# Essays in Finance and Macroeconomics: Household Financial Obligations and the Equity Premium

by

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A Dissertation Presented in Partial Fulfillment
of the Requirements for the Degree
Doctor of Philosophy

Approved March 2017 by the
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ARIZONA STATE UNIVERSITY

May 2017

ABSTRACT

This dissertation is a collection of three essays relating household financial obligations to asset prices. Financial obligations include both debt payments and other financial commitments.

In the first essay, I investigate how household financial obligations affect the equity premium. I modify the standard Mehra-Prescott (1985) consumption-based asset pricing model to resolve the equity risk premium puzzle. I focus on two channels: the preference channel and the borrowing constraints channel. Under reasonable parameterizations, my model generates equity risk premiums similar in magnitudes to those observed in U.S. data. Furthermore, I show that relaxing the borrowing constraint shrinks the equity risk premium.

In the Second essay, I test the predictability of excess market returns using the household financial obligations ratio. I show that deviations in the household financial obligations ratio from its long-run mean is a better forecaster of future market returns than alternative prediction variables. The results remain significant using either quarterly or annual data and are robust to out-of-sample tests.

In the third essay, I investigate whether the risk associated with household financial obligations is an economy-wide risk with the potential to explain fluctuations in the cross-section of stock returns. The multifactor model I propose, is a modification of the capital asset pricing model that includes the financial obligations ratio as a "conditioning down" variable. The key finding is that there is an aggregate hedging demand for securities that pay off in periods characterized by higher levels of financial obligations ratios. The consistent pricing of financial obligations risk with a negative risk premium suggests that the financial obligations ratio acts as a state variable.

*To*

*My parents, because I owe it all to you. Many Thanks!*

*My wife, because you are my eternal cheerleader*

ACKNOWLEDGMENTS

First, I would like to express my sincere gratitude to my advisor Rajnish Mehra for continuous support of my Ph.D. study and related research, for his patience, motivation, and immense knowledge. His guidance helped me during the research and writing of this dissertation. I could not imagine having a better advisor and mentor for my Ph.D. study.

Second, I would like to thank the members of my dissertation committee, Sunil Wahal and Kevin Reffett, for their insightful comments and encouragement, and also for the hard questions that inspired me to broaden my research. My sincere thanks also goes to Manjira Datta and Gustavo Ventura, who provided me the opportunity to join the Department of Economics. Without their valued support, it would not have been possible to conduct this research.

I am grateful to my parents, Khosro and Maliheh, and my brothers, Ehsan and Keivan. There were moments during this journey when I could not be with them though they needed me. I am grateful for their love and understanding. I would like to express my gratitude to my parents-in-law for their unfailing emotional support. I thank with love my wife, Sepideh. Understanding me best as a Ph.D. student herself, Sepideh has been my best friend and great companion and has loved, supported, encouraged, entertained, and helped me get through this agonizing period in the most positive way.

And, last but by no means least, I thank Beth Baugh for her outstanding editing and proofreading of my dissertation. Thank you everyone in the Department of Economics. It had been a great journey with all of you these last four years.

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Chapter 1

# INTRODUCTION

This dissertation is a collection of three essays relating household financial obligations to asset prices. In the first essay, I investigate how household financial obligations affect the equity premium. I modify the standard Mehra-Prescott (1985) consumption-based asset pricing model to resolve the equity risk premium puzzle. I focus on two specific channels: the preference channel and the borrowing constraints channel. Under reasonable parameterizations, my model generates equity risk premium similar in magnitudes to the observed ones in US data. Furthermore, I show that relaxing the borrowing constraints, shrinks the equity risk premium.

In my model, preferences are defined over households' consumption relative to their financial obligations. This is the preference channel. The framework is analogous to habit formation models where the utility function depends on consumption relative to some persistence level. However, unlike habit formation models, the persistent level in my model — household financial obligations — is observable, which is a key advantage over habit models in general. Also, in an infinite-horizon aggregate household economy, the financial obligations ratio is used as a proxy for households borrowing constraints. Households can borrow against their income up to a certain level, which is exogenously set by a lending institution arrangement. This is the borrowing constraint channel. The mechanism by which the household financial obligations ratio helps explain the equity risk premium is as follows: in recessions, when income falls, household financial obligations ratios increase and hence lenders become reluctant to lend to the households. The

borrowing constraint binds and makes it more difficult for households to smooth their consumption exactly when they need to do so. However, in these bad times, households also behave in a much more risk-averse fashion because they want to make sure they can repay their debt obligations and financial commitments. This dual mechanism both amplifies the risk premium and makes it time varying.

In the second essay, I test the predictability of excess market returns using the household financial obligations ratio. I show that deviations of the household financial obligations ratio from its long-run mean is a better forecaster of future market returns than several other predicting variables. The results remain significant using either quarterly or annual data and is robust to out of sample tests.

For the last two decades there have been many efforts to identify and establish the existence of time variation in expected asset returns. It is now widely accepted that excess returns are predictable by ratios such as dividend-price, earnings-price, dividend-yield, investment-capital, and other financial indicators. However, these financial variables, though successful at predicting long-horizon returns, are less successful at predicting short-horizon returns. Therefore, we are also interested in examining the linkage between *macroeconomic* variables and financial markets, mostly because expected returns appear to vary with business cycles so that stock market returns should be forecastable by business cycle variables at cyclical frequencies. One macroeconomic business cycle variable that is successful at predicting short-horizon returns is the consumption-wealth ratio (*cay*) proposed by Lettau and Ludvigson (2001) [43]. However, the statistical significance and predictability power of the consumption-wealth ratio is hump-shaped and peaks at around one year. Indeed, the predictability power shrinks over long horizons. In this essay, I show that mean-deviations from the household obligations ratio create

another macroeconomic business cycle variable whose predictability power is significant at short horizons and remains more significant over longer horizons than does the consumption-wealth ratio.

In the third essay, I investigate whether the risk associated with household financial obligations is an economy-wide risk and thus significant at explaining fluctuations in the cross-section of stock returns. The multifactor model proposed is a modification of the capital asset pricing model (CAPM) includes financial obligations ratio as a "conditioning down" variable. The key finding is that there is an aggregate hedging demand for securities that pay off in periods with high financial obligations ratios. The consistent pricing of financial obligations risk with a negative risk premium suggests that the financial obligations ratio acts as a state variable.

The cross-sectional intuition is as follows: in bad times when incomes fall, households' financial obligations ratios increase. Portfolios that pay off in these times are more valuable to investors because these portfolios hedge investors against financial obligations risk. The increase in hedging demand for these portfolios raises the price and hence implies lower expected returns. The negative financial obligation risk premium delivers this lower expected return for asset whose payoffs are positively correlated with the financial obligations ratio.

The macroeconomic variable explored in this research — financial obligations ratio — provides a fresh opportunity to investigate the determinants of asset risk in general. I show that there is systematic risk associated with high levels of the households financial obligations ratio and that this risk is priced across different portfolios. Furthermore, I show that mean deviations from financial obligations ratio are a strong predictor of market returns over business cycle frequencies.

Chapter 2

# HOUSEHOLDS' FINANCIAL OBLIGATIONS RATIO

In this chapter, I define households' financial obligations and financial obligation ratio. Household financial obligations include total debt payments (mortgage debt payments plus consumer debt payments) and total financial commitments (rent, lease, insurance, and property tax payments). Financial obligations dramatically affect households' decisions. There is extensive documentation in the economic psychology literature suggesting that financial obligations are associated with high levels of anxiety and stress (Brown, Taylor, and Price (2005)[5], Richardson, Elliott, and Roberts (2013)[54]). More important, this effect is independent of the poverty with which it is often associated (Jenkins et al. (2008)[35], Meltzer et al. (2011)[51]).

Financial obligations also affect households' budget constraints. In particular, if these obligations are high, relative to income, and it is not possible to roll over the debt, then borrowers have to cut back on expenditures to avoid default. There is evidence that high financial obligations reduce expenditures at the micro level.[1] The households' financial obligations position is important in determining whether they are constrained from optimal consumption smoothing. The fact that a household may have been able to borrow in the past does not imply that it can borrow as much in the future. However, household financial obligations in isolation are not indicative of the household's borrowing capability. Consider two households with the same financial obligations but with different levels of income. Lenders are far more likely willing to

[1]The negative effect of a high debt service burden on consumption of households is shown by Olney (1999)[52], Johnson and Li (2010)[37], Dynan, (2012)[17] and Juselius and Drehmann (2015)[39].

lend to the household with higher income. Hence, I need to define an appropriate variable as a direct proxy of borrowing constraints. I do this using the financial obligations ratio, which I present in next section.

### 2.1 What Is the Financial Obligations Ratio and Why Does It Matter?

Households' financial obligations ratio is defined as households' total financial obligations divided by their total disposable income. The financial obligations ratio consists of two parts:

1. **Total debt service ratio**, which is equal to total debt payments divided by total disposable income. Debt payments include all mortgage debt payments and consumer debt payments including auto loans, student loans, and consumer credit cards.

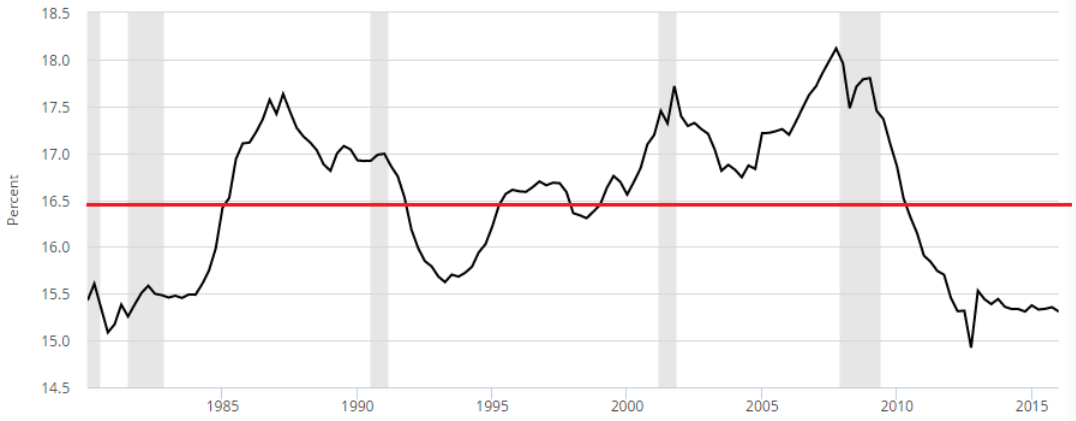
2. **Total financial commitment ratio**, which is equal to total financial commitments divided by total disposable income. Financial commitments include all rent payments, lease payments, insurance, and property tax payments of the homeowners.

### 2.2 Properties of the Financial Obligations Ratio

The Federal Reserve Board has estimated the aggregate household financial obligations ratio for the U.S. economy since 1980.[2] Figure 2.1 shows that the household financial obligations ratio is a time-varying macroeconomic variable that has an average value of 16.65%. As this figure suggests, the financial obligations ratio tends to move counter cyclically over business cycles.

[2]More discussion is provided in Appendix A.

**Figure 2.1:** Household Financial Obligations as a Percentage of Disposable Income




When the economy is good, consumers spend more and increase their financial obligations. Then, when the economy is hit by a negative shock (recession), consumers who have wracked up high financial obligations cannot smooth their consumption exactly when they need to do so. Hence, we observe that the financial obligations ratio is high in the early stage of almost every recession because households are carrying heavy financial obligations from "good old days" and then the ratio decreases as the economy recovers and households delever slowly. A higher financial obligations ratio also implies less investment in risky assets. This is due to the fact that when households are overextended, even a small income shortfall prevents them from smoothing consumption and making new investments (Drehmann and Juselius (2012)[38]). Figure 2.1 shows that after almost every recession, the financial obligations ratio pulls back to lower levels because of the households' higher income during booms.[3] Thus, the financial obligations ratio has a counter-cyclical property. The shaded areas in Figure 2.1 indicate U.S. recession periods. Properties of the financial obligations ratio that are pertinent to this research are as follows:

[3]Another reason is that when household obligations ratios are high and unemployment is rising, lenders may respond to the expected increase in defaults by limiting the availability of credit. This leads to lower aggregate payments and a lower obligations ratio.

- All the components of the financial obligations ratio are observable. Hence, when working with the data, there is no need to come up with questionable proxies for the ratio.

- The financial obligations ratio is directly related to the interest rate. By construction, the higher the interest rate, the higher the payments and the higher the financial obligations. This explicit dependency establishes a direct link between obligation ratios and the predictability of stock market returns.[4]

- The financial obligations ratio captures the burden of obligations on households more accurately than does the established debt-to-GDP ratio. More specifically, the financial obligations ratio accounts for changes in interest rates and maturities that affect households' repayment capacity.

- Drehmann and Juselius (2012)[38] find that the debt service ratio (which is the main part of financial obligations ratio) produces a reliable early warning signal ahead of systemic banking crises. In the context of absolute asset pricing, this is important because I am looking for a conditioning down variable that is correlated with business cycles, especially in bad times.

- The financial obligations ratio can be used as a direct indicator for borrowing constraints. Johnson and Li (2010)[37] test the proposition that a higher debt service ratio increases the likelihood of credit denial.[5] Therefore, a household's obligation ratio is a critical input for lending institutions to decide whether to provide the household with more leverage. Note that this ratio is a better proxy

[4] Juselius and Drehmann (2015)[39] argue that the average lending rate reflects not only current interest rate conditions, but also past money market rates, past inflation and interest rate expectations as well as past risk and term premia. This implies that the lending rate, and hence the debt service ratio, is chiefly influenced by current and past monetary policy decisions.

[5] A household with a debt service ratio in the top two quantiles of the distribution is significantly more likely than other households to have been turned down for credit in previous years (2005-2010).

for borrowing constraints than the traditional debt-to-income ratio because the debt-to-income ratio does not consider debt maturities.

In the following chapters, I first investigate how households financial obligations affect the equity risk premium. I then test the predictability of excess market returns using the financial obligations ratio. Finally, I show that my multifactor model can explain the cross-sectional variations in stock returns using the financial obligations ratio as a "conditioning down" variable.

Chapter 3

# HOW DO HOUSEHOLDS' FINANCIAL OBLIGATIONS AFFECT THE EQUITY RISK PREMIUM?

In this chapter, I study how household financial obligations affect the equity risk premium. The model I develop focuses on two channels: preference channel and borrowing constraints channel. Preferences are defined as households consumption relative to their financial obligations. These preferences allow for time variation in risk aversion. The model also introduces dynamic borrowing constraints, using the household financial obligations ratio as a proxy. In an infinite-horizon aggregate household economy, households can borrow against their uninsurable stochastic endowments. Borrowing limits are exogenously set by an institutional arrangement. A novel feature of the model is that in states of high marginal utility, when income falls, lenders become reluctant to lend, the borrowing constraint binds, making it more difficult for households to smooth consumption. In addition, in these states, households become more risk averse. This dual mechanism both amplifies the risk premia and makes it time varying.

## 3.1 Introduction

Mehra and Prescott (1985)[50] , modify a Lucas (1978)[46] -type exchange economy to reconcile standard neoclassical macroeconomic theory with U.S. data on the equity premium. In a consumption based asset pricing setup, they specify an explicit two-state Markov process for consumption growth and calculate the price of the consumption claim and the risk-free rate. They find that under reasonable parameterization, the model is able, at most, to generate an equity premium of about 0.35 % as opposed to the 6 % premium observed in the data. They term this the "equity premium puzzle" and argue that the mean stock excess return calculated in their calibrated economy is too low, unless the coefficient of relative risk aversion is raised to implausibly high values.

There is no easy way to summarize the huge literature on the equity risk premium puzzle. Nevertheless, there is consensus among researchers that only an absolute asset pricing model can explain the equity risk premium rather than a portfolio-based [1] model. After all, portfolio models are relative asset pricing models and cannot answer questions such as why the average returns are what they are or why the expected market return varies over time. To answer these questions, we need to construct a macroeconomic-based asset pricing model. Note that the most basic absolute pricing model — the standard consumption based model — performs poorly in explaining the historical equity premium puzzle and cross-sectional variations in expected returns. Hence, proposing a macroeconomic-based asset pricing model with the ability to explain the equity risk premium and cross-sectional variations in excess returns has long been the focus of macro finance researchers.

[1] The absolute pricing model refers to asset pricing models that use macroeconomic variables such as consumption, labor income, GDP growth, and interest rate.

In the last 30 years, many efforts have been made to solve the equity premium puzzle. Multiple generalizations have been proposed to address the shortcomings of the standard consumption-based model. To approach the puzzle, it is natural to start with the dependency of the marginal utility of consumption variables other than today's consumption. Employing a non separable utility function allows for this dependency, and indeed this is what has been done in the literature (Eichenbaum, Hansen, and Singleton (1988) [20], Eichenbaum and Hansen (1990) [19], Ait-Sahalia, Parker, and Yogo (2004) [57], Pakos (2004) [47], Piazzesi, Schneider, and Tuzel (2007) [53]). Another generalization is to consider non separability over time (Constantinides (1990) [24], Campbell and Cochrane (1999) [6]) and/or non separability across states of nature (Epstein and Zin (1991) [21]). Relaxing expected utility assumptions (Kreps and Porteus (1978) [41]), incorporating modified probability distributions to admit rare events (Rietz (1988), [55] and Barro (2006) [4]), and considering incomplete markets (Constantinides, Donaldson and Mehra (2002) [15] , Constantinides and Duffie (1996) [14]) are just some risk-based explanations that have been offered to solve the equity premium puzzle.[2]

In my version of the consumption-based asset pricing model, household financial obligations affect the equity risk premium via two channels. The first is the preference channel where individuals' preferences are defined as consumption relative to financial obligations. The framework is analogous to habit formation models where the utility function depends on consumption relative to some habit level (Abel (1990)[1], Constantinides (1990)[24] and Campbell and Cochrane

[2] For more recent literature, look at one of John Cochrane's latest essays based on a talk at the University of Melbourne 2016 Finance Down Under conference [13]. He surveys many current frameworks including habits, long run risks, idiosyncratic risks, heterogenous preferences, rare disasters, probability mistakes, and debt or institutional finance.

(1999)[6]). However, in my model the persistence level is observable, which is an advantage over other standard habit formation models. With this setup, the marginal utility derived from the model is directly related to two components: household consumption and household financial obligations.

The preference channel is important because one possible explanation for consumption variation is the potential impact of a household's debt level on its preferences. If households are averse to holding large amounts of debt relative to income, a decline in income will prompt larger declines in consumption among highly indebted households to restore the desired debt-to-income ratio for a wide range of loss functions (Baker (2015)[3]). Conversely, there is evidence that individuals who are more likely to face income uncertainty or to become liquidity constrained exhibit a higher degree of risk aversion in the presence of uninsurable risk (Guiso and Paiella (2008)[25]). By defining households' preferences as consumption relative to financial obligations, the preference channel captures the time-varying risk aversion behavior of individuals by featuring fluctuations in consumption net of financial obligations over business cycles.

The second channel I investigate is the borrowing constraint channel from which most asset pricing models abstract. [3] In an infinite-horizon aggregate household economy, the financial obligations ratio — defined as total debt payments, housing payments, and auto lease payments divided by total disposable income — act as the borrowing constraint in my model. I show that the equity risk premium implied by the model is sensitive to the financial obligations ratio as a proxy for borrowing constraints.

[3] One example that considers the impact of borrowing constraints on the equity risk premium is Constantinides, Donaldson and Mehra (2002)[15]. They use an overlapping generation model.

But why should the model work? Which elements of the model enables it to explain/generate the observed equity risk premium in U.S. data? The intuition is straight-forward. As consumption rises in good times, households take on more debt and debt payments gradually increase. In bad times, consumption falls and households de-lever slowly. Thus, debt payments move slowly following consumption. Now consider a household that has taken on a specific debt level it must repay. In recessions, as income declines toward this specific level of debt payments, the household reduces its consumption because of risk and risk aversion. Indeed, to ensure that it can make its debt payments, the household exhibits more risk aversion and takes on less risk. This decreases the demand for risky assets and increases the demand for precautionary savings in recessions. During booms, however, consumption moves away from slow-moving financial obligations and hence the household becomes less risk averse and takes on more risk. Thus, the lower ratio of consumption to financial obligations in bad times and the higher ratio in good times directly affect the household's marginal utility and make the pricing kernel more volatile. This is the household time-varying risk aversion that leads to a shift in the composition of its portfolio from risky to risk-free assets, and this is what generates the equity premium.

Also note that in bad times, because of lower income, households face a higher financial obligations ratio. This means that lenders are less willing to lend to them in these states of the economy. Thus, households' borrowing constraints become binding exactly when they want to smooth consumption. Now because they cannot borrow to consume, they must either invest less or liquidate more of their assets. In either case, they start by divesting risky assets rather than risk-free assets (recall that the model features a time-varying risk aversion and people are more risk averse in a recession).

The decrease in demand for risky assets is much faster than the decrease in demand for risk-free assets. This generates a higher risk premium during recessions. This intuition is consistent with my findings when I relax the borrowing constraint. As I let the households borrow more in bad times, the equity risk premium shrinks. Time-varying borrowing constraints are basically factors that make households shift from risky assets to risk-free assets.

In summary, a novel feature of the model is that in states of high marginal utility (i.e., in recessions, when income falls), lenders are reluctant to lend, the borrowing constraint binds, and it becomes more difficult for households to smooth consumption exactly when they need to do so. In addition, in these states, households become more risk averse. This dual mechanism both amplifies the risk premia and makes it time varying.

The economic variable explored in this essay — the financial obligations ratio — provides a fresh opportunity to investigate the determinants of asset risk. As a start, in separate research, I (Jahangiry (2016a)[32]) document that the risk associated with aggregate household financial obligations is an economy wide risk and is significant for explaining the variations in the cross-section of stock returns. Conditioning down on the financial obligations ratio, the FCAPM proposed in Jahangiry (2016a)[32], survives a wide range of classical econometric and diagnostic tests for explaining the variations in average returns across 25 portfolios formed based on size and book-to-market ratio. In another study, I (Jahangiry (2016c)[34]) take one step further and test the predictability of stock returns/excess returns using households obligations ratio. Using U.S. stock market data, I show that the household's debt service ratio can predict market returns at short horizons and over business cycle frequencies. Jahangiry (2016c) argues that mean deviations from the debt service ratio are a

better forecaster of future returns both in sample and out of sample than several other popular forecasting variables.

The rest of this chapter proceeds as follows. In Section 3.2, I set up the model and derive the fundamental equations of asset pricing. In Section 3.3 I discuss the data and estimations. I provide the results of the model in Section 3.4 and the conclusion in Section 3.5.

## 3.2 The Model

### *3.2.1 Environment*

I consider an infinite horizon endowment economy in which the agents are endowed with an uninsurable stochastic income at each period. The agents in this economy are:

1. Large number of homogeneous households and
2. A lending institution.

I use a modified version of the Greenwook-Hercowitz-Huffman utility structure that enables me to represent aggregate households with a representative agent. Therefore, I have a representative agent environment with an external supply of debt provided by lending institutions. This can be thought of as a small open economy. Assume that the lending institution is aware of the income distribution of the representative agent. There are three assets and two markets in this economy. The assets are one perishable consumption good and two durable assets: an inside security and an outside security (debt instrument). The inside security provides dividends according to an exogenous stochastic process and the outside security is exogenously supplied by the lending institution. The environment is summarized in Table 3.1. The two markets in the model are:

1. The **capital market**, which is a market for allocating idiosyncratic risk among households. In this capital market everything is in zero net supply. Households can trade contingent claims (inside securities) among themselves but because all households are identical, the prices are shadow prices for a no-trade equilibrium.
2. The **debt market**, in which there is an outside supply of debt (outside security) provided by a lending institution. I do not model the supply side of

the debt market. The lending rate is an exogenously specified rate. Households can borrow from this lending institution to a certain amount defined by their financial obligations ratio.

**Table 3.1:** The Model Environment

| **Capital Market** | **Debt Market** |
|---|---|
| Households | Households + a lending institution |
| One inside security | One outside security |
| Zero net supply | Positive net supply |
| No trade equilibrium | Exogenous prices |

### *3.2.2 Constraints*

In each period, households are endowed with stochastic exogenous income that they can either consume or invest in an inside security. They are allowed to borrow against their stochastic income and use it only for consumption purposes. The model has a non stationary environment because of the non stationarity of the stochastic aggregate income and the exogenous dividends of the inside security. However, the exogenous stochastic borrowing rates are stationary. Consumption, financial obligation, dividends, aggregate disposable income, prices of the equity, and the risk-free bond are all denominated in units of the consumption good.

In the model, for each period $t$, $C_t$ is aggregate consumption, $Y_t$ is aggregate disposable income, and $X_t$ represents the dividends generated by the inside security. By making the wage income process exogenous, I abstract from the labor-leisure trade-off. This means that labor is supplied inelastically and the labor-leisure choice is not modeled. $D_t$ is the debt service level borrowed by the representative agent at the gross rate $R_t^d$. I assume that the lending institutional arrangement can issue

and redeem debt instruments. This lending institution exogenously set a lending cap $\theta_t$ on each household. Hence, $\theta_t$ provides a state-dependent upper bound for the household's borrowing capacity. $Z_t$ is a non-negative amount of investment in an inside security with an ex-dividend price of $p_t$ at time $t$. The agent faces the following constraints:

1. Budget constraint

$$C_t + p_t Z_t + D_{t-1} R^d_{t-1} \leq Y_t + (p_t + X_t) Z_{t-1} + D_t \tag{3.1}$$

2. Financial obligations constraint

$$D_t R^d_t \leq \theta_t Y_t, \quad D_t \geq 0. \tag{3.2}$$

Inequality (3.1) is the budget constraint the agent faces in each period. The agent comes into the period with stochastic wage income $Y_t$. There is also income from securities purchased in the last period. The agent can liquidate $Z_t$ amount of inside security at price $p_t$ with dividend $X_t$. Further-more, the agent can borrow against his stochastic income at amount $D_t$. These are the resources of funds. Now, the left-hand side of (3.1) shows how the agent spends the available funds. First he consumes $C_t$, then he can purchase an inside security to take to the next period, and finally he must pay interest on the debt he borrowed. $R^d_{t-1}$ is the gross return on the debt instrument, and it means that the agent has to pay back whatever he has borrowed in last period plus the interest. At $t = 0$, the representative agent begins debt free $D_{-1} = 0$, and he is endowed with nothing but stochastic income $Y_0$ i.e., $Z_0 = 0$.

