} \tag{2.15}$$

We see that $Q^* > 0 \Leftrightarrow \frac{r_0}{e_P} \frac{E_P^*}{\theta_P + E_P^*} > 1$, provided that $E_P^* > 0$. We let

$$\mathcal{R}(\theta_P, E_P^*) \equiv \frac{r_0}{e_P} \frac{E_P^*}{\theta_P + E_P^*} \tag{2.16}$$

denote the influence reproductive number. Consequently, the existence of $Q^* > 0$ requires that $\mathcal{R}(\theta_P, E_P^*) > 1$, with $E_P^* > 0$. $\mathcal{R}(0, E_P^*) = \frac{r_0}{e_P}$ with $\frac{r_0}{e_P} > 1$ being a necessary condition for $Q^* \geq 0$. Analogously to the analysis of Model 1, we obtain the following results:

$$\mathcal{R}(\theta_P, E_P^*) > 1 \Longleftrightarrow 0 \leq \theta_P < E_P^* \left(\frac{r_0}{e_P} - 1\right) \text{ and so, we always require that } \frac{r_0}{e_P} > 1.$$

We define the critical level of delay influence of the poor educated as $\theta_P^c \equiv E_P^* \left(\frac{r_0}{e_P} - 1\right)$. When $0 < \theta_P < \theta_P^c$, the effectiveness of influence acts fast enough for $Q^* > 0$ while if $\theta_P > \theta_P^c$, the effectiveness of influence operates too slow, and we have that $Q^* \leq 0$. Figure 2.11 presents the connection between the basic influence number $\mathcal{R}(\theta_P, E_P^*)$ and, the environment Q^*, as a function of θ_P. Q^* may be also interpreted as the influence function and if we wish, we could restrict it to the range $Q^* \in [-1, 1]$ to interpret it as an index. There is a possibility to experience negative influence but the equilibrium result for E_P^* and Q^* present a quadratic solution and to establish a range over $\mathcal{R}(\theta_P, E_P^*)$ to obtain this index is not straightforward. In order to explain the equilibrium through the most simple terms, we focus on $Q^* > 0$. If we do not place restrictions on Q, there exist also levels of $Q^* < 0$ that guarantee (numerically) the existence of a second root of Q^* (orange segment), unstable equilibrium, since $Q^* \to 0$, whenever $\mathcal{R}(\theta_P, E_P^*) < 1$, as $t \to \infty$. We can observe this possibility of negative influence in, for example, Figure 2.11.

Figure 2.11.a shows that if influence is strong enough (lower than critical value), there is a force of change generated by the environment Q over the *status quo* and hence, $E_P^* > E_P^0$. If influence is weak, (larger than critical value), *status quo* prevails. With enough influence, environment activates $Q^* > 0$. If not, $Q^* < 0$ (see Figure 2.11.b).

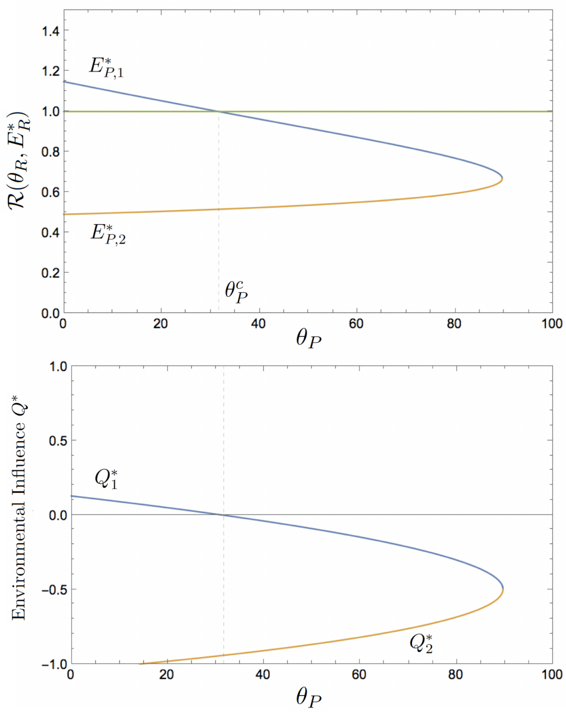


Figure 2.11: Influence Reproductive Number and Influence Delay θ_P Coupled with Environment.

Figure 2.12 highlights the equilibria for educated poor as a function of the influence delay. The intersection of the equilbrium E^*_P with the *status quo* equilibrium E^0_P occurs at the critical level of influence given by θ^c_P. No restrictions on Q^* or on the relationships between $\alpha_P > \beta_P$ generate (numerically) the stable (blue) and unstable (orange) equilibrium curves. Figure 2.12 summarizes the equilibria and the space where stability of equilibria occurs. Whenever $\theta_P < \theta^c_P$ and $E^*_P > E^0_P$, influence equilibrium is stable, $E^*_{P,1}$ in this particular case. If $\theta_P > \theta^c_P$, *status quo* equilibrium is stable and E^*_P becomes unstable. This figure guides our results for the equilibria in the rest of this section.

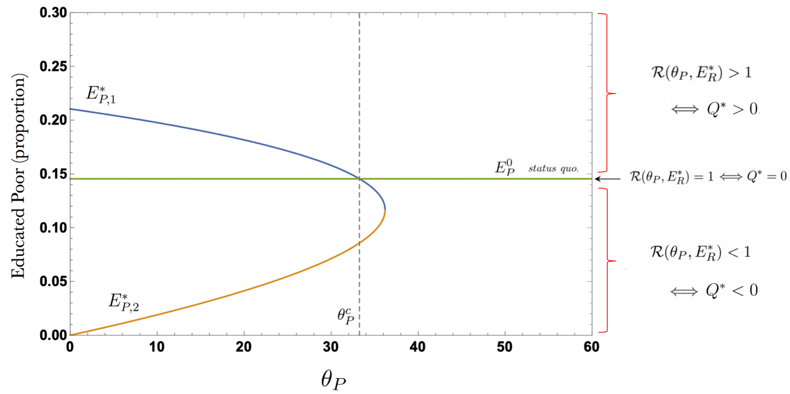


Figure 2.12: Educated Poor at Equilibrium and Influence Delay θ_P.

Replacing the expression for the equilibrium Q^* and U_P^* into the above expressions leads to the following relationship

$$E_P^* = \frac{1}{\mu}\left(\alpha_P + \beta_P Q^*\right)\frac{q_p\Lambda}{\alpha_p + \beta_p Q^* + \mu}, \tag{2.17}$$

or, equivalently, to

$$E_P^* = \frac{1}{\mu}\left[\alpha_P + \beta_P\left(1 - \frac{e}{r_0}\frac{\theta + E_P^*}{E_P^*}\right)\right]\frac{q_p\Lambda}{\alpha_p + \beta_p\left(1 - \frac{e}{r_0}\frac{\theta + E_P^*}{E_P^*}\right) + \mu}, \tag{2.18}$$

which leads to a quadratic equation for E_P^*.

Solutions are of the form $E_P^* = \frac{-B \pm \sqrt{B^2 - 4AC}}{2A}$, where $A = \mu(\beta_p(r_0 - e) + r_0(\mu + \alpha_R))$, $B = \theta\beta_P e\mu - q_p\Lambda(\beta_P(r_0 - e) + r_0(2\mu + \alpha_R))$ and $C = r_0 q_P^2 \Lambda^2$. If $B^2 - 4AC > 0$, two real roots exist. Two real solutions are obtained when the intercept is positive $C > 0$, first derivative of the function evaluated at zero is negative (that is, the parabola open upwards, $A > 0$) and, finally, that the vertex of parabola is negative. Whenever $C > 0$, the intercept of the quadratic function is always positive. Then, we compute the first derivative of the quadratic function and confirm that it is negative and compute the second derivative and confirm that it is positive. If $f(E_P)$ equals the quadratic equation 2.18, we define

$$f(E_P^*) \equiv \frac{1}{\mu}\left[\alpha_P + \beta_P\left(1 - \frac{e_P}{r_0}\frac{\theta_P + E_P^*}{E_P^*}\right)\right]\frac{q_p\Lambda}{\alpha_p + \beta_p\left(1 - \frac{e_P}{r_0}\frac{\theta_P+E_P^*}{E_P^*}\right) + \mu} - E_P^* \quad (2.19)$$

and observe that, the first derivative is:

$$f'(0) = -\theta_P\beta_p e_P\mu + q_p\Lambda[\beta_p(e - r_0) - r_0\alpha_p]$$
$$-\theta_P\beta_P e_P\mu + q_p\Lambda[\beta_p(e_P - r_0) - r_0\alpha_p]$$
$$\text{which is negative} \Leftrightarrow \frac{\beta_p}{\beta_p + \alpha_p}\frac{q_p\Lambda - \theta_P\mu}{q_p\Lambda} < \frac{r_0}{e_P}$$

that is,

$$\frac{\beta_P}{\beta_P + \alpha_P}\left[1 - \frac{\theta_P}{q_p\frac{\Lambda}{\mu}}\right] < \frac{r_0}{e_P} \quad (2.20)$$

Since by assumption we have that $\frac{r_0}{e_P} > 1$, then we need $q_p\frac{\Lambda}{\mu} > \theta_P$. Also,

$$f''(E_p) = 2\mu(\beta_p(r_0 - e_P) + r_0(\mu + \alpha_P))$$
$$2\mu(\beta_p(r_0 - e_P) + r_0(\mu + \alpha_p))$$
$$\text{which is positive} \Leftrightarrow \frac{r_0}{e_P} > \frac{\beta_p}{\beta_p + \alpha_p + \mu}$$

Finally, the vertex of the parabola has to be a negative value for the parabola to cross the x-axis and form the two positive roots. The vertex of the parabola gives origin to the new threshold point that serves to understand the new stability conditions. It will be referred to as $\mathcal{R}_c = -\frac{B}{2A} = W + Z\mathcal{R}(\theta_P, E_P^*)$, where $W = \frac{\beta_P}{2\mu}\frac{e_P\mu\theta_P + q_P\Lambda(r_0 - e_P)}{\beta_P(r_0 - e_P) + r_0(\mu + \alpha_P)}$ and $Z = \frac{e_P}{r_0}\frac{\mu\theta_P(\mu + \alpha_P) + q_P\alpha_P\Lambda}{\mu + \alpha_P}$.

Figure 2.13 shows the equilibrium of educated poor E_P^* and the environmental influence Q^*, as functions of the influence reproductive number $\mathcal{R}(\theta_P, E_P^*)$. Figure 2.13.a shows that when $\mathcal{R}(\theta_P, E_P^*) < 1$, *status quo* equilibrium is stable. When $\mathcal{R}(\theta_P, E_P^*) > 1$, $E_{P,1}^*$ is stable and *status quo* is unstable with $E_{P,1}^* > E_P^0 > E_{P,2}^*$.

Under these conditions, $E^*_{P,2}$ is always unstable. Figure 2.13.b presents the equilibrium and stability diagram for environmental influence, which is analogous to the E^*_P equilibrium. For this particular case, $Q^* > 0$ represents the force of change in the form of environmental influence, making $E^*_{P,1} > 0$. When $\mathcal{R}(\theta_P, E^*_P) < 1$, only $Q^* = 0$ is the stable solution (*status quo*). If $\mathcal{R}(\theta_P, E^*_P) > 1$, Q^*_1 is stable and Q^*_2 is unstable, with $Q^*_1 > Q^0 > Q^*_1$.

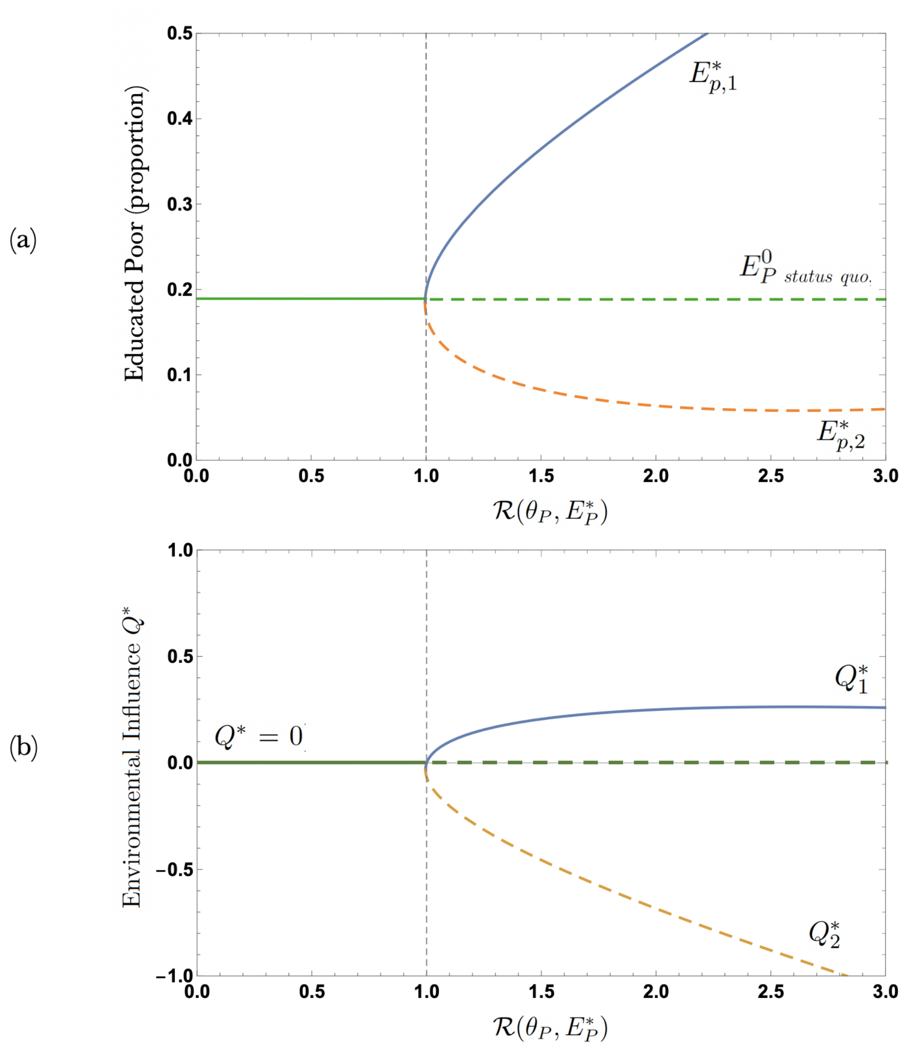


Figure 2.13: Influence Equilibrium as a Function of the Influence Reproductive Number.

Recall that when $\theta_P < \theta_P^c$, the threshold value $\mathcal{R}(\theta_P, E_P^*) > 1$. When the critical influence is relatively "weak", there is a delay in the impact on the educated poor, making the influence lower and unable to generate an effective environmental influence. The critical level of influence can be expressed as follows, given that we already obtained a solution for E_P^0:

$$\theta_P^c = q_P \frac{\Lambda}{\mu} \frac{\alpha_P}{\alpha_P + \mu} \left(\frac{r_0}{e_P} - 1 \right) \tag{2.21}$$

This necessary level of influence is equivalent to the proportion of poor individuals that are getting education multiplied by the net positive maximal influence of the environment $\left(\frac{r_0}{e_P} - 1 \right)$. The net maximal influence means that we take into account the result of increasing influence minus the loss of influence with $\frac{r_0}{e_P} > 1$.

Another interpretation of the critical level of influence is by expressing the condition 2.15 in terms of E_P^*, to obtain $E_P^c = \frac{\theta_P e_P}{r_0 - e_P}$. This expression may be defined as the critical mass educated poor individuals needed to begin the activation of environmental influence. One educated poor individual may become a valuable mentor in the community, but having 1000 educated poor may produce a higher impact in the community. Figure 2.14 highlights the threshold needed (critical mass) that guarantees $Q^* \equiv Q^*(E_P) > 0$, which is a condition equivalent to $\theta_P < \theta_P^c$ that we use to focus the analysis.

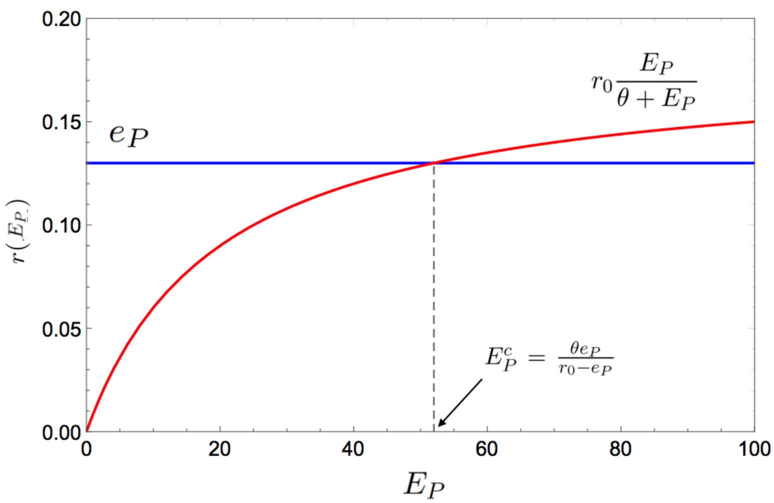


Figure 2.14: Critical Mass for Educated Poor

Here, we describe in general terms, the result of three possible outcomes for the equilibria and stability of E_P^* and Q^*. Figure 2.15 presents two bifurcation diagrams (subcritical type) to explore the change in the qualitative behavior of the system. In Figure 2.15.a, the vertex of the bifurcation coincides with the *status quo* equilibrium. When the vertex coincides with *status quo* equilibrium, $E_P(\mathcal{R}_c) = E^0$, if $\mathcal{R}(\theta_P, E_P^*) > 1$, $E_{P,1}^*$ is stable, with $E_{P,1}^* > E_P^0$; $E_{P,2}^*$ is unstable, and also Q_1^* is stable and Q_2^* unstable. If $\mathcal{R}(\theta_P, E_P^*) < 1$, E_P^0 and Q^0 are stable.

When the vertex is lower than the *status quo*, $E_P(\mathcal{R}_c) < E_P^0$, that is, $\mathcal{R}_c < \mathcal{R}(\theta_P, E_P^*) = 1$, $E_{P,1}^*$ is stable and $E_{P,2}^*$ and E_P^0 are unstable. $\mathcal{R}_c > \mathcal{R}(\theta_P, E_P^*)$, only E_P^0 is stable and E_P^* unstable.

When vertex is greater than the *status quo*, $E_P(\mathcal{R}_c) > E_P^0$, the *status quo* E_P^0 is stable if $\mathcal{R}(\theta_P, E_P^*) < 1$ and is turns to be unstable when $\mathcal{R}(\theta_P, E_P^*) > 1$. $E_{P,1}^*$ is a stable equilibrium for $\mathcal{R}(\theta_P, E_P^*) > \mathcal{R}_c$. The other equilibrium $E_{P,2}^*$ is a always unstable for $\mathcal{R}(\theta_P, E_P^*) > \mathcal{R}_c$.

Figure 2.15.b presents a backward bifurcation with respect to parameter $\mathcal{R}(\theta, E_P^*)$.

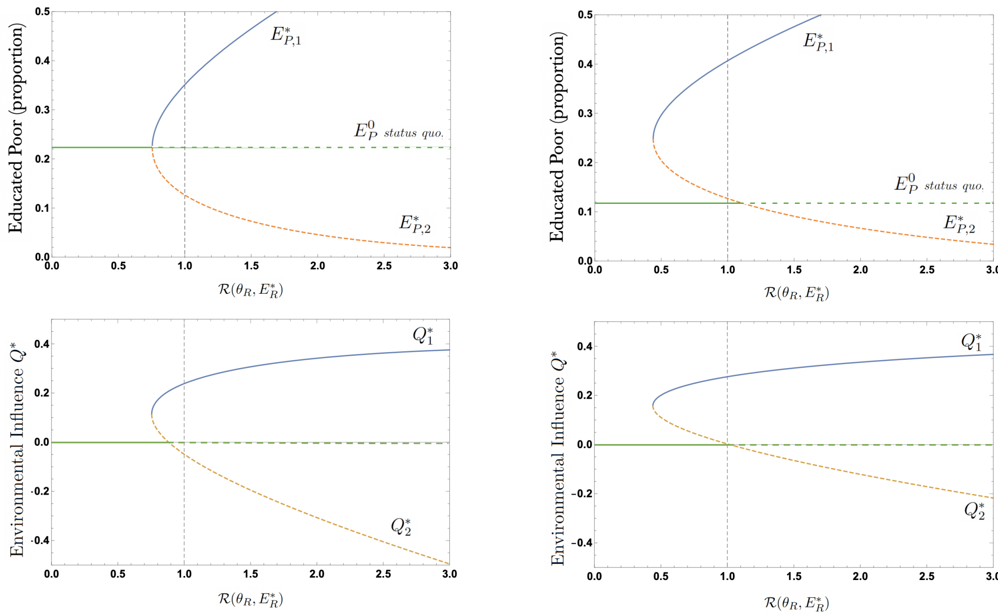


Figure 2.15: Bifurcation Diagram of Steady States for Positive Influence Equilibrium as a Function of the Influence Reproductive Number. (a) $\theta_P = 62.2, e_P = 0.30, \beta_P = 0.087, \alpha_P = 0.0019$. (b) $\theta_P = 68.8, e_P = 0.30, \beta_P = 0.047, \alpha_P = 0.0058$.

As to whether a point where $\mathcal{R}_c < \mathcal{R}(\theta, E^*_P)$ is stable or not, it depends on the initial conditions of that particular situation and Figure 2.16 shows this case more closely. If the point is over the area represented by letter A, the equilibrium eventually tends to the *status quo* , reducing the number of educated poor. However, if the initial condition point is situated within range $\mathcal{R}_c < \mathcal{R}(\theta, E^*_P) < 1.5$ and above equilibrium $E^*_{p,2}$ (orange dashed line), the equilibrium will converge to the upper equilibrium since $E^*_{p,1}$, which is the stable equilibrium. Even though the system might be below the threshold level $\mathcal{R}(\theta, E^*_P) < 1$, there are still educated poor people generating a “loop” of influences that keep adding new educated poor individuals. In summary, the system converges to stable equilibrium depending on the initial conditions.

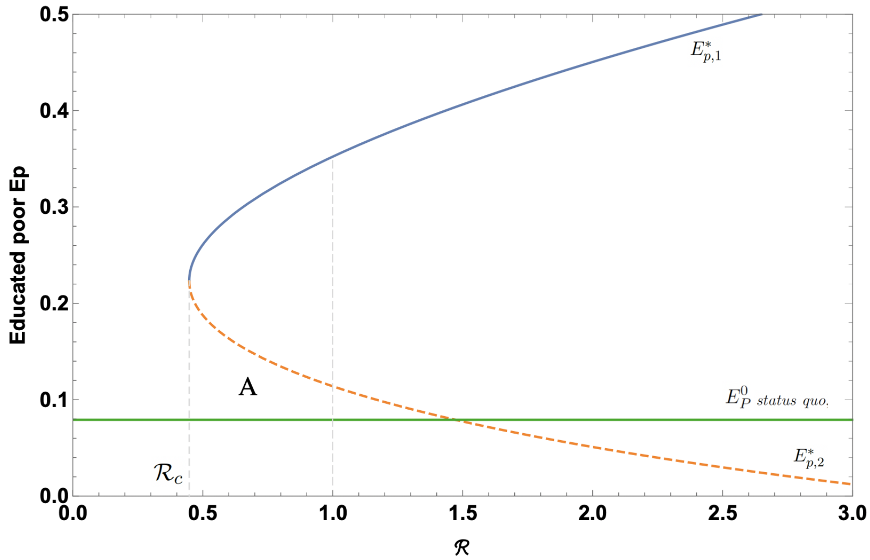


Figure 2.16: Backward Bifurcation Diagram of Educated Poor with Respect to Influence Reproductive Number $\mathcal{R}(\theta_P, E_P^*)$.

Stability

The stability of equilibria is obtained from the linear approximation. This matrix is calculated by obtaining the first derivative of the equations in the system with respect to each state variable [75]. This allows to estimate the localized behavior of solutions and identify if a small disturbance causes a large change in the solution. The Jacobian matrix is given by:

$$J = \begin{pmatrix} -\mu - \alpha_R & 0 & 0 & 0 & 0 \\ 0 & -\beta_P Q - \mu - \alpha_P & 0 & 0 & -\beta_P U_P \\ \alpha_R & 0 & -\mu & 0 & 0 \\ 0 & \beta_P Q + \alpha_p & 0 & -\mu & \beta_P U_P \\ 0 & 0 & 0 & \frac{(1-Q)Qr_0}{\theta+E_P} - \frac{E_P(1-Q)Qr_0}{(\theta+E_P)^2} & -e + \frac{E_P(1-Q)r_0}{\theta+E_P} - \frac{E_P Q r_0}{\theta+E_P} \end{pmatrix}$$

Then, we evaluate each equilibrium at the Jacobian matrix and obtain eigenvalues to determine stability. We conclude that the *status quo* equilibrium is stable as long as the term $\mathcal{R}(\theta, E_P^*) < 1$. For the two influence equilibria, $E^*_{P,1}$ and $E^*_{P,2}$, we used numerical approximations considering several initial conditions to study the stability of solutions.

Figure 2.17 includes the equilibria $E^*_{P,1}$, $E^*_{P,2}$ and *status quo* E^0_P. We first start with trajectories of solutions with reference to the critical influence delay. When $\theta_P < \theta^c$, the trajectories of equilibrium approach the equilibrium 1 $E^*_P > 0$ asymptotically.

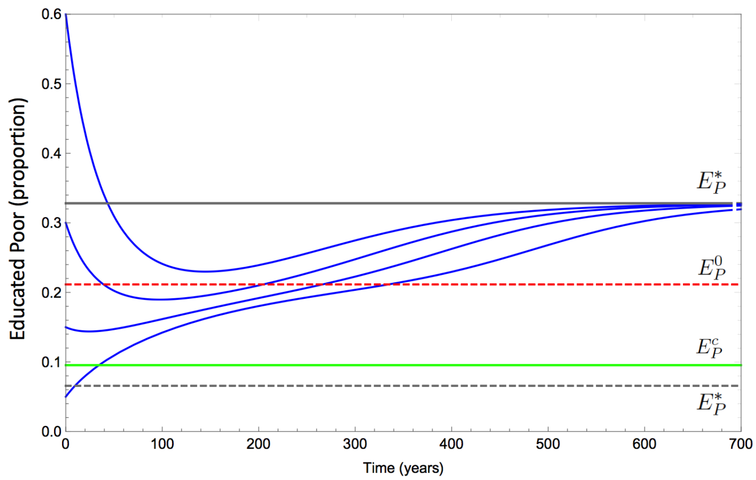


Figure 2.17: Educated Poor and Stability when $E^0_P > E^c_P$, with Initial Conditions $E_P(0) = \{0.05, 0.25, 0.3, 0.6\}$.

Figurer 2.18 shows that the influence equilibrium $E^*_{P,2}$ is unstable when $\mathcal{R}(\theta_P, E^*_P) < 1$. That is, When $\theta_P > \theta^c$, the equilibrium approaches the *status quo* equilibrium. the trajectories asymptotically approach the *status quo* equilibrium. In summary, if the system maintain stability at equilibrium $E^*_{P,2} > 0$, the equilibrium approaches back the *status quo*, E^0_P.

Sensitivity Analysis

Figure 2.19 also presents the sensitivity analysis for educated poor with respect to the parameters of the model using the partial rank correlation coefficient, as it was done in the first model. The coefficients for this case are higher, for the loss of influence e_P and also for the intrinsic growth rate r_0, suggesting that this second model captures more perturbations with respect to the abundance of educated poor as parameters

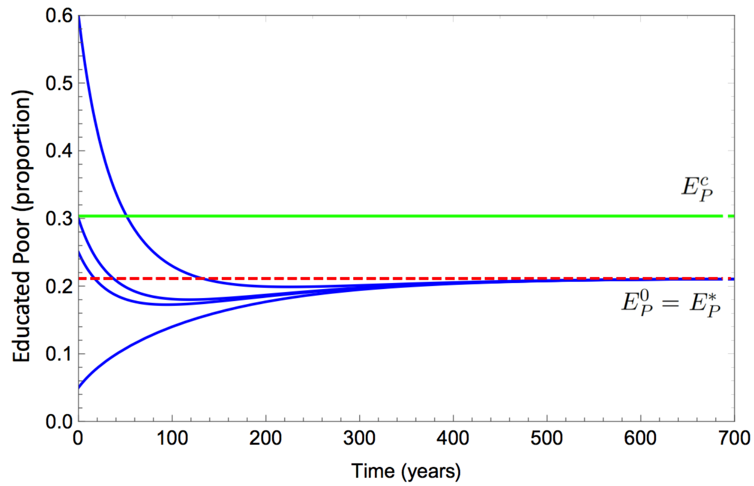


Figure 2.18: Educated Poor and Stability when $E_P^0 < E_P^c$, when $\mathcal{R}(\theta_P, E_P^*) < 1$.

vary.

2.2.3 Simulations

Numerical simulations of the proportion of educated poor over time help understanding the time scales needed to reach a goals in this artificial community. Recall that the rate of growth of influence for this model is given by $r(\theta_P, E_P) = r_0 \frac{E_P}{\theta_P + E_P}$, and the influence delay equal to zero corresponds to the immediate influence which reaches the highest level of influence growth, that is, $r(0, E_P) = r_0$. Figure 2.21 includes the simulations for both educated rich (blue) and educated poor (red) for various levels of influence delay θ_P.

