

The Economics
of Need-based Transfers

by

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A Dissertation Presented in Partial Fulfillment
of the Requirements for the Degree
Doctor of Philosophy

Approved April 2018 by the
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May 2018

ABSTRACT

Need-based transfers (NBTs) are a form of risk-pooling in which binary welfare exchanges occur to preserve the viable participation of individuals in an economy, e.g. reciprocal gift-giving of cattle among East African herders or food sharing among vampire bats. With the broad goal of better understanding the mathematics of such binary welfare and risk pooling, agent-based simulations are conducted to explore socially optimal transfer policies and sharing network structures, kinetic exchange models that utilize tools from the kinetic theory of gas dynamics are utilized to characterize the wealth distribution of an NBT economy, and a variant of repeated prisoner's dilemma is analyzed to determine whether and why individuals would participate in such a system of reciprocal altruism.

From agent-based simulation and kinetic exchange models, it is found that regressive NBT wealth redistribution acts as a cutting stock optimization heuristic that most efficiently matches deficits to surpluses to improve short-term survival; however, progressive redistribution leads to a wealth distribution that is more stable in volatile environments and therefore is optimal for long-term survival. Homogeneous sharing networks with low variance in degree are found to be ideal for maintaining community viability as the burden and benefit of NBTs is equally shared. Also, phrasing NBTs as a survivor's dilemma reveals parameter regions where the repeated game becomes equivalent to a stag hunt or harmony game, and thus where cooperation is evolutionarily stable.

ACKNOWLEDGMENTS

Thank God for giving me living hope and working for my good in all things.

I want to thank Dr. Armbruster for his tireless and precious support. As I look back at the different stages of research, conference talks, dissertation writing, and applications for awards and positions, you helped everything to go so much better than I could have asked for. Thank you.

To my wife, without your incredible strength and spirit, I wouldn't be able to be father to two beautiful girls, Mary and Johanna. Sarah, I can't imagine a more suitable helper; you've supported me so tremendously in all that I've done that I truly feel like I've been held up by you the whole time. Mary and Johanna, thank you for invaluable doses of pure joy.

Thank you to my family who frequently visited and always supported and encouraged me. Thank you to other faculty who helped me academically and practically, Marilyn Carlson, Dr. Gel, Dr. Lampert, Mark Ashbrook, Katie Kolossa, the swarms group, and many more. Thank you to the UBF community in Tempe and across the world, who faithfully prayed for my studies, mission, and family. Thank you to the Human Generosity Project for introducing me to the concept of osotua, and to Drs. Herty and Düring for supporting and encouraging my research. And thank you to my peers and colleagues who shared in all the challenges with me and helped me to stay optimistic.

TABLE OF CONTENTS

	Page
LIST OF TABLES	v
LIST OF FIGURES	vi
CHAPTER	
1 INTRODUCTION	1
1.1 Overview	1
1.2 Statement of Problem	3
1.3 Results	5
2 BACKGROUND	7
2.1 Agent-based Models of Need-based Transfers	7
2.2 Kinetic Exchange Models	10
2.3 Repeated Prisoner’s Dilemma and Survivor’s Dilemma	14
3 SOCIAL OPTIMA OF NEED-BASED TRANSFERS	19
3.1 Transient Wealth Evolution with Need-based Transfers Model	22
3.1.1 Model Structure and Observables	22
3.1.2 Transfer Policies	26
3.1.3 Optimal Network Topology	34
3.2 Quasi-equilibrium Model	38
4 KINETIC MODELS OF CONSERVATIVE ECONOMIES WITH WELFARE THRESHOLDS	42
4.1 Kinetic NBT Model	42
4.2 Kinetic NBT Policies	50
4.3 Control of NBTs	57
4.4 Kinetic NBT with Diffusion	59
4.5 Central Redistribution Model	64

CHAPTER	Page
5 SURVIVOR'S DILEMMA IN VOLATILE ENVIRONMENTS	68
5.1 2-Player Repeated Survivor's Dilemma	68
5.1.1 Nash Equilibria	72
5.1.2 Evolutionary Stability	74
5.2 3-Player Repeated Survivor's Dilemma	76
5.2.1 Nash Equilibria	80
6 CONCLUSIONS	82
6.1 Future Work	85
6.1.1 Agent-based Model Extensions	85
6.1.2 Kinetic NBTs	87
6.1.3 Survivor's Dilemma	88
REFERENCES	90
APPENDIX	
A IMPACT OF NETWORK PARAMETERS ON TRANSFER POLICY EFFI- CIENCY	93
B IDEA OF PROOF FOR $S^{LR}(1) \geq S^R(1)$ FOR $N \leq 4$	96

LIST OF TABLES

Table		Page
3.1	Volatility Model Description	24
3.2	Transfer Policy Descriptions	27
3.3	Mean Steady State Lifespans for Transfer and Attachment Policies in Quasi- Equilibrium Model	41
5.1	2-Player Repeated Survivor's Dilemma Equivalent Static Games	73
5.2	3-Player Repeated Survivor's Dilemma Equivalent Games	80

LIST OF FIGURES

Figure	Page
2.1 Prisoner’s Dilemma Payoffs	15
3.1 Survival Rates for Transfer Policies.....	29
3.2 Efficiency and Number of Deaths for Transfer Policies	30
3.3 Fraction in Need for Transfer Policies	31
3.4 Log-log and Log-lin Plots of Wealth Distributions for Transfer Policies	32
3.5 Gini Coefficient and Wealth Distributions for Transfer Policies	33
3.6 Survival Rates for Transfer Policies Including Hybrid Policy in Osotua Model	34
3.7 Survival Rates and Degree Standard Deviation for Network Reattachment Policies	35
3.8 Mean Wealth of Community and Exhausted Nodes.....	37
3.9 Mean Lifespans for Transfer Policies wth Random Edge Attachment in Quasi-equilibrium Model	40
4.1 Steady State Cases for Kinetic NBT Model.....	48
4.2 Wealth Distribution Evolution in Kinetic NBT Model	49
4.3 Steady State for Exact-match Kinetic NBT Model	52
4.4 Kinetic NBT Policies	53
4.5 Flat Policy Comparison to Agent-based Simulation	54
4.6 Steady State Distributions and Statistics for Kinetic NBT Policies with Gamma Initial Condition	56
4.7 Steady State Distributions and Statistics for Kinetic NBT Policies with Uni- form Initial Condition	57
4.8 Wealth Distributions for Kinetic NBT Policies and Optimal Control Policy .	60
4.9 Wealth Distribution Evolution in Kinetic NBT with Diffusion Model	64
4.10 Steady State Wealth Distributions of Flat Tax Model	67

Figure	Page
5.1 Extensive Form of 2-Player Repeated Survivor's Dilemma	71
5.2 2-Player Repeated Survivor's Dilemma Equivalent Static Game Parameter Regions.....	74
5.3 Probability of Cooperation in 2-Player Repeated Survivor's Dilemma	74
5.4 Extensive Form for 3-Player Repeated Survivor's Dilemma	79
5.5 3-Player Repeated Survivor's Dilemma Equivalent Game Parameter Regions	81
A.1 One-step Survival Rates for Transfer Policies with Various Initial Wealth Distributions as Initial Population Varies	94
A.2 One-step Survival Rates for Transfer Policies with Various Initial Wealth Distributions as Network Mean Degree Varies	95
B.1 Example Transfers for N=5	97

Chapter 1

INTRODUCTION

1.1 Overview

In Aktipis *et al.* (2011), the East African Maasai tradition of *osotua*, practiced among nomadic herders, is presented. In response to disasters like drought and disease, which reduce cattle counts and threaten the economic viability of herders, the Maasai establish binary *osotua* contracts with one another that promise the gift of cattle from one's surplus to meet a partner's need whenever necessary. There is no record keeping in *osotua*, so the decision to gift is purely based on present herd sizes, with no consideration of the history of transfers. Similar binary welfare or risk-pooling relationships are present among vampire bats as well, where reciprocal gifts of food are shared to insure against starvation resulting from unsuccessful foraging [Refs: Wilkinson (1984, 1988); Carter and Wilkinson (2013)].

As risk pools and welfare practices are often centralized, e.g. modern insurance and government welfare programs, the presence and theoretical implications of decentralized binary risk pooling and welfare is of great interest. The investigation of Aktipis *et al.* (2011) is extended in Hao *et al.* (2015), where a generalized version of such binary welfare is defined as being a need-based transfer (NBT), and the impact of network parameters and limited-information transfer policies is investigated.

NBTs are sensible beyond moral codes and social norms under two assumptions:

1. No fault disasters. NBTs are feasible when needs are not indicative of skill or risk, but of unbiased disasters which are out of control of the victims. In such cases, sharing resources without keeping record is more reasonable as losses are considered random and the risk is shared equally.

2. High overhead cost. NBTs, as opposed to partial giving or centrally pooling resources, make sense when transfer or organizational costs are high. For example, with nomadic herders, bringing all cattle to a central location to divide them up would be much less feasible than having binary transfers, as gathering herds centrally may cause hardship in transportation, greater competition for shared resources, greater risk of disease spreading, etc. Similarly, large scale emergencies in modern societies that disrupt infrastructure often lead to direct need-based transfers without resource pooling.

The studies of Aktipis *et al.* (2011) and Hao *et al.* (2015) utilize agent-based models in which a number of agents (representing herds) are simulated and experience random discrete events where the herd sizes grow or decay according to some probability. In response to needs and surpluses being generated by these events, agents redistribute wealth according to prescribed rules. Ultimately, many simulations are conducted and average statistics like mean survival rate of herds is recorded to provide insight into how the various interaction rules impact the community viability. In Aktipis *et al.* (2011), it is found that with dyads, the osotua form of NBTs results in a greater fraction of the population remaining viable than with no transfers or when the decision to give and amount given are probabilistic. Hao *et al.* (2015) find that on small-world Watts-Strogatz networks the survival rate increases with the population size as well as the mean degree of the transfer network; also, individuals with higher degree have higher survival rates.

In addition to investigating the socially optimal transfer rules and network structures, characterizing the wealth distribution of communities that implement NBTs is also of interest. Given the binary nature of the transfers, it is natural to model an NBT economy using kinetic models. Whereas the Boltzmann equation describes the evolution of the position and velocity distribution of gas particles by utilizing physical rules for the outcomes of collisions, kinetic exchange models of markets, in the area of econophysics, liken binary

collisions to two individuals interacting in a trade and so utilize Boltzmann-like equations to describe the evolution of the macroscopic wealth distribution given microscopic rules for binary trades [Refs: Chakrabarti *et al.* (2013); Pareschi and Toscani (2013)]. In comparison to agent-based models, the integro-differential equation (IDE) structure of Boltzmann-like kinetic equations has greater potential for proving results about wealth distributions. Though a variety of such results exist in the econophysics literature, the concept of a viability or welfare threshold as well as deterministic transfers based on exact need have not yet been discussed.

Reciprocal altruism like the reciprocal gift giving of vampire bats and Maasai herders has long been studied from the game theoretic perspective as well, with much attention being given to the repeated prisoner's dilemma framework. Payoffs in the repeated prisoner's dilemma literature are often considered as additive contributions to reproductive fitness, which accumulate over interactions that continue with constant probability [Refs: Trivers (1971); Axelrod and Hamilton (1981); Boyd (1988); Nowak (2006)] (further background in repeated prisoner's dilemma is provided in Section 2.3). However, when considering survival of communities in volatile environments, payoffs should be in terms of survival probability as cooperative decisions more directly impact immediate survival rather than future reproductive fitness. Growth in knowledge of such games remains of value.

1.2 Statement of Problem

Broadly, the goal of this work is to increase knowledge about the mathematics of binary welfare and risk pooling as implemented in NBTs. Agent-based simulation is used to explore socially optimal transfer policies and network structures for NBT sharing networks, kinetic theory is utilized to describe the evolution of NBT community wealth distributions, and a variant of repeated prisoner's dilemma is used to determine parameter regions where such economic cooperation is feasible.

Using agent-based simulation, Hao *et al.* (2015) consider two different transfer rules or policies where those with wealth below threshold are allowed in random order to seek help from (1) the wealthiest individual of their NBT network or (2) a random member of their network. The result of this hit-or-miss process is that requests are more likely to be satisfied by the richest connected member than a randomly chosen one, and so asking the wealthiest connected member results in higher survival rates. We consider a more well-informed community (all individual wealths are public knowledge), and expand this consideration of asking and giving orderings to consider 16 different transfer policies in an effort to determine which policies result in the highest survival rates. We also consider that individuals who lose a partner to loss of viability may establish a new connection with a still viable member of the network, and investigate which reattachment rules result in increased survival in order to infer about a socially optimal network topology. In addition, we define a model for a sustainable NBT economy as opposed to the transient economies of Aktipis *et al.* (2011) and Hao *et al.* (2015) where exponential growth or decay of individual wealths and a death process absent a birth process result in either finite-time community extinction or unrealistic exponential growth of wealths.

Utilizing kinetic theory, we model NBTs by deriving Boltzmann-like kinetic IDEs that describe the evolution of the wealth distribution. Results given from the agent-based models are sought to be confirmed and expanded upon. This analytical IDE structure can be used to examine collision invariants, consider optimal control, and prove some statements about the evolution of the wealth distribution.

In such a reciprocal gift giving system as NBT, there may be temptation to cheat, e.g. by over-reporting needs or under-reporting surplus; thus it is relevant to investigate when mutual cooperation is feasible. By phrasing a repeated prisoner's dilemma on survival rates rather than on additive contributions to reproductive fitness, we seek to understand

the necessary balance between the cost and benefit of NBTs in order to determine when such decentralized economic cooperation is possible.

1.3 Results

In Section 3.1.2 we find that in the short-term, having needy individuals request help from the connected individual whose surplus most closely matches their deficit works as a cutting-stock optimization heuristic [Ref: Wäscher and Gau (1996)]: closely matching deficits and surpluses reserves larger surpluses to be available for those who really need them and is therefore optimal in maintaining viability. In the long-term, asking the wealthiest for help results in the most fit wealth distribution for maintaining survival (fewer individuals with wealth near threshold and less inequality). Also, in Section 3.1.3, we show that anti-preferential edge reattachments result in a network with low variance in degrees, which is found to be socially optimal.

In order to create a sustainable economy, in Section 3.2, logistic growth for community wealth is used, and along with loss of members due to lack of viability, new members are added by splitting existing individuals whose wealths cross a certain splitting threshold. In this way, eventually steady state is achieved where the total wealth of the population as well as the number of individuals in the economy stabilize. Ultimately the average lifespan of an individual equilibrates and can be used to determine optimal network structure and transfer schemes.

In Chapter 4 We derive a kinetic equation to describe NBT transfer policies and compare results related to the survival rate and the impact of transfer schemes on the wealth distribution with those from the agent-based models. We also characterize collision invariants and define an optimal iterative scheme for bringing the greatest fraction of the population above threshold. Too, another model is constructed where a central wealth re-

distribution agent is used to facilitate welfare; this is an early step in comparing the wealth distributions that result from central or binary welfare.

Finally, in Chapter 5, a variant of repeated prisoner's dilemma is constructed and the repeated game framework is used to find expected lifespan from survival probabilities in order to determine the cost and benefit parameter regions where cooperation is evolutionarily stable or can be coordinated to.

Chapter 2

BACKGROUND

2.1 Agent-based Models of Need-based Transfers

Beginning their collaborative work with Aktipis *et al.* (2011), the co-directors of the Human Generosity Project, Athena Aktipis and Lee Cronk examine the question: Did humans evolve to be generous? This initial study examines a form of risk pooling present among the Maasai of East Africa called *osotua*, a specific form of what Hao *et al.* (2015) refer to as “need-based transfers”. Dyads of herders whose cattle populations experience random growth and significant loss with random disasters are modeled in agent-based simulations to share cattle with each other in response to the threatened viability of their partners caused by disasters. It is assumed that a herder with fewer than 64 cattle can no longer viably participate in the economy as a herder and would have to sell what remains of their cattle and pursue a different profession. With this assumption, it is measured how long the average herder can remain viable using different sharing rules with their partner.

The dyadic *osotua* asking and giving rules are formalized as:

1. *Osotua* asking rule: Individuals ask their partner for cattle only if their current holdings are below the viability threshold, 64 cattle.
2. *Osotua* giving rule: Individuals give what is asked, but not so much as to put their own cattle holdings below the viability threshold.

Their agent-based model obeys the following schedule:

1. Herds grow: Individual herds increase in size according to a growth rate sampled from a normal distribution.

2. Potential disaster strikes: Disasters occur for individuals as a Poisson process on average once every 10 years/simulation rounds, and the disaster results in the herd size decreasing according to a volatility rate, also sampled from a normal distribution.
3. Requests made: Requests are made according to probabilistic or osotua rules. The osotua rule that was formalized earlier in this section resulted in individuals asking for help 33% of the time, and asking for 12 cattle on average. The probabilistic asking rule is defined to have individuals ask for 12 cattle with probability .33.
4. Requests fulfilled: Requests are fulfilled or not according to probabilistic or osotua rules. Probabilistic giving is defined such that individuals give 29% of what is asked, which is the average relative size of amount gifted to amount requested using the osotua asking and giving rules.
5. Check for viable herds: If cattle holdings are below the viability threshold after transfers for two consecutive rounds, the herd is determined to be no longer viable.

It is found that the Maasai practice of osotua results in significantly higher survival rates than with no exchanges, probabilistic exchanges, or combinations of osotua and probabilistic asking and giving rules.

Hao *et al.* (2015) consider such risk-pooling policies in greater detail by studying osotua reciprocal gift giving on small-world Watts-Strogatz sharing networks. Watts-Strogatz networks are undirected networks with N nodes and mean degree K (assumed even) that are constructed as follows [Ref: Watts and Strogatz (1998)]:

1. Construct a ring lattice with N nodes each connecting to $K/2$ neighbors on each side.

2. For every node, rewire an edge connecting to that node with probability β and choose the new end of the edge to connect to another node chosen with uniform probability from among all nodes that would avoid self-loops and link duplication.

The parameter β is called the rewiring coefficient and for small values of β , small-world networks are generated with the properties that average path lengths are short and clustering is high. For $\beta \rightarrow 1$, the graph converges to an Erdős-Rényi random graph [Ref: Erdős and Rényi (1959)]. It is found that survival in osotua networks increases with the mean degree of the Watts-Strogatz network and also with the network size when considering complete graphs; also, an individual's survival rate increases with their network degree [Ref: Hao *et al.* (2015)]. What is significant about these results is that there are diminishing returns, e.g. the increase in survival rate from $N = 5$ to $N = 7$ individuals is considerable, but the difference in survival between when $N = 10$ to $N = 100$ is almost indistinguishable. This implies that when social cost is incorporated, such that there is a cost to increase the size and mean degree of the network, this cost will determine an optimal network size and mean degree.

Since Hao *et al.* (2015) model sharing in groups larger than dyads, it matters who asks for help first and who gives it. The order the askers are chosen is random, and two rules for who is asked are given as follows:

1. Random asking: Individuals make a request to one of their osotua partners with equal probability. This assumes that individuals do not have information on their partners' herd sizes.
2. Selective asking: Individuals pick the wealthiest among all their osotua partners to make a request. This requires that individuals have information on their partners' herd sizes.

The osotua giving rule described in Aktipis *et al.* (2011) is used. As opposed to the dyadic model, in this network model there are multiple potential sources of aid, but it is decided that individuals may only ask once (one individual) for cattle. Hence, the result that the selective asking rule gives higher survival rates intuitively makes sense as the wealthiest individuals are more likely to be able to fulfill a request than a randomly selected individual.

In both of these studies, agent-based models are used with initial cattle amounts of 70 (either in dyadic sharing pairs or networks) and the fraction of the initial population remaining viable is observed as herders' wealths evolve according to random growth and volatility, and need-based transfers.

2.2 Kinetic Exchange Models

The kinetic theory of rarefied gases uses statistical mechanics to study the macroscopic behavior of gases that arises from microscopic interactions. In the 19th century, Ludwig Boltzmann developed the most famous kinetic model, the Boltzmann equation [Refs: Boltzmann (1872); Pareschi and Toscani (2013)]. Considering particles of gas as hard spheres with the same radius that hit each other in elastic (energy-conserving) binary collisions, simple microscopic physical rules govern how the velocities and positions of two colliding spheres evolve. With assumptions of a fixed domain, the number of particles increasing to infinity and the mass of the molecules (and also radii of the particles) decreasing to 0 such that their product approaches a finite value, and other assumptions, we can envision a continuum of space filled with particles that do collide [Refs: Cercignani *et al.* (2013)].

As tracking the position and velocity of every individual gas particle is impossible, what is instead modeled is the probability density of particle positions and velocities. Boltzmann used statistical independence and molecular chaos to simplify from modeling the N -particle distribution to modeling just $f(x, v, t)$, the relative density of particles at posi-

tion $x \in \mathbb{R}^3$ with velocity $v \in \mathbb{R}^3$ at time $t > 0$. This Boltzmann equation reads:

$$\frac{\partial f(x, v, t)}{\partial t} = -v \cdot \nabla_x f(x, v, t) + Q(f, f)(x, v, t) \quad (2.1)$$

where the first term on the right describes the change in position of particles that results from their collision-free motion, and Q is the collisional term, which tracks additions and losses to the density of particles with position x and velocity v as a result of collisions [Refs: Cercignani *et al.* (2013); Pareschi and Toscani (2013)].

About a century after the development of the Boltzmann equation, Angle (1986) constructed an economic model that considers random binary interactions of individuals in an economy that exchange wealth based on an inequality process; this comes close to current kinetic exchange models, which will be introduced momentarily. Then, in the early 1990s the term *econophysics* was coined at a conference in Calcutta [Ref: Stanley *et al.* (1996)] as the methods and theories of statistical physicists were explicitly used to study economic problems [Ref: Pareschi and Toscani (2013)]. Soon Slanina (2004) used a kinetic equation for an economic model, where instead of gas particles colliding and changing velocity, individuals in an economy are considered to collide/interact and exchange wealth according to microscopic binary transfer rules; such models are called kinetic exchange models [Refs: Chakrabarti *et al.* (2013); Pareschi and Toscani (2013)]. Rather than using the Boltzmann equation to describe the time evolution of the distribution of particle positions and velocities, a spatially homogeneous Boltzmann-like equation is used to describe the evolution of the distribution of wealth [Refs: Düring *et al.* (2008); Bisi *et al.* (2009)].

Whether kinetic equations are appropriate as economic models has been debated [Ref: Gallegati *et al.* (2006)], but as an a posteriori evaluation of such models, properties observed in empirical wealth distribution data have been recovered by kinetic exchange models of markets [Ref: Chakrabarti *et al.* (2013)]. It has been found that wealth distributions in capitalistic economies can be described as having log-normal distribution of wealth

in the lower majority of the population, with a Pareto tail describing the distribution of wealth of the top 5-10% richest individuals [Refs: Clementi and Gallegati (2005); Silva and Yakovenko (2004); Fujiwara *et al.* (2004)]. With simple assumptions about binary exchange rules, e.g. saving propensities or risky investments, realistic wealth distributions are recovered from kinetic exchange models [Ref: Chakrabarti *et al.* (2013)], and as an indication of social inequality, the fatness of the Pareto tail is commonly the main output observation of interest [Refs: Pareschi and Toscani (2013); Düring *et al.* (2008)].

A Pareto tail refers to a power law distribution for the tail of the wealth distribution, characterized by a Pareto index $\alpha > 0$, such that if $f(x)$ is the relative density of individuals with wealth x , then f is said to have a Pareto tail if $f(x) \sim x^{-\alpha-1}$ for $x \geq c$, where c is some constant greater than 1 [Ref: Pareto (1897); Fujiwara *et al.* (2004)]. The smaller the index α , the fatter the tail is and the greater the fraction of total wealth in the tail. Hence, a small Pareto index or a fat tail corresponds to greater inequality in the wealth distribution.

