Measurement Systems Analysis Studies:

A Look at the Partition of Variation (POV) Method

by

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ABSTRACT

The Partition of Variance (POV) method is a simplistic way to identify large sources of variation in manufacturing systems. This method identifies the variance by estimating the variance of the means (between variance) and the means of the variance (within variance). The project shows that the method correctly identifies the variance source when compared to the ANOVA method. Although the variance estimators deteriorate when varying degrees of non-normality is introduced through simulation; however, the POV method is shown to be a more stable measure of variance in the aggregate. The POV method also provides non-negative, stable estimates for interaction when compared to the ANOVA method. The POV method is shown to be more stable, particularly in low sample size situations. Based on these findings, it is suggested that the POV is not a replacement for more complex analysis methods, but rather, a supplement to them. POV is ideal for preliminary analysis due to the ease of implementation, the simplicity of interpretation, and the lack of dependency on statistical analysis packages or statistical knowledge.

DEDICATION

To my family, friends, and my adoring wife.

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INTRODUCTION

Measurement systems are an integral part of manufacturing and quality control in nearly all industries. A measurement system is the set of measurement tools, operators, parts, etc. that define the true measure of the performance of a manufacturing process. Therefore, it is crucial that the measurement system is accurate, unbiased, and robust. It must truly define the variance of the process itself, rather than variance due to the tool, operator, etc. Multiple methods for analyzing the measurement system have been developed, most common of which is the Gauge R&R Analysis of Variance Method. An already familiar method of Measurement System Analysis (MSA), the Gauge R&R method looks at the average and variance of response partitioned into sub-categories such as part or operator and makes comparisons using the ANOVA (Analysis of Variance) method. The classical method falls short in defining the precise location of the variance in the measurement system. For example, an engineer may find that their wafer production may be producing non-uniform products. However, they may be doubtful that their operators are measuring with minimal bias. With the ANOVA method, they would be able to find that the operators are a significant source of the variance. However, it would be of more value to them to know *which* of the operators are causing the most variance. While there are secondary methods to find this source with the ANOVA method, a faster and simpler analysis method could be useful. Little and Brekke (1995) introduced a more streamlined method, which they designated as the Partition of Variance (POV). It is noted that although Little and Brekke are co-authors of the article "Partition of variation: A new method for σ Reduction," Dr. Thomas Little is credited as the sole inventor of the POV method. The POV method aims to reduce the complexity of measurement system analysis in order to make the process simple and easy to understand while maintaining the robustness of more complex methods. This method uses the variance of means and the means of the variances partitioned in varying dimensions of interest. It provides simple comparisons for engineers to find the true source of the variance in a measurement system. The POV method is more of a qualitative, rather than quantitative approach to defining the measurement system. It is important to note that no tests of significance will be approached in this paper. One added benefit to the POV method is its lack of dependency on complex statistical programs. POV analysis can be easily be implemented through proper set-up of a simple spreadsheet program.

Even though the set-up is simple, the method still yields less volatile variance estimates when compared to the ANOVA method. Engineers and statisticians are often concerned with the normality of their data and the confidence of their estimates. While non-normality isn't usually a large concern for most techniques, the difficulty found in Gauge R&R studies are due to a lack of "samples" in certain dimensions. For example, if a particular study has 3 tools, 2 operators, and 10 parts, the estimators will have sample sizes of 3, 2 and 10, respectfully. If the study included multiple measures, those degrees-of-freedom will be soaked up into the estimate for error. Thereby decreasing the confidence of the estimators. This concept is explored further in Vardeman and VanValkenburg (1999). It is also important to note that in order to increase the sample size for operators or tools would mean hiring more operators, or purchasing more tooling. This is highly impractical in nearly all situations.

In this exploratory article, the methodology of the POV method is introduced and fully explained. Then, example calculations for the POV method are demonstrated. The results of the POV method are then compared to the ANOVA method. The ability of the POV to further break down the variance is explored. Finally, a simulation is outlined and executed. The results of the simulation are then outlined and compared.

POV MATHEMATICS

It is understood by most engineers and statisticians that the global variance is the sum of all the subsequent variances within the system. This can be represented with the general equation:

$$\sigma^2{}_{total} = \sigma^2{}_1 + \sigma^2{}_2 + \cdots + \sigma^2{}_n \qquad (1)$$

where the subscripts 1 through *n* represent different subsets of the global data formed into logical categories. Extending this equation into the measurement system framework, we can apply categories pertinent to the problem:

$$\sigma^2{}_{total} = \sigma^2{}_{part} + \sigma^2{}_{operator} + \sigma^2{}_{tool} + \cdots + \sigma^2{}_{etc.} \qquad (2)$$

The POV approach is partitioned into different levels of complexity, depending on the problem. POV_{II} is concerned with two-dimensional problems, POV_{III} for three dimensional problems, and so on. When constructing a dataset, an engineer or statistician would assign the columns of the sheet to be the operator, tool, etc. and the rows to be individual measurements. An example of a spreadsheet design is found in Table 1.

Table 1. Spreadsheet Design of POV II Using Parts and Operators

POV II	Operator 1	Operator 2	…	Operator n
Part 1	part 1, operator 1	part 1, operator 2	…	part 1, operator n
Part 2	part 2, operator 1	part 2, operator 2	…	part 2, operator n
…	…	…	…	…
Part n	part n, operator 1	part n, operator 2	…	part n, operator n

Therefore, in this problem, we would construct the variance equation to be:

$$\sigma^2{}_{total} = \sigma^2{}_{part} + \sigma^2{}_{operator}. \tag{3}$$

Or, to put the equation in terms of a spreadsheet:

$$\sigma^2{}_{total} = \sigma^2{}_{rows} + \sigma^2{}_{columns}. \tag{4}$$

The subsequent equations will be analyzed and explained in further detail later. POV II can easily be expanded into higher-level dimensions. Let's say we would like to add the dimension of a tool to the system, such that there are *n* measuring devices. We can construct POV_{III} similarly to POV_{II} by adding the block of tool. An example sheet is found in Table 2.

Table 2. Spreadsheet Design of POV III Using Parts, Operators, and Tools.

POV III	Operator 1				Operator 2				…	Operator n
	Tool 1	Tool 2	…	Tool n	Tool 1	Tool 2	…	Tool n	…	
Part 1	part 1, operator 1, tool 1	part 1, operator 1, tool 2	…	part 1, operator 1, tool n	part 1, operator 2, tool 1	part 1, operator 2, tool 2	…	part 1, operator 2, tool n	…	part 1, operator n, tool n
Part 2	part 2, operator 1, tool 1	part 2, operator 1, tool 2	…	part 2, operator 1, tool n	part 2, operator 2, tool 1	part 2, operator 2, tool 2	…	part 1, operator 2, tool n	…	part 1, operator n, tool n
…	…	…	…	…	…	…	…		…	…
Part n	part n, operator 1, tool 1	part n, operator 1, tool 2	…	part n, operator 1, tool n	part n, operator 2, tool 1	part n, operator 2, tool 2	…	part 1, operator 2, tool n	…	part n, operator n, tool n

Thus, the equation for system variance is defined as:

$$\sigma^2{}_{total} = \sigma^2{}_{part} + \sigma^2{}_{operator} + \sigma^2{}_{tool}. \tag{5}$$

Furthermore, to extend the equation to the spreadsheet version:

$$\sigma^2{}_{total} = \sigma^2{}_{rows} + \sigma^2{}_{columns} + \sigma^2{}_{blocks} \tag{6}$$

where blocks are defined as the sub columns of tool 1 as block 1, tool 2 as block 2, and so on.

Computation of POV is straightforward. The foundation is based on comparing the relative magnitudes of the variance partitions to the overall variance through simple proportions. All computations of variance are based upon the equation:

$$\sigma^2 = \frac{1}{n}\sum_{i=1}^{n}(x_i - \bar{x})^2 \tag{7}$$

where

$$\bar{x} = \frac{1}{n}\sum_{i=1}^{n}(x_i). \tag{8}$$

Equation (7) and Eq. (8) are used to find the variance of the entire dataset. This is considered the estimate of the population variance. Discussion as to why this is used over sample variance will be discussed in succeeding sections. In the instance mentioned earlier, this formula could be applied to all rows and columns. Modifying this equation to our spreadsheet, we get:

$$\sigma^2{}_{total} = \frac{1}{(n_i * n_j)}\sum_i \sum_j (x_{ij} - \bar{x}_{ij})^2 \tag{9}$$

where

$$\bar{x}_{ij} = \frac{1}{(n_i * n_j)}\sum_i \sum_j (x_{ij}) \tag{10}$$

such that *i* and *j* are representative of the row and column indices, respectfully. Next, we would like to complete our partitions. In order to do this, we want to find the mean and variance over the dimension of interest. For example, if an engineer were looking at three tools, he would want to find the mean response and variance within the tool itself. Again, applying this to the spreadsheet, this would mean finding the mean and variance within the column. Thus, for each tool, we would find the variance to be:

$$\sigma^2{}_{column\ j} = \frac{1}{(n_i)} \sum_i (x_{i,column\ j} - \bar{x}_{i,column\ j})^2, \quad (11)$$

with the mean equivalent to:

$$\bar{x}_{i,column\ j} = \frac{1}{(n_i)} \sum_i (x_{i,column\ j}). \quad (12)$$

Next, we would like to find the variance of the means. By doing this, it allows us to find the variance between the columns. This will be used in further comparisons. This is found by:

$$\sigma^2{}_{Between\ j} = \frac{1}{(n_j)} \sum_j (\bar{x}_j - \bar{x}_{total})^2 \quad (13)$$

where $\bar{x}_{total}$ is Eq. (9) and $\bar{x}_j$ is Eq. (12) for each column.

Table 3. Summary of Formulas Used in POV_{II}

Condition	Formula	Description
$\sigma^2{}_{total}$	$\sigma^2{}_{total} = \frac{1}{(n_i * n_j)} \sum_i \sum_j (x_{ij} - \bar{x}_{ij})^2$	Total variance
$\sigma^2{}_{within\ column}$	$\sigma^2{}_{within\ j} = \frac{1}{(n_j)} \sum_j \sigma^2{}_{column\ j}$	Average of column variances
$\sigma^2{}_{between\ column}$	$\sigma^2{}_{between\ j} = \frac{1}{(n_j)} \sum_j (\bar{x}_j - \bar{x}_{total})^2$	Variance of column averages

Table 4. Summary of Formulas Used in POV_{III}

Condition	Formula	Description
${\sigma^2}_{total}$	${\sigma^2}_{total} = \frac{1}{(n_i * n_j)} \sum_i \sum_j (x_{ij} - \bar{x}_{ij})^2$	Total variance
${\sigma^2}_{within\ column}$	${\sigma^2}_{within\ j} = \frac{1}{(n_j)} \sum_j {\sigma^2}_{column\ j}$	Average of column variances
${\sigma^2}_{between\ column}$	${\sigma^2}_{between\ j} = \frac{1}{(n_j)} \sum_j (\bar{x}_j - \bar{x}_{total})^2$	Variance of column averages
${\sigma^2}_{within\ block}$	${\sigma^2}_{within\ k} = \frac{1}{(n_k)} \sum_k {\sigma^2}_{block\ k}$	Average of block variances where k are block indices
${\sigma^2}_{between\ blocks}$	${\sigma^2}_{between\ k} = \frac{1}{(n_k)} \sum_k (\bar{x}_k - \bar{x}_{total})^2$	Variance of column averages where k are block indices

Comparisons

Once partitions are calculated, the analyst would now move toward calculating the comparison statistics. The calculations are straightforward; presented here is the Percent Effect, a statistic that represents the proportion of the total variance contributed by the various partitions in terms of percentage. It is simply the proportion of the variance of interest with respect to the total variance. For example in POV_{II}:

$$\%Effect = 100 * \frac{{\sigma^2}_{Column}}{{\sigma^2}_{total}}. \tag{14}$$

The statistic is a percentage with the range $0 \leq \%\ Effect \leq 100$. This can be shown true through Eq. (1). Moving the global variance from the left side of the equation:

$$1 = \frac{{\sigma^2}_1 + {\sigma^2}_2 + \cdots + {\sigma^2}_n}{{\sigma^2}_{total}}. \tag{15}$$

After simplification:

$$1 = \frac{\sigma^2{}_1}{\sigma^2{}_{total}} + \frac{\sigma^2{}_2}{\sigma^2{}_{total}} + \cdots + \frac{\sigma^2{}_n}{\sigma^2{}_{total}}. \tag{16}$$

Thus, we see that the sum of the proportions is equivalent to unity.

