Computer Tool for Comparison of Classical and Non-Conventional Lightning Protection
Designs for Electric Substations
by

Vinit Marathe

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Approved December 2015 by the Graduate Supervisory Committee:

George Karady, Chair
Raja Ayyanar
Keith Holbert

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#### Abstract

Electric substation physical plans are developed with consideration given to lightning protection. To develop these plans utility design engineers use various methods. This thesis focuses on developing a computer program for two methods/models for substation shielding against direct lightning strokes. The first method is being used currently in the industry to protect the substation structures. The second model is a new and more physics based approach towards lightning phenomenon. Both the methods consider only direct lightning strikes that can hit the substation equipment. Hence, the travelling waves, indirect strokes or over-voltage arriving at the substation equipment are not considered.

The Electro-Geometric method (EGM) based Rolling Sphere Method (RSM) is used to develop first part of the program. The aim of the program is to design the protection system for the substation equipment quickly and error free. The protection system uses lightning masts and/or shield wires to protect the station equipment. These are grounded solidly with low impedance to earth. The MATLAB based program gives a two dimensional visual representation of the zone of protection and therefore helps utility engineers to position shielding system. As this program is converted further into an executable file, it can be used on any computer to produce the results without need of any other software. The second part of the thesis focuses on developing the MATLAB code for protection of substation equipment using the Rizk model which is not used as of now for shielding system design in industry. Using more physics based model, simulation of downward lightning leader and connecting upward leader is shown.


Finally both the methods are compared. This includes consideration of a 220 kV substation layout arrangement. The equipment are protected using shielding masts and the comparison
is made in terms of number of the protective equipment needed. It is found that the classical rolling sphere model gives more conservative results than the physics based model. Hence the results shows that it is possible to use present methods and still protect the equipment sufficiently.

Dedicated to my Parents

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## CHAPTER 1

## INTRODUCTION

### 1.1 Lightning - Overview

### 1.1.1. Phenomenon

Lightning is a conducting channel of air plasma. The lightning is caused due to electrostatic charges accumulated in clouds during thunderstorms. There are different types of lightning based on where the strike takes place [1]. The most important of all, as far as electric systems are considered is cloud to ground discharges. As the name suggests the discharge starts as a movement of charges from the cloud towards ground.

The phenomenon of generation and propagation of lightning during a thunderstorm event is discussed below. Strong winds moving in upward direction carries water droplets upward where they are cooled between temperatures of -10 to -20 degree Celsius. The collision of the super cooled water droplets with ice crystals forms a soft ice-water mixture. The collisions result in positive charge on ice crystals and a negative charge on soft ice-water mixture. The ice crystals are less heavy and therefore carried on the top portion of the cloud whereas the soft ice water mixture being heavier stays at the bottom of the cloud. This causes a charge separation within the cloud with positive charge at top of the cloud and negative at the base of the cloud. The negative charge at the bottom of the cloud produces a positive charge on the earth ground beneath it. Due to the separation of the charge within the cloud and between cloud base and earth, electric field is generated. Figure 1 explains the charge accumulation and electric field generation process in clouds and earth during thunderstorm event.


Figure 1 Charge Distribution in Cloud and Ground during Lightning [2]

Since we are concerned with lightning strikes to objects and structures on the surface of the earth and nearly $85-95 \%$ of all ground strikes are negative cloud to ground lightning [3], for the purpose of this discussion only negative cloud-to-ground lightning is described.

As the charge builds up in the cloud and on the earth due to cloud, a point is reached where strength of electric field is sufficient to cause air breakdown which has breakdown strength of approximately $30 \mathrm{kV} /$ centimeter. This field generates electron avalanche which joined together forms streamer. When tip of the leader exceeds thermal ionization threshold it propagates with high speed. The streamer propagates as its head is charges continuously seeking least resistance path. Streamers moves 30-100 meters and stops and some successful streamers move towards earth in series of steps. Due to this structures on the ground produce upward streamers. When this two discharges are joined together, an ionized path is formed which leads to a high magnitude of current from earth to the cloud. This is the
current that causes damage to the structures. As this current superheats the air to plasma generating a shock wave of thunder.

### 1.1.2. Important Lightning Parameters

1. Electrical Fields Generated by Thunderclouds

The thunderclouds generate electric field and they are an important factor to consider in following ways:

- Electric field causes sharp, grounded tips and pointed leaves of vegetation to go into corona, which generates space charge.
- The electric fields generated by thunderclouds at ground level are responsible for the initiation of upward flashes.
- The electric fields generated by thunderclouds can be used in issuing warnings on the threat of lightning strikes.


## 2. Distribution of Magnitudes of Current in Lightning

The first stroke of the flash normally contains the highest crest current. An AIEE working group published the crest current distribution [4] as given in Figure 2. This curve includes both positive and negative flashes. The curve is approximated by a lognormal distribution with 15 kA as median current and standard deviation of 0.98 .

Later on Anderson analyzed the crest current and gave an alternate distribution with median current of 46.5 kA . It had log standard deviation of 0.71 for currents above median current and 0.41 for currents below median current.

In the recent CIGRE working group report [5] it was shown that median value of current is 34 kA with $\log$ standard deviation of 0.74 . The minimum crest current was 3 kA and maximum of 100 kA .


Figure 2 AIEE Lightning Stroke Current Distibution [4]

IEEE working group uses distribution as given by Popolansky and Anderson with 31 kA as the median current. The probability function is given by,

$$
\begin{equation*}
\mathrm{P}(\mathrm{I})=\frac{1}{1+\left(\frac{\mathrm{I}}{31}\right)^{2.6}} \tag{1.1}
\end{equation*}
$$

Here, $\mathrm{P}(\mathrm{I})$ is the probability that the current in the lightning will be greater than I , where I is in kA . This IEEE curve and the CIGRE curve agree with each other except for some discrepancy at end of the distribution. Both the curves are shown in Figure 3.


Figure 3 IEEE and CIGRE Lightning Stroke Current Distribution [5]

## 3. Lightning Incidence

The number of flashes that could end up in hitting transmission line or substation is given by lightning incidence. The quantity known as ground flash density (GFD) is the basic measure of the lightning incidences on the structure. It is denoted generally as $\mathrm{N}_{\mathrm{g}}$ with units of flashes $/ \mathrm{km}^{2}$-year. The best method of this is direct measurement of the lightning activity. As it is difficult to measure all the lightning flashes accurately, an empirical formula is given as,

$$
\begin{equation*}
\mathrm{N}_{\mathrm{g}}=\mathrm{kT}_{\mathrm{d}}{ }^{\alpha} \tag{1.2}
\end{equation*}
$$

Where, $T_{d}=$ Number of thunderstorm days per year also known as keraunic level. The values of k and $\alpha$ varies according the region concerned. For example, Sweden uses $\mathrm{k}=$ 0.0046 with $\alpha=2$. Whereas South Africa which has more number of lightning incidents
used $\mathrm{k}=0.04$ and $\alpha=1.25$. CIGRE has also suggested to use number of thunderstorm hours instead of days which some researchers believe provides better estimate of GFD. Vaisala's U.S. National Lightning Detection Network (NLDN) is the most scientifically accurate and reliable lightning information system, monitoring total lightning activity across the continental United States, 24 hours a day, and 365 days a year. Corresponding GFD graph is shown in Figure 4.


Figure 4 Vaisala National Lightning Detection Network Flash Density Map -
USA[6]

More recently CIGRE has used flash counters which has range of up to 300 to 400 km to measure ground flash density.

### 1.2 Substation and Lightning

Nearly $11.3 \%$ of the blackouts in USA are due to lightning related events [7]. Due to increase in demand of electricity, reliable operation of the electric system is necessary. Substations are the point of connections that help direct flow of electricity. Many transmission
lines are connected and substation equipment failure will result in shut down of some or whole part of substation. Lightning protection of substation equipment is crucial in electric utilities since the lightning strike can cause transformer and equipment damage and therefore longtime substation outages. In many cases if the lightning hits the transformer, the oil catches fire which can lead to other failures.

There are two types of substation arrangements, Air insulated substation (AIS) and Gas insulated substations (GIS). Out of these AIS being air insulated are prone to natural factors and weather conditions. The lightning being the most hazardous. Due to electric supply reliability and expensive equipment, they must be protected from such adverse conditions. It costs a lot of money, time to repair or replace the damaged equipment. Hence utilities protect and invest a lot of money in the area of protection. For example protecting transformers or substations from lightning can cost anywhere from $\$ 20,000$ to $\$ 150,000$ and more, depending on the size and intricacies of the facility. Some large facilities whose equipment is valued at $\$ 7$ to $\$ 10$ Million have spent $\$ 150,000$ and up to protect their installations [8]. The direct stroke protection to substation is provided through mast or static shield wires. Both are grounded solidly and placed physically above the equipment. This is to make sure that the lightning strikes first such protection system and the energy is carried safely to the ground without any equipment damage. The placing of substation is done by methods as described by standards. These standards are IEEE 998-2012, NFPA 780 and IEC 62305. Such placement of protection system is done during design phase of substation plans [9].

### 1.3 Objective and Scope of Research

There are two objectives for the thesis. First is to develop two computer programs that can calculate zone of protection against direct lightning to the substations for two different methods. The first method is rolling sphere method and second is the Rizk model method. The other objective is to determine the number of protective masts needed to protect same area of the substation. The comparison will help to distinguish any differences between the conventional method and more physics based approach. In the end a conclusion can be drawn if it is really needed to change the shielding plans according to the new method or current plans are sufficient to protect equipment against direct lightning strike.

### 1.4 Thesis Outline

Chapter 1 provides an overview of process of lightning generation and propagation. The important lightning parameters are discussed which are of engineering interest. The rest of the thesis is organized as follows.

Chapter 2 presents a detailed literature review of types of transients generated due to lightning and corresponding protection methods. Also the existing methods for lightning protection are reviewed. This involves methods used by utilities for many years. Also more recent models which are based on physics of lightning are discussed. Existing software tools for lightning protection, their limitations and need of developing a simple program is discussed. Finally, the need to determine if the existing methods are sufficient for lightning protection of substation is discussed.

Chapter 3 presents development of MATLAB program for one of the conventional methods that is being used by the utilities namely rolling sphere method. A test case for addition of a substation transformer is shown.

Chapter 4 presents MATLAB program development for the Rizk model. Simulation of negative downward leader and connecting upward positive leader is described. Criterion for final jump and inception of upward leader from ground object is discussed. Also criterion for shielding failure is presented.

In chapter 5 a standard layout of 220 kV substation is taken. The calculations for number of static masts required to protect substation are performed. Both the methods are applied and results are presented.

Chapter 6 draws a conclusion based on the results of the program and presents future work. Appendix A shows detailed and commented MATLAB code for the Rolling Sphere Method (RSM). Appendix B shows detailed MATLAB code for Rizk model with commenting.

## CHAPTER 2

## LITERATURE REVIEW

2.1 Transients Generated by Lightning - Consideration for Substations

When a lightning strike hits any object or its surroundings, it generates transient voltage and current pulses. Following section presents important distinction amongst these and which of it is considered as a part of this thesis. The effects of cloud to ground discharges can be broadly grouped into two categories, namely: (1) indirect strokes which are known as induced overvoltage and (2) direct strokes. Following sections describes them in more detail.

