Early Wing Structural Design for Stiffness and Frequency Response

by

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ABSTRACT

This paper describes an effort to bring wing structural stiffness and aeroelastic considerations early in the conceptual design process with an automated tool. Stiffness and aeroelasticity can be well represented with a stochastic model during conceptual design because of the high level of uncertainty and variability in wing non-structural mass such as fuel loading and control surfaces. To accomplish this, an improvement is made to existing design tools utilizing rule based automated design to generate wing torque box geometry from a specific wing outer mold-line. Simple analysis on deflection and inferred stiffness shows how early conceptual design choices can strongly impact the stiffness of the structure. The impacts of design choices and how the buckling constraints drive structural weight in particular examples are discussed. The model is then carried further to include a finite element model (FEM) to analyze resulting mode shapes and frequencies for use in aeroelastic analysis. The natural frequencies of several selected wing torque boxes across a range of loading cases are compared.

This Thesis is dedicated to my wife for her patience and support as I pursued my graduate studies. Thank you for your love and support when I needed it most.

ACKNOWLEDGMENTS

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LIST OF SYMBOLS

E	Modulus of Elasticity
EI	Flexural Rigidity
F_{ty}	Tensile Yield Strength
F_{cy}	Compressive Yield Strength
F_{su}	Shear Ultimate strength
I_{xx}, I_{yy}	Area moment of inertia
J	Polar moment of area (Torsion Constant)
M	Spanwise bending moment
m	Mass
x	Spanwise displacement
Δx	Displacement increment
ρ	Density
μ	Deflection
$\Lambda_{c/4}$	Sweep at indicated chord location
i (subscript)	Variable number
ω_{sp}	Longitudinal short period frequency
ω_{dr}	Dutch-roll frequency

INTRODUCTION

In typical aircraft design, engineers commonly use empirical weight estimation methods in the preliminary design process. As challenging as it can be to estimate the structural weight of a wing, these empirical methods do not capture important effects such as the impact of design strategy upon wing stiffness. Aeroelastic design considerations are usually left off the table until they are determined through high-fidelity analysis in detail design; this occurs after many design choices have already been fixed.

This often leads to problems with aeroelasticity of wings, which is patched up with a fix late in detailed design rather than mitigated throughout the design process. Stiffness analysis is difficult early in the design process because of the level of uncertainty in the design.

The author believes that existing empirical models simply do not provide the answer.

In this work, I seek to understand and document the impact of important preliminary design decisions upon wing structural weight and aeroelastic stiffness. This paper extends the legacy of several rapid, rule-based structural design systems, developed previously at Arizona State University. See works by Takahashi & Lemonds[1][2], Anderson[3] and Allyn[4].

In this work, I enhance an existing wing structural synthesis model[1] with a lumped-mass / beam-element model to estimate structural stiffness, static load-deflection and frequency / modal analysis. It is then possible verify the static and dynamic behavior using higher-fidelity structural analysis tools. The early results of this model enhancement limited to load and deflection were published in a collaborative work with the author and Takahashi[5] earlier in 2018, leading up to the culmination of this work. The novel contributions of this culminating work include the following:

1. Automating inputs of Excel-based design tool in modeling software *ModelCenter* to allow enumeration of many designs for comparison,
2. Addition of wingtip-deflection calculator to design tool
3. Addition of torque-box approximate calculation of stiffness parameters I_{xx}, I_{yy}, and J, and
4. Generation of multiple beam-element with lumped-mass stick models for modal analysis.

Results and conclusions from the novel contributions and how they build upon past works are discussed.

An empirical weights approach to conceptual and preliminary design requires the detailed design team to engineer a production structure to a strict, allocated weight requirement. The ability for a structural team to achieve these goals varies from project to project. The story of the Lockheed C-5 is the most famous example of a system-level program failure stemming from a desire to meet weight targets. The original C-5 wing was designed to meet the allocated weight target at the expense of a usable fatigue life. At great expense, the government[6] re-winged the entire early C-5 production run by swapping out the original light-weight wing for a heavier, more structurally sound wing.

To improve the design process of a wing structure to include stiffness analysis and predictions, a rule-based structural synthesis tool is employed, enumerating many possible torque box configurations.

Initial modeling by enumeration of a multitude of feasible design choices such as rib and stiffener spacing will later be incorporated with uncertain non-structural mass estimates to provide a prediction of the aeroelastic effects.

The end goal is to examine the impacts of:

1) rib spacing
2) stiffener spacing strategy
3) design aerodynamic loads
4) non-structural mass (inertial relief)

on the predicted structural mass of the primary wing torque box, its transverse stiffness, and its structural response to forces (simple oscillatory loads and aeroelastic) for any given wing geometry (planform and thickness distribution).

By developing this capability, I will document:

1) system level trends in weight and overall stiffness
2) the detailed structural layout resulting from a specific material, rib and stiffener spacing strategy
3) the spanwise distribution of "stiffness" of the equivalent beam formed by the torque box
4) the free natural frequencies and mode shapes exhibited by this beam under various loading conditions

This innovation lets us use rapid computational methods and modeling tools to provide insight that allow the engineer to "design for stiffness" and hence "design around aeroelastic constraints" earlier in the system life cycle.

The need for structural analysis in conceptual design with a rapid turnaround time is apparent because it is simply impractical to work the time-intensive detailed design on more than a very few discrete designs. Schedule and cost limitations on most engineering programs will limit even large design teams from developing more than a few designs out to reasonable confidence. Hence, a good conceptual design which can incorporate structural elements will prevent the burden on aircraft designers being locked into a point-design where any weight shortfall will directly and negatively impact system performance.

LITERATURE SURVEY - PRIOR ART

It is not a new idea to use a computer to help produce a reasonable structural layout based upon a simple, conceptual sketch of an aircraft.

Over the years, many programs have been developed to help streamline structural design of aerospace structures.

General Aerospace structural design and optimization methods

First, aerospace vehicles are expected to WORK – that is that they need to be strong enough to support all anticipated flight loads (aerodynamic and inertial) without structural failure while being light enough so that the machine can actually fly and accomplish its intended mission.

Secondly, aerospace vehicles are expected to LAST – that is, with reasonable preventative maintenance, that they should not degrade due to age and environment. For example, the United States Air Force[7] expects to operate its B-52 fleet on into the

2050's when the youngest airframe will be over 80 years old. Durability implies a resistance to corrosion and a resistance to structural failure from fatigue stress.

Thirdly, aerospace vehicles are expected[8] to be INSPECTED and REPAIRED – that is, they need to be able to be taken apart and put back together again. While small subsystems may be considered "disposable" - i.e. if it breaks, take it out and install a "new" one - large scale aerospace structures are expected to be repaired in-place.

Empirical Weight Estimation for Conceptual & Preliminary Design

Weight estimation in aircraft design and production is not an easy task, as described by Takahashi in Volume 1 of his book *Aircraft Performance and Sizing*[9]. However difficult it may be, it is equally important, because many of the performance parameters are based on weight. The amount of fuel on board to accomplish a mission, the range of allowable missions, the maximum payload, and many other performance parameters will largely depend on the Operational Empty Weight (OEW) of the aircraft. It may seem simple enough, but for the fact that this weight estimation is needed far before the time when the aircraft weight can be accurately determined by scale. When a structural weight estimation is incorrect, and the actual weight comes in heavier than expected the negative impacts ripple through the performance measures.

Past aircraft manufacturers have published their production aircraft weight breakdowns. Empirical prediction models have been constructed from this publicly available history of aircraft weights. One of the famous sets of equations, and one seen as particularly useful by Takahashi, are the equations first published by Torenbeek[10] and later revisited in Niu's[11] famous text, *Airframe Structural Design*. Included in these equations, and of particular interest to this work is the prediction equation for the overall weight of a wing, see equation 1.

$$W_{WING} = 0.81 \cdot \left(\frac{ULF * MTOW * Sref^{0.6}}{TCE}\right) * \left(1 - \frac{W_{BMR}}{MTOW}\right) * (1 + TR)^{0.4} * \left(\frac{AR^{0.5}}{TS^{0.2} * \cos(\Lambda_{c/2})^{1.2}}\right) + 3.3 * S_{spoiler} + 3.28 * \left(S_{LEflaps}\right)^{1.13} \quad (1)$$

Where:

ULF = Ultimate Load Factor Nzmax multiplied by the design factor of safety
MTOW = Maximum Takeoff Weight in lbm
Sref = Wing Planform Area in ft^2 = b*(1/2)(ctip+croot)
b = Wing Span in ft (tip to tip)
ctip = Wing Tip Chord in ft
croot = Wing Root Chord in ft
TCE = Effective Thickness Ratio = 4/5 (t/croot + t/ctip) * 100 (typically a value between 120 and 250)
WBMR = Weight of inertial bending-moment-relief items in lbm - Includes the weight of wing mounted engines and pylons, the weight of wing stored fuel, the weight of wing mounted landing gear (an amount that can easily comprise 30% of the MTOW of the aircraft).
TR = Wing Taper Ratio = ctip / croot
AR = Wing Aspect Ratio = b / ((1/2)(ctip+croot))
TS = Design Tension Yield Stress of the Wing (Typically 48,000 lbf/in^2)
Lambda_{c/2} = Wing Half Chord Sweep Angle (remember to convert from degrees to radians)
Sspoiler = Spoiler Area in ft^2 for the entire wing
S_{LEflaps} = Leading Edge Flap Area in ft^2 for the entire wing

Although it is clear from the complicated form of this regression-based equation that it does not have a direct physical background, Takahashi does point out the following trends in his text;

> "the wing grows heavier as 1) the planform area increases, 2) the aircraft grows heavier, 3) the wing thickness decreases, 4) the aspect ratio increases, 5) the wing sweep increases, 6) the aerodynamic complexity increases (bigger spoilers and larger, more complex flaps)."[9]

In the results section I will examine how Torenbeek's weight estimator applied to one test case compares with the results from our model.

Physics Based Detailed Structural Design

Current advances in technology allow for improvements to the design process in the aerospace industry. Automated models which allow for design trades and choices to be rapidly analyzed can improve the design process and reduce the need for later and costly rework once basic parameters have been locked down. Anderson[3] summarizes some of the historical advances beginning from Dr. Richard H. MacNeil's[12] approach to automated structural design in 1956 through current work from aerospace industry leaders such as Lockheed Martin and the National Aeronautics and Space Administration (NASA).

The focus of these automated design aid processes and tools is not to complete detailed design of structures, but rather to provide for further analysis to inform trade studies and decisions. Similarly, the work of this paper is not intended to provide the final layout of a wing structure, but rather to provide early stiffness predictions of a wing box. By determining how early input decisions affect the outcome, I aim to look forward and design to aeroelastic effects rather than apply late wing fixes when wing flutter problems are discovered.

Rule Based automated structural design

The wing structure design and analysis tool is an evolution of the rule based automated structural design tool created by Takahashi & Lemonds[1][2], also described in chapter two of *Aircraft Performance & Sizing Volume 2*[13]. In rule based automated structural design, the wing torque box is sized based on the shear, tension, compression, and buckling that the wing is expected to experience throughout the flight envelope, to include both high gee loading and hard landing, with transport category required factor of safety applied. As described by Takahashi & Lemonds, skin thickness, transverse wing stringers, and wing spar caps are selected to carry the different loading cases at a minimum weight design. These parameters are changed on a bay-by-bay basis, to remain constant between sequential rib sets.

One of the challenges in designing a wing structure is understanding the loading conditions, especially considering the highly interdependent nature of the loads with wing structural weight and total aircraft MTOW. The basic inputs to the design tool are the aircraft geometry such as wing planform, taper, *t/c* distribution. However, to capture the structural requirements, the designer must also provide aerodynamic and inertial flight loads. In addition to flight loads, the ground loads due to the limiting case of hard landing, which load the wing structure in the opposite direction, must also be considered. Additional loads working as inertial relief in the flight loaded case include lumped masses representing non-structural mass on the wing such as engines, control surfaces, and fuel weight. Additionally, material properties of tensive and compressive yield strength, modulus of elasticity, and density must be provided, as can be found in Military Handbook 5J[14] and are enumerated in a later section of this work. Material yield properties will be de-rated by the required factor of safety (1.5) as defined in the Title 14 CFR § 25.303[15].

