Characterization of the Mathematical Theoretical Biology Institute as a Vygotskian-Holzman Zone of Proximal Development

by

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ABSTRACT

The Mathematical and Theoretical Biology Institute (MTBI) is a summer research program for undergraduate students, largely from underrepresented minority groups. Founded in 1996, it serves as a 'life-long' mentorship program, providing continuous support for its students and alumni. This study investigates how MTBI supports student development in applied mathematical research. This includes identifying of motivational factors to pursue and develop capacity to complete higher education.

The theoretical lens of developmental psychologists Lev Vygotsky (1978, 1987) and Lois Holzman (2010) that sees learning and development as a social process is used. From this view student development in MTBI is attributed to the collaborative and creative way students co-create the process of becoming scientists. This results in building a continuing network of academic and professional relationships among peers and mentors, in which around three quarters of MTBI PhD graduates come from underrepresented groups.

The extent to which MTBI creates a Vygotskian learning environment is explored from the perspectives of participants who earned doctoral degrees. Previously hypothesized factors (Castillo-Garsow, Castillo-Chavez and Woodley, 2013) that affect participants' educational and professional development are expanded on.

Factors identified by participants are a passion for the mathematical sciences; desire to grow; enriching collaborative and peer-like interactions; and discovering career options. The self-recognition that they had the ability to be successful, key element of the Vygotskian-Holzman theoretical framework, was a commonly identified theme for their educational development and professional growth.

Participants characterize the collaborative and creative aspects of MTBI. They reported that collaborative dynamics with peers were strengthened as they co-created a learning environment that facilitated and accelerated their understanding of the mathematics needed to address their research. The dynamics of collaboration allowed them to complete complex homework assignments, and helped them formulate and complete their projects. Participants identified the creative environments of their research projects as where creativity emerged in the dynamics of the program.

These data-driven findings characterize for the first time a summer program in the mathematical sciences as a Vygotskian-Holzman environment, that is, a `place’ where participants are seen as capable applied mathematicians, where the dynamics of collaboration and creativity are fundamental components.

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INTRODUCTION

Motivation for Study

According to a 2013 National Science Foundation report on "Women, Minorities, and Persons with Disabilities in Science and Engineering: 2013", it is estimated that by 2050, half of the United States (U.S.) residents will be minorities. Unfortunately, the percent of underrepresented minorities earning degrees in engineering and physical sciences has remained about the same since 2000, and the percent of degrees in Mathematics has been decreasing.

Increasing the number of underrepresented individuals in the Mathematical Sciences is essential if we are to remain competitive and preserved economy security (National research council, 2007, Friedman, 2005). The national council has called for national efforts in human, research and capital investments and recommended focusing on improving the education from K-12 to higher education in Mathematics and Science, commitment to research, careers, jobs, innovation in the Science Technology Engineering and Mathematics (STEM) fields, and the development and retention of students in the STEMs fields, if we are to remain competitive and be innovators in the world (2007). The National Academy of Sciences (NAS) has also recommended to invest in human and financial capital, as well as in research on STEM in order to remain competitive and prosperous. In addition, the National Academy has called for national efforts in broadening participation from all groups, including underrepresented minorities as it is vital in improving the nation's global economic leadership (The National Academy of Sciences, 2005, 2010).

Hurtado (2007) has made a call for advancing research and practice while linking diversity and higher education as a way to promote social progress. She explains that in linking diversity to higher education we "must focus on a set of democratic outcomes that recognizes difference as a constructive part of a democracy, promotes student's ability to work with diverse people and viewpoints, and builds student self-efficacy for change" (p. 191).

Tough (2015) observes that over the last twenty years, the U.S. has fallen from the top of international list of nations given access to higher education and it now ranks 12th in the world of young people earning a college degree. He also noted that more well-to do students were doing very well in college while middle and low-income students were struggling to graduate. He argues, that for the U.S. to regain its global competitiveness there has to be a national effort to motivate the youth raised in working class families while promoting a national environment that uses open and selective access models to education, models not based on the super elite selection of young talent; models that include and provide with the necessary tools needed to be successful.

The numbers of underrepresented groups in higher education earning PhDs are not encouraging. A recent report on doctoral recipients in the mathematical sciences reported, that 1,926 people earned their PhDs from July 2013 to June 2014. Of these, 48% were U.S. citizens, and only 28% were female; with approximately 7% of the doctoral recipients earned by individuals from underrepresented groups, which includes American Indian, Black, Latino, and Hawaiian (Velez, Maxwell, and Rose, 2014).

In 2009, the Obama administration launched an STEM national initiative involving not only the government but also public and private companies, non-profits, and science and engineering entities. This initiative stresses the importance of working together to improve the quality of STEM Education (The White House, n.d.).

In 2010, the U.S. department of commerce reported that 7.6 million people of the labor force worked in STEM jobs. It projects that over the next 10 years, the percentage STEM jobs will be three times greater than of non-STEM. And so, we need to prepare a new set of young minds that can take advantage of the growth in STEM related jobs. The advantages of a STEM education were clearly spelled, for example, this report noted that STEM workers tend to earned more money and less likely to experience a job loss (U.S. Department of commerce, 2011).

There is a great national need to increase the numbers in STEM and given the changing demographics of our nation, it is especially important that the issues of underrepresentation in STEM get addressed so that we remain not only globally competitive but that also we serve the needs of our people fulfilling the promises of our democracy for every American. Changing the degree of underrepresentation in the Mathematical sciences requires committed individuals, resources from national agencies, and foundations and university administrators. The Mathematical Theoretical Biology Institute (MTBI) offers a model for tackling such a problem (Castillo-Chavez, and Castillo-Garsow, 2008) through its open access to all participants who are eager to learn independently of their initial training.

The program has been recognized nationally for successfully mentoring individuals and its role in ensuring that America remains competitive in the Mathematical sciences (ASU News, 2011). Castillo-Garsow, Castillo-Chavez and Woodley (2013) noted that as February 2012, MTBI has had 83 MTBI alumni completed a PhD in the

Mathematical sciences, and of these 79% were underrepresented minorities. As of May 2013, the MTBI program has been successful in training and mentoring students for 18 years, with at least 149 students graduating from higher education in the Mathematical sciences.

Research Questions

In this study, we focused our work with MTBI participants who eventually obtained their PhD, that is the subgroup of 101 doctoral recipients, at the time of writing this dissertation. There are two reasons for doing this: 1) the PhD group size is bigger than the 45 Masters students (and convenience too), and 2) these PhDs have continued further on their educational development and expanded on their educational and research experiences. We only looked at participants who were at some point new students, advanced students or mentored teaching assistants (including mentored graduate assistants) in the program.

Our investigation focus on characterizing the ways MTBI is developing these students to do applied mathematical research and their motivating factors or learning experiences to continue with higher education and training. Specifically, the goal of this study is to gain insights on:

1) What characteristics of MTBI, including relationships built in the program, helped undergraduate participants grow or change in their educational and professional careers?

2) What did participants learn in collaboration or creative imitation?

- **What did they learn in collaboration that helped them in their career beyond MTBI?**

To further understand the dynamics of learning and development that takes place during MTB, we used Vygotsky's Zone of Proximal Development (ZPD) on social learning and development, and in particular focused on Holzman's perspective. The ZPD is a theoretical framework of social learning and development introduced by Lev Vygotsky. According to Vygotsky, (1978, 1987) development based on collaboration and imitation is the source of all consciousness and development. We believe as Holzman (2010) does, that the ZPD is a process and an activity, and the focus is on the activity that people do together, as well as how the activity (learning environment) looks. In this case, we are seeing activity as ways of relating to people, not necessarily as events.

The Goal of the Study

My motivation for this study is to investigate the ways in which the MTBI program support students' professional and educational development. We argue that MTBI's ability to produce and help develop PhD students from less advantaged URM backgrounds can be attributed to the Zone of Proximal Development created as students learn to do research. Hence, this was our hypothesis as we started investigating the ways MTBI was creating a learning environment for its participants.

In this study, we see development similar to Vygotsky's metaphor of *becoming taller than himself* within the Zone of Proximal Development *(ZPD),* and Holzman's *methodology of becoming*. We define MTBI mentorship or development as a relationship between the overall program activities (as described in this paper) and the person, its relationship to Zone of Proximal Development (refers its relationship to collaboration and creative imitation), and relationship to informal and formal structural support. It is in this sense that we characterized the mentoring and development from MTBI. Here, we further define development as human development and not in its usual economic and psychological meaning (Fulani, 2013). Generally speaking, we see development as a creative activity where humans shape and change their relationship to themselves and others by becoming a *head taller* than them selves (Vygotsky, 1978, 1987, Holzman, 2010).

Castillo-Garsow, et. al. (2013), identified some factors that could potentially explain the success of the program, and hence participants' success and educational development. Factors such as the intellectual and cultural diversity of participants, the challenging homework sets; student directed research, the learning community and the role of returning participants. We also used other hypothesized factors from literature on undergraduate research experiences in the sciences to explore question one. We were interested from the perspective of the students and its relationship to the MTBI program, were these same factors important in their educational development? In our attempt to answer our questions on MTBI supporting student's educational development, we also asked two questions:

- **What learning experiences or factors motivated you to pursue graduate school in the Mathematical sciences?**
- **What learning experiences or factors contributed to your continuation and completion of a PhD?**

For the second question, we wanted to investigate what they learn in collaboration and/or in creative imitation in MTBI? We wanted to identify instances where students saw examples or experiences in relation to collaboration and creative imitation. These are two important characteristics in Vygotsky's and Holzman work on learning and development. We had a sub question where we also wanted to know what participants learn in collaboration that helped them in their career beyond MTBI? With this sub question, we wanted further investigate in what ways MTBI enhanced their development experiences and the type of choices they were able to accomplish or do after the program.

Investigating these questions is important because we will gain insights on how MTBI has supported this students but also because it also provide insights on what has helped in student development within the STEM field. This study will contribute to the literature on STEM, undergraduate research experiences (UR), development of undergraduates into applied mathematics. Other programs may want to launch their own mentorship program and they can use our findings to get a sense of some the factors and ways that have been useful in mentoring minority students' interest in the STEM field.

Organization of Dissertation Chapters

This study in organized in seven chapters. An introduction to our research question is provided in Chapter 1. The background overview on MTBI, and related Vygotskian-Holzman environments are presented in Chapter 2. Chapter 3 is divided into three sections. An overview of our theoretical work on the ZPD by Vygotsky and Holzman is presented in 3.1. A practical and theoretical overview is given in 3.2. A methods description is in 3.3 Results are presented in Chapter 4, 5, and 6. Results from online survey are presented in chapter 4 and 5. Results from interviews are presented in chapter 6. The discussion chapter in relation to our findings and concluding remarks are in Chapter 7.

CHAPTER 2

BACKGROUND

In this study, we examine MTBI as a developmental environment for undergraduate students. One core question then is: what aspects make MTBI developmental? We propose that MTBI learning community is one that resembles a Vygotskian-Holzman ZPD. Vygotsky's ZPD consist of collaboration and creative imitation. In this chapter we provide 1) a description of the history, environment and mentoring in MTBI, 2) an overview on learning environments which seemed to resemble Vygotskian-Holzman ZPD, 3) improvisation and its connections to the Zone of Proximal Development (ZPD) and 4) brief overview on undergraduate research experiences.

MTBI

The institute was held for six years at Cornell University from 1996 to 2002 then, it was transferred to Los Alamos National Laboratory for three years until 2005. After which it was moved to Arizona State University (ASU), where the institute now resides. The transfer of MTBI to ASU allowed the institute to expand and merge with another successful institute, the Arizona State University's Strengthening Understanding of Math and Science Institute (SUMS), and joined efforts in increasing the number of under-represented minorities in the Mathematical Sciences. Both MTBI and SUMS are part of the Simon A Levin Mathematical, Computational, Computation and Modeling Sciences Center (SAL-MCMSC). A center that focuses on establishing research and training across all disciplines, especially in providing solutions to problems in biology, environmental and social sciences (SAL-MCMSC, n.d.). In its current form, SUMS/MTBI is a mentorship program, which provides mentorship from the high school level and up to graduate school.

Structure of the MTBI Program

The MTBI program consists of two parts: learning mathematics in applied mathematics, and learning or doing applied mathematics research. For the first three weeks of the program, students attend lectures in mathematical biology, population genetics, stochastic processes, statistics, inverse problems, sensitivity analysis, ODEs and discrete models including. Students learn, for example, in the context of dynamical systems, stability methods such as phase plane, cobwebbing for discrete systems, and the next generator approach method, useful in calculating thresholds in epidemic systems. After lectures, students go to problem sessions where they get help to assist them in completing their daily homework. In the afternoons, students also attend computer lab sessions in Matlab, LaTex, and Mathematica. The program usually offers a review of linear algebra and probability for those students that may need it. There is also tutoring available until late hours of the night if students have questions about their homework.

In the first part of the summer session, students are encouraged to collaborate in groups in order to finish their homework and lab assignments. Otherwise, it is unlikely that the students will finish the amount of work assigned by just working individually given the limited time and the fast pace of the program. Students learn that collaboration among themselves is important if they are to finish the program successfully. As students go into the second part of the program, they must choose a group to work with on a topic and question of their own interest. It is a research question of their design, and it is not predetermined from a set of questions or from a pre-assigned list of problems.

A NSF website reports 62 associated programs, REUs in Mathematics, focused in various areas of mathematics including, algebra, discrete mathematics, mathematical biology, combinatorics, etc. Most summer programs that provide a research experience for undergraduates do it via a predetermined research question. Very few will give students the opportunity to initiate their own research project like MTBI. We found three, including MTBI, on the following website that give students the opportunity to initiate their own research project (National Science Foundation, n.d.).

Once groups have a research question, students must present their initial ideas in front of faculty, visiting professors, and postdoctoral and graduate students. The purpose of these meetings is to make sure that not only do the teams of students have a good research question and model, but that it can be done in a period of four weeks. Each group gets assigned a faculty advisor, and/or a graduate student mentor to assist them in their research experience. Students continue to work hard until late hours in the night to make sure they are making progress each day so that they can complete their project at the end of four weeks. In addition to attending lectures and working in groups, students attend weekly seminars where they are exposed to research in Mathematical Biology. Usually summer faculty, postdocs or visiting professors and some MTBI alumni attend these seminars. At the end of the program, students give an oral presentation of their work, submit a technical report, and prepare and present a poster prior to the final celebratory banquet.

During their MTBI participation, first year students, and MTBI faculty lived within close-living arrangements, which allowed them to be in continuous proximity to interact and work. Therefore, faculty are usually available to work with students at any time of the day. Everyday students and faculty have mentoring dinners. During these dinners there is an opportunity to discuss research and socialize within the overall MTBI community - made up of students, faculty and visiting faculty.

There are also planned outdoors activities for the students to have fun during the weekend besides doing work and research. For example, in Arizona, participants are usually taken to see the Grand Canyon and attend an Arizona baseball game. Sometimes these are mandatory activities. Otherwise they might not take a break or socialize beyond the work. We believe their lives become driven by a passionate determination to answer their research questions.

First year students also have a chance to return to MTBI to continue with their educational development. In fact, they can return for multiple years. The ones we classify as advanced students involves only returning undergraduates. Many return as graduate students serving as tutors, mentors, research and graduate assistants while working on their own or collaborative research. Returning students also attend regular lectures on mathematical biology topics. Returning students, regardless of the number of times they return, serve as supporting mentors to MTBI ``freshmen". The advanced and graduate students run computer labs, support problem-solving sessions on homework or related topics, tutor participants and grade assignments and labs; they also participate in research and typically co-author a technical report every summer. The purpose of the homework is to reinforce mathematical concepts. If students get problems from the assignment incorrect, they must redo them until they are understood well. If there is a topic that they find interesting, returning undergraduate students form groups with the first year students, or work on their own research project. Graduate students in the program are expected to work on their own research project and present their research the end of the program.

The continued recruitment of a few past participants has allowed this selected group of students to attend the program in different capacities. They have played multiple roles. We believe this has been beneficial to their educational careers. A nontrivial number of participants have come in as new undergraduate participants, advanced students, graduate students, and junior faculty mentor.

In summary, MTBI brings students to an environment where they are encouraged to work together and solve a question of interest to them. All involved in the program, including new participants, returning participants, and faculty benefit from the experience as can be seen by the quality of the technical reports (MTBI technical repots, n.d.), which have generated a large number of publications in referred journals.

National Recognition of the Program

A measure of the program's success in mentoring minority students has been its national recognition by the U.S. government. In 1997, the program Director Carlos Castillo Chavez was recognized with the presidential award in Science Mathematics and Engineering Mentoring. More recently in 2011, MTBI was recognized because of their commitment to higher education and student mentorship. The White House announced "through their commitment to education and innovation, these individuals and organizations are playing a crucial role in the development of our 21st century work force. Our nation owes them a debt of gratitude for helping ensure that America remains the global leader in science and engineering for years to come" (ASU News, 2011)

It is worthy to note that the previous program SUMS director, Joaquin Bustoz Jr., as well as the institute SUMS itself also received the Presidential Award in Science Mathematics and Engineering Mentoring, in 1996, and 2003, respectively. SUMS/MTBI together have received 4 presidential awards in recognition to their work and mentoring of underrepresented minorities in Science Technology Engineering and Mathematics (STEM).

Summer program alumni have the opportunity to present their research findings at different national conferences, such as the Mathematical Association of America and the American Mathematical Society (AMS/MAA) annual meeting or the Society for Advancement of Chicanos and Native Americans in Science (SACNAS). From summer 1998 to May 2013, MTBI students have received a total of 51 national award recognitions. There have been 22 awards given to students for their poster presentations at the join meetings at the Mathematical Association of America and the American Mathematical Society (AMS/MAA) annual meetings, 14 at the Society for Advancement of Chicanos and Native Americans in Science (SACNAS), and 14 at several other Mathematical Science research or symposium organizations (SAL-MCMSC, n.d.) .

The success of the program and the structure and learning community has been previously documented (Castillo-Chavez, C., and Castillo-Garsow, C., 2007; Castillo-Garsow, Castillo Chavez & Woodley, 2013, Camacho, et. al, 2013, Castillo-Garsow, and Castillo-Chavez, 2015 forthcoming). Camacho et. al (2013) provided an analysis of MTBI successful mentoring through its research, its devoted faculty, and returning and graduate participants. The authors describe the program as a multi level mentoring, where undergraduate or post-baccalaureates students are given the opportunity to participate in MTBI and then return for one or more years, as well as mentor while encourage to graduate school. Graduate students also receive help on their dissertation projects while getting the opportunity to mentor, first year or returning participants. Junior faculties receive professional training in Mathematical Biology, and in the mentoring model.

The mentorship model involves sequential research experiences, which reinforce students' academic and research exposure. Camacho et. al (2013), specifically analyzed the type of projects generated via student driven research questions, and found that some of the most common main research topics coming are: ecology, infectious diseases, wild life diseases, epidemiology and social dynamics.

JBMSHP and Need to Study Other Components in SAL-MCMSC

In a 2012 case study, Escontrias explored the experiences and interests of seven undergraduate Latina and Latino Arizona state students attending the Joaquin Bustoz Math Science Honors Program (JBMSHP) in the Mathematical sciences, and how the program has enhanced their math and science interests. Escontrias found that K-12 educational experiences and family's support and higher education influence were the two factors that kept their interest in the mathematical sciences. Secondly, Escontrias also found that after JBMSHP students better defined their educational objectives with all but one continuing on to a STEM related field. The program positively enhanced their lives and gave them the opportunity to be part of a community through the rigorous courses taken during the summer. The program also better prepared them for college life. The program's continued support through communication provided them with the help needed to get financial support, while providing academic year round tutoring.

Escontrias (2012) pointed out that further investigation was needed to determine how the Simon A. Levin- Mathematical, Computational & Modeling Sciences Center (SAL-MCMSC) at ASU might positively enhance the Math and Sciences experiences from high school up to doctoral degree. Other components of the center to investigate include its undergraduate degree in Applied Mathematics in the Life and Social Sciences, the research experiences through MTBI, and the graduate degree in Applied Mathematics for the life Sciences. Of interest to us is the role of SUMS and MTBI and their positive effects in increasing the number of STEM students. As of Summer 2013, and over its 28 years of existence, SUMS had mentored 2,534 students through its Joaquin Bustoz Math Science Honors Program (JBMSHP), with 75% under-represented minorities, that is, African American, Hispanics and Native American (JB-MSHP, 2013*)*. For this study, we are concentrating exclusively on MTBI's role in mentoring undergraduates that not only continue into graduate school but that also earned a PhD. Our goal is to add insight into the beneficial mentoring practices in MTBI by looking at it through the responses of PhD recipients who had participated as undergraduates and graduate students.

We next provide a picture of the demographic data from the participants in the program from the summer 1996 to summer 2013. The picture that we provide is limited since, the data maintained on past participants were limited. We also provide empirical data on student success in the program, using the number of Masters and PhD degrees completed after MTBI as a key measure.

Demographics and Academic Achievements of Students in MTBI From 1996 – 2013

We counted the total number of participants, ethnicity, citizenship and gender over from 1996-2013. As of the Summer 2013, there have been a total of 402 first time (or regular) participants to the MTBI program. There have been 82 returning students. Some return for a second and third time. There has been 141 TAs, which in most cases are also returning students from previous years or graduate students. The numbers of returning or TAs may be less given that sometimes the same student returned multiple times over the 17 years. In other words, the 141 number overestimates the number of distinct graduate students.

Out of the 402 first time students, 50% were female, and 50% were males. From the Advanced group, 41% were females, and 59% were males. From the TA group, 42% were females and 58% were males. From first time participants, there has been an equal number of participants from either gender. But for the advanced students and TA there was 16% more male participation than females (*Figure 1*).

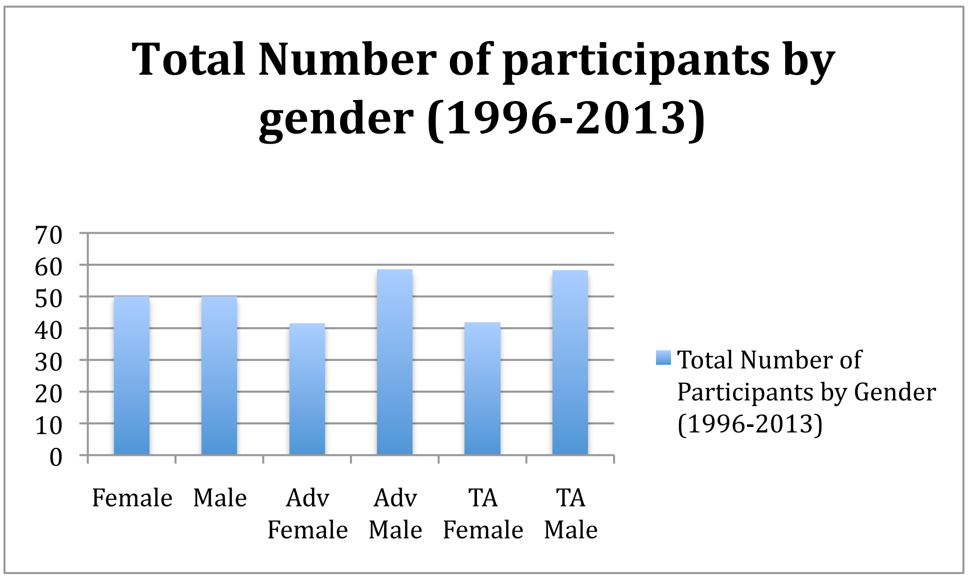

Figure 1. General Statistics on Total Number of Participants by Gender in MTBI 1996-2013

Table 1 has URM information on all U.S. and International MTBI participants across type, where *reg* stands for regular, *adv* for advanced, and *TA* for teaching assistant.

We can see that on average MTBI accepts 13.61 URM students per year since 1996. Advanced students have 1.89 URM per cohort year. TAs have 9.79 URM per cohort year.

Table 1

URM in MTBI from 1996 to 2013 by Gender Female (F), Male (M) Across Type.

