Hosting Capacity for Renewable Generations in Distribution Grids

by

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ABSTRACT

Nowadays, the widespread introduction of distributed generators (DGs) brings great challenges to the design, planning, and reliable operation of the power system. Therefore, assessing the capability of a distribution network to accommodate renewable power generations is urgent and necessary. In this respect, the concept of hosting capacity (HC) is generally accepted by engineers to evaluate the reliability and sustainability of the system with high penetration of DGs. For HC calculation, existing research provides simulation-based methods which are not able to find global optimal. Others use OPF (optimal power flow) based methods where too many constraints prevent them from obtaining the solution exactly. They also can not get global optimal solution.

Due to this situation, I proposed a new methodology to overcome the shortcomings. First, I start with an optimization problem formulation and provide a flexible objective function to satisfy different requirements. Power flow equations are the basic rule and I transfer them from the commonly used polar coordinate to the rectangular coordinate. Due to the operation criteria, several constraints are incrementally added. I aim to preserve convexity as much as possible so that I can obtain optimal solution. Second, I provide the geometric view of the convex problem model. The process to find global optimal can be visualized clearly. Then, I implement segmental optimization tool to speed up the computation. A large network is able to be divided into segments and calculated in parallel computing where the results stay the same. Finally, the robustness of my methodology is demonstrated by doing extensive simulations regarding IEEE distribution networks (e.g. 8-bus, 16-bus, 32-bus, 64-bus, 128-bus). Thus, it shows that the proposed method is verified to calculate accurate hosting capacity and ensure to get global optimal solution.

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NOMENCLATURE

- λ The parameter that can have different meanings to provide flexibility
- θ_{ik} The voltage angle difference between bus *i* and bus *k*
- θ_{max} The upper bound of voltage angle constraint
- θ_{min} The lower bound of voltage angle constraint
- B_{ik} The element in the bus admittance matrix Y_{Bus} which represents the line susceptance between bus *i* and bus *k*.
- f_j The HC of part j
- G_{ik} The element in the bus admittance matrix Y_{Bus} which represents the line conductance between bus *i* and bus *k*.
- I_{ik} The current flow between bus *i* and bus *k*
- I_{Lmax} The upper bound of thermal limit constraint, $I_{Lmax} = C$

 I_{Lmin} The lower bound of thermal limit constraint, $-I_{Lmin} = C$

- *m* The number of segments
- P_i The active power injection at bus *i*

 $P_{Gi, predicted}$ The predicted active generation at bus *i*

 P_{Gi} The active generation at bus *i*

 $P_{i,\text{solution}}$ The power injection at bus *i* to get the solution of hosting capacity

 $P_{Li,\text{predicted}}$ The predicted active load at bus *i*

- P_{Li} The active load at bus *i*
- Q_i The reactive power injection at bus *i*

- s_j The segment point j
- t_j The local/private variable that appear in only one subsystem

 $V_{i,\text{imag}}$ The imaginary part of V_i as the state variable of the system

 $V_{i,\text{real}}$ The real part of V_i as the state variable of the system

- *V_{max}* The upper bound of voltage magnitude constraint
- *V_{min}* The lower bound of voltage magnitude constraint
- x_j The complicating variable that appear in more than one subsystems

Chapter 1

INTRODUCTION

In recent years, an increasing number of renewable energy-based distributed generators (DGs) come into use in the power system. Although the widespread use of DGs brings great benefits including voltage profile support, loss reduction, and lower capital cost, it also imposes operational challenges [1][2]. Particularly, renewable distributed energy resources raise many reliable operation issues respect to power quality and system control[3]. Many studies show that high-level DG penetration may cause problems such as undesirable voltage flicker or excessive operation of the voltage regulating equipment [4, 5]. Thus, assessing the capability of a distribution network to accommodate renewable power generations is urgent and necessary.

The concept of hosting capacity (HC) is defined as the maximum amount of the power generation that the system can host without violating any operating standards [6]. The study on HC aims to guide utilities and residential PV owners about better DGs installations in the distribution network[7][8]. In addition, system planners and operators can guarantee reliable operation and efficient power use with the accurate evaluation of renewable generation integration limit.

Electric Power Research Institute (EPRI) has significant research in this area

[9, 10, 11]. They proposed a simulation-based method which models each feeder and examines all the power quality and reliability issues with screening tools to determine HC respect to different locations. It keeps increasing PV (photovoltaics) penetration levels in the distribution network and runs the power flow until violations of several operation standards appear. The extreme value of PV generations that comes out before the violation occurs is hosting capacity. For computing simplification, this detailed method is upgraded by the streamlined method. Such a method calculate location-specific HC and make it possible to be visualized as heat map [12]. Based on it, several studies use extensive simulations to analyze different networks [13, 14, 15].

In addition to the simulation-based methods[16], others calculate the hosting capacity of the overall distribution network which is different from the location-specific HC[17]. Such a problem can be formulated as a common OPF (optimal power flow) problem [18, 19]. Existing research also extends the formulation to a multi-period AC–OPF for accuracy but it makes the problem more complicated with so many scenarios [20, 21]. To solve the OPF problem, researchers find correlated factors that impact hosting capacity from the possible operating violations in real systems[18, 22, 23]. The main factors are: 1) voltage profile, 2) the network topology, and 3) the feeder load size. In [24], the authors try to identify the key factor of HC determination from the comparison of the three factors: feeder load, voltage limitations, and different locations of distributed energy resources.

With the factors, different approaches are implemented trying to maximize HC without causing adverse impacts. [20] and [21] utilize both static and dynamic network reconfigurations to increase HC. [20] uses active network management that can better quantify HC. Three commonly used control strategies including active power generation curtailment, smart reactive power absorption control, and OLTC (on-load tap-changers) technology are evaluated with real-time information for the increase of HC [24, 25, 26, 27, 28]. [29] presents the algorithm of adding soft open points, which is a power-electronics technique to connect two networks. The above researchers do not care about economic aspects, a cost-benefit analysis is provided to guarantee profitability while determining maximum HC in [30].

