Time Metric in Latent Difference Score Models

by

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ABSTRACT

Time metric is an important consideration for all longitudinal models because it can influence the interpretation of estimates, parameter estimate accuracy, and model convergence in longitudinal models with latent variables. Currently, the literature on latent difference score (LDS) models does not discuss the importance of time metric. Furthermore, there is little research using simulations to investigate LDS models. This study examined the influence of time metric on model estimation, interpretation, parameter estimate accuracy, and convergence in LDS models using empirical simulations. Results indicated that for a time structure with a true time metric where participants had different starting points and unequally spaced intervals, LDS models fit with a restructured and less informative time metric resulted in biased parameter estimates. However, models examined using the true time metric were less likely to converge than models using the restructured time metric, likely due to missing data. Where participants had different starting points but equally spaced intervals, LDS models fit with a restructured time metric resulted in biased estimates of intercept means, but all other parameter estimates were unbiased, and models examined using the true time metric had less convergence than the restructured time metric as well due to missing data. The findings of this study support prior research on time metric in longitudinal models, and further research should examine these findings under alternative conditions. The importance of these findings for substantive researchers is discussed.

DEDICATION

This dissertation is dedicated to my husband, Gene, and to my best friend, Amy, for their patience, love, and unconditional support over the past six years. I never would have made it through without you both.

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Introduction

Longitudinal data analysis requires a temporal design that allows for accurate examination of the true process in the data (Collins, 2006). There are many considerations for temporal design, such as whether each participant has a similar or different timing schedule (that is, measurements are spaced uniquely for each participant), and whether the design is balanced (the spacing and number of measurement waves are the same across participants) (Card & Little, 2007; Singer & Willett, 2003). The time span of a study is also important when investigating developmental processes (Card & Little, 2007). Temporal design is defined as the “rationale underlying the sampling of times of measurement” (Collins & Graham, 2002). In longitudinal studies, temporal design choices should have a strong theoretical or empirical basis; however, these choices rarely have strong support in practice (Collins & Graham, 2002; Selig & Preacher, 2009).

Most researchers use relatively simple time metrics when examining latent difference score (LDS) models even if a complex time metric is more appropriate. For example, researchers may use measurement occasion data instead of using age (a more complex time metric that is potentially more sensitive to capturing the true change process) because their data are already structured with respect to measurement occasion (Small, Dixon, McArdle, & Grimm, 2012). Researchers may not be willing to use the more sensitive time metric because it can be tedious to change the data structure and to re-program the LDS model to accommodate the more complex time metric. Furthermore, when using a more complex time metric, the organization of the data may lead to a very sparse data structure, which can affect model convergence.

However, analyzing data structured with a time metric that does not capture the true process of change to estimate LDS models can affect parameter estimation and the conclusions drawn from the study. Specifically, coupling parameters (the time-dependent effects of one construct on successive change in another construct; McArdle, 2001, 2009) in the bivariate dual change LDS model have different interpretations based on the time metric of the data. Furthermore, model fit may be worse when using the time metric that does not capture the true process of change, and models may not converge if the time metric used is incorrect for the process of interest. Finally, covariance coverage may be very low for the later measurement occasion variables when data are structured by measurement occasion because of random and non-random attrition (i.e. a pattern of attrition). The consideration of time metric in LDS models is an extension of work examining the importance of time metric for other longitudinal models such as growth models (Hoffman, 2015).

It is important to examine the role of time structure in LDS models because the time metric must represent change as accurately as possible in the LDS model. Research has not yet addressed accuracy of change in the LDS model with respect to time, and there has been no simulation work investigating different time metrics with LDS models. This lack of discussion in the literature would indicate that researchers using the LDS model might not understand the consequences of using different time metrics for their data. This study is an important contribution for substantive researchers because it can inform researchers about the benefits of using more appropriate time metrics when fitting LDS models. The aim of this study is to determine whether different time metrics influence the estimation and interpretation of model parameters, model convergence, and conclusions drawn from LDS models.

Background Literature

Latent Growth Curve (LGC) Models

Latent growth curve (LGC) models account for intraindividual change over time and take into account predictors of this intraindividual change. LGC models can be fit in either the structural equation modeling (SEM) or multilevel modeling (MLM) frameworks (Muthén & Curran, 1997; Selig & Preacher, 2009; Singer & Willett, 2003). The LGC model can be represented as a restricted common factor model. Using matrix notation, the LGC model can be written as

$$\mathbf{y}_n = \boldsymbol{\Lambda}\eta_n + \mathbf{u}_n \tag{1}$$

where $\mathbf{y}_n$ is the t x 1 vector of observed scores with t repeated measures for participant n, $\boldsymbol{\Lambda}$ is the t x R factor loading matrix defining the latent growth factors (R = 1 for no growth, R = 2 for linear growth), η_n is the R x 1 vector with latent factor scores for participant n, and $\mathbf{u}_n$ is the t x 1 vector of unique scores for participant n.

Latent Difference Score (LDS) Models

The latent difference score (LDS) model (also known as the latent change score or LCS model) allows researchers to investigate time-sequential effects with multivariate longitudinal data (Ferrer & McArdle, 2003; Hamagami & McArdle, 2001; McArdle, 2001, 2009; McArdle & Nesselroade, 1994). The difference between LDS and LGC models is that LGC models assess the growth rates of the variable over time, and LDS models assess change between two time points for several measurement waves. The LDS and LGC models answer different substantive research questions about change over time

(Grimm, 2007). Growth models investigate change from start to finish, and LDS models focus on dynamic change where change between two time points is based prior status (and potentially prior rates of change). While LGC models provide information about how growth in variables is related over time, the LDS model provides information about dynamic relations between variables that the LGC model does not (McArdle, 2009). The LDS model represents dynamic change using difference scores (McArdle, 2001).

The LDS model is often fit in the SEM framework and models change between each time point and the previous time point as a latent difference between two common factor scores (McArdle, 2001). LDS models address questions of causal and dynamic change that prior SEMs did not address (McArdle, 2009). The LDS model represents difference scores as latent variables meaning it is assumed that the differences represented by the scores are measured without error. Representing difference scores using latent variables addresses the problem of measurement error in difference scores (Cronbach & Furby, 1970). One advantage of the LDS model over the LGC model is that the LDS model allows predictors of change to differ at different measurement waves. The LDS model is also useful when researchers are interested in intraindividual change, but when that change may be different at different waves of measurement (Selig & Preacher, 2009). With respect to time, the LGC and LDS models differ in that LGC models often treat time as a predictor, LDS models handle time by changing the data structure before fitting the model.

To understand a latent difference score, it is necessary to understand the decomposition of an observed score following classical test theory (Ferrer & McArdle, 2003; McArdle 2001; Grimm, An, McArdle, Zonderman, & Resnick, 2012). An observed score Y is the sum of a true score and a unique score:

$$Y[t]_n = y[t]_n + e[t]_n \tag{2}$$

where $Y[t]_n$ is the observed score for individual n at time t, $y[t]_n$ is the true (latent) score, and $e[t]_n$ is the unique score. True scores are related over time and unique scores are not. In LDS models, true scores have fixed unit autoregressive relations over time, so a true score $y[t]_n$ is a function of the true score at the previous time, $y[t - 1]_n$ plus true score change from time $t – 1$ to t, $\Delta y[t]_n$:

$$y[t]_n = y[t - 1]_n + \Delta y[t]_n \tag{3}$$

Rearranging the terms from Equation 3, the latent difference score is represented by the difference between consecutive latent scores at time t and time t - 1:

$$\Delta y[t]_n = y[t]_n - y[t - 1]_n \tag{4}$$

Like LGC models, LDS models have a trajectory equation for each observed variable. However, LDS model trajectory equations place focus on the latent difference scores instead of on latent true scores. The trajectory equation for a LDS model is as follows:

$$y[t]_n = g_{0n} + \sum_{r=2}^{r=t} (\Delta y[r]_n) \tag{5}$$

where g_{0n} is the initial true level and $\sum_{r=2}^{r=t} (\Delta y[r]_n)$ is the sum of latent changes up to time t such that $r = t$.

For a single observed variable Y, there are three commonly specified LDS models. These models can represent time-dependent change based on determinants of the LDSs

(Grimm, 2012; Grimm et al., 2012; McArdle & Grimm, 2010). The first model is the constant change model, $\Delta y[t]_n = \alpha \cdot g_{1n}$, where g_{1n} is a constant change component for participant n (meaning it varies for individuals) with a mean of μ_{g1} and variance of σ^2_{g1} and α is a fixed parameter usually equal to 1. The second model is the proportional change model, $\Delta y[t]_n = \beta \cdot y[t-1]_n$, where β is a fixed parameter that does not vary for individuals, meaning each time-dependent change is proportional to the previous true score. This proportional change parameter represents the effect of y on itself over time. The third model is the dual change model, which combines the constant change and proportional change models. In the dual change model, $\Delta y[t]_n = \alpha \cdot g_{1n} + \beta \cdot y[t-1]_n$, so time-dependent changes have both a constant change component and depend on the previous true score. The dual change model is named as such because it includes both systematic constant change from the linear slope (α) and systematic proportional change over time (β) (McArdle, 2001, 2009).

LDS models were developed to address change in multiple variables over time (Grimm et al., 2012). The composition of true and unique scores (Equation 1) and the trajectory equation (Equation 4) can also be used for a second observed variable X. When examining a bivariate dual change LDS model, the relationship between two variables over time is of interest and so we examine coupling effects (i.e., the effects of each variable on the other over time; McArdle, 2001). The following equations are the LDSs for X and Y for a bivariate dual change model:

$$\Delta y[t]_n = \alpha \cdot g_{1n} + \beta_y \cdot y[t-1]_n + \gamma_{yx} \cdot x[t-1]_n \quad (6)$$

$$\Delta x[t]_n = \alpha \cdot h_{1n} + \beta_x \cdot x[t-1]_n + \gamma_{xy} \cdot y[t-1]_n \quad (7)$$

where g_{1n} and h_{1n} are the constant change components for participant n on X and Y, β_x and β_y are the proportional change parameters for X and Y that represent the effects of the variables on themselves over time, and γ_{xy} and γ_{yx} are the coupling parameters for X and Y that represent the effects of each variable on the other over time. Non-zero coupling parameters are typically less than 1.

Figure 1 (Appendix A, p. 54) shows a LDS model with five measurement waves for X and five measurement waves for Y. Although the model in Figure 1 includes LDSs for X and Y across five time points, in general LDS models can include LDSs for any number of time points greater than 1.

In Figure 1, μ_{g0} and μ_{h0} are the means for the initial levels (intercepts) of Y and X and μ_{g1} and μ_{h1} are the means for the constant change components (slopes) of Y and X. The variances σ^2_{g0} and σ^2_{h0} are the variances for the initial levels (intercepts) of Y and X and σ^2_{g1} and σ^2_{h1} are the variances for the constant change components (slopes) of Y and X. A full list with definitions for each term in Figure 1 is given in Appendix B.

Assumptions of LDS models. Like most SEMs, the LDS model requires several assumptions about observed data and latent variables. Five key assumptions of the LDS model are: 1) change in the model applies only to the true (latent) scores, 2) the proportional change component does not vary for individuals, however the constant change component may vary for individuals, 3) time interval is constant, 4) difference equations which approximate differential equations are used to represent change, and 5) means, variances, and covariances of observed variables over time are given a structure in order to fit SEMs (Hamagami & McArdle, 2007).

Time Metric

Time metric is defined as the time scale over which change occurs (Cicchetti, 2016). The scale of a time metric can range from seconds to millennia, although most studies of longitudinal processes in the social sciences use a scale ranging from weeks to years. Example time metrics commonly found in longitudinal research are measurement occasion, chronological time from beginning of study, or age in years or months. When researchers are interested in investigating change over time, it is important to select a time metric that is appropriate for examining the process (Hoffman, 2015). Hoffman (2013) argued that an important missing step in conducting longitudinal data analysis is determining what the metric of time should be that matches the process of interest, and also determining how time should be modeled when there are individual differences in the time metric (Bell, 1953; McArdle & Bell, 2000). For example, participants may begin a study at different time points, or may have different measurement intervals. A time metric that is scaled such that it closely matches the longitudinal process of interest may be more sensitive to detecting true effects than other potential time metrics. A more sensitive time metric may have more or fewer time points than less sensitive time metrics depending on the process of interest.

Time and Longitudinal Models

Timing and spacing of measurement waves is an important consideration for longitudinal studies that can influence the interpretation of study results. Participants may all be on the same assessment schedule so that every participant is assessed at the same time (i.e., time-structured data) or the assessment schedule of a study may vary for different participants (i.e., time-unstructured data; Singer & Willett, 2003). For both time-structured and time-unstructured data, measurement waves may be equally or unequally spaced. For some models, spacing of measurement waves is critical because relations between variables can differ at different measurement intervals (Collins & Graham, 2002). Many times, longitudinal researchers use calendar time to determine timing and spacing of measurements, however calendar time is not always an appropriate choice (Collins, 2006; Lerner, Schwartz, & Phelps, 2009; Little, Card, Preacher, & McConnell, 2009). As an example from developmental data, growth spurts in height during adolescence mean that regular interval measurements and time-structured data are not nuanced enough to capture the true change process; however, developmental researchers rarely address choice of time metric when examining height (Lerner et al., 2009). Models do exist that handle different measurement intervals in a single data set by treating measurement interval as a predictor (Selig, Preacher, & Little, 2012). In general, for a study with a given length of time, measurement waves that are spaced too far apart can lead to erroneous interpretations, and more measurement waves spaced closer together provide more accurate conclusions (Collins & Graham, 2002).

Time and Growth Models

Preacher (2010) gave a list of the components that are necessary for investigating growth and one of the most important items on the list was the description of the metric of time, second only to an appropriate substantive theory underlying the model. A benefit of LGC models is that they can accommodate any time metric, from milliseconds to millennia (Ram & Grimm, 2007), but it is important to know the metric underlying the process of interest. Referring to Equation 1, the LGC model is based on *t* repeated measures. The contents of each vector and matrix differ for different time metrics, because the value of *t* differs (and therefore the number of vector and matrix rows). Time metrics for growth models can be discrete or continuous, with continuous time metrics referred to as individually-varying time metrics, although there is not a straightforward approach to growth models with individually-varying time metrics (Grimm, Ram, & Estabrook, 2016). For LGC models, the timing and spacing of measurement waves affects inference differently than for other models. The coding of slope factor loadings determine intercept location and slope scale in LGC models (Grimm, 2012). It follows that the treatment of the timing variable in longitudinal studies using LGC models can affect the interpretation of intercept and slope parameter estimates (Biesanz, Deeb-Sossa, Papadakis, Bollen, & Curran, 2004; Grimm, 2012; Mehta & West, 2000) and accurate choice of lag spacing is essential for capturing the true trajectory of change (Selig & Preacher, 2009). Larger intervals between measurements can result in oversimplified growth curves that do not reflect the true trajectory of change (Collins & Graham, 2002). More specifically, the selection of the unit of time may affect both the precision of estimates and power to detect effects in LGC models, and can affect the interpretation of functional form of relationships (Biesanz et al., 2004). Furthermore, when time intervals differ for participants or when participants vary widely in age at first measurement point, traditional SEM approaches to LGC models will produce biased estimates of intercept and slope covariances (Coulombe, Selig, & Delaney, 2015; Mehta & West, 2000). However, when time intervals are different for participants, the LGC model can also be used to investigate individual trajectories of change using a definition variable approach (Sterba, 2014). For nonlinear growth, using more measurement occasions to model growth will increase accuracy of parameters up to a point, and concentrating measurements at extremes or around areas of predicted greatest nonlinear change will improve accuracy and efficiency of estimates (Timmons & Preacher, 2015). Timing is also important when making decisions about model intercepts in LGC models. For the LGC model, the time point that is equal to “time 0” (the reference point) changes the interpretation of effects related to the intercept.

There has been debate in the literature about how to handle time metric in growth models. Some researchers argue that occasion should be used as the time metric for growth models, while including time-related information (such as age) as a predictor or covariate in the model (Hoffman, 2015, p. 442). Others argue that depending upon the process of interest, it is more appropriate to use time-related information as the actual time metric for the growth model (for example, using age as the time metric instead of including age as a predictor). These discussions about handling of time metric extend to the LDS model as well.

Time and Latent Difference Score Models

As mentioned above, LDS models make certain assumptions about measurement intervals, specifically that the time between all pairs of latent scores of interest (that is, the time between t and $(t - 1)$) has a constant interval such that Δt is equal to 1 (McArdle, 2001, p. 348). The specific assumption is that the time interval between each set of latent variables is equal to the time interval between every other set of latent variables in the model, even if time interval is not equal for observed scores (the observed data are unbalanced) (Hamagami & McArdle, 2001, 2007; McArdle, 2009). Making the equal intervals assumption allows latent difference scores to be interpreted as rates of change, where $\Delta y[t]_n = \Delta y[t]_n / \Delta t$, and this assumption extends to bivariate dual change models.