	Reg U.S		**Reg Int'l**			**Adv U.S**		**Adv Int'l**			**TA U.S**		**TA Int'l**		
URM	F	M	F	M	**Total**	F	M	F	M	**Total**	F	M	F	M	**Total**
1996	16	14	1	5	**30**	2	4	0	0	**6**	0	1	0	0	**1**
1997	9	8	4	4	**17**	1	4	0	1	**5**	0	0	0	0	**0**
1998	7	6	5	2	**13**	2	5	0	2	**7**	1	0	0	0	**1**
1999	8	9	3	4	**17**	1	2	0	0	**3**	0	2	0	0	**2**
2000	11	6	2	2	**17**	0	1	0	2	**1**	2	0	0	0	**2**
2001	12	9	0	1	**21**	3	1	2	0	**4**	0	1	0	2	**1**
2002	9	7	2	1	**16**	2	3	0	0	**5**	1	3	0	1	**4**
2003	5	7	1	0	**12**	2	3	0	0	**5**	3	2	0	1	**5**
2004	0	10	1	1	**10**	3	1	0	0	**4**	7	5	0	2	**12**
2005	6	7	0	1	**13**	0	1	0	0	**1**	2	3	0	2	**5**
2006	3	6	1	2	**9**	0	1	0	1	**1**	3	4	0	1	**7**
2007	4	7	2	1	**11**	2	2	0	1	**4**	2	6	0	1	**8**
2008	7	2	1	1	**9**	0	2	0	0	**2**	3	3	0	1	**6**
2009	1	4	0	0	**5**	1	1	1	0	**2**	4	8	0	1	**12**
2010	5	5	0	2	**10**	1	0	0	0	**1**	3	6	0	1	**9**
2011	3	7	0	2	**10**	1	1	0	0	**2**	3	5	0	0	**8**
2012	7	7	0	2	**14**	0	2	1	0	**2**	2	4	0	1	**6**
2013	6	5	0	0	**11**	0	0	0	0	**0**	2	2	0	0	**4**
Total	119	126	23	31	**245**	21	34	4	7	**55**	38	55	0	14	**93**
AVE	6.61	7			13.61	1.17	1.89			1.89					9.79

Of 402 participants, 343 were U.S citizens or residents. Of the 82 advanced students, 71 were U.S citizens or residents. Of 141 TAs, 107 were U.S. citizens or residents. Using these numbers and table 1, we see that 71.43 % (or 245/343) of U.S. participants were URM. URM were 77.47% (or 55/71) of advanced students, and 86.92% (93/107) of TAs. URM were mostly male across all groups, even for the cohort whose incoming students were 50% female and male. (*Figure 2* below)

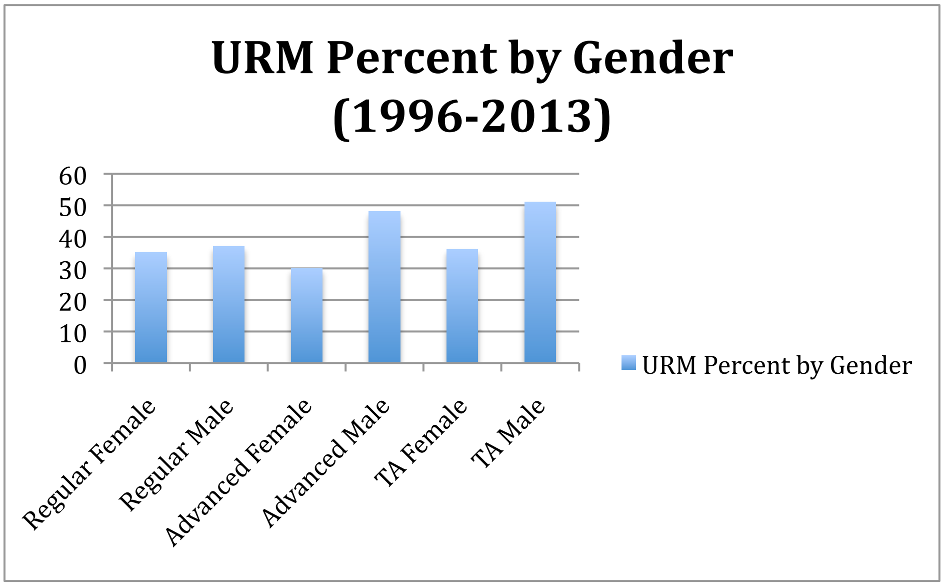

Figure 2. URM Percent by Gender in MTBI 1996-2013

The total number of URM numbers per cohort is given in *Figure 3*, which shows 1996 with the highest number of URM. This is expected given that all participants from 1996 cohort year were from Hispanic backgrounds. The year 2009 has the lowest number. This is also the smallest class across the years, with only 10 first time participants.

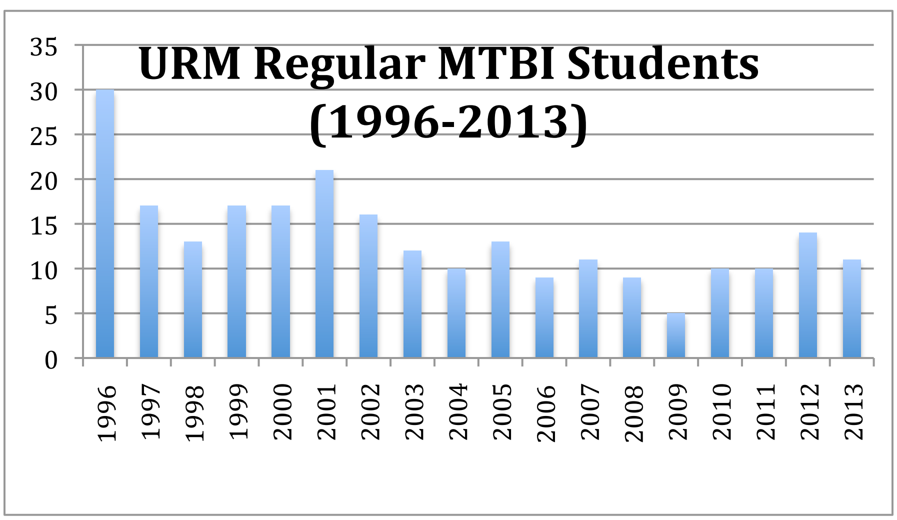

Figure 3. URM Regular MTBI Students from 1996 -2013

Masters and PhDs in MTBI 1996-2013

This section contains more detailed information on demographics for those participants who competed a master and PhD degrees. We only looked at those students who had already completed their degrees. The numbers do not capture students who may have entered graduate school but not yet completed their degrees. Out of the 402 first-time students, at least 45 past participants earned a Masters degree and 102 earned a PhD degree *(Table 2* below*)*.

Table 2

All Masters and PhDs per cohort year (1996 -2010)

MTBI year	**Masters**	**PhD**	**Total**
1996	7	10	17
1997	5	7	12
1998	3	5	8
1999	5	8	13
2000	2	9	11
2001	4	14	18
2002	2	11	13
2003	3	4	7
2004	6	5	11
2005	4	15	19
2006	2	5	7
2007	2	4	6
2008	0	0	0
2009	0	3	3
2010	0	2	2
Total	45	102	147
Average/year	**3.00**	**6.80**	**9.80**

From the available information, all of the 45 students who earned a master degree were U.S. citizens, while 84 of the students who earned a PhD were U.S. residents or citizens. From the 45 who earned a Masters degree, 26 were female and 19 were male.

From the 84 U.S. PhDs, 44 were females and 40 were males.

On average from 1996 to 2010, there have been about 3 students earning a Masters and 6.80 students earning a PhD per cohort year. In total, there is an average of 9.80 students earning a graduate education degree per year (T*able 2* on previous page). MTBI accepts an average of 22 students per cohort year. This seems to indicate that close to 45% of participants per each cohort end up completing their degree.

Numbers of PhDs in MTBI

There were 18 International PhD students, 10 (or 56%) of these were females and 8 (or 44%) were males. From the U.S. PhD group, 52% were female, and 48% were males. In *figure 4* we show a graphical representation of the same numbers of PhDs and Masters as in *Table 2.*

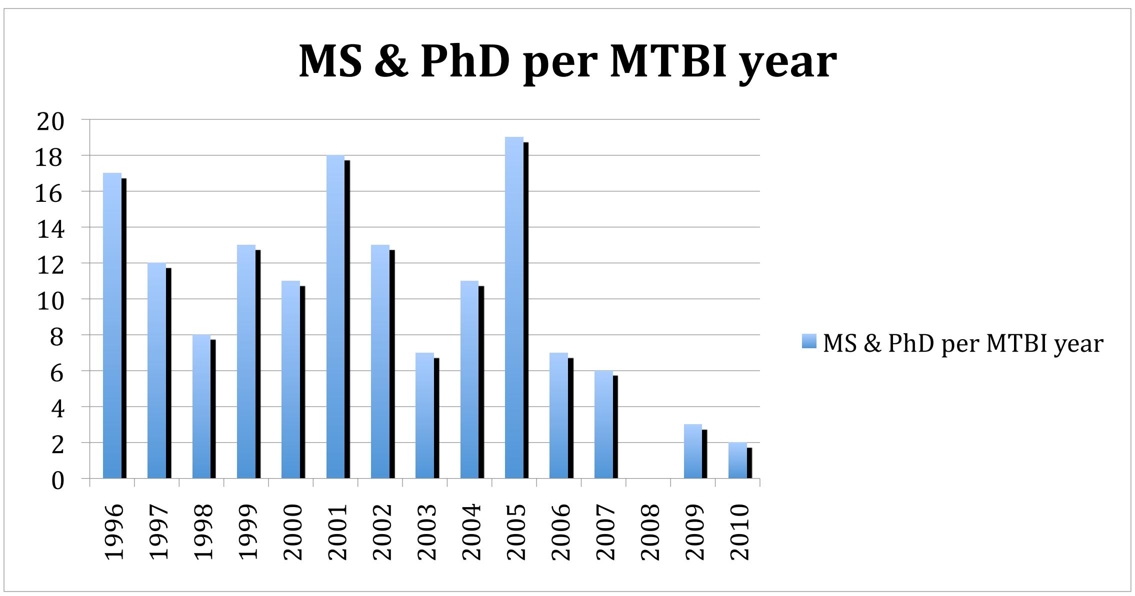

Figure 4. MS and PhD per MTBI Cohort Year (Including U.S. residents or citizens and International participants)

In *figure 5*, we show a graphical representation of the number of all PhDs per cohort year, including U.S. and international. The years 2001 and 2005 had the greater amount of students completing their PhD, with 13.73% and 14.71%, respectively. The next MTBI cohort with a relatively high percent of students earning a higher education degree was in the 2002 with 10.78% PhD. Given the students from the first years of the program were undergraduates, the first PhDs were not awarded until 4 to 5 years after they had attended the program. The first PhD awarded to an MTBI alumni was given in 2003.

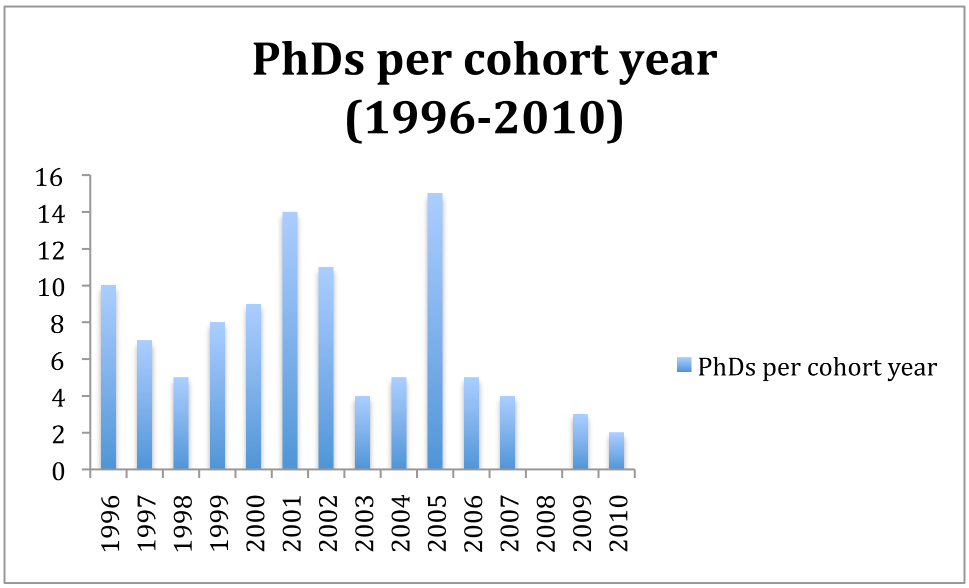

Figure 5. The Number of All PhDs per Cohort Year

A Theoretical Model on Cooperative Learning and MTBI

Crisosto, Kris-Zaleta, Castillo-Chavez and Wirkus (2010) created a theoretical mathematical model of cooperative learning (we are using the word cooperative learning and collaborating interchangeably) to investigate its role or impact on a group of community of learners, as well as to investigate what role individuals have on communities that value collaborative learning. The authors studied the mechanism for the growth and persistence of large cooperative learning communities.

In their model, using the concepts of evolutionary biology as a metaphor, it was assumed that collaborative learning would not only increase the "fitness" (or intellectual development) of the group but it could also improve the fitness of the individual member in the group, a perspective consistent with the Vygotskian views on learning, that is, things are first social then individual. Interestingly, this paper was the brainchild of a student who participated in MTBI a few years before. This theoretical work investigated the sustainability of programs involving a community of learners that value and practice collaboration.

The mathematical results showed that the establishment of a cooperative learning culture depended on the initial investment of highly cooperative individuals. If the highly cooperative individuals were mentoring and increasing the number of novices and moderately collaborative participants engaged in a collaborative culture, then a learning community would be established where all participants benefited from each other. Otherwise, if not enough mentors from the highly cooperative class were available or if recruiting from moderately collaborative learning individuals was weak, then a collaborative culture could not be established.

MTBI was cited as an empirical example where collaborative learning was value, and where a key number of MTBI mentors and experienced participants had been responsible for the survival and success of the MTBI program. The authors explained that at the beginning of each program it is not known how likely new participants are willing to be part of the learning community, but the return of core mentors allows for the survival of MTBI and its culture of collaborative learning. In fact, there are faculty and students who have been associated with MTBI since its beginning. At least three faculty have continuously been involved with the program. One faculty member has missed only 1 year of the summer programs. This empirical reference in relation to the core collaborators is consistent with the hypothesis of the model.

For this research, we are focusing on PhDs that are MTBI alumni and community members. The alumni may or may not be cooperative learners or experienced mentors. The criteria was based on their participation in MTBI and their completion of a PhD. This project investigates via their reflections the role of MTBI in their careers.

Vygotskian-Holzman Development: Creating Growthful Performances

MTBI, a learning community that values cooperative work, has established learning environments that seem to resemble a Vygotskian-Holzman ZPD. The following learning environments along with relevant ideas of development and performance provide examples of how such learning environments foster individual growth.

In its 2011 report, the All Stars argued that one way to resolve the education crisis, with children failing in school, is for everyone in the school system (teachers, administrators) and parents to pretend that all students are good learners and high achievers (Fulani and Newman, 2011). Their emphasis is on learning that you can learn. According to the authors, "poor children fail in school because they are culturally, emotionally and experientially underdeveloped. Learning takes development and if kids are underdeveloped, they do not become learners" (p. 2). The All Stars is an out school developmental program for poor communities. It is mainly composed of professionals who volunteer with the community (Gordon, Bowman, and Mejia, 2003). According to their website their mission is to “transform the lives of youth and poor communities using the developmental power of performance, in partnership with caring adults” (All Starts Project, n.d.).

Newman and Fulani (2011) argue that children in the classroom do not do well because there is a “development gap” between these children who are failing and the ones who have an opportunity to experience the world, such as travel or getting to know different cultures. She argues that development is not an issue addressed by schools, achievement is. Children do not learn that they can learn. They are taught instead that they can not learn and therefore fail to learn. Fulani (2013) attributed this underdevelopment to being a by-product of poverty. Similar to the problem that occurred with the ending of slavery in America, Fulani (2013) cites the sociologist historian W.E.B Dubois. The problem of twentieth century was the problem of the color line. She observes that the problem of the twenty-first century is the “problem of the development line”. Development in the sense of human development, where individuals have the ability to make choices and shape their circumstances, and become learners. Unfortunately, the education gap in achievement between rich and poor children has grown (Tavernise, 2012).

This problem of the development gap is also reflected in the low numbers of underrepresented minorities in STEM. Minorities do not always see that they are able to be successful in STEM or they do not see how to be successful in STEM fields. This inability to see possibility can be seen as a development gap. In this study, we focus on how MTBI supports the development of its participants who are predominantly minorities and often raised in economically disadvantaged communities.

The All Stars program attempts to fight poverty, which is about 15.9% of the U.S. total population, and its subsequent underdevelopment by broadening children's experiences about the world outside of school. The program introduces them to new areas of New York given that the participants rarely travel outside their poor neighborhoods. These students are also introduced to the "cosmopolitan world" such as the arts, travel or foreign languages. The youth will experience work and cultural environments that they had never been exposed to, or were beyond their life experiences. They are supported to "perform" as part of the environment. In fact, they are explicitly taught what the performance of different social settings are and asked to perform or pretend. This pretending is such an important intervention when young children fail in school or to learn because the intervention is supports human development, their creativity and appreciation to become learners. It is also creates an opportunity to reflect on who they are in the world, and to reflect on the their choices for the growth they want in their lives (Fulani, 2013).

In this way, young children have been exposed to theater, performing arts or leadership training (with corporate professionals) to further develop their talents and passions. Holzman (2009) indicates that these out of school programs such as theatre, performing arts have been found to be beneficial in helping the youth develop emotionally, socially, culturally, intellectually and in creating a positive self worth perception of themselves (Holzman, 2009).

Children who participate on the All Stars are not being tested for specific learning outcomes, but more for children's ability to perform (or pretend) as capable learners. The All Stars focus on development, not acquisitional learning. When children participate in this type of environment, where they are seen as learners even before they know that they know how to learn, it allows them to engage with others, and to appreciate learning.

"If you can perform on stage, you can perform in life"- is a mantra Pamela Lewis, vice president for Youth Program in the All Stars, follows as she tries to help young people develop in their educational and personal growth. They learn that there are things beyond the experiences available in their poor communities, and that they are capable of learning and participating in them.

Newman and Fulani (2001), advocated for learning and development outside of a classroom settings, given that within the school systems, most educators are pressured to make sure kids do well on tests, or satisfy other school requirements. The All Stars started in 1981, and the understanding of how the program supported development came after the programs were created. The All Stars say that the reason they are successful is that they commit to relating to the students as capable learners. Other projects like Harlem Children's Zone (HCZ), the Knowledge is Power Program (KIPP) schools, Eagles Academies, and other charter schools (Newman and Fulani 2011) have also created programs where individuals are related to as capable learners. Inspired by this community approach, in 2010, the U.S. government also launch a program called the *Promise Neighborhood* program to improve education and fight poverty (Department of Education, 2010).

Development and Performing

Lois Holzman uses the theoretical understanding of Vygotsky to look at and advance the work of social therapy, the All Stars programs, and a professional development company. Holzman together with Newman co-founded the East Side

Institute, an “international educational, training and research center for developing and promoting alternative and radically humanistic approaches in psychology, therapy, education and community building” (East Side Institute, n.d.). Holzman and Newman are one of the first psychotherapists to use Vygotsky to further understand human development as performance or improvisation. Holzman (2010) argued that performing creates new ways of seeing and being. She states that,

> “Performing—creating who we are by performing who we are not –is ‘a new way to be’ that does not prioritize thought (or perception) over action, cognition over emotion, or being over becoming. It is activity that is social, communal, reflexive and reconstructive, wherein lies its potential for qualitative socio-cultural transformation” (Holzman, 2010, p. 115).

Social Therapeutics was originally developed by Fred Newman, in which he saw it as

> “a radical psychotherapy in the 1970s, it has evolved into a practical human development methodology with broad applications across the life span, cultures and environments” (Social Therapeutics, n.d.).

Social therapeutics understands or sees people as performers and life as improvisational. Importantly, it sees human development and learning as first social and then individual. Hence performance is an activity through which people create their lives and transform. Holzman states,

> "human development is an activity and a method that advances because the participants are 'co-builders of new practices'.

In trying to understand development, we see individuals as performers, where they can perform a collectively create a “scene."

Improvisation and Creativity: Characteristics of ZPD

Improvisation and performance in adult learning and communities is relatively recent area of study (Goffman, 1956; Martin, Towers, and Pieri, 2006). This model of development has been used in therapy, education, outside school programs, corporate organizations (Holzman, 2009, Young, 2013), computational science (Holmes, 2010), and medical schools (Watson, 2011). Practitioners report transformation in which individuals grow and develop collectively and individually. This is because through improvisation supportive learning environments (a physical, emotional, and temporal environment) can be created where individuals can learn, grow, and accomplish something that they did not know how to do at the beginning. Furthermore, individuals come to understand that they are learners and are capable of "changing", "growing", and doing new things (Vygotsky, 1978; Lobman, Carrie, and Lundquist, 2007; Holzman, 2010). In many arenas, creativity research, education, business youth and community development, we are seeing that improvisational activity and improvisational theater offer new ways of seeing and doing our professional lives (Sawyer, 2004; Barrett, 1998; Farmer, 2008; Gordon, Bowman, & Mejia, 2003).

Holmes (2011) introduced improvisational theater to a group of computational scientists. Through workshops students and faculty learned to listen to each other, to contribute to the ongoing conversation and discuss how to improvise and be creative in their computational biology field. Holmes reported that "improvisation creates a performance that is collaborative and unscripted" and allows for creativity. One narrow definition is the "ability to produce numerous or novel suggestions". Using a pre and post survey, Holmes found that on average participants did not show a significant change in their view of individual creativity or collaborative work, but some recognized the value of creativity, communication and the environment when using improvisation in their field. However, through interviews Hug and Holmes (2012) saw that the workshops gave the computing scientists a chance to see dialogues and interacting in a new way as they learned to perform. These studies informed our thinking and suggest that people in MTBI are working collaboratively without a script, especially in the research projects where people may not know how to translate their research interest into a mathematical model

We believe these improvisational practices characterize what we see in MTBI participants as they experience research and become learners of applied mathematics. This understanding and belief sets a foundation for the research we present here.

Undergraduate Research Experiences in the Sciences

Our work focuses on characterizing the ZPD created as students learn to do mathematical research in summer programs. We provide a brief overview of related literature on research experiences in science and similar to MTBI.

Studies indicate that student development and advancement in science is one of the benefits of undergraduate research experiences (Hunter, Laursen and Seymour, 2006; Laursen, Newman, Tran and Chang, 2010; Caudill, Lester, et al., 2010; Seymour, Hunter, 2012; Hurtado, Castillo-Garsow and Castillo-Chavez, 2015 forthcoming). Research that exposes or encourages students in interdisciplinary research such mathematics and biology also positively benefits the students (Duncan, Bishop, and Lenhart, 2010; Feser, Vasaly, Herrera, 2013). Although it can be beneficial for students and faculty, it can also create tensions for faculty in managing their teaching and research responsibilities and personal lives (Seymour, Hunter, 2012).

In a 2006 study, Hunter, Laursen and Seymour provided a comparative study about student and faculty's perspectives on the type of gains students obtain from participating in an undergraduate science research programs. The study involved 76 undergraduate students, 80 faculty and administrator members doing research at four liberal arts colleges. Authors showed that students became participants in undergraduate research (UR) and as a whole became part of a community of practice, where there is mutual engagement, participation, and support from an expert.

Overall, both students and faculty agreed that UR experiences were highly beneficial for students, with positive gains in cognitive and personal growth. There were four areas where gains were reported: profound personal experience about the growth of their own identities, careers, and professional development in becoming scientists; clarification for their career and school intentions (whether to continue or change career paths); other gains were in communication, laboratory skills, work collaboration, and time management. Interestingly, students only saw their own personal and professional growth without realizing they were becoming scientists, part of a community of science professionals, which is how faculty viewed them. This inspired us to wonder if we would find similar results when surveying the MTBI participants.

Tsui (2007) calls for researchers and evaluators to investigate what strategies are effective for underrepresented minorities, as their needs are different than other groups. In another study, Hurtado, Newman, Tran and Chang (2010) presented insights from a national project on the factors that contribute to the retention of science oriented students from underrepresented groups in college to degree completion and advancement to graduate studies. These factors were: adjusting to social and academic environment, student involvement of undergraduate science research, persistence through a supportive institution and aspiration to go to graduate school that would help in the overall development of their science identities.   Another factor was how collaborative or competitive undergraduate college settings empowered (inspired to do better) or could disempowered (negative environment) students. Financial need was another factor affecting their academic choices. The authors identified the factors in improving the rate of success of underrepresented students in the sciences, and argued that further research was needed on this factors and issues. Our goal was to further investigate the role of research experiences and collaboration.

In 1997, Smith, Haarer and Confrey published a paper on mathematical learning via an effort at the interface of mathematics and biology. The authors investigated how two different groups, one of applied mathematicians and a second of biologist come to understand and model dynamic population models within their own practices. Two groups from a biological modeling class were observed and interviewed as they created a mathematical model, and the perspectives of a mathematician and a biologist were used to analyze their modeling process. The authors wanted to investigate the final mathematical model and the interactions and resources used between people from both practices when they got together to work on a mathematics biology project.