However, studies above start with many constraints which prevent them to find exact solutions and provide direct connections between the important factors and HC calculation. Therefore, our work focus on analyzing the accurate interconnections that determine HC.

First, we follow the OPF problem model and make several changes to improve. Our objective function is the summation of each bus power injection in a distributed network multiplied by a coefficient, which can present the different requirements for buses with flexibility. To preserve convexity as much as possible, we gradually add realistic constraints. As the nonlinear function types in power flow equations break convexity, we transfer them to rectangular coordinate for functional reduction. Then during the process to find the solution, we provide the geometric illustration of a toy model and visualize the feasible region with several limits. At this point, a unique pattern is observed to achieve global optimal. We generalize into a theorem for arbitrary radial network and show the mathematical proof to support it. Also, the voltage angle constraint and thermal limit are found to have considerable impacts on the results. So, we provide a piecewise discussion to analyze the problem with these constraints.

When applying the method to real systems, the solving time is fairly long as so many complex system information and constraints are involved. To deal with it, we implement the distributed optimization tool. It helps partition the large model into several subsystems and solve them simultaneously. Thus, the computational efficiency is improved.

The performance of the method is verified by extensive simulations on typical IEEE distribution networks (8-bus and 123-bus). The results show that our problem formulation is able to find accurate HC in different scenarios. Also, the distributed optimization model is validated for computational reduction.

The rest of the thesis is organized as follows: Chapter 2 shows the problem formulation to calculate hosting capacity. Chapter 3 provides the geometric illustration of the HC problem and proposes a generalized theory of maximum HC solution. Chapter 4 shows the mathematical proof to support the theory. Section Chapter 5 illustrates the distributed optimization problem model. Section Chapter 6 tests the theory in extended systems and Section Chapter 7 make a conclusion of the thesis and show future work.

Chapter 2

PROBLEM FORMULATION

2.1 Two Ways to Describe Power Flow Equations

2.1.1 Polar Coordinate-based Power Flow Equations

The commonly used power flow equations are

$$P_i = \sum_{k=1}^n |V_i| |V_k| (G_{ik} \cos \theta_{ik} + B_{ik} \sin \theta_{ik}),$$
$$Q_i = \sum_{k=1}^n |V_i| |V_k| (G_{ik} \cos \theta_{ik} - B_{ik} \sin \theta_{ik}),$$

where *n* is the bus number of the electrical system; P_i is the active power injections at bus *i*; V_i is the voltage magnitude at bus *i*; G_{ik} and B_{ik} are the elements in the bus admittance matrix Y_{Bus} which represents the line admittance between bus *i* and bus *k*.

2.1.2 rectangular Coordinate-based Power Flow Equations

The polar coordinate-based power flow equations involve two types of nonlinear functions: polynomials and sinusoids, which makes the power flow equations difficult to analyze. The nonlinear functions should be eliminated or reduced for simplification. Therefore, we change from the polar coordinate to rectangular coordinate to describe the power flow equations[31]. Let V_i be the complex bus voltages which is $|V_i| \angle \theta_i$, then we think of the real and imaginary parts of V_i as the state variables of the system,

$$V_{i,\text{real}} = Re(V_i), V_{i,\text{imag}} = Im(V_i).$$

This choice of state variables retains full information about the power flow equations, and makes it easier to analyze for our topic. The power flow equations described in the rectangular coordinate system then becomes

$$P_{i} = t_{i,1} \cdot V_{i,\text{real}}^{2} + t_{i,2} \cdot V_{i,\text{real}}^{2} + t_{i,1} \cdot V_{i,\text{imag}}^{2} + t_{i,3} \cdot V_{i,\text{imag}}^{2},$$
$$Q_{i} = t_{i,4} \cdot V_{i,\text{real}}^{2} - t_{i,3} \cdot V_{i,\text{real}}^{2} + t_{i,4} \cdot V_{i,\text{imag}}^{2} + t_{i,2} \cdot V_{i,\text{imag}}^{2},$$

where

$$t_{i,1} = -\sum_{k \in N_i} G_{ki}, t_{i,2} = \sum_{k \in N_i} (G_{ki}V_{k,\text{real}} - B_{ki}V_{k,\text{imag}})$$
$$t_{i,3} = \sum_{k \in N_i} (V_{k,\text{real}}B_{ki} + V_{k,\text{imag}}G_{ki}), t_{i,4} = \sum_{k \in N_i} B_{ki}.$$

In the power flow equations, P and Q have similar expressions where the only difference is the plus or minus sign of a term. So if we analyze P clearly, Q can also be analyzed in a similar way. The apparent power S is often used in real distribution grids. It has the relationship with active and reactive power: $S = \sqrt{P^2 + Q^2}$. The power factor is $PF = \frac{P}{S}$. Therefore, reactive power can be represented as $Q = \sqrt{(\frac{1}{PF} - 1) \cdot P^2}$. It means the reactive power is calculated if we know active power and power factor. We can set the power factor to be a fixed value like the

unity power factor setup in other papers which means there is no reactive power to be concerned. The power factor can also be set up as a constraint in order to control the ratio of active power and reactive power. Here in the first place, we use the unity power factor setup to ignore the reactive power. Because mathematically the different setup of PF is only to change the complexity of the problem model.

2.2 A Flexible Objective Function in the Optimal Power Flow

In this thesis, we use an optimization tool to calculate hosting capacity. The problem formulation of HC calculation consists of two parts: the objective function and several constraints. Compared to others, we provide an objective function with more flexibility.