### 2.1.1. Indirect Strokes - Induced Overvoltage

Even if a lightning strikes near an object, tower, phase conductor it can be a cause of overvoltage. Such surges are produced by induction. These arise in two different ways, one by electrostatic induction and other by electromagnetic induction. The return stroke of the lightning discharge is responsible factor for the induced voltages. Calculation of such induced stroke are done in two parts [10]:
i. The return stroke model with its associated electric field effects. This is a model of return stroke current in time and space. The return stroke is assumed vertical
ii. The voltage induced on phase resulting from the interaction of above model. For calculating induced voltages, coupling models are used. The Rusck's model is used for calculating induced voltages flashover rate.

The distance within which a cloud-to-ground lightning discharge can cause an induced voltage flashover is generally within 200 meters of the stroke. The Rusck's formula is given by [11], for calculation of induced voltage is as follows,

$$
\begin{equation*}
\mathrm{V}_{\max }=\frac{\mathrm{Z}_{0} \mathrm{I}_{0} \mathrm{~h}}{\mathrm{y}}\left(1+\frac{\mathrm{v}}{\sqrt{2} \mathrm{v}_{0} \sqrt{1-\frac{1}{2}\left(\frac{\mathrm{v}}{\mathrm{v}_{0}}\right)^{2}}}\right) \mathrm{kV} \tag{2.1}
\end{equation*}
$$

Where, $\mathrm{Z}_{0}=30$ ohms, $\mathrm{I}_{0}$ is the lightning peak current in $\mathrm{kA}, \mathrm{h}$ is the average height of the power line over ground, y is the distance between line and the lightning strike in meters, v is return stroke velocity and $\mathrm{v}_{0}$ is speed of light in free space in $\mathrm{m} / \mathrm{sec}$. Generally assumed value of v varies between 0.3 and $1.5 \times 10^{8} \mathrm{~m} / \mathrm{sec}$. The number of induced flashovers decreases as a function of BIL. Also major factor while considering induced overvoltage is steepness of the pulse. Such overvoltage can cause phase-to-ground or phase-phase flashover.

Therefore static overhead shielding system is useless in these kind of strokes. This effect of induced overvoltage is prominent in distribution lines where the insulation level is low and where overhead ground wires are normally not employed. Also since, low and medium voltage distribution networks have heights less than the surrounding environment, induced overvoltage is of major concern. Therefore for substation design of protection, these can be eliminated as far as use of static shielding system is considered.

### 2.1.2. Direct Stroke

Another effect of lightning is due to direct lightning stroke where the discharge path, that is, the path of the current is directly from cloud to the object struck. A lightning CG discharge like this hitting power-line, building structure or even a person is far more dangerous than indirect stroke. It can result in significant physical damage and have associated fire hazards. In the case of buildings it can result in cracks in the masonry work. In case of power lines and substation equipment, it can cause flashovers and it is fatal if it hits a
person. The injected voltages and currents associated with direct strokes are much higher compared to indirect strokes. As in this thesis we are concerned about power system, the focus is on power lines and substation equipment and not on the personnel safety. Most of the times the direct stroke terminate on the overhead ground wire or onto the structure that holds power conductor, like a tower. As both of these are grounded the surge travels into the ground but generates potential rise of the tower. The amount of overvoltage generated by the stroke is product of surge current and the impedance it encounters till perfect ground. For example a 30 kA strike current with equivalent tower resistance and footing resistance of $200 \Omega$ can generate voltage of the magnitude of 6 million volts. This back flashover phenomenon is another effect of lightning caused overvoltage. But as substation ground grid resistance is very low, nearly equal to $1-2 \Omega$, compared to tower footing resistance of the transmission line and therefore this is not considered while designing direct stroke protection of substations.

Another important aspect of lightning generated direct stroke is travelling waves generated on nearby transmission lines travelling till the substation. When a lightning strikes a transmission lines that are connected to the substation, a travelling wave of voltage and current is generated. This wave travels towards the substation and may cause a considerable damage if adequate protection is not provided. As the wave travels along the line, the wavefront above the corona inception voltage is reduced in magnitude by corona loss. Skin effect on line conductors also causes further attenuation due to high frequency nature of the surge. Therefore it is usual to consider lightning strikes that are close in (approximately 3 km ) when assessing the surge arrestor installation requirements. Therefore in this cases too, shielding wires are ineffective and not used.

### 2.2 Lightning Protection Practices for Substations

After careful review of transients generated by lightning and its dangerous effects of to the substation, methods of preventing damages from such overvoltage are discussed for substation.

### 2.2.1. Shield wires and masts protection

This type of protection scheme is a focus of this thesis, along with it some other protection methods are mentioned in following sections. Shield wires and masts installation are two methods are commonly employed in a substation. Overhead ground wire are bonded to the primary earthing system. They are sized for maximum lightning stroke current in the area and maximum fault current level. Preference is to use ACSR for overhead earth wires. In most of the substations, the overhead earth wires do not cross substation equipment beneath it [12] since an anchor failure which holds the shield wire can cause faults in the equipment. Sometimes lightning mast are the preferred method of lightning mitigation in smaller stations over overhead earth wires because they provide a greater security of supply and better maintenance accessibility as well as the ability to minimize the overall visual profile of the substation. The lightning masts are positioned in the substation in such a way that they do not obstruct electrical clearances and maintenance access to other equipment within the switchyard. These masts are connected directly to the substation's earth grid. Ground grids are installed at a depth such that the currents flowing in from the shield wire are easily dissipated into the earth. Ground rods at strategic locations are drilled to a depth where the soil resistivity is low. Connecting the ground grid to the rods so that the grid can access the low resistivity soil. The ground grid and depth of the ground rods are governed by standards
and utility practices. At locations where the soil resistivity is high, ground wells are accessed.

### 2.2.2. Surge arrestors protection

Surge arresters are placed in substations, transmission or distribution lines to provide the protection to the equipment connected against voltage surges. Arrestors appear as a very high impedance at normal operating voltages and a very low impedance on the arrival of a high voltage surge resulting from lightning or switching activity. Every equipment has a BIL and BSL level. Arrestors will make sure that incoming surge is either clamped below this value or grounded without reaching and damaging the equipment for which it is intended to protect. Metal-Oxide Surge Arrester (MOSA), with resistors made of zinc oxide $(\mathrm{ZnO})$ blocks, or gapped type with resistors made of Silicon-Carbide (SiC) are used. These arresters have extremely non-linear voltage-current or V-I characteristic, low power losses. Insulation coordination studies are conducted in order to decide the placement and rating of such arresters. For switching over voltages studies, the surge arresters can be represented by their nonlinear V-I characteristic as shown in Figure 5 for Siemens surge arrestor [13]. Lightning arresters are rated by the peak current they can withstand, the amount of energy they can absorb, and the breakover voltage that they require to begin conduction.


Figure 5 V-I characteristics of Typical MO Arrestors in a Solidly Eatherd 420 kV
System [13]

### 2.2.3. Communication and Electronics Equipment Protection

Even if substation is grounded properly, communication equipment are victim of the Ground potential rise (GPR) [14]. Communication equipment are grounded at some point in the substation. When a lightning strikes and potential of this ground changes and raises up. If the equipment is grounded at same points, this potential rise is same and do no affect operation of communication equipment. But wire-line telecommunications are connected through equipment bonded to the substation's ground grid and also terminated at another end by copper pair. Therefore during such events of potential rise, current will flow through the equipment and wire-line. To resolve this issue, an isolator device is placed. It functions such a way that it will allow the communication signals to pass through but not fault currents through the phone lines The standards used for designing this are IEEE Standard 487-

2000-Guide for the protection of wire-line communication facilities serving electric power stations and ANSI/IEEE Standard 80-2000-Guide for safety in AC substation grounding.

### 2.3 Classical Models for Lightning Protection of Substations

Impact of a lightning strike varies depending upon if it is an indirect stroke or a direct one.
The type of protection to be offered depends upon the equipment to be protected and type of stroke. As mentioned in previous sections, the direct stroke protection of substation equipment is based on protection offered by shield wires and masts, the methods and models that have been used by industry as of now to design these systems are discussed in detail in following section.

### 2.3.1. Fixed Angle Method

The first concept for lightning protection assumed that there is a protected zone into which neither the lightning channel nor its effect can penetrate to cause damage. According to this concept structure inside this zone is protected against direct lightning strikes. This angle is at the top of air terminal. The border surface can be produced by moving a straight line which has a constant angle to the vertical. The motion is, for example, rotation around to the vertical rod or parallel translation long a horizontal conductor. To apply the concept for shielding wires in substation, a cross section can be taken at desired location. Protective angle ranging from 30 to 50 degrees can be used for designing the protection system. As it can be seen from the Figure 6, lesser the angle, lesser is the protective angle provided by the lightning rod. Therefore more number of lightning masts will be required for low angles. The most important conflict of the fixed angle concept is with observed lightning strikes which penetrated possible protection zone. Typical cases are the lightning strikes on the sides of thin high objects.


Figure 6 Protection Angle Method with $\alpha$ as Protective Angle [15]

### 2.3.2. The Mesh Method

Air terminals are positioned around the edge of the roof and on high points. A network of conductors follows the external perimeter of the roof. This network is completed with transverse elements. The mesh size is between 5 and 20 meters depending upon the effectiveness required. The top of the down conductors fitted to the walls are connected to the roof mesh, and the bottom to dedicated earthing systems. The distance between two down conductors is between 10 and 25 meters depending upon level of protection required. The majority of lightning current is conducted and dissipated by the conductors and earthing systems closest to the point of impact of the lightning strike.

### 2.3.3. The Empirical Curve Method

The empirical curves are derived experimentally and then used as a 'scaled' model. The first step is to determine what equipment needs to be protected. After this, the exposure level has to be selected. This can be from $0.1 \%, 1 \%$ etc. Figure 7 shows protection offered by a single lightning mast to object. The height of mast is $h$ meters and that of protected
object is $d$ meters. The distance between these two is $x$. There are six curves showing various exposure levels from 0.1 to $15 \%$.


Figure 7 Empirical Curves for Single Mast Protecting One Object [16]
Some of the assumption of this method are mentioned as follows. All lightning strikes are assumed to propagate vertically downward and earth resistivity is considered very low. The method is independent of voltage level and only depends on the geometric relationship between the shield or mast, the equipment, and the ground. It does not take into account factors such as surge impedance, insulation level. Also stroke current magnitude, and the probability of lightning. Although not much numerically difficult it has some limitations. Since it ignores almost all important factors as mentioned above its application is doubtful. The modified curves are not user friendly and time consuming for design purposes. It is seen by experience that this method is not recommended for shielding design for EHV substations.

### 2.3.4. The Electro-Geometric Model

During early 1950's transmission lines were protected by protection angle and empirical methods. It was observed that outage rate was much higher than expected. Therefore, E. R. Whitehead developed EGM in 1963 after an extensive research. Later in 1976, Mousa developed a program called Subshield to use this method. In 1977 Ralph H. Lee developed rolling sphere method for shielding buildings and industrial plants. Which was then extended by J.T. Orrell [17] for use in substation design. It uses stroke current, surge impedance and BIL level of the equipment to calculate zone of lightning protection for substation. Figure 8 gives visual difference between protection angle, mesh method and rolling sphere method.


Figure 8 Difference between Rolling Sphere and Protection Angle Method [18]

### 2.4 Physics Based Models

The final attachment of lightning strike to structures is dependent on downward and upward moving leader. These models try to explain how this mechanism takes place and try to simulate the leaders to find location of final strike.

