Modeling Multifaceted Constructs in Statistical Mediation Analysis:

A Bifactor Approach

by

Oscar Gonzalez

A Thesis Presented in Partial Fulfillment
of the Requirements for the Degree
Master of Arts

Approved May 2016 by the
Graduate Supervisory Committee:

David P. MacKinnon, Chair
Kevin Grimm
Yi Zheng

ARIZONA STATE UNIVERSITY

August 2016

ABSTRACT

Statistical mediation analysis allows researchers to identify the most important the mediating constructs in the causal process studied. Information about the mediating processes can be used to make interventions more powerful by enhancing successful program components and by not implementing components that did not significantly change the outcome. Identifying mediators is especially relevant when the hypothesized mediating construct consists of multiple related facets. The general definition of the construct and its facets might relate differently to external criteria. However, current methods do not allow researchers to study the relationships between general and specific aspects of a construct to an external criterion simultaneously. This study proposes a bifactor measurement model for the mediating construct as a way to represent the general aspect and specific facets of a construct simultaneously. Monte Carlo simulation results are presented to help to determine under what conditions researchers can detect the mediated effect when one of the facets of the mediating construct is the true mediator, but the mediator is treated as unidimensional. Results indicate that parameter bias and detection of the mediated effect depends on the facet variance represented in the mediation model. This study contributes to the largely unexplored area of measurement issues in statistical mediation analysis.

DEDICATION

This document is dedicated to my late advisor, Dr. Roger Millsap, who believed in me and gave me the opportunity to attend graduate school. You gave me the inspiration to pursue the area of psychometrics and to think about what we measure in the field of psychology. We all miss you. This document could not have been possible without my extended network of friends who were always there to support me. Amber, James, Matt, and Peter, I thank you deeply for keeping me sane. I also dedicate this document to the kid that almost two years ago thought about quitting graduate school and running away because his mentor was gone. Ted, from *How I Met Your Mother,* once said something like, "…eventually, over time, we become our own doppelgangers… you know, these completely different people who just happen to look like us." Well, the kid did not quit. He won a fellowship, finished this project, and is ready to do more science.

…y no me puedo ir sin agradecerle a mis padres! ¡¡Papá, mamá, lo hicimos!!

Small victories, be humble, move forward.

ACKNOWLEDGMENTS

I thank my committee chair, Dr. David P. MacKinnon, Dr. Yi Zheng, and Dr. Kevin Grimm for their help in the development of this document. Their comments were challenging and made the project better. My research is supported by the National Science Foundation Graduate Research Fellowship under Grant No. DGE-1311230.

TABLE OF CONTENTS

Page

LIST OF TABLES

Table Page

Table Page

Table Page

Table Page

Table Page

LIST OF FIGURES

Introduction

The goal of statistical mediation analysis is to uncover the intermediate causal mechanisms (known as mediators) through which an independent variable brings about a change on an outcome (Baron & Kenny, 1986; MacKinnon, 2008). Statistical mediation is relevant in prevention research where interventions are designed to target mediators that are thought to be causally related to an outcome (MacKinnon, Fairchild, & Fritz, 2007). Beyond testing the success of an intervention, researchers can save resources if they investigate which aspects of the mediator do not contribute to a change in the outcome (Cox, Kisbu-Sakarya, & MacKinnon, 2012; MacKinnon & Dwyer, 1993; Weiss, 1997). Identifying the true mediator in the causal process can be thought of as a *measurement problem*, where a framework to *distill* the mediating variable is needed to find the underlying mediating construct (MacKinnon, 2008, p.4).

An assumption in mediation analysis is the accurate characterization of the construct underlying the mediator. This assumption is relevant when researchers measure multifaceted constructs. Facets are subordinate concepts of a construct that could be measured independently from the *general* construct (Carver, 1989). A construct is considered *general* when it is defined by aggregating its facets. On the other hand, a construct is considered *specific* when it is defined by only one facet of a *general* construct. Multifaceted constructs are challenging because the specific aspects of a construct might relate differently to an outcome. Therefore, the mediator can be incorrectly characterized by including non-predictive aspects of the construct in the mediation model which could lead to inaccurate conclusions on mediation.

In this study, I propose a framework to *distill* the mediator by modeling the construct's *general* variance and *specific* facet variance with a bifactor measurement model. First, I will describe statistical mediation and multifaceted constructs. Next, I will review the latent variable approaches to represent multifaceted constructs in mediation analysis. I will then describe the properties of the bifactor measurement model as a way to *distill* multifaceted constructs. Finally, Monte Carlo simulation results are presented on the properties of the mediated effect when one of the *specific* facets of a construct is the true mediator in the causal process, but the structure of the mediator is misspecified. The rationale of the study is that by modeling the facets of the mediator researchers could obtain more power to detect mediated effects.

Statistical Mediation Analysis

Statistical mediation analysis addresses the question of *how* two variables are related by considering mediators (*M*) to explain the relationship between an independent (*X*) and a dependent variable (*Y*; see Figure 1; MacKinnon, 2008). The model can be conceptualized into three regression equations:

$$\hat{Y} = i_1 + cX + e_1 \quad (1)$$

$$\hat{M} = i_2 + aX + e_2 \quad (2)$$

$$\hat{Y} = i_3 + c'X + bM + e_3 \quad (3)$$

Equation 1 represents the total effect of the independent variable (*X*) on the dependent variable (*Y*; *c* coefficient). Equation 2 represents the effect of the independent variable (*X*) on the mediator (*M*; *a* coefficient). Equation 3 represents the effect of the mediator (*M*) on the dependent variable (*Y*), controlling for *X* (*b* coefficient) and the effect of the independent variable (*X*) on the dependent variable (*Y*), controlling for *M* (*c'* coefficient).

Finally, the mediated effect, the indirect influence of X on Y through M, is captured by the product of the a and b parameters. Moreover, the standard error for ab can be derived through the multivariate delta method (Sobel, 1982; 1986) to test for statistical significance. However, the Sobel test of significance assumes that the distribution of the product of two random variables is normally distributed and this is rarely the case. Methods for calculating asymmetric confidence intervals with the distribution of the product method and resampling techniques have been developed to accurately test for the mediated effect (MacKinnon, Lockwood & Williams, 2004).

Several assumptions are also needed to accurately test for mediation (MacKinnon, 2008). First, the functional form and temporal precedence among the three variables has to be correctly specified. Also, no relevant variables have been excluded from the model. Independent and identically distributed residuals across values of the predictors are also assumed. Finally, X, Y, and M are reliable and valid measures of their respective constructs. This study focuses on this last assumption due to the complications of representing multifaceted mediators.

The Complexity of Multifaceted Constructs

Typically, when researchers are interested in studying a construct, they hypothesize that multiple facets encompass the construct (Chen, West, & Sousa, 2006). Carver (1989, p. 577) indicates that multifaceted constructs "are composed of two or more subordinate concepts, each of which can be distinguished conceptually from the others and measured separately, despite being related to each other both logically and empirically." Furthermore, Carver (1989) offers two competing arguments about multifaceted constructs. Examining *specific* facets of the construct and how they relate to an external criterion might be more accurate because a *general* construct might mask the differential contributions of the facets to prediction. On the other hand, the *interaction* of the facets as a whole might be the construct of interest, where *the whole is greater than the sum of its parts* (see Bagozzi & Heatherton, 1994). Much of the psychometric work suggests that there are situations in which individual facets are important. Examples include alexithymia (trouble expressing emotion; Haviland, Warren & Riggs, 2000), self-monitoring (Briggs, Cheek & Buss, 1980), the big five factors of personality (Chen et al., 2012), general intelligence (Brunner, 2008) and well-being (Chen et al., 2013). According to Reise (2012), multifaceted constructs are complex because items measuring a *specific* facet are not interchangeable indicators of the *general* construct, and each facet might relate differently to external criterions. In other words, the *specific* facets could make a theoretically important contribution to prediction beyond the *general* construct (Chen, West, Sousa, 2006). Therefore, the representation of multifaceted mediating constructs as *general* or *specific* could compromise accurate conclusions from statistical mediation.

Representing Multifaceted Constructs with Latent Variables

One of the approaches to test for statistical mediation is to use covariance structure analysis to investigate relationships between the three variables in the model. These methods evaluate how well a model represents the data by comparing an expected covariance matrix among the variables to the observed covariance matrix among the variables (Bollen, 1989). Variables in the model can either be represented as manifest or latent. Chen and colleagues (2012) review the manifest variable approaches to represent multifaceted variables and suggest that they suffer from many disadvantages, such as not controlling for unreliability of the construct. The latent variable model approach consists of measuring the individual facets of the construct and estimating the extent to which the facets are related to each other (Bollen, 1989). The latent variable cannot be directly measured, but it is indicated by its manifestations, such as the responses to the administered items (indicators). The relationships between indicators and latent variables are estimated through confirmatory factor analysis (CFA; Brown, 2014). In this study the indicators are assumed to be continuous and linearly related to the latent variable.

A challenge of latent variable modeling is to choose a priori which measurement model represents the data better. Ideally, this decision would be backed-up with theory. However, researchers often overuse unidimensional models, assuming that only a single common factor accounts for the relationships among all of the items. Below, two measurement models used in this study and their priority in modeling the *general* construct or *specific* facets are discussed.

Measurement Models

One-factor model. Proposed by Charles Spearman (1904; Figure 2) to explain the structure of intelligence, the one-factor model assumes that correlations among facets and individual differences in the test can be explained by a single, *general* factor (Reise et al., 2010). This model does not take in consideration the *specific* facets of the construct. The variance of each indicator is influenced by two sources of variance: common variance shared by all the indicators due to the *general* construct and the unique variance of each indicator. The unique variance is comprised of reliable *specific* facet variance not shared among the other facets and unreliable variance due to measurement error (Brunner et al., 2012). This model assumes that the unique factors are uncorrelated with each other because all common variance is accounted for by the single common factor. A violation of unidimensionality could show up in the model through “correlated uniquenesses,” where some or all of the unique factors of the indicators still share variance after accounting for the common factor. If the data violates the unidimensional assumption, alternative measurement models need to be considered.

Bifactor model*:* The bifactor model was recently “rediscovered” (Reise, 2012) after being introduced almost 80 years ago (Holzinger & Harman, 1938; Holzinger & Swineford, 1937) as an option to modeling construct-relevant multidimensionality. Researchers in personality assessment have used the bifactor model to help conceptualize psychological constructs and the bifactor model is starting to be considered as a competing model with the higher-order model and correlated-factor model (Reise, 2012).

The bifactor model specifies that relationships among the items can be explained by a *general* factor that reflects the common variance among the indicators, and by several *specific* factors (group factors; Reise, 2012) reflecting the common variance of indicators with highly similar content not accounted by the *general* factor (Figure 3). The *general* factor represents the broad construct that the scale intends to measure and the *specific* factors incorporate the multifaceted aspect of the construct by influencing the indicators that represent the facets of the broad construct. Also, indicators are influenced by their own unique factor. Therefore, the bifactor model can separate the *general*, *specific*, and unique variance of each of the indicators.

According to Chen, West, and Sousa (2006), the bifactor model provides many advantages over conventional models, such as the higher-order factor model, when researchers want to test the unique contributions of the facets in prediction. When modeling *specific* facet variance, a higher-order model is prone to mask the lack of variability in a facet by including a non-significant disturbance in a lower-order factor, while the bifactor model would have problems converging due to factor overextraction. By modeling *specific* factors, researchers can test for measurement invariance in the facets, calculate latent means, and study relationships between facets and outcomes beyond the *general* factor.

Overall, if a researcher is only interested in the *general* construct, other models are more parsimonious than the bifactor model. Yet, if the interest is on how *specific* facets of a construct carry the influence of the independent variable to the outcome, a bifactor model for the mediator provides a promising approach to study the influence of facets on a criterion.

Distilling a Mediator with the Bifactor Model in Statistical Mediation

If a researcher fits multidimensional data in a unidimensional model, the model misspecification might lead to biased parameters and inaccurate results. This problem is relevant in statistical mediation when the true mediator is only one facet of a multifaceted construct. Reise et al. (2013) conducted a simulation study to determine if indices of model fit or indices of factor strength predict structural bias when multidimensional data (generated with a bifactor model) are treated as unidimensional when predicting an outcome. Reise et al. (2013) concluded that indices of factor strength, such as the explained common variance (ECV; the variance explained by general common factor over the total common variance explained in the model) and the percent of uncontaminated correlations (PUC; percentage of unique correlations among the indicators that are not confounded by both the *general* and *specific* factors) predict structural bias. This study expands on Reise et al. (2013) by carrying out a simulation to investigate if the mediated effect can be *distilled* with the bifactor model when one of the *specific* facets of a multifaceted construct is the true mediator, but the mediator is treated as unidimensional. This study evaluates bias, power, Type I error, and confidence interval coverage of the mediated by analyzing simulation datasets with four different models. Two of the models ignore the *specific* facet variance and one of the models ignores the *general* factor variance of the bifactor mediator.

Study Hypotheses

The data-generating model used to test all of the hypotheses is shown in Figure 4 (**Model 1**). The mediator has a bifactor structure and one of the three *specific* factors of the construct is specified to be the true mediator. The four data-analysis models are described next.

Finite-sample bias. The first data-analysis model was identical to the data-generating model (**Model 1;** see Figure 4). It was hypothesized that conditions with higher sample sizes will have higher statistical power, *adequate* Type 1 errors, lower bias, and *adequate* confidence interval coverage than conditions with lower sample sizes (**Hypothesis 1**). Specifically, as sample size increases, there will be lower bias in the mediated effect (**Hypothesis 1.1**) Furthermore, as the loadings on the *general* factor increase, there will be lower bias in the mediated effect (**Hypothesis 1.2**). Also, as the loadings on the *specific* factor increase, there will be lower bias in the mediated effect (**Hypothesis 1.3**).

Also, a sample size increases, it would be more likely for the mediated effect to be statistically significant, have *adequate* Type 1 errors, and for the true estimate to be covered in the confidence intervals (**Hypothesis 1.4**). Moreover, as the loadings on the *general* factor increase, it would be more likely for the mediated effect to be statistically significant, have *adequate* Type 1 errors, and for the true estimate to be covered in the confidence intervals (**Hypothesis 1.5**). Finally, as the loadings on the *specific* factor increase, it would be more likely for the mediated effect to be statistically significant, have *adequate* Type 1 errors, and for the true estimate to be covered in the confidence intervals (**Hypothesis 1.6**). It was also hypothesized that the combination of small sample sizes and low *specific* factor loadings will have convergence problems owing to a non-positive definite covariance matrix (**Hypothesis 2**).

Ignoring the general construct. The effect of ignoring the *general* aspect of the mediating construct was evaluated with **Model 2** (Figure 5). The mediator has a unidimensional structure, where only the indicators of the true facet mediator are included and specified to load on a single factor. **Model 2** represents a situation where the researcher believes that a *specific* part of a construct, such as a subscale, is the mediator. It was hypothesized that mediated effect estimates will be attenuated and have lower power, *inadequate* Type 1 errors, and *inadequate* confidence interval coverage (**Hypothesis 3**). Specifically, as sample size increases, there will be lower bias in the mediated effect (**Hypothesis 3.1**). Also, as the loadings on the *general* factor increase, there will be higher bias in the mediated effect (**Hypothesis 3.2**) because the variance of the mediator will have variance from the *general* and *specific* factors. Finally, as the loadings on the *specific* factor increase, there will be lower bias in the mediated effect (**Hypothesis 3.3**).

Furthermore, as sample size increases, it would be more likely for the mediated effect to be statistically significant, have *adequate* Type 1 errors, and for the estimate to be covered in the confidence intervals (**Hypothesis 3.4**). As the loadings on the *general* factor increase, it would be less likely for the mediated effect to be statistically significant, have *adequate* Type 1 errors, and for the estimate to be covered in the confidence intervals (**Hypothesis 3.5**). Finally, as the loadings on the *specific* factor increase, it would be more likely for the mediated effect to be statistically significant, have adequate Type 1 errors, and for the estimate to be covered in the confidence intervals (**Hypothesis 3.6**).

Ignoring specific facets. The effect of ignoring the multidimensionality of the mediator was evaluated with the data-analysis models in Figure 6 (**Model 3**) and Figure 7 (**Model 4**). In **Model 3**, the mediator has a unidimensional structure where all of the indicators of load on a single factor. **Model 3** represents a situation where the researcher believes that the *general* construct is the true mediator and facets are not important. Anticipating poor fit of **Model 3**, **Model 4** also assumes a unidimensional mediator but indicators of the same facet had correlated uniquenesses. It is hypothesized that mediated effect estimates in **Models 3** and **4** will have negative bias, low power, *inadequate* Type 1 errors, and *inadequate* confidence interval coverage (**Hypothesis 4**) because the variance of the true mediator is only shared by a third of the indicators, so a unidimensional model will not accurately capture all the *specific* factor variance. In this case, as sample size increases, there will be lower bias in the mediated effect (**Hypothesis 4.1**). Furthermore, as the loadings on the *general* factor increase, there will be higher bias in the mediated effect (**Hypothesis 4.2**) because more *general* factor variance is reflected on the latent variable. But, as the loadings on the *specific* factor increase, there will be lower bias in the mediated effect (**Hypothesis 4.3**).

As sample size increases, it would be more likely for the mediated effect to be statistically significant, to have *adequate* Type 1 errors, and for the estimate to be covered in the confidence intervals (**Hypothesis 4.4**). Also, as the loadings on the *general* factor increase, it would be less likely for the mediated effect to be statistically significant, to have *adequate* Type 1 errors, and for the estimate to be covered in the confidence intervals (**Hypothesis 4.5**). Finally, as the loadings of the *specific* factor increase, it would be more likely for the mediated effect to be statistically significant, have *adequate* Type 1 errors, and for the estimate to be covered in the confidence intervals (**Hypothesis 4.6**).

Finally, it was hypothesized that ignoring the *general* construct (**Model 2**) will have lower bias, higher power, more *adequate* Type 1 errors and more *adequate* confidence interval coverage than ignoring the *specific* facets of the construct (**Model 3** and **4; Hypothesis 5**). The mediator in **Model 2** reflects the most *specific* factor variance from the true mediator in **Model 1**.

Method

Data-Generating Model

The statistical package *R* (R Core Team, 2013) and Mplus 7.1 (Muthen & Muthen, 1998-2011) were used to conduct the simulation. The equations below represent the data-generating model **(Model 1)**, specifying a bifactor model for *M* and the structural model for *X*, *M*, and *Y*.

Measurement Model for the Mediator

$$\boldsymbol{M} = \boldsymbol{\Lambda_m \eta} + \boldsymbol{\epsilon}, \quad where: \tag{4}$$

$$\boldsymbol{M} = \begin{bmatrix} M_1 \\ M_2 \\ M_3 \\ M_4 \\ M_5 \\ M_6 \\ M_7 \\ M_8 \\ M_9 \end{bmatrix} \quad \boldsymbol{\Lambda}_m = \begin{bmatrix} 1 & 1 & 0 & 0 \\ \lambda_{g2.1} & \lambda_{s2.1} & 0 & 0 \\ \lambda_{g3.1} & \lambda_{s3.1} & 0 & 0 \\ \lambda_{g4.1} & 0 & 1 & 0 \\ \lambda_{g5.1} & 0 & \lambda_{s5.2} & 0 \\ \lambda_{g6.1} & 0 & \lambda_{s6.2} & 0 \\ \lambda_{g7.1} & 0 & 0 & 1 \\ \lambda_{g8.1} & 0 & 0 & \lambda_{s8.3} \\ \lambda_{g9.1} & 0 & 0 & \lambda_{s9.3} \end{bmatrix} \quad \boldsymbol{\eta} = \begin{bmatrix} \eta_{g1} \\ \eta_{s1} \\ \eta_{s2} \\ \eta_{s3} \end{bmatrix} \quad \boldsymbol{\epsilon} = \begin{bmatrix} \epsilon_1 \\ \epsilon_2 \\ \epsilon_3 \\ \epsilon_4 \\ \epsilon_5 \\ \epsilon_6 \\ \epsilon_7 \\ \epsilon_8 \\ \epsilon_9 \end{bmatrix}$$

Structural Model for Mediation

$$\text{X} \sim \text{N}(0,1) : \text{x} \geq \tilde{x}=1; \text{x} < \tilde{x}=0 \tag{5}$$

$$\eta_{s1} = \text{aX} + e_2 \tag{6}$$

$$\text{Y} = \text{c'X} + \text{b}\eta_{s1} + e_3 \tag{7}$$

$$\sigma^2_{\eta_{s1}} = 1 \tag{8}$$

$$\sigma^2_Y = 1 \tag{9}$$

$$\sigma_{e2e3} = 0 \tag{10}$$

$$\sigma_{\epsilon i\, \epsilon j} = 0 \text{ ; for } i \neq j \tag{11}$$

In this case, *M* is the mediator and it has nine indicators. *M* has a bifactor measurement structure, where the common variance among the indicators is explained by one *general* factor (η_g) and three *specific* factors (η_{s1}, η_{s2}, η_{s3}) influencing the indicators that represent the facets of the construct. The *general* factor and all of the *specific* factors are uncorrelated with each other. The identification of the bifactor measurement model is similar to Thoemmes et al., (2010) specification for Monte Carlo power analysis in Mplus. *X* is a binary experimental condition randomly assigned and was determined by a conditioning on $\tilde{x}$, which is the mean of the normal distribution ($\tilde{x}$ = 0; see Equation 5).

Finally, *Y* is a normally-distributed continuous outcome. The true mediated effect (*ab*) is the influence of *X* on outcome *Y* through the *specific* factor η_{s1}.

Simulation Procedure

The true covariance matrix for the distillation of the mediated effect was analytically derived with RAM matrices in Symbolic Python (SymPy; SymPy Developing Team, 2014) and presented in Appendix D. Population values were then generated corresponding to simulation conditions hypothesized to influence the detection of the mediated effect (explained at the beginning of the **Results** section).

The R package *MplusAutomation* (Hallquist & Wiley, 2013) was used to produce and analyze the Mplus syntax files in the study. Each syntax file represents a condition along with 1,000 replications of that condition. The data analysis models were estimated with maximum likelihood under the structural equation modeling framework. Monte Carlo datasets and estimated results were saved and processed by the R package *RMediation* (Tofigui & MacKinnon, 2011) to compute confidence intervals using the distribution of the product, Monte Carlo method, and asymptotic normal theory methods. Appendix E shows a flow chart with the simulation procedure steps. Appendix F, G, and H show an *MplusAutomation* template file, Mplus Monte Carlo syntax, and Mplus syntax for the analysis of each replication, respectively.

Parameter Bias

Three measures of bias were used to evaluate point estimation in the simulation study. Raw bias in the mediated effect estimate was calculated by the difference between the estimate and the population true value of the mediated effect.

$$Bias(\hat{\theta}) = \hat{\theta} - \theta \qquad (12)$$

Second, relative bias in the mediated effect was calculated by the difference between the estimate and the population true value, divided by the population true value.

$$RBias(\hat{\theta}) = \frac{\hat{\theta}-\theta}{\theta} \tag{13}$$

Finally, standardized bias in the mediated effect was calculated by the difference between the estimate and the population true value, divided by the standard deviation of the estimates across replications.

$$SBias(\hat{\theta}) = \frac{\hat{\theta}-\theta}{SD(\hat{\theta})} \tag{14}$$

Standardized bias gives a magnitude of bias when a population value is equal to zero, which is not possible to compute with Equation 14. An estimator was considered *unbiased* when the relative and standardized bias were less than .10 (Flora & Curran, 2004).

Statistical Power and Type 1 Error

Type 1 error rates were calculated by the proportion of times across all replications within a condition that a mediated effect estimate was statistically significant when the population value was zero. Power was the proportion of times across all replications within a condition that a mediated effect estimate was statistically significant when the population value was nonzero. The best estimator will have the highest power across simulation conditions.

Confidence Interval Estimation

Confidence interval coverage was the proportion of times across all replications within a condition that each confidence interval contains the true value of the mediated effect.

Distribution of the product. Asymmetric confidence intervals based on the non-normal distribution of the product of two random variables that represent the mediated effect (*ab*; Mackinnon et al., 2007) were computed.

Monte Carlo confidence intervals. To build Monte Carlo confidence intervals (MacKinnon et al., 2004), the *a*- and *b*-path estimates and their standard errors were used to generate a sampling distribution of *ab*, with the replication estimates as true values of the distribution. The lower and upper confidence limits for the mediated effect for each replication were the values in the sampling distribution in the 2.5% and 97.5% percentiles.

Asymptotic normal theory. The asymptotic normal confidence interval is $ab \pm 1.96 \times SE(ab)$, where $SE(ab)$ is the standard error of the mediated effect derived by the equation below:

$$\mathbf{SE(ab)} = \sqrt{\left(\boldsymbol{a}\left(\boldsymbol{SE}(\boldsymbol{b})\right)\right)^2 + \left(\boldsymbol{b}\left(\boldsymbol{SE}(\boldsymbol{a})\right)\right)^2 + 2\boldsymbol{ab}\boldsymbol{\rho}_{\boldsymbol{ab}}\boldsymbol{SE}(\boldsymbol{a})\boldsymbol{SE}(\boldsymbol{b}) + \boldsymbol{SE}(\boldsymbol{a})^2\boldsymbol{SE}(\boldsymbol{b})^2 + \boldsymbol{SE}(\boldsymbol{a})^2\boldsymbol{SE}(\boldsymbol{b})^2\boldsymbol{\rho}_{\boldsymbol{ab}}^2}\,. \tag{15}$$

Data Analysis Models

Finite model. The simulated datasets were analyzed using the true population model (**Model 1**; Figure 4) to get information about sample size bias, power, and Type 1 error in parameter estimates and confidence interval estimation.

Ignoring the general construct. The equations below estimate the *facet model* (**Model 2)**, where the mediator has a unidimensional structure and only the indicators of the true facet are modeled.

$$\boldsymbol{M} = \boldsymbol{\Lambda}_{\boldsymbol{m}}\boldsymbol{\eta} + \boldsymbol{\epsilon}, \quad where: \tag{16}$$

$$\boldsymbol{M} = \begin{bmatrix} M_1 \\ M_2 \\ M_3 \end{bmatrix} \quad \boldsymbol{\Lambda}_m = \begin{bmatrix} 1 \\ \lambda_{s2.1} \\ \lambda_{s3.1} \end{bmatrix} \boldsymbol{\eta} = [\eta_{s1}] \; \boldsymbol{\epsilon} = \begin{bmatrix} \epsilon_1 \\ \epsilon_2 \\ \epsilon_3 \end{bmatrix}$$

η_{s1}= aX + e_2 (17)

Y = c'X + bη_{s1} + e_3 (18)

For this model, the mediated effect is the influence of *X* on the outcome *Y* through the *specific* factor η_{s1} and calculated by the product of *ab*. The *a* parameter represents the effect of *X* on the *specific* factor η_{s1}. The *b* parameter represents the effect of η_{s1} on *Y*, adjusting for *X*. The effect of *X* on *Y*, adjusting for η_{s1}, is represented by the *c'* parameter.

Ignoring specific facets. The equations below estimate the *unidimensional model* (**Model 3)**, where the mediator is unidimensional and all indicators are included.

$\boldsymbol{M} = \boldsymbol{\Lambda_m \eta} + \boldsymbol{\epsilon}, \quad where$: (19)

$$\boldsymbol{M} = \begin{bmatrix} M_1 \\ M_2 \\ M_3 \\ M_4 \\ M_5 \\ M_6 \\ M_7 \\ M_8 \\ M_9 \end{bmatrix} \quad \boldsymbol{\Lambda}_m = \begin{bmatrix} 1 \\ \lambda_{g2} \\ \lambda_{g3} \\ \lambda_{g4} \\ \lambda_{g5} \\ \lambda_{g6} \\ \lambda_{g7} \\ \lambda_{g8} \\ \lambda_{g9} \end{bmatrix} \boldsymbol{\eta} = [\eta_g] \; \boldsymbol{\epsilon} = \begin{bmatrix} \epsilon_1 \\ \epsilon_2 \\ \epsilon_3 \\ \epsilon_4 \\ \epsilon_5 \\ \epsilon_6 \\ \epsilon_7 \\ \epsilon_8 \\ \epsilon_9 \end{bmatrix}$$

η_g= aX + e_2 (20)

Y = c'X + bη_g + e_3 (21)

For this model, the mediated effect is the influence of *X* on the outcome *Y* through the *general* factor η_g and calculated by the product *ab*. The *a* parameter represents the effect of *X* on the *general* factor η_g. The *b* parameter represents the effect of η_g on *Y*, adjusting for *X*. The *c'* parameter represents the effect of *X* on *Y*, adjusting for η_g.

With the same parameter interpretations as **Model 3**, the equations below estimate the *correlated factor model* (**Model 4),** where the mediator is unidimensional and the unique factors of indicators that measure the same facet are correlated, represented by $\boldsymbol{\Theta_\delta}$.

$$\boldsymbol{M} = \boldsymbol{\Lambda_m \eta} + \boldsymbol{\epsilon}, \quad where: \tag{22}$$

$$\boldsymbol{M} = \begin{bmatrix} M_1 \\ M_2 \\ M_3 \\ M_4 \\ M_5 \\ M_6 \\ M_7 \\ M_8 \\ M_9 \end{bmatrix} \; \boldsymbol{\Lambda}_m = \begin{bmatrix} 1 \\ \lambda_{g2} \\ \lambda_{g3} \\ \lambda_{g4} \\ \lambda_{g5} \\ \lambda_{g6} \\ \lambda_{g7} \\ \lambda_{g8} \\ \lambda_{g9} \end{bmatrix} \; \boldsymbol{\eta} = [\eta_g] \; \boldsymbol{\epsilon} = \begin{bmatrix} \epsilon_1 \\ \epsilon_2 \\ \epsilon_3 \\ \epsilon_4 \\ \epsilon_5 \\ \epsilon_6 \\ \epsilon_7 \\ \epsilon_8 \\ \epsilon_9 \end{bmatrix} \; and$$

$$\boldsymbol{\Sigma}(\boldsymbol{M}) = \boldsymbol{\Lambda_m \eta \Lambda'_m} + \boldsymbol{\Theta_\delta}, where{:}\, \boldsymbol{\Theta_\delta} = \begin{bmatrix} \delta_{11} & \delta_{12} & \delta_{13} & 0 & 0 & 0 & 0 & 0 & 0 \\ \delta_{21} & \delta_{22} & \delta_{23} & 0 & 0 & 0 & 0 & 0 & 0 \\ \delta_{31} & \delta_{32} & \delta_{33} & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \delta_{44} & \delta_{45} & \delta_{46} & 0 & 0 & 0 \\ 0 & 0 & 0 & \delta_{54} & \delta_{55} & \delta_{56} & 0 & 0 & 0 \\ 0 & 0 & 0 & \delta_{64} & \delta_{65} & \delta_{66} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & \delta_{77} & \delta_{78} & \delta_{79} \\ 0 & 0 & 0 & 0 & 0 & 0 & \delta_{87} & \delta_{88} & \delta_{89} \\ 0 & 0 & 0 & 0 & 0 & 0 & \delta_{97} & \delta_{98} & \delta_{99} \end{bmatrix}$$

$$\eta_g = aX + e_2 \tag{23}$$

$$Y = c'X + b\eta_g + e_3 \tag{24}$$

Results

Presentation Strategy

The results are organized as follows. First, simulation conditions are summarized. Second, convergence information is used to decide which conditions are analyzed. Third, fit information per model is reported. Fourth, the influence of the simulation factors on the bias, power, Type 1 error, and confidence interval coverage of the mediated effect are described for each of the four data-analysis models. Finally, simulation outcomes are compared across models.

Simulation Conditions

There were 864 conditions (with 1,000 replications per condition) examined under four data-analysis models. The simulation factors are summarized in Table A below and were sample size (*small*=200, *medium*=500, *large*=1,000); factor loadings on the *general* factor, referred to as general factor variance (*small*=.3, *medium*=.5, *large*= .7); factor loading on the *specific* factor, referred to as specific factor variance (*small*=.3, *medium*=.45, *large*=.6); *a*-path effect size (*zero, small*, *medium*, *large*); *b*-path effect size (*zero, small*, *medium*, *large*); and *c'*-path effect size (*zero*, *small*). The label for the sizes of the simulation conditions (*zero, small*, *medium*, *large*), the model labels (*finite, correlated, unidimensional*, and *facet*), the label *simulation factors* to refer to the set of predictors, and the hypotheses numbers are used through this section.

Table A. *Summary of Simulation Factors*

Symbol	***Interpretation***	***Simulated Values***	***Levels***
n	Sample size	200, 500, 1000	3
a-path	Effect size of *a*-path (zero, small, medium, large)	0, .28, .72, 1.02	4
b-path	Effect size of *b*-path (zero, small, medium, large)	0, .14, .36, .51	4
c'-path	Effect size of the direct effect (zero, small)	0, .283	2
gen	Factor loading on general factor	.3, .5, .7	3
spec	Factor loading on specific factor	.3, .45, .6	3

Convergence Statistics

There were differences in convergence rates across the 864,000 estimated models (864 conditions times 1000 replications). Replications did not converge when they had a non-positive covariance matrix or Mplus default iteration limit of 1,000 iterations was exceeded. Nonconverged replications were dropped from the analysis.

Table B. *Convergence summary for data-analysis models*

	Finite Factor	Corr Factor	Facet Factor	Unidim Factor
Initial Replications	864,000	864,000	864,000	864,000
Convergence	835,097	577,643	852,574	863,264
Non-convergence	28,903	286,357	11,426	736
Non-convergence %	3.34	33.14	1.32	0.08

Nonconvergence was investigated as a function of the simulation conditions and described below. Conditions with less than a 70% convergence rate were *problematic conditions* and excluded from analyses. There were 357 *problematic conditions* out of 3,456 total conditions (864 conditions times four models) in the simulation.

Unidimensional model. There were no *problematic conditions* analyzed with the *unidimensional* model. The 736 nonconverged replications were dropped from the simulation.

Facet model. There were no *problematic conditions* analyzed with the *facet* model. The 11,426 nonconverged replications were dropped from the simulation.

Finite model. There were eight *problematic conditions* analyzed with the *finite* model dropped from the simulation. Six of those conditions had a small sample size, small general factor variance, small specific factor variance, a zero effect on the *a*-path, and a zero or small effect on the *b*-path. The other two conditions had a medium general factor variance, a small specific factor variance, small sample size, and zero effects for the *a*- and *b*-paths. The nonconvergence patterns are consistent with **Hypothesis 2**, which hypothesized that a combination of small sample sizes and low factor loadings on the *specific* factor would have convergence problems. The 28,903 nonconverged replications were also dropped from the simulation.

Correlated factor model. There were 349 *problematic conditions* analyzed with the *correlated* factor model dropped from the simulation. Nonconvergence rates are discussed below. The 286,357 nonconverged replications were dropped from the simulation.

Small general factor variance. There were 224 *problematic conditions.* Shaded cells in Table C indicate nonconverged conditions, averaged over the *c'*-path. Most *problematic conditions* had a medium or large *b*-path effect size or a medium or large *a*-path effect size with a zero or small *b*-path (144 and 72 conditions, respectively).

Table C. *Convergence table for the* Correlated Factor *model with small general factor variance*

	Spec loading	s-.3			s-.45			s-.6		
	N	200	500	1000	200	500	1000	200	500	1000
a-zero	b-zero									
	b-small									
	b-med									
	b-large									
a-small	b-zero									
	b-small									
	b-med									
	b-large									
a-med	b-zero									
	b-small									
	b-med									
	b-large									
a-large	b-zero									
	b-small									
	b-med									
	b-large									

Medium general factor variance. There were 120 *problematic conditions*. Shaded cells in Table D indicate nonconverged conditions, averaged over the *c'*-path.

Most *problematic conditions* had a large *b*-path or a large *a*-path and a medium/large specific factor variance (48 conditions and 36 conditions, respectively).

Table D. *Convergence table for the* Correlated Factor *model with medium general factor variance*

	Spec loading	s-.3			s-.45			s-.6		
	Sample Size	200	500	1000	200	500	1000	200	500	1000
a-zero	b-zero									
	b-small									
	b-med									
	b-large									
a-small	b-zero									
	b-small									
	b-med							s		
	b-large	s								
a-med	b-zero									
	b-small									
	b-med									
	b-large									
a-large	b-zero									
	b-small									
	b-med									
	b-large									

Note: split cells with the letter "s" indicate that only the conditions with a small c' path did not converge

Large general factor loading. There were five *problematic conditions* that had a large specific factor variance and a combination of large *a*- and *b*-path effect sizes.

Summary for the convergence statistics. The *unidimensional* and *facet factor* models do not have *problematic conditions*. The *finite factor* model has problems converging when the model structure is the weakest, i.e., small general and specific factor variance, mediation paths of zero to small effect size, and small sample size. Finally, the *correlated factor* model has *problematic conditions* when the *a*- and *b*-path effects are large and when the general factor variance is not large. The correlated errors capture the *specific* factor variance, leading to negative residuals in the indicators when *X* predicts the mediator, which pulls the factor. Table E shows the number of replications and conditions in the simulation retained for analysis and differs from Table B above because *problematic conditions* have been dropped, reducing the converged replications.

Table E. *Convergence summary after problematic conditions have been dropped*

	Finite Factor	Corr Factor	Facet Factor	Unidim Factor
Convergence	829,763	493,994	852,574	863,264
Non-convergence	26,237	21,006	11,426	736
Conditions post-deletion	856	515	864	864

Model Fit

The RMSEA, the SRMR, and the CFI were used to evaluate how well the models represent the data (Hoyle, 2012). Conventional thresholds for the RMSEA and SRMR are .05 for *perfect* fit and .08 for *adequate* fit. The thresholds for the CFI are .95 and .90, respectively. Table F shows the number of times each of the fit indices per replication were below the thresholds.

Table F. Fit indices for data-analysis models

	Finite Factor	Corr Factor	Facet Factor	Unidim Factor
RMSEA.05	824,884	302,626	655,309	599
	99.4%	61.3%	76.9%	0.06%
RMSEA.08	829,763	451,339	775,406	53,596
	100%	91.4%	90.9%	6.2%
SRMR.05	789,664	404,823	845,642	1,911
	95.1%	81.9%	99.2%	0.22%
SRMR.08	829,610	493,049	852,569	271,441
	99.9%	99.8%	99.9%	31.44%
CFI.95	824,213	462,324	829,048	48
	99.3%	93.6%	97.2%	0.01%
CFI.90	829,302	493,336	851,039	897
	99.9%	99.9%	99.8%	0.10%

The *finite factor* model fits the datasets perfectly across replications. The *correlated* and the *facet factor* models fit the data well given *adequate* fit criteria. The *unidimensional* model had the poorest model fit because it does not account for the multidimensionality of the mediator. The *unidimensional factor* model is nested under the *correlated factor* model, so the influence of the correlated uniqueness on fit is described by the percentage change across replications, where the RMSEA increased by 60%, SRMR by 80%, and the CFI by 92%.