2.2.1 The Objective Function of other models

In [32], the objective function is the normalized ADC (available delivery capability) under the given load and renewable generation. It represents that each bus in the system has a predicted value of power injection and a coefficient λ is used to scale the power up or down, which is shown below.

$$\sum_{i=1}^{n} \lambda \cdot P_{i,\text{predicted}} = \sum_{i=1}^{n} \lambda \cdot (P_{Gi,\text{predicted}} - P_{Li,\text{predicted}}),$$

where λ is the variable and $P_{i,\text{predicted}}$ is the given value. This choice of objective function helps simplify the process to solve the problem. However, to assume all the buses increase/decrease power injection with the same ratio λ under fore-

casted renewable generation and the load may not be accurate in real systems.

2.2.2 The Objective Function with Flexibility

Since we aim to maximize the summation of all the power injections, meanwhile, investigate the dynamic interaction between buses related to HC, our objective function changes into

$$\sum_{i=1}^n \lambda_i \cdot P_i.$$

It provides the flexibility to the setup of the objective function. λ_i is a binary parameter: $\lambda_i = 1$ if bus *i* has renewable generations; $\lambda_i = 0$ if not. It can also be a non-negative parameter to represent importance or price of each bus: $\lambda_i \in \mathbb{R}+$. Thus, we better satisfy various requirements compared to other methods.

In this thesis, we first set λ_i to be 1 for simplification and P_i is the variable,

$$\sum_{i=1}^{n} \lambda_i \cdot P_i = \sum_{i=1}^{n} P_i.$$

Real power generation without the load is considered in the problem model. It's attributed to that the difference between load and generation in power flow equations is only the plus or minus sign of the power value. Therefore, each bus can have the different generation.

2.3 Operation and Network Constraints

Another part is the constraints that limit the objective function. The existing literature shows the voltage violation is the most important operational limit, so we add voltage constraints firstly. The voltage angle constraint and the thermal limit constraint are also concerned. The following problem formulation is proposed for hosting capacity calculation.

$$\max\sum_{i=1}^{n} \lambda_i P_i \tag{2.1}$$

s.t.
$$P_i = \sum_{k=1}^n |V_i| |V_k| (G_{ik} \cos \theta_{ik} + B_{ik} \sin \theta_{ik}),$$
 (2.2)

$$V_{min} \le |V_i| \le V_{max},\tag{2.3}$$

$$\theta_{min} \le \theta_i \le \theta_{max},$$
(2.4)

$$I_{Lmin} \le I_{ik} \le I_{Lmax}.\tag{2.5}$$

Equation (2.1) is the objective function which is hosting capacity in per unit; equation (2.2) specifies that the solution must satisfy the parameterized power-flow equations; (2.3) and (2.4) specify that the solution must satisfy the operational constraint where voltage magnitudes of all nodes must lie within the specified per unit range (0.95 to 1.05 p.u.) and the voltage angle of bus *i* must lie within the specified lower and upper bounds; equation (2.5) represents the thermal limit where the branch current flow of all distribution lines must lie within specified range.

All the buses except the infinite bus in the distribution network are set to be PV buses because we model DGs with renewable generation as the power sources on each bus. Therefore, P and V are the main factors corresponding to hosting capacity.

Chapter 3

GEOMETRIC ILLUSTRATION OF THE PROBLEM MODEL

Existing research lacks an exact understanding of the interaction inside the network that impacts the hosting capacity. To find out the correlation, we simplify the model firstly.

3.1 Model Simplification

Since the complex system information (e.g., the branch impedance and bus voltage) and constraints make the model difficult to analyze, we set specific conditions. The voltage angle, as well as the line reactance, is set to be zero, which means the network is resistive and there is no constraint on the angle difference. Therefore, the problem formulation is

$$\max \sum_{i=1}^{n} P_i$$

s.t. $P_i = \sum_{k=1}^{n} |V_i| |V_k| G_{ik},$
 $V_{min} \le |V_i| \le V_{max}.$

3.2 Visualization of Toy Models

In order to investigate the large-scale HC, We shift the perspective to think over this problem from a small start point contrary to others. Considering the complexity of power systems, toy models are chosen to analyze the correlation of factors.

3.2.1 Power and Voltage Correlation of the 2-bus Model

Figure 3.1 shows a 2-bus system which is consist of a reference bus 0 whose voltage is $1\angle 0^\circ$ per unit and a PV bus 1 that grows from the infinite bus. In this figure, the resistance of the branch is set to be R = 1 per unit (G = 1). The voltage angle is 0.



Figure 3.1: Compared to the classical definition of PQ load bus, we set each node to be a PV bus because of renewable generations. We start from a simplified 2-bus system. (Bus 0 is the reference bus and bus 1 is a PV bus. The line resistance is set to be 1.) We add one more bus each time to analyze the interconnection

corresponding to HC and generalize the theory to the n-bus system



Figure 3.2: The normally used PV curve where bus 1 is a load bus.

The normally used PV (Power-voltage) curve is plotted in Figure 3.2. As in our model, bus 1 is a generator bus, the PV curve transfer to Figure 3.3. When apply our model to the 2-bus network, we show the PV curve with regard to bus 1 in Figure 3.4. The dashed line means voltage magnitude is negative which is unrealistic. The segmentation marked in red is the feasible region with voltage magnitude constraint.

As shown in the PV curve, both voltage and active power injection have a large range of values from negative to positive. It means when the commonly used power flow equation is the only basis to be considered for this sample without



Figure 3.3: The PV curve of bus 1 when it is a generator bus.

any limitation, the power injection achieves the minimum value of -0.25 p.u, indicating the loadability of bus 1. Meanwhile, its maximum value can be positive infinite which is the hosting capacity of this small system. In other words, infinite renewable energy power can be generated at bus 1 with valid unconstrained power flow equations. However, the voltage needed to get this maximum power is also infinite, which is unpractical in real life.

