The Use of

Proportional Reasoning and Rational Number Concepts

by Adults in the Workplace

by

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ABSTRACT

Industry, academia, and government have spent tremendous amounts of money over several decades trying to improve the mathematical abilities of students. They have hoped that improvements in students' abilities will have an impact on adults' mathematical abilities in an increasingly technology-based workplace. This study was conducted to begin checking for these impacts. It examined how nine adults in their workplace solved problems that purportedly entailed proportional reasoning and supporting rational number concepts (cognates).

The research focused on four questions: a) in what ways do workers encounter and utilize the cognates while on the job; b) do workers engage cognate problems they encounter at work differently from similar cognate problems found in a textbook; c) what mathematical difficulties involving the cognates do workers experience while on the job, and; d) what tools, techniques, and social supports do workers use to augment or supplant their own abilities when confronted with difficulties involving the cognates.

Noteworthy findings included: a) individual workers encountered cognate problems at a rate of nearly four times per hour; b) all of the workers engaged the cognates primarily via discourse with others and not by written or electronic means; c) generally, workers had difficulty with units and solving problems involving intensive ratios; d) many workers regularly used a novel form of guess & check to produce a loose estimate as an answer; and e) workers relied on the social structure of the store to mitigate the impact and defuse the responsibility for any errors they made.

Based on the totality of the evidence, three hypotheses were discussed: a) the binomial aspect of a conjecture that stated employees were hired either with sufficient

mathematical skills or with deficient skills was rejected; b) heuristics, tables, and stand-ins were maximally effective only if workers individually developed them after a need was recognized; and c) distributed cognition was rejected as an explanatory framework by arguing that the studied workers and their environment formed a system that was itself a heuristic on a grand scale.

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CHAPTER 1

INTRODUCTION

I have a keen interest in understanding similarities and differences between school mathematics and workplace mathematics. This interest was initially piqued by recurring claims that large numbers of people leave school without sufficient mathematical abilities to be successful in a world based on technology. This study focused on how employees used proportional reasoning and rational number concepts in a work environment that regularly required their use. The study's results added to the understanding of how adult employees solve authentic problems that entail proportional reasoning.

Overview of the Study

This study had two specific purposes. The first was to identify the ways workers encountered and utilized cognates while on the job, and then contrast them with school mathematics problems and solution techniques. The second was to document the mathematical difficulties workers experienced while solving problems that required the cognates, along with the tools, techniques, and social supports they used to augment or supplant their own abilities. To answer the research questions, I used two qualitative methods based on a cognitive constructivist epistemology and a post-positivistic theoretical perspective.

Chapter 1 is an introduction to the topic of the proposed study. It contains background information and the rationale, as well as the research perspective, and further develops the purposes of the study. The background literature that informed the study is reviewed in Chapter 2. An overview of historical developments in the field is also included in the second chapter. Chapter 3 documents the methods used in the design, collection, and analysis of the data. The results of the study, including specific answers to the research questions, are included in Chapter 4. I discuss the results in Chapter 5, along with the limitations of the study and ideas for further research.

Background

The data from large-scale assessments (e.g., National Assessment of Educational Progress [NAEP], Trends in International Mathematics and Science Study [TIMSS]) have indicated that the U.S. education system adequately prepares students in mathematics through grade 4 (Reyna & Brainerd, 2007). However, after fourth grade the scores of U.S. students have typically begun a downward trend (TIMSS, 1996, 1999, 2003a, 2003b, 2011; Program for International Student Assessment [PISA], 2012) when compared to their peers in other countries. For example, on the third TIMSS (1996), U.S. students were ninth in the world in grade 4, fifteenth in the world by grade 8, and near the bottom of all participating countries by grade 12. It is important to note that, at about grade 4, instructional emphasis transitions from learning basic arithmetic facts to understanding rational-number concepts and problem solving using proportional reasoning.

Proportional reasoning and related rational-number concepts (hereinafter referred to as 'cognates') have been claimed as the most complex and important mathematical ideas developed in presecondary school (Behr, Lesh, Post, Silver, 1983; Karplus, Pulos, Stage, 1983; Silver, 2000). Researchers have made various claims concerning the importance of being able to reason using the cognates (Behr et al., 1983; A. Hoffer & S. Hoffer, 1988; Silver, 2000; Welder, 2007; Lamon, 2012). Any synopsis of these claims should include the four principle benefits of mastering the cognates. Together, they form: a) a practical basis for understanding and dealing with a vast number of real-world situations; b) a psychological structure that provides "a rich arena within which children can develop and expand the mental structures necessary for continued intellectual development" (Behr, Reiss, Harel, Post, & Lesh, 1986, p. 91); c) the mathematical groundwork upon which algebra is introduced; and c) the basis of measurement and thus the basis of science.

Although the cognates are vitally important, three types of studies have indicated the existence of shortcomings in peoples' understanding and use of them: a) periodic studies of students, b) studies of adults, and c) studies of teachers. Periodically administered studies have repeatedly determined that U.S. students did not adequately understand the cognates and found they were not able to consistently use them (e.g., NAEP, TIMSS, & PISA). Research focused on adults found that cognate skills did not improve with age. Rather, the continued lack of understanding in relation to the cognates became the source of poor decision making in various contexts, such as consumer decision making (Capon & Davis, 1984), democratic citizenship (Rose, 1991), health care and diet (Rothman et al., 2006; Ancker & Kaufman, 2007), and risk and investment (Reyna & Brainerd, 2007; Christelis, Jappelli, & Padula, 2010). Studies have provided evidence that a significant number of teachers who were responsible for teaching the cognates were themselves unable to consistently use proportional reasoning and tended to make the same sorts of errors as their students (Cramer, Post, & Currier, 1993; Simon & Blume, 1994; Johnson, 2013).

Poor results by U.S. students on large-scale international assessments (e.g., TIMSS, 1996, 1999; PISA, 2012) have led to several different responses from government entities, academic institutions, industry, and the public. One response, in particular, gained traction and was accepted by many reform advocates. It was based on results from studies and programs such as the NRC (1989) report, *Everybody Counts*, and Izaak Wirszup's work on the UCSMP. The response emphasized movement away from purely abstract mathematical problems, and towards real-world problems and modeling. Current curricular examples include *Everyday Math's* focus on solving problems found in common situations and *Connected Math Project's* focus on problems in context.

Closely related to solving real-world problems and modeling has been a trend towards aligning both the content and the methods of K-12 mathematics instruction with adult workplace problems. This trend has two premises: 1) a primary goal of mathematics education is success in the adult world of work; and 2) pedagogical effectiveness in obtaining this goal will increase with greater alignment between K-12 mathematics instruction and real-world adult mathematical usage (NCTM, 2000, 2003, Gainsburg, 2005). Research literature has often couched the first premise in terms of seeking success for the nation's economy (NRC, 1989), achieving national security (NAS, 2007), and allowing for personal success of the individual in finding employment and having upward mobility (NRC, 1990). The second premise was clearly stated by some entities (NCTM, 2000, 2003) while others embraced it without explicitly stating it. For example, the NRC (1990) argued that students should engage mathematical ideas in meaningful and actual contexts, such as business, science, and community events. Further, it argued that, "[t]he primary goal of instruction should be for students to learn to use mathematical tools in contexts that mirror their use in actual situations" (p. 38).