Often times there is problem associated with a process that leads to the need for process characterization. The POV method allows an engineer to prioritize their efforts in variance reduction by outlining the relative magnitude of the contributing components of the global variance. For instance, if in a particular study the part-to-part (between) variance was found to be 85%, and the within part variance was 15%, it would be safe to conclude that much of the variance was stemming from part-to-part variance and not from variance due to operator or tool. Furthermore, if the study found part-to-part variance to be 15%, within part variance to be 85%, and operator-to-operator variance to be 95%, the engineer would conclude that his operators are highly varying. This would cause each part to appear to have high within variance. The engineer would conclude that he would need to train operators in order to reduce the operator-to-operator variance.

Introduction of Interaction Effect

Readers with more statistics training may be questioning where the variance due to interaction fits into the POV method. It is the author's belief that interaction effects are rare in MSA studies. However, this could be explored in greater depth in the future. One advantage the POV method has is its simplicity. Many analysts with little to no statistical background may find the concept of interaction difficult to grasp. Therefore, if it is necessary to keep the analysis simple, one may leave this calculation out. The POV method will still be able to characterize much of the variance and only severe cases of interaction effects will impair the POV method. That being said, if interaction effects are a concern, or the analyst feels confident in their understanding of interaction, the calculation is straightforward. It is calculated much the same way as the ANOVA method. If we include the interaction effect into the POV II mentioned earlier, Eq. (3) now becomes:

$$\sigma^2{}_{total} = \sigma^2{}_{between\ part} + \sigma^2{}_{between\ operator} + \sigma^2{}_{between\ interaction}. \qquad (17)$$

There are estimates for $\sigma^2{}_{total}$, $\sigma^2{}_{between\ part}$, and $\sigma^2{}_{between\ operator}$. It is important to note that for this calculation, the analyst must use the "between" variance estimators. The interaction effect is calculated through simple algebra:

$$\sigma^2{}_{between\ interaction} = \sigma^2{}_{total} - \sigma^2{}_{between\ part} - \sigma^2{}_{between\ operator}. \qquad (18)$$

It should be clear that the POV III and upward are calculated the same way. However, once 3 or more factors are included, there is no way to partition the interactions. For example, in the case of POV III, interaction would be calculated as:

$$\sigma^2{}_{between\ interaction} = \sigma^2{}_{total} - \sigma^2{}_{between\ part} - \sigma^2{}_{between\ operator} - \sigma^2{}_{between\ tool}. \qquad (19)$$

Also, because the within and between variance must sum to the total variance, the $\sigma^2{}_{within\ interaction}$ can be calculated as:

$$\sigma^2{}_{within\ interaction} = \sigma^2{}_{total} - \sigma^2{}_{between\ interaction}. \qquad (20)$$

The interaction effect is not broken into operator-part interaction, operator-tool interaction, part-tool interaction, or operator-part-tool interaction. Thus, if the interaction were deemed significant, there would be no way for the engineer to validate the precise interaction effect. He would need to employ secondary analysis methods to characterize this variance. This is one drawback of the POV method. Further research could be explored in this subject.

EXAMPLE – POV VS. GAUGE R&R

Gauge R&R Results

In order to demonstrate its effectiveness, the dataset from Montgomery and Runger (1993) was analyzed using the POV method. The dataset is found in Appendix A. Details of the methods they proposed could be explored through their paper. A summary of their results is provided in Table 5.

Table 5. ANOVA Table of Results From Appendix A Dataset

Source of Variability	Sum of Squares	Degrees of Freedom	Mean Square	F_0	P-value
Operators	2.62	2	1.31	1.85	0.1711
Part	1185.43	19	62.39	87.63	0.0001
Operator by Part	27.05	38	0.71	0.72	0.4909
Repeatability	59.50	60	0.99		
Total	1274.60	119			

Through looking at the resulting p-values, it is revealed that the parts are contributing significantly to the variance of the overall dataset. However, operators and the operator-part interactions are not significant. This would lead an engineer to conclude that there is something faulty with the equipment leading to further root cause analysis for contributing factors. The engineer will no longer need to be concerned with improving the operator's ability to measure the parts correctly.

POV Calculations

In order to implement the POV method, the data is organized into the rows and columns outlined as it is in Appendix A. First, calculate the average and variance of the operators, parts, and measurements. Rely on Table 4 to execute the calculations. In order to find the within-operator variance for operator 1, first find the average for all the measures executed by operator 1:

$$\bar{x}_{operator\ 1} = \frac{1}{n\ operator\ 1\ measurements} \sum (each\ operator\ 1\ measurement),$$

$$\bar{x}_{operator\ 1} = \frac{1}{40}(892) = 22.30.$$

Then calculate the variance of operator 1:

$$\sigma^2{}_{operator\ 1} = \frac{1}{n\ operator\ 1\ measurements} \sum (each\ measurement - \bar{x}_{operator\ 1})^2,$$

$$\sigma^2{}_{operator\ 1} = \frac{1}{40}(392.4) = 9.81.$$

After completing these calculations for each operator, repeat the process with each part. For part 1:

$$\bar{x}_{part\ 1} = \frac{1}{n\ part\ 1\ measurements} \sum (each\ part\ 1\ measurement),$$

$$\bar{x}_{part\ 1} = \frac{1}{6}(121) = 20.17.$$

Then calculate the variance of part 1:

$$\sigma^2{}_{part\ 1} = \frac{1}{n\ part\ 1\ measurements}\sum(each\ part\ 1\ measurement - \bar{x}_{part\ 1})^2,$$

$$\sigma^2{}_{part\ 1} = \frac{1}{6}(2.83) = 0.47.$$

After completing calculations for each part, repeat the process with each measurement. For measurement 1:

$$\bar{x}_{measurement\ 1} = \frac{1}{n\ measurement\ 1}\sum(all\ measurement\ 1),$$

$$\bar{x}_{measurement\ 1} = \frac{1}{60}(1342) = 22.37.$$

Then calculate the variance of measurement 1:

$$\sigma^2{}_{measurement\ 1} = \frac{1}{n\ measurement\ 1}\sum(each\ measurement\ 1 - \bar{x}_{measurement\ 1})^2,$$

$$\sigma^2{}_{measurement\ 1} = \frac{1}{60}(577.67) = 10.90.$$

It is pertinent at this point to find the average and variance across the entire dataset:

$$\bar{x}_{total} = \frac{1}{n\ total\ measurements}\sum(each\ measurement),$$

$$\bar{x}_{total} = \frac{1}{120}(2687) = 22.39.$$

Then calculate the variance of the dataset:

$$\sigma^2{}_{total} = \frac{1}{n\ total\ measurements}\sum(each\ measurement - \bar{x}_{total})^2,$$

$$\sigma^2{}_{total} = \frac{1}{120}(1274.59) = 10.62.$$

Below is a summary of the mean and variance for each operator, part, measurement, and grand total.

Table 6. Average and Variance for Operator

Operator:	Average	Variance
Operator 1	22.30	9.81
Operator 2	22.28	11.10
Operator 3	22.60	10.89

Table 7. Average and Variance of Each Part

Part:	Average	Variance
1	20.17	0.47
2	23.67	0.22
3	20.50	0.92
4	27.17	0.47
5	18.83	1.14
6	22.33	1.22
7	21.83	1.47
8	18.50	0.92
9	23.83	0.47
10	24.67	0.89
11	20.33	0.22
12	18.33	0.56
13	24.67	0.56
14	24.17	0.47
15	29.67	0.89
16	25.83	0.47
17	19.83	0.14
18	20.33	2.22
19	25.00	0.33
20	18.17	0.81

Table 8. Average and Variance for Measurement

Measurement:	Average	Variance
Measurement 1	22.37	10.90
Measurement 2	22.42	10.35

Table 9. Average and Variance of Entire Dataset

Grand Total	Average	Variance
Total	22.39	10.62

The researcher must now find the variance of averages, and the average of variances for each dimension of interest. For the purposes of this example, the average of the variances of each operator equates to the average within-operator variance. The variance of the averages equates to the between-operator variance. Therefore, the average variance is calculated as such:

$$\overline{\sigma^2}_{within\ operator} = \frac{1}{number\ of\ operators} \sum (each\ operator\ variance),$$

$$\overline{\sigma^2}_{within\ operator} = \frac{1}{3} \sum (31.80) = 10.60.$$

And the variance of averages to be:

$$\overline{\sigma^2}_{between\ operator} =$$

$$\frac{1}{number\ of\ operators} \sum (each\ operator\ average - grand\ average)^2,$$

$$\overline{\sigma^2}_{between\ operator} = \frac{1}{3}(0.07)=0.02.$$

The same method would be employed with the parts and measurements. At this point, if the analyst chooses to include interaction, the between interaction effect can be calculated employing Eq. (19):

$$\sigma^2_{between\ interaction} =$$

$$\sigma^2_{total} - \sigma^2_{between\ part} - \sigma^2_{between\ operator} - \sigma^2_{between\ measurement},$$

$$\sigma^2_{between\ interaction} = 10.62 - 9.88 - 0.02 - 0.00 = 0.72.$$

And within-interaction can be calculated with Eq. (20):

$$\sigma^2{}_{within\ interaction} = \sigma^2{}_{total} - \sigma^2{}_{between\ interaction},$$

$$\sigma^2{}_{within\ interaction} = 10.62 - 0.72 = 9.90.$$

Table 10 summarizes the calculations.

Table 10.Within and Between Variance Estimations for Each Dimension

	Within	Between
Operator	10.60	0.02
Part	0.74	9.88
Measurement	10.62	0.00
Interaction	9.90	0.72

The reader should keep note that the estimation of between measurement variance is zero due to rounding.

POV Comparisons & Interpretation

Once complete, the researcher is ready to begin comparisons. The researcher now calculates the %Effect (Eq. 14) for each dimension of interest. Therefore, for the calculation for %Effect for within operator are as follows:

$$\%\ Effect_{Within\ operators} = 100 * \frac{\sigma^2{}_{Within\ operators}}{\sigma^2{}_{Total}},$$

$$\%\ Effect_{Within\ operators} = 100 * \frac{10.60}{10.62} = 99.8\%.$$

And for between operators:

$$\%\ Effect_{\ Between\ operators} = 100 * \frac{\sigma^2{}_{Between\ operators}}{\sigma^2{}_{Total}},$$

$$\%\ Effect_{Between\ operators} = 100 * \frac{0.02}{10.62} = 0.2\%.$$

Therefore, the table below outlines the %Effect for each variance component.

Table 11. %Effect for Operator

Operator	Variance	%Effect
Within Operator	10.60	99.79%
Between Operator	0.02	0.21%
Total	10.62	100.00%

Table 12. *%Effect for Part*

Part	Variance	%Effect
Within Part	0.74	7.00%
Between Part	9.88	93.00%
Total	10.62	100.00%

Table 13. %Effect for Measurement.