Usually, voltage magnitudes of nodes lie within specified per unit range (e.g., 0.9 to 1.1 p.u. or 0.95 to 1.05 p.u.) in power system operation. Hence, we add voltage magnitude constraint (0.95 to 1.05 p.u.) to the 2-bus system and get the



Figure 3.4: The unconstrained PV curve of bus 1 in the simplified 2-bus system. P_{G1} below zero is seen as a load to assume power and P_{G1} above zero means bus 1 generates power. As we set the voltage constraint to be [0.95, 1.05], only a segment is feasible which is marked with red. The highest point denotes the operation point with maximum generation, where the voltage magnitude is 1.05 p.u. and the generation is 0.0525 p.u.



3-bus PV curve (2D)

Figure 3.5: The PV (total power-voltage at bus 1) curve of the 3-bus network.

red part in Figure 3.2. The max power injection of bus 1 turns to 0.0525 p.u. when the voltage magnitude equals 1.05 p.u. The loadability also decreases from 0.25 to 0.0475 due to the increasing lower bound of the voltage constraint. It can be observed that there is a positive correlation between power and voltage at bus 1. With the voltage at bus 1 increasing, the power generation capability at bus 1 is larger.

3.2.2 Total Power and Individual Voltage Interaction in the 3-bus Model

Adding one PV bus after the 2-bus system above to get the 3-bus model. In the simplified model, P_i is formed by the relevant variables V_i . P-V curves of bus 1



Figure 3.6: A 3D plot of $P_1 + P_2$, V_1 , and V_2 that illustrate the P-V (total power-individual voltage) relevancy. The value range of both V_1 and V_2 respect to the total power makes it a surface. It is corresponding to the 2D line in Figure 3.4.

in Figure 3.4 illustrate the power-voltage interaction, which is similar to the total power-individual voltage plot of the 3-bus system (Figure 3.5 and 3.6).

Figure 3.5 shows the positive correlation between total power and voltage at bus 1. It leads to the peak total power appearing at the upper bound of V_1 . Figure 3.6 is a 3D plot of $P_1 + P_2$, V_1 , and V_2 that illustrates the P-V relevancy. The figures provide a geometric explanation of the solution for the problem model.

It is worth mentioning that P_2 is negative at the solution point which means

bus 2 doesn't generate any power, accordingly it acts as a load. This is interesting because to achieve the total maximum, each bus should make contributions, which is a normal thinking. In this small system, the total peak hosting capacity is under the condition that bus 1 voltage is the highest while bus 2 voltage is the lowest. Only bus 1 generates power and bus 2 consumes part of power from bus 1 generation. Power is generated by the voltage difference. It leads us to think about how interconnection between voltages of different buses correlated with the hosting capacity.

3.2.3 Pairwise Power Correlation in the 3-bus Model

The PV (power-voltage) curve in Figure 3.4, 3.5 and 3.6 presents the variation of power generation for bus 1, where we find the HC of the 2-bus model. Since hosting capacity in this toy example is defined as the maximum total generation of bus 1 and bus 2, the interconnection observed from the plots of P_1 vs P_2 may relate to the HC value.

It should be noticed that power flow equations are quadratic functions including four variables: P, Q, V, and θ . We set Q and θ to zero for simplification to focus on P and V. To find out the correlation of P_1 and P_2 , the voltages still need to be taken into concern. In Figure 3.7, the area under voltage bounds is the feasible solution region towards the problem model. All the points in the area are feasible solutions under power flow equations. Our goal is to maximize the total amount which is geometrically illustrated as the summation of the *x*-axis value and the



Figure 3.7: The plots of P_1 vs P_2 . The voltage constraints bound a feasible region under power flow equations. We use a figure line: $HC = \max(P_1 + P_2)$ to cut the maximum value. The solution point is found when $V_1 = 1.05$ and $V_2 = 0.95$. It denotes that bus 1 generates 0.1575 p.u. power while bus 2 consumes 0.095 p.u.

power.

y-axis value. So, we use the line $y = P_1 + P_2$ to find the maximum value.

3.3 The Generalized Theory

When adding one bus in a forward direction each time in the existing system, it grows to an *n*-bus single line network or extend to an *n*-bus radial network (tree model), the optimization results show that neighbor buses are in the form of "high-low voltage" to get the maximum generation. In spite of the infinite bus, each

odd bus works on the maximum voltage while the even buses next to them work on the minimum voltage. The 2 buses next to each other are a pair of "high-low voltage". When the distribution lines impedance are not fixed at specified number, the unique solution stay unchanged.

3.3.1 The Theorem of the Simplified Model

Theorem 1. For a simplified single-line feeder or a complicated radial distribution network with resistive or inductive line impedance, when the voltage magnitude constraint is the only limit concerned, the maximum power is generated under the condition that odd buses reach the highest voltage while even buses voltages are the lowest. Hosting capacity depends on voltage magnitudes difference of neighbor buses.

Figure 3.7 provides a geometric illustration to validate the solution in the simplified 3-bus network. The solution point ($V_2 = 0.95$ p.u. and $V_1 = 1.05$ p.u.) for the problem model to find maximum ($P_1 + P_2$) is marked in the figure.

For real system analysis, the voltage angles are not fixed at zero. Thermal limit is also a significant factor to be concerned. With the added constraints, we improve Theorem 1 and get Theorem 2.

3.3.2 The Theorem of the Model with Added Constraints

Theorem 2. Based on the problem model in Theorem 1, when we add voltage angle and thermal limit constraints. First, we consider the voltage angle constraint: $\theta_{min} \leq \theta_i \leq$ $\theta_{max}, \theta_{min} = 0$. The piecewise solution is shown below.

$$\theta_{ik} = \min[\pi, \theta_{max}],$$

$$|V| = \begin{cases} V_{max} \text{ for all buses, when } \theta_{max} > 0.3098, \\ V_{max} \text{ for bus } i, \text{ when } 0 < \theta_{max} \le 0.3098, \\ V_{min} \text{ for bus } k, \text{ when } 0 < \theta_{max} \le 0.3098, \end{cases}$$

where bus *i* is the odd bus, the distance between bus *i* and the infinite bus is odd. Similarly, bus *k* is even. Then we consider the thermal limit constraint: $I_{Lmin} \leq I_{ik} \leq$ $I_{Lmax}, I_{Lmax} = -I_{Lmin}$. We compare the solution above with thermal limit. If the constraint is loose, the solution remains the same; if it is tight, the solution refers to (*) in the proof section.