Inequality (3.2) is the borrowing constraint. It indicates how much the agent can borrow against his labor income. This financial obligations ratio constraint (3.2) is a type of constraint that we observe in the economy. Interest payments on debt over

income is a number that lenders would like see below some certain levels such as 1/3 or 1/4, and this number varies over time. Inequality (3.2) is a constraint I impose in this model and it is one of the innovations of the model, the borrowing constraint channel [4] . Note that income $Y_t$, dividends $X_t$, and obligation ratio cap $\theta_t$ are all exogenous stochastic variables. For calculation purposes, I work with detrended income $y_t$[5] and dividend growth $x_t$ which are determined by the following Markov processes:

$$y_{t+1} = (1 - \rho_y)\bar{y} + \rho_y y_t + \epsilon^y_{t+1}, \tag{3.3}$$

$$x_{t+1} = (1 - \rho_x)\bar{x} + \rho_x x_t + \epsilon^x_{t+1}. \tag{3.4}$$

In equations (3.3) and (3.4), $\bar{y}$ and $\bar{x}$ are the averages of detrended aggregate income and dividend growth, respectively. $\rho_y$ and $\rho_x$ are the auto-correlations, and epsilons are the relevant shocks associated with $y$ and $x$. Finally, the exogenous process for financial obligations ratio cap $\theta_t$ is defined as in (3.5):

$$E_t(\theta_{t+1}|y_t) = f(y_t). \tag{3.5}$$

Equation (3.5) implies that the process of $\theta$ is totally pinned down by the process of $y$. This assumption is intuitive as the lending institution sets $\theta$ exogenously and it is aware of the household's income distribution. Timing of the constraints is as follows. At time $t$, the agent knows at what rate he can borrow, so $R^d_t$ is measurable with respect to time $t$. According to the financial obligations constraint in (3.2), $D_t$ is also measurable at time $t$. Thus, the only random variables here are $Y$ and $\theta$. Note that

[4]The innovation is introducing a state dependent time-varying borrowing constraint. I make the borrowing capacity to be state dependent. In bad times, the financial obligations ratio is higher (because of negative income shock) and the agent's borrowing capability shrinks thereafter. This forces the agent to further reduce his consumption in bad times.

[5]In our sample, since the income follows a deterministic trend, we detrend the income by time-detrending.

there is no default in this model. Under my parameterization, the agent can always reduce his consumption such that he has positive net worth. In other words, he never has a realization of $Y$ so low that he cannot pay off his debt by reducing consumption. In addition, the lender chooses $\theta$ conditional on some expectation of future income of the agent. Therefore, if the lender's conditional expectations of future income are low, it will lower $\theta$ to make sure the agent can pay off his debts. Hence, the lender is building expectations of $\theta_{t+1}$ based on $y_t$. It means that when expected income is low, the lender reacts by decreasing $\theta_{t+1}$ conditional on $y_t$ such that the conditional expectation of $\theta$ is lower than it's expected value, that is, $E_t(\theta_{t+1}|Y_t) \leq \bar{\theta}$.

In short, in this model, the lender is the one monopolist and everybody else is a price taker. This monopolist has some expectations of agents' income and is going to reduce the amount that agents can borrow, precisely when they would like to borrow next period. This is the building block of my model and it shows how the model generates a more volatile marginal utilities. In bad times, effective consumption (consumption net of financial obligations) is smaller and in good times it is bigger than the standard consumption in a Mehra-Prescott (1995) world. Therefore, with this set-up, everything is conditional on $Y_t$. The variations in the marginal rate of substitution determine the returns on the inside security, and hence, these are the extra variations in dividends that will generate higher equity premiums.

### *3.2.3 Preferences*

The utility function presented here shows how the financial obligations ratio affects the equity risk premium via the preference channel. In my model, the agent's preferences are defined as consumption relative to financial obligation $G$. This is a behavioral set-up that is analogous to habit formation models. In the sense that,

where as in habit formation models, *the distance from the consumption habit* gives the agent utility, in my model *the distance from the financial obligation* does the same job. More specifically, I use the simple power utility function defined as the representative agent's effective consumption $C^*$, where $C^*$ is consumption net of financial obligations incorporating the distance from the financial obligations. Effective consumption is defined as $C_t^* = C_t - G_t$ where $G_t = D_{t-1}R_{t-1}^d$ is the financial obligations the agent carries over to period $t$ from the last period. Hence, the utility function of the agent is:

$$U(C_t^*) = \frac{C_t^{*(1-\gamma)}}{1-\gamma} = \frac{1}{1-\gamma}\left(C_t - G_t\right)^{(1-\gamma)} \tag{3.6}$$

Equation (3.6) suggests that a household with lower financial obligations has a higher effective consumption and hence receives a higher utility. This behavioral setup is chosen based on insights from economic psychology literature on the psychological impact of being in debt. Financial obligations are associated with high levels of anxiety and stress (Brown, Taylor, and Price (2005)[5], Richardson, Elliott, and Roberts (2013)[54]). And more important, this impact is independent of the poverty with which it is often associated (Jenkins et al. (2008)[35], Meltzer et al. (2011)[51]). This is the *behavioral* rational to include financial obligations in the utility function.

There is also a *structural* reason. Specifically, I use difference form $(C - G)$(Constantinides (1990)[24], Campbell, and Cochrane (1999)[6], rather than the ratio form $(\frac{C}{G})$(Abel (1990)[1], because the difference form generates a time-varying relative risk aversion. [6] Time-varying risk aversion plays an important

[6] However, there is no consensus on the pro cyclicality or counter cyclicality of relative risk

role in determining the equity premium, especially during recessions, because "recessions are phenomena of risk premiums, risk aversion, risk bearing capacity and desires to shift the composition of a portfolio from risky to risk-free assets, a flight to quality, not a phenomenon of intertemporal substitution, a desire to consume more tomorrow vs. today." [7] Also note that in this model, $C_t - G_t$ is always positive. People slowly develop financial obligations, so consumption is always greater than debt obligations (no default assumption); indeed, financial obligations form the trend in consumption.

With this specification in (3.6), the coefficient of relative risk aversion is:

$$RRA = -C\frac{u''(C)}{u'(C)} = \gamma\left(\frac{1}{\frac{C-G}{G}}\right) = \frac{\gamma}{S},$$

where $S = \frac{C-G}{C}$ is the consumption surplus. This is analogous to the Campbell and Cochrane (1999) habit model, with financial obligations replacing the consumption habit. It is important to note that the persistence level in my model is observable, which is an advantage to over Campbell and Cochrane's external habit model in which the habit level is not observable. However, the idea is the same: in bad times, as consumption or the surplus consumption ratio $S$ decreases, the agent's relative risk aversion rises; that is, the same proportional risk to consumption is a more fear-inducing event when consumption starts closer to financial obligations $G$.

The link between consumption surplus $S$ and the financial obligations ratio is straight-forward. Financial obligations $G$ is like a slow-moving habit in this model. In recessions, when a negative shock to the aggregate income is realized, it increases the current financial obligations ratio of the representative agent. According to the

aversion. Risk aversion is counter cyclical in habit formation models (in recessions, the consumption surplus ratio is lower so risk aversion is higher) and pro cyclical in happiness maintenance models (in good times, individuals become more risk averse as they wish the good days will never end).

[7] John Cochrane, Macro-Finance, Feb 2016 [13]

budget constraint(3.1) and the borrowing constraint (3.2), the agent has no option but to decrease his consumption $C_t$[8] . This moves consumption closer to slow-moving obligation $G$ and hence reduces consumption surplus $S$. Therefore, in recessions (negative income shocks), the financial obligations ratio is high, consumption surplus is low, and relative risk aversion is high. This enables the model to deliver a time-varying, recession-driven equity risk premium.

### *3.2.4 Stochastic Sequential Problem*

The representative agent maximizes the following sequential problem :

$$Max \quad E_0 \left\{ \sum_{t=0}^{t=\infty} \beta^t U(C_t^*) \right\},$$

where $U(C_t^*) = \frac{C_t^{*1-\gamma}}{1-\gamma}$ and $C_t^* = C_t - G_t$ subject to the budget constraint (3.1), borrowing constraint (3.2), exogenous stochastic processes (3.3)-(3.5), and the the non-negativity constraints:

$$G_t = D_{t-1} R_{t-1}^d$$

$$C_t + p_t Z_t + D_{t-1} R_{t-1}^d \leq Y_t + (p_t + X_t) Z_{t-1} + D_t$$

$$D_t R_t^d \leq \theta_t Y_t$$

$$y_{t+1} = (1 - \rho_y)\bar{y} + \rho_y y_t + \epsilon_{t+1}^y$$

$$x_{t+1} = (1 - \rho_x)\bar{x} + \rho_x x_t + \epsilon_{t+1}^x$$

$$E_t(\theta_{t+1}|y_t) = f(y_t)$$

$$C_t \geq 0, \quad D_t \geq 0, \quad Z_t \geq 0$$

$$\text{Given} \quad X_0, Y_0, R_0^d \quad , \quad Z_0 = D_{-1} = 0$$

[8]This is true because: (1) there is a no-trade equilibrium — the representative agent does not have the option of liquidating assets; (2) it is implicitly assumed that the agent cannot default on his debt payments so he must pay back $D_{t-1} R_t^d$ in full; and (3) in bad times the financial obligations ratio is already capped so the agent cannot leverage any more.

For simplification, I assume the exogenous lending rate is constant and equal to $R_t^d = \bar{R}^d$. The transversality condition holds for financial obligations, which implies that the shadow value of debt service must be equal to zero in the limit. I also assume Inada conditions on effective consumption. The economy is completely specified by the preference parameters $\beta$, $\gamma$, and realization of the stochastic processes followed by $\Lambda_t = (y_t, x_t, \theta_t)$. Equilibrium is defined as the sequences of consumption $\{\tilde{C}_t\}$, investment $\{\tilde{Z}_t\}$, borrowing decisions $\{\tilde{D}_t\}$ of the representative agent, and prices $p_t$ such that:

1. Taking the prices and exogenous vector $\Lambda$ as given, the sequences of consumption, investment, and borrowing decisions optimizes households' lifetime expected utility.

2. Consumption, capital, and debt markets clear in all periods.[9]

### *3.2.5 Pricing Kernel*

Given the utility function in (3.6), the intertemporal marginal rate of substitution (the pricing kernel) is: [10]

$$M_{t+1} = \beta \frac{U_c(C_{t+1}^*)}{U_c(C_t^*)} = \beta \left(\frac{C_{t+1}}{C_t}\right)^{-\gamma} \left(\frac{S_{t+1}}{S_t}\right)^{-\gamma}, \tag{3.7}$$

where $S_t = \frac{C_t - G_t}{C_t}$. The pricing kernel is related to consumption growth and consumption surplus, which depends on financial obligations and is an implicit state

[9]Note that there is no need to clear the debt market; as mentioned earlier, debt service rates are exogenously determined.

[10]The log-linearized version of the pricing kernel and interpretation of the interest rate is provided in Appendix B.

variable. I can now calculate moments of the marginal rate of substitution (3.7) and find asset prices. Taking the first-order conditions (FOCs) with respect to $C_t$, $D_t$, and $Z_t$ and combining the results gives the "fundamental equations of asset pricing."

From FOC ($[C_t], [Z_t]$) the price of the inside security is:

$$p_t = E_t \left( \beta \left( \frac{C_{t+1}}{C_t} \right)^{-\gamma} \left( \frac{S_{t+1}}{S_t} \right)^{-\gamma} (p_{t+1} + X_{t+1}) \right) \tag{3.8}$$

With equation (3.8) the price of any inside security can be derived given its dividend. To calculate the equity risk premium the price of risk-free bond $q_t$ is needed. Using (3.8) and the fact that no-coupon Treasury bonds are traded in discounted values, the price of risk-free bond is equal to:

$$q_t = E_t \left( \beta \left( \frac{C_{t+1}}{C_t} \right)^{-\gamma} \left( \frac{S_{t+1}}{S_t} \right)^{-\gamma} 1 \right) \tag{3.9}$$

Market-clearing conditions imply that $Z_t = 0 \quad \forall t \geq 0$; this is because every household is the same and the equilibrium outcome must be the no-trade outcome. I am interested in finding the prices that support this no-trade outcome. From market clearings and budget constraints, the equilibrium consumption sequence is:

$$C_t = Y_t + D_t - D_{t-1} R_t^d. \tag{3.10}$$

Only one step remains to finding an explicit-form solution for the equity price and the risk-free bond price. Fortunately, it is easy to show that the debt service ratio constraint in (3.2) is binding.[11] This implies that the representative agent will cap

[11] Note that at each period $t$, the utility function $U_t$ is strictly increasing in consumption $C_t$, so the

the amount of borrowing and paves the way for calculating the equity premium and the risk-free rate implied by the model. Using the equilibrium consumption path in (3.10) along with equations (3.8) and (3.9), gives the fundamental equations of asset pricing at the equilibrium.

Note that it is convenient to define $w_t = \frac{p_t}{X_t}$ as the price-dividend ratio because it allows the equilibrium equity returns to be written down in terms of dividend growth, which is stationary, and not the dividend itself, which is not stationary. Therefore, by dividing both sides of (3.8) by $X_t$, (3.8) can be written as:[12]

$$w_t = \beta E_t \left\{ \left( \frac{C_{t+1}}{C_t} \right)^{-\gamma} \left( \frac{S_{t+1}}{S_t} \right)^{-\gamma} (1 + w_{t+1})(\frac{X_{t+1}}{X_t}) \right\}, \tag{3.11}$$

where $\frac{X_{t+1}}{X_t}$ is dividend growth. Equations (3.9) and (3.11) can be used to solve for the risk-free rate $R_t^f$, equity returns $R_t^e$, and consequently the equity premium $EP_t$.

$$R_t^f = \frac{1}{q_t} = \frac{1}{\beta E_t \left( \left( \frac{C_{t+1}}{C_t} \right)^{-\gamma} \left( \frac{S_{t+1}}{S_t} \right)^{-\gamma} \right)} \tag{3.12}$$

$$R_{t+1}^e = E_t \left( \frac{p_{t+1} + X_{t+1}}{X_t} \right) = E_t \left( \frac{w_{t+1} X_{t+1} + X_{t+1}}{w_t X_t} \right) = E_t \left\{ \left( \frac{X_{t+1}}{X_t} \right) \left( \frac{1 + w_{t+1}}{w_t} \right) \right\}, \tag{3.13}$$

where $w_t$ is defined as in (3.11).

Finally, the equity premium is simply defined as (3.13) minus (3.12):

budget constraint and the borrowing constraint bind to guarantee that the agent is maximizing his preferences. However, one may argue that as financial obligations increase, effective consumption decreases. This is not true here, as current obligation $G_t$ is a function of last-period debt service $D_{t-1}$ and not the current borrowing $D_t$.

[12]The right-hand side of (3.8) is a *conditional* expectation; by applying the Law of iterated expectations $X_t$ can be taken into the conditional expectation.

$$EP_t = E_t(R^e_{t+1}) - R^f_t \tag{3.14}$$

In the end, if the exogenous variables follow Markov processes, then solving functional equations in (3.12) and (3.13) is simply solving a finite system of linear equations. Thus defining the exogenous stochastic processes for aggregate real income and the dividend growth rate, the model can be tested by comparing the observed equity premium and risk-free rates in U.S. data to the rates implied by the model. Note that the model deals with a *non stationary* environment because of the non stationarity of aggregate dividends and aggregate income. This enables the model to generate a non stationary equilibrium consumption path, non stationary equity prices, and a stationary equity premium, which is consistent with the data.[13]

## 3.3 Data and Estimations

The motivation of this chapter is to compare the equity risk premium observed in U.S. data with the equity risk premium generated by the model presented in this chapter. I use annual data from 1980 to 2015 to report the equity premium observed in the data. Table 3.2 summarizes the data source.

Next, the equity premium implied by the model is estimated in a calibrated endowment economy. Generally, the solutions to asset pricing equations (3.8) and (3.9) are not available in an analytically simple closed form. However, there are instances where calculation of the exact solution, or a good approximation, is possible. In my model the binding borrowing constraint does the job and makes it

[13]Note that the risk-free bond prices generated by (3.9) are stationary because $q_t$ depends only on consumption growth and financial obligations growth, which are both stationary.

**Table 3.2:** Data Source

| Variables | Data source (1980-2015) |
|---|---|
| S&P composite prices and dividends | Robert J. Shiller data |
| One month T-Bill returns | Center for Research in Security Prices |
| Consumer Price Index | U.S. Bureau of Labor Statistics |
| Aggregate income per capita | National Income and Product Accounts |
| Household financial obligations ratio | Federal Reserve Bank of St. Louis |

possible to solve for closed-form solutions. Indeed, solutions to functional equations (3.12) and (3.13) depend on values of three exogenous stochastic processes for $y_t, x_t$, and $\theta_t$. Note that $R_t^d$ has already been assumed to be constant and equal to $\bar{R}^d$. More specifically, using the weighted average annual rate of commercial bank interest rate on credit cards and one-year adjustable rate mortgages, the average gross borrowing rate is equal to 1.085.

A typical problem is how to characterize the price of an asset, where the laws of motion for exogenous stochastic state variables are AR(1) processes. Therefore, the next step is to check whether the stochastic processes for $y$ and $x$, are stationary over time and then estimate them with an AR(1) process. Table 3.3 summarizes the results of stationarity tests for detrended aggregate income and dividend growth using two methods: augmented Dickey-Fuller unit root test (ADF)(1979)[16], and the Kwiatkowski-Phillips-Schmidt-Shin (KPSS)[42] stationarity test.

As Table 3.3 suggests, the ADF test statistics for detrended income $y$ and aggregate dividend growth $x$ are equal to -5.343 and -5.042, respectively, meaning that the null hypothesis can be rejected (null: $y_t$ and $x_t$ exhibit unit root

**Table 3.3:** Stationarity Test

| | **ADF Test Statistic** | | **KPSS Test Statistic** | |
|---|---|---|---|---|
| | $y_t$ | $x_t$ | $y_t$ | $x_t$ |
| | **-5.343** | **-5.042** | **0.243** | **0.273** |
| 1 % level | -3.643 | | 0.739 | |
| 5 % level | -2.954 | | 0.463 | |
| 10 % level | -2.615 | | 0.347 | |

properties). Also, using the KPSS test, the null hypothesis that the variables are stationary over time cannot be rejected as the test statistics are smaller than the critical values. Thus, it is reasonable to estimate the stochastic processes for income and dividend growth with an autoregressive process AR(1).

For deriving a numerical closed-form solution and generating the equity risk premium implied by the model, the last step is to discretize AR(1) processes for income and divided growth. I use the Rouwenhorst (1995)[56] technique to discretize the AR processes. In this study, the Rouwenhorst method is prefered to the Tauchen (1986)[58] approach for the following reasons. First, the residuals of both AR(1) processes pass the autoregressive conditional heteroskedasticity (ARCH) test; that is, the heteroskedasticity of residuals can be rejected. Second, I discretize the economy with a two-state Markov process for each $y$ and $x$. When the number of states is small (equal to two here), the Rouwenhorst technique outperforms the Tauchen approach. Recall that the AR(1) stochastic processes for income and dividend growth are as follows:

$$y_{t+1} = (1 - \rho_y)\bar{y} + \rho_y y_t + \epsilon^y_{t+1}$$

$$x_{t+1} = (1 - \rho_x)\bar{x} + \rho_x x_t + \epsilon^x_{t+1}$$

. In my model, the stochastic process for the financial obligations cap is fully

determined by the process for aggregate income. Table 3.4 summarizes the estimated parameters of AR(1) processes.

**Table 3.4:** AR(1) Estimations

| Parameter/Variable | | **Description** |
|---|---|---|
| $y$ | Normalized labor income per capita (detrended) | $\bar{y} = 1$ , $\rho_y = 0.64$ , $\epsilon_y = 0.08$ |
| $x$ | Dividend growth | $\bar{x} = 1.032$ , $\rho_x = 0.61$ , $\epsilon_x = 0.05$ |
| $\theta$ | Financial obligations ratio cap | $\bar{\theta} = 0.165$, $\sigma_\theta = 0.009$ |

The economy is completely specified by the realization of the joint stochastic process followed by aggregate real income and dividend growth. I model the joint process of aggregate income and dividend growth as a time-stationary Markov chain with a nondegenerate, unique, stationary probability distribution. Starting with real aggregate income, I assume that $y_t$ follows a two-state Markov chain $(y, Q, \pi^Y)$ where $y$ is the state vector, $Q$ is a $2X2$ transition matrix, and $\pi^Y$ is the probability distribution. The two states are high (H) and low (L), which stand for high and low aggregate income during booms and recessions, respectively. Therefore, I define the states vector $y$ as:

$$y = (y^H, y^L) = (\mu^Y + \delta^Y, \mu^Y - \delta^Y),$$

where $\mu^Y$ is long-run aggregate income and $\delta^Y$ is its standard deviation. The probability distribution $\pi^Y$ is defined as :

$$\pi^R_{ij} = Pr(y_{t+1} = y_j | y_t = y_i)$$

Then the transition matrix $Q$ is symmetric with:

$$Q = \begin{bmatrix} q_{11} & q_{12} \\ q_{21} & q_{22} \end{bmatrix} = \begin{bmatrix} \pi^Y_{11} & 1 - \pi^Y_{11} \\ 1 - \pi^Y_{22} & \pi^Y_{22} \end{bmatrix} = \begin{bmatrix} \phi^Y & 1 - \phi^Y \\ 1 - \phi^Y & \phi^Y \end{bmatrix},$$

where $\phi^Y$ is the long-run auto correlation coefficient of $y$.

Next, I assume that dividend growth $x_t$ also follows a Two-states Markov process $(x, P^k, \pi^X)$ where $x$ is the state vector and $P^k$ is a $2X2$ transition matrix where $k$: $H, L$, meaning that the transition matrix for $x$ depends on whether $y$ is in a high or low state. Also $\pi^X$ is the probability distribution. Again the two states are high (H) and low (L), standing for high and low dividend growth. I define the states vector $x$ as:

$$x = (x^H, x^L) = (\mu^X + \delta^X, \mu^X - \delta^X),$$

where $\mu^X$ is long-run aggregate dividend growth and $\delta^X$ is its standard deviation. The probability distribution $\pi^X$ is defined as:

$$\pi^X_{ij} = Pr(x_{t+1} = x_j | x_t = x_i).$$

Then the transition matrix $P^k$ is symmetric with:

$$P^k = \begin{bmatrix} \pi^X_{11} & 1 - \pi^X_{11} \\ 1 - \pi^X_{22} & \pi^X_{22} \end{bmatrix} = \begin{bmatrix} \phi^X_k & 1 - \phi^X_k \\ 1 - \phi^X_k & \phi^X_k \end{bmatrix},$$

where $\phi^X_k$ is the long-run auto-correlation coefficient of $x$ conditional on whether $y$ is in its high $(k : y_H)$ or low $(k : y_L)$ states. Therefore, the exogenous joint stochastic processes for $y$ and $x$ follow a four-state coupled Markov chain with an 1-by-4 probability distribution $\pi$ and a $4X2$ state matrix $yx$:

$$yx = \begin{bmatrix} yx_{HH} \\ yx_{HH} \\ yx_{HH} \\ yx_{HH} \end{bmatrix} = \begin{bmatrix} y_H & x_H \\ y_H & x_L \\ y_L & x_H \\ y_L & x_L \end{bmatrix},$$

and a $4X4$ transition matrix $S_{YX}$:

$$S_{YX} = \begin{array}{c} \\ HH \\ HL \\ LH \\ LL \end{array} \begin{array}{c} \begin{array}{cccc} HH & HL & LH & LL \end{array} \\ \begin{pmatrix} s_{11} & s_{12} & s_{13} & s_{14} \\ s_{21} & s_{22} & s_{23} & s_{24} \\ s_{31} & s_{32} & s_{33} & s_{34} \\ s_{41} & s_{42} & s_{43} & s_{44} \end{pmatrix} \end{array},$$

where $s_{ij}$ can be calculated directly from transition matrices $Q$ and $P$. For example, $s_{12} = S_{YX_{HL,HH}}$ and is derived by:

$$s_{12} = S_{yx_{HL,HH}} = \mathbf{Pr}(yx_{t+1} = yx_{HL} | yx_t = yx_{HH}) = \mathbf{Pr}\left(\begin{array}{c|c} y_{t+1} = y_H & y_t = y_H \\ x_{t+1} = x_L & x_t = x_H \end{array}\right)$$

$$s_{12} = \mathbf{Pr}(x_{t+1} = x_L, | x_t = x_H, y_t = y_H) * \mathbf{Pr}(y_{t+1} = y_H, | y_t = y_H) = P^H_{12} * q_{11},$$

where $P^H_{12}$ is the 1, 2 element of the matrix $P^H$. Similar reasoning leads to the following transition matrix for $S_{YX}$:

$$S_{YX} = \left(\begin{array}{cc|cc} P^H_{11}q_{11} & P^H_{12}q_{11} & P^L_{11}q_{12} & P^L_{12}q_{12} \\ P^H_{21}q_{11} & P^H_{22}q_{11} & P^L_{21}q_{12} & P^L_{22}q_{12} \\ \hline P^H_{11}q_{21} & P^H_{12}q_{21} & P^L_{11}q_{22} & P^L_{12}q_{22} \\ P^H_{21}q_{21} & P^H_{22}q_{21} & P^L_{21}q_{22} & P^L_{22}q_{22} \end{array}\right) = \begin{pmatrix} P^H q_{11} & P^L q_{12} \\ P^H q_{21} & P^L q_{22} \end{pmatrix}$$

It is easy to confirm that $S_{YX}$ is a bona fide transition matrix. In summary, to solve the model the parameters in Table 3.5 need to be estimated.