Equal intervals are a key assumption for every LDS model because the use of fixed unit coefficients allows any trajectory equation to be defined from a starting change equation (McArdle, 2009). One traditional approach to using LDS models with unequal spacing of intervals is to include additional "incomplete" intervals representing gaps in the data such that there are unmeasured latent variables representing missing time points, allowing a model with data and incomplete data where Δt is equal to 1 (McArdle, 2001). More recently, a model termed the triple change score (TCS) model was proposed to relax the requirements of equal time intervals between the LDSs by including latent basis coefficients of change into the dual change model (McArdle & Nesselroade, 2014), but the TCS model is not widely used.

Choice of interval size is important to the interpretation of effects in the LDS model because the interpretation of a difference score is dependent upon the lag between the two measurement waves used to calculate the difference score (Selig & Preacher, 2009). As a reminder, in LDS models a fixed parameter α that is usually set to 1 scales the constant change component g_{1n}. Changing the value of the fixed parameter α changes the interpretation of the constant change parameter by changing the metric of t. For example, changing α to 2 would mean that g_{1n} can be interpreted with a .5-unit change in t as opposed to a 1-unit change. Consequently, the mean and variance of g_{1n} both change to reflect the α scaling, and the covariance between g_{1n} and the intercept may be influenced as well (Grimm, 2012). Finally, as with LGC models, time is an important factor for LDS models when making decisions about intercept coding and centering. Recently, LDS models have been developed that allow for intercept centering at any measurement wave (Grimm, 2012). Moving the intercept requires changing the LDS weights to maintain the assumption of invariant time intervals. Changing the location of the intercept influences interpretation of intercept parameters, but not other parameters or model fit.

Hypotheses

Time metric is important for the interpretation of estimates in models assessing longitudinal change. It is expected that time metric will influence interpretation and accuracy of estimates for the bivariate LDS model as well. Specifically,

1) Given a set of true parameters and a time metric that reflects the true process of change, bivariate LDS models using the original time metric will yield estimates closer to the defined true parameters than bivariate LDS models using a restructured time metric that provides less information about the process of change.
2) Given a set of true parameters, coupling parameters will be more accurate in bivariate LDS models with data structured according to the original time metric, whereas bivariate LDS models with a restructured time metric will yield biased estimates of coupling parameters.
3) Given a set of true parameters, bivariate LDS models using the original time metric will have better model convergence than bivariate LDS models using a restructured time metric.

Method

Simulation Conditions and Procedure

Data for a bivariate dual change LDS model were simulated in SAS 9.4 with random seed set to the current time using the following equations:

$$Y[t]_n = y[t]_n + e[t]_n \qquad (8)$$

$$X[t]_n = x[t]_n + e[t]_n \quad (9)$$

Where

$$y[t]_n = y[t - 1]_n + \Delta y[t]_n \quad (10)$$

$$x[t]_n = x[t - 1]_n + \Delta x[t]_n \quad (11)$$

$$\Delta y[t]_n = \alpha \cdot g_{1n} + \beta_y \cdot y[t - 1]_n + \gamma_{yx} \cdot x[t - 1]_n \quad (12)$$

$$\Delta x[t]_n = \alpha \cdot h_{1n} + \beta_x \cdot x[t - 1]_n + \gamma_{xy} \cdot y[t - 1]_n \quad (13)$$

In the above equations, $e[t]_n$ is the product of σ^2_e and a random variate distributed $N(0, 1)$. Conditions for this study were created based on differing number of time points (t), number of observations (n), and values of coupling parameters (γ_{yx} and γ_{xy}). Parameter bias, parameter variability, and model convergence were used to assess performance for each model.

Parameter values for the simulations were selected based on a real data example of an LDS model examining reading and IQ (Ferrer, Shaywitz, Holahan, Marchione, & Shaywitz, 2010). The data were originally taken from the Connecticut Longitudinal Study (Shaywitz, Shaywitz, Fletcher, & Escobar, 1990) and comprised a sample of N = 232 students measured from grades 1-12 (ages 6-18). The data included information on several time metrics, including age and grade; Ferrer et al. (2010) used grade as the time metric for their LDS model. In the Ferrer et al. example, X was a reading score composite that included subtests of sound-letter correspondence, word identification, and general reading comprehension from the Woodcock-Johnson Psycho-Educational Battery (Woodcock & Johnson, 1977). The Y measure was the Wechsler full scale IQ, assessed at each grade using the WISC-R (Wechsler, 1981). The Wechsler measure of IQ encompasses several measures of cognition and is expected to fluctuate over time. There were three groups examined: typical readers (N = 142), compensated readers (N = 28), and persistently poor readers (N = 62). The parameters from the typical readers group were selected based on sample size and the presence of significant coupling for that group. The parameters for the typical readers group are as follows: μ_{g0} = 0.220, μ_{g1} = 0.550, μ_{h0} = 0.570, μ_{h1} = 1.410, β_y = -0.274, β_x = -0.549. All parameters from Ferrer et al. 2010 were standardized. The intercept mean parameters indicated that at baseline (grade 1), typical readers had above-average reading and IQ means ($\mu_{h0} = \mu_{read0}$ = .570 and $\mu_{g0} = \mu_{IQ0}$ = .220). The other parameters indicated that yearly changes from 1st to 12th grade were a function of a positive constant slope (positive slope mean parameters $\mu_{h1} = \mu_{read1}$ = 1.410 and $\mu_{g1} = \mu_{IQ1}$ = .550) with inertia (negative proportional change parameters $\beta_x = \beta_{read}$ = -0.549 and $\beta_y = \beta_{IQ}$ = -0.27) representing the proportional effect of each variable's previous value on its changes at the next measurement, and positive coupling ($\gamma_{yx} = \gamma_{read \rightarrow IQ}$ = 0.130 and $\gamma_{xy} = \gamma_{IQ \rightarrow read}$ = 0.401). The positive coupling parameters indicated that there was a mutual, positive relationship between reading and IQ over time. These parameters did not vary over conditions.

All correlations between intercepts and slopes were set to 0.5. For coupling parameters, models had either positive non-zero coupling or no coupling, with non-zero coupling parameters coming from the typical readers group (γ_{yx} = 0.130 and γ_{xy} = 0.401) and for no coupling, coupling parameters were set to zero. When coupling parameters were set to zero, the error variance of X was adjusted 0.5 to produce appropriate trajectories. Individuals were assumed to have the same process of change (that is, variation in change was not measured at multiple levels). For number of observations, models were examined at two sample sizes, N = 200 and N = 1000. These samples sizes were selected as representations of relatively small and large sample sizes in developmental research.

Two time structures were investigated. The first (referred to as the *time-occasion* structure) compared two time metrics with varied numbers of time points, 20 time points restructured to 10 time points and 20 time points restructured to 4 time points. The metric with 20 time points, the true time metric used to generate the data, can be considered a measure such as chronological time (for example, 20 months). This time metric will be referred to as the original time metric. The metric with fewer time points (10 or 4 time points) that was the restructured time metric can be considered an occasion metric. In the original time metric, participants were measured for a total of 10 or 4 occasions over 20 time points where each participant had consecutive measurement occasions such that interval lag was equal within and across participants, but participants had different starting points. The restructuring of the data resulted in restructured time metrics with 4 or 10 total time points that ignored participants' different starting points. A figure elaborating the metrics of the *time-occasion* structure with t = 20 restructured to t = 4 is shown in Figure 2 (Appendix A, p. 55). The second time structure (referred to as the *age-grade* structure) compared two time metrics with 24 time points restructured to 6 time points, and 12 time points restructured to 6 time points. The time metric with more time points (24 or 12 time points) can be considered a measure such as age measured in quarter- or half-years. For example, 24 time points could represent age measured in quarter-years from age 7 to age 12. The time metric with fewer time points that was the restructured time metric can be considered a less informative age metric, for example a measure of school grade that only allows measurement once per grade. In the original time metric, participants were measured for a total of 6 occasions over 12 or 24 time points where each participant had different intervals between measurements such that interval lag was different within and across participants, and participants had different starting points as well. The restructuring of the data resulted in a restructured time metric with 6 total time points that ignored participants' different lags between measurements and differing starting points. A figure elaborating the metrics of the *age-grade* structure with t = 24 restructured to t = 6 is shown in Figure 3 (Appendix A, p. 56). For both the *time-occasion* and *age-grade* structures, the true time metric used to generate the data was the metric with more time points (t = 20 for *time-occasion* and t = 12 or 24 for *age-grade*), however a metric with more time points will not always be the true time metric in applied research. More time points in the true time metric meant that there were complete data for all participants for the restructured time metric, but necessarily missing values for each participant for the true time metric.

These condition variations resulted in $2^3 = 8$ conditions (N = 200 or 1000, coupling or no coupling, and two sets of time points) for each time structure. For each of the eight conditions, 500 replications were simulated. For each replication, data were simulated in SAS 9.4 with the true time metric, and then restructured to a second, less informative time metric (two data sets per replication). The two data sets from each replication were then read into Mplus 7 and appropriate bivariate dual change LDS models were fit to each data set. Models did not include starting values and maximum number of iterations was set to 10000 for each replication. Because models were fit to the true and restructured time metrics for each condition, 8*2 = 16 conditions were examined each for the *time-occasion* and *age-grade* structures. Appendix C gives sample code for data generation in SAS and Appendix D gives sample code for the bivariate dual change LDS model in Mplus.

Dependent Variables

This study assessed performance of parameter estimates for the slope and intercept means, proportional change parameters, and coupling parameters by calculating parameter bias and parameter variability. Accuracy of parameter estimates was also examined using box plots. This study also examined model convergence and inadmissible parameter estimates. For the full simulation, some models provided all fit indices and some models provided only a subset of fit indices due to lack of information on the alternative hypothesis log-likelihood, and consequently it was difficult to examine model fit across conditions. Therefore, parameter estimate performance, model convergence, and presence of inadmissible parameter estimates were used as a proxy to determine adequacy of each model instead of model fit indices. If parameter estimates were unbiased and efficient, the model converged, and inadmissible parameter estimates were not present, a model was deemed adequate.

Parameter bias was assessed in three ways, using raw, relative, and standardized parameter bias. Raw parameter bias was calculated by taking the difference between the true value of the parameter and the simulated parameter estimate. Relative parameter bias was calculated by dividing raw parameter bias by the true value of the parameter. Standardized parameter bias was calculated by dividing raw parameter bias by the standard deviation of the averaged parameter estimate by condition. Parameter variability was assessed by examining the standard deviation of the averaged parameter estimates by condition.

Model convergence was assessed by examining the proportion of times a model converged for a given condition. Conditions with a higher proportion of model convergence were considered to have better model convergence. Model convergence was coded for each replication as a binary variable where 0 = non-convergence and 1 = convergence. Inadmissible parameter estimates were assessed by examining the variance of the intercepts and slopes (σ^2_{g0}, σ^2_{h0}, σ^2_{g1}, and σ^2_{h1}). If any of the intercept or slope variance parameter estimates were negative, they were considered inadmissible. Inadmissible parameter estimates were coded by creating four binary variables, one for each of the four variances, where 0 = positive value (admissible) and 1 = negative value (inadmissible). For each replication, there were four variables coding inadmissible estimates: one variable each for σ^2_{g0}, σ^2_{h0}, σ^2_{g1}, and σ^2_{h1}.

Statistical Analyses

Analyses examined the impact of condition on the dependent variables of interest. Analyses were conducted at the replication level, where each replication was considered one observation. All analyses were conducted in SAS 9.4. For performance of estimates, ANOVA was used to examine the effect of study condition on parameter bias and parameter variability. Factors representing study conditions included in each ANOVA were *n* (200, 1000), *t* (6, 12, and 24 for *age-grade* structure; 4, 10, and 20 for *time-occasion* structure), and coupling (positive non-zero coupling or no/zero coupling, also referred to as no coupling "NC" or yes coupling "YC"). All results reported from ANOVAs were computed using Type III sums of squares. For model convergence, logistic regression assessed predictors of a binary outcome where model non-convergence = 0 and model convergence = 1. Logistic regression analyses were also used for assessment of inadmissible parameter estimates, where 0 = admissible and 1 = inadmissible for the four variances of interest (negative values of intercept and slope variance).

For each analysis, lower-order significant effects were not reported if higher-order interactions were significant as well. Because the large number of replications included in each analysis could lead to multiplicity (i.e. the multiple comparisons problem) and thus inflated Type I error rates, only effects with odds ratios (ORs) above 1.5 or below .5 or partial η^2 effect sizes greater than 0.09 (roughly a medium partial η^2) were considered meaningfully significant effects and were discussed.

Results for Age-Grade Structure

Parameter Estimate Bias

For each measure of raw bias, an analysis of variance (ANOVA) was conducted with n (n = 200 or 1000), t (t = 6, 12, or 24), and coupling (positive non-zero coupling or "yes coupling" ("YC") compared to "no coupling" ("NC")) as predictors. All interactions between predictors were included in each analysis. Effects were deemed meaningfully significant if $p < .001$ and $\eta_p^2 \geq .09$. Partial η^2 effect sizes are reported for each effect of interest. Table 1 (Appendix A, p. 42) shows results of these ANOVAS. For raw bias of the intercept mean parameter estimates μ_{g0} and μ_{h0}, t was a meaningfully significant predictor of both parameters ($p < .0001$ and $\eta_p^2 = .62$ for μ_{g0}, $p < .0001$ and $\eta_p^2 = .72$ for μ_{h0}). For raw bias of the slope mean parameter estimates μ_{g1} and μ_{h1}, the highest order meaningfully significant interaction was the interaction between the t and coupling predictors ($p < .0001$ and $\eta_p^2 = .08$ for μ_{g1} which bordered on meaningful significance, $p < .0001$ and $\eta_p^2 = .09$ for μ_{h1}), while t was also a meaningfully significant predictor of the slope means ($p < .0001$ and $\eta_p^2 = .48$ for μ_{g1}, $p < .0001$ and $\eta_p^2 = .52$ for μ_{h1}). For raw bias of the proportional change parameters β_y and β_x, the highest order meaningfully significant interaction was the interaction between the t and coupling predictors ($p < .0001$ and $\eta_p^2 = .108$ for β_x and $p < .0001$ and $\eta_p^2 = .28$ for β_y), and t was also a meaningfully significant predictor of β_x ($p < .0001$ and $\eta_p^2 = .43$) and coupling was also a meaningfully significant predictor of β_y ($p < .0001$ and $\eta_p^2 = .11$). For the coupling parameters γ_{yx} and γ_{xy}, the highest order meaningfully significant interaction was the interaction between the t and coupling predictors ($p < .0001$ and $\eta_p^2 = .15$ for γ_{yx}, and $p < .0001$ and $\eta_p^2 = .21$ for γ_{xy}), and t was a meaningfully significant predictor as well ($p < .0001$ and $\eta_p^2 = .23$ for both coupling parameters). Because the parameters used in the simulations were based on real data and therefore different values with differing variability, results were not the same for parameters related to X compared to parameters related to Y (specifically β_x and β_y).

To investigate the raw bias significant interactions further, box plots of raw bias were created separated by condition. Box plots of raw bias by condition for all parameters of interest are shown in Figures 4 through 11 (Appendix A, pp. 57-64). In each Figure, “YC” indicates “yes coupling” and “NC” indicates “no coupling”. Sample sizes are represented by “200” or “1000”, time structure is represented by “24-6” or “12-6”, and number of time points for that condition is represented by “6”, “12”, or “24”. For all parameters, average raw bias was close to zero for true time metrics and was non-zero for the restructured time metric. The difference in bias increased when the true time metric had more time points (24-6 compared to 12-6), and when the 24-6 time structure had non-zero coupling. The bias values were also more stable (and slightly larger) at n =

1000 than at n = 200 for conditions with large bias. For the slope parameters, bias was positive and comparatively very large with non-zero coupling for the restructured time metric when 24 time points were restructured to 6 time points. For the proportional change and coupling parameters, bias was comparatively very large with non-zero coupling for the restructured time metric when 24 time points were restructured to 6 time points, and was negative for β_x and γ_{yx} and positive for β_y and γ_{xy}.

For each measure of relative bias, ANOVAs were conducted with n, t, and coupling as predictors and all interactions between predictors included. For the measures of relative bias for γ_{yx} and γ_{xy}, analyses were conducted for only those conditions with non-zero coupling, as relative bias is undefined for conditions where coupling is zero. These analyses included n, t, and the interaction between n and t as predictors. The same pattern of results held for measures of relative bias as for measures of raw bias for the intercept and slope mean parameter estimates and proportional change parameter estimates in terms of meaningfully significant effects with similar p values for all effects. For relative bias of estimates for γ_{yx} and γ_{xy}, all interaction and lower-order effects were significant for both measures.