The authors discussed that models will not necessarily represent reality, but the theoretical models can still be helpful within the practice of biology. Both the mathematician and the biologist agreed that models can represent to some extent the dynamics and biology, modeling allows experimenting with different possible scenarios, and advancing the biological knowledge within a system. The authors concluded that this was dialectical relationship that made it relevant for the mathematics and biology students because knowledge grew form their own knowledge base.

Our work focuses less on the specific conceptual models learned than on the experience of learning, collaboration and development. This work specifically contributes to understanding the characteristics and dynamics of the MTBI summer research environment in which students from poor communities and communities of color develop as professionals in the Mathematical Sciences.

CHAPTER 3

THEORETICAL PERSPECTIVE

3.1 Theoretical Perspective

In this chapter, we describe the Theoretical perspectives used for the study. We choose two social theoretical learning lenses from which to analyze the extent of collaboration, creativity, and the developmental growth of the participants in the program. We use Vygotsky's work on the Proximal Development (ZPD) and Holzman's work on *methodology of becoming*, a perspective on human development based on Vygotsky's work. Next we provide a brief review on: a previous theoretical view of MTBI that sets a foundation for our work, and views of Vygotsky that help contextualize our work.

A Preliminary Theoretical Analysis on MTBI

In Castillo-Garsow et. al (2013)'s preliminary theoretical analysis of the MTBI program the researchers used two learning theories and a behavioral theory when investigating the success of the program.

The authors first used Piaget to examine the role of individuals understanding within the MTBI learning community. The authors explained that individuals come with different cultural and educational backgrounds and understandings, and given the challenging homework sets and research questions, those understandings might fail. However, through the conversation and collaboration with the learning community the individuals can advance their understanding beyond what they would do individually.

The authors explained that Bandura's work from a behavioral standpoint could explain the behavioral changes of MTBI participants, as well as their academic success. First, they described that MTBI builds pro-social behavior because they genuinely need each other to accomplish certain activities, such as selected challenging assignments. They explained, one of MTBI's goals is the recruitment and retention of graduate students into the mathematical sciences. When students can work on challenging homework problems, self selected and self directed research projects, with some help from mentors, the students feel accomplished, and self efficient such that they set more challenging goals, including publishing papers or writing a dissertation.

The authors used Vygotsky's work to analyze the human interactions and collaboration the program. Citing Vygotsky's work on the Zone of Proximal Development and the importance of working in collaboration with their peers and faculty mentors, the authors also argued that the challenging homework problems and research projects allowed individuals to become members of an existing society. The authors define the Zone of Proximal Development as the difference in the level of development of a student working on their own compared to working with assistance. In this interpretation of the ZPD, the instructor assists the students to achieve a level of development beyond what they would on their own.

Vygotsky's Work on the Zone of Proximal Development

Vygotsky (1978, 1987) studied how children develop. We believe his understanding on social and individual learning are visible in MTBI as indicated by Castillo-Garsow et al (2013). We now provide a brief overview of the two theoretical lenses from which we analyzed the ways MTBI is mentoring students.

According to Russian psychologist Lev Vygotsky (1978, 1987) acquisition of knowledge and development happens at the social level first, and then at the individual level. We can see this when he stated the following about development:

> "Every function in the child's cultural development appears twice: first on the social level and later on the individual level; first between people (interpsychological) and then inside the child (intrapsychological). This applies equally to all voluntary attention, to logical memory, and to the formation of concepts. All the higher mental functions originate as actual relations between people" (Vygotsky, 1978, p. 57).

According to Vygotsky (1987), the meaning of a concept or word develops. They are not taken in complete form when first introduced to a person. He reports that there are different forms (spontaneous and scientific) of development when learning a new concept. Vygotsky developed these concepts and the ZPD theoretical framework when he worked with young children as they learned to speak with adults. However, our focus is not on this learning and development of concepts but rather human development. One common interpretation among researchers is the attention given to the developmental transition from what a child can do in collaboration with a teacher or an adult compared to what a child can do without help or guidance. Vygotsky stated that

> "Development based on collaboration and imitation is the source of all the specifically human characteristic of consciousness that develop in the child. Development based on instruction is a fundamental fact. Therefore, a central feature for the psychological study of instruction is the analysis of the child's potential to raise himself to a higher intellectual level of development through collaboration, to move from what he has to what he does not have through imitation" (p. 210).

Hence, collaboration and imitation are key characteristics in helping a child develop intellectually, and to accomplish and/or learn things he has not done yet.

Within this interpretation of the Zone of Proximal Development, a developmental transition happens through collaboration and imitation with the help of an adult or instructor. For example, Vygotsky described that with assistance through demonstration, leading questions or initial elements on how to solve a task, an eight –year old child can solve a problem characteristically for a nine or 12-year old. Furthermore, he stated, "what the child is able to do in collaboration today he will be able to do independently tomorrow" (p. 211).

Vygotsky stated that there is a difference between intellectual meaningful imitation vs. imitation as blindly imitating without any without structure. According to Vygotsky (1987), "the child can imitate only what lies within his zone of his own intellectual potential...to imitate, there must be some possibility from what I can do to what I cannot" (p. 209). Hence, there must be potential for creative intellectual imitation.

The ZPD and Holzman's Methodology of Becoming

Educational researchers, psychologist, among others can be said to interpret the Zone of Proximal Development into three main categories of development: Individual, dyadic and collective (Holzman, 2009). The ZPD has been interpreted as an individual characteristic of a child's development, where the ZPD is seen as "being –or producing -- a measure of a child's potential" development. The ZPD is also interpreted as a dyadic form of development where learning happens socially and usually between two people, and in most cases it is assumed that it happens between a child or novice and an adult or expert. In its third interpretation, the ZPD is also seen as collective form of development, where learning is not necessarily between two people but among a group of people who are working together.

Influenced by Vygotsky's work on the ZPD, Holzman interprets the ZPD and sees it as a "process rather than as a spatio-temporal entity, and as an activity rather than a zone, space or distance. Furthermore, I offer the ZPD activity as the simultaneous creating of the zone (environment) and what is created (learning –leading-development)" (p. 6). Holzman interprets the ZPD as a process and an activity that creates a learning environment where learning will lead development, similar to Vygotsky (1978)' s belief that a useful instruction needs to "lead development". Further expanding on the ZPD, Holzman (2010) defined development as

> "[The] practice of a methodology of becoming (*a head taller than himself*) – in which people shape and reshape their relationships to themselves, each other and to the material and psychological tools and objects of their world" (p. 7).

In this process of becoming "a head a taller than himself", Holzman is referring to Vygotsky's position that "in play a child always behaves beyond his average age, above his daily behavior; in play it is as though he were taller than himself". Both Vygotsky and Holzman believed that a child is capable of behaving (or acting, doing) beyond what she or he is currently capable of doing.

It is through activity that both the creation of a developmental environment and development happens which according to Holzman defines the *methodology of becoming* (Holzman, 2009, 2010). In this context of trying to understand human development, the activity is seen as a "search for a method" that "generates both a tool-and-result at the same time and as continuous process" (Holzman, 2009, p. 9). It is a theory of "what is becoming ", and not just "what is" the state of being. What is becoming, includes the state of “what is”. Holzman (2009) added that it is in this way social “activity is a cultural-historical phenomenon that emerges and transforms [...]. It is how human beings transform the existing circumstances, develop as individuals and as species, and create culture” (Holzman, p. 16). Using this methodology, Newman and Holzman interprets Vygotsky’s work with children as the simultaneous creation of developmental environment and development (this tool-and-results), in which caregivers provide an environment where kids are related as speakers, children speak before they know how to, and as a result development happens (children become speakers).

In the methodology of becoming, imitation and completion are two important concepts in creating the ZPD. In this case, imitation (or creative imitation) goes beyond the act of mimicking. It is a social relational activity in it self. It creates in collaboration, in social units, the ZPD by imitating creatively beyond what is currently possible. Besides imitation, there is also an act of completion that helps create a ZPD. Completion is the social process of thinking and speaking as one activity, speaking-thinking, where speaking completes thinking.

Holzman (2010) further explains Vygotsky making a distinction between “play” ZPD and learning instructing ZPD, where imagination and pretend become important characteristics in play, and therefore a better developmental process than direct instruction. However, Holzman believes that both play and instruction are needed in learning and creating a ZPD. She did not like that,

> “Schooling transforms not knowing into a deficit; creative imitation into individualized accomplishments, rote learning and testing, and completion into correction and completion” (p. 15) .

Holzman is an advocate for play to become center stage in schools, so that individuals can create a collective form of working together (or ZPD); so that individuals are freed from knowing (not expected to know everything); they are creative imitators and being completed by their social units.

Holzman also interprets play as,

> "[Play] as both appropriating culture and creating culture, a performing of who we are becoming (Newman and Holzman, 1993; Holzman, 1997, 2009). I see creative imitation as a type of performance. When they are playing with language very young children are simultaneously performing – becoming- themselves. In the theatrical sense of the word, performing is a way of taking "who we are" and creating something new- in this case a newly emerging speaker, on the stage a newly emerging character, in an outside school program a skilled dancer or athlete –through incorporating 'the other' " (p. 17).

This interpretation and concept of becoming is observed in Vygotsky(1987)'s work on "The development of scientific Concepts in Childhood", where young children and their caretaker together create a developmental environment where the young children do not how to speak but are treated as if they are, and in this process the young children are both learning how to speak and becoming speakers in the process through the ZPD.

For our project, we see MTBI undergraduate students performing and becoming "newly emerging" applied mathematics researchers. We use the ZPD interpretation where the collective form of development is among a group of people. Nevertheless, we argue that regardless what interpretation you are using from the ZPD, in the end, the head taller than themselves metaphor is always present, which allows us to see people's potential growth and development through their social learning and relational activities with people or events.

ZPD as an Improvisational Activity

In this study we are not using ZPD as teaching technique for the sake of learning a new concept or a mathematics concepts but as a creative, and improvisational activity (Lobman, Carrie, and Lundquist, 2007; Holzman, 2010). That is, we frame the ZPD as an activity that fosters and creates an improvisational learning environment, where individuals learn how to work as a group. Improvisation, commonly referred to as Improv, is a theatrical genre with actors creating a scene as they perform in it. According to Lobman and Lundquist (2007), everyone can improvise, that is, everyone can "choose to create something (usually with other people) by making use of whatever is available" (p. 2). A class or group can learn and are capable of doing an activity that they do not know how to do, and through being improvisational they can create an environment where learning happens. Lobman and Lundquist, further state, "Improv is about learning to take risks and, even more important, it's about learning how to support other people to take risks" (p. 1).

Holzman (2010) stated that Improv or improvisation could be interpreted as what she called, theatrical play, where ordinary individuals improvise (or perform) and collectively create a situation with rules that emerge as individuals participate in the pretend play. Holzman (2009) sees improvisers whether in school or in life as ZPD creators. They are: "creating the 'zone' for their social performance of going beyond what they know and know how to do. I saw Improv as the socially completive activity of making meaning" (p. 62).

Through improvisation, human beings can together create a learning environment where everyone learns to listen, to think, to make mistakes, and to work as a group. Improvisers take risks and get involved in doing an activity that they do not already know how to do. In attempting to do something that the individual does not know how to do (searching for method), they grow and develop.

MTBI as a ZPD

We focus on the learning and development in relation to overall training and research in MTBI, the ZPD characteristics, and the students. MTBI is developing people who otherwise would not be viewed as capable applied mathematics researchers, but they are given the opportunity to do novel and original research in applied mathematics through a collaborative process. From this Vygotskian-Holzman perspective, we argue that as students become a learning community, they are asked to improvise. Each individual is given the opportunity to learn, ask questions together and to "complete" each other as they communicate their ideas and understandings.

MTBI is an environment with high expectations for all participants, students and faculty. Students are expected to learn mathematical content and research skills that might be beyond what they may currently be able to do. The learning environment is one where collaborative learning is valued and expected, and where students are expected to learn and do research whether or not the students knows how to do it already. Everybody relates to them and acts (pretends) like they can. The research is mainly student directed. There is an assigned advisor who is expected to help these students but both are co-creators of the final mathematical model and analysis. Even though there are certain assumptions in place the learning environment (and development) in MTBI is created as students and faculty come together. A second goal of this study is to highlight a few particular elements: Creative imitation and collaboration.

Similar to the All Stars Project, KIPP schools among others, MTBI seems to be creating a Vygotskian ZPD, without calling it such. An environment is built where students can perform as learners and do research even when they may not necessarily know how to. In MTBI, there are no traditional tests to assess specific learning outcomes. The emphasis is more on student's development and ability to do applied mathematical research. Challenging homework assignments are assigned and graded, but if students do not do well, they are asked to redo the problems until they get it right. So getting the right answer is not the goal but understanding, learning and growing.

3.2. A Practical and Theoretical Perspective

In this subsection of the chapter, we focus our analysis on MTBI from a practical and theoretical perspective. In other words, we examine what we see as improvisational specifically in the MTBI program. This includes completion and creative imitation, and how we see the building of the ZPD. The following reported instances are drawn from the author's experiences within the program as a participant in different roles: as student, advanced student, and graduate student. We highlight elements of completion, creative imitation and instances of improvisation that we see as central to the ZPD.

MTBI 2004: Improvisational Development of Research Projects

We focus our analysis on the dynamics among all participants (faculty, postdoctoral students, graduate student mentors, and undergraduates) during group presentations as student research projects are developed. I describe a project from my third year of participation in MTBI. I attended as an advanced and graduate student in 2004. Prior to that I had attended MTBI twice as an undergraduate student.

I report my experience of the student group and faculty interactions as they relate to creative imitation, completion (a social process of thinking and speaking as one activity as explain in Chapter 3.1) and principles of improvisation that are common across research groups. Similar instances can be observed in other years. Here I only highlight these interactions within this specific year when we worked on a project related to obesity in the U.S. We see this example as an improvised unscripted and collaborative performance (Holmes, 2011) and as the creation of an improvisational learning environment (Lobman and Lundquist, 2007). In this context, we describe an *ensemble* performance as described by Holzman (2010), that is we focus on the collective creation of a group in "performing" and learning mathematical research.

In MTBI, it is critical that each group's research question is formulated and a mathematical model well defined in the first week of the research section in order for the projects to be completed by the end of the program. Once a question and model are in hand, each group works with their assigned advisor to make progress and obtain results. Each project is formed through presentations and discussions within the MTBI community over the first week.

My group was composed of three other students. Inspired by "Super Size Me" documentary, our group wanted to model obesity in the U.S.A and fast food consumption. We found data from the National Health and Nutrition website, which contained sample weights and demographic variables (gender, age, race/ethnicity, education and income). Our initial goal was to create a model using this data (Evangelista, et., al, 2004).

We presented our initial ideas in the group presentation to the MTBI community. Any of the 10 to 15 faculty and graduate students listening to us might ask a question or argue for or against approaches to our research question. The order of discussion is not pre-defined, and it is unknown who will speak most during the meeting. A typical scene develops like this:

> Four students stand in front of a screen with slides projected ready to present. The MTBI faculty and graduate students listen as one undergraduate begins to present the topic. The program director stops the student and asks a second student to continue. "Okay Arlene, you tell us what approach you are going to take." Arlene now describes the project's background- what is known about obesity and fast food consumption, and possible ways to model the real world situation.

The focus of the discussion is on crafting the best approach to answering the students desired research questions. What the project will be is not known before hand. It is created in the conversation that takes place.

> "So, what are you trying to do? What question do you want to answer?" This question is asked more than once and answered differently each time. The students want to incorporate the data sample into the model. They also want to model the spread of fast food consumption leading to obesity. The faculty says, "Use a peer pressure rate. Check the previous technical papers. There is some paper where peer pressure was used in the model." As the first session ended, the project became: to study the potential role of peer pressure in fast food consumption and its effect on an individual's weight.

We see this moment as the MTBI community acting as an improvisational ensemble that completes for-with the students. No one knew what approach should be taken. The room worked collectively to develop a direction that the students would try. The direction to use peer pressure rate was something that would model spread of the disease. Hence, the spread of obesity became 'the role of peer pressure" on fast food consumption. What the students said and what the faculty asked was listened to and used by both to give some shape to the otherwise unformed project.

Discussion about possible parameter values and model occur in subsequent group meetings, followed by suggestions on how to go about and continue the analysis of the model. Together with graduate student and faculty advisors, we worked late into the night trying different strategies to incorporate the data sample. We developed a mathematical model to analyze the progression rate from normal weight individuals (**N**) to overweight (**O1**) and obese (**O2**) individuals. An adapted compartmental model from the 2004 technical report is provided next in *Figure 6*:

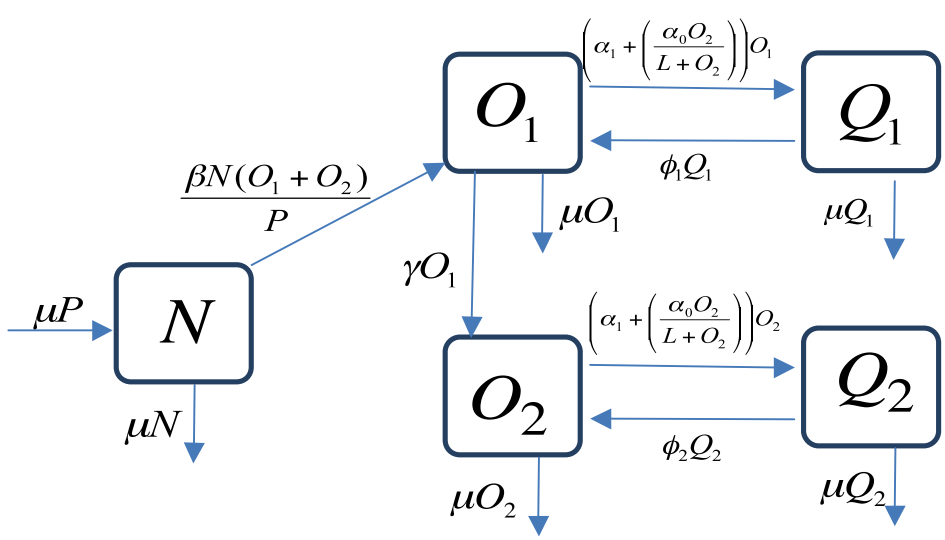

Figure 6. Obesity Model from 2004 Technical Report

The classification of individuals as N, **O1**, and **O2** was based on their body mass index (BMI). The progression from normal to overweight individuals is measured by incorporating a peer pressure, β**,** by which individuals started eating at fast food restaurants. Our decision to include peer pressure made it impossible to incorporate the sample data directly. But we were able to use the data for statistical analysis. The statistical analysis did not contribute to the model, but it allowed us to do correlation analysis on different demographic factors in relation to individual's weight.

Everyone, the novices, other faculty or postdoctoral members in the common room (where all group members work, present, and have mentoring meals) contributed to the creation of the model. It was through this overall discussion that we incorporated another peer pressure, α_1, by which individuals stop eating at fast food restaurants.

As we continued and tried to do an analytical analysis of our model, we found that it was algebraically challenging. We did not know how to proceed. During our next group presentation, faculty recommended that we simplify our model with another feature (nonlinear quitting class but without relapse) that allowed us to find equilibrium points we needed for the system. We continued working with our assigned advisor and together developed three scenarios: a nonlinear quitting model (our original model), a nonlinear quitting model without relapse and a linear model with relapse. Our simulations showed that even though both peer pressures (α_1, β) play a key role in controlling the obesity, α_1 would be a better strategy in controlling the disease given that we can reach more people through health awareness programs compared to reducing the rate β since it depends on media, and individual's social and economic status.

Creative Imitation and Improvisation

Here we see instances of creative imitation and improvisation supporting students to become mathematical researchers. We as students creatively imitated others to make the first mathematical model, using models from the technical reports as guides from previous work done in 2001. We did not mimic the model. We creatively figured out how to imitate its structure and elements to make our own unique mathematical model. We were working well “over our head”. Faculty input directed us to simplify the model, which allowed us to do algebraic analysis and develop two additional models.

We see creative imitation happening in MTBI in the building of ideas, first within the research – extending or finding ways to fit the model into a new model being created. In creating the Obesity model, we looked at a previous technical report that included peer-pressure dynamics of ecstasy use. In general, as students find and create new projects, they are referred back to previous research (not only technical reports, but all scientific papers) if topics are in a way related.

Connecting Practice to Theoretical Framework

According to Vygotsky (1978, 1987) development through collaboration and imitation is the source of all consciousness and development. He further states that a developmental transition can happen through collaboration and imitation with the help of an adult or teacher (an interpretation of the Zone of Proximal Development). Similar to Holzman (2010), we also see that in this development transition, that being a “head taller” is an ensemble performance involving collaboration, imitation and completion.

Prior to attending MTBI, I had no research experience, nor had I taken too many courses on applied mathematics. Everything was new, and well “over my head”.

However, MTBI provided an environment conducive for learning and creating collaborations even though we all came from different intellectual and educational backgrounds. Returning to MTBI, gave me an opportunity to fully learn the previous mathematical concepts taught. For example, returning a second and third time, gave an opportunity to fully understand the different methods for solving and analyzing dynamical systems. As an advanced or graduate student, we needed to provide tutoring to the new students, as well as grade their assignments, and/or help them during lab sessions. This gives us the opportunity to further reflect on the conceptual interpretation of the mathematical methods, and/or theory.

In the end, students are expected to know everything about their projects within the group context. This includes an expectation of individual responsibility for learning. As students present their project in front of faculty, they are related to as good presenters of their work. Yet, these students are also in the process of learning how to be good presenters, how to organize and present graphical analysis. There is both the creation of the applied mathematics researcher and the person capable of presenting or becoming a researcher. The success of the group projects is the collective responsibility of students, mentors, and faculty.

3.3 Methods

We used Holzman (2009, 2010) and Vygotsky (1978, 1987)'s work on the ZPD development to explore our research questions. The two conceptual frameworks (Vygotskian-Holzman) described above guided our survey, interview questions, interpretation and analysis of our results. We next provide more detailed information on survey, interviews and analytical methods.

Data sources: Survey and Interviews

The survey was sent to 99 MTBI past participants who had already completed a PhD. These alumni had participated in MTBI as new students, advanced students and/or as graduate student, and for whom we found a current public email. The email was sent over a period of six weeks (not including holiday break). We also sent about 4 reminders in that period. The survey was anonymous and volunteers had the option of skipping questions if they wished to do so. The number of participants that responded to the survey was 70.

The survey consisted of 34-item questions (see appendix B), 32 of them were multiple-choice or ranking questions, and two were open-ended questions. These questions were created to find out from the students' perspective about their MTBI experience, their undergraduate and graduate career, postdoctoral and/or other professional and educational experiences. More specifically, the 34 survey questions (plus an additional question asking them if they would like to volunteer in the second part of study) were divided into six major sections:

- Demographic questions (i.e. gender, race, work sector)
- MTBI and undergraduate experiences (i.e. year participating in MTBI, interest in graduate school, undergraduate degree, and/or factors that motivated them to continue in the Mathematical Sciences)
- Graduate degree and career after graduation (i.e. degree, years in graduate school, postdocs and/or job after graduating)
- Mentoring attributes in relation to their educational development (i.e.

ranking 11 mentoring attributes in relation to their MTBI experience)

- Questions about collaboration, creativity and training as an Applied Mathematician (ranking and agreement questions on whether these concepts were present or not in the program)
- Current educational and professional achievements (ranking questions on their current satisfaction level in their educational and professional achievement, and future plans)

Given these participants may have had a range and variety of experiences influencing their professional and educational development, two open-ended questions to capture this data were included. Specifically, we asked:

15. What learning experiences or factors motivated you to pursue graduate school in the Mathematical sciences?

20. What learning experiences or factors contributed to your continuation and completion of a PhD?

We hoped we could capture other factors that helped in their professional and educational development with these questions. We also asked participants to further include a reflection on the aspects of MTBI that have contributed to their professional and educational situation. Our goal was to discover factors that may have not have been included in the multiple-choice section, including any related mentoring from their MTBI experience

The interview questions consisted of reflective questions about their educational and professional experiences divided into two major parts:

- MTBI learning experiences: questions about specific instances of collaboration, creativity, imitation, and mentoring relationships within the program, as well as a general one about whether MTBI was helpful in learning skills (such as becoming critical thinkers, problem solvers, active listeners, etc.)

- Learning experiences not limited to MTBI: questions about what motivated participants to continue with graduate school, and what factors contributed to their current professional and educational situation, future plans and any further comments.

We administered the survey over a period of 7 weeks, and the interviews over a period of two weeks after survey was completed it. The last question of the survey asked participants to provide their contact information in case they wanted to volunteer for the second part of the study. Based on the people who responded, the number of years participating in MTBI, as well as their work sector, we chose to contact eight participants. We chose participants across different groups to capture a broad perspectives of views. We interviewed people based on years of participation, one time, twice or multiple years; their work sector such as academia, industry or government. The volunteers came from cohort year 1996 to the most recent cohort year in the volunteer pool, which turned out to be from the summer of 2006.