In Düring *et al.* (2008), a unifying approach to analyzing kinetic models of conservative economies is introduced, where binary interactions are described as:

$$v^* = p_1 v + q_1 w, \quad w^* = q_2 v + p_2 w, \quad (2.2)$$

where v, w are the wealths of the two individuals before collision and v^*, w^* are the wealths of the individuals after collision/exchange. The positive weights p_i, q_i are assumed to have fixed laws that are independent of time and initial wealths, and generally involve some form of randomness. Given that $f(v, t)$ is defined as the relative density of agents with wealth v at time $t \geq 0$ (note that for the references mentioned here, $f(v, t) = 0$ for $v < 0$), we can begin to examine how the wealth distribution evolves in time:

$$f(u, t + \Delta t) - f(u, t) = \left\langle \int dv \int dw f(v, t) f(w, t) \times \{[\delta(v^* - u) + \delta(w^* - u)] - [\delta(v - u) + \delta(w - u)]\} \right\rangle \quad (2.3)$$

where $\langle \cdot \rangle$ denotes the average over the random coefficients p_i, q_i [Refs: Slanina (2004); Chakrabarti *et al.* (2013)]. Essentially, we find the change in density at wealth u as the sum of gains (individuals ending with wealth u after a collision) minus the losses (individuals who initially had wealth u but have a different wealth after collision). Observing that the loss terms just become $f(u, t)$ because our assumption about the weights p_i, q_i being positive gives that every individual wealth changes in a collision, Equation 2.2 can be rewritten in the form of a homogeneous Boltzmann equation as:

$$\partial_t f(v, t) = -f(v, t) + Q_+(f, f)(v, t) \quad (2.4)$$

where $Q_+(f, f)(v, t)$ is used to denote the bilinear collisional gain operator. It can be seen that the collisional gain operator acts on test functions $\varphi(v)$ as

$$Q_+(f, f)[\varphi] = \frac{1}{2} \int_0^\infty \int_0^\infty \langle \varphi(v^*) + \varphi(w^*) \rangle f(v) f(w) dv dw, \quad (2.5)$$

where $\langle \cdot \rangle$ denotes the mean with respect to the random coefficients p_i, q_i [Ref: Düring *et al.* (2008)]. For the collision operator to conserve wealth, it must be that $\langle p_1 + q_2 \rangle = \langle p_2 + q_1 \rangle = 1$. Some examples of models that fit this framework are the model by Cordier *et al.* (2005) and the model by Chatterjee *et al.* (2004).

Introducing redistribution to these kinetic exchange models, Bisi *et al.* (2009) implement a tax to each collision. An example of such a collision tax is

$$v_\epsilon^* = (p_1 - \epsilon)v + q_1 w, \quad w_\epsilon^* = p_2 v + (q_2 - \epsilon)w. \quad (2.6)$$

where constant $\epsilon \in (0, 1)$ can be set to satisfy $\epsilon \leq \delta < \min_{i=1,2} \{p_i, q_i\}$ given that p_i, q_i are bounded below by a constant δ . Hence, the amount being taxed from the individual with wealth v is ϵv and similarly for the individual with wealth w . This gives that v^*, w^* are nonnegative, but wealth conservation is lost such that

$$\langle v_\epsilon^* + w_\epsilon^* \rangle = (1 - \epsilon)(v + w). \quad (2.7)$$

This results in the total mean wealth decreasing exponentially in time,

$$m(t) = \int_{\mathbb{R}^+} v f(v, t) dv = m(0) \exp(-\epsilon t). \quad (2.8)$$

In order to make the model overall wealth conserving, Bisi *et al.* (2009) use a redistribution operator that independently increases the mean wealth at the same rate the collision/tax operator extracts it. They assume this operator to be of the form:

$$R_{\chi}^{\epsilon}(f)(v, t) = \epsilon \frac{\partial}{\partial v} [(\chi v - (\chi + 1)m(t))f(v, t)], \quad (2.9)$$

where the real parameter χ determines the way in which wealth is distributed. Some challenges with this type of redistribution are that the redistribution term is complicated, it is unclear how this could be described at the microscopic level, and it is possible for the redistribution parameter to actually take further money away (beyond the tax). Still, the impact of χ can be characterized as follows: (i) for $\chi > 0$, agents of large wealth are taxed more and wealth is redistributed to poor agents, (ii) for $\chi = 0$ the yield from taxation is equally distributed, (iii) for $\chi < -1$ the poorest are taxed further and the rich benefit, and for $-1 < \chi < 0$ the poor and rich benefit at the expense of the middle class. In summary, the taxation/redistribution kinetic equation is given as:

$$\frac{\partial f(v, t)}{\partial t} = Q_{\epsilon}(f, f)(v, t) + R_{\chi}^{\epsilon}(f)(v, t) \quad (2.10)$$

where here the collisional gain/loss terms are combined into the operator $Q_{\epsilon}(f, f)$.

2.3 Repeated Prisoner's Dilemma and Survivor's Dilemma

The previously introduced practices of the Maasai herders as well as vampire bats are examples of reciprocal gift giving. Whether and why individuals would participate in such an economic system are questions best investigated in the game theoretic framework. In Chapter 5, we examine when selfish individuals will cooperate in a repeated survivor's dilemma, which is a variation of the repeated prisoner's dilemma.

		Player 2	
		<i>C</i>	<i>D</i>
Player 1	<i>C</i>	(R, R)	(S, T)
	<i>D</i>	(T, S)	(P, P)

Figure 2.1: Prisoner’s dilemma payoffs using the notation of Rapoport and Chammah (1965): R is the reward for cooperation, T the temptation to cheat, S the sucker’s payoff, and P the punishment for mutual defection. To be considered a prisoner’s dilemma, the payoffs must satisfy $T > R > P > S$.

Trivers (1971) connects repeated symmetric situations like NBTs with the prisoner’s dilemma [Refs: Tucker and Straffin Jr (1983); Rapoport and Chammah (1965)], which has a payoff matrix as described in Figure 2.1. A two-player prisoner’s dilemma is in general a symmetric game where defecting when an opponent cooperates results in a temptation to cheat T , mutual cooperation leads each individual to receive a reward R , mutual defection leads to each individual receiving a punishment P , cooperating while an opponent defects leads to a sucker’s payoff S , and the payoffs obey the following relation: $T > R > P > S$. Because $T > R$ and $P > S$, no matter what Player 2 chooses, it is always best for Player 1 to defect. Thus, in this symmetric game, the temptation to cheat and fear of being a sucker lead to mutual defection as opposed to mutual cooperation, which would be socially optimal.

If such prisoner’s dilemma decisions are made at the timescale of weeks, then over years these games may be repeated many times, leading to an iterated or repeated prisoner’s dilemma as popularized by Axelrod and Hamilton (1981). While Triver’s drowning man example [Ref: Trivers (1971)] was originally phrased as having cooperation impact survival probability, much attention has instead been given to treating payoffs as additive contributions to reproductive fitness, accumulated over interactions that continue with

a fixed probability, called the “shadow of the future” [Refs: Axelrod *et al.* (1988); Oye (1985)]. Symbolically, for constant continued interaction probability w , an individual’s fitness would be found as:

$$F = \sum_{t=1}^{\infty} w^{t-1} U_t = U_1 + wU_2 + w^2U_3 + \dots \quad (2.11)$$

where U_t is the payoff the individual gets on the t -th time the two individuals interact and play the game.

Yet in the examples of bats and herders, cooperation directly impacts survival probability and thus the probability of future interactions w should be a function of player decisions rather than independent of them; this is especially important when survival events occur on a shorter timescale than reproduction. In Chapter 5 we develop a model with the following features: (i) the prisoner’s dilemma is on survival rates with overall fitness defined as expected lifespan, i.e. a survivor’s dilemma, (ii) there is an infinite horizon, i.e. the game has an unknown ending time, (iii) we consider survival rates determined by a space of parameters β, b, c , which represent survival in isolation, the benefit of cooperation, and the cost of cooperation respectively, and (iv) we scale our model from a 2-player game to a 3-player game.

Axelrod *et al.* (1988) acknowledge the potential of variation in continued interaction probability w to affect observed patterns of cooperation. Feldman and Thomas (1987) and Thomas and Feldman (1988) consider behavior-dependent contexts for RPD where, for example, one individual determines whether the game will continue based on their strategy such that if this individual cooperates the game will continue with probability w and if the player defects they continue to play with probability u . This however lets only one individual determine whether the game continues, and not as related to survival.

A better model for the vampire bats and Maasai would be what is called a survival game or survivor’s dilemma [Ref: Garay (2009); Garay and Varga (2011)]. In Eshel and

Weinshall (1988), rather than encounters determining additive contributions to fitness, the authors phrase a game such that interaction payoffs are survival probabilities and the overall goal of the supergame is to maximize expected lifespan. Eshel and Weinshall (1988) consider that the survival rates generated by an encounter have a positive probability of being of the prisoner's dilemma type, but also a positive probability of having mutual cooperation be of immediate self reward. They find that cooperation can be favored at a present disadvantage to a player in order to preserve their partner's survival for a potential beneficial selfish mutual cooperation in the future; this concept is called partnership [Ref: Eshel and Shaked (2001)]. We consider an iterated game that is always a prisoner's dilemma with survival rates determined by fixed parameters rather than random payoff functions.

The survival game model of Lima (1989) considers that with some probability an individual can regain a partner when their current partner dies; the author also considers a known end time for the interactions and generates a table of cooperation probabilities. We will instead consider that in the course of a single pairing a deceased partner may not be replaced. Also, we consider a sort of "shadow of the future" model where the end of the game (signaled by the death of both players) is unknown.

Garay (2009) and Garay and Varga (2011) model a survivor's dilemma which considers whether individual's will cooperate to defend each other against a fixed number of attacks. Again, we consider an unknown stop to the game or infinite horizon; also, there are some differences between the threat of a predator and the threat of e.g. starvation for a bat. For example, a model of Garay and Varga (2011) considers that in an attack, if one individual defends itself and the other flees, the one who flees survives with probability 1. In contrast, the metaphorical mouth of starvation is large enough to not be filled with one prey, and taking the life of one bat does not prevent starvation from taking the life of another simultaneously.

What is of interest in our survival game is whether cooperative strategies can be selected for in a context of repeated survivor's dilemmas. The iterated prisoner's dilemma tournaments of Axelrod and Hamilton (1981) had a surprisingly simple strategy as their victor: tit-for-tat, which would involve initial cooperation and then mimicking the other player. Especially for relatively simple species like vampire bats, it is of interest to understand evolutionarily how cooperation could have been selected for with similarly simple interactive strategies.

While concepts such as Nash equilibria predict the behavior of rational agents, expecting bats to calculate expected lifespans from survival rates based on environmental parameters is unreasonable. Instead, it is more logical to assume that bats are genetically or otherwise coded to either be cooperators or not, and then determine how the prevalence of both strategies evolve according to reproductive competitiveness. One standard treatment to investigate evolutionary stability of strategies is to model the evolution of strategy densities using replicator equations [Ref: Nowak (2006); Hofbauer and Sigmund (1998)]. Where x_i is the fraction of the population of type i , $x = (x_1, \dots, x_n)$, $f_i(x)$ is the fitness of strategy i given the densities of all strategies, and $\phi(x) = \sum_{j=1}^n x_j f_j(x)$ is the average fitness, the evolution of x_i is given by:

$$\dot{x}_i = x_i[f_i(x) - \phi(x)]. \quad (2.12)$$

Standard tools from systems of ordinary differential equations can then be used to determine equilibria and stability.

Chapter 3

SOCIAL OPTIMA OF NEED-BASED TRANSFERS

In this chapter, we define a general structure for transient agent-based models of need-based transfers (NBTs), and explore socially optimal transfer policies and network topology as well as develop a quasi-equilibrium agent-based model of NBTs.

NBTs are a form of risk pooling where binary exchanges of some currency occur to ensure the economic viability of a community, especially in volatile environments. A specific form of this has been studied by Aktipis *et al.* (2011) and Hao *et al.* (2015) who modeled the East African Maasai tradition of *osotua*, which involves nomadic herders gifting cattle to a member of the tribe with whom they have a special relationship. Here, the assumption is that any member of an *osotua* relationship gives the amount of cattle needed by another member, whenever possible, in order to maintain the receiving member's economic viability. This reciprocal relationship provides a form of insurance, or risk pooling in times of disaster like drought, flood, famine, or disease.

The studies by Aktipis *et al.* (2011) and Hao *et al.* (2015) use transient agent-based models, and examine the survival rate, or fraction of the initial population remaining viable at some time. Aktipis *et al.* (2011) show that the *osotua* model of need-based transfer, where requests are fulfilled whenever possible, results in significantly higher survival rates than with no exchanges or probabilistic exchanges when used with isolated dyads. Hao *et al.* (2015) show for small-world Watts-Strogatz networks that survival rates increase with the size of the population and mean degree of the transfer networks, with individuals with high degree having higher survival rates than individuals with low degree; because the improvement in survival obeys a law of diminishing returns, the social cost of increasing network size and mean degree will determine the optimal network size and degree. Hao

et al. (2015) also examine two different asking rules: *random asking* involves a hit-or-miss request to a random neighbor (connected member of the network), while *selective asking* requests help from the richest individual that the asker has an osotua relationship with. It is found that the selective asking rule results in a higher survival rate, which is intuitively reasonable as the wealthiest individual is more likely to be able to meet a request than some other random connection.

The goal of this study is to develop a more general framework for NBT modeling and investigate social optima. In particular, we (i) examine the impact of informed asking and giving transfer policies, (ii) seek an optimal network structure, and (iii) develop a quasi-equilibrium agent-based simulation model that allows us to study the impact of time-varying policies as well as the influence of changing risk patterns due to climate change on the stability of an economy that is strongly determined by NBTs. To conduct the first two investigations, we extend the osotua model from Aktipis *et al.* (2011) and Hao *et al.* (2015) to a transient model with more volatile wealth evolution; this will encourage more frequent transfers and therefore result in a magnified view of the effects of our various transfer policies.

We find that need-based transfers that focus on short-term benefits can be compared to a cutting-stock optimization problem [Ref: Wäscher and Gau (1996)] where finite sizes of deficits and surpluses are attempted to be matched in a way as to reduce waste and keep as many individuals viable as possible. In the long-term, the rules for need-based transfers not only have a direct impact on the survival of an individual, they have a secondary effect of changing the wealth distribution. Specifically, rules that increase the contributions from the richest members of the group lead to a fitter wealth distribution (less inequality and fewer individuals near threshold) which results in higher long-term survival rates. Finally, we also claim that the socially optimal network structure is one that minimizes variance in network degrees, e.g. by conducting anti-preferential network attachment in steady state.

To develop a quasi-equilibrium simulation platform, we have to balance the birth and death processes in the current model. Specifically, in order to have stabilization in the viable community population we change the NBT model from an exponential growth model to logistic growth of the total community wealth, modeling growth limits due to finite environmental resources. In addition, a birth process needs to be introduced that compensates for individuals that have been eliminated from the economy due to loss of viability. We do this by adding a splitting mechanism, whereby an individual that has high wealth splits and transfers a part of their wealth to the new ‘birthed’ individual. We are not concerned with the details of this process yet, but note that there is an opportunity here to connect to anthropological and economic theory of succession and inheritance rules.

To fix ideas, we characterize a need-based transfer: As implied, there has to be some apparent need. When focusing on wealths of individuals, we consider a poverty or viability threshold. Poverty thresholds, like income thresholds for qualifying for welfare are determined such that in order to have a certain quality of life or to be a participating member of an economy, an individual must have wealth above a certain level. Among the Maasai, there is a viability threshold where having less cattle than a certain amount would not allow an individual to viably survive as a herder, and they would therefore have to sell out and pursue another profession. Thus, when we consider some viability threshold, we measure need as the difference between that threshold and an individual’s wealth.

When these needs or deficits occur as the result of major or minor economic disasters in modern society, insurance companies are commonly used to prevent loss of viability. Here individuals pay premiums at rates proportional to their risk and are paid to cover their losses. Through government, many countries provide various forms of welfare, which may be thought of as individuals collecting their money to insure against those who may be driven by circumstance to a lesser quality of life. Need-based transfers, in the context in

which we consider them, are *not central collections of fractional amounts, but are completely binary transfers of entire needs.*

An individual may be connected to multiple others in a need-based transfer network, and if that individual's wealth falls below threshold, that individual will be gifted the complete amount of their need from one of the other individuals in the network whenever possible, or receive nothing. Here, "whenever possible" means that an individual will only be gifted wealth if the entire amount of their need can be met without causing the giver to go below threshold themselves. With these need-based transfers, there is no record keeping, so if an individual has need every year and another individual is able to help every year, help will be given. Beyond moral codes and social norms, such a form of risk pooling is sensible under the assumption of no-fault disasters and high overhead costs; this is discussed in Section 1.1.

3.1 Transient Wealth Evolution with Need-based Transfers Model

As in Aktipis *et al.* (2011) and Hao *et al.* (2015), we begin by considering transient NBT wealth evolution models. These are transient in that individuals may only lose viability; no new individuals are born. Thus, if the growth/disaster rates are such that there is a positive probability of an individual losing viability, eventually all individuals will lose viability. While such an inevitable extinction model may seem unrealistic, the rate at which the community approaches extinction, or the fraction of the population remaining viable at any time are useful indicators of NBT policy decisions.

3.1.1 Model Structure and Observables

The overall structure of a transient NBT simulation model can be described in four steps:

1. Initialization. Assign the number of individuals $N \in \mathbb{N}$, the length of simulation $T > 0$, the number of simulations, the initial wealths for all individuals $\vec{w}(0) \in \mathbb{R}^N$, the viability threshold $\theta \in \mathbb{R}$, and a network adjacency matrix $X(0) \in \{0, 1\}^{N \times N}$.
2. Growth/decay process. The wealth of individual i grows/decays at rate $R_i(t)$, such that $\hat{w}_i(t) = (1 + R_i(t))w_i(t)$ for all $1 \leq i \leq N$.
3. Wealth transfer process. Using a threshold $\theta \in \mathbb{R}$, transfers update the wealth $\hat{w}_i(t)$ to the new wealth after transfer, $w_i(t + 1)$. Let $w_i \in \mathbb{R}$ be the wealth of individual i (note that we may consider negative wealth as debt), and W_i be the set of wealths of individuals it is connected to. If $\hat{w}_i < \theta$, then individual i 's wealth after transfers becomes

$$w_i(t + 1) = \begin{cases} \theta & \text{if there exists } \hat{w} \text{ in } W_i \text{ such that } \hat{w} - (\theta - \hat{w}_i) \geq \theta \\ \hat{w}_i(t) & \text{else.} \end{cases}$$

A giver of help loses the wealth that is transferred and individuals that are not part of a transfer maintain their wealths. Such transfers are conducted until all viable members with need have had a chance to request help; the order of requests is provided by some prescribed rule.

4. Viability check. An individual i 's viability at time $t+1$ is determined to be either non-viable, $v_i(t + 1) = 0$, or viable $v_i(t + 1) = 1$, depending on whether the individual's wealth $w_i(t + 1)$ is below threshold or not, respectively. The adjacency matrix is updated to remove all connections to nonviable nodes. Edge reattachments may be performed to determine $X(t + 1)$.

Hence, $w_i(t)$ is individual i 's wealth at the beginning of a simulation round, $\hat{w}_i(t)$ represents the wealth after the growth/decay step, and $w_i(t + 1)$ gives the wealth after all

transfers have been conducted. To complete one simulation step, the individual's viability $v_i(t+1)$ is calculated, and its connections are updated. Different transient NBT models are constructed by changing the rules for steps 2-4.

	Volatility model
Initialization	$N = 100, T = 300, 10^5$ simulations, $\vec{w}(0) = \{70\}^N$, $\theta = 64, X_{ij}(0) = \mathbb{1}_{i \neq j}$.
Growth/decay	$R_i(t) \sim \mathcal{N}(0, 0.3^2)$.
Transfer policy	$\star \leftarrow \star$ (see Table 3.2).
Viability check	$v_i(t+1) = \begin{cases} 0 & \text{if } w_i(t+1) < \theta \\ 1 & \text{else} \end{cases}$ $X_{ij}(t+1) = \begin{cases} 1 & \text{if } X_{ij}(0) = 1 = v_i(t+1) = v_j(t+1) \\ 0 & \text{else.} \end{cases}$

Table 3.1: Description of the volatility model. The initial network is a complete graph. $\mathcal{N}(\mu, \sigma^2)$ denotes a normal distribution with mean μ and variance σ^2 .

We illustrate this structure in Table 3.1, defining what we will call the volatility model. The model obeys Gibrat's law [Ref: Mansfield (1962)] in that the multiplicative growth/decay rates are independent of wealth. We choose the specific threshold value and initial wealths in order to maintain comparability with the osotua model from Aktipis *et al.* (2011) and Hao *et al.* (2015). However, whereas the osotua model decomposes the growth/decay rate into a growth rate $g_i(t)$ and disaster volatility $l_i(t)$ with disasters generated as a Poisson process, the volatility model simplifies this into an annual growth rate that may be bigger or smaller than 0. Also, the variance in the growth/decay rate in the volatility model is

chosen to be much larger than what is typical for cattle herders in order to magnify the effects of the policy decisions.

The main observable for the transient models is the survival rate, i.e. the fraction of the population remaining viable at any time,

$$S(t) = \frac{1}{N} \sum_{i=1}^N v_i(t). \quad (3.1)$$

$S(t)$ is the cumulative result of death/loss of viability over time. The number of deaths occurring in a simulation round,

$$D(t) = \sum_{i=1}^N (v_i(t) - v_i(t+1)), \quad (3.2)$$

gives an instantaneous measure of the loss of viability in time.

Wealth inequality will be measured by the Gini coefficient [Ref: Dorfman (1979)] based on the Lorenz curve. The Lorenz curve $y = L(x)$ describes the fraction y of the total wealth contained in the fraction x of the total population ordered from poorest to richest. For example, the point $(0.5, 0.3)$ means that 30% of the community wealth is contained in the bottom 50% of the population. Thus, necessarily, $L(x)$ goes through the points $(0, 0)$ and $(1, 1)$. A homogeneous population where wealth is completely equally distributed has a Lorenz curve that is a straight line $L(x) = x$. The Gini coefficient of a wealth distribution then is defined as $G = 1 - 2B$, where B is the area under the associated Lorenz curve. Thus for no inequality, i.e. a Lorenz curve that is a straight line, $G = 0$, while Gini coefficients near 1 correspond to high inequality.

To measure the vulnerability of a population we define the *fraction in need* as

$$FIN(t) = \frac{\sum_{i=1}^N H(\theta - \hat{w}_i(t))v_i(t)}{\sum_{i=1}^N v_i(t)}, \quad (3.3)$$

which is the fraction of the viable population that is below threshold before transfers.

To characterize the effectiveness of a transfer scheme, we consider *efficiency* to be the fraction of wealth that was transferred relative to the amount of wealth that was needed and available:

$$EFF(t) = \frac{\sum_{\{i:w_i(t+1)=\theta\}} v_i(t)H(\theta - \hat{w}_i(t))(\theta - \hat{w}_i(t))}{\min \left\{ \sum_{i=1}^N v_i(t)H(\theta - \hat{w}_i(t))(\theta - \hat{w}_i(t)), \sum_{i=1}^N v_i(t)H(\hat{w}_i(t) - \theta)(\hat{w}_i(t) - \theta) \right\}}. \quad (3.4)$$

3.1.2 Transfer Policies

In Hao *et al.* (2015), individuals ask for help in random order, and whom they ask is determined in two different ways: (i) *random asking* implies that an individual asks for help from a random osotua member and if that specific osotua member cannot help, the asker loses viability; (ii) *selective asking* means that help is sought from the asker's richest osotua member. Thus, the random asking is a hit-or-miss procedure that obviously 'misses' more often than the *selective asking* procedure and therefore performs worse.

For our generalized asking rules, we assume that individuals in our communities have complete and accurate information on the wealths of the other members of their community and may use that information to different degrees. Among the Massai this is reasonable as cattle are visible and may be counted. In other situations, public information of wealths may be available, or communication among sharing networks may provide such information. Thus, in our version of random giving, the asker sensibly requests help from an individual randomly chosen from among only the individuals which are connected to the asker *and* are able to fulfill the request. Hence, an asker will receive help if and only if there exists network member whose surplus is greater than or equal to the asker's deficit at the time of request. Note also that individuals with enough surplus may give help multiple times (to multiple needy individuals) in a single round of transfers.

If multiple individuals need help, not only who is asked, but also the order of the asking is a relevant aspect of an NBT transfer policy. We consider four different asking and four different giving orderings as described in Table 3.2.

Asking order		Giving order	
<i>Poorest</i> (<i>P</i>):	The individual with the greatest need asks for help first, then the individual with second greatest need, etc.	<i>Richest</i> (<i>R</i>):	The individual with greatest wealth is asked for help.
<i>Least Poor</i> (<i>LP</i>):	The individual with the least need asks first, then the individual with the second least need, etc.	<i>Least Rich</i> (<i>LR</i>):	The individual with the smallest surplus who can meet the request is asked.
<i>Random</i> (<i>RA</i>):	The players with need ask in random order.	<i>Random</i> (<i>RG</i>):	The asker randomly selects an individual among those who can help.
<i>Fixed</i> (<i>FA</i>):	The asking order is fixed for all time, e.g. by social status.	<i>Fixed</i> (<i>FG</i>):	The giving order is fixed.

