Smart Building with Predictive Air Conditioning Control: A Knapsack Approach

by

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ABSTRACT

This thesis proposes a policy to control the heating, ventilation and air conditioning (HVAC) systems in an industrial building. The policy designed in this thesis aims to minimize the electricity cost of a building while maintaining human comfort. Occupancy prediction and building thermal dynamics are utilized in the policy. Because every building has a power constraint, the policy balances different rooms' electricity needs and electricity price to allocate AC unit power for each room. In particular, energy costs are saved by reducing the system's power for times when the occupancy is low. Human comfort is preserved by restricting the temperature to a given range when the room occupancy is above a preset threshold. This thesis proposes a greedy policy, with provably good performance bound, to reduce costs for a building while maintaining overall comfort levels. The approximation ratio of the policy is developed and analyzed, demonstrating the effectiveness of this approach as compared to an ideal optimal policy.

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Chapter 1

INTRODUCTION

According to U.S. Energy Information Administration, the electricity consumption in the commercial and residential sectors will be increasing by 0.5% to 0.8% per year from 2013 through 2040 [9]. Additionally, energy prices have been increasing over the last 10 years. Hence, it is of interest for both individuals and industry to consume energy more cost-efficiently for both saving energy and reducing electricity cost. According to U.S. Energy Information Administration, Energy consumption of buildings corresponds to 41% (or 40 quadrillion btu) of the total US energy consumption in 2014 and HVAC systems caused 43% of the energy consumption for commercial and residential buildings [9]. Notably, Google Nest designed a thermostat to optimize residential energy utility. The Nest thermostat learns the temperature people like and the pattern of occupancy in the room to save the energy by turning off the AC unit when there is no occupancy. It can also turn on the AC unit to pre-cool or pre-heat the room when the energy price is low and there is no occupancy. An independent study show that it saved people an average of 10% to 12% on heating bills and 15% on cooling bills.

In this thesis, optimal control of an industrial building HVAC system is considered. The goal is to maintain human comfort and reduce the electricity cost as much as possible. There are four issues related to this optimization problem: 1). The prediction model of occupancy, weather and electricity price. 2). The model used to capture the room's physical thermal dynamics. 3). The model used to meter the human thermal discomfort level. 4). The balance between human discomfort and electricity cost.

Recent work [3] developed a model predictive control technique for HVAC systems to reduce energy consumption while maintaining occupant comfort. However, they didn't take into consideration the discomfort value (temperature deviation from the comfort temperature) in their optimization formulation. [7] presented a similar model predictive control algorithm, which adaptively balances energy consumption and human comfort. Additional constraints are added on the accumulated discomfort due to occupancy mis-predictions. [9] also aimed to reduce the energy consumption in a HVAC system while maintaining the human comfort, and proposed a technique to predict zone's temperature and occupancy based on a deterministic subspace identification method and mobility model, respectively.

A greedy control policy is introduced and its approximation ratio is quantified in this thesis. Both occupancy threshold and restricted temperature set are introduced in the optimization constraint in our control policy, which results in better performance than [3]. The new terms defined in this thesis help compensate for large differences between desired future temperature range and current temperature when the system's prediction model is constrained to few future time steps. This prevents a system with a slow response time from violating the desired temperature bounds, when occupancy abruptly changes from low to high and the temperature between current or desired room temperature is too different so that the AC unit does not have the ability to cool down the room fast. The condition of many rooms in a large building is considered by using discrete optimization (cf., [2], [9] and [7]).

The occupancy prediction model and human discomfort model used in this thesis follow [3]. A Markov state transition matrix is trained by using on-line Bayesian learning in the occupancy prediction model. The human discomfort model is captured by the (occupancy weighted) square of the temperature deviation from the set point. The thermal dynamic model in this thesis is based on [5] and [6]. The optimization formulation for balancing of human discomfort and electricity cost, is inspired by [3] and [9]. The cost function, aiming to minimize the sum of human discomfort and energy consumption, is used in both [3] and [9]. All the models and control policy are presented in details in the following chapters.

Chapter 2

AC UNITS CONTROL IN SMART BUILDING

2.1 Problem Description

Consider a discrete-time optimization problem, where the power modes for the central system of many AC units can be dynamically adapted at the beginning of each time slot of duration T_s. Also assume the electricity price is approximately unchanged within one time-slot. We assume that there are N rooms and there is one AC unit for each room.

The goal is to make AC mode decisions at each time-slot, in order to save the electricity cost while maintaining the human comfort. The objective of the optimization problem is to maximize the negative sum of the total electricity cost of AC units and the total discomfort value of people of all rooms in the building, which is given as:

$$V_i(k) = -(V_{i\ \mathrm{dis}}(k) + V_{i\ \mathrm{ec}}(k)) \tag{2.1}$$

where i denotes the index of room/AC unit; k denotes the index of time slots; $V_i(k)$ denotes the value obtained by room i at time slot k; $V_{i\ \mathrm{dis}}(k)$ denotes the human discomfort in room i at time slot k; $V_{i\ \mathrm{ec}}(k)$ denotes the electricity cost of room i at time slot k. The human discomfort in a given room during time-slot k can be captured by the following:

$$V_{i\ \mathrm{dis}}(k) = f(T_i(k), T_{\mathrm{sp}}, p(k), U_i(k), I(o_i(k) = 1)) \tag{2.2}$$

where $f(.)$ is the discomfort function that quantizes the human thermal perception discomfort in room i during time slot k; $T_i(k)$ is the temperature of room i during time-slot k; $U_i(k)$ is the power load of the AC unit in room i during time-slot k; $p(k)$ is the electricity price during time-slot k and it is assumed as unchanged within any time slot; T_{sp} is the comfort temperature set point, $I(.)$ is an indicator function; $o_i(k) = 1$ indicates that room i is occupied within time-slot k and $o_i(k) = 0$ otherwise. It is assumed that the temperature of zone i is deterministic and unchanged during the time-slot, and satisfies the equation:

$$T_i(k) = g(T_i(k-1), T_{\text{w}}(k), U_i(k)) \tag{2.3}$$

where $g(.)$ is the thermal-dynamic function that captures the temperature of each room at a given time slot; $T_{\text{w}}(k-1)$ is the outdoor temperature during time-slot $k-1$.

In the above model, all rooms are outfitted with the same AC unit. Each AC unit has different power mode choices, which are $0, \Delta_U, 2\Delta_U, ..., U_{\max}$. At the beginning of each time slot, a control policy is applied and the discomfort/cost value of each room is calculated.

2.2 Control Policy

It is clear that, the control decision will not only affect the current time slot, but also future time slots. At each slot, the decision is made by aiming to strike a good balance between human comfort and electricity cost. Next, we define two policies as follows:

Definition 1 *(Finite-horizon policy) A Finite horizon policy is a policy that the power mode at each time slot is determined by optimizing the value over finite time-slots.*

Definition 2 *(Greedy policy) A greedy policy is a policy that the power mode at each time slot is determined by optimizing the value over one time-slot.*

The following optimization is inspired by the study in [3].

$$\max_{U(k_0),U(k_0+1),....,U(T)} \quad -\mathbb{E}\left[\sum_{k=k_0}^{T}\sum_{i=1}^{N}(V_{i\ \mathrm{dis}}(k)+V_{i\ \mathrm{cs}}(k))\right] \tag{2.4a}$$

$$\text{subject to} \quad V_{i\ \mathrm{dis}}(k) = f(T_i(k), T_{\mathrm{sp}}, I(o_i(k)=1)) \tag{2.4b}$$

$$V_{i\ \mathrm{cs}}(k) = p(k)U_i(k) \tag{2.4c}$$

$$T_i(k) = g(T_i(k-1), T_{\mathrm{w}}(k), U_i(k)) \tag{2.4d}$$

$$p(k) = h_{\mathrm{p}}(p(1), ..., p(k-1)) \tag{2.4e}$$

$$I(o_i(k)=1) = h_{\mathrm{o}}(o_1(1), ..., o_i(k-1)) \tag{2.4f}$$

$$U_i(k) \in \{0, \Delta_U, 2\Delta_U, ..., U_{\max}\} \tag{2.4g}$$

$$\sum_{i=1}^{N} U_i(k) \le c, \quad \forall \ k \in \{k_0, ..., T\} \tag{2.4h}$$

where $U(k)$ is a vector which captures power load decision at k for all rooms; c is the power constraint of the building; $h_{\mathrm{q}}(.)$ and $h_{\mathrm{o}}(.)$ are the price and occupancy estimation function, respectively. We assume that the weather can be perfectly forecast in this thesis. At each time slot, the future electricity price and occupancy are predicted. The prediction depth is dictated by the policy. The forecast is based on historical data. $\Gamma_i(k)$ is denoted as the fractional occupancy term to capture the true time that people stay in room i within time slot k. Because the goal is to balance human discomfort and electricity cost, a preset constraint is made to guarantee the humans' thermal perception requirement. Here Γ_h is introduced as follows:

Condition 1: *If* $\Gamma_i(k) > \Gamma_h$, *then* $T_i(k) \in [\tau - \delta, \tau + \delta]$.