The example reform curricula cited above have differences, but the differences between these sorts of reform programs are of degree and not of kind. All of the reform programs cited a) are rooted in constructivism; b) emphasize real-world applications; c) advocate spending a large amount of instructional time on proportional reasoning and rational number concepts; and d) were developed through feedback-based research systems (i.e., experimental classrooms were used to provide feedback into the next iteration of curricular changes that were then tried in experimental classrooms). Although these sorts of curricula exemplify our best pedagogical thinking on learning the cognates, the research on students who have matriculated through these programs is mixed—there is some improvement, but students continue to struggle with the material (Hirschhorn, 1993; Moyer-Packenham, 2006). Additionally, throughout the entire literature review, I was not able to find any longitudinal studies that examined whether students who had matriculated through a reform curriculum had better adult outcomes. It appears that one of the primary goals of reform curricula (better adult outcomes) has remained untested.

Several reasons were identified during the literature review that explained why students and adults struggled with the cognates. One reason is the inherent longitudinal complexity of developing the cognates. That is to say, the cognitive structures necessary to reason proportionally are constructed over a period of several years. Furthermore, these structures must incorporate experiences, skills, and knowledge in only one manner if they are to be used successfully and consistently to solve problems based on a proportional relationship (Confrey, Maloney, Nguyen, Mojica, & Myers, 2009).

Two currently under-researched areas may prove to be fruitful if research is increased. First, there is limited research focused on adults using the cognates; hence, researchers do not know the extent of adult understanding or use of the cognates. Second, there is a lack of research that examines the relationship between early educational interventions (late elementary through secondary school) and adults' knowledge, abilities, and use of the cognates. This may be due to current research methods that tend to study the effectiveness of an educational intervention (e.g., change in curriculum, teacher training, or textbook) by examining students' scores on achievement tests administered temporally close to the intervention. Temporally close assessments are a common characteristic of current educational research because minimizing the passage of time is of the essence in avoiding time-dependent threats to validity, such as maturation, attrition, and treatment diffusion. However, this means that the ultimate result of an intervention is unknown. Knowing the first would help researchers identify specific desired adult outcomes in relation to the cognates. Knowing the second would help researchers design curricula with the potential to make changes in students' understanding of the cognates that then could last through adulthood, rather than temporary changes that last only until their next semester.

Research Perspective

After careful consideration based on Phillips & Burbules (2000) and Crotty (1998), I chose to use a cognitive constructivist perspective for understanding an individual's thinking and learning, but I also suspected that situated cognition would play a substantial role in an authentic workplace environment. Some may argue that a research perspective that places the mind in two places (the head and socially situated) is contradictory. However, the apparent theoretical conflict between constructivism and situated cognition was well addressed by Cobb (1994), who asserted "that the sociocultural perspective informs theories of the conditions for the possibility of learning, whereas theories developed from the constructivist perspective focus on what students learn and the processes by which they do so" (p. 13); hence, they can be complementary and not mutually exclusive.

Rationale and Problem Statement

Many people alive today remember the substantive calls for educational reform due to the successful 1957 launch of Sputnik by the then existent Soviet Union. Twenty-six years later, the call for school reform again made headlines with the publication of *A Nation at Risk* (National Commission on Excellence in Education [NCEE], 1983). School reform again became front page news in 1995 when the results from the third TIMSS study were released. In response to the results of TIMSS, headlines in the U.S. proclaimed: "Poor academic showing hurts U.S. high schoolers" (Henry, USA Today, 1998, p. 1A) and "Hey! We're No. 19!" (Leo, U.S. News & World Report, 1998, p. 14). The call for school reform was kept active between big events by pronouncements from companies and other organizations such as the following New York Times headline attributed to the CEO of IBM, Lou Gerstner: "Our Schools Are Failing: Do We Care?" (May 27, 1994, p. A-27). These events and headlines caused many Americans to begin believing that the U.S. educational system was failing to adequately prepare students.

The belief that the educational system was somehow broken was strengthened by a perception that scientific evidence supported the claims of a failing system (Steen, 1999; Dossey 1997). Anecdotal evidence (e.g., cashiers not able to make change and students' dependence on calculators for basic operations) also reinforced beliefs that the current system was inadequate (Gainsburg, 2005). An ongoing facet of these beliefs brings them into the present—America has not yet turned the corner on this decline and is still "losing ground mathematically" (Gainsburg, p. 4). Support for this facet was found in 2007 when the National Academy of Sciences (NAS) published *Rising Above the Gathering Storm,* once again calling for a major overhaul of the U.S. educational system. This was followed by the establishment of the National Math and Science Initiative (NMSI) in 2007, primarily by businesses aiming to assist in the ongoing implementation of the recommendations of the NAS.

The establishment of the NMSI signaled a change away from calls for general reform and towards specifying reform in mathematics and science. At this point, the reform message communicated by world events, researchers from academia, business leaders, and the military had become one of stressing mathematics and science. For example, in the early years of this trend, the National Research Council (NRC) published a report entitled *Everybody Counts: A report to the nation on the future of mathematics education.* In the report, the authors wrote:

> Mathematics is the key to opportunity. No longer just the language of science, mathematics now contributes in direct and fundamental ways to business, finance, health, and defense. For students, it opens doors to careers. For citizens, it enables informed decisions. For nations, it provides the knowledge to compete in a technological community. To participate

> fully in the world of the future, America must tap the power of mathematics. (NRC, 1989, Pendergast, p. 1)

According to the above passage, it is mathematics, and not language fluency, written literacy, or health, that is the key to opportunity. Large national-level adult literacy studies (n=19,000), such as the National Adult Literacy Survey (NALS, 1985, 1992) and the National Assessment of Adult Literacy (NAAL, 2003), also began measuring quantitative literacy as a part of their studies.

Carefully rereading the calls for educational reforms (e,g., *A Nation at Risk* and *Everybody Counts*) revealed a substantial focus on adult matters, outcomes, and issues that were typically attended to after high school or college. This means that many of the cited reform reports and studies were not interested in just improving students' test scores; rather, they were seeking a change in adult cognition generally, and in particular, mathematics cognition in adults.

During this time, an unstated, untested, and generally unrecognized premise became pervasive throughout the vast majority of mathematics educational research and continues to be used today. The premise assumes that an effective pedagogy based on cognitive constructivism leads to a permanent mental structure, 'effective' in this case being measured by an assessment administered temporally close to the treatment. Since it is an unstated and perhaps unrecognized premise, little research has been conducted to check its veracity. On its face, however, it is a false premise. For example, secondary-level teachers (of any topic) would not expect their former students to perform well on their final examination if it were readministered at their 20 year reunion.

Major study programs (e.g., NAEP, TIMSS, & PISA) have posited that students need improved mathematical abilities to enable them to compete in a world that is undergoing an ever increasing rate of technological expansion, and Hong (2012) argued that a nation's future economic well-being depends on high mathematics scores because of the correlation between GDP and TIMSS scores. However, none of the three study programs (NAEP, TIMSS, and PISA) gathered data to explicitly determine if various education reforms were actually affecting adult cognitive outcomes.