A total of eight people volunteered. Of the these eight, three had participated multiple times (two in academia and one in industry), three had participated one time (government, industry and academia), and two had participated twice (one in academia and another in industry). In terms of ethnicity, five were Hispanics, two were Black, one White. In terms of gender, five were male and three were females. The group description is shown in *Table 3* below.

Table 3

Interviewee Descriptions

	Group description
Multiple	Ethnicity: 3 Hispanic Work sector: 2 in academia and 1 in industry Gender: 1 females and 2 males
Double	Ethnicity: 1 Hispanic, 1 Black Work Sector: 1 in academia and 1 in industry Gender: 2 males
Single	Ethnicity: 1 Hispanic, 1 Black, 1 White Work sector: 1 in academia, 1 in industry, 1 in government Gender: 2 females and 1 male

Method Analysis

We recruited PhD volunteers by email and also by posting on the MTBI website. The selection was at random from any of the students who wanted to complete the survey and then also participate with audio follow up research interviews. The survey was provided online, and the follow up questions were done using Skype or Google chat. We used Microsoft Word to record the audio interview. All video, audio data was saved in a computer with session number, and participants were assigned a number from 1-8. All data was manually transcribed using *StudioCode,* a video coding software.

We used the constant comparative approach (Glaser, 1967) to do our qualitative analysis. This method combines explicit coding of data (using grounded theory methodology) with that of theory development. According to Glaser (1967),

> "The purpose of the constant comparative method of joint coding and analysis is to generate theory more systematically" (p 437).

Glaser explains that the method generates theoretical ideas and allows to test for many hypotheses at different levels of generality.

To begin our qualitative analysis of data coding, we used Grounded theory methodology (Strauss & Corbin, 1996). According to Strauss and Corbin, this methodology is a

> "theory that was derived from data, systematically gathered and analyzed through the research process. In this method, data collection, analysis and eventual theory stand in close relationship to one another" (p. 12).

In this case, data analysis and eventual theory are so closely related because the researcher will develop the emerging theory from the data, and not from an existing theory.

The method for data analysis can be broken into four steps: open *coding, conceptualizing, axial coding* and *theorizing*, which are not necessarily sequential. Open coding is a process where concepts are identified according to their properties (characteristics) and dimensions (the variation along properties). In *open coding*, all data is examined and break into discrete parts by ideas, events, acts and/or meanings. Once discrete parts are identified within the context that is being study, it is label with a concept descriptor. This concept can represent an event, a happening, an object and/or action/interactions and this labeling and abstraction of the data is called conceptualizing.

Next, concepts are compared for similarities and differences so that objects can be grouped into categories when they share similar properties and dimensions (or subcategories too). Categories are important because they are able to explain and predict the phenomenon that is being studied. Next, in *axial coding*, the data is reassembled (and organized) so that categories and subcategories are related and link as a way to further explain what is going on with the phenomena being study. Before theorizing can happen, there is a process of integration and refining among the categories. This integration is a slow process that actually starts as the researcher starts to code, it continues as she or he tries to determine the central categories that explain and predict the phenomena, and it ends when findings can explain what is going on in a general sense (Strauss and Corbin, 1996).

To finalize our analysis we generated themes from the categories created to evaluate our question: does MTBI have a Vygotskian-Holzman environment? The constant comparative method is designed so the analyst can generate "a theory that is integrated, consistent, plausible, close to the data". The method can be describe in four stages:

1) comparing incidents applicable to each category
2) integrating categories and their properties
3) delimiting theory
4) writing the theory

However, the early stages can simultaneously take place until final analysis is completed.

We used both quantitative and qualitative methods to analyzed our data. For our survey (N = 70), we used the programming language *R* to do summary statistical, and relational analysis. For open-ended question 15 (N = 59) and open-ended question 20 (N = 52), as well as for our interviews (N = 8) we used the constant comparative method to generate code and themes.

We used excel to organize coded responses until major themes emerged from the analysis for question 15 and 20. For example, in column *A* we put students actual responses, in Column *B* we put verbatim one-line responses or our interpretation based on their language use in their response. These one-line responses we called *codes.* In most cases, participants listed from one to two factors that have influenced their decision to pursue graduate studies. In the third phase of our analysis we look for commonalities and differences among responses to form patterns among the coded responses that called it *categories*. In the final stage, we group these categories into bigger *themes* based on recurring key concepts and factors that motivated them to pursue graduate school.

In question 15, there were 59 people who responded. Using the grounded theory and constant comparative methodology, we identified 93 distinct codes. From these we further identified 15 categories when comparing the coded lines. In the last stage we identified 5 final themes. It is important to remember that because participants might have listed more than one factor as a motivation, we ended up having 93 distinct codes from 59 people . *Table 4* and *Table 5* below show the different classifications from raw data to themes.

Table 4

Diagram for Coded Analysis on Question 15

raw data	**codes**	**categories**	**Themes**
1 paragraph per person (59 people)	93 distinct lines	15	5

We followed the same procedure in question 20, with 52 people responding to this question. A summary of coded process is again given below:

Table 5

Diagram for Coded Analysis on Question 20

raw data	codes	categories	Themes
1 paragraph per person (52 people)	98 distinct lines	14	4

Where applicable, we classify categories or themes with the same names across the two questions. After general coding was completed for both questions, we went back to the raw data and codes to separate the coded analysis into MTBI mentioned vs. MTBI not mentioned. We this separation because we wanted to capture to what extent MTBI mentoring resources and services played a role in their educational and professional development.

The lengths of their verbal responses vary from one brief paragraph of about 71 words to a full page of about 500 words. We used excel to organize the their responses by listing key ideas/concepts and phrases for each participants. For example on the first column we had participant one through eight, on the second column we match the key descriptors and/or sentences participants responded to collaboration. By match we mean that we identified phrases that could be classified as collaboration with others, and these were our key phrases or codes. Then we repeated the process by going back to the data to clarify, modify our key phrases if they were not representing what we observed in the data. For example in the initial coding of collaboration, it was observed that participants were “working with faculty”, and “working with peers”. Generally we scanned the responses, and we analyzed a phrase, sentence, and/or paragraph, and then we return to a word of phrase that we felt significant. As we when back to the data to compare it against each participant, we expanded and/or discarded concepts or descriptors as needed. For example, in the second and third revising of coding, the “working with faculty change” to “informally working with faculty, all of the time, "thinking"; sitting down with faculty at tables, couches, irrespective of time”. We highlighted the recurring ideas among the coded codes, and made notes of the emerging themes. The same process was done when coding responses from the creativity question.

CHAPTER 4

PROFESSIONAL AND EDUCATIONAL DEVELOPMENT IN MTBI

The goal of this chapter is to identify demographic trends in the MTBI cohort from the participants' responses. The trends include sample distributions based on gender, ethnicity, work sector, type of degree programs, learning and research experiences during their educational career including while at MTBI. The trends were summarized via exploratory data analysis and relational models (linear regression). The results are presented in the form of pie charts, bar graphs, histograms, and box scatter plots. The analysis aims to provide insights into participant's undergraduate through graduate educational and learning experiences.

The results on respondents' experiences and behavior during or before MTBI are mentioned in Section I, the results on respondents' motivational factors to go to graduate school are in Section II, findings related to respondent's experiences with graduate school are in Section III, and Section IV collects results on respondents' experiences after graduate school. Implications to theoretical framework is in section V.

I. Exploratory Analysis on Respondents' MTBI and Prior Learning Experiences

Survey respondents included MTBI participants from summer 1996 to summer 2010, with the exception of 2008 (no respondents from this year). The survey sample size was 70. Among those 48.57% were female and 51.43% were male. The majority, 64.29%, of respondents were one-time participants in MTBI, where as 18.57% participated twice, and 17.14% more than two times in successive years (multiple years).

Data summary shows that 55.71% were Hispanic, 18.57% White, 4.29% Asian, 7.14% Black, 8.57% multiple ethnicity (such as Black and White or Latino and White), and 4.29% decided not to respond. Black could refer to Haitian, African, Cuban, etc. The results clearly reflect that the program recruited minorities and has been able to maintain roughly equal proportions in gender.

When asked how students became interested in the program approximately 51.43% responded that they were recommended by their mentor or advisor to join the MTBI program, and 10% reported that their peers recommended it. 14.29% of the respondents said they were interested in carrying out Applied Mathematics Research experience and found the MTBI program by themselves. Moreover, 12.86% of respondents listed at least two of the three above reasons for their participation in the MTBI program and 4% came to know about the program from Mathematical newsletters.

When asked about any prior participation in Research Experience for Undergraduates (REU), 67.14% responded that MTBI had been their first REU experience, 20% responded they had no prior REU experience, 5.71% did not respond to the question, and 4.28% responded yes and had some prior research experience in their own undergraduate institutions.

When participants were asked if they were interested in graduate school prior to MTBI, 72.86% said yes, and 92.85% remained interested after MTBI. 10% of the respondents were not sure if they were interested, and 15.71% were not interested in graduate school. A 98.57% also responded that after MTBI, they were still interested in the Mathematical Sciences field (*Figure 7).*

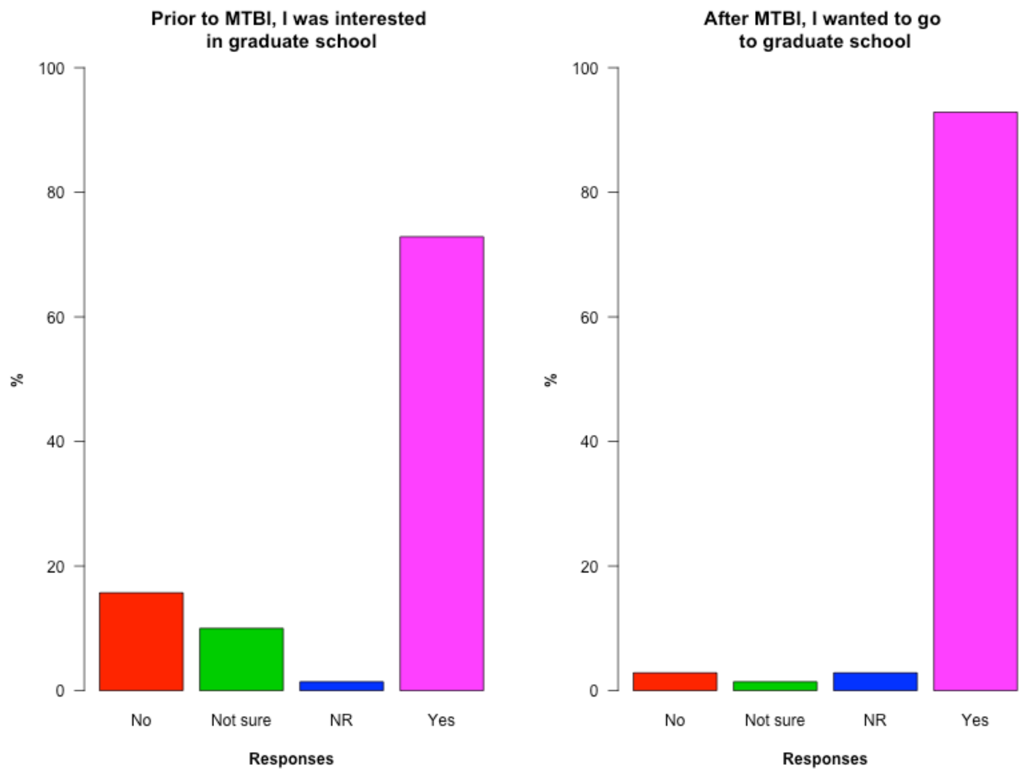

Figure 7. Interest in Graduate School Before and After MTBI (*NR represents No Response)*

In response to the question *"What was your graduate degree?"* 10% listed PhD. as their degree with no listing of their field of study, and 38.57 % and 27.14% of the respondents got a degree in Applied Mathematics and Mathematics, respectively. The rest, 17.14% major in other quantitative sciences fields, such as biostatistics, computer science, and engineering, 2.86% major in philosophy and psychology.

One of the goals of this study was to address a question "*were there any participants who were not interested in graduate school who became interested in MTBI or vice versa?"*

Table 6

Prior and Post MTBI Frequency Numbers for Inclination towards Graduate School

Prior ↓ / Post →	No/ NS	Yes	Total
No/ NS	0	18	18
Yes	2	47	49
Total	2	65	67

We found that the probability that a participant was interested in graduate school after MTBI participation given that the participant was not sure or not interested in graduate school prior to MTBI is 18/18, or 100% (*Table 6*). Our results show that the proportion of students who were interested in graduate school after MTBI given that they were already interested prior to MTBI is 47/49 or about 96% of participants remained interested in graduate after MTBI. This implies that students were mostly encouraged to go to graduate school after going to MTBI.

II. Potential Factors for Development

We explore MTBI development as a function of the relationship among the program elements, the participants and their collective ability to create a ZPD. Castillo-Garsow et al. (2013) identified six potential factors (or attributes) that may explain the success of the MTBI program. We tested if these and other new factors existed in our data. In our survey, we asked about the role of different MTBI training resources, as well as the role of the learning environment of MTBI with respect to students' educational and professional development.

We used factors identified by Castillo-Garsow et al. (2013), other factors hypothesized by us and asked the survey participants to reflect on the reasons for their educational development (question 26 in the survey). The responses allowed us to rank 11 different MTBI related factors. The ranking was taken from the sample percent response when marked as *very important* from the pie charts. The factors and their response percent ratings in order of their ranking are mentioned as follows:

- *Mentoring from MTBI faculty:* 74.29%
- *Student-mentor relationships:* 67.14%
- *Undergraduate-graduate or undergraduate-undergraduate relationships:* 66.2%
- *Student initiated research:* 62.86%
- *High expectations on students:* 61.43%
- *Training on how to become a scientist (like how to do research, write technical report and/or presentations):* 60%
- *Close living arrangement for two months:* 41.33%
- *Becoming a member of the MTBI learning community:* 40%
- *Participants from different academic preparation:* 38.57%
- *Challenging homework sets:* 34.29%
- *Participants from different cultural backgrounds:* 31.43%

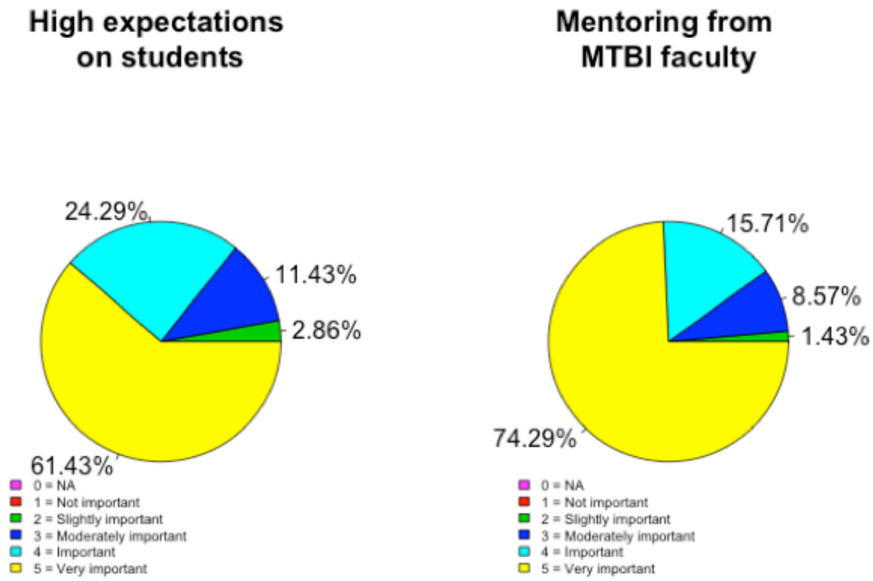

Figure 8. Pie Chart Representation of Participant's Opinion on MTBI Providing High Expectations and Mentoring from MTBI Faculty

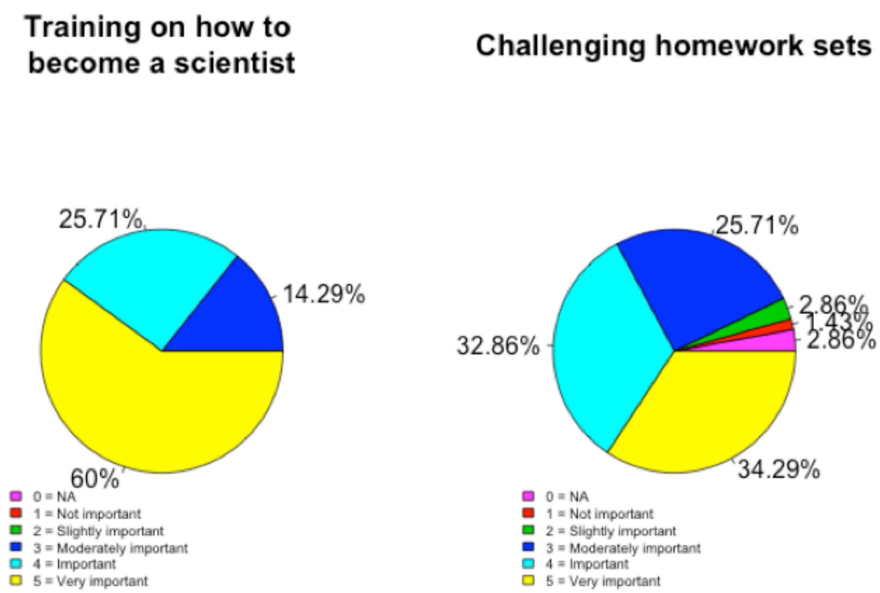

Figure 9. Pie Chart Representation of MTBI Participant's Training Experiences

By looking at the pie charts (only a few of the pie charts are here in *Figure 8, and 9*) and their responses, we found that all of the factors considered by us in the survey were important according to our respondents for their educational development. However, the degree to which they agree in regards to being important in their educational development differs drastically. The attribute with the highest percent was mentoring from MTBI faculty at 74.29%, followed by student-mentor relationships at 67.14%, relationships between students and other participants in the program whether undergraduate or graduate at 66.2% and student initiated research at 62.86%.

In summary, from the perspective of the MTBI alumni, the three top attributes to their development are: mentoring from MTBI faculty, relationships built with others in the program whether with faculty or the MTBI community, and their research projects. Challenging homework sets and participants coming from different backgrounds were ranked the lowest but even then these factors were significant (*very important*) to more than one third of the respondents.

We also analyzed the responses to the same question: *Rank the following attributes of your MTBI experience that you think were most important in your educational development?"* in a different way that included all variations (6 categories from least important to most important) to this question.

We created a 70 x 11 (sample size 70, 11 attributes) score matrix for this question, which corresponds to the different attributes, with each row representing the participant's response and each column consisting one of the 11 MTBI factors. If something is really important to each of them it will have the highest marks (5) from among the choices they had. If it was least important then the participant assigned the lowest mark as 0. The sum for each column was computed and the greatest column sum resulted in the most important factor. The numbers in parenthesis reflect the cumulative score across responses. The highest possible score would have been 5*70 = 350, and the lowest 0.

We found that in order in terms of mentoring resources is as follows:

1st: Mentoring from faculty (324)

2nd: Faculty and student mentor relationships (314)

3rd: Training on how to become a scientist, and high expectation on students (311,312)

4th: Student initiated research (308)

5th: Relationships between undergraduates or graduates or Postdocs (286)

6th: Becoming a member of the MTBI community (282)

7th: Close living arrangements for two month and participants with diverse cultures (271)

8th: Participants with diverse academic preparation (259)

As observed in the ranking list, mentoring from MTBI faculty was the primary attribute, followed by student mentor, training on how to become a scientist and expectations tied at third place, and research at fourth place. The least important were the challenging homework, people with different academic preparation and people from different cultural backgrounds. The only difference from our previous observation in the pie charts is that training on how to become a scientist was in third place. This factor was previously reported as important in undergraduate research experience in science (Hunter, Laursen and Seymour, 2006).

Results From Survey on Collaboration and Creativity

The results on questions on whether or not the MTBI alumni saw "MTBI as a place of collaboration and creativity" as well as whether or not "MTBI fostered students to think like Applied Mathematician and/or to grow intellectually?" were also analyzed (question 28 to question 29). We found that 69.01% strongly agreed, and 26.35% agreed moderately that MTBI provided an environment of collaboration. While 57.75% strongly agreed, and 26.76% agreed moderately that MTBI was a place of creativity (*Figure 10*). In terms of MTBI fostering students to think and work like an Applied Mathematician, 61.97% strongly agreed with 25.35% agreed moderately. And 67.61% strongly agreed (with 23.94% agreed moderately) that MTBI fosters students to grow intellectually.

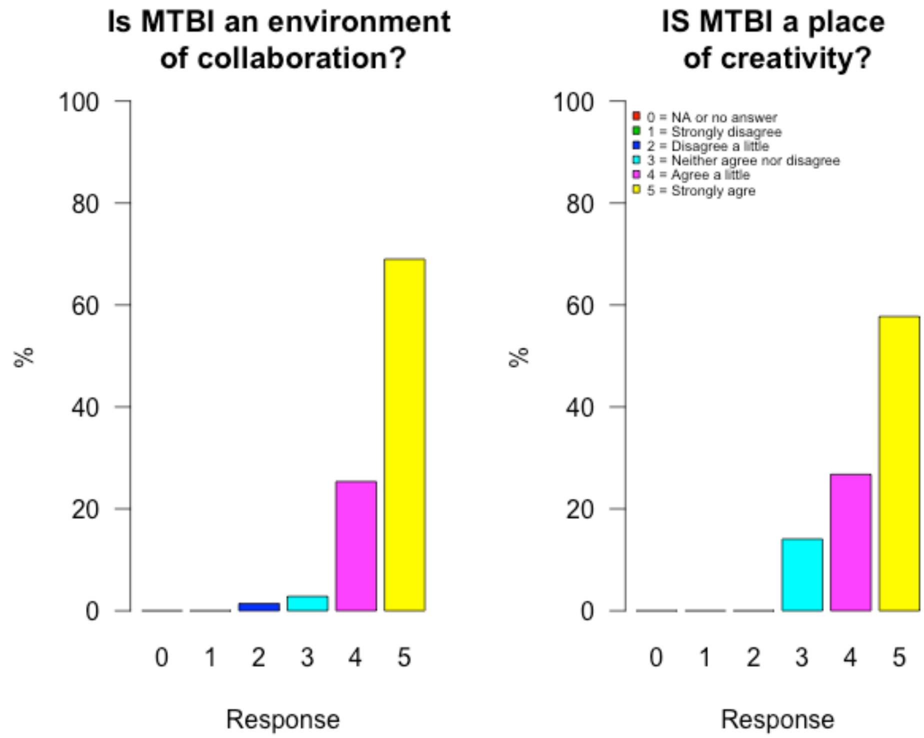

Figure 10. Bar Graphs of Responses on whether MTBI is an Environment of Collaboration or Creativity

It seems all students agreed that collaboration, creavity, working like an Applied Mathematican and intellectual growth were present in MTBI. Approximately, 58% and 69% of the respondents strongly agree to the presence of creativity and collaboration in the MTBI, respectively.

We also computed the rankings of these attributes in order to answer "*Which of these factors are present within the MTBI learning environment: collaboration, creativity, working like an Applied Mathematician and intellectual growth?"* We found the importance of the factors in descending order as follows:

1st: MTBI as a place of collaboration (325)

2nd: MTBI fostering Intellectual growth (322)

3rd: MTBI as a place of creativity (311)

4th: MTBI fostering students to think and work like an Applied Mathematician (307)

It seems that students see collaboration as the primary factor present in MTBI, followed by intelletual growth, creavity and fostering students to think and work like an Applied Mathematican. Even though thinking like an Applied mathematician is ranked lowest within MTBI, they did find this attribute to be important to their educational development (Question 26).

III. Educational and Learning Experiences after MTBI

We also surveyed students on what happened after MTBI in terms of their educational and professional development. When asked if MTBI helped them manage graduate school (*Figure 11*), 84.29% responded with agreement, 11.43% conveying disagreement, 1.43% said somewhat, and 2.86% marked other category (*such as they were already in graduate school or no comments*).