Measurement	Variance	%Effect
Within Measurement	10.62	100.00%
Between Measurement	0.00	0.00%
Total	10.62	100.00%

Table 14. %Effect for Interaction

Interaction	Variance	%Effect
Within Interaction	9.90	93.22%
Between Interaction	0.72	6.78%
Total	10.62	100.00%

The researcher can now compare the %Effect for their parts, operators, measurements, and interaction. In this example, the within operator variance is much larger than the between operator variance. This would signal that the operators are not contributing a large portion to the overall variance, and that the operators are of little interest for further analysis. If we turn our attention towards the parts, we see that there is a large portion of the variance being contributed by the part-to-part (between) variance. Therefore, the researcher would conclude that the part-to-part variance is contributing the most to the overall variance. Between measurement and between interaction both have small contributions to the overall variance. Thus, these small contributions validate that the parts are contributing the most to the overall variance, and that there is little contribution elsewhere. These conclusions reduce the number of contributing factors to the overall variance. This then allows the researcher to explore the root cause more precisely and effectively. Further analysis of the tooling, machinery, and basic design of the part itself could lead to discoveries that reduce the overall variance.

Montgomery and Runger (1993) showed that the part-to-part variance was significantly larger in comparison to the other sources of variance. Furthermore, the ANOVA method demonstrated little significance in other sources of variance. Therefore, the POV and ANOVA methods are in agreement. Thus we can see that the POV method is an effective tool in identifying the source of variance in an easily understood presentation.

Population Variance vs. Sample Variance

As mentioned earlier, the POV method estimates the variance using the population variance. It is well understood that as the sample size diminishes, the variance estimate becomes more and more bias to the true variance. However, population variance estimate is chosen to enable the summary table to sum to 100%. This enables interpretation thereof to be simplified. It allows an engineer to visualize with more ease where source of the variance is. If one were to use the sample variance, the total variance of the system will become >100%. Therefore, it is decided to remain with the population variance estimator. Though, this comes at a cost, which will be discussed in succeeding sections.

Further Breakdown

One advantage the POV method has over traditional methods is the ability to quickly break down the variance further. In the example above, part-to-part (between) variance was discovered as the main source of variance for the system. If this is the case, traditional methods of analysis can be performed to isolate if there are one or more parts that are outliers. A P-P plot would be an example method of isolating such cases. However, in the instance of within-part or within-operator variance being significant, there aren't as well established methods to do so. The POV method can help isolate high variance component, as well as the low varying components of the system. This can be calculated as:

$$\% \, Influence_{component\ i} = 100 * \frac{\sigma^2{}_{component\ i}}{\sigma^2{}_{total}}. \tag{21}$$

Thus we see, it is essentially the same calculation as mentioned previously; however, it is now comparing the individual variance of each component of interest to the whole. In Little and Brekke (1995), this analysis technique was demonstrated with the following dataset:

Table 15. POV Data from Little and Brekke (1995)

Location/Wafer	Wafer 1	Wafer 2	Wafer 3	Wafer 4	Wafer 5	Wafer 6
Location 1	24.00	23.00	23.50	22.80	23.00	22.90
Location 2	24.56	22.50	24.07	23.00	22.60	25.00
Location 3	25.04	23.00	24.54	23.00	23.04	23.80
Location 4	23.00	23.00	20.87	22.58	19.60	20.30
Location 5	22.87	22.00	22.41	24.24	21.04	22.70
Average	23.89	22.70	23.08	23.12	21.86	22.94
Variance	0.723	0.160	1.724	0.335	1.804	2.402

The summary of results:

Table 16. POV Results from Little and Brekke (1995)

Partition	Variance	% Effect
Within Wafer	1.1915	76.5%
Between Wafer	0.3659	23.5%
Total	1.5574	100%

In this example, the engineers would like to ensure that each wafer has a uniform amount of material deposition that is constant across all wafers. They sampled 5 wafers from the line, and measured them on the same tool by the same operator in 5 separate locations within each wafer. In the summary table, the larger portion of the variance is coming from within the wafer, rather than between them. These values are calculated the same as demonstrated earlier. However, Eq. (14) is used to calculate the %Influence for each wafer.

For Wafer 1 it is:

$$\%Influence_{Wafer\ 1} = 100 * \frac{\sigma^2{}_{Wafer\ 1}}{\sigma^2{}_{Total}},$$

$$\%Influence_{Wafer\ 1} = 100 * \frac{0.723}{1.5574} = 46.4\%.$$

The summary is as follows:

Table 17. %Influence of Each Wafer from Little and Brekke (1995)

Wafer	Variance	%Influence
Wafer 1	0.723	46.4%
Wafer 2	0.160	10.23%
Wafer 3	1.724	110.7%
Wafer 4	0.335	21.5%
Wafer 5	1.804	115.8%
Wafer 6	2.402	154.3%
Average	1.1915	76.5%

It should be obvious to the reader that the summation of the percentages does not equate to unity. Therefore, the interpretation of this table is slightly different than the comparisons performed earlier. These percentages are representative of the magnitude from the mean. Or rather, it can be considered the influence on the within-wafer variance. Therefore, a value of 150%+ has a large positive influence on the within wafer variance. Furthermore, a value of 50%- has a large negative influence on the within wafer variance. Thus, an engineer would conclude that wafer 6 has a large positive influence on the within-wafer variance. Ideally, the engineer would review the parameters that were in place to cause such a large variance within the wafer. Also, the engineer could look at wafer 2 to see what influenced such a stable deposition. This will direct the engineers to the variables that influence deposition stability and allow them to reduce their problematic variance. This method is much more simple and allows for those with little to no statistical background understand the source of the variance by putting the variance in terms of the overall variance.

Excel Implementation

As it has been mentioned previously, the POV method allows implementation through simple spreadsheet programs. The most commonly used is Microsoft Excel. In order to implement, an analyst would need to employ two simple functions beyond basic algebraic calculations. The needed functions are: VAR.P() and AVERAGE(). It is important that the user chooses the VAR.P() function over the VAR.S() function. The VAR.S function is the calculation of variance of a sample. This means that the denominator will contain *(n-1)* rather than *n*. This will cause the POV method to deteriorate due to reasons mentioned in earlier sections. After identifying the correct functions, the analyst would simply need to identify the blocks of interest. In the example mentioned previously, the analyst would need to calculate VAR.P() (population variance) and AVERAGE() (mean) for each part, operator, and measurement. Once the average and variance have been calculated, the analyst would then use VAR.P() on all the averages to get the between variance, and use AVERAGE() on all the variances to get the within variance. Then, the %Effect can be calculated through simple algebra in the program. Therefore, it is easy to see the simplicity of execution in any spreadsheet program. Further information regarding Excel and other programs are easily found.

Simulation Implementation

The ability to accurately estimate the variance for each component is essential to proper analysis. It is well understood that the assumption of normality is a key component in variance estimation. In an effort to evaluate each method's ability to estimate the variance component in the midst of non-normal distributions, a simulation was created using R software. Scripts were written to calculate variance across parts and operators with both the ANOVA and POV method. The variance estimators from the POV were calculated by the "between" variance formulations. For the ANOVA method, the estimators were calculated the same as Montgomery and Runger (1995). The data structure was similar to the table used in the first example: 3 operators and 20 parts with replicate measures. An operator-part interaction effect was also added.

The normal multivariate matrix "T" was generated by:

$$T = \begin{bmatrix} 1_1 \\ 1_2 \\ \vdots \\ 1_k \end{bmatrix} \otimes \left(X_i * \begin{bmatrix} 1_1 & 1_2 & \dots & 1_j \end{bmatrix} + \begin{bmatrix} 1_1 \\ 1_2 \\ \vdots \\ 1_i \end{bmatrix} * X_j + X_{ij} * \begin{bmatrix} 1_1 \\ 1_2 \\ \vdots \\ 1_i \end{bmatrix} * \begin{bmatrix} 1_1 & 1_2 & \dots & 1_j \end{bmatrix} \right) + \left(X_k * \begin{bmatrix} 1_1 & 1_2 & \dots & 1_j \end{bmatrix} \right) \otimes \begin{bmatrix} 1_1 \\ 1_2 \\ \vdots \\ 1_i \end{bmatrix} \qquad (22)$$

where X_i is the column vector of bias for each part, X_j is the row vector of bias for each operator, X_{ij} is the bias for operator-part interaction, and X_k is the row vector of bias for each measure.

In the case where each bias is independent and normally distributed, i=20, j=3, and k=2, we calculate:

$$T = \begin{bmatrix}1\\1\end{bmatrix} \otimes \left(\begin{bmatrix} \sim N(0,1)_1 \\ \sim N(0,1)_2 \\ \vdots \\ \sim N(0,1)_{20} \end{bmatrix} * \begin{bmatrix}1 & 1 & 1\end{bmatrix} + \begin{bmatrix} 1_1 \\ 1_2 \\ \vdots \\ 1_{20} \end{bmatrix} * \begin{bmatrix} \sim N(0,1) & \sim N(0,1) & \sim N(0,1) \end{bmatrix} \right.$$

$$\left. + \begin{bmatrix} \sim N(0,1)_{1,1} & \sim N(0,1)_{1,2} & \sim N(0,1)_{1,3} \\ \sim N(0,1)_{2,1} & \sim N(0,1)_{2,2} & \sim N(0,1)_{2,3} \\ \vdots & \vdots & \vdots \\ \sim N(0,1)_{20,1} & \sim N(0,1)_{20,2} & \sim N(0,1)_{20,3} \end{bmatrix} \begin{bmatrix} 1_1 \\ 1_2 \\ \vdots \\ 1_{20} \end{bmatrix} * \begin{bmatrix}1 & 1 & 1\end{bmatrix} \right) +$$

$$\left(\begin{bmatrix} \sim N(0,1) \\ \sim N(0,1) \end{bmatrix} * \begin{bmatrix}1 & 1 & 1\end{bmatrix} \right) \otimes \begin{bmatrix} 1_1 \\ 1_2 \\ \vdots \\ 1_{20} \end{bmatrix}.$$

The resulting matrix is 40x3 and is of similar structure to the Montgomery and Runger (1993) dataset analyzed in the first section. In order to add non-normality, skewness and kurtosis were added using two methods: Generalized AutoRegressive Conditional Heteroskedasticity (GARCH) Process and the Johnson S_U distribution. The GARCH method was used to add skewness to the normal distribution and the Johnson distribution added kurtosis. The magnitude of skewness ranged from [1,5] where a skewness of 1 represents the normal distribution. Kurtosis is on the scale of (1,5] where a kurtosis of 3 represents the normal distribution.

The figures below demonstrate the extreme cases for both skewness (Figure 2) and kurtosis (Figure 4 and Figure 5).

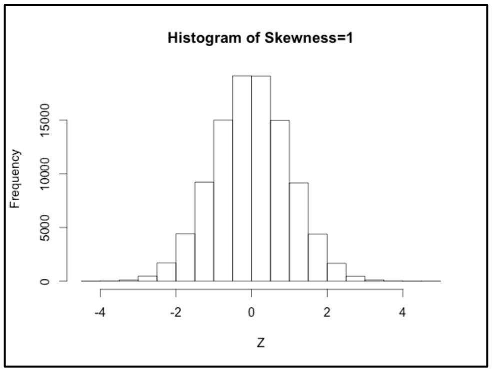


Figure 1. Histogram of simulated data where ~N(0,1) and skewness=1, or normally distributed.

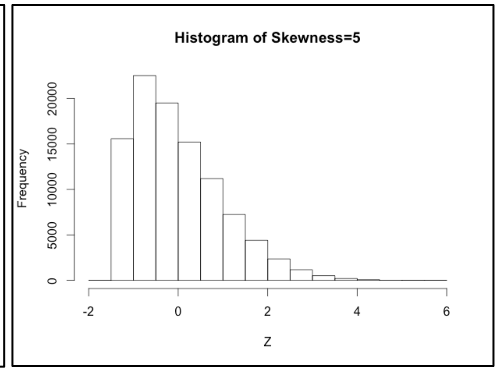


Figure 2. Histogram of simulated data where ~N(0,1) and skewness=5, or a positively skewed distribution.

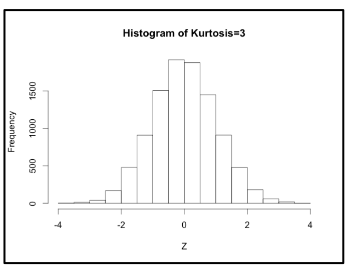


Figure 3. Histogram of simulated data where ~N(0,1) and kurtosis=3, or normally distributed.