The theory is inferred from the visualization and solutions to the problem formulation based on the unique tree structure we set at the beginning. We need mathematical derivation to support the theorem.

Chapter 4

MATHEMATICAL PROOF

In the section above, we show the visualization and analysis from the plots corresponding to the problem model. The validity is supported by mathematical derivation below.

The simplified problem model is

$$\max \sum_{i=1}^{n} P_i$$

s.t. $P_i = \sum_{k=1}^{n} |V_i| |V_k| G_{ik},$
 $V_{min} \le |V_i| \le V_{max}.$

We apply the model to the 3-bus toy example,

 $\max P_{1} + P_{2}$ s.t. $P_{1} + P_{2} = 2V_{1}^{2} - V_{1} - 2V_{1}V_{2} + V_{2}^{2},$ $0.95 \le V_{1} \le 1.05,$ $0.95 \le V_{2} \le 1.05.$ $Y_{3-bus} = \begin{bmatrix} 1 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 1 \end{bmatrix}.$

The solution is

$$\begin{cases} P_1 + P_2 = 0.0625 \ p.u. \\ V_1 = 1.05 \ p.u, \ V_2 = 0.95 \ p.u, \\ P_1 = 0.1575 \ p.u, \ P_2 = -0.095 \ p.u. \end{cases}$$

4.2 The Model with Voltage Magnitude Constraint

We show the results validation of the general model with only voltage constraint below. Apply the problem model to the system,

$$\max \sum_{i=1}^{n} P_i$$

s.t. $P_i = \sum_{k=1}^{n} |V_i| |V_k| \cdot G_{ik},$
 $V_{min} \le |V_i| \le V_{max}.$

the maximum solution speculation is achieved when neighbor buses are in the form of high-low voltage. If the in-equation below is proved to be true, the solution is valid.

$$\sum_{i=1}^{n} P_{i,\text{solution}} \ge \sum_{i=1}^{n} P_{i}.$$

It turns into

$$\sum_{i=1}^{n} P_{i} = \sum_{i=1}^{n} |G_{ik}| \cdot (V_{i} - V_{k})^{2} \le \sum_{i=1}^{n} P_{i,\text{solution}}.$$

According to the voltage limits: $0.95 \le |V_i| \le 1.05$,

$$\sum_{i=1}^{n} |G_{ik}| \cdot (V_i - V_k)^2 \le n \cdot |G_{ik}| \cdot (1.05 - 0.95)^2$$
$$= \sum_{i=1}^{n} P_{i,\text{solution}}.$$

The proof above shows that for a tree (single-line) model with only voltage magnitude constraint, when even buses voltage reach the highest value (1.05 p.u. in this thesis) and odd buses go down to the lowest value (0.95 p.u. in this thesis), the system hosting capacity is the maximum. To observe mathematically, neighbor buses always "work in pairs" as square terms to reach local maximum so that the total maximum value is formed by all of the local maximum values. The conclusion is feasible in both resistive and general inductive network.

4.3 The Model with Voltage Magnitude and Angle Constraints

For the model with voltage magnitude constraint, we get the unique solution. However, real systems are mostly inductive network with varied voltage angles. The two types of sinusoidal functions and polynomials in the polar coordinatebased power flow equations that interact non-linearly make them difficult to analyze. The nonlinear functions should be eliminated or reduced for simplification, we use an intermediate rectangular state space between the polar state space and the lifted state space. Apply the problem model with rectangular coordinate-based power flow equations,

$$\begin{aligned} \max \sum_{i=1}^{n} P_i \\ \text{s.t. } P_i &= -\sum_{k \in N_i} G_{ki} \cdot (V_{i,\text{real}}^2 + V_{i,\text{imag}}^2) \\ &+ \sum_{k \in N_i} (G_{ki} V_{k,\text{real}} - B_{ki} V_{k,\text{imag}}) \cdot V_{i,\text{real}} \\ &+ \sum_{k \in N_i} (V_{k,\text{real}} B_{ki} + V_{k,\text{imag}} G_{ki}) \cdot V_{i,\text{imag}}, \\ V_{min} &\leq |V_i| \leq V_{max}, \\ \theta_{min} \leq \theta_{ik} \leq \theta_{max}, \end{aligned}$$

where bus k is the neighbor bus of bus i. The objective function can be represented

as

$$\sum_{i=1}^{N} P_i = \sum_{i=1}^{N} \sum_{k=1}^{N} (-G_{ik}) \cdot [(V_{i,\text{real}} - V_{k,\text{real}})^2 + (V_{i,\text{imag}} - V_{k,\text{imag}})^2]$$
$$= \sum_{i=1}^{N} \sum_{k=1}^{N} (-G_{ik}) \cdot (V_i \cos \theta_i - V_k \cos \theta_k)^2 + (-G_{ik}) \cdot (V_i \sin \theta_i - V_k \sin \theta_k)^2.$$

To find the maximum solution, we simplify the objective function. Some details involved are shown below.