Table 3.2: Transfer policy descriptions. A policy will thus be defined and referred to as *Asking order* \leftarrow *Giving order*, for example $P \leftarrow LR$ refers to the policy where the *Poorest* asking order is used and the *Least Rich* giving order is used. Once an asker is chosen, a request is made to the connected individual who matches the giving order rule.

The relevance of the asking order is illustrated by this simple example: Let $N = 4$, $\vec{w}(0) = (1, 0, 3, 4)$, and $\theta = 2$. Here the $LP \leftarrow R$ scheme leads to the wealth transfer $(\mathbf{1}, 0, 3, \mathbf{4}) \rightarrow (2, 0, 3, 3) = \vec{w}(1)$, as the least poor individual received help from the richest individual. Since there is no single individual with a surplus of 2 that matches individual 2's deficit, individual 2 will lose viability. However, for the $LP \leftarrow LR$ scheme, two exchanges are possible: $(\mathbf{1}, 0, \mathbf{3}, 4) \rightarrow (2, \mathbf{0}, 2, \mathbf{4}) \rightarrow (2, 2, 2, 2) = \vec{w}(1)$. Here all four individuals remain viable. Note that the choice of the scheme not only impacts viability, it also impacts the distribution of wealth of the survivors. Saving the extra individual comes with the increased vulnerability of all four individuals being exactly at threshold.

All results and figures in this section use the volatility model detailed in Table 3.1. Figure 3.1 shows the mean survival rates $\langle S(t) \rangle$ of extensive simulations of all 16 different combinations of asking and giving rules, and leads to the following two general statements:

1. *In the short-term, survival rates are highest for the $\leftarrow LR$ giving policies and lowest for the $\leftarrow R$ giving policies.*
2. *In the long-term, survival rates are highest for the $\leftarrow R$ giving policies and lowest for the $\leftarrow LR$ transfer policies.*

Attempting to explain these important phenomena, we narrow our policy considerations to those with random asking order, $RA \leftarrow$ and first examine the short-term behavior of our transfer policies. One can prove (see Appendix A) that regardless of the asking order and initial wealth distribution, for $N \leq 4$, $S^{LR}(1) \geq S^R(1)$. Thus, if four individuals are participating in an NBT relationship together, regardless of their wealth distributions, to optimize their survival for any given year, they should ask the least rich to give with priority. However, for $N \geq 5$ there do exist initial wealth distributions for which this is not true anymore (example provided in Appendix A).

It may sound counterintuitive to encourage asking the least rich to give first, but given the rules governing NBTs in Section 3.1.1, maximizing survival for a single exchange round is essentially equivalent to a cutting-stock optimization problem where the heuristic approach of matching the smallest surpluses to deficits first improves efficiency (reduces unutilized surplus), especially by reserving larger surpluses for individuals with larger deficits rather than breaking up those larger surpluses so that individuals with large deficits are left without matches. This is illustrated in Figure 3.2 where in the short-term, while the wealth distributions are still similar (before transfer policies differences have a cumulative effect), the efficiency of a policy primarily determines survival as can be seen by the anti-correlated efficiency and deaths data for $t < 10$.

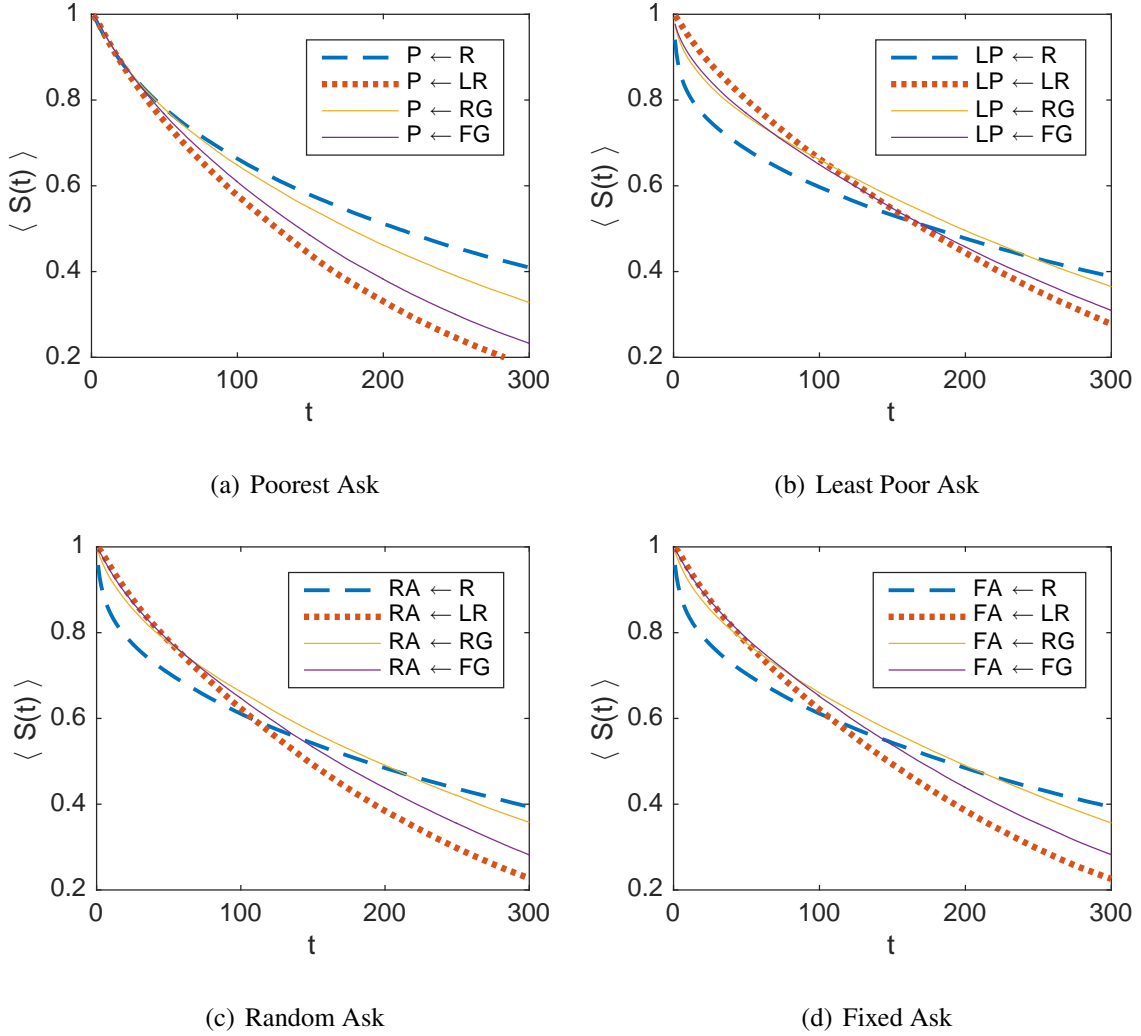


Figure 3.1: Mean survival rates for the volatility model with various transfer policies.

More generally, we find that for randomly chosen initial wealth distributions sampled from uniform, normal, exponential, or power law distributions, there still appears to be a common trend that $\langle S^{LR}(1) \rangle > \langle S^{RG}(1) \rangle > \langle S^R(1) \rangle$, where $\langle \cdot \rangle$ represents the average over multiple simulations (see Appendix A, Fig. A.1 and Fig. A.2). In addition, the gap in survival rates between different transfer policies increases with the initial population N , and mean degree. Naturally, the more NBT partners an individual has, the more significant the difference in wealth of the least rich, richest, and randomly selected giver and thus the more differently the transfer policies can behave.

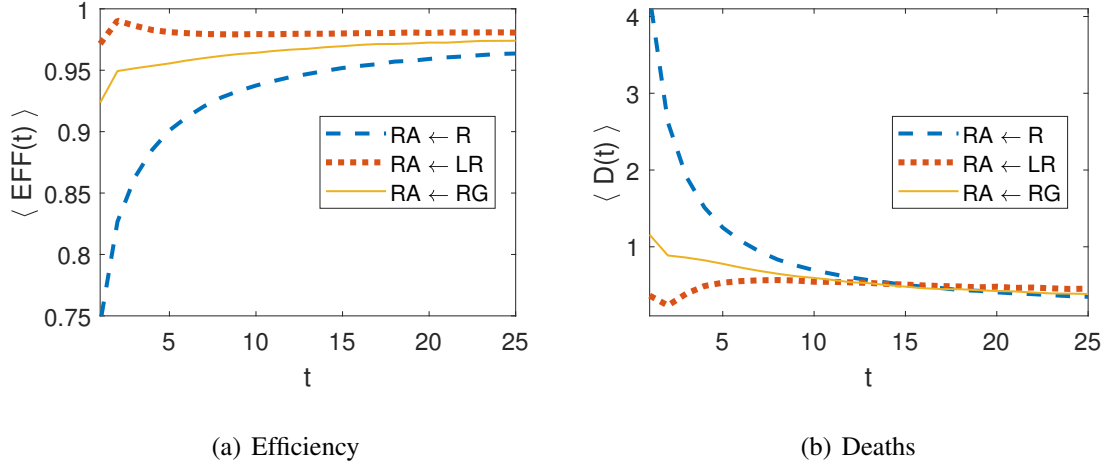


Figure 3.2: Mean efficiency and number of deaths as functions of time for random asking order and various giving orders.

However, whereas $\leftarrow LR$ is found to be optimal and $\leftarrow R$ the worst for short-term survival, we observe the exact opposite behavior in the long-term: $\leftarrow LR$ results in the lowest survival rates and $\leftarrow R$ results in the highest survival rates. This clearly illustrates that transfer policies not only have an immediate direct impact on survival but also an apparent indirect impact. Certainly, redistributing wealth in dramatically different ways results in varying wealth distributions; some wealth distributions are more secure than others in the face of disasters. This was hinted at in the simple example in the beginning of this section with $N = 4$.

Using the Least Rich giving rule and matching surpluses and deficits as closely as possible results in not only the recipient coming to threshold, but the giver also ending up with wealth near threshold. Hence, after every transaction the $\leftarrow LR$ rule generates *two* agents with wealth near threshold whereas the other policies typically generate only one agent with wealth near threshold. Hence, as opposed to the progressive $\leftarrow R$ policy, the regressive $\leftarrow LR$ policy generates a wealth distribution that has many more individuals with wealth near threshold and is thus far more susceptible to disasters. Figure 3.3 illustrates

this, showing the increase of population at risk $FIN(t)$ for the $\leftarrow LR$ rule and decrease for $\leftarrow R$ after just a few iterations.

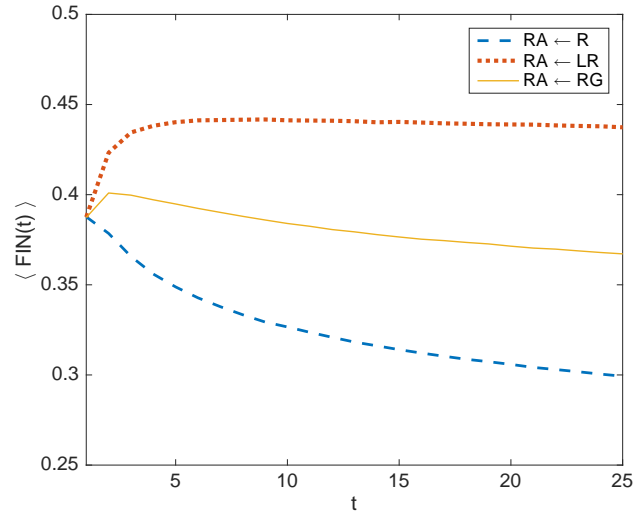


Figure 3.3: Time evolution of mean fraction in need for random asking order and various giving orders.

Increased economic risk is not only caused by many individuals near threshold, but also by the vulnerability of the total community wealth. As depicted in Figure 3.4 (a), in log-log scale the $RA \leftarrow LR$ and $RA \leftarrow RG$ probability density functions appear linear for wealths above 80, revealing a power law or Pareto tail [Ref: Chakrabarti *et al.* (2013)]. However, the smaller negative slope of the $RA \leftarrow LR$ scheme shows that the Least Rich giving order results in a smaller Pareto index and therefore larger portion of very high-wealth individuals. The wealth distribution corresponding to the $RA \leftarrow R$ policy is shown in a log-lin plot in Figure 3.4 (b), fitted to a log-normal distribution. Note that the delta function at the threshold wealth of 64 is not included in the probability density function plots.

All transfer policies are community-wealth preserving; however, by leaving a bulk of the community wealth in the hands of a few very wealthy individuals that are not often called upon to give, the $\leftarrow LR$ policy is proverbially putting all of the community eggs

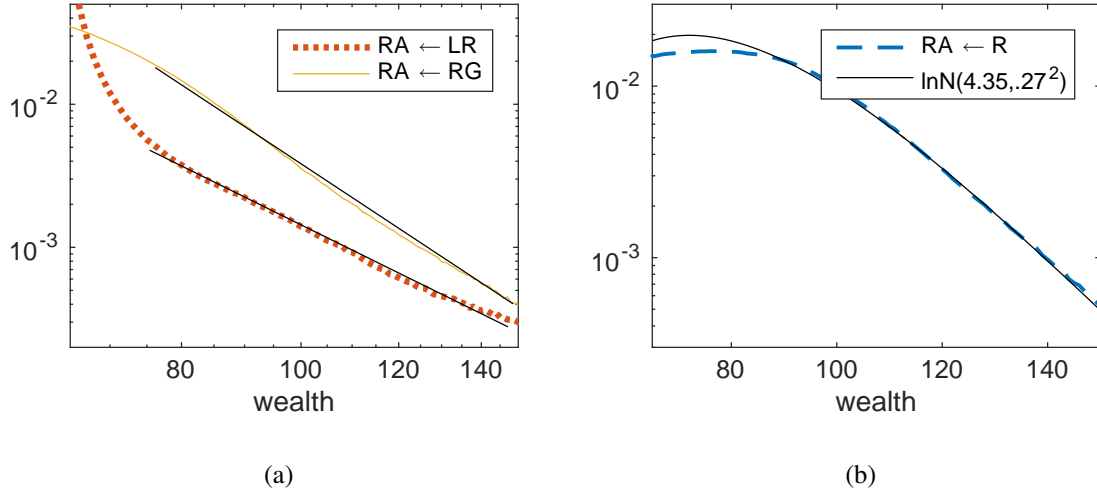


Figure 3.4: (a) Log-log plot of wealth probability density functions for the $RA \leftarrow LR$ and $RA \leftarrow RG$ policies at $t = 20$. (b) Log-lin plot of $RA \leftarrow R$ probability density function at $t = 20$ fitted to a log-normal distribution. $\ln\mathcal{N}(4.35, .27^2)$ is used to denote the probability density function for a log-normal distribution with the associated normal distribution having mean 4.35 and variance $.27^2$.

into a few baskets. Thus, when a very wealthy individual is struck by disaster, the result is devastating to community wealth, and the large fraction of individuals that are at risk are left without a source of support. The $\leftarrow R$ policy, on the other hand, better diversifies community wealth by producing a more equal wealth distribution. The evolution of inequality (high Gini coefficient corresponds to high inequality) as well as the wealth distributions that result from each policy are illustrated in Figure 3.5.

The results of this section are summarized as follows:

1. *The $\leftarrow LR$ policy acts as a cutting-stock optimization heuristic which most efficiently matches deficits to surpluses; this results in the highest short-term survival rate.*
2. *The $\leftarrow LR$ policy leads more individuals having near-threshold wealths, which increases the fraction of the population in need and results in lower long-term survival.*

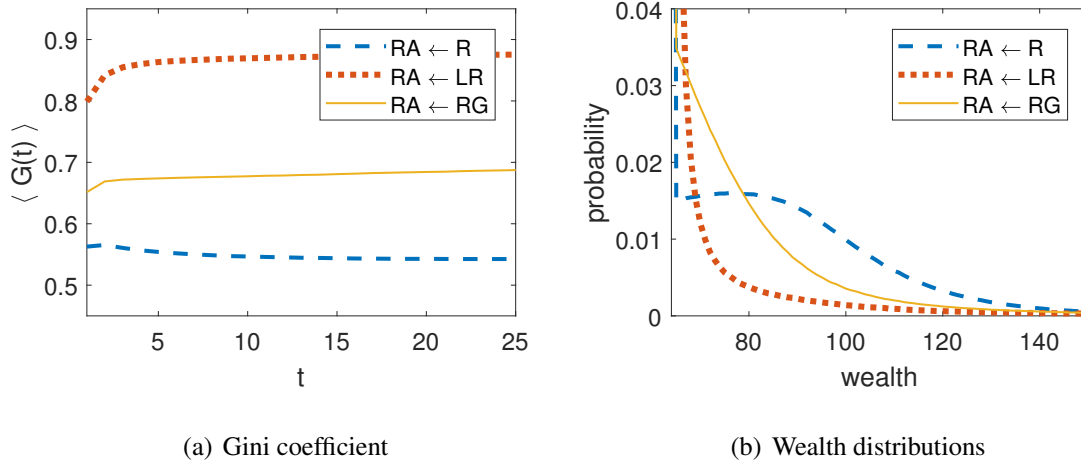


Figure 3.5: (a) Time evolution of mean Gini coefficients with random asking order and various giving orders. (b) Wealth probability density functions for $RA \leftarrow$ policies at $t = 20$. Defining $f_x(64)$ the value of the probability density function for the $\leftarrow x$ giving order at wealth 64, we have $f_R(64) = .26$, $f_{LR}(64) = .56$, and $f_{RG}(64) = .35$. Both figures use post-transfer wealths.

3. *The $\leftarrow R$ policy avoids placing donors near threshold and diversifies community wealth with more equal distribution; this results in the highest long-term survival rates.*

These observations inform the hypothesis that perhaps a hybrid transfer policy, which asks from the least rich only in response to rare major disasters and otherwise asks the richest to give with priority, could seize the benefits unique to each policy while avoiding their flaws.

Such a hybrid policy was tested on the osotua model from Hao *et al.* (2015). Figure 3.6 shows the survival rates for our original transfer policies as well as the hybrid policy, which is defined in the following way: Since individual disasters in Hao *et al.* (2015) were generated as a Poisson process, occurring once every ten years on average, in an average year 10% of the community would be struck by a disaster. The hybrid $\leftarrow H$ policy uses the $\leftarrow R$ policy in general, and uses the $\leftarrow LR$ policy only when 15% or more of the community are struck by disaster (a rare disaster-dense year). Figure 3.6 shows that the osotua model also displays the same short- and long-term behavior as with the volatility

model. Also, the hybrid policy improves upon the $\leftarrow R$ policy and results in the highest long-term survival rates.

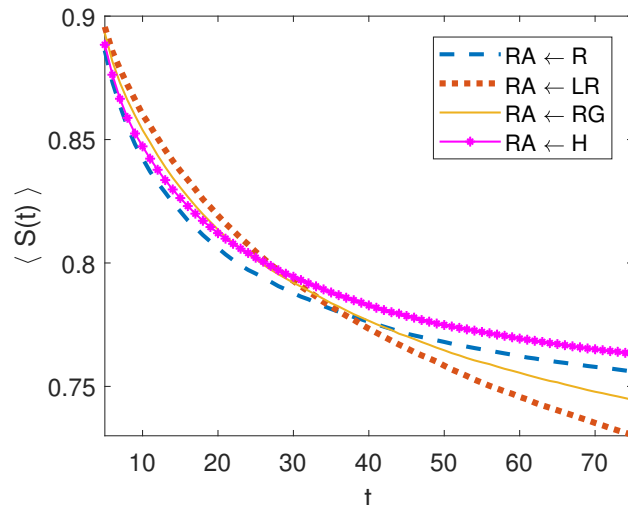


Figure 3.6: Survival rates for *random ask* policies (including the hybrid policy $RA \leftarrow H$) for the osotua model of Hao *et al.* (2015). Networks are Watts-Strogatz with size 100, mean degree 10, and rewiring probability 0.1. Growth/decay rates are $R_i(t) = g_i(t)d_i(t)$, where the growth term is sampled from a Gaussian distribution $g_i(t) \sim \mathcal{N}(.034, .0253^2)$, and the disaster term is generated as a Poisson process occurring for each individual once every ten simulation rounds on average and is sampled from a Gaussian distribution $\mathcal{N}(.7, .1^2)$.

3.1.3 Optimal Network Topology

In order to investigate the optimal NBT network topology, we simulate sharing networks which are small-world Watts-Strogatz networks [Ref: Watts and Strogatz (1998)] and consider an edge reattachment model in which individuals who lose a member of their network to non-viability decide to reconnect to another viable member of the community as a response to increased risk. Not only does this have interesting anthropological implications, but conducting this reattachment in different ways will allow us to create different degree distributions and therefore study their impact on the survival of the community.

To isolate the impact of the degree distributions from varying mean degree we allow only $\left\lfloor \frac{K \sum_{i=1}^N v_i(t) - \sum_{1 \leq i, j \leq N} X_{ij}(t)}{2} \right\rfloor$ reattachments at the end of simulation round t , which is

approximately the number of edges needed to restore the mean degree of the viable network to its initial value, K . Each individual who loses a neighbor will be allowed to reattach that edge to another individual with a probability proportional to the number of neighbors they lost until the prescribed number of edges have been reattached. After all reattachments have been conducted, the network will have gone from $X(t)$ to $X(t + 1)$.

Whom the reattaching individuals connect to is determined according to one of the following reattachment rules: (1) random, (2) preferential: connect to the node with highest degree, (3) anti-preferential: connect to the node with smallest degree, and (4) richest: connect to the node with greatest wealth. In all cases, the reattachment is to the individual that best satisfies the rule, is viable, and is not already connected to the node who is reattaching.

All results in this section use the volatility model detailed in Table 3.1 but with the following modifications: (i) the initial networks are Watts-Strogatz networks of mean degree 10 and rewiring coefficient 0.1, (ii) only the $RA \leftarrow RG$ transfer policy is considered, and (iii) the edge reattachments just described are used.

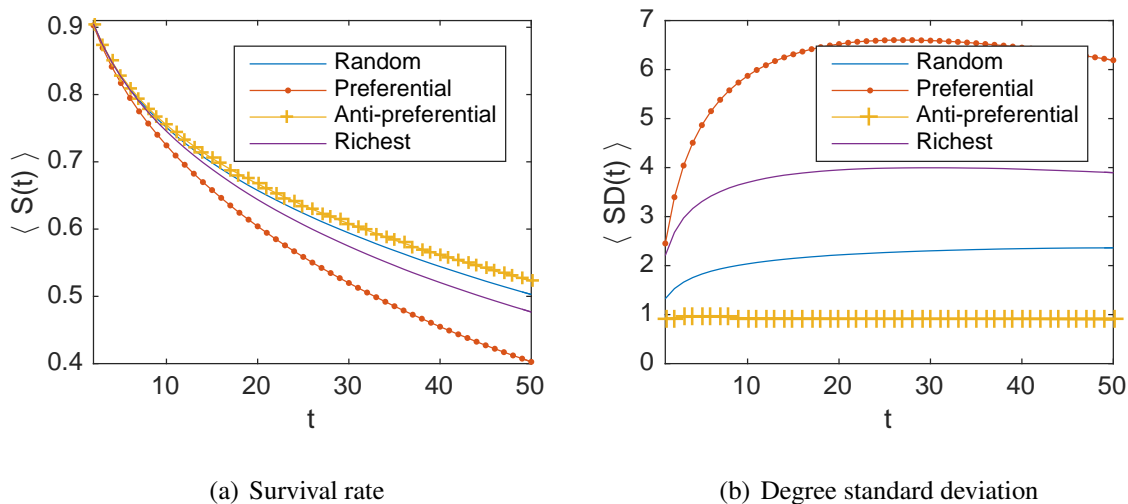


Figure 3.7: (a) Time evolution of mean survival rates for the $RA \leftarrow RG$ transfer policy and various edge reattachment rules. (b) Time evolution of degree standard deviation.