The input geometry, loading, and material properties are then run through automated calculations based on equations presented in Niu's *Airframe Structural Design* textbook[11][16] and well-known structural equations for column, panel, and stringer buckling. The underlying coded algorithm determines if the geometry is feasible and introduces thinner or thicker elements as needed to produce the lightest weight geometry that satisfies all the equations. Further description of the model is contained in the Model section herein.

Summary of Underlying Structural Analysis Principles

Building upon the many precursory tools and methods, I can now take the next step in conceptual design level structural analysis. Firstly, an algorithm for sizing specific wing structure elements from a chosen planform is used to determine the number and sizing of stringers and the skin thickness of the torque box. After these are known, I can begin to analyze the resulting structure for wing stiffness. Moving beyond the basic modeling elements, the specific geometry can then be extended to finite element modeling (FEM) for resulting natural frequency consideration.

Stiffness vs Weight trades

In a paper published in 2014, Bai et. al.[17] presented a software-based method for estimating wing weight during the conceptual design phase. They also use parametric modeling of a wing constrained by strength and stiffness to optimize structural mass. Their method used automated CAD and FEM applying parametric inputs to quickly perform the analyses. In addition to analyzing the wing structure, they also provide for early design of the wing fuel tank as part of their automated modeling. They use advanced but cost-efficient methods for computing the aerodynamic and inertial loads to prepare the finite element model.

The strength and rigidity constraints used in the Bai model are stress and displacement. Although this would seem to provide the necessary checks for the wing, it neglects the important buckling constraints that can drive wing structure design. They do assert that additional failure modes would be considered in future research. For the example case used, comparable to an Airbus A320, the solution may be largely driven by the allowable stress. However, I have found that in smaller wing structures such as the example test case used for this paper, the solution is driven significantly by buckling.

Vibration and Flutter Analysis Using Stick Models

To quickly analyze the structural dynamic properties of the wing, a rapid construction and analysis method for vibration and flutter is needed. The full detail of a finite element model (FEM) may be cost prohibitive in early design. Fortunately, it has been demonstrated that much simpler models can be capable of producing comparably accurate results. A so-called "stick model" derived by reduction of the stiffness matric to grid points representing the wing can be used to obtain structural dynamic response. In a paper by Panza & Suciu[18], a detailed examination of the performance of the historically used "stick model" is performed. They conclude based on a comparison of a swept-wing transport category aircraft FEM and stick model that the stick model is adequate for structural dynamic and flutter analysis for so long as the aspect ratio is medium to high, and not a low aspect ratio or delta wing.

The development of rapid analysis design tools for aircraft using low to medium fidelity modeling is not new. Cavagna, Ricci, & Ricobene[19] present the development of a tool for applying stick model analysis for aeroelastic analysis of aerospace structures. In their work they also detail the necessity of accurately estimating aircraft weight early and the penalties that can arise from weight growth during later design phases. The limitations of the classic analytical weight equations are again stressed, especially in the case of innovative or unconventional configurations. This led to a large-scale software development for accurate early design optimization tools. The Next-generation Conceptual Aero-structural Sizing Suite (NeoCASS) tool takes user inputs to generate an airframe structure model and run preset weight and balance studies, stability, static and dynamic aeroelasticity, and flutter analysis. The software is licensed as Free and Open Source Software (FOSS) and operates in the MATLAB environment. The software was developed at the *Dipartimento di Ingegneria Aerospaziale* of the University "Politecnico di Milano" for research purposes and has been shared with the research community as FOSS available at *neocass.org*[20] . The website also advertises consulting services for training users or specific problem solving, with the recognition that the software is not trivial to use. However, for those capable of applying the full functionality of that toolset, structural models such as the example in Figure 1 are possible.

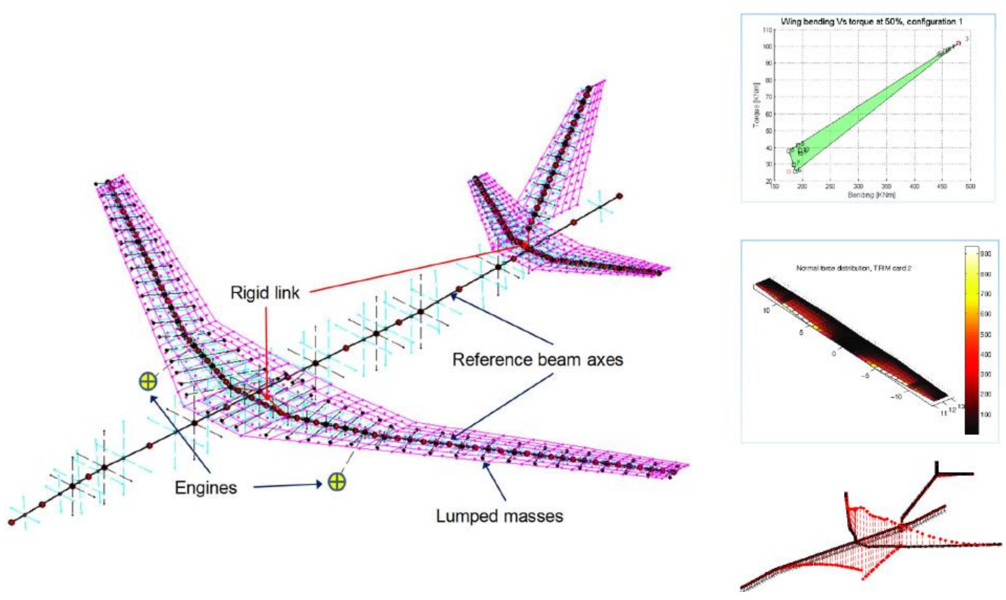


Figure 1: NeoCASS example Structural Stick Model

TEST CASE - FLIGHTERJET PLANFORM

The specific design example case cited in this effort is the *FlighterJet*[21], a student designed light aerobatic jet designed to fill the civilian market gap for a pilot-oriented jet aircraft for those wanting to punch holes in the sky. The design requirements included high speed and high maximum gee loading for high speed maneuvers, as well as relatively long range of 1250 nM. *FlighterJet* is designed to be certified as a commercial transport category aircraft under 14 CFR § 25.

Table 1: FlighterJet Basic Sizing

Parameter	Value	Units
MTOW	22725	lbs.
Sref	472.7	ft^2
Span	50.5	ft
Λ	31.4	deg
TR	0.295	
AR	5.4	
t/c (mean)	10.1%	

The aircraft size and geometry are provided in Table 1. The wing planform being studied for design optimization comes from the preliminary design of the *FlighterJet*, whose basic planform is shown in Figure 2. See also Figure 4 and Figure 5 for the rendering and three view sketches of the concept aircraft.

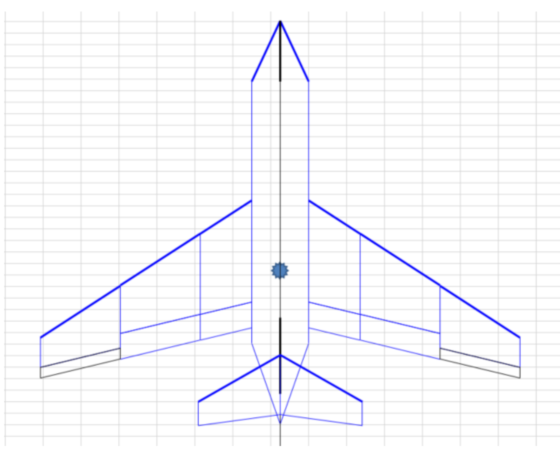


Figure 2: FlighterJet Planform

One critical element that is interconnected in the planform design covered in the capstone project[21] is the thickness to chord ratio. The Swept Wing Korn equation ($M_{cr} = k - 0.1C_L - t/c$)[22] is used to size the thickness of the wing, which is largely driven by transonic effects. The need for high altitude and high maneuverability for the FlighterJet drives the mean thickness of the wing down. Relief from this comes from the trade of design Mach number, wing sweep, and section *CL* to allow for a final thickness choice. As a design driver, a higher wing thickness is desirable to allow for maximum room for fuel storage and landing gear in the wing. The extensive trade performed in the capstone project resulted in the wing thickness to chord distribution shown in Figure 3.