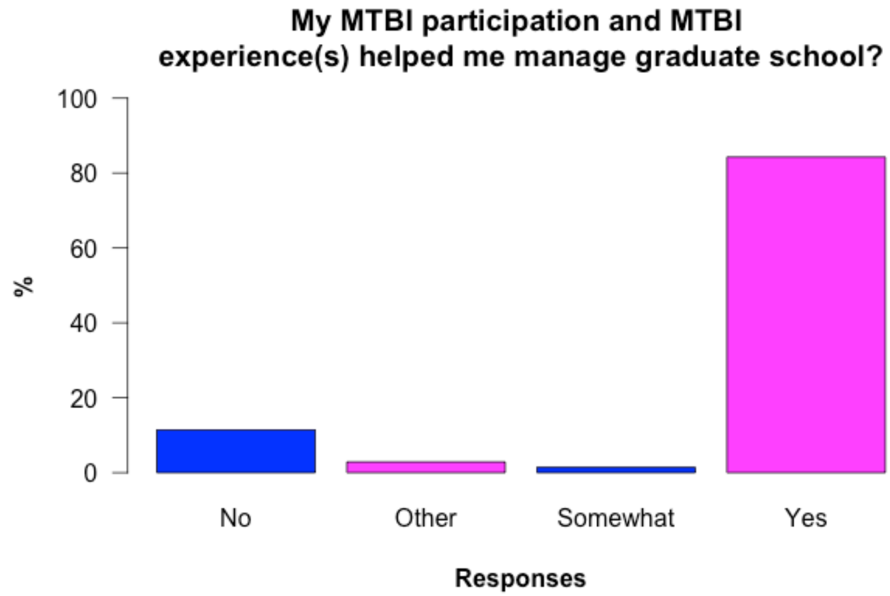

Figure 11. *Bar Graph on whether MTBI Participation and MTBI Experience(s) Help Manage Graduate School*

Given that some research projects by undergraduates have led to journal publications, we also asked if their MTBI participation led to their first journal publication from work carried out at the program, and 30% responded yes, while 70% said no.

In *Figure 12*, we combined MTBI alumni responses in terms of if they were the first in the family to obtain an undergraduate or graduate degree. Only 28.57% were the first in their family to obtained a undergraduate degree (with 71.43% saying no) however, a significantly higher proportion (80%) were the first in the family to get a graduate degree with a PhD.

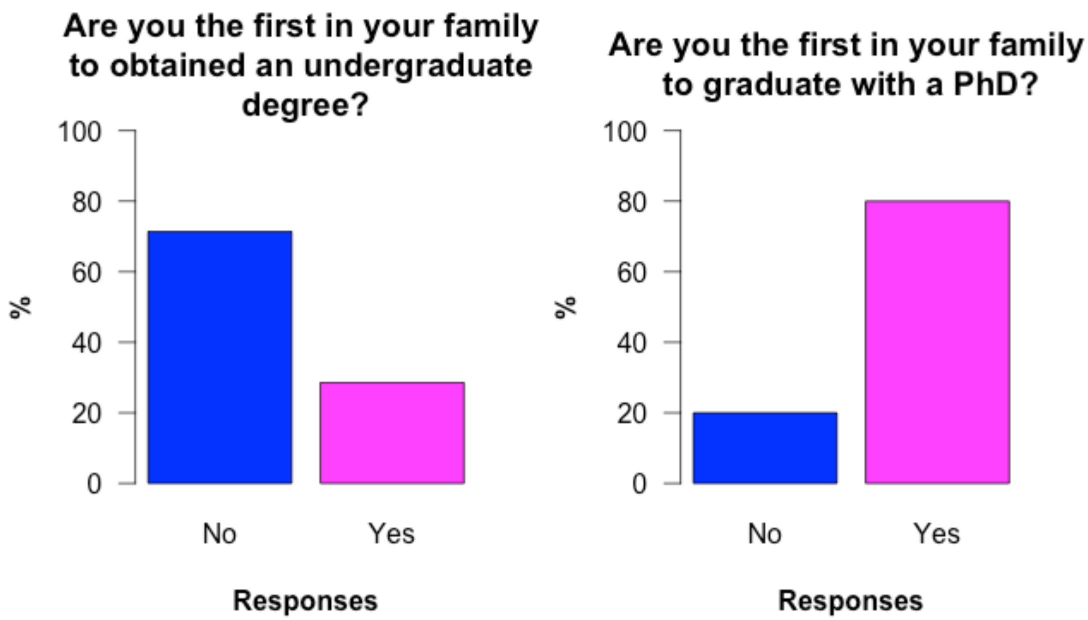

Figure 12. Responses on if Participant was First in Family to have Obtained an Undergraduate Degree and Graduate Degree

According to their responses most participants had a STEM undergraduate degree. More specifically, 8.57% major in Applied Mathematics, 60% in Mathematics, 12.86% had double majors (in Mathematics and some other concentration such as science, technology or social science). An additional 12.86% of participants majored in other non-mathematical concentrations such as engineering, computer science, or classics. 5.71% gave no response.

In terms of graduate educational experiences, a student on an average spent 6.05 years in the graduate school. The average also included their time in masters program, if they completed one before the PhD. The maximum number of years in graduate school for a participant in the sample was 12 years and the minimum was 3 years. The average mean for men was 6.33 years, and for females was 5.75 years (*Figure 13*).

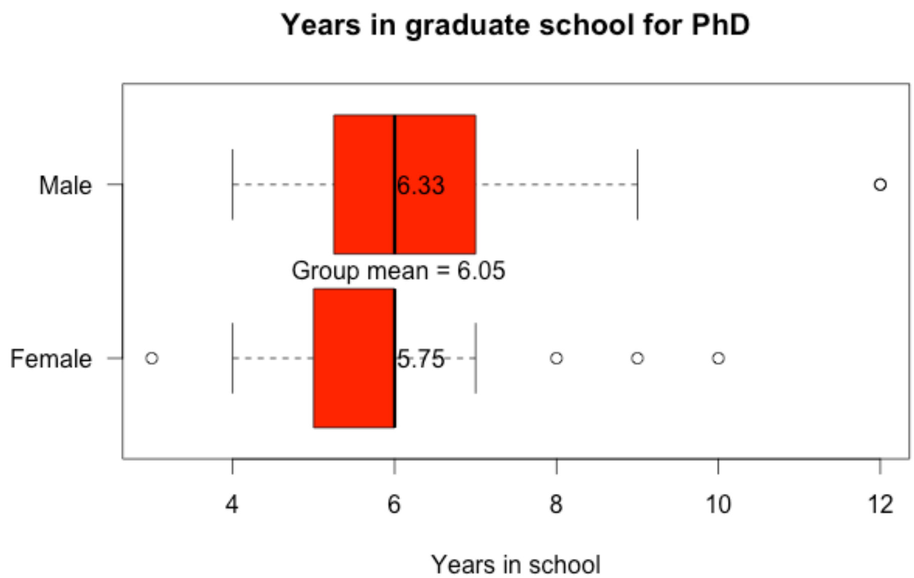

Figure 13. The Numbers of Years Participants Spent in Graduate School by Gender

IV. Educational and Professional Achievements after MTBI

We also asked a set of general questions that reflected students' educational and professional achievements and career. In response to the question (Question 32) "how they felt about their educational achievements?" 75.71% responded they felt very satisfied and 24.29% felt somewhat satisfied. When asked about how they felt about their professional achievements: 62.86% felt very satisfied, 31.43% were somewhat satisfied, 2.86% were both neither satisfied or unsatisfied or were somewhat unsatisfied (*Figure 14*) .

We also analyze their responses on their future career paths. 71.43% wanted to stay in their current sector (academia, industry, or government/public sector). However, 17.14% were not sure, while 5.72% wanted to change either to industry or to academy, and 1.43% wanting to change their career path to something different career option but were not sure of the option.

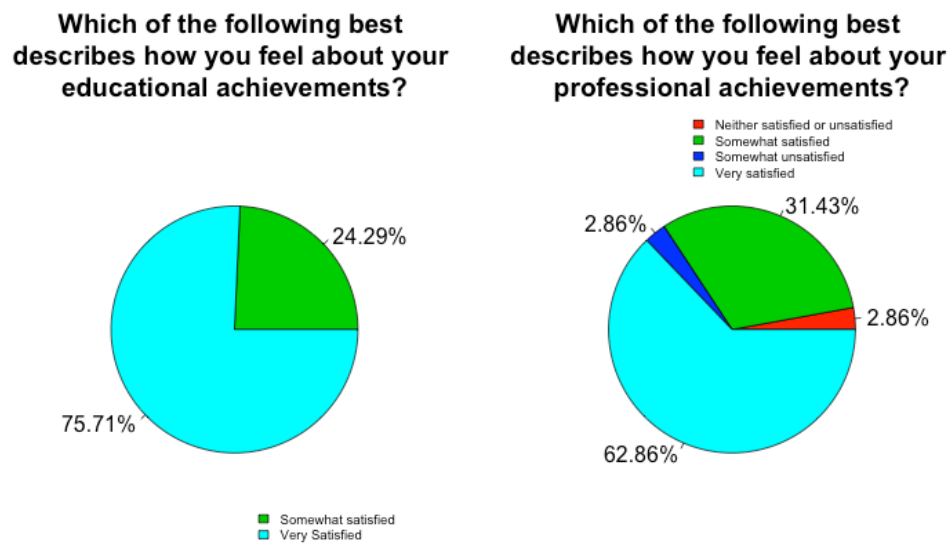

Figure 14. Responses on Participant's Educational Achievements and Professional Achievements

We were also interested in what did they do professionally immediately after completing their PhD? 55.71% got a postdoctoral position with the position lasting an average of 2.83 years, with 42.86% not accepting a postdoctoral position offer and went for other option. From those who responded no postdoctoral position after PhD, 11.43% said they went into academia for an assistant professor position, 4.29% went into banking sector and 1.43% gave no response in relation to the question. Most did their postdoctoral work within the U.S. very few went out the country

We also asked participants to describe their current work sector and 78.57% responded they are in academia, 14.29% in Industry, 5.72% are in government/public sector and 1.43% were currently unemployed (*Figure 15*).

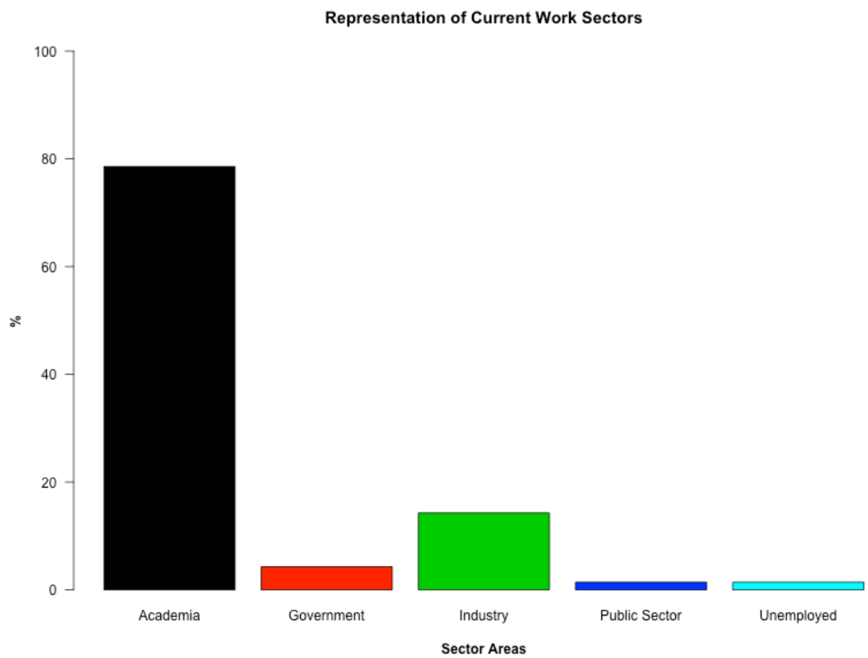

Figure 15. Responses on Participants' Current Work Sector

For those that were in academia, we asked if they were currently in a tenure track position: 42.86% are in tenure-track, 37.14% are in non-tenure track academic position, with 18.57% not in academia (initially 78.57% responded they were currently in academia).

For those going to academia, which was 60% of those surveyed, it took them about 2.35 years to get a faculty position, after getting their PhD, with 9.52% (4 out the 42 who got a professorship) started their faculty responsibilities before graduating.

V. Study Implications Related to Theoretical Framework

We were particularly interested in quantifying variability in participants' response with respect to collaboration and creativity, as well as years in graduate school with respect to program natural improvement over years. Therefore, we focus on participant's responses on time spent in graduate school, and on following questions:

- *Question 28: When thinking about collaboration in the workplace or university setting, do you believe MTBI provides an environment of collaboration?*
- *Question 29: When thinking about creativity as being able to take old ideas and make something new or different, do you believe MTBI can be described as a place of creativity?*

Our results (*Figure 10*) showed that in general participants agreed that MTBI fostered an environment of collaboration and creativity. We run a *chi square test* to identify any differences in the empirical distribution related to the level of agreement between collaboration and creativity among participants. Our results suggests that there is no difference in the two distributions $(\chi^2(28) = 29.15, p < 0.05)$, that is, students were in similar level of agreement for the presence of collaboration (Mean = 4.6), and creativity in the MTBI (Mean 4.44) .

We further explored the extent to which participants, whether in academia or not in academia, agreed to the presence of collaboration in the program. Participants indicated their response from “Strongly disagree” (by marking 1) to “Strongly agree” (by choosing 5). We inspected their responses, on presence of collaboration, by dividing the sample into two different groups, people who work in academia (N = 55; mean response was 4.65) versus those who did not work in academia (N = 15; mean response was 4.67). Using t-test, we found that there was no significant difference in the collaboration related responses for academia (M = 4.65, SD=0.64) and non-academia (M=4.67, SD = 0.51) work sector (t(27.665) = -0.3471, p = 0.7312). Overall, most of them agreed that MTBI was providing an environment of collaboration.

We also analyzed the responses to creativity in question 29 in comparison to the number of years students participated in MTBI, that is, whether they participated one (single) time, twice (double) or multiple (more than two) times. The mean responses on level of agreement on the creativity (5 being maximum and 1 being minimum) according to their respective years of participation in the MTBI program are: Multiple 4.67, Double 4.31, Single 4.42. The responses suggest that most survey participants highly agreed that creativity is present in the program as in seen in *Figure 10.*

We used a t-test to assess whether the means (to the presence of creativity in MTBI) of two different groups within the MTBI participation years were statistically different for each other. We compared three groups: people who participated double vs. multiple, double vs. single, and multiple vs. single. Since the p-value was a greater than significance level (0.05) therefore we rejected the null hypothesis over alternative hypothesis, that is, the means of the two groups may not be different. That is, there was no significance difference in the responses for Double ($M = 4.31$, $SD=0.63$) and Multiple ($M=4.67$, $SD = 0.65$) year participation ($t(22.69)=-1.3981$, $p = 0.1756$).

We created similar comparison for Double vs. single groups and Multiple vs. Single groups. There was no significant difference in the responses between Double ($M = 4.31$, $SD=0.63$) and Single ($M=4.42$, $SD = 0.78$) year participation ($t(23.79) = -0.5448$, $p = 0.591$) and between Multiple (M=4,67, SD=0.65) vs. Single (M =4.42, SD =. 078) participation ((t (20.36) = -0.2167, p=0.2822). In conclusion, in all the three comparisons among the single, double and multiple participants groups, all agreed that creativity was present in the program.

One of the limitations of the analysis and the results is the small sample size when the data is subdivided based on an additional feature. For example, sample sizes of participants who work in academia and non-academia are N = 55, and N =15, respectively) as compared to the study sample of size 70.

From the available data we also investigated whether the number of years a typical MTBI participant spend in the graduate school is related to the participant MTBI cohort year. MTBI cohort years refer to the year when the respondent went through MTBI for the first time as a participant in the summer program. This analysis was carried out using linear regression model with independent variable being MTBI cohort year and dependent variable being number of years in graduate school. To see if the data meets the primary assumptions for the linear regression, residuals and Q-Q plots were constructed. The residuals are found to be randomly distributed around zero (*Figure 16*).

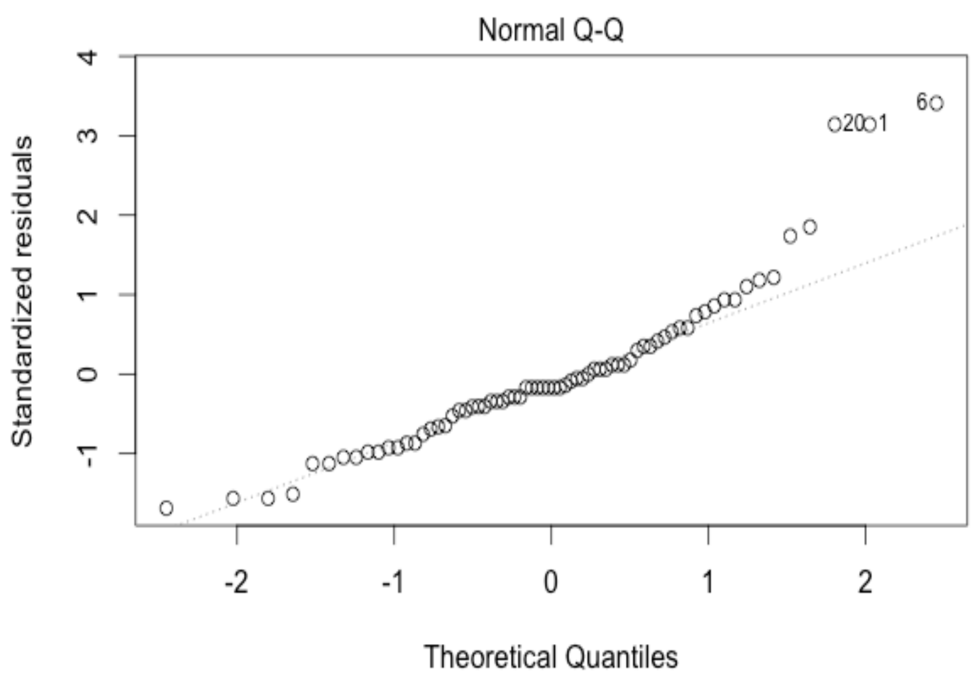

Figure 16. Normal Q-Q for Linear Regression

Q-Q plot shows residuals following linearly against the theoretical quartiles (*Figure 17*). This suggested that the normality assumptions for linear regression models for our data sets are met.

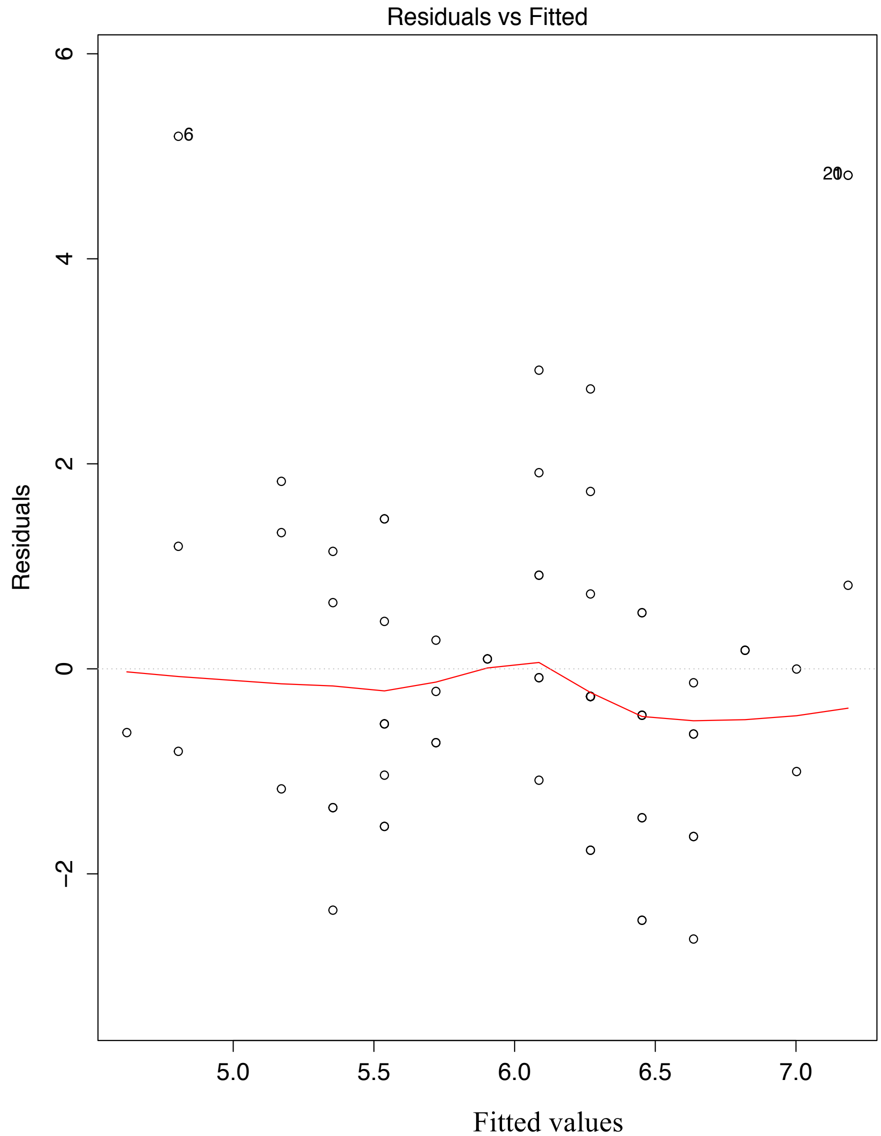

Figure 17. Residuals from Linear Regression.

A significant linear regression was found (F(1, 68)= 10.3, p<0.05) with a $R^2 = 0.1315$ (*Figure 18*). The regression equation is given as

$$y = 372.64 - 0.18x$$

where *x* is cohort 1996 to cohort year 2010. See *table 7* for linear coefficients terms.

Table 7 Linear Coefficients on Years Spent in Graduate School with Respect to MTBI Cohort Year

Coefficients	Estimate	Std error	t value	p value
Intercept	372.64	114.24	3.262	0.00173
Cohort year	-0.18	0.057	-3.209	0.00203

The results suggest that the numbers of years in graduate school decreases by -0.18309 with age of MTBI program (cohort year). In other words in 14 years, there seems a significant improvement in the program training as the average number of years participants took to graduate decreases linearly (*Table 7* and *Figure 18*)

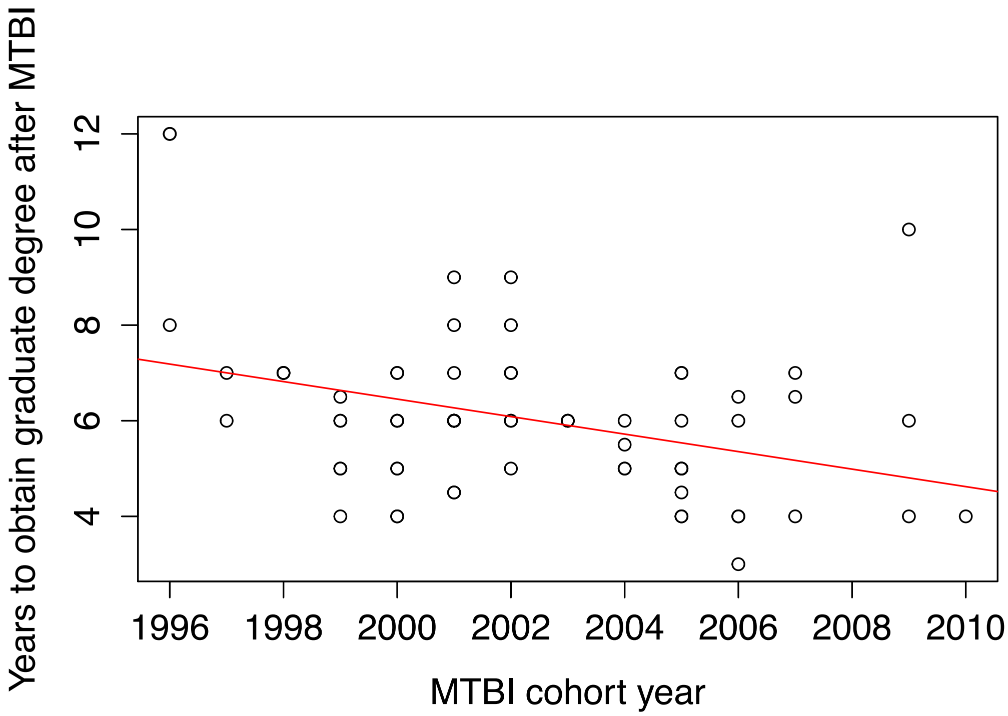

Figure 18. Years Participants Spent in Graduate with Respect to MTBI Cohort Year

A possible reasons for this negative trend could be that the MTBI mentoring resources and infrastructure has been improving over the years, and/or with time the MTBI community and networking are getting larger and cohesive. MTBI participants could be helping each other navigate graduate school or having learning experiences that helped them overall.

Because of the possible outliers (or influential observations) in the data, we also used a robust linear regression diagnostic to study the same question. We used the "rlm" in R programming language to confirm results obtained from linear regression (*Table 7*, *Figure 17*). The robust model also predicted the same trend between number of years participants took to graduate and participant MTBI cohort year.