Currently, studies are conducted to determine which pedagogical techniques are effective in helping students learn a particular topic. The effectiveness of a technique is judged by administering an assessment near in time to the instruction. Rarely, however, is research conducted to determine whether the techniques being used are effective for establishing a lifetime of knowledge, understanding, and usefulness for the students. That is to say, it is left unknown whether a technique permanently changes how students think, thus actually preparing them for a technological future.

The ultimate purpose of mathematical education reform should be to positively influence *adult mathematical abilities* (the real construct), but the vast majority of the studies reviewed have measured the construct of elementary and secondary school *achievement*–and it is not even clear what the research community has meant by *achievement* (e.g., Kupermintz & Snow, 1997). Given that students are exposed to mathematics and continue to learn after the completion of secondary school, are childhood *achievement* and adult *ability* even closely related? Knowing this would be useful.

In summary, it has been argued that individual mathematical achievement as measured by standardized testing is vital to the security and economic well being of any society. Additionally, it has been claimed that upward mobility, productivity, effective daily decision making, and civic participation are all inexorably linked to mathematical achievement (e.g., NCEE, 1983; NRC, 1989; NAS, 2007). Since much of mathematics, beyond simple counting and basic operations, is accessed through proportional reasoning and rational-number concepts, further research into these cognates is vital. Specifically, it is imperative that current longitudinal investigations (e.g., Baccalaureate and Beyond [B&B], Beginning Postsecondary Students Longitudinal Study [BPS], and NELS programs such as High School and Beyond [HS&B]) be expanded, and new studies initiated, to understand the link between school instruction and adult cognitive outcomes. This study is a small step in that direction.

Purpose

The general purpose of this study was to develop an understanding of how adult employees solve authentic mathematics problems involving proportional reasoning and related rational-number concepts. Broader purposes included: (a) adding to the research base of everyday understandings and uses of mathematics as described by Lave (1988) and Bishop (1994); (b) beginning the process of connecting school-level interventions with adult outcomes; and (c) starting a research program which facilitates the development of a cognitive transfer model between mathematics in school and adult use of mathematics.

To achieve the general and broader purposes, I chose two specific purposes as the investigative foci of the study.

1. Identify the ways workers encounter and utilize the cognates while on the job, and contrast them with school mathematics problems and solution techniques.
2. Document the mathematical difficulties workers experience while solving problems that require the cognates, along with the tools, techniques, and social supports used to augment or supplant their own abilities.

To these ends, the study examined adult employees in a work environment that regularly presented problems that ostensibly required the use of the cognates to produce an acceptable solution: retail and commercial sales in the home construction and improvement industry.

Research Questions

The research questions were numbered to facilitate a transparent relationship to the rest of the study. That is, questions 1(a) and 1(b) relate directly to the first purpose, and likewise for the other two questions and the second purpose. This reference structure applies throughout the study. Below are the two purposes of the study with their attending research questions.

1. Identify the ways workers encounter and utilize the cognates while on the job, and contrast them with school mathematics problems and solution techniques.

   1(a) In what ways do workers encounter and utilize the cognates while on the job?

   1(b) Do workers engage cognate problems they encounter at work differently than similar cognate problems found in a textbook?

2. Document the mathematical difficulties workers experience while solving problems that require the cognates, along with the tools, techniques, and social supports used to augment or supplant their own abilities.

   2(a) What mathematical difficulties involving the cognates do workers experience while on the job?

   2(b) What tools, techniques, and social supports do workers use to augment or supplant their own abilities when confronted with difficulties involving the cognates?

CHAPTER 2

LITERATURE REVIEW

Chapter 2 presents a review of the literature on proportional reasoning, along with the rational-number concepts associated with proportional reasoning. The results of the review are organized into four sections.

1. Section one offers a brief history of the cognates and examines current understandings of them with an emphasis on topics important to this research.
2. Section two reports on studies which have examined the ability of adults to solve problems involving the cognates in academic and other formal testing environments.
3. Section three reviews situated research into the understanding and usage of the cognates in solving common problems found at work and at home, meaning situations involving everyday mathematics (Lave, 1988; Bishop, 1994).
4. Section four summarizes and draws four conclusions, in brief: a) mathematics in the workplace is different than mathematics in school; b) much of the mathematics in the workplace is routine; c) adults make the same sorts of errors as children; and d) investigators frequently use situated cognition and a form of observation when conducting research involving adults and mathematics in non-school settings.

The intent of this organizational structure is to start broadly and end narrowly; hence; the first section provides the general vocabulary, definitions, concepts, and history necessary for a non-specialist in the field to access the remainder of the study. The second and third sections narrow the research by specifying adults in particular settings. Section four narrows the review further into a summary with conclusions that were important to this study.

Before proceeding, it is important to note that mathematics research focused on adults has generally been limited in scope. Moreover, as each search filter was added, the amount of available research diminished to near zero in some cases (e.g., everyday mathematics and proportional reasoning). Eventually, it became clear that little research exists that has examined the ability of adults to solve cognate-based problems, and no research was found that connected the use of the cognates by adults with their earlier schooling.

Current Understanding of the Cognates

This section summarizes the current general understanding of the cognates and is separated into three subsections: a) definitions and framework, b) brief history, and c) what is known. The vast majority of the current general understanding of the cognates is a result of research done with K-12 students. A smaller body of research exists that has focused on teacher (adult) understandings of the cognates (e.g., Post, Harel, Behr, & Lesh, 1991; Cramer et al., 1993; Simon & Blume, 1994; Sowder et al., 1998; Johnson, 2013). Two additional research areas contain relevant cognate information but were not focused on the cognates: the medical field has studied patients' health numeracy (e.g., Reyna & Brainerd, 2007), and the ability of consumers to select best buys has been studied (e.g., Capon & Kuhn, 1982).

Definitions and framework. There have been several nuanced definitions of proportional reasoning offered by researchers (e.g., Inhelder & Piaget, 1958; Karplus et al., 1983; Tourniaire & Pulos, 1985). An agreed upon definitional feature found across the research was that students who were reasoning proportionally had the ability to identify and solve problems that fundamentally involved multiplicative relationships as opposed to additive relationships. Piaget and Inhelder (1958) argued that just arriving at the correct answer was not sufficient. They noted that some young children were unable to recognize the structure of the proportion, but were able to determine the correct answer to a problem by using preproportional reasoning. Lamon (1993) warned that this distinction was important and that it should be "implicitly understood that proportional reasoning consists of being able to construct and *algebraically* solve proportions" (p. 41, italics mine).

Another frequently cited requirement (e.g., Puchalska & Semadeni, 1987; Van Dooren, DeBock, Hessels, Janssens, & Verschaffel, 2005) has been distinguishing proportionality from pseudo proportionality. Pseudo proportionality refers to questions that appear to be proportional but are not. For example, if it takes 30 minutes for three shirts to dry on a clothes line, then how long will it take for 6 shirts to dry? The addition of this requirement extended the definition of proportional reasoning beyond the ability to distinguish between multiplicative and additive relationships, to also being able to distinguish whether *any* mathematical operation is warranted.