Figure 3.7 illustrates the following observations:

1. *Anti-preferential reattachment leads to networks that have significantly higher survival rates than networks using preferential attachment.*
2. *Degree variance is anti-correlated with survival rate.*

We find that preferential edge attachment, coupled with relatively low mean degree, results in two detrimental extremes: *isolated nodes* and *exhausted nodes*. With preferential reattachment, the recipient of a relocated edge is prioritized to be the available candidate with highest degree. This results in two phenomena: (i) As high degree nodes are prioritized as the recipients of relocated edges, a sort of “the rich get richer” situation occurs and the result is a few individuals with very high degree while the rest of the population has considerably lower degree. (ii) Because low degree nodes rarely become the recipient of a relocated edge, their degree continues to decrease as their partners die and few new connections are made. So, proverbially, not only do the rich get richer, but also the poor get poorer (in terms of degree).

In the extreme, if the mean degree of the network is low enough, some individuals can be left with 0 connections as a result of preferential reattachment and thus become what we call an isolated node. Neither contributing to any NBT relationship nor benefiting from one, Hao *et al.* (2015) determine that such isolated nodes have lower survival rates than nodes with higher degree.

Exhausted nodes are individuals who are frequently asked to give. Because the preferential reattachment with low mean degree results in a few individuals with very high degree and many individuals with low degree, the high degree individuals have many connections who depend on them and do not have many other sources of aid (note that if the mean degree of the network is high, the higher degree nodes are less strongly depended on by their lower degree connections). As a result, the high degree nodes are frequently asked to give and are consequently unable to accumulate much wealth above threshold. This is

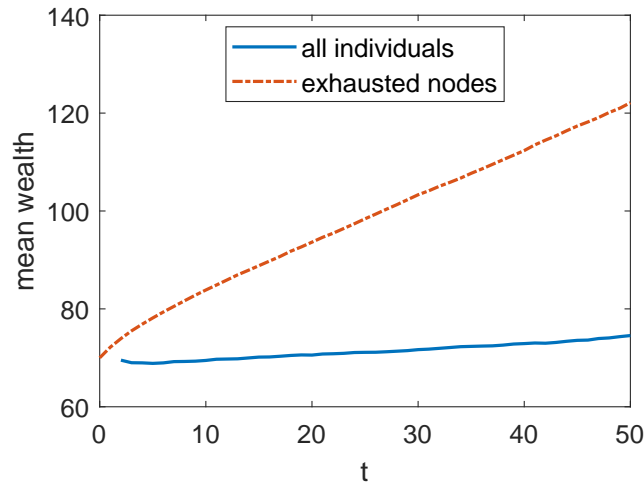


Figure 3.8: Mean wealth of the entire community and mean wealth of exhausted nodes (here exhausted nodes are considered as individuals of degree at least 20) using the $RA \leftarrow RG$ transfer policy and preferential reattachment rule.

illustrated in Figure 3.8, which shows that for preferential reattachment, individuals with high degree have lower wealth on average than the community as a whole.

The consequence is that often the exhausted nodes are not left with enough surplus to be an adequate support for the many connections that depend on them. Thus, while the high degree nodes are very secure, the survival rates of their connections are low on average, which results in lower mean survival rates for the preferential policy than with the other reattachment policies as illustrated in Figure 3.7 (a). The presence of nodes with very high degree and nodes with low degree results in the high degree variance observed in Figure 3.7 (b).

In contrast, the anti-preferential reattachment policy encourages networks to have homogeneous degree distribution (with low degree variance), avoiding both isolated nodes and exhausted nodes. Essentially, the anti-preferential policy effectively diversifies risk by equally distributing the cost and benefit of participating in an NBT relationship throughout the network. The consequence is higher survival rates for the anti-preferential policy, as seen in Figure 3.7 (a).

3.2 Quasi-equilibrium Model

A natural simulation setup, taken e.g. in Aktipis *et al.* (2011), Hao *et al.* (2015), and in Sections 3.1.2 and 3.1.3, is to start with an initial population and observe the survival rate of the population as a function of time under the influence of different policies. As we have shown, such transient models are easy to set up and can lead to interesting insights.

However, such models also have fundamental limitations. With exclusively a death process, either the population stabilizes and individuals live forever with continually increasing herd sizes, or there is finite-time extinction of the entire population. Extinction cases result in a general trend of survival rates for all policies going from 1 to 0, and thus in the time window before extinction, differences in survival can be challenging to observe. Another consequence is that any process that acts on a timescale longer than the mean lifetime of an agent, will encounter very small agent populations when the process becomes relevant. The only way to deal with this problem using transient models is to start with a huge initial agent population and hope to still have a statistically meaningful number of agents at the time the long-term process becomes relevant. There is a similar challenge in studying non-autonomous natural wealth/cattle evolution processes.

To study long-term and/or time-dependent control actions we need to have a model that, at least for a deterministic description, has an economic steady state with total wealth not increasing to infinity nor decaying to zero. To keep the total wealth finite, we assume limited resources and hence consider logistic growth for the total wealth of the community. Specifically, we change the mean value μ of the normal distribution that governs the growth rates to a time-dependent $\mu(t)$ that depends linearly on the total community wealth $W(t)$:

$$\begin{aligned}\mu(t) &= \mu_0 - \mu_1 W(t), \\ W(t) &= \sum_{i=1}^{N(t)} w_i(t).\end{aligned}$$

Note that $W(t)$ is the total wealth (e.g. cattle population) in a community of $N(t)$ members (cattle herders). Simply, the larger the total cattle population, the greater competition for resource and thus the average growth rate decreases; inversely, low cattle populations means less competition for resource and higher growth rates on average.

To prevent the number of herds from going to zero, we balance the loss of individuals caused by the viability threshold θ_d with a birth process, generated through a splitting mechanism. When an individual has wealth above a splitting threshold θ_b , this individual splits his/her wealth, and generates another individual. In general, the wealth splitting process can be governed again by a policy. To keep things simple, we define that the new individual takes half of the wealth of the individual that was split. Thus in order to generate viable new individuals, it should be that $\theta_b > 2\theta_d$. Beyond this constraint, if θ_b is small there will be many individuals with small wealths, and if θ_b is large, there will be fewer individuals, but larger average wealths. Similar statements can be made for the choice of θ_d .

As a proof of concept for the quasi-equilibrium model we repeat the simulations discussed in Section 3.1.2. In particular, we study the impact of different giving orders with an $RA \leftarrow$ asking policy. In addition we use the different reattachment policies discussed in Section 3.1.3 to connect the individuals generated by the splitting process.

Our primary measure of social fitness is mean lifespan:

$$ML(t) = \frac{N(t)}{D(t)} \quad (3.5)$$

is the reciprocal of the probability of dying in simulation round t . Of particular interest is the mean lifespan of individuals when the model equilibrates, which we approximate by averaging $ML(t)$ over 100 rounds after equilibrium is achieved:

$$ML_\infty = \frac{1}{100} \sum_{t=t^*}^{t^*+100} ML(t), \quad (3.6)$$

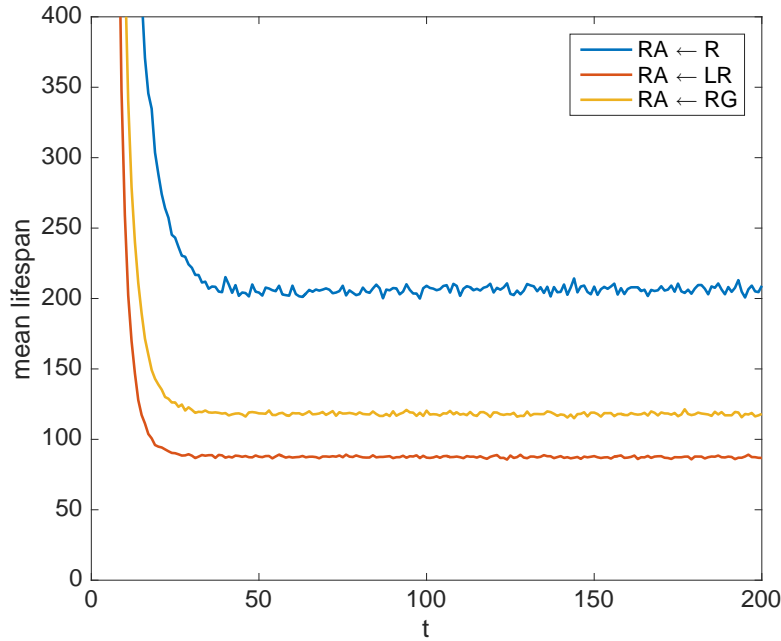


Figure 3.9: Mean lifespan of individuals (defined in Equation 3.5) in the quasi-equilibrium model with random edge attachment.

where t^* is visually identified as the time at which equilibrium is achieved.

We find that the statements discerned from the transient simulations have parallel observations for the quasi-equilibrium model: mean lifespans are largest for the $\leftarrow R$ policy and smallest for the $\leftarrow LR$ policy. Figure 3.9 with random edge attachment is given as an example of the evolution and stabilization of the mean lifespan in the quasi-equilibrium model, and data for each combination of transfer policy and attachment rule are collected in Table 3.3. Mean lifespan in steady state is consistently larger with anti-preferential attachment and smaller with preferential attachment, which is in harmony with the transient statements for the network reattachment policies.

For the simulations in this section we use $N = 100$, $\vec{w}(0) = \{200\}^N$, $\theta_d = 100$, $\theta_b = 300$, $\mu_0 = 1$, and $\mu_1 = 5 \times 10^{-5}$ and fix the variance to sample the growth/decay rates $R_i(t)$ from the distribution $\mathcal{N}(\mu(t), .25^2)$ for all $1 \leq i \leq N(t)$.

	Random attachment	Anti-preferential attachment	Preferential attachment
$RA \leftarrow R$	206.7	242.7	23.7
$RA \leftarrow LR$	87.4	99.6	21.5
$RA \leftarrow RG$	117.9	137.5	22.6

Table 3.3: Mean lifespans for each transfer and reattachment method in steady state of the quasi-equilibrium model, i.e. ML_∞ , which is defined in Equation 3.6.

KINETIC MODELS OF CONSERVATIVE ECONOMIES WITH WELFARE
THRESHOLDS

In this chapter, we utilize the binary nature of NBTs to model the evolution of the corresponding wealth distribution using Boltzman-like kinetic equations. We develop such kinetic exchange models of NBTs in order to (i) compare numerical results from the kinetic model to observations made in the agent-based studies and contribute new observations, (ii) control for desirable wealth distributions, and (iii) potentially prove some results in the integro-differential equation framework.

4.1 Kinetic NBT Model

A brief introduction to kinetic exchange models is given in Section 2.2, where the model of Bisi *et al.* (2009), with wealth redistribution via a collisional transfer tax and redistribution operator is discussed. We develop a kinetic model that conducts wealth redistribution quite differently. Namely, we seek to model NBTs where a fixed welfare threshold determines the relative surplus or deficit of each individual and binary transfers deterministically occur to give from one individual's surplus to cover the other individual's need whenever possible. A microscopic description of NBTs with the welfare threshold $\theta \in \mathbb{R}$ and pre-trade wealths $u, v \in \mathbb{R}$ is thus the following:

$$\begin{aligned} u^* &= u + H(u + v - 2\theta)[(\theta - u)H(\theta - u) - (\theta - v)H(\theta - v)] \\ v^* &= v + H(u + v - 2\theta)[(\theta - v)H(\theta - v) - (\theta - u)H(\theta - u)]. \end{aligned} \quad (4.1)$$

In Equation (4.1), H is the standard Heaviside step function, and as the rule is invariant under the permutation $v \leftrightarrow u, v^* \leftrightarrow u^*$, it allows for the assumption of statistical independence for many identical individuals. Essentially, the rule determines that wealths will

change only if there is enough total wealth for both individuals to be at or above threshold after the transfer. Then, if there is enough total wealth for a transfer to occur, and if one individual has need, the individual with surplus gives from his/her surplus whatever the deficit of the needy individual is.

If we assume that collisions occur at rate 1 and consider $f(w, t)$ as the relative density of individuals with wealth $w \in \mathbb{R}$ at time $t \geq 0$, the Boltzmann-like wealth distribution evolution equation corresponding to the microscopic transfers described in Equation (4.1) is:

$$\partial_t f(w, t) = \int_{-\infty}^{\theta} \int_{2\theta-u}^{\infty} \left[-\delta(w-u) - \delta(w-v) + \delta(w-\theta) + \delta(w-u-v+\theta) \right] \times f(v, t) f(u, t) dv du, \quad (4.2)$$

Note that here wealth is allowed to be negative; this may be understood as individuals having debt.

Equation (4.2) can be understood as u denoting the wealth of a below-threshold individual and v denoting the wealth of a donor (individual with enough surplus to cover the deficit $\theta - u$ and still be above threshold, i.e. $v > 2\theta - u$). Thus, when these individuals interact, there are density losses (sinks) at their pre-trade wealths u, v and density gains (sources) at their post-trade wealths, $u^* = \theta$ and $v^* = v - (\theta - u)$.

In Equation (4.2), the interactions occur at rates proportional to the relative densities of individuals with wealths u and v , i.e. one can think of individuals being selected uniformly at random from the wealth distribution and paired. Hence, (4.2) is a model for the $RA \leftarrow RG$ policy where no preference is considered in how below- and above-threshold individuals are paired. Interactions in this kinetic model are thus naturally more analogous to colliding gas particles than the organized matchings of policies like $P \leftarrow LR$ discussed in Section 3.1.2.

Integrating (4.2) with a well-behaved test function $\phi(w)$ gives the weak form of the Boltzmann equation:

$$\int_{-\infty}^{\infty} \phi(w) \partial_t f(w, t) dw = \int_{-\infty}^{\theta} \int_{2\theta-u}^{\infty} [-\phi(u) - \phi(v) + \phi(\theta) + \phi(u+v-\theta)] \times f(v, t) f(u, t) dv du. \quad (4.3)$$

In particular, we are interested in the evolution of the n -th moments, which are obtained by integrating (4.2) with $\phi(w) = (w - \theta)^n$:

$$M_n(t) := \int_{-\infty}^{\infty} (w - \theta)^n f(w, t) dw. \quad (4.4)$$

These moments are not central moments in that θ is not necessarily the mean wealth; however, since θ is central in determining the shape of our wealth distributions, it will be advantageous to examine moments about the threshold θ rather than the mean wealth.

Lemma 1. *For $f(w, t)$ satisfying (4.2), the n -th moment of f evolves as follows:*

$$\frac{d}{dt} M_n(t) = \int_{-\infty}^0 \int_{-y}^{\infty} [-y^n - x^n + 0^n + (x+y)^n] f(x+\theta, t) f(y+\theta, t) dx dy \quad (4.5)$$

Proof.

$$\begin{aligned} \frac{d}{dt} M_n(t) &\stackrel{(4.4)}{=} \frac{d}{dt} \int_{-\infty}^{\infty} (w - \theta)^n f(w, t) dw \\ &= \int_{-\infty}^{\infty} (w - \theta)^n \partial_t f(w, t) dw \\ &\stackrel{(4.3)}{=} \int_{-\infty}^{\theta} \int_{2\theta-u}^{\infty} [-(u - \theta)^n - (v - \theta)^n + (\theta - \theta)^n + (u + v - 2\theta)^n] f(v, t) f(u, t) dv du \\ &= \int_{-\infty}^{\theta} \int_{\theta-u}^{\infty} [-(u - \theta)^n - x^n + 0^n + (u + x - \theta)^n] f(x + \theta, t) f(u, t) dx du \\ &= \int_{-\infty}^0 \int_{-y}^{\infty} [-y^n - x^n + 0^n + (x + y)^n] f(x + \theta, t) f(y + \theta, t) dx dy \end{aligned}$$

□

Lemma 2. *Let $f(w, t)$ evolve according to Equation (4.2) and have initial condition $f(w, 0) = f_0(w)$, a probability density. Then,*

(i) $M_0(t) = 1$ is constant.

(ii) $M_1(t) = M_1(0)$ is constant.

(iii) $M_2(t)$ is non-increasing.

(iv) $M_3(t)$ is non-increasing.

Proof. The following results utilize Equation (4.5):

(i) $\frac{d}{dt}M_0(t) = \int_{-\infty}^0 \int_{-y}^{\infty} [-1 - 1 + 1 + 1]f(x + \theta, t)f(y + \theta, t)dx dy = 0$. Thus $M_0(t)$ is constant and $M_0(t) = M_0(0) = 1$ since $f_0(w)$ is a probability density.

(ii) $\frac{d}{dt}M_1(t) = \int_{-\infty}^0 \int_{-y}^{\infty} [-x - y + 0 + (x + y)]f(x + \theta, t)f(y + \theta, t)dx dy = 0$. Thus, $M_1(t)$ is constant and $M_1(t) = M_1(0)$ is finite for realistic initial wealth distributions.

(iii)

$$\begin{aligned}\frac{d}{dt}M_2(t) &= \int_{-\infty}^0 \int_{-y}^{\infty} [-x^2 - y^2 + 0 + (x + y)^2]f(x + \theta, t)f(y + \theta, t)dx dy \\ &= \int_{-\infty}^0 \int_{-y}^{\infty} 2xyf(x + \theta, t)f(y + \theta, t)dx dy \\ &\leq 0.\end{aligned}$$

The inequality is a result of f having only non-negative values, and in the last integral $y \leq 0$ and $x \geq -y \geq 0$, so the integrand is non-positive. Hence, $M_2(t)$ is non-increasing.

(iv)

$$\begin{aligned}\frac{d}{dt}M_3(t) &= \int_{-\infty}^0 \int_{-y}^{\infty} [-x^3 - y^3 + 0 + (x + y)^3]f(x + \theta, t)f(y + \theta, t)dx dy \\ &= \int_{-\infty}^0 \int_{-y}^{\infty} 3xy(x + y)f(x + \theta, t)f(y + \theta, t)dx dy \\ &\leq 0.\end{aligned}$$

The inequality here is a result of $f \geq 0$, $y \leq 0$ and $x \geq -y \Rightarrow (x + y) \geq 0$. Thus, since $\frac{d}{dt}M_3(t) \leq 0$, $M_3(t)$ is non-increasing.

□

The significance of Lemma 2(i) is that f remains a probability density. Lemma 2(ii) states that the mean wealth is conserved for model (4.2), and in combination with Lemma 2(i) determines that total wealth is also conserved for this model. Lemma 2(iii) finds the second moment to be non-increasing, which intuitively makes sense as transfers naturally bring wealths more central, i.e. the distance between two agents' wealths decreases in a transfer with all wealths coming closer to θ . $\frac{d}{dt}M_3(t) \leq 0$ means that we expect the left tail to be getting fatter than the right tail. This is also intuitive as individuals to the far left have very few who can help them though individuals to the far right can help many.

By observing that $\frac{d}{dt}M_0(t) = \frac{d}{dt}M_1(t) = 0$, we equivalently notice that $\phi(w) = 1$ and $\phi(w) = w$ are collision invariants, i.e. these test functions result in the collisional term on the right hand side of Equation (4.3) being equal to 0. Actually, linear functions are the only collision invariants for (4.3).

Lemma 3. *The only collision invariants for (4.3) are of the form $\phi(w) = aw + b$, where $a, b \in \mathbb{R}$.*

Proof. From Equation (4.3), collision invariants are functions ϕ such that $[-\phi(u) - \phi(v) + \phi(\theta) + \phi(u + v - \theta)] = 0$. Taking the derivative of this expression with respect to u gives that $\phi'(u) = \phi'(u + v - \theta)$. Since v can vary in the argument of $\phi'(u + v - \theta)$, it must be that ϕ' is constant and therefore ϕ is linear: $\phi(w) = aw + b$ where $a, b \in \mathbb{R}$. □

Lemma 4. *Steady state, f_∞ for Equation (4.2) takes one of the following forms (note that $\text{supp}(f)$ means the support of f):*

- (i) *No individuals with surplus: $\text{supp}(f_\infty) \subseteq (-\infty, \theta]$*

(ii) *No individuals with deficit:* $\text{supp}(f_\infty) \subseteq [\theta, \infty)$

(iii) *Individuals with deficit and surplus but no individuals with enough surplus to match the deficits of below-threshold individuals:* $\max\{\text{supp}(f_\infty)\} \in (\theta, \infty)$ and $(\theta - \max\{\text{supp}(f_\infty) \cap (-\infty, \theta)\}) > (\max\{\text{supp}(f_\infty)\} - \theta)$

Proof. Recall Equation (4.2):

$$\partial_t f(w, t) = \int_{-\infty}^{\theta} \int_{2\theta-u}^{\infty} [-\delta(w-u) - \delta(w-v) + \delta(w-\theta) + \delta(w-u-v+\theta)]$$

$$sss \times f(v, t)f(u, t) dv du.$$

$\partial_t f(w, t) = 0$ when the product $f(v, t)f(u, t) = 0$ for all $u \in (-\infty, \theta), v \in (2\theta - u, \infty)$.

(i) Since $f_\infty(v) = 0$ for all $v > \theta$, $f_\infty(v) = 0$ for all v such that $u \in (-\infty, \theta)$ and $v \in (2\theta - u, \infty) \subset (\theta, \infty)$. Hence, the product in the integrand is 0 and so is the entire right hand side of (4.2).

(ii) Since $f_\infty(u) = 0$ for all $u < \theta$, the right hand side of (4.2) is 0.

(iii) For this case, $\text{supp}(f_\infty) \cap (2\theta - u, \infty) = \emptyset$ for all $u \in \text{supp}(f_\infty) \cap (-\infty, \theta)$ and so the integrand of (4.2) is 0.

Any case where there are individuals above threshold and below threshold but (iii) is not satisfied necessarily has that there exists some open interval about $u \in \text{supp}(f_\infty) \cap (-\infty, \theta)$ such that $\text{supp}(f_\infty) \cap (2\theta - u, \infty)$ contains an open interval where $f(v, t)f(u, t)$ is nonzero and so $\partial_t f(w, t) \neq 0$ and steady state has not been reached. \square

Contextually, Lemma 4(iii) says that if the richest individual's surplus is less in magnitude than the least poor individual's deficit then no one is eligible for a binary transfer and thus the wealth distribution ceases to change. Figure 4.1 illustrates simple examples of the different types of steady state identified in Lemma 4.

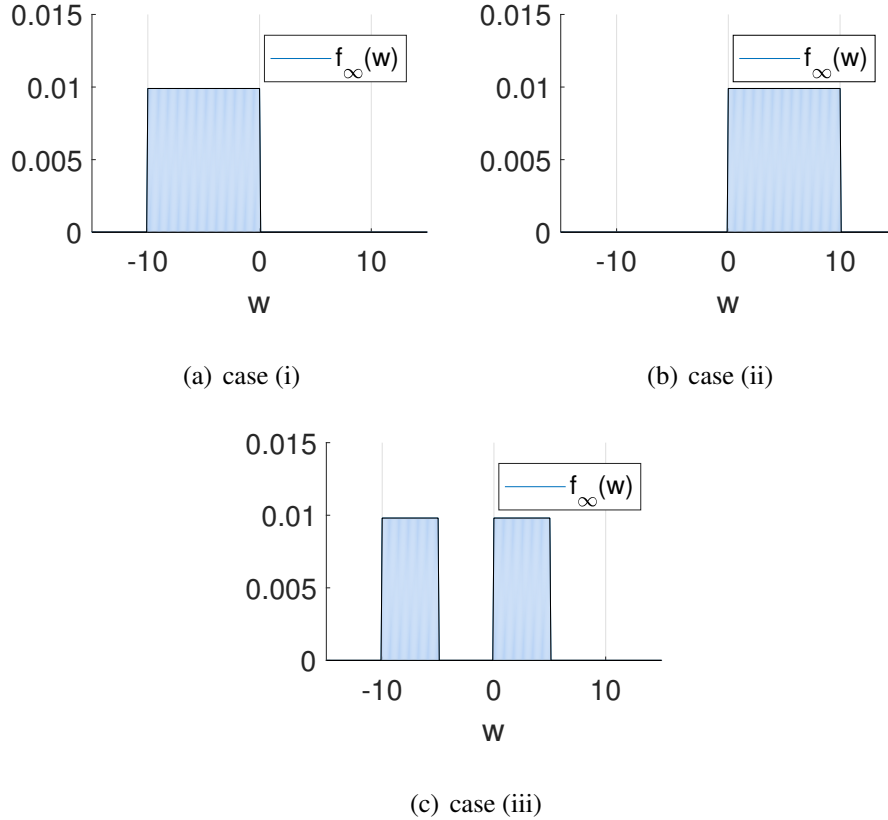
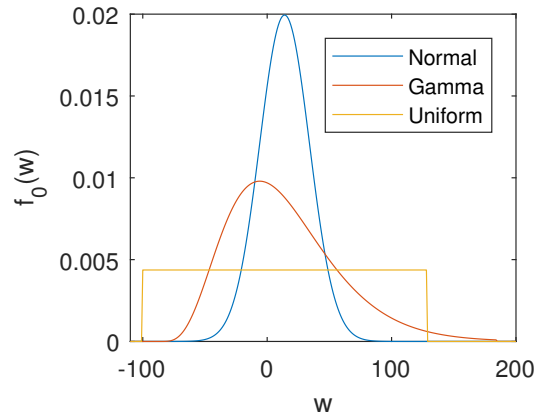


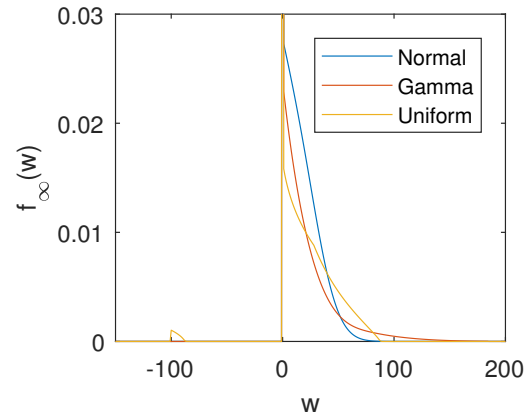
Figure 4.1: Simple examples of the different cases of steady states for Equation (4.2) mentioned in Lemma 4 where $\theta = 0$; the area under the probability density curve is shaded for visibility.