### 2.4.1. Model by Dellera and Garbagnati

The model by Dellera and Garbagnati takes into account the main physical mechanisms defined from studies of discharges in long air gaps as well as studies of lightning channels [19]. It assumes that charge per unit length along the leader channel is equal except that lower few meters have charge of $100 \mu$ Coulombs/meter. The electric field so generated by the leader is calculated based on this charge relationship. It is assumed that downward leader follows electric field lines. Therefore The direction of propagation of the leaders, both the down-coming stepped leader and the upward moving connecting leader, is determined by the direction of the maximum electric field along an equipotential line at a distance from the leader tip equal to the streamer extension. The upward leader speed is assumed to be a function of the mean voltage gradient between the upward and descending leader tips at any instant.

### 2.4.2. Leader progression model of Erikson

Eriksson proposed the improved EGM which took into account the dependence of striking distance on the structure height in addition to the known dependence on peak stroke current [20]. It also considered field intensification factors (FIF) given by Ki. These factors define degree of intensification of the electric field by the structure on the ground. The extension of the Eriksson EGM into a practical, three-dimensional air terminal placement method is referred as the collection volume method (CVM). Ki is depends upon height and width and
radius of curvature of the structure. In the case of vertical masts FIF depends on the height and tip radius of curvature. For horizontal shield wires, similar concepts are applied. For elevated structures, the Ki's are multiplied by a factor that depends on the structure dimensions and the location of the air termination on the structure. Value of Ki in space is calculated by numerical techniques such as the finite element method (FEM). CVM requires extensive electric field modeling in 3D to be carried out and greater weight is given to taller air terminations. Along with this physical criteria for leader inception must be met. It enforces the important concept of competing features which says that all points are considered capable of launching upward leaders and hence must be taken into account in the analysis.

### 2.4.3. Rizk leader progression model

The basis for this model is that an object struck by lightning is an active participant in the attachment process. In this model, upward connecting positive leader and downward connecting negative leader are modelled [21]. First the movement of downward negative leader in space is defined. Then for upward positive leader from ground structures, criterion for inception and propagation are simulated. The final strike condition is checked to see if the successful strike takes place or there is a shielding failure. This model is discussed and applied in detail in chapter 4.

### 2.5 Software Currently Used for Lightning Protection

SESShield-3D [22] is a software package developed by safe engineering services and technologies. It can be used for lightning protection designs of complex 3D environments, including substations, power plants, industrial plants and buildings. It uses rolling sphere method and the Eriksson electro geometric model, the protection angle and the mesh
method. It allows any metallic structures to act as a shielding system. It performs its calculations by first generating a 3- dimensional surface corresponding to all possible positions of the center of a rolling sphere for a specific radius, whose surface is in contact with a vulnerable structure to be shielded. Next, the software generates a 3-dimensional surface corresponding to all possible positions of the center of a similar sphere that is in contact with the shielding structures. A 3D hidden surface algorithm is then used to determine which surfaces corresponding to contact with a vulnerable structure protrude outside surfaces generated by shielding structures. These protruding surfaces represent the locations where lightning strike can hit a vulnerable structure due to inadequate shielding. The shielding structures, then are adjusted and positioned in such a way that the unprotected surface is no longer visible. No utility would spend money on a software that only does dedicated lightning protection calculations. This software is more valuable for complex structures in 3D such as buildings, bridges where geometry is unpredictable. In case of the substations, the geometry of the equipment and corresponding shielding system is not complex. For example, for protecting a transformer or a bus bar, there will be always wires parallel to the equipment and will never cross equipment as breaking of the shield wire support can cause it to fall over energized equipment. Whereas for other complex engineering structures the placement of shielding is not fixed.

Another software provided for lightning protection is by ABB known as Furse StrikeRisk. It is used to calculate risk assessments of a facility against lightning as it automates the complex calculations required by BS EN 62305 [23]. The designer can carry out and view multiple risk assessments under the banner of a single project, build new projects from
previously saved cases and create templates for standard cases. A project case can be created and used for calculations. Each case is a separate risk assessment in its own right. Each Case is used to carry out a series of calculations using relevant formulae to determine the actual risk R for the structure under review. The designer should decide the type of losses relevant to the structure, enter a number of dimensions and various weighting factors relative to the structure, along with various assigned values from the appropriate tables in annexes. Then risk R is calculated and then compared to its corresponding value of RT. If the result shows $\mathrm{R}<=\mathrm{RT}$ then the structure is adequately protected for a particular type of loss. If the result shows $\mathrm{R}>\mathrm{RT}$ then the structure is not adequately protected for the type of loss, therefore protection measures need to be applied. The above steps can be set within each case and by a series of trial and error calculations sufficient protection measures are taken until the risk R is reduced below that of RT .

Primtech is another software that features a powerful 3D lightning protection calculation implementing conventional lightning protection calculation methods [24]. Using lightning protection rods and wires as lightning arresters, it visually illustrates the required results in form of the lightning protection volumes and areas Primtech supports lightning calculations according to the DIN VDE 0101 Standard and the rolling-sphere method.

### 2.6 Conclusion

The conventional models for the lightning protection of substation are based on different concepts, namely the protective angle, empirical curves and the electro-geometrical method. Most of these models, especially protective angle and empirical curves neglect the physics behind the lightning inception and propagation. As it can be seen they are com-
pletely independent of stroke current magnitude, BIL levels of the equipment. These assumptions are oversimplifying and therefore application of such methods is limited. The rolling sphere method on the other hand gives some explanation and engineering evaluation towards lightning phenomenon and electrical parameters. However the advancement in the lightning research during the last several decades has resulted in deeper and more physics based understanding of the lightning attachment process. As explained in section 2.4, many researchers have tried to explain inception and propagation of the lightning leaders with emphasis on upward and downward leaders. Therefore today we have a possibility to simulate such phenomenon on a computer to determine the strike point onto the structure. Looking at the limitations of lightning models currently used, it is an important factor to determine if the current methods are really enough to protect the substation equipment against direct stroke or do we have to switch to new models. Also such complex analysis require software automation, this thesis attempts to focus on development of such computer program.

As seen from section 2.5 , there are many software that are commercially available for calculation of lightning protection of substation and structures. Even though some of the software are powerful 3D visualization tools, in many utilities only two dimensional cross sections are enough to determine the protection level. Also the drawing files of the utilities can be in a different format which is a limiting factor considering costs/price of such commercial software. In many utilities, they already have all most of their static shielding plans developed while designing the substation layout. Whenever new equipment is to be installed at the substation, the adequacy of already present static wire system to protect this equipment needs to be calculated. This include the addition of a transformer, reactor or bus
bar bay. Also, after a careful revision and literature review, it was found that there is no commercial software currently available in the industry that simulates new physics based models into a program. Therefore final aim would be to develop computer program and see if there are any practical differences between the conventional method and new models. Therefore the first objective of the proposed computer program is to develop a computer tool for rolling sphere method that would provide quick results with high accuracy in such a way that it is independent of any CAD software which can run on any computer. Also 'to the scale' visual representation will be provided by the program rather than just numbers. This would help the utility design engineer to position the shielding system faster. The program will also be calculating lightning protection based on Rizk model, which is physical approach towards lightning. A comparison between these models considering actual substation layout will determine if we need to switch to newer methods.

## CHAPTER 3

## ROLLING SPHERE METHOD COMPUTER PROGRAM

### 3.1 Explanation of Rolling Sphere Method and Formulae

Figure 9 shows a cross section of a static shielding system. Two poles are shown at height H above the ground. These can be either lightning masts or points on a static shield wire after taking a cross section. In a substation, it is possible to have unequal height of the masts. D is the distance between two shielding structures. According to rolling sphere method, an imaginary sphere of radius R is rolled over the substation structures. It is supported by masts, shield wires, fencing and all metallic grounded objects that can provide shielding. Starting from the leftmost side in Figure 9, the sphere first touches the ground. After encountering the mast of height H it rolls over and on top of it. Before it goes on the other side and touches the ground, another mast of height H supports it. After that, similar to the first mast it rolls over the second mast and onto the ground on the other side.


Figure 9 Calculation of Protected Zone by Rolling Sphere Method [3]
In Figure 9, all the area under the arcs generated by the sphere is considered as protected. This implies that any equipment having dimensions under this area is protected from lightning strokes whereas any structure that protrudes out of the area of protection is vulnerable
to direct lightning stroke. In such a case, additional shielding is necessary. It can be noticed from Figure 9, the distance between the two masts, D, dictates how much area of protection in generated between the two poles. As they move closer, more area is protected. This principle is used in positioning the shielding system, in order to protect the equipment. The radius of sphere R is also known as the striking distance. According to the rolling sphere method, this depends upon the magnitude of the return stroke current and is given as follows,

$$
\begin{equation*}
\mathrm{R}=8 * \mathrm{k} * \mathrm{I}^{0.65} \tag{3.1}
\end{equation*}
$$

Where, R is strike distance in meters, k equals 1 for wires and ground plane and equals 1.2 for mast. I is return stroke current magnitude in kilo amperes.

Since the stroke current in the lightning is not fixed, it is necessary to find the stroke current magnitude for which protection is required. This current is known as 'allowable stroke current' and it is calculated by following formula

$$
\begin{equation*}
\mathrm{I}_{\mathrm{S}}=\frac{1.1(\mathrm{BIL})}{\left(\frac{\mathrm{Z}_{\mathrm{S}}}{2}\right)}=\frac{2.2(\mathrm{BIL})}{\mathrm{Z}_{\mathrm{S}}} \mathrm{kA} \tag{3.2}
\end{equation*}
$$

Where, BIL-Basic lightning impulse insulation level of equipment to be protected in kilovolts. Zs is surge impedance of the bus in ohms. The calculation of surge impedance is done using equation (3.3).

As seen from (3.1), the striking distance is a function of stroke current and increases as the current increases. If a shield wire protects the equipment for stroke current $I_{1}$ then it will shield for any current $I_{2}>I_{1}$. Therefore, shielding will be effective against any current higher than protected current. Stroke currents less than this value are permitted to enter protected zone since the equipment can withstand stroke generated voltages below its BIL
and would not damage it. The probability that the current in lightning will be more than the calculated allowable stroke current is given by equation (1.1). For example, if allowable stroke current is 10 kA , then probability that a current less than this will be present in a lightning strike is $5 \%$.

### 3.2 Data Preparation for the Program- Entering Electrical Parameters

The substation has different voltage levels with transformers at the voltage interface. Therefore, equipment BIL on each voltage side is different. As seen from (3.1) and (3.2), every BIL has a corresponding allowable stroke current and therefore striking distance. The surge impedance depends upon the height and radius of the bus. Therefore, user of the program is required to enter BIL level, bus height and conductor radius. The surge impedance $(\mathrm{Zs})$ in ohms is given by:

$$
\begin{equation*}
\mathrm{Z}_{\mathrm{S}}=60 \times \sqrt{\ln \left(\frac{2 \mathrm{~h}}{\mathrm{R}_{\mathrm{C}}}\right) \ln \left(\frac{2 \mathrm{~h}}{\mathrm{r}_{\mathrm{s}}}\right)} \tag{3.3}
\end{equation*}
$$

Where, h is average height of the conductor in meters. Rc is the corona radius of the conductor in meters. $\mathrm{r}_{\mathrm{s}}$ is metallic radius of the conductor or equivalent radius in case of bundled conductors in meters. The corona increases the radius of the conductor. The corona radius in (3.3) is calculated using (3.4), which is given below and should be solved iteratively.

$$
\begin{equation*}
\mathrm{R}_{\mathrm{C}} \times \ln \left(\frac{2 \mathrm{~h}}{\mathrm{R}_{\mathrm{C}}}\right)-\frac{\mathrm{V}_{\mathrm{C}}}{\mathrm{E}_{0}}=0 \tag{3.4}
\end{equation*}
$$

Where, Vc is allowable insulator voltage for negative polarity surge having 6 microseconds front in kilovolts (Vc is BIL for post insulators) and Eo is limiting corona gradient taken
equal to $1500 \mathrm{kV} / \mathrm{m}$. The metallic radius of the bus, if bundled, is calculated by (3.5) and substituted in (3.3).

$$
\begin{equation*}
r_{s}=r \times\left(\frac{g \times s}{r}\right)^{\frac{n-1}{n}} \tag{3.5}
\end{equation*}
$$

Where, n - Number of sub-conductors in bundle. g equals 1 for bundles of 1,2 and 3 sub conductors and equals 1.12 for 4 bundle conductor. $r$ is a conductor radius in meters and $s$ is the distance between conductors in meters. Figure 10 represents the flowchart of entering the data into the program developed for rolling sphere method and finding strike distance.


