Fix-and-Optimize Heuristic and MP-based Approaches for Capacitated Lot Sizing Problem with Setup Carryover, Setup Splitting and Backlogging

by

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ABSTRACT

In this thesis, a single-level, multi-item capacitated lot sizing problem with setup carry-over, setup splitting and backlogging is investigated. This problem is typically used in the tactical and operational planning stage, determining the optimal production quantities and sequencing for all the products in the planning horizon. Although the capacitated lot sizing problems have been investigated with many different features from researchers, the simultaneous consideration of setup carryover and setup splitting is relatively new. This consideration is beneficial to reduce costs and produce feasible production schedule. Setup carryover allows the production setup to be continued between two adjacent periods without incurring extra setup costs and setup times. Setup splitting permits the setup to be partially finished in one period and continued in the next period, utilizing the capacity more efficiently and remove infeasibility of production schedule.

The main approaches are that first the simple plant location formulation is adopted to reformulate the original model. Furthermore, an extended formulation by redefining the idle period constraints is developed to make the formulation tighter. Then for the purpose of evaluating the solution quality from heuristic, three types of valid inequalities are added to the model. A fix-and-optimize heuristic with two-stage product decomposition and period decomposition strategies is proposed to solve the formulation. This generic heuristic solves a small portion of binary variables and all the continuous variables rapidly in each subproblem. In addition, the case with demand backlogging is also incorporated to demonstrate that making additional assumptions to the basic formulation does not require to completely altering the heuristic.

The contribution of this thesis includes several aspects: the computational results show the capability, flexibility and effectiveness of the approaches. The average optimality gap is 6% for data without backlogging and 8% for data with backlogging, respectively. In addition, when backlogging is not allowed, the performance of fix-and-optimize heuristic is stable regardless of period length. This gives advantage of using such approach to plan

longer production schedule. Furthermore, the performance of the proposed solution approaches is analyzed so that later research on similar topics could compare the result with different solution strategies.

DEDICATION

To my grandma Jui-Hsun Yeh, who left us too early, your memory will always be in our hearts.

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INTRODUCTION

1.1 Overview of Lot Sizing Problems

Production planning and control is a recurring activity for manufacturing industry and has remained one of the most challenging problems since the development of operations research. Fast development in economics and globalization have elevated the level of competition and continued to put pressure on corporations to squeeze out more productivity. Under this situation, one of the key success strategies for manufacturing companies is to achieve maximum efficiency and utility through the design of production processes. An effective production planning demonstrates the ability of production system to produce products at low costs with faster processes and minimum defective products. Generally, the common objectives for production planning can be stated as the following: minimize total production time and production costs, meet customer requirements and use resources effectively. Depending on the type of production planning, mathematical models are extensively adopted to support the decision making processes in this problem context.

Most production planning problems can be divided into three hierarchical stages: long-term, medium-term and short-term planning. Decision making in long-term planning problems usually focus more on the strategic level of activities, such as facility location, selection of products and necessary equipment investment for production. While in medium-term level, tactical planning decisions include allocation of capacity, determination of the amount of overtime used and the amount of inventory. Short-term level planning usually involves the production scheduling and sequencing of jobs for daily operations, vehicle routing and inventory control activities. As one of the important components in production planning, lot sizing attempts to determine the optimal timing and quantity of production for all the products, so that the total cost of production can be minimized. To be more precise, the ex-

pected output of lot sizing is to give a complete picture of how many units of each product to produce, and how many units to carry in inventory for each period in the planning horizon, so that total relevant costs such as setup costs and inventory costs can be minimized. The early development of lot sizing problem can be traced back to several decades ago. Harris introduced the framework in 1913 for the well-known economical ordering quantity (EOQ) model under the assumption of single-item, stationary demand and infinite planning horizon (Harris, 1990, reprinted version). The model demonstrates a simple tradeoff between order costs and inventory holding costs. Later the model was further analyzed and applied extensively in practice. EOQ does not impose the capacity restriction, which was taken care in the economic lot sizing problem (ELSP) (Rogers, 1958) as in reality the available resource are limited. Starting from the Wagner-Whitin problem (Wagner and Whitin, 1958), named after the two researchers who proposed this model, research direction has been shifted to investigate lot sizing with time-varying demand and finite planning horizon.

Ever since then, lot sizing problem today has many extensions corresponding to different assumptions and characteristics. Other than the type of demand data and length of planning horizon, there are several characteristics of the lot sizing problem that deeply affect the complexity. Production processes can be either be single-level or multi-level. Demand in multi-level production has the predecessor-successor relationship, that is, the output of one operation is the input of another. Capacity is another crucial factor in lot sizing. If the limitation of capacity does not exist, the problem is called uncapacitated, otherwise it is called capacitated. Problem with capacity limitation usually follows with setup time when changeover of products is allowed during the production processes. For a setup to be done within a time period, both setup cost and setup time can be either product-dependent or sequence-dependent. If setup for a product is established in current period and carried into next period to continue the production for similar product, it is often called setup carryover or linked lot sizes. Setup carryover reduces additional setup costs and setup time, thus giving lower cost production plans. It is also possible to perform setups between two

adjacent periods. This setup pattern is referred to as setup splitting or setup crossover (Mohan *et al.*, 2012; Belo-Filho *et al.*, 2013). With setup splitting, period idle capacity can be better utilized because inserting additional setups is possible. In terms of mathematical modeling, whether to launch setups or not is modeled by binary variables, which makes solving the problem even harder. Other features such as allowing demand backlogging or using parallel machines in the production are considered in some lot sizing extensions as well.

Since lot sizing can be classified according to different characteristics, they are commonly divided into two groups: big bucket model and small bucket model. The fundamental assumption for big bucket model is that it allows multiple items to be produced within a single time period. Furthermore, setup operations can be performed multiple times as needed. The time period in big bucket model usually represents the length of weeks or months, so it can be used for medium-term planning. The capacitated lot sizing problem (CLSP) is a typical big bucket model. CLSP allows multiple items to be produced with capacity restrictions. The applicability and realistic assumption of CLSP to industrial practice has triggered a lot of research over the past few decades. On the other hand, small bucket model such as discrete lot sizing problem (DLSP) and continuous lot sizing problem (CSLP), the former has a strict all-or-nothing assumption, indicating the production uses up all capacity to produce one item only per period and incurs setup costs each time when new lots are being produced. The later relaxes this relatively unrealistic condition and allows setup to be preserved and carried into next few periods, resulting in less setup costs. However, the unused capacity each period in CSLP is discarded, another small time bucket model, the proportional lot sizing and scheduling problem (PLSP) allows the model to schedule second item in some period if necessary. Because of the all-or-nothing assumption, short-term daily operational planning usually considers small bucket model. In addition, the solution output of small bucket model could provide production sequence information, while the sequence is not obvious in big bucket model.

Solving lot sizing problem is a challenging task. Commercial production planning tools such as material requirement planning (MRP) or manufacturing resource planning (MRP II) aim to provide a solution for this routine problem. However, both MRP and MRP II lack of certain features (Drexl and Kimms, 1997) so that they cannot satisfy the need from practitioners. MRP does not consider the capacity constraints, whereas criticism for MRP II usually lies in producing schedule with long lead times and backlogging. Researchers have done a lot of investigation in favor of finding more sophisticated solution approaches for lot sizing problems under different complicated assumptions. The computational complexity for lot sizing problem such as ELSP or CLSP are known to be NP-hard (Florian *et al.*, 1980; Bitran and Yanasse, 1982; Hsu, 1983). Including different features into the model will only increases the complexity. The theory of NP-completeness has reduced the hope to solve NP-hard problem within polynomial computational times. Broadly speaking, no dominant solution approaches has been proposed for lot sizing problems so far. Nevertheless, effectively designed heuristic algorithm can be helpful in achieving nearly optimal solution in a reasonable time.

1.2 Motivation and Objectives

One of the recent research trends on big bucket CLSP is to include setup carryover, setup splitting and backlogging. Considering setup splitting either generates a better production schedule in terms of lower costs or removes infeasibility from models with setup carryover only. A possible condition for infeasibility is when the length of setup time is substantially long and might even surpasses the capacity of the time period. Long setup time is ubiquitous in some process manufacturing industries and automobile production processes. Therefore it is necessary to take setup splitting into account in order to have a feasible production schedule. In this thesis, we first study the single-level, multi-item, capacitated lot sizing problem with setup carryover and setup splitting (CLSP-SCSS) and then incorporate demand backlogging in the model. As an extension of the basic CLSP, the CLSP-SCSS is also NP-hard. It is unlikely to solve the problem within reasonable time

limit by traditional exact methods such as branch-and-bound when problem size is getting larger. To address this problem, the goal of this thesis is to focus on developing an efficient solution procedure for the CLSP-SCSS. The contribution of this thesis is to give a comprehensive computational result for CLSP-SCSS and later other similar research can use our work as a point of reference to compare the efficiency of different algorithms. In particular, the objective of this research is to address the following questions.

- What are the factors that make setup splitting take effect?
- What is the overall performance of the proposed heuristic algorithm?
- How does the performance of the proposed algorithm affected by the characteristics of the model, e.g. the number of products, periods, average length of setup time and consideration of demand backlogging?

Reformulations and some valid inequalities are modified from previous research on the CLSP with setup carryover, which can also be implemented if attempts are made to enhance the mathematical formulation. Specifically, a MIP based heuristic called fix-and-optimize heuristic is described and applied to solve the model with long average setup time (e.g. 120% of period capacity), large size data instances with number of 15 products and number of periods 48.

In the following section, we give a detailed literature review on CLSP that considers various types of setup operations, different characteristics and solution approaches. Then we provide an outline to the rest of this thesis.

1.3 Literature Review

Research on capacitated lot sizing problem with different characteristics is plentiful. We do not claim to cover every single article in this area. Therefore we restrict the review to dynamic big bucket CLSP type with different setup operation features, i.e., setup carryover and setup splitting. We refer some excellent research work for reviews with possible

extensions of CLSP. For instance, Drexl and Kimms (1997) gave a comprehensive survey on dynamic, capacitated lot sizing and scheduling. Karimi *et al.* (2003) discussed the sing-level lot sizing together with exact method and heuristic algorithm. Gicquel *et al.* (2008) reviewed the single and multi-level lot sizing on single or multiple resources. They also included exact method, specialized heuristic, mathematical programming based heuristic and meta-heuristic. Quadt and Kuhn (2008) presented a review for CLSP with several features such as backlogging, setup carryover, sequencing and parallel machines. Similarly, various solution approaches have been proposed to tackle CLSP. Our review focus on mathematical programming based approaches, especially the LP based heuristic. Jans and Degraeve (2007) provided an overall review of lot sizing problem solved by meta-heuristic. Buschkühl *et al.* (2010) gave an excellent comprehensive review of multi-level capacitated lot sizing problem (MLCLSP) on both modeling approach and corresponding algorithms during the past four decades. Díaz-Madroñero *et al.* (2014) presented a survey work of 250 articles in the domain of discrete-time tactical production planning. They classified these papers by various attributes, including problem type, production structure, model extensions, solution approaches, modeling languages and application filed. They concluded that master production scheduling with a big bucket is the most widely studied problem type. In addition, they highlighted that backlogging, setup time, parallel machines, overtime and network formulation for model extension being considered the most.

1.3.1 Modeling of Setup Extensions

Two different important setup features have been considered in the capacitated lot sizing problems: setup carryover and setup splitting. We begin to review these features in the following sections and discuss their relation with this thesis.

Setup Carryover

The optimal production schedule from CLSP with setup carryover (CLSP-SC), or known as CLSP with linked lot sizes (CLSPL) could be really different from CLSP. Production

for a specific item in previous period is continued to next period without additional setups, thus saving the setup costs. Free capacity at the end of previous period is also possible to be utilized as a setup for the immediate production in the next period.

Dillenberger *et al.* (1993) is known to be the first one to cover setup carryover in the CLSP. Instead of considering holding costs or setup costs, the objective is to minimize total weighted processing time and backlog units at the end of planning horizon. Also, in their formulation setups can be carried over several periods.

Haase (1994) established the name linked lot sizes for the standard CLSP. The so-called CLSPL considers production on a single resource without setup time. The objective is to minimize total holding and setup costs. Similar to Dillenberger *et al.* (1993), the formulation allows consecutive setup carryover. Later in Haase (1998), he reduced the formulation and restricted setup can only be carried over at most one period.

Gopalakrishnan *et al.* (1995) proposed two types of single resource CLSPL. The first formulation takes product-independent setup time and setup costs into account. Compare to previous research, binary variables are used extensively to model setup carryover. Setup preserved over idle periods is indicated by auxiliary variables. The objective is to minimize total holding and setup costs. The author referred second formulation as an integrated model, which it includes attributes such as parallel machines, resource assignment and tool selection. Gopalakrishnan (2000) modified the first model of CLPSL in Gopalakrishnan *et al.* (1995) by incorporating producti-dependent setup time and setup costs.

Sox and Gao (1999) considered CLSPL on a single resource with product-dependent setup costs, holding costs and variable production costs but without setup time. They also restricted the number of setup carryover to be one across two periods.

Suerie and Stadtler (2003) allowed setup can be carried over several consecutive periods in CLSPL and then extended it as to a multi-level production structure (MLCLSPL). They also assumed production happened on multiple resources with a unique assignment to each resource for every product.

Gupta and Magnusson (2005) developed a single resource CLSPL with sequence-dependent setup costs and setup time. Here again setups can be carried over several periods. Their formulation is similar to Gopalakrishnan *et al.* (1995). The sequence-dependency relationship gives the sequence of products in every period.

Quadt and Kuhn (2009) presented a CLSPL with setup time, backlogging and parallel machines. A subsequent model provides sequence and schedule of products on parallel machines. Their research is motivated from the semiconductor assembly facility.

Setup Splitting

Modeling setup splitting is necessary for some manufacturing industry that the portion of setup time is considerably long and capacity is scarce.