Figure 4.2 illustrates the evolution of the wealth distribution and moments according to Equation (4.2) for a few different initial wealth distributions. We observe steady states of type (ii) for both the Normal and Gamma initial conditions, and steady state of type (iii) for the Uniform initial condition. Also, in agreement with Lemma 2, the second and third moments decrease and then level off as steady state is being reached.

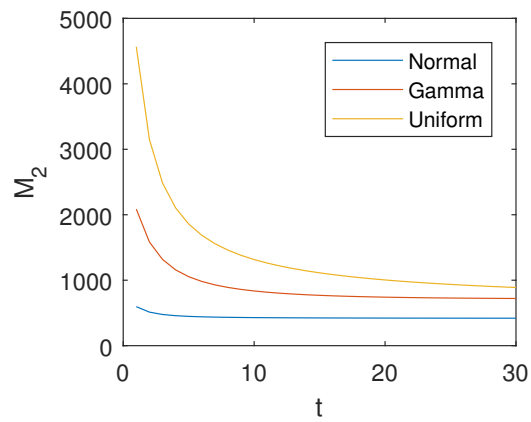
It is apt to comment that while steady state for (4.2) is useful for informing how NBTs impact the wealth distribution, there is currently no natural wealth evolution process in the model. In Section 4.4 we will begin to examine natural wealth evolution by considering wealths to diffuse according to a random walk. Yet, while conceptually we understand that natural wealth evolution and NBT redistribution occur simultaneously, e.g. vampire bats



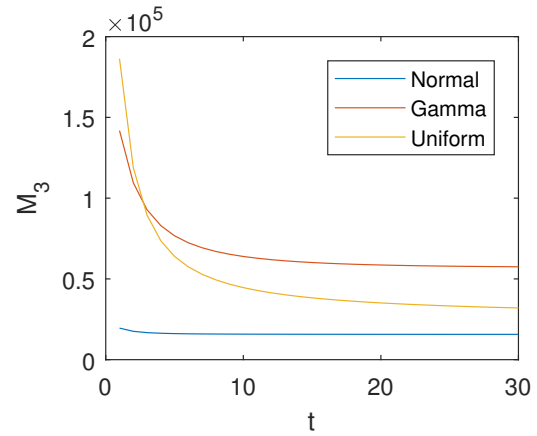
(a) $f_0(w)$



(b) $f_\infty(w)$



(c) $M_2(t)$



(d) $M_3(t)$

Figure 4.2: (a) A few different initial wealth distributions with $M_1 = 14$ that when evolving according to Equation (4.2) with $\theta = 0$ reach steady states in (b). The second and third moment evolutions are shown in (c) and (d) respectively.

digesting, foraging, and sharing on similar timescales, we are at the moment investigating just NBT redistribution detached from a natural evolution process.

4.2 Kinetic NBT Policies

As relating to the transfer policies discussed in Section 3.1.2, Equation (4.2) is a model for the $RA \leftarrow RG$ policy where there is no preference or order given to who ask and who donate. Like gas particles, the kinetic exchange model considers individuals to interact simply at rates proportional to the product of their probability densities, i.e. uniformly at random. However, we are still interested in investigating regressive to progressive transfer policies in the kinetic framework to validate some of the agent-based results and extend our observations.

We first define an exact-match kinetic NBT. Essentially, we fix not only a recipient threshold, $\theta \in \mathbb{R}$, but also a donor threshold, $\theta + \epsilon_0$ where $\epsilon_0 \geq 0$ is not necessarily small, such that a transfer will occur only if the exchange will cause an individual with wealth below θ to end with wealth θ and an individual with wealth above $\theta + \epsilon_0$ to end with exactly $\theta + \epsilon_0$ as a result of the transfer. Hence, the microscopic description of the exact-match kinetic NBT model, given θ, ϵ_0 , is

$$\begin{aligned} v^* &= v + \delta(v + w - 2\theta - \epsilon_0) [(\theta - v)H(\theta - v) - (\theta - w)H(\theta - w)] \\ w^* &= w + \delta(v + w - 2\theta - \epsilon_0) [(\theta - w)H(\theta - w) - (\theta - v)H(\theta - v)], \end{aligned} \quad (4.6)$$

and the macroscopic kinetic equation is

$$\begin{aligned} \partial_t f(w, t) &= \int_{-\infty}^{\theta} \left[-\delta(w - u) - \delta(w + u - \epsilon_0 - \theta) + \delta(w - \theta) + \delta(w - \theta - \epsilon_0) \right] \\ &\quad \times f(\epsilon_0 + \theta - u, t) f(u, t) du. \end{aligned} \quad (4.7)$$

Lemma 5. Given $f(w, t)$ has initial condition $f_0(w) = f(w, 0)$ and evolves according to Equation (4.7), the steady state is given by:

$$\begin{aligned} f_\infty(w) = & f_0(w) - [H(\theta - w) + H(w - \theta - \epsilon_0)] \min\{f_0(w), f_0(2\theta + \epsilon_0 - w)\} \\ & + \delta(w - \theta) \int_{-\infty}^{\theta} \min\{f_0(v), f_0(2\theta + \epsilon_0 - v)\} dv \\ & + \delta(w - \theta - \epsilon_0) \int_{\theta + \epsilon_0}^{\infty} \min\{f_0(v), f_0(2\theta + \epsilon_0 - v)\} dv, \end{aligned}$$

Proof. Since the model restricts transfers to exact symmetric matches across respective recipient donor thresholds, f simply evolves by equally decreasing matching (across respective thresholds) densities and sending that mass to the thresholds; steady state is reached when one of the matching densities reaches 0 in the limit. \square

Figure 4.3 provides an example of how an initial wealth distribution evolves according to exact match redistribution (4.7), and illustrates the steady state.

Choosing a donor threshold $\theta + \epsilon_0$ automatically guarantees that no individuals with wealth below that threshold will be able to give. Thus, a higher donor threshold (larger ϵ_0) corresponds to only wealthier individuals being able to give. In this sense, exact-match transfers with large donor thresholds can be considered as describing more progressive wealth redistribution than policies with smaller donor thresholds. We utilize this exact match structure to develop NBT policies that incorporate what we call donor preference.

Fixing a donor threshold $\theta + \epsilon_0$ determines that potential donors will only give if doing so causes their post transfer wealth to be exactly $\theta + \epsilon_0$. What we consider now is that donors do not have a necessary specific terminal wealth, but instead a varying preference for a range of post transfer wealths. Thus, we assume the existence of a probability density function p such that $p(\epsilon_0)d\epsilon$ is the probability a donor threshold will be selected between $\theta + \epsilon_0$ and $\theta + \epsilon_0 + d\epsilon$. Hence, the donor preference, or transfer policy is defined by p and

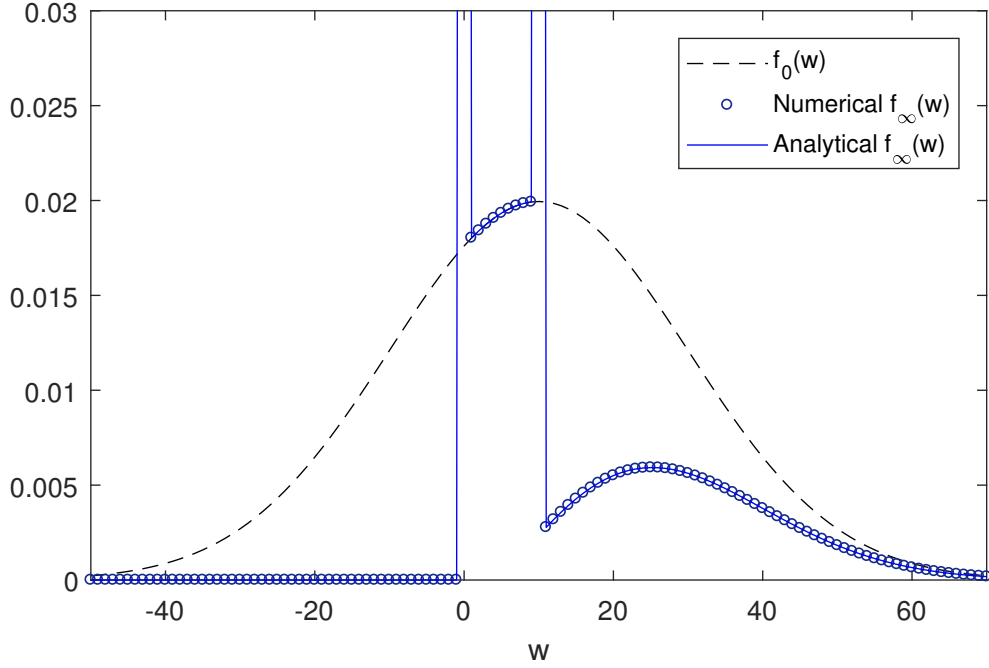


Figure 4.3: Numerical steady state solution to Equation (4.7) as well as analytical steady state solution from Lemma 5 with initial condition $f_0(w)$ a Normal distribution $\mathcal{N}(\mu = 10, \sigma^2 = 20^2)$ and parameters $\theta = 0, \epsilon_0 = 10$.

the corresponding wealth evolution equation is

$$\partial_t f(w, t) = \int_0^\infty p(\epsilon) \int_{-\infty}^\theta [-\delta(w-u) - \delta(w+u-\epsilon-2\theta) + \delta(w-\theta) + \delta(w-\theta-\epsilon)] \times f(\epsilon+\theta-u, t) f(u, t) du d\epsilon. \quad (4.8)$$

Definition 1 (Kinetic policies). *For numerical results, we assume some maximal surplus $L > 0$ and define a parameterized probability density function $p_\alpha : [0, L] \rightarrow (0, \infty)$ as*

$$p_\alpha(\epsilon) = \left(\frac{\alpha}{e^{\alpha L} - 1} \right) e^{\alpha \epsilon}.$$

The policies we will focus on are then defined by their respective α values:

(a) *progressive policy:* $\alpha = 0.05$

(b) *flat policy:* $\alpha = 0$

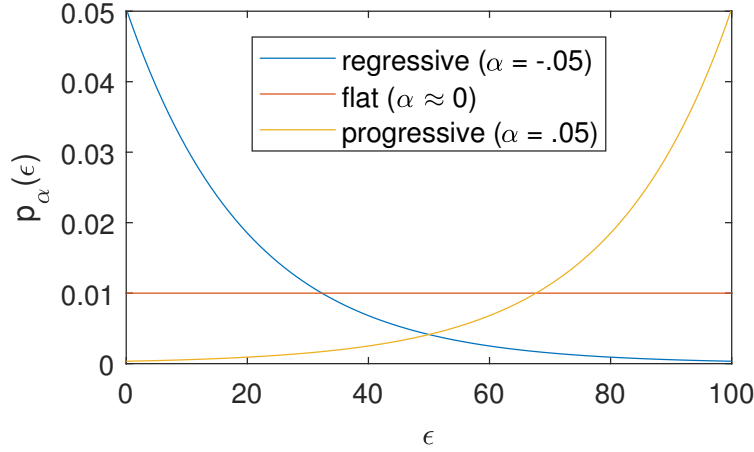


Figure 4.4: Probability densities for probability of choosing donor threshold $\theta + \epsilon$ for regressive, flat, and progressive policies with $\theta = 0$ and maximal wealth $L = 100$. The equation for these parameterized donor threshold probability distributions is given in Definition 1.

(c) regressive policy: $\alpha = -0.05$.

p_α for these policies is illustrated in Figure 4.4, where $L = 100$.

Lemma 6. *The flat policy kinetic equation (Equation (4.8), Def. 1) is equivalent to the random interaction kinetic NBT model (4.2), but with interaction rate $\frac{1}{L}$.*

Proof. Assume maximal surplus $L > 0$, and consider the flat policy $p(\epsilon) = \frac{1}{L}$. We will show that the right hand side of (4.8) for the flat policy is a constant multiple of the right hand side of (4.2).

$$\begin{aligned}
& \int_0^\infty p(\epsilon) \int_{-\infty}^\theta [-\delta(w-u) - \delta(w+u-\epsilon-2\theta) + \delta(w-\theta) + \delta(w-\theta-\epsilon)] f(\epsilon+\theta-u, t) f(u, t) du d\epsilon \\
&= \frac{1}{L} \int_{-\infty}^\theta \int_0^\infty [-\delta(w-u) - \delta(w+u-\epsilon-2\theta) + \delta(w-\theta) + \delta(w-\theta-\epsilon)] f(\epsilon+\theta-u, t) f(u, t) d\epsilon du \\
&= \frac{1}{L} \int_{-\infty}^\theta \int_{2\theta-u}^\infty [-\delta(w-u) - \delta(w-v) + \delta(w-\theta) + \delta(w-u-v+\theta)] f(v, t) f(u, t) dv du
\end{aligned}$$

□

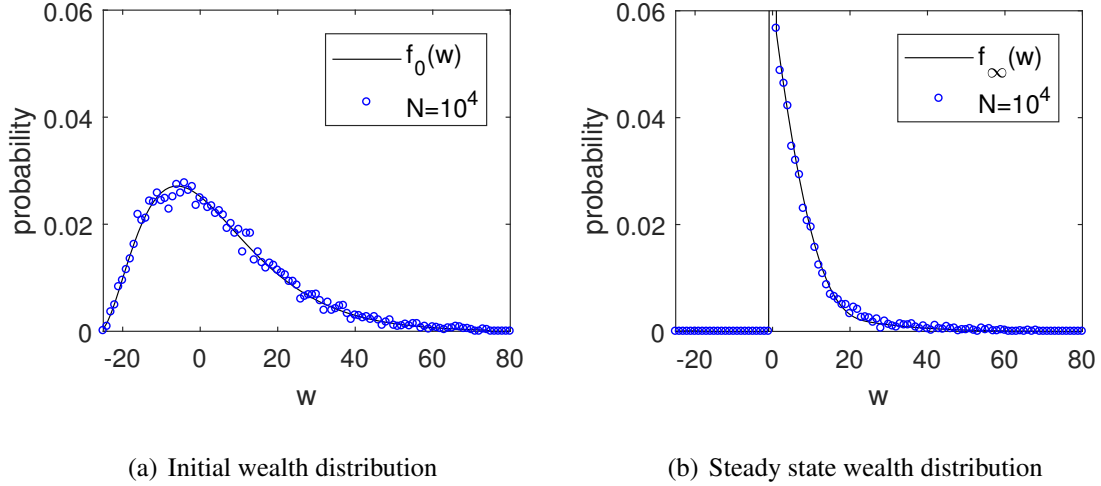


Figure 4.5: Flat policy comparison with agent-based simulation. A gamma initial condition is used for $f_0(w)$ and 10^4 agents are sampled from this distribution as well. Equation (4.8) is used with $\alpha = 0$ to find the steady state solution of the Boltzmann-like equation; for the agents, interactions are randomly generated and transfers are conducted according to the microscopic description of equation (4.1) until all 10^4 agents are at or above threshold.

As illustrated in Lemma 6, the flat policy ($\alpha = 0$) indicates no distinct preference for donor threshold and thus corresponds to the microscopic description of the random interaction kinetic NBT model of Equation (4.1). Figure 4.5 shows a comparison of the numerical steady state solution of the flat policy kinetic IDE and the wealth distribution resulting from agent-based simulation using the microscopic description of equation (4.1); the initial wealth distribution was chosen to be gamma, which is considered to be qualitatively realistic for natural wealth distributions [Refs: Angle (1986); Chakrabarti *et al.* (2013)].

For numerical experiments, $\theta = 0$ is considered and two different initial conditions are used: (i) gamma distribution and (ii) uniform distribution. Again, the gamma distribution is chosen as qualitatively representative of naturally observed wealth distributions; the uniform distribution is chosen because it allows for comparability of effectiveness of each policy in meeting the needs of below-threshold individuals.

For all results in Figures 4.6 and 4.7, ‘steady state’ is considered to have been reached at time T such that $\|f(w, T) - f(w, T - \Delta t)\|_2 < 10^{-5}$; in the tables, T is rescaled by

the minimum T value for comparability. $\Delta t = 1$ is used for simulations. Population below threshold is found as $\int_{-\infty}^0 f(w, T) dw$ for Figure 4.7 and is considered essentially 0 for Figure 4.6. Gini index, a measure of wealth inequality introduced in Section 3.1.1 is defined in Definition 2; smaller Gini index corresponds to less inequality.

Definition 2 (Gini index). *Gini index G for the viable population (individuals with wealth above threshold) is calculated as $G = 1 - 2A$, where A is the area under the Lorenz curve $(x(r), y(r))$. $x(r)$ and $y(r)$ are defined as follows:*

$$x(r) = \frac{\int_{\theta}^r f(s) ds}{\int_{\theta}^{\infty} f(s) ds}, \quad y(r) = \frac{\int_{\theta}^r (s - \theta) f(s) ds}{\int_{\theta}^{\infty} (s - \theta) f(s) ds}$$

as r takes values from θ to the maximal wealth.

Observation 1. *Recalling that decreasing α corresponds to making the policy more regressive, the following numerical observations are inspired by Figures 4.6 and 4.7: As α decreases,*

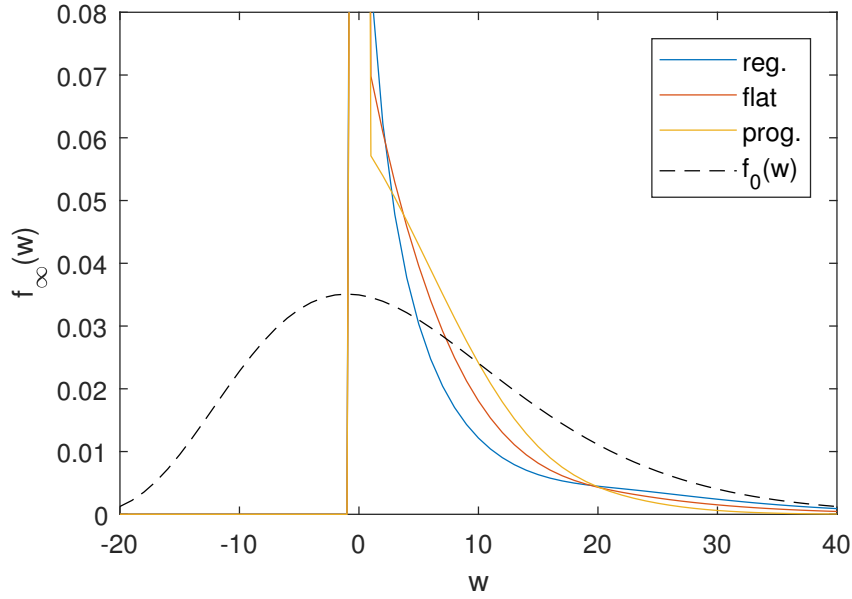
(i) *The Gini index increases (more inequality)*

(ii) *The convergence rate increases*

(iii) *The fraction of the population below threshold at steady state decreases*

In Figure 4.6 and Figure 4.7, the steady state distributions of each policy are qualitatively, and in terms of inequality, predictable or reinforce the regressive/progressive natures of the policies. As expected, a more regressive policy (lower α) results in greater inequality (higher Gini index).

Also, the rate of convergence for the regressive policy is greater than for the progressive policy. For example, with the Gamma initial condition (Figure 4.6), the progressive policy took 71 times as long as the regressive policy to reach a state where the change in f was

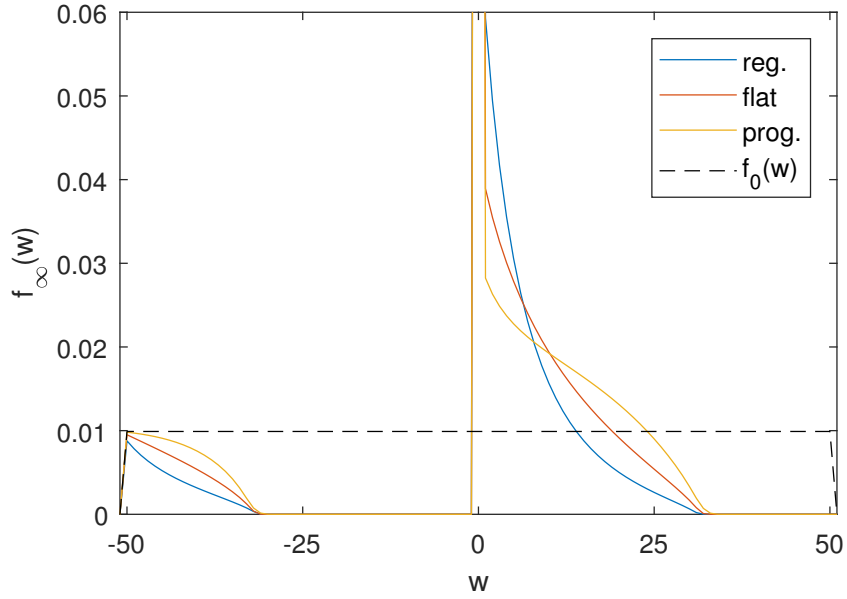


Policy	T	Gini index
Regressive	1	.785
Flat	3.7	.732
Progressive	71.7	.685

Figure 4.6: Steady state distributions and data for parameterized kinetic NBT policies with initial condition $f_0(w) \sim \text{Gamma}$.

less than 10^{-5} . This is a new observation with respect to the work of Aktipis *et al.* (2011, 2016) and Hao *et al.* (2015), but intuitively makes sense as the higher donor thresholds preferred in the progressive model reduce the number of potential donors, making it harder for below-threshold individuals to find a donor.

Figure 4.7 echoes an observation made in Section 3.1.2, where regressive transfers were found to be a sort of cutting-stock optimization heuristic [Ref: Wäscher and Gau (1996)] for best matching all of the deficits to surpluses. We see that here also the regressive policy results in more individuals above threshold in steady state than the other policies. Essentially, by allowing individuals with small surplus to give more frequently, the regressive



Policy	T	Gini index	Pop. below threshold
Regressive	1	.764	.074
Flat	2.2	.728	.105
Progressive	11.3	.704	.133

Figure 4.7: Steady state distributions and data for parameterized kinetic NBT policies with initial condition $f_0(w) \sim \text{Uniform}$.

policy utilizes small surpluses to cover small needs while reserving larger surpluses for larger needs.

4.3 Control of NBTs

In the kinetic NBT policy model (4.8), $p(\epsilon)$ may be considered not to be a fixed distribution of donor thresholds, but rather a time-varying prioritization of donor thresholds, $p(\epsilon, t)$ that could be considered as a control. As such, we define a few natural optimal control problems to consider in Definition 3.