The robust linear model was found to be

$$y = 328.13 - 0.1609x$$

with a similar negative trend in the number of years people took to graduate with respect to their MTBI cohort year, as in the linear regression equation. Linearity assumptions were again met since most of the data is within zero, as seen in *Figure 19.* In addition, in *Figure 20*, the Q-Q shows that residuals were normally distributed.

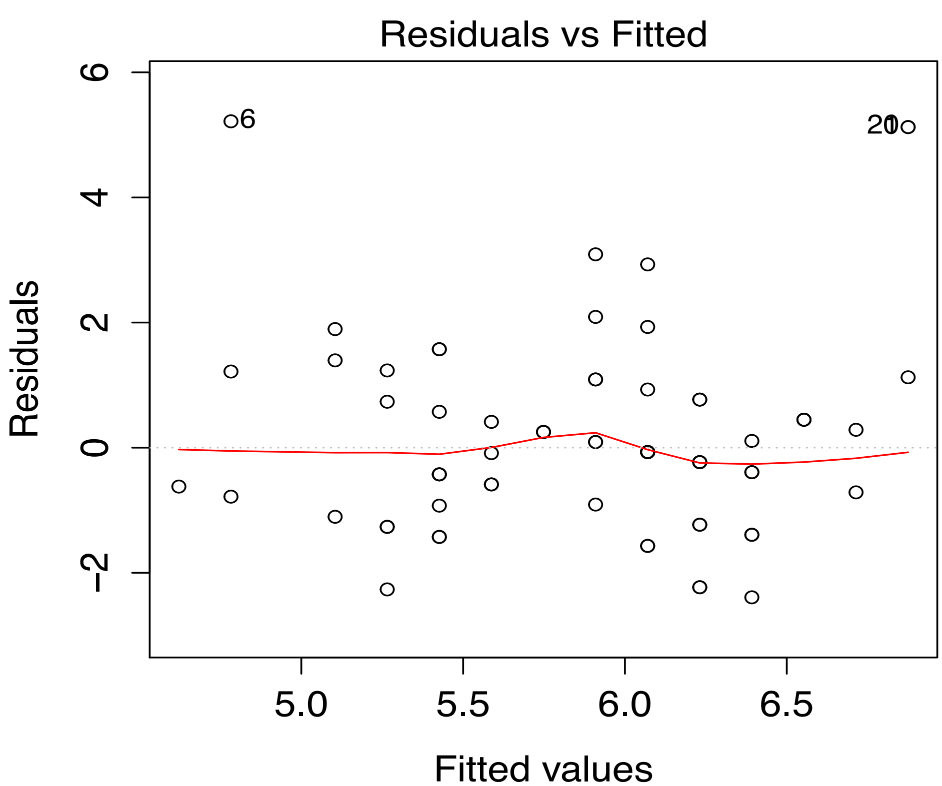

Figure 19. Residuals Graph for Robust Linear Regression

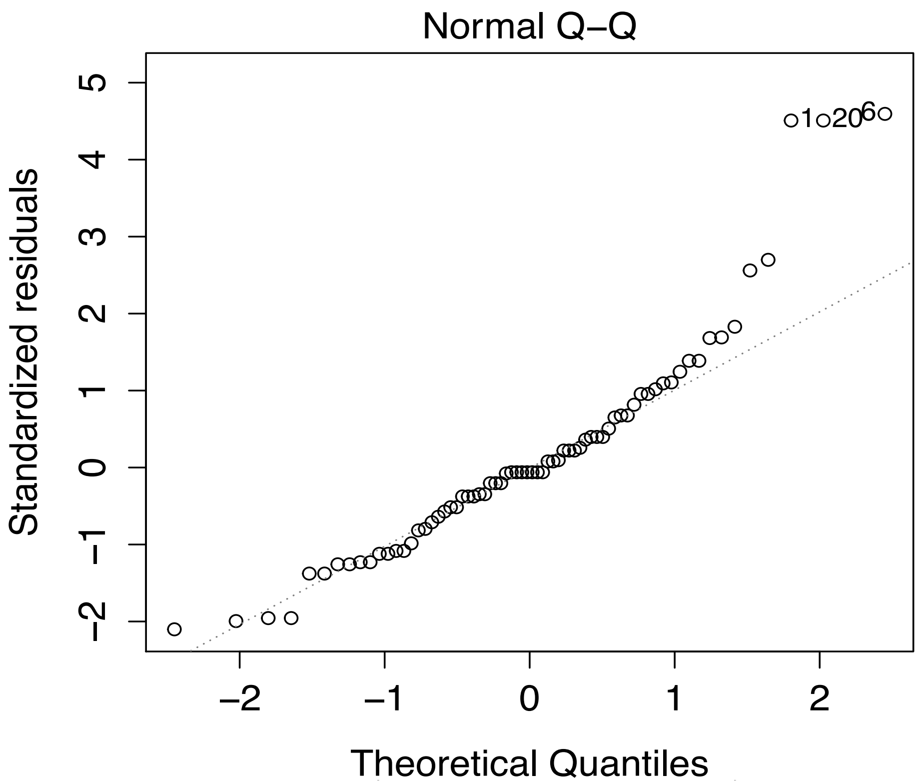

Figure 20. Normal Q-Q Graph for Robust Linear Regression

There may be other factors that may have influenced and resulted in this decrease in number of years participants spent on graduate school with increasing MTBI cohort year. To this end, we run a multiple linear regression checking three other factors: MTBI experience, ethnicity and journal publication out of MTBI.

We created a multiple linear regression comparing number of years spent in graduate school to the responses of their MTBI experience on whether MTBI helped them manage graduate school. Responses were “yes”, “no”, “somewhat “ and “other”. The model found to be significant is

$$y = -0.20x_1 + 400.65x_2 + 400.48x_3 + 400.29x_4 + 400.11x_5$$

where x_1 is MTBI cohort year, x_2 is No response, x_3 is Other response, x_4 is Somewhat, and x_5 is a Yes response, and where y intercept was zero . Otherwise, if y intercept is not zero then other factors get "hidden" into the y-intercept. *Table 8* summarizes the statistical result of the test.

Table 8 Multiple Linear Coefficients on Years Spent in Graduate School with Respect to MTBI Cohort Year and MTBI Experience

Coefficients	**Estimate**	**Std Error**	**t value**	**p value**
Cohort year	-0.1969	0.0608	-3.239	0.00189
No	400.65	121.78	3.29	0.00162
Other	400.48	121.94	3.284	0.00165
Somewhat	400.29	121.74	3.288	0.00163
Yes	400.11	121.65	3.289	0.00163

A significant multiple linear regression was found ($F(5, 65)= 199.7$, $p<0.05$) with a $R^2 = 0.9389$. In addition, all coefficients were statistical significant with $p< 0.05$. This provides another potential factor that may have contributed to the decline in the time spent in graduate school with increase in MTBI cohort year. MTBI helped participants manage graduate school, however we were unable to identify clearly the level of MTBI support that carry on to years in graduate programs.

We further checked if student's publications or ethnicity played a role as other potential factors in decreasing participants' number of years in the graduate school. We found only publication played a role while ethnicity did not. A significant linear regression was found to be (F(1, 68)= 10.3, p<0.05) with a $R^2 = 0.9396$. However, we were not able to determine to which level of agreement or disagreement it influence the model. The information is summarized in the *Table 9.*

Table 9 Linear Coefficients on Years Spent in Graduate School with Respect to MTBI Cohort Year, and Publication

Coefficients	Estimate	Std error	t value	p value
Cohort year	-0.19	0.0569	-3.292	0.00159
No	381.24	113.92	3.346	0.00135
Yes	380.71	113.90	3.343	0.00136

This analysis may suggest that the training by MTBI has been improving over the years and is translating into participants graduating sooner. Another possibility could be that in the 1990's the learning community was still fairly new and in process of forming to support student support.

We also try to see if the negative trend observed in *Figure 17* was present in the whole MTBI alumni of the 102 PhDs. However, from the available information, we only knew the cohort year and time spend in graduate school for 90 of them. The linear model was statistically significant (F(1, 87)=11.4, p< 0.05) with a $R^2 = 0.1159$:

$$y = 435.76 - 0.21x$$

where y is number of years spent in graduate school, and x cohort years from 1996 to 2010 (*Table 10*).

Table 10

Linear Coefficients on Years Spent in Graduate School with Respect to MTBI Cohort Year from Targeted PhD Sample

Coefficients	Estimate	Std-error	t-value	p-value
Intercept	435.76	127.33	3.422	0.0009
Cohort year	-0.21	0.06362	-3.377	0.001

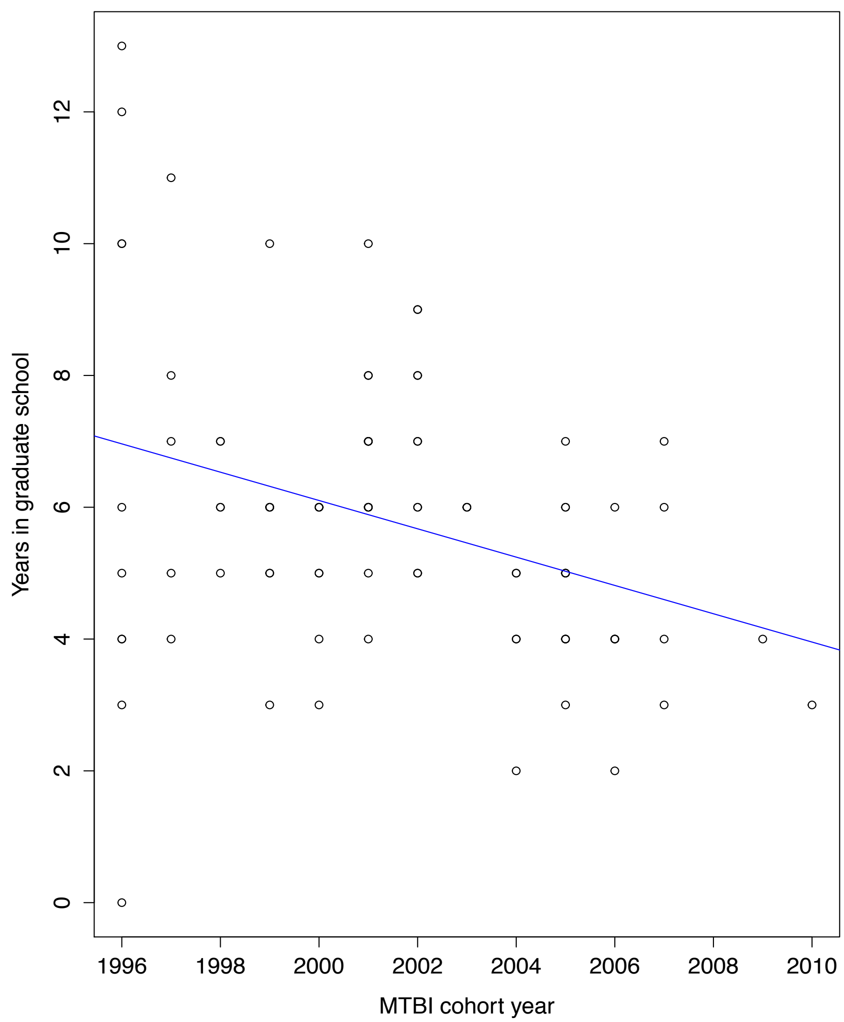

Figure 21. Linear Regression on Data from Targeted PhD Sample

We observed a similar linear negative association as shown in *Figure 21.* In 1996, participants were taking approximately seven years, while by 2010 students are taking an average of 4 years. These findings and our results in *Figure 17* are consistent with each other, and shows the negative trend not only in the people who answered the surveyed but also those we try to recruit from the whole MTBI alumni group.

CHAPTER 5

NEW FACTORS ON DEVELOPMENT

5.1 Newly identify Motivating Factors to Pursue Graduate School

From those 70 participants who answered the survey, 84.29% (59/70) responded to question 15. Five major themes developed from their learning experiences or listed factors motivating them to pursue graduate school in the Mathematical Sciences:

1) *appreciation for the Mathematical Sciences*
2) *desire to grow*
3) *discovery* and *confidence*
4) *beneficial relationships,* and
5) *research experiences*

A description of these themes, including details about their categories follows.

Theme 1: Appreciation for the Mathematical Sciences (18/59)

18 participants stated something in which they described their "love" or curiosity for mathematical sciences, or love of teaching and research. This we classified as an *appreciation for the Mathematical sciences*. Mathematics, applied mathematics, or quantitative sciences we all classified as mathematical sciences. Two categories led to this theme were: *a) interest in the mathematical sciences and b) love for the discipline*.

a) There were thirteen (72.22%) participants who described an interest in the mathematical sciences, and 5 of these 13 (38.46%) also had an interest in research or teaching. We made this distinction because even if applied mathematics might include teaching and research the other 62% did not directly mention them as interests.

b) The other five (27.78%) participants described a passion for the Mathematical sciences by using phrases such as “love”, and ”enjoy”. A clear example is participant 25. Participant 25 was motivated by the “passion for math and research”, while another described how much fun they had with mathematics.

In this theme, none of the participants made a reference to MTBI in providing a reflection on factors to go to graduate school.

Theme 2: Desire for Growth (12/59)

12 participants said something that we classified as *desire for growth*, in which participants described factors of social change or professional growth. Three categories emerged: *a) wanting social change, b) increasing their value in the job market, and c) reaffirmation in pursuing graduate school.*

a) Three (25%) of the participants in this theme described their desire to go to graduate school because they were *wanting social change* by increasing the numbers of the underrepresented or changing people’s lives with math. For example, participant 39 stated that,

> “I didn’t know quite how under-represented we were until hearing her talk [a presentation on the numbers of URMs in STEM] and I wanted to change those figures”.

This participant was inspired and wanted to make a contribution to society and help change the low numbers of URMs. We see this as a *desire for growth* given that the participant wanted to contribute to society.

b) Five (41.68%) of the participants in this theme were motivated because they wanted to increase their value in the job market. These participants wanted to be better prepared or to have better options when looking for jobs. Participant 65 talked about the "need and desire to get a job with better pay and stability".

c) Four (33.33%) participants in this theme made the third category *reaffirmation in pursuing graduate school*. These students were already interested in graduate before they attended MTBI, and then confirmed that graduate school was a choice they wanted to pursue. Interestingly, these four participants all decided to pursue math biology as their graduate school career. For example, participant 49 was already interested in pursuing a master degree in Biology or Mathematics. This participant stated that

> "My research experience and the lectures during MTBI transformed what was at the time a conflict between choosing to pursue a master's in Mathematics or Biology, into a clear desire to pursue a PhD that combined the two".

In this case participating in a research undergraduate experience helped them decide to move forward with this initial interest and helped shape their field of research.

In this theme, four people (25%) made a reference to MTBI in the reaffirmation category, the other eight people and categories did not reference MTBI.

Theme 3: Discovery and Confidence (34/59)

34 participants listed factors that we classified as *discovery and confidence* in which they first became aware of and gained knowledge of graduate school and/or research, and then through this awareness built confidence to pursue graduate studies. Five categories led to this theme: *a) first learning about graduate school, b) becoming aware of ways mathematics could be applied, c) getting familiar with the research process or receiving funding opportunities, d) gaining of confidence and e) learning they can work hard and succeed.*

Twenty (82.35%) of the participants within the group listed their MTBI experience or MTBI influence as playing a role in their motivation to go to graduate school. This theme had the highest percentage of MTBI references compared to the other themes.

a) Five (14.71%) of participants in this theme described how through MTBI they were *first learning of graduate school*. It was when they first realized they had options on what to do with their lives beyond undergraduate work. Participant 4 shared her experience about being exposed to the possibility of graduate school, and then deciding to go. Participant 20 said that MTBI "gave me the first hand experience of how different graduate school would be and experience the degree of intensity required to succeed, especially coming from a small city college".

A different participant stated,

> "I hadn't really considered graduate school until MTBI. MTBI helped me realize the opportunities that could be open for me, and how much I wanted to continue studying math and gave me information on how it could be possible".

b) Nine (32.35%) participants shared about their experiences of getting introduced to the *applications of mathematics*. They talked about becoming excited about the type of work they can do with a PhD in mathematics. Within this category 81.81% referenced MTBI. Participant 40 shared that through MTBI she gained an,

> "Understanding that mathematics is a language applicable to a wide variety of a context, and its power comes from is abstraction was a profound force for me to continue studying mathematics"

In another instance, participant 33 also mentioned that "mainly the realization of the ample applications of mathematics to the life and social sciences" was a factor.

c) Three (11.77%) of the participants in this theme described the importance of *getting familiar with the research process* as a main motivating factor. Participant 8 described that it was helpful getting familiar with research in general such as learning Matlab, LaTex, and how to write technical reports in MTBI. In another instance participant 25 mentioned, “MTBI helped me build the foundations for conducting research”. An example of a non-MTBI reference stated as a motivating factor was “participating in research activities“.

d) A fourth category within this theme was that of *gaining confidence* to further pursue graduate school. All seven participants who contributed to this category shared the overall effect that their MTBI experience had on their lives. They became confident to pursue graduate life. First of all, there was a sense of self-discovery about their abilities to do research and a general feeling of being capable of succeeding in graduate school. Participant 24’s response about her MTBI experience was:

> “The intense 2 months as an MTBI participant probably 'over-prepared' me for life as a graduate student. From MTBI I learned that (1) I had the ability to do research and learn new subjects outside of the classroom and (2) Hard work outweighs intelligence. These lessons led to a successful graduate career”

This participant describes how the MTBI experience led to a discovery on her abilities to do research, as well as being a learner outside of the classroom. No gender inference should be made with individual quotes. We have altered the pronouns throughout the report to a female participant. The demographics are reported in section 3.3.

In another instance participant 21 shared that:

> "MTBI did not play a role helping me decide on what subject to study during grad School, but it helped me mature, gave me confidence and I made excellent contacts that helped me obtain a scholarship to pursue a graduate degree".

In addition, her participation in the program provided the opportunity to get funding to pursue that path. Another participant also listed this same factor of getting funding to attend graduate school.

e) In the last category, five (14.71%) of the participants described a realization about their abilities to work hard and succeed: *they can work hard and succeed*. Participant 66 listed "hard work leading to breakthrough "ah ha" moments". Participant 66 saw hard work as conducive to learning. Participant 24 also stated that through MTBI she learned that "hard work outweighs intelligence" and listed this as one of the factors in helping to have a successful graduate career. It appears participants have associated a relationship of hard work to success and learning. Among this group, three (60%) of the participants made a reference to learning this skill while participating in MTBI.

Theme 4: Beneficial Relationships (14/59)

14 participants listed *beneficial relationships* among peers or role models as motivation. This fourth theme consists of two categories: a*) beneficial peer relationship* and b) *role of mentors* were a motivation for these participants to pursue graduate school. Within this theme, nine (64.28%) made a MTBI reference.

a) Six of seven of the responses on beneficial peer relationships referenced MTBI. The shared sentiments among 85.14% of the participants was the positive effect of meeting other peers, such as peers interested in math or similar interests, enjoyment of learning together or having the same desires of attending graduate school. Participant 11 clearly describes this sentiment,

> "I attended MTBI before my senior year of college and had a great experience. It was a lot of fun working people interested the same topics, learning new areas and learning more about graduate school…"

b) The *role of mentors* also were an important factor cited for the other 50% of the participants as a motivation to pursue graduate school. The participants talked about both their undergraduate mentors or graduate advisors. 57.14% of the participants in that category talked about the "good mentors" they had in their undergraduate career that inspired them to continue to graduate school. The existence of role models motivated them to continue in that path of higher education. 42.86% talked about their admiration for MTBI faculty in their example.

Theme 5: Research Experiences (21/59)

We grouped the responses of 21 participants who listed research experiences as a factor. This included when participants used phrases like "prior research experiences" or "getting introduced to research experiences", or "having research experience". The three included categories were: *a) having prior research experience, b) undergraduate research at undergraduate institutions and c) the introduction of research experience* as a result of their MTBI participation. Within this theme, 62% included an MTBI reference or influence from the program.

a) Five (23.81%) of the participants listed *having research experience* as a factor. Among this category, 40% of these participants listed research training or research experience as a factor. The other 60% mentioned the similar sentiment of gaining research experience in MTBI, and hence becoming motivated to go to graduate school. Participant 31 best summarizes this sentiment when stating that,

> "Having the time to look for what I was interested in, and when found, being able to work full time on that without any other worries was a great experience. Also, realizing that getting a PhD would allow me to work in a similar way as in that summer was one of the main contributions of MTBI".

b) Seven (33.33%) of the participants identified research completed at their undergraduate institution as a factor. An example is participant 65 who "enjoyed doing research in mathematics and theoretical biology at MTBI as well as my university". Participant 65 described the enjoyment out of doing research whether at her undergraduate institution or MTBI. Others just listed general REU experiences as factors, without going into further detail of the influence of this type of research in their educational careers.

c) In the last category nine (2.86%) of the participants described the role that *getting introduced or gaining research experience* as a result of their MTBI participation played in their career. 33.33% of the participants within this sample category described how they learned about the process of doing research, or how they discovered the applied side of mathematics and wanting to pursue that road while attending MTBI. Participant 32 shared that she was inspired to purse of a PhD in Mathematics as result of her MTBI research project. She stated that,

> "Carrying out the research project as part of MTBI introduced me to mathematics research, particularly being exposed to how mathematics can be used to solve real world problems. This inspired me to pursue a PhD in mathematics."

This person felt inspired to pursue a PhD after learning that she can solve real world problem using mathematics.

Three of the other participants listed the importance *of gaining research experience* as a result of their MTBI experience as a motivating factor. These three participants used phrases like "getting introduced", so we did not included them in the category "having research experience".

These participants briefly described that attending or gaining research experience was a factor in motivating them to go to graduate school. For example Participant 63 shared that "doing research as part of MTBI caused me to become interested in doing research, hence grad school".

5.2 Newly Identify Factors on Motivation to Continue and Complete a PhD

In this subsection we provide the findings for question 20, where we asked participants: What learning experiences or factors contributed to your continuation and completion of a PhD? If MTBI played a role, please include a reflection of how aspects of the program contributed.

For question 20, 52/70 participants responded or about 74.29% of those who completed the survey. The responses were similar in size to question 15, from a one-line sentence to a paragraph. Using the constant comparative method, we broke the 52 responses into 98 distinct codes. From these codes we further identified 14 categories. Four main themes emerged that encompass participants' descriptions of their learning experiences or factors that contributed to their continuation and completion of a PhD. The four themes were:

1) *desire for growth*

2) *research experience*s

3) *positive interactions*

4) *developing professional and personal skills*.

After coding was completed, we also noted who made a MTBI reference or not. Description of these themes and associated categories now follows.

Theme 1: Desire for Growth (12/52)

A group of 12 participants described a *desire for growth*, which included professional growth or personal motivation. Three categories emerged in the analysis: a*) interest in working in academia, b) personal motivation, and c) increasing their value in the job market.* Similar to the *desire for growth* theme in 15, we identified categories for increasing job market. The other two categories are unique to question 20.

a) Three (25%) of the participants expressed a desire to go either to academia or do research. Participant 62 stated she "wanted to get a university job".

b) Six (50%) of the respondents listed *personal motivation* as factors they used to grow professionally. One participant talked about the "finish what you start" attitude, another one cited "discipline and hard work", and another one described "commitment to completion and focus" allowed her to persevere and finish. Two other listed confidence or having strong statistics and mathematics background as reasons. Participant 5 listed a desire to create social change and increase numbers in the underrepresentation in STEM (same person as in Q 15).

c) In the third category, 25% of these respondents expressed an interest in increasing their value in the job market as a reason. Participant 6 responded:

“The desire to be in academia and do research but also the economic stability and better chances of getting a job”

In the statement we can see that participant 6 was not only interested in working in academia, but also in better job opportunities. No participant made a reference to MTBI experience in this theme.

Theme 2: Research Experiences (29/52)

29 participants listed research experiences as a motivating factor, which consisted from participants getting introduced to and loving their research experiences. Within this theme of research experiences we had three categories: a*) undergraduate learning experiences, b) getting introduced or gaining research experience, and c) enjoying of research and mentoring*. In addition, 19/28 or 67.86% made a reference to their MTBI experience.

a) In the first category, 11 (37.93%) of the participants described their interest or having undergraduate research experiences as a factor in motivating completion of their PhD. We classified this as u*ndergraduate learning experiences*. Four participants described their interest or desire to do research in general. One mentioned she wanted to do research in mathematics, and another in math biology. The other seven participants mostly credited “having research experience” as contributing to finishing their degree. These responses were mostly short and concise. From this category, 36.36% specifically made reference to the research experience gained as a result of MTBI.

b) In the second category, nine (31.03%) of the participants listed either *getting introduced or gaining research experience* as a shared experience that contributed to the continuation and completion of PhD. All participants provided a reflection that included learning about the research process, and having an experience similar to what graduate life would be like.