In the two small bucket PLSP based formulations, Suerie (2006) clearly defined the starting and ending points of setup operations to handle long setup times. He solved the instances by commercial MIP solver with additional valid inequalities, providing improvement on solution quality compared with Drexl and Haase (1995).

Sung and Maravelias (2008) incorporated sequence independent setups, non-uniform time buckets and long setup time in their formulation, as these features are often founded in process manufacturing industry. They took the solutions without setup carryover and crossover and adjusted the time bucket boundaries to handle long setup time by allowing setup carryover and crossover. They also gave detail description to include idle time, parallel machines, families of products, backlogged demand and lost of sales in the formulation.

Menezes *et al.* (2011) considered sequence-dependent, non-triangular setup, setup carryover and crossover in CLSP. The sequence-dependent feature may result in producing more than one batch for a similar product within a period. They introduced a dynamical method to omit disconnected sub tours, as this is especially important for large problems. The computational test showed that setup crossover is important for obtaining better solution compared with small bucket models.

Kopanos *et al.* (2011) developed a formulation for simultaneous production planning and scheduling, where items are grouped into families. They allowed the changeover of families to be crossover two adjacent periods and applied the model on a large-scale industrial case. The result indicated that the formulation works well since hundred of items could be solved in a reasonable time.

Mohan *et al.* (2012) presented a single-level single machine CLSP-SCSS, which is an extended formulation of CLSPL from Suerie and Stadtler (2003). They included variables to indicate start and finish time when splitting occurs. Based on a small numerical test, they demonstrated that solutions obtained from CLSP-SCSS are either at least equal or better than CLSPL. Furthermore, CLSP-SCSS could find solutions in instances that are considered infeasible in CLSPL.

Belo-Filho *et al.* (2013) proposed two formulations that involve backlogged demand, setup carryover and setup crossover. The first formulation is based on Sung and Maravelias (2008) and reformulated in simple plant location representation. The second uses the disaggregated time-index approach by defining starting and ending point on all the setup operations, resulting in less integer variables. The computational results showed that setup crossover is essential and uses the idle time more efficiently when setup time consumes a significant portion of period capacity.

Fiorotto *et al.* (2014) took the ideas from Menezes *et al.* (2011) and Mohan *et al.* (2012) and reformulate two models in simple plant location with setup crossover only. They developed three methods to model setup crossover without using extra binary variables. Moreover, they added symmetry breaking constraints on two literature models to analyze the relationship among these formulations. The computational test was conducted on instances taken from Trigeiro *et al.* (1989) and they concluded in two aspects: setup crossover is especially worth to have when backlog is allowed and more feasible solutions could be found with the existence of setup crossover.

Summary

From the literature, research consider setup carryover have attracted much attentions. The assumption of allowing setup carryover is more realistic and provide cost-attractive production schedule. Therefore other extensions such as parallel machines, backlogging, multi-level structure as well as many solution approaches have been investigated. On the other hand, unlike the flourish research of CLSP-SC and its variants, the number of literature of CLSP with setup splitting is relatively limited. By far research trend on setup splitting is mainly emphasizing in the formulation technique. The computational test results are based on imposing time limit for MIP solver to evaluate the optimality gap between different formulations. No literature regarding to specific solution technique to handle CLSP with setup splitting have been proposed so far. This thesis aims to provide a computational study and fill this gap in.

1.3.2 Mathematical Programming based Approaches

The property of mathematical programming based approaches is that they are more flexible and general than other solution approaches when it comes to model extensions. This group can be divided into exact method and LP-based heuristic. Since most of the lot sizing problems are NP-hard, heuristic technique are adopted to solve the problem. Exact method such as brand and bound (B&B) enumerates feasible solutions in the whole solution space. Although B&B guarantees to find an optimal solution if it exists, the required computational time would be too long if the problem size is large. While heuristic only examine partial solution space and try to return a good quality solution in a reasonable time. Research trend on solution approaches of lot sizing is either trying to develop a faster heuristic algorithm that leads to nearly optimal solution or combing several different solution techniques together to increase the overall efficiency. This research adopts several mathematical programming based approaches hence we give a review focus on this group.

Reformulations

The efficiency of standard MIP solver is heavily depending on the mathematical formulation. The technique of reformulations redefines the variable and seeks to approximate the convex hull of the integer programming model. Moreover, reformulations are often used to increase the efficiency for standard MIP solver and to tighten the LP relaxation since the bound generated from LP relaxation of CLSP are often very loose. Two reformulation techniques have been proposed so far. Eppen and Martin (1987) introduced the shortest route (SR) formulation for standard CLSP. Tempelmeier and Helber (1994) included capacity constraints and multi-level structure in this formulation. Stadtler (1996, 1997) proposed a improved SR formulation for MLCLSP with reducing some constraints in contrast to Tempelmeier and Helber (1994). Another reformulation is called simple plant location (SPL) formulation. Rosling (1986) first introduced it to model a multi-level uncapacitated lot sizing problem, which is an extension of Krarup and Bilde (1977). Later Maes *et al.* (1991) added capacity constraints to it. Stadtler (1996) again proposed a SPL formulation for a general bill-of-material structure. The author observed that the LP bound for both formulations is identical. This observation is proved in Denizel *et al.* (2008)such that LP bound of both SR and SPL for CLSP with setup time are exactly the same.

Valid Inequalities

A common way to tighten the formulation is by adding valid inequalities so the irrelevant part of solution space can be cutoff. The success of dealing traveling salesman problem by cutting plane method in 1980 has motivated researchers to develop problem-specific valid inequalities for lot sizing problems as well (Pochet and Wolsey, 2006). Barany *et al.* (1984) derived valid inequalities to describe the convex hull for single-item uncapacited lot sizing problem. Miller *et al.* (2000) considered the capacitated case. Another extension on multi-item based on Barany *et al.* (1984) is made by Pochet and Wolsey (1991). Belvaux and Wolsey (2000) presented a framework called bc-prod to generate pre-processing, lot

sizing specific inequalities and general cutting plane lot sizing problems. Later in Belvaux and Wolsey (2001) they included cases with start-up, changeover and switch off conditions. Suerie and Stadtler (2003) extended the MLCLSPL formulation by making the idle-period indicator variables from time-dependent become product-dependent. Three set of valid inequalities were embedded to the branch-and-cut or cut-and-branch processes for finding first feasible solution, so heuristic can use it as a starting point.

Rounding Heuristic

Many research on lot sizing problems use rounding heuristic (RH) to generate initial feasible solution, it belongs to constructive heuristic. After solving the LP relaxation, the fractional binary variables are rounded afterwards. With the capacity constraints, the setup patterns might not be feasible if backlogging or overtime is not allowed. However, the solution can serve as a starting point and combined with other heuristic. Eppen and Martin (1987) introduced RH with B&B together to solve the SR formulation of CLSP. Maes *et al.* (1991) presented some RH to solve the SPL formulation of MLCLSP without considering setup time. Akartunalı and Miller (2009) combined RH and relax-and-fix (see next section) to solve MLCLSP with valid inequalities added. The RH is used to determine initial feasible solution and upper bound cutoff values for later window, the experiment showed that it improved the solution process.

Relax-and-fix Heuristic

Relax-and-fix (R&F) heuristic divide the binary variables into three subsets, resulting in less binary to be treated simultaneously hence reducing the computational effort. The binary variables are fixing at current value, relaxing to continuous value or solving to optimality in the given interval. Dillenberger *et al.* (1993) applied R&F heuristic with B&B algorithm for the CLSPL. The binary setup variables are fixed to all the prior periods and relaxed to current periods until the end of planning horizon. The branching order is determined by the sequence of periods. Stadtler (2003) proposed a overlapping rolling time window heuristic

for MLCLSP. The length of time window, later period where binary variables are relaxed and number of overlapping periods when move to next iteration are controlled by separate parameters. As the planning time window move forward, the setup variables are optimized except the last period in current window is relaxed. Also, binary in previous time windows are all fixed except the overlapping periods. This method is also used in Suerie and Stadtler (2003) for the MLCLSPL. Federgruen *et al.* (2007) also adopted R&F for the CLSP with joint setup costs.

Fix-and-optimize Heuristic

The general idea of fix-and-optimize (F&O) heuristic is to decompose the binary variables into two disjoint sets $\mathcal{K}$ and $\mathcal{L}$. A subproblem consists of the set $\mathcal{K}$ to be fixed, while the set $\mathcal{L}$ and all other continuous variables are optimized in the algorithm in an iterative manner. Given an initial solution, the algorithm update the incumbent solution in each iteration whenever better solution is found until the terminating condition is met.

Sahling *et al.* (2009) first proposed the iterative F&O heuristic for the multi-level capacitated lot sizing problem with setup carryover (MLCLSPL). They presented three principles to determine the search direction in the solution space and relative number of binary variables optimized for the heuristic: product decomposition, resource decomposition and process decomposition. Their experiment showed the cost of each product affect the solution founded by the heuristic therefore they presented a cost-approximation formula to decide the product decomposition order. They concluded that the selection of subset of products and periods is a critical part for the heuristic. Later in Helber and Sahling (2010), they applied the same F&O approach again for the multi-level case with lead time. They pointed out that one could obtain higher solution quality with unfixing more binary variables in the algorithm.

Goren *et al.* (2012) embedded the F&O heuristic into the genetic algorithm for the capacitated lot sizing problem with setup carryover. Unlike Sahling *et al.* (2009) and Helber and Sahling (2010) set all the setup variables to construct an initial solution, they used GA

to determine the initial solution then applied F&O heuristic to guide the GA jump out from local minima to new solution space. They compared the result with Gopalakrishnan *et al.* (2001) and using pure GA only. Overall the proposed hybrid approach is out performed the others.

Lang and Shen (2011) considered the capacited lot sizing problem with sequence-dependent setups and substitutions (CLSD-S) by using Relax-and-Fix (R&F) and F&O heuristic. The initial fixation of all setup variables does not always guarantee a feasible solution in the CLSD-S. They took the idea from Asymmetric Traveling Salesman Problem to determine the efficient product sequence for generating initial solution. The computational results showed that F&O approach combined with time window decomposition is better then product decomposition. Furthermore the F&O approach surpasses the R&F approach.

Xiao *et al.* (2013) examined CLSP with sequence-dependent setup time (CLSD), time window, machine eligibility and preference constraints. They applied fix-and-optimize to solve the problem based on a decomposition strategy called randomized lease flexible rule, which fixed the binary variables with the assignment of machines. The computational experiment demonstrated that this strategy improve the previous fix-and-optimize algorithm especially for data with high machine flexibility and demand variation.

Summary

Generally, mathematical programming based approaches provide two aspects for solving the problem: the constructive phase and improvement phase. First, in the process of constructive heuristic, one can simply impose a time limit in the branch-and-bound process to obtain a solution, using this initial solution as a starting point. The purpose is to generate a good quality initial solution and hopefully it would lead to good suboptimal or even true optimal solution. However, the solution quality is highly depending on the formulation. Reformulation technique and valid inequalities help reduce the feasible solution space but still include all feasible integer solutions. Using rounding heuristic is intuitive and easy to implement, however, the rounded setup variables might not be able to generate a feasible

schedule as the capacity might be violated. Including overtime in the model can solve this problem. R&F heuristic also seek to generate a initial feasible solution by using rolling time window, however, Helber and Sahling (2010) pointed out the implementation of R&F heuristic used in Stadtler (2003) is quite time-consuming and often fail to find first feasible solution within the given time limit.

Secondly, improvement heuristic such as F&O heuristic divides the solution space into different neighborhoods. The solution moves from one neighborhood to another for obtaining better solution in the iterative procedure until reach the local optimum. Recall the time-consuming issue mentioned about R&F above, F&O does not have such disadvantage, as in each iteration all the continuous variable are optimized with a small subset of setup binary variables, the capacity infeasibility can be removed quickly. Because the F&O only considers the subset of binary variables and treats all the continuous variables equally, adding extra constraints without modifying the procedure is possible. The flexibility of implementation also allow F&O to be embedded to other algorithm or combine it with other methods. It is worth to understand the initial solution, the size of $\mathcal{K}$ and $\mathcal{L}$ and the decomposition strategies are all crucial factors to effect the computational time and solution quality of F&O algorithm.

Given the analysis mentioned of each method above, we combine the reformulations, valid inequalities ,rounding heuristic to construct a good initial solution and fix-and-optimize heuristic together to solve the problem.

1.4 Organization of the Thesis

The remainder of this thesis is organized as follows. Chapter 2 includes the standard formulation of CLSP and then discuss the extension with setup carryover and setup splitting, respectively. A reformulation of CLSP-SCSS based on the simple plant location (SPL-SCSS) representation is presented. We also introduce the extended formulation and three sets of valid inequalities are modified specifically for SPL. The last section of this chapter describes the modification of allowing backlogging to the model both in standard formulation and

simple plant location representation. In chapter 3, we describe the implementation of fix-and-optimize heuristic, including the selection of initial feasible solution and two types of decomposition methods: product and period decomposition. Chapter 4 gives the results and analysis of computational test. As summarized in chapter 5, our studies promote the effectiveness of using the chosen heuristic together with mathematical-programming based approaches to solve CLSP-SCSS. We also suggest further research on applying different algorithms to improve or solve the extension of this problem.

Chapter 2

MODEL FORMULATION

In this chapter we present the basic formulation of single-level, multi-products capacitated lot sizing problems and its two extensions: the capacitated lot sizing problem with setup carryover (CLSP-SC) and the capacitated lot sizing problem with setup carryover and setup splitting (CLSP-SCSS). CLSP-SCSS is the base formulation to solve problem, later we reformulate CLSP-SCSS by using simple plant location formulation SPL-SCSS to estimate the lower bound. Valid inequalities are developed to SPL-SCSS. The final section gives the necessary modification of allowing backlogging to both basic and strengthened formulation. We first discuss the characteristic and general assumption of the model.