Figure 10 Calculation of Strike Distance

### 3.3 Defining Axis Orientation for the Program

As RSM is a geometry based method, dimensions of the shielding system and of protected equipment are required. The coordinate system is established to enter geometrical data. Figure 11 shows a top view of a shielding system and equipment in a typical substation.


Figure 11 Coordinate System for the Program
In Figure 11, OP is a shield wire denoted by dotted line. Similar to OP, there are two more shield wires running parallel to this wire. They too are shown by dotted lines. Point ' O ' is considered as the origin for this coordinate system. This origin can be any point on which a shielding wire rests. In this particular example, the origin is one of the poles of static wire OP but can be taken either at Q or R . The Y axis runs parallel to the shielding system and the X axis is perpendicular to it. The Z axis comes out of XY plane and is not visible in this two dimensional view. $Z$ axis represents height of the equipment or shield wire. Typical circuit breaker, disconnect switch and a bus bar are shown between shield wires. AB represents the cross section taken at distance ' S ' away along the Y axis. Similarly, different vertical cross sections can be taken along the Y axis. Once the axes orientation is defined, coordinates corresponding to the equipment and shielding wires can be entered.

Following section describes how to enter coordinates of the shielding system and equipment. The static wire is supported at two ends by poles or dead end structure. The shield
wire can be resting on poles with either equal or unequal heights. Therefore, to define a shield wire we need ( $\mathrm{x}, \mathrm{y}, \mathrm{z}$ ) coordinates of the topmost point on two poles. Here, z coordinates is the height of the pole. A hanging wire supported at two ends with only its own weight as the force acting on it, takes the shape of a catenary. Shield wires are approximated by single straight wire. Once the equation of the wire is known, only one co-ordinate is enough to find the other two along the curve. When a cross section ' $S$ ' distance away on $y$ axis is defined, $x$ and $z$ coordinates can be known by substituting $y=S$ in the equation. The equipment is approximated by a rectangular parallelepiped. Only two opposite corner coordinates are enough in three dimensional space to define a parallelepiped. Hence the user is required to enter only ( $\mathrm{x}, \mathrm{y}, \mathrm{z}$ ) coordinates of the bottom corner and diagonally opposite top corner. Z coordinate of bottom corner is always 0 since it lies on the ground and that of top corner is height of the equipment. When cross section at ' $S$ ' is defined, corresponding x and z coordinates can be found.

### 3.4 Data and Executable File

Data files are created from where the program reads the data. Equipment data, bus data and shield wire data are the three files created that store coordinates of the respective elements and electrical parameters. Since user already knows the position of the existing shielding system, he can enter coordinates corresponding to shield wires. Also the placement of the equipment is fixed since it is decided on factors like connection to other buses or ease of installation. Therefore equipment coordinates are fixed and can be entered. The bus data and BIL levels can be entered by knowing at which voltage level the equipment is operating. Once the user runs the program and finds that existing system is inadequate, repositioning can be done easily. The visual representation of the program gives exact location
where additional or repositioning of shielding is required. The repositioning is achieved by changing the coordinates of the shield wire appropriately. Since it involves only minor changes in the data file, user need not enter all other data again hence making calculations faster.

The other goal of the program is to make it independent of any CAD tool used by the utility. This is achieved by creating an executable file of the program. MATLAB compiler allows to create an executable files that can run on any machine. Figure 12 shows the executable program file output. The program finds the strike distance and waits for user to enter the desired cross section. The specifications of the bus are the same as mentioned in Table I for a 69 kV substation


Figure 12 Executable File Output

### 3.5 Result and Test Case

The program calculates the surge impedance of the bus using (3.3). To verify that calculated values are correct, they are verified with the values given in [25] and shown in Table I.

| Bus Names | Height of <br> Bus <br> (Feet) | BIL <br> $(\mathrm{kV})$ | Diameter of <br> Bus <br> (Inches) | Surge imped- <br> ance by IEEE <br> 998 (Ohms) | Surge Impedance <br> by Program <br> (Ohms) |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 69 kV Substation |  |  |  |  |  |
| Bus A | 14 | 350 | 4.5 | 300 | 308 |
| 230 kV Substation |  |  |  |  |  |
| Bus B | 39 | 900 | 5.5 | 336 | 335 |
| 500 kV Substation |  |  |  |  |  |
| Bus C | 55 | 1800 | 4.5 | 336 | 336 |

Table I : Surge Impedance Verification
Since the values match very closely, the program calculates them correctly and can be used further for finding stroke current and strike distance.

Now a case for addition of a station transformer in parallel to existing transformer is presented to show usability of the program. Figure 13 shows addition of a transformer in a substation. The proposed transformer is shown in dotted rectangle. The smaller rectangle is the actual transformer whereas outer one is a degasifier unit.


Figure 13 Adding New 69/230 kV Station Transformer.

The rating of the transformer is $69 / 230 \mathrm{kV}$. The transformer has a height of 32 feet, 30 feet of length and 20 feet width. The height of the transformer includes foundation, bushing and phase connection height. The existing shielding system is shown by dotted lines. Wires $\mathrm{OP}, \mathrm{QV}, \mathrm{RT}$ are three shield wires and are resting on their respective poles. Other structures such as bus bars, breakers are also seen in the diagram. But we are only interested in shielding system that protects the transformer and the transformer itself. We should now determine if these three wires are enough to protect a newly added transformer. The pole O is chosen as the reference with $(\mathrm{x}, \mathrm{y})$ as $(0,0)$. Poles $\mathrm{O}, \mathrm{P}, \mathrm{Q}, \mathrm{R}, \mathrm{V}$ are at height of 60 feet. Poles T and U are 55 feet tall and they are on 69 kV side of the transformer.

Once all the data is entered into the program, it finds out strike distance and plots out the zone of protection, which is marked by circles in Figure 14 and equipment as rectangular section. The three vertical masts are basically heights of the shield wires at cross section specified. The cross section specified is 202 feet away from the origin where the transformer is placed. Since the transformer width is 20 feet, it ends at $y=222$ feet.


Figure 14 Visualization of Existing Shielding System.

Figure 14 shows a portion of the equipment protruding out of the sphere on the rightmost side. Therefore shielding is inadequate and we must reposition static wire RT. Figure 15 shows repositioning of wire RT to the new wire, RU. With this, we must check if the transformer is protected by running the program again and changing the T coordinate of wire to U coordinate in the data file. All the other parameters are same as in Figure 13.


Figure 15 Repositioning the Shield Wires for Protection
The program is run again and used to verify if the repositioning gives adequate lightning protection zone. Two different sections are shown in Figure 16 and Figure 17, one at 202 feet away and another at 222 feet away respectively. These are the end coordinates of the transformer. If these are protected then all cross sections in between are also protected


Figure 16 Visualization after Repositioning Shielding System with Section at $\mathrm{y}=$

$$
202 \text { feet }
$$



Figure 17 Visualization after Repositioning Shielding System with Section at $\mathrm{y}=$ 222 feet

We can see from Figure 16 and Figure 17, the transformer is perfectly protected for both the sections and no area of it protrudes out of the rolling sphere arcs. Hence, repositioning is correct and there is no need to reiterate the steps.

## CHAPTER 4

## RIZK LEADER INCEPTION AND PROPAGATION MODEL IN MATLAB

The following section describes how Rizk model is simulated in MATLAB. This involves simulation of downward negative leader as a vertical charge column in space, inception of positive leader from the ground object and propagation of upward leader towards negative leader. In the end successful meeting of these two leader will be described and simulated. The program flow is described and various test cases are shown in order to explain program visually.

### 4.1 Modeling Descending Downward Leader

The downward leader is modeled as a vertical negative charge column in space. As the leader moves down towards earth, it carries this charge along with it. The charge center is assumed to be at the base of the cloud from which leader originates. A finite charge is supplied from this cloud base to the leader.

There are various models to define how exactly charge in the leader is distributed. For modelling the leader in Rizk method, the linear charge decay is used to define charge distribution. The mathematical modelling in MATLAB goes as follows. The charge density in the leader at ground level is assumed to be $\rho_{0}$ coulomb/meter. This charge density decreases gradually and becomes zero as one moves from tip of the leader to base of the cloud. This charge column is modeled by linear charge density column and therefore charge density along the column with height ' $Z$ ' of the leader can be written mathematically as,

$$
\begin{equation*}
\rho(\mathrm{z})=\rho_{0} \times\left(1-\frac{\mathrm{Z}}{\mathrm{H}_{\mathrm{CL}}}\right) \frac{\text { Coloumb }}{\text { meter }} \tag{4.1}
\end{equation*}
$$

Both Z and Hcl are in meters. For a small segment of charge length, say dz, the charge contained in it can be written as,

$$
\begin{equation*}
\mathrm{dq}=\rho(\mathrm{z}) \times \mathrm{dz} \tag{4.2}
\end{equation*}
$$

Therefore total charge contained in the downward lightning column is given by integrating whole column of the leader from ground level (that is $\mathrm{z}=0$ ) till cloud base $\left(\mathrm{z}=\mathrm{H}_{\mathrm{cl}}\right)$,

$$
\begin{equation*}
\text { Total Charge }=\int_{0}^{\mathrm{H}_{\mathrm{cl}}} \rho(\mathrm{z}) \times \mathrm{dz} \tag{4.3}
\end{equation*}
$$

Substituting (4.1) in (4.3),

$$
\begin{align*}
& \text { Total Charge }=\mathrm{p}_{0} \times \int_{0}^{\mathrm{H}_{\mathrm{CL}}}\left(1-\frac{\mathrm{Z}}{\mathrm{H}_{\mathrm{CL}}}\right) \mathrm{dz}  \tag{4.4}\\
& \text { Total Charge }=\frac{\mathrm{H}_{\mathrm{CL}} \times \rho_{0}}{2} \text { Coulombs } \tag{4.5}
\end{align*}
$$

Once total charge is described in terms of $\rho_{0}$ as in (4.5), it is important to know charge in terms of stroke current. It is well known that the charge is related to stroke current in the downward leader I. There are many empirical formulae that predict the total charge in the descending downward leader. The empirical formula in [26] is used to describe total charge in leader as follows,

$$
\begin{equation*}
\mathrm{Q}=76 \times \mathrm{I}^{0.68} \times 10^{-3} \text { Coulombs } \tag{4.6}
\end{equation*}
$$

In above equation I is in kA. Figure 18 shows variation of current in negative leader versus charge density at ground level.