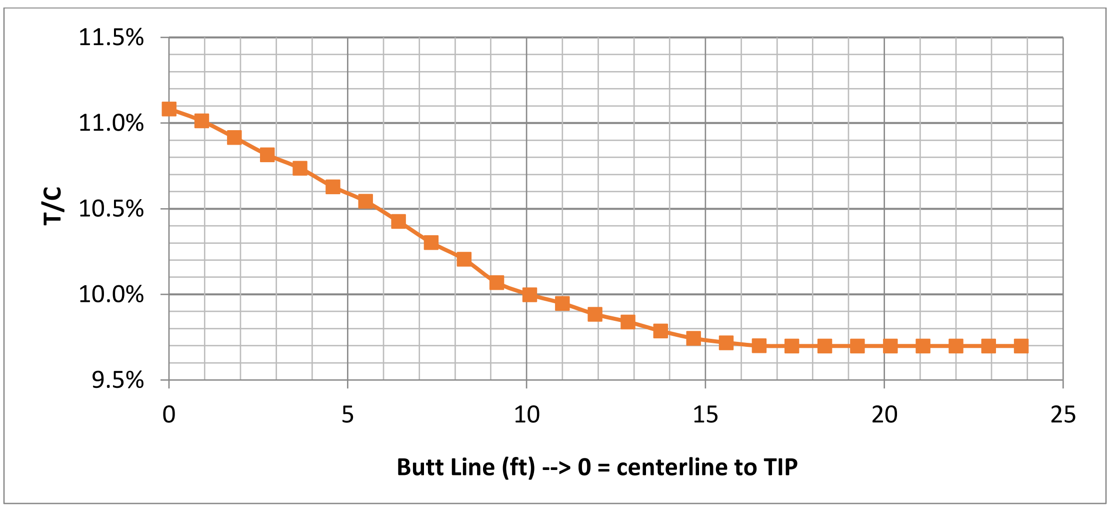


Figure 3: Element-wise Thickness to Chord Distribution

Figure 4: 3D rendering of FlighterJet

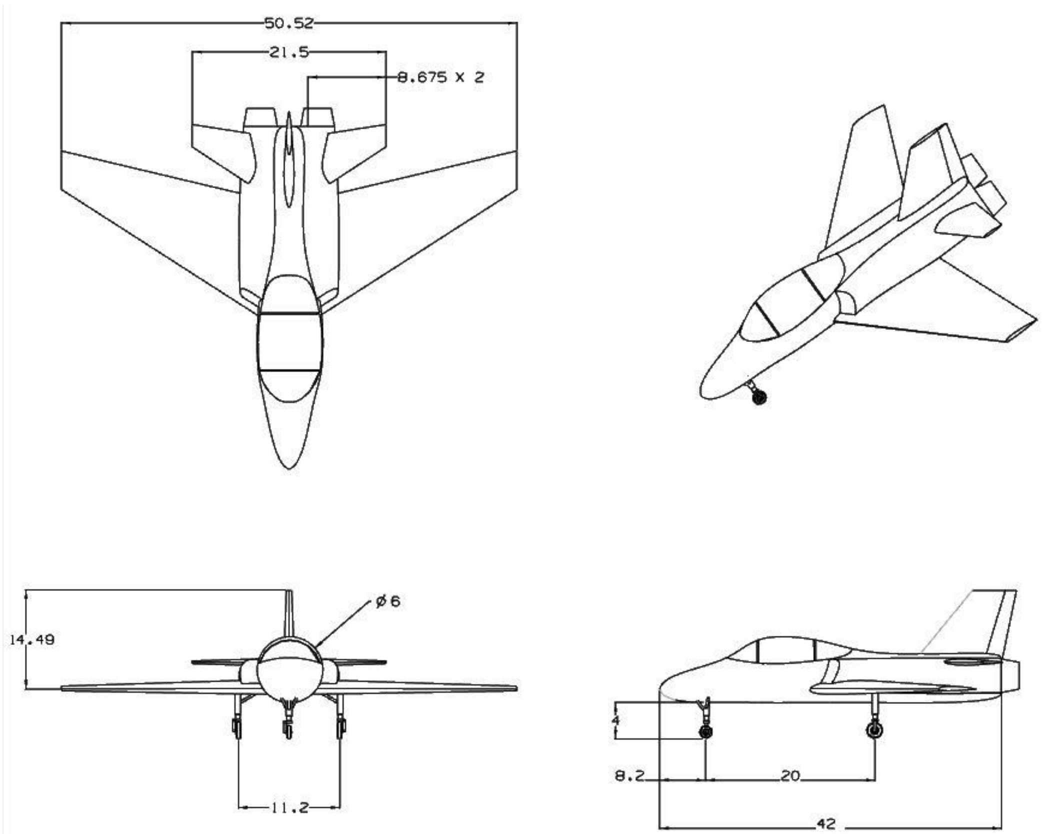


Figure 5: Three view sketch of FlighterJet

MODEL

Takahashi & Lemonds Model

The modeling tool which provides structural geometry takes inputs of customary wing planform variables as well as inertial loading and landing gear information to set up the wing design. Additionally, materials choice and resulting properties such as elastic modulus, yield strength and density are also needed. A choice of rib and stringer spacing is also taken as a user input for conceptual design. Figure 6 contains an Airbus wing structure example, demonstrating a configuration of rib and stiffener (or stringer) density. The algorithm leverages rule-based methods discussed to find a plausible wing design with the lowest weight, unless none can be found, which the tool is capable of reporting. When a design does exist, the tool provides the geometry of the wing structure to include the skin thickness, spar cap sizing, and number and size of stringers.

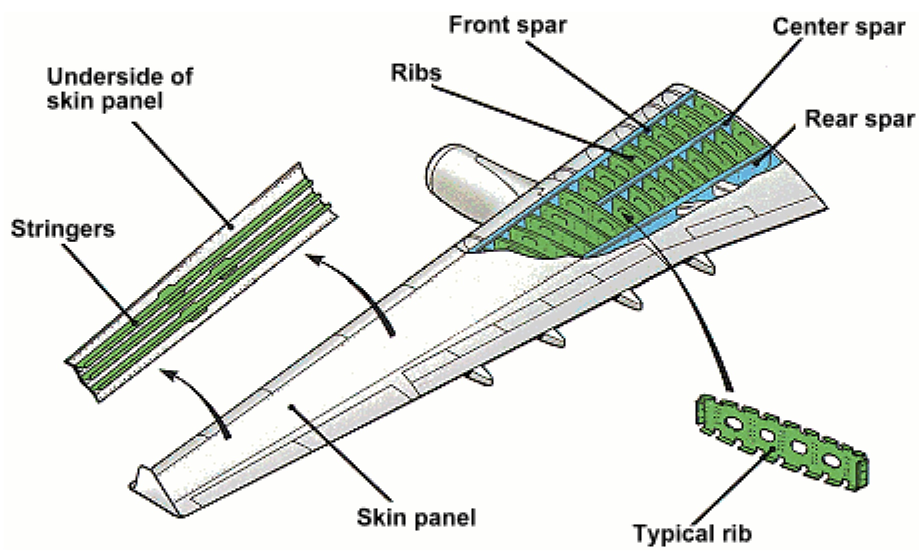


Figure 6: Example Wing Structure from Airbus. Retrieved from http://www.aerospaceengineeringmagazine.sae.org/aeromag/techupdate/06-1999/06.htm

The wing geometry is defined in primary variables of the wing span (tip to tip), reference area (*Sref*), taper ratio (*TR*), and mid-chord sweep ($\Lambda_{C/2}$). These constraints provide the basic trapezoidal wing planform. The geometry is further defined by secondary variables including the span-wise thickness to chord ratio, location of landing gear, and placement and mass of engines and fuel weight providing inertial relief on the structure. Further structural parameters which impact the resulting design are left open as trade variables, including the spacing of ribs and stiffeners. These are selected for trade because they are otherwise free from directly impacting the flight characteristics, where the other variables would be selected for their direct performance characteristics. The purpose of the tool is then to size the structural elements of the wing, based on the inputs, to minimize the structural mass that will withstand the three limiting load cases: ultimate yield strength of the selected material in both compression and tension (de-rated by necessary factors of safety), local strip buckling, and stiffened panel buckling.

The limiting loading conditions considered by the design tool are guided by the regulations set forth in 14 CFR § 25.337 for maneuvering load factors. These include both the flight loads at MTOW scaled by the load factors of positive or negative gee forces (based on the design flight envelope) and the ground loads at MLW applied at the landing gear locations.

The critical buckling load constraints used in the design tool come from Niu's *Airframe Structural Design*[11].

For stiffened panel buckling:

$$P_{cr.stiffened.panel} = \pi^2 E \left(\frac{\rho^2}{L^2}\right) A_{stiffenedpanel} \qquad (2)$$

where

$$A_{stiffenedpanel} = thickness_{skin} dist_{spar.to.spar} + \#_{stg}\left(height_{stg} thickness_{str}\right)$$

$$L = \ dist_{rib.to.rib}$$

and the radius of gyration, ρ, is described by

$$\rho^2 = \frac{\left(b^2\left(\frac{d}{b}\right)^3\left(\frac{Ts}{t}\right)\right)}{\left(12\left(1+\left(\frac{d}{b}\right)\left(\frac{Ts}{t}\right)^2\right)\right)\left(4+\left(\frac{d}{b}\right)\left(\frac{Ts}{t}\right)\right)^2}$$

with the following relationships

$$b \qquad = dist_{spar.to.spar}/(\#_{str}+1)$$

$$d = height_{str}$$

$$Ts = thickness_{str}$$

$$t = thickness_{skin}$$

The local strip buckling, also recommended in Niu[11], utilizing the Euler Strip Buckling Equation with NASA factors for a strip with all edges clamped and fully supported:

$$P_{cr.strip} = 6.3E\left(\frac{t^3W}{b^2}\right)$$

(3)

where

$$W = dist_{spar.to.spar}/\left(\#_{stringers}+1\right)$$

$$t = thickness_{skin}$$

$$b = dist_{rib.to.rib}$$

The model "builds" up the wing structure bay-by-bay to design the lightest weight wing that satisfies all constraints. Lemonds[1] gives a fully detailed description of the algorithmic rule-based design approach utilized.

Model of Wingtip deflection

The new addition to the wing design model is a prediction of wingtip deflection. Basic beam theory was used to characterize the stiffness of the wing with available design choice data. Based on design choices, the wing weight estimator tool as previously described provides wing structure geometry along span-wise increments. Along each of these increments, or bays, the structure is constant. Because these bays are relatively small compared to the overall span of the wing, they can be used as constant beam elements to numerically approximate the deflection of the wing tip under load.

The calculation begins with Eq. (4) from Hibbler[23] which assumes constant flexural rigidity (EI) along the length of the beam increment.

$$EI\frac{d^2\nu}{dx^2} = M(x) \tag{4}$$

Next, Eq. (4) is applied numerically along the semi-span, from centerline to tip, and recognize that the deflection and angle boundary conditions at the centerline are both theoretically zero.

$$\left(\frac{d\nu}{dx}\right)_i = \frac{M}{EI}\Delta x + \left(\frac{d\nu}{dx}\right)_{i-1}; \quad \left(\frac{d\nu}{dx}\right)_0 = 0 \tag{5}$$

$$\nu_i = \frac{M}{EI}\Delta x^2 + \left(\frac{d\nu}{dx}\right)_{i-1} + \nu_{i-1}; \quad \nu_0 = 0 \tag{6}$$

Finally, Eqs. (5) and (6) are applied numerically to determine the deflection at the wingtip as an additional output for trade study examination.

Wing Stick Model

To perform structural dynamic analysis, at least a basic stick model of the wing is needed. For the purposes of this research the low fidelity stick model which represents the wing section by section as an equivalent beam will be used. This low fidelity is considered acceptable in this case for two reasons. The first is that it is necessarily simple to enable multiple designs. In the previous case of the wing tip deflection, tens of designs can be rapidly analyzed, yet the amount of effort required to build a full FEM model for that many designs would be restrictive. A model that can be generated for dozens of designs will be most beneficial for this research. The second reason this simplified model is suitable is that at the early stage of design, the wing non-structural mass is largely still unknown.

There is a considerable amount of expected variability in the total wing mass, even when a single wing structure is selected, and the structural mass is fixed. Known variation in the wing non-structural mass includes the large fluctuation in the fuel storage expected in standard operation. In the case of the *FlighterJet* application, over 5000 pounds of fuel will be stored in the wings. There is also significant unknown mass variation due to the lack of design maturity. This includes the subsystems housed in the wing, including anti-icing systems, high-lift devices as leading and trailing edges, and control surfaces, each with complex mechanical and electrical components whose weights can only be estimated. Although significantly less than the fuel weight, this is still a considerable effect, with one estimation formula from Torenbeek[10] putting the wing surface controls as heavy as 827 pounds if the wing includes leading edge flaps and air brake spoilers in addition to the standard necessary controls surfaces of flaps and ailerons (see eqn. 7).

$$W_{control\,surfaces} = 0.64 * MTOW^{0.67}(1 + 0.2LEF + 0.15SP) + 110 \qquad (7)$$

where

LEF = logical 0 or 1 to include leading edge flaps

SP = logical 0 or 1 to include spoilers

The necessary information to represent the wing structure as an equivalent beam in the simplest of stick models is the material properties, the geometry to include cross section parameters of area moment of inertia at each wing station, and the equivalent lumped masses with locations. Material properties are selected for a nominal aircraft material, Aluminum 2024[24].

Table 2: Aluminum 2024-T3 Properties

Property	**Value**	**Units**
F_{ty}	44000	lbf/in^2
F_{cy}	44000	lbf/in^2
F_{su}	38000	lbf/in^2
E	$1.07*10^7$	lbf/in^2
ρ	1.00	lbm/in^3

The lumped mass model used in this work has the limitation of being based on point mass only, not lumped moment of inertia. This is a “clean” wing, with no wing mounted engines, external fuel tanks, or (as in the case for military aircraft) weapon systems. Therefore, much of the mass will be relatively close to the position represented along the midline, however, it is expected that this simplification will limit the accuracy of the torsional mode calculation – it will be driven to higher-than-true frequencies. If designing a wing with a heavy wing mounted engine, this could be represented as an off-center lumped mass which would dominate the twisting effect of the wing.

The calculation of the area moment of inertia is not trivial. Complex geometries and the area integration required is one of the driving factors for CAD and FEM software tool development. However, a simplified and representative geometry of a collection of rectangles to represent the torque box allows for a feasible analytical calculation. The simplified geometry torque box representation is shown in Figure 7.

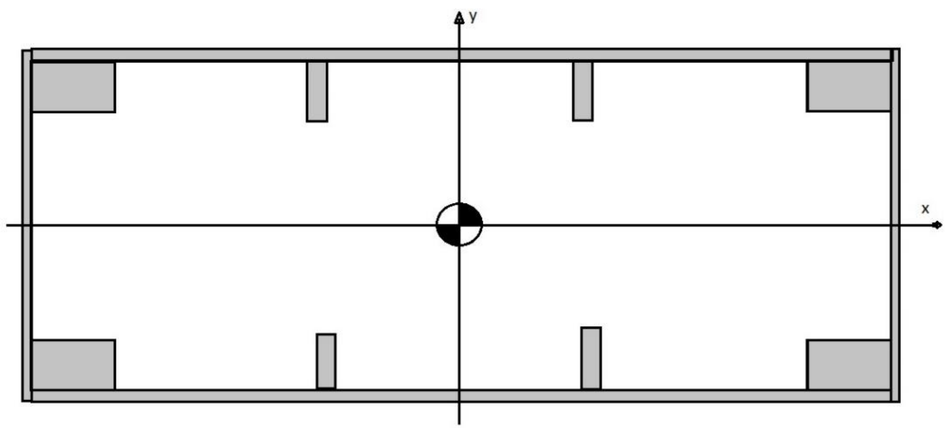


Figure 7: Simplified Torque Box Cross-Section

The Takahashi & Lemonds[1] tool for calculating wing structures provides the dimensions of each rectangular component. The largest simplifying assumption taken is the treatment of the skin thickness as a rectangle. The actual shape of the upper and lower skin is determined by airfoil choice, which has not been selected. However, the maximum thickness at each station is defined, and an average torque box height can be estimated using a knowledge of the common wing shape. Customary placement of the torque box places the front spar at 15% chord length, back spar at 65% for a total spar spacing of

50% of the chord. NACA thickness form then gives the height of the front spar to be 84% of maximum thickness, with the rear spar at 60%. I will take a simple average and generalize the rectangular height of the torque box to 72% of the maximum thickness at each wing station.