Unidimensional model. Shaded cells in Table G describe the conditions where less than 70% of the replications did not meet the adequate fit criterion, averaged over the *c'*-path. There was *adequate* fit in conditions with a small specific factor variance. As the general factor variance and the *a*- and *b*-paths increase, the model fits worse because not enough of the true mediator variance is represented by the *unidimensional* model.

Table G. Unidimensional *models where fit indices did not suggest adequate fit 70% of the time*

	Gen loading	g-.3			g-.5			g-.7		
	Sample Size	200	500	1000	200	500	1000	200	500	1000
a-zero	b-zero									
	b-small									
	b-med							■		
	b-large				■	■		■	■	
a-small	b-zero									
	b-small									
	b-med				z ■			■		
	b-large				■	■	■	■	■	s ■
a-med	b-zero							■		
	b-small							■		
	b-med				■	■	s ■	■	■	s ■
	b-large				■	■	■	■	■	■
a-large	b-zero				■	■		■	■	
	b-small				■	■	s ■	■	■	
	b-med				■	■	■	■	■	■
	b-large	■			■	■	■	■	■	■

Note: split cells with the letter *s* indicates small c' path and letter *z* indicates zero c' path

Overview of the Analyses of Simulation Outcomes

To assess which factors were associated with simulation outcomes in the mediated effect, OLS regression analyses were conducted for continuous outcomes and logistic regressions for binary outcomes. Models with at least a small effect for both the *a*-path and *b*-path in the data-generating model are referred to as *models with nonzero mediated effects,* and models where either the *a*- or *b*-path (or both) had a zero effect are referred to as *models with zero mediated effects*. Unless specified, all simulation factors were dummy-coded and included in the regression model along with all possible interactions. Given that factors have three levels, interactions with the largest magnitude of the factor are reported. For OLS regression analyses, models with an R-squared value above .01 and a partial η^2 above .005 for a predictor were further investigated. For logistic regression analyses, statistically significant predictors with at least a small effect size in the transformation of odd ratios into Cohen's d (Chinn, 2000) were investigated. A log odds ratio of .362 (OR= 1.44) represents a small effect size, a log odds ratio of .905 (OR= 2.47) represents a medium effect size, and a log odds ratio of 1.448 (OR=4.25) represents a large effect size. The transformation equation is shown below:

$$Cohen's\ d = \frac{\ln(Odds\ Ratio)}{\frac{\pi}{\sqrt{3}}} \tag{25}$$

Bias in the mediated effect. Raw, relative, and standardized bias were used as continuous outcomes in *models with nonzero mediated effects*. Preliminary analyses indicate that the mediated effect in *models with zero mediated effects* were *unbiased* and simulation factors do not account for variance. These results are not presented. Tables are provided for relative and standardized bias in *models with nonzero mediated effects* because of the interpretable metric.

Power to detect the mediated effect. The binary outcome variable for power in *models with nonzero mediated effects* was coded 1 if the 95% confidence interval did not contain zero or 0 otherwise. Statistical power above .80 was considered *adequate*.

Methods to assess statistical power. Differences in power by method were used to understand the best method to detect the mediated effect in the *finite factor* model. The power difference per condition through the distribution of the product method and the Monte Carlo method was never more than .01. According to Table 6-1 to Table 6-3, power of the asymptotic normal theory confidence intervals was never higher than the power from the distribution of the product confidence intervals, with differences up to .30. The discrepancy in the power decreased as the simulation factors increased. Only the distribution of the product method was used for the analyses of statistical power, Type 1 error, and confidence interval coverage.

- Insert Table 6-1 to Table 6-3 about here-

Type 1 error. The binary outcome variable for empirical Type 1 error in *models with zero mediated effects* was coded 1 if the 95% confidence interval did not contain zero or 0 otherwise. Type 1 error rates between .025 and .075 (Bradley, 1978) were considered *adequate*.

Confidence interval coverage and interval width. The binary outcome variable for coverage was coded as a 1 if the 95% confidence interval contained the true value of the mediated effect, and coded 0 otherwise. Coverage rates between .925 and .975 were considered *adequate*. Coverage in *models with nonzero mediated effects* need to be interpreted with caution due to estimate bias in the misspecified models. The interval width outcome was the difference between the lower and upper confidence interval limit. Smaller width suggests more precision.

Comparisons across analysis models. For power and confidence interval coverage, a binary indicator that indexed discrepancies in conclusions on the mediated effect per replication was used as a dependent variable in a logistic regression. Standardized bias, relative bias, and Type 1 error rate were compared across models through summary tables. Comparing simulation outcomes in misspecified models need to be with caution because the mediated effects across models are theoretically different.

Analysis of the Finite Factor Model

Bias in the finite model. The bias in the mediated effect decreased as sample size, the *a*- and *b*-paths, and the specific and general factor variance increased, supporting Hypotheses **1.1**, **1.2**, and **1.3**. Conditions with 500 cases, medium general factor variance and medium *a*- and *b*-paths were *unbiased*. Detailed analyses of bias outcomes are found below.

***Standardized bias in the finite model with nonzero mediated effects*.** Table 1A-1 to Table A1-3 show that conditions with low sample size, general factor variance, specific factor variance, and large *a*- or *b*-paths had standardized bias above .10. The variance explained in the regression predicting standardized bias from the simulation factors was $R^2 = .003$. Figure 1A-1 suggests that standardized bias decreased as the specific factor variance, general factor variance, and sample size increased.

- Insert Table 1A-1 to Table 1A-3 and Figure 1A-1 about here-

***Raw bias in finite model with nonzero mediated effects*.** The variance explained by the regression predicting raw bias from the simulation factors was $R^2 = .021$. As sample size increased, the raw bias in the mediated effect decreased (b=-.022, t=-4.972, p<.05, partial η^2=.006), supporting **Hypothesis 1.1**. Figure 1A-2 shows that raw bias decreased as sample size, general factor variance, and specific factor variance increased. Raw bias increased as the *a*- and *b*-paths increased.

- Insert Figure 1A-2 about here-

Relative bias in the finite model with nonzero mediated effects. Table 1A-4 to Table 1A-6 show that conditions with small sample size, specific factor variance, and general factor variance had relative bias above .10. The variance explained by the regression predicting relative bias from the simulation factors was R^2= .014. No predictors met the η^2 criterion. Figure 1A-3 shows that relative bias decreased as simulation factors increased.

- Insert Table 1A-4 to Table 1A-6 and Figure 1A-3 about here-

Power in the finite model. The power to detect the mediated effect increased as sample size, the *a*- and *b*-paths, and the specific and general factor variance increased. Conditions with 500 cases, medium general factor variance and medium *a*- and *b*-paths were *adequately* powered (Table 1B-1 to Table 1B-3 and Figure 1B-1). Power was assessed for conditions with a sample size of 200. Detailed analyses of the power outcome are found below.

-Insert Table 1B-1 to Table 1B-3 and Figure 1B-1 about here-

Power to detect the mediated effect with a small sample size. Figure 1B-2 shows that power increased as the simulation factors increased. There was a significant interaction among all of the predictors (large effect size; b=2.352, z=3.803, p<.05). Power increased faster for conditions with larger *a*- and *b*-paths and larger general and specific factor variances than with smaller *a*- and *b*-paths and smaller general and specific factor variances. Power increased as the general factor variance ($\chi^2(2, N=152,659) = 50.42$, p<.05), specific factor variance ($\chi^2(2, N=152,659) = 98.53$, p<.05), *a*-path ($\chi^2(2, N=152{,}659) = 14.148$, $p<.05$), and *b*-path ($\chi^2(2, N=152{,}659) = 249.145$, $p<.05$) increased, supporting **Hypotheses 1.4, 1.5** and **1.6**.

- Insert Figure 1B-2 about here-

Type 1 error in the finite model. The Type 1 error in the mediated effect approached .05 as sample size, the *a*- or *b*-path, and the specific and general factor variance increased. Type 1 error was *adequate* for conditions with a medium *a*- or *b*-path. Detailed analyses follow.

Type 1 error in the mediated effect. As show in Table 1C-1 to Table 1C-3, conditions where the *a*- and *b*-path had a zero effect had empirical Type 1 errors close to zero. Conditions analyzed had one nonzero *a*- or *b*-path and sample size at or above 500.

- Insert Table 1C-1 to Table 1C-3 about here-

Nonzero effect size in the a- and b-path: Figure 1C-1 and Figure 1C-2 show that the Type 1 error approached .05 as the simulation factors increased. For conditions with a nonzero *a*-path, there was a significant interaction between the general and specific factor variance and the *a*-path (large effect size; $b=1.912$, $z=-2.375$, $p<.05$). The Type 1 error approached .05 faster as the *a*-path increased for conditions with smaller general and specific factor variance than conditions with larger general and specific factor variance. Type 1 errors approached .05 as sample size ($\chi^2(1, N=140{,}596) = 31.228$, $p<.05$), general factor variance ($\chi^2(2, N=140{,}596) = 39.469$, $p<.05$), specific factor variance ($\chi^2(2, N=140{,}596) = 39.469$, $p<.05$), and the *a*-path ($\chi^2(2, N=140{,}596) = 18.888$, $p<.05$) increased. For conditions with a nonzero *b*-path, there were significant interactions among the *b*-path and the general and specific factor variance (medium effect size; $b=.943$, $z=-1.972$, $p<.05$) and among the *b*-path, sample size, and general factor variance (medium effect size; b=.968, z=2.041, p<.05). Type 1 errors approached .05 faster as the specific factor variance, general factor variance, and sample size increased for smaller *b*-paths than for larger *b*-paths. Type 1 errors approached .05 as sample size (χ^2(1, N=140,426) = 18.930, p<.05), general factor variance (χ^2(2, N=140,426) = 11.347, p<.05), specific factor variance (χ^2(2, N=140,426) = 18.858, p<.05), and the *b*-path (χ^2(2, N=140,426) = 57.199, p<.05) increased, supporting **Hypotheses 1.4, 1.5** and **1.6**.

-Insert Figure 1C-1 and Figure 1C-2 about here-

Coverage and interval width in the finite model. The 95% confidence interval coverage of mediated effect approached .95 as sample size, the *a*- and *b*-paths, and the specific and general factor variance increased. Conditions with 500 cases, medium *a*- and *b*-paths, and medium general factor variance were *adequately* covered. Detailed analyses are found below.

Confidence interval coverage of the mediated effect. Table 1D-1 to Table 1D-3 show that the confidence interval coverage was mostly *adequate*. Ten conditions with small sample size were outside of the robust criterion of coverage. Figure 1D-1 shows that coverage approached .95 as simulation factors increased. There was a significant interaction between sample size, general factor variance, specific factor variance, and the *b*-path (medium effect size; b= 1.172, z=2.283, p<.05). Coverage approached .95 faster as the general factor variance increased for conditions with a larger specific factor variance, sample size, and *b*-path, than for conditions with smaller specific factor variance, sample size and *b*-path. Coverage approached .95 as sample size (χ^2(2, N= 474,754) = 50.634, p<.05), specific factor variance (χ^2(2, N= 474,754) = 38.040, p<.05), *a*-path (χ^2(2, N= 474,754) = 24.853, p<.05), *b*-path (χ^2(2, N= 474,754) =26.070, p<.05) and general factor variance ($\chi^2(2, N= 474{,}754) = 13.141$, $p<.05$) increased, supporting **Hypotheses 1.4**, **1.5**, and **1.6**.

Confidence interval width. Figure 1E-1 shows that confidence interval width for the mediated effect decreased as the sample size, general and specific factor variance increased, and as the *a*- and *b*-path decreased. The variance explained by the regression predicting interval width from the simulation outcomes was $R^2= .027$. There was a significant interaction among all of the predictors (b=.648, z=3.110, $p<.05$, partial η^2 =.008). The interval width increased at a faster rate as the *a*- and *b*-path increased for conditions with smaller sample size, specific and general factor variance than for larger sample size, specific and general factor variance

-Insert Table 1D-1 to Table 1D-3 and Figure 1D-1 to Figure 1E-1 about here-

Summary of the finite factor model. The mediated effect in models with a bifactor mediator structure (**Model 1**) is *unbiased*, *adequately* powered, and *covered* by 95% confidence intervals in conditions with 500 cases, medium general factor variance and medium *a*- and *b*-paths. The Type 1 error approached .05 when one of the paths had a medium effect size. For the other conditions, as the simulation factors increased, bias decreased, power increased, coverage approached .95, and Type 1 error approached .05. Finally, the model does not converge with a sample size of 200, zero or small *a*- and *b*-paths, and small specific and general factor variance.

Analysis of the Facet Factor Model

Bias in the facet model. The mediated effect was underestimated. Bias decreased as the specific factor variance increased and as sample size, the *a*- and *b*-paths, and the general factor variance decreased. Conditions with large specific factor variance and small general factor variance, *a*- and *b*-paths and sample size had the least bias. Detailed analyses are found below.

***Standardized bias in the facet model with nonzero mediated effects*.** Table 2A-1 to Table 2A-3 and Figure 2A-1 show that standardized bias decreased as the specific factor variance decreased and other simulation factors increased. The variance explained by the regression predicting standardized bias from the simulation factors was R^2 = .856. There were significant interactions between sample size and the *a*- and *b*-paths (b=-2.313, t=-35.155, p<.05, partial η^2= .04) and between the general factor variance and the *a*- and *b*-paths (b=-1.256, t=-17.086, p<.05, partial η^2= .026). Standardized bias increased faster as sample size and general factor variance increased for conditions with larger *a*- and *b*-paths than smaller *a*- and *b*-paths, supporting **Hypotheses 3.2**, and **3.3**.

-Insert Table 2A-1 to Table 2A-3 and Figure 2A-1 about here-

***Raw bias in the facet model with nonzero mediated effects*.** Figure 2A-2 shows that raw bias decreased as the specific factor variance increased, as the general factor variance and *a*- and *b*-paths decreased, and not influenced by sample size. The variance explained by the regression predicting raw bias from the simulation factors was R^2 = .822. There was a significant interaction between the general factor variance and the *a*- and *b*-paths (b=-.031, t=-11.281, p<.05, partial η^2=.006). The raw bias increased as the general factor variance increased faster for conditions with larger *a*- and *b*-paths than for smaller *a*- and *b*-paths, supporting **Hypothesis 3.2**.

-Insert Figure 2A-2 about here-

***Relative bias in the facet model with nonzero mediated effects*.** Table 2A-4 to Table 2A-6 and Figure 2A-5 show that relative bias decreased as the general factor variance and *a*-path decreased and the specific factor variance increased. The variance explained by the regression predicting relative bias from the simulation factors was R^2 = .11. Relative bias decreased as the specific factor variance increased (b=.097, t=10.683, p<.05, partial η^2=.033), and as the *a*-path (b=-.116, t=-12.306, p<.05, partial η^2= .012) and general factor variance (b=-.229, t=-25.069, p<.05, partial η^2=.070) decreased, supporting **Hypotheses 3.2** and **3.3**.

-Insert Table 2A-4 to Table 2A-6 and Figure 2A-3 about here-

Power in the facet model. The power to detect the mediated effect increased as sample size, the *a*- and *b*-paths, and the specific factor variance increased, and as the general factor variance decreased, supporting **Hypotheses 3.4**, **3.5**, and **3.6**. Conditions with 500 cases and medium *a*- and *b*-paths were *adequately* powered (Tables 2B-1 to Table 2B-3). Power was assessed for conditions with a sample size of 200. Detailed analyses are found below.

Power to detect the mediated effect with a small sample size. Figure 2B-1 shows that power was not influenced by the general factor variance, and increased as the other simulation factors increased. There was a significant interaction among all the predictors (medium effect size; b=-1.269, z=-2.787, p<.05). Power increased as the specific factor variance increased faster for conditions with smaller *a*- and *b*-paths than with larger *a*- and *b*-paths. Power increased as the *b*-path (χ^2(2, N=157,074) = 328.93, p<.05), specific factor variance (χ^2(2, N=157,074) = 38.55, p<.05), and *a*-path (χ^2(2, N=157,074) = 94.04, p<.05) increased, but power did not significantly increase as the general factor variance increased (χ^2(2, N=157,074) = 3.29, p=.19). Evidence supports **Hypotheses 3.4** and **3.6**.

-Insert Table 2B-1 to Table 2B-3 and Figure 2B-3 about here-

Type 1 error in the facet model. The Type 1 error in the mediated effect approached .05 as sample size, the *a*- or *b*-path, and the specific factor variance increased, and not influenced by the general factor variance, supporting **Hypotheses 3.4** and **3.6**. Type 1 errors were *adequate* for conditions with a medium *a*- or *b*-path (Table 2C-1 to Table 2C-3). Conditions analyzed had a nonzero *a*- or *b*-path and a zero effect in the other path. Detailed analyses are found below.

Type 1 error in the mediated effect for nonzero a- or b-paths. Figure 2C-1 and Figure 2C-2 shows that the Type 1 error approached .05 as the simulation factors increased. For conditions with a nonzero *a*-path, there was a significant interaction between the general factor variance, specific factor variance, and sample size (large effect size; b= 1.449, z=2.126, p<.05). Type 1 error approached .05 faster as the sample size increased for conditions with a smaller general and specific factor variance than for a larger general and specific factor variance. Also, as the *a*-path increased, Type 1 error approached .05 (large effect size; b=2.540, z= 4.890, p<.05). Type 1 error rate approached .05 as sample size (χ^2(2, N=159,830) = 53.313, p<.05), specific factor variance (χ^2(2, N=159,830) = 12.209, p<.05), and *a*-path (χ^2(2, N=159,830) = 47.828, p<.05) increased, and was not significantly influenced by the general factor variance (χ^2(2, N=159,830) = 5.629, p=.06). For conditions with a nonzero *b*-path, there was a significant interaction between sample size and the *b*-path (large effect size; b= -1.822, z=-4.371, p<.05). Type 1 error approached .05 faster as the sample size increased for conditions with a smaller *b*-path than for a larger *b*-path. Type 1 error approached .05 as sample size (χ^2(2, N=159,706) = 27.874, p<.05) and *b*-path (χ^2(2, N=159,706) = 62.810,

$p<.05$) increased, and was not significantly influenced by the general factor variance ($\chi^2(2, N=159{,}706) = 1.217$, $p=.54$) and specific factor variance ($\chi^2(2, N=159{,}706) = 2.983$, $p=.23$). Analyses supported **Hypotheses 3.4** and **3.6**.

-Insert Table 2C-1 to Table 2C-3 and Figure 2C-1 and Figure 2C-2 about here-

Coverage and interval width in the facet model. The 95% confidence interval coverage of the mediated effect approached .95 as the specific factor variance increased and as the sample size, the *a*- and *b*-paths, and general factor variance decreased, supporting **Hypotheses 3.5** and **3.6**. Conditions with large specific factor variance and small general factor variance, *a*- and *b*-paths, and sample size had coverage closest to .95. Detailed analyses follow.

Confidence interval coverage of the mediated effect. Table 2D-1 to Table 2D-3 show that all conditions had coverage rates below 92.5%. Conditions with a medium or large *a*- and *b*-path had zero coverage. Confidence interval coverage was assessed for small sample size conditions. Figure 2D-1 shows that coverage approached .95 as the specific factor variance increased, but decreased as the general factor variance and the *a*- and *b*-paths increased. There was a significant interaction between the general factor variance and the *a*- and *b*-paths (large effect size; $b= -2.322$, $z=-7.747$, $p<.05$). As the general factor variance decreased, the confidence interval coverage approached .95 at a faster rate for conditions with larger *a*- and *b*-paths than with smaller *a*- and *b*-paths. Coverage approached .95 as the general factor variance ($\chi^2(2, N= 157{,}074) = 113.312$, $p<.05$), *a*-path ($\chi^2(2, N= 157{,}074) = 69.962$, $p<.05$), and *b*-path ($\chi^2(2, N= 157{,}074) =28.913$, $p<.05$) decreased, and was not influenced by the specific factor variance ($\chi^2(2, N= 157{,}074) = 2.254$, $p=.28$), supporting **Hypothesis 3.5**.

Confidence interval width. Figure 2E-1 shows that the confidence interval width increased as the *a*- and *b*-paths and the general factor variance increased and not influenced by the specific factor variance. The variance accounted for by the regression predicting confidence interval width from the simulation factors was $R^2 = .614$. There was a significant interaction between the *a*- and *b*-path (b=.648, t=.311, p<.05, partial η^2=.064). As the *a*-path increased, the interval width increased faster for conditions with smaller *b*-paths than for larger *b*-paths.

-Insert Table 2D-1 to Table 2D-3 and Figure 2D-1 to Figure 2E-1 about here-

Summary of the facet factor model. When the bifactor model is misspecified by ignoring the *general* construct (**Model 2**), the mediated effect is underestimated and has confidence interval coverage below .95. Conditions with a large specific factor variance and small general factor variance, *a*- and *b*-paths, and sample size have the least bias and the highest coverage. Bias decreased and coverage approached .95 as the specific factor variance increased and the rest of the simulation factors decreased. Conditions with 500 cases and medium *a*- and *b*-paths for models with *zero* and *nonzero mediated effects* had *adequate* power and Type 1 error rates. Conditions approached *adequate* power and Type 1 error rates as the simulation factors increased, except for the general factor variance. All models met the *adequate* fit criteria

Analysis of the Unidimensional Model

As previously mentioned, only conditions with a small specific factor variance and *a*- and *b*-path less than a large size were analyzed for the *unidimensional* model.

Bias in the unidimensional model. The mediated effect was negatively biased (Table 3A-1 and Table 3A-2). Bias decreased as the general factor variance, sample size, and the *a*- and *b*-paths decreased, supporting **Hypothesis 3.2**. Conditions with large general factor variance, *a*- and *b*-paths, and sample size had the least bias. Detailed analyses are found below.

Standardized bias in the unidimensional model with nonzero mediated effects. Figure 3A-1 shows that standardized bias increased as the simulation factors increased. The variance explained by the regression predicting standardized bias from simulation factors was $R^2 = .976$. There was a significant interaction among all of the predictors (b=-5.524, t=-59.506, p<.05, partial η^2=.051). As the general factor variance increased, standardized bias increased faster for conditions with a larger sample size and *a*- and *b*-paths than with a smaller sample size and *a*- and *b*-paths. Evidence supports **Hypothesis 4.2**.

-Insert Table 3A-1 and Figure 3A-1 about here-

Raw bias in the unidimensional model with nonzero mediated effects. Figure 3A-2 shows that raw bias decreased as the simulation factors increased, except for sample size. The variance explained by the regression predicting raw bias from the simulation factors was $R^2 = .908$. There was a significant interaction among the general factor variance and the *a*- and *b*-path (b=-.028, t=-18.567, p<.05, partial η^2 = .019). As the general factor variance increased, raw bias increased faster for conditions with larger *a*- and *b*-paths than for smaller *a*- and *b*-paths, supporting **Hypothesis 4.2**.

-Insert Figure 3A-2 about here-

Relative bias in the unidimensional model with nonzero mediated effects. Figure 3A-3 show that relative bias decreased as the general factor decreased. The variance explained by the regression predicting relative bias from the simulation factors was R^2 =

.219. There was a significant main effect of the general factor variance on relative bias (b=-.279, t=-35.957, p<.05, partial η^2=.218). As the general factor variance increased, relative bias increased, supporting **Hypothesis 4.2**.

-Insert Table 3A-2 and Figure 3A-3 about here-

Power in the unidimensional model. The power to detect the mediated effect increased as sample size and the *a*- and *b*-paths increased, and as the general factor variance decreased (Figure 3B-1). Conditions with 1,000 cases, medium *a*- and *b*-paths, and small general factor variance were *adequately* powered (Table 3B-1). Detailed analyses are found below.

Power in the mediated effect. There was a significant interaction among all the simulation factors (large effect size; b=-3.737, z=-4.547, p<.05). As sample size increased, power increased faster for conditions with a smaller general factor variance and large *a*- and *b*-paths than larger general factor variance and smaller *a*- and *b*-paths. Power increased as the *b*-path (χ^2(1, N=69,022) = 147.18, p<.05), sample size (χ^2(2, N=69,022) = 751.18, p<.05), and *a*-path (χ^2(2, N=69,022) = 82.830, p<.05) increased, but power decreased as the general factor variance increased (χ^2(2, N=69,022) = 15.800, p<.05), supporting **Hypotheses 4.4, 4.5,** and **4.6**.

-Insert Figure 3B-1 and Table 3B-1 about here-

Type 1 error in the unidimensional model. The Type 1 error in the mediated effect approached .05 as the *a*- or *b*-path and sample size increased, and general factor variance decreased (Figure 3C-1 to Figure 3C-2) supporting **Hypotheses 3.4, 3.5,** and **3.6**. Type 1 errors were *adequate* for conditions with a medium *a*- or *b*-path, a sample size of 500, and small general factor variance (Table 2C-1). Conditions analyzed had a nonzero *a*- or *b*-path and a zero effect in the other path. Detailed analyses are found below.

Type 1 error in the mediated effect. Table 3C-1 shows that all conditions have Type 1 errors below .05; conditions with zero *a*- and *b*-paths had Type 1 errors close to zero. Conditions analyzed had a nonzero *a*- or *b*-path and a zero effect in the other path. For conditions with a nonzero *a*-path, there was a significant interaction among the predictors (large effect size; b=2.847, z=3.058, p<.05). Type 1 error approached .05 faster as the general factor variance increased for conditions with smaller *a*-paths and larger sample size than for larger *a*-paths and smaller sample size. Type 1 error approached .05 as the *a*-path (χ^2(1, N=35,155) = 83.211, p<.05) and sample size (χ^2(2, N=35,155) = 20.218, p<.05) increased. The general factor variance (χ^2(2, N=35,155) = 1.356, p=.508) did not influence Type 1 error rates. For conditions with a nonzero *b*-path, Type 1 error approached .05 as the *b*-path increased (large effect size; b=3.858, z=4.416, p<.05). Type 1 error approached .05 as the *b*-path increased (χ^2(3, N= 35,153) = 134.117, p<.05), but sample size (χ^2(2, N= 35,153) = 4.320, p=.116) and general factor variance (χ^2(2, N= 35,153) = 0.006, p=.997) did not influence Type 1 error.

-Insert Figure 3C-1 to Figure 3C-2 and Table 3C-1 about here-

Coverage and interval width in the unidimensional model. The 95% confidence interval coverage of mediated effect approached .95 as the general factor variance, sample size and the *a*- and *b*-paths decreased. Conditions with small general factor variance, small *a*- and *b*-paths, and small sample size had coverage closest to .95. (Table 3D-1). Detailed analyses are found below.

Confidence interval coverage for the mediated effect. Table 3D-1 shows that all conditions had a coverage rate below 92.5 %; conditions with medium or large sample sizes had zero coverage. Confidence interval coverage was assessed for conditions with 200 cases. As shown in Figure 3D-1, coverage approached .95 as sample size and general factor variance decreased. There was a significant interaction between the general factor variance and *a*- and *b*-paths (large effect size; b=-2.661, z=-2.636, p<.05). As the general factor variance decreased, coverage approached .95 faster for conditions with a small *a*- and *b*-paths than for conditions with a large *a*- and *b*-paths. Confidence interval coverage approached .95 as the general factor variance ($\chi^2(2, N= 21{,}092) = 488.89$, p<.05), the *a*-path ($\chi^2(1, N= 21{,}092) = 141.98$, p<.05), and the *b*-path ($\chi^2(1, N= 21{,}092) =144.01$, p<.05) decreased, supporting **Hypothesis 4.5**.

*Confidence interval width***.** Figure 3E-1 shows that interval width decreased as sample size and general factor variance increased and as the *a*- and *b*-paths decreased. The variance explained by the regression predicting interval width from the simulation factors was $R^2 = .754$. There was a significant interaction between the *a*-path and the general factor variance (b=-0.013, t=-10.890, p<.05, partial η^2=.051). As the general factor variance increased, the interval width decreased faster for conditions with a smaller *a*-path than for a larger *a*-path.

-Insert Table 3D-1 and Figure 3D-1 to Figure 3E-1 about here-

Summary of the unidimensional model. When the bifactor model is misspecified by only modeling one dimension (**Model 3**), only conditions with a small specific factor variance *adequately* fit the data. The mediated effect is negatively biased and has coverage below .95. Conditions with a small sample size, general factor variance and *a*- and *b*- paths had the least bias and highest coverage. Bias increased and coverage decreased as the rest of the simulation factors increased. Also, conditions with 500 cases, small general factor variance, and medium *a*- and *b*-paths for models with *zero* and *nonzero mediated effects* had *adequate* power and Type 1 errors. Other conditions approached *adequate* power and Type 1 errors as the *a*- and *b*-path increased. Power also increased as sample size increased and general factor variance decreased.

Analysis of the Correlated Factor Model

As previously mentioned, only conditions with a large general factor variance, except conditions with a large *b*-path, were analyzed for the *correlated* model.

Bias in the correlated factor model. The mediated effect was negatively biased (Table 4A-1 and Table 4A-2). Bias decreased as the specific factor variance increased and as sample size and the *a*- and *b*-paths decreased. Conditions with large specific factor variance and small *a*- and *b*-paths and small sample size had the least bias. Detailed analyses are found below.

Standardized bias in the correlated factor model with nonzero mediated effects. Figure 4A-1 shows that standardized bias decreased as the specific factor variance increased and as the *a*- and *b*-paths and sample size decreased. The variance explained by the regression predicting standardized bias from the simulation factors was $R^2 = .982$. There was a significant interaction among all of the predictors (b=8.833, t= 97.365, p<.05, partial η^2=.083). As the specific factor variance increased, standardized bias increased slower for larger sample sizes and *a*- and *b*-paths than for the smaller sample sizes and *a*- and *b*-paths, supporting **Hypothesis 4.3**.

-Insert Figure 4A-1 and Table 4A-1 about here-

Raw bias in the correlated factor model with nonzero mediated effects. Figure 4A-4 shows that raw bias was not influenced by sample size or specific factor variance. The variance explained by the regression predicting raw bias from the simulation factors was R^2 = .971. There was a significant interaction between the *a*- and the *b*-path (*b*=-1.574, *t*=-192.150, *p*<.05, partial η^2= .747). As the *a*-path increased, raw bias increased faster for a medium *b*-path than a small *b*-path.

-Insert Figure 4A-2 about here-

Relative bias in the correlated factor model with nonzero mediated effects. Figure 4A-5 show that relative bias decreased as the specific factor variance increased. The variance explained by the regression predicting relative bias from the simulation factors was R^2 = .018. As the specific factor variance increased, the relative bias decreased, (*b*=.041, *t*=8.721, *p*<.05, partial η^2=.016), supporting **Hypothesis 4.3**.

-Insert Table 4A-2 and Figure 4A-3 about here-

Power in the correlated factor model. The power to detect the mediated effect increased as sample size, the *a*- and *b*-paths, and the specific factor variance increased (Figure 4B-1). Conditions with 1,000 cases, large *a*- and *b*-paths, and large specific factor variance were *adequately* powered (Table 4B-1). Detailed analyses are found below.

Power to detect the mediated effect. There were significant main effects of the *a*-path (large effect size; *b*=1.668, *z*=3.053, *p*<.05), sample size (large effect size; *b*=1.453, *z*=2.612, *p*<.05), and *b*-path (medium effect size; *b*=1.257, *z*=2.216, *p*<.05). As the sample size and *a*- and *b*-paths increased, the power increased. Power increased as the *b*-path (χ^2(1, *N*=107,086) = 5.509, *p*<.05), sample size (χ^2(2, *N*=107,086) = 9.066, *p*<.05), and *a*-path (χ^2(2, *N*=107,086) = 13.134, *p*<.05) increased, but power did not significantly increase as the specific factor variance increased ($\chi^2(2, N=107{,}086) = 3.29$, $p=.416$), supporting **Hypothesis 4.4**.

-Insert Table 4B-1 and Figure 4B-1 about here-

Type 1 error in the correlated factor model. The Type 1 error in the mediated effect approached .05 as the *a*- or *b*-path increased (Figure 4C-1 to Figure 4C-2). Type 1 errors were *adequate* for conditions with a medium *a*- or *b*-path, a sample size of 1,000, and large specific factor variance (Table 4C-1). Conditions analyzed had a nonzero *a*- or *b*-path and a zero effect in the other path. Detailed analyses are found below.

Type 1 error in the mediated effect. Table 4-C1 show that all conditions had empirical Type 1 error rates below .075. Conditions with zero *a*- and *b*-paths had Type 1 errors close to zero. In the model predicting Type 1 error rate from the *a*-path, specific factor variance, and sample size, Type 1 error approached .05 as the *a*-path increased (large effect size; $b=1.614$, $z=2.083$, $p<.05$). Type 1 error approached .05 as the *a*-path increased ($\chi^2(2, N=53{,}870) = 6.235$, $p<.05$), but sample size ($\chi^2(2, N=53{,}870) = 2.972$, $p=.226$) and the specific factor variance ($\chi^2(2, N=53{,}870) = 3.245$, $p=.197$) did not influence Type 1 error rates. Similarly, in the model predicting Type 1 error rate from the *b*-path, sample size, and specific factor variance, there was a significant interaction between the *b*-path and sample size (large effect size; $b=1.868$, $z=2.624$, $p<.05$). As sample size increased, Type 1 error approached .05 faster for conditions with a larger *b*-path than with smaller *b*-path. The Type 1 error approached .05 as the *b*-path ($\chi^2(2, N=53{,}997) = 3.307$, $p<.05$) increased, but specific factor variance ($\chi^2(2, N=53{,}997) = 0.389$, $p=.197$), and sample size ($\chi^2(2, N=53{,}997) = 0.527$, $p=.226$) did not significantly influence Type 1 error rates.

-Insert Table 4C-1 and Figure 4C-1 to 4C-2 about here-

Coverage and interval width in the correlated factor model. The 95% confidence interval coverage of mediated effect approached .95 as the specific factor variance increased and the *a*- and *b*-paths and sample size decreased (Figure 4D-1). Conditions with large specific factor variance, and small *a*- and *b*-paths and sample size had coverage closer to .95. (Table 4D-1). Detailed analyses are found below.

Confidence interval coverage and width for the mediated effect. Table 4D-1 shows that all conditions had a coverage rate below 92.5%. Conditions with a medium *a*- and *b*-path and 500 cases had coverage of zero. Confidence interval coverage was examined only for conditions with a sample size of 200. There were significant interactions between the specific factor variance and the *a*-path (medium effect size; b= -1.252, z=-3.368, p<.05) and between the specific factor variance and the *b*-path (medium effect size; b=.569, z=3.310, p<.05). As the specific factor variance increased, confidence interval coverage approached .95 faster for conditions with a smaller *a*- and *b*-paths than for conditions with a larger *a*- and *b*-paths. Across all conditions, confidence interval coverage approached .95 as the specific factor variance increased (χ^2(2, N= 35,390) = 19.91, p<.05), and the *a*-path (χ^2(2, N= 35,390) = 1,335.06, p<.05) and *b*-path (χ^2(2, N= 35,390) =742.90, p<.05) decreased, supporting Hypothesis **4.6**.

*Confidence Interval Width***.** Figure 4E-1 shows that interval width increased as the specific factor variance and *a*- and *b*-paths increased, and as the sample size decreased. The variance explained by the regression predicting confidence interval width from the simulation factors was R^2 = .611. There was an interaction between the *a*-path and the specific factor variance (b=.025, z=19.281, p<.05, partial η^2=.014). As the specific factor variance increased, interval width increased faster for conditions with larger *a*-path than for their smaller *a*-path.

-Insert Table 4D-1 and Figure 4D-1 to Figure 4E-1 about here-

Summary of the correlated factor model. When the bifactor model is misspecified by only modeling one dimension with correlated uniquenesses (**Model 4**), most conditions with a large general factor variance converged. The mediated effect was negatively biased and had coverage below .95. Conditions with small *a*- and *b*-paths, small sample size, and large specific factor variance had the least bias and highest coverage. Bias decreased and coverage approached .95 as the specific factor variance increased and the other simulation factors decreased. Also, conditions with 1,000 cases, large *a*- and *b*-paths, and large specific factor variance in models with *zero or nonzero mediated effects* had *adequate* power and Type 1 error rates. Other conditions approached *adequate* power and Type 1 error as the *a*- and *b*-paths increased, and were not influenced by the specific factor variance. Only power increased as the sample size increased.

Model Comparisons

Comparisons of the different analysis models need to be done with caution because the mediated effects across misspecified models are theoretically different.

Correlated v. unidimensional factor model. The *unidimensional* model is nested under the *correlated factor* model, so the influence of the correlated uniqueness on the simulation outcomes was investigated. Only conditions with large general factor variance and a small specific factor variance converged and fit both models. First, Table 5A-1 and Table 5A-2 show that the *correlated* model had slightly higher bias than the *unidimensional* model. The variance explained by the regression predicting the difference in the standardized bias between the models was R^2=.906. There was an interaction among the *a*- and *b*-paths and sample size (b=0.081, t= 5.004, p<.05, partial η^2=.008). The difference in standardized bias increased faster as the sample size increased for conditions with larger *a*- and *b*-paths than for smaller *a*- and *b*-paths. Second, Table 5B-1 shows that both models had power around .05 for the analyzed conditions, so no statistical tests were performed. Conditions with the highest power had a medium effect size in the *a*-path and the *b*-path and 1,000 cases. Third, Table 5C-1 shows that both models had Type 1 errors below .075. Type 1 error rates were only *adequate* for the *unidimensional* model in conditions of 1,000 cases. Finally, Table 5D-1 shows that confidence interval coverage was similar for both models. Conditions with small sample size and small *a*- and *b*-paths had coverage closest to .95.

-Insert Table 5A-1, Table 5A-2, Table 5B-1, Table 5C-1 and Table 5D-1 about here-

Facet v. finite factor model. First, bias in the mediated effect from the *facet* model is always negative and higher than the bias from the *finite* model. The most biased condition in the *finite* model has a small sample size, small general factor variance, small specific factor variance, and small *a*- and *b*-paths. Those conditions had the least bias in the *facet* model, which increased as the specific factor variance decreased and the other simulation factors increased.

Second, conditions with medium *a*- and *b*-paths and a sample size of 500 had *adequate* power for both models. The *facet* model only had more power in conditions where the general factor variance is small or medium, sample size is small, and one of the paths is small. For conditions with a small sample size, there was a significant interaction among the predictors (small effect size; b=.802, z=.326, p<.05). The difference in power decreased faster as the specific factor variance increased for conditions with larger *a*- and *b*-paths and smaller general factor variance than smaller *a*- and *b*-paths and larger general factor variance.

Third, when the *a*- or *b*-path had nonzero effects and a small general factor variance, the *facet* model had Type 1 errors closer to .05. The *finite* model had Type 1 errors closer to .05 when the general factor variance and sample size increased and the *a*- and *b*-paths were small.