**Table 3.5:** Parameters/Variables to Be Estimated

| **Item** | **Description** |
|---|---|
| $\beta$ | Constant discount factor, time preference |
| $\gamma$ | Coefficient of relative risk aversion |
| $\theta$ | Exogenous predetermined upper bound for the FOR |
| $\mu^Y$ | Long-run average annual aggregate income (detrended) $y$ |
| $\delta^Y$ | Standard deviation of $y$ (annual) |
| $\phi^Y$ | First-order autocorrelation coefficients of $y$ (annual) |
| $\mu^X$ | Long-run average aggregate dividend growth $x$ |
| $\delta^X$ | Standard deviation of $x$ (annual) |
| $\phi_k^X$ | First-order autocorrelation coefficients of $x$ for $k$: $H, L$. (annual) |

The acceptable value for $\beta$ based on different macroeconomic models ranges from 0.95 to 0.99. I set $\beta = 0.98$ and let the coefficient of relative risk aversion $\gamma$ vary from 1 to 3. In the "Results" section of this chapter I show that, unlike in the standard consumption based model, my model can get close to the equity risk premium observed in U.S. data even for small values of risk aversion. The rest of the parameters and variables are estimated as illustrated in Table 3.6.

## 3.4 Results

In this section, I show that the model can get close to the equity risk premium observed in U.S. data while keeping the risk-free rate low. Table 3.7 summarizes the results for different specifications of the model. As Table 3.7 suggests, the model

**Table 3.6:** Estimated Values for Markov Processes

| **Parameter** | **Estimation** |
|---|---|
| $\mu^Y$ | 1.00 |
| $\delta^Y$ | 0.06 |
| $\phi^Y$ | $= \frac{1+\rho_y}{2} = \frac{1+0.64}{2} = 0.82$ |
| $\mu^X$ | 1.04 |
| $\delta^X$ | 0.07 |
| $\phi_k^X$ | $= \frac{1+\rho_x}{2} = \frac{1+0.61}{2} = 0.805$ |

presented in this chapter outperforms the standard consumption model in explaining the equity risk premium observed in the data. The equity premium puzzle states that the mean excess return calculated in the standard consumption-based model is too low unless the coefficient of relative risk aversion is implausibly high. For the standard model this number is 20, which makes no economic sense. However, in this study, the model generates an equity premium of 4.62% and the utility curvature is only 3. This is consistent with the intuition of my model. In bad times, as consumption gets closer to a household's financial obligation, people become more risk averse (as they have to pay back their obligations) and take on less risk. This leads to less investment in the risky asset and eventually a higher equity risk premium.

Another advantage is that unlike the standard model, my model keeps the risk-free rate relatively stable and low. For relative risk aversion between 1 and 3, the risk-free rate induced by the model varies between 0.56% and 1.87%. This is true because in my setup, as the coefficient of relative risk aversion increases, the precautionary savings dominate the intertemporal substitution effect faster than in the standard model and generate lower interest rates.

**Table 3.7:** Results: Annual Returns

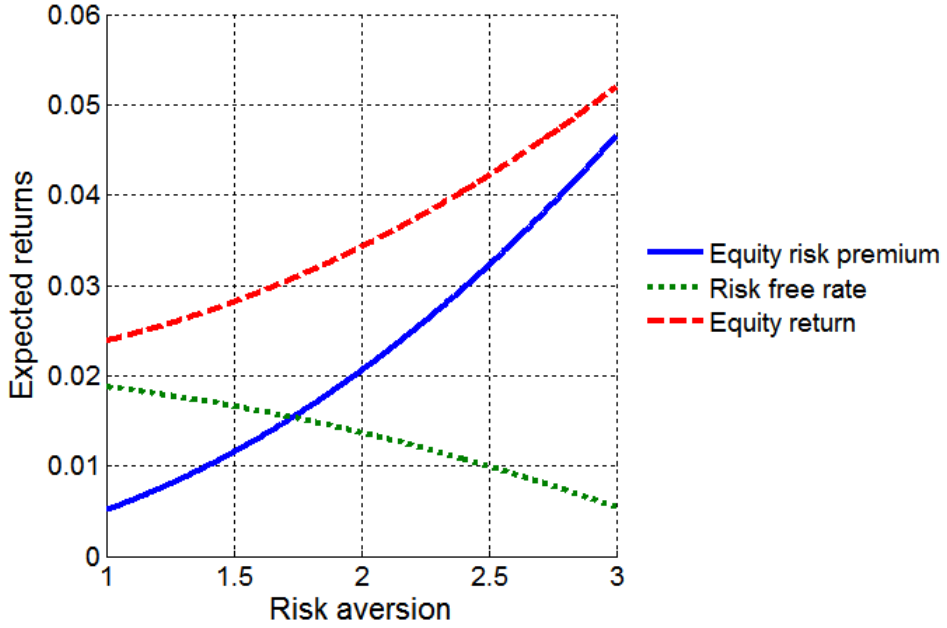
| | Risk Aversion | Risk-Free Rate | Equity Returns | Equity Premium |
|---|---|---|---|---|
| Data | 1 | 0.80 % | 5.60 % | **4.80** % |
| Mehra-Prescott model (1985) | 1 | 4.82 % | 4.95 % | 0.13 % |
| | 3 | 7.97 % | 8.46 % | **0.49** % |
| Our model | 1 | 1.87 % | 2.38 % | 0.51 % |
| | 3 | 0.56 % | 5.17 % | **4.62** % |

### *3.4.1 Returns and the Utility Curvature*

Figure 3.1 shows how the equity premium, risk-free rate, and equity returns vary with the utility curvature $\gamma$. As $\gamma$ increases, agents become more risk averse to any bet. In this model, precautionary savings play an important role because households are afraid of bad times during which the financial obligations ratio is high. Because households are restricted by the borrowing constraint and cannot leverage further because of the already-capped financial obligations ratio, they demand more of precautionary savings (Treasury bond investments) to smooth their consumption for bad states of the economy. This higher demand for risk-free bonds increases the bond price $q_t$ and thus decreases the risk-free rate. The dotted line in Figure 3.1 shows the risk-free rates for different values of risk aversion between 1 and 3.

Having invested more in Treasury bonds, the demand for equity investment decreases and the expected equity returns increase for higher values of $\gamma$. The dashed line shows the equity returns for different values of risk aversion. Hence, by generating higher equity returns and lower risk-free rates, the model gets close to the equity risk premium observed in the data. The solid line shows the equity risk premium generated by the model versus the different values for utility curvature.

**Figure 3.1:** Expected Returns and the Coefficient of Risk Aversion

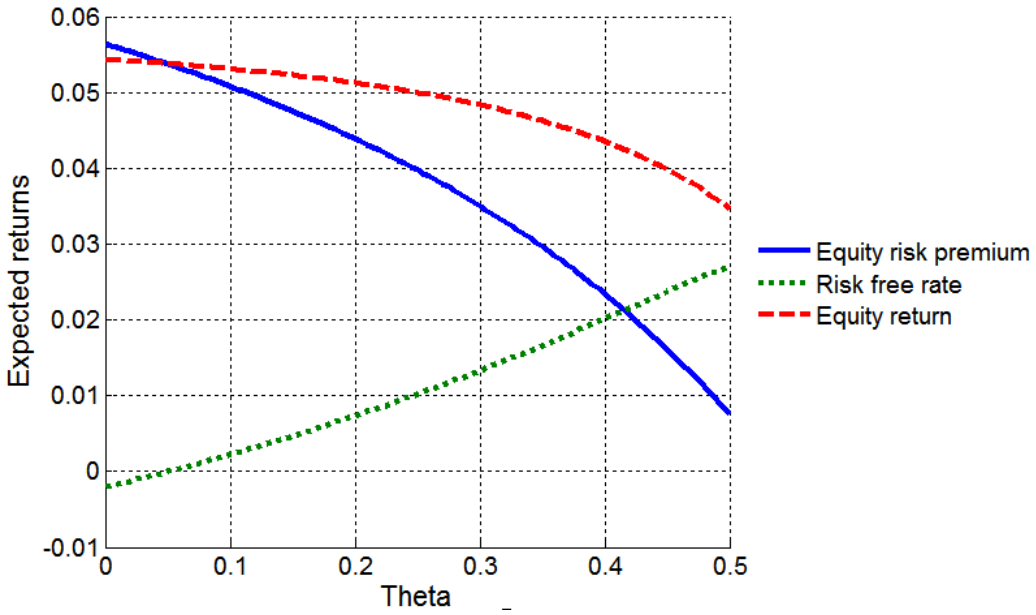



### *3.4.2 Expected Returns and the Borrowing Constraints*

Figure 3.2 shows how the equity premium, the risk-free rate, and equity returns vary with the household's obligation ratio $\theta$. Recall that the financial obligations ratio is a direct indicator of the borrowing constraint in my model, so relaxing the borrowing constraint is equivalent to increasing the household financial obligations ratio cap $\theta$. Figure 3.2 indicates that as the borrowing constraint is relaxed, the equity risk premium shrinks, which is numerically consistent with results in Constantinides, Donaldson, and Mehra (2002)[15].

The intuition is straight-forward. According to the financial obligations ratio constraint (the borrowing constraint) in equation (3.2), households choose a debt service level that caps their financial obligations ratio at anytime. Hence, as this $\theta$ increases, agents can smooth their consumption much more easily via larger borrowings. This leads to a decrease in the demand for precautionary savings

**Figure 3.2:** Expected Returns and the Household Obligations Ratio




(Treasury bond investments), driving down bond prices $q_t$ and increasing the risk-free rate. This is shown by the dotted line in Figure 3.2. Conversely, as $\theta$ increases, households have more funding resources for their consumption purposes and they can make more investments. The level of equity investment increases (as more borrowing is consumed and the investment portfolio is more heavily weighted toward equity investment rather than bond investment), leading to lower equity returns. The dashed line shows equity returns for different values of $\theta$. These two effects decrease the equity premium as the level of $\theta$ increases, shown by the solid line in Figure 3.2.

## 3.5 Conclusion

This chapter addresses how, in an infinitely lived representative agent endowment economy, household financial obligations affect the equity risk premium. The effect is studied under two channels: the preference channel and the borrowing constraint channel. The financial obligations ratio is a counter cyclical indicating variable that affects agents' marginal utility of consumption and reinforces its counter cyclicality over business cycles. This is the driving force behind the model. I specify an explicit Markov process for consumption growth in a non stationary environment, derive the expected returns on equity and the risk-free bond and calculate the equity risk premium in equilibrium. I show that in a reasonably calibrated economy, my model can generate the equity premium observed in U.S. data.

Chapter 4

# FINANCIAL OBLIGATIONS RATIO AND THE PREDICTABILITY OF MARKET RETURNS

In this chapter, I test the predictability of excess market returns using the household financial obligations ratio. Using U.S. stock market data from 1980 to 2015, I show that deviations of the household financial obligations ratio from its long-run mean are a better forecaster of future market returns than the dividend-price ratio, dividend-yield, earnings-price ratio, investment-capital ratio, and other popular forecasting variables. The results remain significant using either quarterly or annual data and are robust to out-of-sample tests.

## 4.1 Introduction

The household financial obligations ratio is defined as total debt payments, housing payments, and auto lease payments divided by total disposable income. It is a macroeconomic counter cyclical variable that helps explain the equity risk premium observed in U.S. data (Jahangiry, (2016b)[33]). Jahangiry (2016b) shows that households' financial obligations affect the equity risk premium via two channels: the preference channel and the borrowing constraint channel. In his set-up, individuals preferences are defined as consumption relative to financial obligations. The framework is analogous to habit formation models where the utility function depends on consumption relative to some habit level (Abel (1990)[1] , Constantinides (1990), [24] and Campbell and Cochrane (1999)[6]). Conversely, in an infinite-horizon aggregate household economy, the financial obligations ratio represents the borrowing constraint because the agents' borrowing capability is limited by their financial obligations ratio in the model.

The mechanism by which the financial obligations ratio helps explain the equity risk premium is straight-forward. In bad times when consumption is low, mostly because of lower income and more borrowing incentives, the financial obligations ratio is high. This dynamic borrowing constraint becomes binding in states of the economy in which agents want to smooth consumption. Turning to the preference channel, in good times when consumption is high, households slowly take on more debt. But in bad times when consumption falls, households de-lever slowly. Thus, debt moves slowly, following consumption, much like a slow-moving habit. Now imagine that an agent has taken on specific level of debt he must repay. In recessions, as income declines toward this specific level of debt, to make sure he can repay the debt, the agent becomes more risk averse and takes on less risk. This decreases the demand

for risky assets and increases the demand for precautionary savings in recessions. During booms, however, consumption moves further away from slow-moving financial obligations and hence the agent becomes less risk averse and takes on more risk. Thus, lower ratio of consumption to financial obligation in recessions and the higher ratio in good times directly affect the marginal utility and make the pricing kernel more volatile. These two channels are the key elements of Jahangiry's (2016b) model that explain the equity risk premium.

It has also been documented that the risk associated with aggregate households financial obligations is an economy wide risk and is significant in explaining variations in the cross-section of stock returns (Jahangiry, (2016a)[32]). Conditioning down on the financial obligations ratio, the FCAPM proposed by Jahangiry (2016a), survives a wide range of classical econometric and diagnostic tests when explaining the variations in average returns across 25 portfolios formed based on size and the book-to-market ratio. The consistent pricing of financial obligations risk with a negative risk premium suggests that the financial obligations ratio acts as a state variable. The cross-sectional intuition is as follows: in bad times, the financial obligations ratio is high and the marginal utility of consumption is also high. Portfolios that pay off in these times are more valuable assets to investors. The increase in hedging demands for these portfolios raises the prices and hence implies a lower expected return. The negative risk premium delivers this lower expected return.

In this study, I investigate the predictability of market returns/excess returns using households' obligations ratios, namely, debt service ratio and the financial obligations ratio. For the last three decades there have been many efforts to identify and establish the existence of time variation in expected asset returns; it is now

widely accepted that excess returns are predictable by variables such as the dividend-price ratio, earnings-price ratio, dividend-yield ratio, investment-capital ratio, and other financial indicators. These financial variables have been successful at predicting long-horizon returns but less successful at predicting short-horizon returns. The dividend-price ratio, earnings-price ratio and all other predictive variables are *financial* variables.

We are also interested in the relation between *macroeconomic* variables and financial markets, mostly because expected returns appear to vary with business cycles so that stock market returns should be forecastable by business cycle variables at cyclical frequencies. One macroeconomic business cycle variable that is successful at predicting returns at shorter horizon is the consumption-wealth ratio (*cay*) proposed by Lettau and Ludvigson (2001)[43]. Lettau and Ludvigson study the role of fluctuations in the aggregate consumption-wealth ratio for predicting stock returns. Using U.S. quarterly stock market data, they find that these fluctuations in the consumption-wealth ratio are strong predictors of both real stock returns and excess returns over a Treasury bill rate. However, the statistical significance and predictive power of the consumption-wealth ratio is hump-shaped and peaks at around one year. Indeed, the predictive power shrinks over long horizons. In this study I show that mean deviations from the household financial obligations ratio is another macroeconomic business cycle variable whose predictive power is significant at short horizons and remains more significant over longer horizons than does the consumption-wealth ratio.

But why should mean deviations from the household financial obligations ratio have any predictive power? The economic intuition is as follows: in the early stages of recessions, when returns are expected to be lower in the near future, households face

the possibility that their obligation ratios will go above their long-run average. This is because households' obligations behave like a slow-moving habit (with a no-default assumption, of course): in good times households taken on more financial obligations for which they are responsible in bad times as well. This, along with negative income shocks in recessions, eventually leaves households with a higher financial obligations ratios. The opposite is true for good times. During booms, households receive positive income shocks, which allows them to experience lower financial obligations ratios. [1] Hence, lower expected future returns followed by higher financial obligations ratios and vice versa. This suggests that deviations in the financial obligations ratio from its long-run mean should be negatively correlated with future returns, which is consistent with what I find.

The rest of this chapter proceeds as follows. In Section 4.2, I present the data and summary statistics. In Section 4.3 I test the predictability of market returns with mean deviations from households financial obligations ratios. Sections 4.4 and 4.5 document the findings on long-horizon forecasts and on out-of-sample tests. Section 4.6 concludes.

## 4.2 Data and Summary Statistics

A key advantage with respect to the data is that all the variables are directly observable and there is no need to work with proxies. The data include stock market returns and dividends per share from the Standard & Poor's Composite Index. We also consider returns on the value-weighted Center for Research in Security Prices (CRSP) Index as it provides a better and broader proxy for total asset wealth than

[1] Note that households' obligations may increase during booms but higher income shocks will offset these effects and the overall financial obligations ratio will be lower.

does the $S\&P$ Composite Index. The data sources are summarized in Table 4.1. Let $r^{SP}$ and $r^{vw}$ denote the market returns using the $S\&P$ Composite Index and the value-weighted CRSP Index, respectively. Market excess return is denoted by $(r^{SP} - r^f)$ where $r^f$ is the risk-free rate or the return on the one-month Treasury bill.

**Table 4.1:** Data Source

| **Variables** | **Data Source** |
|---|---|
| $r^{SP}$, $r^{vw}$, $r^f$ | Center for Research in Security Prices |
| $d/p$, $d/y$, $e/p$ | Standard & Poor's |
| $cay$ | Sdyney Ludvigson's website |
| $eqis$, $i/k$ | Amit Goyal's website |
| $DSR$, $FOR$, $MKV/GDP$ | Federal Reserve Bank of St. Louis |

Some of the most successful in-sample predictors that I compare with mean deviations from the debt service ratio ($DSR$) and the financial obligations ratio ($FOR$) are as follows. The dividend-price ratio ($d/p$) is the ratio of divided per share over price. The dividend-yield ($d/y$) is the ratio of dividends over lagged prices. [2] The earnings-price ratio ($e/p$) is the ratio of earnings over prices. I also consider a successful corporate issuing activity variable, namely, percent equity issuing ($eqis$), which is the ratio of equity issuing activity as a fraction of total issuing activity. This variable is proposed in Baker and Wurgler (2000)[2]. $MKV/GDP$ is the ratio of market value to GDP. $i/k$ is the investment-to-capital ratio, which is the ratio of aggregate (private nonresidential fixed) investment to aggregate capital for the whole economy. This variable is proposed in Cochrane

[2] $d/p$ and $d/y$ studied in many articles (Ball (1978), Campbell (1987), Campbell and Shiller (1988a, 1988b), Campbell and Viceira (2002), Campbell and Yogo (2006), the survey in Cochrane (1997), Fama and French (1988), Hodrick (1992), Lewellen (2004), Menzly, Santos, and Veronesi (2004), Rozeff (1984), Shiller (1984)).

(1991)[9]. Finally, ($cay$) which is the Consumption-wealth ratio proposed in Lettau and Ludvigson (2001)[43].

The properties of the above variables are well known, so I focus on the debt service ratio ($DSR$) and the financial obligations ratio ($FOR$). Table 4.2 presents summary statistics for these variables using annual data between 1980 and 2015. Table 4.3 reports the same summary statistics using quarterly data between 1980Q1 and 2015Q4.

**Table 4.2:** Summary Statistics (Annual Data: 1980-2015)

| | $r_t^{sp}$ | $r_t^{sp} - r_t^f$ | $DSR_t$ | $FOR_t$ |
|---|---|---|---|---|
| Panel A : Correlation Matrix | | | | |
| $r_t^{sp}$ | 1.00 | 0.977 | -0.274 | -0.221 |
| $r_t^{sp} - r_t^f$ | | 1.00 | -0.246 | -0.187 |
| $DSR_t$ | | | 1.00 | 0.967 |
| $FOR_t$ | | | | 1.00 |
| Panel B: Univariate Summary Statistics | | | | |
| Mean | 0.126 | 0.083 | 0.113 | 0.164 |
| Standard error | 0.166 | 0.161 | 0.009 | 0.009 |
| Autocorrelation | -0.012 | -0.044 | 0.861 | 0.813 |

**Table 4.3:** Summary Statistics (Quarterly Data: 1980Q1-2015Q4)

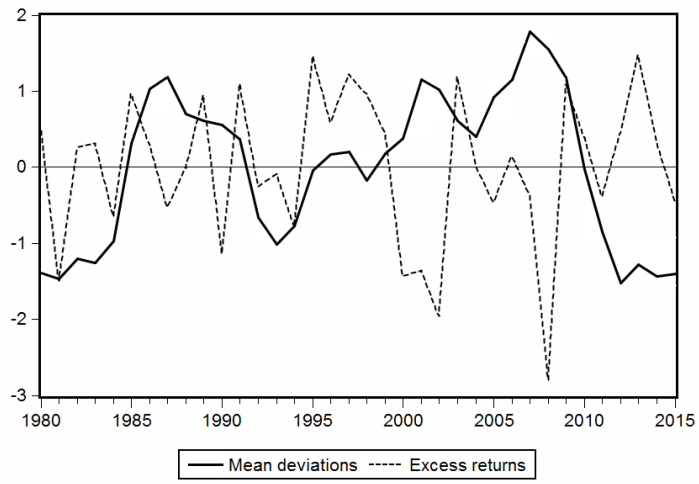
| | $r_t^{sp}$ | $r_t^{sp} - r_t^f$ | $DSR_t$ | $FOR_t$ |
|---|---|---|---|---|
| Panel A : Correlation Matrix | | | | |
| $r_t^{sp}$ | 1.00 | 0.993 | -0.182 | -0.151 |
| $r_t^{sp} - r_t^f$ | | 1.00 | -0.169 | -0.135 |
| $DSR_t$ | | | 1.00 | 0.966 |
| $FOR_t$ | | | | 1.00 |
| Panel B: Univariate Summary Statistics | | | | |
| Mean | 0.052 | 0.039 | 0.113 | 0.164 |
| Standard error | 0.082 | 0.081 | 0.009 | 0.008 |
| Autocorrelation | 0.100 | 0.077 | 0.978 | 0.966 |

In both Tables 4.2 and 4.3, $DSR_t$ and $FOR_t$ are negatively correlated with excess

stock market returns. The means of $DSR_t$ and $FOR_t$ are equal to 11.38% and 16.48%, respectively, with standard deviations close to 1%. The debt service ratio and the financial obligations ratio are persistent and the autocorrelation is high.

Figure 4.1 plots the standardized mean deviation of $FOR_t$ and the standardized excess return on the $S\&P$ Composite Index between 1980 and 2015. As discussed in the previous section, large positive mean deviations preceded large negative excess returns and vice versa. This trend is perceptible during U.S. recession periods, namely: January 1, 1990 to March 1, 1991, March 1, 2001 to November 1, 2001, and December 1, 2007 to June 1, 2009. [3]

**Figure 4.1:** Excess Returns and Mean Deviations




A detailed discussion on derivation of $DSR$ and $FOR$ is provided in Chapter 2 and Appendix A. It is important to check that mean deviations from the financial obligations ratio are stationary over time because expected returns appear to be stationary over time, and the variables predicting these returns should be stationary as well. Table 4.4 shows the ADF unit root test [16] and the KPSS test to confirm this stationarity of the predicting variable.

[3]The standardized mean deviations of $DSR_t$ are substantially the same as those $FOR_t$.

**Table 4.4:** Stationarity Tests for the Financial Obligations Ratio

| | ADF Test Statistic | Prob.* | KPSS Test Statistic |
|---|---|---|---|
| | **-3.522** | 0.014 | **0.1492** |
| 1 % level | -3.671 | | 0.739 |
| 5 % level | -2.963 | | 0.463 |
| 10 % level | -2.621 | | 0.347 |

As the ADF test statistic suggests, we can reject the null hypothesis (at p-values greater than 1.5% at least), meaning that we can reject it if the financial obligations ratio has a unit root property. The KPSS test statistic also confirm the stationarity of this ratio as we could not reject the hypothesis that the financial obligations ratio is stationary over time. [4]

## 4.3 Forecasting Regressions

In this section, I investigate the predictive power of mean deviations from households' obligation ratios for asset returns/ excess returns. The dependent variables are market returns ($S\&P_{500}$) and market excess returns. I also consider returns on the value-weighted CRSP Index as it provides a better and broader proxy for total asset wealth than does the $S\&P$ Composite Index. The independent variable is the financial obligations ratio. For comparison purposes, I assess the forecasting power of the most successful predicting variables using my sample data. These variables are listed in Table 4.1, namely, dividend-price ratio ($d/p$), dividend-yield ratio ($d/y$), earnings-price ratio ($e/p$), percent equity issuing ($eqis$), investment-to-capital ratio ($i/k$), market-to-GDP ratio ($MKT/GDP$), and consumption-wealth ratio ($cay$). Table 4.5 summarizes the regression results and

[4]The results hold for the debt service ratio as well. Both the ADF and the KPSS tests for $DSR$ confirms the stationarity of the debt service ratio.

reports one-period-ahead forecasts of the stock market returns. I correct for generalized serial correlation of the residuals using the Newey-West (1987) correction to the $t$-statistics.