For each measure of standardized bias, ANOVAS were conducted with n, t, and coupling as predictors and all interactions between predictors included. Table 2 (Appendix A, p. 43) shows results of these ANOVAS. Results for analyses of standardized bias were similar to analyses of raw bias, with several additional meaningfully significant effects. For μ_{g0} and μ_{g0}, the interaction between n and t approached meaningful significance (p < .0001 and η_p^2 = .08 for both). For μ_{g1}, the interaction between n and t was meaningfully significant (p < .0001 and η_p^2 = .10) and the interaction between t and coupling was not meaningfully significant for either slope mean parameter. For β_x, the interaction between n and t was meaningfully significant ($p <$ .0001 and $\eta_p^2 = .09$) and the interaction between t and coupling was not meaningfully significant. For μ_{g0}, μ_{h0}, β_y, γ_{yx}, and γ_{xy}, the same predictors were meaningfully significant in the analyses of standardized bias as for raw bias.

Parameter Estimate Variability

Parameter variability (or stability) for the *age-grade* time structure was examined using the standard deviations of the averaged parameter estimates for each condition. The standard deviations of all parameter estimates of interest for each condition are shown in Table 3 (Appendix A, p. 44). For the intercept means, parameter estimates in conditions with the true time metric had larger variability (and thus less stability) than parameter estimates in conditions with the restructured time metric, and variability did not vary widely across conditions with coupling compared to no coupling. For the slope means, proportional change, and coupling parameters, parameter estimates in conditions with the restructured time metric had larger variability (and thus less stability) than parameter estimates in conditions with the true time metric. Also for the slope means, proportional change, and coupling parameters, parameter estimates in conditions with the restructured time metric at n = 200 with 24 time points restructured to 6 had much larger variability than parameter estimates in all other conditions, and variability was largest in the coupling condition. Variability was also large in the condition with the restructured time metric, no coupling, n = 1000 with 24 time points restructured to 6. The larger variability in intercept mean parameter estimates in the true time metric was likely due to the missing data and individually differing intercepts in the true time metric.

Convergence

For model convergence in the *age-grade* time structure, a logistic regression was conducted at the condition level with *n*, coupling, and the interaction between *n* and coupling as predictors. The *t* predictor was excluded from analysis as non-convergence occurred in non-zero coupling conditions only when $t = 24$, and there were too few instances of non-convergence at $t = 6$ and $t = 12$ to include *t* as a predictor. The outcome was coded as 1 = convergence and 0 = non-convergence, so this logistic regression predicted the group coded zero (or instances of non-convergence). Table 4 (Appendix A, p. 45) shows results of this logistic regression. For model convergence, all effects were significant at $p < .001$, however, only the coupling predictor had an odds ratio (OR) above 1.5. The coupling predictor had an OR of 3.581. Coupling was coded as 0 = no coupling and 1 = coupling, so the interpretation of this OR is that the odds of non-convergence were larger in replications with coupling compared to replications with no coupling. To elaborate these results further, Table 5 (Appendix A, p. 46) shows frequency tables for convergence separated by *t* and coupling. For conditions with the restructured time metric ($t = 6$) and non-zero coupling, models converged for all 2000 replications, and for conditions with the restructured time metric and no coupling, models did not converge for 10 of 2000 replications. Models converged for all 1000 replications for conditions with the true time metric ($t = 12$) and non-zero coupling, and models did not converge for 4 of 1000 replications for conditions with the true time metric ($t = 12$) and no coupling. The majority of models that did not converge were in conditions with the true time metric where $t = 24$. For conditions with the true time metric ($t = 24$), models did not converge for 49 of 1000 replications in conditions with non-zero coupling, models did not converge for 301 out of 1000 replications in conditions with no coupling.

Inadmissible Parameter Estimates

A check for inadmissible parameter estimates was conducted by examining descriptive statistics for variances of intercepts and slopes. For all conditions in the *age-grade* time structure, there were no inadmissible parameter estimates, meaning all intercept and slope variances were non-negative at the replication level.

Results for Time-Occasion Structure

Parameter Estimate Bias

For each measure of raw bias, an ANOVA was conducted with n (n = 200 or 1000), t (t = 4, 10, or 20), and coupling (positive non-zero coupling or “yes coupling” (YC) compared to “no coupling” (NC)) as predictors, and all interactions between predictors included. Effects were deemed meaningfully significant if $p < .001$ and $\eta_p^2 \geq .09$. Partial η^2 effect sizes are reported for each effect of interest. Table 6 (Appendix A, p. 47) shows results of these ANOVAS. For raw bias of the intercept mean parameter estimates μ_{g0} and μ_{h0}, meaningfully significant predictors were the t predictor ($p < .0001$ and $\eta_p^2 = .72$ for both) and the coupling predictor ($p < .0001$ and $\eta_p^2 = .14$ for both). There were no meaningfully significant predictors of raw bias of any other parameter estimates.

To investigate significant predictors of raw bias further, box plots were created to examine raw bias separated by condition for the intercept mean parameter estimates. Examination of box plots of raw bias for the slope mean parameter estimates μ_{g1} and μ_{h1}, and proportional change and coupling parameter estimates β_x, β_x, γ_{yx}, and γ_{xy} showed that average raw bias was very close to zero across conditions, but that raw bias tended to have larger variability (and thus was less stable) with smaller *n* and restructured time metrics (t = 4 and t = 10). Box plots for the intercept mean parameter estimates μ_{g0} and μ_{h0} (the only parameter estimates with conditions that had non-zero average raw bias) are shown in Figures 12 and 13 (Appendix A, pp. 65-66). In these two Figures, “YC” indicates “yes coupling” and “NC” indicates “no coupling”, sample sizes are represented by “200” or “1000”, time structure is represented by “20-10” or “20-4”, and number of time points for that condition is represented by “4”, “10”, or “20”. For the intercept mean parameter estimates μ_{g0} and μ_{h0}, average raw bias was close to zero for the true time metric and there was non-zero bias for the restructured time metric. This difference in bias increased when the restructured time metric had fewer time points (20-4 compared to 20-10), and for conditions with non-zero coupling. The bias values were also more stable at n = 1000 compared to n = 200.

For each measure of relative bias, *n*, *t*, and coupling were used as predictors in a series of ANOVAs with all interactions between predictors included. For relative bias of γ_{yx} and γ_{xy}, analyses were conducted only for those conditions with non-zero coupling, as relative bias is undefined where coupling is zero. Analyses for relative bias of γ_{yx} and γ_{xy} included *n*, *t*, and the interaction between *n* and *t* as predictors. The same pattern of results held for measures of relative bias as for measures of raw bias of the intercept mean parameter estimates in terms of meaningfully significant predictors between predictors with similar *p* values for the interactions.

Table 7 (Appendix A, p. 48) shows results from ANOVAS for each measure of standardized bias with *n*, *t*, and coupling as predictors and all interactions between predictors included. For standardized bias of the intercept mean parameter estimates μ_{g0} and μ_{h0}, the highest order meaningfully significant interaction was the two-way interaction between *n* and *t* ($p < .0001$ and $\eta_p^2 = .10$ for μ_{g0} and μ_{h0}) and the *n* and *t* predictors were meaningfully significant as well. For the slope mean parameter estimates μ_{g1} and μ_{h1}, proportional change parameter estimate β_x, and coupling parameter estimates γ_{yx} and γ_{xy}, there were no significant predictors of standardized bias at $p < .0001$ with $\eta_p^2 \geq .09$.

Parameter Estimate Variability

Standard deviations of the averaged parameter estimates for each condition were used to examine parameter variability (or stability) for *the time-occasion* time structure. The standard deviations for each condition for all parameter estimates of interest are shown in Table 8 (Appendix A, p. 49). For the intercept means, conditions with the true time metric had parameter estimates with larger variability (less stability) than conditions with restructured time metrics, and parameter estimate variability was larger for conditions with non-zero coupling. For the slope means, proportional change, and coupling parameter estimates, conditions with restructured time metrics had parameter estimates with larger variability (and thus less stability) than parameter estimates in conditions with the true time metric. Also for slope means, proportional change, and coupling, parameter estimates in conditions with restructured time metrics at $n = 200$ with 20 time points restructured to 4 had larger variability than parameter estimates in all other conditions, and variability was largest in the no coupling condition. Variability was also large in the condition with the restructured time metric, no coupling, $n = 1000$ with 20 time points restructured to 4. As for the *age-grade* time structure, the larger variability noted in intercept mean parameter estimates in the true time metric was likely due to the missing data and individually differing intercepts in the true time metric.

Convergence

A logistic regression was conducted at the condition level with *n*, *t*, and coupling as predictors to assess model convergence in the *time-occasion* time structure. All interactions between predictors were included in the analysis. The outcome was coded as 1 = convergence and 0 = non-convergence, so this logistic regression predicted the group coded zero (or instances of non-convergence). Results from this logistic regression are shown in Table 9 (Appendix A, p. 50). For model convergence, all effects had odds ratios (ORs) very close to 1 except for the effect of coupling on convergence, which had an OR of 7.345. Coupling was coded as 0 = no coupling and 1 = coupling, so the interpretation of this OR is that the odds of non-convergence were larger in replications with coupling than in replications with no coupling.

To investigate these results further, Table 10 (Appendix A, p. 51) shows frequency tables for convergence separated by coupling. For conditions with no coupling, models did not converge for 601 of 4000 replications, and for conditions with coupling, models did not converge for 1156 of 4000 replications. Conditions with coupling had models with no convergence in almost twice the number of replications as conditions with no coupling. Although the effect of the interaction between *t* and coupling was not significant, conditions with *t* = 20 had the majority of models that did not converge, and within *t* = 20 conditions with coupling had twice as many replications with no convergence as conditions with no coupling.

Inadmissible Parameter Estimates

A check for inadmissible parameter estimates was conducted by examining descriptive statistics for variances of intercepts and slopes. For the *time-occasion* time structure, there were no inadmissible parameter estimates for the *y* intercept and slope variances (σ^2_{g0} and σ^2_{g1}), however negative parameter estimates were detected for the *x* intercept and slope variances (σ^2_{h0} and σ^2_{h1}). These inadmissible parameter estimates were investigated by examining frequency tables of each of the binary variables created to code negative variances of h_{0n} and h_{1n}. Negative estimates of *x* intercept variances occurred in replications for two conditions: where $n = 200$, no coupling, and $t = 20$, for 20 time points restructured to 4 (11 replications), and for 20 time points restructured to 10 (2 replications). Negative estimates of *x* slope variances occurred in replications for three conditions: where $n = 200$, no coupling, and 20 time points restructured to 4 for 1) $t = 4$ (12 replications); for 2) $t = 20$ (2 replications); and for 3) $n = 1000$, no coupling, 20 time points restructured to 4, and $t = 4$. Frequencies of inadmissible parameter estimates by *t*, *n*, and coupling for *x* slope and intercept variances are shown in Tables 11 and 12 (Appendix A, pp. 52-53). There did not appear to be a pattern of results with regard to conditions containing inadmissible parameter estimates, however all inadmissible parameter estimates were estimates relating to *x* slope and intercept. These results should thus be investigated further before conclusions can be drawn.

Discussion

Summary of Results

The purpose of this study was to investigate the influence of time metric on interpretation, model estimation, and model convergence for bivariate dual change LDS models. Results for the *age-grade* time structure (24 or 12 time points restructured to 6) indicated that parameter estimates in models with the restructured time metric (t = 6) had more bias and less stability than estimates from the models with the true time metric (t = 24 or t = 12). When the true time metric had more time points, the difference in bias and stability was even larger, indicating that more information about the change process was lost when models were fit using the restructured data. This difference in bias and stability was most apparent in conditions with non-zero coupling, where bias and variability of all parameter estimates were comparatively greatly increased for conditions with the restructured time metric where the true time metric had more time points. The finding that parameter estimates with the true time metric have less bias and less variability indicates that when the restructured time metric was used, model parameter estimates did not capture the true process of change in the data.

In contrast to the improved accuracy of the true time metric above, there was less convergence in the *age-grade* structure for models in conditions where the true time metric had more time points. The models with 24 time points may have had less convergence due to the fact that each participant had only 6 time points and was missing data on all other time points, so covariance coverage was very low, which resulted in more models that did not converge. Covariance coverage is the proportion of cases that contribute values used to calculate each variance or covariance between variables, and ranges from 0 to 1 with 1 being 100% of cases used and 0 being 0% of cases used (Geiser, 2013). When a small proportion of cases are used to calculate variances and covariances and thus covariance coverage is low, this can lead to model non-convergence. This lack of convergence is one limitation of using data with a true time metric that has nontrivial missing data for LDS models. Inadmissible parameter estimates were not present for any conditions examined using the *age-grade* time structure, regardless of true or restructured time metric.

For the *time-occasion* structure, all parameters estimates excluding the intercept means were unbiased regardless of the time metric used. However, although bias was near zero across conditions, all parameter estimates from models with the restructured time metric except the intercept means were less stable than estimates from models with the true time metric. The high variability of the parameter estimates indicates that using a time metric that does not match the true process of change, but has lags equal to the true time metric, may result in unstable parameter estimates even though the estimates produced using the restructured time metric are generally unbiased.

For the intercept mean parameter estimates, the same pattern of results for bias held in the *time-occasion* structure as for all parameters in the *age-grade* structure. Intercept mean parameter estimates from models with the restructured time metric were more biased than estimates from the models with the true time metric and that difference in bias was increased when the true time metric had more time points (20 vs. 10), particularly with non-zero coupling. However, the intercept mean parameter estimates in models with the restructured time metric were also more stable than in models with the true time metric.

There were less instances of convergence for conditions in the *time-occasion* structure for models with coupling, especially in models with the true time metric and more time points (t = 20). As with the *age-grade* structure, non-convergence was likely due to the covariance coverage being near zero and presents a potential problem when the true time metric requires many missing data points Some replications in the *time-occasion* conditions also had negative variances of intercepts and slopes for both the true and restructured time metrics. Negative slope and intercept variances indicate that parameter estimates from some of these models were untrustworthy. It is possible that the negative variances appearing in the *time-occasion* structure were due to the combination of parameter estimates used in the simulation, and the inadmissible parameter estimates should be investigated further with other combinations of parameters.

The *time-occasion* structure in the time study differed from the *age-grade* structure in that for the *time-occasion* structure, participants had different starting points in the data for the true time metric but each participant had the same time lag between measurements and the measurement intervals were equal for both time metrics. In the *age-grade* structure, participants had different starting points and different lags between time points in the true time metric, but starting points and lags were the same across participants and lags were equal in the restructured time metric. For both time structures, the difference in participants' starting points in the true time metric resulted in biased estimates of intercept means for the restructured time metric. The bias and instability of all other parameter estimates for models with the restructured time metric in the *age-grade* structure was likely due to the differing lags between measurements in the true time metric, which were not accounted for when models were fit to the data using the restructured time metric.

Fit with Earlier Literature

In previous research on time in longitudinal models, time metric, intercept placement, and interval spacing influenced model interpretation and accuracy. The results of this study confirm that time metric also influenced model interpretation and accuracy, including intercept and interval spacing, for the LDS model. These results support findings in existing literature and extend these findings to the LDS model as well. In addition, results from this study shed light on how time metric influences those parameters that are unique to LDS models. In LDS models, the unique parameters of interest that reflect dynamic change are the proportional change and coupling parameters. When a time metric that does not reflect true process of change was used to fit LDS models, the proportional change and coupling parameters were biased.

Limitations

The difference in meaningfully significant predictors of bias of the proportional change and coupling parameter estimates was likely due to the different values of population parameters that were used for X and Y. This study did not vary parameter estimates over conditions, but used one set of parameter estimates based on a LDS model from a substantive example. Use of alternative parameter estimates with otherwise equivalent conditions could provide explanation for the difference in meaningful significance. Furthermore, the parameter values were taken from a subgroup with a sample size of 142. These parameters were then used to simulate data with sample sizes of N = 200 and N = 1000. The difference in sample size could account for some of the parameter variability results. The lack of literature on optimal parameter estimates to use when investigating LDS models using analytical work and simulations is a general limitation of methodological work on LDS models. Future research should determine realistic or typical parameters and parameter variances and covariances that would allow for more detailed examination of LDS models via analytical and simulation work.

Future Directions

Future research should simulate these conditions using alternative parameter values to determine that the results generalize to other values of parameters. Future research should also investigate importance of time metric under conditions where the true time metric has fewer time points than the restructured (less informative) time metric. For example, a process that unfolds more slowly over time may require less time points to represent change accurately. If a process takes place over several years, it is unnecessary to measure participants at monthly intervals to investigate the change process, and doing so could result in inaccurate or unstable parameter estimates from LDS models. Also, time metric in the LDS model should be examined including additional predictors such as moderators and mediators of change to examine how time metric influences these additional effects. Finally, work on time metric in LDS models can be extended to investigate the question: Which time metrics most accurately capture true change processes in LDS models with variables commonly used in developmental research?