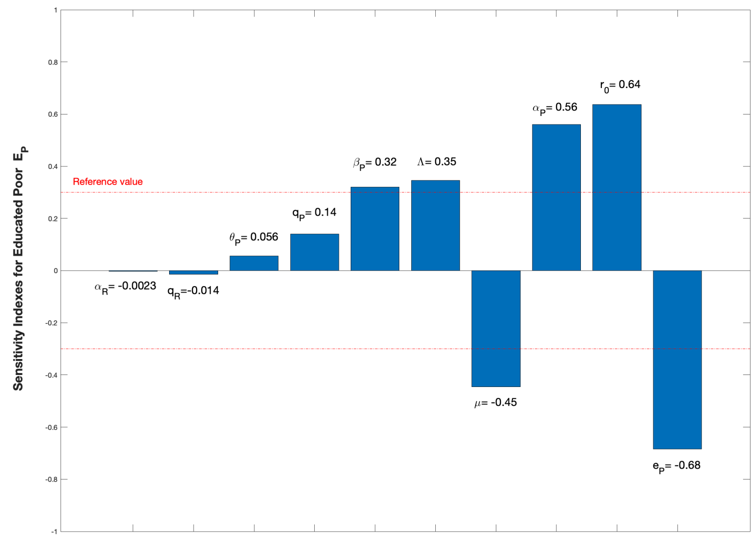


Figure 2.19: Sensitivity Analysis of Parameters Affecting E_P^*.

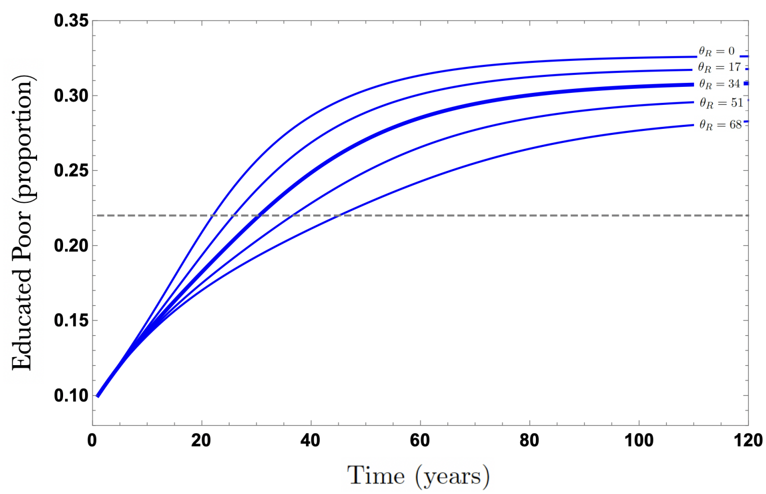


Figure 2.20: Simulations of Model of Proportion of Educated Poor with Influence of the Poor Educated $Q \equiv Q(E_P)$

The simulations illustrate the time it takes for education contagion effects to change the proportion of the poor population seeking education. For instance, under an immediate influence of the educated poor, it will take 27 years to reach the 22% goal. When the influence reaches $\theta_P = 17$, the goal is achieved in 35 years and if the influence get slow as in the case of a $\theta_P = 34$, the goal is achieved approximately in 52 years.

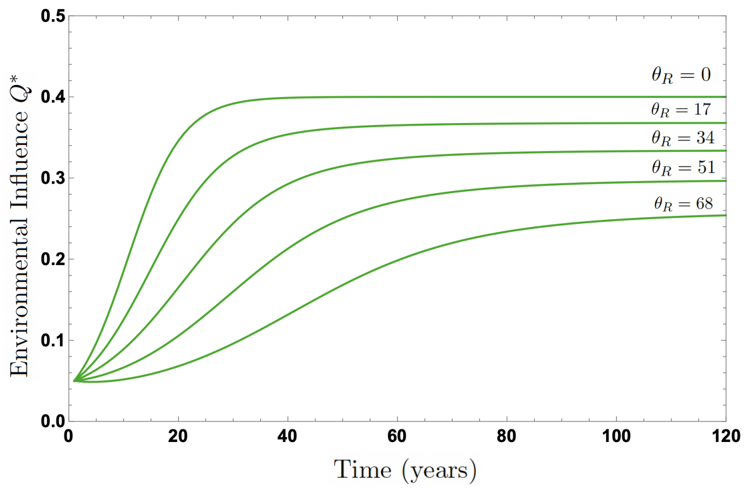


Figure 2.21: Simulations of Environment Influenced by Different Levels of Influence Delay.

One of the subsequent aspects of interest is to determine which group has more substantial influence over the uneducated poor. Figure 2.22 merges the equilibria results for educated poor when influence is generated by educated rich $Q \equiv Q(E_R)$ (Model 1) and when the influence is originated by educated poor, $Q \equiv Q(E_P)$.

Influence from the peer class (poor influencing poor) has a larger effect than the class effect (rich influencing poor). Proximity of conditions or similarity of personal events may have a larger connotation in the life of the community, rather than the demonstration effect coming from the more privileged classes. Figure 2.22 shows that being influenced by the peers, the 22% goal of educated poor is achieved at year $t = 30$, 6 years earlier in contrast to the influence from the rich class, that is, when $Q \equiv Q(E_R)$. If the influence is instantaneous, with $\theta_P = 0$, this goal can be achieved at year 21.

Being influenced by a person from the same class, has a larger effect in contrast to an elite class and the difference can be clearly identified after a year 90 in Figure 2.22.

Figure also suggests that during the first generation, approximately 20 years, the proportion of poor educated is similar, in-distinctively of whom creates the environmental influence. However, after year 90, there is a separation between the influences in which, the influence of the educated poor dominates always the influence of the rich. The figure also presents the environmental influence under both cases, where the influence decays overtime when $Q \equiv Q(E_R)$ but, it maintains logistic saturating shape when $Q \equiv Q(E_P)$ due to the feedback loop created in the model.

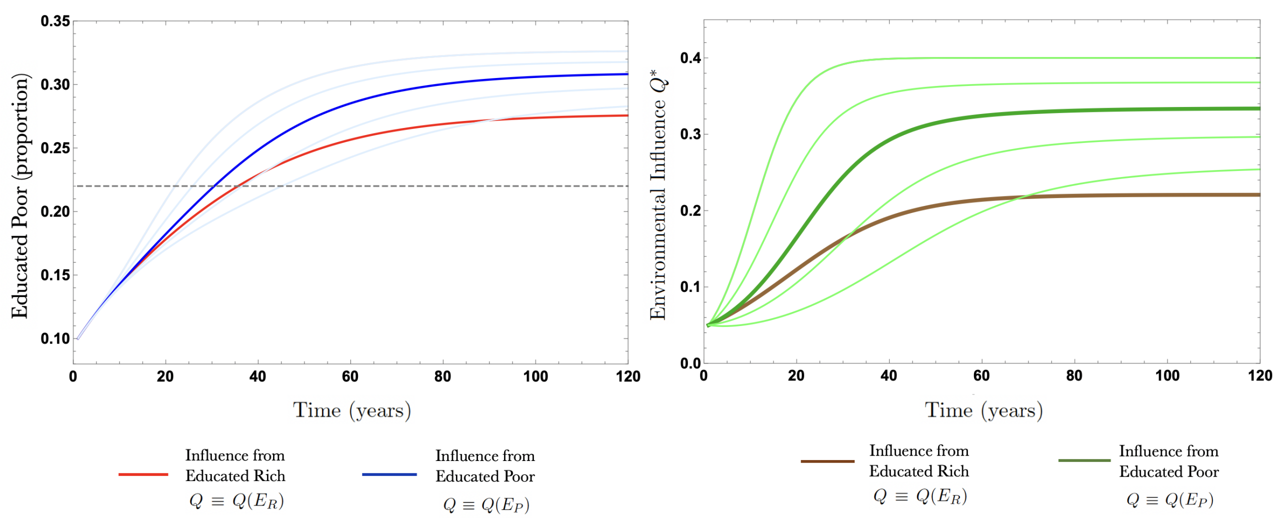


Figure 2.22: Educated Poor and Environmental Influence Generated by the Influence of Educated Rich (Model 1) and Educated Poor (Model 2)

2.3 Model 3: Rich-Poor and Influence from Rich and Poor Educated

2.3.1 Model Derivation

The third artificial community representing the model for rich and the poor, now includes a variation in the influence from the environment. The rest of the model assumptions are kept such that total population equals $N = U_R + U_P + E_R + E_P$ and rates of transition between compartments also maintain their per capita rates α_R for the rich and α_P for the poor with $\alpha_R > \alpha_P$. The environmental influence is state-dependent of a linear combination of both the presence of rich and poor educated, that is, $Q \equiv Q(E_R, E_P, l)$, where l is a proportion. The equations regarding the population compartments used in Models 1 and 2 are maintained. The intrinsic rate of growth now takes the form $r(E_R, E_P) = r_0 \frac{lE_R + (1-l)E_P}{\theta + lE_R + (1-l)E_P}$, that is, we include a single delay θ for simplicity. The system of differential equations is now given by,

$$\frac{dU_R}{dt} = q_R\Lambda - \alpha_R U_R - \mu U_R, \tag{2.22}$$
$$\frac{dU_P}{dt} = q_P\Lambda - \alpha_P U_P - \beta_P U_P Q - \mu U_P, \tag{2.23}$$
$$\frac{dE_R}{dt} = \alpha_R U_R - \mu E_R, \tag{2.24}$$
$$\frac{dE_P}{dt} = \alpha_P U_P + \beta_P U_P Q - \mu E_P, \tag{2.25}$$
$$\frac{dQ}{dt} = \left(r_0 \frac{lE_R + (1-l)E_P}{\theta + lE_R + (1-l)E_P} \right) Q(1-Q) - eQ, \tag{2.26}$$

where, $l \in [0, 1]$ represents the relative importance of influence of the rich with respect to the poor. If $l = 1$, it corresponds to the Model 1 where rich educated influence the poor uneducated; $l = 0$ represents the other extreme, that is, the case when the poor educated are the unique source of influence (Model 2).

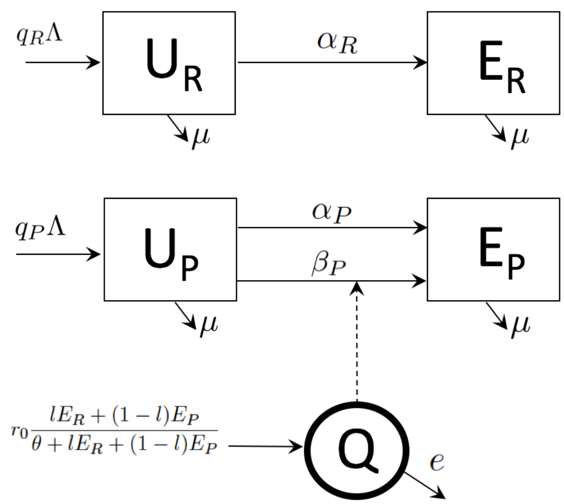


Figure 2.23: Flow Diagram of Compartmental Model

2.3.2 Mathematical Analysis

Equilibria

The model supports three equilibria. The first equilibrium corresponds to the *status quo* ($Q^* = 0$), has identical expressions to those obtained in models 1 and 2.

$$U_R^0 = \frac{q_R\Lambda}{\alpha_R + \mu}$$
$$U_P^0 = q_P\frac{\Lambda}{\mu}$$
$$E_R^0 = q_R\frac{\Lambda}{\mu}\frac{\alpha_R}{\alpha_R + \mu}$$
$$E_P^0 = q_P\frac{\Lambda}{\mu}\frac{\alpha_P}{\alpha_P + \mu}$$
$$Q^0 = 0$$

The additional equilibria corresponds to the case when the non trivial solutions of $\frac{dQ}{dt} = 0$ are considered, that is, when $Q^* > 0$. Hence, we analyze the solution of Q^* for $r_0\frac{lE_R+(1-l)E_P}{\theta+lE_R+(1-l)E_P}(1 - Q) - e = 0$,

$$Q^* = 1 - \frac{e}{r_0}\frac{\theta + lE_R^* + (1-l)E_P^*}{lE_R^* + (1-l)E_P^*} > 0 \tag{2.27}$$

Using Expression 2.27, we identify the threshold expression:

$$\mathcal{R}(\theta, E_R^*, E_P^*) = \frac{r_0}{e}\frac{lE_R^* + (1-l)E_P^*}{\theta + lE_R^* + (1-l)E_P^*} \tag{2.28}$$

Expression 2.28 is now defined as the influence reproductive number.

With equilibrium solution for Q^*, we can substitute in the the following expression to obtain the rest of the equilibria:

$$U_P^* = \frac{q_p\Lambda}{\alpha_p + \beta_p Q^* + \mu}$$
$$E_P^* = \frac{1}{\mu}\left(\alpha_p + \beta_p Q^*\right)\frac{q_p\Lambda}{\alpha_p + \beta_p Q^* + \mu}$$

Additionally, $U_R^* = \frac{q_R\Lambda}{\alpha_R+\mu}$ and $E_R^* = q_R\frac{\Lambda}{\mu}\frac{\alpha_R}{\alpha_R+\mu}$.

With a simple calculation, from the result of $Q^* > 0$, we obtain the critical value of the parameter representing the critical level of influence:

$$(lE_R^* + (1-l)E_P^*)\left(\frac{r_0}{e} - 1\right) > \theta^c$$

2.3.3 Simulations

Figure 2.24 presents numerical simulations under various values of the proportion l, including when influence is originated entirely from poor educated ($l = 0$) and rich educated only ($l = 1$).

Figure 2.24 corresponds to the equilibria over time when $\mathcal{R}(l, \theta, E_P, E_R) > 1$. Under the different cases, the dashed lines represent different combinations of the extreme cases in straight lines. The influence reproductive number is now also a function of the parameter l, representing the importance of rich and $(1-l)$ of for the case of the poor. For $\mathcal{R}(l = 0, \theta = 74, E_P, E_R = 0) = 1.10676$, $\mathcal{R}(l = 1, \theta =$

$74, E_P = 0, E_R) = 1.10518$, $\mathcal{R}(l = 0.25, \theta = 74, E_P, E_R) = 1.10637$, $\mathcal{R}(l = 0.5, \theta = 74, E_P, E_R) = 1.10597$ and, $\mathcal{R}(l = 0.75, \theta = 74, E_P, E_R) = 1.10558$. Figure 2.24.b presents the environmental influence Q^* at equilibrium under these cases.

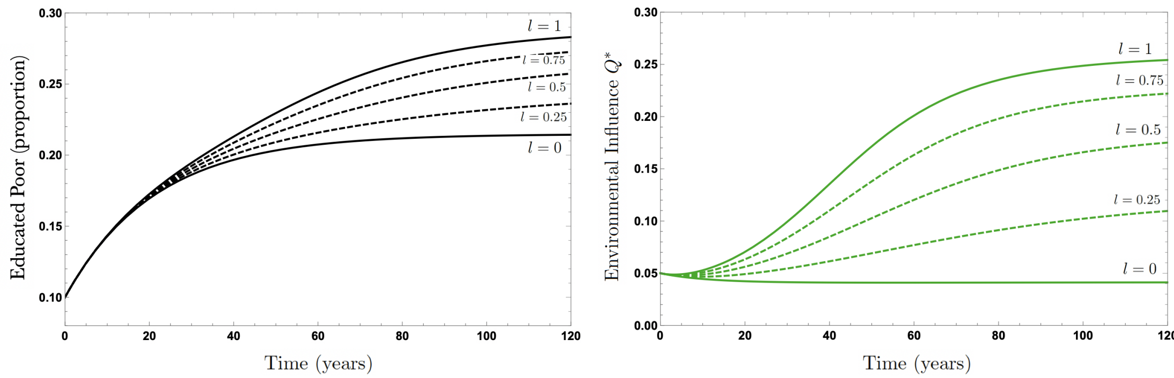


Figure 2.24: Environmental Influence when $\mathcal{R}(l, \theta, E_P, E_R) > 1$ and Proportion of Educated Poor with Different Proportions $l = 0, l = 0.25, l = 0.5, l = 0.75, l = 1$

Figure 2.25 present the equilibrium over time for the educated poor and the environmental influence when $\mathcal{R} < 1$, without no additional condition. The educated poor reach a positive equilibrium which behaves more slowly over time, due in fact that there is a higher time delay given by θ. The environmental influence tends to zero after a long period of time and $\alpha < \beta$. For different values of parameters, we have that $\mathcal{R}(l = 0, \theta = 100, E_P, E_R = 0) = 0.97783$, $\mathcal{R}(l = 1, \theta = 100, E_P = 0, E_R) = 0.976678$, $\mathcal{R}(l = 0.25, \theta = 100, E_P, E_R) = 0.977545$, $\mathcal{R}(l = 0.5, \theta = 100, E_P, E_R) = 0.977258$ and, $\mathcal{R}(l = 0.75, \theta = 100, E_P, E_R) = 0.976969$.

Explicit results for equilibria and stability become more complex in Model 3 but numerical simulations help exploration of different combinations of educated rich and poor that produce environmental influence. The simulations suggest that any changes in the importance of the group, given by the proportional weight l, may be treated as a perturbation from the base model, so the results and intuition hold also for this

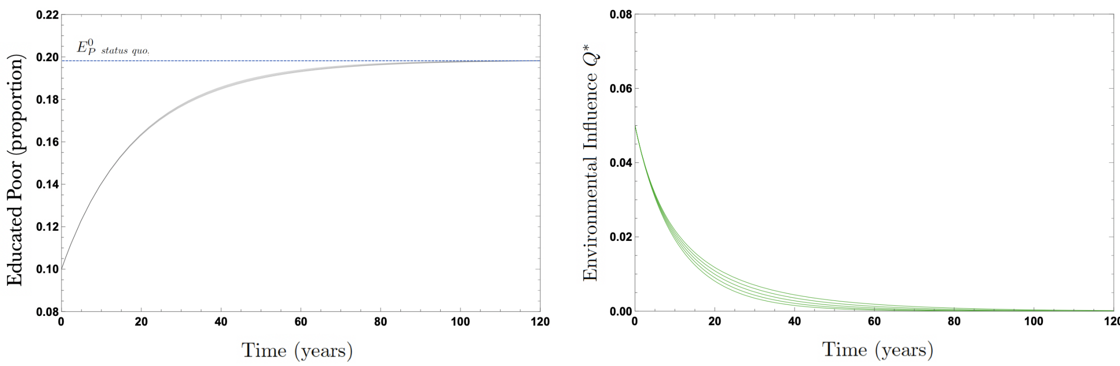


Figure 2.25: Environmental Influence when $\mathcal{R}(l, \theta, E_P, E_R) < 1$ and Proportion of Educated Poor with Different Proportions $l = 0, l = 0.25, l = 0.5, l = 0.75, l = 1$

combined perspective. There is a larger effect realized from the poor educated over the uneducated poor potentially explained due to proximity of peers and maybe, class consciousness, a topic that requires deeper and further research.

This chapter explores a simple base model that considers poor and rich classes in combination with educational status to understand the impact that educational access and completion may have on altering observed mobility patterns. The role of the environment is modeled through the use of a modified version of the invasion/extinction model of Richard Levins, with a rate of growth determined by the density of successful individuals, that is, college educated. There is always a *status quo* equilibrium that is viable. When the environmental influence is activated, an influence equilibrium emerges and remains stable as long as there is a sufficiently strong level of influence. Peer influence, poor influencing the poor, exert a larger effect over the underprivileged groups (uneducated poor). Simulations show that peers generate an environmental influence that accelerates the achievement of a specific goal by 20 years in contrast to class influence. Results would remain equal if the influence of the class effect acts immediately.

2.4 Paremeters and Calibration

Parameters were calibrated using information of the United States. The majority of information was obtained from official sources such as the Census Bureau, Bureau of Labor Statistics, Federal Reserve, the National Center for Education Statistics (NCES), the Centers for Disease Control and Prevention (CDC); and also from international organizations such as, the World Bank, UNESCO, World Health Organization, etc.

The average educational time window is considered to be 30 years, a rough estimate of the time that a person is likely to use to pursue a college degree. From this estimate, the per capita rate of system exit is $\mu = \frac{1}{30} = 0.033$.

The recruitment rate Λ that maintains population constant is given by $\Lambda = \mu N = 33.34$, when we consider a community with $N = 1000$ individuals.

A crude approximation is to use mean income in the US and assume these are the figures of a community. According to an estimation using information from the US Census Bureau, 60 % of households in the US have income below the national average in 2018, so then, this percentage of population will be roughly considered as non-rich and 40% as rich. The proportion of individuals according to socio-economic class may take two groups of quantities. For the Rich-Poor community, $q_R = 0.1$ and $q_P = 0.9$ to capture that the large proportion of new comers are from the poor class. When we consider a middle class, the proportions are assumed to be $q_R = 0.1$, $q_M = 0.6$ and $q_P = 0.3$. Almost 50% of Americans are part of the middle class according to the Pew Research Center. This percentage has decreased considerably since the 1970, when it was approximately 61% [76]. Figure 2.27 presents a idea of the the share of adults living in households according to their income group in the United States.

Enrollment rate. The total college enrollment was approximately 2.9 million

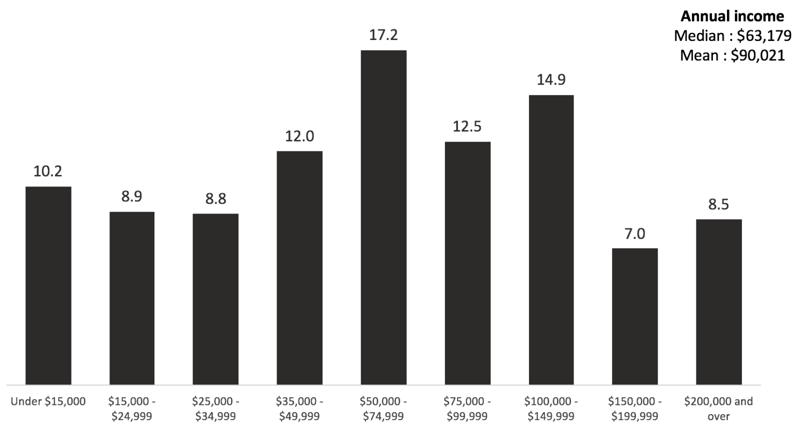


Figure 2.26: Distribution of Household Income the US, 2018. Source: United States Census Bureau

students in 2018, that is, approximately 8.7 per thousand individuals per year, according to the National Center for Education Statistics. We base the estimates of rates of progression for"regular" education, given by α_R, α_M, and α_P. Figure 1.5 presents the percentages of college attainment according to family income, categorized in quartiles. The average proportion of rich, taken as the 25% richest and the 75% highest income percentile, has an average proportion of approximately 56%. This proportion of people becoming educated is represented by $\frac{\alpha_R}{\alpha_R+\mu}$ and we can obtain that $\alpha_R \approx 0.044$. There exists approximately 4 times more graduated people in the non-poor segment with respect to poor, hence, we can assume that $\alpha_R \approx 4\alpha_P$, consequently, $\alpha_P = 0.011$. Finally, an average value for middle class gives $\alpha_M = 0.02$.

Effectiveness of influence β_p. The per capita rate of effectiveness of the influence may require elaborated estimation methods. However, we try to simplify this estimation with intuitive idea to capture the parameter's facilitation for mobility. We may set this value as a proportion of the largest rate of education (corresponding to the

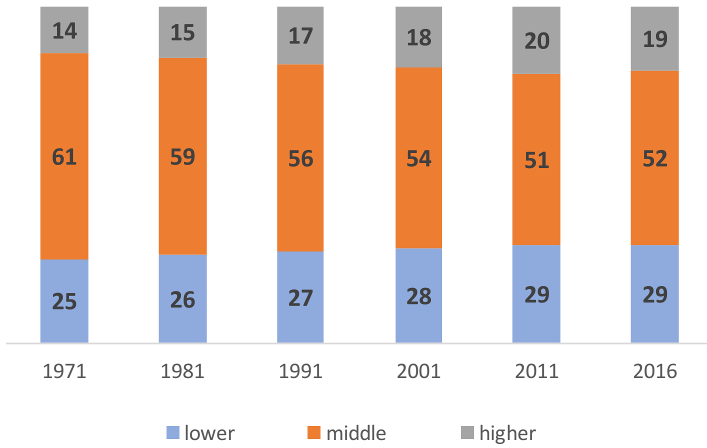


Figure 2.27: Share of Population by Income Classes. Source: Pew Research Center

rich) and establish different scenarios of sensitivity. Then, $\beta_p = p\alpha_R$, where $p \in [0, 1]$. We can approximate the proportion by having an idea of what influences going to college, other than the parental ability to send children to the school. We proposed a simple estimation using a study conducted by Phinney [1], where 713 university freshmen participated in a survey about the reasons to go to college. Low socio-economic status was identified with the conditions of parents with respect to unskilled occupations and lack of elementary school. Phinney constructed seven motivation categories explaining college attendance: Career/Personal, Humanitarian, Default Motivation, Expectation, Prove Worth, Encouragement and Help Family Motivation. Encouragement motivation is the third most important factor for college attendance for European American students, fourth for African American and Latinos and fifth for Asians Americans. Motivation from others to attend college is the closest idea connecting an environmental influence with education. Students from minority groups mentioned that a particular mentor encouraged them to attend and complete college. Encouragement was notably important for ethnic minority students who "are unlikely to receive the same amount of support for attending college as White middle-class students" [77]. For parameter β_p representing an enrollment rate-like sponsored by the environment, we scaled factors with respect to the rate of college enrollment referring to the career and personal motivation. β_p is then assumed to be a fraction of the education rate of the rich, and this fraction is represented by the factor Encouragement divided by Career which, is close to 0.5, then, $\beta_p = 0.9\alpha_R$. For an initial exercise we establish $\beta_P = 0.04$. The results of Phinney are summarized in table 2.2.

	Asian American	**African American**	**Latino**	**European American**
Help family	4.21	4.16	4.28	3.09
Career/personal	4.05	4.2	4.19	4.01
Humanitarian	3.77	4.07	3.97	3.94
Encouragement	3.67	3.99	3.9	3.68
Prove worth	3.6	3.55	3.79	2.6
Expectation	3.71	3.36	3.36	3.31
Default	2.42	2.29	1.94	2.02
Encouragement/Career	0.91	0.95	0.93	0.92

Table 2.2: Means for Reasons to Attend College by Ethnicity. Source: Phinney et al. [1]

We set per capita rates of growth $r_0 = 0.5$ and $e = 0.3$ for modeling purposes since the average time of influence $\frac{1}{e}$ is considered to be a little bit more than 3 years, satisfying the condition that $r_0 > e$.

par.	Description	Value	Units	Source
μ	per capita system exit	0.033	years	assumed
$\frac{1}{\mu}$	average educational-time window	30	$\frac{1}{years}$	assumed
α_R	per capita rate education for rich	0.044	$\frac{1}{years}$	[78]
α_M	per capita rate education for middle-class	0.020	$\frac{1}{years}$	[78]
α_P	per capita rate education for poor	0.011	$\frac{1}{years}$	[78]
Λ	recruitment rate	33.34	individuals	assumed
θ	influence impact delay	34	individuals	assumed
q_R	proportion of rich recruited	0.1	uniteless	[76]
q_P	proportion of middle class recruited	0.6	uniteless	[76]
q_P	proportion of poor recruited	0.3	uniteless	[76]
β_P	effectiveness of environmental influence	0.03	$\frac{1}{years}$	assume
r_0	intrinsic rate of growth of environ. influence	0.5	$\frac{1}{years}$	assumed
e	rate of loss of influence	0.3	$\frac{1}{years}$	assumed

Table 2.3: Variables and Parameters of the Combined Model.

One of the most difficult aspects to conduct research is the poor availability of information. In the United States, there are data available across a wide range of time. However, in other countries, such information is poorly recorded or simply, does not exist. I am using mostly data from the US to highlight the fact that in many countries there is inability to make assessments or test for potential changes in policy because they do not collect adequate information.

Chapter 3

MODELS INCLUDING MIDDLE CLASS

So far, we have explored the dynamics of mobility within a simplified model composed of two classes of origin of individuals, the rich and the poor. We have assumed that education may significantly improve mobility as measured by average income under assumptions in the average ($\hat{x}$) income satisfying the relationship $\widehat{U}_P < \widehat{E}_P \leq \widehat{U}_R << \widehat{E}_R$ or that $\widehat{U}_P < \widehat{U}_R \leq \widehat{E}_P << \widehat{E}_R$. We also have operated under the demographic assumptions supported by the recruitment per capita q_R and q_P, satisfying $0 < \frac{q_R}{q_P} = s << 1$, that is, it is assumed that the poor population is much greater than the rich, namely, $P(0) >> R(0) > 0$. We may also assume, for example, that $0 < \frac{E_R(0)}{U_R(0)} = l_R < 1$, with $l_R = 5, 6, 7, ...$; and $0 < \frac{E_P(0)}{U_P(0)} = l_P < 1$, with $l_P = 4, 5, ...$ Hence, if we assume, for example, that the African Americans are predominantly born "poor" and that White Americans are predominantly born "rich", then an appropriate choice for the value of l could be used to capture population size differences.