A significant, but often unstated, difference in definitions has been whether students' proportional reasoning should be thought of as a relationship between two relationships (i.e., a second-order relationship), normally expressed in the form $\frac{a}{b} = \frac{c}{d}$, or as a linear equation with a zero intercept written in the form $y = mx$. Usually, this difference has been correlated with the age of the students involved in the study. Studies involving younger students have typically focused on second-order relationships while older students who were beginning algebra garnered the linear equation form (e.g., Tourniaire & Pulos, 1985; Ben-Chaim, Fey, Fitzgerald, Benedetto, & Miller, 1998; Misailidou & Williams, 2003).

Defining rational number concepts that support proportional reasoning has been difficult because of the complex web of distinct yet related notations and meanings. Researchers, with minor disagreements (e.g., Kieren, 1976; Novillis, 1976; Behr et al., 1983), identified six ways in which rational numbers can be interpreted: part-whole, quotient, decimal, ratio, operator (transformer), and measure (continuous or discreet). These six descriptors were often referred to as subconstructs in the literature. Behr et al. posited that this complexity of meanings suggested "one obvious reason why complete comprehension of rational numbers is a formidable learning task" (p. 92). Furthermore, Kieren (1976) claimed that understanding rational numbers required knowing the meaning of the subconstructs *and* how they interrelate. Behr et al. added to this by suggesting that a different cognitive structure had to be built for each subconstruct and its interactions. The following are explanations of the six subconstructs:

- Part-whole aligns with the common notion of fraction; hence, $\frac{3}{4}$ means 3 parts out of the 4 that make up the whole with all parts being equal in size. The whole is always unity (1) in common usage, meaning a whole 'one' pizza or a whole 'one' dollar—not a whole 'four pizzas' or a whole 'five dollars'.
- Quotient refers to the arithmetic operation of division; hence, $\frac{3}{4}$ means 3 divided by 4. Here the “whole” can be any quantity greater than, equal to, or less than unity (1).

- The execution of division may lead to a number on a number line (also known as decimal notation), which in this case is equal to 0.75. This is typically understood as an extension of the decimal notation system, and not as fractional numbers per-se.
- Ratio, typically written as 3:4 or $\frac{3}{4}$, is a comparison. In this example, it could mean there are 3 boys for every 4 girls in a classroom. A ratio, when written with the fraction bar notation, has often been confused with part-whole because of its comparable form. Continuing this example, a part-whole would be $\frac{3}{7}$, indicating there are 3 boys for every 7 students (boys + girls) in the classroom.
- The operator notation transforms the size of geometric figures or the cardinality of sets, and is the algebraic equivalent to: $y = \mathrm{m}x$.
- The measure subconstruct refers to fractional measurements obtained by the use of measurement tools (e.g., ruler, tape measure, and graduated cylinder). Behr et al. (1983) explained that the fractional measure subconstruct "represents a reconceptualization of the part-whole notion of fraction. It addresses the question of how much there is of a quantity relative to a specified unit of that quantity" (p. 9). Other researchers consider the measure subconstruct to be a reconceptualization of the quotient subconstruct where, instead of partitioning a whole into parts, a set of parts are appended to each other to make up a whole (Kieren, 1976).

Researchers found that, whenever any one of the subconstructs was applied to a situation with units (e.g., feet, grams, and miles per hour), then student understanding of extensive and intensive quantities became particularly important (e.g., Siegler, Strauss, & Levin, 1981; Schwartz, 1988; Nunes, Desli, & Bell, 2003; Howe, Nunes, & Bryant, 2010). Extensive quantities consist of a single unit, and the quantity changes when the size of the system changes. Examples include mass, length, and volume. Intensive quantities are typically a ratio of two extensive quantities such as density, speed, or pressure. They usually consist of multiple units and involve the concepts of direct and inverse proportionality. Intensive quantities do not change when the size of the system changes. That is, the density of a material does not change based on the amount of material, but density itself is directly proportional to mass and inversely proportional to volume. A common manifestation of this occurs with the operator subconstruct. That is, the slope (the m in $y = mx$) is often an intensive quantity and is invariant as the operator.

The mental processes involved in identifying and solving proportional tasks have also occasionally been included in the definition. Post, Behr, and Lesh (1988) agreed that a definition of proportional reasoning should include "a sense of co-variation, multiple comparisons, and the ability to mentally store and process several pieces of information" (p. 79). Due to the complex mental processes involved, various cognitive learning theories, particularly constructivist theory, were used as the theoretical framework in the reviewed research. However, some researchers (Sfard, 1998; Hynd, 1998; Ormrod & Davis, 2004) have rejected boundaries between cognitive perspectives. Ormrod and Davis wrote that combining ideas from three perspectives—information processing theory, contextual views theory (situated cognition), and constructivism—along with parts from Piaget's and Vygotsky's theories would "give us a more complete understanding of human cognition than any single approach offers alone" (p. 181).

Proportional reasoning and the rational-number concepts that support it are a surprisingly broad set of related topics. They are the bedrock of higher mathematics and make modern science possible—this is not hyperbole. Although it is an immense field, the vocabulary and framework which define it have fundamentally been agreed upon, with only a few nuances to still be settled. To fully appreciate this agreement, it is necessary to review the story of how we arrived at it.

Brief history. An early (circa 400 BC) and often cited example of proportional reasoning is the duplication of a square in the Socratic dialogue *Meno* written by Plato. A slave was asked to double the area of a given square and at first incorrectly proposed doubling its side. Socrates, through a series of questions, helped the slave correctly solve the problem. This purportedly proved that the slave did not learn a geometric principle. Rather, he spontaneously recovered his memory of a geometric principle. The Babylonians, 1600 years before Plato, were already using a form of whole-part relationships and a sexagesimal precursor to our decimal-fraction (Kieren, 1976). At about the same time, the Egyptians were using unit fractions, meaning that 16/63 would be expressed as1/7 + 1/9 (Mainville, 1969). Another example, some 2000 years after Plato, is Chevalier de Méré's problem of gambling using dice. He was a gambler who believed that the probability of rolling a 6 in 4 throws of a die was equal to rolling a pair of 6's in 24 throws of a pair of dice. After repeatedly losing bets based on this belief, he wrote to the famous mathematicians, Pascal and Fermat, to help him solve the problem. Their efforts on this problem of proportionality led to some of the first substantial work on and subsequent understanding of probabilities (Debock, Van Dooren, Jansens, & Verschaffel, 2002). Despite several thousand years of development and use, Karplus et al. (1983) wrote that only the 'between' form of proportion was accepted for computation prior to the 14th century, meaning that only comparisons of like units was accepted (see the next section for a full discussion of within, between, and formal forms of computation).

Early modern research (20th century) by Winch (1913) found that British students used a unit measures approach when solving missing value problems, and Polkinghorne (1935) showed that first and second grade students had a 'primitive' understanding of 1/2 and the general partitioning of wholes.