Figure 18 Variation of Charge Density at Ground versus Current in Downward

## Leader

After assuming the charge distribution in the negative leader, it is important to describe how the voltage at the tip of the leader is calculated. It is possible to calculate the potential along the length of the leader. The total voltage or the voltage at the tip of the negative leader is sum of the potential along the leader channel plus the voltage of the corona shell at the tip of the leader. Therefore,

$$
\begin{equation*}
\text { Voltage }_{\text {tip }}=\sum \text { Voltage along the leader channel }+ \text { Voltage }_{\text {corona }} \tag{4.7}
\end{equation*}
$$

Now, the voltage along the channel is defined by the voltage gradient of the leader. This gradient is given to be 60 volts/cm as given in [27]. Therefore, total voltage depends on length of the leader, which in turn depends the height at which cloud base is assumed. For example for one kilometer of a leader will have total voltage along the leader channel of 6 megavolts. As the cloud base is assumed at 2.5 km above the earth. Therefore total voltage drop across the leader length is nearly 15 megavolts.

The voltage of the at corona tip of is calculated by knowledge of corona shell. The field at outer surface of this shell is $\mathrm{E}=30 \mathrm{kV} / \mathrm{cm}$. The radius of this corona shell can be given by,

$$
\begin{equation*}
\mathrm{r}=\frac{\mathrm{Q}}{2 \pi \varepsilon \mathrm{E}} \text { meters } \tag{4.8}
\end{equation*}
$$

In this equation, Q is the charge density at ground level of the leader column.

### 4.2 Inception of Positive Leader from Structure

Once detailed modelling of the downward leader is done, following section presents MATLAB coding for upward leader inception. As the negative leader travels downward it carries charge and thereby generates surrounding electric field. Therefore, at some point there is enough space potential around ground objects that a successful inception of the upward positive leader takes place from the ground object. This is given for mast/slender structure and wire/horizontal conductor as follows [28],

$$
\begin{gather*}
\mathrm{U}_{\text {POS_CRIT_MAST }}=\frac{1556}{1+\left(\frac{3.89}{\mathrm{~h}}\right)} \mathrm{kV}  \tag{4.9}\\
\mathrm{U}_{\text {POS_CRIT_WIRE }}=\frac{2247}{1+\left(\frac{5.15-5.49 \times \log (\mathrm{a})}{\mathrm{h} \times \log \left[\frac{2 \mathrm{~h}}{\mathrm{a}}\right]}\right)} \mathrm{kV} \tag{4.10}
\end{gather*}
$$

In (4.9), h is the height of mast in meters and in (4.10) h is height of shield wire in meters and ' $a$ ' is radius of the shield wire in meters. Figure 19 shows variation of inception voltage for both mast and wires. Radius ' $a$ ' is taken to be 5 centimeters which is close to radius of a shield wire.


Figure 19 Effect of Height of Mast/Shield Wire on Upward Leader Inception Voltage

Once the critical inception voltage for each of the structure is known, the condition to reach corresponding voltage is calculated by simulating the downward leader and its effect on the structure as it approaches near the ground. The voltage due to any charge Q at some distance away is given as follows,

$$
\begin{equation*}
\text { Voltage }=\frac{1}{4 \pi \varepsilon_{0}} \times \frac{\text { Total Charge }}{\text { distance }} \tag{4.11}
\end{equation*}
$$

Consider a leader approaching ground structure at R distance away from the object and height of the grounded object is $h$. As the downward leader moves towards ground, the charge carried produces voltage on the ground object. By method of image of charge and substituting (4.2) in (4.11) we get voltage at the tip of the mast due to this leader as,

$$
\begin{equation*}
\mathrm{V}_{\text {tip }}=\frac{1}{4 \pi \varepsilon_{0}}\left[\int_{\mathrm{h}}^{\mathrm{H}_{\mathrm{CL}}} \frac{\rho(\mathrm{z}) \mathrm{dz}}{\sqrt{(\mathrm{z}-\mathrm{h})^{2}+\mathrm{R}^{2}}}+\int_{\mathrm{h}}^{\mathrm{H}_{\mathrm{CL}}} \frac{-\rho(\mathrm{z}) \mathrm{dz}}{\sqrt{(\mathrm{z}+\mathrm{h})^{2}+\mathrm{R}^{2}}}\right] \tag{4.12}
\end{equation*}
$$

Substituting (4.1) in (4.10) we get,

$$
\begin{equation*}
\mathrm{V}_{\text {tip }}=\frac{\rho_{0}}{4 \pi \varepsilon_{0}}\left[\int_{\mathrm{h}}^{\mathrm{H}_{\mathrm{CL}}} \frac{1-\frac{\mathrm{x}}{\mathrm{H}_{\mathrm{CL}}}}{\sqrt{(\mathrm{x}-\mathrm{h})^{2}+\mathrm{R}^{2}}}-\frac{1-\frac{\mathrm{x}}{\mathrm{H}_{\mathrm{CL}}}}{\sqrt{(\mathrm{x}+\mathrm{h})^{2}+\mathrm{R}^{2}}}\right] \tag{4.13}
\end{equation*}
$$

Figure 20 shows variation of height of inception versus the current in downward leader. It is assumed that mast is 50 feet tall with location $(0,0)$ and negative leader at $(5,20)$ in coordinate axis.


Figure 20 Variation of Height of Inception with Stroke Current Magnitude

### 4.3 Propagation and Leader Potential of Positive Leader

The propagation of the positive leader is in such a way that it tries to meet the tip of the negative leader. Once the positive leader inception takes place from the ground object, the vector motion of the upward positive leader seeks tip of negative leader and the negative leader maintains its position in $x-y$ plane but reducing its $z$ (vertical) coordinate.

As the upward leader moves its length increases till it either meets the downward leader or is unsuccessful in the strike. The voltage at the tip of the leader as it moves is calculated by empirical formula [29] by Rizk as,

$$
\begin{equation*}
U_{\text {POS }}=(L) * E_{\text {inf }}+x_{0} * E_{\text {inf }} * \ln \left(\frac{E_{i}}{E_{\text {inf }}}-\frac{E_{i}-E_{\text {inf }}}{E_{\text {inf }}} \times e^{\frac{-L}{x_{0}}}\right) k V \tag{4.14}
\end{equation*}
$$

Where, $\mathrm{E}_{\mathrm{i}}$ and $\mathrm{E}_{\mathrm{inf}}$ are the initial and ultimate values of the leader gradient. It is assumed that $E_{i}=400 \mathrm{kV} / \mathrm{m}$ and $\mathrm{E}_{\mathrm{inf}}=50 \mathrm{kV} / \mathrm{m} . \mathrm{x}_{0}$ is the product of velocity of the upward leader and time step in meters. $L$ is length of positive leader in meters.

The propagation of the upward leader is in such a way that its tip always seeks the downward leader tip. In the program, it is assumed that both leaders move with a step of $2 \mu$ seconds during each iteration. Therefore for each iteration the downward leader moves 1.5 (leader velocity) $* 2($ single iteration time step) $=3$ centimeters down. Correspondingly positive leader moves up in the space.

### 4.4 Validation of the Program Using a Test Case

Following section describes various test cases to show working and validity of the program. Initially a single mast is considered with downward leader approaching ground. This case shows successful interception of lightning strike by the mast. The downward leader is located at location ( 5 meters, 5 meters). The mast is 50 feet tall and located at ( 0 meters, 0 meters). As the downward leader approaches ground, the upward leader is initiated when tip of the downward leader is at 73.95 meters. The current magnitude is considered to be 15 kA . Figure 21 shows how a positive leader is incepted and seeks negative leader. It shows a point in space when initiation of upward leader starts and when the leader potential
gradient reaches $500 \mathrm{kV} /$ meter and the simulation stops. At this point negative leader is at 52.4 meters and positive leader is at 36.54 meters above the ground.


Figure 21 Single Mast With Upward and Downward Leader - Successful Strike Now a case is considered where the mast is unable to intercept the lightning stroke. The lightning mast is located at $(0,0)$ meters and the leader is travels down with location at $(15$, 15) meters. The leader current is now 5 kA . The inception takes place at 31.35 meters. It is therefore clear that as lightning stroke goes far away from the mast or if the current in the leader reduces, the mast is not able to intercept the current. In this case, by the time leader gradient reaches $500 \mathrm{kV} /$ meter, the negative leader is at 16.83 meters and positive leader at 19.22 meters. That height of positive leader has already crossed that of negative leader leading to shielding failure. Figure 22 clearly explains this case.


Figure 22 Unsuccessful Interception of Lightning Strike
Now two masts are considered, one located at $(10,10)$ and other at $(-10,10)$. One mast is 50 feet and other is 60 feet tall. The leader is assumed to carry 8 kA of current and located at $(0,0)$ in space. Figure 23 explains this case.


Figure 23 Successful Interception of Stroke by One of the Mast - Two Mast Case

As the leader approaches ground the upward leader is incepted at different time and at different height of the downward leader. Once upward leaders are initiated and start travelling toward negative leader, the condition for final strike has to be checked. In this case upward leader for 50 feet mast is unsuccessful in interception the negative leader since the leader for 60 feet mast meets the final strike criteria prior in time.

Now, there can be a case where both the upward leaders fail to intercept negative leader and this criteria is crucial in shielding failure. Therefore, for a case where both the upward leader reach the same height as the downward leader but still do not reach gradient of $500 \mathrm{kV} /$ meter, the shielding failure occurs. In this case designer must change the coordinates of the static shielding mast and recalculate for that particular stroke.

## CHAPTER 5

## COMPARISON OF TWO METHODS USING SUBSTATION LAYOUT

### 5.1 Substation Layout and Geometry of the Equipment

Before starting lightning protection design of the substation, it is important to know layout of the substation bus work. Figure 24 shows a physical layout of a 220 kV substation. As shown in the figure, there are three buses in the substation. Two of the buses are 28 feet high and other is 20 feet high. The bus spans and length of the sections in the substation are shown too.


Figure 24220 kV Substation Layout Top View

### 5.2 Rolling Sphere Method to Design Protection Shielding System

The lightning protection should be designed for every bus. Therefore initially bus 1 is considered. The bus is to be protected by positioning the lightning masts using developed computer program. To start with the designing, two lightning masts are placed on either side of the bus with an objective that the rolling sphere will not touch the bus bar and go over and
on top of it. Height of the bus is 28 feet and width is 30 feet as shown in figure 24 . The BIL level corresponding to 220 kV bus is 900 kV . Once these electrical parameters of the bus are entered into the program, allowed stroke current and strike distance for the rolling sphere is calculated. This strike distance is radius of the rolling sphere. The value of allowable stroke current is 6.27 kA . This corresponds to radius of rolling sphere equal to 86.04 feet. While positioning the lightning masts, a two dimensional cross section of the bus is used. Now the two lightning masts are positioned 37 feet away from either side of the bus. The height of lightning mast is selected to be 50 feet. The program is run and output is shown in figure 25 .


Figure 25 Protection for Bus 1 using Two Masts on Both Sides
As we can see, the bus is perfectly protected in between two masts without rolling sphere touching it. If we try to move masts farther away from the bus, the rolling sphere will touch the equipment and therefore this position of the masts is now fixed. Now the next step for placement of the masts is to place a mast next to current mast and in parallel to the equipment. To decide how far the mast need to be placed, it is made sure that the rolling sphere
stays on top of the bus for diagonal section of the bus. Initially, the two masts is placed at 37 feet from the current masts on either sides. Figure 26 shows the placement of the old masts and the new two masts for this diagonal two dimensional section.


Figure 26 Cross Section for Placement of New Mast Location
As seen from the figure, the distance between two masts, one already placed and other new mast, is the diagonal distance which equals 110.4 feet and length of the bus (diagonal) in between is 31.84 feet. Entering this data into the program, the output is shown in figure 27.