Panels A, B, and C in Table 4.5 summarize the results of ordinary least squares (OLS) Single regressions using $S\&P$ Composite Index returns, market excess returns, and value-weighted market returns, respectively. As the table suggests, the financial obligations ratio alone is significantly able to predict one-period-ahead market returns. Note that the coefficients of $DSR$ and $FOR$ are both negative and significant at 1% level in all three panels. These negative coefficients are consistent with the economic intuition that laid out in "Introduction" section. As the economy is hit by a negative income shock, the increasing financial obligations ratio is followed by lower expected returns. The $R^2$s are not worse than those generated by the most successful competitive predicting variables in the literature. Indeed, both obligation ratios have the highest $R^2$s among all variables after the *cay* variable. In next section I show that obligation ratios are even better than *cay* when doing long-run regressions.

In Table 4.5, both the constants and coefficients of $DSR$, $FOR$, and *cay* are significant at 1% level in panels A-C. The coefficient of the dividend-price ratio ($d/p$), earnings-price ratio ($e/p$), and $MKV/GDP$ are significant at the 10% level in Panels A and C. The rest of the coefficients are not significant using either market returns or market excess returns.

**Table 4.5:** Forecasting One-Period-Ahead Returns (Single Regressions)

This table reports estimates from OLS single regressions of stock returns on variables named in the column headings. The $t$-stats are Newey-West (1997) corrected. Significant coefficients at 1% level are highlighted in bold-face. Regressions use data from 1980 to 2015.

| | $DSR$ | $FOR$ | $d/p$ | $d/y$ | $e/p$ | $eqis$ | $i/k$ | $MKV/GDP$ | $cay$ |
|---|---|---|---|---|---|---|---|---|---|
| Panel A: Market Returns (SP500) | | | | | | | | | |
| Constant | **0.124** | **0.123** | 0.013 | 0.042 | 0.027 | 0.087 | 0.350 | **0.185** | **0.100** |
| $t$-stat | 6.719 | 5.862 | 0.158 | 0.577 | 0.391 | 1.593 | 0.916 | 6.535 | 3.522 |
| Coefficient | **-5.380** | **-4.743** | 4.101 | 2.757 | 1.633 | 0.230 | -6.396 | -0.052 | **4.451** |
| $t$-stat | -3.362 | -2.038 | 1.798 | 1.577 | 1.857 | 1.057 | -0.555 | -1.869 | 2.936 |
| $R^2$ | 0.078 | 0.052 | 0.081 | 0.046 | 0.055 | 0.015 | 0.020 | 0.090 | 0.194 |
| Adj.$R^2$. | 0.050 | 0.023 | 0.053 | 0.017 | 0.027 | -0.015 | -0.010 | 0.062 | 0.170 |
| Panel B: Market Excess Returns | | | | | | | | | |
| Constant | **0.083** | **0.082** | 0.021 | 0.053 | 0.036 | 0.074 | 0.418 | **0.111** | **0.064** |
| $t$-stat | 8.025 | 5.085 | 0.260 | 0.730 | 0.518 | 1.420 | 1.192 | 4.025 | 2.185 |
| Coefficient | **-4.425** | -3.745 | 2.281 | 0.970 | 0.786 | 0.049 | -9.452 | -0.025 | **3.668** |
| $t$-stat | -2.976 | -1.741 | 1.039 | 0.551 | 0.890 | 0.224 | -0.897 | -0.880 | 2.306 |
| $R^2$ | 0.054 | 0.033 | 0.026 | 0.006 | 0.013 | 0.001 | 0.044 | 0.021 | 0.136 |
| Adj.$R^2$. | 0.026 | 0.004 | -0.004 | -0.024 | -0.017 | -0.030 | 0.015 | -0.009 | 0.110 |
| Panel C: Market Returns (CRSP_vw) | | | | | | | | | |
| Constant | **0.127** | **0.126** | 0.014 | 0.044 | 0.027 | 0.087 | 0.341 | **0.189** | **0.103** |
| $t$-stat | 6.791 | 5.963 | 0.171 | 0.635 | 0.393 | 1.595 | 0.881 | 6.548 | 3.606 |
| Coefficient | **-5.456** | **-4.845** | 4.190 | 2.782 | 1.679 | 0.245 | -6.063 | -0.053 | **4.482** |
| $t$-stat | -3.454 | -2.103 | 1.925 | 1.685 | 1.905 | 1.115 | -0.519 | -1.889 | 2.965 |
| $R^2$ | 0.078 | 0.053 | 0.083 | 0.046 | 0.057 | 0.016 | 0.017 | 0.090 | 0.193 |
| Adj.$R^2$. | 0.050 | 0.025 | 0.055 | 0.017 | 0.029 | -0.014 | -0.013 | 0.063 | 0.169 |

To check the robustness of the results, as additional controls, Table 4.6 reports the regressions of market returns and market excess returns on the variables shown in the column headings of the table. Panel A reports estimates from OLS multiple regressions of stock market returns on different combinations of the variables in the table. The highest adjusted $R^2$ is equal to 26.7% and is in row (4) where the right-hand-side variables include the debt service ratio, dividend-price ratio and consumption-wealth ratio. However, in row (4) the constant is not significant. The only forecasting multiple regression in which all the coefficients are significant and the constant is also significant at 1% level is where $DSR$ and *cay* are the predictors (row(1) in Panel A). Using these two variables alone generates an adjusted $R^2$ of 23.51%, which is significantly higher that the $R^2$ estimated by using either $DSR$ or *cay* individually.

**Table 4.6:** Forecasting One-Period-Ahead Returns (Multiple Regressions)

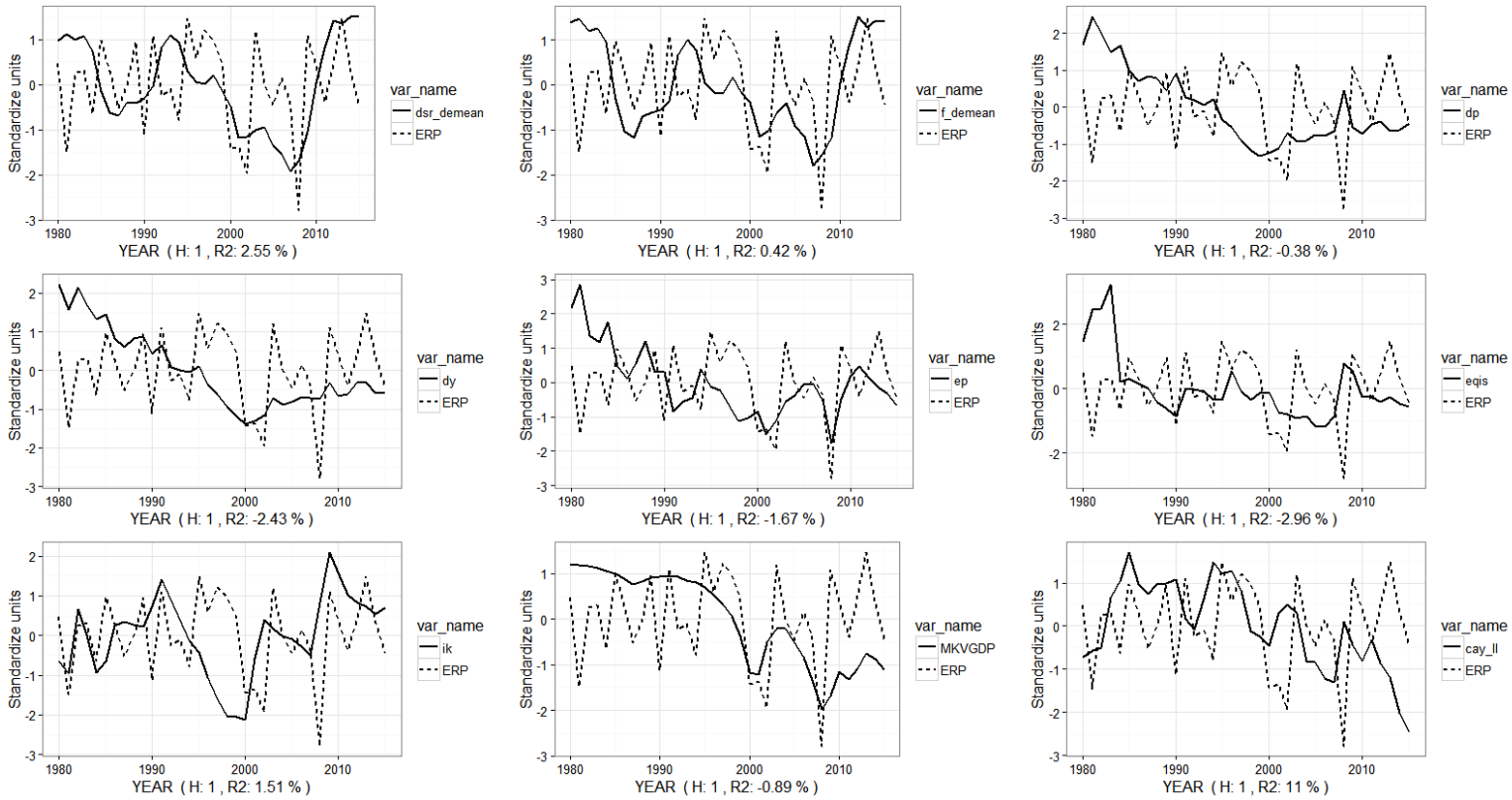
This table reports estimates from OLS multiple regressions of stock returns on variables named in the headings column. The $t$-stats are Newey-West (1997) corrected. Regressions use data from 1980 to 2015. ***, **, and * indicate significance at the 1%, 5%, and 10% levels, respectively.

| | Constant | $DSR$ | $d/p$ | $e/p$ | $eqis$ | $i/k$ | $MKV/GDP$ | $cay$ | $R^2$ | Adj.$R^2$. |
|---|---|---|---|---|---|---|---|---|---|---|
| | | | | | Panel A: Additional Controls; Market Returns | | | | | |
| (1) | 0.101*** | -5.652** | | | | | | 4.543*** | 0.2801 | 0.2351 |
| (2) | 0.081 | -5.249* | 0.790 | | | | | 4.373*** | 0.2824 | 0.213 |
| (3) | -0.044 | -6.828*** | 2.815 | | | | 0.053 | 5.784*** | 0.313 | 0.222 |
| (4) | 0.110 | -7.078*** | 10.816*** | | | | | 4.772** | 0.353 | 0.267 |
| (5) | 0.070 | -4.893** | -0.269 | 0.657 | | | | 4.519*** | 0.285 | 0.189 |
| (6) | 0.436* | -5.055* | -1.355 | 1.256 | -0.115 | -10.013 | | 5.061** | 0.333 | 0.191 |
| (7) | 0.218 | -6.621** | 1.136 | 1.359 | -0.232 | -7.259 | 0.049 | 6.175*** | 0.355 | 0.188 |
| | | | | | Panel B: Additional Controls; Excess Returns | | | | | |
| (1) | 0.064** | -4.649* | | | | | | 3.744** | 0.195 | 0.145 |
| (2) | 0.084 | -5.045* | -0.776 | | | | | 3.911*** | 0.198 | 0.120 |
| (3) | -0.106 | -7.451** | 2.308 | | | | 0.082* | 6.062** | 0.273 | 0.176 |
| (4) | 0.119 | -7.259*** | 11.361*** | | | | | 4.395** | 0.305 | 0.212 |
| (5) | 0.084 | -5.027* | -0.828 | 0.032 | | | | 3.918*** | 0.198 | 0.091 |
| (6) | 0.566** | -5.201* | -2.364 | 0.836 | -0.136 | -13.206* | | 4.64** | 0.283 | 0.129 |
| (7) | 0.241 | -7.535*** | 1.348 | 0.989 | -0.310 | -9.103 | 0.074 | 6.307*** | 0.333 | 0.161 |

In Panel B of Table 4.6, market excess returns are regressed on variations of the predictors. As in Panel A, row (4) produces the highest adjusted $R^2$ but again the constant is not significant. Using $DSR$ and $cay$ alone eventuates in adjusted $R^2 =$ 14.5% and significant coefficients and constant (row (1)). In both panels, when all predicting variables are included in row (7), the debt service ratio and consumption-wealth ratio are the only forecasting variables that remain significant in one-period-ahead multiple regressions. This reveals that $DSR$ and $cay$ contain information about future asset returns that is not included in other forecasting variables.

It is well known that some of the variables in Table 4.6 typically perform better at forecast horizons in excess of two years. Thus I also examine the long-run analysis and report the results in the "Long-Horizon Forecast" section in this chapter. Figure 4.2 plots the normalized —standard deviations of unity— market excess returns (ERP) versus the nine variables listed in Table 4.5. The forecast horizon $H$ is equal to 1 in all graphs, indicating that the regressions are one-year-ahead forecasting regressions.

**Figure 4.2:** Excess Returns and Predicting Variables




Normalized market excess returns (ERP) versus the nine variables listed in Table 4.5. The forecast horizon $H$ is equal to 1 in all graphs, indicating that the regressions are one-year-ahead forecasting regressions. The $R^2$s are also reported separately beneath each graph.

## 4.4 Long-Horizon Forecasts

In earlier sections, I show that the household's financial obligations ratio is directly related to the interest rate. By construction, the higher the interest rate, the higher the household's financial obligations. This direct dependency suggests that the financial obligations ratio should track longer-term fluctuations in asset markets returns rather than provide accurate short-term forecasts. Further-more, the summary statistics in Table 4.2 indicate that financial obligations ratios are highly persistent, supporting the idea that these ratios should provide a more accurate signal of long-term trends in asset returns than of short-term movements. [5]

Table 4.7 reports the results of single regressions of $H$-period market returns and market excess returns on different lagged forecasting variables over horizons spanning one to seven years. The table presents estimated coefficients on the included explanatory variables, $R^2$s and the adjusted $R^2$s, and the Newey-West (1997) corrected $t$-statistics. The significant coefficients at 5% level are in bold-face.

In Panel A, and Panel C, the $H$-period market returns are predicted by lagged values of forecasting variables listed in the column labeled "Regressors." The predictive power of financial obligations ratio (either $DSR$ or $FOR$) is hump-shaped and peaks at around period 6 with an adjusted $R^2$ of 27.9%. The coefficients remain almost constant and significant until period 6. This suggests that financial obligations ratio is a relatively stronger predictor in long horizons than are other hump-shaped variables such as the consumption-wealth ratio ($cay$), which peaks at

[5] This is like the dividend-price ratio and consumption-wealth ratio, which are both more accurate in predicting long-term stock market returns.

around period 2. As expected, the dividend-price ratio, dividend-yield ratio, and earnings-price ratio perform significantly better in longer horizons; however, their predictive power in one-year-ahead forecasts is weak (not a significant coefficients with small adjusted $R^2$). Panel B, and Panel D, reports the $H$-period forecasts using market excess returns. As these panels suggest, the obligations ratios are not as strong predictors as in panels A and C at very long horizons. The predictive power of obligations ratio peaks at period 3 with an adjusted $R^2$ equal to 12.30%. The coefficients are significant at the 5% level until period 4. For the consumption-wealth ratio, the results are the same as in panels A and C. The predictive power peaks at period 2 and the coefficients are significant only up to this period with an adjusted $R^2$ of 21.20%. For the rest of the predicting variables, none of the coefficients are significant at 5% level over any period between 1 and 7 years. In my sample, the only variable that provides significant predictive power for excess returns in longer horizons is the investment-capital ratio.

To test the robustness of the results, I consider additional controls by including other predicting variables in the long-horizon regressions. I regress market returns and market excess returns on variables including the household financial obligations ratio ($DSR$), dividend-price ratio ($d/p$), earnings-price ratio ($e/p$), percent equity issuing ($eqis$), investment-capital ratio ($i/k$), market value to GDP ($MKV/GDP$), and consumption-wealth ratio ($cay$). Table 4.8 reports the long-run multiple-regression estimates. The table reports estimates from OLS multiple regressions of stock returns on variables listed in the column labeled "Regressors." As Table 4.8 suggests, in all panels A-D, when including all the forecasting variables together, the household obligations ratio ($DSR$) is the only predicting variables that remains significant in all horizons spanning 1 to 7 periods. Whereas some variables

are significant at shorter horizons, such as $cay$, others are significant at longer horizons, such as $d/p, e/p, eqis$, and $i/k$. As in short-horizon regressions, this significance of the debt service ratio in all horizons reveals that $DSR$ contains information about future asset returns that is not included in other forecasting variables.

**Table 4.7:** Long-Horizon Forecasts (Single Regressions)

**Annual Data**: 1980-2015

| | | | Forecast Horizon $H$ | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| Row | Regressors | | 1 | 2 | 3 | 4 | 5 | 6 | 7 |
| | | | Panel A: Stock Market Returns | | | | | | |
| | | Coefficient | **-5.380** | **-5.712** | **-5.932** | **-5.557** | **-5.271** | -4.567 | -3.939 |
| 1 | $DSR$ | $t$-statistics | -3.362 | -5.501 | -3.466 | -3.476 | -2.767 | -1.831 | -0.994 |
| | | Adj.$R^2$ | 0.050 | 0.135 | 0.223 | 0.235 | 0.261 | 0.279 | 0.240 |
| | | Coefficient | **-4.743** | **-4.905** | **-5.067** | **-4.576** | **-4.525** | -3.973 | -3.477 |
| 2 | $FOR$ | $t$-statistics | -2.038 | -3.020 | -2.945 | -2.578 | -2.369 | -1.739 | -1.471 |
| | | Adj.$R^2$ | 0.023 | 0.076 | 0.132 | 0.126 | 0.156 | 0.172 | 0.150 |
| | | Coefficient | 4.101 | **4.112** | 3.910 | **4.045** | **4.300** | **4.031** | **3.979** |
| 3 | $d/p$ | $t$-statistics | 1.798 | 2.367 | 1.714 | 2.180 | 3.237 | 3.550 | 5.659 |
| | | Adj.$R^2$ | 0.053 | 0.134 | 0.194 | 0.282 | 0.417 | 0.517 | 0.611 |
| | | Coefficient | 2.757 | **3.071** | **3.268** | **3.278** | **3.217** | **3.257** | **3.218** |
| 4 | $d/y$ | $t$-statistics | 1.577 | 1.936 | 1.979 | 2.398 | 3.664 | 4.265 | 5.507 |
| | | Adj.$R^2$ | 0.017 | 0.085 | 0.169 | 0.232 | 0.287 | 0.426 | 0.509 |
| | | Coefficient | 1.633 | **1.521** | **1.544** | **1.417** | **1.285** | 1.105 | 1.211 |
| 5 | $e/p$ | $t$-statistics | 1.857 | 2.399 | 2.307 | 2.415 | 2.519 | 1.771 | 1.249 |
| | | Adj.$R^2$ | 0.027 | 0.067 | 0.123 | 0.138 | 0.145 | 0.153 | 0.236 |
| | | Coefficient | 0.230 | 0.327 | 0.341 | 0.341 | **0.330** | 0.310 | **0.264** |
| 6 | $eqis$ | $t$-statistics | 1.057 | 1.745 | 1.745 | 1.911 | 2.092 | 1.720 | 1.969 |
| | | Adj.$R^2$ | -0.015 | 0.029 | 0.069 | 0.099 | 0.123 | 0.161 | 0.136 |
| | | Coefficient | -6.396 | -7.603 | -9.457 | -10.435 | **-10.039** | **-9.974** | **-11.619** |
| 7 | $i/k$ | $t$-statistics | -0.555 | -0.671 | -1.033 | -1.873 | -2.583 | -3.009 | -4.167 |
| | | Adj.$R^2$ | -0.010 | 0.025 | 0.099 | 0.172 | 0.199 | 0.260 | 0.360 |
| | | Coefficient | -0.052 | -0.047 | -0.042 | -0.039 | -0.039 | -0.039 | -0.046 |
| 8 | $MKV/GDP$ | $t$-statistics | -1.869 | -1.777 | -1.424 | -1.370 | -1.432 | -1.203 | -1.515 |
| | | Adj.$R^2$ | 0.062 | 0.117 | 0.144 | 0.160 | 0.196 | 0.271 | 0.401 |
| | | Coefficient | **4.451** | **4.389** | **3.647** | **3.190** | **2.288** | **1.662** | 1.282 |
| 9 | $cay$ | $t$-statistics | 2.963 | 2.761 | 2.284 | 2.404 | 2.098 | 2.989 | 1.398 |
| | | Adj.$R^2$ | 0.170 | 0.298 | 0.291 | 0.275 | 0.166 | 0.107 | 0.064 |

## **Annual Data**: 1980-2015

| Row | Regressors | | Forecast Horizon $H$ 1 | 2 | 3 | 4 | 5 | 6 | 7 |
|---|---|---|---|---|---|---|---|---|---|
| | | | Panel B: Market Excess Returns | | | | | | |
| | | Coefficient | **-4.425** | **-4.435** | **-4.336** | -3.546 | -2.983 | -2.200 | -1.645 |
| 1 | $DSR$ | $t$-statistics | -2.976 | -4.496 | -2.149 | -1.806 | -1.279 | -0.752 | -0.277 |
| | | Adj.$R^2$ | 0.026 | 0.079 | 0.123 | 0.097 | 0.087 | 0.066 | 0.034 |
| | | Coefficient | -3.745 | **-3.631** | -3.530 | -2.607 | -2.272 | -1.639 | -1.236 |
| 2 | $FOR$ | $t$-statistics | -1.741 | -2.581 | -1.839 | -1.543 | -1.085 | -0.631 | -0.535 |
| | | Adj.$R^2$ | 0.004 | 0.033 | 0.059 | 0.028 | 0.027 | 0.014 | -0.002 |
| | | Coefficient | 2.281 | 2.376 | 2.228 | 2.403 | 2.713 | 2.503 | **2.497** |
| 3 | $d/p$ | $t$-statistics | 1.039 | 1.357 | 0.898 | 1.124 | 1.876 | 1.836 | 3.131 |
| | | Adj.$R^2$ | -0.004 | 0.029 | 0.051 | 0.100 | 0.196 | 0.262 | 0.338 |
| | | Coefficient | 0.970 | 1.392 | 1.627 | 1.693 | 1.695 | 1.798 | 1.798 |
| 4 | $d/y$ | $t$-statistics | 0.551 | 0.860 | 0.853 | 1.040 | 1.599 | 1.686 | 1.805 |
| | | Adj.$R^2$ | -0.024 | -0.005 | 0.025 | 0.051 | 0.080 | 0.161 | 0.214 |
| | | Coefficient | 0.786 | 0.750 | 0.826 | 0.726 | 0.594 | 0.419 | 0.551 |
| 5 | $e/p$ | $t$-statistics | 0.890 | 1.167 | 1.158 | 1.136 | 1.044 | 0.575 | 0.441 |
| | | Adj.$R^2$ | -0.017 | -0.005 | 0.018 | 0.021 | 0.015 | 0.002 | 0.046 |
| | | Coefficient | 0.049 | 0.154 | 0.175 | 0.185 | 0.180 | 0.162 | 0.105 |
| 6 | $eqis$ | $t$-statistics | 0.224 | 0.814 | 0.944 | 1.068 | 1.396 | 1.243 | 1.058 |
| | | Adj.$R^2$ | -0.030 | -0.016 | -0.002 | 0.013 | 0.025 | 0.039 | 0.003 |
| | | Coefficient | -9.452 | -10.034 | -11.301 | **-11.803** | **-10.968** | **-10.302** | **-10.947** |
| 7 | $i/k$ | $t$-statistics | -0.897 | -1.123 | -1.706 | -3.352 | -5.528 | -6.033 | -4.905 |
| | | Adj.$R^2$ | 0.015 | 0.077 | 0.182 | 0.282 | 0.323 | 0.405 | 0.481 |
| | | Coefficient | -0.025 | -0.020 | -0.016 | -0.014 | -0.015 | -0.017 | -0.023 |
| 8 | $MKV/GDP$ | $t$-statistics | -0.880 | -0.714 | -0.500 | -0.463 | -0.547 | -0.453 | -0.559 |
| | | Adj.$R^2$ | -0.009 | -0.001 | -0.003 | -0.002 | 0.011 | 0.041 | 0.127 |
| | | Coefficient | **3.668** | **3.599** | 2.843 | 2.378 | 1.506 | 0.989 | 0.734 |
| 9 | $cay$ | $t$-statistics | 2.306 | 2.096 | 1.521 | 1.611 | 1.220 | 1.618 | 0.922 |
| | | Adj.$R^2$ | 0.110 | 0.212 | 0.191 | 0.172 | 0.077 | 0.035 | 0.012 |