- A set of different products {1,...,N} is manufactured on a single resource and the production processes is assumed to be single level.
- For a single level production processes, there is no successor of the end item. Initial inventory and ending inventory are assumed to be zero. In practical continuous and multi-level production environment, initial inventory and ending inventory are considered in each stage. We simplify the assumption here.
- Excessive items produced are stored as inventory and incur inventory costs.
- Planning horizon is divided into several discrete time periods {1,..,T}.
- Both setup time and setup costs are product-dependent, setups consume capacity and incur setup costs.
- The demand of each product is time-variant. For data without backlogging, zero demand are inserted in the first several periods to ensure the feasibility. Such modification is not required for data with backlogging.

- Overtime production is allowed and incurs penalty costs if period capacity is not sufficient. In practice, a production schedule with overtime is never desired . We include the overtime to indicate whether the solution output gives an acceptable schedule without generating high penalty costs.

The symbols and notation used in the formulation of the next couple of sections is presented in Table 2.1.

Symbol	Definition
Index	
$\mathcal{N}$	set of products {1,...,N}
$\mathcal{T}$	set of time periods {1,...,T}
Parameters	
B_i	capacity consumed for processing one unit of product i
H_i	inventory holding cost per unit of product i
SC_i	cost for setting up the facility to produce product i
ST_i	time for setting up the facility to produce product i
C_t	total available capacity in period t
D_{it}	demand for product i in period t
PC_t	penalty cost for overtime capacity used in period t
M_{it}	cumulative demand of product i from period t up to T, where $M_{it} = \sum_{j=t}^{T} d_{ij} \quad \forall i, t.$
Variables	
x_{it}	production quantity for product i in period t
I_{it}	inventory of product i at the end of period t
z_{it}	1 if a complete setup is done in period t for product i, 0 otherwise
O_t	overtime consumed in period t

Table 2.1: Notation for the Base Model

2.1 Basic: CLSP (I&L Representation)

We begin with the formulation of basic CLSP in this section. Trigeiro *et al.* (1989) first considered to incorporate setup time in the CLSP and stated that the finding a feasible solution for the multi-item CLSP with setup time is NP-complete. This formulation is also referred to as inventory and lot sizing model (Stadtler, 1996) as it explicitly indicates the

inventory of item i at the end of period t and the associated production quantities.

$$\text{Minimize:} \quad \sum_{i=1}^{N}\sum_{t=1}^{T}(SC_i \cdot z_{it} + H_i \cdot I_{it}) + \sum_{t=1}^{T} PC_t \cdot O_t \tag{2.1}$$

Subject to:

$$x_{it} + I_{i(t-1)} = D_{it} + I_{it} \quad \forall i, t \tag{2.2}$$

$$\sum_{i=1}^{N}(x_{it} \cdot B_i + z_{it} \cdot ST_i) \leq C_t + O_t \quad \forall t \tag{2.3}$$

$$x_{it} \leq M_{it} \cdot z_{it} \quad \forall i, t \tag{2.4}$$

$$x_{it}, I_{it}, O_t \geq 0, \quad I_{i0} = I_{iT} = 0, \quad z_{it} \in \{0, 1\} \tag{2.5}$$

The objective function (2.1) is to minimize overall setup costs, holding costs and penalty cost. Constraints (2.2) indicate that the demand could be satisfied from current production or previous inventory. The total production and setup time consumed in each period is less or equal to the corresponding capacity and overtime used, as stated in constraints (2.3). Constraints (2.4) make sure production quantity is restricted to a large number M_{it} and setup is launched. The upper bound of the large number M_{it} is equal to the cumulative demand for product i from period t up to T, where $M_{it} = \sum_{j=t}^{T} d_{ij} \quad \forall i, t$. Constraints (2.5) define the range of the decision variables. Without loss of generality, we use the similar description of M_{it} for the following sections.

2.2 CLSP with Setup Carryover (CLSP-SC)

The first extension regarding to setup operations is to include the setup carryover feature in the basic CLSP. In some literature, the setup carryover is also referred to linked lot sizes (CLSPL) (Haase, 1994; Gopalakrishnan *et al.*, 1995). It links adjacent periods and continue the production for identical items, hence, removing one unnecessary setup and save in setup time and setup costs. CLSP does not provide sequence information yet CLSP-SC is a partial sequence model since the first and last production item can be identified if setup carryover

is allowed. The concept of setup carryover is illustrated in Figure 2.1. Without setup carryover, the sequence of productions in CLSP can be arbitrary. When setup carryover is included, product 4 has to be the last production in period t and the beginning of period $t+1$ so that setup state can be preserved. To model setup carryover, we introduce the following binary variables.

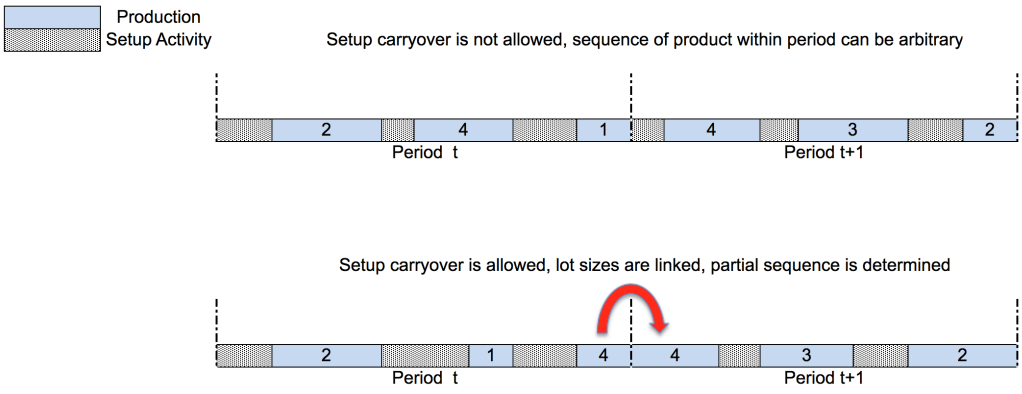


Figure 2.1: Gantt Chart of Allowing Setup Carryover

Variable	**Definition**
u_{it}	$u_{it} = 1$ if setup is carried over from period $(t-1)$ to period t for product i, 0 otherwise
Q_t	$Q_t = 1$ if there is no complete setup activity and at most one product is produced in period t

$$\text{Minimize:} \quad \sum_{i=1}^{N}\sum_{t=1}^{T}(SC_i \cdot z_{it} + H_i \cdot I_{it}) + \sum_{t=1}^{T} PC_t \cdot O_t \tag{2.1}$$

Subject to:

$$x_{it} + I_{i(t-1)} = D_{it} + I_{it} \quad \forall i, t \tag{2.2}$$

$$\sum_{i=1}^{N}(x_{it} \cdot B_i + z_{it} \cdot ST_i) \leq C_t + O_t \quad \forall t \tag{2.3}$$

$$x_{it} \leq M_{it} \cdot (z_{it} + u_{it}) \quad \forall i, t \tag{2.6}$$

$$\sum_{i=1}^{N} u_{it} \leq 1 \quad \forall t \tag{2.7}$$

$$u_{it} \leq z_{i(t-1)} + u_{i(t-1)} \quad \forall i, t \tag{2.8}$$

$$z_{it} + Q_t \leq 1 \quad \forall i, t \tag{2.9}$$

$$u_{i(t+1)} + u_{it} \leq 1 + Q_t \quad \forall i, t \tag{2.10}$$

$$x_{it}, I_{it}, O_t, Q_t \geq 0, \quad I_{i0} = I_{iT} = 0, \quad u_{i1} = 0, \quad z_{it}, u_{it} \in \{0, 1\} \tag{2.11}$$

The definition of objective function (2.1), inventory balance constraints (2.2) and capacity balance constraints (2.3) do not change in CLSP-SC. Constraints (2.6) restrict the production quantity to cumulative demand M_{it} such that setup is launched or carried from previous period. Constraints (2.7) secure that setup could be carried across over two periods for at most one item only. Constraints (2.8) state that setup can be carried from period $t-1$ to t only if there is a complete setup in period $t-1$ or setup is already carried over in $t-1$. In (2.9), Q_t is the idle period indicator so that it is mutually exclusive with the complete setup variables z_{it}. Note here Q_t does not have to declare as binary variables. Constraints (2.10) make sure that setup can be carried over two consecutive periods if $Q_t = 1$. Constraints (2.11) define the range of the decision variables.

2.3 CLSP with Setup Carryover and Setup Splitting (CLSP-SCSS)

Setup is not only can be preserved and carried into next period, it is also able to be partially finished in one period and continued in next period. This feature is referred to as setup splitting or setup crossover (Mohan *et al.*, 2012; Belo-Filho *et al.*, 2013). Modeling setup splitting is necessary and practical to tackle long setup time, as long setup time are often exceed the available capacity and can not be solved by CLSP or CLSP-SC. The concept of setup splitting is illustrated in Figure 2.2. With setup splitting, the idle capacity can be utilized, the production of product 2 is moving forward, producing more units to

satisfy the demand. Also, if the setup activity exceeds the capacity, the setup splitting allows possibility to perform the setups in two adjacent periods. Setup splitting is allowed to be carried over to the next period if needed. To model the setup splitting feature, several variables are added to the formulation.

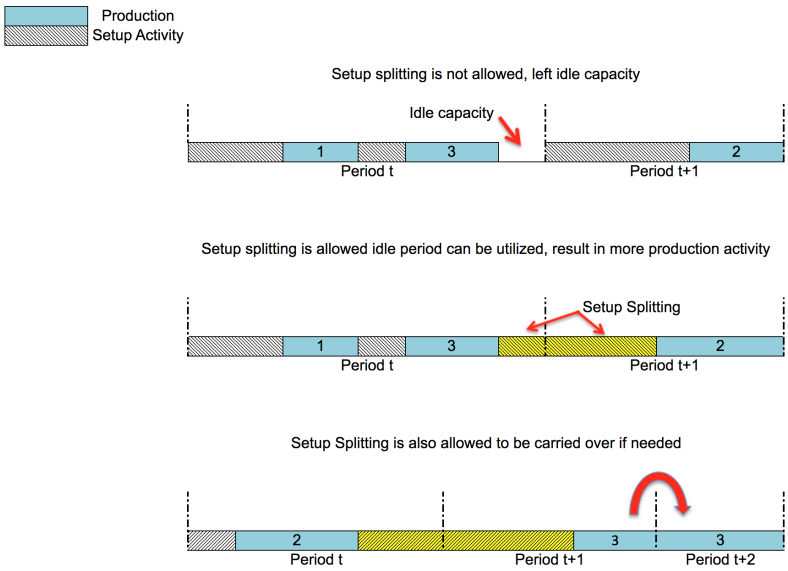


Figure 2.2: Gantt Chart of Allowing Setup Splitting

Variable	**Definition**
l_{it}	duration of the setup for product i completed at the end of period t
f_{it}	duration of the setup for product i completed at the beginning of period t
v_{it}	$v_{it} = 1$ if setup is split between period $(t-1)$ and period t for product i with the splits being $l_{i(t-1)}$ and f_{it} respectively, 0 otherwise

$$\text{Minimize:} \quad \sum_{i=1}^{N}\sum_{t=1}^{T} SC_i \cdot (z_{it} + v_{it}) + \sum_{i=1}^{N}\sum_{t=1}^{T} H_i \cdot I_{it} + \sum_{t=1}^{T} PC_t \cdot O_t \tag{2.12}$$

Subject to:

$$x_{it} + I_{i(t-1)} = D_{it} + I_{it} \quad \forall i, t \tag{2.2}$$

$$z_{it} + Q_t \leq 1 \quad \forall i, t \tag{2.9}$$

$$u_{i(t+1)} + u_{it} \leq 1 + Q_t \quad \forall i, t \tag{2.10}$$

$$\sum_{i=1}^{N} (x_{it} \cdot B_i + z_{it} \cdot ST_i + f_{it} + l_{it}) \leq C_t + O_t \quad \forall t \tag{2.13}$$

$$x_{it} \leq M_{it} \cdot (z_{it} + u_{it} + v_{it}) \quad \forall i, t \tag{2.14}$$

$$f_{it} + l_{i(t-1)} = v_{it} \cdot ST_i \quad \forall i, t \tag{2.15}$$

$$\sum_{i=1}^{N} (u_{it} + v_{it}) \leq 1 \quad \forall t \tag{2.16}$$

$$u_{it} \leq z_{i(t-1)} + u_{i(t-1)} + v_{i(t-1)} \quad \forall i, t \tag{2.17}$$

$$u_{i(t+1)} + v_{it} \leq 1 + Q_t \quad \forall i, t \tag{2.18}$$

$$z_{it} + u_{it} + v_{it} \leq 1 \quad \forall i, t \tag{2.19}$$

$$x_{it}, I_{it}, O_t, Q_t, f_{it}, l_{it} \geq 0, \quad I_{i0} = I_{iT} = 0, \quad u_{i1} = v_{i1} = 0, \quad z_{it}, u_{it}, v_{it} \in \{0, 1\} \tag{2.20}$$

The definition of inventory balance constraints (2.2), idle period indication constraints (2.9) and two-periods carryover constraints (2.10) are similar to previous section. Here the objective function (2.12) minimizes the total holding costs, penalty costs and setup costs with the corresponding complete setup and setup splitting. Constraints (2.13) ensure that the capacity consumed by production and setups is less or equal to available capacity and overtime used in a period. Constraints (2.14) allow production activity only if either complete setup, setup carryover or setup splitting is done in a period. Constraints (2.15) ensure that the setup splitting time is the sum of two splitting indicator variables. Constraints (2.16) state that either one setup carryover or setup splitting is allowed for at most one product in a period. Constraints (2.17) make sure that setup carryover can only be done in period

t if there is either a complete setup or setup carryover or setup splitting in period $t-1$. Constraints (2.18) indicates that in an idle period, at most one product is being produced because of partially setup and this setup state can be carried into next period. For each production activity, constraints (2.19) ensure that only one setup condition (complete, carryover or splitting) or no production in that period. Constraints (2.20) define the range of the decision variables.