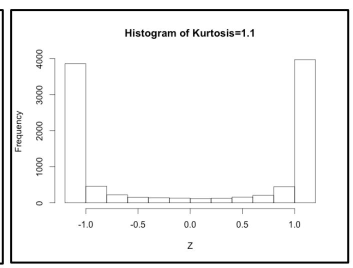


Figure 4. Histogram of simulated data where ~N(0,1) and kurtosis=1.1, or an extremely platykurtic distribution.

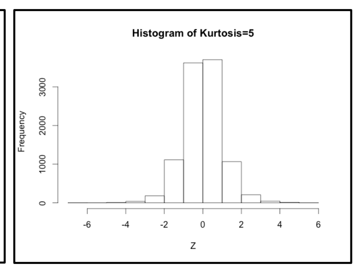


Figure 5. Histogram of simulated data where ~N(0,1) and kurtosis=5, or an leptokurtic distribution.

All iterations of the simulation assumed a standard deviation of 1 for each variance component. The simulation cycled through different magnitudes of skewness and kurtosis, while still holding standard deviation at 1. Each point on the following graphs represents 100,000 randomly generated data sets. The same data sets were simultaneously fed into each solver (POV and ANOVA) at each iteration, allowing for a direct comparison of performance.

Metrics of Performance

In order to compare the methods, several measures of performance were decided upon. The mean, 95% quantiles, mean squared error (MSE), Average Absolute Percent Difference (AAPD), and ratios of each interval were calculated and plotted. The calculation of the mean was straightforward. It is just the average response of all 100,000 iterations at each degree of skewness or kurtosis. The MSE was calculated as the square of the standard deviation of all iterations at each point. The quantiles were the 5th, 50th, and 95th percentage of the vector of results. This equates to the median (50th quantile) and a bootstrapped 95% confidence interval (CI) with the 5th quantile representative of the lower CI and the 95th quantile being the upper CI. With 100,000 iterations, this confidence interval is assumed to be extremely stable. The AAPD is a variation of the Mean Absolute Percent Error (MAPE) metric. AAPD is defined as:

$$AAPD = \frac{\sum_i \left| \frac{{\sigma^2}_i - {\sigma^2}_{true}}{{\sigma^2}_{true}} \right|}{n}. \tag{23}$$

Since ${\sigma^2}_{true}$ in these examples is 1, the equation simplifies to:

$$AAPD = \frac{\sum_i \left| {\sigma^2}_i - 1 \right|}{n}. \tag{24}$$

This metric was decided upon because it adjusts the magnitude of deviation from the true variance in terms of percentage from it.

Lastly, ratios of each metric were calculated as:

$$Ratio = \frac{ANOVA_i}{POV_i} \tag{25}$$

where $ANOVA_i$ is the performance metric (mean, MSE, AAPD) for each level of skewness or kurtosis *i* for the ANOVA method, and POV_i is the same, but with the POV metric. The reasoning behind this ratio was to be able to compare the performance of both methods as the degree of non-normality changes. The ratio is unitless in order to ease interpretation.

Simulation Results

This section outlines the results with a series of graphs, with a summary of the results for each non-normal type (skewness and kurtosis). Each graph has the x-axis being the varying degrees skewness or kurtosis, and the y-axis being one of the performance measures mentioned in the previous section. It is organized as follows: charts of varying degrees of skewness, organized into operator, part, measurement, and interaction estimates. Charts of varying degrees of kurtosis are also organized into operator, part, measurement, and interaction estimates. The performance measures are in the order of mean, quantiles, MSE, and AARP; ordered from left to right, top to bottom. After each performance metric group, a summary of the ratios for each metric is organized into one figure. There is one ratio figure for each partition. A summary and discussion of the results for skewness and kurtosis are after their respective charts.

Output from skewness alterations.

Operator variance estimation.

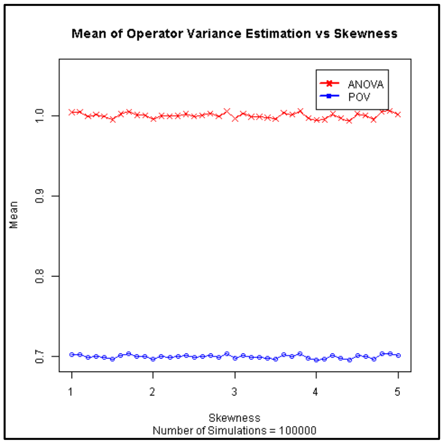


Figure 6. Average of operator variance estimation of 100,000 iterations for varying degrees of skewness for both the ANOVA (red) and POV (blue) methods.

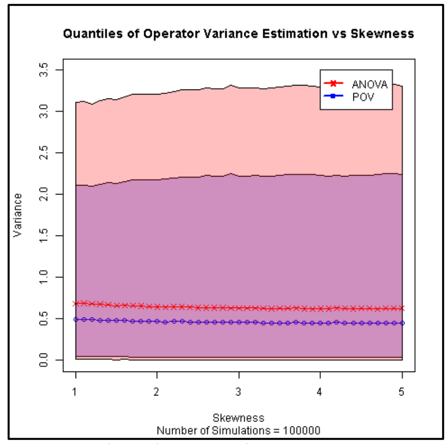


Figure 7. The 5^{th}, 50^{th}, and 95^{th} quantiles of operator variance estimation of 100,000 iterations for varying degrees of skewness for both the ANOVA (red) and POV (blue) methods.

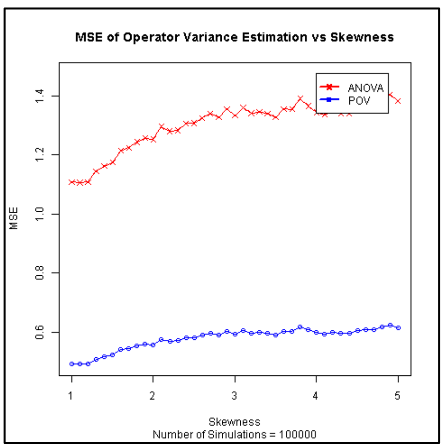


Figure 8. Mean Squared Error (MSE) of operator variance estimation of 100,000 iterations for varying degrees of skewness for both the ANOVA (red) and POV (blue) methods.

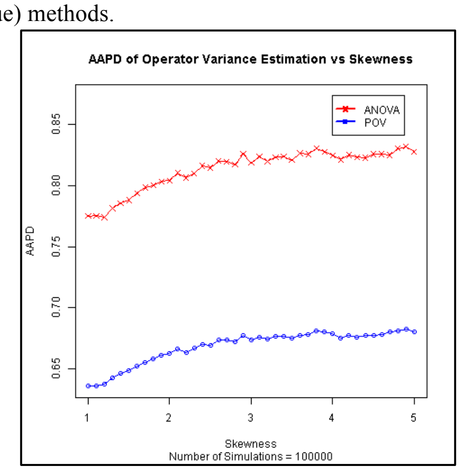
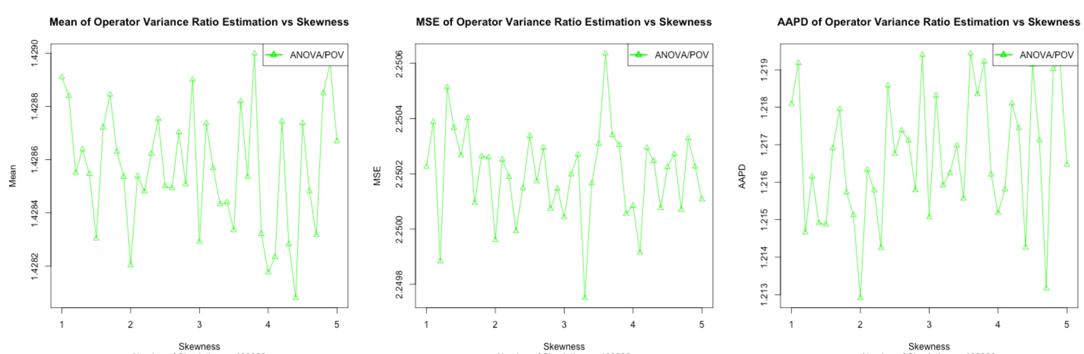


Figure 9. Average Absolute Percent Difference (AAPD) of operator variance estimation of 100,000 iterations for varying degrees of skewness for both the ANOVA (red) and POV (blue) methods.



(a) *(b)* *(c)*

Figure 10. ANOVA/POV ratio comparison for mean (a), MSE (b), and AAPD (c) for operator vs. skewness.

Part variance estimation.

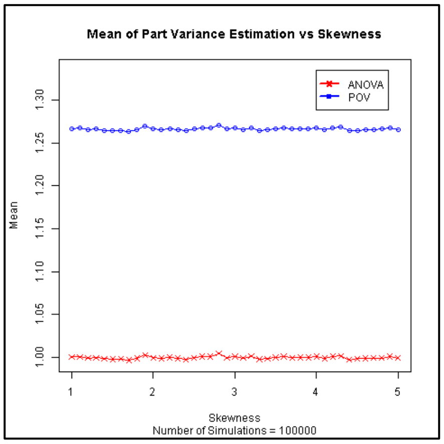


Figure 11. Average of part variance estimation of 100,000 iterations for varying degrees of skewness for both the ANOVA (red) and POV (blue) methods.

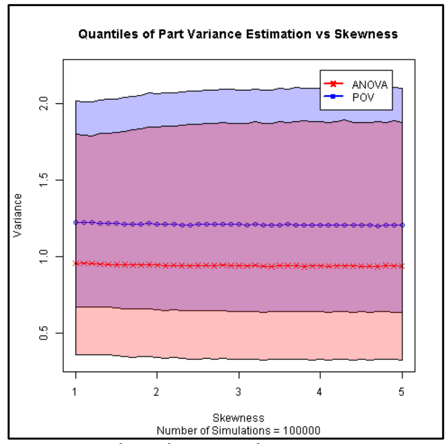


Figure 12. The 5th, 50th, and 95th quantiles of part variance estimation of 100,000 iterations for varying degrees of skewness for both the ANOVA (red) and POV (blue) methods.

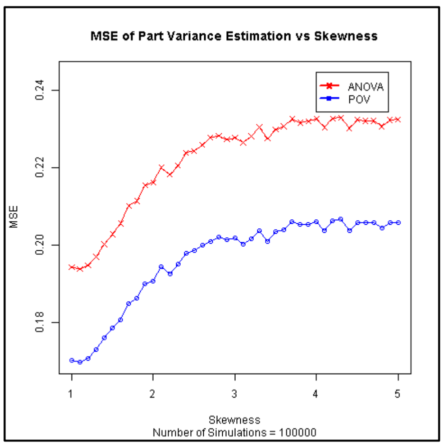


Figure 13. Mean Squared Error (MSE) of part variance estimation of 100,000 iterations for varying degrees of skewness for both the ANOVA (red) and POV (blue) methods.