The most important finding of this study was that time metric influenced bias and interpretation of the proportional change and coupling parameters in the LDS model. Another finding was that assuming the same starting point for all participants resulted in biased intercept mean parameter estimates when participants had different starting points in the true time metric. These results for the influence of time metric on LDS models are important for planning in research design. Substantive researchers planning longitudinal designs with focus on dynamic change must consider the time metric that will most accurately represent the true process of change. Specifically, results from this study indicate that when the true process of change is most accurately represented by a time metric where participants have different starting points, if researchers use a time metric that does not take into account the different starting points to estimate a LDS model this will result in biased intercept mean parameter estimates. If researchers use a time metric that does not account for different lags between measurements both between and within participants to estimate a LDS model, this will result in biased constant change (slope), proportional change, and coupling parameter estimates. Researchers must also make special considerations for coupling, a parameter that is unique to LDS models. When the two variables of interest have a mutual relationship over time, researchers should closely examine the intercept mean parameter estimates (for time metrics with different starting points but equal intervals) or all parameter estimates (for time metric with different starting points and unequal intervals) for bias if it is suspected that the time metric does not accurately represent the change process. Bias will be increased compared to a research scenario where two variables do not have a mutual relationship over time. Researchers must also be aware of the potential for non-convergence if data are structured with a time metric that accurately represents change in the data, but results in many missing values for each participant. Failure to consider the true process of change will result in inaccurate conclusions about dynamic change from LDS models. In conclusion, consideration of time metric is essential for interpreting dynamic change processes in LDS models.

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APPENDIX A

TABLES AND FIGURES

Tables

Table 1

Analyses of Variance for n, t, and Coupling on Raw Bias of Parameter Estimates for Age-Grade Time Structure

		df	F	p	η_p^2
μ_{g0}					
	n	1	9.07	0.0026	0.00044
	t	2	6438.57	<.0001	0.6177*
	coupling	1	39.57	<.0001	0.0019
	n*t	2	5.78	0.0031	0.00055
	n*coupling	1	0.07	0.7961	3.2E-06
	t*coupling	2	60.51	<.0001	0.0058
	n*t*coupling	2	0.13	0.8746	1.3E-05
μ_{h0}					
	n	1	13.64	0.0002	0.00047
	t	2	10384.4	<.0001	0.71945*
	coupling	1	58.57	<.0001	0.00203
	n*t	2	8.23	0.0003	0.00057
	n*coupling	1	0	0.9758	3.2E-08
	t*coupling	2	68.01	<.0001	0.00471
	n*t*coupling	2	0.59	0.5525	4.1E-05
μ_{g1}					
	n	1	0.8	0.372	3.6E-05
	t	2	5375.34	<.0001	0.48492*
	coupling	1	680.06	<.0001	0.03068
	n*t	2	1.33	0.2658	0.00012
	n*coupling	1	1.01	0.3147	4.6E-05
	t*coupling	2	887.17	<.0001	0.08003
	n*t*coupling	2	1.3	0.2724	0.00012
μ_{h1}					
	n	1	0.78	0.377	2.9E-05
	t	2	6936.71	<.0001	0.51858*
	coupling	1	933.86	<.0001	0.03491
	n*t	2	1.87	0.1546	0.00014
	n*coupling	1	1.48	0.2245	5.5E-05
	t*coupling	2	1237.15	<.0001	0.09249*
	n*t*coupling	2	1.18	0.3064	8.8E-05
β_x					
	n	1	4.77	0.0289	0.00021
	t	2	4878.15	<.0001	0.43141*
	coupling	1	917.87	<.0001	0.04059
	n*t	2	9.12	0.0001	0.00081
	n*coupling	1	7.88	0.005	0.00035
	t*coupling	2	1227.79	<.0001	0.10858*
	n*t*coupling	2	6.32	0.0018	0.00056
β_y					
	n	1	7.65	0.0057	0.00039
	t	2	201.15	<.0001	0.02047
	coupling	1	2127.09	<.0001	0.10822*
	n*t	2	12.31	<.0001	0.00125
	n*coupling	1	5.43	0.0198	0.00028
	t*coupling	2	2726.21	<.0001	0.2774*
	n*t*coupling	2	8.36	0.0002	0.00085
γ_{yx}					
	n	1	4.53	0.0334	0.00026
	t	2	1973.3	<.0001	0.23004*
	coupling	1	1013.29	<.0001	0.05906
	n*t	2	7.71	0.0005	0.0009
	n*coupling	1	4.09	0.0431	0.00024
	t*coupling	2	1304.44	<.0001	0.15207*
	n*t*coupling	2	5.82	0.003	0.00068
γ_{xy}					
	n	1	8.4	0.0038	0.00036
	t	2	2696.59	<.0001	0.23048*
	coupling	1	1854.33	<.0001	0.07924
	n*t	2	14.08	<.0001	0.0012
	n*coupling	1	12.06	0.0005	0.00052
	t*coupling	2	2445.92	<.0001	0.20905*
	n*t*coupling	2	10.65	<.0001	0.00091

* indicates $\eta_p^2 \geq .09$.
Note: n = 200, 1000; t = 6, 12, 24; coupling = yes coupling (YC), no coupling (NC).
Note: t = 12 and 24 are the true time metrics, and t = 6 is the restructured time metric.

Table 2

Analyses of Variance for n, t, and Coupling on Standardized Bias of Parameter Estimates for Age-Grade Time Structure

μ_{g0}		*df*	*F*	*p*	η_p^2
	n	1	806.79	<.0001	0.032742
	t	2	6184.26	<.0001	0.501947*
	coupling	1	27.03	<.0001	0.001097
	*n*t*	2	1025.55	<.0001	0.083239
	*n**coupling	1	5.9	0.0152	0.000239
	*t**coupling	2	41.1	<.0001	0.003336
	*n*t**coupling	2	8.36	0.0002	0.000679
μ_{h0}					
	n	1	1055.3	<.0001	0.033292
	t	2	8169.56	<.0001	0.515458*
	coupling	1	303.07	<.0001	0.009561
	*n*t*	2	1342.25	<.0001	0.084689
	*n**coupling	1	45.54	<.0001	0.001437
	*t**coupling	2	392.19	<.0001	0.024745
	*n*t**coupling	2	60.75	<.0001	0.003833
μ_{g1}					
	n	1	4832.24	<.0001	0.037866
	t	2	44283.1	<.0001	0.694008*
	coupling	1	746.05	<.0001	0.005846
	*n*t*	2	6287.21	<.0001	0.098534*
	*n**coupling	1	22.16	<.0001	0.000174
	*t**coupling	2	976.45	<.0001	0.015303
	*n*t**coupling	2	30.34	<.0001	0.000476
μ_{h1}					
	n	1	1571.99	<.0001	0.027734
	t	2	15479.9	<.0001	0.546218*
	coupling	1	1578.88	<.0001	0.027856
	*n*t*	2	2049.02	<.0001	0.072301
	*n**coupling	1	107.09	<.0001	0.001889
	*t**coupling	2	2062.33	<.0001	0.072771
	*n*t**coupling	2	127.18	<.0001	0.004488
β_x					
	n	1	2901.67	<.0001	0.034088
	t	2	24747.4	<.0001	0.581444*
	coupling	1	1951.88	<.0001	0.02293
	*n*t*	2	3821.63	<.0001	0.08979*
	*n**coupling	1	186.34	<.0001	0.002189
	*t**coupling	2	2575.29	<.0001	0.060507
	*n*t**coupling	2	158.67	<.0001	0.003728
β_y					
	n	1	323.84	<.0001	0.003351
	t	2	2078.03	<.0001	0.043007
	coupling	1	13140.2	<.0001	0.135974*
	*n*t*	2	375.9	<.0001	0.00778
	*n**coupling	1	2240.57	<.0001	0.023185
	*t**coupling	2	16598.8	<.0001	0.343525*
	*n*t**coupling	2	2974.58	<.0001	0.061561
γ_{yx}					
	n	1	1631.06	<.0001	0.023392
	t	2	14128.6	<.0001	0.405256*
	coupling	1	4099.74	<.0001	0.058797
	*n*t*	2	2246.59	<.0001	0.06444
	*n**coupling	1	445.18	<.0001	0.006385
	*t**coupling	2	5089.96	<.0001	0.145997*
	*n*t**coupling	2	607.1	<.0001	0.017414
γ_{xy}					
	n	1	1322.83	<.0001	0.015313
	t	2	11937.9	<.0001	0.276385*
	coupling	1	7590.91	<.0001	0.087872*
	*n*t*	2	1774.1	<.0001	0.041074
	*n**coupling	1	1212.92	<.0001	0.014041
	*t**coupling	2	9844.15	<.0001	0.227911*
	*n*t**coupling	2	1256.92	<.0001	0.0291

* indicates $\eta_p^2 \geq .09$.
Note: n = 200, 1000; t = 6, 12, 24; coupling = yes coupling (YC), no coupling (NC).
Note: t = 12 and 24 are the true time metrics, and t = 6 is the restructured time metric.

Table 3

Standard Deviations of Parameter Estimates by Condition for Age-Grade Time Structure

Condition	$SD(\mu_{g0})$	$SD(\mu_{h0})$	$SD(\mu_{g1})$	$SD(\mu_{h1})$	$SD(\beta_x)$	$SD(\beta_y)$	$SD(\gamma_{yx})$	$SD(\gamma_{xy})$
NC, n = 200, 12-6, t = 6	0.0509	0.0670	0.0548	0.0925	0.0809	0.0426	0.0426	0.0867
NC, n = 200, 12-6, t =12	0.0588	0.0938	0.0430	0.0858	0.0655	0.0238	0.0290	0.0604
NC, n = 200, 24-6, t = 6	0.0506	0.0615	0.1388	0.2137	0.1723	0.0824	0.1078	0.1343
NC, n = 200, 24-6, t = 24	0.0850	0.1371	0.0718	0.1177	0.0780	0.0235	0.0415	0.0599
NC, n = 1000, 12-6, t = 6	0.0239	0.0302	0.0266	0.0442	0.0366	0.0195	0.0204	0.0376
NC, n = 1000, 12-6, t = 12	0.0281	0.0445	0.0194	0.0378	0.0277	0.0105	0.0129	0.0246
NC, n = 1000, 24-6, t = 6	0.0225	0.0270	0.0496	0.0700	0.0545	0.0283	0.0361	0.0469
NC, n = 1000, 24-6, t = 24	0.0384	0.0621	0.0314	0.0539	0.0345	0.0098	0.0175	0.0254
YC, n = 200, 12-6, t = 6	0.0533	0.0499	0.0586	0.0545	0.0603	0.0547	0.0503	0.0690
YC, n = 200, 12-6, t = 12	0.0583	0.0610	0.0327	0.0401	0.0386	0.0306	0.0280	0.0423
YC, n = 200, 24-6, t = 6	0.0527	0.0459	0.2737	0.2987	0.2841	0.2690	0.2565	0.2996
YC, n = 200, 24-6, t = 24	0.0713	0.0841	0.0467	0.0560	0.0489	0.0378	0.0370	0.0510
YC, n = 1000, 12-6, t = 6	0.0218	0.0214	0.0279	0.0272	0.0298	0.0265	0.0246	0.0340
YC, n = 1000, 12-6, t = 12	0.0241	0.0270	0.0138	0.0134	0.0126	0.0134	0.0122	0.0138
YC, n = 1000, 24-6, t = 6	0.0241	0.0212	0.1436	0.1580	0.1534	0.1452	0.1378	0.1620
YC, n = 1000, 24-6, t = 24	0.0324	0.0353	0.0210	0.0233	0.0207	0.0167	0.0164	0.0217

Note: "NC" indicates no coupling conditions and "YC" indicates coupling conditions.
Note: 12-6 indicates 12 time points restructured to 6, and 24-6 indicates 24 time points restructured to 6.
Note: t = 24 and 12 are the true time metrics, and t = 6 is the restructured time metric.

Table 4

Logistic Regression with n and Coupling Predicting Model Convergence for Age-Grade Time Structure at t = 24

	B	*S.E.*	Wald	*p*	OR
n	-0.0021	0.0005	18.1184	<.0001	0.9980
coupling	1.2756	0.2378	28.7857	<.0001	3.581*
*n**coupling	0.0016	0.0005	9.5504	0.0020	1.0020

Note: * indicates OR > 1.5 or OR < .5.
Note: Convergence was coded such that 0 = non-convergence and 1 = convergence; *n* = 200, 1000; coupling was coded such that 0 = no coupling and 1 = coupling.

Table 5

Frequencies of Model Convergence by t and Coupling for Age-Grade Time Structure

t = 6 (restructured)		No Convergence	Convergence	Total
	No Coupling	0	2000	2000
	Coupling	10	1990	2000
	Total	10	3990	4000
t = 12 (true)				
	No Coupling	0	1000	1000
	Coupling	4	996	1000
	Total	4	1996	2000
t = 24 (true)				
	No Coupling	49	951	1000
	Coupling	301	699	1000
	Total	350	1650	2000

Note: t = 12 and t = 24 are the true time metrics, and t = 6 is the restructured time metric.

Table 6

Analyses of Variance for n, t, and Coupling on Raw Bias of Parameter Estimates for Time-Occasion Time Structure

		df	*F*	*p*	η_p^2
μ_{g0}					
	n	1	2.73	0.0987	0
	t	2	431479	<.0001	0.7204*
	coupling	1	166414	<.0001	0.1389*
	*n***t*	2	0.02	0.9833	0
	*n**coupling	1	0.41	0.5243	0
	*t**coupling	2	46905.8	<.0001	0.0783
	*n***t**coupling	2	0.26	0.7676	0
μ_{h0}					
	n	1	0.58	0.4454	0
	t	2	372212	<.0001	0.7211*
	coupling	1	144250	<.0001	0.1397*
	*n***t*	2	3.02	0.0491	0
	*n**coupling	1	1.06	0.3043	0
	*t**coupling	2	39681.9	<.0001	0.0769
	*n***t**coupling	2	0.09	0.9172	0
μ_{g1}					
	n	1	0.88	0.3486	0.0001
	t	2	0.45	0.6347	0.0001
	coupling	1	0.54	0.4605	0.0001
	*n***t*	2	0.84	0.4309	0.0003
	*n**coupling	1	0.01	0.9335	0
	*t**coupling	2	0.3	0.7418	0.0001
	*n***t**coupling	2	0.06	0.9374	0
μ_{h1}					
	n	1	0.68	0.4092	0.0001
	t	2	0.18	0.839	0.0001
	coupling	1	0.06	0.7991	0
	*n***t*	2	0.48	0.6206	0.0002
	*n**coupling	1	0.58	0.4475	0.0001
	*t**coupling	2	0.11	0.8995	0
	*n***t**coupling	2	0.14	0.8656	0
β_x					
	n	1	0.61	0.4339	0.0001
	t	2	3.66	0.0258	0.0012
	coupling	1	1.01	0.3155	0.0002
	*n***t*	2	0.92	0.3985	0.0003
	*n**coupling	1	0.05	0.8205	0
	*t**coupling	2	2.64	0.0716	0.0008
	*n***t**coupling	2	1.29	0.2765	0.0004
β_y					
	n	1	1.25	0.2633	0.0002
	t	2	14.98	<.0001	0.0048
	coupling	1	0.71	0.3991	0.0001
	*n***t*	2	1.15	0.3156	0.0004
	*n**coupling	1	6.55	0.0105	0.001
	*t**coupling	2	0.53	0.5865	0.0002
	*n***t**coupling	2	6.6	0.0014	0.0021
γ_{yx}					
	n	1	0.4	0.5293	0.0001
	t	2	8.54	0.0002	0.0027
	coupling	1	1.09	0.2966	0.0002
	*n***t*	2	0.39	0.6766	0.0001
	*n**coupling	1	5.23	0.0223	0.0008
	*t**coupling	2	0.8	0.4498	0.0003
	*n***t**coupling	2	5.11	0.0061	0.0016
γ_{xy}					
	n	1	1.59	0.2074	0.0003
	t	2	4.71	0.009	0.0015
	coupling	1	0.97	0.3251	0.0002
	*n***t*	2	1.52	0.2189	0.0005
	*n**coupling	1	0.4	0.5276	0.0001
	*t**coupling	2	2.86	0.0574	0.0009
	*n***t**coupling	2	1.64	0.1947	0.0005

* indicates $\eta_p^2 \geq .09$.

Note: *n* = 200, 1000; *t* = 4, 10, 20; coupling = yes coupling (YC), no coupling (NC).

Note: *t* = 20 is the true time metric, and *t* = 4 and 10 are the restructured time metrics.