4.3.1 Simplification of the Objective Function

The objective function based on rectangular coordinate power flow equations is

$$\sum_{i=1}^{N} P_i = \sum_{i=1}^{N} \sum_{k=1}^{N} (-G_{ik}) \cdot [(V_{i,\text{real}} - V_{k,\text{real}})^2 + (V_{i,\text{imag}} - V_{k,\text{imag}})^2]$$
$$= \sum_{i=1}^{N} \sum_{k=1}^{N} (-G_{ik}) \cdot (V_i \cos \theta_i - V_k \cos \theta_k)^2 + (-G_{ik}) \cdot (V_i \sin \theta_i - V_k \sin \theta_k)^2,$$

For proof simplification, set $|V_i| = a$, $|V_k| = b$, $\cos \theta_i = a_1$, $\sin \theta_i = a_2$, $\cos \theta_k = b_1$, $\sin \theta_i = b_2$. The formulation above converts to

$$\max \sum_{i=1}^{n} (-G_{ik}) \cdot \left[(aa_1 - bb_1)^2 + (aa_2 - bb_2)^2 \right]$$

s.t. 0.95 \le a \le 1.05, 0.95 \le b \le 1.05,
$$a_1^2 + a_2^2 = 1, \ b_1^2 + b_2^2 = 1.$$

Substitute the equality constraint into the objective function and we get

$$a^{2} + b^{2} - 2ab \cdot (a_{1}b_{1} + a_{2}b_{2}) = a^{2} + b^{2} - 2ab \cdot \cos(\theta_{i} - \theta_{k}).$$

$$\max (-G_{ik}) \cdot [a^2 + b^2 - 2ab \cdot \cos(\theta_i - \theta_k)]$$

s.t. 0.95 \le a \le 1.05, 0.95 \le b \le 1.05.

In the distribution network, G_{ik} is negative which means $-G_{ik}$ is a positive coefficient. To solve this problem, we provide a piecewise discussion for θ in chapter 4.

4.3.2 Proof of the Solution When
$$\theta \in [0, \theta'], \theta' < 0.3098$$

The solution is

$$a = 1.05, b = 0.95, \theta_i - \theta_k = \theta_{max}.$$

When the upper bound θ' changes from 2π to 0, a change of voltage magnitudes is observed, odd buses voltages drop downward to 0.95, even buses voltages keep the value 1.05 and it shows the same result as the simplified network when θ' is relatively small. The rule of voltage angles stay the same during this process. To support the observation results by theoretical basis, we try to prove it mathematically.

$$a^{2} + b^{2} - 2ab \cdot \cos(\theta_{i} - \theta_{k})$$

= $a^{2} + b^{2} - 2ab + 2ab \cdot (1 - \cos(\theta_{i} - \theta_{k}))$
= $(a - b)^{2} + 2ab \cdot (1 - \cos \theta_{ik}),$
 $\cos \theta' \le \cos \theta_{ik} \le 1.$

If θ' is relatively small, then

$$\cos \theta' \simeq 1,$$

 $1 - \cos \theta' \simeq 0$

Therefore,

$$(a-b)^2 + 2ab \cdot (1-\cos\theta_{ik}) \approx (a-b)^2,$$

the solution is proved.

Then a new variable θ in the inductive network needs to be concerned. We provide a piece wise discussion below.

• $\theta \in [0, 2\pi]$

When there is no voltage angle constraint, θ can be any value between 0 and 2π . Therefore, $-1 \leq \cos(\theta_i - \theta_k) \leq 1$. As *a* and *b* are positive, we get the maximum when $\cos(\theta_i - \theta_k) = -1$,

$$a^2 + b^2 - 2ab \cdot \cos(\theta_i - \theta_k) = (a+b)^2$$

The solution of the problem model is a = b = 1.05, $\theta_i - \theta_k = \pi$. We get the maximum hosting capacity under the condition that all the bus voltage magnitudes are the maximum and the difference between neighbor buses voltage angles is π . It means to get the biggest hosting capacity, we need to change the voltage angle of π within one branch interval, which is impossible in the real system. Meanwhile, the unconstrained voltage angle is unstable observed

from several attempts. As a result, the next step is to add an appropriate constraint on the voltage angle.

• $\theta \in [0, \theta'], 0.3098 \le \theta' \le \pi$

The solution is

$$a = b = 1.05, \theta_i - \theta_k = \theta_{max}.$$

• $\theta \in [0, \theta'], \theta' < 0.3098$

$$a = 1.05, b = 0.95, \theta_i - \theta_k = \theta_{max}$$

When the upper bound θ' changes from 2π to 0, a change of voltage magnitudes is observed, odd buses voltages drop downward to 0.95, even buses voltages keep the value 1.05 and it shows the same result as the simplified network when θ' is relatively small. The rule of voltage angles stay the same during this process. Proof shown in appendix supports the solution.

From the variation of the results when the voltage angle constraint is narrowed down, it is shown that the system acquires power generation prior to the difference of neighbor bus voltage angles. In the real systems, the angle difference between a 100km line is approximately six degrees. θ' can be set to be a relatively small value below 0.04π , under which circumstance the system voltage magnitudes correspond with the theorem.

4.4 The Model with Added Thermal Limit Constraint

The problem model with thermal limit constraint is

$$\max \sum_{i=1}^{n} \lambda_i P_i$$

s.t. $P_i = \sum_{k=1}^{n} |V_i| |V_k| (G_{ik} \cos \theta_{ik} + B_{ik} \sin \theta_{ik}),$
 $V_{min} \le |V_i| \le V_{max},$
 $\theta_{min} \le \theta_i \le \theta_{max},$
 $I_{Lmin} \le I_{ik} \le I_{Lmax},$

for two-way power flow, $I_{Lmax} = -I_{Lmin} = C$.