The rectangular components of each wing station torque box are given the naming convention as depicted in Figure 8. By applying the second area moment of inertia for a rectangular shape as well as the parallel axis theorem, I can build the composite shape stiffness parameters via the equations shown below.

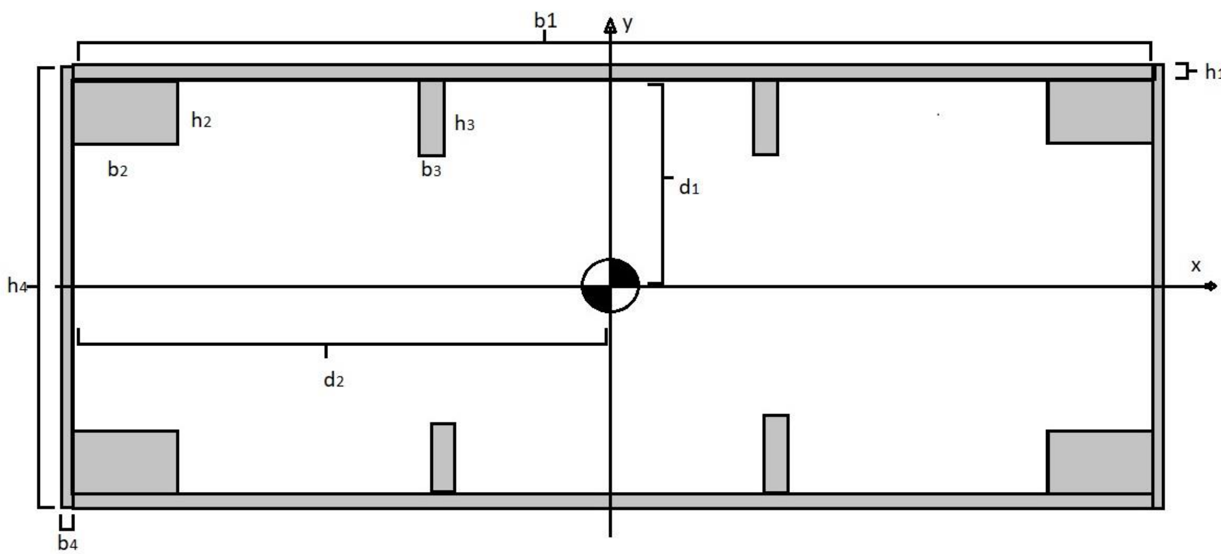


Figure 8: Dimension assignment to torque box features

$I_{rectangle} = \frac{bh^3}{12}$, rectangular solution to second area moment (8)

$I_{x\prime} = I_x + Ad^2$, parallel axis theorem (9)

$$I_{xx} = 2\left(\frac{b_1h_1^3}{12} + b_1h_1d_1^2\right) + 4\left(\frac{b_2h_2^3}{12} + b_2h_2d_1^2\right) + n_{stringers}\left(\frac{b_3h_3^3}{12} + b_3h_3d_1^2\right) + 2\left(\frac{b_4h_4^3}{12}\right)$$

(10)

$$I_{yy} = 2\left(\frac{h_1 b_1^3}{12}\right) + 4\left(\frac{h_2 b_2^3}{12} + b_2 h_2 d_2^2\right) + 2\left(\frac{h_4 b_4^3}{12} + b_4 h_4 d_2^2\right) \quad (11)$$

$$I_{xy} = 0, based\ on\ assumption\ of\ symmetry\ about\ centroid \quad (12)$$

$$J = I_{xx} + I_{yy} \text{ , perpendicular axis theorem} \quad (13)$$

Note that in the calculation of I_{yy} the contribution of the stringers must be neglected because the distance for parallel axes in the *y* direction is not defined and will change based on the number of stringers and stringer spacing. The stringers have a smaller area compared to the other components, and they will also have a smaller distance from the centroid than the spar caps or spar web, so the neglected contribution is accepted as being a small.

One observation at this stage is that the contribution of the skin dominates the calculation of the stiffness parameter I_{xx}, the stiffness of the wing to resist bending in the primary loading conditions during flight. For a wing configuration nominally chosen with a rib to rib spacing of 12 inches and spar spacing of 10 inches, the skin contribution makes up an average 64% of the stiffness in the x direction, see Table 3 for details. This reveals an interesting aspect of this complex design problem. The choice of spacing and sizing of the minor torque box component have a small contribution on the stiffness, but they will also interact with the optimized skin thickness based on buckling constraints, which will then strongly drive the wing stiffness. This is cross coupling of the results makes this research and design tool necessary, because predicting these complicated interactions becomes challenging.

Table 3: Torque box *Ixx* contributions

Wing Station	**I_{xx} Skin**	**I_{xx} Spar Caps**	**I_{xx} Stringers**	**I_{xx} Spar Web**
1	62%	18%	17%	3%
2	61%	20%	16%	4%
3	63%	17%	17%	4%
4	62%	19%	15%	4%
5	62%	20%	13%	5%
6	61%	21%	13%	5%
7	61%	21%	14%	5%
8	62%	22%	14%	1%
9	63%	23%	12%	2%
10	64%	22%	13%	2%
11	65%	20%	13%	2%
12	67%	17%	14%	2%
13	67%	16%	15%	2%
14	66%	17%	16%	2%
15	68%	18%	13%	2%
16	67%	20%	12%	2%
17	65%	21%	12%	2%
18	63%	24%	11%	2%
19	64%	23%	12%	1%
20	66%	25%	8%	1%
21	61%	31%	7%	1%
22	68%	23%	8%	1%
23	64%	29%	6%	1%
24	73%	17%	8%	2%
25	66%	27%	5%	2%
Average	64%	21%	12%	2%

Finally, the values needed for the equivalent beam model can be used to analyze the frequency response of a modal analysis of the wing. The stiffness and point mass parameters at each wing station are added along the midline representation of the wing. The FEM software used to generate the stick model (*Creo Parametric*) generates a rectangular shape to represent each ideal beam element, shown in Figure 9. The root of the wing is constrained to be fixed in translations and rotations to represent the center of the wing running through the bottom of the aircraft fuselage. Modal analysis is then performed collecting natural frequencies associated with the first 8 modes shapes. The designer would then have these natural frequencies to perform aeroelastic analysis and design early to avoid destructive flutter of the wings.

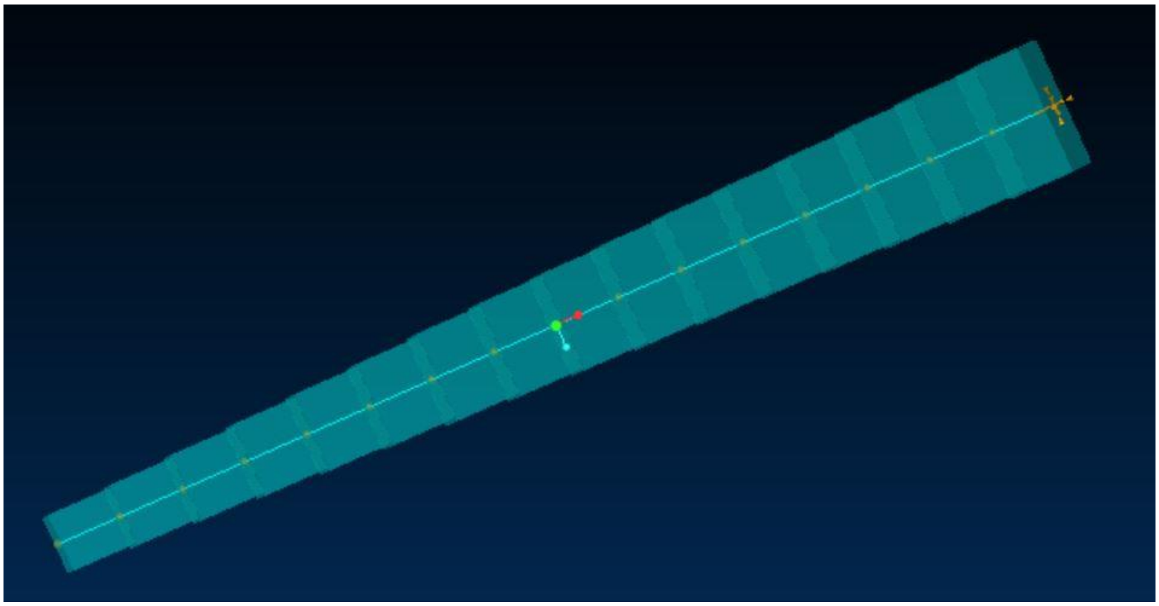


Figure 9: Stick Model - Idealized Beam Elements

RESULTS

Wing Loading

The original tool as developed by Takahashi & Lemonds[1] took into consideration wing aerodynamic and inertial loading and designed to the limiting cases of high gee maneuvers (*Nzmax*) and hard landing. The load-up of the wing for various conditions also considers inertial relief from non-structural mass, such as the fuel in the wing. This non-structural mass configuration is another source of variability for the resulting loading and deflection. The following plots show the subjected loading that is used to determine the needed structural elements and further is used in determining the deflection of the wing. The wing loading cell size follows the wing stations from rib to rib. The total volume of fuel available in the torque box is significantly more than is needed, so there is flexibility in the fuel tank positioning. The max fuel weight volumetrically only uses approximately 55% of the estimated available volume of the torque box.

The decision to place the fuel in the 15-55% span location (see Figure 10) and not at the wing root takes into consideration that under fuselage storage space is limited in this small business jet aircraft, based on cabin layout and main landing gear storage. The fuel tank placement choice could also be considered as a source of design variability, though it is not pursued in this work. The resulting bending torque is shown in Figure 11 for the two design loading cases; "*NzMin Full*" representing a heavy plane landing, "*NzMax Empty*" representing the high gee maneuver without fuel weight bending relief.