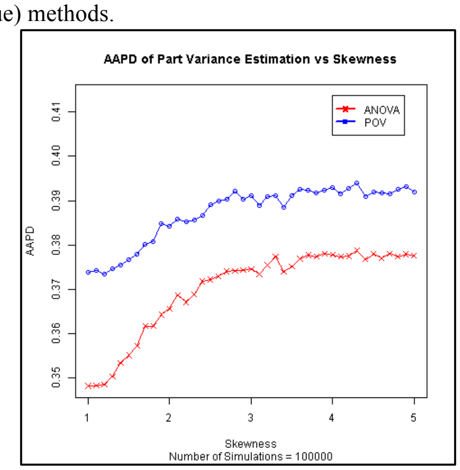
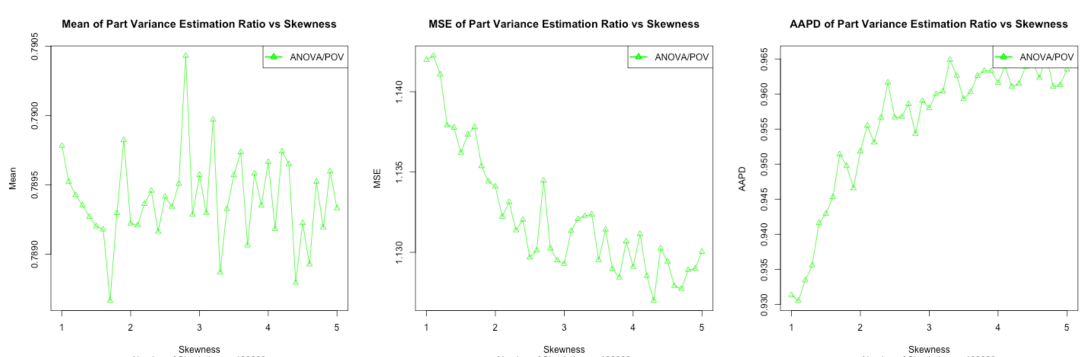


Figure 14. Average Absolute Percent Difference (AAPD) of part variance estimation of 100,000 iterations for varying degrees of skewness for both the ANOVA (red) and POV (blue) methods.



(a) *(b)* *(c)*

Figure 15. ANOVA/POV ratio comparison for mean (a), MSE (b), and AAPD (c) for part vs. skewness.

Measurement variance estimation.

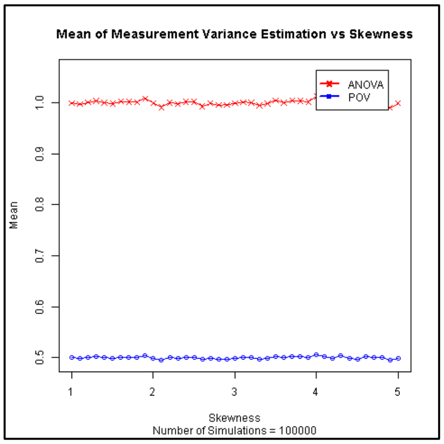


Figure 16. Average of measurement variance estimation of 100,000 iterations for varying degrees of skewness for both the ANOVA (red) and POV (blue) methods.

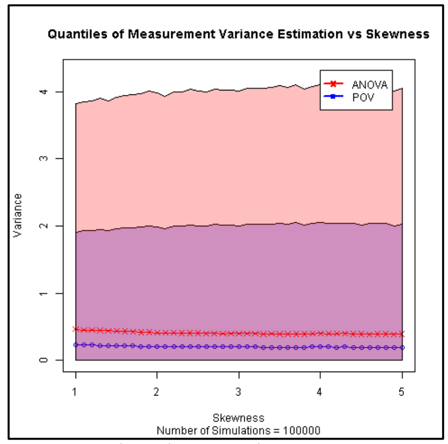


Figure 17. The 5th, 50th, and 95th quantiles of measurement variance estimation of 100,000 iterations for varying degrees of skewness for both the ANOVA (red) and POV (blue) methods.

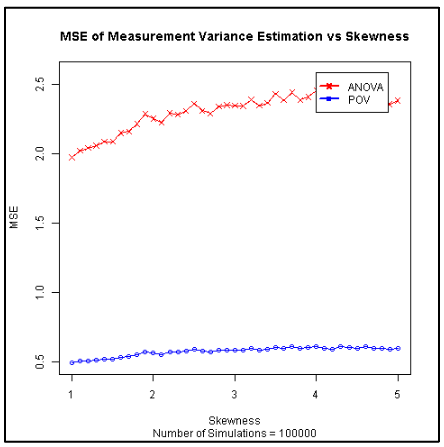


Figure 18. Mean Squared Error (MSE) of measurement variance estimation of 100,000 iterations for varying degrees of skewness for both the ANOVA (red) and POV (blue) methods.

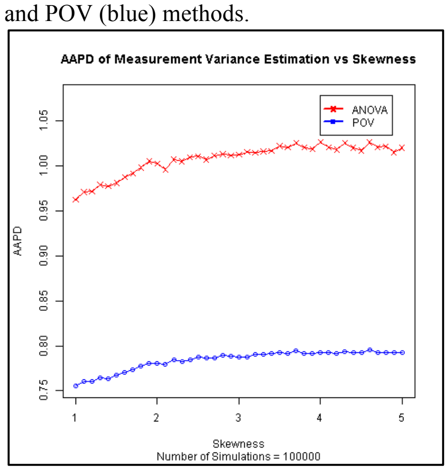
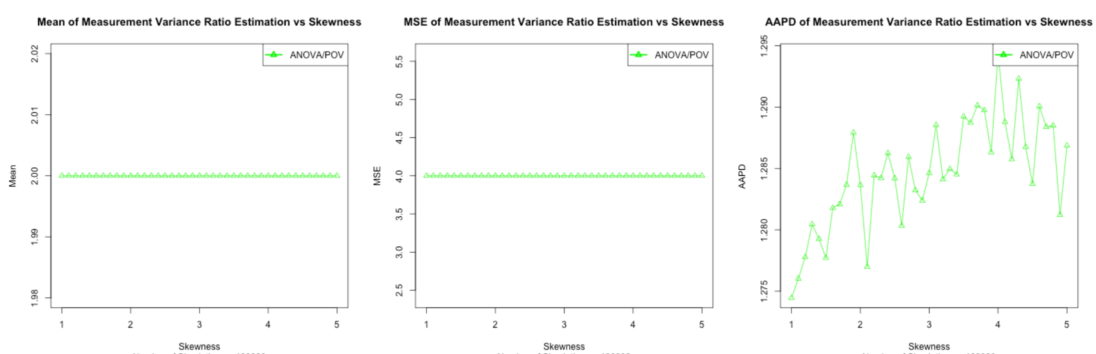


Figure 19. Average Absolute Percent Difference (AAPD) of measurement variance estimation of 100,000 iterations for varying degrees of skewness for both the ANOVA (red) and POV (blue) methods.



(a) *(b)* *(c)*

Figure 20. ANOVA/POV ratio comparison for mean (a), MSE (b), and AAPD (c) for measurement vs. skewness.

Interaction variance estimation.

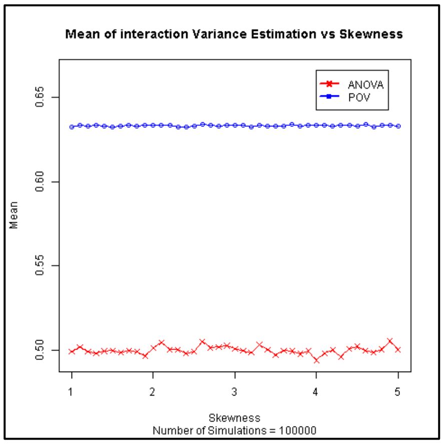


Figure 21. Average of interaction variance estimation of 100,000 iterations for varying degrees of skewness for both the ANOVA (red) and POV (blue) methods.

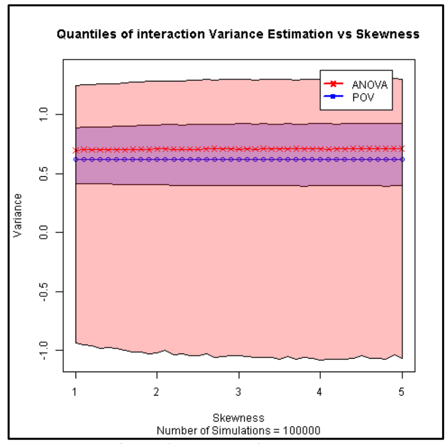


Figure 22. The 5th, 50th, and 95th quantiles of interaction variance estimation of 100,000 iterations for varying degrees of skewness for both the ANOVA (red) and POV (blue) methods.

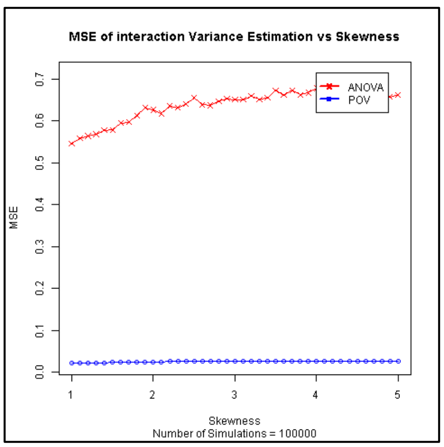


Figure 23. Mean Squared Error (MSE) of interaction variance estimation of 100,000 iterations for varying degrees of skewness for both the ANOVA (red) and POV (blue) methods.

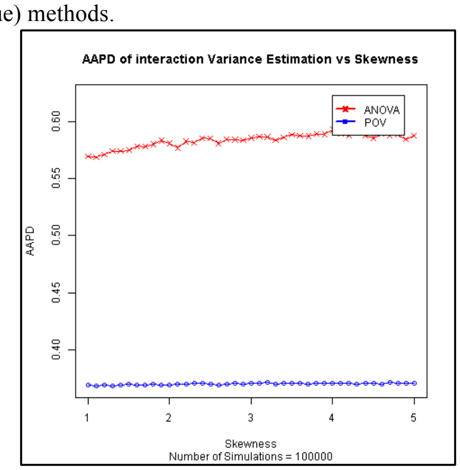
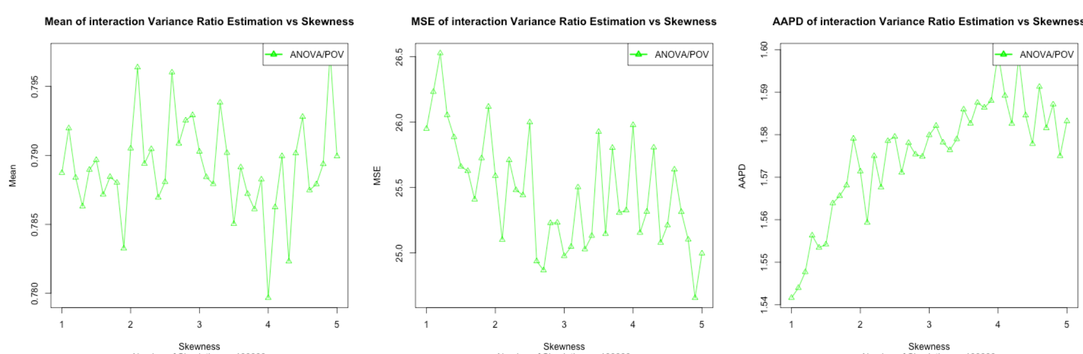


Figure 24. Average Absolute Percent Difference (AAPD) of interaction variance estimation of 100,000 iterations for varying degrees of skewness for both the ANOVA (red) and POV (blue) methods.



(a) *(b)* *(c)*

Figure 25. ANOVA/POV ratio comparison for mean (a), MSE (b), and AAPD (c) for interaction vs. skewness.

Skewness results summary.

Looking at Figure 6, the POV method underestimates the variance on average. This is due to the discussion earlier on the population variance versus the sample variance. The ANOVA method corrects for bias due to small sample size by reducing the denominator to *(n-1)*. However, if attention is turned to Figure 7, the 95% CI overlap to a great extent. Thus, it can be stated that there is no significant difference in the estimators. The MSE (Figure 8) is much smaller for the POV method. Again, this is due to the POV underestimating the variance. On one hand, the POV method underestimates the variance; On the other hand, it is more stable in the aggregate. The AAPD (Figure 9) shows the relatively same information as the MSE. Though, the scale is more straightforward. The ANOVA method is roughly 78-83% off on average from the true variance, where the POV method is about 63-68% different than the true variance. Figure 10 demonstrates that the two methods are relatively stable with respect to each other over the severity of skewness. It is observed that both methods remain relatively stable with even large deviations from normality with respect to skewness.

Looking to the part estimate, the POV estimate for part variance is overestimating the true variance. This is due to the fact that there is part-operator interaction added. Furthermore, the interaction effect is not removed from the part-to-part variance in the POV method. This is not the case for the ANOVA method, as the calculation corrects for the interaction effect by removing it from the part variance (Montgomery and Runger (1993)).