Behaviorism dominated instructional theory through the 1960s. This meant that student work on proportional problems consisted of rote memory and doing "computational exercises" rather than "solution exercises" (Kieren, 1976, p. 104). However, Piaget and Inhelder (1955, 1958, 1959, 1971, 1975) had already begun working on their seminal research of proportional reasoning using various apparatus: balance beams, projection of shadows, paper folding, and probabilities (two-set alternative choice test). By 1964, Piaget, Inhelder, and Szeminska had identified four stages which students typically move through as they learn to reason proportionally (these are discussed fully in the section *What is Known - Knowledge of Solution Techniques and Errors*). The clear empirical evidence for these stages was a challenge for the behaviorists to explain, and although the cognitive theory of learning was gaining traction, a theory for explaining the complicated thoughts of students who were engaged in true multiplicative reasoning was still missing.

In 1976, Kieren offered seven interpretations of rational numbers. This research, along with Dienes' (1967) *Fractions: An Operational Approach,* moved research on proportionality into the realm of cognitive constructivism. Eventually, Kieren's original list was modified to six subconstructs (described above in *Definitions and Framework*) and was canonized in 1979 by the Rational Number Project (RNP). The RNP is an ongoing National Science Foundation (NSF) supported program of research which examines the development of children's understandings of rational numbers in grades 2-8, and how various representations (e.g., manipulatives, spoken language, abstract symbols) help or hinder the acquisition of rational-number concepts. The primary researchers are currently Cramer, Harel, Lesh, and Post. Behr (deceased 1995) was an early primary researcher.

Since the 1950s, a steady stream of research has been expanding our knowledge of the cognates, particularly with children. Confrey et al. (2009) produced a database of studies with more than 600 entries related specifically to research on reasoning with rational numbers. I posit that a meta-view of the research reveals a pattern of research waves, each lasting approximately 20 years. The first wave (1950 to 1970) established that the cognates were appropriately studied using cognitive learning theory with an emphasis on constructivism. Later eddies on this wave made room for social and cultural influences. The next wave (1970 to 1990) defined the field and identified the student and problem (item) variables that affect measurement and achievement. The wave of accountability began in the 1990s and turned the focus towards understanding teachers' knowledge of the cognates. Also in the 1990s, researchers were able to begin suggesting improvements to instruction based on growing foundational research. Research into pseudo proportionality and other forms of misidentification of linear relationships (2000 to present) has been adding to the original research into student errors. In the medical field, research focused on adult-patient errors due to faulty numeracy has been increasing. It is part of the health literacy movement and has included a spattering of findings that involved the cognates.

The history of the cognates is long and interesting. It is filled with colorful stories of great construction projects, colorful gamblers, medical malpractice, and orange juice. What has been learned? Are there any claims that can be made with certainty?

What is known. It is not surprising that much is known about rational numbers and proportional reasoning since they are arguably the two most widely researched areas in mathematics education (Confrey et al., 2009). General consensus exists in four areas: a) definitions, which have already been discussed; b) students' solution techniques and their errors; c) the temporally lengthy journey and barrier-like nature encountered while learning the cognates; and d) current instructional techniques used to negotiate the barriers. Of these, the first two are directly applicable to this research. The first has already been reviewed; the second is discussed next.

Knowledge of Solution Techniques and Errors. The solution techniques being used by students align with the four proportional reasoning stages first theorized by Piaget, Inhelder, and Szeminska in 1964: 1) primitive reasoning, 2) difference and additive relations about relations, 3) build-up strategy, and 4) true multiplicative relations. Other labels and subcategories for these stages have been used in the literature but have described similar concepts. For example, the primitive reasoning stage was subcategorized into guessing and pre-proportional reasoning, with guessing being the most primitive (Tourniaire & Pulos, 1985).

Primitive reasoning is based on the most obviously available information. Students who are focusing on the difference in weights on a balance beam and ignoring distances from the fulcrum are using primitive reasoning. This form of reasoning emphasizes simple less-than and greater-than relationships.

Difference and additive relations about relations can be conceptualized as two modes of thought. The first is based on comparing the differences between two ratios. That is, given A/B = C/D, students will check if A–B is greater than, less than, or equal to C–D, rather than checking the multiplicative relationship of the four values. The second mode is based on simple addition. For example, if students are shown a 3 × 5 rectangle and are asked to enlarge it to having a base of 7, but keeping the same general shape, then students will add 2 to both sides, making it a 5 x 7 rectangle.

Students using a 'build-up' strategy extend a table of values or add equivalent ratios to arrive at an answer (e.g., Hart, 1980, Ricco, 1982). For example, given the missing value problem of $\frac{2}{3} = \frac{x}{12}$, a student adds $\frac{2}{3} + \frac{2}{3} + \frac{2}{3} + \frac{2}{3}$ to arrive at $\frac{8}{12}$ (Karplus & Peterson, 1970; Hart, 1980). This strategy can lead to a correct answer, although the reasoning is erroneous. Piaget and Inhelder (1958) argued that this is not true proportional reasoning. This technique is usually used by young students, but only when the ratios are integer multiples of each other, meaning, in this case, that 12 is an integer multiple of 3 (four 3s equal 12).

Students who use true multiplicative relations base their solutions on second-order relations between two ratios. There are three strategies available that use second-order relations: within, between, and formal. Given the relationship $\frac{a}{b} = \frac{c}{d}$, students use a 'within' strategy when their solution process primarily consists of dividing *a* by *b* and *c* by *d,* and then comparing the results. Similarly, students use a 'between' strategy if they first compare *a* to *c* and *b* to *d,* and then compare those two results.

Researchers have used slightly different definitions of the within and between strategies (Karplus et al., 1983, p. 221; Freudenthal, 1983). Succinctly, Noelting (1980) claimed: "Within strategies are those where terms within states serve as a basis of an operation" and "Between strategies are those where terms between states are the terms of an operation" (p. 334). Freudenthal (1977) wrote that within strategies used internal ratios and between strategies used external ratios. Students use the 'formal strategy' (Freudenthal, 1978) if they are formally (algebraically) manipulating all of the variables to arrive at a solution state while avoiding any intermediate arithmetic computation, meaning that only after algebraic manipulation will they input pertinent data and perform necessary calculations.

Given these solution techniques, it is theorized that multiple paths leading to failure exist (Confrey et al., 2009). However, in summary, they can be placed into one of two categories: construction related or experience related. Students must properly construct useful and correct cognitive structures. Failure to do so may lead to didactically simplistic memorization such as 'just cross-multiply' which itself leads to misapplications and reliance on heuristics (Choi & Hannafin, 1995; Griffin, 1995; Silver, 2000; DeBock et al., 2002; Dooley, 2006). Another facet of faulty construction stems from not explicitly attending to knowledge structure, notation, and understanding (e.g., a fraction as part-whole versus as a ratio). This may lead to misapplications and misconceptions (Kieren, 1976; Schoenfeld, 1985; DeBock et al., 2002).