The analysis of the models of Chapter 2 highlights the impact of the nonlinear influence function $Q(\cdot)$, modeling environmental influence on the education success. Special scenarios are used to explore the impact of such influence function on mobility, defined as the rate of progression $U \to E$ under a weighted influence function, $Q(\cdot)$. The models in Chapter 2 always support a *status quo* equilibrium, that is, the state of the system in the absence of environmental influence ($Q^* = 0$). When influence comes into play ($Q^* \neq 0$), an influence equilibrium emerges that remains stable as long as the influence effect is fast enough, that is, if $\mathcal{R}(E_P, E_R, \theta) > 1$. We also found that there is the possibility of negative influence ($Q^* < 0$) that it may occur under the assumption $\frac{1}{2} < \mathcal{R}(E_P, E_R, \theta) < 1$ and $\alpha_P > \beta_P$ with $E_P^0 > E_P^* > 0$. When influence is negative, E_P^* is unstable, and any perturbation from E_P^* leads back to the *status quo* equilibrium.

In this chapter, we increase the level of heterogeneity by introducing a middle class (M) in the model. The role of this middle class is used to assess analytically and numerically the influence of the environment, as defined by $Q(\cdot)$. The middle class represents the largest proportion of the population in the United States with an estimate of 52% of individuals belonging to this group (Pew Center for Research, [76]). The association of a stable and large middle class with economic growth and working democracy has been at the center of economic debates on the evaluation of prosperity. In general, the existence and size of a middle class provide a solid measure of the well being of a community.

The commonly associated rewards linked to the middle class include stable jobs, access to educational opportunities, and the comforts of decent housing. Hence, increasing the size of the middle class is a priority for most societies. Economic historians have highlighted the relevance of a middle class in development: Landes talked about of an ideal model society as that supporting a "relatively large middle class." His prototypical example was based on the observation that industrialization emerged in England first and that its success was driven by the existence of a relatively large middle class [79] . The middle class has been recognized as a source of increased consumption and investment. The middle class is a generator of GDP growth and entrepreneurial activities [80, 81]. Lipset defended a modernization process brought by the coupling of a liberal middle class and a democratic political system [82]. Huntington contended that the foundation for a democratic society is linked to the presence of continuous economic development and a growing middle class [83]. The theory of stratification developed by Max Weber in 1905 [84] highlighted a model of society with a middle class that reflected wealth, power and prestige. The middle class was viewed as the source of entrepreneurs, creators of wealth, productivity, accumulation of capital and, a reservoir of a body of consumers that kept economic activity vibrant [85].

The fact remains that there is no accepted a unified criterion used to define a middle class. An attempt to identify income ranges to define a middle class have been introduced [86]. Agencies use surveys of households with daily per capita expenditures valued at a purchasing power parity of between \$2 to \$10; an approach followed by a group of developing countries [87]. This *ad hoc* definition was introduced as a way of recognizing individuals living "much better" than the poor. In the case of the United States, the Pew Research Center considers a family to be middle class whenever its income ranges from 67% to 200 % of the national median income, which was \$61,937 in 2018 [88].

Easterly [80] defined the middle class as that group of individuals within the 20th and 80th percentiles of the consumption distribution. His classic economic work on "the existence of a Middle Class Consensus" was used to explain differences in development across nations. He observed that countries with a relatively large middle class tend to grow faster and exhibit "higher levels of public goods." Middle class size matters when attempts are made to identify differences in college enrollment or health outcomes such as life expectancy, infant mortality, low birth weight, or DPT and polio immunization [80].

3.1 Model 4: Rich-Middle Class-Poor and influence from the Rich Educated

This chapter looks at the impact of the educated rich class, that is, E_R, on increases on the classes of uneducated (born) poor U_P and uneducated (born) middle class, U_M. It is assumed that the following conditions are met $P \equiv U_P + E_P$;

$M \equiv U_M + E_M$ and $R \equiv U_R + E_R$ and $N \equiv P + M + R$. It is further assumed that $R(0) << P(0) < M(0)$; $U_P(0) >> E_P(0)$; $U_M(0) >> E_M(0)$, $E_R(0) >> U_R(0)$.

When it comes to average income, it is assumed that $\hat{P}(0) < \hat{M}(0) << \hat{R}(0)$ ($\hat{x}$ denoting the average of x). Further, it is assumed that $\hat{E}_P(0) \geq \hat{U}_M(0)$, $\hat{U}_P(0) < \hat{U}_M(0) << \hat{U}_R(0)$, $\hat{E}_M(0) \geq \hat{E}_P(0) \geq \hat{U}_M(0)$ and that $\hat{E}_R(0) \geq \hat{E}_M(0)$, and that either $\hat{U}_R(0) \geq \hat{E}_M(0) \geq \hat{E}_P(0)$ or $\hat{E}_M(0) \geq \hat{U}_R(0) \geq \hat{E}_P(0)$.

In addition, we have that $q_R + q_M + q_P = 1$ with $q_M > q_P >> q_R > 0$. For example, we may have that $q_R = 0.1, q_M = 0.6, q_P = 0.3$. We assume that $P(0) = q_P \frac{\Lambda}{\mu}, R(0) = q_R \frac{\Lambda}{\mu}, M(0) = q_M \frac{\Lambda}{\mu}$. In addition, the rates of "natural" per capita progression (access) of the successful educated are denoted by α_R, α_M and α_P, with $\alpha_R > \alpha_M >> \alpha_P$. The environmental influence function $Q(\cdot)$ is now assumed to operate on U_P and U_M with "effectiveness" rates denoted by β_P and β_M, respectively, with $\beta_M \geq \beta_P$. The assumptions and definitions lead to the following nonlinear model when $Q \equiv Q(E_R)$:

$$\frac{dU_R}{dt} = q_R\Lambda - \alpha_R U_R - \mu U_R, \tag{3.1}$$
$$\frac{dU_M}{dt} = q_M\Lambda - \alpha_M U_M - \beta_M U_M Q - \mu U_M, \tag{3.2}$$
$$\frac{dU_P}{dt} = q_P\Lambda - \alpha_P U_P - \beta_P U_P Q - \mu U_P, \tag{3.3}$$
$$\frac{dE_R}{dt} = \alpha_R U_R - \mu E_R, \tag{3.4}$$
$$\frac{dE_M}{dt} = \alpha_M U_M + \beta_M U_M Q - \mu E_M, \tag{3.5}$$
$$\frac{dE_P}{dt} = \alpha_P U_P + \beta_P U_P Q - \mu E_P, \tag{3.6}$$
$$\frac{dQ}{dt} = r_0 \frac{E_R}{\theta_R + E_R} Q(1 - Q) - e_R Q \tag{3.7}$$

The population is asymptotically constant, that is, $N \to \frac{\Lambda}{\mu}$, as $t \to \infty$ and so we set $N(0) \equiv \frac{\Lambda}{\mu}$. The transitions into different groups are depicted in Figure 3.1:

System is solved simultaneously in order to obtain formulae for the equilibria. There exists two equilibria solutions of the system 3.10- 3.16. The first equilibrium

variable	Description
U_R	number of rich uneducated individuals
U_M	number of middle class uneducated individuals
U_P	number of poor uneducated individuals
E_R	number of rich individuals with college degree
E_R	number of middle class individuals with college degree
E_P	number of poor individuals with college degree
Q	influence probability function

parameter	Description
q_R	proportion of rich recruited
q_M	proportion of middle class recruited
q_P	proportion of poor recruited
α_R	per capita rate to become educated for the rich
α_M	per capita rate to become educated for the middle class
α_P	per capita rate to become educated for the poor
β_P	effectiveness of environmental transmission rate for the poor
β_M	effectiveness of environmental transmission rate for the middle class
r_0	intrinsic rate of growth of Q
θ_R	influence impact delay
e_R	rate of loss of influence
μ	per capita system exit, $\frac{1}{\mu}$: average educational-time window
Λ	constant recruitment rate

Table 3.1: Variables and Parameters of the Model Rich-Middle-Poor

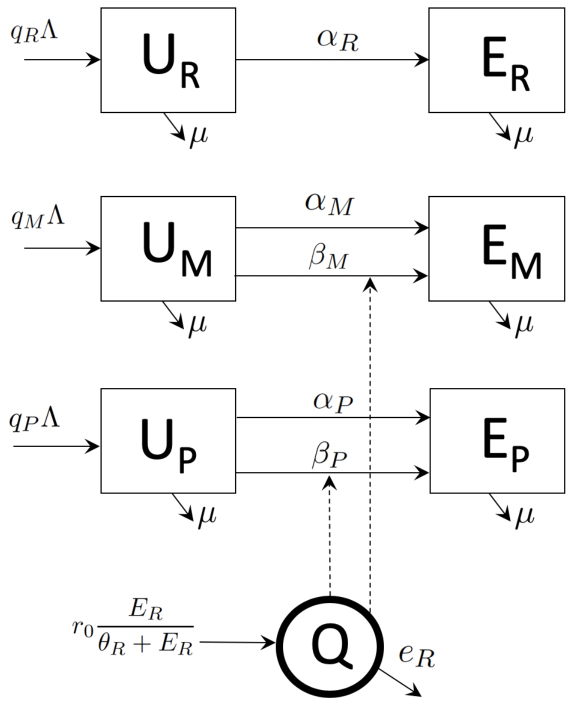


Figure 3.1: Flow Diagram of Compartmental Model Rich-Middle-Poor with $Q \equiv Q(E_R)$

denotes the *status quo*, that is, the situation when there is negligible environmental influence, that is when $Q^* = 0$. *Status quo* is given by:

$$
\begin{aligned}
U_R^0 &= \frac{q_R\Lambda}{\alpha_R + \mu}, \\
U_M^0 &= \frac{q_M\Lambda}{\alpha_M + \mu}, \\
U_P^0 &= \frac{q_P\Lambda}{\alpha_P + \mu}, \\
E_R^0 &= q_R\frac{\Lambda}{\mu}\frac{\alpha_R}{\alpha_R + \mu}, \\
E_M^0 &= q_M\frac{\Lambda}{\mu}\frac{\alpha_M}{\alpha_M + \mu}, \\
E_P^0 &= q_P\frac{\Lambda}{\mu}\frac{\alpha_P}{\alpha_P + \mu}, \\
Q^0 &= 0.
\end{aligned}
$$

The influence equilibria, when $Q^* \neq 0$, is given by:

$$\begin{aligned}
U_R^* &= \frac{q_R \Lambda}{\alpha_R + \mu}, \\
U_M^* &= \frac{q_M \Lambda}{\alpha_M + \beta_M Q^* + \mu}, \\
U_P^* &= \frac{q_P \Lambda}{\alpha_p + \beta_p Q^* + \mu}, \\
E_R^* &= q_R \frac{\Lambda}{\mu} \frac{\alpha_R}{\alpha_R + \mu}, \\
E_M^* &= q_M \frac{\Lambda}{\mu} \frac{\alpha_M + \beta_M Q^*}{\alpha_M + \beta_M Q^* + \mu}, \\
E_P^* &= q_P \frac{\Lambda}{\mu} \frac{\alpha_P + \beta_P Q^*}{\alpha_P + \beta_P Q^* + \mu}, \\
Q^* &= 1 - \frac{e_R}{r_0} \frac{\theta_R + E_R^*}{E_R^*}.
\end{aligned}$$

We observe that $Q^* > 0$, as long as $\frac{r_0}{e_R} \frac{E_R^*}{\theta_R + E_R^*} > 1$, so we always have that $E_R^* > 0$. The addition of the middle class does not affect the analysis but it can have a substantial impact on the class distributions of the middle class and poor. We let $\mathcal{R}(\theta_R, E_R^*) \equiv \frac{r_0}{e_R} \frac{E_R^*}{\theta_R + E_R^*}$ denote, as before, the influence reproductive number and restrict $Q \in [-1, 1]$, with Q denoting the function that models environmental influence. Under positive influence $Q^* > 0$, $\mathcal{R}(\theta_R, E_R^*) > 1$, and the influence equilibrium is " positive" and stable.

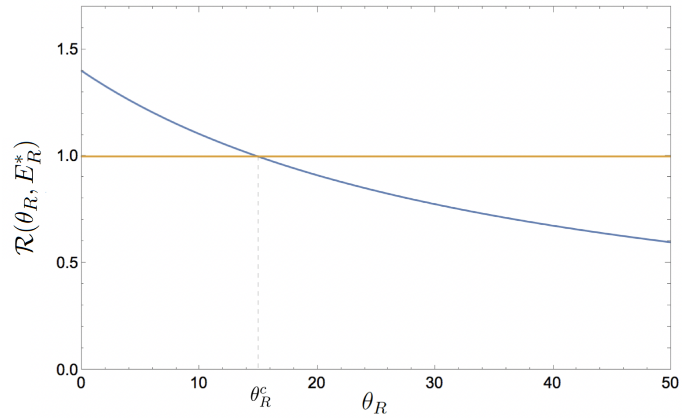
We may include a condition that restricts the influence to be negative. We have, for example, that $Q^* < 0$ when $\frac{1}{2} < \mathcal{R}(\theta_R, E_R^*) < 1$, and as long as, $\alpha_P > \beta_P$ and $\alpha_M > \beta_M$. In this case, we restrict $Q \in [-1, 0]$. Under these conditions, E_P^* is unstable and *status quo* is larger than influence equilibrium $(0 < E_P^* < E_P^0)$.

There exists a critical influence level θ_R^c, analogous to that of Chapter 2 and Figure 3.2 presents the relationship between $\mathcal{R}(\theta_R, E_R^*)$, and Q^* as a function of θ_R. If $\theta_R < \theta_R^c$, environmental influence acts fast enough on the group of uneducated poor and middle classes, a condition that is equivalent to $\mathcal{R}(\theta_R, E_R^*) > 1$. On the other hand, if $\theta_R > \theta_R^c$, influence takes longer to operate, making $\mathcal{R}(\theta_R, E_R^*) < 1$ and $Q^* < 0$.

Figure 3.3 connects the equilibrium for E_P^* and E_M^* with respect to the influence delay θ_R. Whenever $\theta_R^c > \theta_R$, the influence equilibrium is greater than the *status quo* equilibrium for both poor and middle class educated. When influence impact takes longer, that is, $\theta_R^c < \theta_R$, the influence equilibrium is lower than *status quo*, $E_P^* < E_P^0$ and $E_M^* < E_M^0$. Analogously to Model 1, the influence may have a negative value, when $\frac{1}{2} < \mathcal{R}(\theta_R, E_R^*) < 1$ and $\alpha_P > \beta_P$ for $E_P^* > 0$ and, $\alpha_M > \beta_M$ for $E_M^* > 0$.

The *status quo* intersects the influence equilibrium at the critical level, defined at $\theta_R^c \equiv E_R^* \left(\frac{r_0}{e_R} - 1 \right)$, with always $\frac{r_0}{e_R} > 1$. Figure 3.3 shows this intersection at the critical value from which the corresponding analysis of stability is henceforth derived.

(a)



(b)

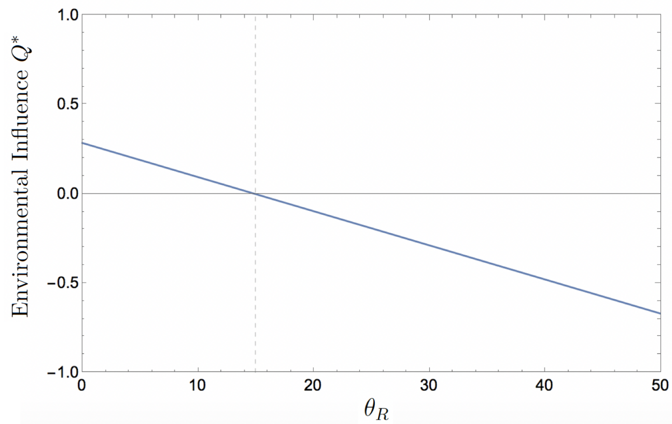


Figure 3.2: Influence Reproductive Number $\mathcal{R}(\theta_R, E_R^*)$ and Influence Delay Coupled with the Environment Q.

We assume that $\beta_M > \beta_P$, implying that middle class individuals may possess or have more tools that help to "absorb" the influence. The assumption may be justified on the result of differences in average income, measured by $\hat{U}_M > \hat{U}_P$. Affordability may provide a plausible explanation for such a difference. When β_M increases, the educated middle class increases as well. The curve E_M^* in Figure 3.3 shifts up on the vertical axis, denoting increases in the proportion of educated in the middle class and pivots around the intersection point between the *status quo* $E_M^{)}$ and E_P^* at the critical level θ_R^c.

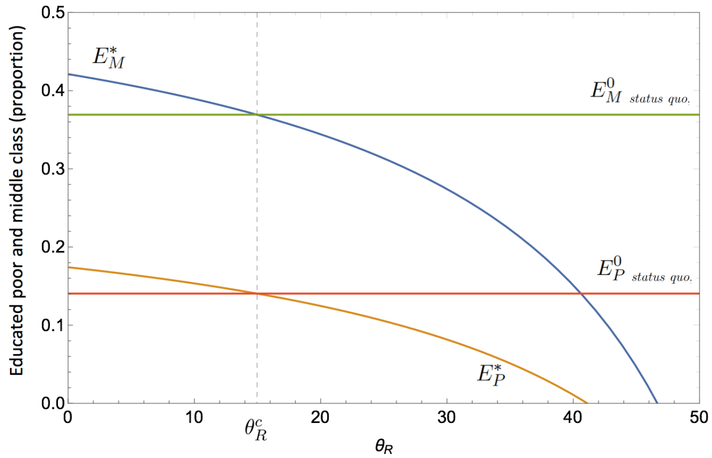


Figure 3.3: Educated Poor and Educated Middle Class as a Function of the Influence Delay.

Figure 3.4 summarizes the equilibrium paths for E_M^* and E_P^* and their corresponding stability spaces with respect to the influence reproductive number $\mathcal{R}(\theta_R, E_R^*)$. When $\mathcal{R}(\theta_R, E_R^*) > 1$, $E_M^* > 0$ and $E_P^* > 0$ are stable and larger than their corresponding *status quo* equilibria. This influence equilibria is a result of the activation of the environment, which begins to exert a positive influence $(Q *^{>} 0)$ when $\mathcal{R}(\theta_R, E_R^*) > 1$.

When $\mathcal{R}(\theta_R, E_R^*) < 1$, influence equilibria for both poor and middle class fall below their respective *status quo* equilibria, meaning $E_M^* < E_M^0$ and $E_P^* < E_P^0$. *Status quo* become stable and influence equilibria become unstable. Any small variation from equilibria makes the trajectories of the system approach the *status quo*, that is, equilibria go from $E_P^* \to E_P^0$ and $E_M^* \to E_M^0$. Under weak levels of influence, only the *status quo* is maintained with resulting lower proportions of educated poor and middle class, possibly a sign that mobility has stagnated.

E_P^* and E_M^* can be explicitly expressed as functions of $\mathcal{R}(\theta_R, E_R^*)$ as follows:

$$E_P^* = q_P \frac{\Lambda}{\mu} \left[\frac{\alpha_P \mathcal{R}(\theta_R, E_R^*) + \beta_P(\mathcal{R}(\theta_R, E_R^*) - 1)}{(\alpha_P + \beta_P + \mu)\mathcal{R}(\theta_R, E_R^*) - \beta_P} \right] \tag{3.8}$$

$$E_M^* = q_M \frac{\Lambda}{\mu} \left[\frac{\alpha_M \mathcal{R}(\theta_R, E_R^*) + \beta_M(\mathcal{R}(\theta_R, E_R^*) - 1)}{(\alpha_M + \beta_M + \mu)\mathcal{R}(\theta_R, E_R^*) - \beta_M} \right] \tag{3.9}$$

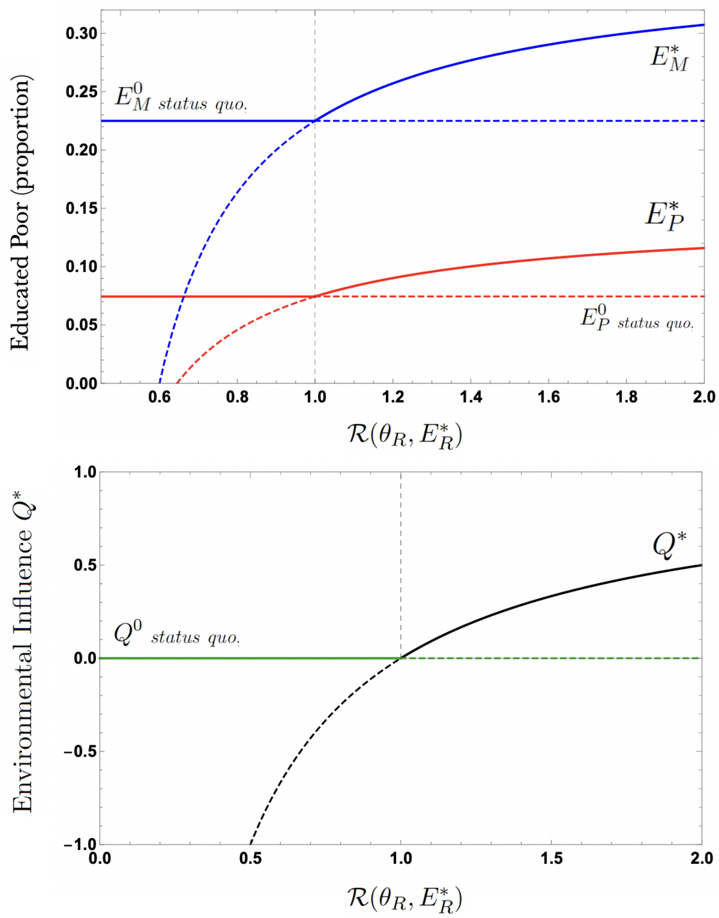


Figure 3.4: Educated Poor and Educated Middle Class as a Function of $\mathcal{R}(\theta_R, E_R^*)$.

3.1.1 Simulations

Figure 3.5 presents simulations with the proportion of uneducated and educated individuals according to their socio-economic status for two scenarios: rich-middle-poor (R-M-P) and rich-poor (R-P) models. First, in the R-P world, poor educated E_P^* increased 2.6 times from 10% to 26% of the population. Rich educated decreased from 10% to 6%. Adding heterogeneity through the middle class, poor educated increased 1.3 times, from 10% to 13%. Educated middle class decreased from 30% to 18% and the rich educated went from 5% to 6%. Uneducated individuals, in the R-P model go from 70% to 64% in the case of uneducated poor and from 10% to 5% for uneducated rich. In the R-M-P model, the uneducated poor increases from 20% to 32%, middle class uneducated from 30% to 27% and rich uneducated drop from 5% to 4%.

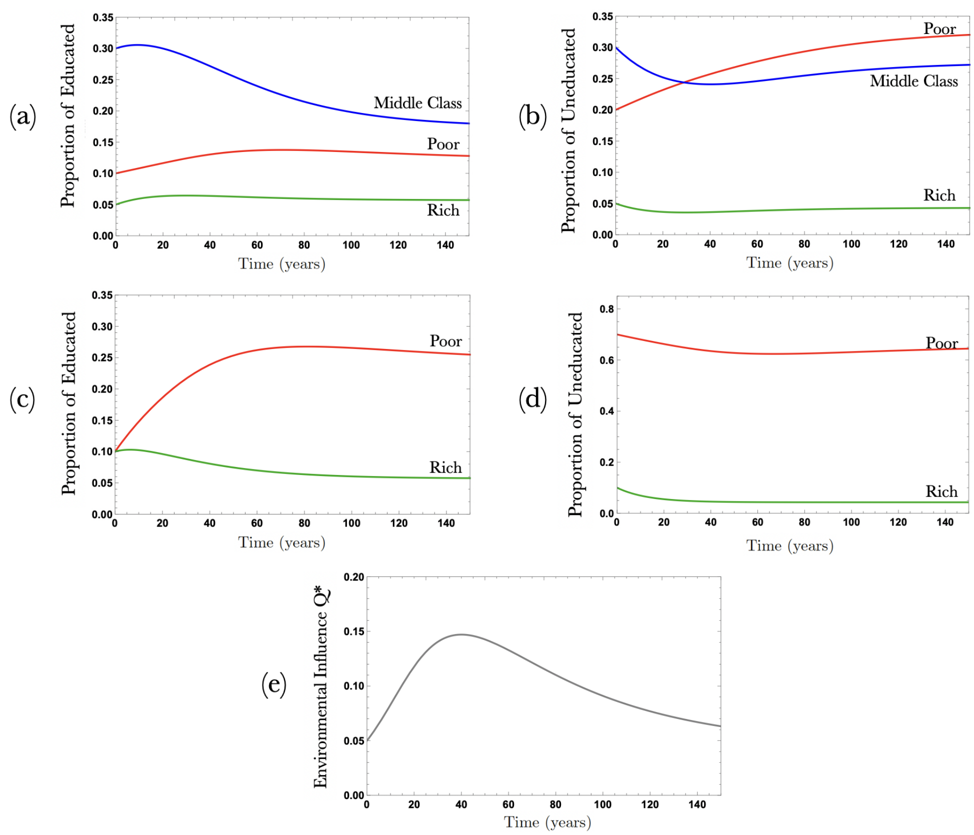


Figure 3.5: Proportion of Educated and Uneducated in R-M-P Model (a and b) and in R-P Model (c and d) and Environmental Influence (e) when $Q \equiv Q(E_R^*)$.

Figure 3.6 shows the estimates of the terms $\beta_P U_P Q$ of model R-P and $\beta_P U_P Q + \beta_M U_M Q$ of Model R-M-P. These terms correspond to the new educated individuals who have obtained college education through the influence generated by the environment. The total number of new educated may be interpreted as the number of individuals experiencing mobility over time due to the particular influence of the social environment generated by the educated rich ($Q \equiv Q(E_R)$). The terms $\beta_P U_P Q|_{R-P} \neq \beta_P U_P Q|_{R-M-P}$, since the proportion U_P differ between models R-P and R-M-P, as it is shown in Figure 3.5. The evolution of this terms is summarized in Figure 3.6. Adding heterogeneity through middle class results in higher degree of mobility, as it is seen by the larger number of "mobilized" individuals due to the environmental influence in the R-M-P model in contrast to the additional educated in the R-P model.

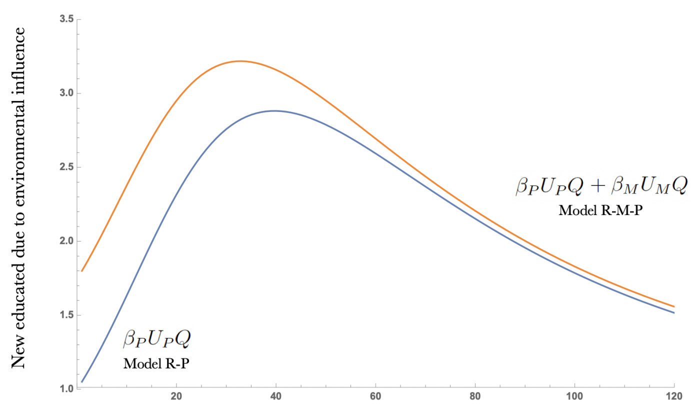


Figure 3.6: New Educated Individuals from the Environmental Influence in R-P and R-M-P Models.

New educated from the born poor and educated from those born in the middle class represent the contribution of the environment towards the generation of mobility. The differences in the final proportion of E_P and E_M are determined directly by the effectiveness of the environmental influence, given by β_P and β_M. The larger the ratio $\frac{\beta_M}{\beta_P}$, the higher the difference between E_M and E_P. Figure 3.7 is an attempt to capture this differences of effectiveness of influence and their consequence over the equilibria. With different combinations of β_P and β_M, this figure estimates the ratio $\frac{E_M}{E_P}$. The upper left corner shows values that denote relatively close effectiveness, that is $\beta_P \approx \beta_M$ and, hence, the ratio $\frac{E_M}{E_P}$. reaches the lowest possible level. However, the bottom right corner shows combinations of effectiveness parameters where there are large differences, that is, $\beta_P << \beta_M$ where, the highest ratio $\frac{E_M}{E_P}$ is achieved, that is, if $E*_P = 10\%$, hence the educated middle class E_M would represent 33% of the population.

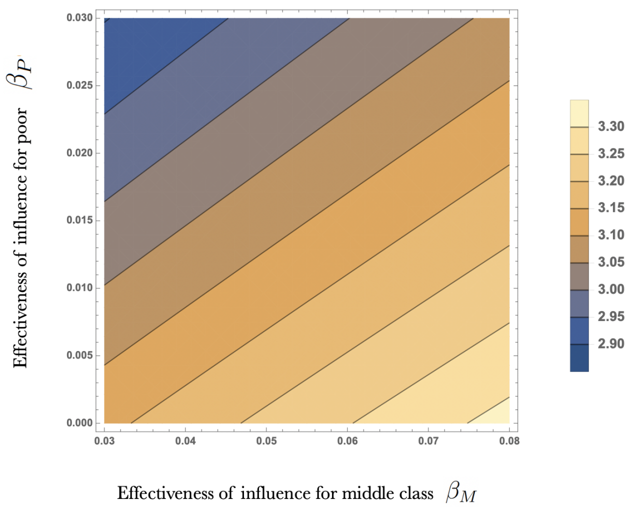


Figure 3.7: Ratio of College Educated Middle Class and Poor at Equilibrium $\frac{E^*_M}{E^*_P}$ within Combinations of Effectiveness of Influence for Middle Class β_M and Poor β_P

Here, it is important to re-state that the term poor, middle class and rich are used to characterize the class of origin. We, in fact, expect that born poor individuals that have moved to the educate (born) poor class would have in fact become members, from an economic perspective, of the middle class. These assumptions are captured on the dynamics of average income of each class, as given by $\hat{E}_P(t)$ and $\hat{E}_M(t)$. Indeed, we could have allowed for transitions of the $E_P \rightarrow E_M$ but that complicates the model and the original intent to use as simple model as possible to capture some of the effects of heterogeneity. In order to provide a general comparison between the two models, we estimate a weighted average income for each community R-P and R-M-P, using the corresponding proportions with respect to the total population. Average income for every class is constant over time.