## **Quarterly Data**: 1980Q1-2015Q4

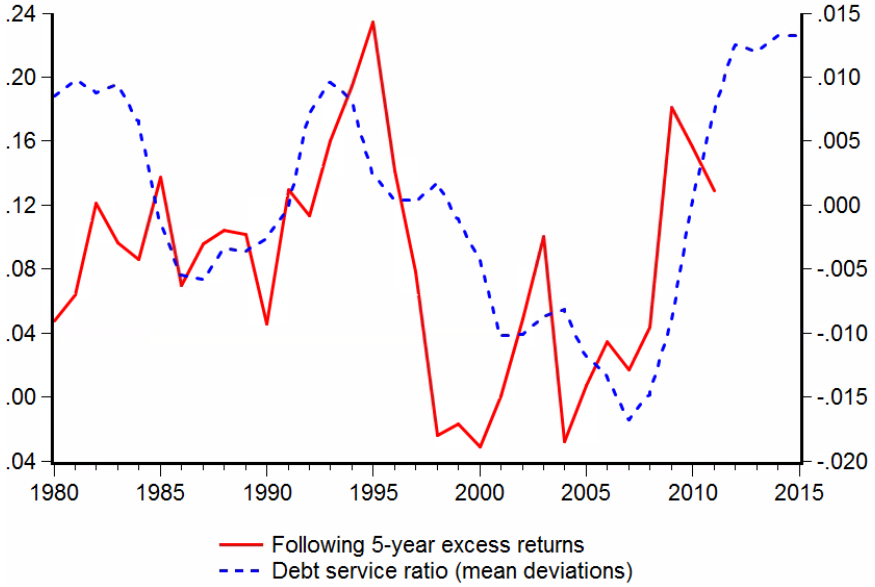
| | | | Forecast Horizon $H$ | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| Row | Regressors | | 1 | 2 | 4 | 8 | 12 | 24 | 36 | 48 |
| | | | Panel C: Stock Market Returns | | | | | | | |
| | | Coefficient | **-1.805** | -1.753 | -1.876 | -1.987 | **-2.027** | **-1.690** | -1.188 | -0.842 |
| 1 | $DSR$ | $t$-stat | -1.978 | -1.764 | -1.127 | -1.701 | -2.533 | -3.061 | -1.473 | -0.819 |
| | | Adj.$R^2$ | 0.029 | 0.054 | 0.123 | 0.251 | 0.331 | 0.389 | 0.210 | 0.103 |
| | | Coefficient | -1.678 | -1.613 | -1.766 | -1.866 | **-1.898** | **-1.629** | -1.121 | -0.781 |
| 2 | $FOR$ | $t$-stat | -1.696 | -1.619 | -1.006 | -1.877 | -2.704 | -2.681 | -1.068 | -0.607 |
| | | Adj.$R^2$ | 0.020 | 0.038 | 0.094 | 0.193 | 0.257 | 0.319 | 0.171 | 0.100 |
| | | Coefficient | **2.050** | **2.040** | **1.871** | 1.692 | **1.700** | **1.557** | **1.324** | **1.136** |
| 3 | $d/p$ | $t$-stat | 3.136 | 3.057 | 2.094 | 1.791 | 2.169 | 5.322 | 5.815 | 4.875 |
| | | Adj.$R^2$ | 0.080 | 0.150 | 0.247 | 0.384 | 0.522 | 0.824 | 0.867 | 0.817 |
| | | Coefficient | **2.077** | **1.982** | **1.754** | **1.588** | **1.633** | **1.476** | **1.280** | **1.093** |
| 4 | $d/y$ | $t$-stat | 3.456 | 3.362 | 2.240 | 2.067 | 3.212 | 8.781 | 8.171 | 6.494 |
| | | Adj.$R^2$ | 0.087 | 0.149 | 0.228 | 0.356 | 0.510 | 0.784 | 0.855 | 0.798 |
| | | Coefficient | **0.866** | **0.814** | 0.733 | **0.613** | **0.629** | 0.512 | **0.544** | **0.458** |
| 5 | $e/p$ | $t$-stat | 2.272 | 2.044 | 1.626 | 2.393 | 2.455 | 1.936 | 2.986 | 3.021 |
| | | Adj.$R^2$ | 0.067 | 0.112 | 0.179 | 0.240 | 0.346 | 0.446 | 0.670 | 0.664 |
| | | Coefficient | -1.515 | -1.655 | -1.828 | -2.317 | -2.726 | -2.809 | **-2.860** | -2.090 |
| 6 | $i/k$ | $t$-stat | -0.805 | -0.751 | -0.709 | -0.482 | -0.866 | -0.935 | -1.995 | -0.857 |
| | | Adj.$R^2$ | -0.003 | 0.003 | 0.015 | 0.061 | 0.119 | 0.202 | 0.262 | 0.192 |
| | | Coefficient | **0.747** | **0.810** | **0.917** | 1.030 | 0.895 | 0.380 | 0.226 | 0.109 |
| 7 | $cay$ | $t$-stat | 2.568 | 2.867 | 2.367 | 1.880 | 1.534 | 1.128 | 0.548 | 0.234 |
| | | Adj.$R^2$ | 0.020 | 0.050 | 0.120 | 0.257 | 0.244 | 0.070 | 0.025 | -0.003 |

## **Quarterly Data**: 1980Q1-2015Q4

| | | | Forecast Horizon $H$ | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| Row | Regressors | | 1 | 2 | 4 | 8 | 12 | 24 | 36 | 48 |
| | | | Panel D: Market Excess Returns | | | | | | | |
| | | Coefficient | -1.637 | -1.559 | -1.634 | -1.657 | **-1.616** | -1.100 | -0.657 | -0.434 |
| 1 | $DSR$ | $t$-stat | -1.845 | -1.726 | -1.048 | -1.650 | -2.811 | -1.693 | -1.079 | -0.644 |
| | | Adj.$R^2$ | 0.023 | 0.043 | 0.099 | 0.200 | 0.259 | 0.243 | 0.099 | 0.040 |
| | | Coefficient | -1.481 | -1.386 | -1.491 | -1.514 | **-1.481** | -1.040 | -0.626 | -0.431 |
| 2 | $FOR$ | $t$-stat | -1.606 | -1.435 | -0.854 | -1.604 | -2.712 | -1.652 | -1.000 | -0.533 |
| | | Adj.$R^2$ | 0.014 | 0.027 | 0.070 | 0.145 | 0.192 | 0.191 | 0.082 | 0.045 |
| | | Coefficient | **1.522** | **1.513** | 1.355 | 1.211 | 1.251 | **1.169** | **0.964** | 0.820 |
| 3 | $d/p$ | $t$-stat | 2.193 | 2.062 | 1.354 | 1.184 | 1.316 | 4.233 | 3.170 | 1.931 |
| | | Adj.$R^2$ | 0.042 | 0.082 | 0.135 | 0.224 | 0.349 | 0.692 | 0.746 | 0.712 |
| | | Coefficient | **1.558** | **1.461** | 1.243 | 1.111 | **1.187** | **1.090** | **0.928** | **0.783** |
| 4 | $d/y$ | $t$-stat | 2.374 | 2.204 | 1.373 | 1.288 | 2.037 | 5.564 | 4.624 | 3.748 |
| | | Adj.$R^2$ | 0.047 | 0.081 | 0.119 | 0.198 | 0.332 | 0.637 | 0.729 | 0.685 |
| | | Coefficient | 0.628 | 0.575 | 0.498 | 0.398 | 0.434 | 0.340 | **0.395** | **0.321** |
| 5 | $e/p$ | $t$-stat | 1.595 | 1.273 | 1.019 | 1.443 | 1.554 | 1.382 | 2.446 | 1.875 |
| | | Adj.$R^2$ | 0.033 | 0.055 | 0.085 | 0.113 | 0.202 | 0.291 | 0.574 | 0.545 |
| | | Coefficient | -2.508 | -2.623 | -2.719 | -3.028 | -3.257 | -2.853 | -2.612 | -1.773 |
| 6 | $i/k$ | $t$-stat | -1.349 | -1.231 | -1.142 | -0.754 | -1.222 | -3.009 | -2.177 | -1.060 |
| | | Adj.$R^2$ | 0.005 | 0.018 | 0.047 | 0.128 | 0.217 | 0.315 | 0.358 | 0.234 |
| | | Coefficient | 0.584 | **0.651** | 0.762 | 0.888 | 0.758 | 0.249 | 0.165 | 0.119 |
| 7 | $cay$ | $t$-stat | 1.986 | 2.177 | 1.914 | 1.669 | 1.359 | 0.996 | 0.505 | 0.348 |
| | | Adj.$R^2$ | 0.010 | 0.031 | 0.087 | 0.219 | 0.217 | 0.042 | 0.021 | 0.005 |

Figure 4.3 plots the five-year market excess returns versus the lagged debt service ratio mean deviations. Note that in this figure, the $DSR$ mean deviations have been flipped because the $DSR$ coefficient is negative (as expected) in long-horizon regressions. This figure shows how successful the household's debt service ratio is in predicting market excess returns over five-year horizon.

**Figure 4.3:** Debt Service Ratio: 1980-2015 Following Five-Year Returns



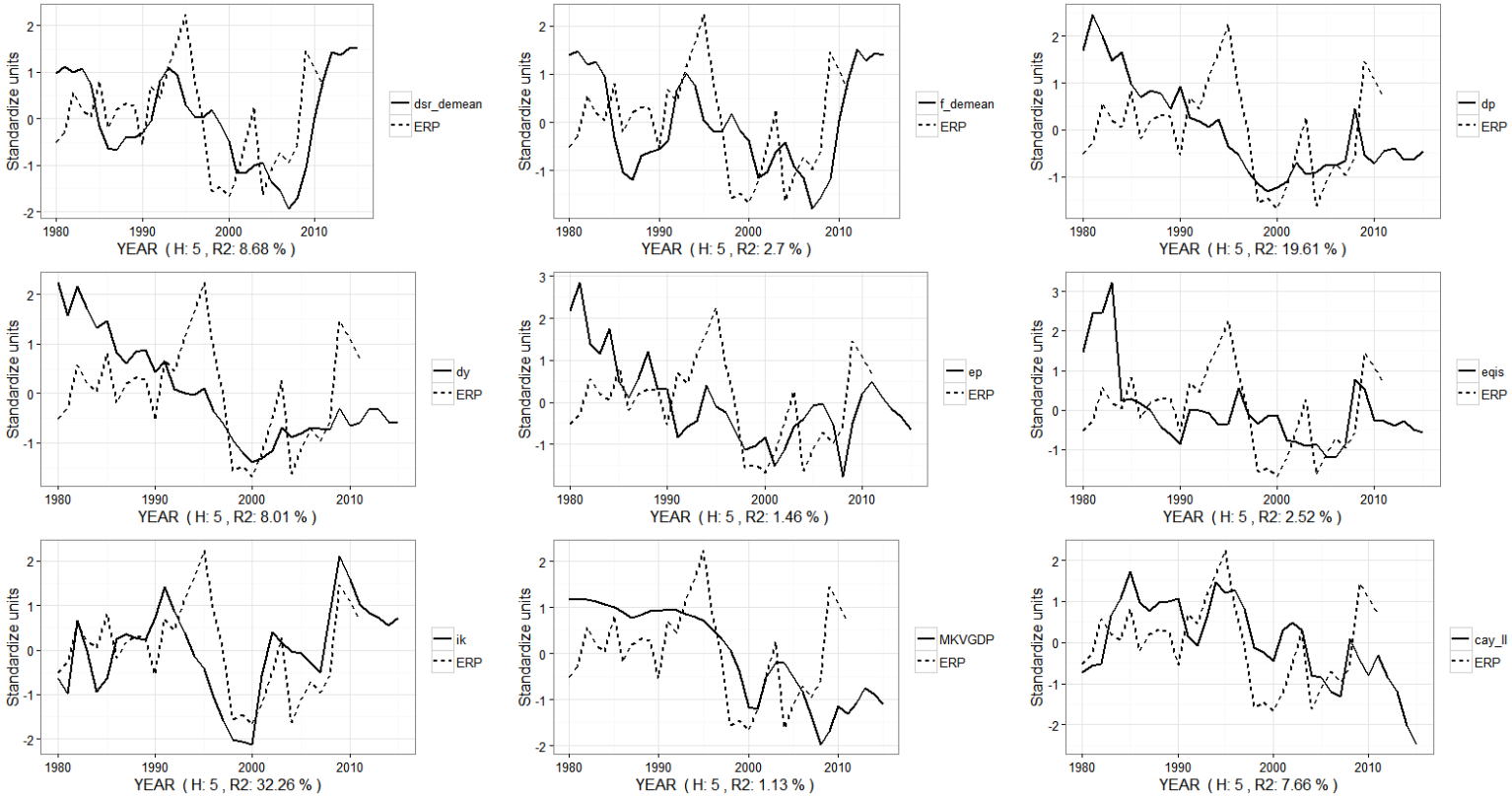

Finally, to see how the financial obligations ratio performs relative to other variables in predicting long-horizon average returns, Figure 4.4 plots the normalized market returns (SP500) versus the nine variables listed in Table 4.7. The forecast horizon $H$ is equal to 5 in all graphs, indicating that the regressions are the five-year-ahead forecasting regressions.

**Table 4.8:** Forecasting $H$-period-Ahead Returns (Multiple Regressions)

This table reports estimates from OLS multiple regressions of stock returns on variables listed in the "Regressors" column. The $t$-stats are Newey-West (1997) corrected. Regressions use data from 1980 to 2015. ***, **, and * indicate significance at the 1%, 5%, and 10% levels, respectively.

| | Forecast Horizon $H$ | | | | | | |
|---|---|---|---|---|---|---|---|
| Regressors | 1 | 2 | 3 | 4 | 5 | 6 | 7 |
| Panel A: Additional Controls; Market Returns | | | | | | | |
| Constant | 0.219 | 0.281 | 0.392** | 0.367*** | 0.267*** | 0.302*** | 0.275*** |
| $DSR$ | -6.622** | -4.834*** | -3.953*** | -4.433** | -7.889*** | -6.602*** | -3.839*** |
| $d/p$ | 1.136 | 0.025 | -1.258 | 1.311 | 6.008*** | 5.819*** | 5.549*** |
| $e/p$ | 1.360 | 1.261 | 1.473 | 0.666 | -0.554 | -0.975*** | -0.897** |
| $eqis$ | -0.232 | 0.043 | 0.135 | 0.010 | -0.342** | -0.301** | -0.290*** |
| $i/k$ | -7.260 | -8.790 | -11.052** | -10.361*** | -7.733*** | -7.228*** | -5.727*** |
| $MKV/GDP$ | 0.050 | 0.040 | 0.026* | 0.031*** | 0.053*** | 0.032** | 0.006 |
| $cay$ | 6.176*** | 5.700*** | 4.571*** | 3.461** | 1.807** | 0.552 | -0.117 |
| Adjusted $R^2$ | 0.188 | 0.468 | 0.630 | 0.697 | 0.778 | 0.839 | 0.835 |
| Panel B: Additional Controls; Market Excess Returns | | | | | | | |
| Constant | 0.242 | 0.280 | 0.379** | 0.361*** | 0.268*** | 0.302*** | 0.278*** |
| $DSR$ | -7.536*** | -5.642*** | -4.588*** | -4.884*** | -7.987*** | -6.258*** | -3.446*** |
| $d/p$ | 1.348 | 0.235 | -1.180 | 1.098 | 5.524*** | 5.215*** | 4.807*** |
| $e/p$ | 0.990 | 1.005 | 1.309 | 0.575 | -0.579* | -0.992*** | -0.841** |
| $eqis$ | -0.311 | -0.042 | 0.055 | -0.053 | -0.386*** | -0.327*** | -0.314*** |
| $i/k$ | -9.104 | -10.099 | -12.017*** | -11.320*** | -8.729*** | -8.061*** | -6.680*** |
| $MKV/GDP$ | 0.074 | 0.065** | 0.050*** | 0.051*** | 0.070*** | 0.045*** | 0.019 |
| $cay$ | 6.307*** | 5.790*** | 4.579*** | 3.355*** | 1.657** | 0.425*** | -0.144 |
| Adjusted $R^2$ | 0.161 | 0.444 | 0.596 | 0.648 | 0.726 | 0.750 | 0.688 |

**Figure 4.4:** Excess Returns and $H$-Horizon Predicting Variables




Normalized market returns (SP500) versus the nine variables listed in Table 4.7. The forecast horizon $H$ is equal to 5 in all graphs, indicating that the regressions are the five-year-ahead forecasting regressions. The $R^2$s are also reported separately beneath each graph.

## 4.5 Out-of-Sample Tests

In this section, I study the out-of-sample performance of the household financial obligations ratio by comparing the mean-squared error from one-period-ahead out-of-sample forecasts obtained from a forecasting regression that includes the household financial obligations ratio as the only forecasting variable, to forecasting regressions that do not include it.

I need to choose the periods over which a regression model is estimated and subsequently evaluated. Although any choice is necessarily ad hoc, the criteria are clear. It is important to have enough initial data to get a reliable regression estimate at the start of evaluation period, and it is important to have an evaluation period that is long enough to be representative. Because annual data are limited, I investigate quarterly periods as well. More specifically, we use one-third of the data to estimate the regression models and the rest of the data to report out-of-sample results. I consider a benchmark model and compare it with the out-of-sample performance of financial obligations ratios, and then do some non-nested analysis comparing the performance of the debt service ratio with other predicting variables mentioned earlier in this chapter. The benchmark model is the historical mean benchmark.

In the historical mean benchmark, a constant is the sole explanatory variable for excess returns. It has been documented (Welch and Goyal (2008)[59]) that most of the predicting variables in the literature have no ability to predict out-of-sample returns relative to a historical mean model despite their ability to do so in sample. Therefore, in this chapter, I also produce Welch-Goyal type figures to see the out-of-sample performance of the financial obligations ratio relative to historical mean models.

### 4.5.1 Out-Of-Sample Empirical Procedure

I closely follow Welch and Goyal's (2008)[59] empirical procedure. The out-of-sample forecast uses only the data available up to the time at which the forecast is made. Let $e_B$ denote the vector of expanding out-of-sample errors from the benchmark model and $e_A$ denote the vector of expanding out-of-sample errors from the OLS conditional model. The out-of-sample statistics are:

$$R^2 = 1 - \frac{MSE_A}{MSE_B}$$

$$\Delta RMSE = \sqrt{MSE_B} - \sqrt{MSE_A}$$

$$\textit{MSE-F} = (T - h + 1)\left(\frac{MSE_B - MSE_A}{MSE_A}\right).$$

$R^2$ is the out-of-sample R-squared (OOS-$R^2$). $MSE_A = E[e_A^2]$ is the mean-squared forecasting error from the relevant conditional model. $MSE_B = E[e_B^2]$ is the mean-squared error from the benchmark model. $RMSE$ is the root mean-squared error and $\Delta RMSE$ is the difference between the benchmark forecast and the conditional forecast for the same sample/forecast period. Positive numbers for $\Delta RMSE$ indicate superior out-of-sample conditional forecast. MSE-F is a test statistics designed to determine whether the one-step-ahead forecasting performance from the benchmark model is statistically different from the conditional model. It is an out-of-sample F-type test developed by McCracken (2007)[48]. $h$ is the degree of overlap. The MSE-F test is a test of equal mean-squared forecasting error. The null hypothesis is that the conditional model (model 1) and the benchmark models (model 2) have equal mean-squared error. The alternative hypothesis is that the benchmark model (model 2) has higher mean-squared error than conditional model (model 1). When the financial obligations ratio models are compared with other conditional models, the debt service ratio model is model 1 and each of the other predicting variable models is model 2

separately.

I compare the out-of-sample performance of financial obligations ratio with other conditional models by providing figures such as those in Welch and Goyal (2008)[59]. These figures graph the in-sample and out-of-sample performance of conditional models.

*For the in-sample regressions, the performance is the cumulative squared demeaned returns minus the cumulative squared regression residual. For the out-of-sample regressions, this is the cumulative squared prediction errors of the prevailing mean minus the cumulative squared prediction error of the predictive variable from the linear historical regression. In the figures, whenever a line increases, the conditional model predicted better; whenever it decreases, the historical mean model predicted better. The units in the graphs are not intuitive, but the time series pattern allows diagnosis of years with good or bad performance. Indeed, the final* $\Delta SSE$ *statistic in the OOS plot is sign-identical with the* $\Delta RMSE$ *statistic in our tables. In these figures, we can easily adjust perspective to see how variations in starting or ending date would impact the conclusion by shifting the graph up or down (redrawing the y = 0 horizontal zero line). The plots have also vertically shifted the IS errors, so that the IS line begins at zero on the date of our first OOS prediction.*[6]

Table 4.9 summarizes the out-of-sample results. Panels A and B use annual data, the initial estimation period begins in 1980 and ends in 2000. The out-of-sample estimation period is equal to 20, which is equal to the number Welch and Goyal (2008) used in their tables. The model is recursively re-estimated until the 2015. Out-of-sample tests are performed for three overlapping horizons. I consider two-year

[6]Welch and Goyal 2008[59]

and three-year overlapping horizons to capture business cycle fluctuations. Panels C and D report quarterly data. The out-of-sample estimation period here is equal to 48, which is one-third of the number of periods in the full sample. In Panels C and D, I report out-of-sample results for one-quarter, four-quarter, and eight-quarter overlapping horizons.

As Table 4.9 reports, the mean-squared forecasting error of the household financial obligations ratio model (either $DSR$ or $FOR$) is always lower than that of the historical mean benchmark model except for the column where out-of-sample market excess returns are predicted using one-year returns ($H = 1$).[7] In Panels C and D, where quarterly data are used, OOS-$R^2$ is always positive and the *MSE-F* statistic strongly rejects the null hypothesis. This suggests that information about the aggregate household financial obligations ratio consistently improves forecasts over models that use only a constant as a predictive variable. In all panels, as the number of overlapping horizons is expanded, both in-sample and out-of-sample $R^2$s increase and *MSE-F* statistics become stronger. This is consistent with what is found in the long-horizon analysis in the previous section.

Table 4.10 compares statistics on the out-of-sample performance of the debt service ratio versus other conditional models using different predicting variables, namely, $d/p$, $d/y$, $e/p$, $i/k$, $eqis$, $MKV/GDP$, and $cay$. [8] The comparison is done using stock market return (Panel A for annual frequencies and Panel C for quarterly frequencies) and market excess return (Panel B for annual frequencies and Panel D for quarterly frequencies) forecasts. A ratio of mean-squared errors *MSE1/MSE2* less than one

[7]The *MSE-F* tests are not significant when using annual data and $H = 1$. This is expected as the number of out-of-sample periods and the valuation periods do not seem to be sufficiently large.

[8]I do not have quarterly data for $eqis$ and $MKV/GDP$ so I exclude these variables when reporting results using quarterly data (Panels C and D of Table 4.10).

**Table 4.9:** Out-of-Sample Tests

This table presents statistics on forecast errors in-sample (IS) and out-of-sample (OOS) for stock market return (Panels A for annual frequencies and Panel C for quarterly frequencies) and market excess return (Panel B for annual frequencies and Panel D for quarterly frequencies) forecasts. $H$ is the number of overlapping horizons in each panel. OOS-$R^2$, $\Delta RMSE$, and *MSE-F* are explained in the text.

Annual Data: 1980-2015, OOS Estimation Period=20

| | $H$=1 | | $H$=2 | | $H$=3 | |
|---|---|---|---|---|---|---|
| | $DSR$ | $FOR$ | $DSR$ | $FOR$ | $DSR$ | $FOR$ |
| Panel A: Stock Market Returns | | | | | | |
| IS $R^2$ | 0.050 | 0.023 | 0.135 | 0.076 | 0.223 | 0.132 |
| OOS $R^2$ | 0.019 | 0.015 | 0.143 | 0.095 | 0.293 | 0.196 |
| $\Delta RMSE$ | 0.002 | 0.001 | 0.011 | 0.007 | 0.019 | 0.013 |
| MSE-F | 0.310 | 0.239 | 2.329 | 1.468 | 4.969 | 2.932 |
| Panel B: Market Excess Returns | | | | | | |
| IS $R^2$ | 0.026 | 0.004 | 0.079 | 0.033 | 0.123 | 0.059 |
| OOS $R^2$ | -0.040 | -0.043 | 0.061 | 0.026 | 0.158 | 0.093 |
| $\Delta RMSE$ | -0.004 | -0.004 | 0.004 | 0.002 | 0.009 | 0.005 |
| MSE-F | -0.617 | -0.662 | 0.917 | 0.377 | 2.246 | 1.237 |

Quarterly Data: 1980Q1-2015Q4, OOS Estimation Period=48

| | $H$=1 | | $H$=4 | | $H$=8 | |
|---|---|---|---|---|---|---|
| | $DSR$ | $FOR$ | $DSR$ | $FOR$ | $DSR$ | $FOR$ |
| Panel C: Stock Market Returns | | | | | | |
| IS $R^2$ | 0.029 | 0.020 | 0.123 | 0.094 | 0.251 | 0.193 |
| OOS $R^2$ | 0.029 | 0.022 | 0.156 | 0.122 | 0.249 | 0.189 |
| $\Delta RMSE$ | 0.001 | 0.001 | 0.004 | 0.003 | 0.005 | 0.004 |
| MSE-F | 2.886 | 2.191 | 16.620 | 12.492 | 27.165 | 19.067 |
| Panel D: Market Excess Returns | | | | | | |
| IS $R^2$ | 0.023 | 0.014 | 0.099 | 0.070 | 0.200 | 0.145 |
| OOS $R^2$ | 0.023 | 0.017 | 0.131 | 0.096 | 0.212 | 0.154 |
| $\Delta RMSE$ | 0.001 | 0.001 | 0.003 | 0.002 | 0.004 | 0.003 |
| MSE-F | 2.295 | 1.637 | 13.540 | 9.603 | 21.998 | 14.905 |

indicates that the mean-squared forecasting error of the $DSR$ is lower than that of the conditional model. Annual data and quarterly data both suggest that when *MSE-F* is significant, the DSR forecasting model contains information that produces (almost always) superior forecasts to those produced by any of the other models. For longer overlapping horizons, this is always the case, suggesting that forecasts using the financial obligations ratio are consistently superior to forecasts using other popular forecasting variables.