Experience related failure is the product of inadequate thoughtful practice with broad and numerous problem sets. The typical results of this failure are poor transfer skills, use of fall-back techniques, and reliance on heuristics (Karplus et al., 1983; Brown, Collins, & Duguid, 1989; Carraher & Schliemann, 2002b). Some examples are:

- the situation was not understood by the student (e.g., the student has never mixed juice or played on a teeter-totter) (Schliemann & Carraher, 2002b);
- the student misapplied a usually successful strategy due to misreading the problem or not understanding the limitations of the strategy (Ricco, 1982);
- when faced with a noninteger ratio, the student used a building-up strategy on some integer part of the number and then fell back to using a constant difference technique on the decimal part of the ratio (Tourniaire, 1984); and
- the student misidentified a situation as being proportional. For example, a student believed that it would take twice as long for two shirts to dry outside as one shirt (DeBock et al., 2002).

The last point illustrates the well researched issues of pseudo proportionality and overuse of linearity by students and adults (e.g., DeBock et al., 2002; Van Dooren, DeBock, Hessels, Janssens, & Verschaffel, 2004; Reyna & Brainerd, 2008; Modestou & Gagatsis, 2013). A study which used in-depth interviews offered four possible proximate causes of pseudo proportionality use in students: a) intuitive reasoning, b) the illusion of linearity, c) shortcomings in geometrical knowledge, and d) inadequate habits and beliefs (Debock et al., 2002). The interviewers in the study attempted to create a cognitive conflict in stages if a student offered an incorrect answer. That is, the interviewer was at first subtle in suggesting the answer was not correct but became more overt through five stages if the student was not willing to reconsider the answer given. On some occasions, the interviewers were not able to create cognitive conflict severe enough to cause the students to change their reasonings. Based on general interview responses, the researchers concluded that the ultimate cause of flawed proportional reasoning was that the errors were not available for introspection by the students because the proportional answers were 'obvious' and the non-proportional answers were 'illogical'.

Overall, proportional reasoning elicits a limited number of regularly appearing solution techniques and errors. This limited number does not make research any easier because the number of interactions among the variables is combinatorially complex. This means that the possible number of outcomes due to the feedback loops between the variables is not only astronomic, but literally unknowable.

Adult Research in Academic and Formal Settings

This section summarizes current understanding of the ability of adults to solve problems involving the cognates in academic and other formal testing environments, meaning that researchers had participants solve predetermined problems in a formal setting as opposed to impromptu situated problems. It is divided into two parts: a) adults involved with academics, and b) adults in the general population.

Research which has specifically investigated the cognates and adults is limited. Some research has been done with younger adults who typically have not yet entered the permanent work force, such as pre-service teachers and other college students. The reasons for this lack of research are not clear, but Coben et al. (2003) argued that adult numeracy has been generally under-researched for three reasons: a) it is under-theorized; b) school-based research on numeracy continues to have a priority; and c) it has only recently become a concern of governments and other funders of research. Carpentieri, Litster, and Frumkin (2009) warranted Coben's claims, but argued that a "new growing interest in the field [was] attributable to concerns, both in the UK and elsewhere, about a numeracy ‘skills deficit’ which limits individuals' life chances while also impacting negatively on national productivity" (p. 5); hence, the field of adult numeracy research has been growing.

Adults involved with academics. Adults who are involved in academics include teachers, pre-service teachers, and students attending college. Recent studies on teachers and pre-service teachers (e.g., Livy & Herbert, 2013; Fernandez, Llinares, & Valls 2013; Lobato, Orrill, Druken, & Jacobson 2011) have found results like those described two decades ago (e.g., Post et al., 1991; Cramer et al., 1993; Simon & Blume, 1994). That is, the research continues to report that the same misconceptions and errors that plague students are also prevalent among upper elementary and middle school teachers (Lobato et al., 2011). Moreover, Lobato et al. claimed that studies "suggest that many elementary and middle grades teachers and prospective teachers lack a deep understanding of proportional reasoning and rely too heavily on rote procedures such as the cross-multiplication algorithm" (p. 3).

Pre-law college students were examined by Lloyd and Frith (2013). They found that approximately 50% of the students were able to successfully reason when confronted with word problems that required proportional reasoning and that only 3% to 11% (dependent on the question) were able to successfully solve and explain their reasoning. The problems focused on well known social issues (e.g., income disparity) that had been previously discussed in class. The students were given graphs, tables, and other sources of information that quantified two elements of the problem: the relative changes and absolute sizes of the quantities involved. The primary error committed by the students was attending to only one of the elements, meaning that the students used a single ratio rather than a comparison of ratios to reach their conclusions. Results of the Lloyd and Frith study, as a percentage, were somewhat lower than those found three decades earlier by a different set of studies. The earlier studies had determined that between 25% and 50% of the assessed college students did not regularly solve problems involving proportionality successfully (Renner & Paske, 1977; Adi & Pulos, 1980; Thornton & Fuller, 1981).

In the realm of academics, only limited research was found that examined proportional thinking in students who routinely used higher level mathematics, such as students majoring in engineering, physics, and mathematics. This was also true of research that targeted working professionals who would have arguably taken higher level mathematics classes (e.g., doctors, engineers, and architects). Interestingly, I did not find any research that examined the proportional reasoning of university professors. Other researchers found the same dearth of research directed at professionals who were presumed to use higher-level mathematics or formal mathematical theory (Gainsburg, 2005, p. 10). The limited research that was focused on professionals found many of the same issues and errors as had been found among other adults; however, the environments in which these professionals worked tended to mitigate harmful outcomes from these errors. That is, the high degree of distributed cognition found in some of these professions (e.g., nursing and architecture) checked the potentially harmful outcomes due to an individual's error in proportional reasoning because the error was spotted by others and corrected before it caused harm (Hutchins, 1995; Kaushal et al., 2001).

Adults tested in the general population. Adults in the general population have recently started to become participants in numeracy testing being conducted by governments (Rashid & Brooks, 2010). In the research, 'numeracy testing' has usually referred to basic competency in performing daily activities. The mathematics at this level have typically been simple arithmetic computation and the comprehension of numerical information (Rashid & Brooks, 2010). Adult numeracy was often referred to as ‘quantitative literacy’ in U.S.-based studies and was defined by the 2003 National Assessment of Adult Literacy (NAAL) to be:

> The knowledge and skills required to perform quantitative tasks, (i.e., to identify and perform computations, either alone or sequentially, using numbers embedded in printed materials). Examples include balancing a checkbook, figuring out a tip, completing an order form or determining the amount. (http://nces.ed.gov/naal/literacytypes.asp)

By either name, quantitative literacy testing has not focused on the cognates. However, both national and international research rated quantitative literacy results in bands of typical abilities that allowed me to estimate proportional reasoning percentages. For example, the Adult Literacy and Life Skills Survey (ALL:2005) states, as part of its *Level 3* band, "Skills required involve number and spatial sense, knowledge of mathematical patterns and relationships and the ability to interpret proportions, data and statistics embedded in relatively simple texts where there may be distractors" (p. 17).