In the above, τ is T_{sp} and δ is a temperature restriction. The definition of Γ_h can be interpreted as follows. When $\Gamma_i(k) \le \Gamma_h$, there is no restriction of the balance of human discomfort and electricity cost. When $\Gamma_i(k) > \Gamma_h$, the balance should satisfy a temperature constraint to ensure the human thermal satisfaction. Consider the finite horizon policy aiming to solve the following optimization at each time slot:

$$\max_{U(k_0), U(k_0+1), ...U(T)} \quad -\mathbb{E}\left[\sum_{k=k_0}^{T}\sum_{i=1}^{N}(V_{i\ \text{dis}}(k) + V_{i\ \text{cs}}(k))\right] \tag{2.5a}$$

$$\text{subject to} \quad V_{i\ \text{dis}}(k) = f(T_i(k), T_{\text{sp}}, \Gamma_i(k)) \tag{2.5b}$$

$$V_{i\ \text{cs}}(k) = p(k)U_i(k) \tag{2.5c}$$

$$T_i(k) = g(T_i(k-1), T_w(k), U_i(k)) \tag{2.5d}$$

$$U_i(k) \in \{0, \Delta_U, 2\Delta_U, ..., U_{\max}\} \tag{2.5e}$$

$$\sum_{i=1}^{N} U_i(k) \leq c \tag{2.5f}$$

$$T_i(k)\mathbf{1}_{(\Gamma_i(t)>\Gamma_h)} \in [\tau-\delta, \tau+\delta] \tag{2.5g}$$

$$T_i(k) \in T^{(T-k_0+1)}, \quad \forall \ k \in \{k_0, ..., T\} \tag{2.5h}$$

The constraint (2.5g) is introduced to satisfy condition 1. Further, $T^{(n)}$ in constraint (2.5h) is defined by:

***Condition 2**: For any $T_w(k), T_w(k+1), ..., T_w(m)$, if $T_i(k-1) \in T^{(m-k+1)}$, there is $\{U_i(k), U_i(k+1), ..., U_i(m)\}$ such that $T_i(m) \in T^{(0)}$ where $T^{(0)} = [\tau-\delta, \tau+\delta]$.*

When $T - k_0$ is big enough, the constraint (2.5h) can be discarded. The system model is shown in Fig 2.1. In this thesis, the performances of both greedy policy and finite-horizon policy will be analyzed.

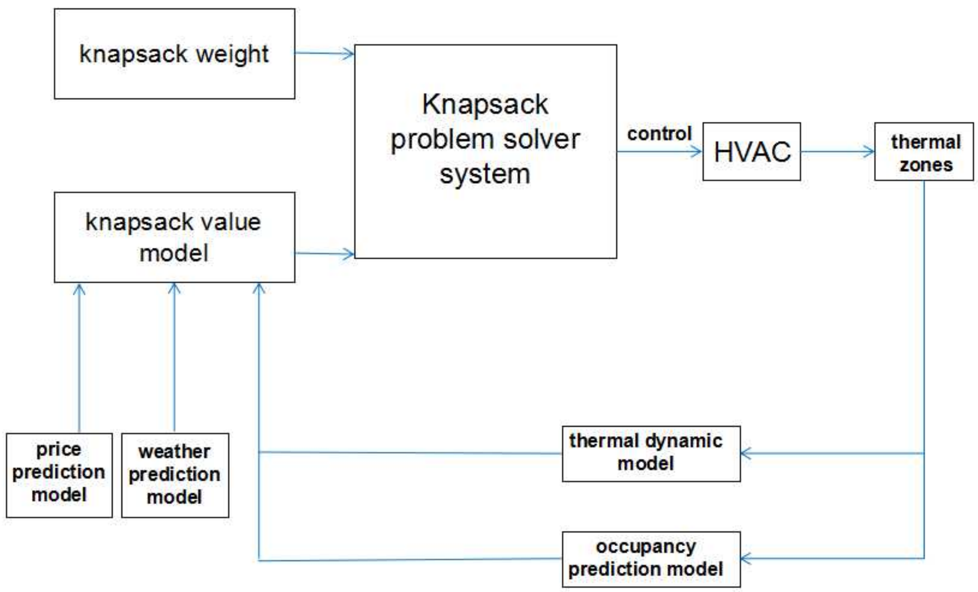

Figure 2.1: System Model

Chapter 3

PREDICTION MODEL FOR OCCUPANCY

One of the key challenges in the optimal control for the HVAC system is occupancy prediction. We report to the stochastic occupancy model in [2], which is a prediction model based on a two-state Markov chain.

3.1 Occupancy Transition Probability

In the model, there are two possibilities, namely the room is either occupied or unoccupied at a given time. In this chapter, $\gamma(t)$ denotes the occupancy statement at time t. $\gamma(t) = 1$ means the room is occupied and $\gamma(t) = 0$ otherwise. Because only the occupancy state within a time-slot matters in our model, we use Γ_k to capture the occupancy state within a time-slot:

$$\Gamma(k) = \frac{1}{T_s} \int_{kT_s}^{(k+1)T_s} \gamma(t)dt \tag{3.1}$$

where $\Gamma(k)$ represents the fractional occupancy and $\Gamma(k) \in [0, 1]$. $\Gamma(k) = 1$ means the room is occupied for the whole time-slot k and $\Gamma(k) = 0$ means the room is unoccupied for the time-slot k. The following equation holds by the law of large numbers (LLN):

$$\Gamma(k) = \mathbb{E}[o(k)] = \mathbf{P}\{o(k) = 1\} \tag{3.2}$$

In what follows, we use $o(k) = 1$ (occupied) and $o(k) = 0$ (unoccupied) for two states in the occupancy Markov chain model. At each time slot, we determine the

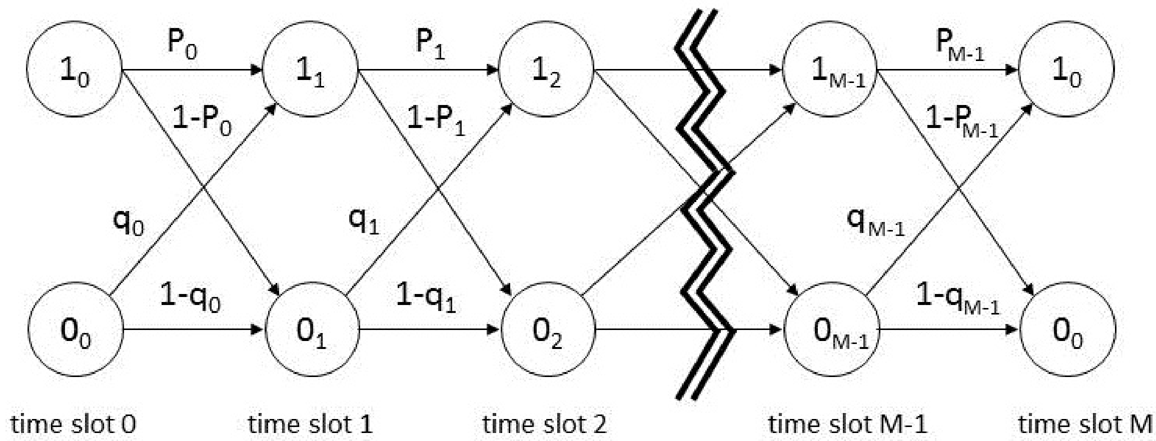

Figure 3.1: Time-varying Periodic Markov Chain Structure

probability of future occupancy. We want to predict the probabilities:

$$p_k = \mathbf{P}\{o(k+1) = 1 | o(k) = 1\} \tag{3.3}$$

$$q_k = \mathbf{P}\{o(k+1) = 1 | o(k) = 0\} \tag{3.4}$$

where p_k is the transition probability from an occupied state within time-slot k to an occupied state within time-slot $k+1$; q_k is the transition probability from an unoccupied state within time-slot k to an occupied stage within time-slot $k+1$.

The transition probabilities of this time-varying Markov chain are periodic; with the period being 24 hours. To visualize the periodicity, we unroll the Markov chain into $2M$ states, where $M = \frac{24 \text{ hours}}{T_s}$ (Fig.3.1). If the current time-slot is the last one for a day, namely $k = M$, the next time-slot returns to the first time-slot, which means $p_M = \mathbf{P}\{o(1) = 1 | o(M) = 1\}$ and $q_M = \mathbf{P}\{o(1) = 1 | o(M) = 0\}$.

At any given time slot, there are two possible states, so we need to maintain two distributions per time slot:

$$\bar{p}_k = \mathbf{P}\{o(k+1) = 1 | o(k) = 1\} = \mathbb{E}[f_k(p_k)] \tag{3.5}$$

$$\bar{q}_k = \mathbf{P}\{o(k+1) = 1 | o(k) = 0\} = \mathbb{E}[g_k(q_k)] \tag{3.6}$$

we denote $f_k(p_k)$ and $g_k(q_k)$ as the density functions, respectively. $\bar{p}_k$ and $\bar{q}_k$ are the expectations, which is the prediction of the occupancy of the next time slot.

3.2 Bayesian Learning of Occupancy Transition Probability

A online Bayesian learning algorithm is used in training of the occupancy transition probability. To get the occupancy transition probabilities, we need to train p_k and q_k respectively for all $k \in \{1, ..., M\}$. Because of the definition of the transition probability, only one probability density function, either $f_k(p_k)$ or $g_k(q_k)$, will be updated using both adjacent time-slots training data. Additionally, $f_k(p_k)$ will be updated if we observe $o(k) = 1$ and $g_k(q_k)$ will be updated if we observe $o(k) = 0$. Observe when data d has an occupied state within time-slot k, the new updated $f_{k,d}(p_k)$ and $g_{k,d}(q_k)$ using training data d are:

$$f_{k,d}(p_k|o_d(k+1)\ , o_d(k) = 1) = \frac{f_{k,d-1}(p_k)\Phi[o_d(k+1)\ , p_k]}{\int_0^1\ f_{k,d-1}(p_k)\Phi[o_d(k+1)\ , p_k]\ dp_k} \tag{3.7}$$

$$g_{k,d}(q_k|o_d(k+1)\ , o_d(k) = 1) = g_{k,d-1}(q_k) \tag{3.8}$$

and if we observe data d has an unoccupied state within time-slot k, we will get:

$$g_{k,d}(q_k|o_d(k+1)\ , o_d(k) = 0) = \frac{g_{k,d-1}(q_k)\Phi[o_d(k+1)\ , q_k]}{\int_0^1\ g_{k,d-1}(q_k)\Phi[o_d(k+1)\ , q_k]\ dq_k} \tag{3.9}$$

$$f_{k,d}(p_k|o_d(k+1)\ , o_d(k) = 0) = f_{k,d-1}(p_k) \tag{3.10}$$

where $f_{k,d-1}(p_k)$ and $g_{k,d-1}(q_k)$ are the probability density functions trained by data 0,1,...,d-1. $\Phi[.]$ is a likelihood function, which satisfies:

$$\Phi[o_d(k+1)\ , p_k] = p_k^{I(o_d(k+1)=1)}(1-p_k)^{I(o_d(k+1)=0)} \tag{3.11}$$

$$\Phi[o_d(k+1)\ , q_k] = q_k^{I(o_d(k+1)=1)}(1-q_k)^{I(o_d(k+1)=0)} \tag{3.12}$$