Table 7

Analyses of Variance for n, t, and Coupling on Standardized Bias of Parameter Estimates for Time-Occasion Time Structure

$g0_n$		*df*	*F*	*p*	η_p^2
	n	1	479888	<.0001	0.1841*
	t	2	852621	<.0001	0.6542*
	coupling	1	9191.1	<.0001	0.0035
	*n***t*	2	125845	<.0001	0.0966*
	*n**coupling	1	2623.17	<.0001	0.001
	*t**coupling	2	2426.81	<.0001	0.0019
	*n***t**coupling	2	1138.48	<.0001	0.0009
$h0_n$					
	n	1	791065	<.0001	0.1839*
	t	2	1414300	<.0001	0.6574*
	coupling	1	11983.9	<.0001	0.0028
	*n***t*	2	205167	<.0001	0.0954*
	*n**coupling	1	5626.88	<.0001	0.0013
	*t**coupling	2	3353.89	<.0001	0.0016
	*n***t**coupling	2	1930.26	<.0001	0.0009
$g1_n$					
	n	1	1.84	0.1746	0.0003
	t	2	0.36	0.6997	0.0001
	coupling	1	0.93	0.334	0.0001
	*n***t*	2	0.1	0.9084	0
	*n**coupling	1	1.42	0.2335	0.0002
	*t**coupling	2	0.14	0.8724	0
	*n***t**coupling	2	0.54	0.5827	0.0002
$h1_n$					
	n	1	0.39	0.5348	0.0001
	t	2	0.75	0.4733	0.0002
	coupling	1	0.53	0.4659	0.0001
	*n***t*	2	0.54	0.5847	0.0002
	*n**coupling	1	1.78	0.182	0.0003
	*t**coupling	2	0.46	0.6316	0.0001
	*n***t**coupling	2	0.37	0.6911	0.0001
β_x					
	n	1	0.03	0.8642	0
	t	2	0.28	0.7528	0.0001
	coupling	1	0.85	0.3572	0.0001
	*n***t*	2	0.22	0.8028	0.0001
	*n**coupling	1	1.22	0.2697	0.0002
	*t**coupling	2	0.54	0.584	0.0002
	*n***t**coupling	2	1.43	0.2384	0.0005
β_y					
	n	1	1.74	0.1876	0.0003
	t	2	2.69	0.068	0.0009
	coupling	1	0	0.9879	0
	*n***t*	2	0.1	0.9013	0
	*n**coupling	1	0.73	0.3939	0.0001
	*t**coupling	2	0.2	0.8191	0.0001
	*n***t**coupling	2	3.09	0.0457	0.001
γ_{yx}					
	n	1	1.88	0.1709	0.0003
	t	2	2.27	0.1034	0.0007
	coupling	1	0.29	0.5876	0
	*n***t*	2	0.11	0.897	0
	*n**coupling	1	0.43	0.5137	0.0001
	*t**coupling	2	0.2	0.8195	0.0001
	*n***t**coupling	2	2.69	0.0679	0.0009
γ_{xy}					
	n	1	0.08	0.7738	0
	t	2	0.77	0.4628	0.0002
	coupling	1	3.73	0.0535	0.0006
	*n***t*	2	0.15	0.8633	0
	*n**coupling	1	0.68	0.4079	0.0001
	*t**coupling	2	0.93	0.3956	0.0003
	*n***t**coupling	2	1.44	0.2366	0.0005

* indicates $\eta_p^2 \geq .09$.

Note: $n = 200, 1000$; $t = 4, 10, 20$; coupling = yes coupling (YC), no coupling (NC).

Note: $t = 20$ is the true time metric, and $t = 4$ and 10 are the restructured time metrics.

Table 8

Standard Deviations of Parameter Estimates by Condition for Time-Occasion Time Structure

Condition	$SD(\mu_{g0})$	$SD(\mu_{h0})$	$SD(\mu_{g1})$	$SD(\mu_{h1})$	$SD(\beta_x)$	$SD(\beta_y)$	$SD(\gamma_{yx})$	$SD(\gamma_{xy})$
NC, n = 200, 20-10, t = 10	0.0665	0.0677	0.1519	0.2867	0.2000	0.0663	0.1013	0.1412
NC, n = 200, 20-10, t = 20	0.1218	0.2262	0.0729	0.1398	0.0860	0.0250	0.0406	0.0582
NC, n = 200, 20-4, t = 4	0.0608	0.0602	0.5828	0.8076	0.7578	0.6073	0.5645	0.9647
NC, n = 200, 20-4, t = 20	0.1425	0.2972	0.1193	0.1747	0.1142	0.0546	0.0750	0.0850
NC, n = 1000, 20-10, t = 10	0.0280	0.0299	0.0441	0.1032	0.0665	0.0198	0.0281	0.0493
NC, n = 1000, 20-10, t = 20	0.0551	0.0964	0.0300	0.0570	0.0339	0.0099	0.0160	0.0246
NC, n = 1000, 20-4, t = 4	0.0292	0.0289	0.1570	0.2719	0.3630	0.2646	0.2103	0.4822
NC, n = 1000, 20-4, t = 20	0.0676	0.1303	0.0461	0.0792	0.0532	0.0244	0.0307	0.0394
YC, n = 200, 20-10, t = 10	0.1081	0.1150	0.0908	0.0791	0.0694	0.0820	0.0791	0.0725
YC, n = 200, 20-10, t = 20	0.0984	0.1191	0.0594	0.0484	0.0419	0.0495	0.0486	0.0441
YC, n = 200, 20-4, t = 4	0.1184	0.1197	0.3425	0.1886	0.2528	0.5018	0.4079	0.3099
YC, n = 200, 20-4, t = 20	0.1749	0.1822	0.1402	0.0930	0.0951	0.1260	0.1222	0.1037
YC, n = 1000, 20-10, t = 10	0.0466	0.0491	0.0350	0.0272	0.0240	0.0323	0.0307	0.0254
YC, n = 1000, 20-10, t = 20	0.0438	0.0520	0.0256	0.0213	0.0184	0.0217	0.0211	0.0194
YC, n = 1000, 20-4, t = 4	0.0519	0.0521	0.1120	0.0596	0.0672	0.1437	0.1141	0.0868
YC, n = 1000, 20-4, t = 20	0.0657	0.0742	0.0385	0.0227	0.0171	0.0294	0.0300	0.0183

Note: “NC” indicates no coupling conditions and “YC” indicates coupling conditions.
Note: 20-10 indicates 20 time points restructured to 10, and 20-4 indicates 20 time points restructured to 4.
Note: t = 20 is the true time metric, and t = 4 and 10 are the restructured time metrics.

Table 9

Logistic Regression with n, t, and Coupling Predicting Model Convergence for Time-Occasion Time Structure

	B	*S.E.*	Wald	*p*	OR
n	-0.0041	0.0006	43.1325	<.0001	0.996
t	0.0658	0.0124	28.0997	<.0001	1.068
coupling	1.9940	51.7296	0.0015	0.9693	7.345*
*n***t*	0.0001	0.0000	8.7462	0.0031	1
*n**coupling	-0.0274	0.2586	0.0112	0.9157	0.973
*t**coupling	-0.0630	2.5865	0.0006	0.9806	0.939
*n***t**coupling	0.0015	0.0129	0.0126	0.9107	1.001

Note: * indicates OR > 1.5 or OR < .5.
Note: Convergence was coded such that 0 = non-convergence and 1 = convergence; *n* = 200, 1000; *t* = 4, 10, 20; coupling was coded such that 0 = no coupling and 1 = coupling.
Note: *t* = 20 is the true time metric, and *t* = 4 and *t* = 10 are the restructured time metrics.

Table 10

Frequencies of Model Convergence by Coupling for Time-Occasion Time Structure

	No Convergence	Convergence	Total
No Coupling	601	3399	4000
Coupling	1156	2844	4000
Total	1757	6243	8000

Table 11

Frequencies of Inadmissible Parameter Estimates for μ_{h0} by n, t, and Coupling for Time-Occasion Time Structure

			Variance ≥ 0	Variance < 0	Total
n = 200	No Coupling	*t* = 4	500	0	500
		t = 10	500	0	500
		t = 20	987	13	1000
	Coupling	*t* = 4	500	0	500
		t = 10	500	0	500
		t = 20	1000	0	1000
n = 1000	No Coupling	*t* = 4	500	0	500
		t = 10	500	0	500
		t = 20	1000	0	1000
	Coupling	*t* = 4	500	0	500
		t = 10	500	0	500
		t = 20	1000	0	1000

Note: *t* = 20 is the true time metric, and *t* = 4 and *t* = 10 are the restructured time metrics.

Table 12

Frequencies of Inadmissible Parameter Estimates for μ_{h1} by n, t, and Coupling for Time-Occasion Time Structure

			Variance ≥ 0	Variance < 0	Total
n = 200	No Coupling	*t* = 4	488	12	500
		t = 10	500	0	500
		t = 20	998	2	1000
	Coupling	*t* = 4	500	0	500
		t = 10	500	0	500
		t = 20	1000	0	1000
n = 1000	No Coupling	*t* = 4	499	1	500
		t = 10	500	0	500
		t = 20	1000	0	1000
	Coupling	*t* = 4	500	0	500
		t = 10	500	0	500
		t = 20	1000	0	1000

Note: *t* = 20 is the true time metric, and *t* = 4 and *t* = 10 are the restructured time metrics.

Figures

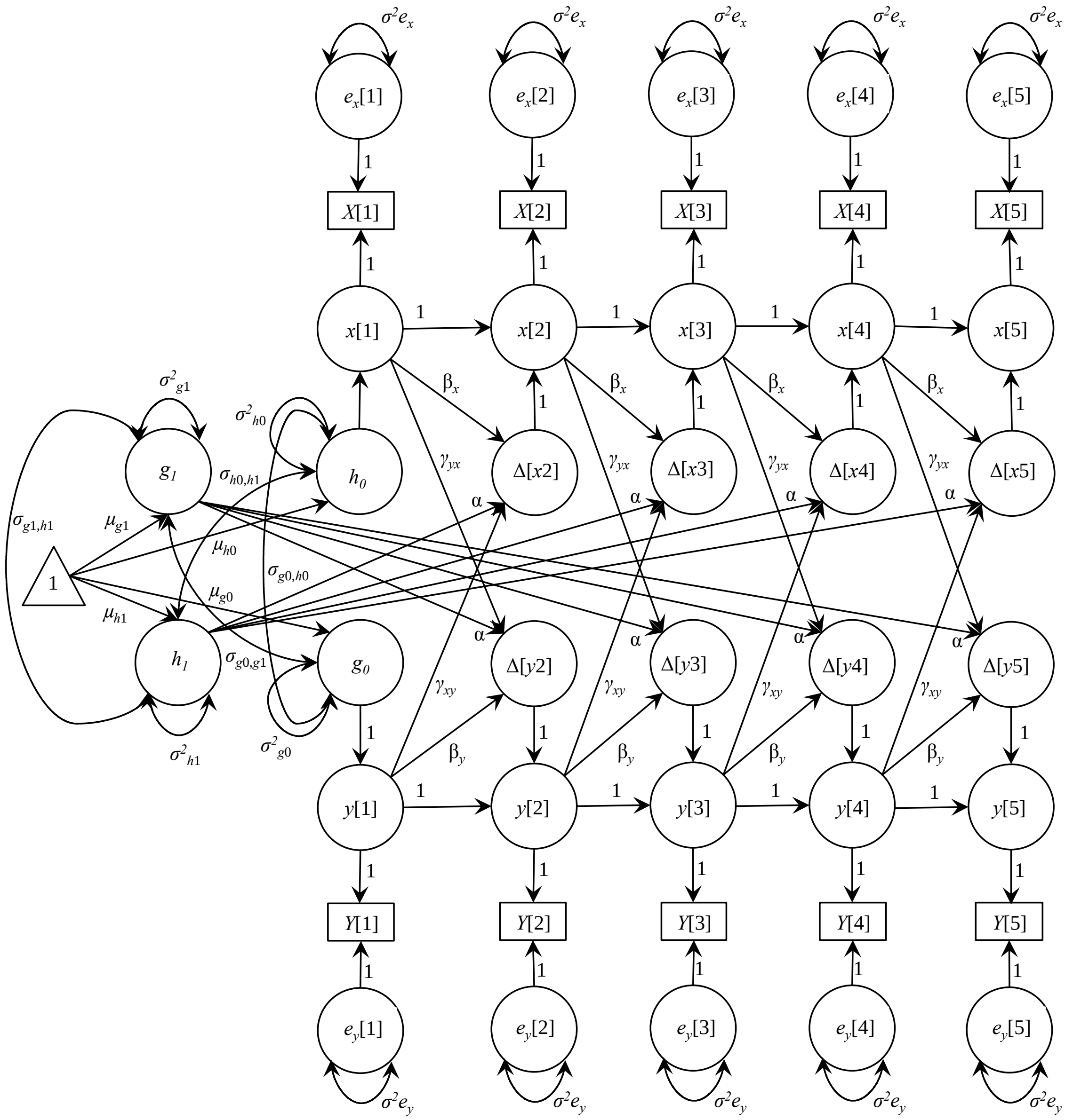


Figure 1. Latent difference score model with five waves. Adapted from McArdle (2009).

True Time Metric (Chronological Time)

	T 1	T 2	T 3	T 4	T 5	T 6	T 7	T 8	T 9	T 10	T 11	T 12	T 13	T 14	T 15	T 16	T 17	T 18	T 19	T 20
ID 1	.	.	.	x	x	x	x	.	.	.	.	.	.	.	.	.	.	.	.	.
ID 2	.	.	.	.	.	.	.	.	.	.	x	x	x	x	.	.	.	.	.	.
ID 3	.	.	.	.	.	.	.	x	x	x	x	.	.	.	.	.	.	.	.	.
ID 4	x	x	x	x	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
ID 5	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	x	x	x	x	.

Restructured Time Metric (Occasion)

	T 1	T 2	T 3	T 4
ID 1	x	x	x	x
ID 2	x	x	x	x
ID 3	x	x	x	x
ID 4	x	x	x	x
ID 5	x	x	x	x

Figure 2. Time metrics compared in *time-occasion* time structure where true time metric has t = 20 time points and restructured time metric has t = 4 time points.

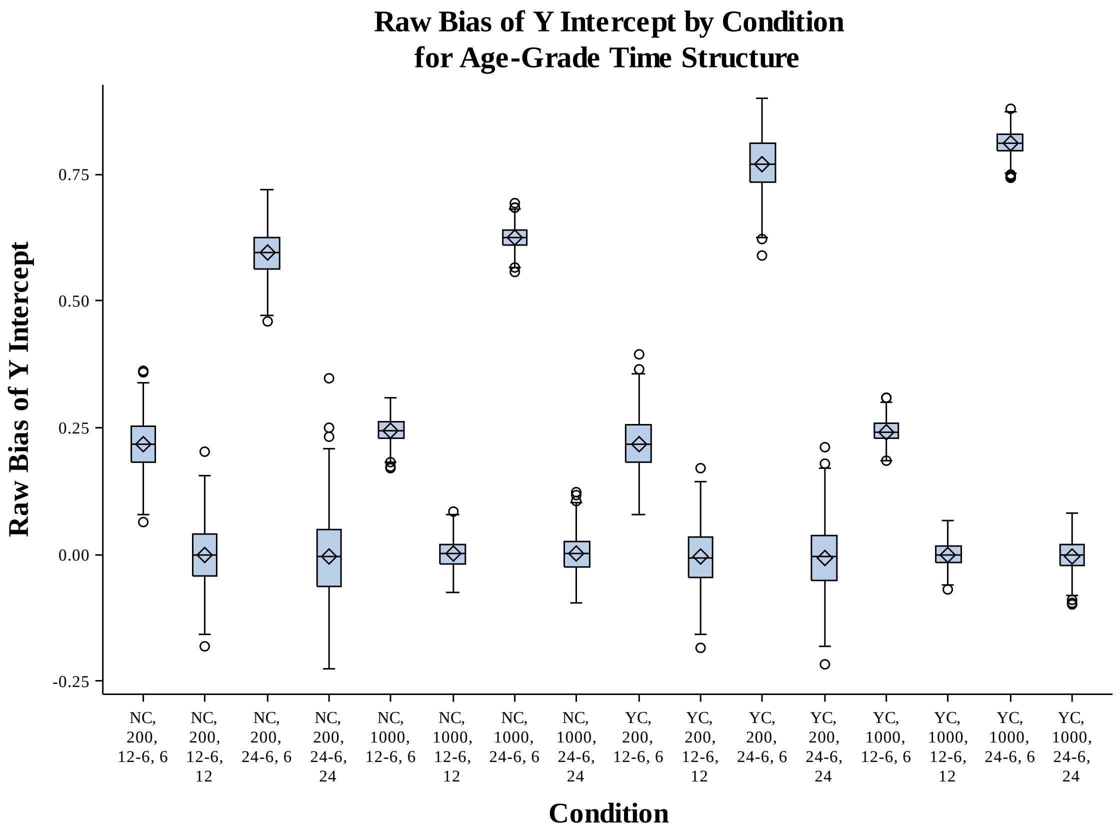
True Time Metric (Age)