Definition 3 (Control problems). *Subject to $f(w, t)$ evolving according to (4.8), initial condition $f_0(w) = f(w, 0)$, and the control $p(\epsilon, t)$ a probability, the following are contextually natural optimal control problems to consider:*

(a) *Given $T > 0$, minimize $\int_{-\infty}^{\theta} f(w, T)dw$.*

(b) *Minimize $\frac{d}{dt} \int_{-\infty}^{\theta} f(w, t)dw$.*

(c) *Minimize $\frac{d}{dt} \int_{-\infty}^{\theta} w f(w, t)dw$.*

Problem (a) set's an end time for transfers and attempts to choose $p(\epsilon, t)$ throughout that time interval to minimize the terminal number of individuals below threshold. Problem (b) seeks at any time t to identify $p(\epsilon, t)$ that reduces the number of individuals below threshold as quickly as possible. Problem (c) has the goal of reducing the deficit (total below-threshold wealth) as quickly as possible.

Lemma 7. *Let $f(w, t)$ evolve according to (4.8), and consider policies of the form $p(\epsilon, t) = \delta(\epsilon - \epsilon_0(t))$.*

- *The solution to the control problem of Definition 3 (b) is achieved by*

$$\epsilon_0(t) = \operatorname{argmin}_{\epsilon \geq 0} \left\{ - \int_{-\infty}^{\theta} f(w, t) f(2\theta + \epsilon - w, t) dw \right\}. \quad (4.9)$$

- *The solution to the control problem of Definition 3 (c) is achieved by*

$$\epsilon_0(t) = \operatorname{argmin}_{\epsilon \geq 0} \left\{ - \int_{-\infty}^{\theta} w f(w, t) f(2\theta + \epsilon - w, t) dw \right\}. \quad (4.10)$$

Proof. • For control problem (b),

$$\begin{aligned} \frac{d}{dt} \int_{-\infty}^{\theta} f(w, t)dw &\stackrel{(4.8)}{=} - \int_0^{\infty} \int_{-\infty}^{\theta} p(\epsilon, t) f(w, t) f(2\theta + \epsilon - w, t) dw d\epsilon \\ &= - \int_0^{\infty} \int_{-\infty}^{\theta} \delta(\epsilon - \epsilon_0(t)) f(w, t) f(2\theta + \epsilon - w, t) dw d\epsilon \\ &= - \int_{-\infty}^{\theta} f(w, t) f(2\theta + \epsilon_0(t) - w, t) dw \end{aligned}$$

- For control problem (c),

$$\begin{aligned}
\frac{d}{dt} \int_{-\infty}^{\theta} w f(w, t) dw &\stackrel{(4.8)}{=} - \int_0^{\infty} \int_{-\infty}^{\theta} p(\epsilon, t) w f(w, t) f(2\theta + \epsilon - w, t) dw d\epsilon \\
&= - \int_0^{\infty} \int_{-\infty}^{\theta} \delta(\epsilon - \epsilon_0(t)) w f(w, t) f(2\theta + \epsilon - w, t) dw d\epsilon \\
&= - \int_{-\infty}^{\theta} w f(w) f(2\theta + \epsilon_0(t) - w, t) dw
\end{aligned}$$

□

Figure 4.8 illustrates how the optimal control policy corresponding to Definition 3 (b) impacts the wealth distribution in comparison to the previously discussed policies. One may observe that the above-threshold wealth distribution in Figure 4.8 appears to have approached a uniform distribution. This absolutely makes sense, because the rate at which successful transfers occur in Equation (4.9) is proportional to the product of the densities of potential matches. Thus, ensuring the most successful matches means targeting the highest densities of donors.

This is interesting because in the agent-based studies, transfers were conducted instantly in a single simulation round without any consideration for time limitations of arranging such matches. The control policy corresponding to (4.9) focuses directly on the time limitations of below-threshold individuals meeting donors in a random interaction environment.

4.4 Kinetic NBT with Diffusion

Here, we consider diffusion to model the natural evolution of wealths, that individuals' wealths increase and decrease by random additive amounts over time. This may be true of the majority of the population of modern societies [Ref: Silva and Yakovenko (2004)]. Multiplicative diffusion would model the evolution of wealth for the top 5%, but adding geometric Brownian motion would not conserve wealth, which is something that

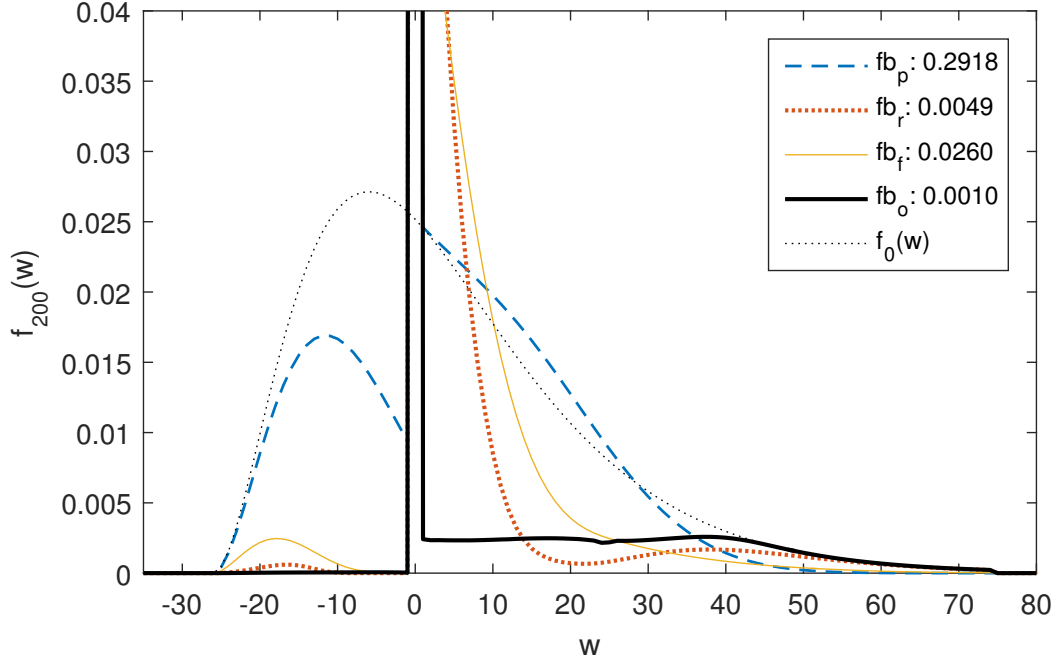


Figure 4.8: Wealth distributions at $t = 200$ for various policies. The notation used in the legend is such that $fb_p: 0.2918$ means that for the progressive policy (p), the fraction of the population below threshold (fb_p) is equal to 0.2918. The initial condition is chosen to be a Gamma distribution. fb_o identifies the optimal policy corresponding to (4.9).

we would like to do. So, let's now consider the following kinetic model where $D > 0$ and $Q(f, f)(w, t)$ is the kinetic NBT interaction operator from the right hand side of Equation (4.2):

$$\partial_t f(w, t) = Q(f, f)(w, t) + D \partial_w^2 f(w, t) \quad (4.11)$$

Lemma 8. *Let f evolving according to (4.11) and $f_0(w)$ a probability density. With reasonable assumptions about the end behavior of f , we have the following observations related to the moments of f :*

- (i) $M_0(t) = 1$ is constant.
- (ii) $M_1(t) = M_1(0)$ is constant.
- (iii) $\frac{d}{dt} M_2(t) = S_2(f, f)(t) + 2D$ where S_2 is non-positive.

(iv) $\frac{d}{dt}M_3(t) = S_3(f, f)(t) + 6D(M_1 - \theta)$, where S_3 is non-positive.

Proof. From Lemma 2, we already know the moment evolutions corresponding to the interaction operator Q ; what remains is to examine the moment evolutions corresponding to the diffusion term. Well,

$$\begin{aligned} \int_{-\infty}^{\infty} (w - \theta)^0 \partial_w^2 f(w, t) dw &= \lim_{z \rightarrow \infty} [\partial_w f(z, t)]_{-z}^z \\ &= 0 \end{aligned}$$

$$\begin{aligned} \int_{-\infty}^{\infty} (w - \theta)^1 \partial_w^2 f(w, t) dw &= \int_{-\infty}^{\infty} w \partial_w^2 f(w, t) dw - \theta \int_{-\infty}^{\infty} (w - \theta)^0 \partial_w^2 f(w, t) dw \\ &= \int_{-\infty}^{\infty} w \partial_w^2 f(w, t) dw - 0 \\ &= \lim_{z \rightarrow \infty} [w \partial_w f(w, t) - f(w, t)]_{-z}^z \\ &= 0 \end{aligned}$$

$$\begin{aligned} \int_{-\infty}^{\infty} (w - \theta)^2 \partial_w^2 f(w, t) dw &= \int w^2 \partial_w^2 f - 2\theta \int w \partial_w^2 f + \theta^2 \int \partial_w^2 f \\ &= \int w^2 \partial_w^2 f - 0 + 0 \\ &= w^2 \partial_w f - 2 \int w \partial_w f \\ &= \lim_{z \rightarrow \infty} \left[w^2 \partial_w f - 2wf + 2 \int f \right]_{-z}^z \\ &= 0 - 0 + 2 \end{aligned}$$

$$\begin{aligned} \int_{-\infty}^{\infty} (w - \theta)^3 \partial_w^2 f(w, t) dw &= \int w^3 \partial_w^2 f - 3\theta \int w^2 \partial_w^2 f + 3\theta^2 \int w \partial_w^2 f - \theta^3 \int \partial_w^2 f \\ &= \int w^3 \partial_w^2 f - 6\theta + 0 - 0 \\ &= \lim_{z \rightarrow \infty} \left[w^3 \partial_w f - 3w^2 f + 6 \int wf \right]_{-z}^z - 6\theta \\ &= 0 - 0 + 6M_1 - 6\theta, \end{aligned}$$

where assuming f decays at least quadratically, e.g. $f \propto w^{-2-\alpha}$, $\alpha > 0$, allows for the limiting behavior terms to equal 0.

Combining this with the proof of Lemma 2 gives

- (i) $\frac{d}{dt}M_0(t) = 0 + 0$, and so $M_0 = 1$ is constant.
- (ii) $\frac{d}{dt}M_1(t) = 0 + 0$, so $M_1(t) = M_1(0)$ is constant.
- (iii) $\frac{d}{dt}M_2(t) = S_2(f, f)(t) + 2D$ where S_2 is the non-positive expression from Lemma 2(iii).
- (iv) $\frac{d}{dt}M_3(t) = S_3(f, f)(t) + 6D(M_1 - \theta)$, where S_3 is the non-positive expression from Lemma 2(iv).

□

Conjecture 1. *For the model (4.11),*

- (a) *In Lemma 8 (iii), $\frac{d}{dt}M_2(t)$ eventually approaches a positive constant.*
- (b) *In Lemma 8 (iv), $\frac{d}{dt}M_3(t)$ eventually approaches a negative constant.*
- (c) *A large open set of initial distributions $f_0(w)$ are attracted to some manifold. The attraction happens on a fast timescale and the manifold evolves to a flat manifold on a diffusive timescale in a non-symmetric way.*

Evidence:

- (a) From Lemma 8 (iii), we know that $\frac{d}{dt}M_2(t) = S_2(f, f)(t) + 2D$ where S_2 is non-positive. From numerical simulation in Figure 4.9 (c), $M_2(t)$ appears to become linear with positive slope. It makes sense that $S_2(f, f)(t)$, which gives the reduction

in variance resulting from matches coming closer to threshold, will approach a negative constant of lesser magnitude than $2D$ such that $\frac{d}{dt}M_2(t)$ approaches a positive constant.

(b) From Lemma 8 (iv), we know that $\frac{d}{dt}M_3(t) = S_3(f, f)(t) + 6D(M_1 - \theta)$, where S_3 is non-positive. From numerical simulation in Figure 4.9 (d), $M_3(t)$ appears to become linear with negative slope. It makes sense that $S_3(f, f)(t)$, which gives the increase in left skew resulting from individuals diffusing to the left who have difficulty finding matches, will approach a negative constant of greater magnitude than $6D(M_1 - \theta)$ such that $\frac{d}{dt}M_3(t)$ approaches a negative constant.

(c) From Figure 4.9 (a) and (b), we see that qualitatively different initial conditions converge to the same curve. In simulations this occurs relatively quickly compared to the very slow decay of that manifold. The asymmetric decay occurs as diffusion pushes mass to the left that is not matched with a transfer as quickly as mass diffused to the right. Thus, the tail to the left gets fatter as mass escapes to the left more quickly than matches can be generated to the right.

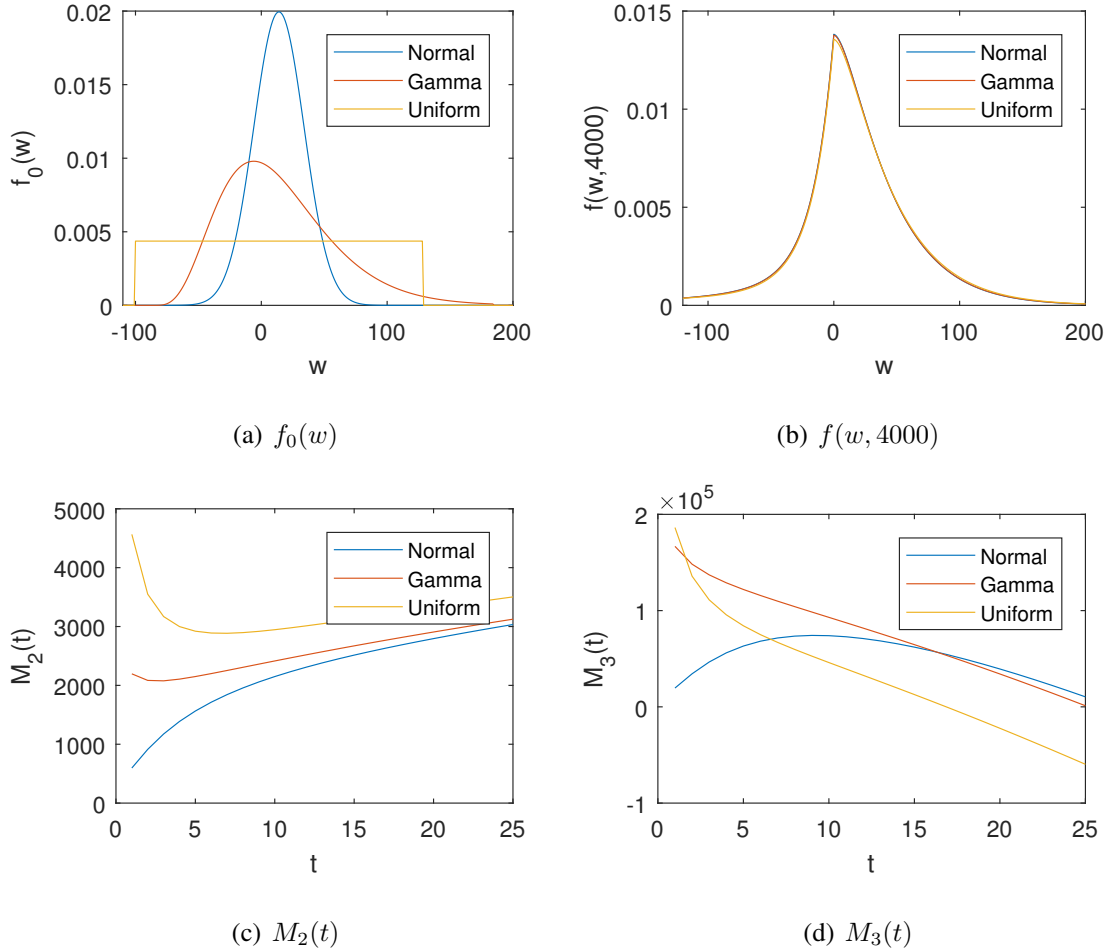


Figure 4.9: (a) A few different initial wealth distributions with $M_1 = 14$, that when evolving according to Equation (4.11) with $\theta = 0$ approach an attractor manifold (b). Note that three curves are present in (b), but they are overlapping. The second and third moment evolutions are shown in (c) and (d) respectively.

4.5 Central Redistribution Model

In order to compare NBT redistribution to more conventional central wealth redistribution, we develop a central redistribution model with an ODE that describes the amount of money in the welfare budget B and a kinetic equation which describes the taxation and welfare distribution. Here, individuals above threshold pay taxes into the welfare budget and from that budget, poor individuals' needs are met.

Trying to keep the model somewhat general, we say that the tax collector will take $T(f, w)$ dollars from an individual with wealth $w > \theta$ at rate $\lambda_1(B, f)$, and give $\theta - w$ to a person with wealth $w < \theta$ at rate $\lambda_2(B, f)$. Again, B is the amount of money in the welfare budget. We assume that $T(f, w) \leq (w - \theta)$ such that an individual may not be taxed to be brought below threshold. We then have the following system:

$$\begin{aligned} \frac{d}{dt}B &= \lambda_1(B, f) \int_{\theta}^{\infty} T(f, w)f(w)dw - \lambda_2(B, f) \int_{-\infty}^{\theta} (\theta - w)f(w)dw \\ \partial_t f(w) &= \lambda_1(B, f)H(w - \theta) \left(-f(w) + \int_w^{\infty} \delta(w' - T(f, w') - w)f(w')dw' \right) \\ &\quad + \delta(w - \theta)\lambda_2(B, f) \int_{-\infty}^{\theta} f(w')dw' - H(\theta - w)\lambda_2(B, f)f(w) \quad (4.12) \end{aligned}$$

Lemma 9. *For system (4.12), the population and total wealth are conserved.*

Proof. Integrating the kinetic equation of (4.12) with respect to w , we observe

$$\begin{aligned} \frac{d}{dt} \int_{-\infty}^{\infty} f(w, t)dw &= \int_{-\infty}^{\infty} \partial_t f(w, t)dw \\ &= -\lambda_1 \int_{\theta}^{\infty} f(w, t)dw + \lambda_1 \int_{\theta}^{\infty} \int_w^{\infty} \delta(w' - T(f, w') - w)f(w')dw' dw \\ &\quad + \lambda_2 \int_{-\infty}^{\theta} f(w', t)dw' - \lambda_2 \int_{-\infty}^{\theta} f(w)dw \\ &= -\lambda_1 \int_{\theta}^{\infty} f(w, t)dw + \lambda_1 \int_{\theta}^{\infty} \int_{\theta}^{w'} \delta(w' - T(f, w') - w)f(w')dw dw' \\ &= -\lambda_1 \int_{\theta}^{\infty} f(w, t)dw + \lambda_1 \int_{\theta}^{\infty} f(w, t)dw \\ &= 0. \end{aligned}$$

Hence, $\int_{-\infty}^{\infty} f(w, t)dw$ is constant, i.e. the total population remains constant in time. For total wealth to be conserved, we need that $\frac{d}{dt} \int_{-\infty}^{\infty} wf(w, t)dw + \frac{d}{dt}B = 0$. Well,

$$\begin{aligned}
\frac{d}{dt} \int_{-\infty}^{\infty} wf(w, t)dw &= \int_{-\infty}^{\infty} w\partial_t f(w, t)dw \\
&= -\lambda_1 \int_{\theta}^{\infty} wf(w, t)dw + \lambda_1 \int_{\theta}^{\infty} \int_w^{\infty} w\delta(w' - T(f, w') - w)f(w')dw' dw \\
&\quad + \lambda_2 \int_{-\infty}^{\infty} \delta(w - \theta)w \int_{-\infty}^{\theta} f(w', t)dw' dw - \lambda_2 \int_{-\infty}^{\theta} wf(w)dw \\
&= -\lambda_1 \int_{\theta}^{\infty} wf(w, t)dw + \lambda_1 \int_{\theta}^{\infty} \int_{\theta}^{w'} w\delta(w' - T(f, w') - w)f(w')dw dw' \\
&\quad + \lambda_2 \int_{-\infty}^{\theta} \theta f(w')dw' - \lambda_2 \int_{-\infty}^{\theta} wf(w)dw \\
&= -\lambda_1 \int_{\theta}^{\infty} wf(w, t)dw + \lambda_1 \int_{\theta}^{\infty} (w' - T(f, w'))f(w')dw' \\
&\quad + \lambda_2 \int_{-\infty}^{\theta} (\theta - w)f(w)dw \\
&= -\lambda_1(B, f) \int_{\theta}^{\infty} T(f, w)f(w, t)dw + \lambda_2 \int_{-\infty}^{\theta} (\theta - w)f(w)dw \\
&= -\frac{d}{dt}B
\end{aligned}$$

□

A major difference with this redistribution model as opposed to NBT is that many more individuals can receive help. Since the tax money is pooled to be given to needy individuals rather than asked for from a single individual, it is much more likely that the budget will have enough funds to help an individual far below threshold than it is that any one individual could meet that need.

Central redistribution example: Flat Tax Model A simple example of choices for T, λ_1, λ_2 in (4.12) is the following:

- $T(f, w) = rw, r \in (0, 1)$ (a flat tax)

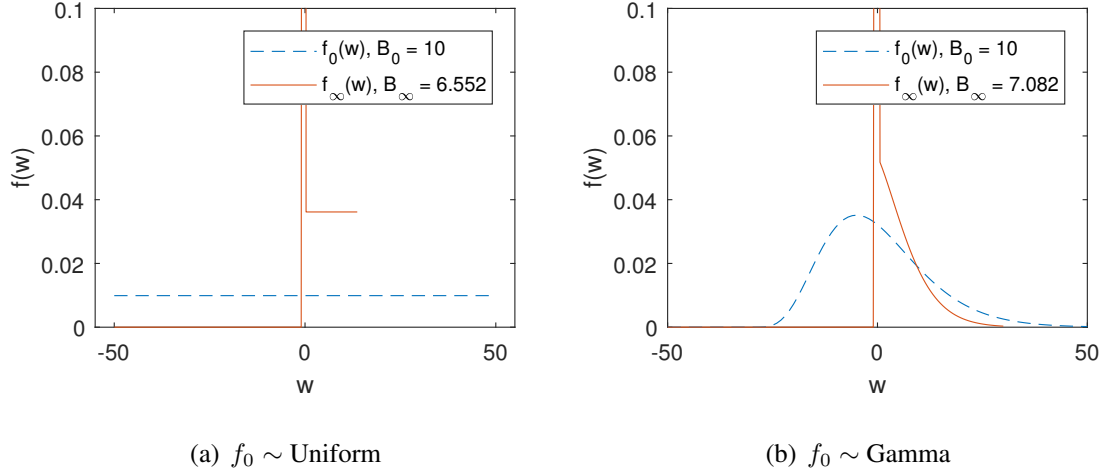


Figure 4.10: Steady state solutions of (4.12) for the Flat Tax Model with uniform and gamma initial conditions. The initial budget is $B_0 = 10$, and the size of the budget in steady state is identified as B_∞ .

- $\lambda_1(B, f) = - \int_{-\infty}^0 w f(w, t) dw$
- $\lambda_2(B, f) = B$.

Note that these choices are sensible for (4.12) in that they satisfy the following:

- $T(f, w) \leq w$, i.e. an individual is not taxed so much as to make their post-tax wealth below threshold.
- $\lambda_1(B, f) \rightarrow 0$ as $\int_{-\infty}^0 w f(w, t) dw \rightarrow 0$, i.e. taxation is reduced to a stop as the deficit reduces to 0.
- $\lambda_2(B, f) \rightarrow 0$ as $B \rightarrow 0$, i.e. the rate at which aid is given to below-threshold individuals reduces to 0 as funds in the budget reduce to 0.

Figure 4.10 shows the initial condition and steady state solution for the wealth distribution that results when f and B evolve according to the Flat Tax Model of (4.12).

Chapter 5

SURVIVOR'S DILEMMA IN VOLATILE ENVIRONMENTS

In this chapter we examine the viability of socioeconomic cooperation with reciprocal gift giving practices like NBTs from a game theoretic perspective. Because participating in such a relationship involves a cost (sharing your surplus when another individual has need) and a benefit (receiving from a partner's surplus when you are in need), there may be a temptation to cheat. This may take the form of under-representing surplus in order to not give support when requested, over-representing need in order to grow one's surplus, or not putting effort into maintaining one's own viability so that a cooperative partner has to bear the load.

Thus, we consider that an individual has a choice, to cooperate (C) or defect (D), where cooperation in a risk pool has a benefit b , but also a cost c ; cheating in this case would be when a player defects against a cooperative partner, i.e. they get the benefit of a cooperative partner, but aren't contributing anything or incurring any cost.

In volatile environments, especially where decisions to cooperate or not are life-and-death decisions that occur on a much shorter timescale than reproduction, the benefit and cost of cooperation should be interpreted as increasing or decreasing survival probability rather than additively increasing or decreasing some bank account that will later competitively determine reproductive fitness. Thus, as described in Section 2.3, the types of cooperation we consider are survivor's games.