As we know from above, either $f_k(p_k)$ or $g_k(q_k)$ will be updated if the room is occupied or unoccupied within the time-slot k. Note that:

$$\begin{aligned}
f_{k,d}(p_k) =& f_{k,d}(p_k|o_d(k)=1)\mathbf{P}\{o_d(k)=1\} \\
&+ f_{k,d}(p_k|o_d(k)=0)\mathbf{P}\{o_d(k)=0\} && (3.13a) \\
=& [f_{k,d}(p_k|o_d(k+1)=1, o_d(k)=1)\mathbf{P}\{o_d(k+1)=1\} \\
&+ f_{k,d}(p_k|o_d(k+1)=0, o_d(k))\mathbf{P}\{o_d(k+1)=0\}]\mathbf{P}\{o_d(k)=1\} \\
&+ f_{k,d}(p_k|o_d(k)=0)\mathbf{P}\{o_d(k)=0\} && (3.13b)
\end{aligned}$$

$$\begin{aligned}
g_{k,d}(q_k) =& g_{k,d}(q_k|o_d(k)=1)\mathbf{P}\{o_d(k)=1\} \\
&+ g_{k,d}(q_k|o_d(k)=0)\mathbf{P}\{o_d(k)=0\} && (3.14a) \\
=& [g_{k,d}(q_k|o_d(k+1)=1, o_d(k)=0)\mathbf{P}\{o_d(k+1)=1\} \\
&+ g_{k,d}(q_k|o_d(k+1)=0, o_d(k)=0)\mathbf{P}\{o_d(k+1)=0\}]\mathbb{P}\{o_d(k)=0\} \\
&+ g_{k,d}(q_k|o_d(k)=1)\mathbf{P}\{o_d(k)=1\} && (3.14b)
\end{aligned}$$

it follows that $f_{k,d}(p_k)$ and $g_{k,d}(q_k)$ can be updated for each observation using equations (3.1)-(3.2) and (3.5)-(3.12).

3.3 Occupancy Prediction

Note that the Markov chain in our model has $2M$ states totally for each AC unit i, which are $o_i(k) = 1$ and $o_i(k) = 0$ for all $k \in \{1, ..., M\}$. We denote $\pi_i(k) \in \mathbb{R}^{1\times 2M}$ as the occupancy probability of zone i within time-slot k. The first M elements in $\pi_i(k)$ represents for $\mathbf{P}\{o_i(k) = 1\}$ in M time slots and the last M elements in $\pi_i(k)$ represents for $\mathbf{P}\{o_i(k) = 0\}$ in M time slots. It is obvious that only two elements, the k^{th} and $(M + k)^{\text{th}}$ elements, can be non-zeros in vector $\pi_{i,k}$, because $\pi_i(k)$ can only capture the k^{th} time slot's occupancy, where $\pi_i(k)$ is represented as:

$$\pi_i(k) = [\Gamma_i(k)\mathbf{1}^k_{1\times M} \quad (1-\Gamma_i(k))\mathbf{1}^k_{1\times M}] \tag{3.15}$$

where $\mathbf{1}^k_{1\times N}$ is a M elements row vector with all zeros except the k^{th} element is 1. If $\pi_{i,k}$ is available, $\Gamma_i(k)$ can be obtained by:

$$\Gamma_i(k) = \pi_i(k)[\mathbf{1}_{1\times M} \quad \mathbf{0}_{1\times M}]^T \tag{3.16}$$

where $\mathbf{1}_{1\times M}$ is a M elements row vector with all 1s and $\mathbf{0}_{1\times M}$ is a M elements row vector with all 0s.

At time slot $k+1$, the observation of time-slot k and $f_{k,d}(p_k)$ and $g_{k,d}(q_k)$ are available, and we can use the following equation to compute the occupancy state prediction:

$$\begin{aligned}\mathbf{p}[o(k+1)=1] =& \mathbf{P}\{o(k)=0\}\mathbf{P}[o(k+1)|o(k)=0] \\ &+ \mathbf{P}\{o(k)=1\}\mathbf{P}[o(k+1)=1|o(k)=1] \quad (3.17a)\\ =&[1-\Gamma(k)]\bar{q}_k + \Gamma(k)\bar{p}_k \quad (3.17b)\end{aligned}$$

The transition matrix is defined by $P_i \in \mathbb{R}^{2M\times 2M}$. The k^{th} row of P_i means what the occupancy probability will be within time-slot $k+1$ of zone i if the zone is occupied within time-slot k. We can generate P_i by the following steps:

$$P^{(1,1)}_{i,st} = \begin{cases} \bar{p}_s & \textit{if } t = s+1 \textit{ mod } M \\ 0 & \textit{otherwise} \end{cases} \tag{3.18}$$

$$P^{(1,2)}_{i,st} = \begin{cases} 1-\bar{p}_s & \textit{if } t = s+1 \textit{ mod } M \\ 0 & \textit{otherwise} \end{cases} \tag{3.19}$$

$$P^{(2,1)}_{i,st} = \begin{cases} \bar{q}_s & \textit{if } t = s+1 \textit{ mod } M \\ 0 & \textit{otherwise} \end{cases} \tag{3.20}$$

$$P^{(2,2)}_{i,st} = \begin{cases} 1-\bar{q}_s & \textit{if } t = s+1 \textit{ mod } M \\ 0 & \textit{otherwise} \end{cases} \tag{3.21}$$

$$P_i = \begin{bmatrix} P_i^{(1,1)} & P_i^{(1,2)} \\ P_i^{(2,1)} & P_i^{(2,2)} \end{bmatrix} \tag{3.22}$$

where s and t are the row and column indices of P_i respectively.

To get the occupancy probability prediction of time-slot $k+j$ based on the zone i's occupancy probability state $\pi_{i,k}$, we can use the following equations:

$$\mathbb{E}[o_i(k+j)] = \mathbf{P}[o_i(k+j) = 1] \tag{3.23a}$$

$$= \Gamma_i(k+j) \tag{3.23b}$$

$$= \pi_i(k+j)[\mathbf{1}_{1\times M} \quad \mathbf{0}_{1\times M}]^T \tag{3.23c}$$

$$= \pi_i(k)P_i^j[\mathbf{1}_{1\times M} \quad \mathbf{0}_{1\times M}]^T \tag{3.23d}$$

Chapter 4

SMART BUILDING AC UNITS CONTROL:A KNAPSACK APPROACH

4.1 Thermal Dynamics Model

Consider the air conditioner thermal dynamic model captured by the following equation, which has been used in [1], [5] and [6].

$$T_i(k+1) = T_i(k) + (1-\alpha)[T_{\mathrm{w}}(k+1) - \beta U_i(k) - T_i(k)] \tag{4.1}$$

where $T_i(k)$ is the indoor temperature of room i at time slot k. All the other rooms' temperature effect on room i is ignored in the above equation. α is dictated by the time slot duration T_s and satisfies the equation $\alpha = e^{-T_s/T_c}$, where T_c is a time constant. β is determined by the performance of heat exchange of the building material and air conditioner.

Based on thermal dynamics function and Condition 1, the following lemma is obtained:

Lemma 1: *if* $T_i(k+1) = T_i(k) + (1-\alpha)[T_w(k+1) - \beta U_i(k) - T_i(k)]$, *then* $\delta \geq \frac{1}{2}\beta(1-\alpha)\Delta_U$.

Proof: $\delta = \frac{1}{2}\beta(1-\alpha)\Delta_U - \epsilon$ $(0 \leq \epsilon < \frac{1}{2}\beta(1-\alpha)\Delta_U)$ is invalid when $T_i(t-1) = \tau + \frac{1}{2}\beta(1-\alpha)\Delta_U$ and $T_w(t) = \tau + \frac{1}{2}\beta(1-\alpha)\Delta_U$, because $T_i(t) = \tau + \frac{1}{2}\beta(1-\alpha)\Delta_U - \beta(1-\alpha)U_i(t) \notin [\tau-\delta, \tau+\delta]$. If $\delta \geq \frac{1}{2}\beta(1-\alpha)\Delta_U$, assume $\alpha T_i(k) + (1-\alpha)T_{\mathrm{w}}(k+1) \in [\tau + \frac{1}{2}n\beta(1-\alpha)\Delta_U, \tau + \frac{1}{2}(n+1)\beta(1-\alpha)\Delta_U]$, then we can always choose $U_i(k) = (n+1)\Delta_U$. The result next time slot temperature is $T_i(k) \in [\tau - \frac{1}{2}\beta(1-\alpha)\Delta_U, \tau + \frac{1}{2}\beta(1-\alpha)\Delta_U]$, which means $T_i(t) \in [\tau-\delta, \tau+\delta]$.

According to equation (4.1), it can be observed that the temperature deviation between two time slots is determined by $T_w(k+1) - \beta U_i(k) - T_i(k)$. It is difficult to cool the room, when the room's temperature is low, meanwhile, the weather temperature is high. We define a parameter Δ_τ to capture the air conditioner's cooling ability.