	T 1	T 2	T 3	T 4	T 5	T 6	T 7	T 8	T 9	T 10	T 11	T 12	T 13	T 14	T 15	T 16	T 17	T 18	T 19	T 20	T 21	T 22	T 23	T 24
ID 1	.	.	x	.	x	.	.	.	.	x	.	.	.	.	.	x	x	.	.	.	.	.	x	.
ID 2	x	.	.	.	.	x	.	.	x	.	.	.	x	.	.	.	.	x	.	.	.	.	.	x
ID 3	.	x	.	.	.	x	.	.	.	.	x	.	.	.	x	.	.	.	x	.	x	.	.	.
ID 4	x	.	.	.	.	.	x	.	.	.	.	x	.	x	.	.	.	.	.	x	x	.	.	.
ID 5	.	.	.	x	.	.	.	x	.	.	.	x	.	.	.	x	.	.	.	x	.	.	.	x

Restructured Time Metric (Grade)

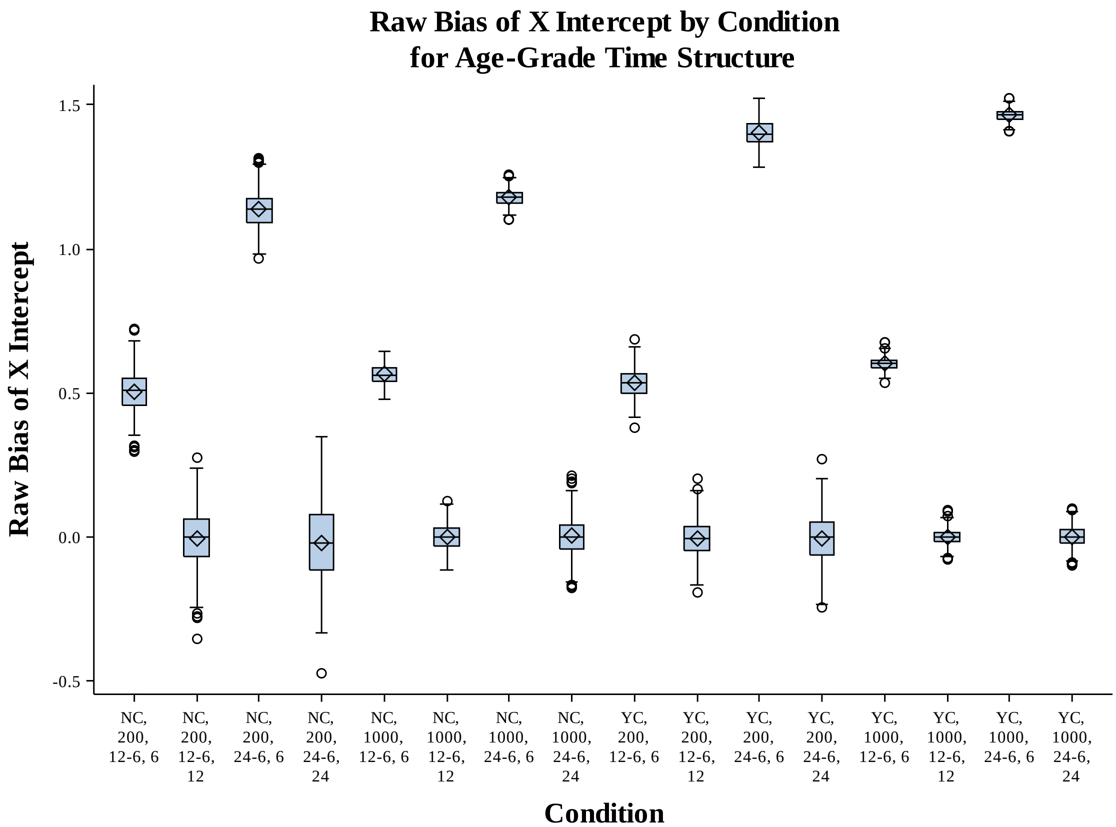
	T 1	T 2	T 3	T 4	T 5	T 6
ID 1	x	x	x	x	x	x
ID 2	x	x	x	x	x	x
ID 3	x	x	x	x	x	x
ID 4	x	x	x	x	x	x
ID 5	x	x	x	x	x	x

Figure 3. Time metrics compared in *age-grade* time structure where true time metric has t = 24 time points and restructured time metric has t = 6 time points.



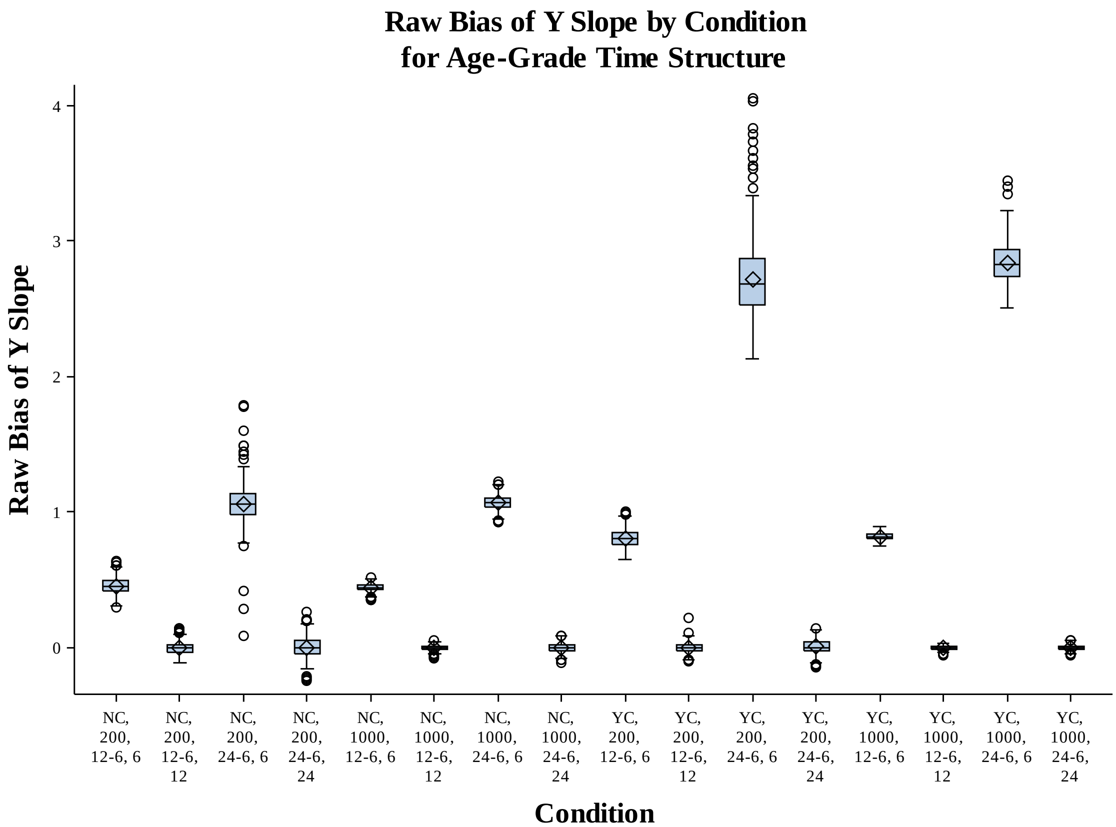
"NC" = no coupling, "YC" = yes coupling
n = 200, 1000
12-6 = 12 time points restructured to 6, 24-6 = 24 time points restructured to 6
t = 6,12, 24
Note: *t* = 12 and 24 are the true time metrics, and *t* = 6 is the restructured time metric.

Figure 4. Box plots of raw bias for the intercept parameter μ_{g0} by condition for age-grade time structure in time study.



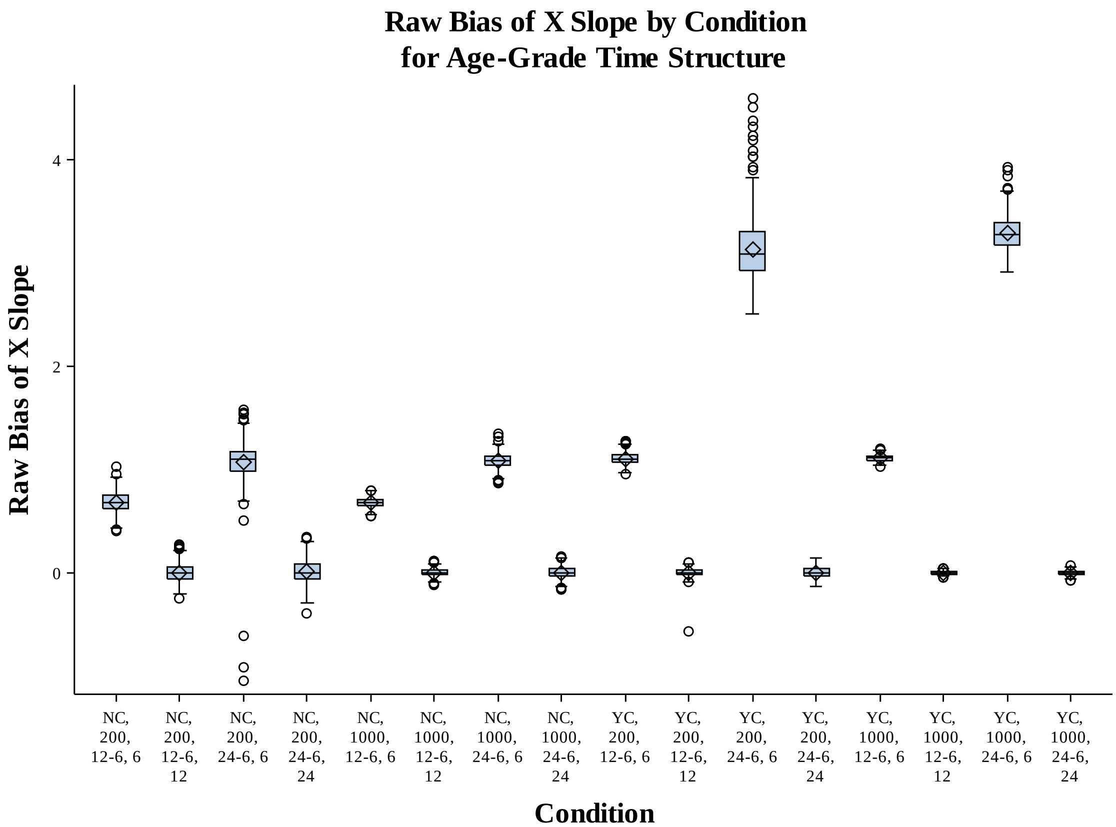
"NC" = no coupling, "YC" = yes coupling
n = 200, 1000
12-6 = 12 time points restructured to 6, 24-6 = 24 time points restructured to 6
t = 6,12, 24
Note: *t* = 12 and 24 are the true time metrics, and *t* = 6 is the restructured time metric.

Figure 5. Box plots of raw bias for the intercept parameter μ_{h0} by condition for age-grade time structure in time study.



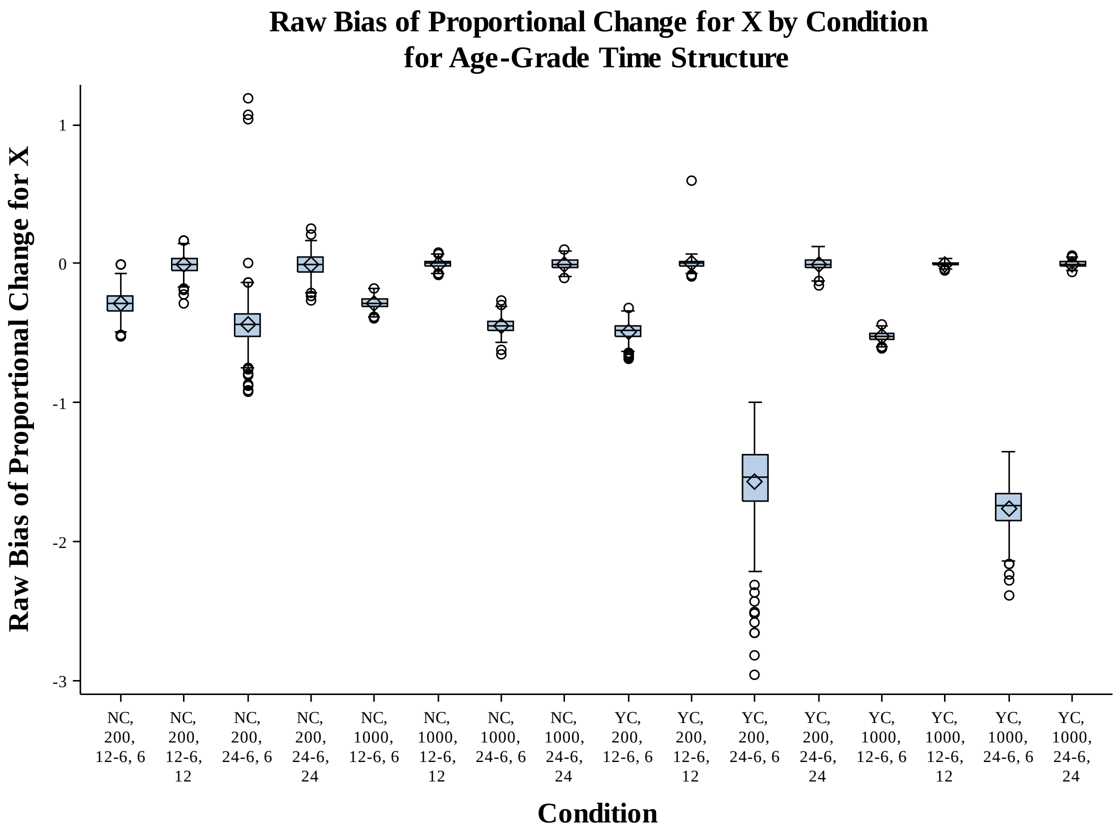
"NC" = no coupling, "YC" = yes coupling
n = 200, 1000
12-6 = 12 time points restructured to 6, 24-6 = 24 time points restructured to 6
t = 6,12, 24
Note: *t* = 12 and 24 are the true time metrics, and *t* = 6 is the restructured time metric.

Figure 6. Box plots of raw bias for the slope parameter μ_{g1} by condition for age-grade time structure in time study.



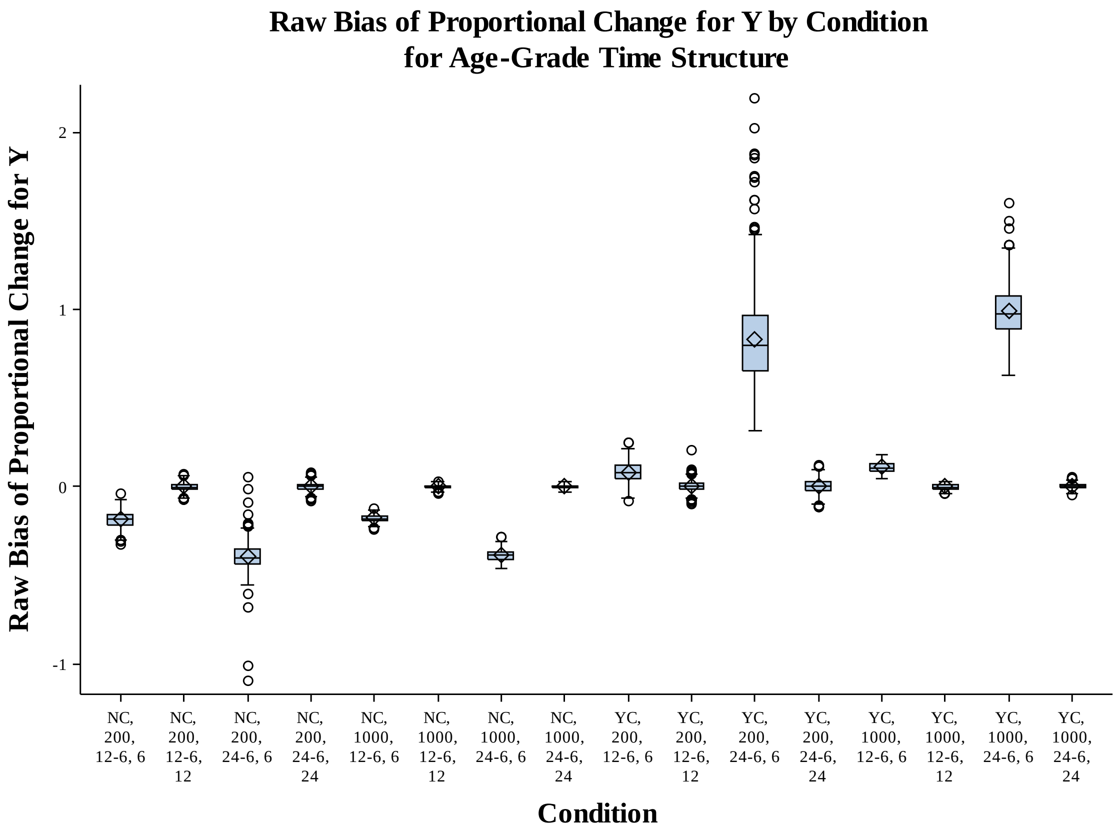
"NC" = no coupling, "YC" = yes coupling
n = 200, 1000
12-6 = 12 time points restructured to 6, 24-6 = 24 time points restructured to 6
t = 6,12, 24
Note: *t* = 12 and 24 are the true time metrics, and *t* = 6 is the restructured time metric.