Figure 3.8.a shows the average income of the two economies: one comprised of only R-P and the second that includes more heterogeneity with R-M-P. Adding more heterogeneity may increase the overall well-being of the community, as measure by the average income only. Note how both cases present a decline in average incomes, in part, due to the decreasing proportion of educated born middle class (12 percentage points) and also the considerable increase in uneducated poor by also 12 percentage points, driving weighted average income to decrease. Figure 3.8.b shows the ratio of average incomes between these two economies to recognize levels of inequality over time. Higher the level, more income inequality is produce in between these two communities. It seemed that efforts to close the gap existed during the first 20 years but after this period, disparity between average income increased again.

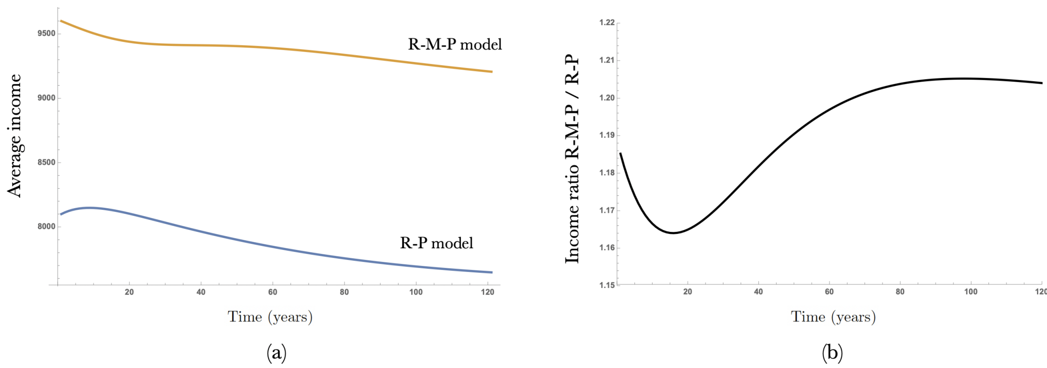


Figure 3.8: Average Income of the Rich-Poor and the Rich-Middle-Poor Communities (a). Total Average Income Ratio between the Rich-Middle-Moor and the Rich-Poor Communities.

Finally, Figure 3.9 presents the weighted average incomes for the two economies, R-P and R-M-P, under different strength of influence given by parameter θ_R that represents the influence delay. As the influence is more instantaneous ($\theta_R = 0$), Figure3.9.a shows that average income seems to maintain higher levels. However, as influence is delayed ($\theta_R = 68$), average income tends to decrease over time as it is shown in Figures 3.9b-c. Also, income disparity seems to be higher when influence acts faster, as it is show in Figure 3.9.f.

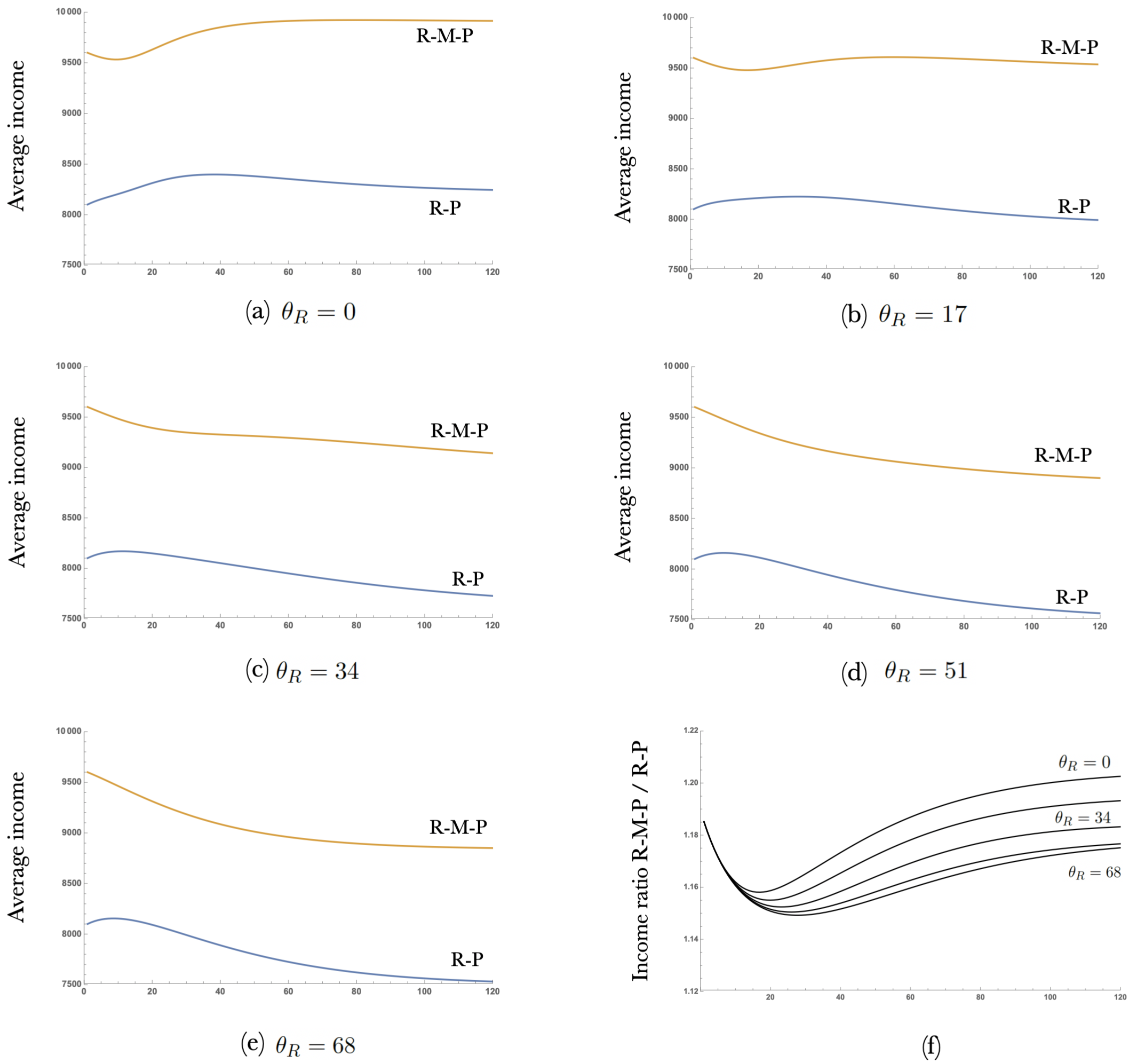


Figure 3.9: Average Income of the Rich-Poor and the Rich-Middle-Poor Communities (a). Total Average Income Ratio between the Rich-Middle-Poor and the Rich-Poor Communities.

3.2 Model 5: Rich-Middle Class-Poor and Influence from the Poor Educated

The world comprised of Rich-Middle-Poor individuals is now assumed to be influenced by an environment generated by changes in the poor educated, that is, $Q \equiv Q(E_P)$. The influence is exerted over both middle class and poor uneducated.

The system is represented by the following nonlinear model:

$$\frac{dU_R}{dt} = q_R\Lambda - \alpha_R U_R - \mu U_R, \tag{3.10}$$
$$\frac{dU_M}{dt} = q_M\Lambda - \alpha_M U_M - \beta_M U_M Q - \mu U_M, \tag{3.11}$$
$$\frac{dU_P}{dt} = q_P\Lambda - \alpha_P U_P - \beta_P U_P Q - \mu U_P, \tag{3.12}$$
$$\frac{dE_R}{dt} = \alpha_R U_R - \mu E_R, \tag{3.13}$$
$$\frac{dE_M}{dt} = \alpha_M U_M + \beta_M U_M Q - \mu E_M, \tag{3.14}$$
$$\frac{dE_P}{dt} = \alpha_P U_P + \beta_P U_P Q - \mu E_P, \tag{3.15}$$
$$\frac{dQ}{dt} = r_0 \frac{E_P}{\theta_P + E_P} Q(1 - Q) - e_P Q \tag{3.16}$$

with, $q_R + q_M + q_P = 1$ with $q_M > q_P >> q_R > 0$. Further assumptions are $P(0) = q_P\frac{\Lambda}{\mu}, M(0) = q_M\frac{\Lambda}{\mu}$, and $R(0) = q_R\frac{\Lambda}{\mu}$. Rates of "natural" per capita progression of education α_R, α_M and α_P, with $\alpha_R > \alpha_M >> \alpha_P$, with "effectiveness" of influence rates β_P and β_M, with $\beta_M \geq \beta_P$. Figure 3.10 is a flow diagram representing the population compartments and rates of progression:

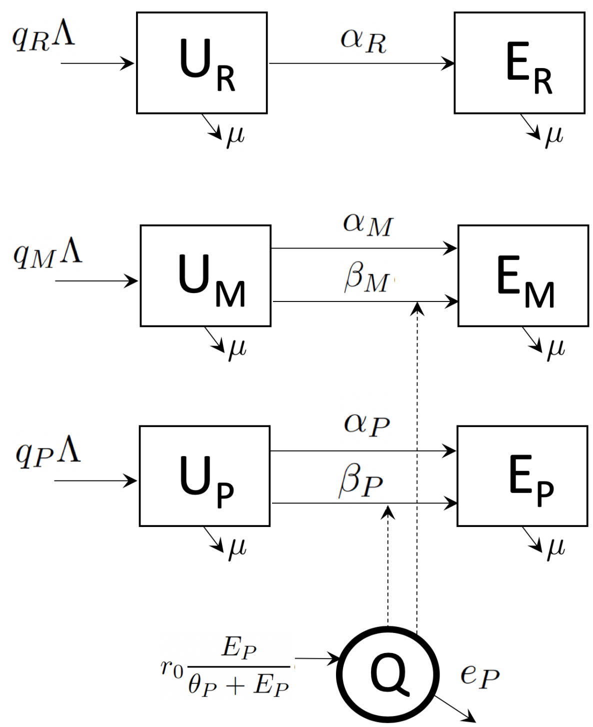


Figure 3.10: Flow Diagram of Compartmental Model 5

This system of equations sustains three equilibrium. The *status quo* equilibrium is always supported by the system when $Q^* = 0$, and it is given by:

$$
\begin{aligned}
U_R^0 &= \frac{q_R\Lambda}{\alpha_R + \mu},\\
U_M^0 &= \frac{q_M\Lambda}{\alpha_M + \mu},\\
U_P^0 &= \frac{q_P\Lambda}{\alpha_P + \mu},\\
E_R^0 &= q_R\frac{\Lambda}{\mu}\frac{\alpha_R}{\alpha_R + \mu},\\
E_M^0 &= q_M\frac{\Lambda}{\mu}\frac{\alpha_M}{\alpha_M + \mu},\\
E_P^0 &= q_P\frac{\Lambda}{\mu}\frac{\alpha_P}{\alpha_P + \mu},\\
Q^0 &= 0.
\end{aligned}
$$

When $Q^* \neq 0$, they system can be solved simultaneously to obtain the influence equilibria:

$$\begin{aligned}
U_R^0 &= \frac{q_R \Lambda}{\alpha_R + \mu}, \\
U_M^* &= \frac{q_M \Lambda}{\alpha_M + \beta_M Q^* + \mu}, \\
U_P^* &= \frac{q_p \Lambda}{\alpha_p + \beta_p Q^* + \mu}, \\
E_R^0 &= q_R \frac{\Lambda}{\mu} \frac{\alpha_R}{\alpha_R + \mu}, \\
E_M^* &= \frac{1}{\mu} \left(\alpha_M + \beta_M Q^*\right) \frac{q_M \Lambda}{\alpha_M + \beta_M Q^* + \mu}, \\
E_P^* &= \frac{1}{\mu} \left(\alpha_P + \beta_P Q^*\right) \frac{q_p \Lambda}{\alpha_p + \beta_p Q^* + \mu}, \\
Q^* &= 1 - \frac{e_P}{r_0} \frac{\theta_P + E_P^*}{E_P^*}
\end{aligned}$$

When expression E_P^* is evaluated at the the environment equilibrium $Q^* = 1 - \frac{e_P}{r_0} \frac{\theta_P + E_P^*}{E_P^*}$, the result is a quadratic equation for E_P^* (Expression 3.17, equal to the expression 2.18, obtained in Model 2. Additionally, when the resulting equilibrium E_M^* is also evaluated at $Q^* \neq 0$, the number of educated middle class can be expressed as a function of the educated poor: (Expression 3.18

$$E_P^* = \frac{1}{\mu} \left[\alpha_P + \beta_P \left(1 - \frac{e}{r_0} \frac{\theta_P + E_P^*}{E_P^*}\right)\right] \frac{q_p \Lambda}{\alpha_p + \beta_p \left(1 - \frac{e}{r_0} \frac{\theta_P + E_P^*}{E_P^*}\right) + \mu}, \tag{3.17}$$

$$E_M^* = \frac{1}{\mu} \left[\alpha_M + \beta_M \left(1 - \frac{e_P}{r_0} \frac{\theta_P + E_P^*}{E_P^*}\right)\right] \frac{q_M \Lambda}{\alpha_M + \beta_M \left(1 - \frac{e_P}{r_0} \frac{\theta_P + E_P^*}{E_P^*}\right) + \mu}, \tag{3.18}$$

As in Model 2, there is a positive influence $Q^* > 0$ when $\frac{r_0}{e_P} \frac{E_P}{\theta_P + E_P} > 1$ from which, we define $\mathcal{R}(\theta_P, E_P) \equiv \frac{r_0}{e_P} \frac{E_P}{\theta_P + E_P}$. This is equivalent to the condition $\theta_P^c \equiv E_P \left(\frac{r_0}{e_P} - 1\right)$. If influence acts strong enough, then $\mathcal{R}(\theta_P, E_P) > 1$ implying $Q^* > 0$. When influence works weakly, then $\mathcal{R}(\theta_P, E_P) < 1$ implying $Q^* < 0$.

Figure 3.11 depicts the bifurcation diagrams with equilibrium for educated poor under environment Q^*. The *status quo* equilibrium is stable whenever $0 < \mathcal{R}(\theta_P, E_P^*) < 1$ and influence equilibrium $E_{P,1}^*$ is stable for $\mathcal{R}(\theta_P, E_P^*) > 1$. When $0 < \mathcal{R}(\theta_P, E_P^*) < 1$, negative influence may occur. In fact, when $\frac{1}{2} < \mathcal{R}(\theta_P, E_P^*) < 1$, we may have $Q \in (-1, 0)$. Equilibrium $E_{P,1}^*$ or $E_{P,2}^*$ are unstable whenever the *status quo* is larger ($E_P^0 > E_{P,1}^*$ or $E_P^0 > E_{P,2}^*$). When $\mathcal{R}(\theta_P, E_P^*) < 1$, any minimum perturbation from the equilibrium $E_{P,1}^*$ or $E_{P,2}^*$ makes the system approach the *status quo* equilibrium.

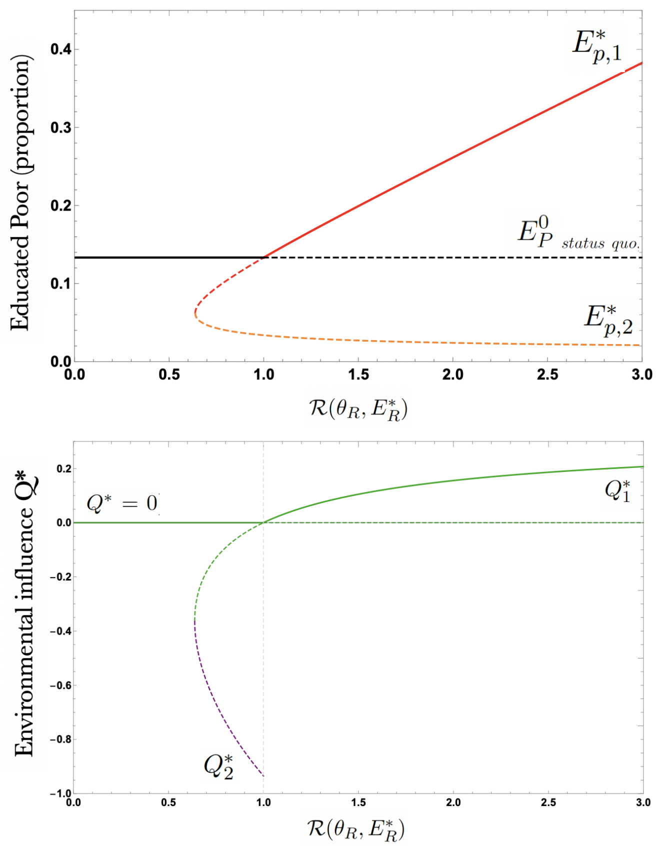


Figure 3.11: Educated Poor and Environmental Influence as a Function of $\mathcal{R}$

Figure 3.12 presents the equilibria for educated middle class with respect to the influence reproductive number. The *status quo* equilibrium is stable when $\mathcal{R}(\theta_P, E_P^*) < 1$ and an influence equilibrium $E_{M,1}^* > E_M^0$ is stable when $\mathcal{R}(\theta_P, E_P^*) > 1$ (straight line). The equilibria are unstable whenever $E_M^0 > E_{M,1}^*$ and $E_M^0 > E_{M,2}^*$. As in the case of equilibrium for educated poor, the influence equilibrium for educated middle class does not hold if $\mathcal{R}(\theta_P, E_P^*) < 1$ and it reaches a higher level of educated middle class $E_{M,1}^* > E_M^0$ when $\mathcal{R}(\theta_P, E_P^*) > 1$.

Figures 3.11 and 3.12 provide a helpful way to compare marginal gains of educated individuals according increases in the probability of influence of education. For instance, the "normal" state of educated population (*status quo*) is 13% and 39% for the educated poor and middle class, respectively. When the likelihood of influence $\mathcal{R}(\theta_P, E_P^*)$ increases from 1 to 1.2, the new educated proportion reaches 16% and 39.5%, for poor and middle class, so then, it suggests that the poor educated gain a larger fraction of educated individuals per additional increase in probability of influence, in other words, $\frac{\Delta E_P}{\Delta \mathcal{R}} = 1.15$, where ΔE_P corresponds to the percentage change of educated poor and $\Delta \mathcal{R}$ represents the percentage change of $\mathcal{R}(\theta_P, E_P^*)$. In case of the middle class, the ratio of variations is $\frac{\Delta E_M}{\Delta \mathcal{R}} = 0.06$ so then, for every 1 percent increase in the probability of environmental influence, the proportion of educated middle class increases by 0.06 %. Therefore, the educated poor realize proportionally higher gains for every additional unit of potential environmental influence.

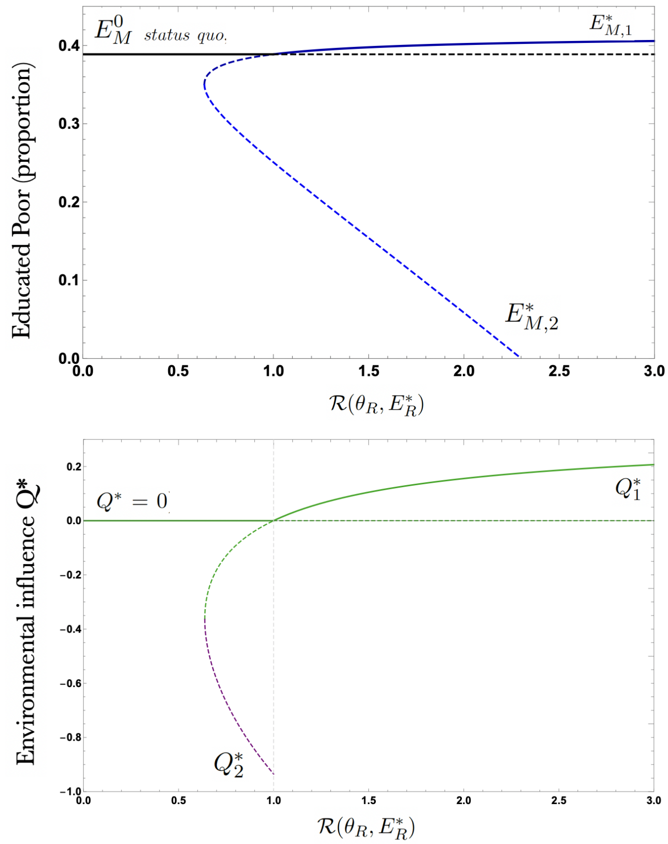


Figure 3.12: Educated Poor as a Function of $\mathcal{R}$

The equilibria for the environment are also depicted in Figure 3.11 and 3.12. The environment begins to exert a positive influence, that means, $Q^* > 0$, when $\mathcal{R}(\theta_P, E_P^*) > 1$ for which we obtain a stable influence equilibrium $Q_1^* > 0$. When $\mathcal{R}(\theta_P, E_P^*) < 1$, the equilibrium $Q^* = 0$ is stable. Any negative form of equilibria Q^* are unstable. This makes the no influence equilibrium ($Q^* = 0$) prevail under the segment $\mathcal{R}(\theta_P, E_P^*) < 1$, as it is shown in the figures.

3.2.1 Simulations

Figure 3.13 shows the proportion of educated and uneducated individuals according to the socio-economic status for the models R-M-P (a and b) and R-P (c and

d) and the environmental influence (e), when the influence is originated by the educated poor $Q \equiv Q(E_P)$. The figures include in a corresponding lighter color, the results under the same parameter values, of the same proportions when the influence is produced by the educated rich ($Q \equiv Q(E_R)$), only for easiness of comparisons. Peer influence (poor influencing poor) produces higher proportion of educated poor and middle class. Educated poor increase now from 10% to 17%, in contrast to just 13% of the previous model ($Q \equiv Q(E_R)$). The drop in educated middle class is not just from 30% to 22%, not 18% as in the previous model ($Q \equiv Q(E_R)$). In the R-P world, the increase in educated poor goes from 10% to 35% when $Q \equiv Q(E_P)$ rather than 26% when $Q \equiv Q(E_R)$. Finally, the effect on the environment looks also more sustainable: there is an increase in Q^* over time that additionally, is maintained at a saturating level $Q^* \approx 0.34$.

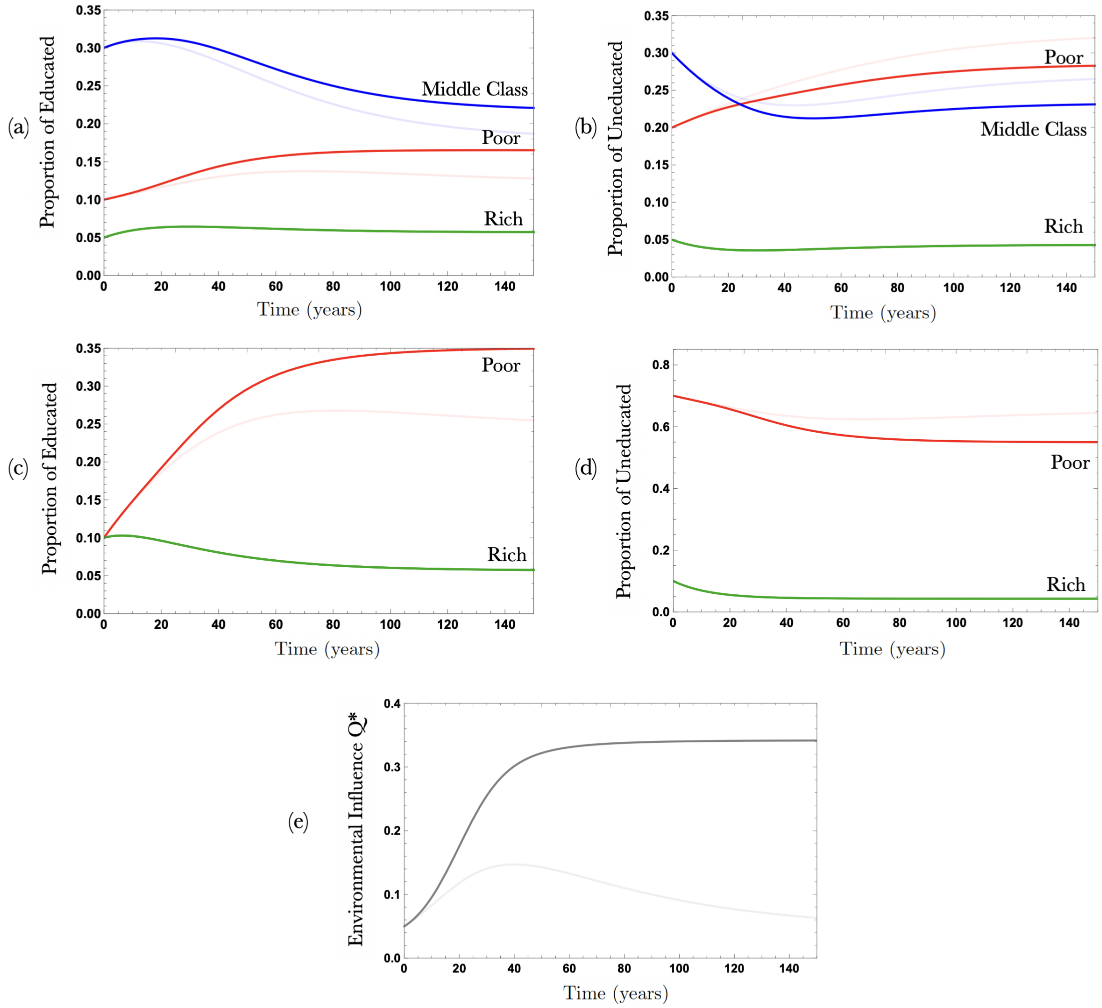


Figure 3.13: Proportion of Educated and Uneducated in R-M-P Model (a and b) and in R-P Model (c and d) and Environmental Influence (e) when $Q \equiv Q(E_P^*)$.

Analogously to previous model, Figure 3.14 shows the number of new educated individuals produced by the influence of the environmental pressure. Note that the shape of the new cases generated over time changes drastically, as a sign of the influence that is maintained over time in the case of $Q \equiv Q(E_P^*)$. A simple calculation of the ratio of this two quantities is 1.1075 with peer influence (poor influencing others) versus the 1.1298 with class influence (rich influencing others). Under the conditions and parameters used, the model suggests that peer influence may reduce the relative gains in mobility (new educated). When the rich educated influenced others, for every 1000 new educated generated in the R-P model, 1130 were generated in the R-M-P model. Now, when poor influence others, this marginal gain is just 1106.

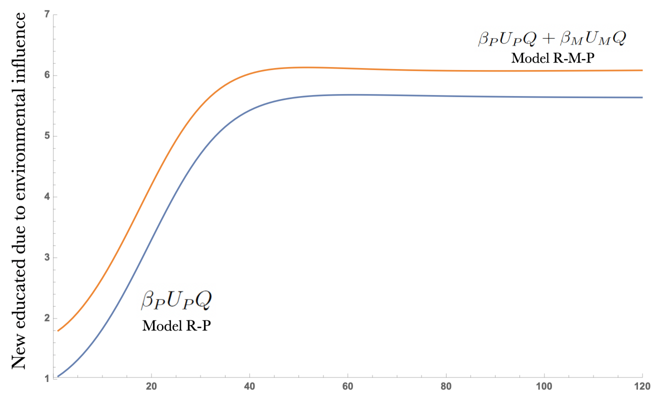
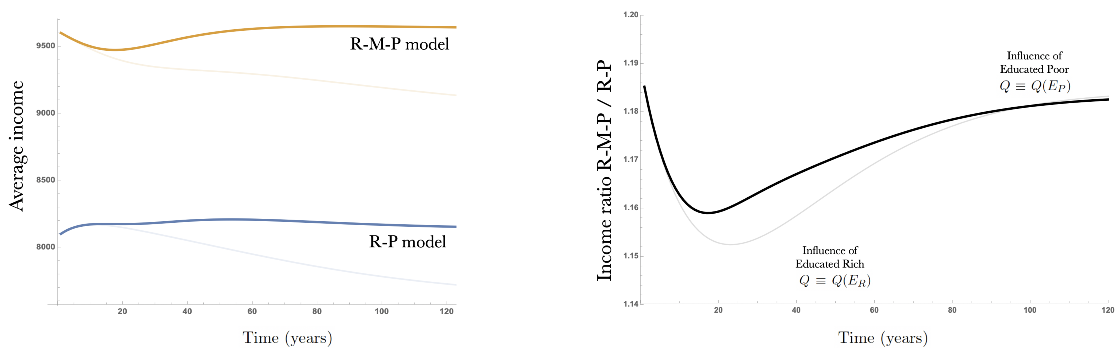


Figure 3.14: New Educated Individuals from the Environmental Influence in R-P and R-M-P Models.

Finally, we also estimate a general metric of the well-being of the community with the weighted average income. Figure 3.15.a shows this average income over time for the two types of communities. The average income is larger and it sustains more steadily when influence is produced by peers $Q \equiv Q(E_P^*)$, in contrast to average income when there is class influence $Q \equiv Q(E_R^*)$ that shows to be decreasing, included in corresponding lighter color. Figure 3.15.b depicts the ratio of average incomes over time. Recall that this is a crude measure of income inequality, the higher the value representing the higher the income inequality. Poor educated influencing uneducated individuals may cause similar levels of average income inequality in the long term. This influence does not allow decrease inequality as much as with the class influence, hence, inequality may present higher levels in the short term.