2.4 Reformulations of the CLSP-SCSS

It is true that how a model is formulated has a significant impact on the solver performance. The purpose of reformulation is to generate a tighter formulation such that computational burden can be reduced. The concept is illustrated in Figure 2.3. Suppose there is a pure integer programming problem with variables x and y. Obviously the formulation of convex hull conv(x) which contains all integer solution is an ideal formulation. Formulation $\mathcal{P}$ and $\mathcal{Q}$ are also both feasible since they do not exclude any integer solutions. Here we can say formulation $\mathcal{P}$ is better than formulation $\mathcal{Q}$ as $\mathcal{P} \subset \mathcal{Q}$. It is very difficult to obtain the convex hull formulation directly for many hard problems in general. However, one can try to find a tighter formulation of the original problem to help reduce the computational effort. Two well-known reformulation methods on capacitated lot sizing problem can be considered, the shortest route representation and simple plant location formulation, as stated in section 1.3.2. In this research we choose the simple plant location (SPL) formulation to strengthen our CLSP-SCSS model.

2.4.1 Simple Plant Location Formulation (SPL)

The simple plant location problem (SPL) is to determine a plant location so that a set of customer demand nearby can be all satisfied while minimizing the transportation costs and fix costs for placing such facilities. This problem is also referred to as fixed-charge facility location problem. Analogously, placing a facility on a location is similar to launch setup operation in a certain period since both activities incur fix costs. The

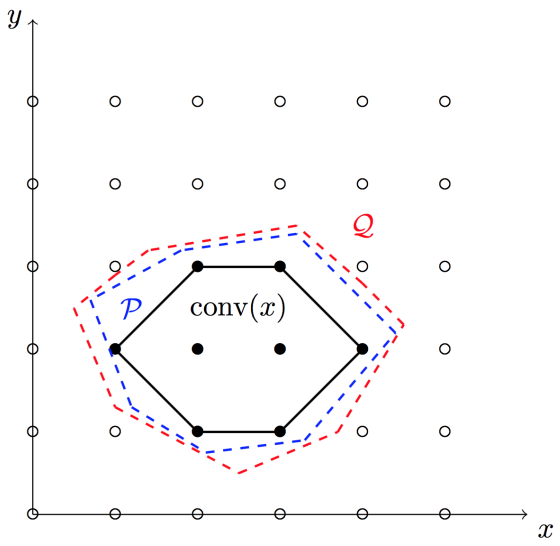


Figure 2.3: Different Formulation for a Problem

transportation costs can thus be interpreted as inventory holding costs. Krarup and Bilde (1977) show that for a uncapacitated case, the LP relaxation of lot sizing problem in simple plant location formulation will provide optimal solutions with integer setup variables. Since our model is capacitated, the formulation would be stronger compared to uncapacitated case, which generates better lower bound. Figure 2.4 illustrates the concept of simple plant location problem. We first give the definition of variables P_{its} and then substitute the

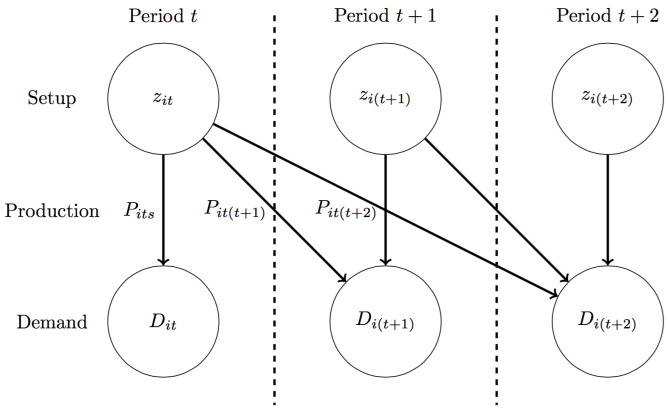


Figure 2.4: Simple Plant Location Problem

production quantity variables x_{it} by variables P_{its} based on equation (2.21). P_{its} represent the proportion of demand of product i in period t to satisfy demand in period s, where $s \geq t$.

$$x_{it} := \sum_{s=t}^{T} D_{is} \cdot P_{its} \quad \forall i, t \tag{2.21}$$

Since the definition of variable P_{its} here is to represent the proportion of demand, the range of variables is bounded in the range $[0, 1]$.

$$\text{Minimize:} \quad \sum_{i=1}^{N} \sum_{t=1}^{T} \sum_{s=t}^{T} h_i \cdot (s-t) \cdot D_{it} \cdot P_{its} + \sum_{i=1}^{N} \sum_{t=1}^{T} SC_i \cdot (z_{it} + v_{it}) + \sum_{t=1}^{T} PC_t \cdot O_t \tag{2.22}$$

Subject to:

$$z_{it} + Q_t \leq 1 \quad \forall i, t \tag{2.9}$$

$$u_{i(t+1)} + u_{it} \leq 1 + Q_t \quad \forall i, t \tag{2.10}$$

$$f_{it} + l_{i(t-1)} = v_{it} \cdot ST_t \quad \forall i, t \tag{2.15}$$

$$\sum_{i=1}^{N} (u_{it} + v_{it}) \leq 1 \quad \forall t \tag{2.16}$$

$$u_{it} \leq z_{i(t-1)} + u_{i(t-1)} + v_{i(t-1)} \quad \forall i, t \tag{2.17}$$

$$u_{i(t+1)} + v_{it} \leq 1 + Q_t \quad \forall i, t \tag{2.18}$$

$$z_{it} + u_{it} + v_{it} \leq 1 \quad \forall i, t \tag{2.19}$$

$$\sum_{i=1}^{N} \sum_{s=t}^{T} B_i \cdot D_{is} \cdot P_{its} + \sum_{i=1}^{N} (z_{it} \cdot ST_i + f_{it} + l_{it}) \leq C_t + O_t \quad \forall t \tag{2.23}$$

$$P_{its} \leq z_{it} + u_{it} + v_{it} \quad \forall i, t, s = t, ..., T \tag{2.24}$$

$$\sum_{s=1}^{t} P_{ist} = 1 \quad \forall i, t \quad \text{provided} \quad D_{it} > 0 \tag{2.25}$$

$$P_{its}, O_t, Q_t, f_{it}, l_{it} \geq 0, \quad u_{i1} = v_{i1} = 0, \quad z_{it}, u_{it}, v_{it} \in \{0, 1\} \tag{2.26}$$

Compared to CLSP-SCSS formulation, there are only several differences. The production quantity variables x_{it} in the objective function (2.12) and constraints (2.13) are substituted

with the proportional variables P_{ist} in (2.22) and (2.23) respectively. Constraints (2.24) state that production take place only if there is either a complete setup, setup carryover or setup splitting of product i in period t. Constraints (2.25) must be added to ensure the range of the proportional variables. All other constraints remain the same from previous formulation CLSP-SCSS. The SPL formulation of CLSP-SCSS (SPL-SCSS) is consists of $3NT$ binary variables, at most $NT(\frac{T+1}{2})$ continuous variables and $NT(\frac{T+1}{2}) + 7NT + 2T$ constraints.

2.4.2 Extended Formulation

Even though the SPL model is a tighter formulation already, we can further extend the formulation. We here adopted the similar idea presented by Suerie and Stadtler (2003). They redefine the idle indicator variables Q_t by making it become product-dependent variables QQ_{it}.

Variable	**Definition**
QQ_{it}	$QQ_{it} = 1$ means a product i in period t is produced purely in an idle period and the setup state is carried into next period.

Similar to Q_t, QQ_{it} here does not necessarily to be declared as binary variables ($QQ_{i1} = QQ_{iT} = 0$). The definition of variable QQ_{it} can replace constraints (2.9) in the SPL formulation with the following:

$$z_{it} + \sum_{i} QQ_{it} \leq 1 \quad \forall i, t \tag{2.27}$$

We subtract the right-hand-side by $v_{it} + u_{it} - QQ_{it}$, resulting in (2.28).

$$z_{it} + u_{it} + v_{it} + \sum_{k \neq i} QQ_{kt} \leq 1 \quad \forall i, t \tag{2.28}$$

Constraint (2.28) is valid because of the following reasons: a product i in period t can either have a complete setup ($z_{it} = 1$) or a carried over from previous period ($u_{it} = 1$) or a

splitting setup ($v_{it} = 1$), or any other single-item production for any $k \neq i$ in period t (one $QQ_{kt} = 1$). The possible condition discussed here are obviously mutually exclusive. Next we can replace constraint (2.17) with (2.29) since the variable QQ_{it} indicate for an item i the setup can be carried over from previous complete setup ($z_{i(t-1)} = 1$) or the setup has already carried over from $t-2$ to $t-1$, which also implies $QQ_{i(t-1)} = 1$.

$$u_{it} \leq z_{i(t-1)} + QQ_{i(t-1)} \quad \forall i, t = 2, ..., T \tag{2.29}$$

Constraints (2.10) and (2.18) now can be dropped as they are dominated by (2.28) and (2.29) now. In addition, we need to add constraints (2.30) and (2.31) to restrict the range of QQ_{it} variables.

$$QQ_{it} \leq u_{is} + v_{is} \quad \forall i, \quad t = 2, .., T-1, \quad s = t, ..., t+1. \tag{2.30}$$

$$QQ_{it} \geq 0 \quad (QQ_{i1} = 0, QQ_{iT} = 0) \quad \forall i, t \tag{2.31}$$

Finally, we included constraints (2.32) for the following reasons. For a product i in period t, if there is a complete setup in period $t-1$, it is unnecessary to perform a setup splitting for the production of product i in period t.

$$v_{it} + z_{i(t-1)} \leq 1 \quad \forall i, \quad t = 2, .., T \tag{2.32}$$

2.5 Valid Inequalities for CLSP-SCSS

Another common way to obtain a tighter formulation is by adding the valid inequalities. The purpose of adding valid inequalities is to eliminate the feasible region of LP relaxation but not integer feasible solution in solution space and approximate the convex hull of the problem, reducing the search time. Figure 2.5 demonstrates this concept. Many commercial solvers such as CPLEX has built-in gomory cuts generator to help speed up the branch-and-bound process. However, for a NP-hard problem like CLSP, developing problem-specific valid inequalities is beneficial to solve the problem more efficiently. Suerie and Stadtler (2003) developed a series of valid inequalities for their multi-level CLSP with linked lot sizes (MLCLSPL) to find the first feasible solution within time limit, we modified the inequalities and added all of them to our model to increase the solution efficiency.

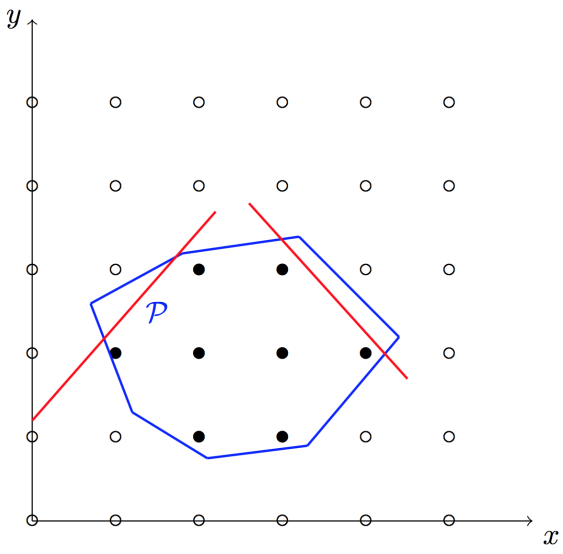


Figure 2.5: Valid Inequalities for the Problem

2.5.1 Pre-processing Inequalities

This concept of this group of valid inequalities is derived by observing the data and also used in Maes et al. 1991. The argument is that within the interval length R, if cumulative slack capacity up to period $t-1$ is less than the requirement of single item production in the interval $[t;\, t+R-1]$, then at least two products will have to be produced in this interval. This also implies that at least one setup will be performed, which means not all periods of this interval can have single-item production. This can help to restrict the range of values of variable QQ_{it}.

$$\sum_{s=t}^{t+R-1} QQ_{is} \leq R-1 \quad \forall i, \quad t=2,...,T-R+1, \quad R=1,..,3 \tag{2.33}$$

if the following condition holds:

$$\sum_{s=1}^{t-1} C_s - \sum_{s=1}^{t-1}\sum_{i=1}^{N} B_i \cdot D_{is} - \sum_{i=1}^{N} ST_i - \sum_{s=t}^{t+R-1}\sum_{j\in N, j\neq i}^{N} B_i \cdot D_{js} < 0 \tag{2.34}$$

The first two components of constraint (2.34) calculate the cumulative slack capacity up to period $t-1$, then subtract the term 3 minimum required setup time. Term 4 calculates the capacity consumption in period t and then subtracted from cumulative slack capacity.

Note that when $R = 1$, QQ_{it} are forced to 0 if slack capacity condition is fulfilled. As the effect of these constraints disappear gradually when R increases, Suerie and Stadtler (2003) formulated these constraints for $R \leq 3$. Here we adopt the same idea with them for our extended formulation.

2.5.2 Inventory/Setup Inequalities

If $z_{it} = u_{it} = v_{it} = 0$ for a product i in period t, there is no production of i in t. Thus the demand for product i in period t (D_{it}) must be satisfied from previous inventory ($I_{i(t-1)}$). The same condition holds in interval $[t; t+p]$ as well, if there is no carryover into the beginning of interval ($u_{it} = 0$) and no setup activities throughout the interval (either $z_{it}, ..., z_{i(t+p)}$ or $v_{it}, ..., v_{i(t+p)}$=0). This leads to the following valid inequalities:

$$I_{i(t-1)} \geq \sum_{s=t}^{t+p} D_{is} \cdot (1 - u_{it} - \sum_{r=t}^{s} z_{ir} - \sum_{r=t}^{s} v_{ir}) \quad \forall i, \quad t = 1, ..., T-1 \quad p = 1, ..., T-t \quad (2.35)$$

Constraints (2.35) can be further formulated with the proportional variables P_{its} in SPL formulation.

$$\sum_{s=t}^{r} P_{isr} \leq u_{it} + \sum_{s=t}^{r} z_{is} + \sum_{s=t}^{r} v_{is} \quad \forall i, \quad r = 1, ..., T \quad t = 1, ..., r \quad (2.36)$$

2.5.3 Single-item Production Inequalities

The last set of valid inequalities is the combination of capacity balance constraints and single-item production variables XQ_{it}. We first define the new variable XQ_{it}.