Condition 3: $\tau - \Delta_\tau = T_w^{max} - \beta U^{max}$.

where Δ_τ is a parameter that measures the air conditioner cooling ability during the hottest weather and $\Delta_\tau \geq 0$ is assumed to hold, which means the AC unit can at least keep the comfort temperature during the hottest weather. According to Condition 2, the following lemma can be developed:

Lemma 2: $T^{(1)} = \{T | \tau - \frac{\delta}{\alpha} \leq T \leq \tau + \frac{\delta}{\alpha} + (\frac{1}{\alpha} - 1)\Delta_\tau\}$

Proof: $T^{(1)}_{min}$ and $T^{(1)}_{max}$ are denoted as the min and max value in set $T^{(1)}$. $T^{(1)}_{min}$ occurs when $T(k) = \tau - \delta$, $T_w(k) = \tau$ and $U(k) = 0$, namely, $T(k) = f(T^{(1)}_{min}, \tau, 0) = \tau - \delta$, which leads to $T^{(1)}_{min} = \tau - \frac{\delta}{\alpha}$. $T^{(1)}_{max}$ occurs when $T(k) = \tau + \delta$, $T_w(k) = T_w^{max}$ and $U(k) = U_{max}$. Hence, $T(k) = f(T^{(1)}_{max}, T_w^{max}, U_{max}) = \tau + \delta$, which leads to $T^{(1)}_{max} = \tau + \frac{\delta}{\alpha} + (\frac{1}{\alpha} - 1)\Delta_\tau$ by Condition 3.

4.2 Knapsack Value Function

The value of each AC unit's power mode consists of two important parts, which are human discomfort and electricity cost of the AC unit. The value of the AC unit in room i within time-slot k is captured by equation (2.1). Electricity cost of ac unit i within time-slot k can be calculated by equation 2.4(c). We have defined human discomfort function $f(.)$ as the penalty function of deviation from the set point. To capture $f(.)$, we use the cost function developed by [3]:

$$f(T_i(k), T_{\text{sp}}, \Gamma_i(k))) = \mu \Gamma_i(k)[T_i(k) - T_{\text{sp}}]^2 \tag{4.2}$$

where μ is the penalty on discomfort during occupancy and may depend on the number of the occupants in the room by [9]. We assume μ is a constant and it can be eliminated by shrinking $p(k)$ in the optimization. Above all, the value of each item i

within time-slot k is:

$$V_i(k) = \Gamma_i(k)[T_i(k) - T_{\text{sp}}]^2 + p(k)U_i(k) \tag{4.3}$$

4.3 Multiple Choice Knapsack Problem Formulation

The building's AC units control is modeled as a knapsack problem within each time-slot. In this knapsack problem, the knapsack capacity is the building's power load capacity. For each room, one of AC unit's power mode will be selected to run within a given time-slot, which is modeled by choosing a weight for each item to insert into the knapsack. We denote $U_i(k) \in \{0, \Delta_U, 2\Delta_U, ..., U_{max}\}$ as the weight set of item i. The negative sum of the human discomfort value for room i and electricity cost of AC unit i, corresponding to the choice of power mode of AC unit i, is modeled as the value for the selected item's weight. According to equations (4.1) and (4.3), the corresponding value to the selected item weight $U_i(k)$ is:

$$\begin{aligned} V_i(k) =& \Gamma_i(k)[\alpha T_i(k-1) + (1-\alpha)T_{\text{w}}(k) - \beta(1-\alpha)U_i(k) - T_{\text{sp}}]^2 \\ &+ p(k)U_i(k) \end{aligned} \tag{4.4a}$$

Based on the above, this is a stochastic multiple choice knapsack problem with deterministic weight and stochastic value. Different from a deterministic knapsack problem, the stochastic knapsack problem aims to find a preference distribution of the knapsack value (see, e.g., [8]). [4] designed a dynamic programming algorithm to select items sequentially depending on the realizations of their stochastic values. Unfortunately, the true value of each selected item cannot be observed immediately after its selection, because the random variables within the time slot may change during the time slot. In this thesis, we optimize the expectation of the total value in a knapsack. The greedy policy for the underlying optimization formulation is the solution to the following problem:

$$\max_{U_i(k)} \quad -\mathbb{E}[\sum_{i=1}^{N} V_i(k)] \tag{4.5a}$$

$$\text{subject to} \quad T_i(k)\mathbf{1}_{(\Gamma_i(k)>\Gamma_h)} \in T^{(0)} \tag{4.5b}$$

$$T_i(k) \in T^{(1)} \tag{4.5c}$$

$$U_i(k) \in \{0, \Delta_U, 2\Delta_U, ..., U_{\max}\} \tag{4.5d}$$

$$\sum_{i=1}^{N} U_i(k) \leq c \tag{4.5e}$$

It can be shown that:

$$\begin{aligned}\mathbb{E}[V_i(k)] =& \mathbb{E}[\Gamma_i(k)][\alpha T_i(k-1) + (1-\alpha)T_w(k) \\ &- \beta(1-\alpha)U_i(k) - T_{set\ point}]^2 + T_s\mathbb{E}[p(k)]U_i(k)\end{aligned} \tag{4.6a}$$

where

$$\mathbb{E}[\Gamma_i(k)] = \pi_{i,k-1}P[\mathbf{1}_{1\times N} \quad \mathbf{0}_{1\times N}]^T \tag{4.7}$$

$$\mathbb{E}[p(k)] = p(k-1) \tag{4.8}$$

4.4 A Solution Based on Recursion

The greedy policy can be obtained based on the following two steps:

- Step 1: Use recursion to solve the stochastic multiple choice knapsack problem at each given time-slot, based on the input of the control system, the observations of the last time-slot.

- Step 2: Adapt the power mode for AC units based on the solution in step 1 for the next time-slot, and use the occupancy and temperature state as the input to the control problem.

We denote $\rho^{max}[i,c]$ as the optimal knapsack value; $V[U_i(k)]$ is the value of the objective function accordingly. We have the following recursion rule:

$$\rho^{max}[i,c] = \max_{U_i(k)} \quad \rho^{max}[i-1, c-U_i(k)] + V[U_i(k)] \tag{4.9a}$$

$$\text{subject to} \quad \rho^{max}[0,c] = 0 \quad \text{for} \;\; c \geq 0 \tag{4.9b}$$

$$\rho^{max}[i,c] = -\infty \;\; \text{for} \;\; c < 0$$

$$\text{or} \;\; T_i(k)\mathbf{1}_{(\Gamma_i(k)\geq\Gamma_h)} \notin T^{(0)} \;\; \text{or} \;\; T_i(k) \notin T^{(1)} \tag{4.9c}$$

$$U_i(k) \in \{0, \Delta_U, 2\Delta_U, ..., U_{\max}\} \tag{4.9d}$$

Based on the above optimization, the solution of $\rho^{max}[N,c]$ is obtained at each time slot.

Chapter 5

PERFORMANCE ANALYSIS

In this chapter, the performance of the greedy policy is analyzed by computing its approximation ratio. The approximation ratio is upper bounded by calculating the ratio of the lower bound value of greedy policy and the upper bound value of finite-horizon policy.

5.1 Greedy Policy

Consider the following problem:

$$\underset{U_i(k)\ \forall i}{\text{maximize}} \quad -\sum_{i=1}^{N}[U_i(k)p(k)+\Gamma_i(k)(T_i(k)-\tau)^2] \tag{5.1a}$$

$$\text{subject to} \quad T_i(k)=\alpha T_i(k-1)-\beta(1-\alpha)U_i(k)+(1-\alpha)T_{\mathrm{w}}(k) \tag{5.1b}$$

$$\mathbf{1}_{(\Gamma_i(k)>\Gamma_h)}(T_i(k)-\tau)\in[-\delta,\delta] \tag{5.1c}$$

$$T_i(k)\in T^{(1)} \tag{5.1d}$$

$$U_i(k)\in\{0,\Delta_U,2\Delta_U,...,U_{\max}\} \tag{5.1e}$$

$$\sum_{i=1}^{N}U_i(k)\le c \tag{5.1f}$$

The constraint (5.1d) is necessary, because $T_{\mathrm{w}}(k+1)$ and $\Gamma_i(k+1)$ are independent with the determination of $U_i(k)$ at time slot k. If (5.1d) doesn't exist, the temperature of a lower occupancy room can go unbounded.

When the constraint (5.1b-f) cannot be satisfied together, we let the objective value be negative infinity for that case. That's mainly due to the minimum of the total power satisfying (5.1b-e), has exceeded the building capacity. We assume that the capacity meets the requirement, namely the optimization objective function value always be finite.

Lemma 3: *If* $c \geq max_k\{\sum_{j=1}^{4} N_j(k)c_j(T_w(k))\}$ *holds, then the optimal value of objective function at any time slot is finite, when greedy policy is applied.*

Proof: When the greedy policy is applied, the temperature of a given room must either be in $T^{(0)}$ or $T^{(1)}$ during a given time slot, where $T^{(0)} = \{T | \tau - \delta \leq T \leq \tau + \delta\}$. Consider room i at time slot k, there are four possibilities, which are determined by $\Gamma_i(k-1)$ and $\Gamma_i(k)$:

1). $T_i(k-1) \in T^{(1)}$ and $T_i(k) \in T^{(1)}$.

2). $T_i(k-1) \in T^{(1)}$ and $T_i(k) \in T^{(0)}$.

3). $T_i(k-1) \in T^{(0)}$ and $T_i(k) \in T^{(0)}$.

4). $T_i(k-1) \in T^{(0)}$ and $T_i(k) \in T^{(1)}$.

We denote $N_1(k)$, $N_2(k)$, $N_3(k)$ and $N_4(k)$ as the number of rooms in the above four situations at time slot k, respectively. Let $U_i^\star(k)$ be the minimum power for AC unit i at time slot k, satisfying the temperature constraints (5.1b-e). The upper bound of $U_i^\star(k)$ is analyzed as follows:

In situation 1, the possible largest $U_i(k)$ occurs when $T_i(k-1) = T_{\max}^{1}$. It is clear that $U_i^\star(k) = \min\{U_i(k) | T_{\min}^{(1)} \leq \alpha T_{\max}^{(1)} - \beta(1-\alpha)U_i(k) + T_w(k) \leq T_{\max}^{(1)}\}$. Hence, $U_i^\star(k) \leq \max\{0, \frac{T_w(k) - T_{\max}^{1}}{\beta} + \Delta_U\}$.