(a) (b)

Figure 3.15: Average Income of the Rich-Poor and the Rich-Middle-Poor Communities (a). Total Average Income Ratio Between the Rich-Middle-Poor and the Rich-Poor Communities.

This chapter focused on the inclusion of a sizable middle class in the base model comprised of only rich and poor to determine the consequences of more heterogeneous communities. There exists a critical level of influence that affects both educated poor and educated middle class, which defines stability of equilibria and whether the influence from environment is positive or negative. There is always a viable *status quo* equilibrium, stable as the influence is relatively weak. If influence is weak, the equilibria still exists but cannot be maintained since any perturbation leads the results to be at least at the *status quo* level. Stable equilibria for poor and middle class can be generated when influence is strong enough, that means, when is below the threshold critical level that makes the above mentioned scenarios possible. Educated poor may receive limited gains once the middle class is introduced in the model, but having a middle class already changes positively the overall well-being of the community, as measured by the average weighted income.

Chapter 3 includes a large middle class to study the role of social mobility in the presence of higher heterogeneity of the population. More heterogeneity may ease mobility of groups, meaning that more people, poor and middle class in this particular case, become educated. The results depend on initial conditions. Average well-being of the community, measured in terms of average weighted income may improve. Also, income disparities may show to decrease over time at a larger extent with peer influence.

Chapter 4

BASE MODEL WITH LOGISTIC GROWTH

In Chapter 2, we introduced a model that presented a dichotomous view of the world and explored the role of environmental influence, using the size of the college educated class as a proxy, and the possible additional impact that may have on mobility, here defined as a moving from $U_P \to E_P$. In Chapter 3, we added a middle class and observed the impact of environmental influence, using the density for the college educated class again as a proxy on mobility, here defined as moving from $U_P \to E_P$ or $U_M \to E_M$. The simulations have been carried out under the assumption that the total population was constant, that is, $N(0) = \frac{\Lambda}{\mu} = N(t)$.

In this chapter, we assume that the population growth follows a logistic model, as presented in Castillo et al. [89], that is, we let

$$\frac{dN}{dt} = g(\cdot)N,$$
$$N(0) = N_0,$$

with N denoting the total population and $g(\cdot)$ a per capita growth rate that depends on the size of the population. The simplest version of logistic growth, first introduced by Verhulst in 1838 [90], is given by:

$$\frac{dN}{dt} = g_0\left(1 - \frac{N}{K}\right)N,$$

where g_0 is the intrinsic growth rate. Three aspects are worth highlighting from the logistic growth equation: population reaches a carrying capacity as $t \to \infty$, that is, $lim_{t\to\infty}N(t) = K$, the per capita growth rate $\frac{dN}{dt}\frac{1}{N} = g_0\left(1 - \frac{N}{K}\right)$ decreases linearly when N increases. It reaches zero when $N = K$, that is, possibilities for growth are exhausted when carrying capacity is approached; and its maximum growth is achieved at $N = \frac{K}{2}$. The chapter includes population growth to understand how the mobility may change, under demographic pressure.

4.1 Model 6: Logistic Growth and Influence of the Rich

The version of the model in Chapter 2 under logistic growth is given by the following nonlinear system of differential equations:

$$\frac{dU_R}{dt} = f_0 U_R \left(1 - \frac{U_R}{K_R}\right) - \alpha_R U_R - \mu U_R, \tag{4.1}$$

$$\frac{dU_P}{dt} = g_0 U_P \left(1 - \frac{U_P}{K_P}\right) - \alpha_P U_P - \beta_P U_P Q - \mu U_P, \tag{4.2}$$

$$\frac{dE_R}{dt} = \alpha_R U_R - \mu E_R, \tag{4.3}$$

$$\frac{dE_P}{dt} = \alpha_P U_P + \beta_P U_P Q - \mu E_P, \tag{4.4}$$

$$\frac{dQ}{dt} = r_0 \frac{E_R}{\theta_R + E_R} Q(1 - Q) - e_R Q. \tag{4.5}$$

It is assumed that individuals are born as college-uneducated and that their populations eventually reaches their carrying capacities in the absence of mobility, K_P for the poor and K_R for the rich, with $K_P >> K_R$ by assumption. We could have used a single carrying capacity, however, given that the gap between the haves and have nots continues to increase, we expect the steady state size of the poor population to be substantially larger than the steady state size of the rich population and, it seemed reasonable to assume that $K_P >> K_R$. We also assume that the intrinsic growth rates for the poor (g_0) and rich (f_0) differ, with $g_0 > f_0$. The environmental influence function is given by $Q(\cdot)$, where $Q \equiv Q(E_R)$, and it is modeled using our modified version of Levins' model [63]. Under mobility, the population reaches a total carrying capacity. That is, $N \to \frac{1}{\mu}\left[K_P(\alpha_P + \mu)\left(1 - \frac{\alpha_P+\mu}{g_0}\right) + K_R(\alpha_R + \mu)\left(1 - \frac{\alpha_R+\mu}{f_0}\right)\right]$ as $t \to \infty$.

variable	Description
U_R	number of rich uneducated individuals
U_P	number of poor uneducated individuals
E_R	number of rich individuals with college degree
E_P	number of poor individuals with college degree
Q	influence probability function

parameter	Description
f_0	intrinsic growth rate of Uneducated Rich
g_0	intrinsic growth rate of Uneducated Poor
K_R	carrying capacity for Uneducated Rich
K_P	carrying capacity for Uneducated Poor
α_R	per capita rate to become educated for the rich
α_P	per capita rate to become educated for the poor
β_P	effectiveness of environmental transmission rate for the poor
r_0	intrinsic rate of growth of Q
θ_R	influence impact delay
e_R	rate of loss of influence
μ	per capita system exit, $\frac{1}{\mu}$: average educational-time window

Table 4.1: Variables and Parameters of the Model

The flow diagram for this model resembles that of Figure 2.1, the constant recruitment rates $q_R\Lambda$ and $q_P\Lambda$ that are now replaced by the logistic growth functions.

4.1.1 *Mathematical Analysis*

Equilibria

System of equations, 4.1 - 4.5 supports up to six equilibria points. There exist 4 *status quo* equilibria, occurring under no environmental influence ($Q^* = 0$), namely:

$$\mathcal{E}_1 = (U_R^* = 0, U_P^* = 0, E_R^* = 0, E_P^* = 0, Q^* = 0),$$
$$\mathcal{E}_2 = (U_R^* = 0, U_P^* > 0, E_R^* = 0, E_P^* > 0, Q^* = 0),$$
$$\mathcal{E}_3 = (U_R^* > 0, U_P^* = 0, E_R^* > 0, E_P^* = 0, Q^* = 0),$$
$$\mathcal{E}_4 = (U_R^* > 0, U_P^* > 0, E_R^* > 0, E_P^* > 0, Q^* = 0),$$

The first equilibrium $\mathcal{E}_1$ corresponds to the "extinction" equilibrium, that is, when there is no people. $\mathcal{E}_2$ represents the poor only *status quo* equilibrium, $\mathcal{E}_3$ is the *status quo* with only rich class equilibrium and, finally, $\mathcal{E}_4$ represents the *status quo* equilibrium. We have that,

$$\begin{aligned}
U_R^* &= \left(1 - \frac{\alpha_R + \mu}{f_0}\right) K_R, \\
U_P^* &= \left(1 - \frac{\alpha_P + \mu}{g_0}\right) K_P, \\
E_R^* &= \frac{\alpha_R}{\mu}\left(1 - \frac{\alpha_R + \mu}{f_0}\right) K_R, \\
E_P^* &= \frac{\alpha_P}{\mu}\left(1 - \frac{\alpha_P + \mu}{g_0}\right) K_P,
\end{aligned}$$

which is feasible as long as $\frac{g_0}{\alpha_P+\mu} > 1$ and $\frac{f_0}{\alpha_R+\mu} > 1$. The educated born poor and educated born rich are directly proportional to the uneducated born poor and born rich with proportional factors $\frac{\alpha_P}{\mu}$ and $\frac{\alpha_R}{\mu}$, respectively. The rates α_P and α_R denote how likely are poor and rich individuals to enroll and be successful in college over the average educational time-window $\left(\frac{1}{\mu}\right)$. The existence of the "all-aboard" equilibrium require that all $\frac{f_0}{\alpha_R+\mu}$ and $\frac{g_0}{\alpha_P+\mu}$ are greater than 1, where $f_0 > g_0$ and $\alpha_R > \alpha_P$. These conditions need to be met to assure that the community receives more individuals than it loses in the uneducated compartments.

When environmental influence takes place, that is, when $Q^* > 0$, the following equilibria are feasible:

$$\mathcal{E}_5 = (U_R^* > 0, U_P^* = 0, E_R^* > 0, E_P^* = 0, Q^* > 0),$$
$$\mathcal{E}_6 = (U_R^* > 0, U_P^* > 0, E_R^* > 0, E_P^* > 0, Q^* > 0)$$

with $\mathcal{E}_5$ corresponding to a community with only rich individuals; and the equilibrium $\mathcal{E}_6$ is the "all-aboard" equilibrium. The equation for the uneducated and educated rich comprises an independent system that can be solved separately, resulting in equilibrium expressions that correspond to the only-rich *status quo*. The rest of expressions of equilibrium $\mathcal{E}_6$ are obtained by solving the System 4.3, 4.4 and 4.5, from which we obtain:

$$\begin{aligned}
U_R^* &= \left(1 - \frac{\alpha_R + \mu}{f_0}\right) K_R, \\
E_R^* &= \frac{\alpha_R}{\mu}\left(1 - \frac{\alpha_R + \mu}{f_0}\right) K_R, \\
U_P^* &= \left(1 - \frac{\alpha_P + \beta_P Q^* + \mu}{g_0}\right) K_P, \\
E_P^* &= \frac{1}{\mu}\left(\alpha_P + \beta_P Q^* + \mu\right) U_P^*, \\
Q^* &= 1 - \frac{e_R}{r_0}\frac{\theta_R + E_R^*}{E_R^*}.
\end{aligned}$$

Under environmental influence ($Q^* > 0$), there is always a positive equilibria for educated rich, that is, $E_R^* > 0$. As in Model 1, environmental influence exists ($Q^* > 0$), as long as $\mathcal{R}(\theta_R, E_R) = \frac{r_0}{e_R}\frac{E_R}{\theta_R + E_R} > 1$ with $E_R^* > 0$.

$Q^* > 0$, is equivalent to $\frac{f_0}{\alpha_R + \mu} > 1$ with $\mathcal{R}(\theta_R, E_R) > 1$. As stated in Chapter 2, there is a critical threshold for the influence delay that maintains $Q^* > 0$. The critical influence is represented by $\theta_R^c \equiv E_R^*\left(\frac{r_0}{e_R} - 1\right)$ for all $\frac{r_0}{e_R} > 1$ and $E_R^* > 0$.

Figure 4.1 presents the relationship between $\mathcal{R}(\theta_R, E_R)$, and Q^* with respect to the influence delay θ_R. When $\theta_R < \theta_R^c$, the resulting threshold is $\mathcal{R}(\theta_R, E_R) > 1$ and environment activates, that is, $Q^* > 0$. On the other hand, when $\theta_R > \theta_R^c$, then $\mathcal{R}(\theta_R, E_R) < 1$. There exists a possibility to have negative influence, occurring when, for example, as in Chapter 2, $\frac{1}{2} < \mathcal{R}(\theta_R, E_R) < 1$. We focus only in the case $Q^* > 0$.

The critical level of influence deserves more attention because the solutions for the logistic model are different than those in the base model. Recall that for logistic growth, $E_R^* = \frac{\alpha_R}{\mu}\left(1 - \frac{\alpha_R+\mu}{f_0}\right)K_R$, and for the case with no growth, $E_R^* = q_R\frac{\Lambda}{\mu}\frac{\alpha_R}{\alpha_R+\mu}$. Figure 4.1 includes the level of critical influence for both the model under no grow and with logistic growth.

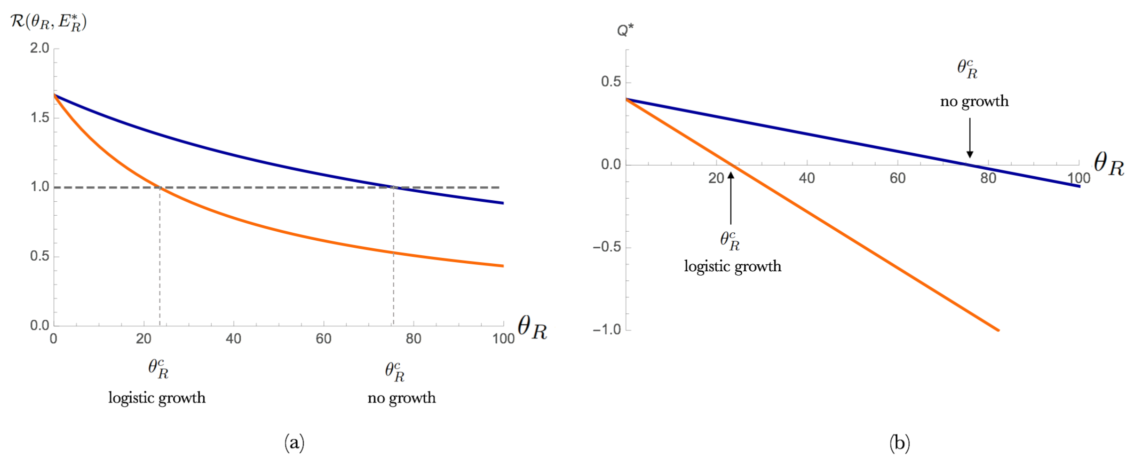


Figure 4.1: (a) Basic Influence Number $\mathcal{R}(\theta_R, E_R)$ and (b) Environmental Influence Q^* as Function of Delay Influence θ_R, Including Two Critical Levels of Influence θ_R^c for Logistic (Model 6) and No Growth (Model 1).

Figure 4.1.a shows $\mathcal{R}(\theta_R, E_R)$ as a function of θ_R for the model with growth $\mathcal{R}(\theta_R, E_R)_{growth}$ and with no growth $\mathcal{R}(\theta_R, E_R)_{\text{no growth}}$. $\mathcal{R}(\theta_R, E_R)_{growth}$ decreases faster (orange) than $\mathcal{R}(\theta_R, E_R)_{\text{no growth}}$ (blue). Population growth intersects $\mathcal{R}(\theta_R, E_R) = 1$ earlier than $\theta^c_{R,growth} < \theta^c_{R,\text{no growth}}$, hence, growth results (under the same param-

eters) on a lower critical influence, that is, $\theta^c_{R,growth} < \theta^c_{R,\text{no growth}}$. When population increases, influence to transition educated poor is more limited. For instance, when $\theta = 40$, $\mathcal{R}(\theta_R, E_R)_{growth} < 1$, limiting E^*_P to the *status quo* equilibrium and $\mathcal{R}(\theta_R, E_R)_{\text{no growth}} > 1$, allowing an influence equilibrium.

Figure 4.1.b shows the relationship between Q^* and θ_R and it indicates that environmental influence is lower under population growth. It seems that environmental influence loses strength in its "ability" to generate educated poor when we consider demographic pressure.

Figure 4.2 shows the equilibrium for educated poor with respect to the critical influence θ^c_R when no demographic growth is included (blue line) and with demographic growth (orange). E^*_P is larger than the *status quo* equilibrium whenever $\theta_R < \theta^c_R$ and *status quo* equilibrium prevails over E^*_P when $\theta_R > \theta^c_R$. After the critical level, the *status quo* equilibrium dominates, meaning $E^*_M < E^0_M$ and $E^*_P < E^0_P$, but before the critical level, when level of influence shows that is strong enough to act over the uneducated poor, the educated poor follow a decreasing function with respect to the influence delay θ_R.

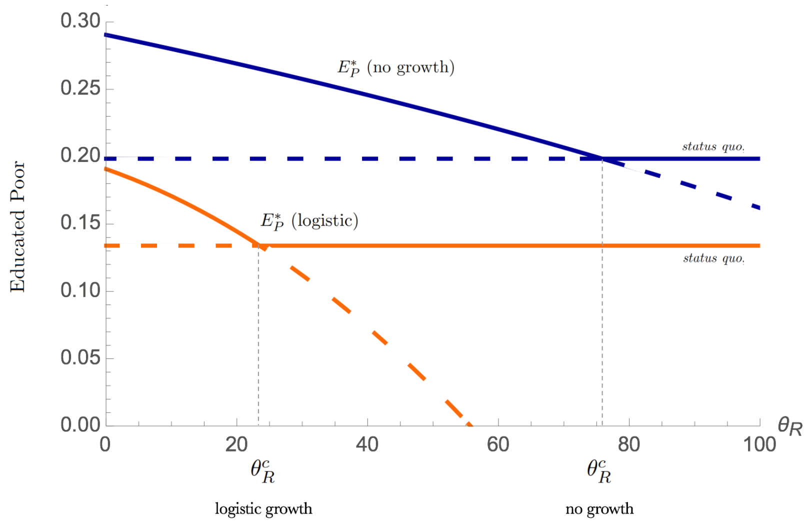


Figure 4.2: Proportion of Educated Poor and Critical Level of Influence θ_R^c for Logistic (Model 6) and No Growth (Model 1) Models.

The carrying capacities for both poor and rich also deserved a little attention to provide appropriate values for simulations. Consider the rate of growth of uneducated rich $\frac{dU_R}{dt}$ and for uneducated poor $\frac{dU_P}{dt}$, when modeled as follows:

$$\frac{dU_R}{dt} = f_0 U_R \left(1 - \frac{U_R}{K_R}\right) - \alpha_R U_R - \mu U_R$$

$$\frac{dU_R}{dt} = f_0 U_R \left(1 - \frac{\alpha_R + \mu}{f_0}\right) \left(1 - \frac{U_R}{K_R\left(1 - \frac{\alpha_R+\mu}{f_0}\right)}\right)$$

$$\frac{dU_P}{dt} = g_0 U_P \left(1 - \frac{U_P}{K_P}\right) - \alpha_P U_P - \mu U_P$$

$$\frac{dU_P}{dt} = g_0 U_p \left(1 - \frac{\alpha_P + \mu}{g_0}\right) \left(1 - \frac{U_P}{K_P\left(1 - \frac{\alpha_P+\mu}{g_0}\right)}\right)$$

We observed a re-scaled version of the carrying capacities for the system, considering inflows (f_0, g_0) and outflows (α_R, α_P), and μ. Hence, the adjusted carrying capacities are $\kappa_R = K_R\left(1 - \frac{\alpha_R+\mu}{f_0}\right)$ and $\kappa_P = K_P\left(1 - \frac{\alpha_P+\mu}{g_0}\right)$, for rich and poor uneducated, respectively. Carrying capacities increase when the inflows increase and also when exit rates decrease. These expressions are used to guide the calibration of carrying capacity parameters.

4.1.2 Stability

The local stability of the equilibria is obtained through the Jacobian of the system where the following conditions are observed:

1. No influence - No Individuals equilibrium ($\mathcal{E}_1$) is stable if $\frac{g_0}{\alpha_P+\mu} < 1$ and $\frac{f_0}{\alpha_R+\mu} < 1$. This equilibrium will always be unstable provided that $\frac{g_0}{\alpha_P+\mu} > 1$ and $\frac{f_0}{\alpha_R+\mu} > 1$.

2. Only poor equilibrium ($\mathcal{E}_2$) is locally asymptotically stable (l.a.s.), if $\frac{g_0}{\alpha_P+\mu} > 1$ and $\frac{f_0}{\alpha_R+\mu} > 1$.

3. Only rich equilibrium ($\mathcal{E}_3$) is l.a.s. if: $\frac{g_0}{\alpha_P+\mu} < 1$, $\frac{f_0}{\alpha_R+\mu} > 1$ and $\mathcal{R}(\theta_R, E_R) < 1$. However, since we established that $\frac{g_0}{\alpha_P+\mu} > 1$, $\mathcal{E}_3$ in an unstable equilibrium.

4. Rich-poor *status quo* equilibrium ($\mathcal{E}_4$) is l.a.s. if $\frac{g_0}{\alpha_P+\mu} > 1$, $\frac{f_0}{\alpha_R+\mu} > 1$ and $\mathcal{R}(\theta_R, E_R) < 1$.

5. Influence equilibrium with only rich ($\mathcal{E}_5$) is l.a.s if $\frac{f_0}{\alpha_R+\mu} > 1$, $\mathcal{R}(\theta_R, E_R) > 1$ and $\frac{g_0}{\alpha_P+\mu} > 1$, provided that $\mathcal{R}(\theta_R, E_R) > \frac{\beta_P}{\beta_P+\mu+\alpha_P-g_0}$.

6. Finally, influence equilibrium ($\mathcal{E}_6$) is l.a.s if $\frac{f_0}{\alpha_R+\mu} > 1$ and $\mathcal{R}(\theta_R, E_R) > 1$ and $\frac{\beta_P}{\beta_P+\mu+\alpha_P-g_0} > \mathcal{R}(\theta_R, E_R)$. This equilibrium is l.a.s. as long as $\frac{g_0}{\alpha_P+\mu} > 1$.

4.1.3 Simulations

We use selected simulations to determine in some cases (numerically) the time needed to increase the proportion of educated poor from 10% to 22% of the population under a model **with** and **without** demographic growth, that is, we contrast the equilibrium of Models 1 and 6. Model 1 assumes no population growth, that is, $N \equiv \frac{\Lambda}{\mu} = N(0)$, and Model 6 assumes population growth that is governed by a logistic function.

Figure 4.3 compares the equilibrium trajectories of educated poor and environmental influence between the growth (black line) and no growth (red line) models. Gray lines represent different equilibrium of growth model under various values of influence delay, $\theta_R = \{0, 17, 34, 51, 68\}$ and the are included to illustrate different equilibrium scenarios.

Simulations show, for example, that under average values ($\theta_R = 34$), it may take 40 years to reach 22% of E_P under the model without growth (red) and 62 years with growth (black). Under growth and weak influence ($\theta_R = 68$), the 22% of educated poor is achieved after 90 years. When the influence is instantaneous, ($\theta_R = 0$), the goal is achieved within 44 years almost the same time as the model without demographic growth (red).

Simulations show that a larger number of the poor may become educated in the long term when populations grows albeit the proportion of poor educated may decrease. This proportion may be larger in contrast to a constant population but proportion of educated goals may not be met faster: when population grows, the goal of 22% may be reached almost 20 years than in contrast to that reached with model with constant population. Models intersect at year $t = 75$ when $E_P = 0.25$.

Environmental influence is modeled in the right side of Figure 4.3. On average terms ($\theta_R = 34$), environmental influence increases monotonically when demographic growth is included (purple line). Without population growth, environmental influence reaches a maximum at year 41 and then it decays over time.

So far, we have observed proportions of educated in the poor and rich classes under

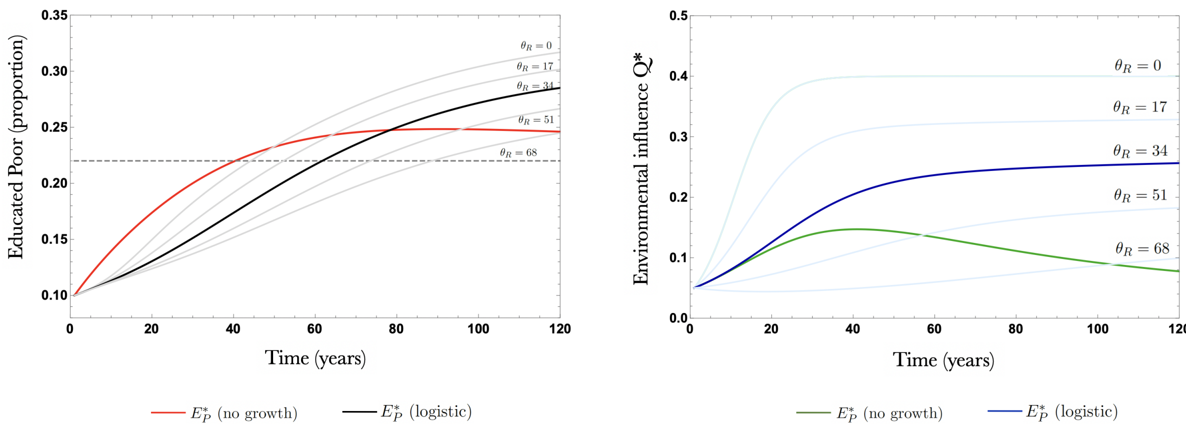


Figure 4.3: Educated Poor and Environmental influence with Demographic Growth (Model 6), Without Demographic Growth (Model 1) and Various Levels of Influence (θ_R). For Calibration of Parameters, and with $\mu = \frac{1}{30}, \Lambda = 33.34, q_R = 0.1, q_P = 0.9, r_0 = 0.5, e_R = 0.3, \theta_R = 34, \alpha_R = 0.044, \alpha_P = 0.011, \beta_P = 0.018, f_0 = 0.08, g_0 = 0.09$ with carrying capacities $k_R = 5000$ and $K_P = 6000$. $\frac{P}{R} = 4$

demographic growth and lack of demographic growth. The proportions of rich and poor, educated and not educated change over time. We may think about alternative, general measures, in order to explore the benefits or losses that a community experiencing demographic changes experiences under different models and scenarios. We propose the weighted average income of this population under the assumption that the college educated earns more than the uneducated. We look at the situation when the uneducated poor earn the lowest within all groups, $\hat{U}_P < \hat{E}_P << \hat{U}_R < \hat{E}_R$. Estimating an average income incorporates economic aspects that may help to develop and test comparable and tractable measure of well-being of a community. We include average income for every group and make the assumption that average income stays constant over time. This assumption may be adjusted in further simulations. The weighted average income for the poor $P = U_P + E_P$ is given by $\hat{P} = \frac{U_P}{P}\hat{U}_P + \frac{E_P}{P}\hat{E}_P$. The average income for the rich $R = U_R + E_R$ is $\hat{R} = \frac{U_R}{R}\hat{U}_R + \frac{E_R}{R}\hat{E}_R$. Figure 4.4

provides a comparison of weighted average income for the poor and the rich under the cases of no growth and logistic growth.

For illustrative purposes, we analyze three different income ratio scenarios between the income received by educated and uneducated, that is, $\frac{\hat{E}_P}{\hat{U}_P} = 2$, $\frac{\hat{E}_P}{\hat{U}_P} = 5$, and $\frac{\hat{E}_P}{\hat{U}_P} = 10$. In the United States, the income ratio between the richest 10% and the poorest 10% has increased from 8.7 in 1968 to 12.7 in 2018 [88].

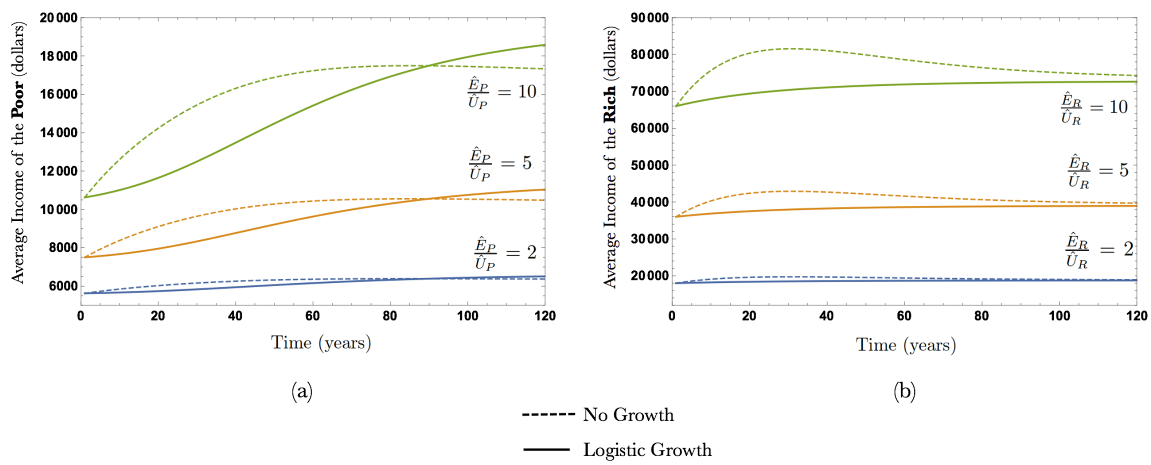


Figure 4.4: Average Weighted Income of the Poor (a) and the Rich (b) under No Growth and Logistic Growth of the Population for Selected Degrees of Income Disparity when $Q \equiv Q(E_R)$.

The average income of the poor is presented in Figure 4.4.a. Before year $t = 90$, average income for poor is higher when population is constant. Also, income differences increase between models (growth vs no growth) increase when income inequality is higher, income inequality understood as the educated-uneducated average income ratio. We illustrate with an example, with year $t = 30$ as a reference. Under a relatively low level of income disparity, $\frac{\hat{E}_P}{\hat{U}_P} = 2$, average income of the poor is approximately \$6160.46 versus \$5833.03 under no growth and growth models, respectively. This represents a 5.3% decrease. As income inequality increase to $\frac{\hat{E}_P}{\hat{U}_P} = 5$, the percentage drop is 13.1%, going from \$9641.85 to \$8332. Finally, at the highest level of income disparity given by $\frac{\hat{E}_P}{\hat{U}_P} = 10$, average income changes from \$15444.2 to \$12497.2, 19.0% decrease.