Variable	**Definition**
XQ_{it}	single-item production quantity of product i in period t, where period t is an idle period and setup state is linked from previous period.

In (2.37), the range of XQ_{it} is restricted to production quantity. Constraints (2.38) reduce XQ_{it} to 0 if there is no single-item production ($QQ_{it} = 0$). If there is a single-item

production in period t, capacity consumption is restricted to production of XQ_{it}, on the other hand, if there is no single-item production, then constraints (2.39) become just the capacity constraint of the original CLSP-SCSS model.

$$XQ_{it} \leq x_{it} \quad \forall i, t = 2, ..., T-1 \tag{2.37}$$

$$XQ_{it} \leq min(\frac{C_t}{B_i}, \sum_{s=t}^{T} D_{is}) \cdot QQ_{it} \tag{2.38}$$

$$\sum_{i=1}^{N}(x_{it} \cdot B_i + z_{it} \cdot ST_i + f_{it} + l_{it}) \leq C_t \cdot (1 - \sum_{i=1}^{N} QQ_{it}) + \sum_{i=1}^{N} XQ_{it} \cdot B_i + O_t \quad \forall t = 2, ..., T-1 \tag{2.39}$$

2.6 Inclusion of Backlogging

Another realistic consideration happens often in industrial practice is to allow the possibility of backlogging in CLSP-SCSS. Basically, the demand now can be met from previous inventory, current production and later production with additional penalty costs. To address this feature, the modification in the basic model and SPL formulation is essential. Literature regarding to this matter can be found in Wu and Shi (2009, 2011) and Wu *et al.* (2013). In this section, we present the inclusion of backlogging to CLSP-SCSS. Furthermore, we present the strong formulation of including backlogging in the SPL-SCSS.

2.6.1 CLSP-SCSS with Backlogging

First, we introduce the definition of backlogging variables and the associated backlogging cost. Then replace constraints (2.2) with (2.40) and add constraints (2.41) to make sure no backlogging unit is allowed in the last period. The corresponding backlogging costs incur because of allowing backlogging in the production flow. The objective function of CLSP-SCSS in (2.12) now becomes the new one (2.42). Only these modifications are needed in the original CLSP-SCSS model.

Variable	Definition
bg_{it}	backlogging unit of product i in period t
BC_i	backlogging cost for product i

$$x_{it} + I_{i(t-1)} + bg_{it} - bg_{i(t-1)} = D_{it} + I_{it} \quad \forall i, t \tag{2.40}$$

$$bg_{iT} = 0 \tag{2.41}$$

$$\text{Minimize:} \quad \sum_{i=1}^{N}\sum_{t=1}^{T} SC_i \cdot (z_{it} + v_{it}) + \sum_{i=1}^{N}\sum_{t=1}^{T} H_i \cdot I_{it} + \sum_{i=1}^{N}\sum_{t=1}^{T} BC_i \cdot bg_{it} + \sum_{t=1}^{T} PC_t \cdot O_t \tag{2.42}$$

2.6.2 SPL-SCSS with Backlogging

To modify the SPL-SCSS for taking backlogging into account, the equivalence equation of production quantity and the definition of proportion variables in equation (2.21) need to be changed with the following equation. Also, with this new definition, the concept of allowing backlogging is demonstrated in Figure 2.6. The new variables P'_{its} represent the proportion of demand of product i in period t to satisfy demand in period s, as stated in equation (2.43).

$$x_{it} := \sum_{s=1}^{T} D_{is} \cdot P'_{its} \quad \forall i, t \tag{2.43}$$

In equation (2.43), the proportion of demand of product i in period t now can be used to satisfy demand either in previous or later period with associated backlogging costs or holding costs, respectively. The objective function of SPL-SCSS in (2.22) can be substituted with the following.

$$\text{Minimize:} \quad \sum_{i=1}^{N}\sum_{t=1}^{T}\sum_{s=t}^{T} h_i \cdot (s-t) \cdot D_{is} \cdot P'_{its} + \sum_{i=1}^{N}\sum_{t=1}^{T}\sum_{s=1}^{t-1} BC_i \cdot (t-s) \cdot D_{is} \cdot P'_{its} + \sum_{i=1}^{N}\sum_{t=1}^{T} SC_i \cdot (z_{it} + v_{it}) + \sum_{t=1}^{T} PC_t \cdot O_t \tag{2.44}$$

According to the definition of proportional variables P'_{its}, several constraints of SPL-SCSS need to be adjusted as well. In constraints (2.45), (2.46) and (2.47), the range of index s is now extended to whole planning horizon. These three constraints together with the new objective function list above can replace constraints (2.23), (2.24) and (2.25) in SPL-SCSS, here we refer this strengthened model with backlogging as SPL-SCSS-BL. This formulation is consists of $3NT$ binary variables, $NT^2+2NT+T$ continuous variables and $NT^2+7NT+2T$ constraints.

$$\sum_{i=1}^{N}\sum_{s=1}^{T} B_i \cdot D_{is} \cdot P'_{its} + \sum_{i=1}^{N}(z_{it} \cdot ST_i + f_{it} + l_{it}) \leq C_t + O_t \quad \forall t \tag{2.45}$$

$$P'_{its} \leq z_{it} + u_{it} + v_{it} \quad \forall i, t, s \tag{2.46}$$

$$\sum_{s=1}^{T} P'_{ist} = 1 \quad \forall i, t \quad \text{provided} \quad D_{it} > 0 \tag{2.47}$$

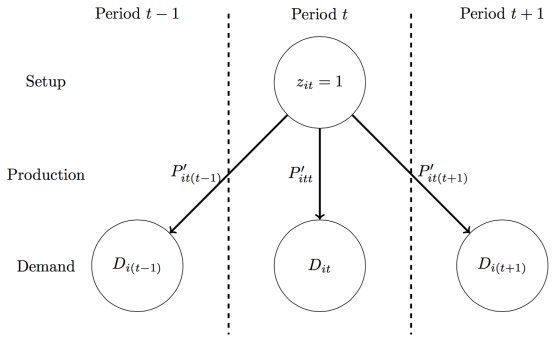


Figure 2.6: Simple Plant Location Problem with Backlogging

Chapter 3

SOLUTION APPROACH

The complexity of CLSP with setup time has been shown to be NP-hard, so does the CLSP-SC and CLSP-SCSS. In mixed integer programming, the number of binary variables is typically a major indicator of the computational time, that is, the more binary variables, the more likely the problem would take longer time to solve. In other words, the binary setup state variables in our model would decelerate the branch-and-bound process. To obtain the solution in a reasonable amount of time, we applied the fix-and-optimize (F&O) heuristic to solve the model. First, all the variables of complete setup z_{it}, setup carryover u_{it} and setup splitting v_{it} are separated into two sets $\mathcal{K}$ and $\mathcal{L}$. In each iteration, only a small subset of z_{it}, u_{it}, v_{it} in $\mathcal{L}$ and other decision variables are solved to optimality, while the rest of binary variables in $\mathcal{K}$ are fixed to exogenous value. Two decomposition methods are used to determine the subproblem: the product decomposition and period decomposition. Through the iteration guided by the decomposition, the solution is updated when lower costs are found because of better combination of setup state until no lower costs can be found. The description of initialization as well as decomposition are discussed in the following respectively, also a framework of the algorithm is presented.

3.1 Construction of Initial Solution

First we need to select the initial feasible solution for the SPL-SCSS. Typically, there are many heuristic available to construct the initial feasible solution without spending too much computational time. For instance, truncated B&B method imposes a time limit for solver to stop during branch-and-bound process, the incumbent found according to the stopping criteria can serve as a starting point. The more time allowed the better solution quality can be found. However, the target of constructive heuristic is not spend too much computational

time, truncated B&B might not be a suitable method if the problem size is getting larger since the time limit would be too short to produce a good solution in terms of optimality gap, or even no feasible solution can be found. In capacitated lot sizing problem, it is trivial that having all the complete setups $z_{it} = 1$ is a feasible schedule to satisfy all the demand. Nonetheless, this cost is not appealing as the setup carryover does not take effect to remove unnecessary setups and extra setup costs. In certain cases the period capacity is insufficient for total setup time, resulting in high penalty costs due to overtime used. Instead of using this trivial solution at beginning, we solved the LP relaxation of SPL-SCSS and then used rounding heuristic to round up all the fractional complete setup variables. Given this setup pattern, we solved the problem again without relaxing the integrality constraints to obtain corresponding setup pattern for setup carryover and setup splitting. This initialization is then passing to the algorithm. During the experiment, we examined the solution quality for several instances and found out that compared to use trivial solution of setting all complete setups, this initialization gives better solution quality and even true optimal solution can be found for small size instances. This test result is given in the next chapter.

3.2 Two-stage Decomposition Strategies

We state the major factor of computational time is the number of binary variables at the beginning of this chapter. The selection of binary variable in set $\mathcal{K}$ and $\mathcal{L}$ is therefore important to the performance of the fix-and-optimize algorithm. Three decomposition strategies were proposed in Sahling *et al.* (2009) to treat the multi-level CLSP problem: product decomposition, period decomposition and process decomposition. They also claimed a combination of these decomposition strategies is beneficial to obtain high quality solutions. Here we used the two-stage decomposition methods: first product decomposition then period decomposition, as single-level production processes is one of our assumptions. Detail description for the decomposition methods will be presented in the following.

3.2.1 Product Decomposition

For product decomposition, the subproblem is defined as the product with relaxed binary variables in the current iteration. Sahling *et al.* (2009) relaxed complete setup variables z_{it} for a single product i and all carryover variables u_{it} for every product across the planning horizon. Considering their test instances are smaller than the one we used, in addition, our model has one more group of binary variables v_{it}, we applied the product decomposition in a slightly different way. Here in our SPL-SCSS model, all the binary variables z_{it}, u_{it} and v_{it} are optimized with other real-valued decision variables for a single product i over the planning horizon. Figure 3.1 demonstrates this concept. At each iteration, the binary variables of one product is optimized with all other continuous variables. According to Sahling *et al.* (2009) and Helber and Sahling (2010), it is worth to break down those products with higher costs. A cost function is thus established in equation (3.1) to determine the holding, setup and overtime costs for each product in descending order. By doing so, the order of decomposition follows this cost information to optimize the product with highest costs first, then the second and so on. Note equation (3.1) is based on the CLSP-SCSS to show the concept of allocating relevant costs to each product. One should modify the equation for SPL-SCSS during the experiment.

$$\sum_{t=1}^{T}[SC_i \cdot (z_{it}^{rel} + v_{it}^{rel}) + H_i \cdot I_{it}]+ \sum_{t=1}^{T}(PC_t \cdot O_t) \cdot \frac{\sum_{t=1}^{T}(x_{it} \cdot B_i + z_{it}^{rel} \cdot ST_i + l_{it} + f_{it})}{\sum_{k=1}^{N}\sum_{t=1}^{T}(x_{kt} \cdot B_k + z_{kt}^{rel} \cdot ST_k + l_{kt} + f_{kt})} \quad \forall i \quad (3.1)$$

In equation (3.1), z_{it}^{rel} and v_{it}^{rel} represent the LP relaxation solution of the corresponding z_{it} and v_{it}. The actual costs for each product is not known before solving the problem, therefore an approximation by solving LP relaxation is used. Note that the first term in equation (3.1) is the sum of setup costs and holding costs. Because it is difficult to argue which product charges more overtime capacity in the production, the second term is to allocate the penalty costs proportionally to a specific product based on the capacity consumed over total capacity. Also, for SPL-SCSS-BL, the backlogging costs should be

included in equation (3.1).

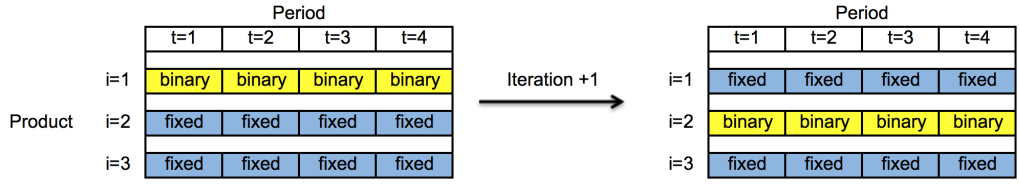


Figure 3.1: Illustration of Product Decomposition

3.2.2 Period Decomposition

Originally, the period decomposition aims to separate the machines available in multi-level CLSP problem and use a time-window forwarding strategy to determine the subproblem. For each machine, the setup variables of all products are released over a short term period. Sahling *et al.* (2009) applied a rolling time window with four consecutive periods for z_{it} and u_{it} and one extra period for u_{it} only in case carryover is needed. Start from beginning, the subproblem is defined as the time window contains released setup variables of all products being optimized. Note that each subproblem has two overlapping periods and no extra period for carryover in the last time window over the planning horizon. We apply the same principle for considering one resource only in our heuristic. Figure 3.2 shows the concept of period decomposition. In first iteration, all the binary variables from period 1 to period 4 and u_{it} in period 5 only are optimized. In next iteration, two overlapping periods are considered. Hence period 3 and period 4 are optimized again in this iteration.