In the inductive radial network, $I_{ij} = |G_{ik} + jB_{ik}| \cdot (V_i - V_k)$. The constraint is represented as

$$-C \le |G_{ik} + jB_{ik}| \cdot (V_i - V_k) \le C,$$
$$-C \le |G_{ik} + jB_{ik}| \cdot (V_{i,r} + jV_{i,i} - V_{k,r} - jV_{k,i}) \le C,$$

which is equal to

$$|(V_{i,r} - V_{k,r})^2 + (V_{i,i} - V_{k,i})^2| \le C^2 / |G_{ik}^2 + B_{ik}^2|.$$

From the proof above,

$$|(V_{i,r} - V_{k,r})^2 + (V_{i,i} - V_{k,i})^2| = a^2 + b^2 - 2ab \cdot \cos \theta_{ik}$$
$$\leq C^2 / |G_{ik}^2 + B_{ik}^2|.$$

We compare the piecewise maximum solution above with the thermal limit constraint. If $C^2/|G_{ik}^2 + B_{ik}^2| > \text{maximum solution}$, the thermal limit has no impact on the solution. In contrast, when $C^2/|G_{ik}^2 + B_{ik}^2| \leq \text{maximum solution}$, the solution is

$$\sum_{i=1}^{n} P_i = \sum_{i=1}^{n} C^2 / |G_{ik}^2 + B_{ik}^2|,$$
$$a^2 + b^2 - 2ab \cdot \cos \theta_{ik} = C^2 / |G_{ik}^2 + B_{ik}^2|. \tag{*}$$

To get the values of $a(V_i)$, $b(V_k)$ and θ_{ik} , this three dimensional cubic equation need to be solved.

4.5 Sensitivity Analysis

The sensitivity of a local bus i is defined as how much impact it has on total hosting capacity when changing the bus voltage. If the sum of the difference between bus i and each bus next to it is positive which shown as the derivative is greater than 0, the system hosting capacity becomes larger with the increasing of bus i voltage. It is positively correlated with hosting capacity.

$$f = \sum_{i=1}^{n} P_i = \sum_{i=1}^{n} \sum_{k=1}^{n} (V_i - V_k)^2 \cdot |G_{ik}|$$
$$\frac{df}{dV_i} = \sum_{k=1}^{n} 2(V_i - V_k) \cdot |G_{ik}|$$

On the contrary, if the summation is less than zero, bus *i* voltage has a negative correlation with hosting capacity. Take a look back at the 3-buses toy model, the degraded curves of the 3D PV plot present visualized demonstration of sensitivity discussed above.

Chapter 5

THE DISTRIBUTED OPTIMIZATION PROBLEM MODEL

With the proof above, we verify the correctness of the theorem. However, real systems are usually very large. Plenty of variables and constraints cause computational difficulties. To solve this problem, we implement the distributed optimization with model segmentation.



Figure 5.1: The system is segmented into parts for easier computation. s_j is the segment bus. The coupling of bus s_j and bus $s_j + 1$ through the line is included in the posterior part.

The original problem model is partitioned into several subsystems. As shown in Figure 5.1, s_j represents the segment point and we totally have m parts. Accord-

ing to the model structure and power flow equations, the variables are divided into two groups: local/private variables (t_j) that appear in only one subsystem and complicating variables (x_j) appear in more than one subsystems because two buses are connected by a line.

The distributed problem model is

$$\max \sum_{j=1}^{m} f_j(t_j, x_j, x'_j)$$

s.t. $x'_j = x_{j-1}, V_{min} \le |V_i| \le V_{max},$
 $\theta_{min} \le \theta_{ik} \le \theta_{max}, I_{Lmin} \le I_{ik} \le I_{Lmax}.$

The model is divided into *m* parts and s_j is the segment bus. f_j means the HC of part *j*; t_j represents the local variables — bus voltages that only appear in one part; x_j and x'_j represent the complicating variables — bus voltages that appear in more than one parts. The objective function loose the coupling as it changes from the summation of power injections for all buses to the summation of HC for all subsystems. The coupling is shifted to the constraints where the segmentation is true only if $x'_j = x_{j-1}$.

For each part, the objective function is

$$f_{j} = \sum_{i=s_{j-1}}^{s_{j}} \sum_{k=s_{j-1}}^{s_{j}} (-G_{ik}) \cdot [(V_{i,\text{real}} - V_{k,\text{real}})^{2} + (V_{i,\text{imag}} - V_{k,\text{imag}})^{2}] (s_{0} = 1)$$
$$= \sum_{i=s_{j-1}}^{s_{j}} \sum_{k=s_{j-1}}^{s_{j}} (-G_{ik}) \cdot (V_{i} \cos \theta_{i} - V_{k} \cos \theta_{k})^{2} + (-G_{ik}) \cdot (V_{i} \sin \theta_{i} - V_{k} \sin \theta_{k})^{2}.$$

According to the proof above, this formulation can be solved to obtain HC. Therefore, the solution of the total system is the same as the summation of all the segments. We are able to reduce computational complexity and improve efficiency as we calculate all the segments simultaneously.

Chapter 6

NUMERICAL VALIDATION

In this section, we apply our problem model shown above to the IEEE 8-bus and 123-bus distribution networks. We perform extensive simulations and the intention is to: 1) determine the hosting capacity of the system and 2) verify if the simulation results are in perfect accord with our theory. During the process, we use the optimization tool — FMINCON and the MATPOWER package in Matlab. Here we demonstrate some representative examples.

6.1 The Flexible Objective Function

As mentioned in the problem model description, all buses in the IEEE 8-bus network are modeled as PV type bus which means each bus can generate power and there are no initial loads. To calculate the hosting capacity of the total system, $\lambda_i = 1$ for all 8 buses is set. Results of the problem subject to several limits are shown in Table 6.1, we present comparisons of the HC under different voltage angle constraints and thermal limits.