5.1 2-Player Repeated Survivor's Dilemma

Definition 4 (2-Player Survivor's Dilemma). *Let $\beta \in (0, 1)$, be an individual's survival probability per unit time in isolation. Let $b \in (0, 1 - \beta)$ be the contribution to survival*

that a cooperative partner imparts, and $c \in (0, \min\{b, \beta\})$ be the cost of cooperating in a reciprocal gift giving risk pool. Then, where C represents the strategy to cooperate and D represents the strategy to defect, payoffs (as survival probabilities) of the 2-player Survivor's Dilemma are as follows:

$$\begin{array}{cc}
 & \text{Player 2} \\
 & \begin{array}{cc} C & D \end{array} \\
 \text{Player 1} & \begin{array}{cc} C \left(\begin{array}{cc} w_R = \beta + b - c & w_S = \beta - c \end{array} \right) \\ D \left(\begin{array}{cc} w_T = \beta + b & w_P = \beta \end{array} \right) \end{array} \end{array} \quad (5.1)$$

In (5.1), only Player 1's survival probabilities are shown, and in a notation that maintains intuitive connection to the "shadow of the future" continued interaction probability w [Ref: Axelrod *et al.* (1988)] and the prisoner's dilemma notation of Rapoport and Chammah (1965) (R, S, T, P). Since the game is symmetric, one understands that, for example, if Player 1 cooperates and Player 2 defects, Player 2's survival rate is the temptation payoff $w_T = \beta + b$.

The following observations about the parameter space and payoffs of (5.1) are important:

- (i) For the given assumptions about the parameter space, $\beta \in (0, 1)$, $b \in (0, 1 - \beta)$, $c \in (0, \min\{b, \beta\})$, each payoff remains a probability, i.e. a value between 0 and 1.
- (ii) It is reasonable to consider that participating in a cooperative relationship results in a greater benefit than cost ($b > c$) because a life-saving gift from one individual's surplus should be worth more to the rescued recipient than the donor who provides it from their surplus. For the vampire bats it is additionally true that time until starvation increases more when an amount of food is added to an empty stomach than

when that same amount is added to a full stomach [Ref: Wilkinson (1984)]. The benefits of cooperation among the Maasai have also been illustrated [Refs: Aktipis *et al.* (2011); Hao *et al.* (2015)].

- (iii) $w_T > w_R > w_P > w_S$, so the game is indeed of the prisoner's dilemma type.
- (iv) $w_R > (w_T + w_S)/2$, so this game is eligible to qualify as a repeated prisoner's dilemma in the sense that there is not the incentive to take turns receiving the sucker and temptation payoffs.

Consider that two individual's continue to repeatedly have interactions of (5.1) at every unit time interval as long as they live. The result of the first round of the game for a given player is that either (i) the player dies, (ii) their partner dies and they live in isolation, or (iii) they both live and the game repeats. Once an individual becomes isolated, their remaining expected lifespan is:

$$u^* := \sum_{n=1}^{\infty} \beta^n = \frac{\beta}{1 - \beta}. \quad (5.2)$$

Definition 5 (2-Player Repeated Survivor's Dilemma). *The extensive form representation of the 2-Player Repeated Survivor's Dilemma for all-C and all-D time-invariant strategy pairings is given by Figure 5.1, which shows survival probabilities and payoffs from Player 1's perspective. The notation $F_{C|D}$ represents the payoff to a player with strategy all-C (always cooperate) who is paired with an all-D (always defect) partner. The two-player supergame payoff matrix is the following:*

$$\begin{array}{c}
 \text{Player 2} \\
 \text{all-C} \quad \text{all-D} \\
 \text{Player 1} \quad \text{all-C} \quad \begin{pmatrix} F_{C|C} & F_{C|D} \\ F_{D|C} & F_{D|D} \end{pmatrix}. \quad (5.3) \\
 \text{all-D}
 \end{array}$$

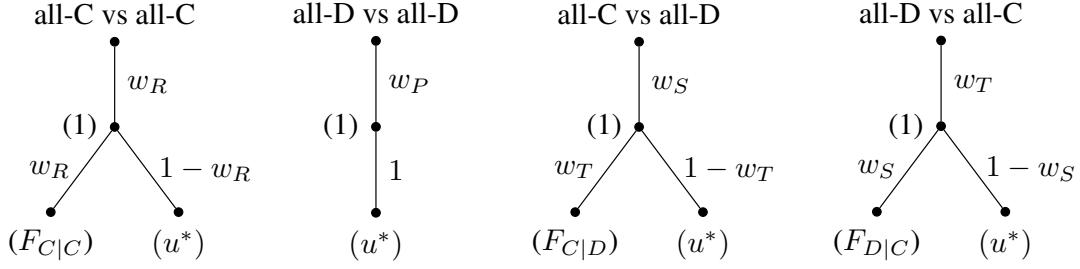


Figure 5.1: Extensive form representation of 2-Player Repeated Survivor's Dilemma for all-C and all-D strategy pairings. Only Player 1's perspective and utilities are shown (in parentheses) since the game is symmetric. u^* is defined in (5.2). The label $F_{C|D}$ identifies the payoff of the 2-player all-C vs all-D game; this appears at the branch where both individuals have survived (in the all-C vs all-D game) since the expected payoff whenever the two individuals are alive and resume interacting with those strategies is again $F_{C|D}$. It is similar for the other strategy pairings.

Lemma 10. *Given the survivor's dilemma survival rates of (5.11), the supergame utilities or fitnesses/lifespans of (5.3) are:*

$$F_{C|C} = \frac{w_R[1 + (1 - w_R)u^*]}{1 - w_R^2} \quad (5.4a)$$

$$F_{C|D} = \frac{w_S[1 + (1 - w_T)u^*]}{1 - w_S w_T} \quad (5.4b)$$

$$F_{D|C} = \frac{w_T[1 + (1 - w_S)u^*]}{1 - w_T w_S} \quad (5.4c)$$

$$F_{D|D} = u^*. \quad (5.4d)$$

Proof. The fitnesses are found by utilizing the repeated game structure. For example, in the all-C vs all-D game, the original game will repeat if both players survive, so, with Figure 5.1 as a reference we have the following:

$$F_{C|D} = w_S[(1) + w_T(F_{C|D}) + (1 - w_T)(u^*)].$$

Solving for $F_{C|D}$ gives (5.4c). For all-D vs all-D,

$$F_{D|D} = w_P[1 + u^*] = \beta \left(1 + \frac{\beta}{1 - \beta} \right) = \beta \left(\frac{1}{1 - \beta} \right) = u^*$$

□

5.1.1 Nash Equilibria

Interestingly, for different β, b, c parameter regions, the static game of (5.3) takes either the form of (i) a prisoner's dilemma, or (ii) a stag hunt.

Definition 6 (Stag Hunt). *Originally a story told by Rousseau (1984), the stag hunt involves deciding whether to maintain hiding positions to eventually kill a deer, or give up your position in pursuit of a visible hare [Ref: Skyrms (2004)]. If one goes after the hare while the other waits for a stag, the one who waits will be left with nothing, while the one who gave away their position will get the hare; if two players coordinate though, they can share the stag or the hare. The 2-player stag hunt as a static game has the following payoff matrix:*

$$\begin{array}{cc}
 & \text{Player 2} \\
 & C \quad D \\
 \text{Player 1} \quad C & \left(\begin{array}{cc} a & b \end{array} \right) \\
 & D \left(\begin{array}{cc} c & d \end{array} \right)
 \end{array} \tag{5.5}$$

satisfying $a > c > d > b$ [Ref: Skyrms (2004)].

Lemma 11. *The pure strategy Nash equilibria for the stag hunt (5.5) are (C, C) and (D, D) , and there is a mixed strategy Nash equilibrium where the probability of cooperating is given by*

$$p^* = \frac{d - b}{a - c + d - b} \tag{5.6}$$

Proof. This is a well known result; see Skyrms (2004). □

Corresponding to Definition 6 and Lemma 11, for parameters β, b, c such that the repeated survivor's dilemma supergame is equivalent to a stag hunt, $F_{C|C} > F_{D|C} > F_{D|D} > F_{C|D}$, and (all-C, all-C) and (all-D, all-D) are pure strategy Nash equilibria. However, as

Game	Payoff orderings	NE strategies, \vec{p}
Prisoner's Dilemma	$F_{D C} > F_{C C} > F_{D D} > F_{C D}$	(0, 0)
Stag Hunt	$F_{C C} > F_{D C} > F_{D D} > F_{C D}$	(0, 0), (1, 1), (p^* , p^*)

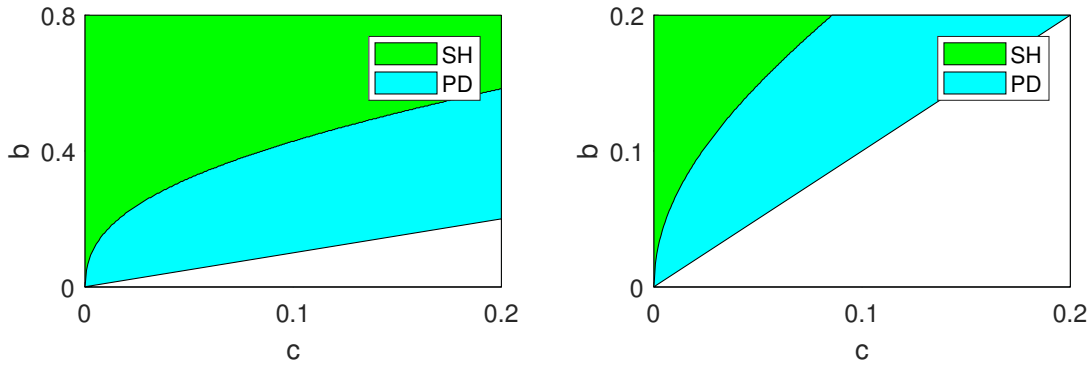
Table 5.1: 2-Player Repeated Survivor's Dilemma supergame equivalent static games for different parameter regions. Recall that the payoffs are functions of the parameters B, b, c . p^* is given in (5.7).

$F_{C|C} > F_{D|D}$, coordination would rationally lead to mutual cooperation, or we could expect that even without communication rational agents would choose to cooperate according to focal point theory [Ref: Schelling (1980)]. In terms of the expected lifespans of (5.4a) - (5.4d), the mixed strategy Nash equilibrium for the repeated survivor's dilemma stag hunt is where the probability of choosing all-C is given by

$$p^* = \frac{F_{D|D} - F_{C|D}}{F_{C|C} - F_{D|C} + F_{D|D} - F_{C|D}}. \quad (5.7)$$

A summary of results related to Nash equilibria of the 2-Player Repeated Survivor's Dilemma is given in Table 5.1. Also, for $\beta = 0.2, 0.8$, the (c, b) parameter regions where the supergame is a stag hunt or prisoner's dilemma are illustrated in Figure 5.2. Finally, Nash equilibria (expressed as probabilities of choosing all-C) for $\beta = 0.8, c = 0.02$, and $b \in (0.02, 0.2)$ are illustrated in Figure 5.3.

Contextually, the significance of Figure 5.2 is that when b is significantly greater than c , individual's will cooperate. The temptation to cheat is reduced because cheating increases the probability a cooperative partner will die and therefore no longer be around to provide the significant benefit. Essentially the long-term benefit of continued cooperation outweighs the short-term benefit of cheating. This is the concept of partnership [Ref: Eshel and Shaked (2001)].



(a) $\beta = 0.2$

(b) $\beta = 0.8$

Figure 5.2: Equivalent 2-player static supergame regions depending on parameters for the 2-player repeated survivor’s game. The region shaded green represents the parameter values (c, b) where the game would be equivalent to a Stag Hunt (SH). The region shaded light blue corresponds to a Prisoner’s Dilemma (PD). The unshaded region is outside of the parameter space.

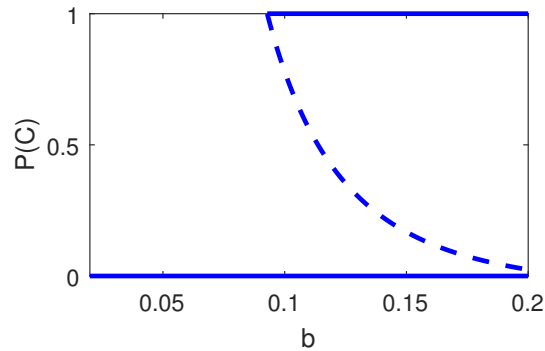


Figure 5.3: For $\beta = 0.2$, $c = 0.02$, and $b \in (0.02, 0.2)$, the probability of choosing all-C is shown. The dashed line appears where the supergame changes from a prisoner’s dilemma to a stag hunt; the dashed line illustrates the mixed strategy Nash equilibrium, p^* of (5.7).

5.1.2 Evolutionary Stability

For vampire bats or herders to identify which parameter region they are in and calculate expected lifespans is unlikely; the evolutionary game theoretic approach is more meaningful for these examples.

Definition 7 (Evolutionary 2-Player Repeated Survivor’s Dilemma). *Let x be the fraction a population that uses strategy all-C, such that $(1 - x)$ is the fraction that uses all-D. Then,*

with 2-player repeated survivor's dilemma payoffs (5.4a)-(5.4d), the evolutionary fitness of all-C and all-D are respectively given by

$$f_{all-C}(x) = xF_{C|C} + (1-x)F_{C|D}, \quad f_{all-D}(x) = xF_{D|C} + (1-x)F_{D|D}. \quad (5.8)$$

Then, where $\phi(x) = xf_{all-C}(x) + (1-x)f_{all-D}(x)$ is the average fitness of the population, the evolution of strategy densities is given by the replicator equation:

$$\dot{x} = x(f_{all-C}(x) - \phi(x)). \quad (5.9)$$

As (5.9) defines a dynamical system, concepts of stability from dynamical systems are applicable and relevant. However, in game theory, there is an additional stability notion which defines an evolutionary stable strategy (Def. 8).

Definition 8 (Evolutionary Stable Strategy). *An evolutionarily stable strategy (ESS) is a strategy which, if adopted by a population in a given environment, cannot be invaded by any other strategy that is initially rare.*

How stability in dynamical systems and evolutionarily stable strategies are related can be read from Sigmund (2011). Henceforth, unless, ESS is explicitly mentioned, one should understand stability to refer to asymptotic stability of the dynamical system.

Lemma 12. *For (5.9), in the stag hunt parameter regions, there is bistability with $x = 0$ and $x = 1$ stable equilibria and $x^* = \frac{F_{D|D}-F_{C|D}}{F_{C|C}-F_{D|C}+F_{D|D}-F_{C|D}}$ an unstable equilibrium. In the prisoner's dilemma parameter regions, $x = 1$ is an unstable equilibrium and $x = 0$ is stable.*

Proof. The dynamics for such a system as (5.9) and (5.3), with 2-player symmetric evolutionary fitnesses, are well known [Ref: Nowak (2006)]. Since $F_{C|C} > F_{D|C}$ and $F_{D|D} > F_{C|D}$, all-C ($x = 1$) and all-D ($x = 0$) are bistable, and there exists an unstable equilibrium

$x^* = \frac{F_{D|D} - F_{C|D}}{F_{C|C} - F_{D|C} + F_{D|D} - F_{C|D}}$ in $[0, 1]$. If the initial condition $x(0) < x^*$, the system will converge to all-D, but if $x(0) > x^*$, the system will converge to all-C.

In the prisoner's dilemma region, since $F_{C|C} < F_{D|C}$ and $F_{D|D} > F_{C|D}$, all-D dominates all-C with $x = 0$ a stable equilibrium and $x = 1$ an unstable equilibrium. \square

Lemma 13. *For (5.9), in the stag hunt parameter regions, all-C and all-D are evolutionarily stable strategies (ESS). For the prisoner's dilemma regions, all-D is an ESS.*

Proof. Since we identified the game regions with strict inequalities, the Nash equilibria we discussed were *strict* Nash equilibria, which means they are also ESS [Ref: Nowak (2006)]. \square

Lemmas 12 and 13 tell us that in the stag hunt parameter space, if the initial fraction of individuals who are all-C is greater than x^* , then the population will become completely cooperative. Since x^* is equivalent to p^* from (5.7), Figure 5.3 allows us to make the relevant observation that for fixed β, c , increasing b reduces the size of the initial fraction of the population necessary to be all-C in order for all-C to dominate. This intuitively makes sense that as the benefit of cooperation increases, the value of a cooperative partner increases. Thus, wanting to maintain a cooperative partner's viability and continued support by also cooperating with them (partnership) is evolutionarily advantageous as b increases.

5.2 3-Player Repeated Survivor's Dilemma

As discovered with respect to the Maasai, increasing the degree of the cooperative network had significant impact on survival [Ref: Hao *et al.* (2015)]. Food sharing among vampire bats also occurs in roosts larger than just dyads [Ref: Wilkinson (1984)]. Thus, naturally, it is of interest to examine the impact of increasing the number of individual's participating in the repeated survivor's dilemma.

How cooperation evolves when increasing the number of participants is certainly connected to how the individual costs and benefits scale with the group size. In order to maintain survival rates as probabilities and not change the parameter space of Definition 4, we determine survival rates for a given round to be calculated as follows:

$$w = \begin{cases} \beta + \frac{N_C-1}{N-1}b - c, & \text{if } C \\ \beta + \frac{N_C}{N-1}b, & \text{if } D, \end{cases} \quad (5.10)$$

where N is the size of the group, and N_C is the number of cooperators. Reasonably, as the number of cooperators increases, the size of the benefit imparted increases. In (5.10), the cost to participate in such a cooperative risk pool is independent of the group size and number of cooperators.

Definition 9 (3-Player Survivor's Dilemma). *Let $\beta \in (0, 1)$, $b \in (0, 1-\beta)$, $c \in (0, \min\{b, \beta\})$.*

Survival rates for the 3-Player Survivor's Dilemma are given by:

		<i>Total # C</i>			
		0	1	2	3
<i>Player 1</i>	<i>C</i>	·	$w_{C 3} = \beta + b - c$	$w_{C 2} = \beta + \frac{b}{2} - c$	$w_{C 1} = \beta - c$
	<i>D</i>	$w_{D 0} = \beta$	$w_{D 1} = \beta + \frac{b}{2}$	$w_{D 2} = \beta + b$	·

(5.11)

where $w_{C|2}$ represents the survival rate for a cooperator when there are a total of 2 cooperators.

Definition 10 (3-Player Repeated Survivor's Dilemma Supergame). *Let N be the number of living group members. Assume that if the group size reaches 2, the two remaining individuals coordinate to cooperate when (5.3) is a stag hunt; both defect when (5.3) is a prisoner's dilemma. Thus, $F^{(2)}$, an individual's payoff when $N = 2$ is determined by the*

parameters β, b, c and is either $F_{C|C}$ or $F_{D|D}$. Then, assume rational agents choose pure strategies and calculate expected lifespans via backward induction. Since strategies when $N = 2$ are determined by parameters, decisions can be understood to be made only at the $N = 3$ level, with time-invariant strategy triples for the 3-Player Repeated Survivor's Dilemma Supergame being

$$\bar{p}^{(3)} = (p_1^{(3)}, p_2^{(3)}, p_3^{(3)}) \in \{0, 1\}^3 \quad (5.12)$$

where $p_i^{(3)}$ is the probability Player i will play C whenever $N = 3$. Payoffs are then expressed as

$$\begin{array}{c} \text{Total \# all-C} \\ 0 \quad 1 \quad 2 \quad 3 \\ \text{Player 1} \quad \text{all - C} \left(\begin{array}{cccc} \cdot & F_{C|3} & F_{C|2} & F_{C|1} \\ F_{D|0} & F_{D|1} & F_{D|2} & \cdot \end{array} \right). \end{array} \quad (5.13)$$

An example extensive form illustration of how the 3-Player repeated game evolves is given in Figure 5.4.

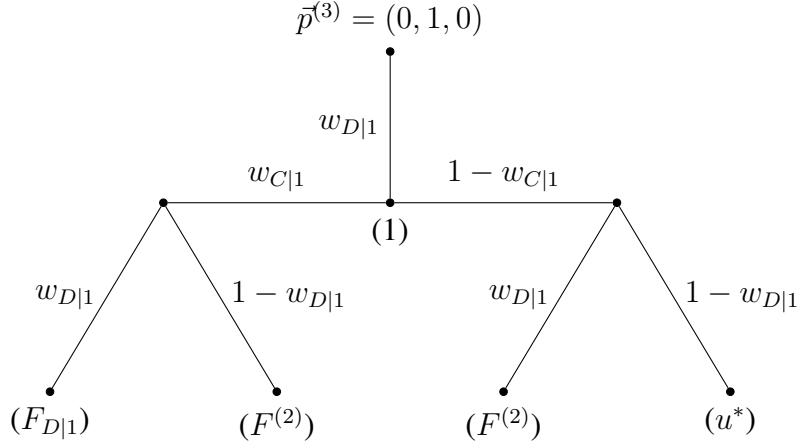


Figure 5.4: Extensive form representation 3-Player Repeated Survivor's Dilemma Supergame with strategy triple $\bar{p}^{(3)} = (0, 1, 0)$, i.e. all-D vs all-C vs all-D. Edge labels are probabilities. Values in parentheses are payoffs if the node is reached: $F_{D|1}$ is defined in (5.13), $F^{(2)}$ represents the expected lifespan once the group size reduces to 2, and u^* is the expected lifespan once in isolation.

Lemma 14. *Payoffs for the 3-Player Repeated Survivor's Dilemma Supergame are as follows:*

$$F_{C|3} = \frac{w_{C|3}[1 + 2w_{C|3}(1 - w_{C|3})F^{(2)} + (1 - w_{C|3})^2u^*]}{1 - w_{C|3}^3} \quad (5.14a)$$

$$F_{C|2} = \frac{w_{C|2}\{1 + [w_{C|2}(1 - w_{D|2}) + (1 - w_{C|2})w_{D|2}]F^{(2)} + (1 - w_{C|2})(1 - w_{D|2})u^*\}}{1 - w_{C|2}^2w_{D|2}} \quad (5.14b)$$

$$F_{C|1} = \frac{w_{C|1}[1 + 2w_{D|1}(1 - w_{D|1})F^{(2)} + (1 - w_{D|1})^2u^*]}{1 - w_{C|1}w_{D|1}^2} \quad (5.14c)$$

$$F_{D|0} = \frac{w_{D|0}[1 + 2w_{D|0}(1 - w_{D|0})F^{(2)} + (1 - w_{D|0})^2u^*]}{1 - w_{D|0}^3} \quad (5.14d)$$

$$F_{D|1} = \frac{w_{D|1}\{1 + [w_{C|1}(1 - w_{D|1}) + (1 - w_{C|1})w_{D|1}]F^{(2)} + (1 - w_{C|1})(1 - w_{D|1})u^*\}}{1 - w_{C|1}w_{D|1}^2} \quad (5.14e)$$

$$F_{D|2} = \frac{w_{D|2}[1 + 2w_{C|2}(1 - w_{C|2})F^{(2)} + (1 - w_{C|2})^2u^*]}{1 - w_{D|2}w_{C|2}^2} \quad (5.14f)$$

Proof. The payoffs are found in the same way as for the proof of Lemma 10, utilizing the repeated game structure. For example, with Figure 5.4 as a reference,

$$F_{D|1} = w_{D|1}\{1+w_{C|1}w_{D|1}F_{D|1}+[w_{C|1}(1-w_{D|1})+(1-w_{C|1})w_{D|1}]F^{(2)}+(1-w_{C|1})(1-w_{D|1})u^*\}.$$

Then, solving for $F_{D|1}$ yields (5.14e). □

5.2.1 Nash Equilibria

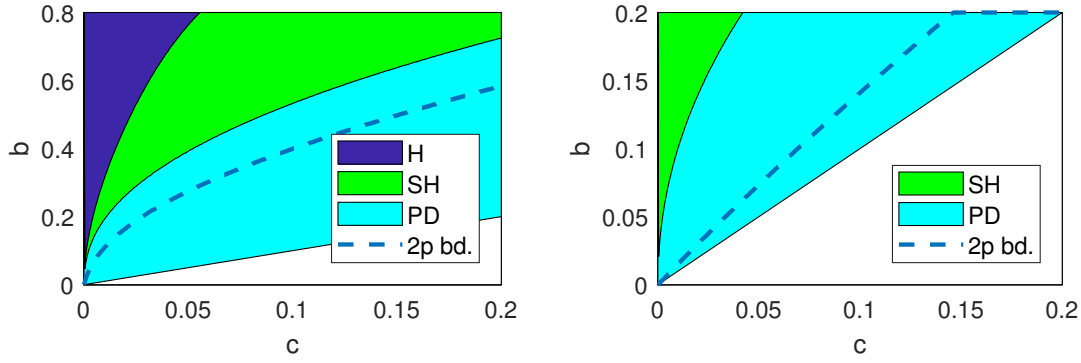
Interestingly, when the survivor's dilemma is scaled up to 3 players according to Definition 10, not only are there parameter regions where the decisions at the 3-player level are equivalent to a stag hunt or prisoner's dilemma; there is additionally a parameter region where the 3-player supergame is equivalent to a Harmony game, i.e. mutual cooperation is the only Nash equilibrium. A description of these three static games is given in Table 5.2. An illustration of the parameter regions where these games occur for $\beta = 0.2, 0.8$ is given in Figure 5.5.