Population growth seems to complicate the realization of income goals, in the sense that it takes longer to reach a specific income over time. For instance, when $\frac{\hat{E}_P}{\hat{U}_P} = 10$, an average income of \$ 14000 is reached in 18 years when there is no demographic growth. However, with population growth, the same average income is reached after 26 more years, at $t = 44$. The case of a population without growth may resemble the case of a developed country whereas growth may be the case of a developing country. In fact, average population growth rate is 2.6 for low income countries and 0.4 for high income countries according to the World Bank. Average income under population growth model "catches up" with no growth model at year 90, that is, the time when trajectories of income for both models intersect. As with all models, the simplifications can be misleading. Population with no growth, like Japan, experience an aging work force and the increase expenses in retirement and health costs. Clearly, demographic growth may have a large positive effects, depending on the circumstances.

The Figure 4.4.b shows the average income of the rich. Intersection of average income under both models takes longer than 90 years. Considering the same reference of year $t = 30$, differences of average income between models (no growth vs growth) are 6.3%, 11.6%, and 13.7%. The rich experience smaller income changes than the poor, suggesting that income vulnerability for the rich is lower under a case of population growth.

Figure 4.5 shows average income ratios between the rich and the poor. This provides a first picture of the evolution of income inequality over time. The ratio indicates, for instance, at time $t = 0$, that for every dollar a poor person has, the rich obtain 6.2 dollars. The lower the value, the lower the level of inequality. When a population does not experience growth, income inequality decreases faster in contrast to population growth case. However, differences are not so notorious before year $t = 47$. After year 47, population growth seems to produce lower income inequality. Without growth, inequality decreases more slowly and stabilizes at $\frac{\hat{R}}{\hat{P}} = 4.15$. Under population growth, rich-poor ratio decreases faster, and it reaches a stable path at $\frac{\hat{R}}{\hat{P}} = 3.01$. That is, in the long term, we would expect, under the same parameters, that population growth may be beneficial for decreasing average income inequality, $\frac{\hat{R}}{\hat{P}}_{\text{growth}} < \frac{\hat{R}}{\hat{P}}_{\text{no growth}}$.

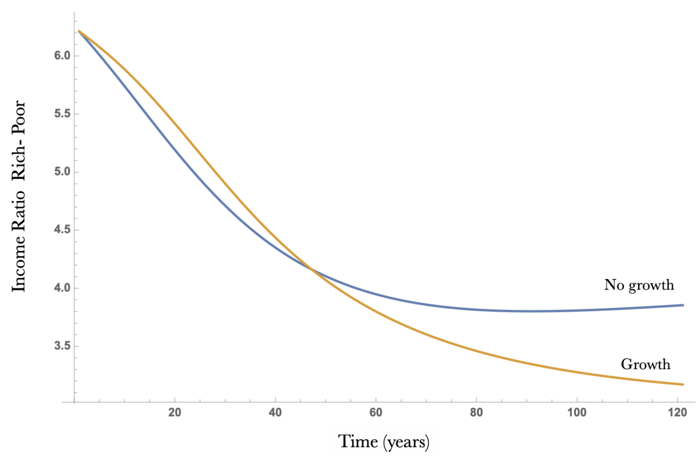


Figure 4.5: Rich-Poor Average Income Ratio for the Highest Educated-Uneducated Ratio $\frac{\hat{E}_P}{\hat{U}_P} = \frac{\hat{E}_R}{\hat{U}_R} = 10$.

The analysis of total wealth in an economy is relevant because the proportion devoted to rich and the poor may be used or should be used to define how egalitarian a society is or should be. The total wealth of a community will be explained by the income generated by all members, that is, we take their salary and multiply by the number of people and, sum up those resources together. Hence, total wealth in this simplified community is a product of price (of labor) and quantity. We assume that the price of labor corresponds to income, which is constant over time. Our simulations provide the changing distributions of individuals and now, with this, we explore how this changes determined the new distribution of wealth in this artificial community.

One of the most common measures of income inequality is the Gini coefficient. This coefficient measures the deviation of the distribution of resources in a country with respect to a perfectly equal distribution [91]. This distribution is depicted by a curve known as the Lorenz curve. We construct the Gini coefficient of this community under the model with and without demographic growth. Figure 4.6.a presents a typical Lorenz curve that associates proportional levels of wealth according to the proportion of the population. For instance, the figure shows a point where 50% of the population owns 18% of the cumulative wealth of the community. The 45 degrees line corresponds to the case of perfect equality, this is, where the 50% of the wealth would be distributed to the 50% of the population. The Gini coefficient results from the proportion of areas $\frac{A}{A+B}$. This coefficient *per se* does not mean anything but a value close to 0 represents an equal society and approaching 1, corresponds to a case of total inequality. Just for illustrative examples, the United States has a Gini coefficient of 0.415, Mexico 0.483, Denmark 28.2, South Africa 0.63 for year 2016 [92]. Hence, a Lorenz curve further separated from the 45 degrees line represents a more unequal society.

Figure 4.6.b shows the estimated curves for the logistic (blue) and no growth (orange) cases. This curves correspond to an approximation of a Lorenz curve with only have two groups, poor ($P = U_P + E_P$) and rich ($R = U_R + E_R$). The highlighted point shows that 86% of the population receives 35.1% of the wealth of the community. We define wealth as the total amount of income generated, that is, number of individuals times its corresponding income. The income refers to the average income estimated in the above simulation presented in Figure 4.4. Just for modeling purposes, the un-

educated had 60% of the salary of the educated and the rich educated had 15 times more the income of the poor, so, this is the case of an highly unequal community measure by the income ratio. The estimated Gini coefficient under logistic growth was 0.483 and under no growth was 0.512. With the rest of parameters constant, a term known in economics as *ceteris paribus*, under population growth, there may be a decrease in income inequality by approximately 0.03 points.

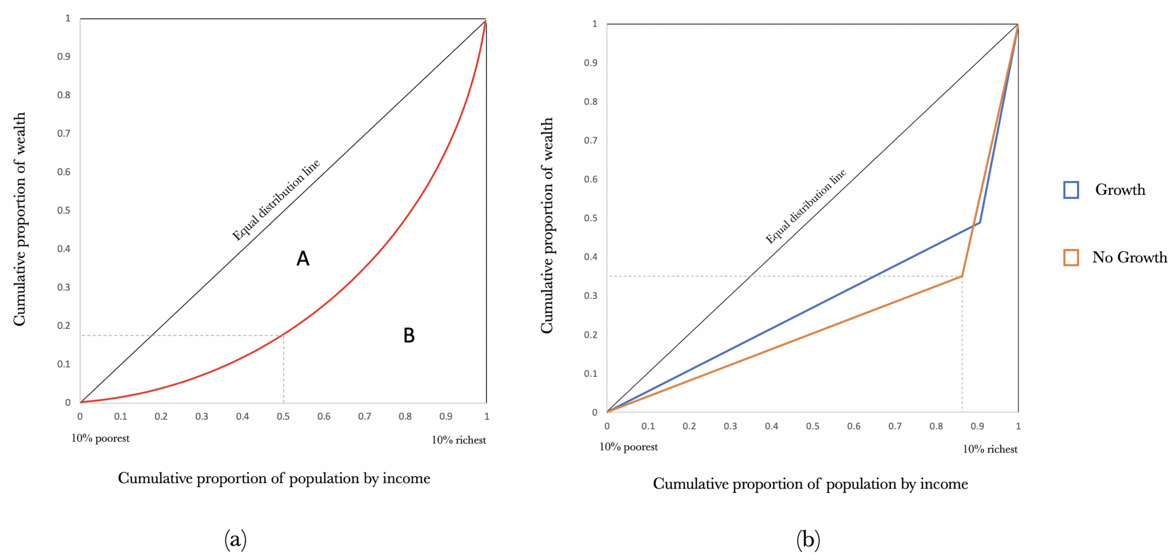


Figure 4.6: Lorenz Curves and Gini Coefficient under No Growth and Growth.

4.2 Model 7: Logistic Growth and Influence of the Poor

The model with logistic growth of the population is now explored considering that the dynamic "probability" given by the environmental influence Q is dependent on the number of poor educated, that is, $Q \equiv Q(E_P)$. Born rich and born poor individuals incorporated through a basic form of logistic equation, with K_P and K_R, as the carrying capacities for the poor and the rich, respectively, assuming that $K_P >> K_R$. Intrinsic growth rates differ among groups, $g_0 > f_0$. Per capita rate of influence "effectiveness" on U_P equals β_P. Per capita rate of growth of Q is given by $r(\theta_P, E_P) = r_0 \frac{E_P}{\theta_P + E_P}$. r_0 is the intrinsic growth rate of Q; $\frac{1}{e_P}$ is the average time influence of Q on U_P; θ_P determines whether or not there is a delay on the impact of E_P on Q; α_R and α_P are the rates of progression from U_R to E_R, and from U_P to E_P, respectively. The average time-span allowing selection of higher education is given by $\frac{1}{\mu}$. The population reaches a limiting value given by $N \to \frac{1}{\mu}\left[K_P(\alpha_P + \mu)\left(1 - \frac{\alpha_P+\mu}{g_0}\right) + K_R(\alpha_R + \mu)\left(1 - \frac{\alpha_R+\mu}{f_0}\right)\right]$ as $t \to \infty$. These definitions and assumptions lead to the following nonlinear system of differential equations:

$$\frac{dU_R}{dt} = f_0 U_R \left(1 - \frac{U_R}{K_R}\right) - \alpha_R U_R - \mu U_R, \tag{4.6}$$

$$\frac{dU_P}{dt} = g_0 U_P \left(1 - \frac{U_P}{K_P}\right) - \alpha_P U_P - \beta_P U_P Q - \mu U_P, \tag{4.7}$$

$$\frac{dE_R}{dt} = \alpha_R U_R - \mu E_R, \tag{4.8}$$

$$\frac{dE_P}{dt} = \alpha_P U_P + \beta_P U_P Q - \mu E_P, \tag{4.9}$$

$$\frac{dQ}{dt} = r_0 \frac{E_P}{\theta_P + E_P} Q(1 - Q) - e_P Q, \tag{4.10}$$

4.2.1 Mathematical Analysis

The system of equations 4.6-4.10 sustains up to 10 boundary equilibrium points. There exists 4 *status quo* equilibria expressed by:

$$\mathcal{E}_1 = (U_R^* = 0, U_P^* = 0, E_R^* = 0, E_P^* = 0, Q^* = 0),$$

$$\mathcal{E}_2 = (U_R^* = 0, U_P^* > 0, E_R^* = 0, E_P^* > 0, Q^* = 0),$$

$$\mathcal{E}_3 = (U_R^* > 0, U_P^* = 0, E_R^* > 0, E_P^* = 0, Q^* = 0),$$

$$\mathcal{E}_4 = (U_R^* > 0, U_P^* > 0, E_R^* > 0, E_P^* > 0, Q^* = 0),$$

where the positive components of equilibria are given by:

$$\begin{aligned}
U_R^* &= \left(1 - \frac{\alpha_R + \mu}{f_0}\right) K_R, \\
E_R^* &= \frac{\alpha_R}{\mu}\left(1 - \frac{\alpha_R + \mu}{f_0}\right) K_R, \\
U_P^* &= \left(1 - \frac{\alpha_P + \beta_P Q^* + \mu}{g_0}\right) K_P, \\
E_P^* &= \frac{1}{\mu}\left(\alpha_P + \beta_P Q^* + \mu\right) U_P^*, \\
Q^* &= 1 - \frac{e_P}{r_0}\frac{\theta_P + E_P^*}{E_P^*},
\end{aligned}$$

When environmental influence takes place ($Q^* \neq 0$), there exists 6 equilibria points:

$$\mathcal{E}_5 = (U_R^* = 0, U_P^* > 0, E_R^* = 0, E_P^* > 0, Q^* > 0),$$

$$\mathcal{E}_6 = (U_R^* = 0, U_P^* > 0, E_R^* = 0, E_P^* > 0, Q^* > 0),$$

$$\mathcal{E}_7 = (U_R^* = 0, U_P^* > 0, E_R^* = 0, E_P^* > 0, Q^* > 0),$$

$$\mathcal{E}_8 = (U_R^* > 0, U_P^* > 0, E_R^* > 0, E_P^* > 0, Q^* > 0),$$

$$\mathcal{E}_9 = (U_R^* > 0, U_P^* > 0, E_R^* > 0, E_P^* > 0, Q^* > 0),$$

$$\mathcal{E}_{10} = (U_R^* > 0, U_P^* > 0, E_R^* > 0, E_P^* > 0, Q^* > 0),$$

where the equilbria are represented by the solutions of the equations given by:

$$U_R^* = \left(1 - \frac{\alpha_R + \mu}{f_0}\right) K_R, \tag{4.11}$$

$$E_R^* = \frac{\alpha_R}{\mu}\left(1 - \frac{\alpha_R + \mu}{f_0}\right) K_R, \tag{4.12}$$

$$Q^* = 1 - \frac{e_P}{r_0}\frac{\theta_P + E_P^*}{E_P^*}, \tag{4.13}$$

$$g_0\left(1 - \frac{U_P}{K_P}\right) - \alpha_P - \beta_P\left(1 - \frac{e_P}{r_0}\frac{\theta_P + E_P^*}{E_P^*}\right) - \mu = 0, \tag{4.14}$$

$$\alpha_P U_P + \beta_P U_P\left(1 - \frac{e_P}{r_0}\frac{\theta_P + E_P^*}{E_P^*}\right) - \mu E_P = 0. \tag{4.15}$$

Equilibria for U_P^* and E_P^* can be obtained from the simultaneous solutions of the third degree polynomials given by expressions 4.14 and 4.15.

The complex nature of this model may be explained by the combination of distinct sources of dynamics, namely, the nonlinear human-environment interaction, the feedback system generated by educated poor and now, the population pressure expressed by the logistic growth. We observed explicit solutions in Chapters 2 and 3 but this section focuses on the qualitative behavior of numerical simulations.

4.2.2 Simulations

Simulations show the effects of poor influencing the poor with and without growth. Figure 4.7.a shows the proportions of educated poor with and without demographic growth when $Q \equiv Q(E_P)$. Without growth, community reaches 22% of educated poor after 34 years and with growth, after 50 years. In the long term, population growth provides a higher proportion of educated poor in comparison to no growth of the population but when population grows, it takes longer to reach a specific goal. If population is constant, the goal may be obtain earlier.

Poor influencing the poor may accelerate the achievement of the goal by 12 years, decreasing the time from $t = 62$ when $Q \equiv Q(E_R)$ to $t = 50$ when $Q \equiv Q(E_P)$. Proximity, closer realities, similar backgrounds seem to have a higher effect on motivation to go to college, a situation that is additionally maintained over time due to the feedback $E_P \to Q \to E_P$. This sustained feedback can be seen in the environmental influence Q^* from Figure 4.7.b that presents a saturating probability of influence.

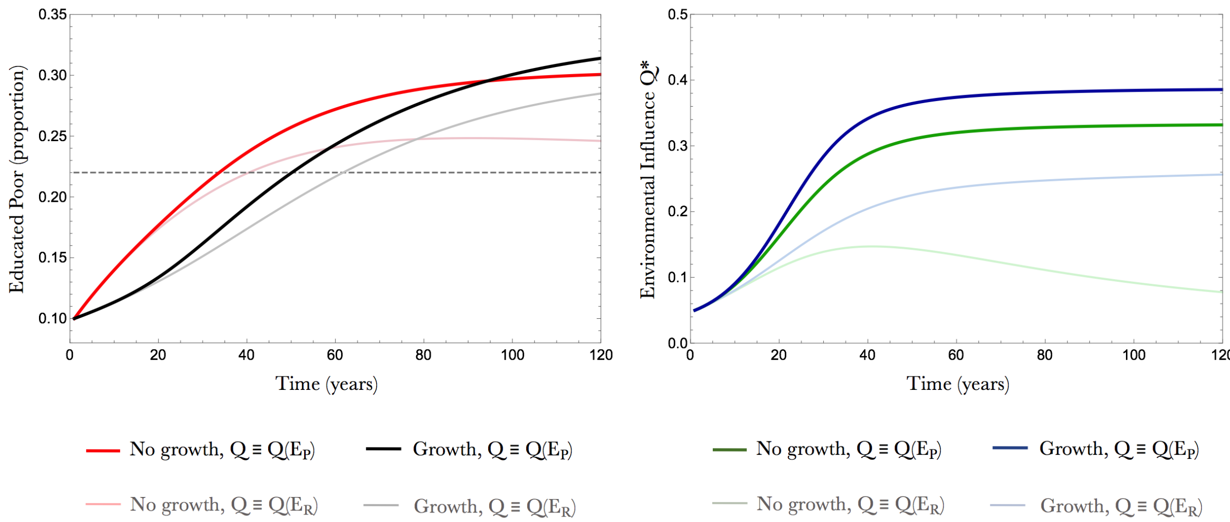


Figure 4.7: Educated Poor and Environmental Influence with No Growth of Population (Model 2) and Logistic Growth (Model 7).

Figure 4.8 shows the average income of the community for poor and rich. Average income for the poor is higher when influence is produced by peers because the proportion of educated poor increases, knowing that average income of the educated poor is larger than uneducated poor $\hat{E}_P > \hat{U}_P$. Also, the difference in average income between models with and without growth become narrower. Average income of the rich are maintained as in previous model the case when $Q \equiv Q(E_R)$. Poor influencing the poor (peer influence) may contribute to closing the income gap, consequently decreasing income disparity.

Finally, Figure 4.9 is an estimation of the Lorenz curve for these two communities (with and without growth). When population grows, the estimated Gini coefficient is

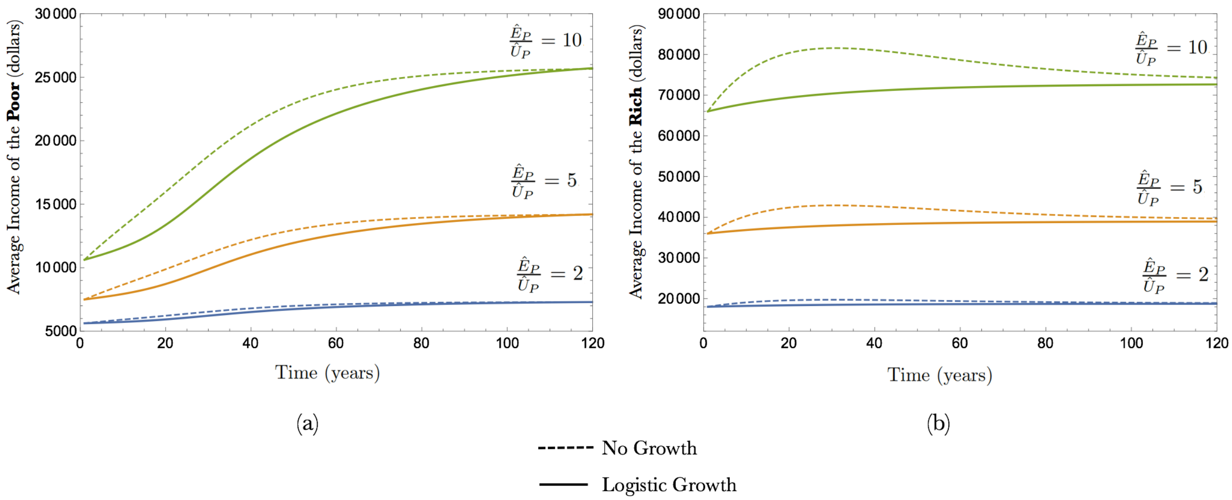


Figure 4.8: Average Weighted Income of the Poor (a) and the Rich (b) under No Growth and Logistic Growth of the Population for Selected Degrees of Income Disparity when $Q \equiv Q(E_P)$.

0.4142. With constant population, Gini is 0.5018, a difference of 0.08756. Population growth may contribute to the decrease of income inequality.

This contribution seems to be larger when poor influence the poor, in contrast to rich influencing the poor: peer effect is superior than class effect. These simulations and conclusions may provide further hints for further and deeper research about mechanisms to reduce inequalities. The introduction of demographic growth permits to evaluate how having "real" increases in mobility affect when a population is growing. The recognizable increases are measured by the proportion that moves from U_P to E_P. Population growth may seem to complicate the attainment of goal, at least in the short term. Population growth may be a source of support to decrease income inequality under special circumstances. Simulations show that goal to achieve a specific percentage of educated poor may delay by approximately one generation (approximately 17 years). Average income of the community increases under con-

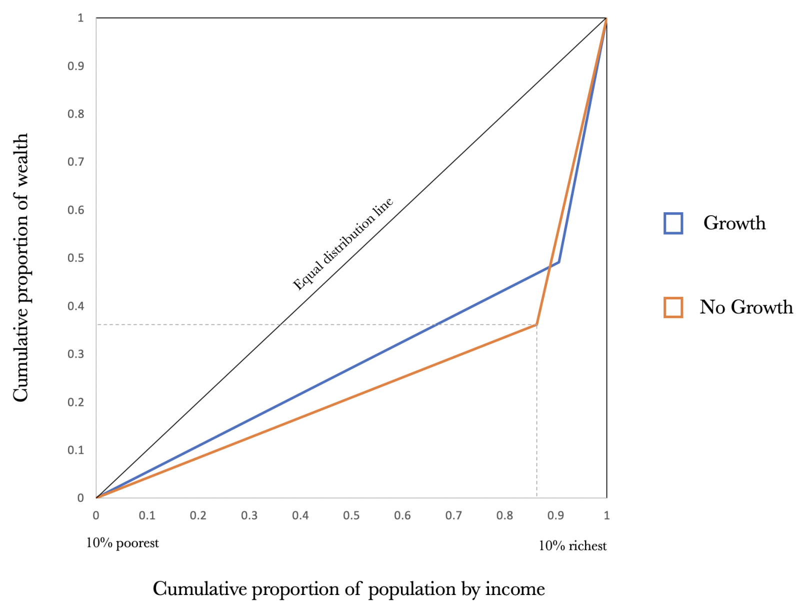


Figure 4.9: Average Weighted Income of the Poor (Uneducated and Educated) under No Growth and Logistic Growth for Selected Degrees of Income Disparity.

stant population until year 90, finally, demographic growth may favor lower income disparities.

Chapter 5

DISCUSSION AND CONCLUSIONS

This dissertation made use of simple models to study the factors that facilitate social mobility. A prototypical community is categorized by level of education and socio-economic status. The nonlinear dynamics may be altered by the impact of education, which is a proxy for mobility. The environment is modeled by a modified version of the Levins' equation. The rate of growth of the influence is tied in to the proportion of successful individuals, that is, college educated, and their status at birth. Finally, the environmental influence takes into account handling time, the time that it takes the influence to take effect. This framework helps explore the nonlinear effects of the environmental influence on underprivileged groups in the community.

There are no studies focusing on the nonlinear dynamics of environmental influence, a function of education, on mobility, a gap that this dissertation intends to address. The majority of research linking the quality of the surroundings with educational and mobility outcomes is concentrated on statistical studies, seeking to answer how much, for example, the income of children is explained by parental income. The relevant economic and social literature was highlighted in the introductory chapter.

Factors affecting the upward social mobility may be internal or external. Internal factors may come from peer group effects, that is, poor influencing the poor, or from class effect, rich influencing the poor and various other combinations. External factors may may come from schools, universities; or from economic, including, inflation, migration, GDP variation, unemployment, etc.

This framework focuses on the internal factors affecting mobility. We found that there exists, possibly substantial, social mobility differentials depending on the generating group of the influence. The impact of certain groups can be minimal or it can be significant. Peer effects may be larger than class effects. We also observed that there exists a critical level of influence that determines a threshold point from which, influence is effective or not. The critical level of influence may take different forms with respect to size and origin. For instance, if one individual initiates a protest in front of Congress, this might be an unperceived event during the day but, if 2 million protesters gather around the Capitol in Washington D.C., will with a high probability, make the news headlines at 7 pm. Also the origin of the influence matters. Every one of us may be influenced by the life and work of Albert Einstein, but other groups may tend to be more influenced by Rosa Parks, David Blackwell or Martin Luther King Jr., in the case of the African American population. Some particular communities of U.S. Latinos may be more effectively influenced by the work of Cesar Chavez, Ellen Ochoa, Richard Tapia or Carlos Castillo-Chavez. The level of proximity and background resemblance may play a larger effect on the formation and persistence of influence of role models in society.

Influence might also gender driven. Women may generate sustained influence over female peers, as we have seen from entrepreneurial mentors: successful business women seem to influence young women more than business men. Female mentors exert a stronger influence to maintain successful communities of other women working on small businesses, as it is the case of the Grameen Bank communities that started in Bangladesh and spread around Asia [93].

Heterogeneity poses a great challenge for modelers. In our simplified scheme, the community was divided into rich-poor (defined by birth) and college uneducated-educated for simplicity. We added a middle class to explore the dynamics of mobility in a community with more heterogeneity. Heterogeneity arises from many sources including the age structure of the population, income differences, ethnic or religious background, immigration status, and more. The United States, a "melting pot" includes 8 broad groups of self-identified races, 50 states, and a multitude of nationalities. The inclusion of some levels of heterogeneity may reveal the lack of robust dynamics. Heterogeneity should not be just added to complicate a model but rather to explore the robustness of dynamical outcomes in response to specific questions that demand the consideration of higher levels of heterogeneity.

Social systems may not be reproducible and most of times, are recognized as out-of-equilibrium or context-dependent, meaning that time, space, initial conditions, external shocks, lack of resources and other factors. Tools of analysis have become limited to address the large dimensionality of factors affecting social systems, and social science "rarely extends beyond linear regression analysis or basic statistics" [94]. Not surprisingly, nonlinear dynamics in the context of social upward mobility brings additional challenges; further complicated since social systems lack the data needed.

Nonlinear dynamics show that the whole is not necessarily the sum of its parts. Nonlinear systems are more complex to be analyzed than linear systems because they cannot be broken down into pieces and added in order to reach a final result.

The use of simulations presented additional challenges when population growth is included. Cities or any other human conglomerate have needs that may depend on size. The introduction of logistic growth of the population just touches on these challenges. If the community is, for instance, New York City, does the influence affect to the same extent as a small town in Wisconsin? If the population of poor doubles, do we need to double the influence to make a difference? If wealth becomes more and more concentrated in fewer families, are the rich successfully influencing the poor or will the rich become a source of social discontent? How does the community respond to a change in size? These are still challenging questions to be addressed.

Geoffrey West puts it in a friendly colloquial way in his book “Scale: the universal laws of growth, innovation, sustainability, and the pace of life in organisms, cities, economies, and companies” [95]: “an animal twice the size of another, requires probably 75% more food and energy per day.” Bigger the element of analysis, probably less it is needed per unit of size. Is there economies of scale in environmental influence? The larger the community tends to create the need for more relationships, increases communication and connectivity so then, the strength of influence may be amplified by different channels such as social media.

Limitations of the Model

The models are by their own nature widely limited and usually useful when tied in to specific questions in specific settings. Our models, though they conveniently capture the environmental influence of education on mobility, they do not incorporate, for example, the age-structure of the population, albeit the framework could be easily expanded to include population structure. The more structure that we add the less tractable the model becomes. The issues arising with an inverted population pyramid, as it might be the case of Japan, which faces faster aging of its population should be addressed in the context of studies the impact of aging in mobility. Figure 5.1 presents the age structure population for 5 selected countries, which would require different considerations for analysis.

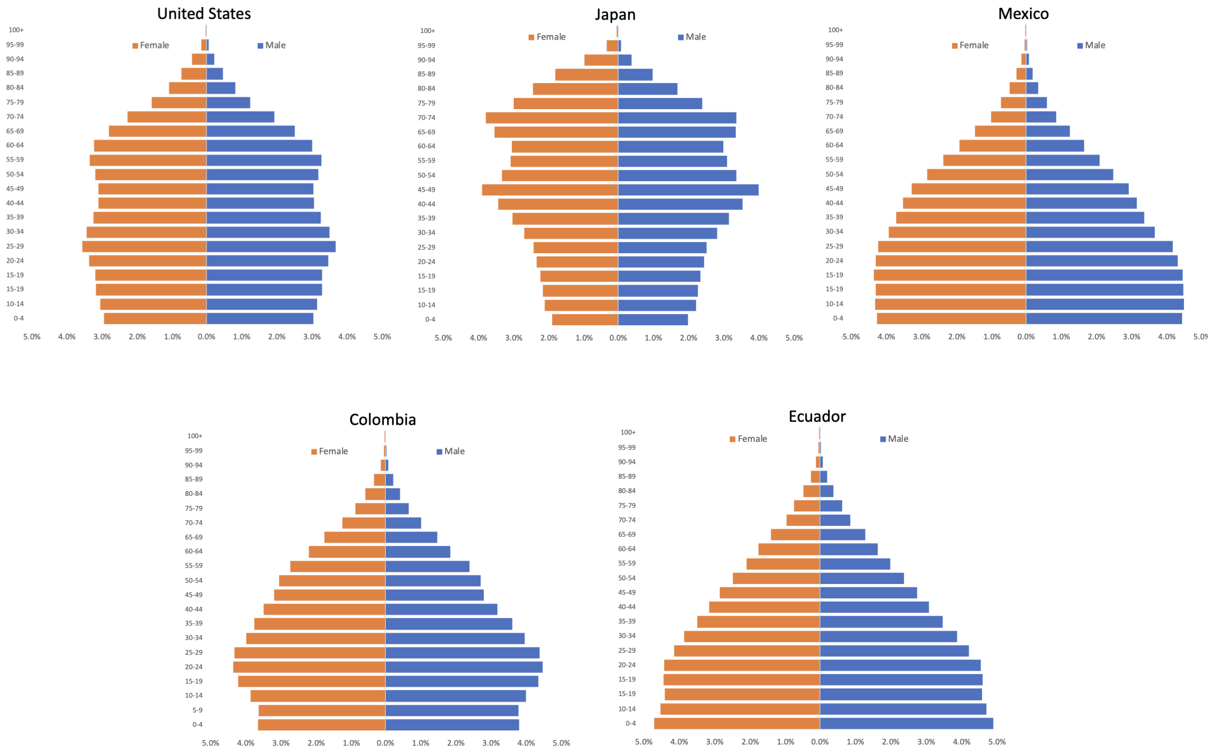


Figure 5.1: Population Pyramids for Selected Countries, 2019. Source: Population Pyramid Net, UN [4].