In the table, we see a series of changes in hosting capacity as well as voltage magnitudes and angles. As presented in the first row of results, hosting capacity is fairly large without voltage angle constraint. When the voltage angle constraint is gradually narrowed down, the maximum power generation decrease. It

$ heta_{max}$	ILik Impact	V	$ heta_{ik}$	HC		
2.00	No	$V_{\rm all} = 1.05$	π	704.73		
2.00π	Yes	V*		518.26		
	No	$V_{\rm all} = 1.05$	π	704.73		
1.50π	Yes	V^*	$ heta^*$	518.26		
	No	$V_{\rm all} = 1.05$	θ_{max}	704.73		
1.00π	Yes	V^*	$ heta^*$	518.26		
	No	$V_{\rm all} = 1.05$	θ_{max}	480.87		
0.50π	Yes	V^*	$ heta^*$	316.62		
0.10	No	$V_{\rm all} = 1.05$	θ_{max}	75.96		
0.10	Yes	V^*	θ^*	62.43		
0.2002	No	$V_{\rm all} = 1.05$	θ_{max}	74.77		
0.3098	Yes	V^*	$ heta^*$	60.21		
0.07	No	$V_i = 1.05 V_k = 0.95$	θ_{max}	51.75		
0.07	Yes	V^*	$ heta^*$	30.44		
	No	$V_i = 1.05 V_k = 0.95$	$ heta_{max}$	29.45		
0.047	Yes	V^*	θ^*	21.25		
V^* and θ^* are related to the solution of (*) in Section IV.						

Table 6.1: 8-bus network simulation results

slightly changes from generating through angles differences to magnitudes differences, which is corresponding to the theorem. In the real system, the acceptable voltage angle gap through one branch is usually below 0.04π . Therefore, if the constraint for θ is fixed to be a relatively small range, the maximum generation is achieved under the condition that neighbor buses have "high-low" voltages. Thus, Theorem 2 is verified in IEEE 8-bus system.

Sometimes only a few buses or areas are concerned, we set $\lambda_i = 0$ for the nonconcerned bus and the partial hosting capacity is calculated. We take the 8-bus system as an example. Bus 3 is a customer with a load and bus 2, 4, 8 have main PV generators which are expected to produce more generation. To satisfy the requirements, we set: $\lambda_{2,4,8} = 0$ and $\lambda_{1,3,5,6,7} = 0$.Then, the objective function is turned into $\sum_{i=2,4,8} P_i$. A nonlinear constraint is added to fix the load at bus 3: $P_3 = -L_3$ (the value of L_3 can be changed under demand). As the demand increases, bus 2 which is close to bus 3 generates more. The maximum load at bus 3 is limited by the thermal constraint because line flow has an upper bound. Bus 4 and 8 generate as much as possible because they are the main generator buses concerned. The voltage magnitudes compared to those of the original problem model is shown in Figure 6.1.



Figure 6.1: Comparison of voltage magnitudes with different sets of λ_i .

6.2 The Distributed Problem Model

When we extend to a larger system, it still shows perfect validation of the Theorem 2 as we expected in IEEE 123-bus network (Table 6.2)

However, the solving time is much longer as the structure is much more complicated (Figure 6.3) for computation.

$ heta_{max}$	ILik Impact	V	$ heta_{ik}$	HC	Solving Time/s
2.00π	No	$V_{\rm all} = 1.05$	π	6.3460×10^{5}	1594.6
1.50π	No	$V_{\rm all} = 1.05$	π	6.3285×10^{5}	1280.3
1.00π	No	$V_{\rm all} = 1.05$	θ_{max}	5.9878×10^{5}	1130.5
0.50π	No	$V_{\rm all} = 1.05$	θ_{max}	2.9895×10^{5}	525.1
0.10π	No	$V_{\rm all} = 1.05$	θ_{max}	1.5764×10^4	439.9
0.3098	No	$V_{\rm all} = 1.05$	θ_{max}	1.5327×10^4	371.3
0.07π	No	$V_i = 1.05 V_k = 0.95$	θ_{max}	8.1794×10^{3}	297.8
0.04π	No	$V_i = 1.05 V_k = 0.95$	θ_{max}	3.8301×10^{3}	236.7

Table 6.2: IEEE 123-bus network simulation results



Figure 6.2: IEEE 123-bus network structure.

Part	Odd Buses		Even Buses			No. of
No.	V	θ	V	θ	HC	Bus
1(1-20)	1.05	0.04π	0.95	0	599.72	20
2(21-56)					1135.80	36
3(57-77)					576.08	21
4(78-123)					1518.50	46
Total					3830.10	123

Table 6.3: IEEE 123-bus network piecewise simulation results

Therefore, we segment the tree structure into 4 parts according to the distributed model in section V. The segmentation is shown in Figure 6.2 with different marked colors. Key nodes No.16 and No.73 are chosen to partition the tree.



Figure 6.3: Comparison of solving time for different systems with different segmentation.

In Table 6.3, the numerical result of each part presents the same pattern in order to generate the most power with constraints. We can see that the HC of the total system is the same as the HC summation of all the parts. The piecewise calculation costs much less time (Figure 6.3) as we solve all the parts simultaneously.

Chapter 7

CONCLUSION AND FUTURE WORK

7.1 Conclusion

In this thesis, we provide a problem formulation to calculate HC. The method is improved based on commonly used methods. We firstly add flexibility to the objective function. To preserve convexity, we then use rectangular coordinate based power flow equations and add several realistic constraints in sequence.

During the process to obtain optimal solution with proposed problem model, the geometric demonstration is presented together with observing a unique pattern to achieve global optimal. We generalize them into a theory and support it with clear mathematical proof. A distributed optimization tool is used for computational reduction. Finally, Our theory insights is validated by numerical results.

7.2 Future Work

Although we present the good performance of the proposed method on IEEE cases, the robustness of application to complicated networks still needs to be proved. In this case, more factors are concerned. Future work includes the complicated problem model based on more relevant operation limits and extension of the network topology.

Also, our research till now is a deterministic problem model with physical laws like power flow equations. However, we have source of datasets regarding the hosting capacity in distribution network (e.g. solar power generation and solar power voltage respect to different time periods and locations). The datasets are valuable as we can create probabilistic problem model and apply data analytic tools such as popular machine learning methods to obtain meaningful information which deterministic methods are not able to find out. We currently collect data and make visualizations for basic analysis. The implementation of new method and comparison of two methods can be investigated in the future.

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