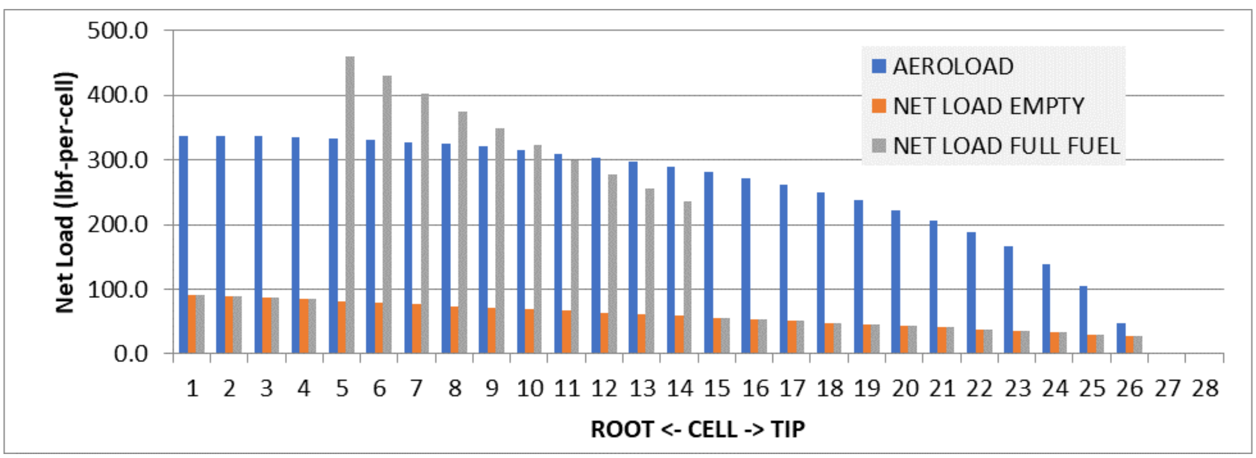


Figure 10: Loading case for the wing with full fuel load

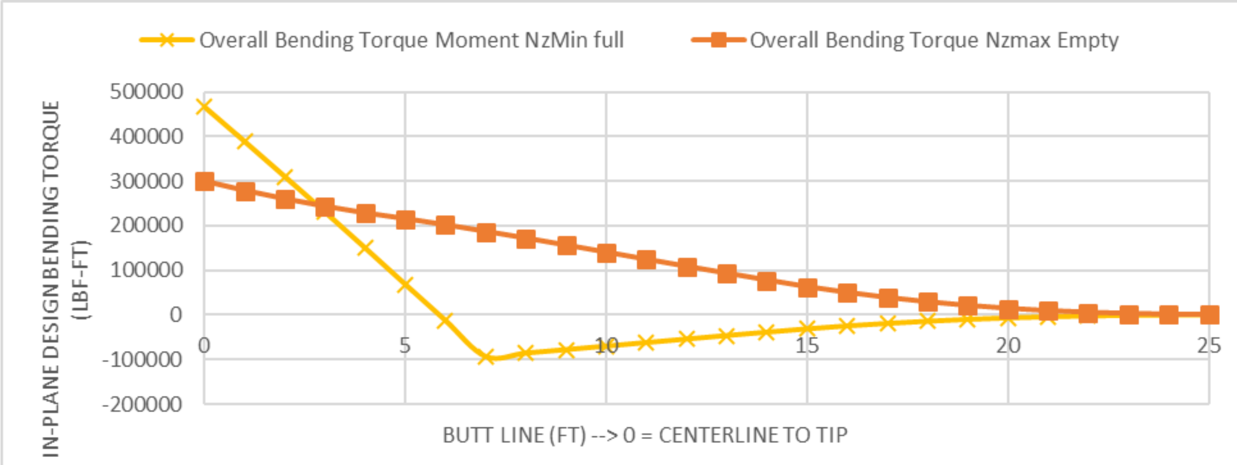


Figure 11: Resulting Bending Torque for the Nzmax limiting case

The rule-based design tool produces the specific geometry optimized for that wing setup, and the resulting structural cover area is given in Figure 12. This cover area comparison shows how much structure would be necessary if tension force and material yield where the only constraints, compared to the actual design constrained by buckling.

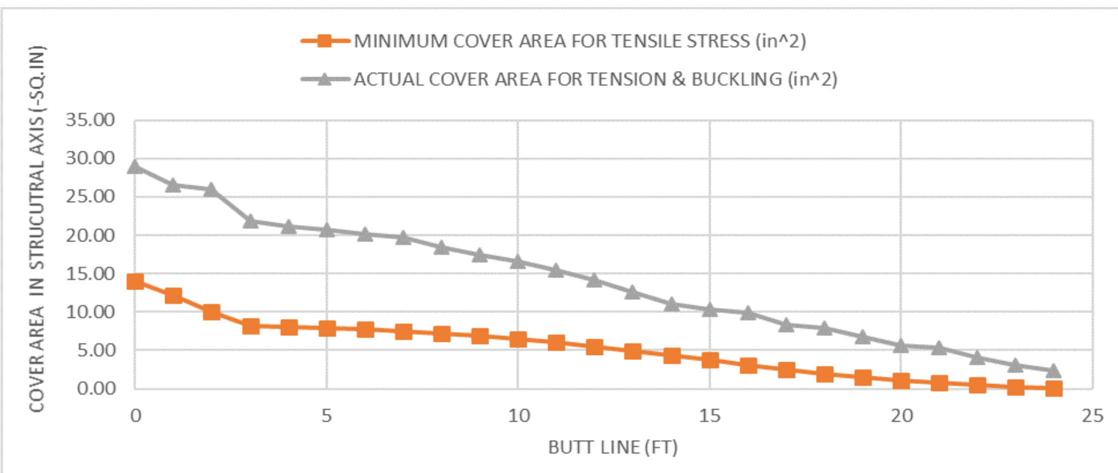


Figure 12: Element-wise cover area of Structural Elements

If the tensile only solution is considered as the "ideal" structural weight, then the ratio of the actual weight to the ideal can be used as a measure of how much the structure is driven by buckling. In this case, a moderately lightweight aircraft with an MTOW below

25,000 lbs., the wing structure is dominated by buckling as seen in this actual-to-ideal ratio which is in the range of 200-300%.

Deflection and Stiffness

Enumeration of 42 different conceptual structural designs, varying only two simple choices, the rib spacing from 9 to 14 inches and the stiffener spacing from 6 to 12 inches produces 42 “optimized” designs, meaning a geometry which can carry the necessary loads with the lowest weight. However, the weights vary widely among the designs for such seemingly trivial choices as the spacing of elements. However, the newly added deflection calculation reveals that those varying weights are closely tied with the wingtip deflection, and therefore the inferred wing stiffness. The reader can finally begin to see how simple conceptual design choices made early can largely impact the potentially problematic stiffness of the wing.

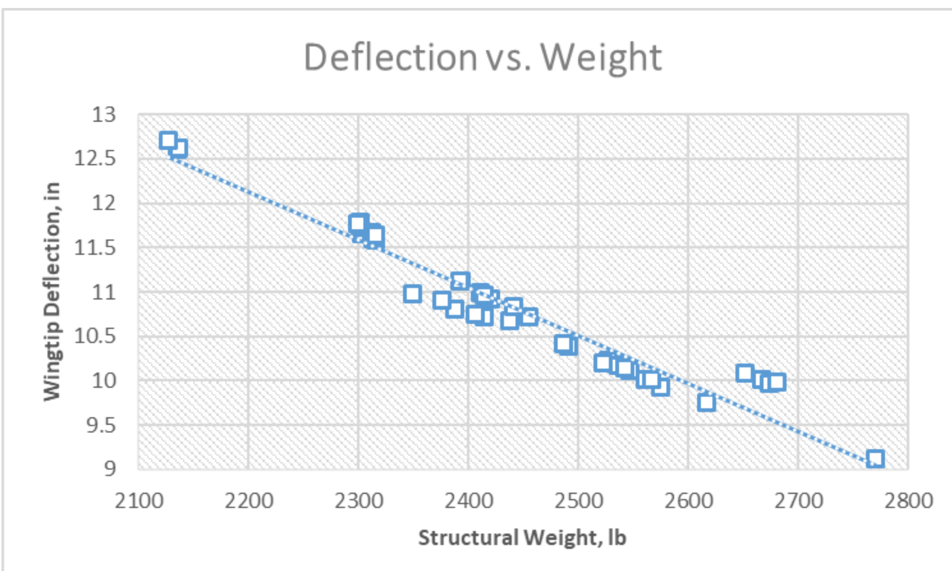


Figure 13: Deflection vs Weight - There is a distinct relationship between the structural weight and the stiffness of the wing from a variety of designed wings.

As shown in Figure 13, those designs which lead to a lower overall weight also allow more wingtip deflection. Although this is not a surprising result (heavy/more material expected to be stiffer), now one can begin to understand which parameters will drive the stiffness and predict the outcome to guide decisions.

Of the 42 designs shown in Figure 13, there is a variation of 26% in structural weight and 33% in stiffness as expressed in terms of tip deflection. The comparison of the variability is summarized in Table 4.

Table 4: Stiffness Variability

Value	Deflection (in)	Weight (lbs.)
Mean	10.8	2440
St. Dev	0.8	150
Min	9.12	2127
Max	12.7	2770
Difference	3.59	642
% Difference	33.1%	26.4%

Observed Trends

An additional scan of the design space was performed, running the model for designs with stiffeners spaced at 4 to 14 inches by one inch, and rib spacing by 8 to 20 inches by one inch. By examining the resulting ten lightest and heaviest wings and their associated design details, several general trends are revealed. See Table 5 for the ten lightest and heaviest configurations.

To build a light wing, one should:

1) Build with larger stiffener spacing resulting in fewer stiffeners,
2) Build with smaller rib to rib spacing resulting in more ribs,

The resulting solution will then have the following characteristics:

3) Smaller spar caps,

4) Thinner skin,

5) More equal balance in local and stiffened panel buckling.

The wing geometry and cell loading for the lightest wing is provided in Figure 24 contained in the Appendix. The wing structural mass is 2039 lbs. and the resulting wingtip deflection is 10.03 inches

Table 5: Ten lightest and heaviest wing structure configurations

Stiffener Spacing	Rib Spacing	Actual Weight (lbs.)	Wingtip Deflection (in)	Actual / Ideal ratio
14	11	2039.1	10.038	2.041
11	14	2099.1	9.712	2.233
14	10	2146.3	9.707	2.167
12	13	2188.2	9.563	2.224
14	13	2191	9.54	2.227
13	13	2196.3	9.563	2.232
11	9	2222.8	9.125	2.157
14	9	2231.8	9.061	2.166
10	9	2241.7	9.05	2.175
...	...	...	...	...
14	19	2812.3	6.531	2.937
4	20	2818.3	7.437	2.864
6	9	2846	6.855	2.762
4	11	2863.2	6.702	2.866
10	19	2871.4	6.397	2.999
4	10	2909.4	6.713	2.938
4	17	2939.6	6.427	3.216
6	19	2972.6	6.223	3.105
7	19	3001.7	6.175	3.135
5	18	3080.4	5.898	3.32

On the other hand (and in the test case the two are found to be mutually exclusive), to build a stiff wing, one should:

1) Build with smaller stiffener spacing resulting in more stiffeners,
2) Build with greater rib to rib spacing resulting in fewer ribs,

The solution will have the characteristics:

3) Large spar caps,
4) Sizing dominated by local buckling.

It is not clear from the current study if these trends are also true of larger wings, on the scale of transport aircraft with an MTOW of greater than 100,000 lbs. However, in the test case of this business jet sized aircraft, the wing structure tends to be driven by buckling constraints.

A closer look at the structural solution on the heavy end of the spectrum shows that the weight seems to be a result of the larger spacing between ribs, leading to an increase in structure needed to prevent local buckling. The heavy wing sample results in a structural weight of 3080 lbs. and is relatively stiff with a wingtip deflection of 5.9 inches. The wing parameters and cell loading are shown in Figure 25 contained in the Appendix.

Torenbeek Weight Estimation

In this model I demonstrated an array of wing structural weights based on a few conceptual design choices to get a better weight estimation while also considering wing stiffness. By comparison, the empirical wing weight formula mentioned in the literature survey does not take into consideration the wing structural choices but can provide a rough order of magnitude weight estimate to begin with. The classic Torenbeek[10] equation (Eqn. 1 from this work) applied to this test case results in a relatively light weight prediction for the overall wing weight, at 2520 lbs. when a full fuel bending relief term is used. However, remove the fuel weight relief and the estimate jumps to over 3200 lbs., with no indication of stiffness or structural layout change. In this case, the equation is overly sensitive to fuel weight change because the engines are not mounted on the wing, so nearly all the bending relief term is in the fuel. From the more detailed rule-based design and weight-by-volume model demonstrated here, a range of roughly 2000 to 3000 pounds for the wing structure based on the layout of ribs and stiffeners is feasible, with the resulting stiffness consequences based on design decisions. This allows for a more informed conceptual wing design phase over the classical empirical estimators.