Figure 4.5 graphs the in-sample and out-of-sample performance of the debt service ratio augmented model using annual data (the three graphs on the left) and quarterly data (the three graphs on the right) for different overlapping horizons. For the in-sample regressions, the performance is the cumulative squared de-meaned returns minus the cumulative squared regression residual. For the out-of-sample regressions, the performance is the cumulative squared prediction errors of the prevailing mean minus the cumulative squared prediction error of the predictive variable from the linear historical regression. In this figure, whenever a line increases, the *DSR*-augmented model predicts better; whenever it decreases, the historical mean model predicts better. The final $\Delta SSE$ statistic in the out-of-sample plot is sign-identical to the $\Delta RMSE$ statistic in our tables. The figure adjusts the perspective to see how variations in the starting or ending date affects the conclusion by shifting the graph up or down (redrawing the $y = 0$ horizontal zero line). The plots also vertically shift the in-sample errors so that the in-sample line begins at zero on the date of the first out-of-sample prediction (1980 for annual data and 1980'0 for quarterly data). As Figure 4.5 suggests, the performance of the *DSR*-augmented model is consistent with Table 4.9. As the number of overlapping horizons is increased, the out-of-sample performance become closer to in-sample performance.

**Table 4.10:** Out-of-Sample Comparisons

This table compares statistics on OOS performance of debt service ratio versus a conditional model. The comparison is done using stock market return (Panel A for annual frequencies and Panel C for quarterly frequencies) and market excess return (Panel B for annual frequencies and Panel D for quarterly frequencies) forecasts.

Annual Data: 1980-2015, OOS Estimation Period=20

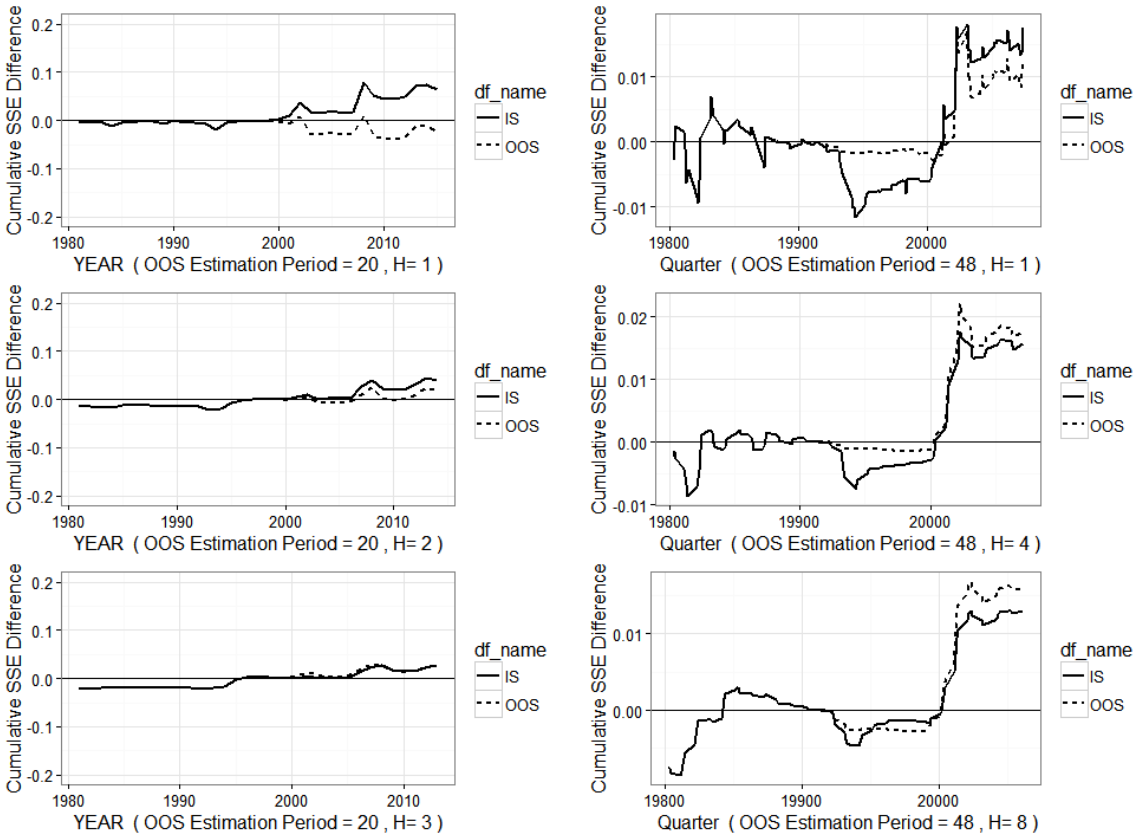
| | $H$=1 | | $H$=2 | | $H$=3 | |
|---|---|---|---|---|---|---|
| | *MSE1/MSE2* | *MSE-F* | *MSE1/MSE2* | *MSE-F* | *MSE1/MSE2* | *MSE-F* |
| Panel A: Stock Market Returns | | | | | | |
| $DSR$ vs. $d/p$ | 1.052 | -0.794 | 0.980 | 0.286 | 0.920 | 1.046 |
| $DSR$ vs. $d/y$ | 0.928 | 1.244 | 0.873 | 2.042 | 0.867 | 1.849 |
| $DSR$ vs. $e/p$ | 1.039 | -0.599 | 0.926 | 1.118 | 0.838 | 2.321 |
| $DSR$ vs. $eqis$ | 0.983 | 0.276 | 0.904 | 1.491 | 0.805 | 2.901 |
| $DSR$ vs. $i/k$ | 0.927 | 1.267 | 0.804 | 3.422 | 0.774 | 3.495 |
| $DSR$ vs. $MKV/GDP$ | 0.800 | 4.001 | 0.769 | 4.197 | 0.674 | 5.803 |
| $DSR$ vs. $cay$ | 1.198 | -2.649 | 1.195 | -2.287 | 1.044 | -0.509 |
| Panel B: Market Excess Returns | | | | | | |
| $DSR$ vs. $d/p$ | 0.980 | 0.329 | 0.896 | 1.627 | 0.850 | 2.117 |
| $DSR$ vs. $d/y$ | 0.865 | 2.504 | 0.804 | 3.404 | 0.791 | 3.169 |
| $DSR$ vs. $e/p$ | 1.002 | -0.035 | 0.907 | 1.430 | 0.848 | 2.154 |
| $DSR$ vs. $eqis$ | 0.974 | 0.419 | 0.908 | 1.411 | 0.847 | 2.164 |
| $DSR$ vs. $i/k$ | 1.003 | -0.046 | 0.947 | 0.786 | 1.079 | -0.882 |
| $DSR$ vs. $MKV/GDP$ | 0.721 | 6.183 | 0.691 | 6.250 | 0.636 | 6.872 |
| $DSR$ vs. $cay$ | 1.104 | -1.511 | 1.068 | -0.894 | 0.953 | 0.587 |

Quarterly Data: 1980Q1-2015Q4, OOS Estimation Period=48

| | $H$=1 | | $H$=4 | | $H$=8 | |
|---|---|---|---|---|---|---|
| | *MSE1/MSE2* | *MSE-F* | *MSE1/MSE2* | *MSE-F* | *MSE1/MSE2* | *MSE-F* |
| Panel C: Stock Market Returns | | | | | | |
| $DSR$ vs. $d/p$ | 1.044 | -4.085 | 1.113 | -9.148 | 1.198 | -13.549 |
| $DSR$ vs. $d/y$ | 1.049 | -4.475 | 1.119 | -9.575 | 1.183 | -12.700 |
| $DSR$ vs. $e/p$ | 1.046 | -4.192 | 1.033 | -2.896 | 0.975 | 2.083 |
| $DSR$ vs. $i/k$ | 0.950 | 5.056 | 0.837 | 17.568 | 0.759 | 26.037 |
| $DSR$ vs. $cay$ | 0.998 | 0.188 | 1.013 | -1.166 | 1.052 | -4.086 |
| Panel D: Market Excess Returns | | | | | | |
| $DSR$ vs. $d/p$ | 1.003 | -0.291 | 0.991 | 0.815 | 1.008 | -0.626 |
| $DSR$ vs. $d/y$ | 1.004 | -0.388 | 0.994 | 0.546 | 0.992 | 0.686 |
| $DSR$ vs. $e/p$ | 1.008 | -0.767 | 0.951 | 4.679 | 0.886 | 10.579 |
| $DSR$ vs. $i/k$ | 0.968 | 3.213 | 0.909 | 9.028 | 0.878 | 11.432 |
| $DSR$ vs. $cay$ | 0.987 | 1.296 | 0.984 | 1.505 | 1.036 | -2.872 |

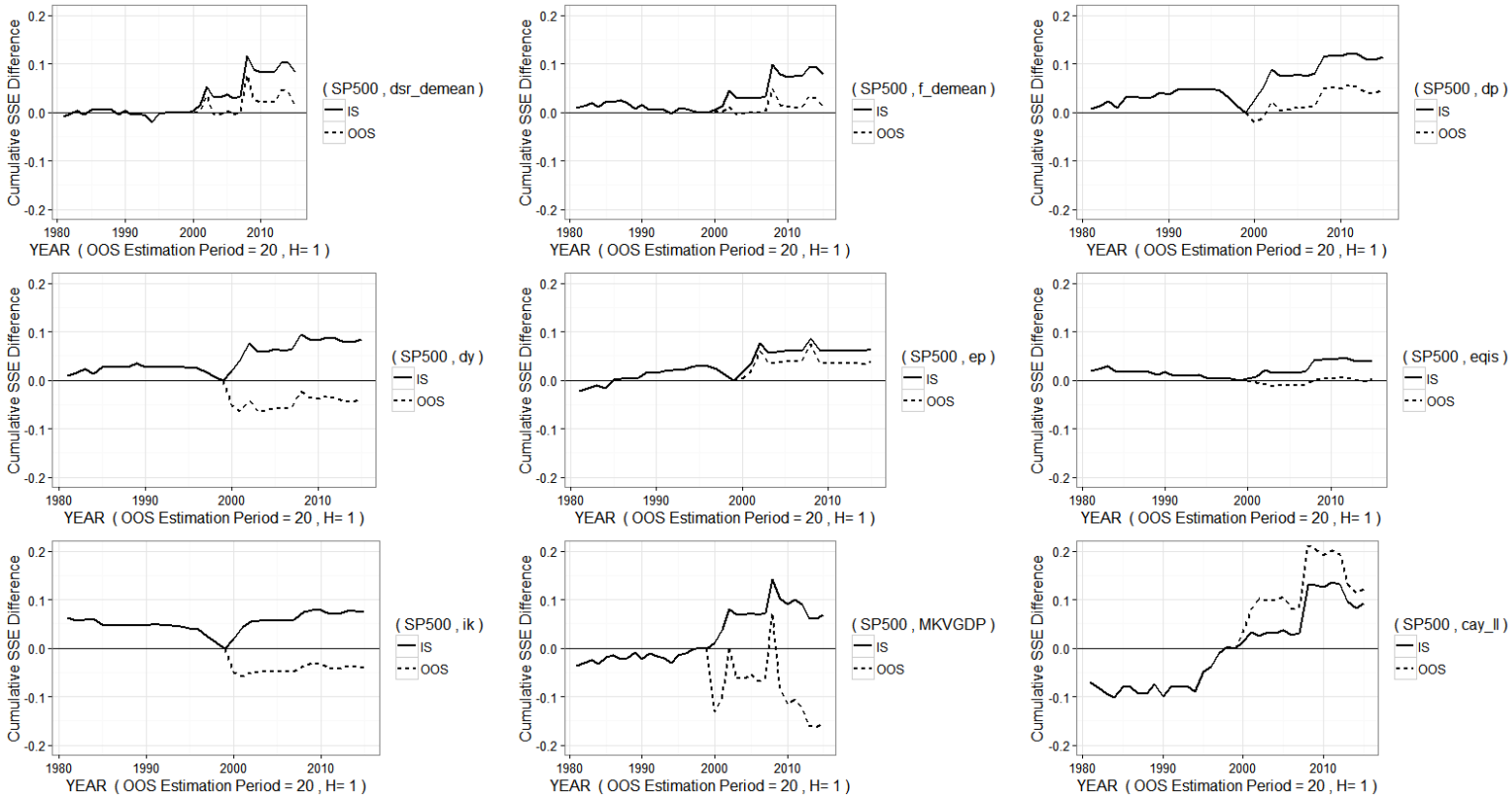
Figure 4.6 graphs the in-sample and out-of-sample performance of each conditional model separately. The conditional model relies on predictive variables noted in each graph. The benchmark is the historical mean model. The interpretation of the increase and decrease in the lines is the same as in Figure 4.5. In Figure 4.6, I show only the graphs where stock market annual returns (SP500) are used, the out-of-sample estimation period equals 20, and the overlapping horizon equal one ($H$=1). The H performance of each model is closer to it's in-sample performance if the solid line and the dashed line are closer to each other. The figure suggests that financial obligations ratio, $cay$, $e/p$, and $eqis$ augmented models perform relatively better than the other conditional models.

**Figure 4.5:** Out-of-Sample Performance of the *DSR*-Augmented Model




These graphs plot the in-sample and out-of-sample performance of the *DSR*-augmented model. Each line is cumulative squared prediction errors of the historical mean model minus the cumulative squared prediction error of the *DSR*-augmented model. The in-sample prediction-relative performance is the solid line and the out-of-sample prediction-relative performance is the dashed line. An increase in a line indicates better performance of the named model; a decrease in a line indicates better performance of the historical mean model.

**Figure 4.6:** Out-of-Sample Performance of Conditional Models




These figures plot the in sample and out-of-sample performance of different conditional models. The lines are the cumulative squared prediction errors of the benchmark model (historical mean) minus the cumulative squared prediction error of the conditional model. The conditional model is a model that relies on predictive variables noted in each graph. The in-sample prediction-relative performance is the solid line and the out-of-sample prediction-relative performance is the dashed line. An increase in a line indicates better performance of the named model; a decrease in a line indicates better performance of the historical mean model.

## 4.6 Conclusion

Using annual and quarterly data from 1980 to 2015, I show that the household financial obligations ratio can predict market returns at short horizons and over business cycle frequencies. The debt service ratio is a macroeconomic business cycle variable that is a better forecaster of future returns both in-sample and out-of-sample than the dividend-price ratio, dividend-yield, earnings-price ratio, investment-capital ratio, and other popular forecasting variables. I conduct multiple regression analyses using some of the most successful predicting variables in the literature for forecasting one-period-ahead returns and find that the debt service ratio and the consumption-wealth ratio are the only forecasting variables that remain significant in these regressions. This indicates that the debt service ratio contains information about future asset returns that is not included in other forecasting variables. I also conduct out-of-sample tests and find that information about the aggregate household financial obligations ratio consistently improves forecasts over models that use only a constant as a predictive variable and over other conditional models that use popular forecasting variables.

Chapter 5

# FINANCIAL OBLIGATIONS RATIO AND THE CROSS-SECTION OF STOCK RETURNS

This chapter examines whether the risk associated with the aggregate households' financial obligations is an economy-wide risk and thus significant for explaining the variation in the cross-section of stock returns. The multifactor model proposed is a modification of the CAPM that includes the financial obligations ratio as a conditioning down variable. The household financial obligations ratio is defined as the ratio of all debt payments and financial commitments over total disposable income. The FCAPM examined in this chapter survives a wide range of classical econometric tests using data from 1980 to 2015. The model performs well in explaining the variation in average returns across 25 portfolios formed based on size and the book-to-market ratio. The Hansen-Jagannathan (1994) distance associated with the FCAPM is calculated and compared to some other conditional and unconditional models. The consistent pricing of financial obligation risk with a negative risk premium suggests that the financial obligations ratio acts as a state variable.

## 5.1 Introduction

In an infinitely lived aggregate household endowment economy, Jahangiry (2016b)[33] explores the effect of a household's financial obligations ratio on the equity risk premium. In the partial equilibrium model he proposes, households' preferences are defined as their consumption relative to financial obligations which also appear as a borrowing constraint in the model. The household financial obligations ratio is defined as the ratio of all debt payments and financial commitments over households' total disposable income. The financial obligations ratio is a counter-business-cycle macroeconomic variable that tends to increase in recessions and decrease in booms (Dynan (2012, 2013), Johnson and Li (2007, 2010)). These obligations affect an individual's marginal utility of consumption and reinforce its counter cyclicality over business cycles. In equilibrium, households' decisions regarding their financial obligation and consumption levels are the driving forces in Jahangiry's model in explaining the observed equity risk premium in the data.

In this chapter, I examine whether the risk associated with the aggregate households' financial obligations is an economy-wide risk and thus significant for explaining the variation in the cross-section of stock returns. As has been well documented, the standard consumption-based asset pricing model performs poorly in explaining the cross-section variations of expected returns.[1] The FCAPM is an extension of the CAPM that uses the financial obligations ratio as a conditioning down variable. The FCAPM can be derived in different ways. One could follow the

[1]However, Jagannathan and Wang (2005)[31] show that, working with Christmas to Christmas consumption data, there is still hope that standard consumption data can explain the cross-sectional variations in excess returns.

conditional CAPM approach of Jagannathan and Wang (1996)[30] to derive the multifactor model proposed in this chapter. The conditioning variable would be financial obligations. Another approach is to approximate the stochastic discount factor (SDF) using a linear model. Either way, one gets the same model.

But what is the intuition behind thinking of financial obligations as a conditioning down factor? Or in other words, why am I willing to show that this *extra risk* associated with financial obligations is priced across different portfolios of assets? In the context of asset pricing, the counter cyclicality of financial obligations ratio is important because I am looking for a conditioning down variable that is correlated with business cycles, especially with bad times. The intuition is straight-forward: financial obligations are contractual obligations and must be made regardless of the realized state of the economy. Only residual income can be used to make consumption-investment decisions. This would make, ceteris paribus, a marginal investor with a higher financial obligations ratio more risk averse compared to a marginal investor with a lower financial obligations ratio. Therefore, the former marginal investor would be willing to pay more for holding assets that pay off when the financial obligations ratio is high; that is, he would demand lower expected returns for the equities that have a positive correlation with financial obligations. Thus, if financial obligations risk turns out to be an economy-wide aggregate risk to which everybody must pay attention, then the price of this risk can be expected to be significant in explaining the expected returns across different assets.

Empirical testing is done by using annual/quarterly data for 1980-2015. I show that differences in exposure to financial obligations risk, along with exposure to market risk, could explain the cross-sectional differences in average excess returns across the 25 benchmark equity portfolios formed based on size and the

book-to-market ratio. The explanatory power of the financial obligations ratio as a conditioning down variable is tested using a wide range of econometric tests and various techniques, including cross-sectional regression (OLS and generalized least squares (GLS)), the Fama-MacBeth (1973) approach, and generalized methods of moments (GMM). However, one caveat of testing factor models is that as the sample size increases, almost every multifactor model is rejected, as the model test statistic blows up with lower standard errors for the cross-sectional residuals. To avoid this issue, I use different diagnostic tests along with the classical econometric test to test the FCAPM. These tests include the "actual versus fitted expected returns plot," "firm-specific characteristics test," and "testing for priced factors."

The rest of this chapter proceeds as follows. In Section 5.2, I set up the model and discuss the derivation of the FCAPM. In Section 5.3, I review the data. I explain estimation procedures in Section 5.4 and present the results in Section 5.5. Section 5.6 concludes.

## 5.2 The Multifactor Model

The FCAPM presented in this chapter can be derived in different ways. One is to follow the conditional CAPM approach used by Jagannathan and Wang (1996)[30]. The conditioning down variable used in this chapter would be the financial obligations ratio. Another way is to start with a no arbitrage assumption and impose a minimal theoretical structure. In the absence of arbitrage there exists an SDF that prices all the traded assets in the economy (Harrison and Kreps (1979))[29]. Then I could approximate the SDF by using a linear model. Using either approach, I get the same multifactor model. Because the focus in this study is on testing the empirical

significance of the financial obligations ratio in the cross-section of expected returns, I do not take a stand on which approach to use.

### 5.2.1 Financial Obligations Ratio as a Conditioning Down Variable

In a conditional model, its parameters vary over time. A classic example is the conditional CAPM where expected returns depend on conditional betas times the conditional factor risk premium:

$$E_t(R^{ei}_{t+1}) = \beta_{it}\lambda_{Mt}, \tag{5.1}$$

where $\lambda_{Mt}$ is the market risk premium. Translating equation (5.1) into discount factor language, there maybe a discount factor $m$ that is a linear function of the market return and the parameters of the model that vary over time:

$$m_{t+1} = a_t + b_t R^M_{t+1} \Longleftrightarrow E_t(m_{t+1}R^{ei}_{t+1}) = 0. \tag{5.2}$$

The problem of working with a model that has time-varying parameters is that when testing the model, equation (5.2) cannot be conditioned down using a managed portfolio, that is,

$$0 = E_t(m_{t+1}R^{ei}_{t+1}) = E_t\left[(a_t + b_t R^M_{t+1})R^{ei}_{t+1}\right] \not\Rightarrow 0 = E\left[(a + bR^M_{t+1})R^{ei}_{t+1}\right].$$

In words, a conditional model does not imply an unconditional model.[2] Conversely, however, an unconditional model implies a conditional model. To test an

[2] Jagannathan and Wang (1996)[30] illustrate this concept through the following example: Consider a hypothetical economy in which the CAPM holds period by period. Suppose that the econometrician considers only two stocks and that there are only two possible types of dates in the world. The betas of the first stock in the two date-types are, respectively, 0.5 and 1.25 (corresponding to an average beta of 0.875). The corresponding betas of the second stock are 1.5 and 0.75 (corresponding to an average beta of 1.125). Suppose that the expected risk premium on the market is 10 percent on the first date and 20 percent on the second date. Then, if the CAPM holds in each period, the expected risk premium on the first stock will be 5 percent on the first date and 25 percent on the second date. The expected risk premium on the second stock will be 15 percent on both dates. Hence, an econometrician who ignores the fact that betas and risk premiums vary over time will mistakenly conclude that the CAPM does not hold, since the two stocks earn an average risk premium of 15 percent, but their average betas differ.

unconditional model, the best I can do is to express the time-varying parameters of the model $(a_t, b_t)$ in terms of a conditioning variable $z_t$, an instrument that is driving the conditioning information. I model the conditioning information as below:

$$m_{t+1} = a(z_t) + b(z_t)R^M_{t+1}, \tag{5.3}$$

where $a(z_t)$ and $b(z_t)$ are assumed to be linear in $Z_t$:

$$a(z_t) = a_0 + a_1 Z_t \qquad b(z_t) = b_0 + b_1 Z_t,$$

The SDF can be rewritten as,

$$m_{t+1} = a_0 + a_1 Z_t + b_0 R^M_{t+1} + b_1 Z_t R^M_{t+1}. \tag{5.4}$$

With this new pricing kernel, to test the model I can take the unconditional expectation from equation (5.2) because $a_0, a_1, b_0, b_1$ are constant. Thus, a conditional model plus one information variable $z_t$ is equivalent to an unconditional multifactor model. More generally, any conditional model can be written as a larger unconditional factor model.[3]

In this chapter, market return is used as a proxy for the universe of risky assets. Note that unlike Jagannathan and Wang (1996)[30], I exclude human capital from the universe of risky assets because the return on human capital is indirectly part

[3]Jagannathan and Wang (1996)[30] work with the conditional CAPM. They show that the conditional CAPM implies an unconditional three-factor model. They call it the premium-labor (P-L) model as it includes human capital as a proxy for the universe of risky assets. The P-L model is given by:

$$E_t(R_{t+1}) = \alpha + \beta_{prem} R^{prem}_t + \beta_{vw} R^{vw}_{t+1} + \beta_L R^L_{t+1},$$

where $R^{prem}$ is the spread between Baa and Aaa bonds and is used as a proxy to capture time variation in the expected market risk premium. $R^{vw}$ is the return on the market and $R^L$ is the return on human capital.

of the model and is captured by the financial obligations ratio.[4] In my model the conditioning down variable is the financial obligations ratio $F_t$. Replacing $Z$ with $F$ in equations (5.3) and (5.4) and using a first-order approximation gives the SDF in (5.5). This is an approximation as I ignore the interactions between the conditioning variable and market returns as the only factor:

$$m_{t+1} \approx \delta_0 + \delta_1 R^M_{t+1} + \delta_2 F_t. \tag{5.5}$$

The multi-beta representation of equation (5.5) is given in (5.6):

$$E_t(R_{i,t+1}) = \alpha_i + \beta_{iM}\lambda_M + \beta_{iF}\lambda_F. \tag{5.6}$$

Here $\lambda_M$ and $\lambda_F$ are the prices of market risk and financial obligations risk, respectively. Although $\beta_{iM}$ is the standard market beta, I define $\beta_{iF}$, the financial obligations beta as below:

$$\beta_{iF} = \frac{Cov(R_{i,t+1}, F_{t+1})}{Var(F_{t+1})}.$$

The intuition behind using the financial obligations ratio as a conditioning down variable has implications for $\lambda_F$, which I explain in detail in the next section.