Specifically, four of 9 participants described how they first learned how graduate school is time consuming; or MTBI research similar to “all but dissertation phase” or also MTBI as a “boot camp” for graduate school. Participant 48 talked about having an understanding of the different aspects in the research process. She stated that MTBI,

> “was "boot camp" for grad school, and gave me a chance to
> experience everything from picking projects to writing up and presenting
> results prior to doing all those things as a graduate student”

Five of the 9 participants stated that learning about the actual research process was a key factor. They talked about getting introduced to the process of scientific research in math bio, or being exposed to collaborating with others. We further see this when participant 48 stated the role research played in her life:

> “My research experiences during MTBI helped me have a realistic idea of how the research process would progress, and helped give me the confidence that I could finish”.

This participant said that that having an idea of the research process and time was a factor that contributed to building confidence in her ability to finish.

c) Nine (31.03%) of participants said that *enjoyment of research and mentoring* others was a contributing factor. Six (or 66.67%) of the participants made a reference to the “great research experiences” or “MTBI as an injection of motivation needed to continue working hard towards my PhD” said an advanced participant. A third one mentioned that she “enjoyed the study and research”. Three (or 33.33%) other participants also described the *enjoyment of research and mentoring* others as a contributing factor. Three people said something like this quote, "I enjoyed the study and research". Three of these 9, shared a sentiment of having had a great experience mentoring students through MTBI, and using this sentiment to complete their PhD. For example, participant 41 listed:

> "[MTBI] it was great experience teaching interdisciplinary subjects to a diverse audience, and mentoring students and guiding the formulation of research questions into a mathematical framework, as well as directing the exploration of those questions".

Theme 3: Beneficial Relationships and Obtaining Support (22/52)

In the third theme 22 participants described the different interactions that helped them continue and complete their PhD. Participants described how having a support from the MTBI community and good mentors was helpful in them continuing and completing their PhD. Three categories emerged: participants felt supported or inspired through the a*) MTBI community, b) getting funding, and c) having a good mentor was helpful.* Within this theme, 72.72% of participants made a MTBI reference.

a) Eight (36.36%) of participants talked about the importance of the MTBI community in providing support to the students. By MTBI community, the participants made references to faculty, graduate students, other graduate peers from institution and/or undergraduate MTBI alumni at different institutions. *Participant 9* shared that:

> "The MTBI community played a big role in the completion of my PhD. It was nice to see that others were struggling just like I was and we all encouraged each other to continue".

For this participant, one of the factors was finding support in the MTBI community when struggling in graduate school, and others among this 36.36% expressed the same sentiment of feeling supported by other peers, or made reference to the new friends they made because the MTBI community. Participant 58 further highlights what she meant on the importance of a supportive community or network community. She talks about:

> "A good network of fellow graduate students. We helped each other a lot and motivated each other. Many of the students that were in my network were also MTBI Alumni. In fact, my support network extended to students not in my PhD. institution. Many of them were MTBI alumni who were in different institutions".

So participant 58 described the importance of motivation and support from peer graduate students at her institution but also support from MTBI alumni at different graduate institution. Interestingly, Participant 58 explicitly described what she meant with a support network: "*a group of people that cared about and believed in me, and that gave me academic and personal advice".* This was a supportive network for getting academic and also personal advice.

b) Seven (50%) of the participants described the great mentorship they received as a positive interaction and experience that helped them finish. Four of these used the word "good mentors", 'helpful advisor', and "advisor support" as a factor. In this group 7 made a clear reference on how the mentoring from MTBI faculty or mentors met through MTBI were helpful. Participants in this group described the mentorship and guidance from faculty as key factors, and 36.36% described the important role the program director played in helping them feel supported in graduate school.

c) In the third category, three (13.64%) of the participants felt that having financial support was a factor in them completing their PhD. In the previous categories most participants received support from the MTBI community but did not mention having monetary support.

Theme 4: Developing Professional and Personal Skills (12/52)

Another major theme coming out of their responses was in reference to participants developing professional and personal skills. In this theme, 12 participants mentioned learning persistence, managing stress and a heavy workloads. Three categories emerged: *a) developing writing skills, b) developing connections in professional settings, and c) learning how to overcome challenges.*

a) Three (25%) of the participants talked about understanding the importance of writing, rewriting, but also developing these skills at MTBI through technical papers. An example is stated by participant 23:

> "Writing is very important. I built up my writing experience developing technical reports with MTBI. I got further training developing journal articles during my graduate career. Journal articles essentially made up my dissertation".

For this participant developing writing stills was helpful in her career development. The participant got initial training and experience in MTBI then from there continued to build and improve.

b) Five (41.67%) of the participants described other professional settings (networking within the Mathematical sciences), such as attending internships, seminars, conference, and summer programs (which could have been REU but they did not specify), or collaboration with external faculty to their institutions.

c) Four (33.33%) of participants described learning how to overcome challenges while in graduate school. One student talked about difficulties finding a suitable advisor and research area, but then eventually finishing. Another one stated that learning to be persistent was a factor. For participant 36,

> "MTBI helped me learn how to manage heavy research work loads as well as managing stress".

In this category 75% made a reference to MTBI and described that because they have had a similar experience in MTBI, they felt they could overcome or manage these type of issues.

Other

There were three respondents that were put in the "other" category because the responses did not directly fit in the other themes nor were they similar enough to create a new theme. The first category was "slow job market and less of a glass ceiling with a doctorate", so in away its also wanting to improve job perspectives. This response could possibly go with *increasing their value in the job market.* Another participant mentioned family support as a factor, and another mentioned that after MTBI she reaffirmed she wanted to go to graduate school, but not in Mathematics.

Discussion on Pre-Determined and Newly Motivating Factors

We previously mentioned that a ZPD is a creative environment in which learning occurs socially and then individually. This learning and development is characterized by creative imitation and collaboration (a co-creation process). We also see the ZPD as a process and activity (Holzman, 2010), in which an activity is seen as "ways of relating to people", and development as a creative process where individuals can change and shape their relationships to themselves or others.

There were two themes that emerged in both results of Q15 and Q20. These were *research experiences* and the *beneficial relationships*. Within the *having research experiences* category of the *research experiences* theme MTBI was reference by four of 9 respondents in Q15, and four of 9 in Q20.

We identified a combined total of 9 themes in Q15 and Q20. We summarize four of these themes and their categories next:

Table 11

Themes and Categories for Question 15 (Q15) and Question 20 (Q20)

Research Experiences (Q15) (Q20) Having research experience (4/21) (4/29) Getting introduced or gaining research experience (9/21)(9/29) Enjoying research and mentoring (NA)(7/29)
Beneficial Interactions (Q15)(20) Positive peer interactions (6/14)(NA) Inspired by role models (3/14) (NA) Having a good mentor (NA)(7/22) Be part of a community (NA)(8/22)
Discover and Confidence (Q15) Discover ways mathematics can be applied (9/34) Understand that they can succeed (3/34) See graduate school as future path (5/34) Getting familiar with research process (3/34) Gaining confidence (7/34)
Developing professional and personal skills (Q20) Developing connections with others (1/12) Developing writing skills (2/12) Learning how to overcome challenges (3/12)

We now describe the implications of *Table 11*. We see the positive interactions as part of the participant's development, and as part of the Zone of Proximal Development. MTBI supports development by supporting students to:

1) *discover new career options*
2) *understand that they can succeed*
3) *experience research first hand*
4) *be part of a community and*
5) *developed as professional applied mathematicians.*

In question 20, 30.77% of all participants described the beneficial peer and faculty relationships as factors in motivating to pursue graduate school. In question 15, 15.25% of participants listed beneficial relationships, 23.08% talked about developing professional and personal skills, and 38.46% talked about learning or enjoying MTBI research experience.

In the discovery and confidence theme in question 15, we can see examples of people getting introduced to the ideas of graduate school, getting familiar with research, and applications of mathematics. This made up 47.46% of all participants, almost half of the people survey. In addition 22% described about getting introduced or gaining research experience through MTBI.

Discover new career options. In this category, participants' decision to pursue graduate school were shaped by information and/or the activities they had participated in (mostly at MTBI). They learned of things they did not know before, and they had an environment to experience and participate in activities and discussions that were not previously part of their lives. Becoming aware of new ideas or career options allowed students to change or modify their expectations, how they see themselves, and how they see their abilities. Participants learned about and recognized opportunities and were able to pursue them. Their ability to pursue opportunities is related to their recognizing that they can become applied mathematicians.

Understand that they can succeed. The students learned about graduate school opportunities, application areas of mathematics, and the research process. They learned that they can do all these different things, and importantly they learned that the things that are required to be successful in graduate school are not beyond them. In MTBI students are expected to succeed at learning and doing research even if at first it may seem daunting. Students may have no prior experience modeling or the topic of research they choose is not well known. Holzman's work on the methodology of becoming based on Vygotsky's ZPD refers to this as the students working a "head taller themselves". In MTBI, this looks like participants not knowing how to research when they come in but learning to do it through collaboration and imitation.

Experience research first hand. A major component for the students is learning to do research at MTBI. Participants experience the whole process of research in applied mathematics including project formulation, analysis and project presentation. These participants experience the research environment as a research activity consisting of a co – created environment because students and the MTBI community are creating the research together and in the process creating new scientists.

Be part of a community and develop as professional applied mathematicians. Participants get initiated as new members of the MTBI community and to becoming new professional applied mathematicians. Whether as a factor to pursue a graduate degree (question 15) or a reason to persist and complete a PhD (question 20), participants described an appreciation for Mathematical sciences, liking the quantitative sciences or loving the research process as a motivating factor. Others described a desire for personal growth or professional interest. Some of these are intrinsic factors, i.e. personal preferences contributing to their interest to go on to complete a higher education degree.

Even though internal factors continued to encourage them when mathematical sciences was their interest, participants gaining knowledge and opportunities such as those provided in REUs, helped them to pursue graduate school. They recognized the possibility of pursuing graduate school and research. This is important for people taking charge of who they want to become. Mentoring efforts to introduce and encourage people already interested in Mathematical sciences are important to change the low numbers of underrepresentation in STEM.

In addition to the factors associated specifically to MTBI, two motivating factors were identified for the pursuit of graduate school: 1) curiosity and love of Mathematical science, and 2) a desire for personal or professional growth. From question 15 (pursuit of graduate school), 18 people listed an interest or love for mathematical sciences, and 12 participant also listed that a desired for grow was factor, whether to increase their value in the job market, wanting social change or interested to working in academia setting. In question 20 (continuation and completion of graduate school), 12 people stated these same motivations.

CHAPTER 6

ON CHARACTERIZING COLLABORATION AND CREATIVITY

6.1 Findings on Collaboration

Based on our theoretical framework, we focus our analysis on how participants observed instances of collaboration and creativity in the program. More specifically, there were two interview questions we focus our work on. These two questions were:

- Can you provide one or two scenarios within your MTBI experience that embodies collaboration? *What aspects of the example are you identifying as collaborative?*
- Do you think MTBI provides an environment that fosters creativity (creativity *as in being able to take old ideas and make something new or different)*? Can you provide one or two scenarios within your MTBI experience that embodies creativity?

All participants experienced collaboration at different times in the program or within different elements. All provided a scenario of collaboration from working on their MTBI research projects, and half of them provided a second example that involved collaboration with peers while doing homework or computer labs in MTBI. All eight described the importance of people sitting down in a common place, brain storming about ideas on learning, and brainstorming on how to develop and refine their mathematical models. They talked about a collaborative process where they met in a "common" place, whether tables, couches, dorm rooms, and started working with their faculty advisor or with their peer collaborators. This "common" place was a space in which all members mutually agreed to work and where they could spend many hours thinking and working on their research projects.

Collaboration with Faculty

Three of the eight examples specifically described when doing research with their faculty mentor. The two participants with an individual project described collaboration in relation to their advisor. One participant worked in a group and referenced the advisors as part of refining the group's work. This person mentioned the collaborative relationship with advisor as well as peers. Her description of collaboration did not exclude the group, nor did she focus on the advisor more than group.

Two of the participants who mentioned working with a mentor were students with individual projects. In this case, one participant was a returning student, and the second was a participant who was an international undergraduate. Most first year students usually have to work in groups, unless they are returning and/or graduate students in the MTBI program. These two participants described the informal way they worked with their project advisors. Similar to a peer-like relationship in part because all involved in the research projects did not know the topic well. The faculty would learn and brainstorm along with the participants. One participant stated that her scenario involved her second year participation in MTBI because she learned a lot of mathematics in that project, and she was “happier” with that project. She remember working very “closely collaborating with Jorge”. She goes on to describe,

> “We used to work all day together, we were discussing, we were actually thinking. I remember people looking at us, we were just seating there on some couch. They were wondering what we were doing, just seating there, and it turns out were

actually thinking on our problem. But that give me a lot…ah, it was a learning experience in collaborating with Jorge in that way. And of course … (no audio) collaborating with someone, thinking a problem with someone else, and working truly, made a really rich experience for me".

Within this response we observe completion happening as the participant described the collaboration happening in the program, similar to as we described in the chapter 3.   This participant talked about the thinking and speaking they were doing together, as both discussed how to proceed with the research project.

A second participant described collaboration with a supervisor or mentor as part of her experiencing collaboration. She stated,

"I think the work with the supervisor was also collaboration, even though perhaps it was supervision in the sense obviously the supervisor had more experience in the area. It was still collaboration because it was a topic that he would not had look at before"

The third participant recalled a general interaction between her group and MTBI faculty. She participated once in the program. Her collaborative example consisted of the iterative process of collaboration between students and faculty in sharing and refining ideas. She recalled,

"… how our student group had to work together to decide on a research project and then we also worked with our mentors, Dr. Steven Wirkus and Dr. Erica Camacho to further refine that research project and also working with Dr. Castillo-Chavez as sort of the final, you know, we would bounce a lot of ideas of each other, of students, and then passed those through you know our mentors and up to Dr. Castillo-Chavez as well. So we were sort of working together on all different levels to hone in on what research project we would tackle.

When we asked for a specific instance of working together, this participant stated that she could not really give one. All she remembered was “working together everyday sitting around in a table coming up with ideas, but other than I can’t”. We can observe in her response, an experience of collaboration of going back between one another among peers, their advisors and then program director.

One main theme that shows up among the three participants is the active role participants and mentors had in collaborating and thinking together to complete the research project.

Collaboration with Peers

The other five participants also provided a scenario of collaboration while working on their research projects but the scenario happened while working with their fellow MTBI peers. All participants described a recollection of collaboration when they were first developing a question: all members from the group contributing to the research process and finishing successfully.

Two participants described a learning process that began in first learning how to collaborate together. For example, a participant discussed the different stages of teamwork, mixed with different intellectual and emotional skills for “norming-storming”. This participant made a reference to a model of team formation in which a collection of strangers united to work together (MindTools, n.d.).

Another participant, described how she learned to frame a question of interest and not just coming up with a theoretical model to do analysis on. She recounted how she created a predator model with a full analysis in the first two-three days research project development. But then one of her teammates did not approved of her approach and redirected the group. Her account on what happened is as follows,

> "….Now, the thing I called collaborative about this, is that, he could have walk away and said, "I don't want be in this group anymore". It was very earliest in the group formation. He did not walk away, he said, "we are not doing this project because there is no reason why we should this project. But we all are going to stick together, and what we have to do now is, let's go our separates ways, let's go think of questions that are interesting to us, and let 's come back together, and let's study each and everybody's questions". We had about 2-3 questions to develop, the why of the question. Let's all talk about the why, and then will see which one is most pertain to all of us. And then we can go ahead and tackle the questions".

This participant later talked of how this experience showed her the importance of "why" of questions.

The other three among this theme of collaboration with peers did not mention how they learned to collaborate together, however their scenario descriptions included similar patterns: there is a research question formulation, a plan and attack strategy to finding a solution. This is all while working in their groups or by dividing tasks, and regrouping.

One participant, for example, gave a collaboration scenario that involved all group members suggesting a research topic, and then her topic being chosen as something to investigate. She stated,

> "we knew how we were going to use a differential equation, but we were just like

what do we put into it…Then someone else in the group was like 'well what are the things [that] are not typical but it would be a different way to look at, how to model disease x[1]'?"

This was their strategy as they starting shaping their research model. The group together with their student mentor connected their topic interest of disease x, and what they were finding from their research literature to create their model.

The participant described and summarized the collaboration experience in this way:

"So for me that's collaboration … when you are able to take from everyone to take parts from everyone to kind of answer the questions that you want to answer".

This specific scenario resembles the type of interactions that will happen among collaboration with peers and mentors, as we described in chapter 3. Even though only one person described the interactions, we can still see the interactions and statements contributing to participants creating a mathematical model for their research question.

Four of the participants experience collaboration happening in MTBI when they worked on their homework or computer labs. These participants described how the help from peers, especially from the more knowledgeable ones about the material, was important to them learning and finishing their assignments.

Two of the participant described working in collaboration while working on lab sessions. One learned from peers how to use LaTex. The second one gave an example on how peer collaboration helped her understand computer programming (in addition to collaboration being present when doing homeworks). She said,

[1] To protect participant's identity we will not disclose what specific disease the group was investigating

> "I have not been that good in computational stuff, and I remember when I was doing the projects, there was always somebody in my group that was stronger in programming, for example. Although I didn't work directly with the programs themselves, they help me to actually understand through a collaborative process how the programs were made. Even I remember, not only for the project but also for the computer labs"

In addition this same participant gave a third scenario of a collaborative process when preparing a manuscript for publication from a project in MTBI. She stated,

> "….the manuscript ends up being very close to the technical reports we did. But also the process of learning how to submit a manuscript to a journal. The faculty was a facilitator in that sense, which would help us to understand how was that process.

Two participants even described how different peers were able to work together, divide tasks depending on their strongest individual academic knowledge, and regroup, similar to what they would later do in the research phase. One of these two participants described the process as,

> "So what we would do is we would split up responsibilities within our group, like please make sure that you understand what's going on the first part of class, is this person might be a little bit more tired from previous jobs or they might have a better grasps on the subject that they are teaching today versus another, so you are going to have to come back in the group and kind of re-explain what is going on. So that's what we did in our group and I know other groups started doing that too, and we would talk in between groups as well."

Reflection on how Collaboration was Useful in their Current Occupation

All participants felt collaboration was useful to them in becoming applied Mathematicians or whichever occupation they are currently working. For the four participants in academia, it gave them an opportunity to develop collaborative skills, and then seek this kind of collaborative opportunities. The others felt collaboration was useful in them becoming successful in their career. One participant stated, “if you don't know how to work with others you can't have successful collaborations in general you do not have successful research”. They also described how they seek to work with others because through a collaborative process they have been able to work on their jobs, mentor students and work on publications. One of these students further stated that, “I don't know if this is a good thing or a bad thing as far as independent research goes. But the research I love to do is collaborative research, and the research I love to do is interdisciplinary research”

For three participants working in industry, the reasons vary. For one participant it was important because she felt the communication skills learn through collaboration were useful in any area (academia, private sector). For a second participant, it was useful as an “initial conditions for greater learning”, and for the third participant, it was useful because she felt she learned how to compromise and push each other. One these participants describe that, “push and challenge each other to make sure that the things that we have make sense and are good. You know those are definitely things that helped in my career and also in my education, being able to think critically”.

In another instance, another participant described how the initial conditions would serve for greater learning, “but it put the whole process, something has to jump start the process right and that was it. Then just not just a mathematician but just as a professional, half of the game as such an industry is knowing how to deal with people and people come from all different backgrounds and skill sets and you know the little funny issues that we deal with as twenty year old kids pretty much are the same as adults it's just that it's just more is at stake so the sooner you learn how to deal with this and win you know come up with a product, the better you are going to be". Hence, it was useful in learning how to deal with people.

For the last participant within this theme of reflections, it helped her reinforce she wanted to be an applied mathematician and become a researcher. She already knew she wanted to be an applied mathematician when she attended the MTBI program.

6.2 Findings on Creativity

We identified creativity as *being able to take old ideas and make something new or different.* We asked for specific scenarios in the interviews to hear from their point of view what they identified as creative or creativity in the program. Seven of the participants agreed that creativity was fostered in the program, while one felt that even though creativity was encouraged, it was hard to implement. We interpret this to mean that she did not think creativity was present. Among the seven participants, four participants agreed to creativity being present in the program, while three agreed very strongly. Based on their agreed response, we observe 1) a description that creativity happened and it was visible in MTBI, and 2) that it was first visible in the research projects. We now provide a detailed account of their responses.

Agreeing

Four people agreed that creativity was present by uttering words "yeah", "I think so” or immediately describing how creativity happened in the research projects. Three of the four participants described how the process started within the initial stages of research development. One of these participants elaborated further on how the program fosters creativity and said that,

> “I know some people would feel that you're kind of directed. You are guided to be creative in a way because you can't just be like “hey be creative” because you can come up with something that is useless. You are given a foundation, a rigorous kind of initial framework of all this kind of homework sets and basic foundation that everybody gets. So automatically you have a brand new kind of developed skill set and then from then you probably get to ponder what you find interesting”. So for this participant creativity was fostered once everyone have the same foundation to ask useful questions.

The fourth participant described how it started in “developing the idea for the research project” along side the mentors, and how “it was very iterative in nature coming up with levels of refinement to the model”.

Two of the four stated that creativity was visible in their own research projects, while the fourth did not provide a specific example. One had a theoretical mathematical background and decided to do a theoretical project based on two competing strains. The other one stated “the first year I chose the engineering project which was different and that got my creativity to the point I ended up doing a master's thesis”. We did not ask why she identified this as creative. It seems she used her MTBI work and then expanded on this to do her master thesis.

The fourth participant stated “I mean this is basically what they do all the time, they take the SIR model and they apply it to everything. They apply it to [one topic] and truck drivers and apply it to violent behavior, you know what I mean so there is a lot of creativity in the sense that they take this standard models that we use for epidemiology and apply it to other areas where it hasn't been applied before".

Strongly Agreeing

Three participants agree strongly that creativity was present in the program. Participants used the phrases "absolutely", "most definitely" and "this is MTBI" as responses and then proceeded to described that creativity was present or fostered in the formulation of the research projects. One of the participants identified when they "use ideas of epidemiological models" to model other "issues from life to social sciences" as creative. In another instance, a participant described how the creativity question

> "encompasses everything that is MTBI. So, when you do research, we were able to choose our own problem. And even though maybe I have some ideas in some of the years that I went, maybe not all those ideas get through a research project, but then I was also able to understand the creativity process of what was going in other students' mind".

This participant described the creative process in choosing their own projects and communicating their ideas to each other. The participant further described how in a returning year she was able to work on a previous published work to follow her own research interest and incorporate new aspects and develop a new mathematical model.

The third participant described that MTBI was her third internship research experience, including another undergraduate research experience and research experience at her home institution. She felt MTBI most definitely fostered creativity. MTBI fostered creativity because undergraduates had influence and control of their research questions. In all her educational experiences from undergraduate to postdoctoral, she felt it was in MTBI that you could ask any question and be like "sure let me ask". She further stated that she always remembers the program director stating,

> “Undergraduates come up with the best type of research questions because they have no inhibition and they don't know what they can't solve".

She felt it was true because at that age the young mind did not worry about the limitations in asking any kind of question. This particular participant described creativity happening when students were given the control over their research questions. She did not describe an example of creativity in particular.

The third participant also saw creativity happening in using SIR models apply to different areas. She stated that in MTBI students are "encouraged to use the ideas from epidemiology to model a lot of different issues from other biological issues that come from differentiation of cells, smoking, or in some sense modeling ecological process as diseases, or even as social issues…”

Not Agreeing

One participant of the eight felt that even though professors and assistants from MTBI encourage and appreciate creativity, the mathematical content taught and limited time of the program did not necessarily allow for much creativity to happen. The participant explained that sometimes is hard to be creative because of the time frame and

> “sometimes is more realistic for students to fall back, and maybe the student isn't being bold enough. I could say, sometimes I was not bold enough. I would fall back and do a predator-prey model because I've seen 20 of them while I was in

MTBI. I go and do an SIR model because I have seen 15 million SIR models during MTBI".

However, the participant did state that it does take someone to be bold and ask, "Ok, this are the methods that I learned, what kind of other questions can I attached these methods too?".

Remarks

Seven out the eight participants agreed that MTBI fosters an environment of creativity. These participants agreed that creativity happened in MTBI and it was visible in the research projects, whether they initiated the research projects or in other extensions of SIR models. We provided participants with a creativity definition within our theoretical lens, and as result their responses were similar. However, the examples of creativity had to do with each participants' MTBI experience. They described what experience matched the definition. One thing we could have done differently was not to give them a definition for creativity and this might have broaden their identification of creativity within the program.