The results give a fairly narrow and consistent result; approximately 70% of the adults tested had inadequate numeracy skills for calculating ratios and proportions. These results are based on the NAAL:2003 with approximately 65%; the ALL:2005 (30 countries) with 67%; and the National Research and Development Centre for Adult Literacy and Numeracy (NRDC) in the United Kingdom with 74% (Carpentieri, Litster, & Frumkin, 2009, p. 12). Furthermore, though the research is limited, this result in the U.K. has been steady since the 1940s (Rashid & Brooks, 2010). It should also be noted that the variation on the ALL:2005 between adults in countries such as Switzerland, Norway, Canada, Italy, and the U.S. was low.

The medical field in the U.S. has also conducted research on adult numeracy (Reyna & Brainerd, 2007). I reviewed several of the instruments used in the medical field and found that, although the rationale for the research was medical, the actual questions were typically not medically framed. That is, specialized medical knowledge or vocabulary was not a necessary prerequisite to successfully solve the posed problems. The list of items tended to have more proportional reasoning tasks than the widely given adult numeracy tests, such as the NAAL:2003 and the ALL:2005, no doubt because of the prevalence of the cognates in medicine (Reyna & Brainerd, 2007). The rationale for the research was to confirm that patients who had higher numeracy scores had better health outcomes (Estrada et al., 2004; Weiss et al., 2005; Reyna & Brainerd, 2007).

The reviewed research in the medical field confirmed the findings of other cited studies such as NAAL:2003. A typical example is from Ancker and Kaufman (2007), who reported that 67% of tested patients determined that "10 in 100 and 10%" were equivalent. Grimes and Snively (1999) reported that 73% of adult patients in a waiting room (n=633) were able to correctly answer that a disease affecting "2.6 per 1000 women" was less common than one affecting "8.9 per 1000 women." However, when the same comparison was presented as "1 in 384 women" and "1 in 112 women," only 56% answered correctly (p. 718).

None of the studies referenced above are equivalent; hence, any sort of comparison, such as from one country to another, is nonsensical. The preponderance of research evidence, however, seems clear; somewhere around 70% of the general adult population of many industrialized nations cannot regularly solve academic or formally presented problems involving the cognates. Additionally, none of the cited studies in this section attempted to examine the current thinking of the participants or how they had come to think in such a manner.

Adult Research in Everyday Settings

This section would ideally summarize the research on adult understanding and usage of the cognates in solving problems found at work and at home, that is, situations involving everyday mathematics. However, as noted earlier, this specific type of research is limited. The only research found that targeted adults in the workplace and focused on the cognates was a small body of research on nurses (e.g., Hoyles, Noss, Pozzi, 2001; Reyna & Brainerd, 2007). Similarly, adults in everyday, non-work settings have primarily been studied as shoppers searching for a best buy (e.g., Capon & Kuhn, 1982; Capon & Davis, 1984). A search of the literature did uncover limited research that included situations involving the cognates as part of a broader study. For example, Christelis, Jappeli, and Padula (2010) compared investment strategies across consumers who had different numeracy scores that were based partially on the cognates.

An often assumed but unstated premise in research studies was that typical workers show up on their first day of employment with at least some basic mathematical knowledge (e.g., counting and arithmetic) upon which they then build situation specific mathematical knowledge. I did not find any research that attempted to determine the origin, nature, or extent of the knowledge that workers brought with them, or how they incorporated their school mathematics with their workplace mathematics. This may be because these sorts of questions have not been asked or because the typical research perspectives and methods selected by researchers for employment-based studies were not designed to make such determinations.

The methods commonly used when researching workers in everyday settings include forms of shadowing and interviewing. As an example, Masingila (1994) spent 140 hours over a three month period shadowing (with informal questioning) carpet-laying employees. Typical for this sort of research, Masinglia reflected several times per week upon the work tasks that had been observed "and made sampling decisions to: (a) observe certain work tasks again, (b) ask specific questions of certain respondents, (c) observe unfamiliar work tasks, and (d) discontinue the observation of work tasks for which I felt I had enough data" (p. 437). Other methods were also occasionally mentioned in the research (e.g., artifact examination and researcher introspection). Formal testing or clinical interviews were seldom used, with the exception of the medical patients discussed above being given brief tests to determine their abilities to understand and manage their medications (Weiss et al., 2005), but these were medical patients and not employees.

The research on shoppers that applied directly to the cognates had interesting results. For example, Capon and Kuhn (1982) studied female shoppers (n=100) at the grocery store. They asked shoppers to compare two differently sized containers of garlic (or deodorant) with different prices to determine which one was the best buy. Approximately 60% of the shoppers were able to determine the best buy. The researchers interviewed the shoppers to determine their rationale. If the correct result was a simple guess or based on faulty reasoning (the equivalent of a lucky guess), then the correct answer was not counted. Lave (1988) found similar results (that 30% failed) when shoppers (n=35) were given paper & pencil tests; however, most of the shoppers (93%), when shadowed and interviewed, convinced Lave (1988) that their answers were correct given the totality of the situation. These results showed that the percentage of shoppers able to select the best buy based only on the numbers closely matched the percentage of patients in a doctor's office who were successful with a more academic set of questions involving the cognates (as discussed above).

The nursing related studies, which applied to the cognates (e.g., Hoyles et al., 2001; Wright 2009; 2010), used shadowing and contingent questioning as methods. The results across the studies found somewhat conflicting results. Nurses rarely made medication errors during their actual work with patients (Noss, 2002). However, errors were common when the nurses were given a 'paper & pencil' test with equivalent medication problems (Pozzi, Noss, & Hoyles, 1998). Researchers who observed the nurses determined that they attempted to solve the two sets of problems (actual work problems versus paper & pencil problems) using two distinct methods. When working with patients, nurses used a multitude of heuristics based on the specific drug, but when solving the paper & pencil problems, they tried using formulaic or textbook methods (Hoyles et al., 2001; Wright, 2009; 2010).

There were other general findings from the research on adults at work or in everyday situations that were relevant to this study, even if they do not at first appear to be related to the cognates. For example, research found that mathematics at work tended to be simple. Frequently, if the work was initially mathematically complicated, then as part of the work process, it was broken into elementary components until it was simple enough to solve without abstract notation and, in many cases, without even having to write anything down (de la Rocha, 1981; Hoyles et al., 2001; Gainsburg, 2005). Another example was the frequent use of estimation (common use of the term) and 'stand-ins' (de la Rocha, 1981; Millroy, 1992; Masingila, 1994). A stand-in replaces a calibrated measuring device: a) a measuring cup replaced by an unmarked container for recurring mixes; b) a mass scale replaced by a 'volume' container, and; c) a tape measure replaced with a piece of lumber, tack strip, or tile. All of these simplified the task by embedding (hiding) the mathematics into the device, thus eliminating the need for numbers or calculations.

Summary and Conclusions Concerning Research on Adults

From the literature review, four findings were revealed that were vital to this study. The first one, however obvious, still seems necessary to state — mathematics at work was found to be different than mathematics in school. Table 1 was adapted primarily from Harris (1991, p. 129) and summarizes the differences.