It is also crucial to compare the relative stability of each estimate as the sample size decreases. Bear in mind that the part estimates have an *p*=20, operators have an *o*=3, and measurements have an *m*=2 in this particular example. As the sample size decreases, the estimate for the POV method becomes more and more bias. However, the ANOVA method remains relatively stable with respect to the mean. Though, it can be seen that the variance of these estimates increase as the sample size decreases for the ANOVA method, whereas the variance for the POV method remains relatively stable. This is one advantage of the POV method over the ANOVA method.

Figure 20 (a) demonstrates the difference between the estimates in terms of the population vs. sample variance estimator. The ratio turns out to be 2 exactly. Because there is no added measurement interaction effect, the sums of squares of the measurement error for both estimates are equal. As a result, the only difference in the two methods is *n* vs. *(n-1)*. Because *m*=2 in the case of measurement, this results in the estimate of variance for ANOVA to consistently be twice as large as the estimate for the POV method. This relationship will not hold true with the addition of measurement interactions, as demonstrated by the operator-part interaction.

Lastly, take particular note of Figure 22. Montgomery and Runger (1993) mention that the ANOVA can have a negative estimate of variance for interaction. On the contrary, POV method not only has a drastically smaller variance, there is no negative estimate of variance due to interaction. This is another advantage of the POV method. Therefore, if there is concern of an interaction effect in the measurement system, it may be deemed prudent to analyze the data using the POV method over the ANOVA method.

Output from kurtosis alterations.

Operator variance estimation.

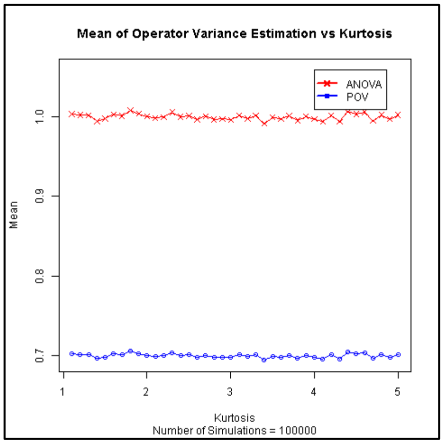


Figure 26. Average of operator variance estimation of 100,000 iterations for varying degrees of kurtosis for both the ANOVA (red) and POV (blue) methods.

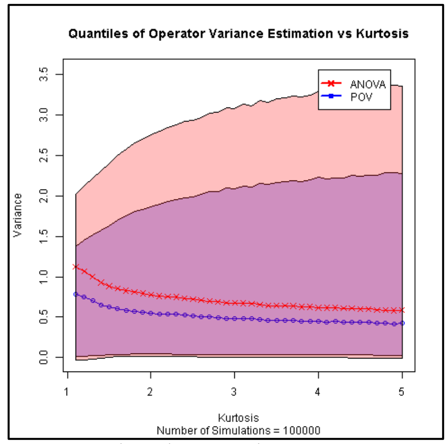


Figure 27. The 5th, 50th, and 95th quantiles of operator variance estimation of 100,000 iterations for varying degrees of kurtosis for both the ANOVA (red) and POV (blue) methods.

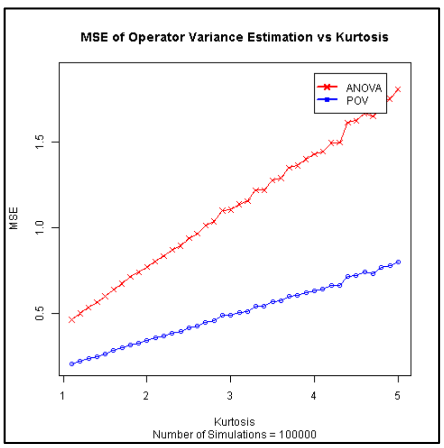


Figure 28. Mean Squared Error (MSE) of operator variance estimation of 100,000 iterations for varying degrees of kurtosis for both the ANOVA (red) and POV (blue) methods.

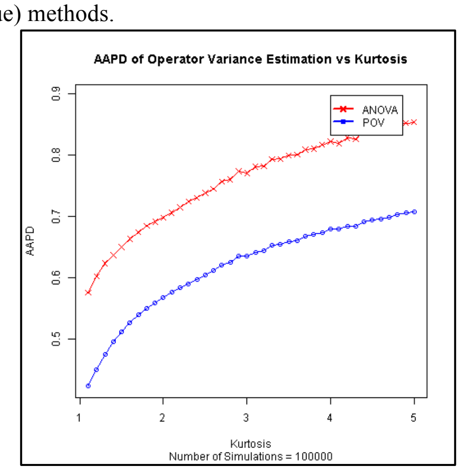
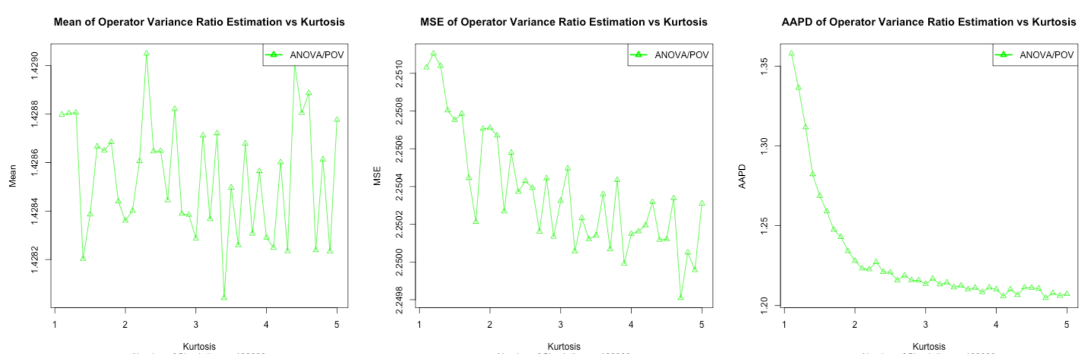


Figure 29. Average Absolute Percent Difference (AAPD) of operator variance estimation of 100,000 iterations for varying degrees of kurtosis for both the ANOVA (red) and POV (blue) methods.



(a) *(b)* *(c)*

Figure 30. ANOVA/POV ratio comparison for mean (a), MSE (b), and AAPD (c) for operator vs. kurtosis.

Part variance estimation.

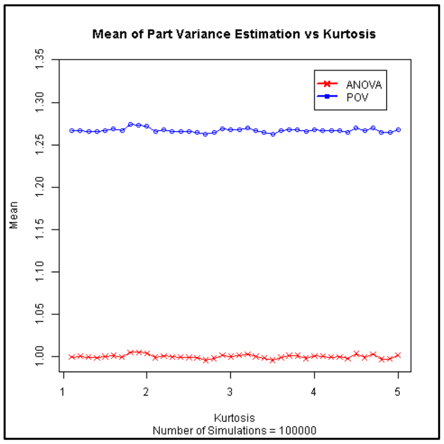


Figure 31. Average of part variance estimation of 100,000 iterations for varying degrees of kurtosis for both the ANOVA (red) and POV (blue) methods.

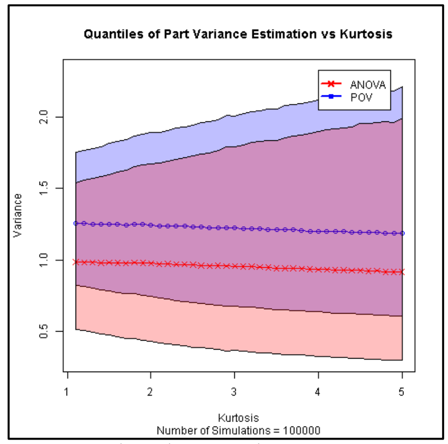


Figure 32. The 5th, 50th, and 95th quantiles of part variance estimation of 100,000 iterations for varying degrees of kurtosis for both the ANOVA (red) and POV (blue) methods.

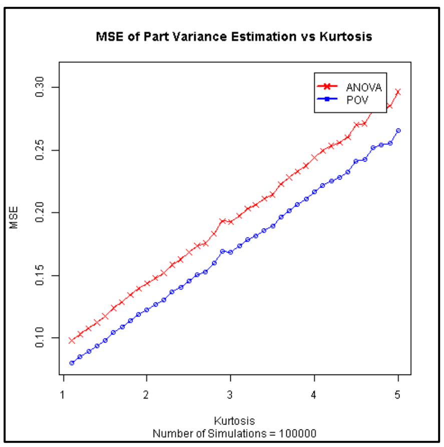


Figure 33. Mean Squared Error (MSE) of part variance estimation of 100,000 iterations for varying degrees of kurtosis for both the ANOVA (red) and POV (blue) methods.

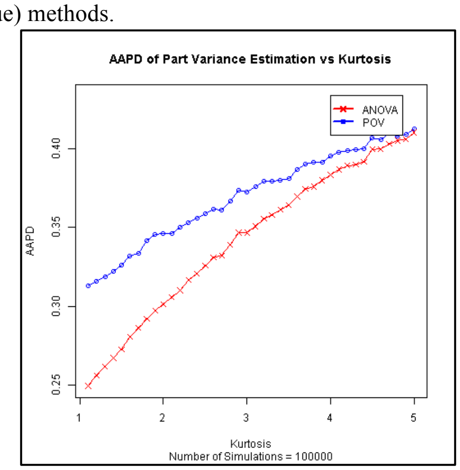
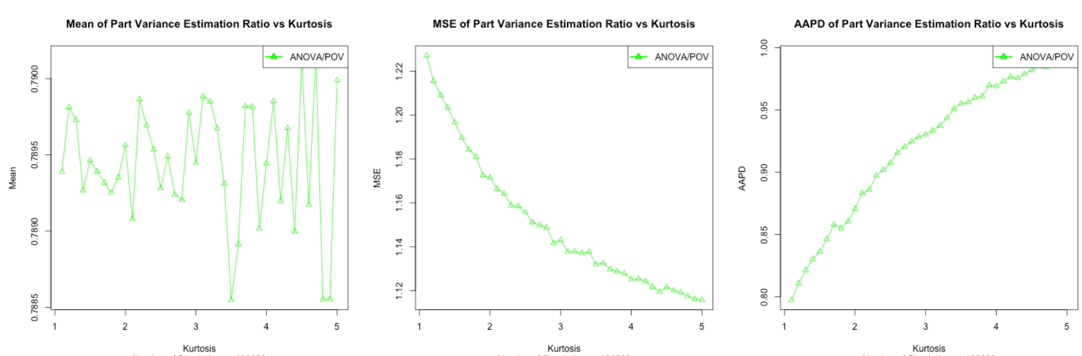


Figure 34. Average Absolute Percent Difference (AAPD) of part variance estimation of 100,000 iterations for varying degrees of kurtosis for both the ANOVA (red) and POV (blue) methods.



(a) *(b)* *(c)*

Figure 35. ANOVA/POV ratio comparison for mean (a), MSE (b), and AAPD (c) for part vs. kurtosis.

Measurement variance estimation.

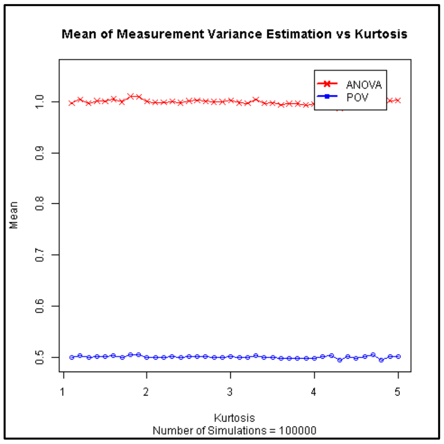


Figure 36. Average of measurement variance estimation of 100,000 iterations for varying degrees of kurtosis for both the ANOVA (red) and POV (blue) methods.