In situation 2, the possible largest $U_i(k)$ occurs when $T_i(k-1) = T_{\max}^{(1)}$. It is clear that $U_i^\star(k) = \min\{U_i(k) | T_{\min}^{(0)} \leq \alpha T_{\max}^{(1)} - \beta(1-\alpha)U_i(k) + T_w(k) \leq T_{\max}^{(0)}\}$. Hence, $U_i^\star(k) < \frac{T_w(k) - \tau + \Delta_\tau}{\beta} + \Delta_U$.

In situation 3, the possible largest $U_i(k)$ occurs when $T_i(k-1) = T_{\max}^{(0)}$. It is clear that $U_i^\star(k) = \min\{U_i(k) | T_{\min}^{(0)} \leq \alpha T_{\max}^{(0)} - \beta(1-\alpha)U_i(k) + T_w(k) \leq T_{\max}^{(0)}\}$. Hence $U_i^\star(k) \leq \max\{0, \frac{T_w(k) - T_{\max}^{(0)}}{\beta} + \Delta_U\}$.

In situation 4, the possible largest $U_i(k)$ happens when $T_i(k-1) = T_{\max}^{(0)}$. It is clear that $U_i^\star(k) = \min\{U_i(k) | T_{\min}^{(1)} \leq \alpha T_{\max}^{(1)} - \beta(1-\alpha)U_i(k) + T_w(k) \leq T_{\max}^{(1)}\}$. Hence, $U_i^\star(k) < \max\{0, \frac{T_w(k)-\tau-\Delta_\tau-\delta(1+\alpha)}{\alpha\beta} + \Delta_U\}$.

We denote $c_1(T_w(k))$, $c_2(T_w(k))$, $c_3(T_w(k))$ and $c_4(T_w(k))$ as follows:

1). $c_1(T_w(k)) = \min\{U_{\max}, \max\{0, \frac{T_w(k)-T_{\max}^1}{\beta} + \Delta_U\}\}$.

2). $c_2(T_w(k)) = \min\{U_{\max}, \frac{T_w(k)-\tau+\Delta_\tau}{\beta} + \Delta_U\}$.

3). $c_3(T_w(k)) = \min\{U_{\max}, \max\{0, \frac{T_w(k)-T_{\max}^{(0)}}{\beta} + \Delta_U\}\}$.

4). $c_4(T_w(k)) = \min\{U_{\max}, \max\{0, \frac{T_w(k)-\tau-\Delta_\tau-\delta(1+\alpha)}{\alpha\beta} + \Delta_U\}\}$.

Based on the above, the minimum total power of a building must satisfy $\sum_{i=1}^N U_i^\star(k) \leq \sum_{j=1}^4 N_j(k) c_j(T_{\text{w}}(k))$. Hence, Lemma 1 is obtained. Lemma 1 can be used, based on the historical weather and occupancy data, to simulate a power constraint of a building. We denote $U_i^{\text{GP}}(k)$ as the power decision when greedy policy is applied, and $T_i^{\text{GP}}(k)$ is the corresponding temperature.

Theorem 1: *When greedy policy is applied, the following inequality holds:*

$$\begin{aligned} & -\sum_{i=1}^N [U_i^{\text{GP}}(k)p(k) + \Gamma_i(k)(T_i^{\text{GP}}(k) - \tau)^2] \\ & \geq -cp(k) - N\max\{\Gamma_h[\frac{\delta}{\alpha} + (\frac{1}{\alpha} - 1)\Delta_\tau]^2, \delta^2\} \end{aligned} \tag{5.2a}$$

Proof: For the electricity cost, the following inequality is obtained:

$$\begin{aligned} & -\sum_{i=1}^N U_i^{\text{GP}}(k)p(k) \\ & = -\, p(k) \sum_{i=1}^N U_i^{\text{GP}}(k) && (5.3a) \\ & \geq -\, cp(k) && (5.3b) \end{aligned}$$

For the human discomfort value, the following inequality is obtained:

$$
\begin{aligned}
& -\sum_{i=1}^{N}\Gamma_i(k)(T_i^{\mathrm{GP}}(k)-\tau)^2 \\
\geq & -\Gamma_h N_{(\Gamma_i(k)\leq\Gamma_h)}[\frac{\delta}{\alpha}+(\frac{1}{\alpha}-1)\Delta_\tau]^2 - N_{(\Gamma_i(k)>\Gamma_h)}\delta^2 && \text{(5.4a)} \\
\geq & -N max\{\Gamma_h[\frac{\delta}{\alpha}+(\frac{1}{\alpha}-1)\Delta_\tau]^2, \delta^2\} && \text{(5.4b)}
\end{aligned}
$$

where $N_{(\Gamma_i(k)\leq\Gamma_h)}$ and $N_{(\Gamma_i(k)>\Gamma_h)}$ denote the total number of rooms with occupancy $\Gamma_i(k)\leq\Gamma_h$ and $\Gamma_i(k)>\Gamma_h$ at time slot k, respectively.

5.2 Finite Horizon Policy

$$
\begin{aligned}
\underset{U_i(0),...U_i(N)\ \forall i}{\text{maximize}} \quad & -\sum_{k=1}^{T}\sum_{i=1}^{N}[U_i(k)p(k)+\Gamma_i(k)(T_i(k)-\tau)^2] && \text{(5.5a)} \\
\text{subject to} \quad & T_i(k)=\alpha T_i(k-1)-\beta(1-\alpha)U_i(k)+(1-\alpha)T_w(k) && \text{(5.5b)} \\
& \mathbf{1}_{\Gamma_i(k)>\Gamma_h}(T_i(k)-\tau)\in[-\delta,\delta] && \text{(5.5c)} \\
& U_i(k)\in\{0,\Delta_U,2\Delta_U,...,U_{\max}\} && \text{(5.5d)} \\
& \sum_{i=1}^{N}U_i(k)\leq C \quad \forall \ k\in\{1,...,T\} && \text{(5.5e)}
\end{aligned}
$$

Consider the above optimization when $T-k$ is big enough. If all $T_w(1),....,T_w(T)$ and $p(1),....,p(T)$ remain static, the upper-bound of (5.4a-e) can be calculated by the following analysis.

If a room with occupancy $\Gamma_i(k)>\Gamma_h$, the temperature should be restricted in the range of $[\tau-\delta,\tau+\delta]$. Otherwise, the room's temperature should be controlled so that it can satisfy $T_w(k')\in[\tau-\delta,\tau+\delta]$, where k' is the first approaching time slot with $\Gamma_i(k')>\Gamma_h$.

Based on the above analysis, the optimization can be reformulated by introducing $\Delta_i(k)$, $a_{i,j}$ and b_i. $\Delta_i(k)$ denotes the deviation of the temperature of room i from the desired temperature τ at time slot k, namely $\Delta_i(k) = T_i(k) - \tau$; $a_{i,j}$ denotes the j^{th} time slot with $\Gamma_i > \Gamma_h$ from the starting time slot; b_i is the the total number of time slots which satisfy $\Gamma_i > \Gamma_h$ from the starting time slot. Then the above optimization can be reformulated as follows:

$$\underset{U_i(1),...U_i(T)\ \forall i}{\text{maximize}} \quad -\sum_{i=1}^{N}\sum_{j=1}^{b_i}\sum_{k=a_{i,j-1}+1}^{a_{i,j}}[U_i(k)p(k) + \Gamma_i(k)\Delta_i(k)^2] \tag{5.6a}$$

$$\text{subject to} \quad \Delta_i(a_{i,j}) = (1-\alpha)\sum_{k=a_{i,j-1}+1}^{a_{i,j}}\alpha^{a_{i,j}-k}[\Delta_w(k) - \beta U_i(k)] + \Delta_i(a_{i,j-1})\alpha^{a_{i,j}-a_{i,j-1}} \tag{5.6b}$$

$$\Delta_{i,a_{i,j}} \in [-\delta, \delta] \tag{5.6c}$$

$$U_i(k) \in \{0, \Delta_U, 2\Delta_U, ..., U_{\max}\} \tag{5.6d}$$

$$\sum_{i=1}^{N} U_i(k) \leq c \quad \forall \ k \in \{1, ..., T\} \tag{5.6e}$$

where $\Delta_w(k) = T_w(k) - \tau$. The temperature of any room is assumed within desirability at the starting time slot, namely $\Delta_i(0) \in [-\delta, \delta]$. From (5.4b), the following equation can be obtained:

$$\Delta_i(k) = \alpha\Delta_i(k-1) - \beta(1-\alpha)U_i(k) + (1-\alpha)\Delta_{\text{w}}(k) \tag{5.7}$$

(5.7) can be interpreted as that, the decision $U_i(k)$ is made to cool the heat caused by $\Delta_{i,k-1}$ and $\Delta_w(k)$. Based on (5.7), the following equation is obtained:

$$\alpha^{-k}\Delta_i(k) = \Delta_i(0) + (1-\alpha)\sum_{s=0}^{k}\alpha^{-s}[\Delta_w(s) - \beta U_i(s)] \tag{5.8}$$

(5.8) can be interpreted as the relationship between the temperature of a given time slot k and the starting time slot. If k is assigned as $\Delta_i(a_{i,j})$ and $\Delta_i(a_{i,j-1})$, respectively, we have:

$$\alpha^{-a_{i,j}}\Delta_i(a_{i,j}) = \Delta_i(0) + (1-\alpha)\sum_{s=0}^{a_{i,j}} \alpha^{-s}[\Delta_w(s) - \beta U_i(s)] \tag{5.9a}$$

$$\alpha^{-a_{i,j-1}}\Delta_i(a_{i,j-1}) = \Delta_i(0) + (1-\alpha)\sum_{s=0}^{a_{i,j-1}} \alpha^{-s}[\Delta_w(s) - \beta U_i(s)] \tag{5.9b}$$

Thus, (5.6b) is obtained, based on (5.9a) and (5.9b).