### *5.2.2 Intuition, Portfolio Perspective*

Let us look at the portfolio logic of how the FCAPM works. Consider two assets, A and B. They have the same means, same standard deviations, and same betas for

[4] Lettau and Ludvigson (2001)[43] also work with the conditional CAPM to explain the cross-section of average returns. They do not include labor income in the proxy for market. They use the log of the consumption-wealth ratio as a conditioning variable and find:

...We demonstrate that such conditional models perform about as well as the Fama-French three-factor model on portfolios sorted by size and book-to-market characteristics. The conditional consumptio CAPM can account for the difference in returns between low-book-to market and high-book-to-market portfolios and exhibits little evidence of residual size or book-to-market effects.

the market portfolio. Therefore, they must yield the same expected returns. For an investor, assets A and B look identical and he could split his portfolio between the two. Assume that in a recession, the investor is likely to face a higher financial obligations ratio, either because of greater likelihood of losing his job or more borrowing incentives to smooth consumption, or both. Also assume that in a recession, stock A goes up and stock B goes down. In time-series regression language,

$$R_t^{ei} = a_i + \beta_{iM} R_t^{eM} + \epsilon_t^i. \tag{5.7}$$

This implies that in a recession, asset A has a positive residual ($\epsilon_t^A > 0$) and asset B has a negative residual ($\epsilon_t^B < 0$); that is, their risks occur at different times. Knowing these conditions, the investor is expected to buy stock A because stocks A and B are no longer identical. A stock that goes up when the financial obligations ratio is high (in bad times), is a good stock to own. Conversely for stock B, the investor wants to get rid of it as soon as possible because he does not want to lose money at the same time he loses his job and faces a higher financial obligations ratio. Now go further and imagine a situation where everybody does this. What is going to happen if many investors try to buy stock A and sell stock B? It will increase the price of stock A and drive down its expected returns. Similarly, because everybody wants to sell stock B, the price of stock B goes down and its expected return goes up.

Therefore, in equilibrium, it is no longer true that the expected returns depend only on the market betas ($E(R^{ei}) = \beta_i \lambda_M$). This tendency to provide insurance against a high financial obligations ratio emerges. Expected returns depend on the market portfolio but also on their tendency to go up and down when financial obligations are high. Therefore, what was $a$ in equation (5.7) now shows up as betas with respect to another factor. In terms of time-series regressions:

$$R_t^{ei} = \beta_{iM} R_t^{eM} + \beta_{iF} F_t + \epsilon_t^i, \tag{5.8}$$

and the expected returns now looks like:

$$E(R^{ei}) = \beta_{iM}\lambda_M + \beta_{iF}\lambda_F + \alpha_i. \tag{5.9}$$

Intuitively, this financial obligations ratio $F$ is a state variable. The news of future high ratios is bad. If investors can identify stocks that reliably go up upon news that there will be a high $F$ in the future, they would want to buy those stocks. One final note is that because I am working with an aggregate financial obligations ratio in this model, there is an aggregate hedging demand for $F$, and therefore it qualifies as a factor to move prices around. In other words, the risk associated with financial obligations is an economy-wide risk. The question I address in this chapter is whether this extra risk is priced in historical data for cross-sectional returns. I use the returns of 25 portfolios formed based on size and the book-to-market ratio to test whether these returns are priced by the model in (5.9).

## 5.3 Data

The data used in this chapter are summarized in Table 5.1. One key advantage of the FCAPM model with respect to data is that all the variables and factors are directly observable.

Portfolios returns are the left-hand-side variables in the model. The portfolios include all stocks listed on the NYSE, NASDAQ, and AMEX. Portfolios are formed using a two-way sort. Size sorting is based on market capitalization. At the end of June each year, stocks are ranked according to their market capitalization quintiles. These quintiles form a set of five portfolios. Book-to-Market sorting is done according to the ratio of book to market equity. The ratio is book equity for the last fiscal year divided by market equity. The book-to-market ratio breakpoints are the 20th, 40th, 60th, and 80th percentiles. These breakpoints are used to form another set of five

**Table 5.1:** Data Source

| Variables | Data Source |
|---|---|
| 25 portfolios formed on size and book-to-market | Kenneth French's website |
| Fama-French (1993) three factors (*Mkt, smb, hml*) | Kenneth French's website |
| S&P composite prices and dividends | Robert J. Shiller data |
| One-month T-bill returns | Center for Research in Security Prices |
| Household financial obligations ratio | Federal Reserve Bank of St. Louis |

portfolios. The intersection of these two sets produces 25 portfolios used as the dependent variables in this study.

The financial obligations ratio and market portfolio returns are used as the right-hand-side variables in the model to explain the cross-section of expected returns across these 25 portfolios. Market returns are value-weighted returns on CRSP stock market indexes (NYSE-AMEX-NASDAQ-ARCA). A detailed discussion of the derivation of financial obligations ratio is provided in Chapter 2 and Appendix A.

It is important to check that my conditioning down variable is stationary because returns are stationary over time and the variables used to explain these returns should be stationary as well. Table 5.2 shows the ADF unit root test along with the KPSS test to confirm the stationarity of the conditioning variable. As the ADF test statistic suggests, the null hypothesis can be rejected at $p$-values greater than 1.5%, implying that it can be rejected that financial obligations ratio has a unit root property. The KPSS test statistic also confirm the stationarity of this ratio, as the hypothesis that financial obligations ratio is stationary over time cannot be rejected.[5]

[5] The ADF null is "$H_0 = F_t$ has a unit root" and the KPSS null is "$H_0 = F_t$ is stationary."

**Table 5.2:** Stationarity Tests for the Financial Obligations Ratio

| ADF Test Statistics | | Prob.* | KPSS Test Statistics |
|---|---|---|---|
| | **-3.522** | 0.014 | **0.1492** |
| 1% level | -3.671 | | 0.739 |
| 5% level | -2.963 | | 0.463 |
| 10% level | -2.621 | | 0.347 |

## 5.4 Estimation Procedure

The model that I estimate in this chapter is:

$$E(R^{ei}) = \gamma + \beta_{iM}\lambda_M + \beta_{iF}\lambda_F + \alpha_i, \tag{5.10}$$

where $\gamma$ is the intercept and $\alpha_i$ are the model errors. $\lambda_M$ is the market risk premium and $\lambda_F$ is the financial obligation risk premium. $\beta_{iM}$ and $\beta_{iF}$ are market beta and financial obligation beta and are defined as:

$$\beta_{iM} = \frac{Cov(R_{i,t+1}, R^M_{t+1})}{Var(R^M_{t+1})}, \quad \beta_{iF} = \frac{Cov(R_{i,t+1}, F_{t+1})}{Var(F_{t+1})}.$$

Finally, $E(R^{ei})$ are the average returns of 25 size and book-to-market portfolios to be explained by the FCAPM in (5.10). In this section, I discuss in detail how to estimate $\alpha, \beta$, and $\lambda$; how to calculate the standard deviations of these estimates; and how to test the central prediction of the model that alphas should be zero. [6] To do so, I use classic linear regression tests, including the cross-sectional approach and the Fama-MacBeth (1973) approach. The model is tested by applying the GMM/SDF approach. Traditionally, time-series regression tests are used, but in my model, because the financial obligations ratio is not a return factor, I cannot test the model using time-series regression implications. In other words, the price of risk

[6] Equivalently, if the quadratic form of the sum of squares of alphas ($\hat{\alpha}'V^{-1}\hat{\alpha}$) is big enough.

for the financial obligations factor is not equal to the mean of the factor, that is, $\hat{\lambda}_F \neq E_T(F)$. I begin with the cross-sectional approach and then conduct the Fama-MacBeth regressions. Afterward, I estimate the model using GMM approach.

### *5.4.1 Cross-Sectional Approach*

The main economic idea is to explain why average returns vary across portfolios. The FCAPM model in (5.10) implies that expected returns of an asset should be high if that asset has high betas; that is, average returns should be proportional to betas. The cross-Sectional approach is a two-step procedure. In the first step, I estimate betas $(\beta_M, \beta_F)$ by conducting a *time-series regression*. The time-series regression formula is:

$$R_t^{ei} = a_i + \beta_{iM} R_t^M + \beta_{iF} F_t + \epsilon_t^i \qquad t = 1, 2...T \quad \forall i. \tag{5.11}$$

Here, $a_i$ are constants, $R_t^M$ and $F_t$ are the right-hand-side variables, and the betas are the regression coefficients. In the second step, I estimate the factor risk premia $(\lambda_M, \lambda_F)$ by conducting a *cross-sectional regression* across portfolios of average excess returns on the estimated betas. Note that in cross-sectional regressions, betas are the right-hand-side variables, lambdas are the regression coefficients, and alphas are the cross-sectional regression residuals, the pricing errors. I run the cross-sectional regression in (5.10) without a constant (i.e, $\gamma = 0$ ) because theory posits that the constant (zero-beta excess return) should be zero.[7] Next, I need to complete three tasks: estimating coefficients, deriving their standard errors, and building a test for the model. Depending on the assumptions imposed, I use the techniques discussed below.

[7] I could also estimate a constant and see whether it is small. This is the trade-off between efficiency and robustness.

**OLS Cross-Sectional Regression**

The OLS cross-sectional regression formula is:

$$E(R^{ei}) = \beta_{iM}\lambda_M + \beta_{iF}\lambda_F + \alpha_i. \tag{5.12}$$

The assumptions I impose are as follows: (1) betas are fixed and do not change over time, (2) regression errors and factors are independent and identically distributed (i.i.d.) over time, (3) residuals are homoskedastic, and (4) factors in cross-sectional regressions are orthogonal to errors from time-series regressions. Using standard OLS formulas along with these assumptions, the following is how the estimates, standard errors, and test of the model are derived.

**Estimates**: Beta hats, $\hat{\beta}$, are estimated from standard time-series OLS regressions. $\hat{\lambda}$ are the slope coefficients in cross-sectional regression, and $\hat{\alpha}$ are the errors of cross-sectional regressions:

$$\hat{\lambda} = (\beta'\beta)^{-1}\beta' E_T(R^e) \tag{5.13}$$

$$\hat{\alpha} = E_T(R^e) - \hat{\lambda}\beta. \tag{5.14}$$

**Standard errors**: Standard errors of betas, $\sigma(\hat{\beta})$, are derived from standard time-series OLS regressions. Accounting for correlated errors, the standard errors of factor risk premia $\lambda$ are calculated as follows:

$$\sigma^2(\hat{\lambda}) = \frac{1}{T}\left[(\beta'\beta)^{-1}\beta'\Sigma\beta(\beta'\beta)^{-1} + \Sigma_f\right]. \tag{5.15}$$

In equation (5.15) $\Sigma$ is the time-series residual covariance matrix $\Sigma = cov(\epsilon_t\epsilon_t')$ and $\Sigma_f$ is the covariance matrix of factors. Finally, the covariance matrix of alphas (cross-sectional regression errors) is derived by

$$cov(\hat{\alpha}) = \frac{1}{T}\left[I - \beta(\beta'\beta)^{-1}\beta'\right]\Sigma\left[I - \beta(\beta'\beta)^{-1}\beta'\right]'. \tag{5.16}$$

**Test**: I test whether all the pricing errors ($\alpha$) are jointly zero, using the following test statistic:

$$\hat{\alpha}' cov(\hat{\alpha})^{-1} \hat{\alpha} \sim \chi^2_{N-K}. \tag{5.17}$$

To conduct the test I first compute the number on the left-hand side of (5.17) and then compare it to the distribution on the right-hand side, which indicates how likely it is to see a number this large if the true alphas are all zero. In other words, if the number on the left-hand is big, there is only a small chance of seeing a number this big if the true alpha is zero. Simply put, by finding big chi-squares, I can reject the model, meaning that the alphas are not jointly equal to zero.

### GLS Cross-Sectional Regression

In the regression model in (5.12) the $\alpha_i$ are correlated with each other. For example, if GM has a low alpha, Ford is also likely to have a low alpha. I run a GLS cross-sectional regression to address this issue.

**Estimates**: The estimate of $\hat{\beta}$ is unchanged. However, $\hat{\lambda}$ and $\hat{\alpha}$ are estimated from standard cross-sectional GLS formulas:

$$\hat{\lambda} = (\beta' \Sigma^{-1} \beta)^{-1} \beta' \Sigma^{-1} E_T(R^e) \tag{5.18}$$

$$\hat{\alpha} = E_T(R^e) - \hat{\lambda}\beta. \tag{5.19}$$

**Standard errors**: Again, the standard errors of betas, $\sigma(\hat{\beta})$, is unchanged. The more efficient $\sigma^2(\hat{\lambda})$ and $cov(\hat{\alpha})$ are derived as below:

$$\sigma^2(\hat{\lambda}) = \frac{1}{T} \left[ (\beta' \Sigma^{-1} \beta)^{-1} + \Sigma_f \right]. \tag{5.20}$$

$$cov(\hat{\alpha}) = \frac{1}{T} \left[ \Sigma - \beta (\beta' \Sigma^{-1} \beta)^{-1} \beta' \right] \tag{5.21}$$

Note that the GLS regression should improve efficiency whereas the OLS regressions are more robust. Hence, I apply both methods and discuss the outcomes in the "Result" section.

**Test**: The test statistics looks like the statistic in (5.17) but with a smaller covariance matrix, reflecting the greater power of the GLS test:

$$\hat{\alpha}'_{gls} cov(\hat{\alpha}_{gls})^{-1} \hat{\alpha}_{gls} \sim \chi^2_{N-K}. \tag{5.22}$$

I also develop an equivalent test that does not require a generalized inverse:

$$T\hat{\alpha}'_{gls} \Sigma^{-1} \hat{\alpha}_{gls} \sim \chi^2_{N-K}. \tag{5.23}$$

**Shanken correction**

So far, I have assumed that betas are constant over time. This turns out to be a matter of concern, even asymptotically. Therefore, I need to use the correct asymptotic standard errors and covariance matrices of the pricing errors. In all of the following formulas there is a multiplicative correction $(1 + \lambda' \Sigma_f^{-1} \lambda)$. This correction is due to Shanken (1992).[8]

$$\sigma^2(\hat{\lambda_{OLS}}) = \frac{1}{T} \left[ (\beta'\beta)^{-1} \beta' \Sigma \beta (\beta'\beta)^{-1} (1 + \lambda' \Sigma_f^{-1} \lambda) + \Sigma_f \right] \tag{5.24}$$

$$\sigma^2(\hat{\lambda_{GLS}}) = \frac{1}{T} \left[ (\beta' \Sigma^{-1} \beta)^{-1} (1 + \lambda' \Sigma_f^{-1} \lambda) + \Sigma_f \right] \tag{5.25}$$

$$cov(\hat{\alpha_{OLS}}) = \frac{1}{T} (I - \beta(\beta'\beta)^{-1}\beta') \Sigma (I - \beta(\beta'\beta)^{-1}\beta')(1 + \lambda' \Sigma_f^{-1} \lambda) \tag{5.26}$$

$$cov(\hat{\alpha_{GLS}}) = \frac{1}{T} \left[ \Sigma - \beta(\beta' \Sigma^{-1} \beta)^{-1} \beta' \right] (1 + \lambda' \Sigma_f^{-1} \lambda). \tag{5.27}$$

The estimates of $\hat{\beta}$ are unchanged. For estimates $\hat{\alpha}$ and $\hat{\lambda}$, I can use either the OLS or GLS approach. I compare the outcomes from different approaches (i.e, OLS cross-

[8] I need to make a simplifying assumption that the errors $\epsilon_t$ are i.i.d. over time and independent of the factors.

sectional regressions using Shanken correction vs. GLS cross-sectional regressions using the Shanken correction) in the "Results" section.

With regard to testing the model, for the OLS cross-section I can use equation (5.17) by employing the corrected covariance of errors in (5.27). For the GLS cross-section, I can use the corrected version of equation (5.23), that is,

$$T(1+\lambda'\Sigma_f^{-1}\lambda)\hat{\alpha}'_{gls}\Sigma^{-1}\hat{\alpha}_{gls} \sim \chi^2_{N-K}. \tag{5.28}$$

As I show in the "Results" section, this correction $(1+\lambda'\Sigma_f^{-1}\lambda)$ is too big to ignore, especially in annual data. Therefore, I need a general method that takes into account all of these issues, namely, correlated cross-sectional errors, generated regressors, and time-varying betas. In next section I will use GMM to tackle these issues.

### Generalized Method of Moments

I need to take into account the fact that betas are "generated regressors." So far, I have been able to relax two assumptions: correlated $\alpha_i$s are corrected by using the GLS method and time-varying $\beta$s are taken care of by the Shanken correction. Now, I take the last step and consider the fact that the betas themselves are estimated. By using GMM, there is no need to assume that factors are independent of error terms and that the factors are uncorrelated over time. Below are the moments that I use in the GMM approach:

$$g_T(b) = \begin{bmatrix} E(R_t^e - a - \beta f_t) \\ E(R_t^e - a - \beta f_t)f_t \\ E(R^e - \beta\lambda) \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}. \tag{5.29}$$

where the top two moment conditions come from time-series regressions and the third condition is the cross-sectional regression of average returns on betas. In equation

(5.29), $a$ is a time-series intercept, $f_t = (R_t^{eM}, F_t)$ is the vector of factors, and beta is the vector of market beta and financial obligations beta, that is, $\beta = (\beta_M, \beta_F)$. In GMM, I treat the moments that generate the regressors $\beta$ (top two moments) at the same time as the moments that generate the cross-sectional regression coefficient $\lambda$ (third-moment conditions). The covariance matrix between the two sets of moments captures the effects of generating the regressors on the standard error of the cross-sectional regression coefficients. Now, all I have to do is map the problem into the GMM notation.[9]

### *5.4.2 Fama-MacBeth Approach*

In this section, I use the Fama-MacBeth (1973) procedure to run the cross-sectional regression and calculate test statistics and standard errors that correct for cross-sectional correlation. The Fama-MacBeth method is a two-step procedure. The first step is to find beta estimates by running a time-series regression. Fama and MacBeth use a rolling five-years regression, but I use full-sample betas. The time-series regressions are the same as in (5.11):

$$R_t^{ei} = a_i + \beta_{iM} R_t^M + \beta_{iF} F_t + \epsilon_t^i \qquad t = 1, 2...T \quad \forall i. \tag{5.30}$$

[9] The parameter vector is: $b' = [a'\beta'\alpha]$. The $a_T$ matrix chooses which moment conditions are set to zero in estimation:

$$a_T = \begin{bmatrix} I_{2N} & 0 \\ 0 & \gamma' \end{bmatrix}.$$

$\gamma$ is the weighting matrix for the cross-sectional regression. The $d$ matrix is the sensitivity of the moment conditions to the parameters,

$$d = \frac{\partial g_T}{\partial b'} = \begin{bmatrix} -I_N & -I_N E(f) & 0 \\ -I_N E(f) & -I_N E(f^2) & 0 \\ 0 & -\lambda I_N & -\beta \end{bmatrix}.$$

Finally, the $S$ matrix is the long-run covariance matrix of the moments. For more details see Cochrane (2009, chap. 12).

The second step is to run a cross-sectional regression at each time period instead of estimating a single cross-sectional regression with the sample averages:

$$R_t^{ei} = \beta_{iM}\lambda_{Mt} + \beta_{iF}\lambda_{Ft} + \alpha_{it} \qquad i = 1, 2, ...N \qquad \forall t. \tag{5.31}$$

Again, as theory suggests, I do not add a constant to the cross-sectional equations. Next, I estimate $\alpha_i$ and $\lambda$, calculate their standard deviations, and build a test of the model. Fama and MacBeth suggest estimating $\lambda$ and $\alpha_i$ as the average of the cross-sectional regression estimates:

$$\hat{\lambda} = \frac{1}{T}\sum_{t=1}^{T}\hat{\lambda}_t, \qquad \hat{\alpha}_i = \frac{1}{T}\sum_{t=1}^{T}\hat{\alpha}_{it}. \tag{5.32}$$

I use the standard deviations of the cross-sectional regression estimates to generate the sampling errors for these estimates (assuming the standard errors are uncorrelated over time):

$$\sigma^2(\hat{\lambda}) = \frac{1}{T^2}\sum_{t=1}^{T}(\hat{\lambda_t} - \hat{\lambda})^2, \qquad cov(\hat{\alpha}) = \frac{1}{T^2}\sum_{t=1}^{T}(\hat{\alpha}_t - \hat{\alpha})(\hat{\alpha}_t - \hat{\alpha})'. \tag{5.33}$$

As for the test statistic, I test whether all the pricing errors are jointly zero using the following test,

$$\hat{\alpha}' cov(\hat{\alpha})^{-1}\hat{\alpha} \sim \chi^2_{N-K}. \tag{5.34}$$

One advantage of the Fama-MacBeth procedure is that it allows for changing betas, which is hard to incorporate in a single, unconditional, cross-sectional regressions or a time-series regression test. As stated earlier, Fama-MacBeth is another way of calculating the standard errors, corrected for cross-sectional correlations. However, Fama-MacBeth standard errors do not correct for serial correlation in the errors. Also note that Fama-MacBeth standard errors do not correct for the fact that $\hat{\beta}$s are generated regressors.

### 5.4.3 GMM in Discount Factor Form

In asset pricing, most data such as stock returns and portfolio returns are characterized by heavy-tailed and skewed distributions. Because GMM does not impose any restriction on the distribution of the data, it is a good alternative to the least squares estimation method. For the GMM estimation, I use the approximated SDF presented in equation (5.5). Here is the equation again:

$$m_{t+1} \approx \delta_0 + \delta_1 R^M_{t+1} + \delta_2 F_t \tag{5.35}$$

The moment condition associated with the SDF approximation is:

$$E_t[(\delta_0 + \delta_1 R^M_{t+1} + \delta_2 F_t) R^i_{t+1}] = 1. \tag{5.36}$$

$R_t = (R^1_t, R^2_t, ..., R^N_t)$ denote the vector of returns on N-portfolios at time $t$. Also, let $\delta = (\delta_0, \delta_1, \delta_2)$ be the vector of unknown parameters and $Z_t = (R^M_t, F_t)$ be the vector of factors. Equation (5.36) implies that the pricing error must be a null vector if the SDF is correctly specified. Denoting the pricing error with $g_t(\delta)$ results in:

$$g_t(\delta) = R_t m_t(\delta) - 1_N. \tag{5.37}$$

I can compare the different model specifications by looking at a quadratic form of the estimated pricing errors implied by the model. Equation (5.38) is one common quadratic form used as a single number to compare different models in the field:

$$Q = E_t[g_t(\delta)]' W E_t[g_t(\delta)], \tag{5.38}$$

where $W$ is a positive definite matrix called the weighting matrix. The choice of $W$ plays an important role in the validity of the results. For example, Hansen (1982)[26] suggests using an optimal weighting matrix (i.e, if errors are i.i.d., then $W = var[g(\delta)]^{-1}$, which can be simply derived from first-stage GMM under the

identity-weighting matrix or a simple OLS). The disadvantage of this $W$ is that it is highly model dependent and a smaller $Q$ could be obtained by simply adding more noise. To avoid this dependency, Hansen and Jagannathan (1994)[27] suggest the weighing matrix, $W_{HJ} = E_t(R_t R_t')^{-1}$. The decision criterion is called the Hansen-Jagannathan (H-J) distance and is estimated as below,

$$Dist_T(\delta) = \left[\min_{\delta} \quad g_T(\delta)' \; W_T^{-1} \; g_T(\delta)\right]^{\frac{1}{2}}. \tag{5.39}$$

where

$$W_T = W_{(HJ)_T} = \frac{1}{T}\sum_{t=1}^{T} R_t R_t'$$

$$g_T(\delta) = \frac{1}{T}\sum_{t=1}^{T}[m_t(\delta)R_t - 1_N].$$

Hansen and Jagganathan (1997)[28] later show that the H-J distance is equal to the pricing error for the portfolio most mispriced by the model. Clearly, $W_{HJ}$ does not depend on model specifications and $Dist_T$ does not reward SDF volatility; hence, it is suitable for model comparison. $Dist_T$ is a measure of model misspecification, meaning that it gives the distance between $m_t(\delta)$ and the nearest point in space of all SDFs that price assets correctly, and it gives the maximum pricing error of any portfolio formed from the N assets. The H-J distance provides a method for comparing models by assessing which is least misspecified.

To compare the FCAPM model with different specifications, the null hypothesis is that financial obligations is relevant for pricing the assets. Under the correct hypothesis, the H-J distance computed with the proposed model must be smaller than the H-J distance of the multifactor model that excludes financial obligations.

### *5.4.4 Alternative Tests*

#### Actual versus Fitted Expected Returns

As the number of observations increases, almost all of the multifactor models proposed so far are rejected. Therefore, I need to do some parallel diagnostic tests. The most common test is to plot the actual returns observed in the data versus the returns predicted by the model. The more the scattered plot is close to the $45^0$ line, the better the model fits the data. In the "Results" section, Figure 5.1 compares realized returns versus the predicted returns of Fama-French's (1993) 25 portfolios using annual data between 1980 and 2015. Each point represents one portfolio.

#### Firm-Specific Characteristics

The question is how to test whether chosen multifactor model is a good approximation for real data. To examine this, I could add a vector of firm-specific characteristics such as size, market capitalization, price-to-earnings ratio, book-to-market ratio, and so on. If the proposed multifactor model is the correct one, firm specific characteristics should play no role in explaining the returns as they represent risks that can be diversified away.

#### Testing for Priced Factors

Can the factor of interest be dropped? I am looking for a statistical procedure to test which factors survive in the presence of others. There are two right ways to answer this question. One is by designing a test in a GMM/SDF framework, and the other is by forming an "orthogonalized factor." In the GMM/SDF framework, such a test is very easy. Consider the following general SDF: $m = \theta_1 f_1 + \theta_2 f_2$. To test the given factor $f_1$, is the factor $f_2$ needed to price assets, or equivalently, does $\theta_2 = 0$?