Finally, about 37.7% of the true mediated effects were covered by confidence intervals in both *facet* and *finite* models. The *facet* model never had higher coverage than the *finite* model. In the prediction of the coverage difference, there was a significant interaction among all of the predictors (large effect size; b=-2.407, z=-4.975, p<.05). The difference in coverage increased faster as sample size increased for conditions with larger *a*- and *b*-paths, larger general factor variance, and smaller specific factor variance than smaller *a*- and *b*-paths, smaller general factor variance, and larger specific factor variance.

Unidimensional v. finite factor model. Only conditions with a small specific factor variance and up to medium *a*- or *b*-paths fit both models. First, bias in the mediated effect for the *unidimensional* model is always negative and higher than for the *finite* model. The most biased condition in the *finite* model had a small sample size, small general and specific factor variance, and small *a*- and the *b*-paths. Those conditions had the least bias in the *unidimensional* model, which increased as simulation factors increased.

Second, the *finite* model had more power that the *unidimensional* model, except in conditions with a small general factor variance and sample size. In the prediction of the difference in power, there was a significant interaction between sample size and the *a*- and *b*-paths (small effect size; b=-0.383, z=-19.372, p<.05). As sample size increased, the difference in power increased faster for conditions with a smaller *a*- and *b*-paths than for larger *a*- and *b*-paths.

Third, the *finite* model has *adequate* Type 1 errors in conditions with 1,000 cases and a small *a*- or *b*-path. The *unidimensional* model had Type 1 error rates below .025 for that condition. Also, the *unidimensional* model had *adequate* Type 1 error rates for conditions with a small sample size, general factor variance and a medium *a*- or *b*-path. The *finite* model had *adequate* Type 1 errors only for conditions with 500 cases.

Finally, the *finite* model had confidence interval coverage closer to .95 than the *unidimensional* model. Discrepancies in coverage increased as the simulation factors increased.

Correlated v. finite factor model. Only conditions with a large general factor variance and up to a medium *b*-path converged for both models. First, bias in the mediated effect for the *correlated factor* model is always negative and higher than for the *finite* model. The most biased conditions in the *finite* model had a small sample size, small general and specific factor variance, and small *a*- and the *b*-paths. Those conditions had the least bias in the *correlated factor* model, which increased as the specific factor variance decreased and the simulation factors increased.

Second, the *finite* model had more power than the *correlated factor* model. In the prediction of the difference in power, there was a significant interaction among all of the predictors (small effect size; *b*=-0.206, *z*=-5.348, $p<.05$). As sample size increased, the difference in power for the models increased faster for conditions with a smaller *a*- and *b*-paths and specific factor variance than for larger *a*- and *b*-paths and specific factor variance.

Third, Type 1 error rates were *adequate* for the *finite* and *correlated factor* models in conditions with a medium *a*- or *b*-path. However, Type 1 error approached .05 for the *correlated factor* model only when the sample size and specific factor variance were medium or large.

Finally, the *finite* model had coverage closer to .95 than the *correlated factor* model. The coverage difference increased as simulation factors increased but as the specific factor variance decreased.

Facet v. unidimensional model. Only conditions with a small specific factor variance and up to a medium *a*- or *b*-path fit both models. First, the mediated effect in the *facet* model was less biased than in the *unidimensional* model. The variance explained by the regression predicting the difference in relative bias from the simulation factors was R^2=.144. There was a significant interaction between sample size and the general factor variance (*b*=0.075, *t*=9.442, $p<.05$, partial η^2=.005). The difference in relative bias decreased faster as sample size increased for conditions with a smaller general factor variance than for a larger general factor variance. Also, the variance explained by the regression predicting standardized bias from the simulation factors was R^2=.993. There was a significant interaction among all of the predictors (*b*=5.555, *t*=95.140, $p<.05$, partial η^2=.124). The difference in the standardized bias increased faster as the sample size increased for conditions with larger general factor variance and *a*- and *b*-paths than with smaller general factor variance and *a*- and *b*-paths.

Second, the *facet* model had more power than the *unidimensional* model. There was an interaction among simulation factors on the power difference (b=2.646, z=4.866, p<.05). As the general factor variance increased, the power difference increased faster for conditions with a larger sample size and *a*- and *b*-paths than for smaller sample size and *a*- and *b*-paths.

Third, conditions with a medium *a*- and *b*-paths had *adequate* Type 1 errors in both models. The *unidimensional* model had *adequate* Type 1 errors for conditions with a small sample size and general factor variance. The *facet* model has *adequate* Type 1 errors only for conditions with medium sample sizes. All of the previous comparisons supported **Hypothesis 5**.

Finally, the *facet* model had coverage closer to .95 than the *unidimensional* model. Coverage differences increased as simulation factors increased but as the general factor variance decreased.

Facet v. correlated factor model. Only conditions with a large general factor variance and up to a medium *b*-path converged for both models. First, the *facet* model was less biased than the *correlated factor* model. The variance explained by the regression predicting standardized bias from the simulation factors was R^2=.992. There was a significant interaction among all the predictors (b=-8.413, t=-124.060, p<.05, partial η^2=.129). The standardized bias difference increased faster as the sample size increased for conditions with smaller specific factor variance and larger *a*- and *b*-paths than with larger specific factor variance and smaller *a*- and *b*-paths.

Second, the *facet* model had more power that the *correlated factor* model. There was an interaction among the all the simulation factors on the power difference (large effect size; b=-1.487, z=-5.602, p<.05). As the sample size increased, the power difference increased faster for conditions with a smaller specific factor variance and larger *a*- and *b*-paths than for a larger specific factor variance and smaller *a*- and *b*-paths.

Third, both models had *adequate* Type 1 errors in conditions with a medium *a*- or *b*-paths, but the *correlated factor* model also needed a large sample size and general factor variance for *adequate* Type 1 error rates.

Finally, the *facet* model had coverage closer to .95 than the *correlated factor* model. The coverage difference increased as the specific factor variance decreased and the other simulation factors increased. All of the previous comparisons supported **Hypothesis 5**.

Discussion

The goal of this Monte Carlo study was to investigate what happens to the mediated effect when a facet of a broad mediating construct is the true mediator, but the mediating construct is misspecified. The simulation study evaluated four latent variable models that included *general* and *specific* aspects of a mediator. The main conclusion is that misspecifying the facets of a mediating construct leads to mediated effect estimates that are too small, though the effect could still be detected under certain conditions. Accurate mediated effect estimation depends on mediator *specific* facet variance. This discussion section describes the contributions of the study, limitations, and future directions.

Summary of the Simulation Results

The mediation model with a bifactor mediator measurement model had *unbiased* and *adequately* powered mediated effects as the *general* and *specific* factor variance in the indicators increased. Conditions with small sample sizes (e.g., N=200), small *general* factor variance, and small *specific* factor variance in the indicators often had models that did not converge.

The mediation model with only mediator facet indicators had small mediated effects, which were more likely to be detected as the *specific* factor variance increased and the *general* factor variance decreased. Recall the facet indicators do not distinguish between the *general* and *specific* factor variance. Not all of the variance in the latent variable is true mediator variance because some is from the *general* factor. The factor loadings on the facet are larger than those from the data-generating model. As a result, the *b*-path and the mediated effect are underestimated and the *c'*-path is overestimated.

The unidimensional mediator model had many model fit problems. Few conditions with a small *general* factor variance, small *specific* factor variance, and small sample size met the conventional fit index thresholds of the RMSEA and the CFI. The model fit the data (with a threshold for the SRMR above .08) when there was 10% *specific* factor variance. This model had small mediated effects and were more likely to be detected as the *general* factor variance decreased because only a small part of the true mediator variance is reflected in the unidimensional latent variable. As the *general* factor variance increased, the latent variable reflects more of the *general* factor variance among the indicators rather than *specific* factor variance.

Adding correlated uniquenesses to the unidimensional mediator model improved model fit. The unidimensional measurement model with correlated uniqueness has the same fit, degrees of freedom, and factor loadings on the *general* factor as the bifactor model. Although model fit improved, the model often did not converge (exceeding iteration limits or having negative residual variances in the indicators) because the correlated uniquenesses captured the true mediator variance. Indicators that did not measure the true mediator had underestimated loadings on the *general* factors when X predicted the mediator because X tried to pull the factor to the right solution. Often, the variance of the mediator and residual variances had improper solutions. Most models with large *general* factor variance converged and had small mediated effects. The mediated effects were more likely to be detected as the *specific* factor variance increased.

Contributions and Implications of the Simulation Results

This study contributes to the important, but largely unexplored area of measurement issues in statistical mediation analysis (MacKinnon, 2008). Previous research investigated the influence of reliability (Hoyle & Kenny, 1999), measurement invariance (Olivera-Aguilar, Kisbu-Sakarya, & MacKinnon, submitted), and confounding-measurement error relationships (Fritz, Kenny, & MacKinnon, submitted) on the mediated effect in the single mediator model. This study described the influence of misspecifying the structure of the mediator on the mediated effect. The misspecifications studied were theoretically valid, alternative measurement models that did not represent the facets of the mediator accurately. This study used the bifactor measurement model to *distill* the indicators of the mediating variable into multiple variance components, obtain a facet latent variable, and use the facet as the mediator.

This study contributes to the psychometric literature of latent variable modeling by investigating how *general* and *specific* factor variance affects the measurement of the mediator (Brunner, Nagy, & Willhem, 2012). One alternative measurement model not studied was the higher-order model. Researchers have historically favored the higher-order factor model over the bifactor model (Reise, 2012). The results of the simulation study demonstrate the viability of the bifactor model because it can simultaneously test relationships of *general* and *specific* aspects of a construct on an outcome (Chen, West & Sousa, 2006). Finally, this study extends Reise et al., (2013), which found that indices of factor strength predicted structural bias when multidimensional data were treated as unidimensional. The results of this study suggest that the strength of the facet factor (*specific* factor variance) was a significant predictor of the bias and power to detect the mediated effect.

This study has implications for how researchers conceptualize mediators. Researchers can apply the latent variable models described in this study to assess measurement structure and how the mediating process occurs. The level of generalization in the latent variable (Gustafsson & Balke, 1993) studied will affect the estimation of the mediated effect. When only the facet indicators were modeled in this study, the latent variable has facet variance of interest and also common variance that does not contribute to the mediation process. When the indicators of a multidimensional mediator are treated as unidimensional, for example, the model did not fit the data. When correlated uniquenesses were used to improve the fit of the unidimensional model, the true mediator variance was hidden in the correlated residuals. However, some misspecified models had sufficient power to detect the mediated effect, depending on the *a*- and *b*-path effect size and the *specific* factor variance in the indicators. Therefore, statistically significant results in models with latent mediators are encouraging but repeated testing of the model is needed to *distill* the true mediator.

There are several challenges to repeated testing to *distill* a mediating process and using the bifactor model in substantive research. First, repeated revisions of the mediator measurement model could inflate Type 1 errors. One solution is to use cross-validation strategies to support the exploratory phase of model building (Bandalos, 1993). If there is a large sample size, using "out-of-bag" cases to test the model or using leave-one-out cross-validation would help researchers overcome inflated Type 1 error rates (Berk, 2008). Second, the bifactor model requires a large sample size and high factor loadings in the *general* and *specific* factors for *unbiased* estimation and *adequate* power. Exploratory (Asparouhov & Muthen, 2009) and Bayesian (Muthen & Asparouhov, 2012) approaches to bifactor modeling could help reduce convergence problems due to low factor loadings, small sample sizes, or the orthogonality of *specific* factors. A Bayesian approach could also be used to update the measurement model as more information is found in the literature. An incomplete bifactor model could also improve estimation when some of the facets do not have reliable variance (Chen, West, & Sousa, 2006). Using model-based measures of reliability, such as coefficient omega or explained common variance, can inform whether the *general* and *specific* factors need to be modeled (Reise et al., 2010). Finally, if the facet is the true mediator and all of the previous recommendations cannot be followed, measuring a few mediator indicators of a theoretical facet will have more power to detect the mediated effect than a unidimensional broad construct.

Limitations and Future Directions

The future directions of this study extend from the limitations. Practical examples with datasets from the field are needed to illustrate the results from this study. More substantive examples of meaningful facets independent from a *broad* construct are also needed. Two examples in the literature are the constructs of depression and work-place vigor. Simms, Gros, Watson, and O'Hara (2008) investigated the bifactor structure of depression, modeling symptom groups as the *specific* factors. They indicate that appetite loss, appetite gain, well-being, and insomnia have high loadings on *specific* factors and that those symptoms relate differently to general distress. Simms et al. (2008) suggest that, if findings are replicated, examining the *specific* factors could provide an index of severity so that symptoms are not weighted the same when diagnosing for general depression (i.e., appetite loss had a higher association with distress than appetite gain). Moreover, Armon and Shiron (2011) fit a bifactor model to study vigor and its facets (physical strength, emotional energy, and cognitive liveliness). They interpreted the emotional energy *specific* factor as variance that "reflects a unique positive energy balance in one's interpersonal relationships, unique in the sense of not being shared with the other two facets of vigor (Armon & Shiron, 2011, pg. 619)." The emotional energy *specific* factor had a .47 stability coefficient across a two-year testing span and was significantly associated with the Big 5 agreeableness factor. DeMars (2013) warns researchers that the interpretation of the *specific* factors reflects information above and beyond the *general* factor score; that a weighted composite of *general* and *specific* factor variance might be more reliable than a *specific* factor score; and that *specific* factors might underestimate the influence of a facet on an outcome.

Also, a limited number of sample size conditions were investigated in this study. The simulation showed that models with a bifactor mediator do not have sufficient power to detect the mediated effect with a sample size of 200, but there was sufficient power in conditions with a sample size of 500. A power curve for the mediated effect is needed to find the exact sample size when power gets to .80 (Fritz & MacKinnon, 2007).

Also, it is difficult to compare the simulation outcomes across misspecified models because they are theoretically different in how they represent the mediator. A criticism of Monte Carlo simulations is that the data-generating model will always be favored by the simulation outcomes. Parameters in misspecified models are expected to be biased and not adequately covered in confidence intervals, but they can still provide information about when an effect can be detected. The goal of this study was to investigate the conditions when a researcher who has some measures of the true mediating construct could find the effect.

For this study, the bifactor mediator model was the data-generating model to study the influence of mediator facets in the presence of *general* factor variance. Data could have been generated under a unidimensional model or a one-factor model for the facet, but the presence of multiple sources of variance in the indicators cannot be studied if all of the sources of variance are not simulated. An alternative model to simulate multidimensional data is the higher-order factor model. The *specific* factor variance from the bifactor model is represented by a lower-order factor disturbance in the higher-order model. It would be interesting to use non-standard structural equation modeling to predict an outcome from a lower-order factor disturbance (see Stacy, Newcomb, & Bentler, 1991). Interpretation is sacrificed when disturbances are used as predictors but results from this model and the bifactor mediator model are comparable.

Another interesting data-generating model is one where some the *general* factor variance *distilled* from the facet factors predicts the outcome. A parallel mediator model would be needed to capture the total mediated effect (O'Rourke & MacKinnon, 2015). The bifactor mediator would have two mediated effects – one through the *general* factor and one through the *specific* facet. It would be interesting to evaluate if a unidimensional model that only includes the facet indicators can fully capture the total mediated effect.

A fully Bayesian approach with informative or diffuse priors can be an alternative model of analysis for statistical mediation (Yuan & MacKinnon, 2009) and might lead to more power and less bias in the mediated effects from misspecified models. A Bayesian approach to bifactor modeling has been proposed (Muthen & Asparouhov, 2012), but its properties for accurate bifactor estimation are yet to be investigated.

Finally, this study assumed that the measurement structure of the mediator was invariant across the binary treatment variable and across time. These hypotheses can be tested if there are pretest measures of the mediator and by comparing the factor structure across groups. Olivera-Aguilar, Kisbu-Sakarya, and MacKinnon (submitted) indicate that violations of scalar invariance in the mediator lead to biased and underpowered estimates of the mediated effect. It would be interesting to evaluate the influence violations of measurement invariance in the data-generating model on mediated effect estimation in misspecified models. Also, the dependent variable in this study was modeled as perfectly reliable and invariant. It would be interesting to evaluate how a bifactor mediator interacts with violations of measurement invariance in the dependent variable.

Conclusions

This study illustrated how identifying the true mediator in the causal process is a *measurement problem.* Incorrect characterizations of multifaceted mediators led to biased and underpowered mediated effects. This study encourages researchers to explore the multidimensionality of their mediators and the influence of facets on outcomes so that they have more power to test for mediation in interventions and other substantive studies.

References

Armon, G., & Shirom, A. (2011). The across-time associations of the five-factor model of personality with vigor and its facets using the bifactor model. *Journal of Personality Assessment*, 93, 618–627.

Asparouhov, T., & Muthén, B. (2009). Exploratory structural equation modeling. *Structural Equation Modeling: A Multidisciplinary Journal*, *16*(3), 397-438.

Bagozzi, R. P., & Heatherton, T. F. (1994). A general approach to representing multifaceted personality constructs: application to state self-esteem. *Structural Equation Modeling: A Multidisciplinary Journal*, *1*(1), 35-67.

Bandalos, D. L. (1993). Factors influencing cross-validation of confirmatory factor analysis models. *Multivariate Behavioral Research*, *28*(3), 351-374.

Baron, R. M., & Kenny, D. A. (1986). The moderator–mediator variable distinction in social psychological research: Conceptual, strategic, and statistical considerations. *Journal of Personality and Social Psychology*, *51*(6), 1173.

Bentler, P. M. (1990). Comparative fit indexes in structural models. *Psychological bulletin*, *107*(2), 238.

Bentler, P. M., & Weeks, D. G. (1980). Linear structural equations with latent variables. *Psychometrika*, *45*(3), 289-308.

Berk, R. A. (2008). *Statistical learning from a regression perspective*. Springer Science & Business Media.

Bollen, K. A. (1989). *Structural equations with latent variables*. New York: Wiley.

Bradley, J. V. (1978). Robustness?. *British Journal of Mathematical and Statistical Psychology*, *31*(2), 144-152.

Briggs, S. R., & Cheek, J. M. (1988). On the nature of self-monitoring: Problems with assessment, problems with validity. *Journal of Personality and Social Psychology*, *54*(4), 663.

Briggs, S. R., Cheek, J. M., & Buss, A. H. (1980). An analysis of the Self-monitoring Scale. *Journal of Personality and Social Psychology*, *38*(4), 679.

Brown, T. A. (2014). *Confirmatory factor analysis for applied research*. Guilford Publications.

Brunner, M. (2008). No g in education?. *Learning and Individual Differences*, *18*(2), 152-165.

Brunner, M., Nagy, G., & Wilhelm, O. (2012). A tutorial on hierarchically structured constructs. *Journal of Personality*, *80*(4), 796-846.

Carmines, E. G., & Zeller, R. A. (1979). *Reliability and validity assessment* (Vol. 17). Sage publications.

Carver, C. S. (1989). How should multifaceted personality constructs be tested? Issues illustrated by self-monitoring, attributional style, and hardiness. *Journal of Personality and Social Psychology*, *56*(4), 577.

Chen, F. F., Hayes, A., Carver, C. S., Laurenceau, J. P., & Zhang, Z. (2012). Modeling general and specific variance in multifaceted constructs: A comparison of the bifactor model to other approaches. *Journal of Personality*, *80*(1), 219-251.

Chen, F. F., Jing, Y., Hayes, A., & Lee, J. M. (2013). Two concepts or two approaches? A bifactor analysis of psychological and subjective well-being. *Journal of Happiness Studies*, *14*(3), 1033-1068.

Chen, F. F., West, S. G., & Sousa, K. H. (2006). A comparison of bifactor and second-order models of quality of life. *Multivariate Behavioral Research*,*41*(2), 189-225.

Cox, M. G., Kisbu-Sakarya, Y., & MacKinnon, D. P. (2012). *Distillation of the mediated effect: Estimating bias in latent variable mediation models.* Poster presented at the annual meeting of the Society for Prevention Research, Washington, DC.

DeMars, C. E. (2013). A tutorial on interpreting bifactor model scores. *International Journal of Testing*, *13*(4), 354-378.

Embretson, S. E., & Reise, S. P. (2000). *Item response theory for psychologists*. Psychology Press.

Fritz, M., Kenny, D., & MacKinnon, D. (2014). *The opposing effects of simultaneously ignoring measurement error and omitting confounders in a single-mediator model*. Tech. rep., Department of Educational Psychology, University of Nebraska, Lincoln, NE. Submitted, Psychological Methods.

Fritz, M. S., & MacKinnon, D. P. (2007). Required sample size to detect the mediated effect. *Psychological Science*, *18*(3), 233-239.

Gustafsson, J. E. (2002). Measurement from a hierarchical point of view. *The role of constructs in psychological and educational measurement*, 73-95.

Gustafsson, J. E., & Balke, G. (1993). General and specific abilities as predictors of school achievement. *Multivariate Behavioral Research*, *28*(4), 407-434.

Hallquist, M. & Wiley, J. (2014). MplusAutomation: Automating Mplus Model Estimation and Interpretation. R package version 0.6-3. http://CRAN.R-project.org/package=MplusAutomation

Haviland, M. G., Warren, W. L., & Riggs, M. L. (2000). An observer scale to measure alexithymia. *Psychosomatics*, *41*(5), 385-392.

Holzinger, K. J., & Harman, H. H. (1938). Comparison of two factorial analyses. *Psychometrika*, *3*(1), 45-60.

Holzinger, K. J., & Swineford, F. (1937). The bi-factor method. *Psychometrika*,*2*(1), 41-54.

Hoyle, R. H. (Ed.). (2012). *Handbook of structural equation modeling*. Guilford Press.

Hoyle, R. H., & Kenny, D. A. (1999). Sample size, reliability, and tests of statistical mediation. *Statistical strategies for small sample research*, *1*, 195-222.

Hull, J. G., Lehn, D. A., & Tedlie, J. C. (1991). A general approach to testing multifaceted personality constructs. *Journal of Personality and Social Psychology*, *61*(6), 932.

Ledgerwood, A., & Shrout, P. E. (2011). The trade-off between accuracy and precision in latent variable models of mediation processes. *Journal of Personality and Social Psychology*, *101*(6), 1174.

MacKinnon, D. P. (2008). *Introduction to statistical mediation analysis*. Routledge.

MacKinnon, D. P., & Dwyer, J. H. (1993). Estimating mediated effects in prevention studies. *Evaluation Review*, *17*(2), 144-158.

MacKinnon, D. P., Fairchild, A. J., & Fritz, M. S. (2007). Mediation analysis. *Annual review of psychology*, *58*, 593.

MacKinnon, D. P., Lockwood, C. M., & Williams, J. (2004). Confidence limits for the indirect effect: Distribution of the product and resampling methods. *Multivariate Behavioral Research*, *39*(1), 99-128.

Muthén, B., & Asparouhov, T. (2012). Bayesian structural equation modeling: a more flexible representation of substantive theory. *Psychological Methods*, *17*(3), 313.

Muthén, B., & Asparouhov, T. (2014). Causal effects in mediation modeling: an introduction with applications to latent variables. *Structural Equation Modeling: A Multidisciplinary Journal*, (ahead-of-print), 1-12.

Muthén, L. K., & Muthén, B. O. (1998-2011). Mplus User's Guide. Sixth Edition. Los Angeles, CA: Muthén & Muthén.

Olivera-Aguilar, M., Kisbu-Sakarya, Y., & MacKinnon, D. P. (2014) *Bias, Type I error rates and statistical power of a mediation model in the presence of violations of invariance*. Manuscript submitted for publication.

O'Rourke, H. P., & MacKinnon, D. P. (2015). When the test of mediation is more powerful than the test of the total effect. *Behavior Research Methods*,*47*(2), 424-442.

R Core Team (2013). R: A language and environment for statistical computing. R Foundation for Statistical Computing, Vienna, Austria. URL http://www.R-project.org/.

Reise, S. P. (2012). The rediscovery of bifactor measurement models. *Multivariate Behavioral Research*, *47*(5), 667-696.

Reise, S. P., Bonifay, W. E., & Haviland, M. G. (2013). Scoring and modeling psychological measures in the presence of multidimensionality. *Journal of Personality Assessment*, *95*(2), 129-140.

Reise, S. P., Moore, T. M., & Haviland, M. G. (2010). Bifactor models and rotations: Exploring the extent to which multidimensional data yield univocal scale scores. *Journal of Personality Assessment*,*92*(6), 544-559.

Reise, S. P., Scheines, R., Widaman, K. F., & Haviland, M. G. (2013). Multidimensionality and structural coefficient bias in structural equation modeling a bifactor perspective. *Educational and Psychological Measurement*,*73*(1), 5-26.

Schmid, J., & Leiman, J. M. (1957). The development of hierarchical factors solutions. *Psychometrika*, 22(1), 53-61.

Simms, L. J., Grös, D. F., Watson, D., & O'Hara, M. W. (2008). Parsing the general and specific components of depression and anxiety with bifactor modeling. *Depression and Anxiety*, *25*(7), E34-E46.

Sobel, M. E. (1982). Asymptotic confidence intervals for indirect effects in structural equation models. *Sociological Methodology, 13*(1982), 290-312.

Sobel, M. E. (1986). Some new results on indirect effects and their standard errors in covariance structure models. *Sociological Methodology*, *16*, 159-186.

Spearman, C. (1904). " General Intelligence," objectively determined and measured. *The American Journal of Psychology*, *15*(2), 201-292.

Stacy, A. W., Newcomb, M. D., & Bentler, P. M. (1991). Personality, problem drinking, and drunk driving: Mediating, moderating, and direct-effect models. *Journal of Personality and Social Psychology*, *60*(5), 795.

SymPy Development Team (2014). SymPy: Python library for symbolic mathematics URL http://www.sympy.org.

Thoemmes, F., MacKinnon, D. P., & Reiser, M. R. (2010). Power analysis for complex mediational designs using Monte Carlo methods. *Structural Equation Modeling: A Multidisciplinary Journal*, *17*(3), 510-534.

Thurstone, L. L. (1947). Multiple-factor analysis; a development and expansion of The Vectors of Mind. Chicago: The Univ. of Chicago Press.

Tofighi, D., & MacKinnon, D. P. (2011). RMediation: An R package for mediation analysis confidence intervals. *Behavior Research Methods*, *43*(3), 692-700.

Valeri, L., Lin, X., & VanderWeele, T. J. (2014).*Mediation analysis in generalized linear models when the mediator is measured with error*. Technical Report.

VanderWeele, T. J., Valeri, L., & Ogburn, E. L. (2012). The role of measurement error and misclassification in mediation analysis. *Epidemiology (Cambridge, Mass.)*, *23*(4), 561.

Weiss, C. H. (1997). Theory-based evaluation: Past, present, and future. *New directions for evaluation*, *1997*(76), 41-55.

Wright, S. (1921). Correlation and causation. *Journal of Agricultural Research*, *20*(7), 557-585.

Yuan, Y., & MacKinnon, D. P. (2009). Bayesian mediation analysis. *Psychological Methods*, *14*(4), 301.

Yung, Y. F., Thissen, D., & McLeod, L. D. (1999). On the relationship between the higher-order factor model and the hierarchical factor model. *Psychometrika*,*64*(2), 113-128.

APPENDIX A

DATA-ANALYSIS MODELS

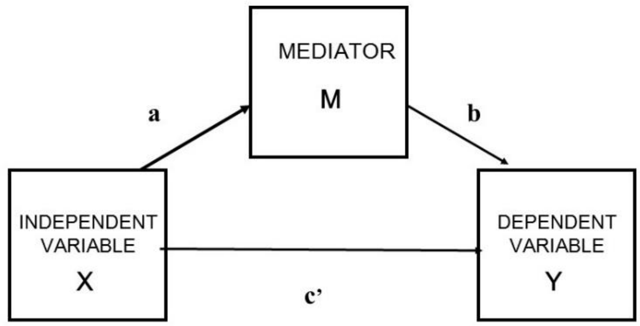


Figure 1. The Single mediator model.

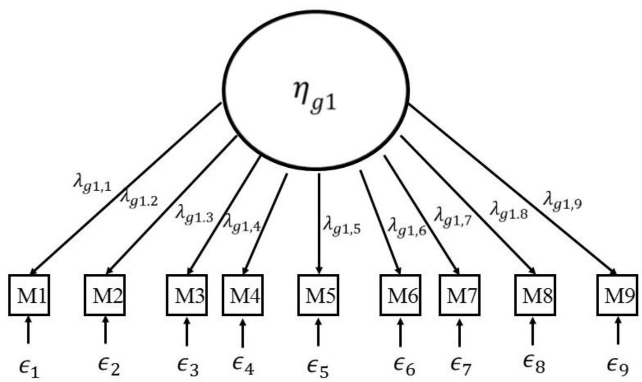


Figure 2. Unidimensional measurement model.

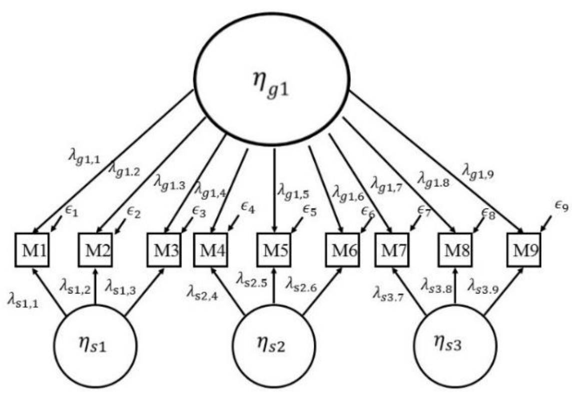


Figure 3. Bifactor measurement model for the mediator.

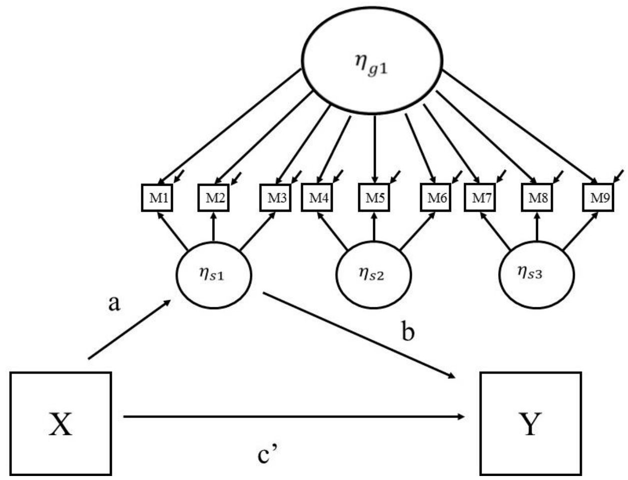


Figure 4. **Model 1 –** Distillation of the mediated effect with the bifactor model (finite-sample model)

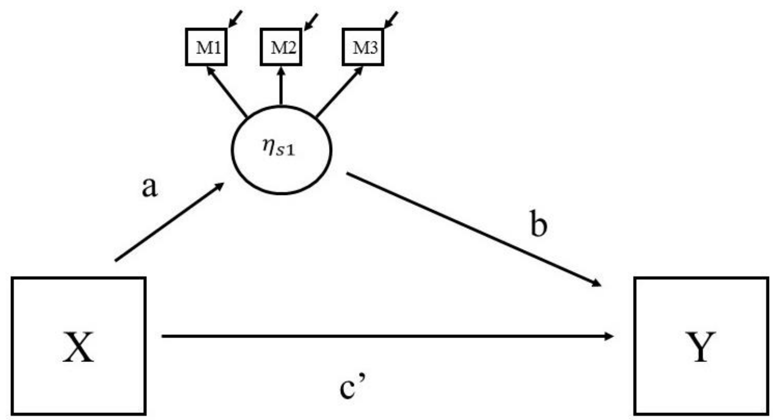


Figure 5. **Model 2** – Facet mediation model

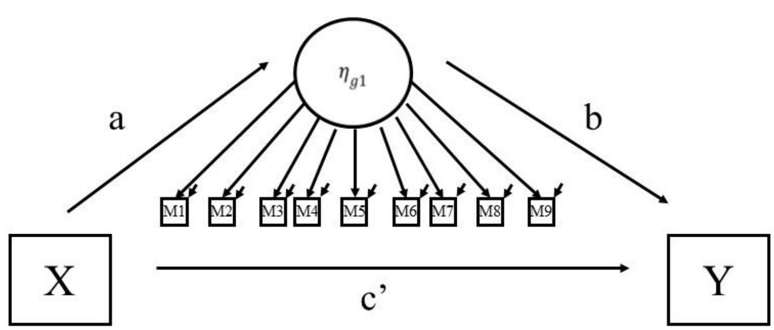


Figure 6. **Model 3** – Unidimensional mediation model

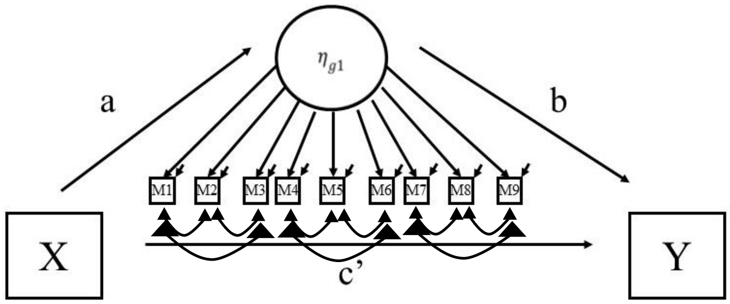


Figure 7. **Model 4 –** Correlated Factor mediation model

APPENDIX B

TABLES

Table 1A-1

Standardized Bias in the mediated effect for the finite model with nonzero effects when general factor variance is .09 (gen λ=.3)

		a-path small			a-path medium			a-path large		
		b-path small	b-path medium	b-path large	b-path small	b-path medium	b-path large	b-path small	b-path medium	b-path large
spec λ	N									
.3	200	0.198	0.169	0.185	0.222	0.266	0.259	0.194	0.252	0.242
.45	200	0.138	0.124	0.121	0.151	0.186	0.194	0.136	0.185	0.194
.6	200	0.113	0.087	0.095	0.112	0.176	0.184	0.119	0.186	0.196
.3	500	0.174	0.137	0.128	0.156	0.168	0.185	0.141	0.179	0.199
.45	500	0.074	0.063	0.068	0.093	0.126	0.144	0.107	0.147	0.166
.6	500	0.045	0.032	0.031	0.079	0.098	0.106	0.088	0.119	0.133
.3	1000	0.112	0.093	0.097	0.106	0.137	0.158	0.108	0.150	0.173
.45	1000	0.075	0.060	0.055	0.086	0.104	0.109	0.089	0.118	0.128
.6	1000	0.062	0.044	0.036	0.075	0.085	0.082	0.079	0.101	0.101

Note: Red are standardized bias above .1.

Table 1A-2

Standardized Bias in the mediated effect for the finite model with nonzero effects when general factor variance is .25 (gen λ=.5)

		a-path								
		small			medium			large		
		b-path								
		small	medium	large	small	medium	large	small	medium	large
spec λ	N									
.3	200	0.198	0.192	0.150	0.190	0.266	0.265	0.172	0.251	0.266
.45	200	0.078	0.060	0.067	0.097	0.120	0.148	0.095	0.143	0.179
.6	200	0.056	0.047	0.054	0.080	0.111	0.121	0.085	0.131	0.145
.3	500	0.153	0.096	0.084	0.114	0.149	0.159	0.102	0.148	0.177
.45	500	0.042	0.023	0.020	0.071	0.086	0.096	0.080	0.110	0.124
.6	500	0.039	0.012	0.004	0.070	0.074	0.073	0.078	0.098	0.104
.3	1000	0.081	0.068	0.072	0.073	0.102	0.122	0.079	0.118	0.139
.45	1000	0.056	0.046	0.040	0.068	0.087	0.089	0.073	0.103	0.109
.6	1000	0.058	0.041	0.030	0.070	0.083	0.077	0.074	0.099	0.098

Note: Red are standardized bias above .1.

Table 1A-3

Standardized Bias in the mediated effect for the finite model with nonzero effects when general factor variance is .49 (gen λ=.7)

		a-path								
		small			medium			large		
		b-path								
spec λ	N	small	medium	large	small	medium	large	small	medium	large
.3	200	0.159	0.125	0.118	0.144	0.212	0.212	0.116	0.198	0.222
.45	200	0.040	0.029	0.032	0.058	0.078	0.091	0.055	0.090	0.113
.6	200	0.022	0.015	0.013	0.047	0.059	0.062	0.053	0.078	0.087
.3	500	0.099	0.066	0.052	0.076	0.104	0.107	0.056	0.090	0.112
.45	500	0.017	0.005	0.001	0.042	0.049	0.049	0.048	0.066	0.072
.6	500	0.015	-0.007	-0.016	0.042	0.039	0.028	0.049	0.058	0.052
.3	1000	0.056	0.042	0.049	0.041	0.064	0.079	0.046	0.075	0.090
.45	1000	0.035	0.031	0.027	0.044	0.060	0.058	0.048	0.073	0.075
.6	1000	0.037	0.027	0.017	0.046	0.058	0.049	0.049	0.071	0.065

Note: Red are standardized bias above .1.

Table 1A-4

Relative Bias in the mediated effect for the finite model with nonzero effects when general factor variance is .09 (gen λ=.3)

		a-path small			a-path medium			a-path large		
		b-path small	b-path medium	b-path large	b-path small	b-path medium	b-path large	b-path small	b-path medium	b-path large
spec λ	N									
.3	200	0.626	0.503	0.514	0.353	0.317	0.348	0.234	0.213	0.205
.45	200	0.262	0.165	0.216	0.173	0.143	0.207	0.132	0.115	0.122
.6	200	0.167	0.122	0.113	0.139	0.104	0.131	0.096	0.092	0.100
.3	500	0.173	0.134	0.136	0.119	0.114	0.125	0.095	0.084	0.102
.45	500	0.055	0.039	0.047	0.073	0.046	0.061	0.057	0.049	0.063
.6	500	0.029	0.017	0.018	0.038	0.028	0.032	0.040	0.030	0.034
.3	1000	0.065	0.044	0.051	0.046	0.041	0.051	0.045	0.041	0.051
.45	1000	0.038	0.024	0.024	0.032	0.023	0.025	0.031	0.024	0.026
.6	1000	0.027	0.016	0.014	0.024	0.016	0.015	0.024	0.016	0.016

Note: Red are relative bias above .1.

Table 1A-5

Relative Bias in the mediated effect for the finite model with nonzero effects when general factor variance is .25 (gen λ=.5)

		a-path small			a-path medium			a-path large		
		b-path small	b-path medium	b-path large	b-path small	b-path medium	b-path large	b-path small	b-path medium	b-path large
spec λ	N									
.3	200	0.424	0.277	0.285	0.225	0.228	0.259	0.174	0.160	0.183
.45	200	0.112	0.061	0.082	0.095	0.071	0.078	0.081	0.062	0.081
.6	200	0.065	0.042	0.057	0.064	0.052	0.070	0.064	0.054	0.062
.3	500	0.122	0.061	0.061	0.065	0.056	0.065	0.054	0.050	0.064
.45	500	0.029	0.013	0.012	0.035	0.025	0.028	0.037	0.028	0.031
.6	500	0.024	0.006	0.002	0.033	0.019	0.019	0.034	0.022	0.022
.3	1000	0.043	0.029	0.033	0.028	0.025	0.031	0.028	0.025	0.030
.45	1000	0.026	0.017	0.016	0.023	0.017	0.017	0.023	0.017	0.018
.6	1000	0.025	0.014	0.011	0.023	0.015	0.013	0.022	0.015	0.014

Note: Red are relative bias above .1.