The optimization (5.5a-e) is upper bounded by (5.6a-e), because (5.6a-e) doesn't include the electricity cost and human discomfort at time slot larger than a_{i,b_i}. Equality holds when the occupancy is 0 after time slot b_i, namely $\Gamma_i(k) = 0$ when $k > b_i$.

$$\underset{U_i(1),\ldots U_i(T)\ \forall i}{\text{maximize}} \quad -\sum_{i=1}^{N}\sum_{j=1}^{b_i}\sum_{k=a_{i,j-1}+1}^{a_{i,j}} U_i(k)p(k) \tag{5.10a}$$

$$\text{subject to} \quad \Delta_i(a_{i,j}) = (1-\alpha)\sum_{t=a_{i,j-1}+1}^{a_{i,j}} \alpha^{a_{i,j}-k}[\Delta_w(k) - \beta U_i(k)] + \Delta_i(a_{i,j-1})\alpha^{a_{i,j}-a_{i,j-1}} \tag{5.10b}$$

$$\Delta_{i,a_{i,j}} \in [-\delta, \delta] \tag{5.10c}$$

$$U_i(k) \geq 0 \quad \forall \ k \in \{1, ..., T\} \tag{5.10d}$$

The optimization (5.6a-e) is upper bounded by (5.10a-d), because:

1) Discomfort value in objective function is discarded in (5.10a).

2) A relaxation of $U_i(k)$ is made and its supremum is discarded in (5.10d).

3) The power constraint is discarded.

Hence, the optimization (5.5a-e) is upper bounded by (5.10a-d). We denote $U_i^{\text{FHP}}(k)$ as the power decision when finite horizon policy is applied, and $T_i^{\text{FHP}}(k)$ is the corresponding temperature.

Theorem 2: *When finite horizon policy is applied, the following inequality holds:*

$$
\begin{aligned}
&-\sum_{k=1}^{T}\sum_{i=1}^{N}[U_i^{\mathrm{FHP}}(k)p(k)+\Gamma_i(k)(T_i^{\mathrm{FHP}}(k)-\tau)^2] \\
&\leq -\sum_{k=1}^{T'}\sum_{i=1}^{N}[\frac{\alpha^{d_i(k)}\Delta_w(k)\mathrm{min}_k\{p(k)\}}{\beta l_i(k)}-\frac{\delta(1+\alpha^{l_i(k)})\alpha^{-d_i(k)}p(k)}{(l_i(k))^2\beta(1-\alpha)}] \qquad (5.11a)
\end{aligned}
$$

where T' is the minimum of the time slot index among all rooms' last high occupancy time slots.

Proof: Based on (5.10a-d), we have:

$$
\begin{aligned}
&\sum_{i=1}^{N}\sum_{j=1}^{b_i}\sum_{t=a_{i,j-1}+1}^{a_{i,j}}U_i(k)p(k) \\
&\overset{\textcircled{1}}{\geq}\sum_{i=1}^{N}\sum_{j=1}^{b_i}\frac{\left[\sum_{k=a_{i,j-1}+1}^{a_{i,j}}\sqrt{\alpha^{a_{i,j}-k}U_i(k)}\right]^2}{\sum_{k=a_{i,j-1}+1}^{a_{i,j}}\alpha^{a_{i,j}-k}p(k)^{-1}} \qquad (5.12a) \\
&\overset{\textcircled{2}}{>}\sum_{i=1}^{N}\sum_{j=1}^{b_i}\frac{\sum_{k=a_{i,j-1}+1}^{a_{i,j}}\alpha^{a_{i,j}-k}U_i(k)}{\sum_{k=a_{i,j-1}+1}^{a_{i,j}}\alpha^{a_{i,j}-k}p(k)^{-1}} \qquad (5.12b) \\
&\overset{\textcircled{3}}{=}\sum_{i=1}^{N}\sum_{j=1}^{b_i}\frac{\frac{1}{\beta}\sum_{k=a_{i,j-1}+1}^{a_{i,j}}\alpha^{a_{i,j}-k}\Delta_w(k)-\frac{\Delta_i(a_{i,j-1})\alpha^{a_{i,j}-a_{i,j-1}}-\Delta_i(a_{i,j})}{\beta(1-\alpha)}}{\sum_{k=a_{i,j-1}+1}^{a_{i,j}}\alpha^{a_{i,j}-k}p(k)^{-1}} \qquad (5.12c) \\
&\overset{\textcircled{4}}{>}\sum_{i=1}^{N}\sum_{j=1}^{b_i}[\frac{\frac{1}{\beta}\sum_{k=a_{i,j-1}+1}^{a_{i,j}}\alpha^{a_{i,j}-k}\Delta_w(k)}{\sum_{k=a_{i,j-1}+1}^{a_{i,j}}\alpha^{a_{i,j}-t}p(k)^{-1}}-\frac{\frac{\delta(1+\alpha^{a_{i,j}-a_{i,j-1}})}{\beta(1-\alpha)}}{\sum_{k=a_{i,j-1}+1}^{a_{i,j}}\alpha^{a_{i,j}-k}p(k)^{-1}}] \qquad (5.12d) \\
&\overset{\textcircled{5}}{>}\sum_{i=1}^{N}\sum_{j=1}^{b_i}[\frac{\frac{1}{\beta}\sum_{k=a_{i,j-1}+1}^{a_{i,j}}\alpha^{a_{i,j}-k}\Delta_w(k)}{\sum_{k=a_{i,j-1}+1}^{a_{i,j}}\alpha^{a_{i,j}-k}p(k)^{-1}} \\
&\qquad -\frac{\delta(1+\alpha^{a_{i,j}-a_{i,j-1}})}{(a_{i,j}-a_{i,j-1})^2\beta(1-\alpha)}\sum_{k=a_{i,j-1}+1}^{a_{i,j}}\alpha^{k-a_{i,j}}p(k)] \qquad (5.12e) \\
&\overset{\textcircled{6}}{>}\sum_{i=1}^{N}\sum_{j=1}^{b_i}\sum_{k=a_{i,j-1}+1}^{a_{i,j}}[\frac{\alpha^{a_{i,j}-k}\Delta_w(k)\mathrm{min}_k\{p(k)\}}{\beta(a_{i,j}-a_{i,j-1})} \\
&\qquad -\frac{\delta(1+\alpha^{a_{i,j}-a_{i,j-1}})\alpha^{k-a_{i,j}}p(k)}{(a_{i,j}-a_{i,j-1})^2\beta(1-\alpha)}] \qquad (5.12f) \\
&\overset{\textcircled{7}}{>}\sum_{k=1}^{T'}\sum_{i=1}^{N}[\frac{\alpha^{d_i(k)}\Delta_w(k)\mathrm{min}_k\{p(k)\}}{\beta l_i(k)}-\frac{\delta(1+\alpha^{l_i(k)})\alpha^{-d_i(k)}p(k)}{(l_i(k))^2\beta(1-\alpha)}] \qquad (5.12g)
\end{aligned}
$$

The inequality of ① is proved by CauchySchwarz inequality, which is $(\sum_{k=n_1}^{n_2} u_k v_k)^2 \leq \sum_{k=n_1}^{n_2} u_k^2 \sum_{k=n_1}^{n_2} v_k^2$. Let $v_k = \sqrt{U_i(k)p(k)}$, $u_k v_k = \sqrt{\alpha^{a_{i,j}-k} U_i(k)}$, $n_1 = a_{i,j-1} + 1$ and $n_2 = a_{i,j}$, then ① can be obtained.

The inequality of ② is obtained by abandoning some positive numbers.

The equality of ③ is obtained by the substitution of equation (5.8b).

The inequality of ④ is got by substitution of $\Delta_i(a_{i,j-1}) = -\delta$ and $\Delta_i(a_{i,j}) = \delta$, because of $\Delta_i(a_{i,j-1}) \in [-\delta, \delta]$ and $\Delta_i(a_{i,j}) \in [-\delta, \delta]$.

The inequality of ⑤ is proved by Arithmetic Mean - Harmonic Mean Inequality, which is $\frac{\sum_{k=n_1}^{n_2} x_k}{n_2 - n_1} \geq \frac{n_2 - n_1}{\sum_{k=n_1}^{n_2} x_k^{-1}}$ when $x_k \geq 0$. Let $x_k = \alpha^{k - a_{i,j}} p(k)$, $n_1 = a_{i,j-1} + 1$ and $n_2 = a_{i,j}$, then ⑤ can be obtained.

The inequality of ⑥ can be proved by using the following inequality:

$$\sum_{k=a_{i,j-1}+1}^{a_{i,j}} \alpha^{a_{i,j}-k} p(k)^{-1}$$

$$\leq \sum_{k=a_{i,j-1}+1}^{a_{i,j}} \alpha^{a_{i,j}-k} \min_k\{p(k)^{-1}\} \tag{5.13a}$$

$$< (a_{i,j} - a_{i,j-1}) \min_k\{p(k)^{-1}\} \tag{5.13b}$$

The inequality of ⑦ is done by substitution of $l_i(k) = a_{i,j} - a_{i,j-1}$, $d_i(k) = a_{i,j} - k$, $T' = \min_i\{a_{i,b_i}\}$. We obtain the inequality by discarding the electricity cost and human discomfort after T'.

The upper-bound in Theorem 2 has two parts, here are the interpretations:

1). The first part is $\sum_{k=1}^{T'} \sum_{i=1}^{N} \frac{\alpha^{d_i(k)} \Delta_{\mathrm{w}}(k) \min_k\{p(k)\}}{\beta l_i(k)}$, which captures the lower bound of the electricity cost in order to maintain the human comfort. $\frac{\Delta_{\mathrm{w}(k)}}{\beta(1-\alpha)}$ captures the power needs to be used in order to make every room satisfy the human comfort requirement. $\frac{(1-\alpha)\alpha^{d_i(k)}}{l_i(k)}$ captures a parameter how the power at different time slot affects the human comfort, because only the high occupancy room has a temperature restriction. $\min_k\{p(k)\}$ means the minimum electricity price. Based on above, we can conclude that this part tells us the lower bound of the electricity cost we need to pay to maintain the human comfort in the room with high occupancy rate.