FEM and Modal Analysis

The next step to be completed in this research will be to generate a simple FEM model to perform modal analysis. Previous works such as Lemonds and Bai have performed FEM analysis for strength constraints, so that will not be the focus of future efforts. Rather, research will focus on obtaining mode shapes and frequencies for prediction and prevention of wing flutter. Additionally, the span-wise distribution of nonstructural lumped masses such as secondary structure, subsystems, and fuel will be included in the model. Because the wing subsystems and control surfaces have not yet been designed, only a basic weight estimate can be given. The design of experiments will include some randomness in the mass distribution to characterize some of the mass uncertainty.

Design of Experiments

The modal analysis setup included choosing a lumped mass distribution to evaluate. To capture the two most significant sources of variation, a scheme was developed to span the range of fuel loading as well as introduce randomness in the non-structural wing elements. Five loading cases were chosen for each wing design, ranging from fuel full to fuel empty by 25% steps. These loading cases are represented in Table 6 and constitute the horizontal axis for Figures 14, 17, and 18.

Table 6: Fuel Level Load Case Definitions

Load Case 1	**Load Case 2**	**Load Case 3**	**Load Case 4**	**Load Case 5**
100% fuel: 2583 lbm per wing	75% fuel: 1937 lbm per wing	50% fuel: 1292 lbm per wing	25% fuel: 646 lbm per wing	0% fuel: 0 lbm per wing

Additionally, the Torenbeek estimate for nonstructural mass is spread across the span weighted according to the structural mass at each wing station. That estimated spread is then scaled up or down by a random variate sampled from a population with a mean of 1 and a standard deviation of 0.3, so the nonstructural mass at a wing station may be slightly larger or smaller, but the total nearly approaches the estimate.

The resulting frequencies across the first 8 mode shapes were collected for each wing design and load case. The results can then be compared for a single wing across loading cases, or for a load case across wing designs.

Modal Results

Three wing designs were chosen for this final analysis. Two represent the opposite extremes of light and heavy, and one is taken from a balanced trade of lightness and stiffness.

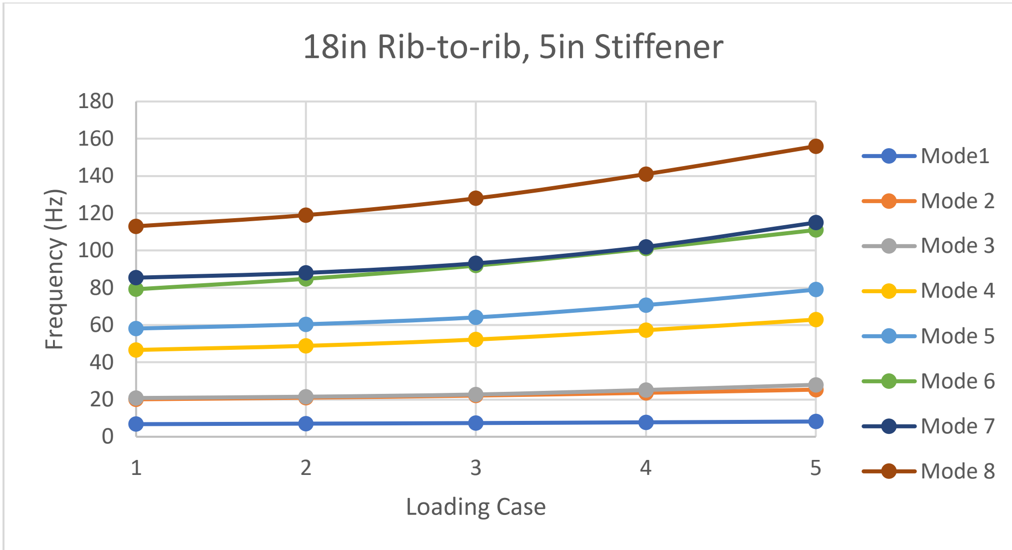
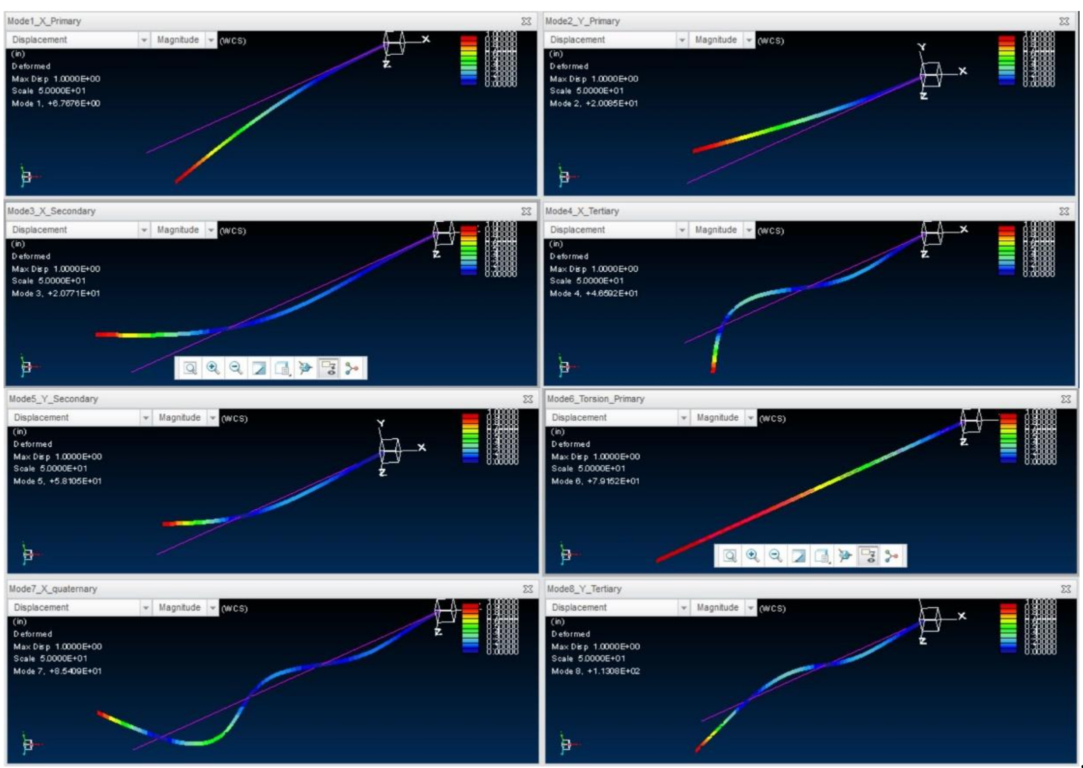


Figure 14: Heavy Wing Modal Analysis

For the first analysis, the “heavy and stiff” wing with a chosen rib spacing of 18 inches and a stiffener spacing of 5 inches was used. The results are summarized in Figure 14. The resulting 8 mode shapes for the full fuel load case (L1) is shown in



, and

the empty fuel load case (L5) in Figure 16. Each window snapshot contains in the title the mode number and what the mode shape is, (X being the primary direction of loading) and the frequency. These identifying mode features, along with the frequencies, are summarized in Table 7. This is to show for a single wing design the amount of variability due to expected fuel mass change.

Table 7: Mode Shape Descriptions for Heavy 18x5 Wing

	"Heavy" wing, Full Fuel (L1)		"Heavy" wing, Empty Fuel (L5)	
Mode	Description	Frequency (Hz)	Description	Frequency (Hz)
1	X-Primary	6.76	X-Primary	8.18
2	Y-Primary	20.1	Y-Primary	25.3
3	X-Secondary	20.8	X-Secondary	27.9
4	X-Tertiary	46.6	X-Tertiary	62.9
5	Y-Secondary	58.1	Y-Secondary	79.0
6	Torsion-Primary	79.2	X-Quaternary	111
7	X-Quaternary	85.4	Torsion-Primary	115
8	Y-Tertiary	113	Y-Tertiary	156

The resulting rising of the natural frequencies across all mode shapes comes as no surprise – as the mass decreases the frequency is expected to increase. At this time it is worth pointing out that the frequency gap seen from load case 1, "full fuel" to load case 5, "empty fuel" grows more significant at the higher mode shapes. This trend is observed across the three wing structures.

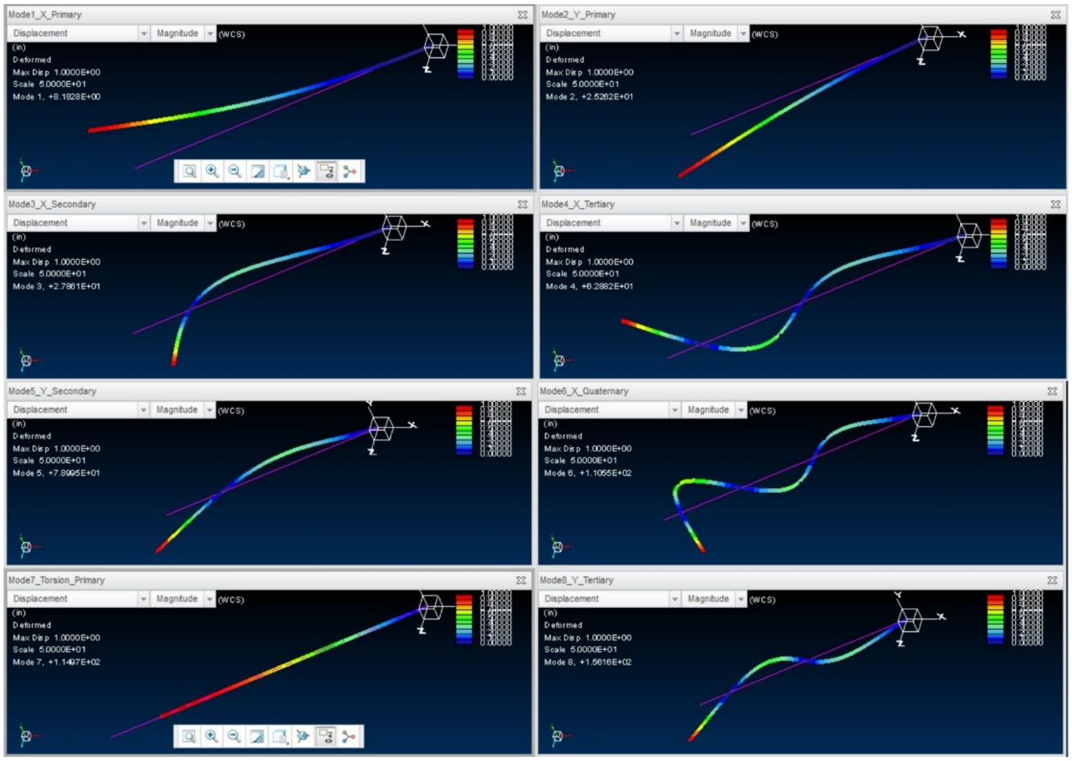


Figure 15: Full Fuel Load Modal Analysis

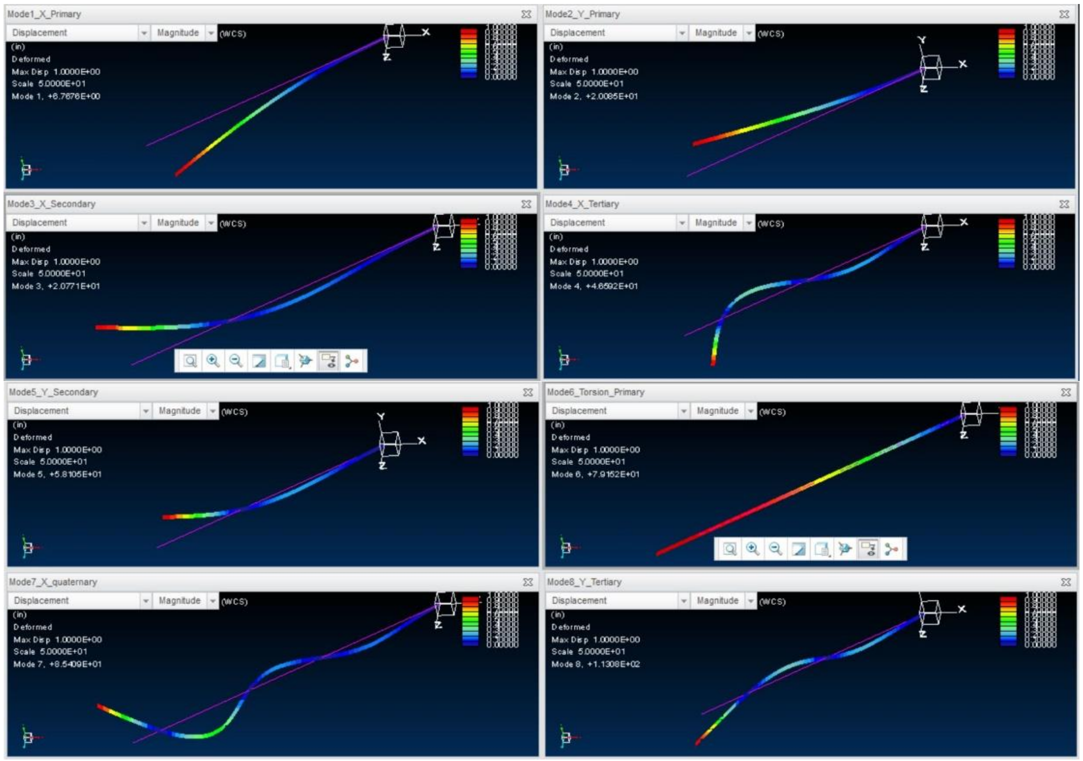


Figure 16: Empty Fuel Load Modal Analysis

Next, the lightweight wing with a chosen rib spacing of 11 inches and a stiffener spacing of 14 inches was used. The results are summarized in

Figure 17.