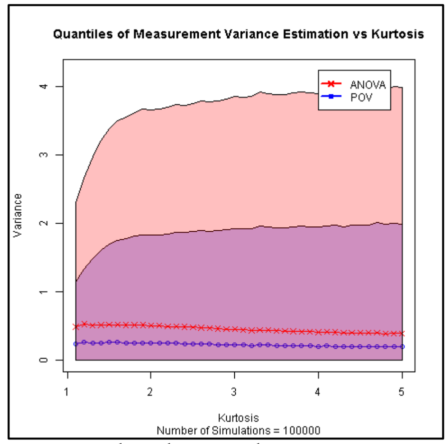


Figure 37. The 5th, 50th, and 95th quantiles of measurement variance estimation of 100,000 iterations for varying degrees of kurtosis for both the ANOVA (red) and POV (blue) methods.

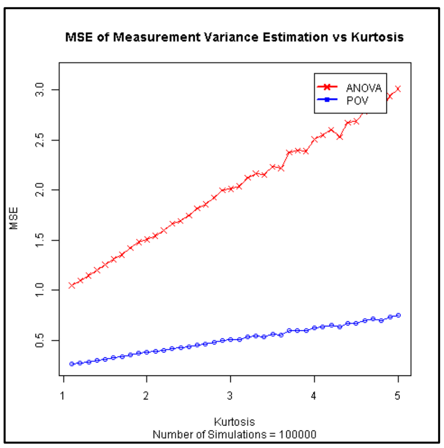


Figure 38. Mean Squared Error (MSE) of measurement variance estimation of 100,000 iterations for varying degrees of kurtosis for both the ANOVA (red) and POV (blue) methods.

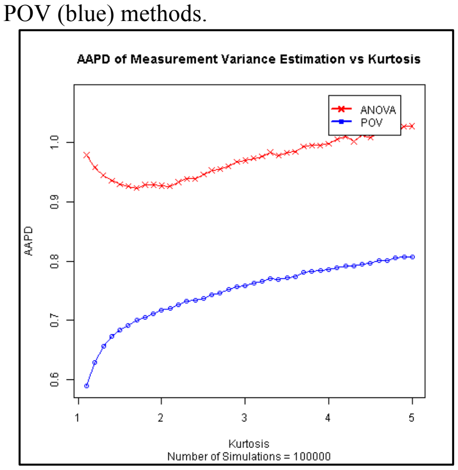
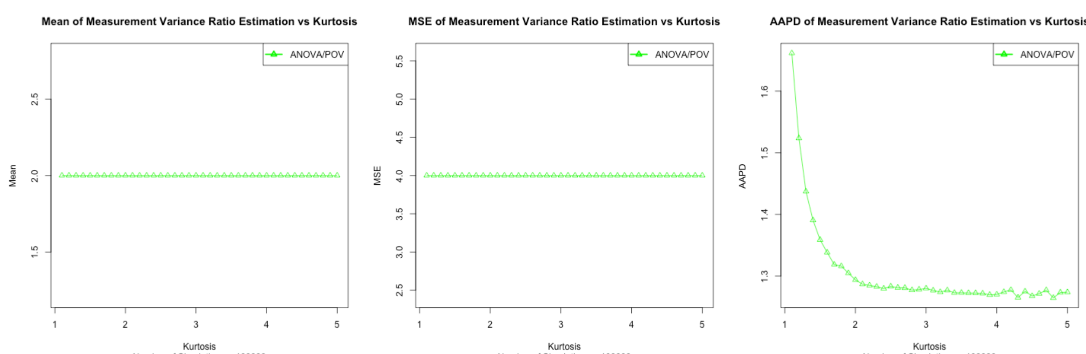


Figure 39. Average Absolute Percent Difference (AAPD) of measurement variance estimation of 100,000 iterations for varying degrees of kurtosis for both the ANOVA (red) and POV (blue) methods.



(a) *(b)* *(c)*

Figure 40. ANOVA/POV ratio comparison for mean (a), MSE (b), and AAPD (c) for measurement vs. kurtosis.

Interaction variance estimation.

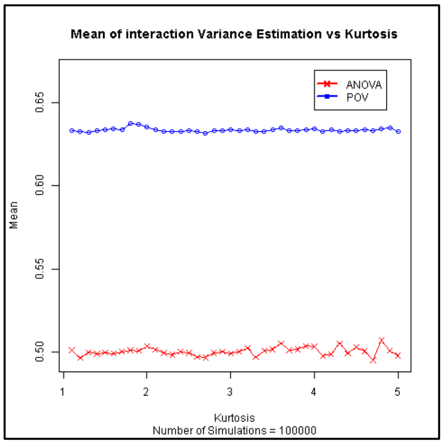


Figure 41. Average of interaction variance estimation of 100,000 iterations for varying degrees of kurtosis for both the ANOVA (red) and POV (blue) methods.

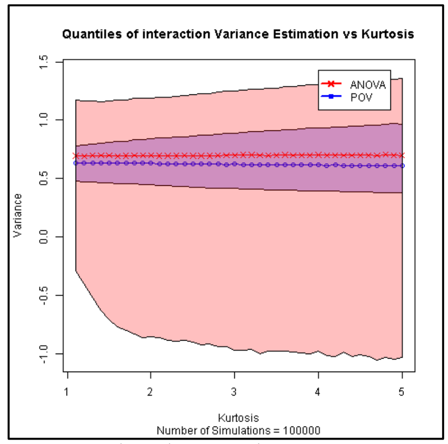


Figure 42. The 5th, 50th, and 95th quantiles of interaction variance estimation of 100,000 iterations for varying degrees of kurtosis for both the ANOVA (red) and POV (blue) methods.

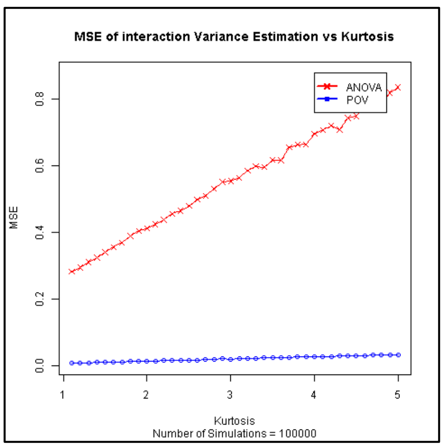


Figure 43. Mean Squared Error (MSE) of interaction variance estimation of 100,000 iterations for varying degrees of kurtosis for both the ANOVA (red) and POV (blue) methods.

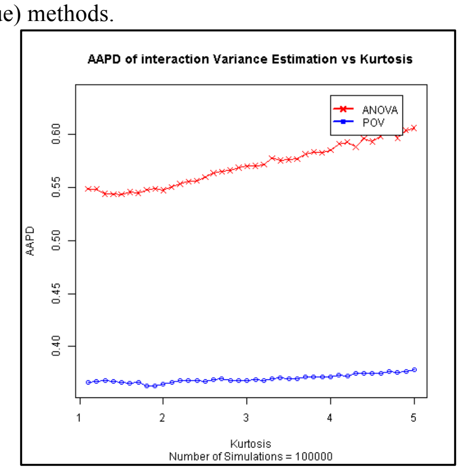
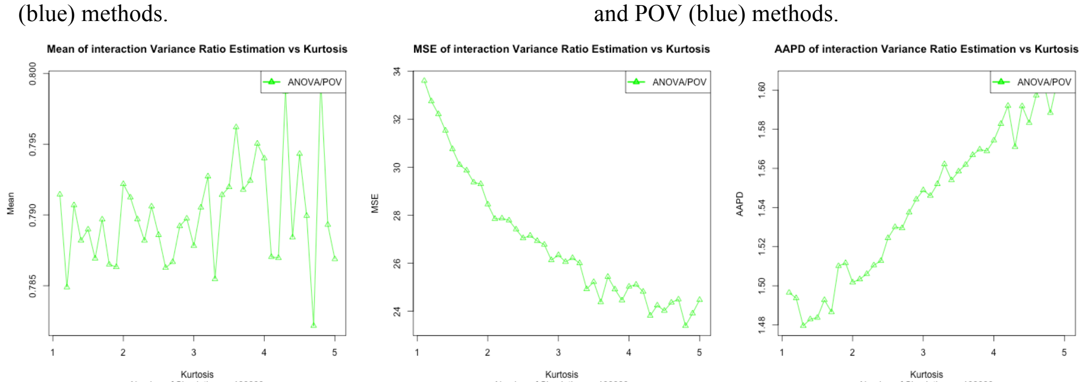


Figure 44. Average Absolute Percent Difference (AAPD) of interaction variance estimation of 100,000 iterations for varying degrees of kurtosis for both the ANOVA (red) and POV (blue) methods.



(a) *(b)* *(c)*

Figure 45. ANOVA/POV ratio comparison for mean (a), MSE (b), and AAPD (c) for interaction vs. kurtosis.

Kurtosis results summary.

The main points mentioned in the summary with skewness still hold true with the addition of kurtosis over skewness. However, there are a few main differences that need to be mentioned: looking at Figure 27, it can be seen that while the median of the estimate lowers as the kurtosis moves from one extreme to the other, the variance increases. It is also observed that the MSE for the POV is less affected by the change in kurtosis. It is also of note that a kurtosis of 1.1 is a heavy tailed (Figure 4) distribution, yet produces the most stable estimate. The variance of all the estimates increases steadily as kurtosis transitions from heavily tailed, to heavily centered. Though, in each case the POV method performs better in terms of variance of the estimate. As the *n* increases, POV estimate variance more closely matches the ANOVA method. Therefore, if highly centered distributions are a concern, the POV method should be considered due to its greater stability.

Increasing sample size of operators.

As mentioned previously, it is highly impractical to hire more operators for the sake of an MSA. In many cases, there are limited numbers of operators that can perform certain measurements. While cross training can alleviate this issue, there are most likely a small number of operators capable of measuring for an MSA. Furthermore, precise measuring tools are often expensive, and this results in a similar issue. Another simulation doubling the number of operators is outlined below in the same as in previous sections. However, only the operator variance estimates are reported. The following charts are the result of changing the number of operators from o=3 to o=6.

Operator variance estimation for skewness where o=6.

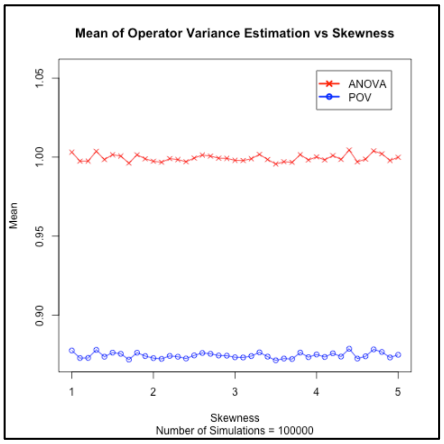


Figure 46. Average of operator variance estimation of 100,000 iterations for varying degrees of skewness for both the ANOVA (red) and POV (blue) methods, where *o*=6.

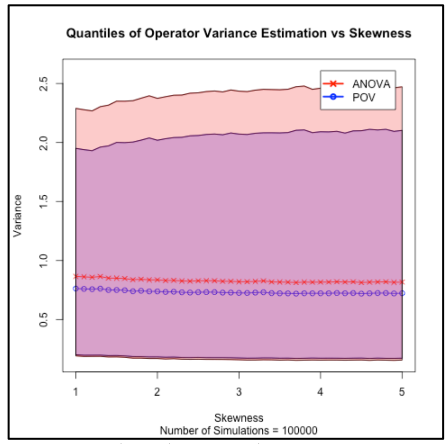


Figure 47. The 5th, 50th, and 95th quantiles of operator variance estimation of 100,000 iterations for varying degrees of skewness for both the ANOVA (red) and POV (blue) methods, where *o*=6.

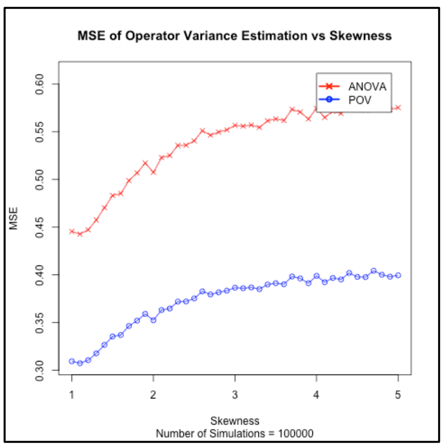


Figure 48. Mean Squared Error (MSE) of operator variance estimation of 100,000 iterations for varying degrees of skewness for both the ANOVA (red) and POV (blue) methods, where *o*=6.