Table 1A-6

Relative Bias in the mediated effect for the finite model when general factor variance is .49 (gen λ=.7)

		a-path								
		small			medium			large		
		b-path								
		small	medium	large	small	medium	large	small	medium	large
spec λ	N									
.3	200	0.201	0.171	0.121	0.119	0.105	0.108	0.086	0.083	0.095
.45	200	0.040	0.022	0.027	0.041	0.030	0.034	0.035	0.028	0.033
.6	200	0.020	0.011	0.010	0.030	0.020	0.020	0.031	0.022	0.021
.3	500	0.065	0.035	0.030	0.036	0.030	0.031	0.024	0.022	0.027
.45	500	0.010	0.002	0.001	0.018	0.011	0.011	0.019	0.013	0.013
.6	500	0.008	-0.003	-0.007	0.016	0.008	0.006	0.018	0.010	0.008
.3	1000	0.025	0.015	0.019	0.014	0.013	0.016	0.014	0.013	0.015
.45	1000	0.014	0.010	0.010	0.013	0.010	0.009	0.014	0.010	0.009
.6	1000	0.014	0.008	0.006	0.013	0.009	0.007	0.013	0.009	0.007

Note: Red are relative bias above .1.

Table 1B-1

Power in the mediated effect through the distribution of product method in the finite model when general factor variance is .09 (gen λ*=.3)*

		a-path small			a-path medium			a-path large		
		b-path small	b-path medium	b-path large	b-path small	b-path medium	b-path large	b-path small	b-path medium	b-path large
spec λ	N									
.3	200	0.009	0.077	0.154	0.028	0.295	0.491	0.019	0.237	0.430
.45	200	0.032	0.204	0.262	0.107	0.609	0.812	0.080	0.521	0.760
.6	200	0.076	0.294	0.352	0.202	0.840	0.942	0.149	0.760	0.915
.3	500	0.126	0.505	0.569	0.229	0.896	0.948	0.158	0.808	0.884
.45	500	0.277	0.629	0.650	0.417	0.993	0.994	0.305	0.970	0.984
.6	500	0.394	0.717	0.732	0.539	0.999	1.000	0.432	0.996	0.998
.3	1000	0.533	0.842	0.856	0.640	0.999	0.999	0.501	0.991	0.995
.45	1000	0.730	0.895	0.906	0.779	1.000	1.000	0.653	1.000	1.000
.6	1000	0.860	0.944	0.949	0.878	1.000	1.000	0.763	1.000	1.000

Table 1B-2

Power in the mediated effect through the distribution of product method in the finite model when general factor variance is .25 (gen λ=.5)

		a-path small			a-path medium			a-path large		
spec λ	N	b-path small	b-path medium	b-path large	b-path small	b-path medium	b-path large	b-path small	b-path medium	b-path large
.3	200	0.019	0.154	0.237	0.075	0.487	0.718	0.047	0.406	0.651
.45	200	0.059	0.290	0.345	0.182	0.825	0.954	0.128	0.734	0.938
.6	200	0.077	0.333	0.376	0.233	0.906	0.976	0.179	0.843	0.983
.3	500	0.228	0.610	0.629	0.375	0.987	0.996	0.295	0.963	0.984
.45	500	0.342	0.659	0.683	0.518	0.999	0.999	0.399	0.998	0.999
.6	500	0.398	0.716	0.719	0.563	1.000	1.000	0.450	0.999	1.000
.3	1000	0.672	0.890	0.893	0.760	1.000	1.000	0.625	1.000	1.000
.45	1000	0.793	0.915	0.923	0.839	1.000	1.000	0.724	1.000	1.000
.6	1000	0.842	0.940	0.949	0.874	1.000	1.000	0.777	1.000	1.000

Table 1B-3

Power in the mediated effect through the distribution of product method in the finite model when general factor variance is .49 (gen λ=.7)

		a-path small			a-path medium			a-path large		
		b-path small	b-path medium	b-path large	b-path small	b-path medium	b-path large	b-path small	b-path medium	b-path large
spec λ	N									
.3	200	0.051	0.302	0.380	0.180	0.821	0.966	0.130	0.758	0.946
.45	200	0.091	0.381	0.421	0.281	0.959	0.996	0.234	0.921	0.997
.6	200	0.128	0.435	0.462	0.335	0.979	0.998	0.275	0.960	1.000
.3	500	0.387	0.703	0.726	0.537	1.000	1.000	0.453	0.999	1.000
.45	500	0.483	0.738	0.768	0.633	1.000	1.000	0.520	1.000	1.000
.6	500	0.547	0.800	0.809	0.677	1.000	1.000	0.593	1.000	1.000
.3	1000	0.852	0.954	0.958	0.881	1.000	1.000	0.784	1.000	1.000
.45	1000	0.898	0.964	0.968	0.915	1.000	1.000	0.845	1.000	1.000
.6	1000	0.938	0.973	0.982	0.946	1.000	1.000	0.891	1.000	1.000

Table 1C-1

Type 1 Error in the mediated effect through the distribution of product method in the finite model when general factor variance is .09 (gen λ=.3)

		a-path						
		zero	small	medium	large	zero	zero	zero
		b-path						
		zero	zero	zero	zero	small	medium	large
spec λ	N							
.3	200	N/A	N/A	0.002	0.004	N/A	0.017	0.030
.45	200	0.000	0.005	0.021	0.019	0.006	0.033	0.048
.6	200	0.001	0.009	0.034	0.036	0.005	0.045	0.050
.3	500	0.000	0.001	0.010	0.011	0.006	0.033	0.042
.45	500	0.000	0.015	0.030	0.028	0.011	0.046	0.046
.6	500	0.000	0.021	0.035	0.034	0.022	0.049	0.050
.3	1000	0.000	0.018	0.030	0.032	0.023	0.043	0.039
.45	1000	0.000	0.034	0.041	0.037	0.027	0.038	0.035
.6	1000	0.000	0.038	0.042	0.040	0.038	0.039	0.038

Table 1C-2

Type 1 Error in the mediated effect through the distribution of product method in the finite model when general factor variance is .25 (gen λ=.5)

		a-path						
		zero	small	medium	large	zero	zero	zero
		b-path						
		zero	zero	zero	zero	small	medium	large
spec λ	N							
.3	200	N/A	0.004	0.011	0.011	0.004	0.023	0.037
.45	200	0.001	0.009	0.033	0.031	0.006	0.042	0.048
.6	200	0.003	0.009	0.041	0.042	0.005	0.046	0.055
.3	500	0.000	0.003	0.015	0.016	0.008	0.036	0.040
.45	500	0.000	0.013	0.034	0.031	0.016	0.042	0.045
.6	500	0.000	0.015	0.027	0.027	0.020	0.040	0.045
.3	1000	0.001	0.024	0.032	0.028	0.028	0.047	0.041
.45	1000	0.001	0.032	0.038	0.035	0.038	0.049	0.044
.6	1000	0.000	0.036	0.038	0.037	0.036	0.043	0.036

Table 1C-3

Type 1 Error in the mediated effect through the distribution of product method in the finite model when general factor variance is .49 (gen λ=.7)

		a-path						
		zero	small	medium	large	zero	zero	zero
		b-path						
		zero	zero	zero	zero	small	medium	large
spec λ	N							
.3	200	0.000	0.009	0.025	0.023	0.007	0.038	0.049
.45	200	0.000	0.020	0.047	0.044	0.011	0.043	0.048
.6	200	0.000	0.018	0.048	0.051	0.019	0.045	0.046
.3	500	0.000	0.016	0.032	0.033	0.017	0.035	0.036
.45	500	0.000	0.021	0.044	0.041	0.027	0.038	0.040
.6	500	0.000	0.024	0.044	0.043	0.025	0.039	0.043
.3	1000	0.001	0.037	0.045	0.045	0.037	0.046	0.048
.45	1000	0.001	0.044	0.048	0.047	0.036	0.049	0.044
.6	1000	0.002	0.046	0.049	0.049	0.040	0.044	0.041

Table 1D-1

Confidence Interval Coverage in the mediated effect through the distribution of product method in the finite model when general factor variance is .09 (gen λ=.3)

		a-path small			a-path medium			a-path large		
		b-path small	b-path medium	b-path large	b-path small	b-path medium	b-path large	b-path small	b-path medium	b-path large
spec λ	N									
.3	200	0.989	0.961	0.967	0.966	0.954	0.951	0.964	0.928	0.924
.45	200	0.970	0.950	0.955	0.949	0.942	0.951	0.949	0.925	0.927
.6	200	0.953	0.950	0.953	0.942	0.943	0.953	0.944	0.931	0.937
.3	500	0.950	0.967	0.962	0.950	0.952	0.954	0.949	0.935	0.932
.45	500	0.947	0.958	0.960	0.951	0.948	0.956	0.958	0.948	0.945
.6	500	0.946	0.953	0.955	0.959	0.955	0.953	0.964	0.956	0.955
.3	1000	0.950	0.963	0.960	0.955	0.953	0.952	0.959	0.946	0.944
.45	1000	0.953	0.952	0.956	0.955	0.962	0.962	0.963	0.961	0.957
.6	1000	0.955	0.949	0.954	0.958	0.953	0.955	0.971	0.965	0.961

Table 1D-2

Confidence Interval Coverage in the mediated effect through the distribution of product method in the finite model when general factor variance is .25 (gen λ=.5)

		a-path small			a-path medium			a-path large		
		b-path small	b-path medium	b-path large	b-path small	b-path medium	b-path large	b-path small	b-path medium	b-path large
spec λ	N									
.3	200	0.989	0.973	0.968	0.970	0.971	0.973	0.968	0.956	0.954
.45	200	0.969	0.962	0.962	0.953	0.953	0.958	0.960	0.943	0.948
.6	200	0.959	0.952	0.954	0.942	0.948	0.957	0.947	0.942	0.948
.3	500	0.961	0.961	0.962	0.951	0.955	0.960	0.952	0.942	0.946
.45	500	0.951	0.951	0.953	0.949	0.951	0.957	0.958	0.951	0.961
.6	500	0.951	0.955	0.949	0.959	0.952	0.956	0.963	0.959	0.958
.3	1000	0.947	0.959	0.959	0.955	0.955	0.960	0.964	0.959	0.956
.45	1000	0.947	0.947	0.950	0.956	0.952	0.953	0.960	0.954	0.957
.6	1000	0.952	0.950	0.954	0.958	0.943	0.947	0.962	0.943	0.944

Table 1D-3

Confidence Interval Coverage in the mediated effect through the distribution of product method in the finite model when general factor variance is .49 (gen λ=.7)

		a-path: small			a-path: medium			a-path: large		
		b-path: small	b-path: medium	b-path: large	b-path: small	b-path: medium	b-path: large	b-path: small	b-path: medium	b-path: large
spec λ	N									
.3	200	0.974	0.971	0.967	0.971	0.979	0.980	0.968	0.974	0.975
.45	200	0.956	0.962	0.963	0.954	0.957	0.964	0.962	0.953	0.962
.6	200	0.953	0.961	0.959	0.949	0.955	0.964	0.947	0.956	0.963
.3	500	0.959	0.964	0.956	0.960	0.960	0.961	0.956	0.960	0.956
.45	500	0.951	0.955	0.955	0.951	0.952	0.954	0.954	0.949	0.949
.6	500	0.951	0.955	0.954	0.950	0.953	0.946	0.951	0.942	0.951
.3	1000	0.956	0.956	0.956	0.948	0.951	0.957	0.951	0.948	0.954
.45	1000	0.949	0.955	0.955	0.949	0.948	0.947	0.951	0.948	0.942
.6	1000	0.943	0.952	0.952	0.946	0.945	0.947	0.947	0.941	0.940

Table 2A-1

Standardized Bias in the mediated effect for the facet model with nonzero effects when general factor variance is .09 (gen λ=.3)

		a-path								
		small			medium			large		
		b-path								
spec λ	N	small	medium	large	small	medium	large	small	medium	large
.3	200	-0.557	-0.876	-0.986	-0.920	-1.918	-2.365	-1.043	-2.419	-3.152
.45	200	-0.532	-0.793	-0.844	-0.825	-1.678	-1.999	-0.895	-2.044	-2.625
.6	200	-0.447	-0.637	-0.667	-0.668	-1.354	-1.613	-0.718	-1.653	-2.144
.3	500	-1.158	-1.661	-1.790	-1.736	-3.484	-4.145	-1.918	-4.323	-5.483
.45	500	-1.027	-1.375	-1.414	-1.447	-2.852	-3.349	-1.539	-3.483	-4.444
.6	500	-0.803	-1.069	-1.098	-1.143	-2.275	-2.677	-1.221	-2.796	-3.598
.3	1000	-1.700	-2.438	-2.630	-2.445	-4.860	-5.830	-2.682	-5.978	-7.608
.45	1000	-1.432	-1.964	-2.044	-1.990	-3.959	-4.698	-2.120	-4.814	-6.167
.6	1000	-1.105	-1.522	-1.587	-1.558	-3.149	-3.751	-1.672	-3.855	-4.985

Table 2A-2

Standardized Bias in the mediated effect for the facet model with nonzero effects when general factor variance is .25 (gen λ=.5)

		a-path								
		small			medium			large		
		b-path								
		small	medium	large	small	medium	large	small	medium	large
spec λ	N									
.3	200	-0.847	-1.326	-1.461	-1.362	-2.895	-3.587	-1.516	-3.618	-4.843
.45	200	-0.796	-1.201	-1.302	-1.239	-2.614	-3.197	-1.357	-3.234	-4.290
.6	200	-0.706	-1.031	-1.103	-1.064	-2.224	-2.697	-1.160	-2.746	-3.620
.3	500	-1.570	-2.232	-2.418	-2.312	-4.893	-6.028	-2.554	-6.167	-8.254
.45	500	-1.450	-2.024	-2.148	-2.105	-4.379	-5.289	-2.291	-5.437	-7.154
.6	500	-1.254	-1.726	-1.814	-1.807	-3.720	-4.456	-1.957	-4.606	-6.028
.3	1000	-2.183	-3.162	-3.481	-3.185	-6.772	-8.418	-3.541	-8.511	-11.382
.45	1000	-2.011	-2.862	-3.070	-2.890	-6.035	-7.331	-3.155	-7.462	-9.822
.6	1000	-1.730	-2.441	-2.595	-2.474	-5.123	-6.190	-2.689	-6.325	-8.289

Table 2A-3

Standardized Bias in the mediated effect for the facet model with nonzero effects when general factor variance is .49 (gen λ=.7)

		a-path								
		small			medium			large		
		b-path								
spec λ	N	small	medium	large	small	medium	large	small	medium	large
.3	200	-1.186	-1.786	-1.848	-1.807	-3.613	-4.151	-1.918	-4.271	-5.388
.45	200	-1.038	-1.562	-1.665	-1.587	-3.309	-4.006	-1.718	-4.047	-5.325
.6	200	-0.933	-1.380	-1.480	-1.420	-2.999	-3.649	-1.552	-3.714	-4.925
.3	500	-2.259	-3.054	-3.082	-3.179	-6.120	-6.871	-3.284	-7.184	-8.772
.45	500	-1.906	-2.631	-2.749	-2.734	-5.597	-6.636	-2.921	-6.843	-8.860
.6	500	-1.673	-2.319	-2.451	-2.405	-5.022	-6.053	-2.605	-6.222	-8.207
.3	1000	-3.179	-4.313	-4.378	-4.391	-8.427	-9.463	-4.541	-9.855	-11.963
.45	1000	-2.650	-3.705	-3.902	-3.754	-7.673	-9.138	-4.024	-9.361	-12.092
.6	1000	-2.303	-3.256	-3.472	-3.293	-6.868	-8.314	-3.578	-8.491	-11.167

Table 2A-4

Relative Bias in the mediated effect for the facet model with nonzero effects when general factor variance is .09 (gen λ=.3)

		a-path small			a-path medium			a-path large		
		b-path small	medium	large	small	medium	large	small	medium	large
spec λ	N									
.3	200	-0.429	-0.444	-0.456	-0.491	-0.495	-0.501	-0.545	-0.547	-0.552
.45	200	-0.391	-0.396	-0.396	-0.443	-0.437	-0.434	-0.485	-0.477	-0.474
.6	200	-0.331	-0.323	-0.316	-0.367	-0.356	-0.350	-0.402	-0.392	-0.387
.3	500	-0.470	-0.482	-0.488	-0.520	-0.523	-0.526	-0.567	-0.566	-0.568
.45	500	-0.423	-0.414	-0.406	-0.451	-0.444	-0.439	-0.487	-0.482	-0.478
.6	500	-0.339	-0.329	-0.321	-0.367	-0.359	-0.352	-0.401	-0.394	-0.389
.3	1000	-0.481	-0.489	-0.491	-0.521	-0.527	-0.528	-0.564	-0.569	-0.570
.45	1000	-0.417	-0.412	-0.405	-0.445	-0.443	-0.438	-0.480	-0.480	-0.477
.6	1000	-0.330	-0.327	-0.321	-0.359	-0.357	-0.352	-0.393	-0.392	-0.388

Table 2A-5

Relative Bias in the mediated effect for the facet model with nonzero effects when general factor variance is .25 (gen λ=.5)

		a-path small			a-path medium			a-path large		
		b-path								
spec λ	N	small	medium	large	small	medium	large	small	medium	large
.3	200	-0.540	-0.547	-0.551	-0.591	-0.591	-0.593	-0.636	-0.634	-0.635
.45	200	-0.497	-0.502	-0.504	-0.551	-0.546	-0.546	-0.593	-0.589	-0.589
.6	200	-0.453	-0.449	-0.448	-0.494	-0.488	-0.486	-0.536	-0.530	-0.529
.3	500	-0.555	-0.556	-0.560	-0.591	-0.593	-0.597	-0.635	-0.636	-0.640
.45	500	-0.512	-0.512	-0.513	-0.549	-0.548	-0.549	-0.592	-0.591	-0.591
.6	500	-0.457	-0.455	-0.453	-0.493	-0.490	-0.488	-0.535	-0.532	-0.530
.3	1000	-0.545	-0.552	-0.558	-0.584	-0.590	-0.596	-0.629	-0.635	-0.640
.45	1000	-0.504	-0.509	-0.511	-0.542	-0.546	-0.547	-0.585	-0.588	-0.590
.6	1000	-0.449	-0.452	-0.452	-0.486	-0.488	-0.487	-0.527	-0.529	-0.529

Table 2A-6

Relative Bias in the mediated effect for the facet model when general factor variance is .49 (gen λ=.7)

		a-path small			a-path medium			a-path large		
		b-path small	b-path medium	b-path large	b-path small	b-path medium	b-path large	b-path small	b-path medium	b-path large
spec λ	N									
.3	200	-0.658	-0.650	-0.634	-0.687	-0.675	-0.660	-0.712	-0.699	-0.690
.45	200	-0.583	-0.582	-0.579	-0.623	-0.617	-0.613	-0.660	-0.653	-0.650
.6	200	-0.526	-0.525	-0.525	-0.567	-0.564	-0.563	-0.610	-0.606	-0.605
.3	500	-0.670	-0.658	-0.641	-0.692	-0.680	-0.664	-0.713	-0.702	-0.690
.45	500	-0.592	-0.588	-0.584	-0.625	-0.620	-0.616	-0.660	-0.656	-0.652
.6	500	-0.533	-0.532	-0.532	-0.569	-0.567	-0.566	-0.610	-0.608	-0.607
.3	1000	-0.664	-0.656	-0.640	-0.686	-0.677	-0.663	-0.706	-0.700	-0.687
.45	1000	-0.585	-0.586	-0.582	-0.617	-0.617	-0.614	-0.653	-0.653	-0.651
.6	1000	-0.525	-0.529	-0.529	-0.561	-0.564	-0.564	-0.602	-0.605	-0.605

Table 2B-1

Power in the mediated effect through the distribution of product method in the facet model when general factor variance is .09 (gen λ*=.3)*

		a-path								
		small			medium			large		
		b-path								
spec λ	N	small	medium	large	small	medium	large	small	medium	large
.3	200	0.029	0.099	0.103	0.156	0.625	0.634	0.226	0.858	0.896
.45	200	0.051	0.175	0.155	0.239	0.789	0.751	0.282	0.938	0.962
.6	200	0.074	0.233	0.203	0.303	0.882	0.849	0.339	0.975	0.994
.3	500	0.185	0.365	0.299	0.520	0.990	0.975	0.527	0.999	1.000
.45	500	0.264	0.448	0.362	0.590	0.998	0.993	0.606	1.000	1.000
.6	500	0.375	0.533	0.447	0.677	1.000	0.999	0.687	1.000	1.000
.3	1000	0.579	0.667	0.563	0.815	1.000	1.000	0.822	1.000	1.000
.45	1000	0.692	0.762	0.649	0.872	1.000	1.000	0.882	1.000	1.000
.6	1000	0.817	0.850	0.756	0.923	1.000	1.000	0.928	1.000	1.000

Table 2B-2

Power in the mediated effect through the distribution of product method in the facet model when general factor variance is .25 (gen λ=.5)

		a-path small			a-path medium			a-path large		
		b-path small	b-path medium	b-path large	b-path small	b-path medium	b-path large	b-path small	b-path medium	b-path large
spec λ	N									
.3	200	0.033	0.126	0.132	0.168	0.654	0.681	0.221	0.859	0.924
.45	200	0.042	0.168	0.158	0.221	0.769	0.745	0.266	0.925	0.958
.6	200	0.065	0.200	0.180	0.268	0.844	0.804	0.310	0.957	0.981
.3	500	0.187	0.380	0.313	0.494	0.994	0.976	0.506	0.999	1.000
.45	500	0.248	0.440	0.347	0.554	0.998	0.990	0.574	1.000	1.000
.6	500	0.312	0.494	0.402	0.621	0.999	0.996	0.632	1.000	1.000
.3	1000	0.558	0.662	0.560	0.794	1.000	1.000	0.811	1.000	1.000
.45	1000	0.667	0.740	0.620	0.851	1.000	1.000	0.864	1.000	1.000
.6	1000	0.741	0.799	0.692	0.893	1.000	1.000	0.900	1.000	1.000

Table 2B-3

Power in the mediated effect through the distribution of product method in the facet model when general factor variance is .49 (gen λ=.7)

		a-path: small			a-path: medium			a-path: large		
		b-path: small	b-path: medium	b-path: large	b-path: small	b-path: medium	b-path: large	b-path: small	b-path: medium	b-path: large
spec λ	N									
.3	200	0.024	0.106	0.128	0.142	0.551	0.621	0.196	0.775	0.883
.45	200	0.039	0.155	0.150	0.199	0.703	0.712	0.239	0.895	0.942
.6	200	0.055	0.201	0.178	0.254	0.829	0.792	0.304	0.950	0.980
.3	500	0.128	0.306	0.259	0.402	0.974	0.951	0.448	0.995	0.999
.45	500	0.209	0.406	0.323	0.527	0.995	0.983	0.530	0.999	1.000
.6	500	0.290	0.482	0.385	0.596	0.999	0.996	0.617	1.000	1.000
.3	1000	0.401	0.570	0.478	0.703	1.000	1.000	0.736	1.000	1.000
.45	1000	0.592	0.686	0.577	0.804	1.000	1.000	0.823	1.000	1.000
.6	1000	0.716	0.775	0.660	0.881	1.000	1.000	0.886	1.000	1.000

Table 2C-1

Type 1 Error in the mediated effect through the distribution of product method in the facet model when general factor variance is .09 (gen λ=.3)

		a-path						
		zero	small	medium	large	zero	zero	zero
		b-path						
		zero	zero	zero	zero	small	medium	large
spec λ	N							
.3	200	0.000	0.003	0.020	0.031	0.005	0.032	0.045
.45	200	0.000	0.006	0.045	0.052	0.009	0.040	0.051
.6	200	0.002	0.012	0.059	0.061	0.010	0.048	0.054
.3	500	0.000	0.010	0.032	0.033	0.010	0.044	0.038
.45	500	0.000	0.018	0.037	0.038	0.015	0.041	0.042
.6	500	0.000	0.022	0.040	0.040	0.020	0.044	0.046
.3	1000	0.001	0.030	0.048	0.048	0.025	0.035	0.037
.45	1000	0.001	0.039	0.050	0.048	0.026	0.036	0.039
.6	1000	0.001	0.043	0.048	0.047	0.036	0.041	0.036

Table 2C-2

Type 1 Error in the mediated effect through the distribution of product method in the facet model when general factor variance is .25 (gen λ=.5)

		a-path						
		zero	small	medium	large	zero	zero	zero
		b-path						
		zero	zero	zero	zero	small	medium	large
spec λ	N							
.3	200	0.000	0.007	0.036	0.046	0.007	0.030	0.042
.45	200	0.001	0.010	0.050	0.058	0.009	0.039	0.049
.6	200	0.002	0.011	0.056	0.060	0.010	0.045	0.052
.3	500	0.000	0.014	0.039	0.040	0.011	0.041	0.043
.45	500	0.000	0.017	0.039	0.038	0.014	0.039	0.042
.6	500	0.000	0.020	0.040	0.040	0.015	0.040	0.045
.3	1000	0.001	0.034	0.052	0.052	0.024	0.036	0.037
.45	1000	0.001	0.040	0.052	0.050	0.028	0.037	0.036
.6	1000	0.001	0.044	0.051	0.052	0.029	0.039	0.038

Table 2C-3

Type 1 Error in the mediated effect through the distribution of product method in the facet model when general factor variance is .49 (gen λ=.7)

		a-path						
		zero	small	medium	large	zero	zero	zero
		b-path						
		zero	zero	zero	zero	small	medium	large
spec λ	N							
.3	200	0.002	0.008	0.038	0.059	0.008	0.034	0.047
.45	200	0.002	0.011	0.048	0.060	0.008	0.038	0.048
.6	200	0.002	0.011	0.055	0.061	0.008	0.042	0.052
.3	500	0.000	0.009	0.040	0.042	0.008	0.045	0.043
.45	500	0.000	0.015	0.040	0.041	0.009	0.040	0.041
.6	500	0.000	0.019	0.041	0.042	0.012	0.041	0.041
.3	1000	0.001	0.025	0.049	0.050	0.021	0.040	0.039
.45	1000	0.001	0.035	0.050	0.051	0.026	0.038	0.039
.6	1000	0.001	0.042	0.048	0.050	0.033	0.040	0.036

Table 2D-1

Confidence Interval Coverage in the mediated effect through the distribution of product method in the facet model when general factor variance is .09 (gen λ=.3)

		a-path								
		small			medium			large		
		b-path								
		small	medium	large	small	medium	large	small	medium	large
spec λ	N									
.3	200	0.877	0.791	0.774	0.826	0.552	0.397	0.813	0.432	0.218
.45	200	0.863	0.807	0.804	0.835	0.612	0.486	0.842	0.500	0.319
.6	200	0.879	0.851	0.847	0.876	0.702	0.615	0.879	0.635	0.454
.3	500	0.761	0.596	0.540	0.647	0.172	0.056	0.589	0.072	0.008
.45	500	0.789	0.685	0.655	0.707	0.289	0.131	0.702	0.159	0.035
.6	500	0.839	0.779	0.759	0.790	0.452	0.276	0.794	0.308	0.105
.3	1000	0.606	0.371	0.286	0.393	0.017	0.001	0.324	0.002	0.000
.45	1000	0.691	0.494	0.448	0.535	0.065	0.009	0.507	0.014	0.000
.6	1000	0.786	0.649	0.621	0.670	0.180	0.063	0.666	0.072	0.005

Table 2D-2

Confidence Interval Coverage in the mediated effect through the distribution of product method in the facet model when general factor variance is .25 (gen λ=.5)

		a-path								
		small			medium			large		
		b-path								
		small	medium	large	small	medium	large	small	medium	large
spec λ	N									
.3	200	0.809	0.714	0.668	0.722	0.300	0.135	0.694	0.154	0.032
.45	200	0.810	0.739	0.704	0.748	0.370	0.198	0.730	0.206	0.055
.6	200	0.829	0.775	0.757	0.785	0.471	0.301	0.781	0.315	0.118
.3	500	0.679	0.456	0.351	0.479	0.022	0.000	0.394	0.002	0.000
.45	500	0.706	0.507	0.418	0.539	0.051	0.005	0.472	0.006	0.000
.6	500	0.744	0.588	0.538	0.622	0.116	0.021	0.578	0.030	0.000
.3	1000	0.494	0.191	0.097	0.198	0.000	0.000	0.117	0.000	0.000
.45	1000	0.542	0.247	0.158	0.261	0.001	0.000	0.187	0.000	0.000
.6	1000	0.611	0.357	0.270	0.377	0.007	0.000	0.320	0.000	0.000

Table 2D-3

Confidence Interval Coverage in the mediated effect through the distribution of product method in the facet model when general factor variance is .49 (gen λ=.7)

		a-path small			a-path medium			a-path large		
		b-path small	medium	large	small	medium	large	small	medium	large
spec λ	N									
.3	200	0.737	0.592	0.557	0.593	0.169	0.053	0.570	0.070	0.008
.45	200	0.767	0.650	0.611	0.664	0.205	0.073	0.631	0.097	0.010
.6	200	0.793	0.694	0.662	0.708	0.268	0.119	0.682	0.140	0.020
.3	500	0.530	0.238	0.182	0.263	0.002	0.000	0.208	0.000	0.000
.45	500	0.607	0.334	0.246	0.371	0.007	0.000	0.291	0.000	0.000
.6	500	0.654	0.417	0.325	0.464	0.014	0.000	0.393	0.000	0.000
.3	1000	0.287	0.045	0.016	0.048	0.000	0.000	0.030	0.000	0.000
.45	1000	0.374	0.095	0.051	0.106	0.000	0.000	0.055	0.000	0.000
.6	1000	0.466	0.152	0.088	0.166	0.000	0.000	0.112	0.000	0.000

Table 3A-1

Standardized Bias in the mediated effect for the unidimensional model with nonzero effects when the specific factor variance is .09 (spec λ=.3)

		a-path			
		small		medium	
		b-path			
		small	medium	small	medium
gen λ	N				
.3	200	-0.971	-1.472	-1.463	-2.773
.5	200	-1.877	-3.208	-3.193	-6.268
.7	200	-2.736	-5.388	-5.321	-11.580
.3	500	-1.837	-2.446	-2.497	-4.653
.5	500	-3.997	-5.843	-5.924	-11.159
.7	500	-6.291	-10.547	-10.568	-21.995
.3	1000	-2.595	-3.385	-3.452	-6.327
.5	1000	-5.960	-8.331	-8.479	-15.653
.7	1000	-9.903	-15.365	-15.547	-31.417

Note: Red are standardized bias above .1.

Table 3A-2

Relative Bias in the mediated effect for the unidimensional model with nonzero effects when the specific factor variance is .09 (spec λ=.3)

		a-path			
		small		medium	
		b-path			
		small	medium	small	medium
gen λ	N				
.3	200	-0.659	-0.645	-0.676	-0.663
.5	200	-0.857	-0.859	-0.873	-0.871
.7	200	-0.938	-0.942	-0.949	-0.950
.3	500	-0.659	-0.634	-0.663	-0.648
.5	500	-0.870	-0.865	-0.880	-0.876
.7	500	-0.947	-0.947	-0.954	-0.954
.3	1000	-0.649	-0.626	-0.655	-0.641
.5	1000	-0.867	-0.864	-0.879	-0.876
.7	1000	-0.945	-0.946	-0.953	-0.954

Table 3B-1

Power in the mediated effect through the distribution of product method for the unidimensional model with nonzero effects when the specific factor variance is .09 (spec λ=.3)

		a-path			
		small		medium	
		b-path			
		small	medium	small	medium
gen λ	N				
.3	200	0.012	0.102	0.071	0.450
.5	200	0.007	0.031	0.024	0.125
.7	200	0.002	0.009	0.010	0.027
.3	500	0.084	0.325	0.265	0.914
.5	500	0.017	0.122	0.085	0.489
.7	500	0.007	0.028	0.027	0.114
.3	1000	0.293	0.626	0.518	0.997
.5	1000	0.069	0.305	0.218	0.861
.7	1000	0.014	0.098	0.075	0.382

.

Table 3C-1

Type 1 Error in the mediated effect through the distribution of product method for the unidimensional model with nonzero effects when the specific factor variance is .09 (spec λ=.3)

		a-path				
		zero	small	medium	zero	zero
		b-path				
		zero	zero	zero	small	medium
gen λ	N					
.3	200	0.001	0.003	0.027	0.007	0.030
.5	200	0.001	0.002	0.008	0.006	0.009
.7	200	0.001	0.002	0.006	0.003	0.006
.3	500	0.000	0.006	0.043	0.006	0.041
.5	500	0.000	0.001	0.029	0.003	0.022
.7	500	0.000	0.001	0.012	0.003	0.008
.3	1000	0.002	0.020	0.055	0.016	0.039
.5	1000	0.001	0.010	0.048	0.008	0.038
.7	1000	0.001	0.002	0.028	0.004	0.029

Table 3D-1

Confidence Interval Coverage in the mediated effect through the distribution of product method in the unidimensional model when specific factor variance is .09 (spec λ=.3)

		a-path			
		small		medium	
		b-path			
		small	medium	small	medium
gen λ	N				
.3	200	0.803	0.620	0.619	0.277
.5	200	0.618	0.278	0.265	0.020
.7	200	0.459	0.101	0.098	0.001
.3	500	0.615	0.382	0.394	0.031
.5	500	0.225	0.024	0.019	0.000
.7	500	0.077	0.000	0.000	0.000
.3	1000	0.389	0.140	0.149	0.000
.5	1000	0.038	0.000	0.000	0.000
.7	1000	0.003	0.000	0.000	0.000

Table 4A-1

Standardized Bias in the mediated effect for the correlated model with nonzero effects when general factor variance is .49 (gen λ=.7)

		a-path small			a-path medium			a-path large		
		b-path small	b-path medium	b-path large	b-path small	b-path medium	b-path large	b-path small	b-path medium	b-path large
spec λ	N									
.3	200	-2.647	-5.361	-6.085	-5.385	-11.019	-12.828	-6.082	-13.346	-15.857
.45	200	-2.373	-4.367	-4.659	-4.409	-8.175	-8.714	-4.686	-9.197	-10.944
.6	200	-2.192	-3.725	-3.851	-3.758	-6.276	-6.908	-3.872	-7.045	n/a
.3	500	-6.389	-11.225	-11.991	-11.359	-22.530	-24.804	-12.327	-26.110	-29.285
.45	500	-5.553	-8.835	-8.670	-8.958	-15.988	-15.173	-9.089	-16.472	-17.035
.6	500	-5.024	-7.433	-6.575	-7.519	-11.767	-10.790	-6.997	-11.395	n/a
.3	1000	-10.749	-17.026	-17.792	-17.344	-33.404	-36.751	-18.340	-38.708	-44.209
.45	1000	-8.883	-13.062	-12.858	-13.345	-23.592	-22.754	-13.448	-24.661	-23.893
.6	1000	-7.873	-10.849	-9.728	-11.092	-17.648	-13.859	-10.365	-15.689	-21.698

Table 4A-2

Relative Bias in the mediated effect for the correlated model with nonzero effects when general factor variance is .49 (gen λ=.7)

		a-path small			a-path medium			a-path large		
		b-path small	medium	large	small	medium	large	small	medium	large
spec λ	N									
.3	200	-0.946	-0.951	-0.948	-0.957	-0.958	-0.956	-0.961	-0.962	-0.961
.45	200	-0.923	-0.924	-0.916	-0.934	-0.931	-0.925	-0.937	-0.935	-0.939
.6	200	-0.904	-0.900	-0.890	-0.911	-0.904	-0.906	-0.911	-0.915	n/a
.3	500	-0.962	-0.960	-0.957	-0.966	-0.964	-0.962	-0.969	-0.968	-0.965
.45	500	-0.940	-0.936	-0.927	-0.944	-0.939	-0.931	-0.945	-0.941	-0.937
.6	500	-0.922	-0.914	-0.894	-0.923	-0.913	-0.902	-0.919	-0.915	n/a
.3	1000	-0.960	-0.961	-0.958	-0.965	-0.965	-0.963	-0.969	-0.969	-0.967
.45	1000	-0.939	-0.937	-0.929	-0.944	-0.942	-0.934	-0.946	-0.943	-0.937
.6	1000	-0.921	-0.915	-0.898	-0.924	-0.917	-0.900	-0.922	-0.914	-0.919

Table 4B-1

Power in the mediated effect through the distribution of product method for the correlated model with nonzero effects when general factor variance is .49 (gen λ=.7)

		a-path small			a-path medium			a-path large		
		b-path small	medium	large	small	medium	large	small	medium	large
spec λ	N									
.3	200	0.002	0.005	0.011	0.007	0.021	0.031	0.019	0.041	0.060
.45	200	0.004	0.009	0.013	0.015	0.044	0.066	0.028	0.086	0.094
.6	200	0.004	0.013	0.026	0.020	0.071	0.085	0.038	0.111	n/a
.3	500	0.007	0.012	0.026	0.015	0.069	0.096	0.037	0.136	0.220
.45	500	0.007	0.028	0.053	0.035	0.148	0.222	0.079	0.313	0.408
.6	500	0.008	0.041	0.077	0.057	0.261	0.328	0.117	0.395	n/a
.3	1000	0.009	0.039	0.065	0.046	0.215	0.267	0.098	0.394	0.510
.45	1000	0.013	0.080	0.102	0.100	0.428	0.511	0.172	0.657	0.793
.6	1000	0.019	0.121	0.145	0.150	0.601	0.666	0.239	0.798	0.867

Table 4C-1

Type 1 Error in the mediated effect through the distribution of product method in the correlated model when general factor variance is .49 (gen λ=.7)

		a-path						
		zero	small	medium	large	zero	zero	zero
		b-path						
		zero	zero	zero	zero	small	medium	large
spec λ	N							
.3	200	0.001	0.001	0.004	0.005	0.003	0.005	0.007
.45	200	0.001	0.001	0.003	0.007	0.003	0.005	0.008
.6	200	0.001	0.000	0.005	0.009	0.004	0.009	0.017
.3	500	0.001	0.001	0.005	0.018	0.003	0.009	0.012
.45	500	0.000	0.002	0.012	0.026	0.003	0.011	0.025
.6	500	0.000	0.001	0.014	0.031	0.002	0.012	0.032
.3	1000	0.001	0.003	0.016	0.034	0.002	0.018	0.030
.45	1000	0.002	0.004	0.025	0.047	0.003	0.024	0.038
.6	1000	0.001	0.004	0.036	0.058	0.005	0.025	0.036