One easy approach to answer this question is the *Wald test*. Because I do have an asymptotic covariance matrix for $\theta = (\theta_1, \theta_2)$,[10] I can form a $t$-test[11] or a $\chi^2$ test with the null $\theta_2 = 0$. The Wald test statistic is:

$$\hat{\theta_2}' \; var(\hat{\theta_2})^{-1} \; \hat{\theta_2} \; \sim \chi^2_{\#\theta_2},$$

where $\#\theta_2$ is the number of elements in the vector $\theta_2$, which in my case is equal to one. The second solution to my earlier question (of whether the extra factor can be dropped?) is to form an orthogonolized factor. Consider this two-factor model: [12] $E(R^{ei}) = \beta_1 \lambda_{f1} + \beta_2 \lambda_{f2} + \alpha_i$. Is the second factor $f_2$ needed? Or equivalently, can $E(R^{ei}) = \beta_1 \lambda_{f1} + \alpha_i$ be written? Apparently $\alpha$ will rise, but will it rise "too much"? To answer these questions I need to run a regression of $f_{2,t}$ on $f_{1,t}$ and take the residuals,

$$f_{2,t} = \alpha_{f_2} + b_1 f_{1,t} + \epsilon_t.$$

Then I can drop $f_2$ from the two-factor model if and only if $\alpha_{f_2}$ is zero. The intuition is straight-forward: if $f_1$ is sufficient to price $f_2$ , it is sufficient to price anything that $f_2$ prices. Hence, $\alpha_{f_2} = 0$ means that all the $\alpha_i$ are the same with or without including the second factor. This procedure is equivalent to forming the following orthogonolized factor:

$$f^*_{2,t} = \alpha_{f_2} + \epsilon_t = f_{2,t} - b_1 f_{1,t}.$$

$f^*_2$ is a cleaned up version of $f_2$ without any correlation with the first factor. Now if $E(f^*_{2,t}) = 0$, then it is okay to drop the second factor.

[10] Note that $\theta$ is the same as $\delta$ estimated in equation (5.36).

[11] This is true if $\theta_2$ is scalar as it is in this chapter.

[12] I use two-factor model to make it comparable to the FCAPM two-factor model presented in this chapter. The idea can be extended to any N-factor model.

## 5.5 Results

The FCAPM model presented in this chapter is estimated and tested with different techniques and diagnostic tests discussed throughout this section. Before looking at the results, look at some characteristics of the data for the sample period between 1980 and 2015. Table 5.3 summarizes the average excess returns of the 25 portfolios constructed based on size and the book-to-market ratio. The annual excess average returns range from 6.24% to 19.86%. Table 5.4 summarizes the market betas for these portfolios, which range from 0.68 to 1.52. As these tables suggest, the average annual excess returns and betas show large variations across 25 portfolios; hence, there is a wide dispersion to explain. On average, small size portfolios and high book-to-market portfolios have higher returns.

**Table 5.3:** Average Portfolio Returns

| | Low | **Book-to-Market** | | | High |
|---|---|---|---|---|---|
| Small | 6.24 | 15.45 | 16.43 | 18.03 | 19.86 |
| | 10.45 | 14.61 | 17.31 | 17.35 | 16.75 |
| **Size** | 12.18 | 15.59 | 15.60 | 16.55 | 1.47 |
| | 14.88 | 14.23 | 14.92 | 15.93 | 16.42 |
| Big | 13.55 | 13.99 | 12.85 | 13.04 | 14.88 |

**Table 5.4:** Market Betas of 25 Portfolios

| | Low | **Book-to-Market** | | | High |
|---|---|---|---|---|---|
| Small | 1.52 | 1.16 | 0.95 | .76 | 1.05 |
| | 1.30 | 0.93 | 0.81 | 0.71 | 0.83 |
| **Size** | 1.21 | 0.89 | 0.68 | 0.83 | 0.73 |
| | 1.14 | 0.83 | 0.94 | 0.82 | 0.94 |
| Big | 1.05 | 0.84 | 0.91 | 0.81 | 0.80 |

### *5.5.1 Cross-Sectional Regressions*

The goal is to estimate the parameters of the model in equation (5.12) and investigate whether the price of market risk ($\lambda_M$) and the financial obligation risk ($\lambda_F$) are statistically significant. I report the results using a wide range of estimation techniques including the OLS/GLS cross-sectional regression, Fama-MacBeth regression, and GMM. I also correct the OLS/GLS estimates using the Shanken (1992) correction and report the results. Table 5.5 and Table 5.6 summarize the estimation results under different techniques, using annual data and quarterly data, respectively.

**Table 5.5:** FCAPM Estimation Results (Annual Data)

This table reports results of testing the financial obligations-capital asset pricing model (FCAPM) using ordinary least squares (OLS) (Shanken (1992)), generalized least squares (GLS (Shanken)), the Fama-MacBeth (1973) regression, and generalized method of moments (GMM). Numbers in parentheses are standard errors and numbers in brackets are $t$-statistics. *** indicates significance at the 1% level.

| | OLS (Shanken) | GLS (Shanken) | FMB | GMM |
|---|---|---|---|---|
| | (1) | (2) | (3) | (4) |
| | 0.128*** | 0.160*** | 0.128*** | 0.138*** |
| $\lambda_M$ | (0.031) | (0.029) | (0.031) | (0.015) |
| | [4.11] | [5.49] | [4.17] | [9.48] |
| | -0.021*** | -0.005*** | -0.021*** | -0.017*** |
| $\lambda_F$ | (0.005) | (0.001) | (0.005) | (0.005) |
| | [-4.21] | [-3.09] | [-4.27] | [-3.49] |
| $J$-test | | | | 6.12 |
| $P$-value | | | | 0.014 |

As Tables 5.5 and 5.6 suggest, different estimation techniques indicate that financial obligations risk is a relevant component of aggregate risk (at least when using annual data) and its price is statistically significant at the 1% level. The $J$-test is the test of the model using the GMM approach. It indicates that the null hypothesis that all the alphas are jointly equal to zero cannot be rejected at 1% level using annual data and at the 5% level using quarterly data. Note that the price of financial obligations risk is negative, which is intuitive. It implies that if a portfolio has a positive correlation with financial obligations, it should worth more and hence have a lower expected return.

**Table 5.6:** FCAPM Estimation Results (Quarterly data)

This table reports results of testing the financial obligations-capital asset pricing model (FCAPM) using ordinary least squares (OLS) (Shanken (1992)), generalized least squares (GLS (Shanken)), the Fama-MacBeth (1973) regression, and generalized method of moments (GMM). Numbers in parentheses are standard errors and numbers in brackets are $t$-statistics. *** indicates significance at the 1% level.

| | OLS (Shanken) | GLS (Shanken) | FMB | GMM |
|---|---|---|---|---|
| | (1) | (2) | (3) | (4) |
| | 0.032*** | 0.031*** | 0.032*** | 0.035*** |
| $\lambda_M$ | (0.008) | (0.007) | (0.008) | (0.002) |
| | [3.77] | [4.34] | [4.32] | [19.86] |
| | -0.013* | -0.002 | -0.013*** | -0.007 |
| $\lambda_F$ | (0.008) | (0.002) | (0.004) | (0.006) |
| | [-1.72] | [-0.7] | [-3.37] | [-0.99] |
| $J$-test | | | | 4.45 |
| $P$-value | | | | 0.034 |

### *5.5.2 Generalized Method of Moments and Stochastic Discount Factor*

I estimate the SDF in equation (5.35) and compare the FCAPM pricing implications with the following models: consumption capital asset pricing model (CCAPM), CAPM, FCAPM, Fama-French (1993) three-factor model (FF3), and Fama-French three-factor model plus financial obligations ratio (FFF). The H-J distance provides a method for comparing models by assessing which is least misspecified. As can be seen in Table 5.7, FCAPM does a good job using both

annual data and quarterly data. The H-J distance associated with FCAPM is smaller than the distance associated with CCAPM, CAPM, and FF3. Comparing FF3 and FFF shows that H-J distance computed with FFF is smaller than the H-J distance of the multifactor model that excludes financial obligations. This suggests that financial obligations risk is important for pricing assets and is a relevant factor.

**Table 5.7:** Hansen-Jagannathan Distance

| Model | Quarterly | Annual |
|---|---|---|
| CCAPM | 0.0851 | 0.1889 |
| CAPM | 0.0746 | 0.1756 |
| FCAPM | 0.0665 | 0.1686 |
| FF3 | 0.0616 | 0.1736 |
| FFF | 0.0563 | 0.1674 |

Using equation (5.35), $m_{t+1} \approx \delta_0 + \delta_1 R^M_{t+1} + \delta_2 F_t$, I normalize $\delta_0$ to be equal to 1. This normalization has no effect on the pricing implications. Next, I estimate $\delta_1$ and $\delta_2$, using iterative GMM. Table 5.8 summarizes the coefficients of market returns and financial obligations. As this table suggests, the estimates are statistically significant and the null hypothesis that the moment conditions are correctly specified cannot be rejected. There are 25 moment conditions and 2 unknown parameters so the degree of freedom of the chi-square test statistic is equal to 23.

**Table 5.8:** GMM and SDF

This table presents results using generalized method of moments (GMM) and stochastic discount factor (SDF). Numbers in parentheses are standard errors and numbers in brackets are t-statistics. *** indicates significance at the 1% level.

| $m_{t+1} = \delta_0 + \delta_1 R^M_{t+1} + \delta_2 F_t$ | |
|---|---|
| | 48.90*** |
| $\delta_1$ | (4.53) |
| | [10.80] |
| | |
| | -13.95*** |
| $\delta_2$ | (4.57) |
| | [-3.04] |
| $J$-test | 25.328 |
| $p$-value | 0.334 |

### *5.5.3 Actual versus Fitted Expected Returns*

Figure 5.1 compares realized returns versus predicted returns of Fama-French's (1993) 25 portfolios using annual data between 1980 and 2015. Each point represents one portfolio sorted by size and book-to-market ratio. The inability of the CCAPM to explain variation in the cross-section of average returns is clear. The adjusted $R^2$ associated with the CCAPM is only 21.75%. The FCAPM model does much better than the CCAPM. The plot of actual returns versus fitted returns is given in Figure 5.2. The fit is better, with an adjusted $R^2$ of 64.22%.

**Figure 5.1:** CCAPM: Actual versus Fitted Returns

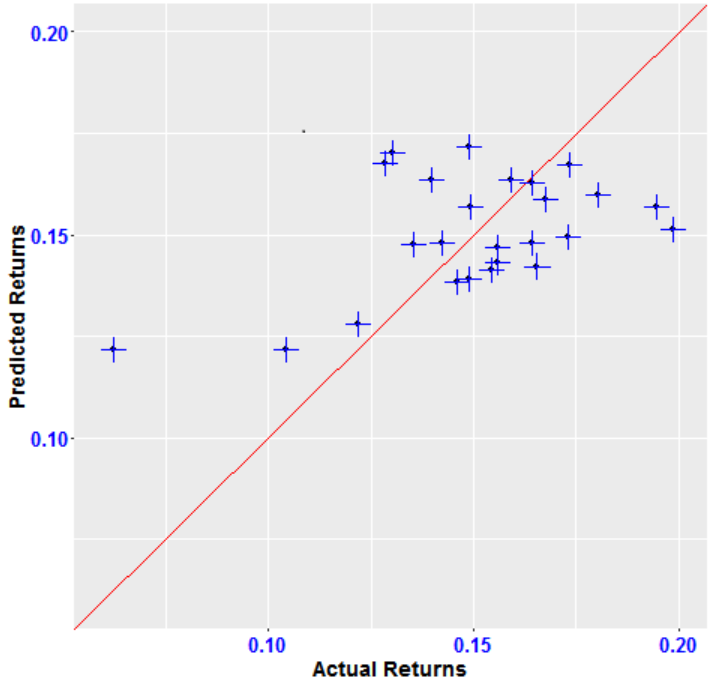
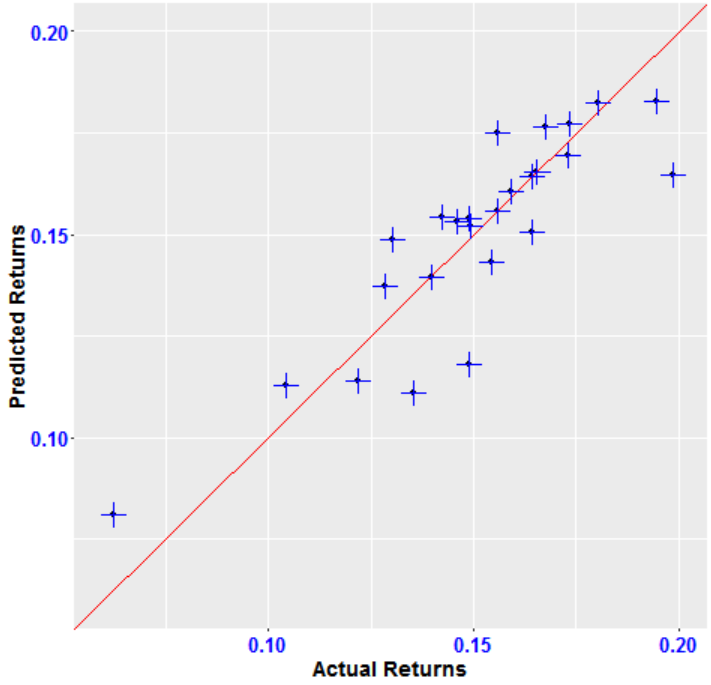



**Figure 5.2:** FCAPM: Actual vs. Fitted Returns



### Firm-Specific Characteristics

One diagnostic test used to see whether the model captures all economy-wide risk is to extend the model and examine whether adding firm-specific characteristics to the model can improve its performance. Therefore, I add the average firm size within each of the 25 portfolios to the FCAPM and estimate the Size-FCAPM model defined as:

$$E(R^{ei}) = \beta_{iM}\lambda_M + \beta_{iF}\lambda_F + size_i\ \lambda_{size} + \alpha_i, \tag{5.40}$$

where the $size_i$ is the average firm size for each portfolio and $\lambda_{size}$ is the price associated with this firm-specific size risk. As can be seen in Table 5.9, firm-specific size plays no role in explaining the portfolio returns: the coefficient is almost zero and the $t$-statistic is not significant. This is not surprising as the risk associated with firm-specific size could be diversified away.

**Table 5.9:** Size-FCAPM Model Estimation

This table presents results for the Size-FCAPM (financial obligations-capital asset pricing model). *** indicates significance at the 1

| $\lambda_M$ | $\lambda_F$ | $\lambda_{size}$ |
|---|---|---|
| 0.125*** | -0.021*** | 4.23E-07 |
| (0.011) | (0.004) | (1.05E-06) |
| [11.68] | [-5.17] | [0.40] |

**Testing for Priced Factors**

To test whether given market returns $R_t^M$ and financial obligations ratios $F_t$ at each time $t$, financial obligations ratio is needed to price assets? Or can $F_t$ be dropped? Using the GMM/SDF approach, this is equivalent to whether $\delta_2 = 0$:

$$m_{t+1} = \delta_1 R_{t+1}^M + \delta_2 F_t.$$

I use the Wald test to answer this question. Using the standard errors of $\hat{\delta}_2$ by applying the GMM approach, the Wald test statistic and the $p$-value of the null hypothesis $H_0 : \hat{\delta}_2 = 0$ are:

$$Wald_{\hat{\delta}_2} = 9.31, \qquad Pvalue = 0.002.$$

Therefore, as the Wald test statistic suggests, the null hypothesis can be rejected, meaning that the financial obligations ratio is priced as a factor and the risk associated with $F_t$ plays an important role in pricing the assets. Another way to check whether the financial obligations ratio can be dropped is the test I proposed in Section 5.4.3. Basically, I run a regression of $F_t$ on $R_t^M$ and take the residuals:

$$F_t = \alpha_F + b_M R_t^M + \epsilon_t. \tag{5.41}$$

Table 5.10 summarizes the estimations of (5.41). As can be seen, the intercept $\alpha_F$ is large and statistically significant. Therefore, $F_t$ cannot be dropped. In other words, market returns alone are not sufficient to price everything that the financial obligations ratio prices.

**Table 5.10:** Can we drop the Financial Obligations Ratio?

This table presents results of estimating equation (5.41). *** indicates significance at the 1% level.

| $\alpha_F$ | $b_M$ |
|---|---|
| 0.167*** | -0.011 |
| (0.002) | (0.007) |
| [99.71] | [-1.34] |

## 5.6 Conclusion

Conditioning down on the financial obligations ratio, the FCAPM proposed in this chapter survives a wide range of classical econometric and diagnostic tests used to explain the variations in average returns across 25 portfolios formed based on size and the book-to-market ratio. I show that the risk associated with aggregate households' financial obligations is an economy-wide risk and is priced across different portfolios of assets. The consistent pricing of financial obligation risk with a negative risk premium suggests that the household's financial obligations acts as a state variable. The intuition is straight-forward. The negative risk premium for the financial obligations ratio implies that a portfolio that pays off in bad times — that is, when the financial obligations ratio is high — is more valuable to investors and there is a hedging demand for it. Hence, this portfolio should have lower expected returns.

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# APPENDIX A

# FINANCIAL OBLIGATIONS RATIO: DATA

From Federal Reserves Board [1]

The household Debt Service Ratio (DSR) is the ratio of total required household debt payments to total disposable income. The DSR is divided into two parts: Mortgage DSR and Consumer DSR. The Mortgage DSR is total quarterly required mortgage payments divided by total quarterly disposable personal income. The Consumer DSR is total quarterly scheduled consumer debt payments divided by total quarterly disposable personal income. The Mortgage DSR and the Consumer DSR sum to the DSR. Quarterly values for the Debt Service Ratio are available from 1980 forward.

The limitations of current sources of data make the calculation of the ratio especially difficult. The ideal data set for such a calculation would have the required payments on every loan held by every household in the United States. Such a data set is not available, and thus the calculated series is only an approximation of the debt service ratio faced by households. Nonetheless, this approximation is useful to the extent that, by using the same method and data series over time, it generates a time series that captures the important changes in the household debt service burden. The series are revised as better data or improved methods of estimation become available. To create the measure, payments are calculated separately for revolving debt and for each type of closed-end debt, and the sum of these payments is divided by disposable personal income as reported in the National Income and Product Accounts. For revolving debt, the assumed required minimum payment is 2-1/2 percent of the balance per month. This estimate is based on the January 1999 Senior Loan Officer Opinion Survey, in which most banks indicated that required monthly minimum payments on credit cards ranged between 2 percent and 3

[1] Data description is provided by the Federal Reserve Board. `https://www.federalreserve.gov/releases/housedebt/about.htm`

percent, a ratio that apparently had not changed substantially over the previous decade.

Payments on closed-end loans, which are calculated for each major category of closed-end loan, are derived from the loan amount outstanding, the average interest rate, and the average remaining maturity on the stock of outstanding debt. Estimates of the amount of mortgage debt are taken from the Federal Reserve Board's Z.1 Financial Accounts of the United States statistical release, and estimates of outstanding consumer debt are taken from the Federal Reserve's G.19 Consumer Credit statistical release. For consumer debt, a more detailed breakdown by type of closed-end loan is obtained using internal Federal Reserve estimates and data from the Federal Reserve's Survey of Consumer Finances (SCF). Interest rates on closed-end consumer loans are obtained from the Federal Reserve Board's G.19 Consumer Credit and G.20 Finance Companies statistical releases, the SCF, and additional proprietary data sources. An estimate of the interest rate on the stock of outstanding debt is obtained by weighting the recent history of interest rates using information on the age of outstanding loans in the SCF. The interest rate on the stock of outstanding mortgage debt is an estimate provided by the Bureau of Economic Analysis. Maturity series for consumer debt are taken from the SCF. Maturity series for mortgage debt are calculated using data from Lender Processing Services and Mortgage Bankers Association.

The financial obligations ratio is a broader measure than the DSR. It includes rent payments on tenant-occupied property, auto lease payments, homeowners' insurance, and property tax payments. These statistics are obtained from the National Income and Product Accounts.

# APPENDIX B

# LINEARIZING THE PRICING KERNEL

In this appendix, I explore how the model proposed in Chapter 3 is able to generate a more volatile kernel while keeping the risk-free interest rate low. The linearized version of the stochastic discount factor is:

$$M_{t+1} = \beta \left(\frac{C_{t+1}}{C_t}\right)^{-\gamma} \left(\frac{S_{t+1}}{S_t}\right)^{-\gamma} = \beta \exp\left\{(-\gamma)\left(\ln \frac{C_{t+1}}{C_t}\right)\right\} \exp\left\{(-\gamma)\left(\ln \frac{S_{t+1}}{S_t}\right)\right\},$$

where $C_t$ is aggregate consumption and $S_t$ is the surplus consumption ratio defined as $S_t = \frac{C_t - G_t}{C_t}$. $G_t$ is aggregate financial obligations in period $t$. Taking the logarithm from both sides and letting $c_t = \ln(C_t)$ and $s_t = \ln(S_t)$ result in the following:

$$\ln M_{t+1} = \ln \beta + (-\gamma)(c_{t+1} - c_t) + (-\gamma)(s_{t+1} - s_t). \tag{B.1}$$

Now, some simplifying assumptions need to be made. For aggregate U.S. data on per capita consumption of nondurables and services, a good approximation to the data is the following model that makes the growth in the log of per capita consumption a random walk with drift:

$$c_t = \mu_c + c_{t-1} + \sigma_c \epsilon_t, \quad \text{where} \quad \epsilon_t \quad i.i.d. \sim N(0,1). \tag{B.2}$$

Assuming that the growth in the surplus consumption ratio also follows a random walk,

$$s_t = \mu_d + s_{t-1} + \sigma_d \epsilon_t, \quad \text{where} \quad \epsilon_t \quad i.i.d. \sim N(0,1). \tag{B.3}$$

Note that $\mu_d$ and $\sigma_d$ are the drift term and standard deviation term of the random walk process for the surplus consumption ratio. Now using (B.2) and (B.3) in (B.1) result in the following:

$$\ln M_{t+1} = \ln \beta + (-\gamma)(\mu_c + \sigma_c \epsilon_t) + (-\gamma)(\mu_d + \sigma_d \epsilon_t). \tag{B.4}$$

Because $\epsilon_t$ is $i.i.d. \sim N(0,1)$, then $\ln M_{t+1}$ is also normally distributed with mean $\mu$ and variance $\sigma^2$:

$$\mu = \ln \beta + (-\gamma)\mu_c + (-\gamma)\mu_d \tag{B.5}$$

$$\sigma^2 = (-\gamma)^2\sigma_c^2 + (-\gamma)^2\sigma_d^2 \tag{B.6}$$

From a property of normal distribution,[1] $E(M_{t+1})$ and $\sigma(M_{t+1})$ can be derived as follows:

$$E(M) = \exp(\mu + \frac{\sigma^2}{2}) \tag{B.7}$$

$$\sigma(M) = E(M)\sqrt{\exp(\sigma^2) - 1}. \tag{B.8}$$

Having $E(M)$ and $\sigma(M)$ in hand, I can now derive an equation for gross risk-free rate $R_F$ and explain the intuition behind:

$$R_f = E(M)^{-1} \Rightarrow \ln(R_f) = \ln(1 + r_f) = -\ln E(M) = -\ln(\exp(\mu + \frac{\sigma^2}{2})) = -\mu - \sigma^2/2$$

Using equations (B.7) and (B.8), the approximate risk-free rate is:

$$r_f \approx -\ln \beta + \gamma\mu_c + \gamma\mu_d - \gamma^2\ \frac{\sigma_c^2}{2} - \gamma^2\ \frac{\sigma_d^2}{2} \tag{B.9}$$

Equation (B.9) has some important implications. There are five terms in this equation that according to the set-up of my model can be interpreted as follows:

[1]Property: If $\log X \sim N(\mu_x, \sigma_x^2)$, then $E(X) = \exp(\mu_x + \frac{\sigma_x^2}{2})$ and $std(X) = E(m)\sqrt{\exp(\sigma^2) - 1}$. Here std denotes a standard deviation.

1. $-\ln\beta$: As $\beta$, the time discount factor decreases, agents become less patient and require higher interest rates to substitute consumption over time. For example, if $\beta$ is calibrated to 0.99, this means that approximately 1% of the risk-free rate is due to time preferences.

2. $\gamma\ \mu_c$: For $\gamma > 0$, this implies that as consumption growth increases, individuals should be compensated with higher interest rates to sacrifice today's consumption for tomorrow's consumption.

3. $\gamma\ \mu_d$: For $\gamma > 0$, this implies that in recessions, when consumption gets close to financial obligations, the surplus consumption ratio decreases and investors require higher interest rates.

4. $-\gamma^2\ \frac{\sigma_c^2}{2}$: Analogous to standard consumption-based models, this part of equation (B.9) can be interpreted as *precautionary savings*. The coefficient of consumption growth volatility is negative, implying that as consumption growth becomes more volatile, precautionary savings push the interest rate down.

5. $-\gamma^2\ \frac{\sigma_d^2}{2}$: This term adds up to the precautionary savings part of equation $(B.9)$ due to economic uncertainties. As the volatility of the surplus consumption ratio increases, demand for safer assets increases which leads to lower interest rates. This is what enables my model — unlike the standard consumption-based model — to generate lower risk-free rates for higher coefficients of risk aversion.

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| | | |
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| | MS | Economics, Simon Fraser University, Vancouver, Canada (2013) |
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**Research**

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Household Financial Obligations and the Cross-section of Stock Returns
Predictability of Stock Returns with Household Obligations Ratio

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**Areas of Expertise**

Macroeconomics, Empirical Asset Pricing, Factor Models, Return Predictability
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Best Comprehensive Exam Award, ASU, 2016
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