Figure 7. Box plots of raw bias for the slope parameter μ_{h1} by condition for age-grade time structure in time study.



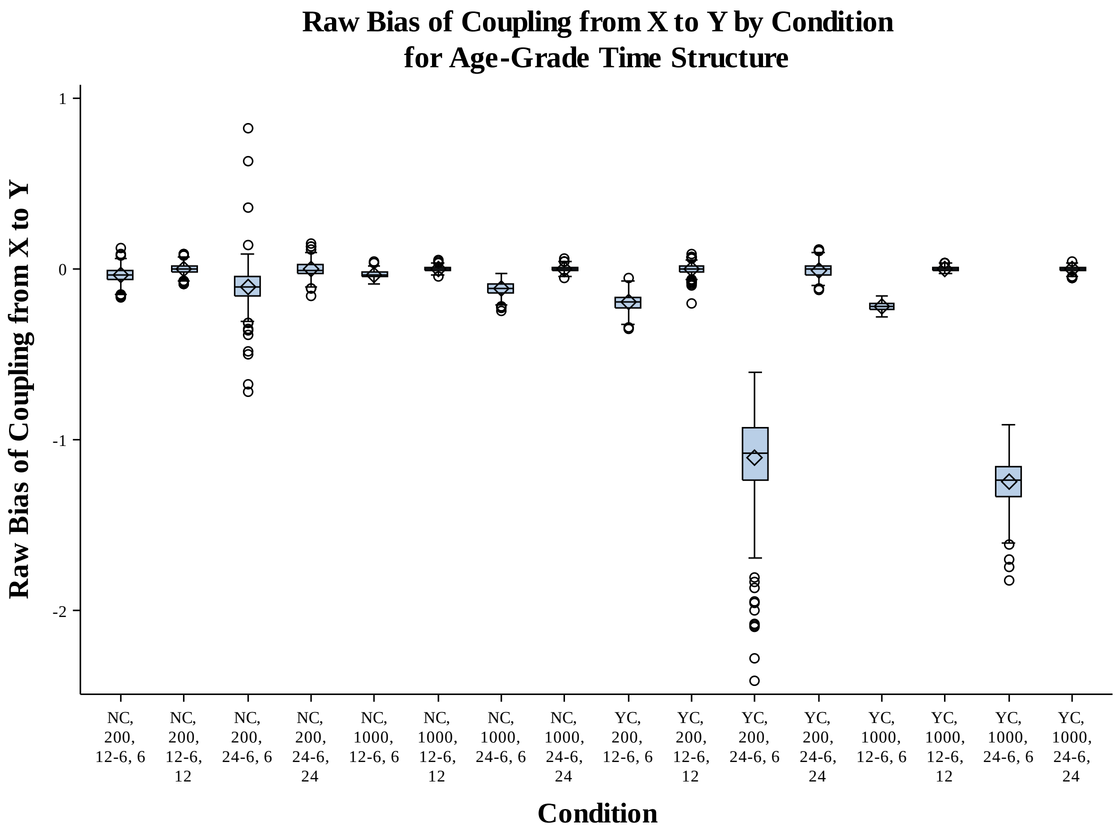
"NC" = no coupling, "YC" = yes coupling
n = 200, 1000
12-6 = 12 time points restructured to 6, 24-6 = 24 time points restructured to 6
t = 6,12, 24
Note: *t* = 12 and 24 are the true time metrics, and *t* = 6 is the restructured time metric.

Figure 8. Box plots of raw bias for the proportional change parameter β_x by condition for age-grade time structure in time study.



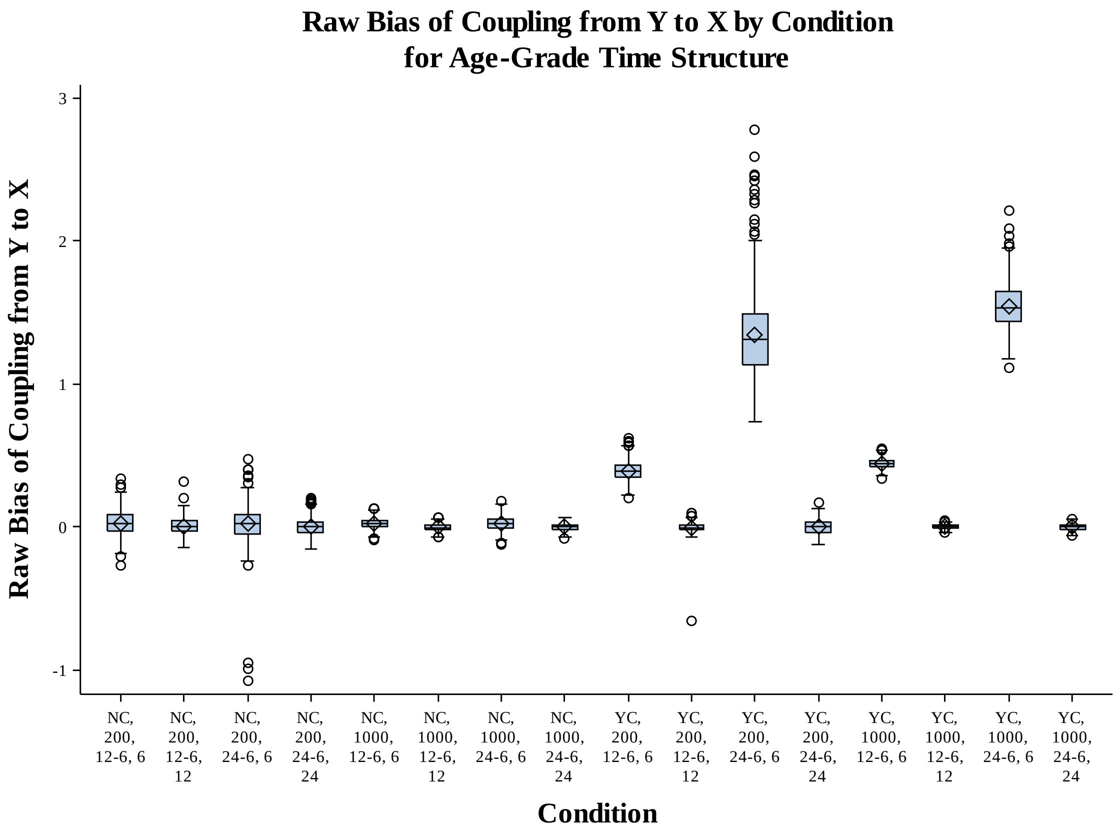
"NC" = no coupling, "YC" = yes coupling
n = 200, 1000
12-6 = 12 time points restructured to 6, 24-6 = 24 time points restructured to 6
t = 6,12, 24
Note: t = 12 and 24 are the true time metrics, and t = 6 is the restructured time metric.

Figure 9. Box plots of raw bias for the proportional change parameter β_y by condition for age-grade time structure in time study.



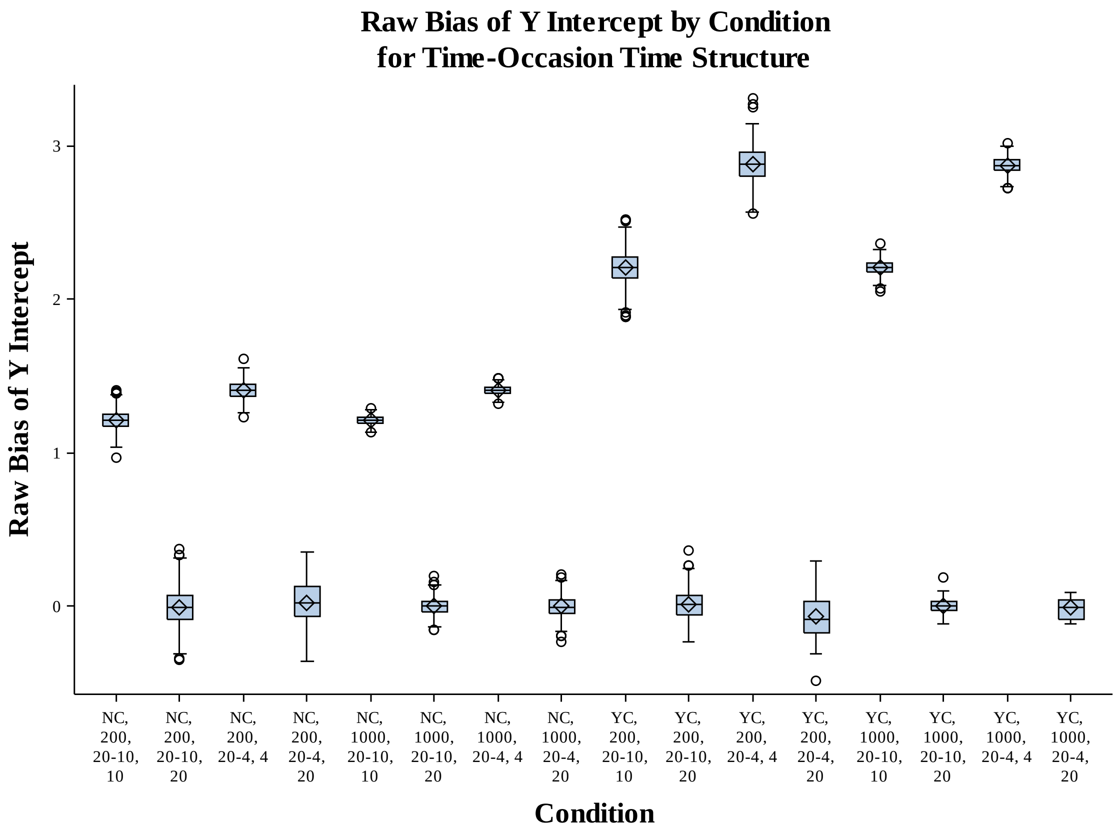
"NC" = no coupling, "YC" = yes coupling
n = 200, 1000
12-6 = 12 time points restructured to 6, 24-6 = 24 time points restructured to 6
t = 6,12, 24
Note: *t* = 12 and 24 are the true time metrics, and *t* = 6 is the restructured time metric.

Figure 10. Box plots of raw bias for the coupling parameter γ_{yx} by condition for age-grade time structure in time study.



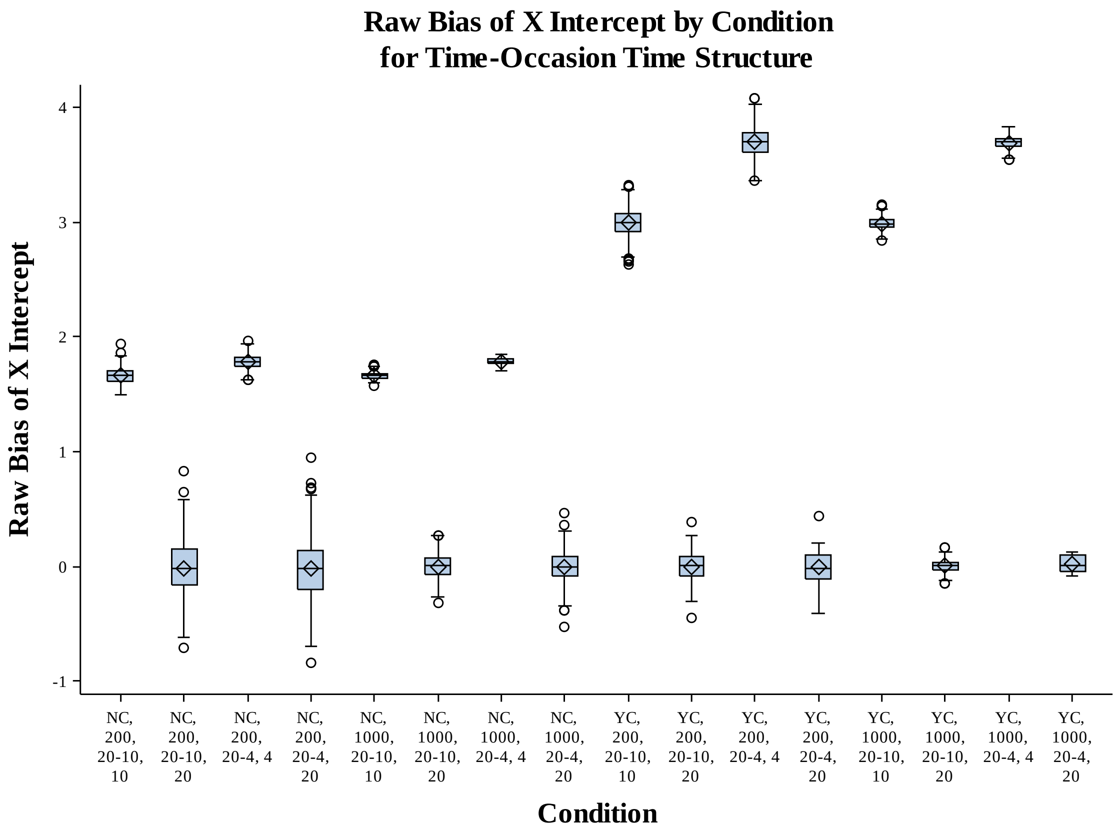
"NC" = no coupling, "YC" = yes coupling
n = 200, 1000
12-6 = 12 time points restructured to 6, 24-6 = 24 time points restructured to 6
t = 6,12, 24
Note: t = 12 and 24 are the true time metrics, and t = 6 is the restructured time metric.

Figure 11. Box plots of raw bias for the coupling parameter γ_{xy} by condition for age-grade time structure in time study.



"NC" = no coupling, "YC" = yes coupling
n = 200, 1000
20-10 = 20 time points restructured to 10, 20-4 = 20 time points restructured to 4
t = 4, 10, 20
Note: *t* = 20 is the true time metric, and *t* = 4 and 10 are the restructured time metrics.

Figure 12. Box plots of raw bias for the intercept parameter μ_{g0} by condition for time-occasion time structure in time study.



"NC" = no coupling, "YC" = yes coupling
n = 200, 1000
20-10 = 20 time points restructured to 10, 20-4 = 20 time points restructured to 4
t = 4, 10, 20
Note: *t* = 20 is the true time metric, and *t* = 4 and 10 are the restructured time metrics.

Figure 13. Box plots of raw bias for the intercept parameter μ_{h0} by condition for time-occasion time structure in time study.

APPENDIX B

DOCUMENT NOTATION

α	A fixed parameter in the latent difference score equation, usually equal to 1.
β	Systematic proportional change over time that does not vary across individuals.
β_x	Proportional change for *X*.
β_y	Proportional change for *Y*.
$\Delta y[t]_n$	The latent difference score for *Y* for an individual *n* at time *t*.
η_n	The *R* x 1 vector with latent factor scores for an individual *n*.
γ_{xy}	The coupling parameter for the effect of *Y* on *X* over time.
$\mathbf{\Lambda}$	The *t* x *R* factor loading matrix defining the latent growth factors.
μ_{g0}	The mean for the initial level of *Y*.
μ_{g1}	The mean for the constant change component of *Y*.
μ_{h0}	The mean for the initial level of *X*.
μ_{h1}	The mean for the constant change component of *X*.
σ^2_{g0}	The variance for the initial level of *Y*.
σ^2_{g1}	The variance for the constant change component of *Y*.
$\sigma_{g0,g1}$	The covariance between the initial level of *Y* and the constant change component of *Y*.
σ^2_{h0}	The variance for the initial level of *X*.
σ^2_{h1}	The variance for the constant change component of *X*.
$\sigma_{h0,h1}$	The covariance between the initial level of *X* and the constant change component of *X*.

$\sigma_{g0,h0}$	The covariance between the initial levels of *X* and *Y*.
$\sigma_{g1,h1}$	The covariance between the constant change components of *X* and *Y*.
$e[t]_n$	Unique score (error) for an individual *n* at time *t*.
g_{0n}	The initial level for participant *n* on *Y*.
g_{1n}	The constant change component for participant *n* on *Y*.
h_{0n}	The initial level for participant *n* on *X*.
h_{1n}	The constant change component for participant *n* on *X*.
N	Number of participants.
n	Denotes an individual.
R	In linear growth model notation, $R = 1$ for no growth, $R = 2$ for linear growth.
t	Number of time points.
$\mathbf{u}_n$	The *t* x 1 vector of unique scores for participant *n*.
$Y[t]_n$	Observed score for an individual *n* at time *t*.
$y[t]_n$	True score for an individual *n* at time *t*.
$y[t - 1]_n$	True score for an individual *n* at the time point previous to *t*.
$\mathbf{y}_n$	The *t* x 1 vector of observed scores with *t* repeated measures for an individual *n*.