Table 4D-1

Confidence Interval Coverage in the mediated effect through the distribution of product method in the correlated model when general factor variance is .49 (gen λ=.7)

		a-path small			a-path medium			a-path large		
spec λ	N	b-path small	medium	large	small	medium	large	small	medium	large
.3	200	0.487	0.102	0.049	0.105	0.004	0.000	0.045	0.002	0.000
.45	200	0.529	0.160	0.112	0.168	0.013	0.007	0.117	0.007	0.001
.6	200	0.557	0.216	0.176	0.215	0.036	0.027	0.161	0.026	n/a
.3	500	0.075	0.001	0.000	0.001	0.000	0.000	0.000	0.000	0.000
.45	500	0.109	0.004	0.002	0.004	0.000	0.000	0.002	0.000	0.000
.6	500	0.142	0.012	0.015	0.011	0.002	0.001	0.019	0.002	n/a
.3	1000	0.003	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
.45	1000	0.007	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
.6	1000	0.012	0.000	0.000	0.000	0.000	0.000	0.001	0.000	0.000

Table 5A-1

Standardized Bias in the mediated effect for the unidimensional and correlated model with nonzero effects when the specific factor variance is .09 (spec λ=.3) and general factor variance is .49 (gen λ=.7)

				a-path			
				small		medium	
				b-path			
Model	gen λ	spec λ	N	small	medium	small	medium
Unidimensional Model	.7	.3	200	-2.736	-5.388	-5.321	-11.580
	.7	.3	500	-6.291	-10.547	-10.568	-21.995
	.7	.3	1000	-9.903	-15.365	-15.547	-31.417
Correlation Model	.7	.3	200	-2.647	-5.361	-5.385	-11.019
	.7	.3	500	-6.389	-11.225	-11.359	-22.530
	.7	.3	1000	-10.749	-17.026	-17.344	-33.404

Table 5A-2

Relative Bias in the mediated effect for the unidimensional and correlated model with nonzero effects when the specific factor variance is .09 (spec λ=.3) and general factor variance is .49 (gen λ=.7)

				a-path			
				small		medium	
				b-path			
Model	gen λ	spec λ	N	small	medium	small	medium
Unidimensional Model	.7	.3	200	-0.938	-0.942	-0.949	-0.950
	.7	.3	500	-0.947	-0.947	-0.954	-0.954
	.7	.3	1000	-0.945	-0.946	-0.953	-0.954
Correlation Model	.7	.3	200	-0.946	-0.951	-0.957	-0.958
	.7	.3	500	-0.962	-0.960	-0.966	-0.964
	.7	.3	1000	-0.960	-0.961	-0.965	-0.963

Table 5B-1

Power in the mediated effect through the distribution of product method for the unidimensional and correlated model with nonzero effects when the specific factor variance is .09 (spec λ=.3) and general factor variance is .49 (gen λ=.7)

				a-path			
				small		medium	
				b-path			
				small	medium	small	medium
Model	gen λ	spec λ	N				
Unidimensional Model	.7	.3	200	0.002	0.009	0.010	0.027
	.7	.3	500	0.007	0.028	0.027	0.114
	.7	.3	1000	0.014	0.098	0.075	0.382
Correlation Model	.7	.3	200	0.002	0.005	0.007	0.021
	.7	.3	500	0.007	0.012	0.015	0.069
	.7	.3	1000	0.009	0.039	0.046	0.215

Table 5C-1

Type 1 Error in the mediated effect through the distribution of product method for the unidimensional and correlated model with nonzero effects when the specific factor variance is .09 (spec λ=.3) and general factor variance is .49 (gen λ=.7)

				a-path				
				zero	small	medium	zero	zero
				b-path				
				zero	zero	zero	small	medium
Model	gen λ	spec λ	N					
Unidimensional Model	.7	.3	200	0.001	0.002	0.006	0.003	0.006
	.7	.3	500	0.000	0.001	0.012	0.003	0.008
	.7	.3	1000	0.001	0.002	0.028	0.004	0.029
Correlation Model	.7	.3	200	0.001	0.001	0.004	0.003	0.005
	.7	.3	500	0.001	0.001	0.005	0.003	0.009
	.7	.3	1000	0.001	0.003	0.016	0.002	0.018

Table 5D-1

Confidence Interval Coverage in the mediated effect through the distribution of product method for the unidimensional and correlated model with nonzero effects when the specific factor variance is .09 (spec λ=.3) and general factor variance is .49 (gen λ=.7)

				a-path			
				small		medium	
				b-path			
				small	medium	small	medium
Model	gen λ	spec λ	N				
Unidimensional Model	.7	.3	200	0.459	0.101	0.098	0.001
	.7	.3	500	0.077	0.000	0.000	0.000
	.7	.3	1000	0.003	0.000	0.000	0.000
Correlation Model	.7	.3	200	0.487	0.102	0.105	0.004
	.7	.3	500	0.075	0.001	0.001	0.000
	.7	.3	1000	0.003	0.000	0.000	0.000

Table 6-1

Difference in power for the mediated effect through the distribution of product method minus the asymptotic normal theory in the finite model when general factor variance is .09 (gen λ=.3)

		a-path								
		small			medium			large		
		b-path								
spec λ	N	small	medium	large	small	medium	large	small	medium	large
.3	200	0.010	0.062	0.127	0.025	0.154	0.199	0.010	0.086	0.111
.45	200	0.027	0.143	0.132	0.059	0.190	0.159	0.031	0.099	0.068
.6	200	0.066	0.143	0.115	0.097	0.130	0.055	0.029	0.067	0.021
.3	500	0.101	0.230	0.189	0.062	0.045	0.028	0.022	0.026	0.013
.45	500	0.187	0.123	0.078	0.068	0.006	0.002	0.030	0.005	0.004
.6	500	0.213	0.080	0.045	0.062	0.000	0.000	0.028	0.000	0.000
.3	1000	0.256	0.091	0.058	0.035	0.000	0.000	0.021	0.001	0.002
.45	1000	0.231	0.031	0.021	0.019	0.000	0.000	0.011	0.000	0.000
.6	1000	0.179	0.012	0.006	0.015	0.000	0.000	0.007	0.000	0.000

Note: Red values are a difference above .01

Table 6-2

Difference in power for the mediated effect through the distribution of product method minus the asymptotic normal theory in the finite model when general factor variance is .25 (gen λ=.5)

		a-path								
		small			medium			large		
		b-path								
spec λ	N	small	medium	large	small	medium	large	small	medium	large
.3	200	0.017	0.127	0.153	0.050	0.204	0.205	0.017	0.105	0.083
.45	200	0.052	0.176	0.142	0.106	0.182	0.078	0.043	0.073	0.028
.6	200	0.061	0.170	0.098	0.106	0.117	0.026	0.049	0.060	0.010
.3	500	0.174	0.185	0.093	0.085	0.015	0.002	0.033	0.010	0.002
.45	500	0.208	0.073	0.051	0.074	0.000	0.000	0.032	0.002	0.000
.6	500	0.205	0.068	0.023	0.060	0.000	0.000	0.030	0.000	0.000
.3	1000	0.283	0.056	0.028	0.030	0.000	0.000	0.015	0.000	0.000
.45	1000	0.196	0.020	0.008	0.021	0.000	0.000	0.014	0.000	0.000
.6	1000	0.167	0.011	0.003	0.013	0.000	0.000	0.009	0.000	0.000

Note: Red values are a difference above .01

Table 6-3

Difference in power for the mediated effect through the distribution of product method minus the asymptotic normal theory in the finite model when general factor variance is .49 (gen λ=.7)

		a-path small			a-path medium			a-path large		
		b-path small	b-path medium	b-path large	b-path small	b-path medium	b-path large	b-path small	b-path medium	b-path large
spec λ	N									
.3	200	0.044	0.203	0.179	0.091	0.179	0.079	0.031	0.083	0.029
.45	200	0.073	0.147	0.076	0.110	0.061	0.007	0.052	0.018	0.002
.6	200	0.099	0.127	0.061	0.111	0.019	0.002	0.035	0.014	0.000
.3	500	0.247	0.103	0.052	0.054	0.000	0.000	0.017	0.001	0.000
.45	500	0.250	0.054	0.034	0.051	0.000	0.000	0.027	0.000	0.000
.6	500	0.232	0.045	0.009	0.026	0.000	0.000	0.017	0.000	0.000
.3	1000	0.212	0.028	0.005	0.014	0.000	0.000	0.009	0.000	0.000
.45	1000	0.140	0.005	0.003	0.010	0.000	0.000	0.004	0.000	0.000
.6	1000	0.099	0.001	0.000	0.007	0.000	0.000	0.002	0.000	0.000

Note: Red values are a difference above .01

APPENDIX C

FIGURES

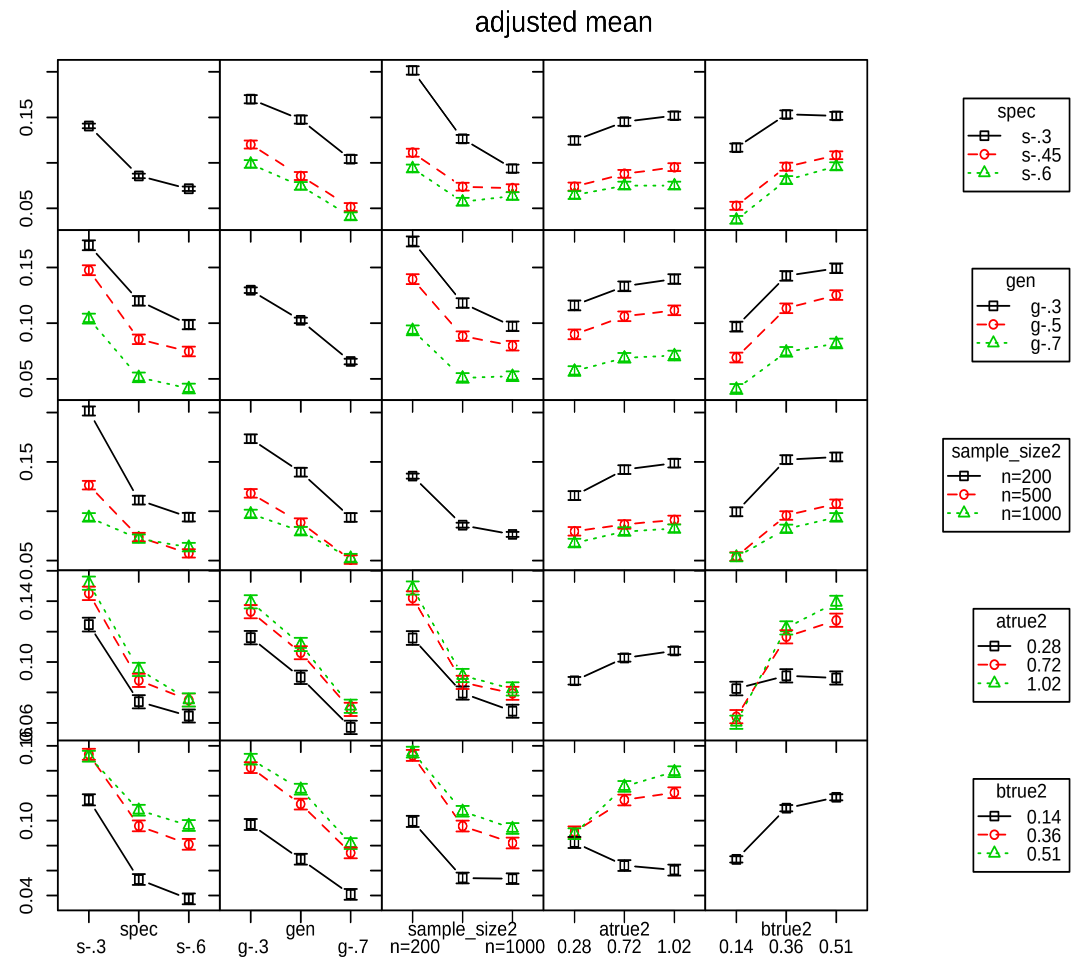


Figure 1A-1. Standardized bias in the mediated effect for the finite model with nonzero mediated effects. spec= specific factor variance; gen=general factor variance; sample_size2=sample size; atrue2=*a*-path effect size; btrue2=*b*-path effect size.

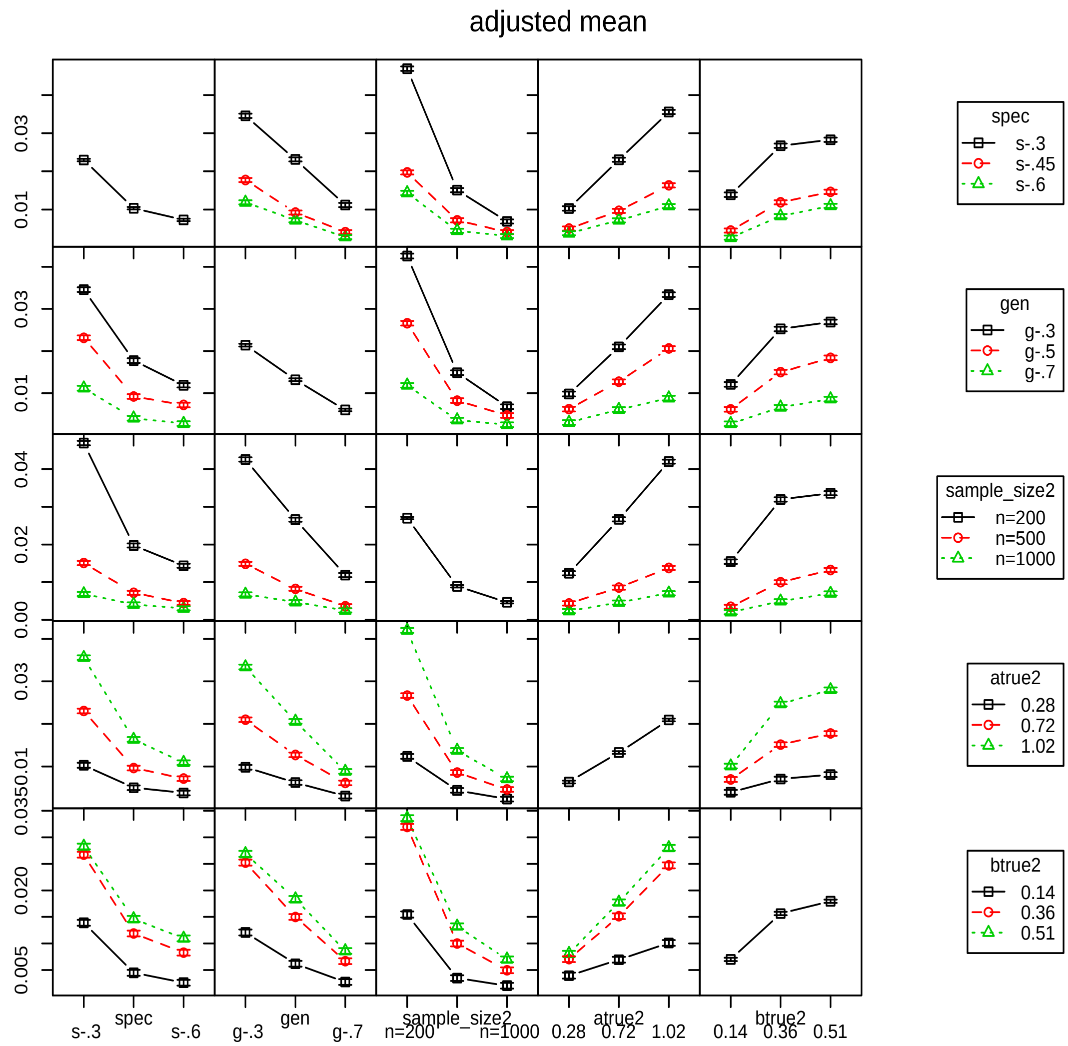


Figure 1A-2. Raw bias in the mediated effect for the finite model with nonzero effects. spec= specific factor variance; gen=general factor variance; sample_size2=sample size; atrue2=*a*-path effect size; btrue2=*b*-path effect size.

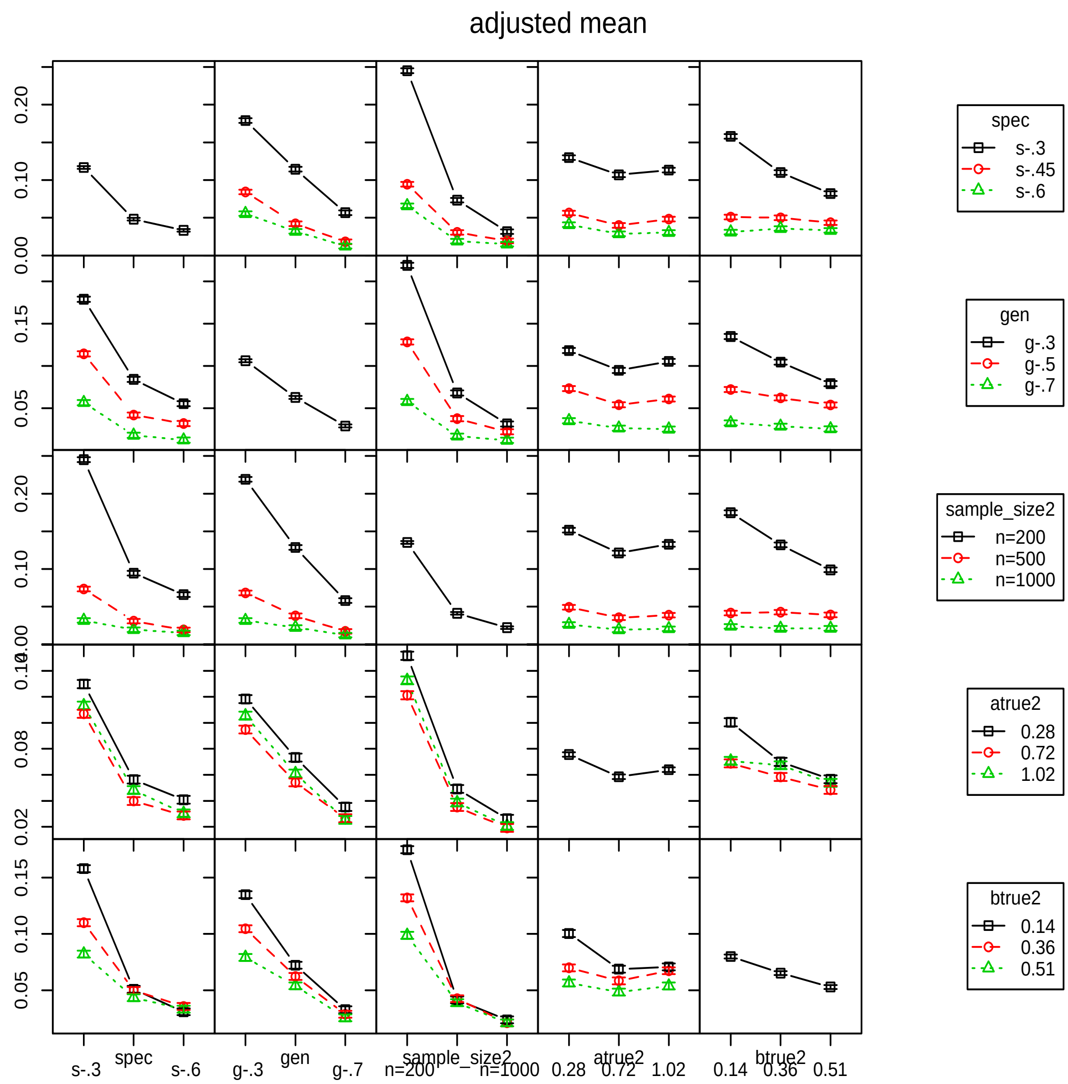


Figure 1A-3. Relative bias in the mediated effect for the finite model with nonzero mediated effects. spec= specific factor variance; gen=general factor variance; sample_size2=sample size; atrue2=*a*-path effect size; btrue2=*b*-path effect size.

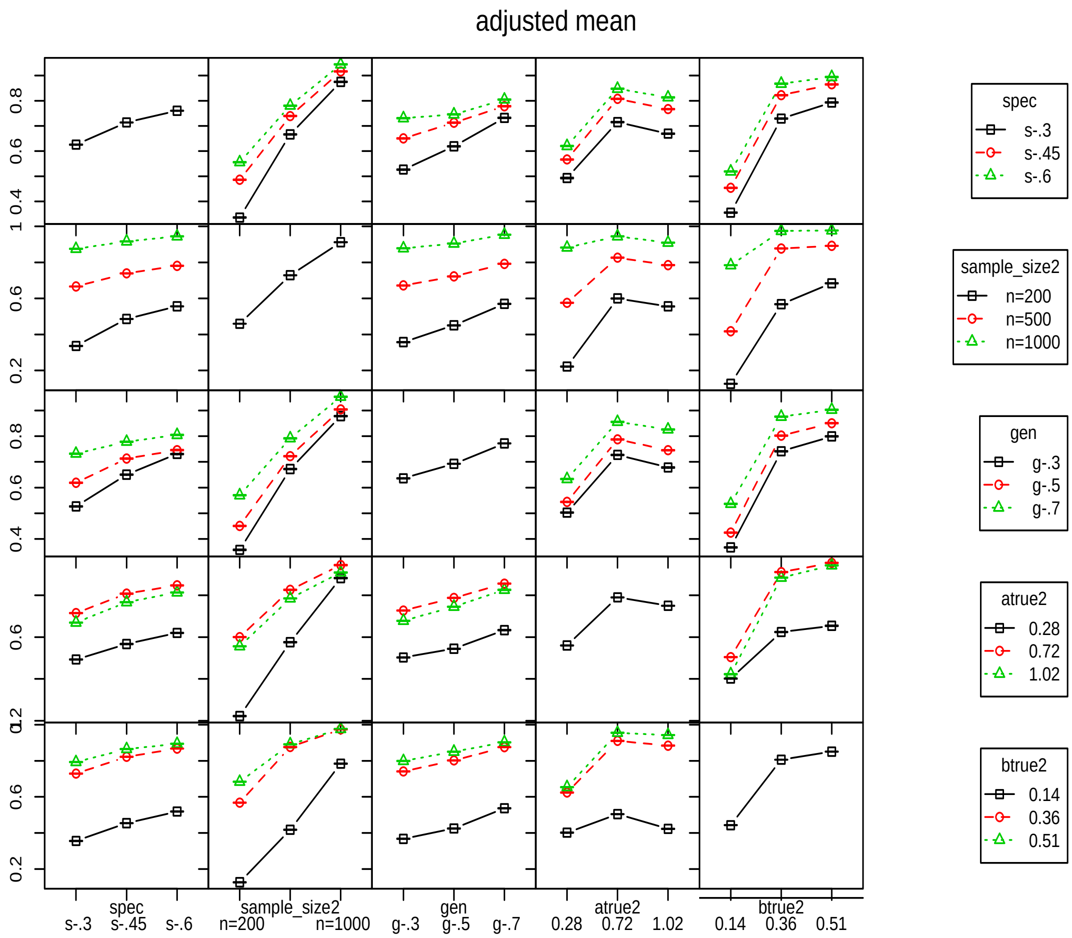


Figure 1B-1. Power in the mediated effect through the distribution of product method in the finite model. spec= specific factor variance; gen=general factor variance; sample_size2=sample size; atrue2=*a*-path effect size; btrue2=*b*-path effect size.

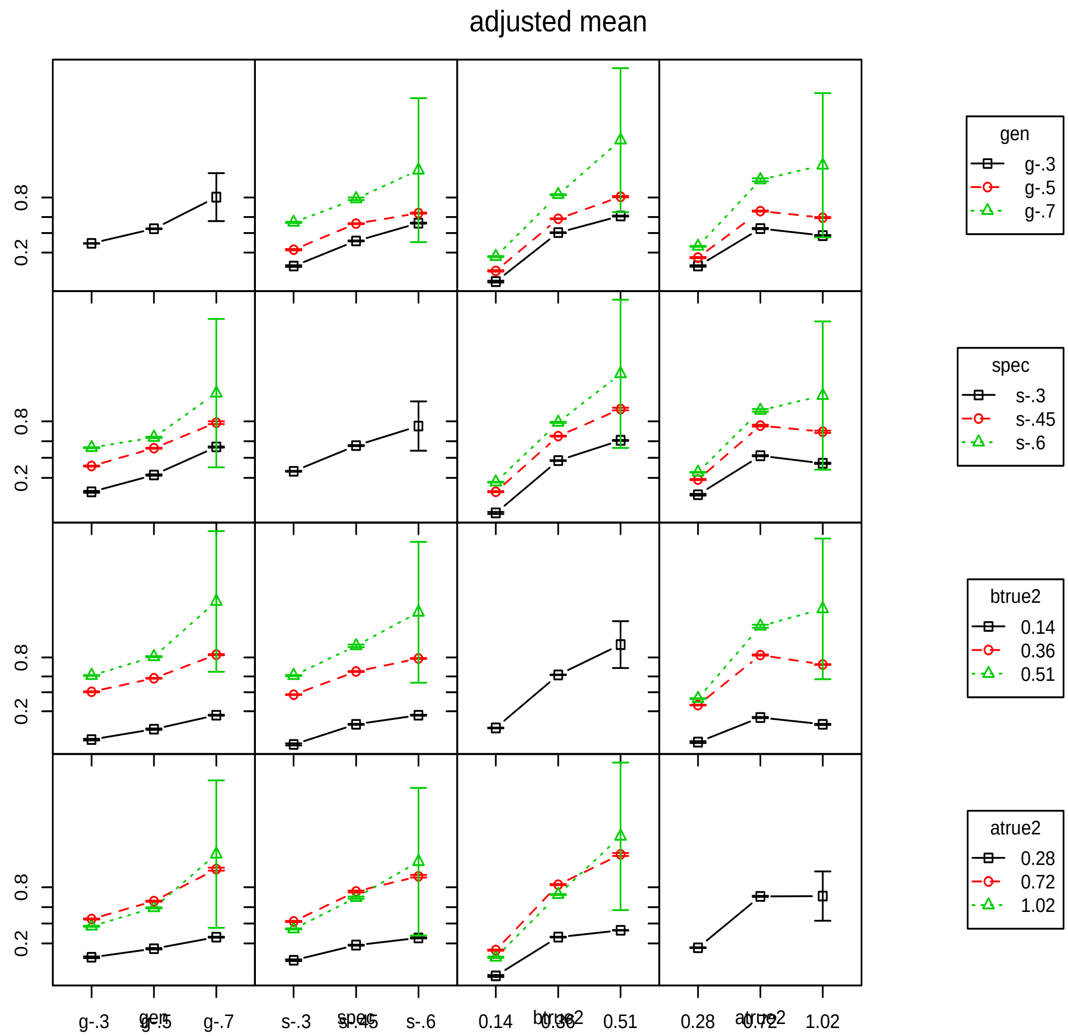


Figure 1B-2. Power in the mediated effect through the distribution of product method in the finite model when the sample size is small. spec= specific factor variance; gen=general factor variance; sample_size2=sample size; atrue2=*a*-path effect size; btrue2=*b*-path effect size.

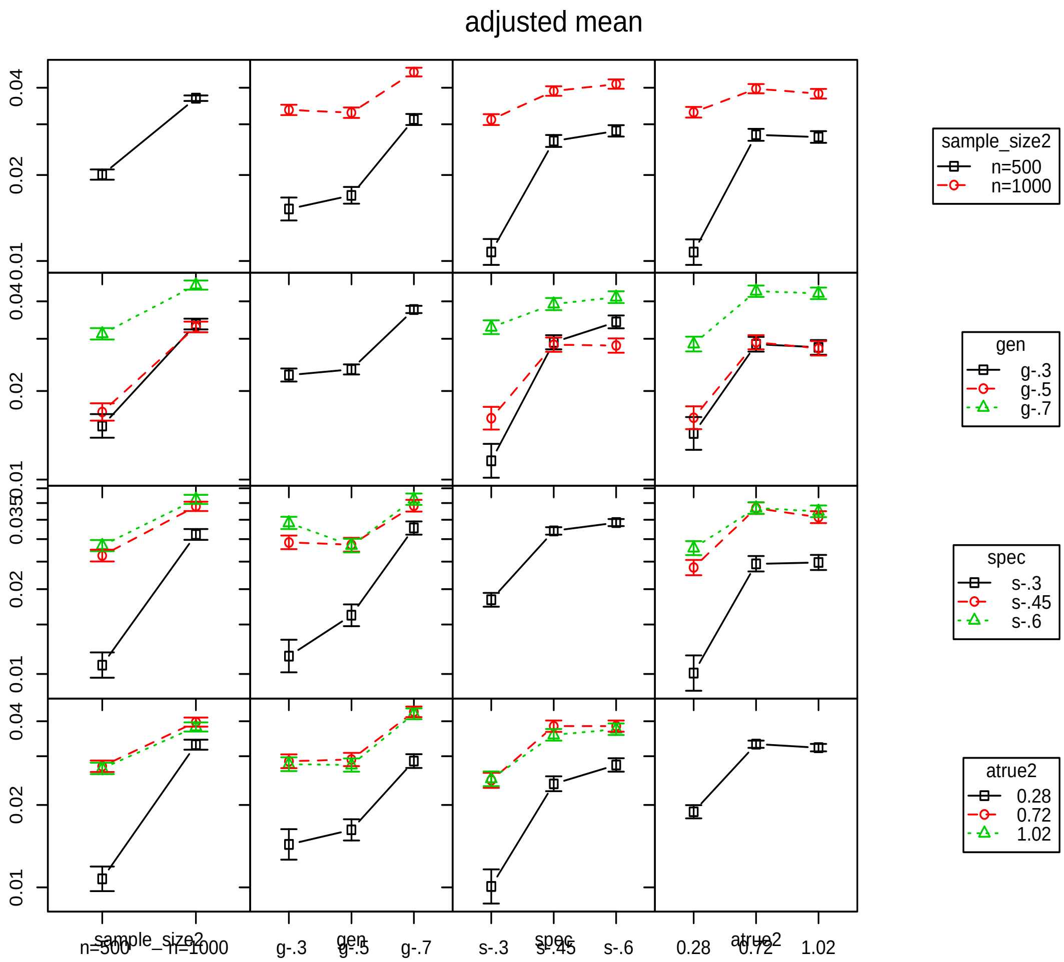


Figure 1C-1. Type 1 error in the mediated effect through the distribution of product method in the finite model when the a-path has a nonzero effect and sample size is greater than 500. spec= specific factor variance; gen=general factor variance; sample_size2=sample size; atrue2=*a*-path effect size; btrue2=*b*-path effect size.

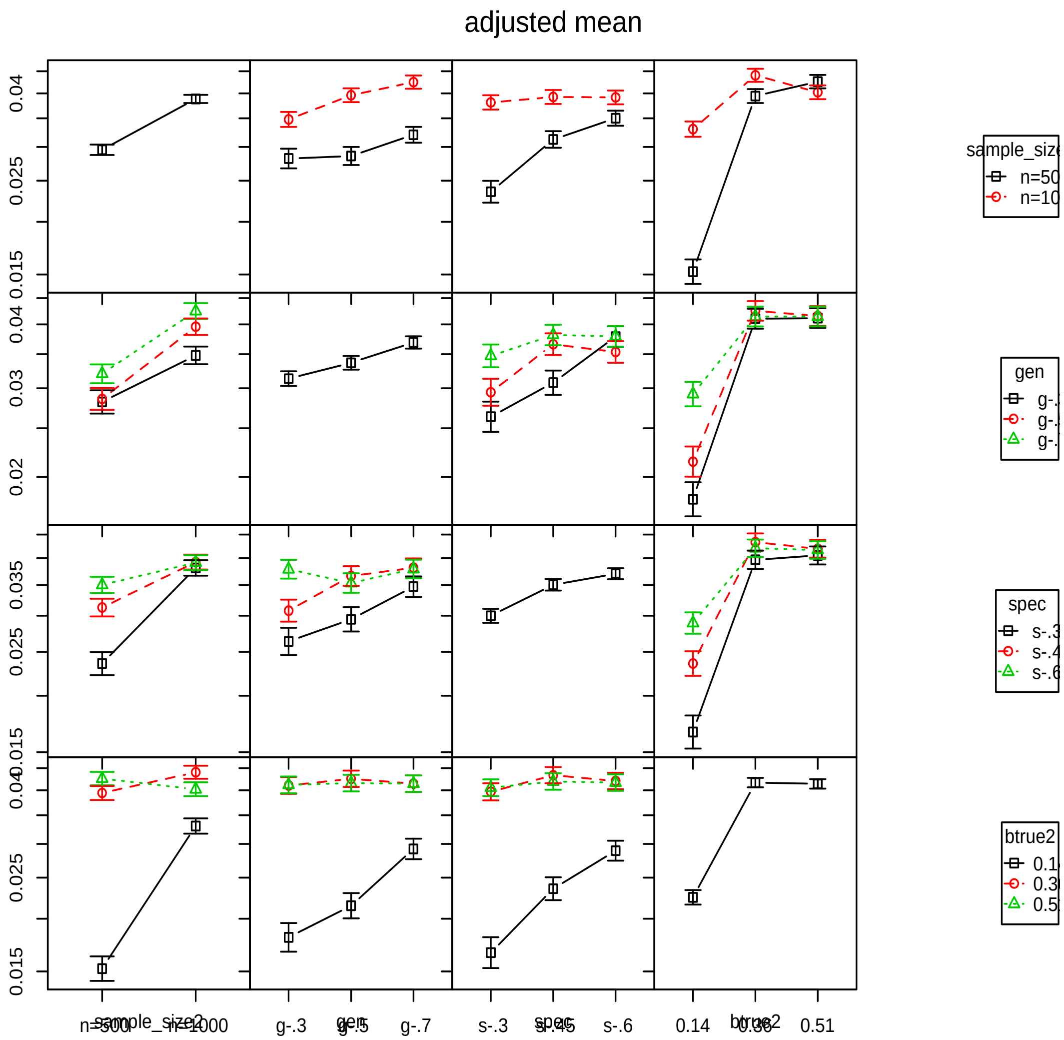


Figure 1C-2. Type 1 error in the mediated effect through the distribution of product method in the finite model when the b-path has a nonzero effect and sample size is greater than 500. spec= specific factor variance; gen=general factor variance; sample_size2=sample size; atrue2=*a*-path effect size; btrue2=*b*-path effect size.

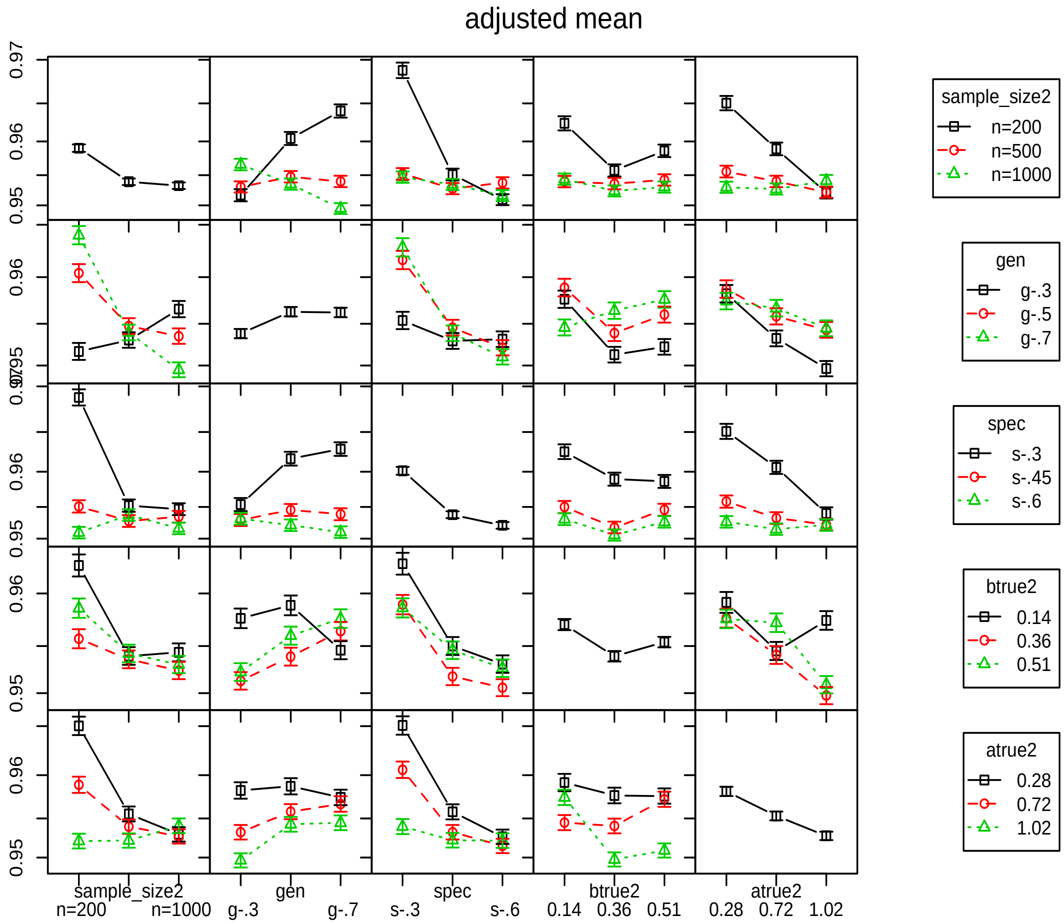


Figure 1D-1. Confidence interval coverage in the mediated effect through the distribution of product method in the finite model. spec= specific factor variance; gen=general factor variance; sample_size2=sample size; atrue2=*a*-path effect size; btrue2=*b*-path effect size.