Since the proportion of binary variables placed in the The percentage of binary variables being optimized θ in each subproblem can be calculated by the number of binary in the set $\mathcal{L}$ over the total number of binary variables. Specifically, θ in product decomposition and period decomposition can be calculated in equation (3.2) and (3.3). In general, θ tend to be smaller when problem size is bigger, the larger the θ is, the more computational time is needed.

$$\theta_{PD} = \frac{3T}{3NT} \tag{3.2}$$

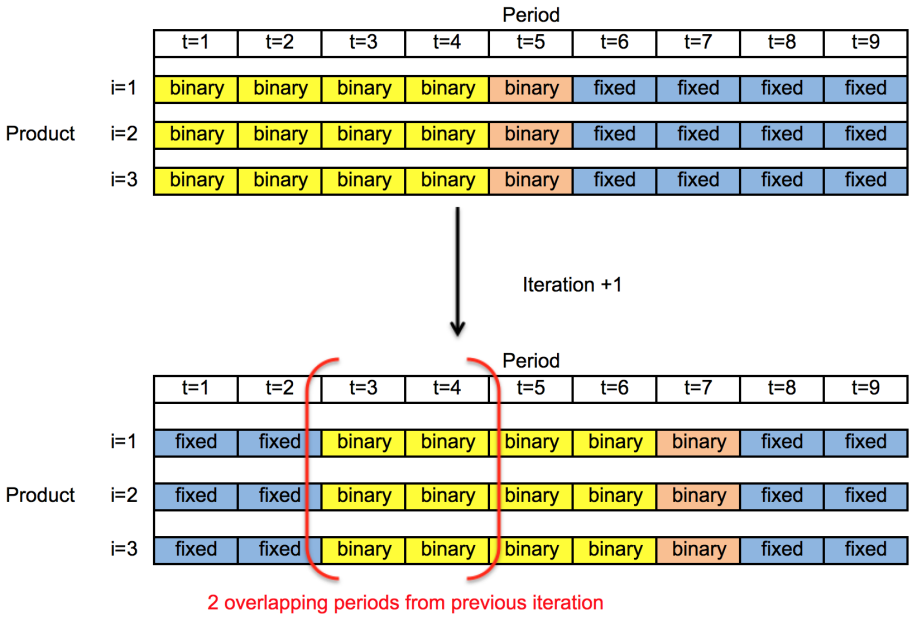


Figure 3.2: Illustration of Period Decomposition

$$\theta_{RD} = \frac{12N + N}{3NT} \tag{3.3}$$

3.3 Framework of the F&O Heuristic

The main structure of the proposed algorithm is illustrated in Figure 3.3 and 3.4. In the first step, the LP relaxation is solved to calculate the approximate costs of each products. Next the fractional z_{it} are setting to 1 and solved again with the integer restriction to obtain corresponding solution of carryover and splitting. This initial solution and objective function is used for the starting point of the algorithm. In each iteration through the product decomposition, if current objective function value is smaller, all binary variables for this specific product is fixed and the objective function value is updated, otherwise moving the iteration to next product. The algorithm examines every product until no better objective function value can be found. The incumbent found by product decomposition is then sent to the period decomposition. With four consecutive periods released, the algorithm starts with period 1-4 contain all binary variables for every product. Note that period 5 is released for carryover u_{it} only. In the next iteration with two overlapping periods,

period 1-2 are now fixed and period 3-6 contain the released binary variables. Similar to product decomposition, the algorithm goes over the whole planning horizon repeatedly until no better objective function value can be found. Sahling *et al.* (2009) and Helber and Sahling (2010) pointed out one can perform multiple iterations or single iterations in each decomposition strategies, the algorithm is terminated by setting an external parameter. In this thesis, we would like to achieve the best performance of the heuristic, hence we terminate the algorithm until no further improvement can be made.

Algorithm 1 Fix & Optimize Heuristics

Initialize the LP relaxation for SPL-SCSS
Let ν = Max number of iterations
Let $z_{it}^{rel} = 1$
Solve SPL-SCSS to determine
objective function obj
$obj^{new} \leftarrow obj$
$\overline{z_{it}} \leftarrow z_{it}$
$\overline{u_{it}} \leftarrow u_{it}$
$\overline{v_{it}} \leftarrow v_{it}$
counter = 0
repeat
  $obj^{old} \leftarrow obj^{new}$
  for each subproblem **do**
    counter = counter + 1
    decompose z, u, v into the fixed set $\mathcal{K}$
    and relaxed set $\mathcal{L}$, respectively
    Solve subproblem to obtain objective
    function obj^{sub}
    if $obj^{sub} < obj^{old}$ **then**
      Update
      $\overline{z_{it}} \leftarrow z_{it}$
      $\overline{u_{it}} \leftarrow u_{it}$
      $\overline{v_{it}} \leftarrow v_{it}$
      $obj^{new} \leftarrow obj^{sub}$
    else
      Restore
      $z_{it} \leftarrow \overline{z_{it}}$
      $u_{it} \leftarrow \overline{u_{it}}$
      $v_{it} \leftarrow \overline{v_{it}}$
    end if
  end for
until $obj^{new} \geq obj^{old}$ or counter = ν

Figure 3.3: Pseudocode of the Algorithm

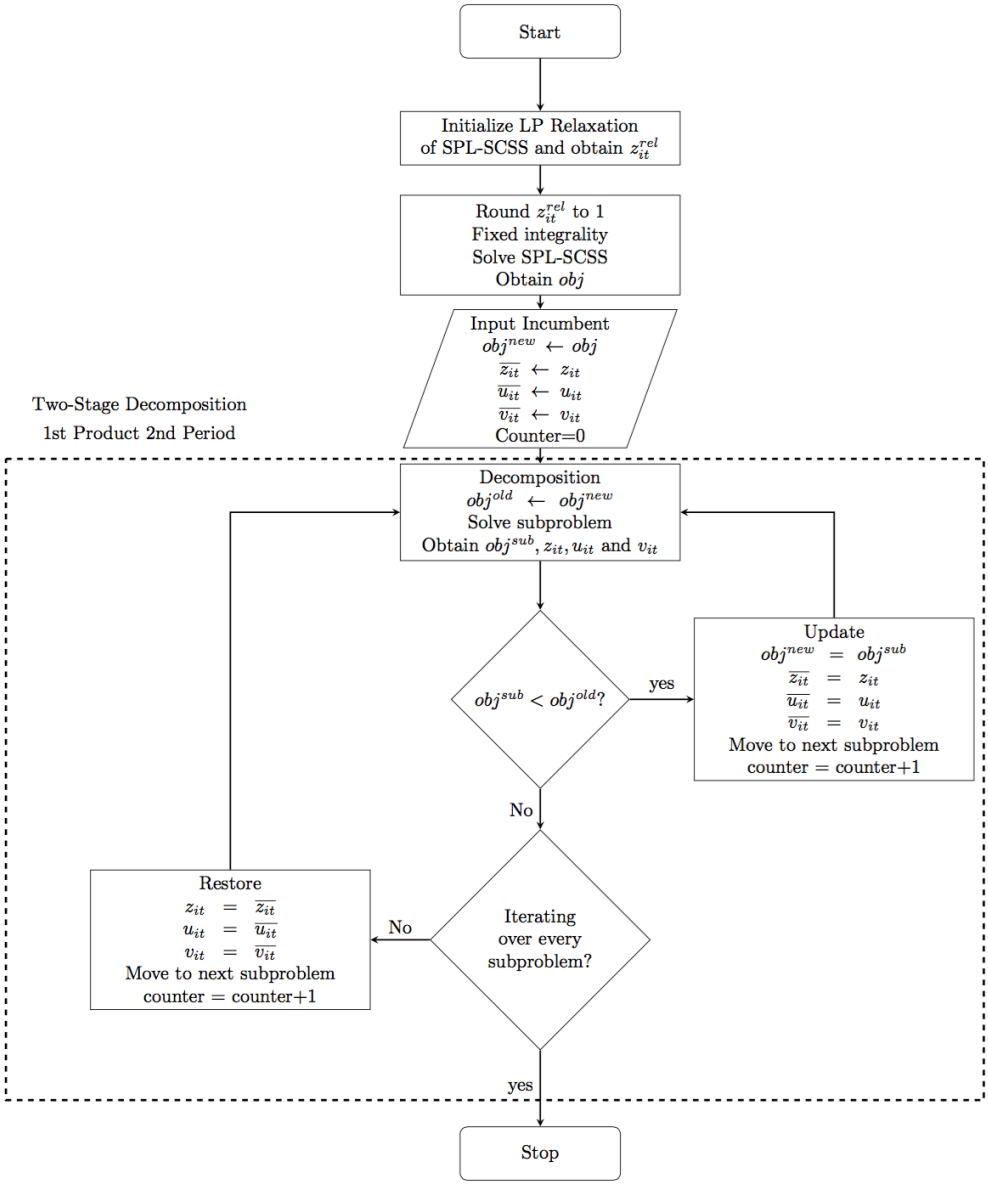


Figure 3.4: Flowchart of the Algorithm

Chapter 4

COMPUTATIONAL TEST

In this section, the result of computational test is reported. In addition, the performance of the algorithm is also analyzed. A Dell Precision T7500 Workstations is used to perform all the computational test. The processor of the workstation is dual six-core Intel Xeon Processor X5690 (4.46 GHz, 12M L3, 6.4 GT=s). The memory of the workstation is 48 GB, 1,333 MHz, DDR3RDIMM, ECC (6DIMMS). The proposed extended formulation and valied inequalities are coded in AMPL and solved by IBM ILOG CPLEX 12.0.6.1.

The fix-and-optimize heuristic is applied on the formulation SPL-SCSS and SPL-SCSS-BL respectively on the corresponding data. All the valid inequalities are added to SPL-SCSS in an attempt to enhance the solution efficiency. But the pre-processing valid inequalities in section 2.5.1 can not be applied to SPL-SCSS-BL since allowing backlogging violates the condition described in this type of valid inequalities.

In Belo-Filho *et al.* (2013), a truncated B&B method is used to obtain solution. To be more precise, they imposed a 600 seconds time limit on the proposed formulation and the parallel mode was active (4 cores) then used MIP solver to obtain final solution. The result shows that the proposed formulation CMLM and DSM give best optimality gap for the data without and with backlogging respectively. The optimality gap is measure by the deviation of incumbent found from LP relaxation. Since the formulation of CMLM and DSM are also based on simple plant location, we list the model size of CMLM, DSM, SPL-SCSS and SPL-SCSS-BL in Table 4.1. Note that their formulation contains one set of variables α to indicate the model size for long setup time. When α long setup time exist, the number of constraints in CMLM becomes $NT^2 + 7NT + (3\alpha + 2)T$, and the number of constraints in DSM becomes $NT^2 + 6NT + (3\alpha + 5)T$ yet this modification is not necessary in our standard formulation.

	Continuous	Binary	Constraints
SPL-SCSS	at most $NT(\frac{T+1}{2})$	$3NT$	$NT(\frac{T+1}{2}) + 7NT + 2T$
SPL-SCSS-BL	$NT^2 + 2NT + 2T$	$3NT$	$NT^2 + 7NT + 2T$
CMLM	$NT^2 + NT$	$3NT + T$	$NT^2 + 7NT + 2T$
DSM	$NT^2 + NT$	$3NT - N + T$	$NT^2 + 6NT + 4T$

Table 4.1: Model Size Comparison

4.1 Data Description

Given the fact that there is no benchmark data considering setup splitting and long setup time, we took the data used in Belo-Filho *et al.* (2013) to test the capability of the proposed solution approach. Three data sets were generated to test the tightness of their formulation, the first one contains 3 products and 20 periods with mean setup time varies from 20% to 250% of total capacity. This data set is used for evaluating the impact and frequency of setup splitting as well as the increasing of cost components when mean setup time is getting longer. For testing the effectiveness of our solution approach, we use the second and third data sets. The only difference between second (BTA-2014-EX) and the third data sets (BTA-2014-BL) is the consideration of backlogging. In each data set, the average setup time of all items is fixed to 40%, 70% or 120% of period capacity (400, 700 and 1200 since period capacity is 1000), also the planning horizon are fixed to 20, 30 and 40 periods. Table 4.2 describes the problem size as follows: small size instances with 5 items (class A, B, C), medium size instances with 10 items (class D, F, F) and large size instances with 15 items (class G, H, I). There are total 270 instances in each data set. It should be noted that models with setup carryover fail to give a feasible planning schedule without using overtime on large size class as the existence of long setup time requires setup splitting to handle it. In addition, there are several zero demand periods inserted at the beginning of planning horizon in the BTA-2014-EX for feasibility, while no adjustment is needed for BTA-2014-BL. Readers who are interested in the generating method can refer

to their research.

Class	Product	Period-BL[1]	Period-EX[2]	Instances
A	5	20	22-24	30
B	5	30	33-35	30
C	5	40	44-46	30
D	10	20	23-25	30
E	10	30	34-36	30
F	10	40	45-47	30
G	15	20	25-27	30
H	15	30	34-36	30
I	15	40	46-48	30

Table 4.2: Classification of Testing Data

4.2 Impact on Initial Solution

In the following we demonstrate the advantage of using rounding heuristic to obtain initial setup patterns in Table 4.3. We tested three data sets with different average setup time (AVST) in class A of data of BTA-2014-EX using the rounding heuristic and trivial solution such that fixes all complete setup variables equal to 1. The solution under these two scenarios is compared with the true optimal solution obtained by branch-and-bound method. In Table 4.3, three types of solution are reported. The "R" column indicates the solution found by using rounding heuristic, the "T" column indicates the solution found by using trivial solution and the "O" column indicates the true optimal solution found by exact branch-and-bound algorithm. First we examine the number of true optimal solutions found by using both scenarios. When AVST equals 40%, optimal solutions are found in 7 out of

[1]Period in BTA-2014-BL

[2]Period in BTA-2014-EX

10 instances by using rounding heuristic, while only 3 optimal solutions are found by using trivial solution. When AVST is 70%, 3 optimal solutions are found in rounding heuristic and 2 optimal solutions are found in trivial solution. None of any true optimal solutions are found when AVST is 120% by either these two initialization scenarios. In addition, we compare the solution quality. There are total 7 out of 10 instances when AVST equals 40%, 7 out of 10 when AVST equals 70% and 8 out of 10 when AVST equals 120%, which achieve better solution quality from rounding heuristic. From this result we can see that using rounding heuristic enhances the solution quality. The LP relaxation provides initial setup patterns, eliminating some unnecessary setup variables. This initial pattern helps the algorithm to terminate at better local optimum and even some global optimum.