We did not ask how creativity was helpful after MTBI. However, one student did mention she used his MTBI research project as basis to her work towards her Masters. A second participant used her two MTBI research experiences to see the creative process involved in developing research projects, which she felt led to her taking a leading role in a project in her third time as a participant, later others joined and together built their research project. From research experiences and educational experiences, a third participant described that sometimes she would not even attempt certain or new problems because of mathematical limitations (such as problems not being well-posed) or funding sources. She is still able to work on research projects, but not necessarily on topics she would love to work on. Despite experiencing the creativity of MTBI, this alumni does not appear to have much room for creativity in her current career.

CHAPTER 7

IMPLICATIONS AND CONCLUDING REMARKS

Previous research hypothesized the success of MTBI (Castillo-Garsow, et. al, 2013), as well as the success of MTBI was due to the ways it mentors students through a collaborative learning environment and supportive mentorship from faculty on the research projects (Camacho et. al, 2013). However, theses studies were observations from faculty and some demographic data. There was no systematic analysis or framework to study questions posed in this research study. For our study, we used data collected from participants of MTBI 1996 to 2010. The goal of the study was to investigate to what extent MTBI could be characterized as a Vygotskian environment, by focusing on Holzman's work on the methodology of becoming and Vygotsky's work on the ZPD. The focus is on the learning environment and the relationships formed from the activities people do together through collaboration and imitation. To this end, we addressed two questions that guided our study:

1) **What characteristics of MTBI, including relationships built in the program, helped undergraduate participants grow or change in their educational and professional careers?**

2) What did participants learn in collaboration or creative imitation?

We assumed participants in the study had different backgrounds and learned in multiple environments so we also asked about other learning experiences or factors motivating them to pursue a higher education. These were the two questions in our survey:

- What learning experiences or factors motivated you to pursue graduate school in the Mathematical sciences? (*If MTBI participation played a role, explain how*)

- What learning experiences or factors contributed to your continuation and completion of a PhD? (*If MTBI played a role, please include a reflection of how aspects of the program contributed)*.

Characterization of Participants

In characterizing participants in MTBI, we found that around 79% of participants remained involved in academia even after completion of their studies. Our preliminary numbers (in chapter 2) indicated that MTBI accepts an equal number of female and males as first time participants, and our results showed that about the same gender ratio of participants have gone to complete their PhDs. This is good news given the underrepresentation of women in STEM as well.

We also found that 71% of participants were underrepresented minorities, and for 67% of these students MTBI was their first REU experience. This is important because MTBI is providing research experience to people who have never had the opportunity, and where the majority are from underrepresented groups. Our data also suggests that a large number of participants were either encouraged or remained encouraged to further their studies and even remain in academia. We also found that the average number of years participants spent in graduate school decreased by 40% in time length, so in recent years the average length is 5 years. The national average for a PhD is 8.2 years (Berger, 2008). This seems to indicate that the MTBI program improved in its mentorship over the years. However, it also could be due to intake of academically better-prepared students in the later years for MTBI. We also speculate that students have developed their leadership abilities. Leadership as described by Camacho, Holmes, and Wirkus (2015):

> "... the learning community in which all individuals provide leadership to their own learning and the learning of others" .

This could be reflected in students being better able to direct their graduate careers and therefore decreasing the length of time to completion of the PhD.

Potential Factors on Educational Development

Castillo-Garsow et. al (2013) had identified six factors (intellectual and cultural diversity of participants, the challenging homework sets, student directed research, the learning community and the role of past participants) that may have contributed to the success of the MTBI program. Our results confirmed their hypothesis on mentoring relationships as critical factors for participants' educational development in addition to being a factor for the success of the program. *Training on how to become a scientist, high expectation on students and student initiated research* were also significant factors, yet their importance was lower than mentoring relationships. These last three factors make reference to the learning environment of MTBI. These findings on how to become a successful applied mathematician, and becoming part of the learning community are similar to Hunter, Laursen and Seymour 2006's finding which identified factors such as: positive growth in their identities, careers, and professional development as scientist from their REU in science.

From the perspective of the MTBI faculty, challenging homework sets are part of the high expectation for students (Castillo-Garsow, et. al, 2013). However, the survey respondents indicated other factors as more significant than this factor in their motivation for continuing to graduate school. This discrepancy is likely due to the different focus of our research and the faculty authored papers written before. In our work, we are interested in identifying the character of MTBI that supports student development. As such we ask them to reflect on the factors that contributed to their overall success in academia. It is reasonable to believe that former participants would not identify homeworks in a particular program as critical to their overall success, regardless of their importance for success within the program. In contrast, the faculty have reported on what the students need to be successful in the program. Thus, faculty who are aware of the skills needed to complete the research projects, would identify the homeworks as a high expectation that is necessary for success in the research program. We see this difference in opinion and experience between students and faculty within MTBI as a function of both the focus of the survey and different roles of the participants in the program.

Additional Hypothesized and Newly Identified Factors on Educational Development

Appreciation for the mathematical sciences and a desire for growth were two factors that motivated participants to pursue graduate school in the Mathematical sciences. We found four additional learning experiences or related factors that helped their educational development as scientists (i.e. Applied Mathematician). Specifically, as identified by respondents, MTBI supported development by supporting students to 1) discover new career options, 2) gain confidence that they can succeed 3) experience from their student initiated research, 4) being part of MTBI community and 5) becoming professional applied mathematician. These experiences of discovering career options, and experiencing research in applied mathematics helped them understand and learned they can succeed and contribute to them wanting to go on to higher education. Overall, these learning experiences resulted in their development, as they are becoming newly mathematicians and researchers. In addition, it provides a characterization of the learning and development environment created in MTBI.

On Collaboration and Creativity during MTBI

Our interview results showed the ways in which participants identified collaboration and creativity in the program, but more for collaboration. Participants identified MTBI as a place for both collaboration and creativity. Participants clearly identified collaboration happening among peers and faculty while working on their research projects, and homework assignments while creativity been foster during the research projects.

In describing the collaboration with faculty, 25% of participants described the peer-like relationship in working with their project advisors. This seems to be a defining characteristic of the MTBI program. In many cases participants do not have a chance to interact on a nearly constant basis with faculty advisors while formulating a research project, let alone develop a peer-like collaborative relationship. This type of relationship allowed them to be co-creators in modeling and the advancement of their research work.

In collaboration with peers, participants explicitly describe working in groups to complete and understand homework assignments. They also experience collaboration in their research groups to get a sense of their research question and then to complete their projects. The participants described dividing tasks, and everyone contributing to complete the project.

Most of the participants described collaboration as everyone contributing some aspect of the research projects, and creating it together. It seems everyone was interested in the success of their group projects as whole, whether working with peers or faculty. This type of collaboration in MTBI seems to resemble this idea of “the whole is greater than the sum of its parts”, that is, the final product or creation from the group is better than the sum of the contribution or work of individuals. Hence, even if a team consisted of some individuals who were very good in mathematics or biology, the group contribution will be much better than the contribution each could produce alone on a given project.

Whether working with faculty or peers, collaboration allowed all involved to be part of the creation process. This also allowed for most participants to see creativity happening in the group and in collaboration process. It also allowed for the groups to be creative as social units.

38% of the participants made reference to the creative process when choosing and defining their research questions in the research groups. They made reference how through a creative process of thinking and co creation they were able to develop mathematical models using epidemiological models. That is, they were creatively imitating the structure and elements of mathematical epidemiology and apply it to other natural or social setting. This is a find similar to what we hypothesized in Chapter 3.2, although their example was not as elaborate.

On Collaboration Beyond MTBI

In investigating our subsequent questions: what participants learn in collaboration (or creative imitation) that helped them beyond MTBI. We found that 3/4 of the participants also stated that collaboration was useful in their current occupation. For those in academia, it help see the usefulness of developing collaborative skills to seek similar experiences to works with others for work or publication. For those in industry, they saw collaboration useful to improve communication skills, and to challenge each other, and for a greater learning which is useful in dealing with people. In addition, 38% of students made reference to the type of research projects they worked on in relation to how they used creativity beyond MTBI. They used them to further advance their educational research career or to choose the type of research they tackle.

Limitations and Future work

This study surveyed participants who had participated in MTBI and completed a PhD by the time of the study. The results in this study do not apply to other MTBI alumni or other students participating in REU or pursuing graduate studies. Participants' responses to collaboration and creativity were made in reference to MTBI, so these responses may be different in other settings or REU programs.

For future research we would like to ask participants who have participated in other REUs before or after MTBI if collaborative, creativity, or REU experiences look different and in what ways these experiences have influenced their educational and professional development. We hoped that when participants provide a scenario of collaboration that it would potently allow us to see more explicitly principles of improvisation such as creative imitation, and completion. Explicitly in the sense of the type of interactions and communication involved between participants and MTBI faculty (including graduate students, postdocs and faculty) when developing the research projects. We provide an example in the practical-theoretical account in Chapter 3. However, these instances did not appear in the interviews. It could be that we did not explicitly ask them to recall specific details in relation to their interactions and communication with peers and faculty as they developed and carried out their research projects. In order to provide a better characterization of the improvisational nature of the research process in MTBI, onsite research should be done as they formulate the research question.

We asked participants to reflect on all their educational and professional experiences (question 7 and 9) during and post MTBI. Specifically, we asked how the participants learned to be an applied mathematician, statistician, analysts or whatever field they may be currently practicing. In question 9, we further wanted to characterize other learning experiences and factors that contributed to their current and professional development. Preliminary analysis shows that for 50% of the participants saying that learning from collaboration among peers and faculty was important for their development as future mentors, and to the type of research agenda they want to build for themselves.

Therefore, a future direction would be to continue to use our Vygotskian-Holzman lens to further examine the characterization on collaboration, creativity, and learning factors associated to their overall success in a broader context of their educational experiences, in addition to MTBI.

In this study, we see *development* as a creative activity where students have the ability to make choices that shape their current growth and future circumstances. For future work, we could use this same metric to investigate the development of other (non-PhD) MTBI alumni, and the type of choices they made to further their educational or personal life. This future sample along with the current sample would provide a further characterization of development through MTBI. We believe that since participants learn how to learn and co-create their research environment, we may also find within the data evidence for their leadership development. In this context, we hypothesize that MTBI provides a platform where participants can build characteristics of leadership. Popularly speaking, leadership may be defined as "taking charge of a situation". Camacho et., al (2015) define leadership similarly to our development definition. Given the shorter time it takes for MBTI students to complete their PhDs characterizing leadership skills learned by MTBI participants is a particularly interesting question to address.

Being able to cultivate productive, collegial working relationships is a skill increasingly vital to good leadership that supports better problem solving and more creative responses to issues (Goldsmith and Eggers, 2004). The results of our interviews point to students developing skills in working with others both during their summer program and ultimately in their career. They reference their ability to work with colleagues and to pursue collaborative and interdisciplinary work. Characteristics described of leaders in other organizations (Tennyson 2005). Building relationships across disciplines is required for successful applied mathematics research. It also helps individuals to develop shared understandings and take on new perspectives when confronted with a problem (Bolden 2011; Gray 2008). In future studies, we would like to collect additional data, including non-PhD alumni, to investigate more directly how the development from MTBI contributes to building leadership.

Concluding Remarks

The findings on this study indicate MTBI mentoring resources do support a Vygotskian-Holzman learning environment as described in the second chapter. We argue that MTBI's ability to produce and help develop PhD students from less advantaged URM backgrounds can be attributed to the Zone of Proximal Development created as students learn to do research, as well as the relationships formed from the overall program activities and among the students. MTBI works with people from disadvantaged backgrounds who might come from economic hardship, first generation college, few resources, underrepresented minorities, who are unlikely to be selected for elite applied mathematics research programs because of some combination of poor grades, low test scores, or being from a "less competitive school". Instead of being placed in remedial groups, or made to work on projects carved from faculty's existing research, they are given the opportunity to do self-initiated original research in applied mathematics through a collaborative and creative process.

The MTBI faculty works alongside with the students exploring questions they have not approached before. As students are introduced to the larger MTBI community, the scientific community and higher education options, they seek this type of research or educational experiences. In fact, we would argue that the low numbers in STEM are less about achievement or interest and more about development. Therefore, we must focus our attention on helping students understand that they can develop, they can learn and shape their circumstances. This is something that Fulani and Kurlander (2009) also advocate for and this is something MTBI participants did as they continue their educational and professional paths.

The MTBI environment is created with collaboration, high expectations, and commitment of program leaders to co-creating a structure and culture in which MTBI supports students to grow intellectually, socially, professionally. MTBI is a developmental program in which its commitment to collaboration (co-creation) is the basis for creating learning experiences where they relate to each other as capable researchers. This intense iterative learning process allows them, especially the students, to go beyond what they are capable of doing given their educational beginnings. MTBI participants experience and learn what is like to carry out applied mathematical research, and learn that there are numerous educational and professional opportunities available for them. They have learn that they are capable of and can succeed and therefore seek to pursue a higher education degree.

Developing communities from disadvantaged backgrounds as accomplished applied mathematicians or engineers is not easy, but it can be done, as shown by MTBI. Change is possible and Fulani (2011) calls for action on the entire educational system in which "we could start pretending that change is happening. And if we do, and if we involved our kids in that performance, it could end the learning crisis in our country". In this study, we describe elements of co-creation and creativity, and developmental factor for interested in graduate school, which are part of the Vygotskian-Holzman Zone of Proximal Development that could be helpful for those interested in creating a model of learning and development like MTBI. We hope the results of this study can be used by others to create social change, as we feel it is important to continue to invest in human capital and access to education for the better future of any country, and its people.

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APPENDIX A

RECRUITMENT SCRIPT

Dear Participant:

I am a graduate student under the direction of professor Anuj Mubayi from the Simon A. Levin Mathematical, Computational and Modeling Sciences Center (SAL MCMSC) at Arizona State University. I am conducting a research study to characterize the ways in which MBTI has helped mentor and develop undergraduate students in their professional career. Hence, this will be a retrospective analysis on your professional and education career.

I am inviting your participation, which has two parts: filling out an online survey and an optional subsequent face-face (in person or online) and recorded audio interview. The questions in the online survey and audio interview are meant to ask about your experience with MTBI, and professional and educational development. The online survey should take about 10 minutes, and the interviews about 70 minutes. For participants wanting to be involved in the second part of the study, you will be ask to provide your contact information so we can contact you later. Your information will be kept confidential. The audio interview will take place on a date and time convenient to you. If you decide to participate in an interview, you will be audiotape through an online audio software system, as you answer a few questions related to your professional and educational career. You have the right not to answer any question, and to stop the interview at any time. I am also requesting permission to collect copies of your vita for research purposes. All participants must be at least 18 years old.

Your participation is important because it will give insights about how MTBI has helped mentor students, possible ideas on how to improve the number of underrepresented minorities in the mathematical sciences. There are no foreseeable risks or discomforts to your participation in the study. Your participation in this study is voluntary. Please let me know if you do not want the interview to be taped; you also can change your mind after the interview starts, just let me know.

Your responses will be kept confidential. They may be used in reports, presentations or publications, but your name will not be used. If you have any questions concerning the research study, please email me at arlene.evangelista@asu.edu.

If you have any questions about your rights as a participant in this research, or if you feel you have been placed at risk, you can contact the Chair of the Human Subjects Institutional Review Board, through the ASU Research Compliance Office, at (480) 965-6788

Sincerely,

Arlene Evangelista

APPENDIX B

SURVEY

---------Survey on your Educational and Professional Development--------

This is survey is voluntary, and anonymous. You can skip questions, and/or decided not participate at any time. Thank you for participating in this survey! The following series of questions are meant to reflect your educational and professional development.

The following questions are demographic questions.

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0. Do you have a PhD?

- Yes
- No

1. Are you male or female?

- Male
- Female

2. What is your race? (optional)

- White
- Black
- Hispanic
- Native American
- Pacific Islander
- Unknown
- Other: ______________

3. Where are you currently working?

- Academia
- Industry
- Own/independent business
- Unemployment
- other: _____

The following questions are related to MTBI and your undergraduate degree

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4. What year(s) did you attend MTBI? (i.e. 2009, 2013, etc.)

5. MTBI was my first Research Experience for Undergraduates (REU) experience (If

your response is “No”, then in the other option type in how many REU you have attended before MTBI and in which area.

- Yes
- Other

6. How did you become interested in the Mathematical and Theoretical Biology Institute? (choose all that apply)

- other people, mentor or friend recommended the program
- Applied Mathematics Research experience
- Exploring graduate options
- Others (explain): _________________

7. Prior to MTBI, I was interested in graduate school

- Yes
- No
- Not sure

8. After MTBI, I wanted to go to graduate school

- Yes
- No
- Not sure

9. After MTBI, I continued to be interested in the Mathematical Sciences

- Yes
- No
- Not sure

10. My MTBI participation and MTBI experience(s) helped me manage graduate school

- Yes
- No

11. My MTBI participation led me to publish a journal article, from research carried out at MTBI

- Yes
- No

12. What was your undergraduate degree? ____________

13. What year did you get your undergraduate degree? ______

14. Are you the first in your family to obtain an undergraduate degree?

- Yes
- No

15. What learning experiences or factors motivated you to pursue graduate school in the Mathematical Sciences? (If MTBI participation played a role, explain how)

The following questions are related to your graduate degree and career after graduation

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16. What was your graduate degree? ____________

17. How many years did it take you to obtain your PhD? Including years in getting your masters degree, if applicable (i.e. 6 years (2 Masters, 4 PhD) ___________

18. What year did you get your PhD? _______

19. Are you the first in your family to graduate with a PhD?

- Yes
- No

20. What learning experiences or factors contributed to your continuation and completion of a PhD? If MTBI played a role, please include a reflection of how aspects of the program contributed.

----

21. After graduation, did you take a Postdoctoral position? (If your answer is no, please write in the job position you obtained. If your answer is yes, write in the subject are of postdoctoral position)

22. How many years did your Postdoctoral last? (Type NA, if not applicable or write "will last x years" if still in progress)

23. What university or institution did you receive your Postdoctoral position? (If not applicable, type "NA")

24. If in academia, are you currently in Tenure Track Faculty position?

- Yes
- No

25. How many years after your PhD did you take a Faculty position? (type "NA" if not applicable) ________

The following is a reflective question about MTBI and your educational development. Please use the following scale when ranking.

0 = NA 1 = Not important

2 = Slightly important 3 = Moderately important

4 = Important 5 = Very important

---

26. Rank the following attributes of your MTBI experience that you think were most important in your educational development?

Participants from different cultural backgrounds

0 1 2 3 4 5

Participants from different academic preparation

0 1 2 3 4 5

Closed living arrangement for two months

0 1 2 3 4 5

Student initiated research

0 1 2 3 4 5

Student-mentor relationships

0 1 2 3 4 5

Undergraduate-graduate or undergraduate-undergraduate relationships

0 1 2 3 4 5

High expectations on students

0 1 2 3 4 5

Mentoring from MTBI faculty

0 1 2 3 4 5

Training on how to become a scientist (like how to do research, write technical report and/or presentations)

0 1 2 3 4 5

Challenging homework sets

0 1 2 3 4 5

Becoming a member of the MTBI learning community

0 1 2 3 4 5

27. In Question #26, you ranked the attributes of your MTBI experience that were most important in your educational development. If there are any attributes not listed in question 26, please list them here:

Instructions: For the following questions, indicate your agreement to each item from "Strongly disagree" to "Strongly agree".

0 =NA or no answer 1 = Disagree strongly 2 = Disagree a little

3 = Neither agree nor disagree 4 = Agree a little 5 = Strongly Agree

---

28. When thinking about collaboration as needing significant contributions from others to complete a project or a task, do you believe MTBI provides an environment of collaboration?

0 1 2 3 4 5

29. When thinking about creativity as being able to take old ideas and make something new or different, do you believe MTBI can be described as a place of creativity?

0 1 2 3 4 5

30. When thinking about your undergraduate and graduate educational path, do you believe MTBI fosters students to think and work like an applied mathematician?

0 1 2 3 4 5

30. When thinking about your undergraduate and graduate educational path, do you believe MTBI fosters students to grow intellectually?

0 1 2 3 4 5

Current Educational and Professional Achievements

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32. Which of the following best described how you feel about your educational achievements?

- Very satisfied
- Somewhat satisfied

- Neither satisfied or not satisfied
- Somewhat unsatisfied
- Unsatisfied

33. Which of the following best described how you feel about your professional achievements?

- Very satisfied
- Somewhat satisfied
- Neither satisfied or not satisfied
- Somewhat unsatisfied
- Unsatisfied

34. Which of the following best described how you feel about your future career path

- Stay in my sector
- Change to industry
- Change to academia
- Change career path
- Not sure
- Other: _________

If are interested in being interviewed about your MTBI experience, educational and professional experience, please provide your first name and contact information.

Please enter your name, and phone number in the "other" so we can contact you.

- No
- Other: _______________

APPENDIX C

INTERVIEW QUESTIONS

------------------------------------- Follow-Up Questions -----------------------------------

This is survey is voluntary, and anonymous. You can skip questions, and/or decided not participate at any time. Thank you for participating in this survey!

The audio interview will last approximately one hour. Your responses will be recorded in order to facilitate future analyses. Your participation in the audio interview is voluntary, which means that you can refuse to answer any question for any reason, or discontinue your participation at any time.

1. *Do you give us permission to audio record your interview?*
2. *Do you have any questions before we begin the interview?*

MTBI: **The following questions are meant to be reflective about your educational and professional experiences with MTBI**

1.When thinking about your educational development, what aspect in MTBI whether related to events, relationships (student-student, student-faculty) or activities do you think were helpful in developing learning skills?

2. Can you provide one or two scenarios within your MTBI experience that embodies collaboration? *What aspects of the example are you identifying as collaborative?*

- Was this collaboration useful in you becoming an Applied Mathematician or your current occupation, if so, how was this useful?

3. Do you think MTBI provides an environment that fosters creativity (*creativity as in being able to take old ideas and make something new or different)*? Can you provide one or two scenarios within your MTBI experience that embodies creativity

4. **Imitation is recognized as an important component for creative learning.** What role did imitating others play in your learning or development at MTBI? Can you identify who or what you imitated and provide a scenario within your MTBI experience that embodies it (*for example including but not limited to imitating a student, faculty, computer lab code, project write up, other?)*

- Was this imitation useful in you becoming an applied Mathematician or your current occupation, if so, how was this useful?

5. When thinking about different kinds of mentoring relationships in MTBI, what role did undergraduate-faculty or undergraduate-graduate/postdoctoral relationships play on your educational growth?

What, if anything, about these relationships with other faculty or students inspired or impacted you?

6. If you returned to MTBI more than once, what role did this play in your educational development?

- Did mentoring relationships look different when coming back?

Current and future situation: The following series of questions are meant to reflect all your educational and professional experiences, including but not limited to your MTBI experience.

7. How did you learn to be an applied mathematician, statistician, analysts or whatever field you maybe practicing now?

- *What choices help you become the professional person you are?*
- *If they mentioned other research experiences ask: how is that similar or different to MTBI?*

8. What learning experiences or factors motivated you to continue (*continue as in motivation to persist and advanced in your field*) with graduate school in the Mathematical sciences? (*State at least 3)*

- Were there any challenges you encountered while pursuing a PhD that you think MTBI prepared you for?

9. In terms of your educational experiences, what factors contributed to your current professional and educational situation?

  - *What role did your MTBI learning experience have to your current situation?*

10. Are you satisfied with your current position, and are you planning to continue in your career path?

- What is your current position and what is your career path?

Other comments

11. Any other observations or comments that you would like to share with me about your life in the Mathematical Sciences or life in general?

APPENDIX D

ASU INSTITUTIONAL REVIEW BOARD APPROVAL

EXEMPTION GRANTED

Anuj Mubayi
Mathematical and Natural Sciences, School of
-
Anuj.Mubayi@asu.edu

Dear Anuj Mubayi:

On 11/14/2014 the ASU IRB reviewed the following protocol:

Type of Review:	Initial Study
Title:	The Mathematical Theoretical Biology Institute as a developmental Model for undergraduates students
Investigator:	Anuj Mubayi
IRB ID:	STUDY00001625
Funding:	None
Grant Title:	None
Grant ID:	None
Documents Reviewed:	• consent_MTBI14_final.pdf, Category: Consent Form; • Protocol_SocBeh_f.pdf, Category: IRB Protocol; • Survey and interview questions, Category: Other (to reflect anything not captured above); • Prof Mubayi's training certificate, Category: Other (to reflect anything not captured above); • Recruitment_2014f.pdf, Category: Recruitment Materials;

The IRB determined that the protocol is considered exempt pursuant to Federal Regulations 45CFR46 (2) Tests, surveys, interviews, or observation on 11/14/2014.

In conducting this protocol you are required to follow the requirements listed in the INVESTIGATOR MANUAL (HRP-103).

Sincerely,

IRB Administrator

cc: Arlene Evangelista
Arlene Evangelista