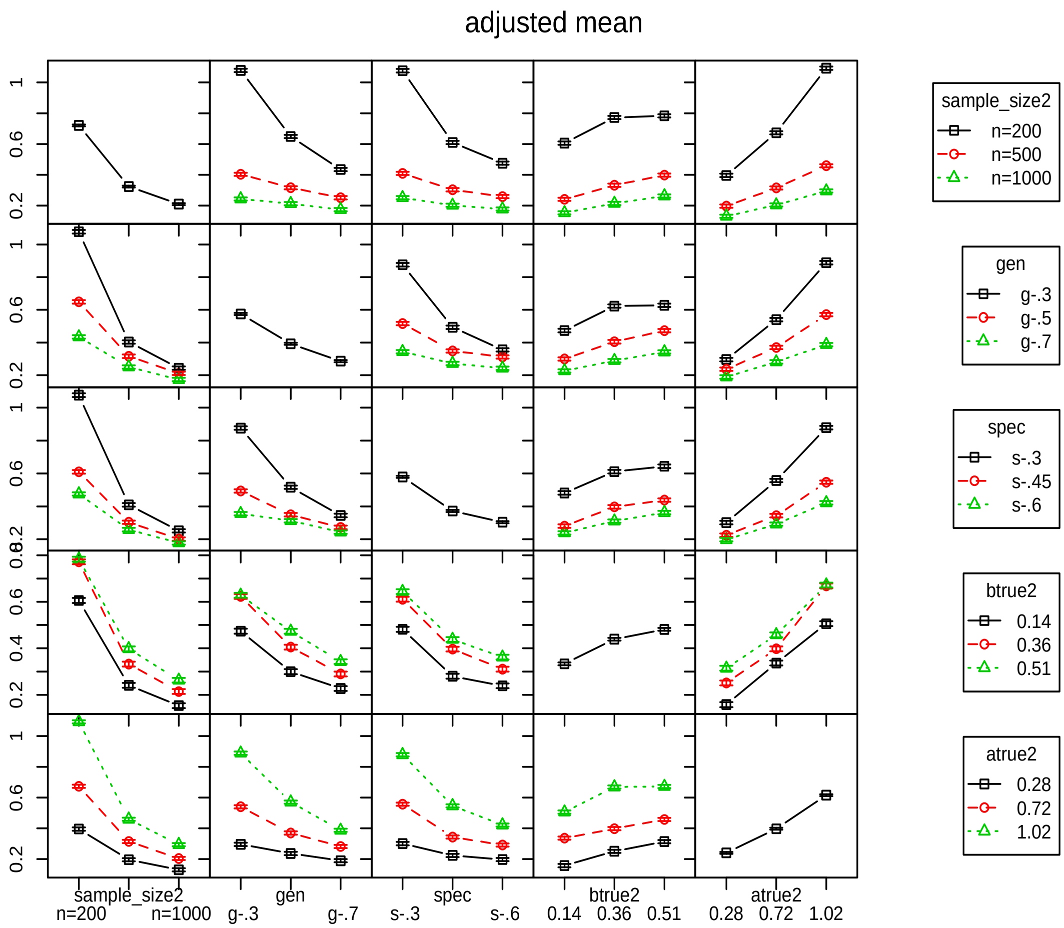


Figure 1E-1. Confidence interval width in the mediated effect through the distribution of product method in the finite model. spec= specific factor variance; gen=general factor variance; sample_size2=sample size; atrue2=*a*-path effect size; btrue2=*b*-path effect size.

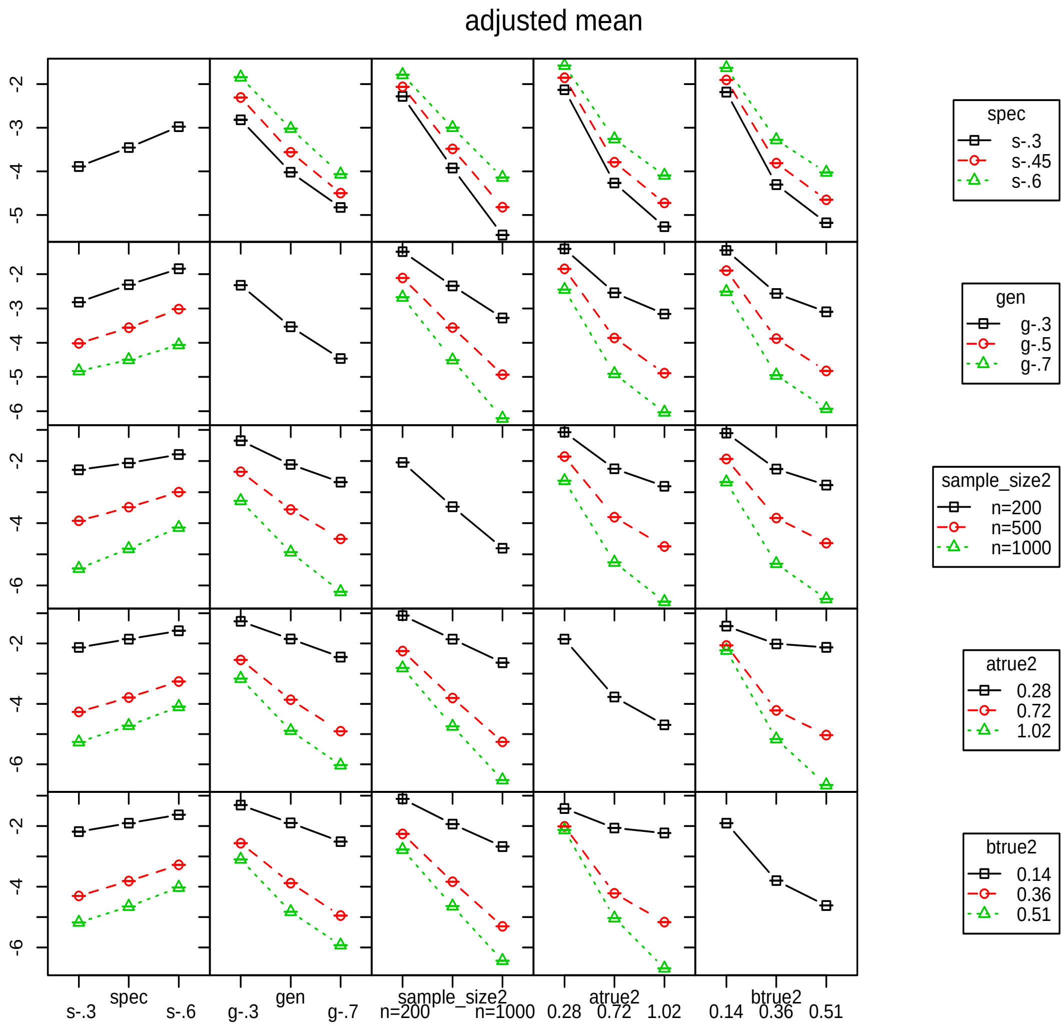


Figure 2A-1. Standardized bias in the mediated effect for the facet model with nonzero mediated effects. spec= specific factor variance; gen=general factor variance; sample_size2=sample size; atrue2=*a*-path effect size; btrue2=*b*-path effect size.

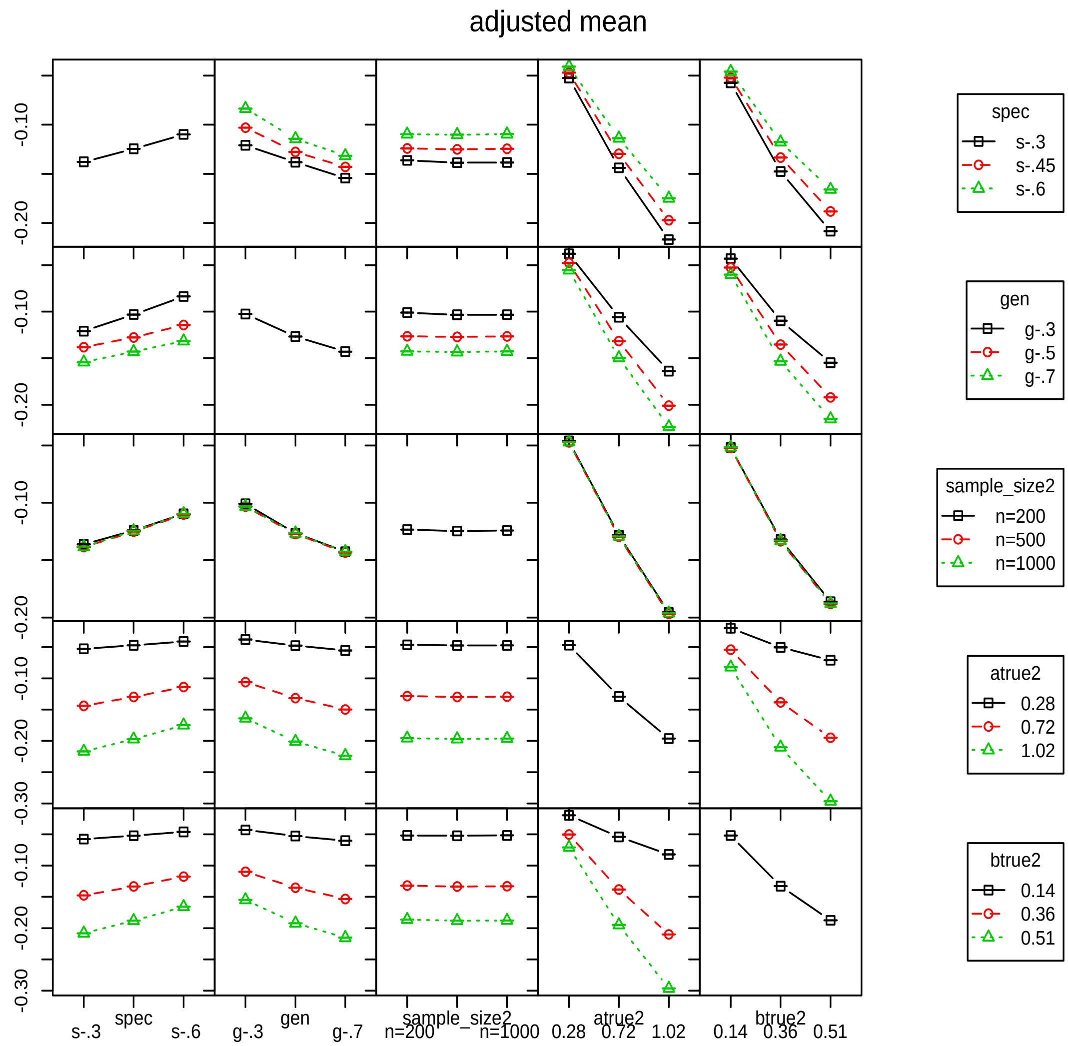


Figure 2A-2. Raw bias in the mediated effect for the facet model with zero mediated effects. spec= specific factor variance; gen=general factor variance; sample_size2=sample size; atrue2=*a*-path effect size; btrue2=*b*-path effect size.

.

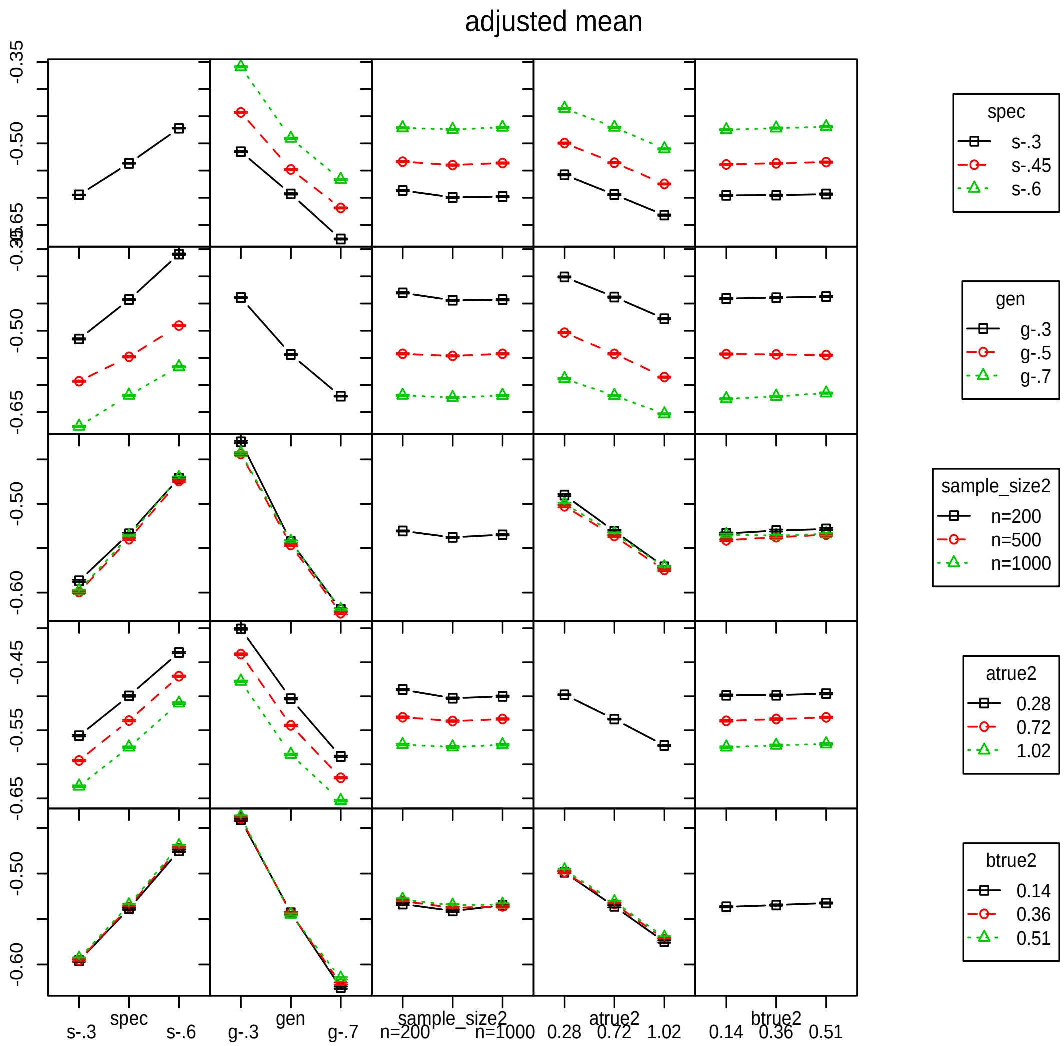


Figure 2A-3. Relative bias in the mediated effect for the facet model with nonzero mediated effects. spec= specific factor variance; gen=general factor variance; sample_size2=sample size; atrue2=*a*-path effect size; btrue2=*b*-path effect size.

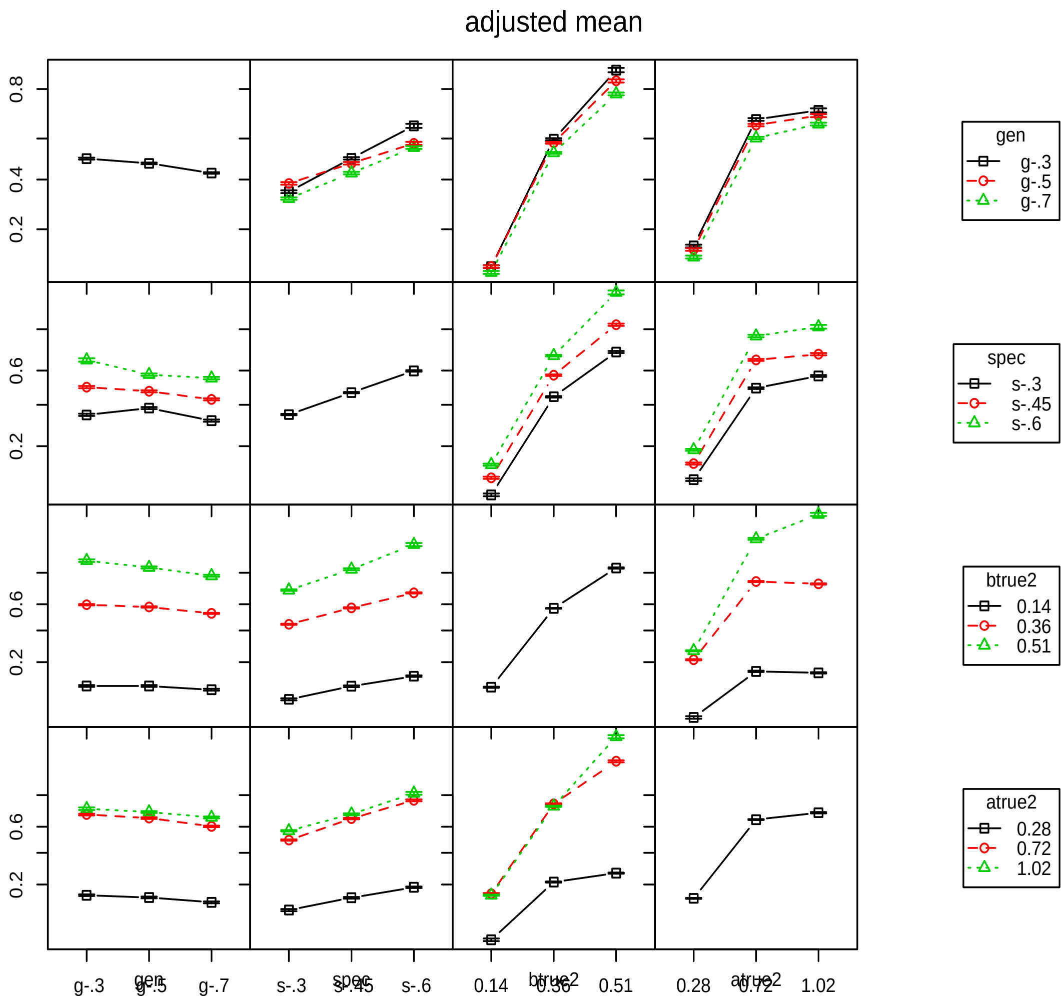


Figure 2B-1. Power in the mediated effect through the distribution of product method in the facet model when the sample size is 200. spec= specific factor variance; gen=general factor variance; sample_size2=sample size; atrue2=*a*-path effect size; btrue2=*b*-path effect size.

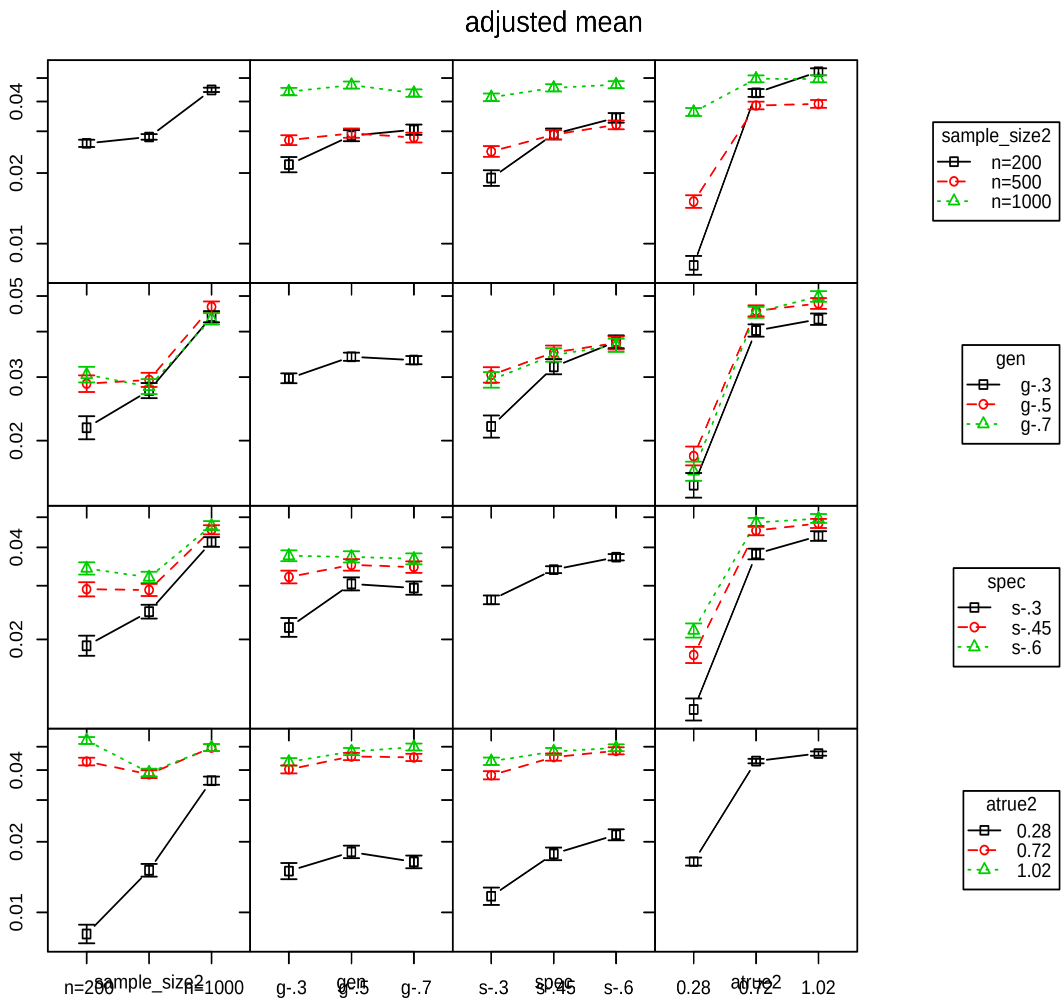


Figure 2C-1. Type 1 error in the mediated effect through the distribution of product method in the facet model when the a-path has a nonzero effect. spec= specific factor variance; gen=general factor variance; sample_size2=sample size; atrue2=*a*-path effect size; btrue2=*b*-path effect size.

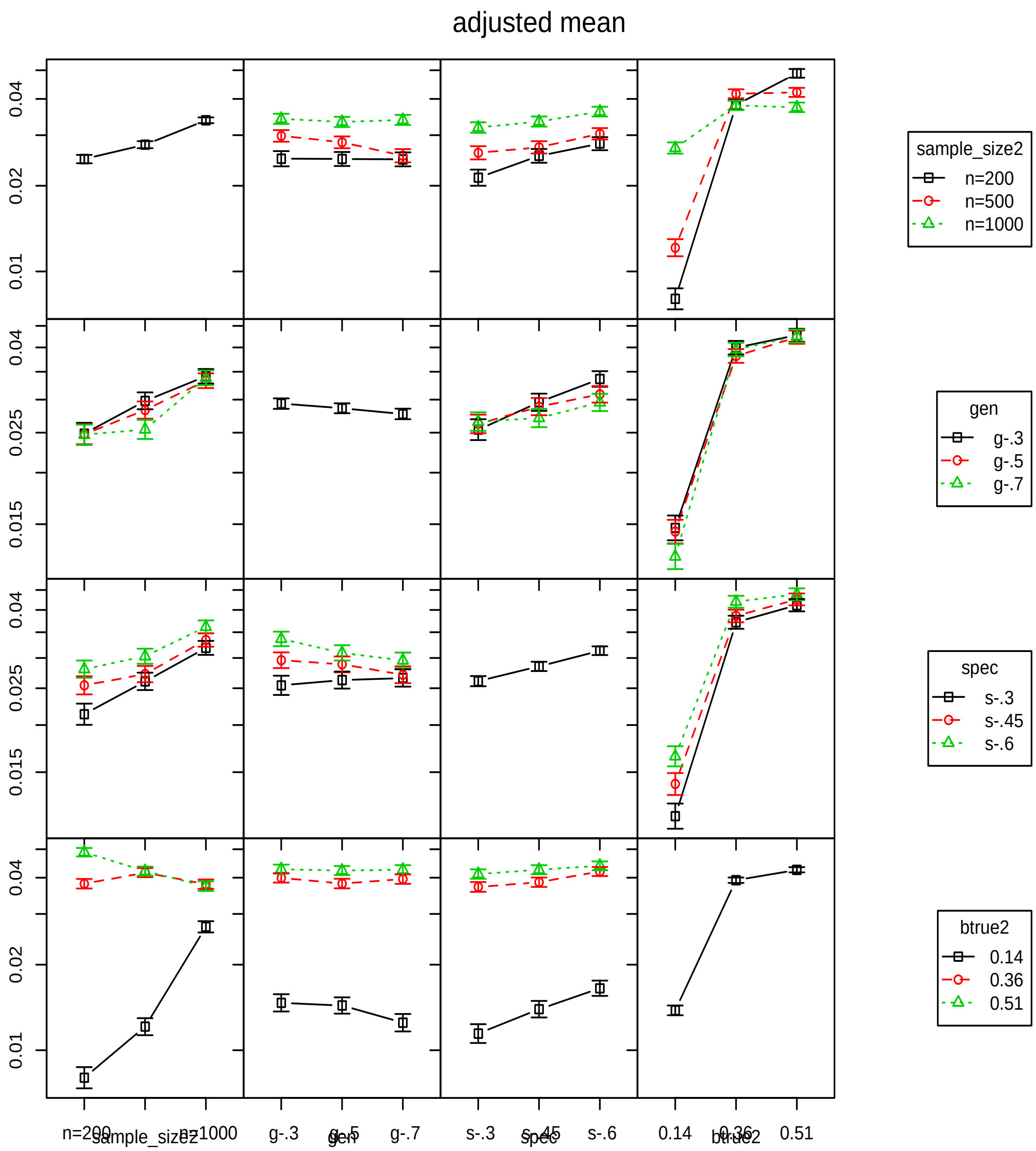


Figure 2C-2. Type 1 error in the mediated effect through the distribution of product method in the facet model when the b-path has a nonzero effect. spec= specific factor variance; gen=general factor variance; sample_size2=sample size; atrue2=*a*-path effect size; btrue2=*b*-path effect size.

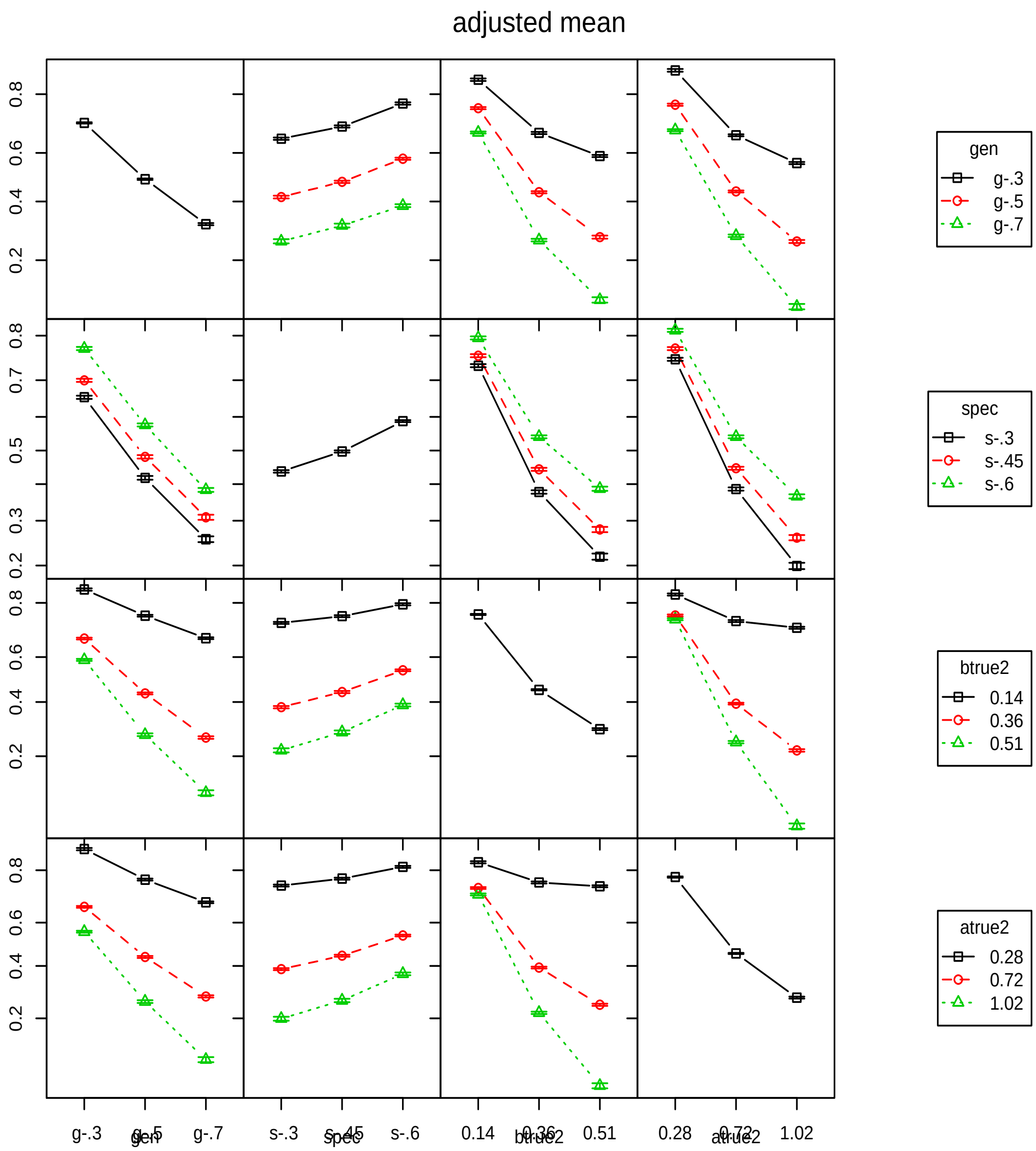


Figure 2D-1. Confidence interval coverage in the mediated effect through the distribution of product method in the facet model when the sample size is 200. spec= specific factor variance; gen=general factor variance; sample_size2=sample size; atrue2=*a*-path effect size; btrue2=*b*-path effect size.

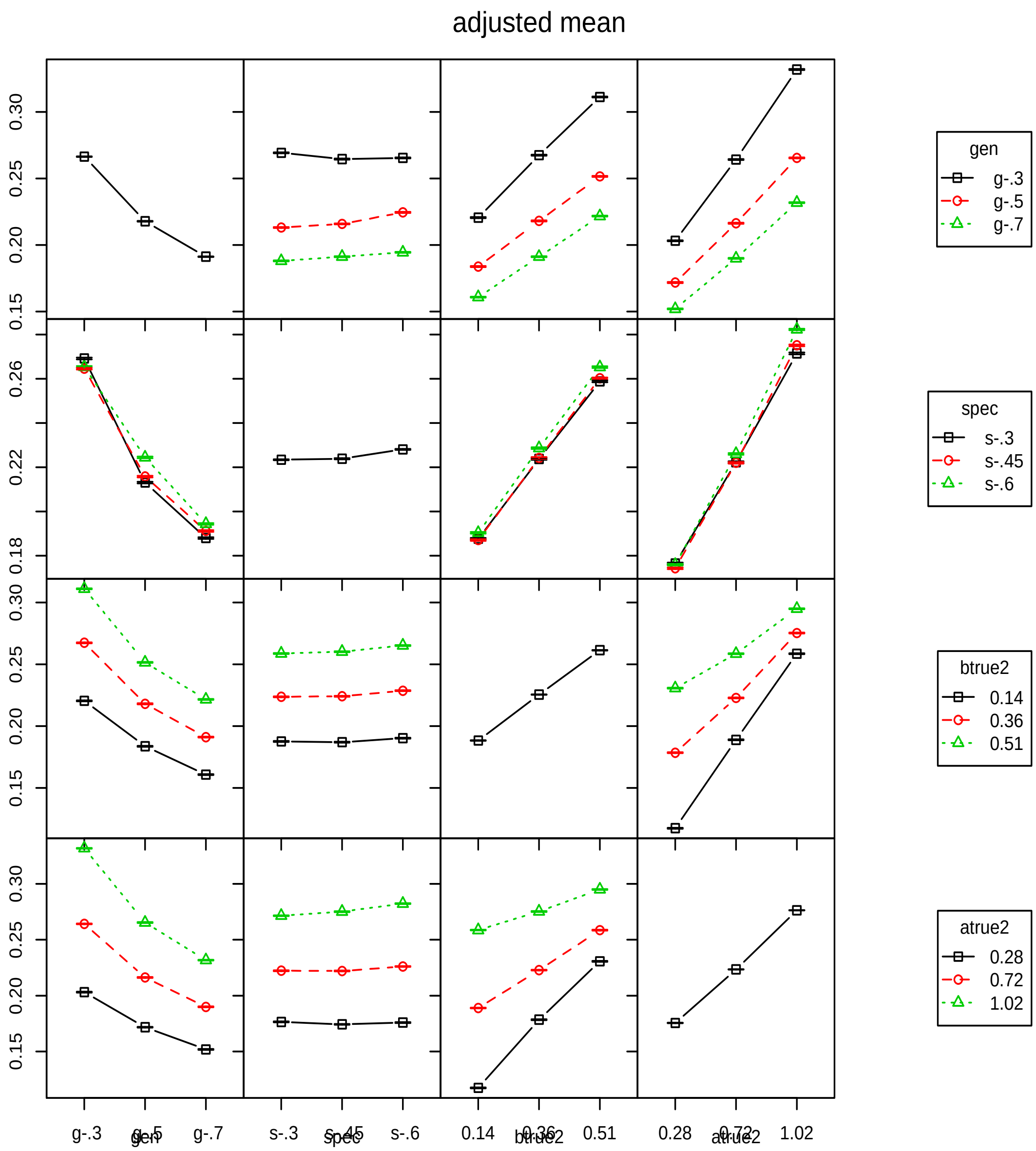


Figure 2E-1. Confidence interval width in the mediated effect through the distribution of product method in the facet model. spec= specific factor variance; gen=general factor variance; sample_size2=sample size; atrue2=*a*-path effect size; btrue2=*b*-path effect size.

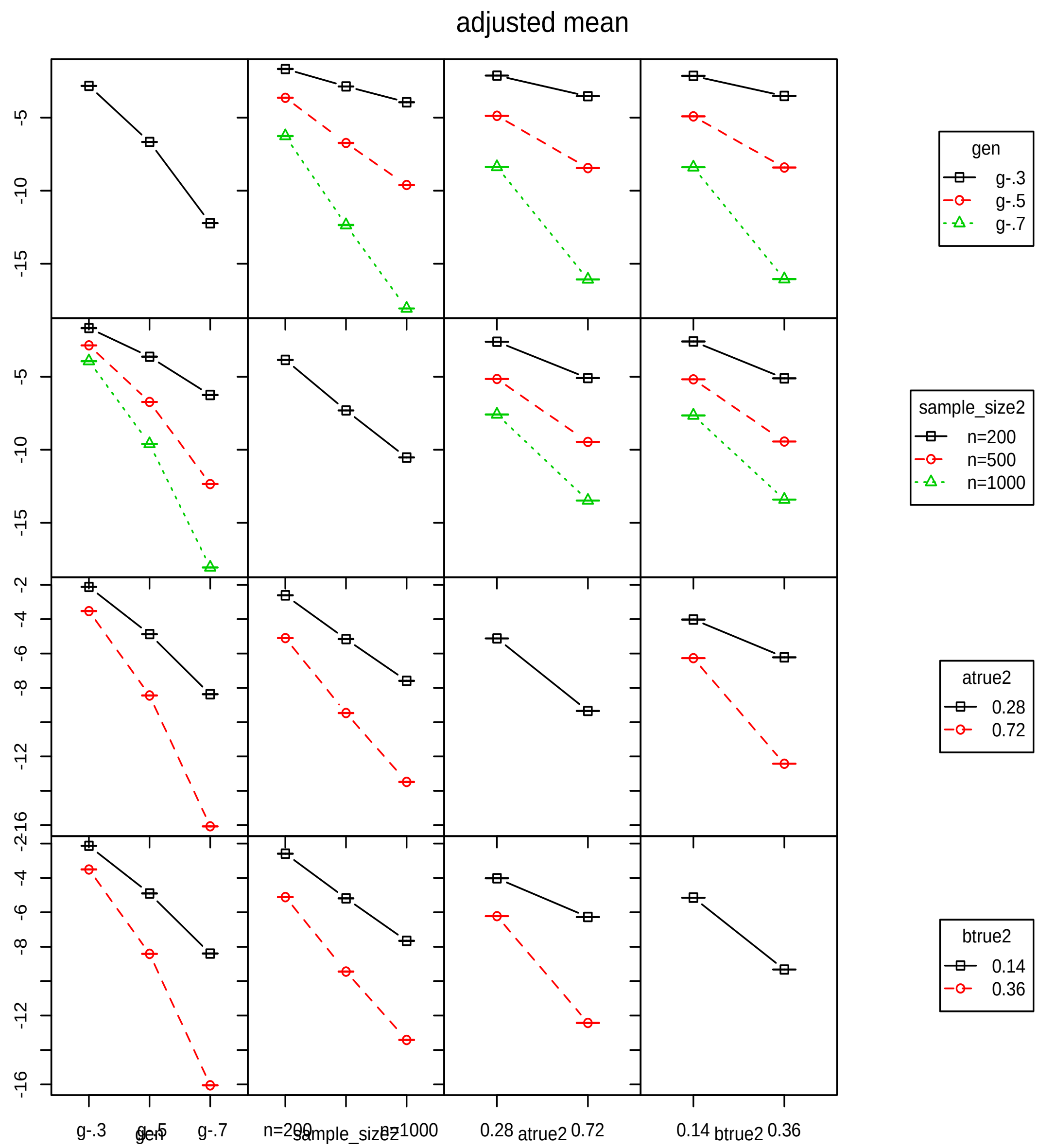


Figure 3A-1. Standardized bias in the mediated effect for the unidimensional model with nonzero mediated effects. spec= specific factor variance; gen=general factor variance; sample_size2=sample size; atrue2=*a*-path effect size; btrue2=*b*-path effect size.

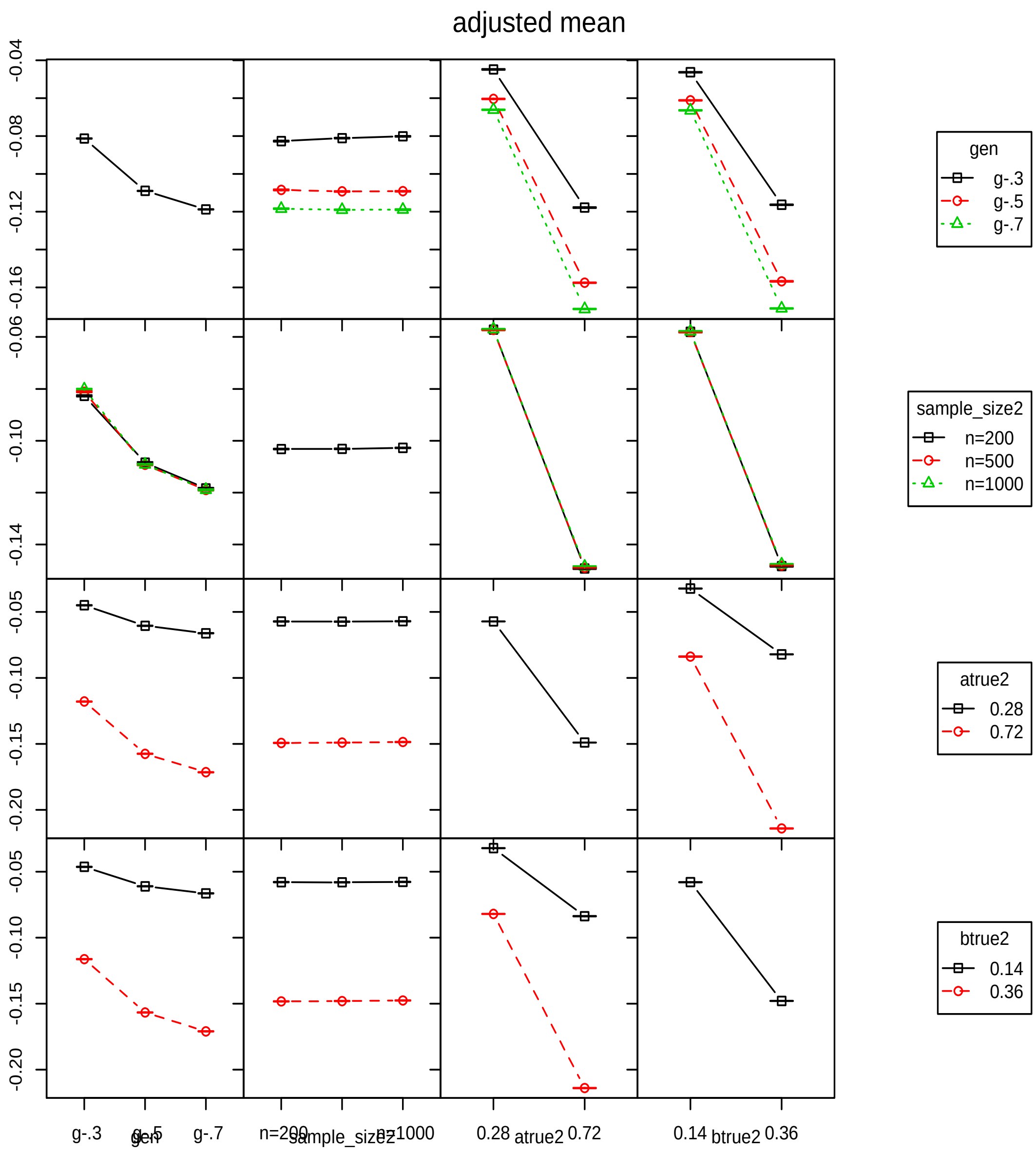


Figure 3A-2. Raw bias in the mediated effect for the unidimensional model with nonzero mediated effects. spec= specific factor variance; gen=general factor variance; sample_size2=sample size; atrue2=*a*-path effect size; btrue2=*b*-path effect size.