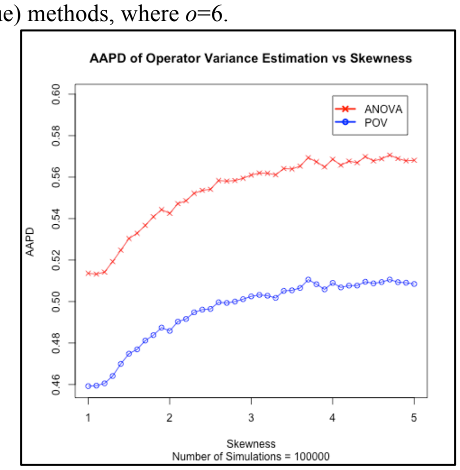
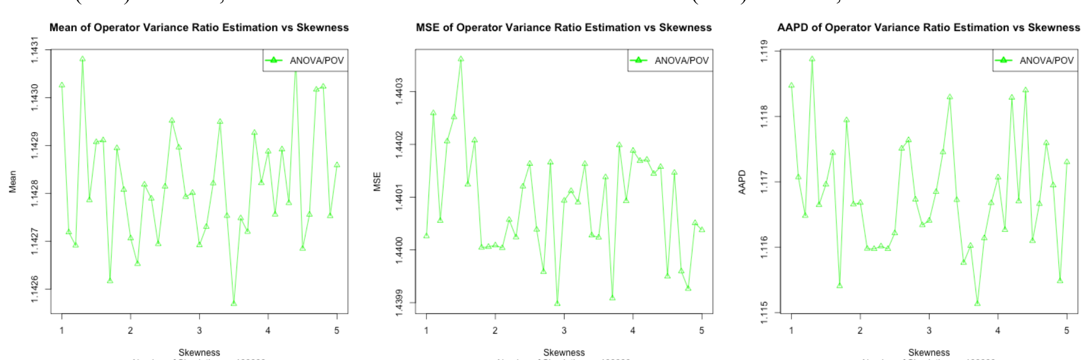


Figure 49. Average Absolute Percent Difference (AAPD) of operator variance estimation of 100,000 iterations for varying degrees of skewness for both the ANOVA (red) and POV (blue) methods, where *o*=6.



(a) *(b)* *(c)*

Figure 50. ANOVA/POV ratio comparison for mean (a), MSE (b), and AAPD (c) for operator vs. skewness for *o*=6.

Operator variance estimation for kurtosis where o=6.

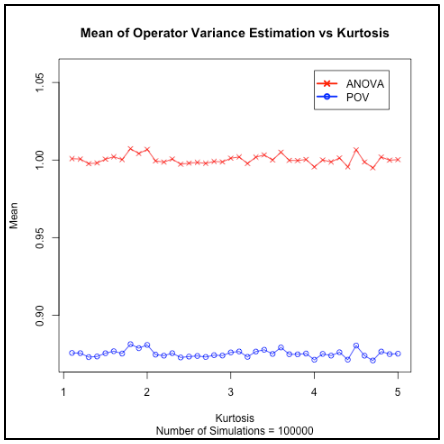


Figure 51. Average of operator variance estimation of 100,000 iterations for varying degrees of kurtosis for both the ANOVA (red) and POV (blue) methods, where *o*=6.

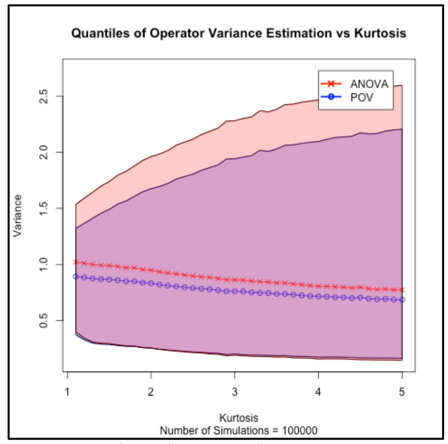


Figure 52. The 5^{th}, 50^{th}, and 95^{th} quantiles of operator variance estimation of 100,000 iterations for varying degrees of kurtosis for both the ANOVA (red) and POV (blue) methods, where *o*=6.

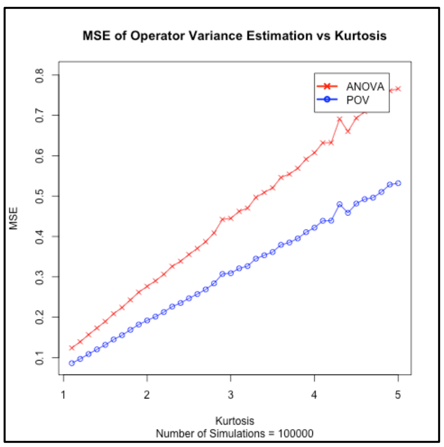


Figure 53. Mean Squared Error (MSE) of operator variance estimation of 100,000 iterations for varying degrees of kurtosis for both the ANOVA (red) and POV (blue) methods, where *o*=6.

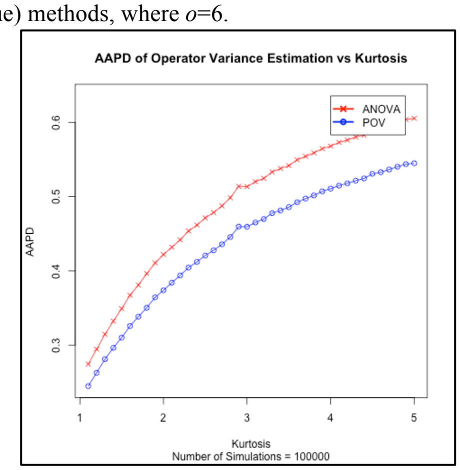
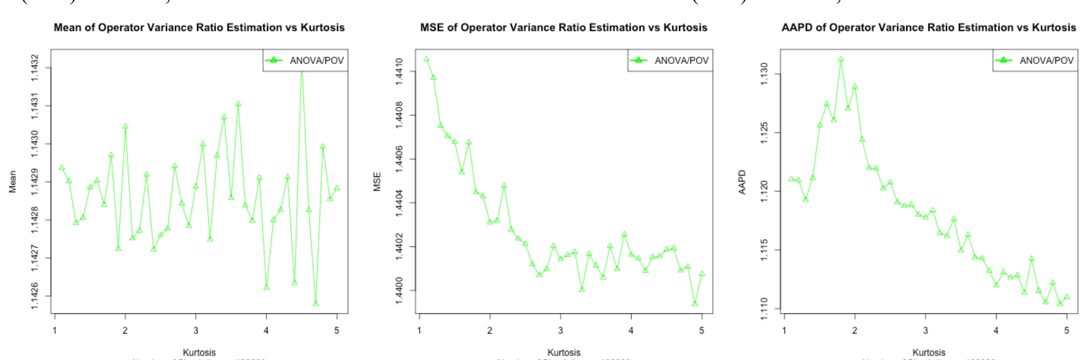


Figure 54. Average Absolute Percent Difference (AAPD) of operator variance estimation of 100,000 iterations for varying degrees of kurtosis for both the ANOVA (red) and POV (blue) methods, where *o*=6.



(a) *(b)* *(c)*

Figure 55. ANOVA/POV ratio comparison for mean (a), MSE (b), and AAPD (c) for operator vs. kurtosis, where *o*=6.

When comparing Figures 46-50 to Figures 6-10, it is shown that the variance of the estimate decreases, paying particular attention to the MSE charts. The same relationship is shown when comparing Figures 51-55 to Figures 26-30. Unfortunately, as demonstrated by Vardeman and VanValkenburg (1999), the addition of more measurements or parts will not strengthen this estimate for operator variance. The only way to do so is to increase the number of operators. This is another advantage of the POV over the ANOVA. The POV method has more stable estimates, even in light of smaller sample sizes.

CONCLUSIONS

The POV method is not meant to define the significance of variance across the dimensions. Instead, this method is meant to be a guide for an engineer to locate the largest source of variance. It is much more qualitative rather than quantitative; it is descriptive rather than interpretive. This interpretive nature is one downfall of the POV method. However, the POV method is simple enough that complex statistical packages are not necessary, and can easily be implemented with simple spreadsheet packages such as Microsoft Excel. The methods demonstrated are also simple enough that engineers and managers with limited statistical backgrounds can identify large sources of variation. This will, in turn, enable them quickly find the variables that are grossly contributing to the variance. This will potentially lead to developments that will help to reduce the overall variance. It can also help identify if more operator training is needed, or if the measuring devices need to be calibrated or updated, depending on how the analysis is organized.

The method correctly identifies the source of variance when applied to the dataset in Montgomery and Runger (1993) reviewing the ANOVA method of the Gauge R&R analysis. Again, the delivery of this information is drastically simplified when demonstrated with the POV method.

The POV method does underestimate the variance, with the exception of strong interaction effects. This underestimation is exaggerated as the sample size diminishes. This can be problematic. However, it was shown that the 95% CI for each estimate were heavily overlapped. Thus, there appears to be no substantial difference between the two methods. Furthermore, the estimates from the POV were shown to be more stable on aggregate than the ANOVA method. In situations where doubt is cast on the normality of the distribution, this can be beneficial, specifically in low sample situations. These low sample situations are common when dealing with gauge studies. Furthermore, when interaction appears to be an issue, the POV method will produce a non-negative, stable estimate of interaction when compared to the ANOVA method. Though, in cases of high dimensionality, the POV method is not capable of separating out the interaction effects. In the case of strong interaction, the POV method may overestimate certain variances. This is because it does not correct the other estimates for interaction.

Lastly, other studies show that the addition of measurements or parts does not strengthen the estimates for static sample size variables, such as operator or tool (Vardeman and VanValkenburg (1999)). The only way to do so would be to increase the number of operators or tools, however, it is highly impractical to hire more operators or purchase more tools for the sake of an MSA. The estimates of the POV method are more stable in low sample situations. Therefore, it could be considered an advantage to use the POV method in situations of low sample sizes.

Ultimately it is up to the analyst what method he chooses. The analyst should weigh the costs versus the benefits. In situations where a quick and dirty analysis is needed to identify the source of where the variance is stemming from, the POV method is ideal. If a more thorough method were deemed necessary, it would be straightforward to analyze the same dataset with more complex methods. More specifically, if significance of the factors must be tested, the ANOVA method should be implemented. In settings where the end user has limited statistical knowledge, the POV method is more suitable. It is simple to train and help others understand when compared to the more complex ANOVA method. Ideally, an engineering manager could train others to implement the POV method. Then, he could have their employees report their findings. If there are findings that are of interest, an ANOVA analysis could be implemented to gain further insight. This can save both time and money. The POV method is not being introduced as a replacement for more complex methods, but rather a supplement to them.

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APPENDIX A

DATA TABLE FROM MONTGOMERY AND RUNGER'S (1993) PAPER

	Operator 1		Operator 2		Operator 3	
	Measurement		Measurement		Measurement	
Part	**1**	**2**	**1**	**2**	**1**	**2**
1	21	20	20	20	19	21
2	24	23	24	24	23	24
3	20	21	19	21	20	22
4	27	27	28	26	27	28
5	19	18	19	18	18	21
6	23	21	24	21	23	22
7	22	21	22	24	22	20
8	19	17	18	20	19	18
9	24	23	25	23	24	24
10	25	23	26	25	24	25
11	21	20	20	20	21	20
12	18	19	17	19	18	19
13	23	25	25	25	25	25
14	24	24	23	25	24	25
15	29	30	30	28	31	30
16	26	26	25	26	25	27
17	20	20	19	20	20	20
18	19	21	19	19	21	23
19	25	26	25	24	25	25
20	19	19	18	17	19	17