AVST=40%[3]			AVST=70%[4]			AVST=120%[5]		
R[6]	T[7]	O[8]	R	T	O	R	T	O
5316.08	5377.34	5232.59	8005.25	8087.75	8005.25	21303.3	21170.6	20397.2
6815.98	6815.98	6815.98	9266.69	9366.91	9266.69	29725.8	30732.3	29655.8
7636	7698	7636	12392	13525	12392	32878.7	34145.4	32829.6
5842.05	5850.29	5842.05	13842.6	13716.1	13716.1	40236.5	40919.7	37632.9
7500.8	7514.76	7487.26	10135	10086	10042.5	33500.1	34238.2	31253.2
6186.15	6186.15	6186.15	7984.3	7999.53	7983.55	58025	69838	57832
5859.98	5845.29	5845.29	11184.9	11185.8	11010.2	35543	35661	35509
7969.38	7984.41	7879.39	12162.6	12765.3	12132.2	33427.9	35878.4	33375.1
6105.7	6210.73	6105.7	10107.3	10051.8	10051.8	45552.2	47740.2	43925
6677	6746	6677	14430.1	15279.1	14342.4	36447.5	36251	33786.5
Optimal found by R:7			Optimal found by R:3			Optimal found by R:0		
Optimal found by T:3			Optimal found by T:2			Optimal found by T:0		

Table 4.3: Comparison of Different Initialization Scenarios

4.3 Impact on Valid Inequalities

In this section, we demonstrate the impact on valid inequalities. Since adding the valid inequalities cut off a part of the feasible region of LP relaxation, it could improve the corresponding lower bound obtained. Again, we chose the three data sets in class A of BTA-2014-EX to see how much improvement we can get by adding valid inequalities. Each data set contains 10 instances. Table 4.4 shows the result of this experiment. We report two types of lower bound from LP relaxation without and with valid inequalities added respectively. The improvement (Ipv.) is computed by the following equation, where LB1 represents the lower bound obtained without valid inequalities, while LB2 is the lower bound obtained with valid inequalities.

$$\text{Improvement} = \frac{(\text{LB2} - \text{LB1})}{\text{LB1}} \times 100\% \tag{4.1}$$

We can see as the average setup time increases, the improvement is significant, especially the average improvement is 83.37% when AVST = 120%. During the experiment, we also notice that when valid inequalities are not added to the formulation, using the rounding heuristic after LP relaxation does not guarantee to find an initial feasible integer solution. This experiment result supports the idea that adding valid inequalities is essential to improve the lower bound obtained and achieve better solution quality.

[3]product=5, period=22

[4]product=5, period=23

[5]product=5, period=24

[6]rounding heuristic

[7]trivial Solution

[8]true optimal solution

AVST=40%			AVST=70%			AVST=120%		
LB1	LB2	Ipv.(%)	LB1	LB2	Ipv.(%)	LB1	LB2	Ipv.(%)
4655.45	5069.83	8.90	6294.62	7484.6	18.90	9531.34	13642.3	43.13
6018.97	6555.68	8.92	6624.2	8412.89	27.00	12933.6	20295.7	56.92
6740.76	7256.76	7.65	9956.16	11217.3	12.67	13155.8	21817.6	65.84
5026.16	5655.3	12.52	9312.04	11844.9	27.20	11009.4	21270.7	93.20
6132.56	7004.91	14.22	8020.35	9101.45	13.48	9273.25	17902.1	93.05
5876.16	6103.83	3.87	7109.45	7748.87	8.99	10860.1	24870.4	129.01
5687.97	5782.72	1.67	6649.42	9829.41	47.82	11355.2	20855.2	83.66
6940.76	7399.81	6.61	6139.01	9412.42	53.32	9602.15	19961.3	107.88
5925.42	6033.52	1.82	7910.13	9358.46	18.31	11936.8	20180.3	69.06
5789.72	6448.85	11.38	8910.71	11982.1	34.47	9619.48	18462.8	91.93
Average Ipv. = 7.76%			Average Ipv. = 26.22%			Average Ipv. = 83.37%		

Table 4.4: Impact on Valid Inequalities

4.4 Improving Lower Bound

The quality of solution is measured by computing the optimality gap in equation (4.2).

$$\text{Gap} = \frac{(\text{heuristic solution} - \text{best obtained lower bound})}{\text{best obtained lower bound}} \times 100\% \tag{4.2}$$

However, the lower bound obtained from LP relaxation (LB2) is still not strong enough even when valid inequalities are added. Therefore we used truncated B&B to generate stronger lower bound (LB3). The solver CPLEX is switched to the "bestbound emphasis mode", which places greater emphasis on the best bound value (Atamtürk and Savelsbergh, 2005). We terminated the solver before it entering the branch-and-bound process, that is, the lower bound is obtained from the corresponding best incumbent solution found by CPLEX in the root node. In Table 4.5, a comparison of using lower bound from two different scenarios LB2 and LB3 on data without backlogging BTA-2014-EX is presented. Similar to previous

section, the improvement is measured by equation (4.3). As the lower bound value LP3 that obtained from bestbound mode is larger, we believe using truncated B&B is a proper modification for us to evaluate the solution quality of the heuristic. We also report the average CPU seconds spent in the truncated B&B method of each category.

$$\text{Improvement} = \frac{(\text{LB3} - \text{LB2})}{\text{LB2}} \times 100\% \tag{4.3}$$

AVST=40%			AVST=70%			AVST=120%		
LB2	LB3	Ipv.(%)	LB2	LB3	Ipv.(%)	LB2	LB3	Ipv.(%)
5069.83	5232.12	3.20	7484.6	7921.76	5.84	13642.3	19862.3	45.59
6555.68	6815.37	3.96	8412.89	9066.48	7.77	20295.7	29211.1	43.93
7256.76	7635.52	5.22	11217.3	12336.5	9.98	21817.6	32552.5	49.20
5655.3	5841.65	3.30	11844.9	13512.3	14.08	21270.7	37294.8	75.33
7004.91	7350.15	4.93	9101.45	9934.4	9.15	17902.1	30846.5	72.31
6103.83	6186.15	1.35	7748.87	7982.79	3.02	24870.4	57262.7	130.24
5782.72	5838.86	0.97	9829.41	10841.8	10.30	20855.2	35032.6	67.98
7399.81	7878.69	6.47	9412.42	11979.1	27.27	19961.3	32803.3	64.33
6033.52	6105.16	1.19	9358.46	10050.8	7.40%	20180.3	43485.3	115.48
6448.85	6676.38	3.53	11982.1	14341.1	19.69	18462.8	33473.7	81.30
Average Ipv. = 3.41%			Average Ipv. = 11.45%			Average Ipv. = 74.57%		
Ave. CPUs = 6.80			Ave. CPUs = 26.27			Ave. CPUs = 52.94		

Table 4.5: Improving Lower Bound by Truncated B&B Method

4.5 Result of Experiment without Backlogging

The first experiment results of using fix-and-optimize heuristic on SPL-SCSS are presented from Table 4.6 to 4.9. From Table 4.6 to 4.8, the first column AVST is the percentage of average setup time of total capacity. The second column ADLB represents the average deviation from lower bound, which is calculated according to the description in section 4.4.

The third column contains the proportion of instances such that splitting takes effect. In each row, 10 instances were tested, therefore a 30% indicate only 3 out of 10 instances use the splitting variables. The last two columns show the average computational time and average number of iterations in the decomposition.

The experiment results demonstrate the following, given the same problem class, the optimality gap become larger when the mean setup time increases. Obviously, splitting variables is essential for finding feasible solution when long setup time exists and problem size become bigger. With an average computational time 73.555 seconds, the average optimality gap achieves 5.56%, while maximum gap is 35.43%. Note that the maximum computational time 439.8032 seconds lies in class I with mean setup time 120%. Table 4.9 summarizes the information of average computational time and optimality gap on each category of BTA-2014-EX. The last column is the result of formulation CMLM from Belo-Filho *et al.* (2013), which achieves the best optimality gap for BTA-2014-EX during their experiment. From this table, we can see that the average optimality gap is stable regardless of the number of periods by using fix-and-optimize, while the performance of truncated B&B deteriorates when the number of products, length of periods and mean setup time increase.

	AVST (%)	ADLB (%)	Splitting (%)	Time (s)	Iterations (#)
Class A (5, 22-24)	40	0.52	30	10.00	35.1
	70	1.41	50	15.90	47.5
	120	4.48	100	19.92	52
Class B (5, 33-35)	40	1.12	30	15.29	55
	70	3.62	70	22.41	66.5
	120	9.47	100	42.24	88.3
Class C (5, 44-46)	40	1.73	50	29.92	78
	70	4.72	80	45.93	91
	120	10.44	100	72.44	118.6

Table 4.6: Test Result for Small Problems without Backlogging

	AVST (%)	ADLB (%)	Splitting (%)	Time (s)	Iterations (#)
Class D (10, 23-25)	40	1.31	50	22.96	61
	70	2.53	100	29.40	62
	120	15.57	100	44.54	102.4
Class E (10, 34-36)	40	2.36	90	42.02	81.5
	70	4.07	80	56.16	98.2
	120	9.76	100	95.52	140.8
Class F (10, 45-47)	40	2.00	80	63.80	108.6
	70	4.65	90	114.00	134.4
	120	11.15	100	157.33	189.2

Table 4.7: Test Result for Medium Problems without Backlogging

	AVST (%)	ADLB (%)	Splitting (%)	Time (s)	Iterations (#)
	40	2.60	90	52.93	84.5
Class G (15, 25-27)	70	3.49	100	68.46	102.9
	120	15.15	100	72.46	148.3
	40	2.54	80	70.32	119.4
Class H (15, 34-36)	70	4.14	100	92.03	126.7
	120	12.74	100	143.26	205.8
	40	3.03	100	153.45	159.3
Class I (15, 46-48)	70	4.03	100	180.00	170.9
	120	11.47	100	253.29	275.6

Table 4.8: Test Result for Large Problems without Backlogging

	Time(s)	SPL-SCSS	CMLM
Maximum	439.8032	35.43 %	57.77 %
Average	73.555	5.56%	14.85 %
Product			
5	30.56	4.17 %	7.95 %
10	69.525	5.93 %	14.73 %
15	120.58	6.58 %	22.19 %
Period			
20	37.397	5.23 %	4.49 %
30	64.473	5.54 %	17.01 %
40	118.79	5.91 %	23.43 %
Setup Time			
40	51.186	1.91 %	3.78 %
70	69.255	3.63 %	16.05 %
120	100.22	11.14 %	25.18 %

Table 4.9: Summary of Optimality Gap and Time for SPL-SCSS

4.6 Result of Experiment with Backlogging

The second experiment result of using fix-and-optimize heuristic to solve SPL-SCSS-BL is presented from Table 4.10 to 4.13. The notation used in these tables are similar to the description in previous section. Here we do not need to modify the heuristic, the only modifications are change of the model, data and the cost equation (3.1). The cost equation now should include backlogging costs to approximate overall total cost of each product. The advantage of using this heuristic when modeling extensions on original formulation is apparently convenient. Since including backlogging increases the size of the model as well as problem complexity, the problem is harder to solve than model without backlogging in general. Hence, the average computational time and average optimality gap of SPL-SCSS-

BL are worse than SPL-SCSS. Also, the computational time and number of iterations in the loop increase as the problem size is getting bigger. For problems with long setup time exceed period capacity, i.e., 120%, having splitting variables is still essential. Take class D and class G as an example, the SPL-SCSS uses splitting variables for every instance when mean setup time is 70% of capacity. On the other hand, the proportion of using splitting variables of SPL-SCSS-EX is lower, only 20% of the instances use splitting variables in class D when mean setup time is 70% of capacity, while in class G there is only 70% of the instances take splitting variables. Table 4.13 summarizes the information of average computational time and optimality gap on each category of BTA-2014-BL. The last column gives the result of DSM formulation in Belo-Filho *et al.* (2013), which achieves best result in terms of optimality gap in the experiment with backlogging. The maximum CPU seconds achieves 5473.788 for a single instance in class I, yet the average computational time is 135.782 seconds. It is true that in some cases, proving optimality requires much longer time than arriving optimal solution. The result of average optimality gap 8.00% suggests that problems with backlogging is harder to solve. Even though, fix-and-optimize still provides a good result in terms of optimaility gap.

	AVST (%)	ADLB (%)	Splitting (%)	Time (s)	Iterations (#)
Class A (5,20)	40	0.88	10	12.37	39
	70	3.16	10	19.26	46.7
	120	12.00	100	22.29	49.5
Class B (5,30)	40	1.84	0	15.42	56.1
	70	5.24	30	29.54	69.8
	120	12.20	100	45.57	76.7
Class C (5,40)	40	2.47	30	29.47	93.4
	70	5.81	50	43.46	90.2
	120	12.77	100	77.40	111.4

Table 4.10: Test Result for Small Problems with Backlogging

	AVST (%)	ADLB (%)	Splitting (%)	Time (s)	Iterations (#)
Class D (10,20)	40	1.50	10	32.49	65.6
	70	6.20	20	53.73	73.8
	120	10.32	100	55.44	89.4
Class E (10,30)	40	3.50	70	58.35	106.2
	70	7.96	60	82.58	112.6
	120	17.51	100	104.92	128.2
Class F (10,40)	40	2.65	30	86.65	123.8
	70	7.96	90	155.72	141.2
	120	14.33	100	257.62	184.2

Table 4.11: Test Result for Medium Problems with Backlogging

	AVST (%)	ADLB (%)	Splitting (%)	Time (s)	Iterations (#)
	40	4.74	50	69.55	107.1
Class G (15,20)	70	13.92	70	110.36	119.6
	120	12.24	100	135.54	122.7
	40	3.25	70	110.19	134.8
Class H (15,30)	70	11.01	60	194.90	147.5
	120	12.30	100	270.91	169.9
	40	3.60	80	243.04	165.9
Class I (15,40)	70	10.55	70	321.11	187.5
	120	6.03	100	1029.21	217.5

Table 4.12: Test Result for Large Problems with Backlogging

	Time(s)	SPL-SCSS-BL	DSM
Maximum	5473.788	29.38 %	65.38 %
Average	135.782	8.00 %	16.75 %
Product			
5	32.75	6.26 %	6.80 %
10	98.61	7.99 %	16.80 %
15	275.98	9.74 %	27.36 %
Period			
20	56.67	7.22 %	6.65 %
30	101.37	8.31 %	19.99 %
40	249.30	8.46 %	24.2 %
Setup Time			
40	73.06	2.71 %	4.3 %
70	112.30	7.98 %	20.03 %
120	221.99	13.30 %	26.71 %

Table 4.13: Summary of Optimality Gap and Time for SPL-SCSS-BL

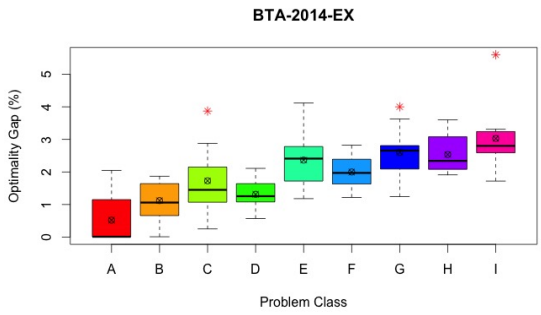
4.7 Performance Analysis

Here we analyze the algorithm performance in terms of optimality gap and computational time. The average, min, max, first, second and third quartiles of the computational time and optimality gap of BTA-2014-EX and BTA-2014-BL in each class is plotted in box plot as shown from Figure 4.1 to Figure 4.3. The box plot can easily identify outliers and show the overall patterns of response for the testing data. These figures are organized by different values of average setup time. The outlier is represented by red asterisk, while the mean value is represented by circle with "X" inside.