Figure 27 Protection Offered by New and Old Mast to Diagonal Section of Bus

As it can be seen from the figure, the bus is perfectly protected. On similar design method two more masts are added in parallel to the existing masts to complete the lightning protection design for bus 1 . The final placement of the masts after designing the protection for bus1 is shown in figure 28. The six circles represent the final placement of lightning masts for bus 1 protection. Each mast is 37 feet away from either side of the bus and distance between two adjacent masts is 37 feet too.


Figure 28 Final Positions of Lightning Masts for Bus 1 Protection
Now since bus 2 has similar dimensions as that of bus 1 which is 30 feet wide and 28 feet high, there is no need to perform the steps for placement of the masts again. The final positioning after designing protection for bus 2 is shown in figure 29 .


Figure 29 Lightning Mast Positions After Bus 2 Protection Design
Bus 3 is protected using one of the mast placed already shown in figure 30 which is 42.2 feet away from the bus and another new mast is added on the other side of the bus at same distance from the bus.


Bus 3

Figure 30 Addition of a New Mast to Protect Bus 3

The zone of protection has to be checked for this placement of the mast. The coordinates of masts and bus are entered into the program and output is shown in figure 31 .


Figure 31 Verification for Correct Positioning of New Mast for Bus 3
As seen from figure 31, the bus is perfectly under arcs generated by rolling sphere and therefore well protected. Now to place new mast next to existing masts, the maximum distance should be calculated. For this diagonal section has to be considered as shown in figure
32.


Figure 32 Cross Section For Placement of New Mast to Protect Bus 3

The coordinate data is entered into the program and figure 33 shows perfect shielding against lightning. Therefore this arrangement of masts is correct and the final arrangement to protect all three buses using rolling sphere method program is shown in figure 34 .


Figure 33 Verification for Correct Positioning of New Mast Location


Bus 3


Figure 34 Final Positioning of Masts to Protect all Three Buses in Substation

### 5.3 Rizk Model based MATLAB Program for Lightning Protection Calculations

From the previous calculations, the rolling sphere method requires 12 masts and the location of masts in shown in figure 34 . With this as a starting point, the calculation for Rizk model are performed. To start with, for bus 1 lightning protection calculations are performed. First, current placement of masts is used to find if they are enough to protect the given area of bus 1. Therefore, coordinates of the six masts located around bus 1 are entered into the program. The design strike current is 6.27 kA which is obtained from rolling sphere method initial calculations. The height of all masts is taken as 50 feet. The output of the Rizk model MATLAB program is shown in figure 35.


Figure 35 Protection of Bus 1 keeping Shielding System same as RSM
The lightning leader travels downward and reaches height of 28 meters ( 92 feet) and the upward leader from mast at $(0,0)$ location travels 24.09 meters( 79 feet) when the final strike happens. Therefore, this arrangement is perfect against direct shielding failure. This is be-
cause, height of positive leader is not greater than negative leader when final strike condition is met. Now since, we have enough margin to reposition the masts away from the substation bus work, the coordinates of the masts are changed without changing height of mast, which is 50 feet ( 15.24 meters).

Instead of six masts only four are placed with distance between adjacent masts as 75 feet and distance of from either side of bus is taken as 47.5 feet away. The output of simulation is shown in Figure 36.


Figure 36 Repositioning of Masts According to Rizk model for Bus 1
The height of inception of upward positive leader for four masts is 37.5 meters above the ground. The height of the negative and positive leader at final strike are 22.3 and 22.29 meters respectively. As it is observed that, any further change in placing the masts will cause negative leader to travel below upward leader which is a condition for shielding failure, the repositioning of the masts is stopped and this is the final position for masts. Figure 37 shows final position of masts in order to protect bus 1.


Figure 37 Placement of Mast after Positioning Masts for Bus 1
Now, protection of bus 2 is considered. Since one mast is placed already near bus 2 , one more mast is placed on other side of bus 2. The distance of both the masts from bus 2 is 36 feet with height 50 feet. The placement of next mast will be on similar lines as for bus 1 and can be done 75 feet away from the first mast. But since there is not enough space the next mast is placed at 60 feet away from the first mast. Therefore final layout is shown in figure 38.


Figure 38 Shielding Mast Positions after Bus 2 Protection

The coordinates of the masts for bus 2 are entered into the program and checked for protection. The detailed result of the simulation are shown in figure 39. It shows four mast placed around bus 2 .


Figure 39 Output of the Program to Validate Correct Shielding Position
It is observed that height of inception for upward leader is 42.85 meters above the ground. At final jump conditions negative leader is at 31.9 meters and positive leader from mast at $(0,0)$ location is at 24.92 meters above ground. Therefore, bus 2 is protected. Now bus 3 already has one mast on one of its sides at 12 feet of distance. Similarly another mast is placed 12 feet away from the bus. Two more masts can be placed on either side of the bus at a distance of 70 feet away from current masts. The final layout is similar to the one shown in figure 40 . As shown by figure 36 and figure 39 , the buses are protected and the design is complete.


Figure 40 Final Placement of Masts to Protect all Three Buses using Rizk Model
Program

## CHAPTER 6

## CONCLUSION AND FUTURE WORK

### 6.1 General Conclusions

- Computer tool is developed for rolling sphere method to design lightning protection of the electric substations. A test case is shown for addition of a new substation transformer using rolling sphere method.
- The program is very easy to use and gives zone of protection calculation quickly. This avoids the design effort by utility design engineers to refer back to drawings and perform calculations by hand. The visualization of zone of protection helps in redesigning protection system by knowing where the rolling sphere touches the equipment.
- The Rizk model program is developed that gives more physical approach towards lightning phenomenon. A detailed simulation of downward, upward leader and their propagation for final jump is programmed into MATLAB. This program too is easy to use, since it involves entering parameters and coordinates similar to rolling sphere method program.
- A 220 kV substation layout is used to compare between these two methods. As the area of the substation is same in both cases, the results obtained give a clear picture of performance of the two methods.
- It is seen that Rizk model requires 10 masts of 50 feet height to protect the bus work whereas rolling sphere method needs 12 masts to protect the area. Although there is not much of a difference between these two, rolling sphere method can therefore be called as conservative method. The main reason that can be attributed to this
difference is the simulation of both upward and downward leaders for final strike in Rizk model. In rolling sphere method, concept of upward leader is not given consideration.
- Therefore it is suggested that utility engineers can still use the conventional lightning protection design methods and there is no need to shift to new models.


### 6.2 Future Work

While developing RSM, some assumptions are made which can be improved.

- The shield wires are installed between two vertical masts. Any structure hanging between two supports would take a shape of a catenary. Therefore, the shield wire will have sag, thereby reducing its height at particular sections. Since RSM is a geometry based method, this would mean that the rolling sphere may dip thereby touching the equipment beneath. Therefore more accurate mathematical catenary equation can be used for shield wires.
- Also, substation equipment is assumed to be a rectangular cross section which is true in case of buses, but transformers, breakers being not ideal shapes cannot be always modeled like this. The geometry can be improved by specifying some points on these equipment so as to model them as closely as possible without sacrificing the program simplicity.

Some of the generalizations made during the development of the Rizk model can be improved.

- The background field during a thunderstorm is neglected. The effect of cloud field and other charges in space different from that of negative leader will induce potential on structures on ground. This will in turn affect the critical upward leader inception voltage. Therefore, the time at which upward leader is initiated can be different than calculated ignoring background electric field.
- The cloud height being a statistical parameter can be varied in order to see its impact on charge density in leader. Although, cloud height is generally given between 2.5 -3 km , this would not make much difference to the final output of the model.
- The proximity effect on one mast due to presence of the other is not considered to simplify the calculations. To consider this, the upward leader inception voltage has to be multiplied by some factor that relates to the geometry between two masts. This will change the height at which upward leader is incepted.


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## APPENDIX A

## MATLAB CODE FOR ROLLING SPHERE METHOD

\%\#\#\#\#\#\#\#\#\#\#\#\#\#\# Main Calling Function \#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\%
\% This program calculates zone of lightning protection for the given
$\%$ arrangment of equipment shield wire positions and bus electrical data.
home;
clear all;
clc;
clf;
disp('Before entering any value, make sure that all the .txt files are in the same folder as this program is.')
disp('Also check if all the information is entered in given units as in the manual and in the respective columns.')
$\operatorname{disp}\left({ }^{\prime} \quad\right.$ )
\%The first step is to calculate surge impedance for given bus structure
\%The function surge_impe() is called which calculates surge impedance Zs
\%and returns R, strike distance for further calculation
$\mathrm{R}=3.28$ *surge_impe_chek();
$\operatorname{str}=[$ 'Strike distance in feet is ' num $2 \operatorname{str}(\mathrm{R})$ ];
disp(str)
hold on;
\% The following piece of code taken in the data of the shield wires
\% It takes in 3-D co-ordinates of the two poles on which shield wire is
$\%$ mounted which are used for further calculation
$\%$ THE CO-ORDINATE SYSTEM HAS ORIGIN $(0,0,0)$ AT ANY POINT ON WHICH SHIELD
\% WIRE IS MOUNTED. SUGGESTED POLE IS ANY POLE ON EXTREME END OF THE CROSS
\% SECTION.
\% WHILE ENTERING CO-ORNITAES START FROM EXTREME END(LEFTMOST) OF THE CROSS
\% SECTION AND MOVE ALONG THE CROSS SECTION
\%fid=fopen('C:/Users/vinitmarathe/Desktop/Lightning Protection/SRP project/SRP_fi-
nal_program/MATLAB code/wire_data.txt');
str=strrep(pwd,'"','/');
str=strcat(str,'/wire_data.txt');
fid=fopen(str);
total_wires=input('How many wires/mast are present?(Note: This should be same as en-
tered in wire data.txt file): ');
for wire_no=1:total_wires
for $\mathrm{i}=1: 2$
tline $=$ fgets(fid);

```
    end
    A=fscanf(fid,'%f');
    x1(wire_no)=A(1);
    yl(wire_no)=A(2);
    zl(wire_no)=A(3);
    x2(wire_no)=A(4);
    y2(wire_no)=A(5);
    z2(wire_no)=A(6);
end
```

\% The following piece of code taken in the data of cross-section of the
\% equipments. It takes in 2-D co-ordinates of leftmost bottom corner and
$\%$ rightmost upper corner which are used for further calculation
\% SINCE CO-ORDINATE SYSTEM IS FIXED AS PER SHIELD WIRE POLE AT
( $0,0,0$ )
\% ENTER THE CO-ORDINATES OF EQUIPMENT WITH RESPECT TO IT.
present_perbus=input('Is there any perpendicular bus in the shielding? Enter Number $1=$
YES and $0=$ NO: ');
if present $\_$perbus==1
\%fid=fopen('C:/Users/vinitmarathe/Desktop/Lightning Protection/SRP project/SRP_fi-
nal_program/MATLAB code/per_bus.txt');
str=strrep(pwd,' ${ }^{\prime}$, '/');
str=strcat(str,'/per_bus.txt');
fid=fopen(str);
$\mathrm{tt}=$ fgets(fid);
P_bus=fscanf(fid,'\%f');
xl_pbus=P_bus(1);
yl_pbus=P_bus(2);
xr_pbus=P_bus(3);
yr_pbus=P_bus(4);
ht_pbus=P_bus(5);
end
total_equipment=input('How many equipments are present Note: This should be same as entered in equipment_data.txt file): ');
\%fid=fopen('C:/Users/vinitmarathe/Desktop/Lightning Protection/SRP project/SRP_fi-
nal_program/MATLAB code/equipment_data.txt');
str=strrep(pwd,' ${ }^{\prime}$, ,'/');
str=strcat(str,'/equipment_data.txt');
fid=fopen(str);
for equipment=1:total_equipment

```
    for i=1:2
        tline= fgets(fid);
    end
    A=fscanf(fid,'%f');
    xl_e(equipment)=A(1);
    zl_e(equipment)=A(2);
    x2_e(equipment)=A(3);
    z2_e(equipment)=A(4);
end
```