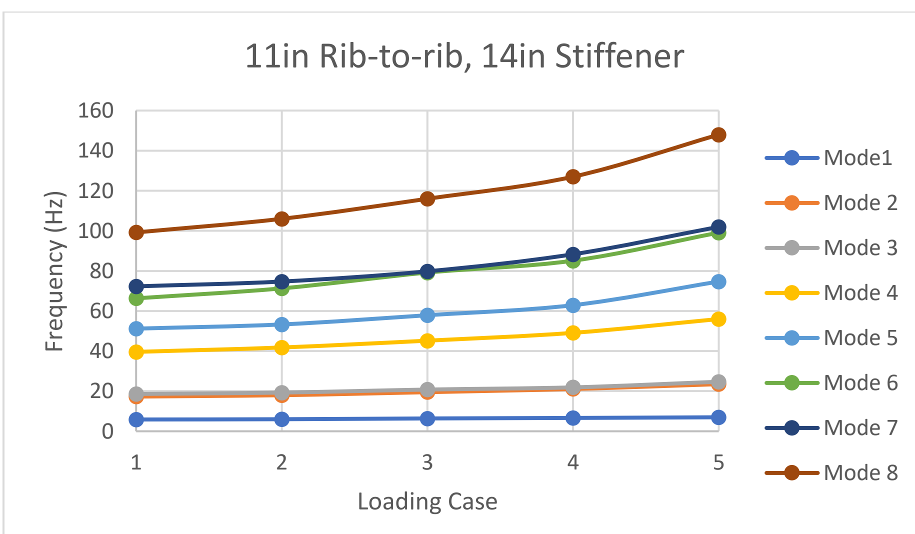


Figure 17: Light Wing Modal Analysis

The “middle-of-the-road” wing structure chosen to represent the balanced selection between weight and stiffness had a rib spacing of 12 inches and stiffener spacing of 10 inches. The results are summarized in Figure 18.

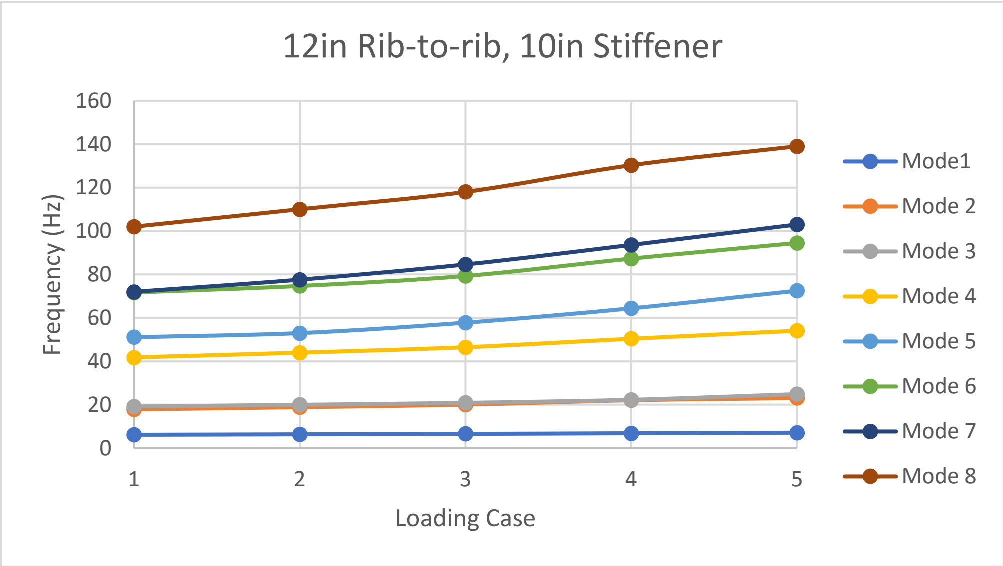


Figure 18: Balanced Weight-Stiffness Wing Modal Analysis

So far, the results have been organized as a single wing design across it’s expected fuel levels. The key observation to note is that above the fourth mode shape at roughly 40 Hz there is no appreciable gap between the frequencies of one mode shape to the next. This means that, regardless of the design of the torque box, at some fuel levels there will be a resulting natural frequency in the range of 40-140 Hz (within the first 8 modes). Hence, if the unsteady aerodynamic forces resonate in that range within the flight envelope, there is potential for flutter.

The next analysis is to show the comparison of the multiple structural designs at a single load case. This is more revealing to what the designer choices can impact.

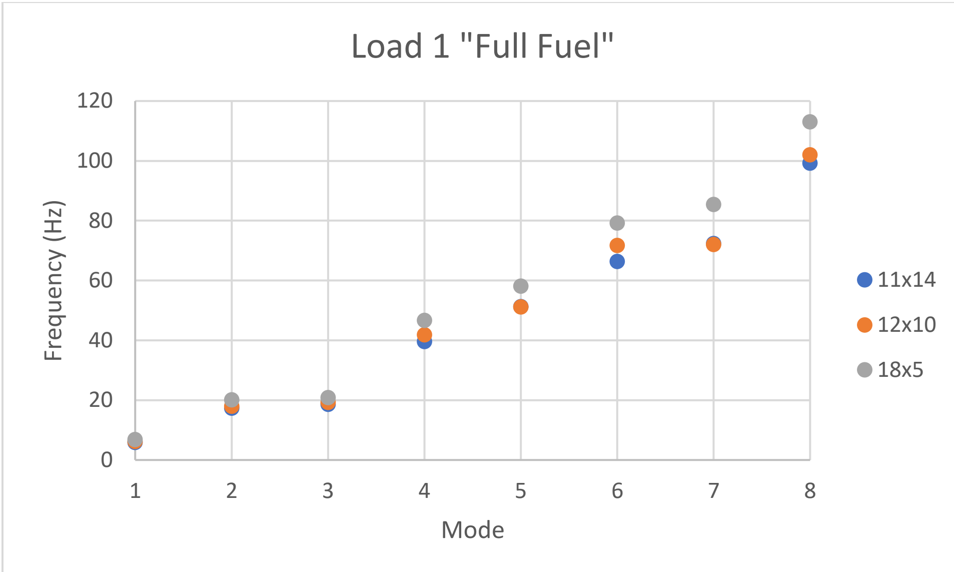


Figure 19: Wing Comparison at Full Fuel

In the first case, the three wings are compared at full fuel weight. The frequencies for the three wings are shown in Figure 19. It is observed that the frequencies are relatively similar, although the weights and stiffnesses vary dramatically. This is possible because the ratio of stiffness to weight, which drives the natural frequency, does not change as much. Also, because the non-structural mass does not scale with the structure changes, the change in stiffness drives the frequency change. In other words, the “heavy” and “stiff” wing has higher frequencies across all mode shapes because although the mass and stiffness both increase, it is only the subset of structural mass that scales, so the stiffness dominates the ratio change. The stiffer and heavier wing has higher natural frequencies than the lighter and less stiff wing.

A similar result is found in Figure 20 and Figure 21 at half fuel and no fuel, respectively.

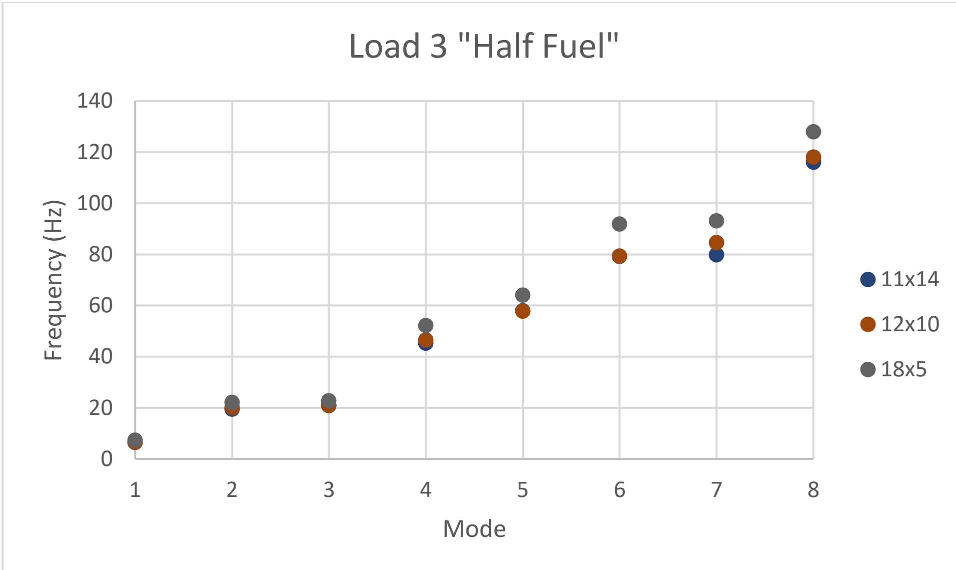


Figure 20: Wing Comparison at Half Fuel

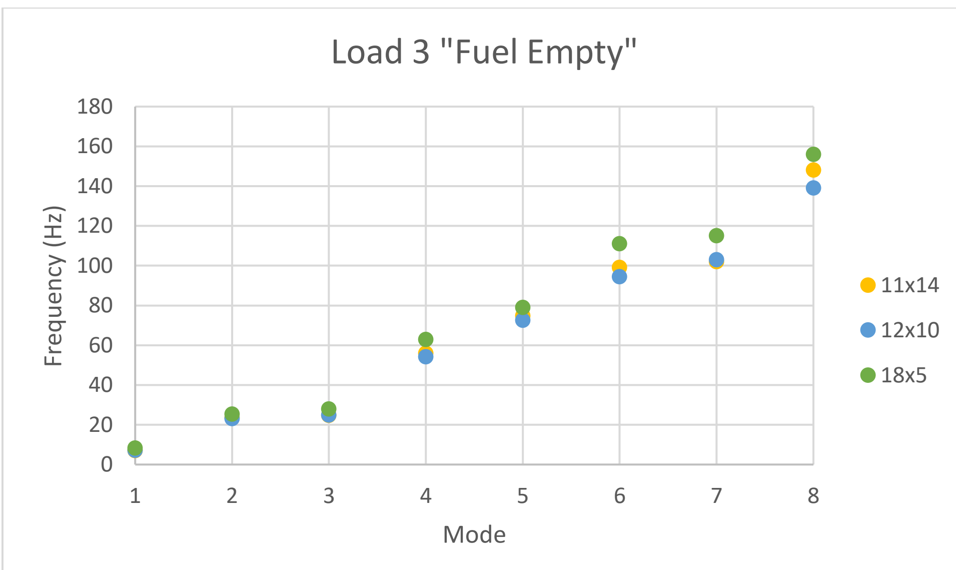


Figure 21: Wing Comparison at Empty Fuel

Finally, Figure 22 gathers the natural frequencies observed on the three wing designs and across the loading conditions. This final view reveals that regardless of design choice or fuel weight, there are pockets of frequencies in the “safe” zone in which there is a relative confidence that flutter will not occur. This includes the low frequencies below the first mode shape, in the range of 0-6 Hz, and the gap between modes three and four, roughly 30-40 Hz. Above that, however, if the unsteady aerodynamic forces drive resonance to a frequency above 40 Hz, there is a high likelihood that at some fuel level, that frequency will result in damaging flutter.