2). The second part is $\sum_{k=1}^{T'}\sum_{i=1}^{N}\frac{\delta(1+\alpha^{l_i(k)})\alpha^{-d_i(k)}p(k)}{l_i^2(k)\beta(1-\alpha)}$, which captures the electricity cost saved by the policy. $\frac{\delta}{\beta(1-\alpha)}$ means the power saved to cool the next high occupancy room, when the beginning high occupancy room's temperature deviation from set point is $-\delta$. $\frac{1+\alpha^{l_i(k)}\alpha^{-d_i(k)}}{l_i^2(k)}$ captures how this starting temperature help the following rooms' power saving. Based on above, we can conclude that this part tells us the upper-bound of the electricity cost saving by the policy.

5.3 Approximation Ratio of Greedy Policy

In the above sections 5.1 and 5.2, we have calculated the lower bound value of the greedy policy and the upper bound value of the finite horizon policy. The approximation ratio of greedy policy is upper bounded as follows:

$$\begin{aligned}
&\frac{\text{Electricity Cost(GP)} + \text{discomfort(GP)}}{\text{Electricity Cost(FHP)} + \text{discomfort(FHP)}} \\
&= \lim_{T\to\infty}\frac{\sum_{k=1}^{T}\sum_{i=1}^{N}U_i^{\text{GP}}(k)p(k)+\sum_{k=1}^{T}\sum_{i=1}^{N}\Gamma_i(k)[T_i^{\text{GP}}(k)-\tau]^2}{\sum_{k=1}^{T}\sum_{i=1}^{N}U_i^{\text{FHP}}(k)p(k)+\sum_{k=1}^{T}\sum_{i=1}^{N}\Gamma_i(k)[T_i^{\text{FHP}}(k)-\tau]^2} && (5.14a)\\
&< \lim_{T\to\infty}\frac{\sum_{k=1}^{T}cp(k)+NT\max\{\Gamma_h[\frac{\delta}{\alpha}+(\frac{1}{\alpha}-1)\Delta_\tau]^2,\delta^2\}}{\sum_{k=1}^{T}\sum_{i=1}^{N}[\frac{\alpha^{d_i(k)}\Delta_w(k)\min_k\{p(k)\}}{\beta l_i(k)}-\frac{\delta(1+\alpha^{l_i(k)})\alpha^{-d_i(k)}p(k)}{(l_i(k))^2\beta(1-\alpha)}]} && (5.14b)\\
&= \frac{\frac{c\mathbb{E}[p(k)]}{N}+\max\{\Gamma_h[\frac{\delta}{\alpha}+(\frac{1}{\alpha}-1)\Delta_\tau]^2,\delta^2\}}{\frac{\min\{p(k)\}}{\beta N}\mathbb{E}[\Delta_w(k)]\sum_{i=1}^{N}\mathbb{E}[\frac{\alpha^{d_i(k)}}{l_i(k)}]-\frac{\delta}{\beta(1-\alpha)N}\mathbb{E}[p(k)]\sum_{i=1}^{N}\mathbb{E}[\frac{(1+\alpha^{l_i(k)})\alpha^{-d_i(k)}}{l_i^2(k)}]} && (5.14c)
\end{aligned}$$

The equation above is deduced by large number theory (LLN). It is assumed that the electricity price, outdoor weather and occupancy are independent with each other.

We can draw a conclusion that the greedy policy's approximation ratio is upper bounded by the ratio in (5.14c). This ratio can be positive by carefully picking up the parameters α, δ, Γ_h.

Chapter 6

SIMULATION RESULTS AND CONCLUSION

6.1 Simulation Setting

A building with five rooms is considered for the simulation. We assume each room is equipped with one AC unit and occupancy sensor. We simulate the building AC units control of cooling injection during summer time. For simplicity, each room contain 1 occupants ($\mu = 1$). For the building thermal dynamic model, a synthetic model is used.

For occupancy prediction algorithm simulation, we used the occupancy data from Mitsubishi Electric Research Laboratory, [10]. Five sensors (sensors index: 214, 309, 310, 338, 356) in the data set are selected to act as the occupancy historical data of the five rooms. Those five sensors' data from July 14, 2006 to October 10, 2006 and October 11, 2006 to January 13, 2007 are chosen to be the testing and training data for the occupancy transition matrix, respectively.

For price data, we use the PJM hourly price data set for September 9, 2016.

For weather data, we get the hourly weather temperature data of Tempe on June, 2016 from the open-source website *'http://weathersource.com/past-weather/weather-history-reports/free'*.

6.2 Simulation Results

In the simulation, we compare the electricity cost and human discomfort value of the greedy policy and a benchmark control policy. The difference between the two policies is that the benchmark control policy always regard all rooms occupied. We chose an arbitrary date of testing data for the occupancy data to simulate Fig 6.1 - Fig 6.4.

From Fig 6.1, we can see that the predicted occupancy line fits the real occupancy star point well, which means that the Markov chain based occupancy model has a good prediction performance.

From Fig 6.2, we can see that the room's temperature when we use the benchmark policy is very close to the set point all day. By comparison, We can see that the room's temperature when we use the greedy control policy is not always close to the set temperature. The most serious temperature deviation occurs from 02:00 to 14:00 due to a low occupancy prediction which is lower than Γ_h. When occupancy prediction is relatively high, for instance during the night, the temperature for each zone still has a small deviation from set point, mainly due to the balance of high price of electricity and electricity needs.

From Fig 6.3, we can see that the discomfort value is bounded around 1 during one day, which means if a room is occupied within a time slot, the temperature deviation from the set point is less than 1°C. The peak of discomfort happens during the night, which is mainly because the price and occupancy prediction are both relatively high at that time and we sacrifice a comfort for a lower electricity cost.

From Fig 6.4, We can also find that the hourly electricity cost is always below the benchmark electricity cost. The most serious difference occurs at the time from 02:00 to 14:00, which is mainly because the occupancy predictions at those time are relatively low. Additionally, the electricity cost is saved around 20:00, which is mainly because we sacrifice the comfort due to a high electricity price.

From Fig 6.5, we can see that the the hourly accumulative electricity cost is always under the benchmark accumulative electricity cost and we can calculate the total electricity cost saving rate is 20.83% for this arbitrary chosen date.

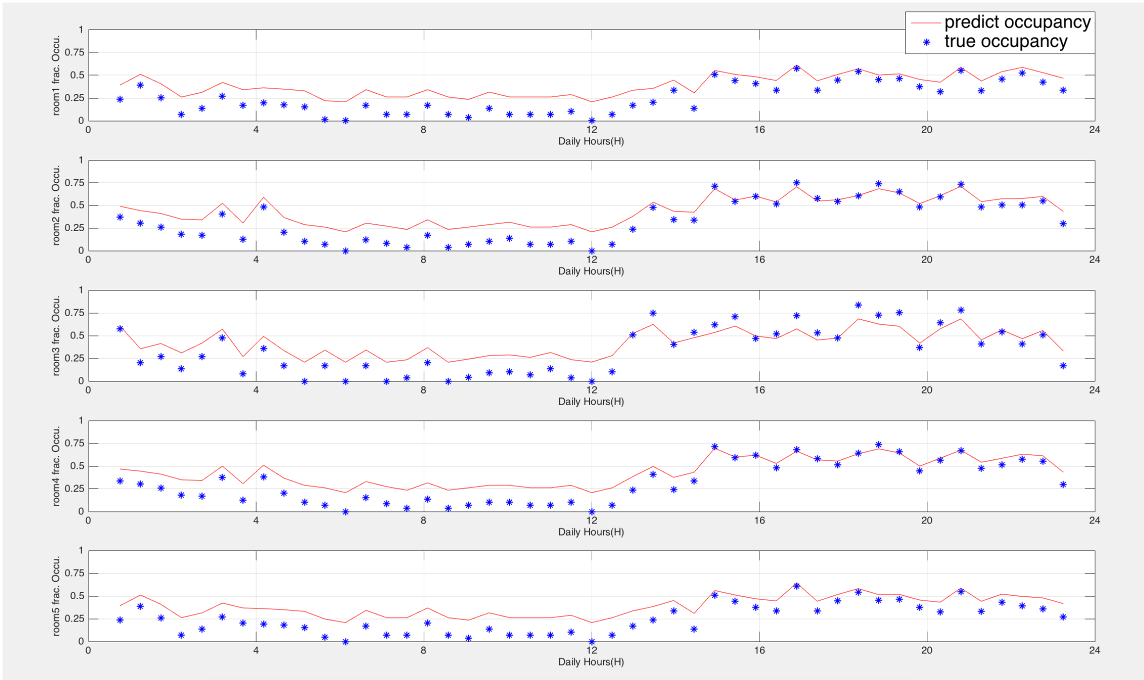

Figure 6.1: Five Zone's Hourly Predicted Occupancy and Real Occupancy within 24 Hours

6.3 Conclusion

We designed a knapsack problem-based control policy, which uses a building thermal dynamic model and occupancy prediction to lower the operating cost of building while maintaining the human comfort levels. Our results show that our system saves the electricity cost when the occupancy is low or when both occupancy and electricity is high, while maintaining the human comfort well. The performance increase stems from tight control of the AC unit and its power level based on reliable occupancy data and predictions.