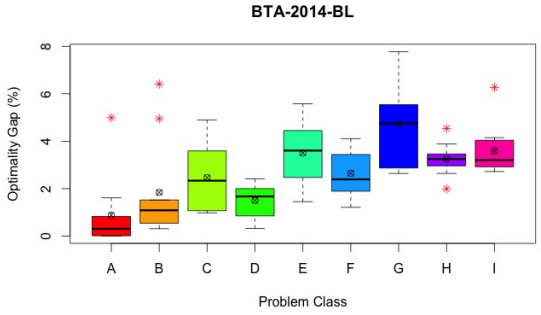
First we have noticed that the algorithm produces less variation in solution quality when period length becomes longer given the same product number. One can observe this

by comparing class A and C, class D and F and class G and I as they share the same number of product but not period length. Class C in picture (b) of Figure 4.1 and class C in picture (a) of Figure 4.2 are the exceptions.

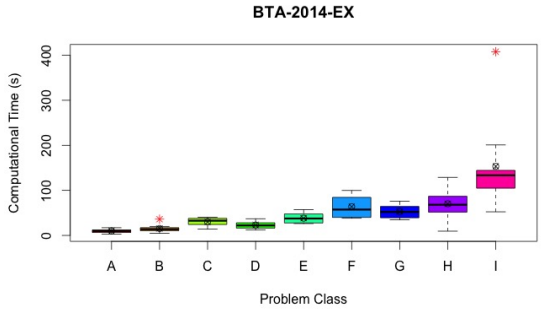
Despite the ascending trend of computational time shows the algorithm requires lots of effort in larger problem size, the mean value is pretty close to median and seldom exceed the 75th percentiles. Besides, computational time in class I takes significant portion of time than all other classes. Using the average might underestimate the performance since the majority of the instances requires less time. Note that in picture (d) of Figure 4.3 we have a extremely high computational time of 5473.788 CPU seconds, this value enhance the average to around 1000 seconds in that class (see Table 4.12). Also, during the experiment we recorded the time spent in different strategies, usually the long computational time is attributed to period decomposition, this is not surprising since equation (3.2) and (3.3) suggest larger number in period decomposition. One can definitely reduce the time window in the algorithm to trade off the time with solution quality if needed.



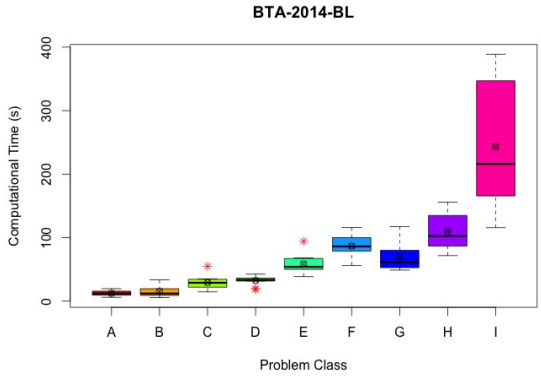
(a) Gap-40-EX



(b) Gap-40-BL

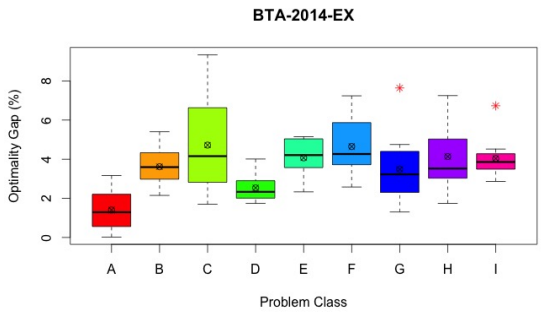


(c) Time-40-EX

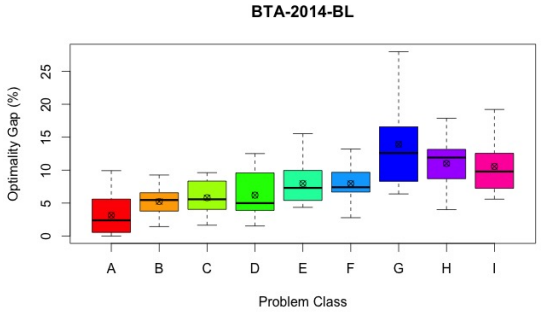


(d) Time-40-BL

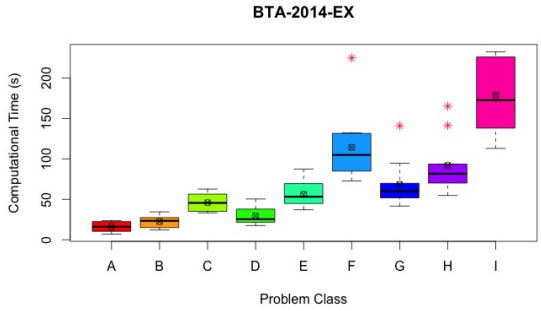
Figure 4.1: Algorithm Performance: Mean Setup Time = 400



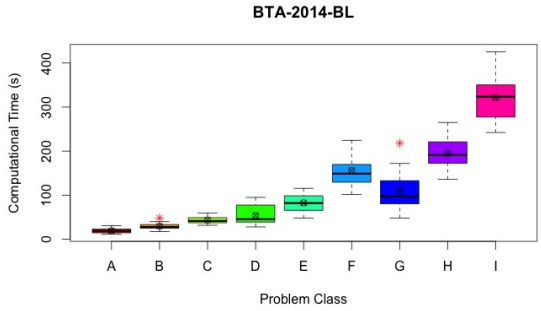
(a) Gap-70-EX



(b) Gap-70-BL

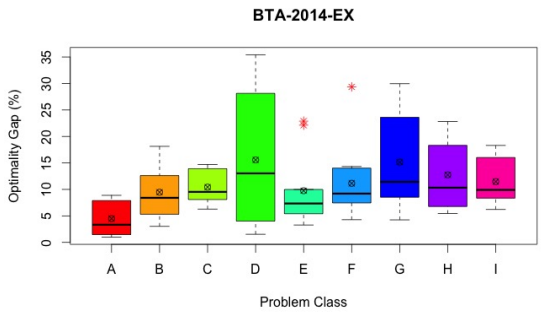


(c) Time-70-EX

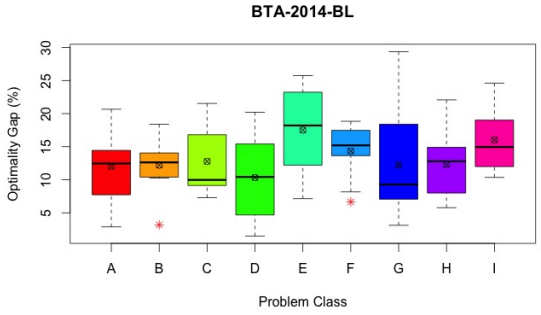


(d) Time-70-BL

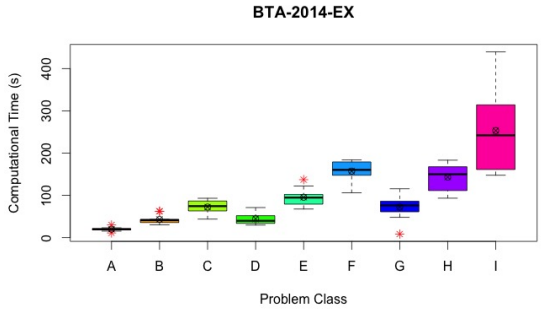
Figure 4.2: Algorithm Performance: Mean Setup Time = 700



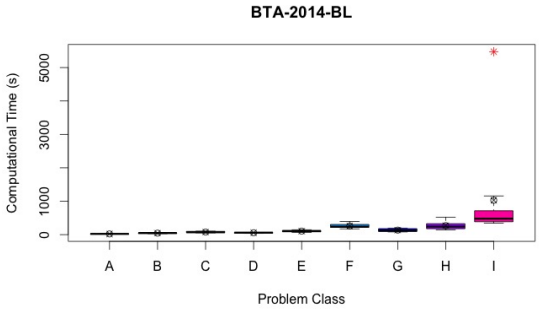
(a) Gap-120-EX



(b) Gap-120-BL



(c) Time-120-EX



(d) Time-120-BL

Figure 4.3: Algorithm Performance: Mean Setup Time = 1200

Chapter 5

CONCLUSION AND FUTURE DIRECTIONS

In this thesis, we study the single level, single machine, multi-item capacitated lot sizing problem with setup carryover, setup splitting and backlogging. To our best knowledge, there has been no such research for lot sizing problems consider setup carryover, setup splitting and backlogging together yet. This thesis aims to fill in this gap.

In chapter 2, we present the formulations of the basic capacitated lot sizing problem and extend it with setup carryover, setup splitting, simple plant location formulation and backlogging consideration. Three sets of valid inequalities from literature are developed and discussed.

In chapter 3, we describe the structure of fix-and-optimize heuristic, including the construction of initial solution by using rounding heuristic and then demonstrate that for some small size problems, using rounding heuristic instead of trivial solution can guarantee to find the true optimal solution. A two-stage decomposition strategies: product and period decomposition are presented. In order to justify how good the solution can be found by this approach, we do not limit the number of iterations and computational time in the algorithm. Instead, the algorithm terminates until no further improvement can be found in each subproblem.

Chapter 4 includes several computational experiments. First, we demonstrate the benefit on using rounding heuristic by showing how many true optimal solution can be found. Then the impact on lower bound of adding valid inequalities is given. Lower bound is further improved by using truncated B&B method. The next two sections summarize the result of the two computational tests on experiments without and with backlogging. From the analysis, the performance of algorithm tends to be stable in terms of average optimality gap regardless of the number of periods when backlogging is not allowed. Such property can be

beneficial for company to extend the planning horizon, which is more common and practical in reality than increasing product types. In addition, less variations of the solution quality are observed in larger problem size given the same product numbers.

Modeling setup features in lot sizing problem usually done by binary variables. Adopting fix-and-optimize heuristic to solve the problem gives advantage as the heuristic mainly decomposes binary variables, resulting in smaller subproblems. We also demonstrate the effectiveness of combining other mathematical programming based approaches including rounding heuristic, reformulation as well as valid inequalities. Another advantage of fix-and-optimize heuristic would be the easiness to implement and flexibility when other features are included in the model, such as backlogging in our case. Other approaches such as Lagrangian relaxation heuristic, which is usually considered very problem-specific, requiring redesign of the whole process when extra constraints are added and does not necessarily produce an easy subproblem after relaxing the hard constraints. To sum up, a comprehensive evaluation with more instances and other potential algorithms would be beneficial.

Several possibilities could be examined in future research. For instance, considering other MIP-based heuristic, metaheuristic or even a hybrid approach to justify the solution quality and computational time on different algorithms. Developing specific valid inequalities for lot sizing problems with setup splitting and backlogging to improve the lower bound from LP relaxation. In addition, other model extension such as multi-level, parallel machine or sequence-dependent setup time and setup costs can be considered in the next step.

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APPENDIX A

ACRONYMS

Notation	Description
B&B	Branch-and-bound
CLSP	Capacitated Lot Sizing Problem
CLSP-SC	Capacitated Lot Sizing Problem with Setup Carryover
CLSPL	Capacitated Lot Sizing Problem with Linked Lot Sizes
CLSP-SCSS	Capacitated Lot Sizing Problem with Setup Carryover and Setup Splitting
CLSD	Capacitated Lot Sizing Problem with Sequence-dependent Setup Time and Setup Costs
CLSD-S	Capacitated Lot Sizing Problem with Sequence-dependent Setup Time and Substitutions
CSLP	Continuous Lot Sizing Problem
DLSP	Discrete Lot Sizing Problem
EOQ	Economical Ordering Quantity
F&O	Fix-and-optimize Heuristic
MIP	Mixed Integer Programming
MRP	Material Requirement Planning
MRP II	Manufacturing Resource Planning
MLCLSP	Multi-level Capacitated Lot Sizing Problem
MLCLSPL	Multi-level Capacitated Lot Sizing Problem with Linked Lot Sizes
PLSP	Proportional Lot Sizing Problem
RH	Rounding Heuristic
R&F	Relax-and-fix Heuristic
SR	Shortest Route
SPL	Simple Plant Location
SPL-SCSS	SPL formulation for CLSP-SCSS
SPL-SCSS-BL	SPL formulation for CLSP-SCSS with Demand Backlogging