Utilizing Figure 5.5, we make the following observations:

- (i) The harmony region appears for $\beta = 0.2$, but not $\beta = 0.8$. Clearly it is harder for an individual to survive in isolation when $\beta = 0.2$ than when $\beta = 0.8$. What the condition of the harmony game, $F_{C|1} > F_{D|0}$, implies is that it is better to be the sole cooperator, i.e. cheated on by the other two players than to essentially have all

Game	Payoff orderings	Pure NE strategies, $\vec{p}^{(3)}$
Prisoner's Dilemma	$F_{D 2} > F_{C 3} \ \& \ F_{D 0} > F_{C 1}$	(0, 0, 0)
Stag Hunt	$F_{C 3} > F_{D 1} \ \& \ F_{D 0} > F_{C 1}$	(0, 0, 0) & (1, 1, 1)
Harmony	$F_{C 3} > F_{D 2} \ \& \ F_{C 1} > F_{D 0}$	(1, 1, 1)

Table 5.2: 3-Player Repeated Survivor's Dilemma supergame equivalent static games for different parameter regions.



(a) $\beta = 0.2$

(b) $\beta = 0.8$

Figure 5.5: Parameter regions for $\beta = 0.2, 0.8$ and corresponding equivalent static 3-player games for the 3-Player Repeated Survivor's Dilemma when all three players are still viable. The region shaded dark blue corresponds to a Harmony game (H), the region shaded green corresponds to a Stag Hunt (SH), the light blue region corresponds to a Prisoner's Dilemma (PD), and the white region is outside of the parameter space. The decision boundary for the 2-player game is shown using a dashed line; when the number of players reduces to 2, above this line is where the game corresponds to SH and below it corresponds to PD.

players be in isolation. This is possible in the harmony region possibly because the value of cooperation in such a volatile environment makes it worth the risk to save another individual, with the certainty of cooperating with them when the number of living players becomes 2. It makes sense for this region to not appear in the $\beta = 0.8$ case as cooperation becomes less precious when survival in isolation is much greater.

- (ii) The 2-player SH/PD boundary (dashed line of Figure 5.5) is below the SH/PD boundary for the 3-player round. This is certainly reflective of how we chose to scale the benefit and cost of cooperation when increasing the number of players in Equation (5.10). For our choice of scaling, a single cooperative partner is of greater value (b) when there are only 2 active players, than when there are three active players ($\frac{b}{2}$). It is thus reasonable that for a given β, b, c , it might be more advantageous to defect at the 3-player level but coordinate to cooperate in a stag hunt at the 2-player level.

CONCLUSIONS

In this study, Need-based Transfers (NBTs), a characterization of the economic cooperation of communities like the Maasai of East Africa and vampire bats, are studied using agent-based simulation, kinetic theory, and game theory. NBTs utilize binary wealth redistribution as a form of welfare or viability risk pooling, and being that transfers are binary rather than centralized like conventional insurance and government welfare, understanding the mathematics of such an economic system is valuable.

With agent-based simulations, we identify that in response to a volatile environment generating a set of surpluses and a set of deficits in a community, how requests for aid and donations are prioritized has significant impact on the survival of NBT communities. This impact is manifested in the short-term and long-term. In the short-term, the giving policy that prioritizes the least rich individuals' donations ($\leftarrow LR$) is found to be similar to a cutting-stock optimal heuristic that results in greatest efficiency and highest short-term survival rates while the progressive policy which prioritizes the donations of the richest individuals ($\leftarrow R$) is least efficient and results in the lowest short-term survival rates. In the long-term, the $\leftarrow LR$ policy generates many at-risk individuals whose loss of viability leads to lower survival rates while the $\leftarrow R$ policy best diversifies wealth, leading to a more risk-averse wealth distribution and higher long-term survival rates. Recognizing the advantages and disadvantages of regressive and progressive binary wealth redistribution in NBTs, we define a hybrid policy which uses the $\leftarrow LR$ policy only in response to rare global disasters when efficiency is vital, and then uses the $\leftarrow R$ policy otherwise. This policy leads to a long-term survival rate that outperforms the previous optimal policy of $\leftarrow R$.

In addition to examining redistribution policies, network topologies are examined as well. Permitting individuals, who lose an NBT partner, to reconnect to the sharing network by connecting to either the highest degree node or lowest degree node results in sharing networks with high variance or low variance respectively. We find that high variance and relatively low mean network degree results in a detrimental phenomenon called *exhausted nodes*, where high degree individuals are depended upon greatly by those connected to them, but because of the large number of requests they receive, they are perpetually close to threshold and often unable to aid their many dependents. Thus, the preferential edge reattachment policy results in high degree variance, which leads to exhausted nodes and decreased community survival. On the other hand, anti-preferential reattachment leads to low degree variance which results in a diversification of risk, with more equally shared burden and support of NBT relationships; this results in improved community survival.

As NBTs consist of binary wealth redistribution, it is natural to consider modeling the evolution of an NBT economy using kinetic exchange models of markets. Moreover, there are multiple advantages of kinetic PDE models over agent-based simulation. In particular, the analytical structure of PDEs allows for proving some results, and even if results are still numerical, the computation time with the PDE model is tremendously reduced compared to the agent-based simulations. Also, the concept of the evolution of the wealth distribution is better captured in the kinetic framework.

We develop such a kinetic model in Chapter 4 and prove some results related to the moments of the wealth distribution, namely that absent a natural wealth evolution process, the second and third moments are non-increasing while the zero-th and first moments are constant. We also incorporate regressive to progressive policies into the kinetic NBT model and observe that as a policy becomes more regressive, (i) the wealth inequality increases, (ii) the rate of successful transfers increases, and (iii) the fraction of the population below threshold at steady state decreases. A time-varying control policy, which seeks to maximize

the rate of successful transfers is also identified and essentially performs by targeting the greatest densities of donors (not necessarily the richest); this leads to a uniform distribution of surpluses. Finally, natural wealth redistribution is modeled using diffusion, and we find that for this model the second and third moments eventually become linear with positive and negative slopes respectively; also, a large set of initial wealth distributions are attracted to some manifold on a fast timescale and then diffuse on a slow timescale in a non-symmetric way. These results contribute to the understanding of how NBTs impact the community wealth distribution.

Reciprocal gift giving or reciprocal altruism has frequently been studied using game theory, with particular attention being given to the repeated prisoner's dilemma framework. Recognizing that the motivational examples of the Maasai and vampire bats feature cooperation that more directly impacts survival than reproductive fitness, we define repeated survivor's dilemma games to determine when such a cooperative system as NBT is viable. We find that with 2 players, cooperation can be coordinated to and equivalently evolutionarily selected for when the benefit of cooperation is high enough and the cost of cooperation is low enough. Agents forgo the short-term benefit of cheating on a cooperative partner in order to preserve their partner's viability and sustain long-term benefits of cooperation; this is partnership. As a third player is introduced, we find that certain parameters lead to a Harmony game where mutual cooperation is the only Nash equilibrium.

In summary, we utilize agent-based simulation, kinetic theory, and game theory to analyze a cooperative economic system called NBT, which features binary wealth redistribution as a form of welfare. We identify socially optimal wealth redistribution policies and network topologies, characterize the evolution of the community wealth distribution, and determine what the relative cost and benefit of cooperation must be in order for such a reciprocal gift giving economic policy to be viable.

6.1 Future Work

6.1.1 *Agent-based Model Extensions*

Quasi-equilibrium NBT model. In Section 3.2, it is addressed that transient agent-based simulations are essentially extinction or wealth blow-up models because of the exponential growth/decay of wealths and a death process absent a birth process. Thus, a quasi-equilibrium model is developed that features logistic growth of wealth, e.g. cattle population limited by resources, and a splitting process where an account or herd may separate into two. This allows for an examination of a stable NBT economy.

While some intuitive observations about the quasi-equilibrium model are made and numerical results are discussed, tools and perspectives from stochastic processes [Ref: Lanchier (2017)] should be used to analyze the behavior of the model. With some simplifying assumptions about the model, we should be able to utilize the mathematics of birth and death processes to determine if the process is recurrent and if there exists a unique stationary distribution, and also calculate the probability of survival. Similarly, we should investigate time to extinction within the logistic growth aspect of the quasi-equilibrium model. This will help to understand how the basic model behaves as well as potentially allow for an analytical determination of how parameters and policies impact survival.

Connection of NBTs to field work and broader application. The Human Generosity Project currently has multiple field studies examining risk pooling implementations that fit the NBT framework. We think that our theoretical analysis could help guide these anthropological and sociological field studies. For instance, it might be investigated which transfer policies are actually used in osotua and perhaps there could be a connection of our theoretical work to data. Also, empirical study of the Maasai or another institution of NBTs

might confirm our optimal network topology findings and determine if the osotua networks naturally evolved to encourage low variance in their degrees.

It is certainly of great interest to discover where else the need-based transfer framework applies. Perhaps large businesses with many locations have the incentive to keep their stores stocked and may replenish low stock in a certain store by transferring it from another nearby store rather than ordering and shipping from some central location which may require significantly more time. As NBT risk pooling networks are logical in settings of no-fault disasters with high overhead costs, finding implementations beyond nomadic pastoralists or low-density populations in volatile ecologies should be possible and meaningful.

Modifications of transient NBT models. There are many more questions that may be investigated within the transient model setup. For example, Aktipis *et al.* (2016) find memoryless NBTs to be more socially optimal than record keeping, i.e. determining whether to give donations based on balancing an account results in lower survival rates than giving donations whenever one is able. Based on the results of Hao *et al.* (2015) where increasing the size and density of sharing networks improves survival, it makes sense that reducing the number of sharing relationships (whether because of unbalanced accounts or whatever reason) will reduce community survival. However, it should also be intuitively true that if there are different skill levels within a community that using record keeping in order to eliminate weaker partners should be more advantageous than continuing to ask the community to support accounts which continually lose community resources and put more skillful individuals at risk. We have developed a simple model that incorporates skill level into an account-keeping versus NBT simulation, which has shown that account keeping can improve survival when different skill levels are present, but this study should be conducted more thoroughly.

For the transient NBT models considered in this paper, we require that donations come from a single source, and we justify this by transfer cost considerations as well by the fear of partial gifts being wasted when a recipient still remains below threshold and potentially loses viability. This restriction of donations to being from only a single individual should be relaxed. Likely, allowing multiple donors to give partial gifts will lead to an even more regressive policy than considered in Section 3.1.2, but with similar results, e.g. greater short-term efficiency but a more unstable wealth distribution that leads to worse long-term survival. However, this should be carefully illustrated and explained to see if the single-donor restriction is unnecessary in revealing the general regressive-progressive short- and long-term observations that have been made in Chapter 3.

6.1.2 Kinetic NBTs

Control for Gini. In Section 4.3, we introduced a control policy that minimized the instantaneous rate of change of the fraction of individuals below threshold. Rather than maximizing the rate at which needs are met, other control goals have meaning in the context of NBTs. In particular, as we observed from the agent-based study that not only the efficiency of transfers but also the resulting fitness of the wealth distribution is important, another control goal to consider would be to prioritize transfers in order to achieve an optimal wealth distribution shape or Gini coefficient. In this dissertation, we use an analytically complicated definition for calculating the Gini index, which measures wealth inequality. However, in Eliazar and Sokolov (2010), we find a more analytically simple expression of the Gini index, which will help in phrasing the optimal control problem.

Attractor manifold. In Section 4.4, a conjecture was made regarding an attractor manifold when diffusion is included in the kinetic NBT model. By considering the system of moment evolution equations, we would like to show fast convergence in the variance per-

haps via a Gronwall inequality. Then, we would like to investigate how the rate of change of the third moment varies for distributions of different shape, e.g. fat tail to left versus right. This further analysis should be done to defend the conjecture related to the attractor manifold and explain what is observed numerically.

Compare central redistribution to NBT. In Section 4.5, we developed a central redistribution kinetic model to better describe conventional welfare. A thorough comparison of NBT versus central wealth redistribution should be conducted where the advantages and disadvantages of each are examined. For example, central redistribution enables meeting the needs of very poor individuals who, if only allowed to be brought above by a single donor, would remain below threshold. However, perhaps organizational costs should be included in the central redistribution model. Also, in many societies, when central or government help becomes inefficient enough, people resort to or rely on binary local sources of aid; a model that pairs central and NBT redistribution and illuminates how the two policies interact would be interesting.

6.1.3 *Survivor's Dilemma*

Evolutionary 3-player stability analysis. The 3-player survivor's dilemma of Section 5.2 reveals that when backwards induction is done and individual's are assumed to coordinate at the 2 player level, then a harmony parameter region appears. A perhaps more natural approach for the evolutionary perspective would be that individuals have a strategy for when there are three players and a strategy for when there are two players, e.g. all-D when 3 players and all-C when 2 players. This results in four different overall strategies whose density evolutions can be studied. A preliminary study of pure strategy local stability has been done, but it remains of interest to further investigate if there are interesting dynamics that occur.

Survivor's dilemma review. The repeated survivor's dilemma is not as popular as the repeated prisoner's dilemma, but there are a number of papers that consider this model. A nice review paper developing a general definition of the repeated survivor's dilemma as well as summarizing the findings of existing studies would be a useful contribution to the game theory literature.

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APPENDIX A

IMPACT OF NETWORK PARAMETERS ON TRANSFER POLICY EFFICIENCY

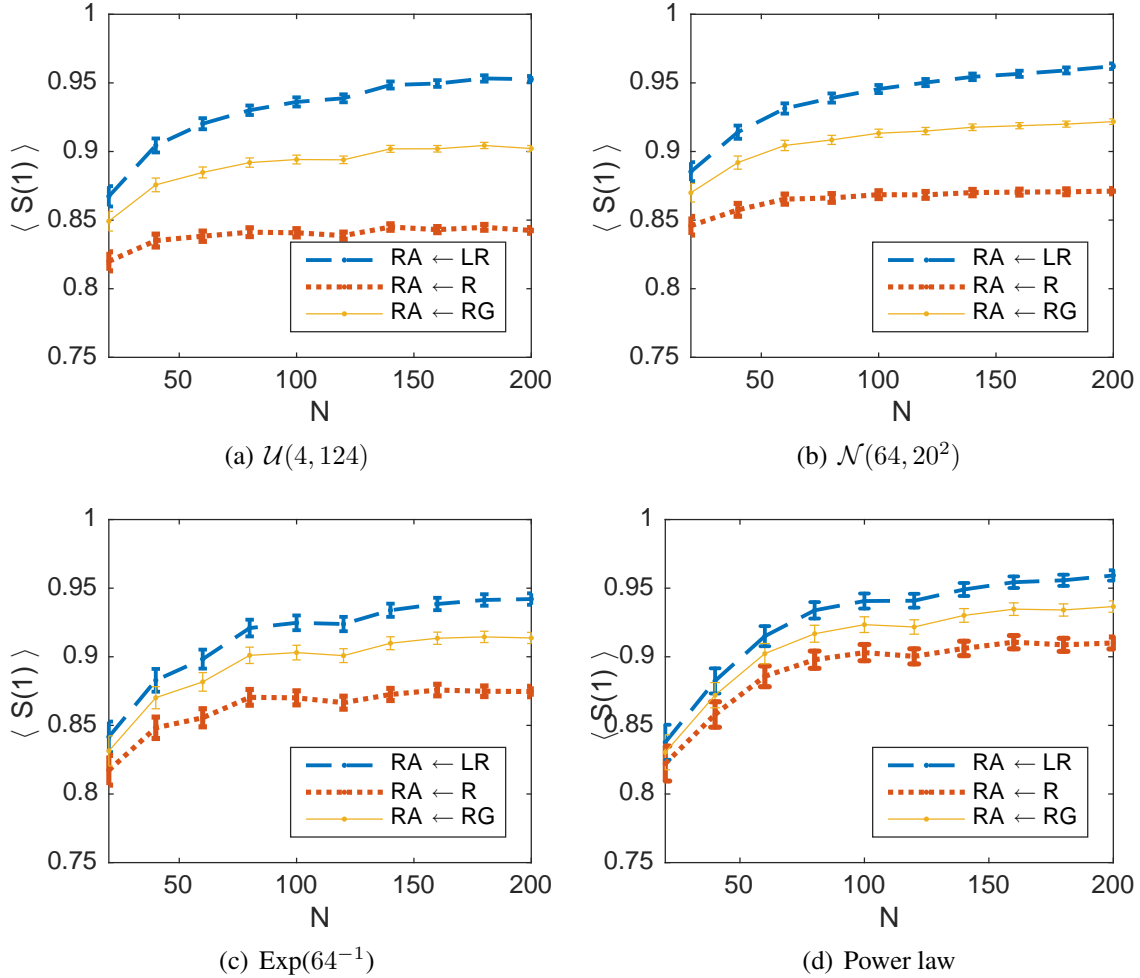


Figure A.1: Mean survival rates after one time step as a function of initial population for all-to-all connected networks. Results are shown for $RA \leftarrow$ transfer policies and initial wealths sampled from uniform, normal, exponential, and power law distributions. 95% confidence intervals are shown at data points.

It was mentioned in Section 3.1.2 that for complete graphs, the gap in transfer policy performance increases with the initial population size N , reflecting the fact that the wealth-based policies can behave more differently if individuals have more choices of wealths to seek exchanges from. Figure A.1 shows $\langle S(1) \rangle$ for 1000 simulations of $RA \leftarrow$ transfer policies and various initial wealth distributions $\vec{w}(0)$ as a function of N . If N is fixed but the initial mean degree K is allowed to vary, this also impacts the performance gap of transfer policies. Note that for the following results in Figures A.1 - A.2 that the probability density function for the power law is $f(x) = [x \ln(400)]^{-1}$, $1 < x < 400$ such that all distributions have a mean of approximately 64.

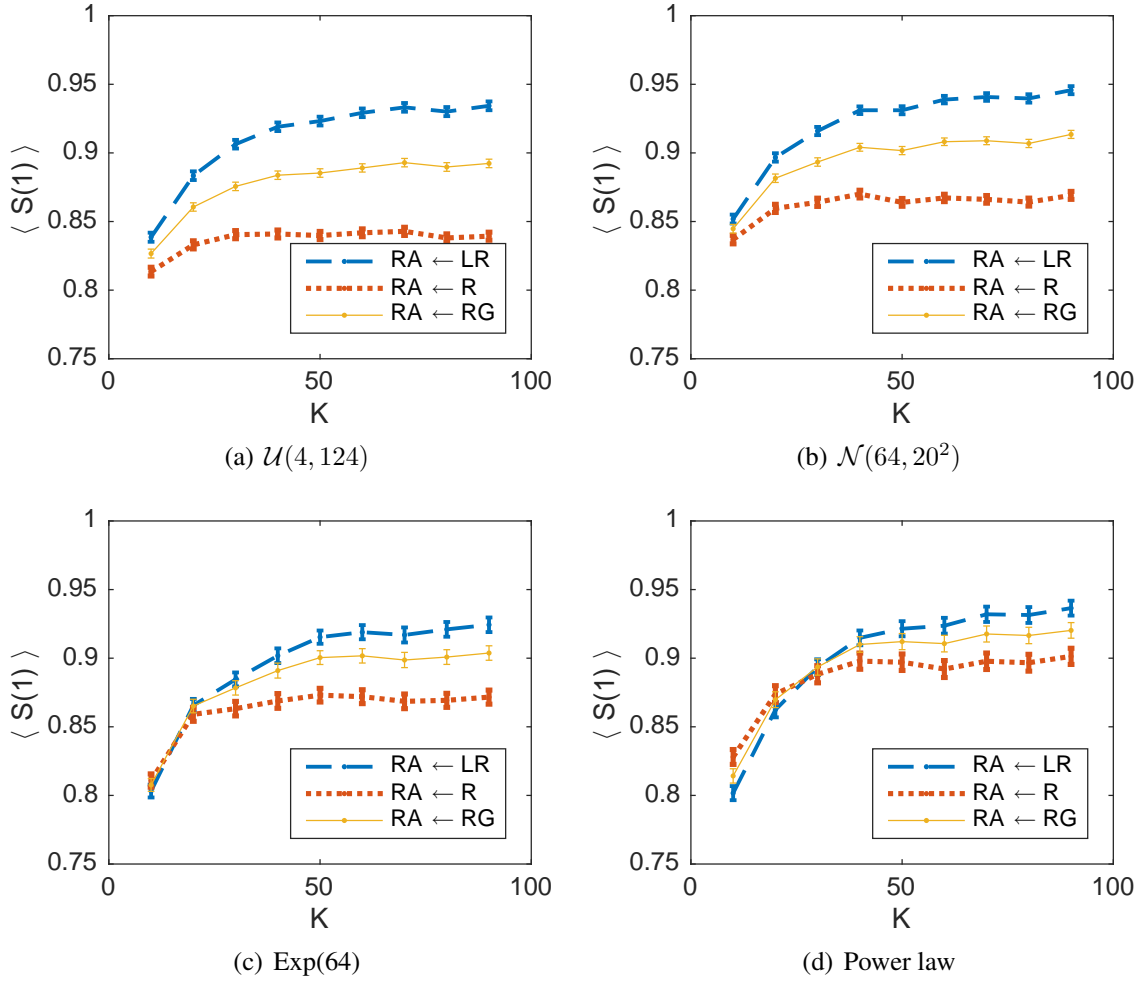


Figure A.2: Mean survival rates after one time step as a function of initial network mean degree for fixed initial population of $N = 100$. Results are shown for $RA \leftarrow$ transfer policies and initial wealths sampled from uniform, normal, exponential, and power law distributions. 95% confidence intervals are shown at data points.

APPENDIX B

IDEA OF PROOF FOR $S^{LR}(1) \geq S^R(1)$ FOR $N \leq 4$

Regardless of the initial wealth distribution, for a given asking order and $N \leq 4$, in every case either $S^{LR}(1) = S^R(1)$ or $S^{LR}(1) > S^R(1)$. Cases $N = 1, 2, 3$ are trivial and for $N = 4$, when either only one individual has surplus or only one individual has a deficit, it is easy to see that both policies result in equal survival rate. When two individuals have deficit and two have surplus, we consider without loss of generality the deficits to satisfy $0 < d_1 \leq d_2$ and the surpluses to satisfy $0 < s_1 \leq s_2$. For the equalities and when $s_2 \geq d_1 + d_2$ it is clear that the survival rates are equal for both policies. So then, we consider potential surpluses of interest as $s_1 = c_1 d_1 + c_2 d_2 + p_1$ and $s_2 = c_3 d_1 + c_4 d_2 + p_2$ where $p_1, p_2 < d_1$ and $\vec{c} = (c_1, c_2, c_3, c_4) \in \{0, 1\}^4$. That leaves 16 cases to consider, but the vast majority are easily eliminated and of the remaining few to consider it becomes apparent that only the case where $\vec{c} = (1, 0, 0, 1)$ results in a different survival rate with $S^{LR}(1) = 1 > 3/4 = S^R(1)$ when the individual with the smaller deficit is helped first.

When $N \geq 5$, it is no longer true that for a fixed asking order $S^{LR}(1) \geq S^R(1)$ regardless of the initial wealth distribution. Figure B.1 gives an example initial wealth distribution that results in $S^{LR}(1) < S^R(1)$ for $\theta = 64$ and a randomly generated asking order.

$RA \leftarrow LR$			$RA \leftarrow R$			
59	64	64	59	64	64	64
78	73	73	78	78	64	64
50	→	50	→	64	→	64
81	81	67	81	76	76	64
52	52	52	52	52	52	64

Figure B.1: $RA \leftarrow LR$ and $RA \leftarrow R$ transfers for $N = 5$ and randomly generated asking order (the asker is highlighted in red).