APPENDIX C

SAMPLE SAS CODE FOR DATA GENERATION OF BIVARIATE DUAL CHANGE LDS MODEL

```
TITLE 'Dissertation 20-4 N=200 couple-Y Occasion example' ;

/**********************************************************************
*****

 THIS FILE GENERATES TWO SETS OF DATA SETS, ONE SET BASED ON 20
OCCASIONS WHERE
 PARTICIPANTS ARE MEASURED 4 TIMES OVER THE 20 MEASUREMENT
OCCASIONS AND
 ONE SET THAT HAS COMPLETE DATA ON THOSE 4 OCCASIONS (THE FIRST
DATA SET,
 RESTRUCTURED. THIS FILE CREATES NUMEROUS DATA SETS FOR EACH
STRUCTURE AND
 OUTPUTS THEM TO TWO SEPARATE FOLDERS, SEPARATED BY DATA
STRUCTURE.
 HOLLY OROURKE 110415

**********************************************************************
*****/

 %MACRO SIMULATION;
 *Covariance Matrix of Levels and Slopes;
 PROC IML;
 * Specify random number seed;
        current = TIME();
        Seed = ROUND(current*10001.5);
 * Specify number of observations for the data matrix;
        n=200;
 * Define Correlation Matrix (add column for systematic variance for moderator);
  R={1.0      0.50   0.50   0.50,
        0.50   1.0           0.50   0.50,
        0.50   0.50   1.0           0.50,
        0.50   0.50   0.50   1.0};
 * Define Vector of Standard Deviations;
 *(comes from Ferrer et al. 2010 - typical readers, p. 97;
 *intercept SDs are .734 & .660 and slope SDs are .118 & .207);
 Ds=Diag({.734 .118 .660 .207});
  * Compute Covariance Matrix;
 S=Ds*R*Ds;
  * Compute Choleski Root for transformation;
 T=Root(S);
  * Initialize data vector with random number seed;
 X=J(n,NRow(S),Seed);
  * Generate Independent Standard Normals for the data matrix;
 X=Rannor(X);
```

```
 * Now transform to have the desired covariance structure;
Y=X*T;
 * Save the data matrix;
CREATE NormalData From Y;
APPEND From Y;
CLOSE NormalData;
 * Done;
QUIT;

DATA newnames;
        SET NormalData;
        RENAME col1 = x0_0;
        RENAME col2 = x1_0;
        RENAME col3 = y0_0;
        RENAME col4 = y1_0;
RUN;

Proc Corr Data=newnames;
Run;


TITLE2 'Generating Simulation Data using random normal theory';
DATA row_DYN1;
        SET newnames;
 ARRAY Ym[20] y01 y02 y03 y04 y05 y06 y07 y08 y09 y10 y11 y12 y13 y14 y15 y16
y17 y18 y19 y20 ;
 ARRAY Xm[20] x01 x02 x03 x04 x05 x06 x07 x08 x09 x10 x11 x12 x13 x14 x15 x16
x17 x18 x19 x20 ;
 ARRAY yl[20] yl01 yl02 yl03 yl04 yl05 yl06 yl07 yl08 yl09 yl10 yl11 yl12 yl13 yl14
yl15 yl16 yl17 yl18 yl19 yl20 ;
 ARRAY xl[20] xl01 xl02 xl03 xl04 xl05 xl06 xl07 xl08 xl09 xl10 xl11 xl12 xl13 xl14
xl15 xl16 xl17 xl18 xl19 xl20 ;
 ARRAY ye[20] ye01 ye02 ye03 ye04 ye05 ye06 ye07 ye08 ye09 ye10 ye11 ye12 ye13
ye14 ye15 ye16 ye17 ye18 ye19 ye20 ;
 ARRAY xe[20] xe01 xe02 xe03 xe04 xe05 xe06 xe07 xe08 xe09 xe10 xe11 xe12 xe13
xe14 xe15 xe16 xe17 xe18 xe19 xe20 ;
 ARRAY dy[20] dy01 dy02 dy03 dy04 dy05 dy06 dy07 dy08 dy09 dy10 dy11 dy12 dy13
dy14 dy15 dy16 dy17 dy18 dy19 dy20 ;
 ARRAY dx[20] dx01 dx02 dx03 dx04 dx05 dx06 dx07 dx08 dx09 dx10 dx11 dx12 dx13
dx14 dx15 dx16 dx17 dx18 dx19 dx20 ;

* setting mathematical parameters -- from Ferrer et al. 2010 typical readers, p. 97;
* gamma_xy=coupling y to x, gamma_yx = coupling x to y;
  mu_y0 = .220;
  mu_ys = .550;
```

```
  sig2_ye = .101;        sigma_ye = SQRT(sig2_ye);

  mu_x0 = .570;
  mu_xs = 1.410;
  sig2_xe =  .025;       sigma_xe = SQRT(sig2_xe);

  alpha_y  = 1;          beta_y =  -.274;
  alpha_x  = 1;          beta_x =  -.549;
  gamma_xy = +.401;              gamma_yx= +.130;

* setting statistical parameters;
  current = TIME();
  Seed = ROUND(current*10001.5);
* generating raw data;
        id = _N_;

        y0 = mu_y0 + y0_0;
   ys = mu_ys + y1_0;
   ye[1] = sigma_ye * RANNOR(seed);
   yl[1] = y0;
   ym[1] = yl[1] + ye[1];

   x0 = mu_x0 + x0_0;
   xs = mu_xs + x1_0;
   xe[1] = sigma_xe * RANNOR(seed);
   xl[1] = x0;
   xm[1] = xl[1] + xe[1];

* latent trajectory ;
   DO t = 2 TO 20;
   dy[t] = alpha_y * ys + beta_y * yl[t-1] + gamma_yx * xl[t-1] ;
        ye[t] = sigma_ye * RANNOR(seed);
   yl[t] = yl[t-1] + dy[t];
   ym[t] = yl[t] + ye[t];

   dx[t] = alpha_x * xs + beta_x * xl[t-1] + gamma_xy * yl[t-1] ;
   xe[t] = sigma_xe * RANNOR(seed);
   xl[t] = xl[t-1] + dx[t];
   xm[t] = xl[t] + xe[t];
        call streaminit(0);
        u=rand("Uniform");
        max=17;
        ranstart=ceil(max*u);
        END;
        OUTPUT;
```

```
        KEEP id ranstart y0 ys x0 xs y01--y20 x01--x20 ;
RUN;
%MEND SIMULATION;

/*%MACRO PLOT;*/
/*PROC GPLOT DATA = col_dyn1 (where = (id<100));*/
/*      SYMBOL1 I=JOIN COLOR=BLACK VALUE=DOT HEIGHT=1.25
WIDTH=1.25 LINE=1 REPEAT=5000;*/
/*      PLOT y*time = id/NOLEGEND;*/
/*RUN;*/
/**/
/*PROC GPLOT DATA = col_dyn1 (where = (id<100));*/
/*      SYMBOL1 I=JOIN COLOR=BLACK VALUE=DOT HEIGHT=1.25
WIDTH=1.25 LINE=1 REPEAT=5000;*/
/*      PLOT x*time = id/NOLEGEND;*/
/*RUN;*/
/*%MEND PLOT;*/

%MACRO TRANSFORM1;
DATA col_dyn1;
        SET row_dyn1;
        y = y01; x = x01;       time = 1;       OUTPUT;
        y = y02; x = x02;       time = 2;       OUTPUT;
        y = y03; x = x03;       time = 3;       OUTPUT;
        y = y04; x = x04;       time = 4;       OUTPUT;
        y = y05; x = x05;       time = 5;       OUTPUT;
        y = y06; x = x06;       time = 6;       OUTPUT;
        y = y07; x = x07;       time = 7;       OUTPUT;
        y = y08; x = x08;       time = 8;       OUTPUT;
        y = y09; x = x09;       time = 9;       OUTPUT;
        y = y10; x = x10;       time = 10;      OUTPUT;
        y = y11; x = x11;       time = 11;      OUTPUT;
        y = y12; x = x12;       time = 12;      OUTPUT;
        y = y13; x = x13;       time = 13;      OUTPUT;
        y = y14; x = x14;       time = 14;      OUTPUT;
        y = y15; x = x15;       time = 15;      OUTPUT;
        y = y16; x = x16;       time = 16;      OUTPUT;
        y = y17; x = x17;       time = 17;      OUTPUT;
        y = y18; x = x18;       time = 18;      OUTPUT;
        y = y19; x = x19;       time = 19;      OUTPUT;
        y = y20; x = x20;       time = 20;      OUTPUT;
        KEEP id ranstart time y0 ys y x0 xs x;
RUN;
%MEND TRANSFORM1;
```

```
%MACRO TRANSFORM2;
DATA col_dyn2;
	SET col_dyn1;
	occ=0;
	if time=ranstart then occ=1;
	if time=(ranstart+1) then occ=2;
	if time=(ranstart+2) then occ=3;
	if time=(ranstart+3) then occ=4;
RUN;

DATA col_dyn2a;
	SET col_dyn2;
	if occ=0 then x=.;
	if occ=0 then y=.;
RUN;

DATA col_dyn3;
	SET col_dyn2;
	if occ=0 then delete;
RUN;
%MEND TRANSFORM2;

%MACRO TRANSFORM3;
proc transpose data=col_dyn3 out=row_dynx prefix=x;
	by id y0 ys x0 xs;
	id occ;
	var x;
run;

proc transpose data=col_dyn3 out=row_dyny prefix=y;
	by id;
	id occ;
	var y;
run;

data row_dyn2;
	merge row_dynx row_dyny;
	drop _name_;
run;

proc transpose data=col_dyn2a out=row_dynxa prefix=x;
	by id y0 ys x0 xs;
	id time;
	var x;
run;
```

```
proc transpose data=col_dyn2a out=row_dynya prefix=y;
        by id;
        id time;
        var y;
run;

data row_dyn2a;
        merge row_dynxa row_dynya;
        drop _name_;
run;

data row_dyn2a;
        set row_dyn2a;
ARRAY Ym[20] y1 y2 y3 y4 y5 y6 y7 y8 y9 y10 y11 y12 y13 y14 y15 y16 y17 y18 y19
y20;
ARRAY Xm[20] x1 x2 x3 x4 x5 x6 x7 x8 x9 x10 x11 x12 x13 x14 x15 x16 x17 x18 x19
x20;
DO t=1 TO 20;
 if Xm[t] =. then Xm[t] = 999;
END;
DO t=1 TO 20;
 if Ym[t] =. then Ym[t] = 999;
END;
KEEP id y0 ys x0 xs x1--x20 y1--y20;
 OUTPUT;
RUN;
%MEND TRANSFORM3;

%MACRO OUTPUT1;
/*Writing Out More-Info Time Structure Data Files for Mplus*/
DATA _NULL_;
        SET row_dyn2a;
        data = &count;
        datastr = PUT(data,4.);
        datastrcomp = COMPRESS(datastr,' ');
        name = 'C:\Users\horourke\Google Drive\Dissertation\more
info\test_'||datastrcomp||'.dat';
        FILE tempfile filevar=name;
        PUT (id y0 ys x0 xs x1-x20 y1-y20) (11.2);
RUN;
%MEND OUTPUT1;

%MACRO OUTPUT2;
/*Writing Out Less-Info Time Structure Data Files for Mplus*/
```

```
DATA _NULL_;
	SET row_dyn2;
	data = &count;
	datastr = PUT(data,4.);
	datastrcomp = COMPRESS(datastr,' ');
	name = 'C:\Users\horourke\Google Drive\Dissertation\less
info\test_'||datastrcomp||'.dat';
	FILE tempfile filevar=name;
	PUT (id y0 ys x0 xs x1-x4 y1-y4) (11.2);
RUN;
%MEND OUTPUT2;

%MACRO ALLJOB;
	%LOCAL count;
	%LOCAL stop;
	%LET stop = 1500;
	%LET count= 1001;
		%DO %WHILE (&count <= &stop);
			%SIMULATION; *Recall local macro - SIMULATION;
			%TRANSFORM1;
			%TRANSFORM2;
			%TRANSFORM3;
			%OUTPUT1;
			%OUTPUT2;
			%LET count = %EVAL(&count+1); *Increment count by 1;
		%END;
%MEND ALLJOB;

%ALLJOB;
```

APPENDIX D

SAMPLE MPLUS CODE FOR BIVARIATE DUAL CHANGE LDS MODEL

```
!********************************************!
!        LDS model for data structure with less info        !
!********************************************!

TITLE: LDS model for age-grade example - less info data structure;
!The data to be analyzed
DATA: FILE IS testage2.DAT;
VARIABLE: NAMES ARE
id y0 ys x0 xs x1 x2 x3 x4 x5 x6 y1 y2 y3 y4 y5 y6 ;
 USEVARIABLE ARE x1 x2 x3 x4 x5 x6 y1 y2 y3 y4 y5 y6 ;
 MISSING are all (999);
 ANALYSIS:
 !TYPE=MEANSTRUCTURE;
 ITERATIONS = 5000;
 COVERAGE = 0;
MODEL:
!For the variable y
!Oberved variables and latent level
ly1 by y1 @1;
ly2 by y2 @1;
ly3 by y3 @1;
ly4 by y4 @1;
ly5 by y5 @1;
ly6 by y6 @1;
!Autoregressive part
ly2 on ly1 @1;
ly3 on ly2 @1;
ly4 on ly3 @1;
ly5 on ly4 @1;
ly6 on ly5 @1;
!Difference score on latent level
dy1 by ly2 @1;
dy2 by ly3 @1;
dy3 by ly4 @1;
dy4 by ly5 @1;
dy5 by ly6 @1;
!Auto-porpotion of difference score on level
! add the starting values after *
dy1 on ly1 * (1);
dy2 on ly2 * (1);
dy3 on ly3 * (1);
dy4 on ly4 * (1);
dy5 on ly5 * (1);
!Model relationship between slope and ds
sy by dy1 @1;
```

```
sy by dy2 @1;
sy by dy3 @1;
sy by dy4 @1;
sy by dy5 @1;
y0 by ly1 @1;
! Set the means and variance to be 0
[y1@0]; [ly1@0]; [dy1@0]; ly1@0; dy1@0;
[y2@0]; [ly2@0]; [dy2@0]; ly2@0; dy2@0;
[y3@0]; [ly3@0]; [dy3@0]; ly3@0; dy3@0;
[y4@0]; [ly4@0]; [dy4@0]; ly4@0; dy4@0;
[y5@0]; [ly5@0]; [dy5@0]; ly5@0; dy5@0;
[y6@0]; [ly6@0];ly6@0;
!beginning the codes for x variables
!For the variable x
!Oberved variables and latent level
lx1 by x1 @1;
lx2 by x2 @1;
lx3 by x3 @1;
lx4 by x4 @1;
lx5 by x5 @1;
lx6 by x6 @1;
!Autoregressive part
lx2 on lx1 @1;
lx3 on lx2 @1;
lx4 on lx3 @1;
lx5 on lx4 @1;
lx6 on lx5 @1;
!Difference score on latent level
dx1 by lx2 @1;
dx2 by lx3 @1;
dx3 by lx4 @1;
dx4 by lx5 @1;
dx5 by lx6 @1;
!Auto-porpotion of difference score on level
! add the starting values after *
dx1 on lx1 * (2);
dx2 on lx2 * (2);
dx3 on lx3 * (2);
dx4 on lx4 * (2);
dx5 on lx5 * (2);
!Model relationship between slope and ds
sx by dx1 @1;
sx by dx2 @1;
sx by dx3 @1;
sx by dx4 @1;
```

```
sx by dx5 @1;
x0 by lx1 @1;
 !Set the means and variance to be 0
[x1@0]; [lx1@0]; [dx1@0]; lx1@0; dx1@0;
[x2@0]; [lx2@0]; [dx2@0]; lx2@0; dx2@0;
[x3@0]; [lx3@0]; [dx3@0]; lx3@0; dx3@0;
[x4@0]; [lx4@0]; [dx4@0]; lx4@0; dx4@0;
[x5@0]; [lx5@0]; [dx5@0]; lx5@0; dx5@0;
[x6@0]; [lx6@0]; lx6@0;

!coupling from y to x
dx1 on ly1*(3);
dx2 on ly2*(3);
dx3 on ly3*(3);
dx4 on ly4*(3);
dx5 on ly5*(3);
!coupling from x to y
dy1 on lx1*(4);
dy2 on lx2*(4);
dy3 on lx3*(4);
dy4 on lx4*(4);
dy5 on lx5*(4);
[y0 x0 sy sx];
y0 x0 sy sx;
!Set all residuals to be equal
y1*1 (5);x1*1 (6);
y2*1 (5);x2*1 (6);
y3*1 (5);x3*1 (6);
y4*1 (5);x4*1 (6);
y5*1 (5);x5*1 (6);
y6*1 (5);x6*1 (6);
! Set the correlated residuals
y1 with x1* (7);
y2 with x2* (7);
y3 with x3* (7);
y4 with x4* (7);
y5 with x5* (7);
y6 with x6* (7);
OUTPUT:  TECH1 TECH4;
SAVEDATA:
RESULTS=testage2results.txt;
```