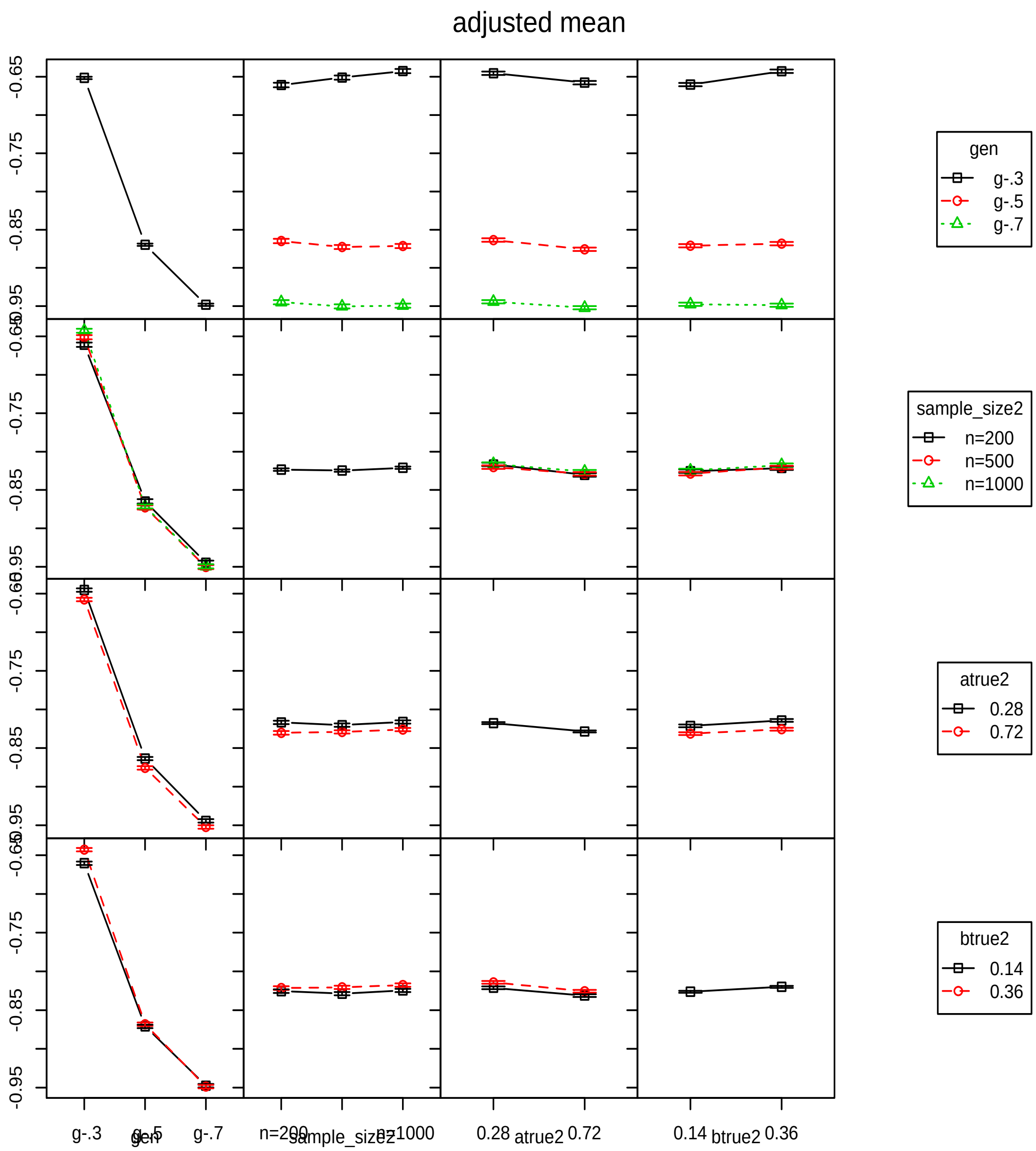


Figure 3A-3. Relative bias in the mediated effect for the unidimensional model with nonzero mediated effects. spec= specific factor variance; gen=general factor variance; sample_size2=sample size; atrue2=*a*-path effect size; btrue2=*b*-path effect size.

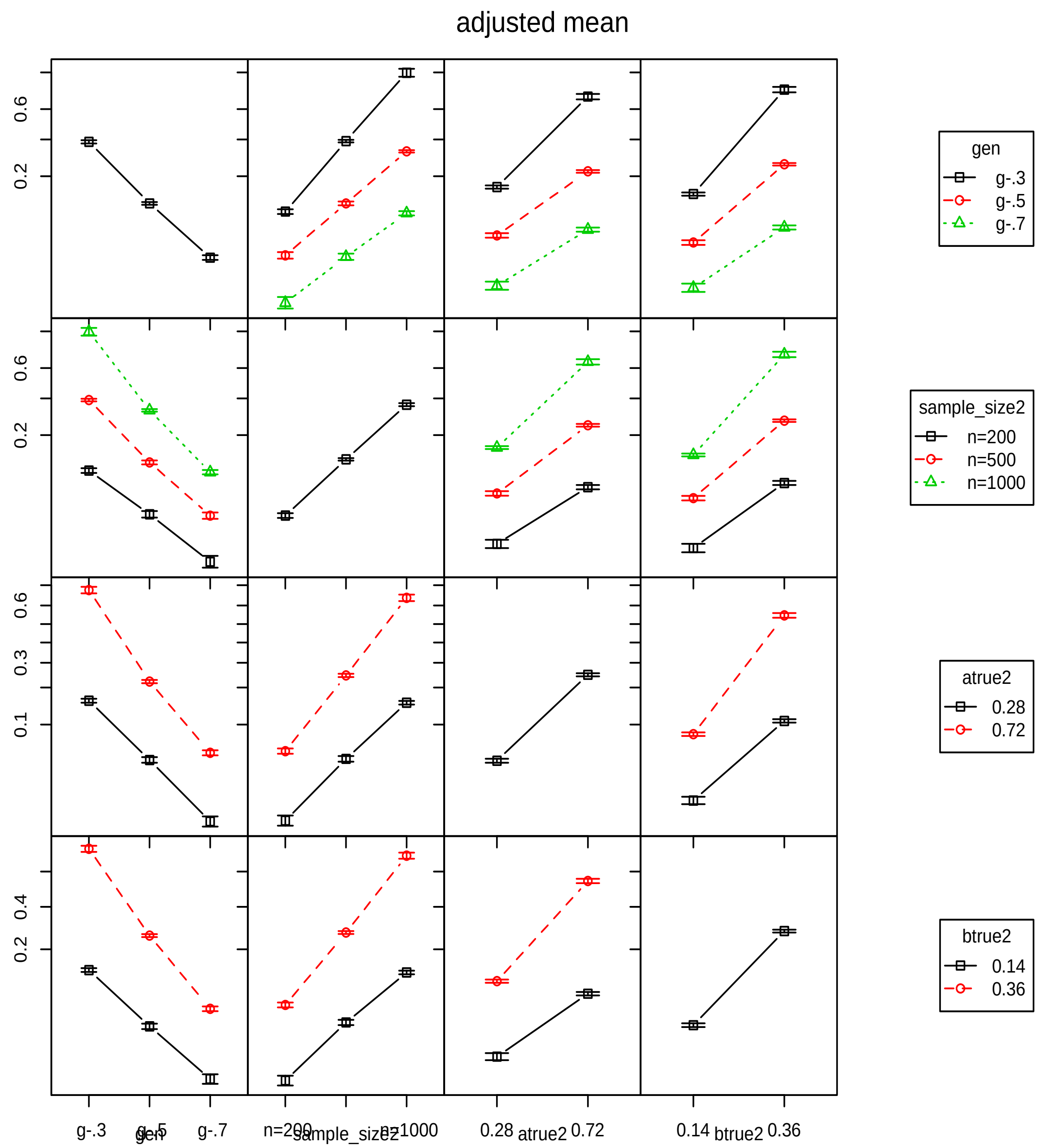


Figure 3B-1. Power to detect the mediated effect through the distribution of product method in the unidimensional model. spec= specific factor variance; gen=general factor variance; sample_size2=sample size; atrue2=*a*-path effect size; btrue2=*b*-path effect size.

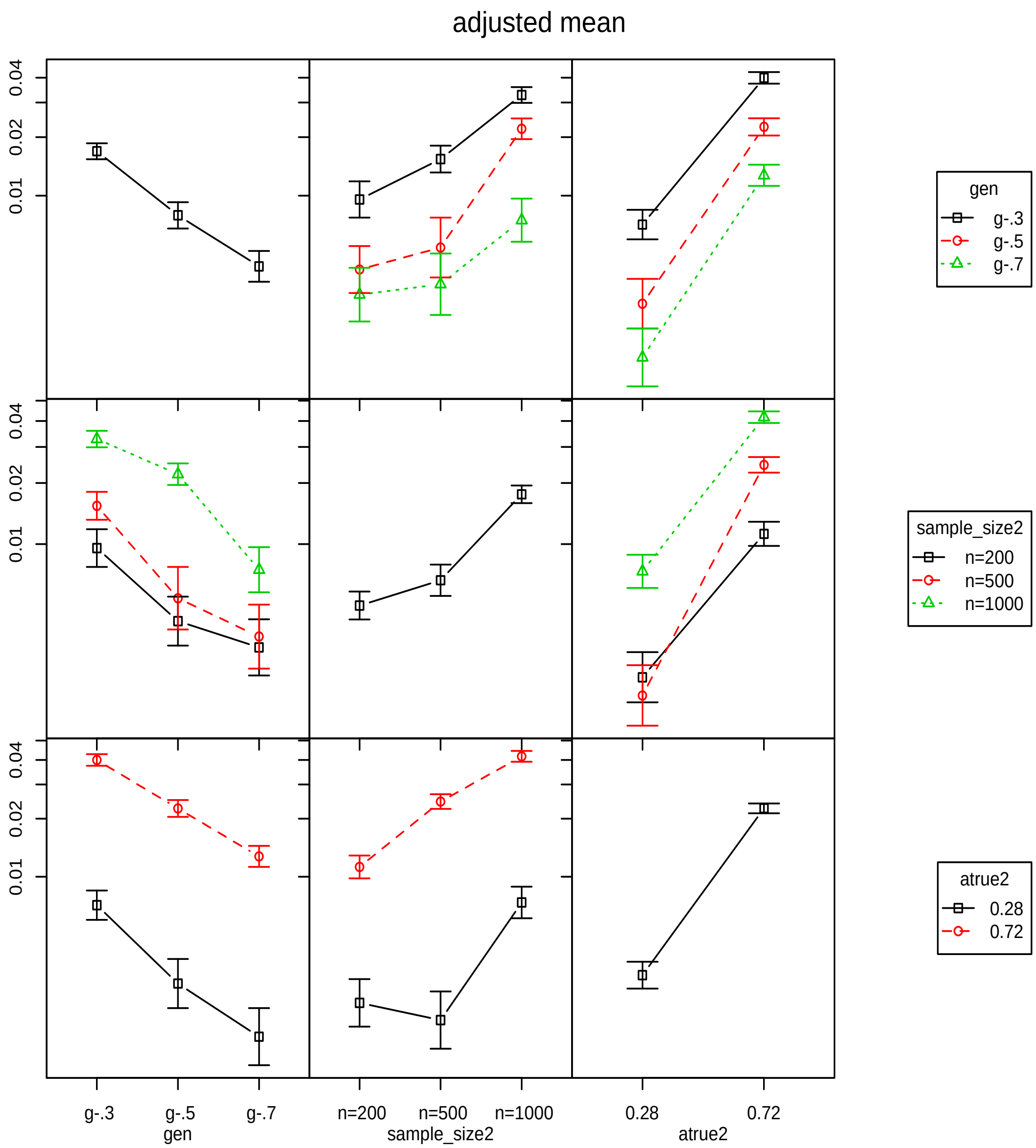


Figure 3C-1. Type 1 error in the mediated effect through the distribution of product method in the unidimensional model when the a-path has a nonzero effect. spec= specific factor variance; gen=general factor variance; sample_size2=sample size; atrue2=*a*-path effect size; btrue2=*b*-path effect size.

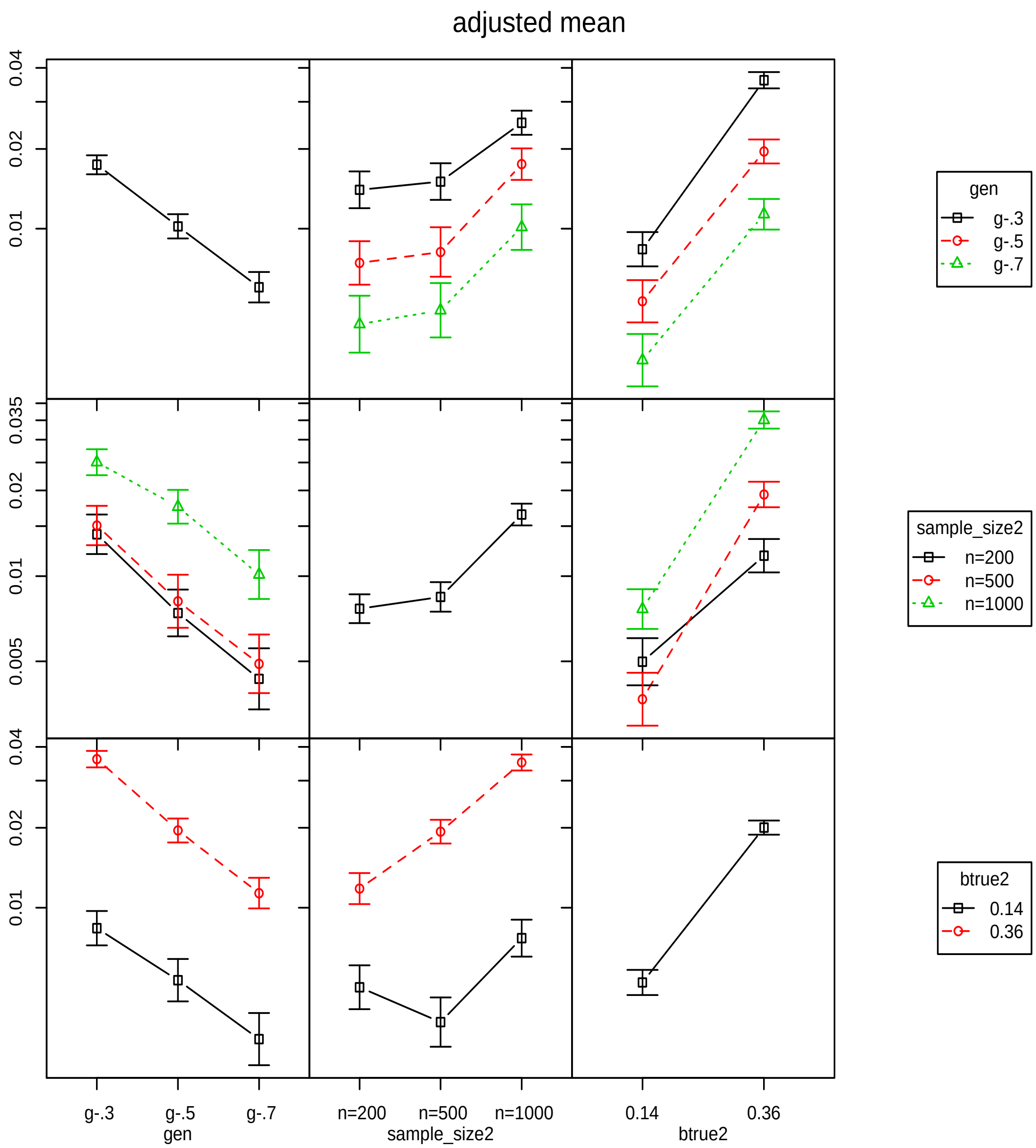


Figure 3C-2. Type 1 error in the mediated effect through the distribution of product method in the unidimensional model when the b-path has a nonzero effect. spec= specific factor variance; gen=general factor variance; sample_size2=sample size; atrue2=*a*-path effect size; btrue2=*b*-path effect size.

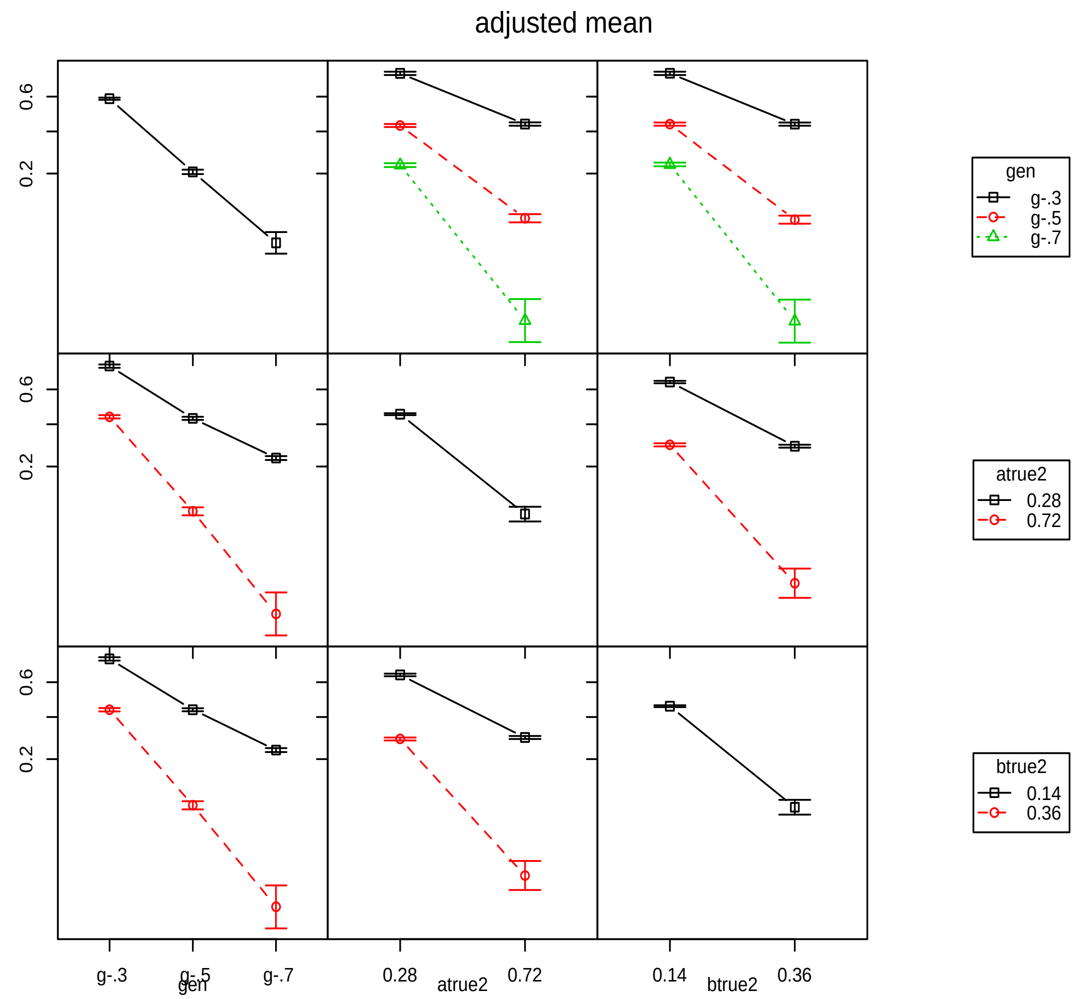


Figure 3D-1. Confidence interval coverage in the mediated effect through the distribution of product method in the unidimensional model, averaged over the effect size of the paths. spec= specific factor variance; gen=general factor variance; sample_size2=sample size; atrue2=*a*-path effect size; btrue2=*b*-path effect size.

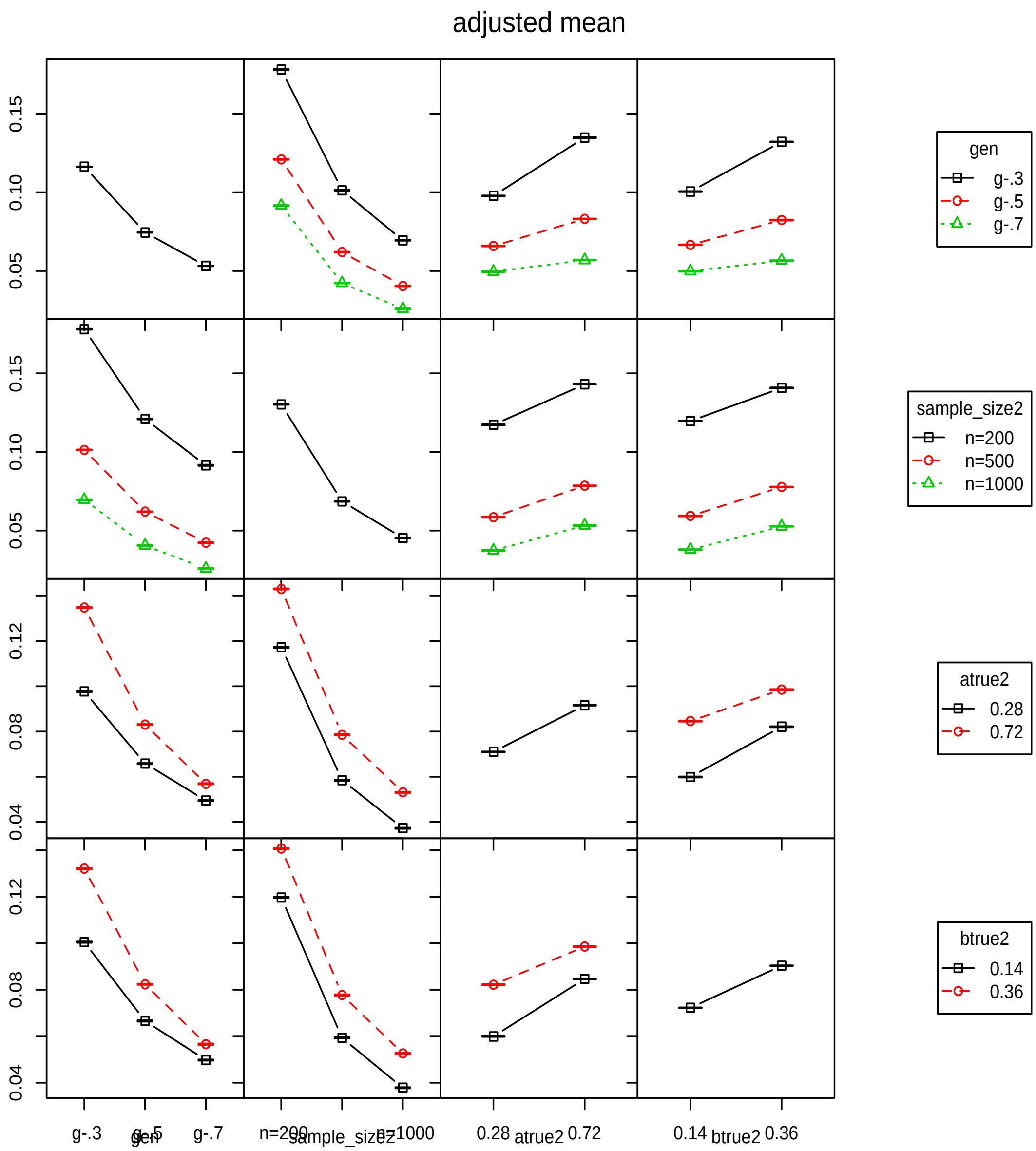


Figure 3E-1. Confidence interval width in the mediated effect through the distribution of product method in the unidimensional model. spec= specific factor variance; gen=general factor variance; sample_size2=sample size; atrue2=*a*-path effect size; btrue2=*b*-path effect size.

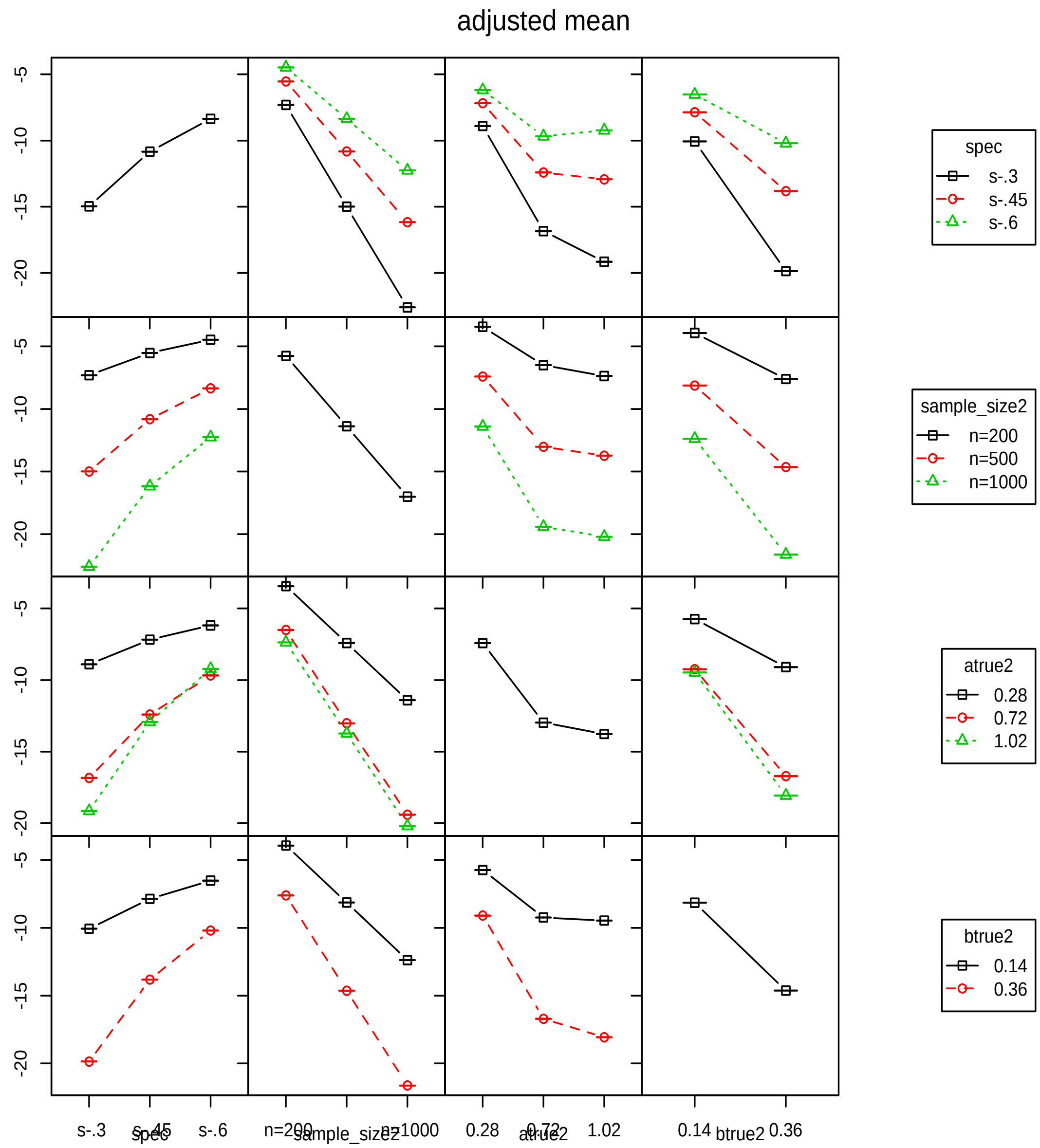


Figure 4A-1. Standardized bias in the mediated effect for the correlated factor model with nonzero mediated effects. spec= specific factor variance; gen=general factor variance; sample_size2=sample size; atrue2=*a*-path effect size; btrue2=*b*-path effect size.

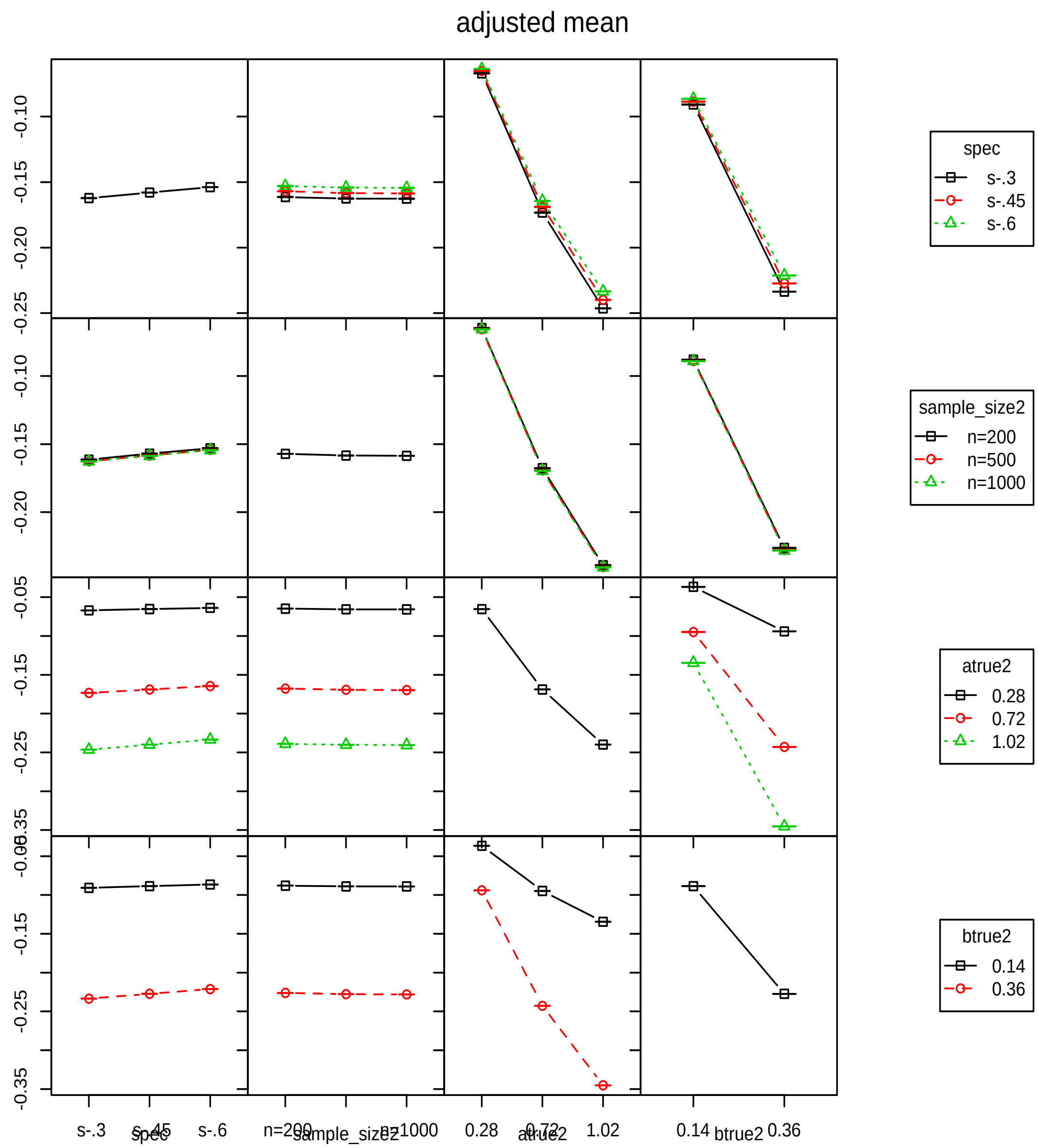


Figure 4A-2. Raw bias in the mediated effect for the correlated factor model with nonzero effects. spec= specific factor variance; gen=general factor variance; sample_size2=sample size; atrue2=*a*-path effect size; btrue2=*b*-path effect size.

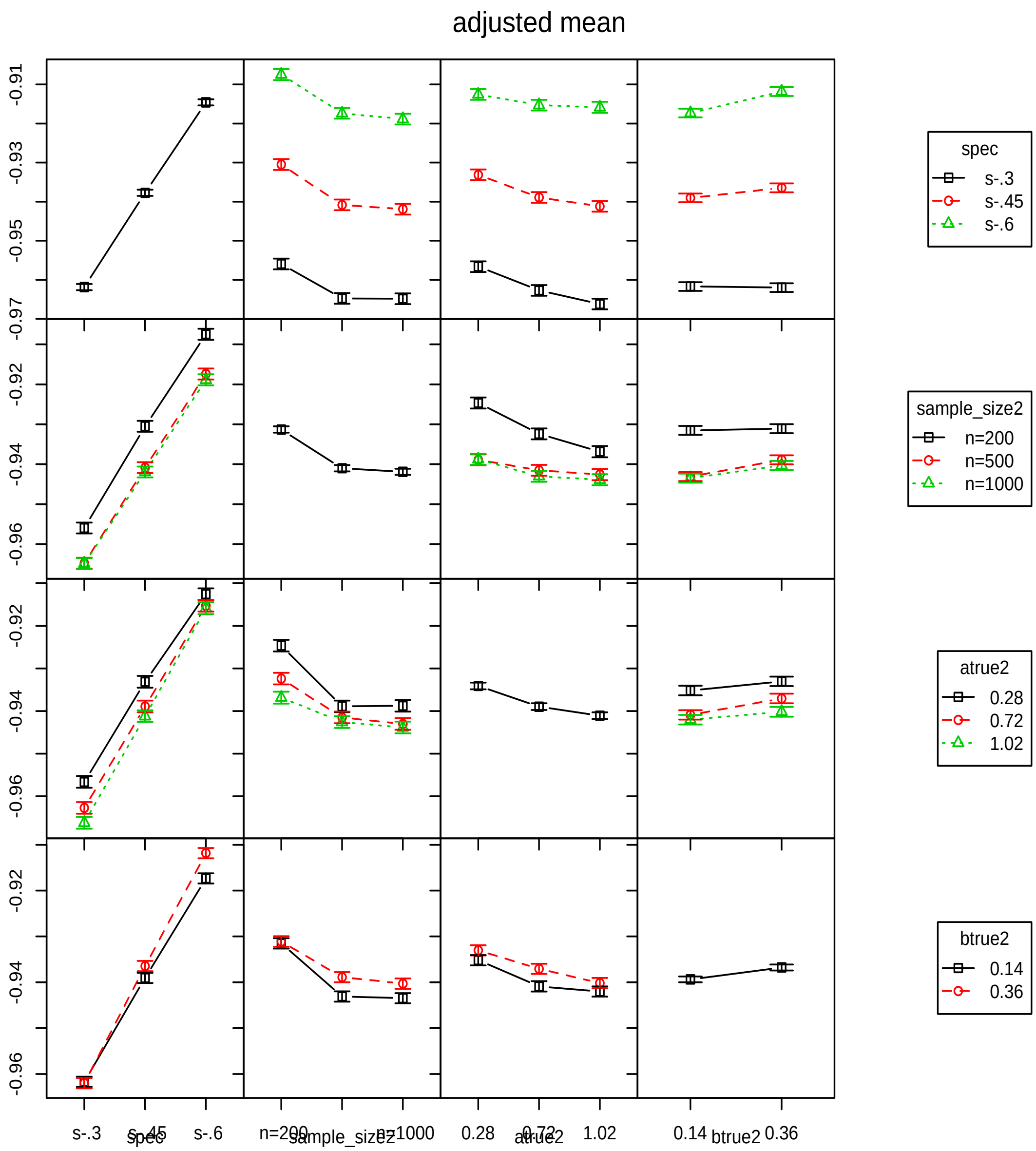


Figure 4A-3. Relative bias in the mediated effect for the correlated factor model with nonzero mediated effects. spec= specific factor variance; gen=general factor variance; sample_size2=sample size; atrue2=*a*-path effect size; btrue2=*b*-path effect size.