Figure 22: Modal Analysis for Three Wing Designs and Three Loading Cases

Expected Excitation

Short of performing unsteady aerodynamic response analysis, there are still options for comparing the natural frequencies with expected forcing functions. Part of the original *FlighterJet* design project included a study on the stability and controllability of the aircraft. In longitudinal stability, the aircraft was designed to achieve Level 1 flight characteristics according to MIL STD-8785C Category A chart. As seen in Figure 23, the stick fixed, short-period longitudinal frequency (ω_{sp}) is shown in relation to pitch responsiveness, *n/α*.

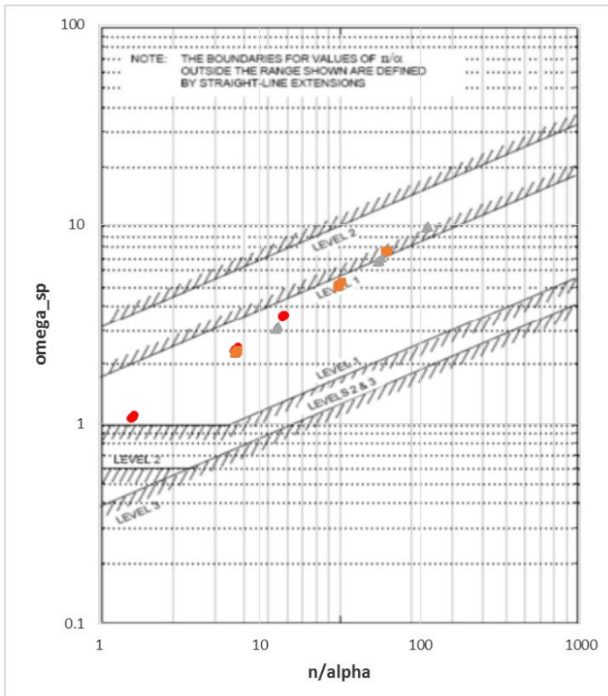


Figure 23: MIL-STD-8785C Category A Chart

Additionally, in lateral-directional stability, the Dutch-roll frequency (ω_{dr}) of 0.42 Hz was calculated for cruise conditions of Mach 0.8 and 48,000 ft. Therefore, the range of the expected rigid body frequencies is seen to be low enough to avoid the wing’s natural frequencies. Additionally, because of the relatively mid-chord sweep ($\Lambda_{C/2}$), there will not likely be significant torque-bending coupling leading to flutter caused by unsteady aerodynamics. Based on this early study, this wing will not likely have any flutter issues, and the design can be focused on reducing weight.

CONCLUSION

In this specific design problem, a moderately lightweight aircraft, the wing structure is constrained by local strip buckling. Therefore, reducing the unsupported length from rib to rib is one of the most significant ways to reduce the coverage area needed to support limiting loads. However, as demonstrated herein, this lighter weight wing will result in a design with increased wingtip deflection compared to alternate designs. As stiffness and aeroelastic effects are balanced with weight targets, a design trade can be made to determine the most appropriate wing structure.

Computer automation and clever planning can greatly improve the conceptual design of airframes. This paper has demonstrated that rule-based automation can be used to enhance the design process to include stiffness results normally left to later design phases. In addition to more accurate stiffness and weight prediction, the added result of basic modal analysis can now be performed. The designer would then perform aeroelastic analysis to determine if the natural frequencies will cause flutter. If so, actions can be taken early to change the stiffness properties of the wing by simple design choices such as rib and stiffener spacing.

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APPENDIX A

WING PARAMETER FULL DETAIL

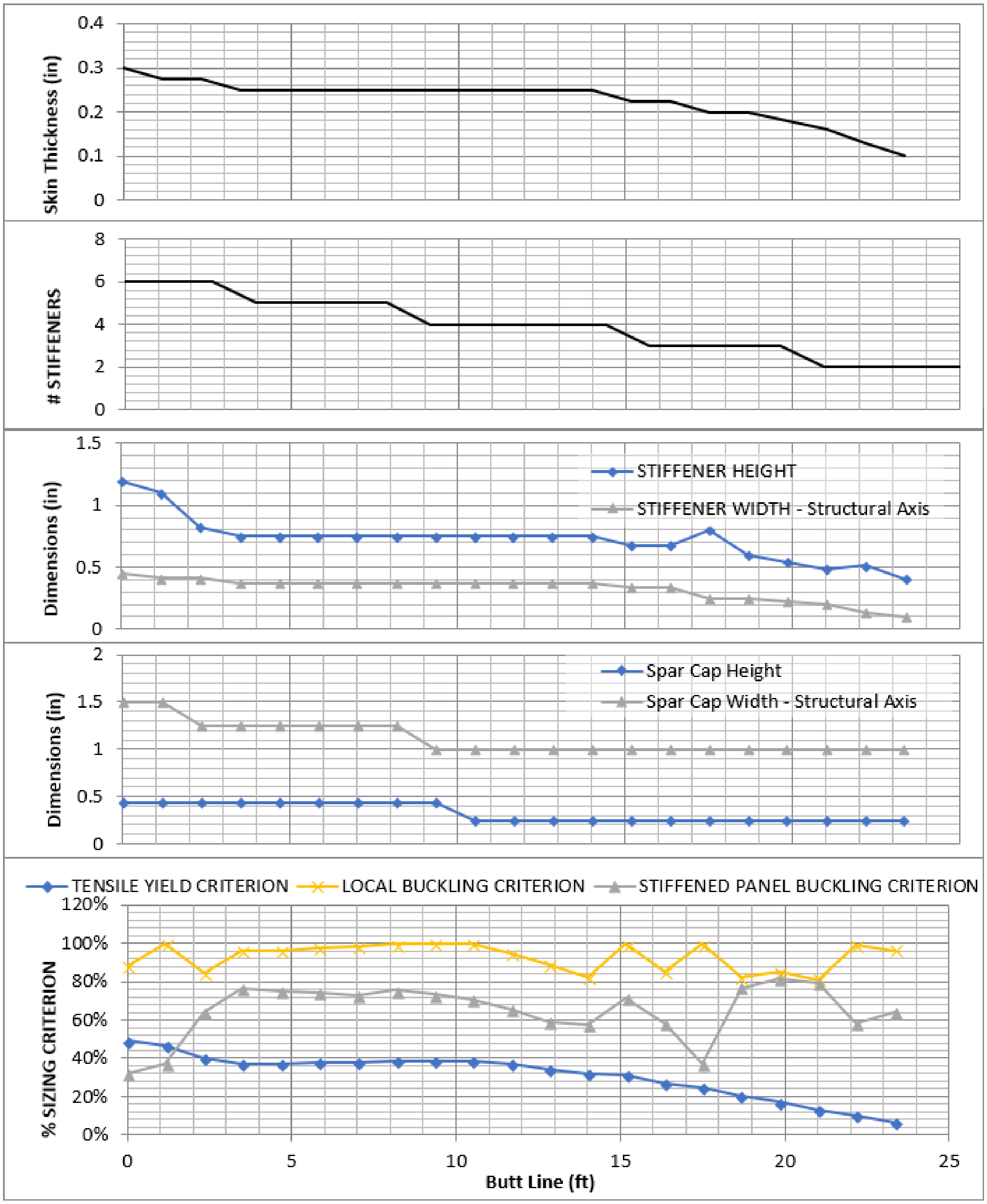

Figure 24: Sample parameters and criterion for a lightweight wing with rib and stiffener spacing of 14 and 11 inches, respectively.

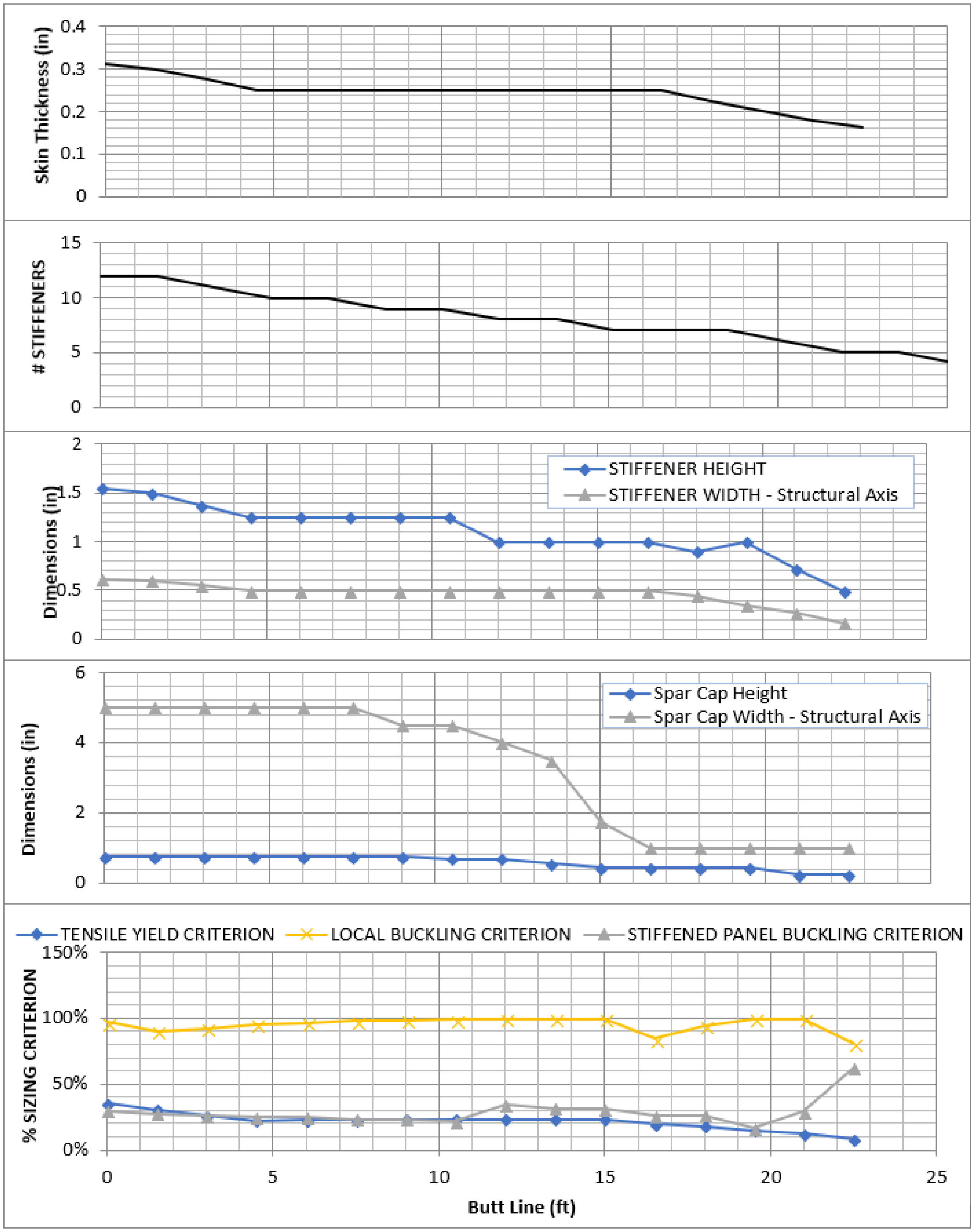

Figure 25: Sample parameters and criterion for a stiff and heavy wing with rib and stiffener spacing of 18 and 5 inches, respectively.