\% This asks user to define sections starting from POLE AT ORIGIN as
$\%$ section at 0 . The number of sections can be changed by changing ' $k$ ' value
$\%$ in following code.
flag $=0$;
if total_wires==1 \&\& $\mathrm{x} 1==\mathrm{x} 2 \& \& \mathrm{y} 1==\mathrm{y} 2 \& \& \mathrm{z} 1==\mathrm{z} 2$
Number_sec $=1$;
flag $=1$;
else
Number_sec=input('How many sections do you want?: ');
end
for $\mathrm{k}=1$ :Number_sec
if flag $==1$
$\mathrm{s}(\mathrm{k})=\mathrm{yl}(1)$;
else
$\operatorname{str}=[$ 'Enter section ' num $2 \operatorname{str}(\mathrm{k})$ ' from origin pole in feet ' $]$;
$s(k)=\operatorname{input}(\operatorname{str})$;
end
end
\% THE FOLLOWING CODE TAKES ALL THE ABOVE ENTERED DATA AND
CONSTRUCTS VISUAL
\% ZONE OF PROTECTION AT 5 DIFFERENT CROSS-SECTIONS
for $\mathrm{k}=1$ :numel(s)
figure;
grid on;
grid on;
xlim('auto')
ylim('auto')

title(str)
if present $\_$perbus==1

```
if s(k)>=yl_pbus && s(k)<=yr_pbus
    rectangle('position',[xl_pbus,0,xr_pbus-xl_pbus,ht_pbus],'FaceColor','g');
    hold on;
end
end
% This code plots the equipments as rectangle by calling rectangle()
% fuction
for equipment=1:total_equipment
rectangle('position',[x1_e(equipment),z1_e(equipment),x2_e(equipment)-x1_e(equip-
ment),z2_e(equipment)-z1_e(equipment)],'FaceColor','b');
hold on;
end
% This code takes in co-ordinates of the wire poles and constructs an
% imaginary wire in 3-D and takes out only two points(x,z) in 2-D at specified
% section 's' to be used for further calculation
for wire no=1:total wires
[x(wire_no),z(wire_no)]=plot_wire(x1(wire_no),y1(wire_no),z1(wire_no),x2(wire_no),y
2(wire_no),z2(wire_no),s(k));
end
total_point=total_wires;
% This code plots the points on shield wire for given cross section by
% calling plot_section_mast(). A point is shown as a static pole of that
% height( since 'seeing' a point is difficult in figure)
for i=1:total_point
    plot_section_mast(x(i),z(i));
end
% After plotting equipments and taking out points of shield wire, following
% piece of code plots actual zone of protection section-by-section in the
% form of circles, depending on the geometry and strike distance 'R'
% calculated at first.This is divided in 3 parts
% Part 1: Checks if the point of the shield wire < 'R' and find where would
% the center of sphere be and stores in 'root'
for i=1:total_point
    if z(i)<R
        root=sort(find_root(x(i),z(i),R));
        root_use(i,1)= =root(1);
        root_use(i,2)=root(2);
    end
end
```

```
% Part 2: After finding roots, this plots out protection zone on either sides of
% first and last points of wire in a section
if }\textrm{z}(1)<\textrm{R
    C_x=root_use(1,1);
else C_x=x(1)-R;
end
vinit_draw_cir(C_x,R,R)
if z(total_point)<R
    C_xl=root_use(total_point,2);
else C_x1=x(total_point)+R;
end
vinit_draw_cir(C_x1,R,R)
% Part 3: After finding roots, this plots out protection zone in between
% the points depending if the sphere 'rests' or 'falls down' on the ground
for i=1:total_point
    if i== total_point
        %disp('breaked')
        break
    else
    c1=[x(i) z(i) R];
    c2=[x(i+1) z(i+1) R];
    % Finds out intersection of two circles
    points=intersectCircles(c1,c2);
    pl=points(2,1);
    p2=points(2,2);
    % if the intersection is not a point(i.e. do not intersect) the sphere
    % is plotted on the ground else with the intersection as centre of
    % sphere, sphere is plotted
    if isnan(points)
        disp('do not intersect');
        if z(i)<R
            C_x=root_use(i,2);
        else
            C_x=x(i)+R;
            end
                vinit_draw_cir(C_x,R,R);
            if z(i+1)<R
                C_x=root_use(i+1,1);
            else
            C_x=x(i+1)-R;
            end
```

```
            vinit_draw_cir(C_x,R,R);
    else
        vinit_draw_cir(p1,p2,R);
    end
    end
end
grid_control()
axis([-100,220,0,155])
end
% This function calculates surge impedance, stroke current and strike distance
% based on the bus height and diameter and BIL Level
% If there are two bus heights and/or two diameters of bus, enter all
% combinations and check for strike distance. Use the smallest value of
% strike distance for conservative reults.
function S=surge_impe_chek()
str=strrep(pwd,'\','/');
str=strcat(str,'/surge_data.txt');
fid=fopen(str);
for i=1:1
    tline= fgets(fid);
end
A=fscanf(fid,'%f');
n=A(1); %Number of conductors in bundle of a phase
d=A(2); %Diameter of one conductor in feet
h=A(3); %Height of bus in feet
Vc=A(4);%BIL Level
if n==1
    s=1;
    g=1;
elseif n==2|3
    s=input('spacing between conductors in feet ');
    g=1;
elseif n==4
    s=input('spacing between conductors in feet ');
    g=1.12;
end
r=(d/2);%in feet
```

$\mathrm{E} 0=1500$;

```
Rc=2; % guess value for corona radius meters
% calculates actual corona radius
for i=1:15
    Rc=Rc-(Rc*log(2*h*0.3048/Rc)-(Vc/E0))/((log(2*h*0.3048/Rc))-1);
end
Rc=3.2808*Rc;
R0=r*(g*s/r)^((n-1)/n);
Rcnew=R0+Rc;
% Surge impedance, stroke current and strike distances are calculated
Zs=60*sqrt(log(2*h/Rc)*log(2*h/R0));
str = ['Surge impedance is ' num2str(Zs) ' ohms'];
disp(str)
Is=(Vc*2.2)/Zs;
str = ['Allowable stroke current is ' num2str(Is) ' KiloAmperes'];
disp(str)
S=8*Is^0.65;
str = ['Strike distance is ' num2str(S) ' meters'];
disp(str)
end
%Plots a circle for given coordinates
function vinit_draw_cir(x1,y1,R)
theta=0:0.01:2*pi; %control smoothness of the circle
[x,y]=pol2cart(theta,R);
plot(x1+x,yl+y,'LineWidth',2,'Color','r');
hold on;
end
function points = intersectCircles(circle1, circle2)
%INTERSECTCIRCLES Intersection points of two circles
% POINTS = intersectCircles(CIRCLE1, CIRCLE2)
% Computes the intersetion point of the two circles CIRCLE1 and CIRCLE1.
% Both circles are given with format: [XC YC R], with (XC,YC) being the
% coordinates of the center and R being the radius.
% POINTS is a 2-by-2 array, containing coordinate of an intersection
% point on each row.
% In the case of tangent circles, the intersection is returned twice. It
% can be simplified by using the 'unique' function.
```

```
% adapt sizes of inputs
n1 = size(circle1, 1);
n2 = size(circle2, 1);
if n1 ~= n2
        if n1>1&& n2 == 1
        circle2 = repmat(circle2, n1, 1);
    elseif n2>1&&n1==1
        circle1 = repmat(circle1, n2, 1);
    else
        error('Both input should have same number of rows');
    end
end
% extract center and radius of each circle
center1 = circle1(:, 1:2);
center2 = circle2(:, 1:2);
r1 = circle1(:,3);
r2 = circle2(:,3);
% allocate memory for result
nPoints = length(r1);
points = NaN * ones(2*nPoints, 2);
% distance between circle centers
d12 = distancePoints(center1, center2, 'diag');
% get indices of circle couples with intersections
inds = d12 >= abs(r1 - r2) & d12 <= (r1 + r2);
if sum(inds) == 0
    return;
end
% angle of line from center1 to center2
angle = angle2Points(center1(inds,:), center2(inds,:));
% position of intermediate point, located at the intersection of the
% radical axis with the line joining circle centers
d1m = d12(inds) / 2 + (r1(inds).^2 - r2(inds).^2) ./ (2* d12(inds));
tmp = polarPoint(center1(inds, :), dlm, angle);
% distance between intermediate point and each intersection point
h = sqrt(r1(inds).^2-d1m.^2);
% indices of valid intersections
```

```
inds2 = find(inds)*2;
inds1 = inds2 - 1;
% create intersection points
points(inds1,:) = polarPoint(tmp, h, angle - pi/2);
points(inds2, :) = polarPoint(tmp, h, angle + pi/2);
%to plot masts
function plot_section_mast(x1,z1)
z_fir=0:1:zl;
x_fir(1:numel(z_fir))=x1;
plot(x_fir,z_fir,'LineWidth',3,'Color','k');
xlabel('x axis');
ylabel('z axis');
zlabel('y axis');
hold on;
end
%% draw your own grid lines
function grid_control()
xrange=300;
yrange=300;
division=10;
for i=1:division:yrange % draw horizontals
    hGRID = plot([-xrange+1 xrange-1],[i-1 i-1],'c-');
    hold on
end
for i=1:division:2*xrange % draw verticals
    hGRID = plot([i-1-xrange i-1-xrange],[0 yrange],'c-');
    hold on;
end
end
```