The United States is slowly losing younger generations, proportionally speaking, and its immigration policies change, aging may become a more pressing issues. According to Social Security Administration estimates, “97 percent of the elderly, aged 60 to 89, receive Social Security payments” in the United States [96]. With almost 64 million Americans receiving benefits in Social Security, representing 1 trillion dollars in 2019, the system will have increasing pressure to support retired and disabled workers, etc.[97]. The costs of health insurance will most likely increase if population becomes older. 8% of Americans did not have any type of health insurance coverage in 2018 [98]. With increasing average annual premiums for family coverage, health expenditures will become a risk for families’ financial stability [99]. Further, a study conducted by Austin [100] on “Medical debt as a cause of consumer bankruptcy”, suggests that 18% to 26% of all consumer bankruptcies are produced by medical debt.

Mexico, Colombia, and Ecuador, 3 Latin American nations present a more triangular version of the pyramid with Colombia starting to develop a concentration on the adult segment of the population, possibly explained by a systematic fall of fertility rates.

Another limitation might be the rate related to likely years when access to education μ is still viable, here considered as constant and equal for all groups. An extension of the model may recognize that groups may have different rates.

The rate of “effectiveness” of influence β is also a parameter that deserves further exploration. The models consider the rate of effectiveness as constant but there is still very little that we know about it. This parameter might be related to the ability of a community to respond to influential factors, for instance, some societies may be more prone to change or to accept change than others.

The framework is our best attempt to explain the dynamics of the environment surrounding individuals with responses to education, following a phenomenological approach. George Box claimed that “All models are wrong but some are useful”. We hope that this framework helps to take small steps to understand the environment-human dynamics.

Contributions and Policy Implications

The economic decision of education is based on the rate of return of becoming educated in comparison to other alternatives, say, get a job. College education is associated to higher income. Increase in income may allow people to improve economic status and this is a recognizable way to distinguish upward mobility. This research considers aspects that go beyond the classic economic theory concerning utility maximization.

Upward mobility works under rather complex circumstances. Characterizing and understanding the environmental effects given by Q, over the less privileged members in society, is the central contribution of this study, under the intuitive idea that "neighborhood matters". This study may provide some elements about educational policy that may enhance mobility issues that are often addressed but without understanding of the role of nonlinear effects. The model has several parameters that may be treated as policy options.

The "effectiveness" of influence β is the first parameter that can provide scenarios for policy. Two country examples may help to illustrate certain practices that can be considered to increase such effectiveness. The first is an experience in China. Liping Ma, a former teacher and director of a school in China, wrote the book " Knowing and Teaching Elementary Mathematics: Teachers' Understanding of Fundamental Mathematics in China and the United States." Liping Ma mentions that math teachers meet with other teachers for several hours every day to learn different approaches and techniques from each other. They take advantage of the experience of the same teachers and the energy and intelligence of the new teachers to improve lessons and, consequently, learning. Supporting this type of initiatives may strengthen the influence generated in the school environment.

The second experience takes place in another Asian country. In the book "The teaching gap: The best ideas of the world's teachers to improve classroom education", Stigler and Hierbert [101] observed that, in Japan, it is common for teachers to meet and develop "lessons of research ", in which they plan a lesson, present it in front of the group, evaluate it and reflect on the best practices. Later, they review the lesson, teach the revised lesson again, evaluate, reflect once more, and share the results.

These systematic practices increase the quality of teaching, the content of knowledge and improve the pedagogy techniques of teachers. It also develops mentoring skills among the group. Hence, the increase in teacher skills is the result of creating learning communities managed by the same teachers. Peer evaluation, read influence, becomes a cultural norm that is adopted by every group of teachers. These initiatives may bring the value of β higher, making influence more effective.

Another parameter of interest for policy is the influence delay, θ. This parameter models the handling time, the time needed by the influence to operate, making influence immediate or delayed. If a community is willing to bring joined efforts to improve the educational influences, through a conscious interest to improve education, the parameter θ may be reduced. For instance, consider a group of high school students that are exposed to a mentoring environment that shows that education is one of the most effective ways out of poverty. Encouragement to go to college on a permanent basis, makes a "strong enough" (small value) θ, increasing the likelihood to become college educated. The opposite might also occur, for instance, when a group of isolated high school students, within an unresponsive community, lacking mentors and permanent influence will tend to have minimum interest to attend college, driving a weak influence, a high value for θ. Different values of this parameter were used during simulation to test the results once the influence is strengthened or delayed. This might be the parameter affected directly by mentoring efforts.

Future Work

The age-structure of the population can be included to divide the community by age-compartments and address the challenges when a population faces an inverted population pyramid. These analyses may be conducted with the use of partial differential equations or the simpler approach developed by Hethcote [102].

The parameters can be also modeled to be dependent on other variables, for instance, income. College graduation rates are determined by income, a discussion that was open earlier based on considerations of affordability. Also, the average rate that individuals spend in the system (educational) given by $\frac{1}{\mu}$ might be dependent on family's income. Family's income is a pertinent discussion in the United States due to the way public schools are funded, which might be enhancing a system of limited mobility. The rate of loss of influence e is another parameter that might deserve attention for future work. The factors affecting this parameter relate to negative aspects that make influence lose its effectiveness. In fact, if this rate is large, it can completely eliminate the influence effect.

Further research can explore additional gains from college education, beyond income. For instance, associated higher levels of life expectancy, improved health indicators, higher creation of entrepreneurial activities, lower crime, conscious patterns of consumption, healthy eating habits, and more.

The need to understand mobility across ethnic groups helps identifying elements that seem to perpetuate low educational attainment, poverty, and inequality. There is still an important breach to be narrowed between ethnic groups. This note comes from a reflection about an updated report of the Kerner commission, a commission established in 1967 by President Lyndon Johnson to investigate the reasons of social unrest that originated riots in the United States and make recommendations about how to avoid a similar scenario in the future [103]. After more than 5 decades since the report, the African American community has improved its socio-economic situation partially [104]. Poverty has decreased from 35% in 1968 to 22% in 2018 but it is still almost three times the rate of poverty of White people. High school graduation has increased from 54% to 92% for the same period but still teenage pregnancy remains high, almost double than Whites'. College graduation rate has increased from 9 to

22.8%: it took almost 50 years to raise college graduation of African Americans by 13.8 percentage points. Unemployment is still a major problem for African Americans. Mean income has increased annually by 0.7% for African Americans vs 0.6% for White population but, initial conditions were dramatically distinct: mean income in White households is still 40% higher than that of Blacks'. In fact, the median household income of Blacks in 2018 is slightly superior to the income of Whites 40 years ago.

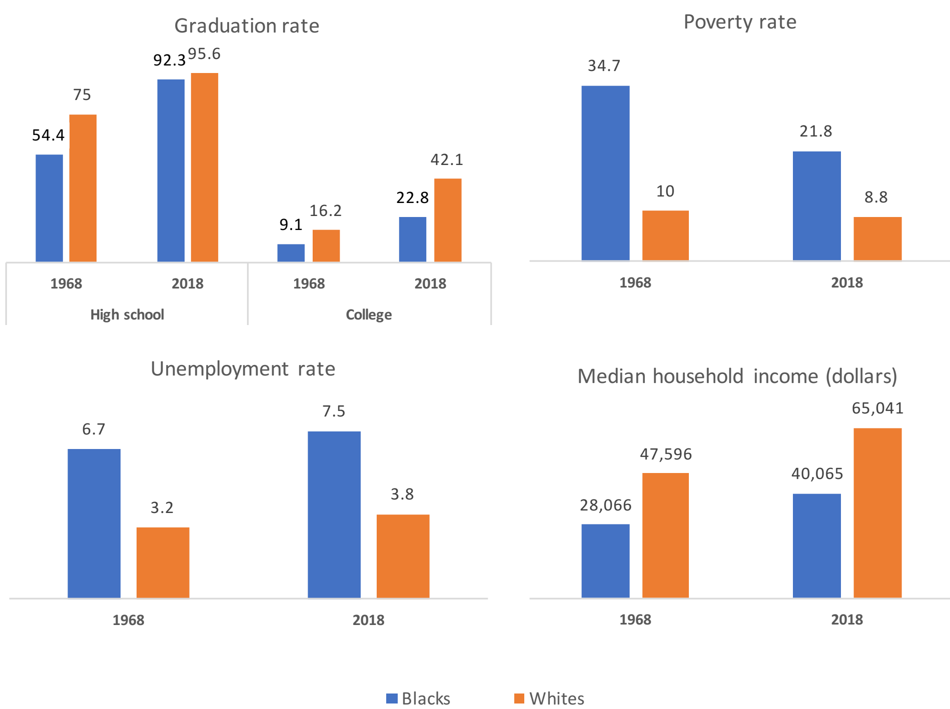


Figure 5.2: Socio-Economic Indicators for Black and White Population in the U.S. Source: Pew Research Center

College success also differs with ethnicity. For 2015, the National Center for Education Statistics reveals that 40% of 18-24 year-old enrolled in a degree-granting institution but rates differ as much as 42% for Whites, 35% for Blacks and 37% for Hispanics with a marked 63% contrast of Asians, as it is shown in Figure 5.3. Also, graduation speed differs among ethnic groups: only within 4 years, Whites had graduation rates of 44%, 21% for Blacks, 30% for Hispanics and 48% for Asians (Figure 5.4).

Unfortunately, as I complete this dissertation, the world is facing a pandemic of coronavirus that most likely change the world, including the role and delivery of education, as well as, the likelihood that we see of the acceleration of mobility over the next decade [105, 106, 107, 108].

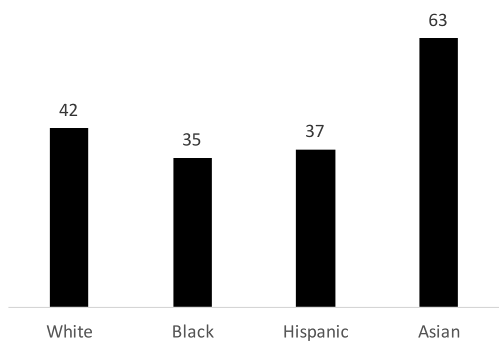


Figure 5.3: College Enrollment Rates of 18- to 24-year-olds by Ethnicity: 2015

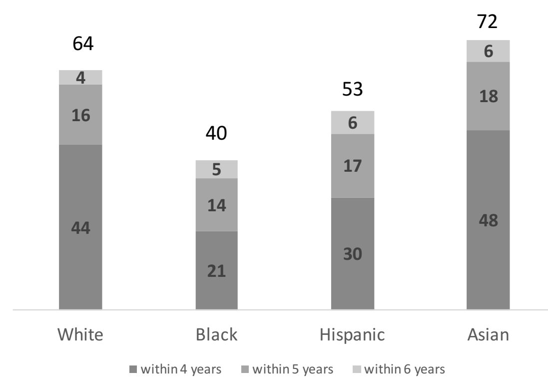


Figure 5.4: Graduation Rates for Full-time Bachelor's Degree-seeking Students by Ethnicity

We raise the concerns over ethnicity because the United States has been tremendously successful in attaining high average income over decades while lifting a large proportion of the population from poverty, but maintaining a strong middle class and closing ethnic gaps are still tremendous challenges that American society faces.

Future work may use and extend the model presented in this framework to analyze the social environmental effect generated by uneducated groups whose social conditions have deteriorated or those groups who have not experienced mobility as high as other groups. These might be a valid exercise to explore the resurge of a backlash effect from discontent groups.

Decoupled Model with an Exogenous Influence

The models in Chapters 2, 3 and 4 contemplate the influence generated by an internal factor given by the the density of successful individuals, namely, poor or rich educated. Education is considered as the proxy that generates social mobility. We emphasized that social upward mobility depends on multiple factors, including the "right" connections and "right" places that serve as circumstances that promote mobility. The framework may be modified to include additional factors that are exogenous, that is, generated outside the system. The system becomes decoupled from this internal influence and now, we assume that an external (economic) factor drives the influence over the uneducated. Hence, the changes in the probability of environmental influence contain this exogenous factor, which can be modeled using the following system of non-autonomous differential equations:

$$\frac{dU_R}{dt} = q_R\Lambda - \alpha_R U_R - \mu U_R, \tag{5.1}$$

$$\frac{dU_P}{dt} = q_P\Lambda - \alpha_P U_P - \beta U_P Q - \mu U_P, \tag{5.2}$$

$$\frac{dE_R}{dt} = \alpha_R U_R - \mu E_R, \tag{5.3}$$

$$\frac{dE_P}{dt} = \alpha_P U_P + \beta U_P Q - \mu E_P, \tag{5.4}$$

$$\frac{dQ}{dt} = r(t)Q(1-Q) - eQ, \tag{5.5}$$

The behavior of such systems depends on time at any given point and $\frac{dQ}{dt}$ captures the non-autonomous feature of the system. Consider, for illustrative purposes, that $r(t)$ represent the per capita growth of unemployment. Figure 5.5.a presents the relationship between unemployment and college enrollment in the United States from 1985 to 2019. During an economic recession, we observe an increase in the unemployment rate. The evidence points out that college enrollment tends to increase when unemployment goes up. When labor market perspectives improved after a recession, less individuals enrolled in college. Figure 5.5.b shows the relationship of unemployment and college enrollment over the same period. Even though the data does not present a clear trend, the information may suggest a positive relationship between enrollment and unemployment (correlation coefficient: 0.2). Economic literature also supports the claim that college enrollment decisions follow the cycle of unemployment. For instance, Barbu [109] found that there is a positive relationship " between the national unemployment rate and undergraduate enrollment... Enrollment of both males and females was found to increase by 1.2 percent and 1.9 percent, respectively, when the national unemployment rate increased by one percent". Also, Long [110] observed that "college attendance levels increased during the recession, especially in the states most affected in terms of rising unemployment and declining home values."

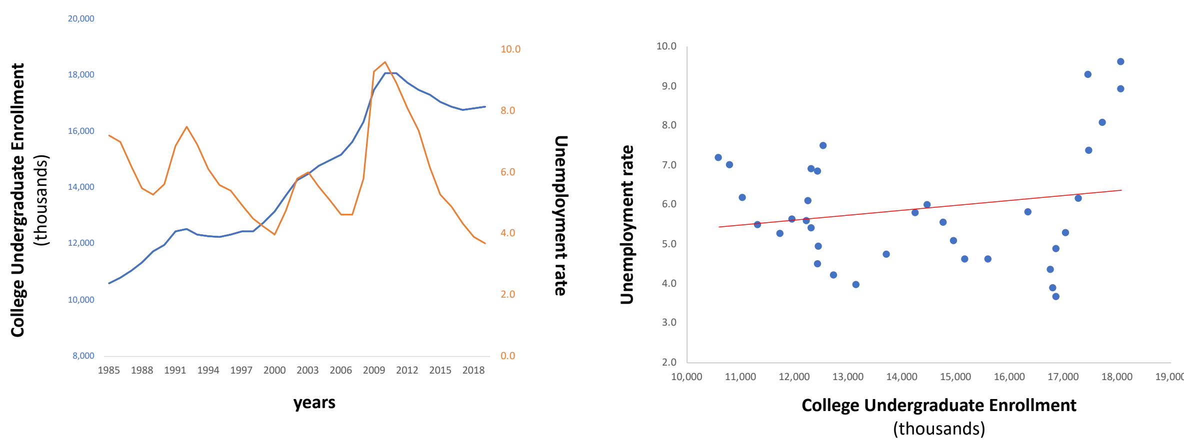


Figure 5.5: Unemployment and College Enrollment Rates in the U.S. Source: Bureau of Labor Statistics and National Center for Education Statistics.

In order to support the oscillating environment, Figure 5.6 compares the annual rate of unemployment in the United States over the period 1970-2020 (2020 is a forecast) with an oscillating function that tries to capture this cyclical evolution of unemployment. The function is represented by the simplest version of the sine function $r(t) = Asin(Bt + D) + C$, where $A = r_0$ gives the amplitude of the function, that is, the maximum variation. For an initial exercise, we approximate the maximum variation with one standard deviation of the series of unemployment, that is $A = 1.6$. The period represents the cycle which establishes the time from one peak to the next peak, period given by $\frac{2\pi}{B}$. For the 1970-2020 period, we can observe a period cycle of approximately 10 years, for which, $B = \frac{\pi}{5}$. C represents the vertical shift and it is parameterized by the average unemployment for the period, that is, $C = 6.15$. D corresponds to the phase shift, with $D = 0$ for an initial approximation.

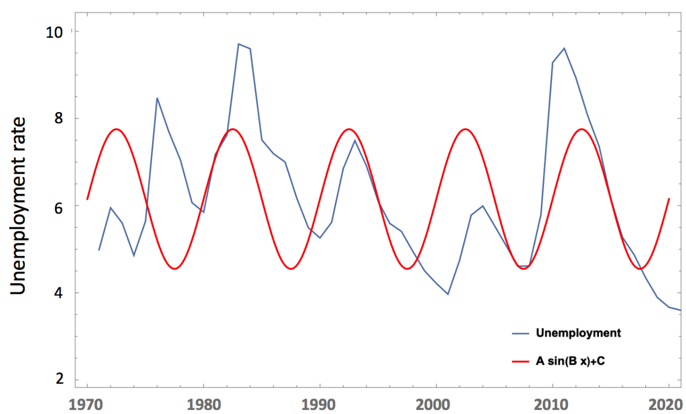


Figure 5.6: Annual Unemployment Rate and Oscillating Function. Source: Bureau of Labor Statistics.

The sine function may be far from being a perfect fit ($R^2 = 0.09$), but it tries only to offer a general idea of the cyclical influence of an exogenous factor, in this case, unemployment. Least squares minimization or summation of sine functions may be used, for instance, to obtain more robust estimates.

Figure 5.7 presents the simulations for both the oscillating environment and the constant population case. Note the oscillating behavior of the environment on figure 5.7.b. This environmental influence suggests that the cyclical behavior of unemployment determines the signaling for individuals to recognize a "good" or a "bad" moment to go to college. The amplitude of the function becomes smaller when the transmission is used to explained proportion of poor individuals that are becoming educated. This result is important because a targeted level may be achieved depending on the state of the economy. The oscillating economy, generated by the unemployment influence, may suggest that it is possible to achieve a goal of 30%, for instance, at year 25 but in eight years, the number increases and falls back to 30%. One could wait this 8 years and might reach the goal in year 33.

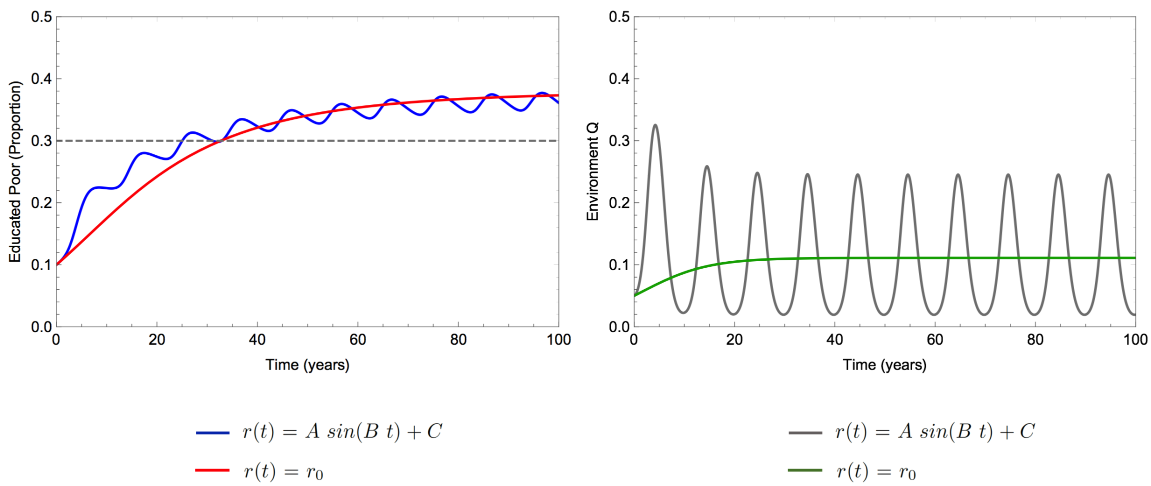


Figure 5.7: Simulation of Educated Poor and Environmental Influence in Non-Autonomous System with Oscillating and Constant Growth Environment.

The oscillating behavior may seem to maintain mean of unemployment which, will not have a major effect on the propensity to go to college. The effect of unemployment may alter this propensity if it changes the mean, that is, when incentives and disincentives are not symmetrical. The key message is that the framework using the modified version of the Levins equation, $\frac{dQ}{dt} = r(t)Q(1-Q) - eQ$, provides a flexible enough mechanism to analyze external factors with the introduction of non-autonomous system that may help to evaluate uncertainty. By providing distribution of variations over time, the model may alert planners to construct scenarios of high and low probability if realistic goals need to be attained.

The United States has used policy capacity to generate macro influences to incentive people to go back to college. Some of this initiatives have been put in place, for example, during the recession of 2014 with the Workforce Innovation and Opportunity Act for job training programs, or the Better Education and Skills Training for America's Workforce Act, or Reagan's Job Training Partnership Act of 1982 [111].

Chapter 6

EPILOGUE: PERSPECTIVES FROM AN ECONOMIST

In this final chapter, I would like to discuss several ideas about the role of mobility in society. Education has been long recognized as one of the most effective vehicles to move people out of poverty and different perspectives coexist on the role of higher education and mobility, with some even assuming that college education is unnecessary. Higher Education is considered by many if not all as a ladder that facilitates mobility. Barriers to higher education include cost, affordability, and the burden or absence of student loans.

Costas Azariadis et al [112] and Bowles et al. [113] identified a source that impedes mobility, namely, "the vicious cycle of poverty", a state perpetuated by low savings rates and poor human and financial capital accumulation. The scarcity of these resources reinforce low income cycles that persist over generations, the so called "poverty traps". We implicitly capture the current scenario by the *status quo* equilibrium, a state that reflects the lack of change in society. Having a low proportion of college graduates may reveal absence of influence. The unused talent not only reduces mobility but impacts the growth in the number of potential biologists, mathematicians, architects or other professionals.

Macroeconomic records do not register the loss of production that results from not having additional professionals. Certainly their loss of influence has rarely been considered. Figure 6.1 provides some multi national evidence. This linear relationship suggests that if the proportion of college graduates increase by 1%, mobility increases by 0.75 points. The deficit of graduates in STEM may provide an additional example of the possibly absence of environmental influence. A report by the U.S. Congress

Joint Economic Committee in 2012 [114] stated that " workforce was falling short of demand in both STEM and non-STEM occupations." The recommendations were to increase annual production of undergraduate STEM graduates by 34 % to match the proyected needs of STEM professionals in the United States [115].

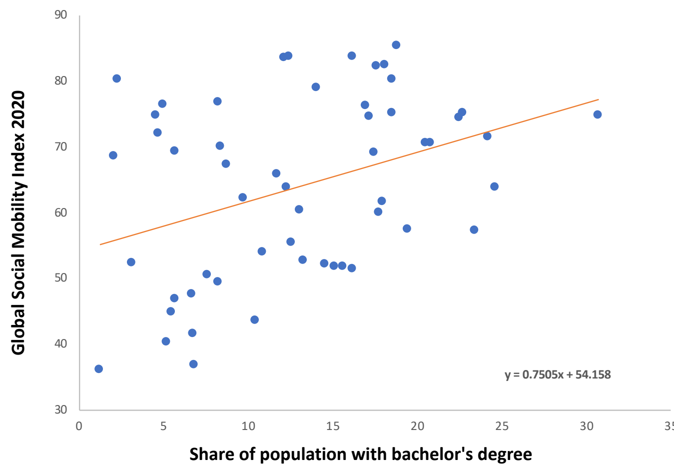


Figure 6.1: Proportion of People with Bachelor's Degree [5] and the Global Social Mobility Index [6].

Heterogeneity is another key factor affecting the dynamics of influence. Cultural and ethnic differences pose modeling challenges, since heterogeneity increases the complexity brought by relationships, as well as the norms and reciprocity by members of society. In addition, for instance, racial discrimination plays a role in the socio-economic proximity (or distance) to other groups. Education brings income differences down and may be a powerful generator of inclusiveness. On the other hand, discrimination has been a pervasive element of countries, a deleterious force to social mobility efforts. The system of casts in India, the discrimination of indigenous communities in Andean countries, African descendant groups or immigrants; are just some examples of the complexity that heterogeneous groups bring to the search of modes for increasing the mobility of all groups.

Alesina et al. [116] constructed an index of fractionalization to quantify the heterogeneity of a country. Indexes were estimated that considered ethnic, linguistic and religious fractionalization, indexes that consider the probability of finding a random individual who is not from the same group. Later, Alesina et al. [117] observed that "public goods provision is less efficient, social participation and trust is lower... and economic success is inferior in U.S. localities with more ethnically fragmented communities," that is, communities with indexes of fractionalization high. Welfare benefits are also found to be low in racially heterogeneous states [118]. Figure 6.2 shows the relationship of mobility with three categories of fractionalization. Countries with higher levels of differentiation in ethnicity (a) and language (b) tend to induce less mobility. Figure 6.2.c shows some evidence that more religious heterogeneity is associated with positive changes in mobility.

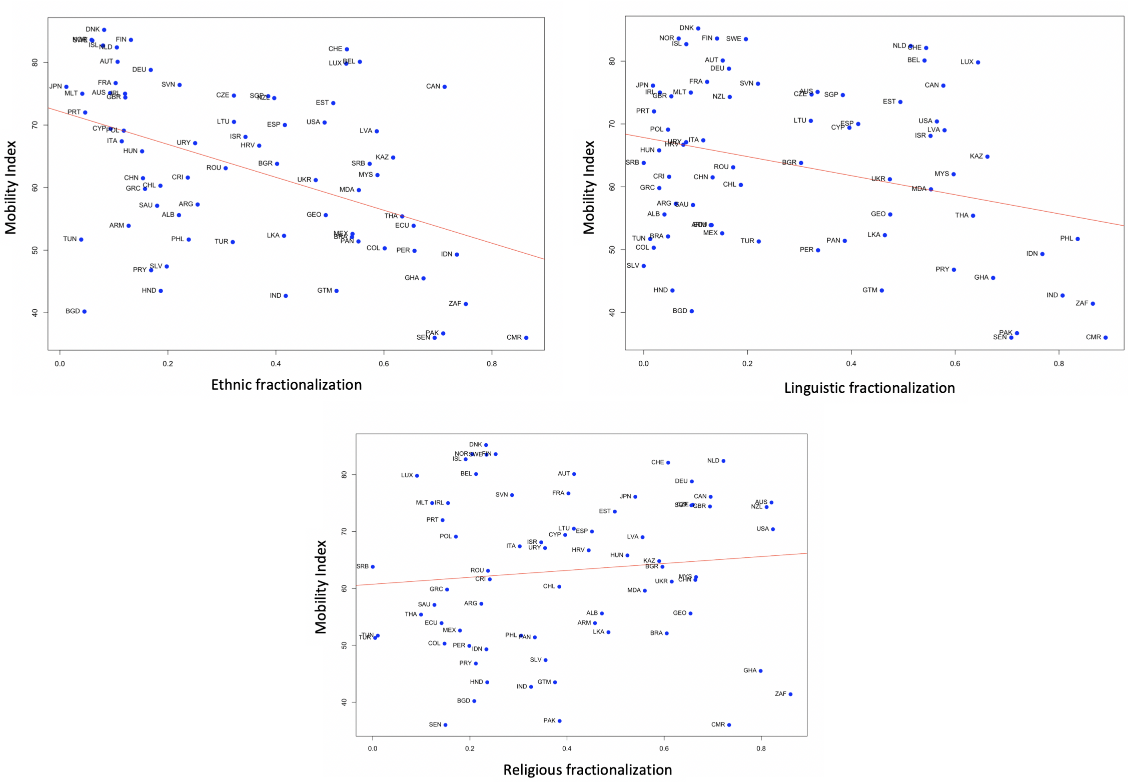


Figure 6.2: Social Mobility Index and Fractionalization Index by Ethnicity (a), Language (b), and Religion (c) [4].

Other forms of “neighborhoods” can be enlarged that strengthen the likelihood influence of college success among diverse groups. Students at Arizona State University may run into professors every day in this academic community. Every Tuesday, Edward Prescott makes the line to buy a coffee for the day. Students might not notice him buy, once he seats, he may have some time to talk about business cycles and macroeconomics or how his work made him recipient of the Nobel Prize of Economics in 2004. Couple of years ago, one could also meet Elinor Ostrom in the same line waiting for the same coffee at Arizona State. Young students would gather around her and ask questions about her experiences as a recipient of Nobel Prize in Economics in 2009. The attention even grew further when students got to know that she became the first woman to receive the award in this field for contributions about institutions and governance of the commons. The "environment" at ASU was intelligently designed so one could thrive in surroundings that promoted the exchange of experiences and ideas.