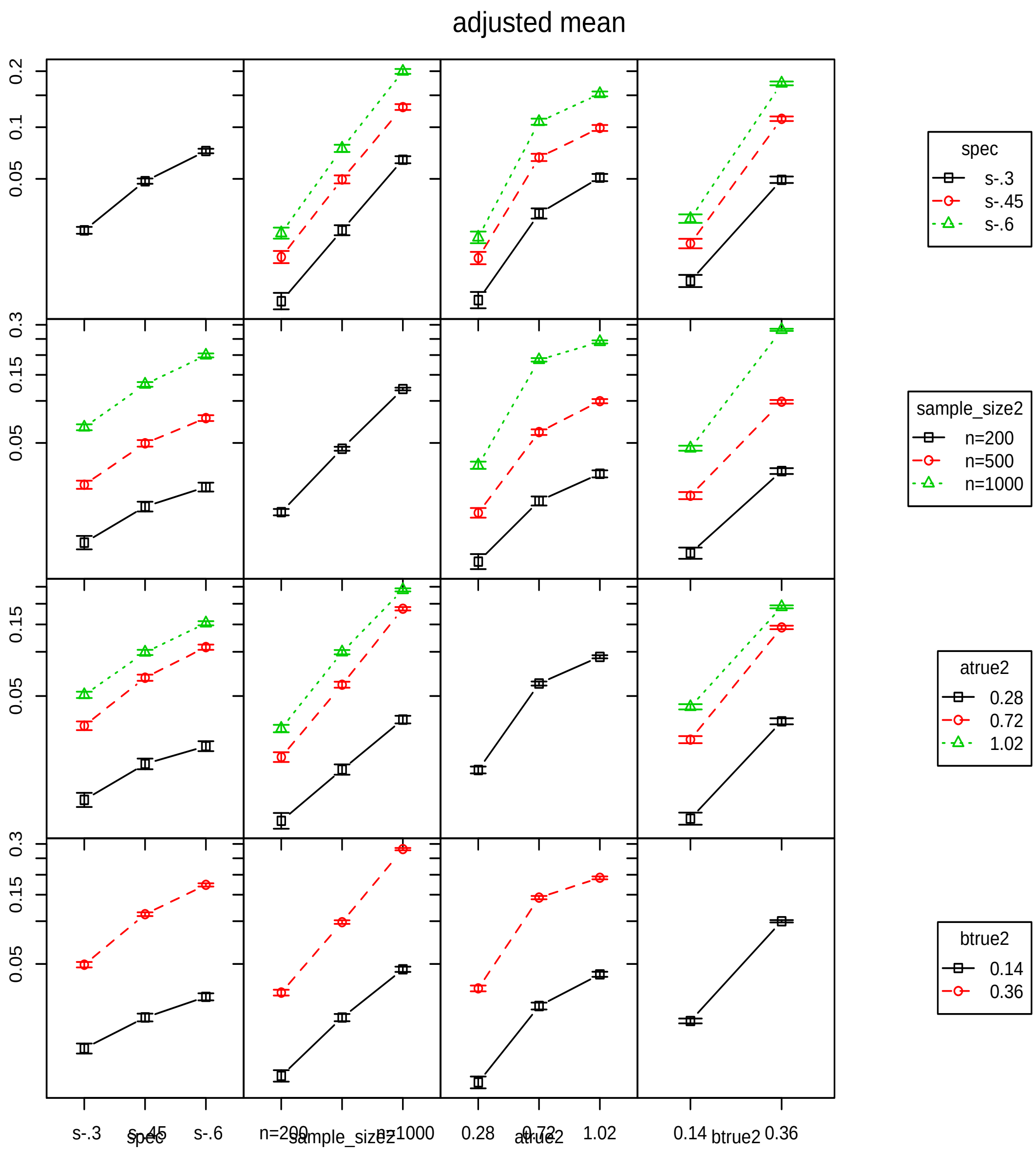


Figure 4B-1. Power in the mediated effect through the distribution of product method in the correlated factor model. spec= specific factor variance; gen=general factor variance; sample_size2=sample size; atrue2=*a*-path effect size; btrue2=*b*-path effect size.

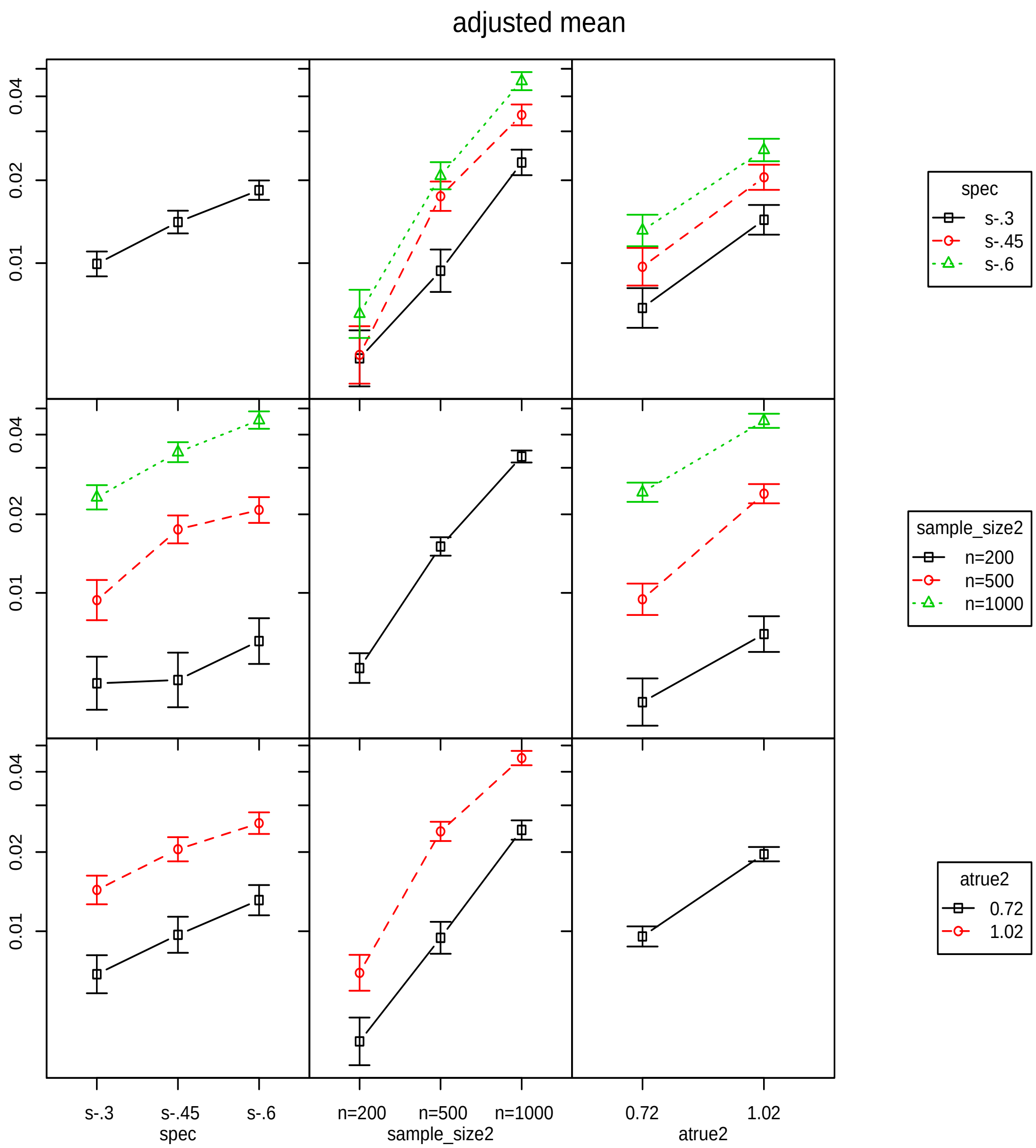


Figure 4C-1. Type 1 error in the mediated effect through the distribution of product method in the correlated factor model when the a-path has a nonzero effect. spec= specific factor variance; gen=general factor variance; sample_size2=sample size; atrue2=*a*-path effect size; btrue2=*b*-path effect size.

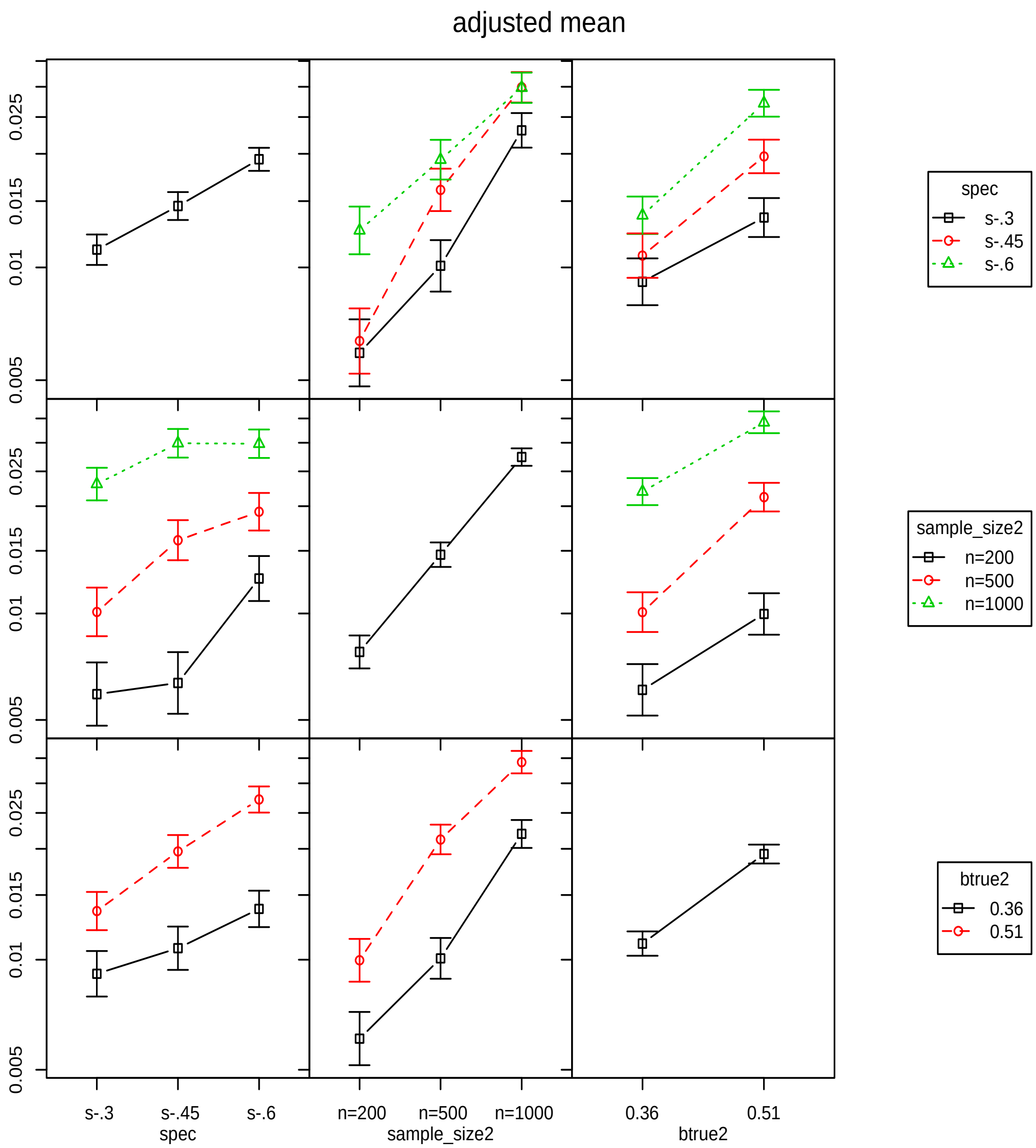


Figure 4C-2. Type 1 error in the mediated effect through the distribution of product method in the finite model when the b-path has a nonzero effect. spec= specific factor variance; gen=general factor variance; sample_size2=sample size; atrue2=*a*-path effect size; btrue2=*b*-path effect size.

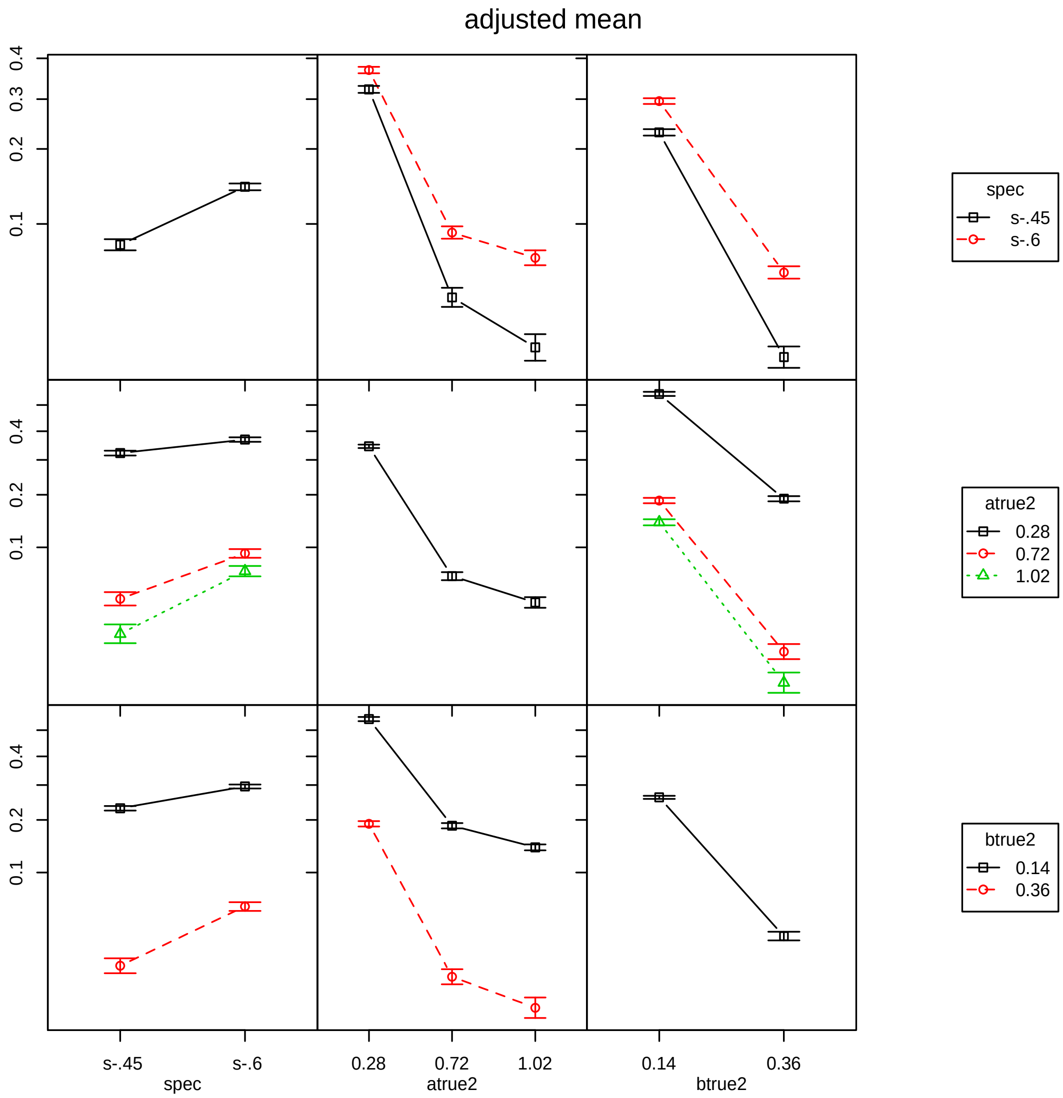


Figure 4D-1. Confidence interval coverage in the mediated effect through the distribution of product method in the correlated factor model. spec= specific factor variance; gen=general factor variance; sample_size2=sample size; atrue2=*a*-path effect size; btrue2=*b*-path effect size.

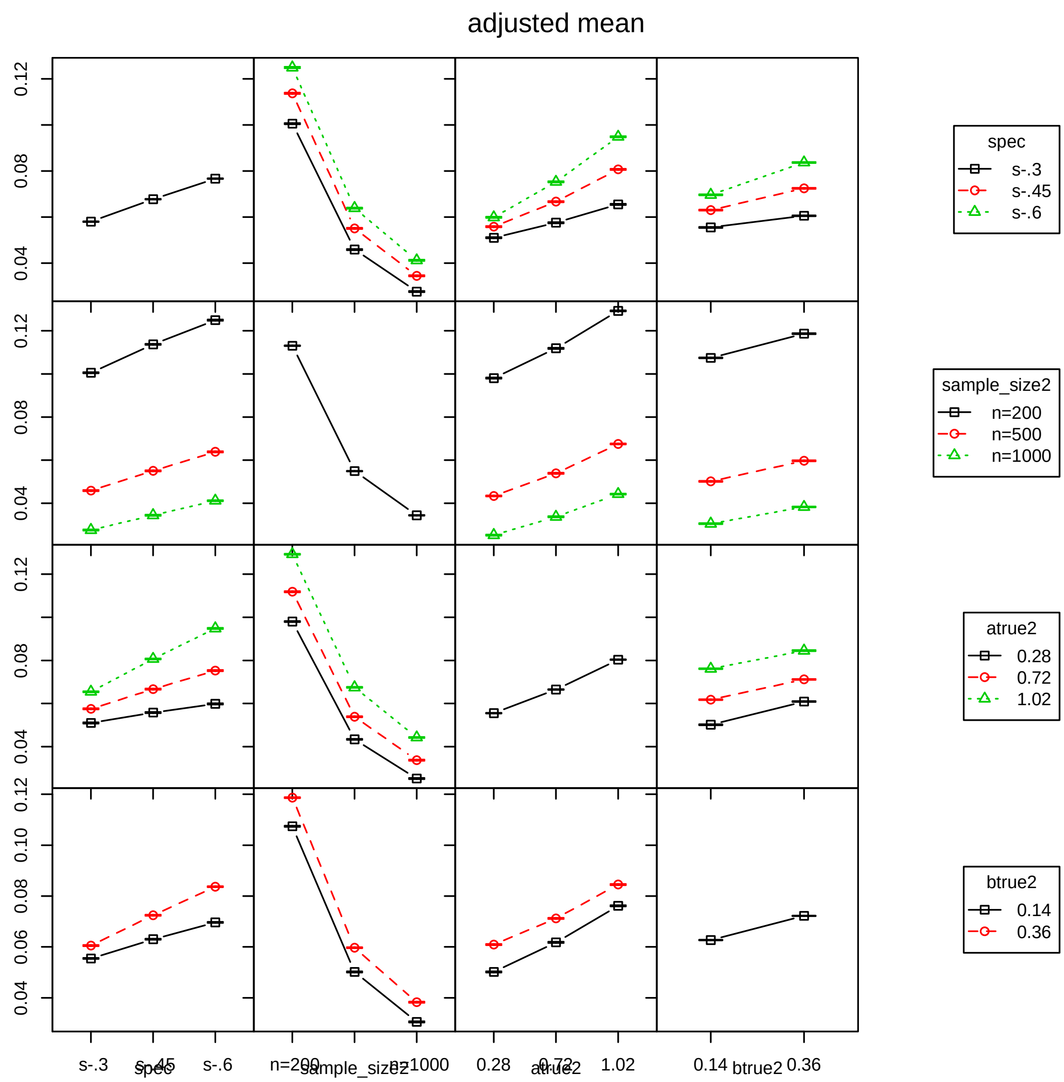


Figure 4E-1. Confidence interval width in the mediated effect through the distribution of product method in the correlated factor model. spec= specific factor variance; gen=general factor variance; sample_size2=sample size; atrue2=*a*-path effect size; btrue2=*b*-path effect size.

APPENDIX D

TRUE COVARIANCE MATRIX FOR THE DISTILLATION OF THE MEDIATED EFFECT

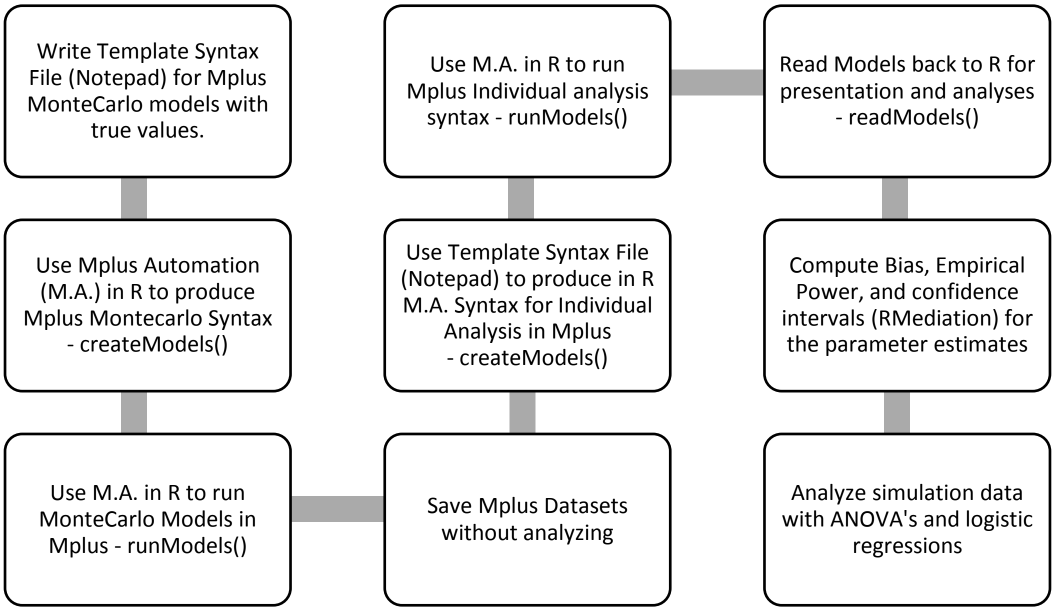
	X	Y	M1	M2
X	σ_x^2	$\sigma_x^2\left(ab+c^{'}\right)$	$a\lambda_{s1.1}\sigma_x^2$	$a\lambda_{s1.2}\sigma_x^2$
Y	$\sigma_x^2\left(ab+c^{'}\right)$	$b^2\zeta_1+e_2+\sigma_x^2\left(ab+c^{'}\right)^2$	$a\lambda_{s1.1}\sigma_x^2\left(ab+c^{'}\right)+b\lambda_{s1.1}\zeta_1$	$a\lambda_{s1.2}\sigma_x^2\left(ab+c^{'}\right)+b\lambda_{s1.2}\zeta_1$
M1	$a\lambda_{s1.1}\sigma_x^2$	$a\lambda_{s1.1}\sigma_x^2\left(ab+c^{'}\right)+b\lambda_{s1.1}\zeta_1$	$a^2\lambda_{s1.1}^2\sigma_x^2+\delta_1\lambda_{g1.1}^2+\epsilon_1+\lambda_{s1.1}^2\zeta_1$	$a^2\lambda_{s1.1}\lambda_{s1.2}\sigma_x^2+\delta_1\lambda_{g1.1}\lambda_{g1.2}+\lambda_{s1.1}\lambda_{s1.2}\zeta_1$
M2	$a\lambda_{s1.2}\sigma_x^2$	$a\lambda_{s1.2}\sigma_x^2\left(ab+c^{'}\right)+b\lambda_{s1.2}\zeta_1$	$a^2\lambda_{s1.1}\lambda_{s1.2}\sigma_x^2+\delta_1\lambda_{g1.1}\lambda_{g1.2}+\lambda_{s1.1}\lambda_{s1.2}\zeta_1$	$a^2\lambda_{s1.2}^2\sigma_x^2+\delta_1\lambda_{g1.2}^2+\epsilon_2+\lambda_{s1.2}^2\zeta_1$
M3	$a\lambda_{s1.3}\sigma_x^2$	$a\lambda_{s1.3}\sigma_x^2\left(ab+c^{'}\right)+b\lambda_{s1.3}\zeta_1$	$a^2\lambda_{s1.1}\lambda_{s1.3}\sigma_x^2+\delta_1\lambda_{g1.1}\lambda_{g1.3}+\lambda_{s1.1}\lambda_{s1.3}\zeta_1$	$a^2\lambda_{s1.2}\lambda_{s1.3}\sigma_x^2+\delta_1\lambda_{g1.2}\lambda_{g1.3}+\lambda_{s1.2}\lambda_{s1.3}\zeta_1$
M4	0	0	$\delta_1\lambda_{g1.1}\lambda_{g1.4}$	$\delta_1\lambda_{g1.2}\lambda_{g1.4}$
M5	0	0	$\delta_1\lambda_{g1.1}\lambda_{g1.5}$	$\delta_1\lambda_{g1.2}\lambda_{g1.5}$
M6	0	0	$\delta_1\lambda_{g1.1}\lambda_{g1.6}$	$\delta_1\lambda_{g1.2}\lambda_{g1.6}$
M7	0	0	$\delta_1\lambda_{g1.1}\lambda_{g1.7}$	$\delta_1\lambda_{g1.2}\lambda_{g1.7}$
M8	0	0	$\delta_1\lambda_{g1.1}\lambda_{g1.8}$	$\delta_1\lambda_{g1.2}\lambda_{g1.8}$
M9	0	0	$\delta_1\lambda_{g1.1}\lambda_{g1.9}$	$\delta_1\lambda_{g1.2}\lambda_{g1.9}$

	M3	M4	M5	M6
X	$a\lambda_{s1.3}\sigma_x^2$	0	0	0
Y	$a\lambda_{s1.3}\sigma_x^2\left(ab+c^{'}\right)+b\lambda_{s1.3}\zeta_1$	0	0	0
M1	$^2\lambda_{s1.1}\lambda_{s1.3}\sigma_x^2+\delta_1\lambda_{g1.1}\lambda_{g1.3}+\lambda_{s1.1}\lambda_{s1.3}\zeta_1$	$\delta_1\lambda_{g1.1}\lambda_{g1.4}$	$\delta_1\lambda_{g1.1}\lambda_{g1.5}$	$\delta_1\lambda_{g1.1}\lambda_{g1.6}$
M2	$^2\lambda_{s1.2}\lambda_{s1.3}\sigma_x^2+\delta_1\lambda_{g1.2}\lambda_{g1.3}+\lambda_{s1.2}\lambda_{s1.3}\zeta_1$	$\delta_1\lambda_{g1.2}\lambda_{g1.4}$	$\delta_1\lambda_{g1.2}\lambda_{g1.5}$	$\delta_1\lambda_{g1.2}\lambda_{g1.6}$
M3	$a^2\lambda_{s1.3}^2\sigma_x^2+\delta_1\lambda_{g1.3}^2+\epsilon_3+\lambda_{s1.3}^2\zeta_1$	$\delta_1\lambda_{g1.3}\lambda_{g1.4}$	$\delta_1\lambda_{g1.3}\lambda_{g1.5}$	$\delta_1\lambda_{g1.3}\lambda_{g1.6}$
M4	$\delta_1\lambda_{g1.3}\lambda_{g1.4}$	$\delta_1\lambda_{g1.4}^2+\epsilon_4+\lambda_{s2.4}^2\zeta_2$	$\delta_1\lambda_{g1.4}\lambda_{g1.5}+\lambda_{s2.4}\lambda_{s2.5}\zeta_2$	$\delta_1\lambda_{g1.4}\lambda_{g1.6}+\lambda_{s2.4}\lambda_{s2.6}\zeta_2$
M5	$\delta_1\lambda_{g1.3}\lambda_{g1.5}$	$\delta_1\lambda_{g1.4}\lambda_{g1.5}+\lambda_{s2.4}\lambda_{s2.5}\zeta_2$	$\delta_1\lambda_{g1.5}^2+\epsilon_5+\lambda_{s2.5}^2\zeta_2$	$\delta_1\lambda_{g1.5}\lambda_{g1.6}+\lambda_{s2.5}\lambda_{s2.6}\zeta_2$
M6	$\delta_1\lambda_{g1.3}\lambda_{g1.6}$	$\delta_1\lambda_{g1.4}\lambda_{g1.6}+\lambda_{s2.4}\lambda_{s2.6}\zeta_2$	$\delta_1\lambda_{g1.5}\lambda_{g1.6}+\lambda_{s2.5}\lambda_{s2.6}\zeta_2$	$\delta_1\lambda_{g1.6}^2+\epsilon_6+\lambda_{s2.6}^2\zeta_2$
M7	$\delta_1\lambda_{g1.3}\lambda_{g1.7}$	$\delta_1\lambda_{g1.4}\lambda_{g1.7}$	$\delta_1\lambda_{g1.5}\lambda_{g1.7}$	$\delta_1\lambda_{g1.6}\lambda_{g1.7}$
M8	$\delta_1\lambda_{g1.3}\lambda_{g1.8}$	$\delta_1\lambda_{g1.4}\lambda_{g1.8}$	$\delta_1\lambda_{g1.5}\lambda_{g1.8}$	$\delta_1\lambda_{g1.6}\lambda_{g1.8}$
M9	$\delta_1\lambda_{g1.3}\lambda_{g1.9}$	$\delta_1\lambda_{g1.4}\lambda_{g1.9}$	$\delta_1\lambda_{g1.5}\lambda_{g1.9}$	$\delta_1\lambda_{g1.6}\lambda_{g1.9}$

	M7	M8	M9
X	0	0	0
Y	0	0	0
M1	$\delta_1\lambda_{g1.1}\lambda_{g1.7}$	$\delta_1\lambda_{g1.1}\lambda_{g1.8}$	$\delta_1\lambda_{g1.1}\lambda_{g1.9}$
M2	$\delta_1\lambda_{g1.2}\lambda_{g1.7}$	$\delta_1\lambda_{g1.2}\lambda_{g1.8}$	$\delta_1\lambda_{g1.2}\lambda_{g1.9}$
M3	$\delta_1\lambda_{g1.3}\lambda_{g1.7}$	$\delta_1\lambda_{g1.3}\lambda_{g1.8}$	$\delta_1\lambda_{g1.3}\lambda_{g1.9}$
M4	$\delta_1\lambda_{g1.4}\lambda_{g1.7}$	$\delta_1\lambda_{g1.4}\lambda_{g1.8}$	$\delta_1\lambda_{g1.4}\lambda_{g1.9}$
M5	$\delta_1\lambda_{g1.5}\lambda_{g1.7}$	$\delta_1\lambda_{g1.5}\lambda_{g1.8}$	$\delta_1\lambda_{g1.5}\lambda_{g1.9}$
M6	$\delta_1\lambda_{g1.6}\lambda_{g1.7}$	$\delta_1\lambda_{g1.6}\lambda_{g1.8}$	$\delta_1\lambda_{g1.6}\lambda_{g1.9}$
M7	$\delta_1\lambda_{g1.7}^2+\epsilon_7+\lambda_{s3.7}^2\zeta_3$	$\delta_1\lambda_{g1.7}\lambda_{g1.8}+\lambda_{s3.7}\lambda_{s3.8}\zeta_3$	$\delta_1\lambda_{g1.7}\lambda_{g1.9}+\lambda_{s3.7}\lambda_{s3.9}\zeta_3$
M8	$\delta_1\lambda_{g1.7}\lambda_{g1.8}+\lambda_{s3.7}\lambda_{s3.8}\zeta_3$	$\delta_1\lambda_{g1.8}^2+\epsilon_8+\lambda_{s3.8}^2\zeta_3$	$\delta_1\lambda_{g1.8}\lambda_{g1.9}+\lambda_{s3.8}\lambda_{s3.9}\zeta_3$
M9	$\delta_1\lambda_{g1.7}\lambda_{g1.9}+\lambda_{s3.7}\lambda_{s3.9}\zeta_3$	$\delta_1\lambda_{g1.8}\lambda_{g1.9}+\lambda_{s3.8}\lambda_{s3.9}\zeta_3$	$\delta_1\lambda_{g1.9}^2+\epsilon_9+\lambda_{s3.9}^2\zeta_3$

APPENDIX E

FLOWCHART FOR SIMULATION PROCEDURES


Write Template Syntax File (Notepad) for Mplus MonteCarlo models with true values.
Use Mplus Automation (M.A.) in R to produce Mplus Montecarlo Syntax - createModels()
Use M.A. in R to run MonteCarlo Models in Mplus - runModels()
Save Mplus Datasets without analyzing
Use Template Syntax File (Notepad) to produce in R M.A. Syntax for Individual Analysis in Mplus - createModels()
Use M.A. in R to run Mplus Individual analysis syntax - runModels()
Read Models back to R for presentation and analyses - readModels()
Compute Bias, Empirical Power, and confidence intervals (RMediation) for the parameter estimates
Analyze simulation data with ANOVA's and logistic regressions

APPENDIX F

MPLUS AUTOMATION FILE FOR MONTE CARLO SIMULATION

Example of an Mplus Automation file for Monte Carlo simulation (Model 1)

```
[[init]]
iterators= g_load s_load n;
n = 200 500 1000;
g_load= 1:2;
s_load= 1;
value#g_load= .6 .7;
fac#s_load= .6;
outputDirectory="C:/Users/ogonza13/Desktop/Montecarlo/sims/sample_size
[[n]]/gen factor[[value#g_load]]/sp factor[[fac#s_load]]";
filename="MC-sample_size [[n]],g-[[value#g_load]],s-[[fac#s_load]]
combination.inp";
[[/init]]

TITLE: MC BIFACTOR MODEL, n=[[n]], g-[[value#g_load]], s-
[[fac#s_load]];

MONTECARLO:
    names are x m1 m2 m3 m4 m5 m6 m7 m8 m9 y;
    ngroups=1;
    nobs=[[n]];
    nreps=1000;
    !seed=2;
    cutpoints=x(0);
    REPSAVE = ALL;
    save= data_rep*.dat;
    results=data_results.txt;


ANALYSIS: !TYPE=MEANSTRUCTURE;
PROCESS=4;
MODEL POPULATION:
!Measurement model
    [m1-m9@0];
    f1 by m1@1 m2-m3@[[fac#s_load]];
    f2 by m4-m6@[[fac#s_load]];
    f3 by m7-m9@[[fac#s_load]];
    m by m1-m9@[[value#g_load]];

[[g_load=1]]
[[s_load=1]]
m1-m9@.28;;  !.6 and .6 loadings
[[/s_load=1]]
[[/g_load=1]]

[[g_load=2]]
[[s_load=1]]
m1-m9@.15; !.6 and .7 loadings
[[/s_load=1]]
[[/g_load=2]]

   m f1 f2 f3 WITH m@0 f1@0 f2@0 f3@0;
    [m @ 0]; [f1@0]; [f2@0]; [f3@0];
```

```
!Structural Model
    [x@0];
    x@.25;
    [y@0];
    y@.813;
    f1@.87;
    f2-f3@1;
    m@1;
    f1 on x@.721;
    y on f1@.36 x@.283;


MODEL:
!Measurement Model
    [m1-m9*0];


[[g_load=1]]
[[s_load=1]]
m1-m9*.28;;  !.6 and .6 loadings
[[/s_load=1]]
[[/g_load=1]]

[[g_load=2]]
[[s_load=1]]
m1-m9*.15; !.6 and .7 loadings
[[/s_load=1]]
[[/g_load=2]]


    f1 by m1-m3*[[fac#s_load]];
    f2 by m4-m6*[[fac#s_load]];
    f3 by m7-m9*[[fac#s_load]];
    m by m1-m9*[[value#g_load]];
    m f1 f2 f3 WITH m@0 f1@0 f2@0 f3@0;
    [m @ 0]; [f1@0]; [f2@0]; [f3@0];


!Structural Model
    [y*0];
    y*.813;
    f1-f3@1;
    m@1;
    f1 on x*.721(a);
    y on f1*.36(b)
    x*.283;
!Mediation

MODEL INDIRECT:
y IND x;

MODEL CONSTRAINT:
NEW(ab*.26);
ab=a*b; OUTPUT: tech3 tech9;
```

APPENDIX G

EXAMPLE OF MPLUS MONTE CARLO SYNTAX

```
TITLE: MC BIFACTOR MODEL, n=200, g-.6, s-.6;

MONTECARLO:
    names are x m1 m2 m3 m4 m5 m6 m7 m8 m9 y;
    ngroups=1;
    nobs=200;
    nreps=1000;
    !seed=2;
    cutpoints=x(0);
    REPSAVE = ALL;
    save= data_rep*.dat;
    results=data_results.txt;

ANALYSIS: TYPE=BASIC;
PROCESS=4;
MODEL POPULATION:
!Measurement model
    [m1-m9@0];
    f1 by m1@1 m2-m3@.6;
    f2 by m4-m6@.6;
    f3 by m7-m9@.6;
    m by m1-m9@.6;

m1-m9@.28;;  !.6 and .6 loadings

   m f1 f2 f3 WITH m@0 f1@0 f2@0 f3@0;
    [m @ 0]; [f1@0]; [f2@0]; [f3@0];

!Structural Model
    [x@0];
    x@.25;
    [y@0];
    y@.813;
    f1@.87;
    f2-f3@1;
    m@1;
    f1 on x@.721;
    y on f1@.36 x@.283;

MODEL:
!Measurement Model
    [m1-m9*0];


m1-m9*.28;;  !.6 and .6 loadings

    f1 by m1-m3*.6;
```

```
    f2 by m4-m6*.6;
    f3 by m7-m9*.6;
    m by m1-m9*.6;
    m f1 f2 f3 WITH m@0 f1@0 f2@0 f3@0;
    [m @ 0]; [f1@0]; [f2@0]; [f3@0];

!Structural Model
    [y*0];
    y*.813;
    f1-f3@1;
    m@1;
    f1 on x*.721(a);
    y on f1*.36(b)
    x*.283;
!Mediation

MODEL INDIRECT:
y IND x;

MODEL CONSTRAINT:
NEW(ab*.26);
ab=a*b;

OUTPUT: tech3 tech9;
```

APPENDIX H

MPLUS SYNTAX FOR THE ANALYSIS OF ONE MONTE CARLO REPLICATION

```
TITLE:INDIVIDUAL BIFACTOR, rep-1, n=200, g-.6, s-.6;
    DATA:   FILE IS data_rep1.dat;
    VARIABLE:  names are m1 m2 m3 m4 m5 m6 m7 m8 m9 y x;


  ANALYSIS: !TYPE=MEANSTRUCTURE;
        PROCESS=4;

  MODEL:
  !Measurement Model
      [m1-m9*0];


  m1-m9*.28;;  !.6 and .6 loadings


      f1 by m1-m3*.6;
      f2 by m4-m6*.6;
      f3 by m7-m9*.6;
      m by m1-m9*.6;
      m f1 f2 f3 WITH m@0 f1@0 f2@0 f3@0;
      [m @ 0]; [f1@0]; [f2@0]; [f3@0];


  !Structural Model
      [y*0];
      y*.813;
      f1-f3@1;
      m@1;
      f1 on x*.721(a);
      y on f1*.36(b)
      x*.283;
  !Mediation

  MODEL INDIRECT:
  y IND x;

  MODEL CONSTRAINT:
  NEW(ab*.26);
  ab=a*b;


  OUTPUT: tech1 tech3;
```

APPENDIX I

CHANGES BETWEEN PROPOSAL AND DATA-MEETING

There were a few differences from what was originally proposed in the prospectus meeting and the final simulation results presented. First, the unidimensional model with correlated uniqueness was added to the simulation. Also, simulation conditions were cut. The table below describes the difference in the conditions:

Simulation Factor	Proposed	Analyzed	Reason
Sample size	200, 500, 1000	200, 500, 1000	Committee Agreed
a-path effect size (zero, small, medium, large)	0, .28, .72, 1.02	0, .28, .72, 1.02	Committee Agreed
b-path effect size (zero, small, medium, large)	0, .14, .36, .51	0, .14, .36, .51	Committee Agreed
c′-path effect size (zero, small)	0, .283	0, .283	Averaged over in analyses
Factor loading on general factor	.3, .4, .5, .6, .7	.3, .5, .7	Committee Agreed
Factor loading on specific factor	.3, .4, .5, .6	.3, .45, .6	Committee Agreed
Percentage of uncontaminated correlations (PUC)	.75, .88., .96	.75	CPU RAM (memory) problems

As shown in the table above, the results from larger models with the “percentage of uncontaminated correlations” conditions had trouble being read from Mplus to R given the large data files. Those conditions also suffered from many non-positive covariance matrices and were not analyzed. Also, the original document proposed to test hypotheses on the individual paths regarding the simulation outcomes. However, due to the wide extent of the project, it was decided to focus on the interpretation of the mediated effect, which was of most interest. Consequently, results were averaged over the *c’*-path conditions.