## APPENDIX B

MATLAB CODE FOR RIZK MODEL
\%\#\#\#\#\#\#\#\#\#\#\#\# Main Calling Function \#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#
home;
clear all;
clc;
clf;
$x l e a=10.97 / 2$; \%Location of leader in space on x axis
ylea $=18.23 / 2 ; \%$ Location of leader in space on y axis
$\mathrm{I}=6.27$; $\quad$ \% Current in KA in downward Negative Leader/Stroke Current
$\mathrm{v} \_$pos $=1.5 / 100$; \% Velocity of upward leader in meters/microsec
v_neg=v_pos; \% Velocity of downward leader in meters/microsec
$\mathrm{Hcl}=2500$; $\%$ Cloud height in meters
$\mathrm{k}=8.984^{*} 10^{\wedge} 9 ; \%$ Value of Constant $1 /\left(4^{*}\right.$ pi*epsilon)
Qim=76* $\mathrm{I}^{\wedge}(0.68)^{*} 10^{\wedge}(-3)$; \%Total charge in the down leader as function of stroke currrent
Rho0 $=2$ * $\mathrm{Qim} / \mathrm{Hcl} ; \quad$ \%Charge density at ground level
total_wire=input('How many protective wires are present? '); \%User input for total shield wires
fid=fopen('C:/Users/vinitmarathe/Desktop/RIZK MODEL_APPROACH/wire.txt'); \%
Opening data file where wire coordinates are located
\%Extracting wire mast locations from data files and storing for program
for wire_no=1:total_wire
for $\mathrm{i}=1: 2$
tline $=$ fgets(fid);
end
A=fscanf(fid,'\%f');
xpw1 (wire_no) $=\mathrm{A}(1)$;
ypwl(wire_no)=A(2);
zpw1(wire_no)=A(3);
xpw2(wire_no)=A(4);
ypw2(wire_no)=A(5);
zpw2(wire_no)=A(6);
end
$\mathrm{a}=0.05$; \%Assuming all wires have radius of 0.05 meters $=2$ inch
\% Find inception potential for all wires
for wire_no=1:total_wire
Upcw(wire_no) $=10^{\wedge} 3 *(2247 /(1+(5.15-$
$5.49 * \log (\mathrm{a})) /($ zpw1(wire_no)* $\log (2 * z p w 1($ wire_no)/a))));
end
\% Find Height of negative leader when upward leader is incepted by the wire
for wire_no=1:total_wire
H_inception(wire_no)=height_of_incep-
tion1(Upcw(wire_no),Hcl,v_neg,Rho0,xpw(wire_no),ypm(wire_no),zpm(wire_no),xlea,y lea,k)
end
total_mast=input('How many protective mast are present? '); \%User input for total shield wires
fid=fopen('C:/Users/vinitmarathe/Desktop/RIZK MODEL_APPROACH/mast.txt'); \%
Opening data file where Mast coordinates are located
\%Extracting mast locations from data files and storing for program
for mast no $=1$ :total mast
for $\mathrm{i}=1: 2$
tline $=$ fgets(fid);
end
A=fscanf(fid,'\%f');
xpm(mast_no) $=\mathrm{A}(1)$;
ypm(mast_no)=A(2);
zpm(mast_no)=A(3);
end
\% Find inception potential for all masts
for mast_no $=1$ :total_mast
$\operatorname{Upcm}($ mast_no $)=10^{\wedge} 3^{*} 1556 /(1+(3.89 /$ zpm(mast_no) $))$;
end
\% Find Height of negative leader when upward leader is incepted by the mast
for mast_no $=1$ :total_mast
H_inception(mast_no)=height_of_incep-
tion1(Upcm(mast_no),Hcl,v_neg,Rho0,xpm(mast_no),ypm(mast_no),zpm(mast_no),xlea ,ylea,k);
end
[H_inception,xpm,ypm,zpm]=Mastsort(xpm,ypm,zpm,H_inception,total_mast) \% Sorting all masts by the heights at which upward leader is incepted.
zlea=H_inception(1); \% Setting negative leader's 'z' in space at height at which first upward leader is incepted
\% Setting positive leaders trajectory first point as indivisual mast's tip
for mast_no $=1$ :total_mast
x _pos(mast_no)=xpm(mast_no);
y_pos(mast_no)=ypm(mast_no);
z_pos(mast_no)=zpm(mast_no);
end
\%Plotting vertical mast in space
len(1:total_mast)=0;
for mast_no=1:total_mast
zpl=0:0.1:zpm(mast_no);
xpl=repmat(xpm(mast_no),1,numel(zpl));
ypl=repmat(ypm(mast_no),1,numel(zpl));
plot3(xpl,ypl,zpl,'linewidth',5);
hold on;
end
$1 \mathrm{v}=1$;
H_inception(total_mast +1 ) $=0$;
InceptedMast=1;
flagattached $=0 ; \%$ Setting a flag that says there is no final strike before simulation starts.
time_interval $=5$; \%Time step simulation takes in microsecond during each iteration
for $\mathrm{t}=0$ :time_interval:100000 \%Setting up simulation time of 0.1 seconds
if H _inception(InceptedMast+1)> zlea InceptedMast=InceptedMast + 1;
end
InceptedMast;
for mast_no=1:InceptedMast
plot3(x_pos(mast_no),y_pos(mast_no),z_pos(mast_no),'Marker','o'); \%Plotting posi-
tive leader trajectory in space
hold on;
plot3(xlea,ylea,zlea,'Marker','*'); \%Plotting negative leader trajectory in space
hold on;
end
\%Positive leader trajectory movement if mast has reached leader inception voltage for mast_no=1:InceptedMast
len(mast_no)=len(mast_no)+v_pos*time_interval;
Upos(mast_no)=Pos_leader_voltage(len(mast_no),time_interval,v_pos); \%Positive
leader potential
$\% 1 \mathrm{v}=\mathrm{lv}+1$;
[x_pos(mast_no),y_pos(mast_no),z_pos(mast_no)]=newcoordi-
nates(x_pos(mast_no),y_pos(mast_no),z_pos(mast_no),xlea,ylea,zlea,v_pos*time_interval);
end
Uneg=Neg_pot(Rho0,Hcl,_zlea,Qim); \% Calculating Negative leader potential using
function Neg_pot()
zlea=zlea-time_interval*v_neg; \% New position of downward negative leader in
spcae
\% finding distance between tips of all positive leaders and a negative leader
for mast_no=1:InceptedMast
tipDistance(mast_no)=Dis-
tance(xlea,ylea,zlea,x_pos(mast_no),y_pos(mast_no),z_pos(mast_no));
end
for mast_no $=1:$ InceptedMast
if $\left(\operatorname{abs}\left(U p o s\left(m a s t \_n o\right)+U n e g\right) /\left(t i p D i s t a n c e\left(m a s t \_n o\right)\right)\right)>=500^{*} 10^{\wedge} 3 \%$ cheking crite-
ria if the potentail gradient has reached $500 \mathrm{kV} /$ meter
stri=['Leaders meet for mast ' num2str(mast_no)]; \% If reached, text message is
displayed and succesful interecption by mast
disp(stri)
flagattached $=1$;
zlea

```
            z_pos(mast_no)
            tipDistance(mast_no)
        break;
        end
    end
    if flagattached==1;
        break;
    end
end
InceptedMast
zlea
z_pos
\%\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\# Function to calculate Height of Inception \#\#\#\#\#\#\#\#\#\#\#\#\#\#
function \(H_{-}\)inception=height_of_incep-
tion1(Upcm,Hcl,v_neg,Rho0,xpm,ypm,zpm,xlea,ylea,k)
Uinduced \(=0 ; \%\) set induced voltage to 0
\(\mathrm{R}=\operatorname{sqrt}\left((\mathrm{xpm}-\mathrm{xlea})^{\wedge} 2+(\mathrm{ypm}-\mathrm{ylea})^{\wedge} 2\right)\)
h=zpm;
time_step \(=10 ; \%\) time step in microsecond
H_inception=Hcl;\%set initial height of inception at height of cloud syms x;
\% Calculate the tip potential
\(\mathrm{A}(\mathrm{x})=\log \left(\mathrm{x}-\mathrm{h}+\left(\mathrm{R}^{\wedge} 2+\mathrm{h}^{\wedge} 2-2^{*} \mathrm{~h}^{*} \mathrm{x}+\mathrm{x}^{\wedge} 2\right)^{\wedge}(1 / 2)\right)-\log \left(\mathrm{h}+\mathrm{x}+\left(\mathrm{R}^{\wedge} 2+\mathrm{h}^{\wedge} 2+2^{*} \mathrm{~h}^{*} \mathrm{x}+\right.\right.\) \(\left.\left.x^{\wedge} 2\right)^{\wedge}(1 / 2)\right)-\left(\mathrm{R}^{\wedge} 2+h^{\wedge} 2-2 * h^{*} x+x^{\wedge} 2\right)^{\wedge}(1 / 2) / H c l+\left(R^{\wedge} 2+h^{\wedge} 2+2 * h^{*} x+\right.\)
\(\left.\mathrm{x}^{\wedge} 2\right)^{\wedge}(1 / 2) / \mathrm{Hcl}-\left(\mathrm{h}^{*} \log \left(\mathrm{~h}+\mathrm{x}+\left(\mathrm{R}^{\wedge} 2+\mathrm{h}^{\wedge} 2+2 * \mathrm{~h}^{*} \mathrm{x}+\mathrm{x}^{\wedge} 2\right)^{\wedge}(1 / 2)\right)\right) / \mathrm{Hcl}-\left(\mathrm{h}^{*} \log (\mathrm{x}-\mathrm{h}+\right.\) \(\left.\left.\left(\mathrm{R}^{\wedge} 2+\mathrm{h}^{\wedge} 2-2^{*} \mathrm{~h}^{*} \mathrm{x}+\mathrm{x}^{\wedge} 2\right)^{\wedge}(1 / 2)\right)\right) / \mathrm{Hcl}\);
while Uinduced \(<\) Upcm \%Iterate till tip potential reaches critical potential
Uinduced \(=\mathrm{k}\) *Rho 0 *double( \(\mathrm{A}(\mathrm{Hcl})-\mathrm{A}\left(\mathrm{H} \_\right.\)inception \()\));
H_inception=H_inception-time_step*v_neg;
if H_inception<=zpm
H_inception \(=0\);
break;
end
Uinduced;
end
\%\#\#\#\#\#\#\#\#\#\#\#\#\#\#\# Function to find Negative leader Potential \#\#\#\#\#\#\#\#\#\#\#\#
function Uneg=Neg_pot(Rho0,Hcl,ht,Qim)
eps \(=8.85^{*} 10^{\wedge}(-12)\);
\(\operatorname{grad}=6^{*} 10^{\wedge} 3 ; \%\) volts/meter
```

```
E=3*10^6;%volts/meter
ul=grad*(ht);% total voltage
Rho0=Rho0*(1-ht/Hcl);
Rad=Rho0/(2*pi*eps*E);
ut=E*Rad;
Uneg=ul+ut;
%Uneg=Uneg*(ht)/Hcl;
end
%############# Function to sort masts/wires according to their inception voltage
###################################
function [H_inception,xpm,ypm,zpm]=Mastsort(xpm,ypm,zpm,H_inception,total_masts)
for i=1:total_masts % creates a matrix of x and z of all wires so that sortrows() func-
tions can be used
    Mat(i,1)=H_inception(i);
    Mat(i,2)=xpm(i);
    Mat(i,3)=ypm(i);
    Mat(i,4)=zpm(i);
end
NewMat=sortrows(Mat,-1); %matlab built-in function that sorts a matriz in DESCEND-
ING order of column 1
for i=1:total_masts
H_inception(i)=NewMat(i,1); % disintegrating the matrix back to x and z to give it
back to the program
xpm(i)=NewMat(i,2);
ypm(i)=NewMat(i,3);
zpm(i)=NewMat(i,4);
end
end
%############## Calculation of Positive leader voltage #############%%%%
function Upos=Pos_leader_voltage(len,time_interval,v_pos)
Ei=400*10^3; % kV/m
Einf=50*10^3; % kV/m
x0=time_interval*v_pos;%meters 0.75
Upos=(len)*Einf+x0*Einf*log((Ei/Einf)-(((Ei-Einf)/Einf)*exp(-len/x0)));
end
% ############# Calculation of new coordinates after every iteration
%################
function [x y z]=newcoordinates(x_pos,y_pos,z_pos,xlea,ylea,zlea,d)
a=xlea-x_pos;
b=ylea-y_pos;
c=zlea-z_pos;
```

$\mathrm{t}=\mathrm{d} / \operatorname{sqrt}\left(\mathrm{a}^{\wedge} 2+\mathrm{b}^{\wedge} 2+\mathrm{c}^{\wedge} 2\right)$;
$\mathrm{x}=\mathrm{x} \_$pos $+\mathrm{a}^{*} \mathrm{t}$;
$\mathrm{y}=\mathrm{y} \_$pos $+\mathrm{b}^{*} \mathrm{t}$;
$\mathrm{z}=\mathrm{z}$ _pos $+\mathrm{c}^{*} \mathrm{t}$;
end