Ideas, knowledge and academic environments are public goods and the extent of public goods is still not well understood. A public good, such as, safety, clean air, an open park, knowledge or ideas; is characterized for being non-excludable and non-rival. They are known as non-excludable because nobody can restrict the use of the good (clean air) and non-rivalry refers to the fact that the consumption of the good does not reduce the amount of the good available for others, for example, an open public park where one can take the family and pets without impeding any neighbor to do the same. George Bernard Shaw put it in a friendly example, saying that "If you have an apple and I have an apple and we exchange these apples then you and I will still each have one apple. But if you have an idea and I have an idea and we exchange these ideas, then each of us will have two ideas." Public goods generate positive influences (an economist would call "positive externalities") that are difficult to price but they have a high value for society.

One of the challenges that societies face is that of developing these public environments, particularly those that influence people to generate changes of states (i.e. from poor to non-poor). The influence should be strong enough and continuous; if possible permanent. Retaining talent is a big challenge. Migration may become a threat, especially for communities that seem not to offer positive perspectives for job opportunities. Young people flocked to the Silicon Tech Valley in California or the environment-friendly business district in Colorado years ago because they foresaw an environment of positive influences. Small actions matter but there is a need for a critical influence. The owner of a house in Mumbai in India got tired that people throw garbage and use the corner of his house as a public toilet every day. The place was completely filthy so, one more person throwing garbage in the place was not a worry for anyone. Instead of becoming a bitter complainer of the situation, the owner decided to completely improve the corner of his house, by painting the place, adding flowers, offering a place that was worth passing by because it really improved all the neighborhood. People changed behavior and began emulating this person. The physical condition of a neighborhood made a difference in people's feelings and state of well-being. Residents of neighborhoods with green and clean areas report lower levels of depression and reductions of feelings of hopelessness or that of being worthless [119]. Influence was necessary to drive the behavior but one person is not sufficient to generate a sustained change. What if 1000 neighbors did exactly the same in a neighborhood?

Environmental aspects become part of the daily landscape and shape the customs of the population. For instance, seeing people throwing garbage in the streets looks sometimes so natural in some developing countries that children get used to it, hence, throwing garbage when they become adults may not be problematic. A long list of aspects that damage the aesthetics of cities can be named such as getting used to street dogs, excessive use of the horn, sexual harassment in the streets, etc. Corruption becomes another environmental influence in many countries. When people grow up in a social environment where it is viewed as "normal" to bribe the police, or to disrespect the authority, the adult generation most likely will develop and continue this destructive practice. Is there a critical influence that can generate the beginning of a change altering the *status quo*?

Society is exposed to forces that shape aspirations, particularly, those of the young generations. Sports, for instance, is one of the forces that bring high aspirations, and hopes that "impossible" dreams are possible to achieve. Art is also another form of these mobilizing forces. So is music. Education, science and technology are also part of these mobilizing forces that create a powerful demonstration effect of influence over communities. Higher education has spillover effects that may be difficult to quantify. But they exist. The non-monetary effects are included in the model as the proxy of a mechanism that generates mobility through the environmental influence of successful college educated people. I would like to introduce the study case of the generation of a massive academic neighborhood in my native Ecuador, and offer some insights about the potential effects that this could have created over the population.

6.0.1 The Environmental Influence of Higher Education in Ecuador

With approximately 17 million inhabitants, Ecuador is a South American country, categorized as an upper middle income country according to the World Bank [120]. Ecuador ranks among countries with high human development in place 85 (out of 189 countries), sharing this position with China. Poverty dropped from 37% in 2007 to 25.5% in 2017 and extreme poverty decreased from 16.5 % to 9.5% during the same period. Income per capita is $ 6,316 for year 2018, in contrast, for example, to $62.600 of the United States.

Ecuador, as well as other Latin American nations, has a long story of inequality and low social mobility. In fact, it is one of the "bottom economies with respect to mobility in Latin America along with Colombia, Guatemala and Panama" [2]. Low mobility suggests that child's future depend at a larger extent on the parents' well-being or what it is known as low opportunity.

Around 11% of the population has a college degree in Ecuador, a relatively low percentage with respect to the region as it is shown by UNESCO estimates in Figure 6.3.a. Population growth and changes in expectations have generated a stronger pressure on the demand for higher education. The proportion of population with a college degree has slowly increased over a decade as it is shown in Figure 6.3.b.

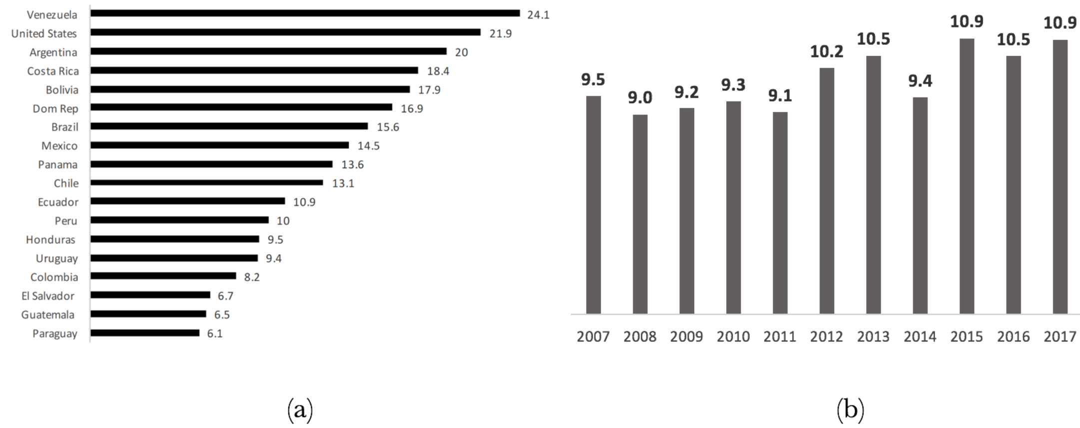


Figure 6.3: (a) Percentage of the Population with College Degree, Last Year Available. (b) Evolution of College Educated in Ecuador. Source: Unesco

Higher Education in Ecuador was characterized by a long tradition of academic stagnation, low levels of innovation and minimum research efforts [121]. Less than 5 % of professoriate at universities hold a PhD degree in year 2008 [122]. Ecuador had one of the lowest rates of graduation 3.5% vs 11% of Colombia, 21% of Cuba, 35% of the United States, according to the Unesco.

In 2008, universities were evaluated to determine the academic quality needed to strengthen higher education. Twenty six universities, out of 70, received a poor evaluation, 24 of them were privately owned [123]. These private universities proliferated during the last decades to absorb the large increase in demand for higher education. The final evaluation declared that their academic and professional strength was seriously questionable [124] and the recommendation was to improve their academic quality. For example, full time professor were less than 15%, there were several universities with 600 students per teacher and some universities did not even have 1 professor with a PhD degree. The majority of these universities developed activities in precarious premises including houses, warehouses or simply offices [124]. Fourteen universities had to close operations.

Figure 6.4 presents the evolution in the number of public and private universities in Ecuador. Twenty-eight private universities were created in the 90's. The sudden decrease in the number of private universities in year 2011 corresponds to the elimination of the 14 universities with the lowest evaluations. The slight increase in the number of public universities in 2012 corresponds to the creation of four universities to cope with the limited offer of higher education and most important, the raise the standard of quality of higher education. These universities were planned specifically to target four areas that have been always neglected, and included pedagogy, arts, mathematical and natural sciences. The plan materialized with their construction starting in 2012. They received the name of emblematic universities.

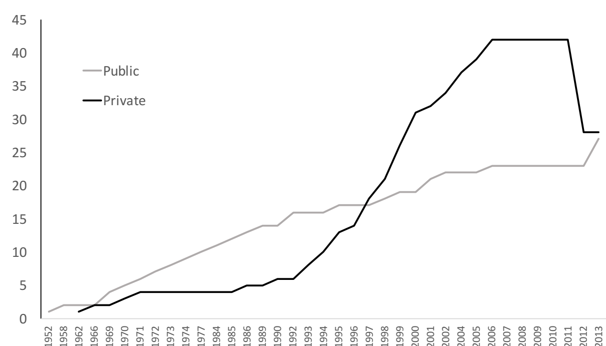


Figure 6.4: Number of Public and Private Universities in Ecuador

Yachay Tech University

Yachay Tech University was one of four universities created as a response to the crisis in higher education. It was aimed to motivate a new economy based on science and technology. One of the main objectives of Yachay Tech University was to enhance the education opportunities in STEM and most important, to set higher standards of science education. For instance, Yachay Tech was the first university in Ecuador that had 100 % of faculty with a doctorate degree, an unprecedented accomplishment in the country.

The Yachay Tech (YT) project was originally designed as a research environment. The idea included the construction of a planned city around the university, a concept that was used for Land Grant Universities in the United States to allow the creation of colleges in the United States to "promote the liberal and practical education of the industrial clases." The Morrill Act was first passed in 1862, signed by President Lincoln [125] and it created the base of a large influence to become educated. The YT project was intended to create a large system of knowledge based on already successful initiatives like the North Carolina Research and Education Network (NCREN) in the United States, the Science and Technology Park Berlin-Adlershof in Germany, the Skolkovo Institute of Science and Technology in Russia, and the Korea Institute of Science and Technology in South Korea.

It is probably too early to evaluate results in the project but after few years since it started, Yachay Tech has obtained important academic recognition, such as second place in the ranking of the Nature index for Ecuador in 2018 and third place in 2017, second place in 2016 in less than 3 years of operations. The university has carried out a large number of project with students that have received attention, including funding from international grants. Students have received international awards such as, Seed for the future of Huawei, and also scholarships in South Korea, Germany and the United States.

Scopus is a database collecting scientific documents that cover areas of physical, life, social, and health sciences. Scopus publications are a reference of scientific generation within the academic community. Figure 6.5 presents the evolution of Scopus documents for the search of Ecuador. The information is limited to articles and conference papers only. The figure shows the beginning of an important increase of research publications in the year when Yachay Tech University initiated activities in 2014. The project alone cannot claim all the research work but this change is associated to the creation of emblematic universities and the state investment in higher education during previous years. The number of research work involving Ecuador multiply by 12 from the period 2007 to 2017. Figure 6.5 may offer an idea that the presence of a university with high standards and highly ambitious scientific goals drove change to the whole higher system of education. Influence can take place at many levels.

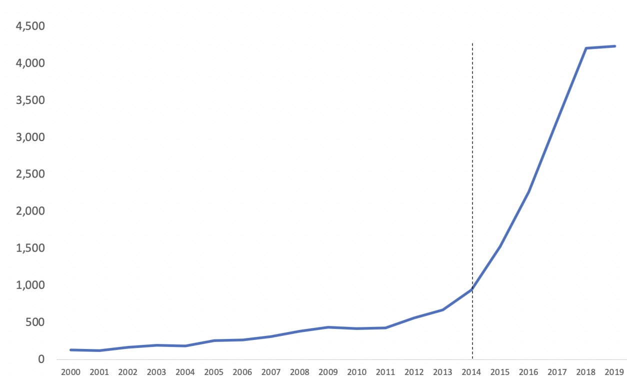


Figure 6.5: Scopus Publications Involving Ecuador.

Before 2007, Ecuador had a university system that became a source of social "immobility". Private universities were costly and public alternatives perpetuated a system that lacked excellence. Additionally, students seeking to study at public universities had to go through a lottery system that assigned a seat to study at university, regardless of the student's academic past. Hence, speaking about meritocracy was not even an option but a utopia.

The original conception of YT University was to break that reality of lack of quality and meritocracy, a result that this dissertation identified it as the *status quo*. The government intended to foster an "... economy of finite resources to the economy of infinite resources, an economy based on human talent, science, technology, ideas" [126]. The university brought international talent in STEM that quickly started to build the first research university. Accepted students to Yachay Tech were offered full funding. To be accepted, students were required to obtain the highest grades at the national exam of admission, an exam that started in 2012. Economic (initial) conditions of students' families were also taken into consideration to consciously promote equality, especially benefiting less privileged groups.

The drivers of mobility changed in Ecuador with emblematic universities. A couple of years ago, migrating to the USA or Europe was considered a realistic path to get out of poverty. After a banking crisis in 1999, consequence of corruption and economic crisis, 2 million Ecuadorians left the country, mainly to the U.S. and Spain. Immigrants became a source of influence for the young generations. The prosperity produced by migrants then became an example of environmental influence to produce non-poor individuals. The mechanism of influence did not focus to create more educated but income-earning individuals. Remittances from migrants peaked as it reached 7.2% of GDP in 2000.

The project YT intended to create another environment of influence, to attract talent and serve as a source to generate mobility. The inauguration of Yachay Tech University on the 31st of March, 2014 with 187 students, brought hope to thousands more of young people who believed that hard work pays off, in a country with large income differences where being successful was determined more by family background. In the initial speech, president Rafael Correa mentioned that the country was "... inaugurating a new era in the university history of the country, on the path to the knowledge society, to overcome poverty, the moral imperative of our time... Ecuador ... a sovereign country, has decided to base its development on the only inexhaustible source of wealth, which is human talent and knowledge. Yachay Tech is not just a university. It will be the City of Knowledge ... serving the development of science and technology. . . . They [politicians] told us that it was impossible. Here, we are proving that it is perfectly possible to have a world-class university."

Yachay Tech generated enough influence to make students aspire a college degree. The international faculty that came to the university also created an environment that fostered scientific research. *Status quo* equilibrium was being broken.

The United States also faces an enormous challenge to increase fairness at colleges. According to the study by Fox et al. [127], the socioeconomic status of the family is more important than scores at the moment to predict graduation from college. Figure 6.6 presents how family background matters more than grades for college graduation. Having the best grades at school and being part of a poor family is not as important as having the lowest scores but being part of a rich family. Students lacking proper initial conditions, say enough financial support, may not complete college even with all the talent that they may have. Further, college students experience a crisis of dropouts that leaves them with a combination of debt and most sadly, without a degree [128]. Barriers are already imposed before the students get to college and; tuition and limited financial options become mechanisms that amplify the gaps.

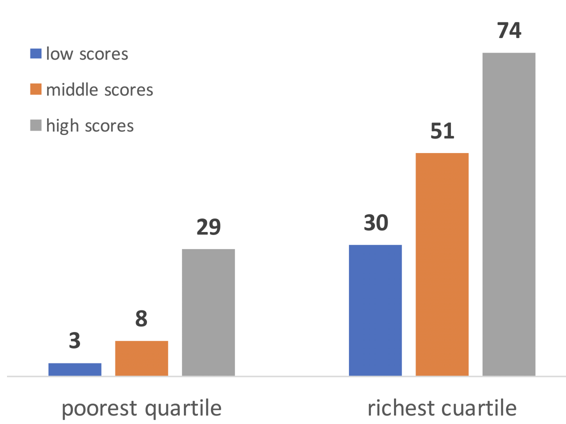


Figure 6.6: College Graduation Rates, Family Background and Test Scores

6.0.2 *The Environmental Influence of the MTBI and JBHSP Programs*

The Joaquin Bustoz Jr. Math-Science Honors Program (JBMSHP) and the Mathematical Theoretical Biology Institute (MTBI) are two examples how policy may enhance the strength of the environmental influence to increase college success. These are two programs based on Arizona State University that served young generations, who are typically underrepresented in STEM.

The JBMSHP is a summer program in mathematics to strengthen the motivation and interest of students before they graduate from high school in careers requiring mathematics, science, or engineering-based coursework. Participants include first-generation college students. They are selected from diverse backgrounds from high schools throughout the State of Arizona, including rural communities and the Navajo Nation. Additionally, they work in groups to complete a research project that is presented in a final oral presentation.

Since 1985, 2,820 students have participated in the JBMSHP. 58% of the participants have been female. 50% are Hispanic, 16% are Native American, 13% are Asian, 13% are Caucasian, and 8% are African American. Students who attend the JBMSHP may earn as many as 11 university credits before first-time freshman in any university. 71% of all JBMSHP alumni currently enrolled in ASU picked majors in Science, Technology, Engineering, or Mathematics. JBMSHP alumni consistently earn higher grade point averages when compared to non-JBMSHP students at Arizona State University, 13% higher within the College of Liberal Arts, 11% higher within the Ira A. School of Engineering, and 14% higher within the Mary Lou Fulton Teachers College, as of fall 2018. 59% of JBMSHP alumni who have earned a degree from ASU are female, 52% are Hispanic, 18% are Asian, 11% are Native American, 12% are Caucasian, and 7% are African American. Since the program initiated in

1985, 73% of all JBMSHP alumni who attended ASU earned a degree from ASU.

The MTBI is a summer program that provides a research experience at the undergraduate and graduate levels, in the field of applied mathematics and its applications to the biological and social sciences. The program consists of 6-8 weeks of residential training involving intense mathematics including a course in mathematical biology. One of the most important aspects of MTBI is the research mentorship training [129].

Since 1996 to 2019, 532 regular first-time undergraduate students and 83 advanced students have participated in MTBI, with 68% coming from underrepresented minorities. 283 out of 420 (67%) of U.S. MTBI student participants had enrolled in graduate or professional school programs. To date 132 US MTBI student participants have completed their Ph.Ds, 97 of whom are URMs; 74% of US MTBI Ph.D. recipients are URMs. 118 from the 132 US MTBI student participants, 90%, who have earned Ph.Ds. are from an underrepresented minority and/or underrepresented group.

Programs like the Joaquin Bustoz Jr. Math-Science Honors Program and the Mathematical Theoretical Biology Institute became a mechanism that serve to enhance the “creation” of new college educated individuals through environmental influence thanks to the creation of a community that supported students with mentoring. MTBI and its founder Carlos Castillo-Chavez has received two awards for the Presidential Awards for Excellence in Science, Mathematics and Engineering Mentoring (PAESMEM) in years 1997 and 2011 and, it has left a legacy in hundreds, if not thousands, of young people [130]. MTBI and JBMSHP are a formidable example that environmental influence may be generated through conscious and institutionalized efforts to change the reality of a *status quo* that has seemed to be difficult to overcome.

6.1 Model of Demonstration Effect: Rich-Middle Class-Poor and Influence from the Rich Educated with Income Mobility

College education operates as a proxy for mobility within classes, that is, from uneducated to educated, this last representing a state where the individual has a higher income. Another potential extension of the model may emphasize mobility between classes, that is, the recognition that an individual may move from the poor educated compartment to the middle class with certain probability and also that, middle class may move to the rich class. The opposite also can be included: rich becoming middle class or poor. Recall that in the U.S., there is a 10% probability that a rich-born White man becomes poor when adult and 21% probability that a rich-born Black man becomes poor. A brief description of a future project is presented in the next subsection.

The models presented until this point considered education as the unique source of mobility. In fact, we referred to mobility as educational mobility and by being more educated, opportunities to increase income come naturally. In this section, we include a slight detail that reinforce the system of communication (feedback) between the socio-economic classes. Consider first, that there exist some scope to move from one class to another class, so then, individuals may potentially change from, for example poor to middle class ($E_P \to E_M$) and from middle class to rich class ($E_M \to E_R$). The reasoning behind is the fact that individuals obtain education to obtain more income, so, there has to be a linkage between them. The central idea is to reinforce the message from the educated class influencing the rest of population. We then, make an assumption about the mechanism to change class. First, we ignore any possibility for the uneducated to become rich or middle class, that is, changes happen only after obtaining college education. However, we are going to assume that there is some possibility for the educated poor to move, at least, to the level of the middle class and also that the middle class may have the possibility to move to the rich class. The model assumes that the mechanism is just a random proportion, as if it is the case of a lottery, then becoming rich (from middle class) or becoming middle class (from poor) happens through a lottery. The individual may become a candidate to be rich at the rate w_R or be middle class at rate w_M. Figure 6.7 contains the flow diagram of the model with class mobility.

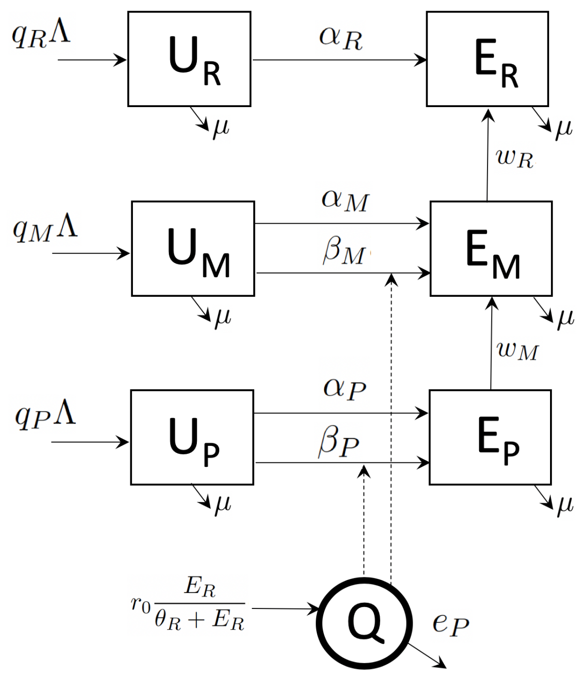


Figure 6.7: Diagram Flow of Model with Class Mobility.

Social systems are driven by more complex mechanisms. For instance, an individual moves from one class to another if it has the "right" connections or if it went to the "right" school, work in the "right" industry or live in the "right" neighborhood. There is an unlimited number of sources that determine mobility and, we do not know who goes to the the "places" but the mechanism is just a proportion. This narrative then makes mobility happen across classes.

The demonstration effect that rich may exert over the other classes becomes higher when mobility across classes is included, since the rich act as a catalyst for emulation. The identifiable success of the rich class contribute to the demonstration effect that may motivate other individuals to replicate the educational behavior. Thornstein Veblen and James Duesenberry recognized a "demonstration effect" on consumption habits of individuals that inspire other to emulate these patterns [131]. Then, the higher the probability to move to a different class, the greater the contagion effect of education. Hence, social mobility within classes occurs through education and mobility between classes happens through the proportion "lottery" mechanism. Becoming educated does not just allow you to enjoy the benefits from just being educated and having more income. When class mobility enters into the model, we make the demonstration effect become bigger and bigger. If such effect is effectively becoming larger due to the demonstration effect, then the target goal of reaching a specific percentage of educated poor can be reduced. Under this considerations, the system of differential equations representing model with class mobility is given by:

$$\frac{dU_R}{dt} = f_0 U_R \left(1 - \frac{U_R}{K_R}\right) - \alpha_R U_R - \mu U_R, \tag{6.1}$$

$$\frac{dU_M}{dt} = h_0 U_M \left(1 - \frac{U_M}{K_M}\right) - \alpha_M U_M - \beta_M U_M Q - \mu U_M, \tag{6.2}$$

$$\frac{dU_P}{dt} = g_0 U_P \left(1 - \frac{U_P}{K_P}\right) - \alpha_P U_P - \beta_P U_P Q - \mu U_P, \tag{6.3}$$

$$\frac{dE_R}{dt} = \alpha_R U_R + w_R E_M - \mu E_R, \tag{6.4}$$

$$\frac{dE_M}{dt} = \alpha_M U_M + \beta_M U_M Q - w_R E_M + w_M E_P - \mu E_M, \tag{6.5}$$

$$\frac{dE_P}{dt} = \alpha_P U_P + \beta_P U_P Q - w_M E_P - \mu E_P, \tag{6.6}$$

$$\frac{dQ}{dt} = r_0 \frac{E_R}{\theta_R + E_R} Q(1 - Q) - e_R Q \tag{6.7}$$

with w_R representing the per capita rate of mobility from middle class to rich and

w_M as the per capita rate of mobility from poor to middle class. These rates open the possibility of social upward mobility between classes and the only idea to be explored in a simple simulation for this case is aimed to show that the feedback system may be larger when individuals have the possibility to reach the rich compartment.

We cannot obtain explicit solutions for equilibria of the system of equations. The analysis will be based on numerical simulations Figure 6.8 presents the proportions of educated individuals according to socio-economic status.

Three simulations are overlaid for every socio-economic compartment. Educational mobility refers to the base model suggesting mobility within classes, that is, $U \rightarrow E$. Class mobility corresponds to base model with progression between classes, that is, $E_P \rightarrow E_M$ and $E_M \rightarrow E_R$. Finally, class mobility including logistic demographic growth. Simulations show that the opportunity of class mobility increases the proportion of individuals reaching the rich compartment. However, population growth complicates the dynamics and the proportion of educated rich may not be as high as with constant population. The proportion of educated poor decrease with class mobility, since people move to middle class expect when we make population grow. Middle class maintains relatively stable proportions except under population pressure, reaching lower proportions. Finally, mobility between classes and population growth creates a larger feedback into the environment.

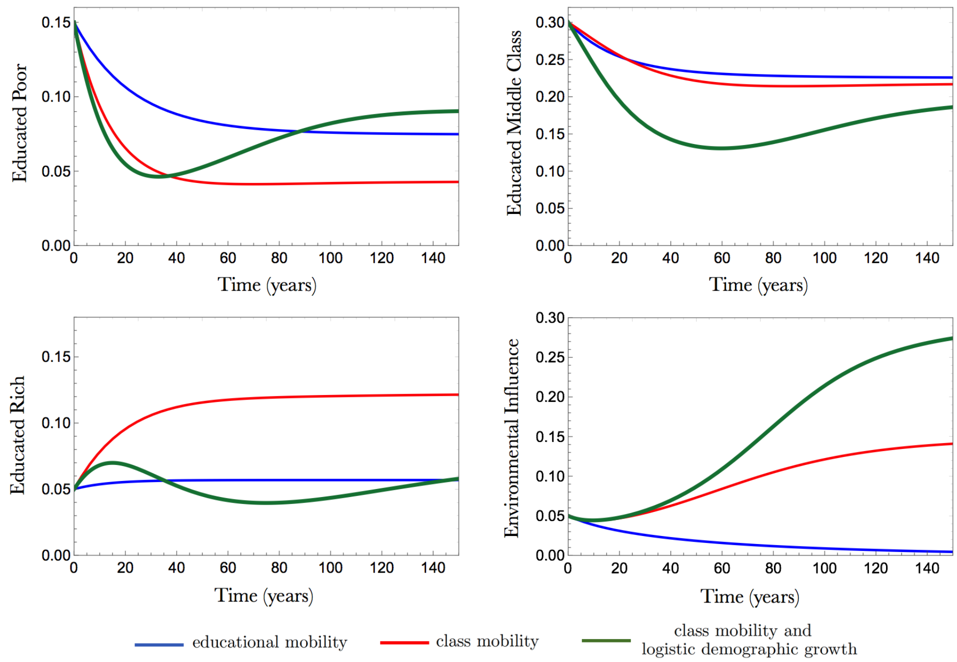


Figure 6.8: Educated as a Proportion of the Population Under Educational Mobility, Class Mobility and Logistic Demographic Growth $w_R = 0.006$ and $w_M = 0.05$.

6.1.1 Final thoughts

Catastrophes interrupt the efforts to increase social upward mobility. Low income individuals are more vulnerable to disasters and they probably experience the hardest consequences during the impact of emergencies. The lack of preparedness, responsiveness, alternative housing, sources of income, safety nets or insurance makes the most vulnerable members of society, the poor, limited in their ability to respond adequately in cases of emergencies.

A magnitude 7.8 earthquake struck Ecuador in April 16, 2016. The death toll reached 673 people. It also costed around 3% of GDP of that year, making one of the most severe crises that Ecuador faced in the last decades. The earthquake affected the poorest areas in the coastal side, especially with physical damage of houses that were poorly constructed with materials of low quality in places that were not suitable for construction. The poor communities lacked a proper system of water and sanitation, with the state of emergency the situation deteriorated even further. The lack of insurance system worsened the socio-economic status of many families that lost their physical properties. The government response tried to ameliorate the crisis with financial support, food supplies, credit and alternatives for housing but families will take years to recover.

Various countries depend largely on oil revenue. The World Economic Forum registered that, for example, in year 2013, oil exports represent 57% of total exports for Ecuador, 88% for Nigeria, 98% for Venezuela [132]. Oil exports connect with oil revenue that the state receives since, in these three countries, the state is the main stakeholder. State revenue faces high risk losses that are tied in to the oil prices. Large variations of oil prices increase the vulnerability of the State to plan infrastructure and social outlays, such as the price drop in year 2016 from $43 to $23 per barrel, almost a 50% decrease that additionally happened amidst the earthquake disaster. Oil prices fell 64% from $ 61.5 in December 2019 to $ 21.9 in March 2020, that will generate a large income fall for the government in the middle of a global health threat due to the coronavirus. The shortage of revenue threatens to decrease social expenditures, particularly education and health, lowering the chances of the continuation of social mobility.

Finally, an increasing global threat developed since December 2019 with the epidemic of coronavirus, now officially categorized as a pandemic by the WHO. As of March 23, 2020, Coronavirus has caused more than 370,000 cases since December 2019, with more than 16,000 deaths [133]. Countries with improved health systems, commonly rich countries, have larger capabilities to respond faster and more effectively. Poor countries face a more delicate situation. The economic cost of social distancing is unprecedented. Poverty and unemployment will increase and most vulnerable individuals will face the hardest consequences, especially those living daily by the paychecks. The coronavirus emergency opened again the need for a massive and urgent net of social security, access to affordable health services, particular for homeless, undocumented immigrants, the elderly and unemployed. The future of young generations was already difficult due to globalization. Now uncertainty has increased and it may affect the perspectives of social mobility.

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