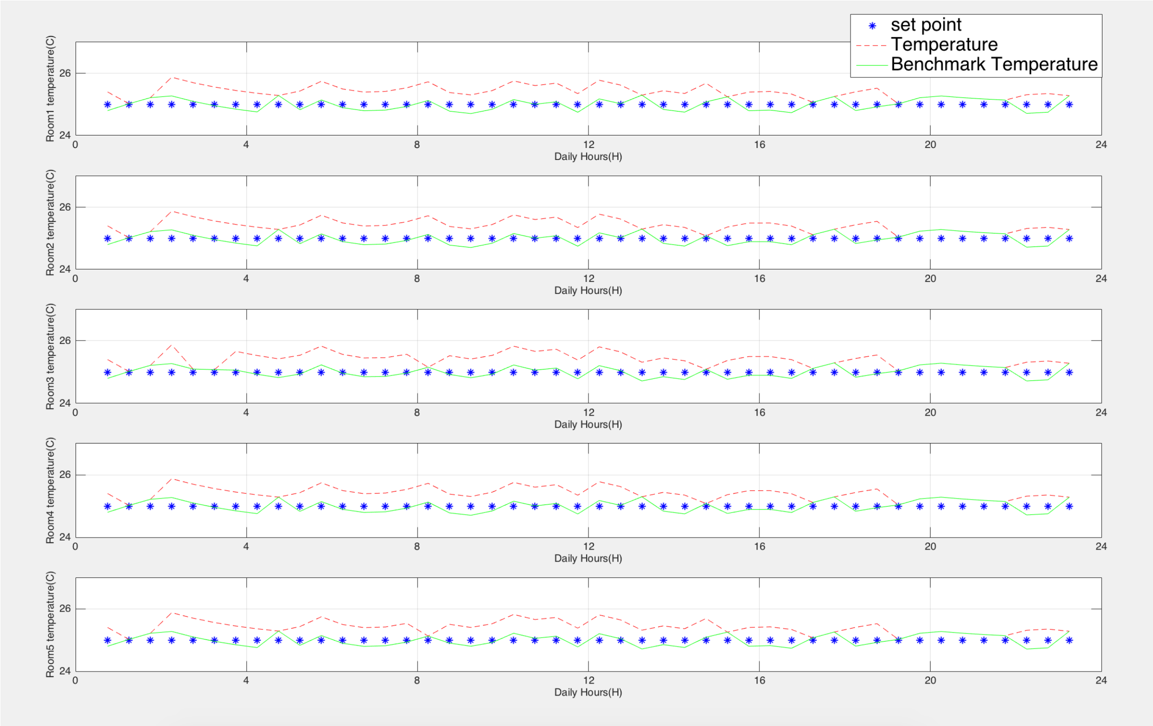

Figure 6.2: Five Zone's Hourly Temperature within 24 Hours by Using Greedy Policy and Benchmark Policy

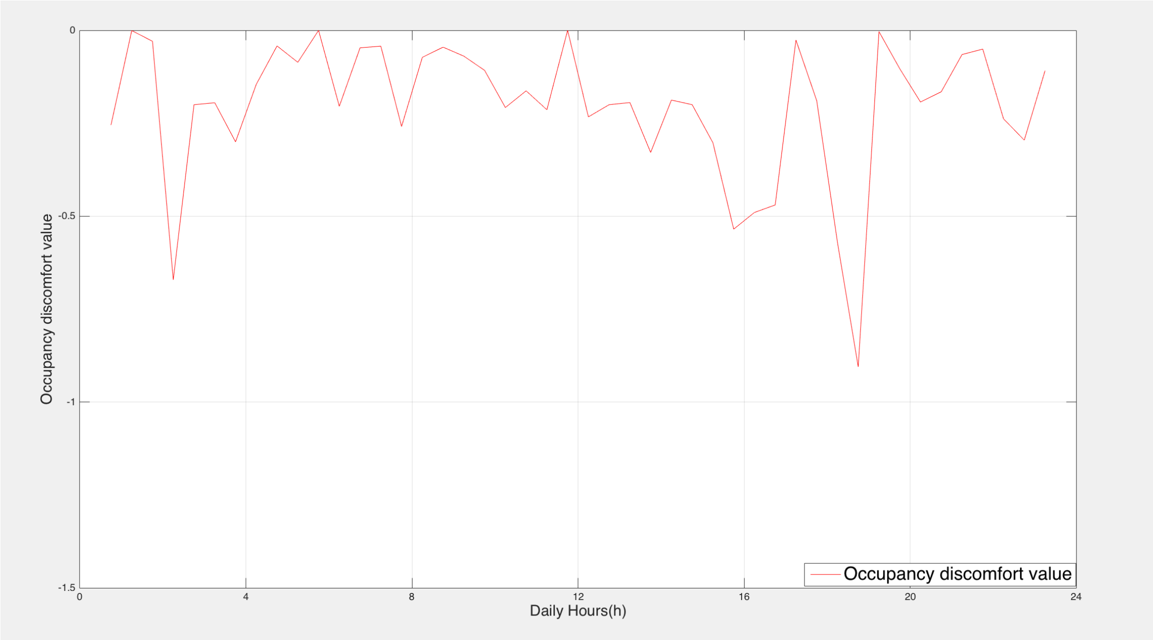

Figure 6.3: The Hourly Discomfort Value within 24 Hours

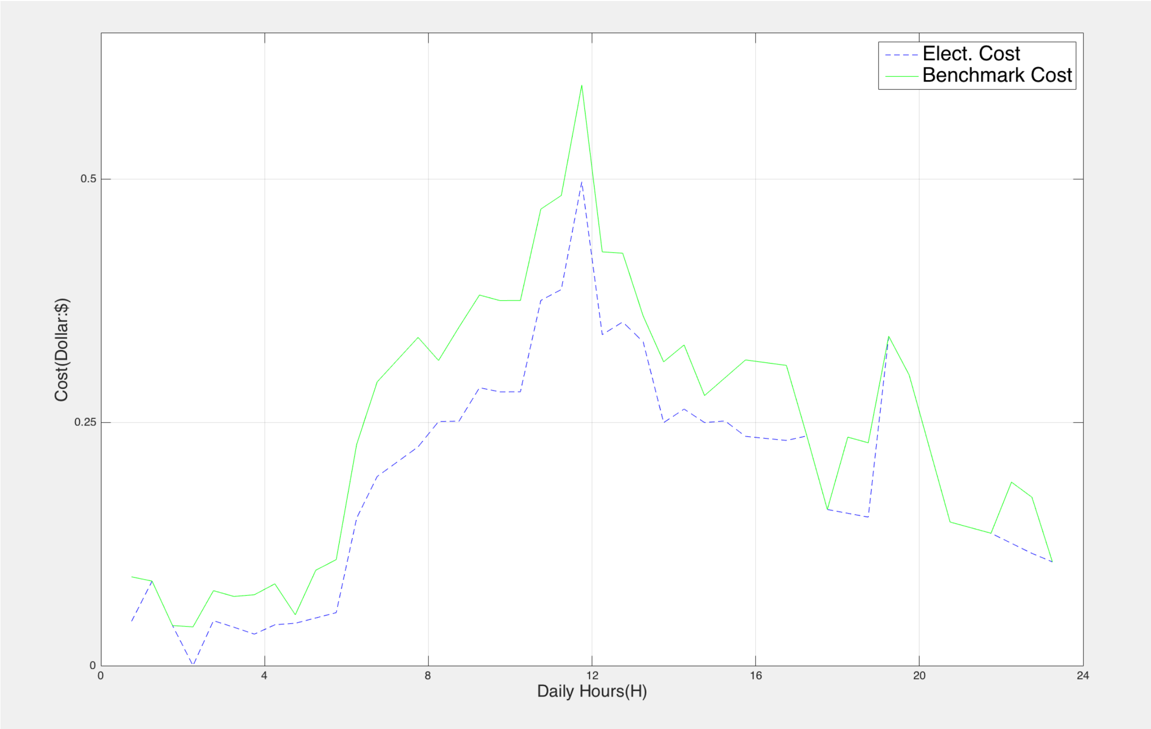

Figure 6.4: The Hourly Electricity Cost within 24 Hours

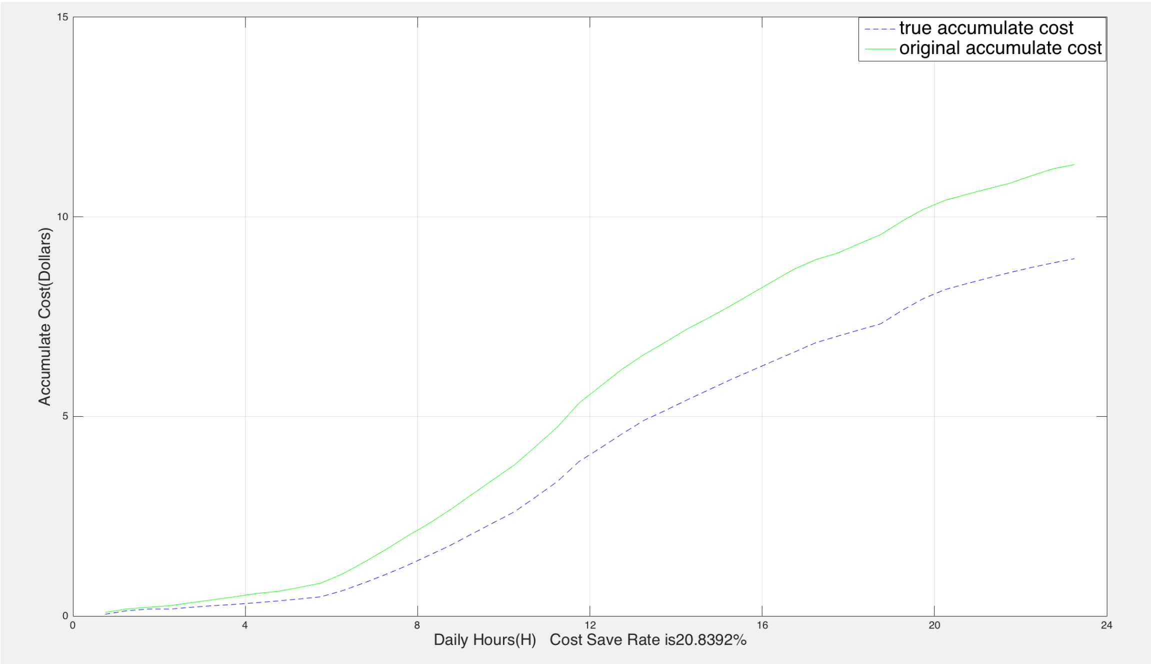

Figure 6.5: The Hourly Accumulative Electricity Cost within 24 Hours

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