Table 1
Mathematics at Work Versus Mathematics at School

Mathematics at work	Mathematics at school
Embedded in task	Decontextualized

Motivation is functional	Motivation is intrinsic
Objects of activity are concrete	Objects of activity are abstract
Processes are not explicit	Processes are named and studied
Data are ill-defined and 'noisy'	Data are well defined and presented tidily
Tasks are particularistic	Tasks are aimed at generalization
Accuracy is defined by situation	Accuracy is assumed or given
Numbers are messy	Numbers are arranged to work out well
Work is collaborative and social	Work is individualistic
Tools, artifacts and stand-ins not restricted	Limited to approved tools (e.g., calculators)
Correctness is negotiable	Answers are right or wrong
Language is imprecise; responds to setting	Language and setting is precisely defined

While none of these differences explicitly relate to the cognates, in their totality, they have played a significant role in this study.

The second finding was that a large amount of the mathematics used in the workplace and everyday settings was "routine". This means that the worker (or shopper) has faced the issue multiple times in the past, and in many instances had been taught a trick (e.g., Millroy, 1992), built a stand-in (e.g., de la Rocha, 1981; Smith, 2002), or developed a heuristic (e.g., Carraher & Schliemann, 2002a) to facilitate solving the issue efficiently and correctly. Research found that these techniques (tricks, stand-ins, and heuristics) were often used by workers, although the workers acknowledged that they did not know why the technique worked (Millroy, 1992; Masingila, 1994; Hoyles, et al., 2001). Another facet of the second finding was that very few errors were made at work as long as the task was routine, but when a quirk occurred, the chance for an error greatly increased (Hoyles, et al., 2001; Kaushal, 2001; Smith, 2002).

The third finding was that adults solving problems in a formal testing (e.g., paper & pencil) setting continued to make errors in much the same way as children, but when confronted with comparable problems in a workplace or everyday setting, they used different and vastly more effective techniques (Capon & Kuhn, 1982; Lave, 1988; Pozzi et al., 1998). Many reasons for this difference in techniques have been proffered; however, no widely accepted consensus or model exists. This finding was important in answering research question 1(b) since the connection between school mathematics and workplace mathematics is unclear. That is, from a constructivist perspective, it is unknown if school mathematics a) form the foundation of, b) are discarded in favor of, c) run parallel to, or d) play some other role in relation to workplace mathematics.

The fourth finding is a determination of research perspectives and methods. The majority of the research studies reviewed utilized a research perspective based on situated cognition. The primary method used was shadowing (or a closely related form) with contingent questioning. A secondary method used was the administration of a formal assessment (e.g., paper & pencil or computer based test). Only a few studies combined these two methods (e.g., Lave, 1988; Pozzi et al., 1998; Wright, 2010). There was, however, an element missing from all of the studies examined during the literature review—any attempt at understanding the cognitive construction flow (if one even exists) in its entirety. I theorized the possibility of two distinct construction flows: one for developers and the other for daily users. The flow for the workers who developed the heuristics and stand-ins might have looked like

School mathematics → Workplace/Everyday mathematics → Heuristics/Stand-ins

whereas the development flow for the daily users of the heuristics and stand-ins might have looked like this:

School mathematics → Heuristics/Stand-ins → Workplace/Everyday tasks

The difference is that developers would experience the mathematical requirements of the workplace in raw form, and then construct cognitive tools to simplify them, whereas the users would be 'taught' how to use the cognitive tools prior to or parallel with experiencing the requirements of the job. I offer an answer to this in the discussion section of Chapter 5.

CHAPTER 3

METHODS

Purpose and Research Questions

This study had two specific purposes:

1. Identify the ways workers encounter and utilize the cognates while on the job, and contrast them with school mathematics problems and solution techniques.
2. Document the mathematical difficulties workers experience while solving problems that require the cognates, along with the tools, techniques, and social supports used to augment or supplant their own abilities.

Four research questions focused and helped to fulfill the purposes of the study:

1(a). In what ways do workers encounter and utilize the cognates while on the job?

1(b). Do workers engage cognate problems they encounter at work differently than similar cognate problems found in a textbook?

2(a). What mathematical difficulties involving the cognates do workers experience while on the job?

2(b). What tools, techniques, and social supports do workers use to augment or supplant their own abilities when confronted with difficulties involving the cognates?

Framework

A qualitative methodology was used to study the mathematical activities of nine individual employees of a company. I interviewed them, assessed their abilities, recorded vignettes about them, examined their workplace environment, and analyzed their actions along with their thoughts. This was not, however, a case study of a group, nor was it a series of multiple case studies of individuals. Rather, it was a study of individuals embedded in a single workplace environment with a singular focus on answering the research questions posed in response to the study's purposes.

Setting

This study was conducted in a large and active home improvement store located in the Phoenix, Arizona metropolitan area. Based on an analysis of demographic data from http://quickfacts.census.gov, the store was located in a zip code that was below the state's average in household median income (≈93% of the state's average) and substantially below in household net worth (≈80% of the state's average). It had a younger demographic (30 versus 36 for the state) and a higher percentage of residents living for five years or more in one location (28% versus 24% for the state average). The median home price was 10% below the average for the state. Overall, this data suggests a stable customer base with moderate levels of income.

The store had approximately 125,000 square feet of retail space, 10 departments, and from 25 to 50 employees working in the store at any given time, depending on the time and day. It was active with combined retail and commercial sales exceeding 42 million dollars annually. During the term of the research, more than 1.5 million dollars of sales took place.

Author Background and Bias

I have an extensive background (over 20 years) in all phases of home improvement and construction. During the course of business, I have spent countless hours at the research site or similar ones, but have never worked as an employee at one.

Over these years, I have witnessed the cognates being properly and improperly used at the research site, and have developed friendships with employees at similar sites. However, none of the participants at the research site were more than business acquaintances prior to this research. All of these factors may have been a source of bias in this study.

Participants

Nine employees were selected from the store to be participants in this study. Ideally, the study would have had three participants from three departments: a) commercial sales, b) building materials, and c) nursery & outdoors. However, only two employees volunteered from the nursery & outdoors department, so one additional employee from flooring & walls was selected.

All participants volunteered, though a small stipend ($25) in the form of a gift card to a local restaurant was given to all of the participants at the end of the study. Potential participants were told about the gift certificates during the recruitment phase of the study. All references to individuals in this study were by pseudonym.

Detailed demographic information was not collected for this study, but some basic information, such as type and length of experience in the industry, any post secondary education, and gender, was recorded. The study had more males (n=7) than females (n=2). However, this gender bias was based purely on the sex of the participants since more men than women worked in the positions of interest to this study. The collected demographic data is in Table 2.

Overview and Schedule

Overall, the fieldwork took 15 days over the period of approximately one month. The first field day was spent planning with the store manager. He granted me access to an interview room and introduced me to key personnel, including the assistant managers and loss prevention employees who would have otherwise seen me as a suspicious character in their store. The full cooperation and support of the store manager was instrumental in making this study possible.

Table 2
Background Information of the Participants

Name	Department	Gender	Age	Relevant retail & other experiences
Caleb	Commercial sales	M	40s	3 years and 20+ years in the building trades
Cindy	Commercial sales	F	40s	8 years and 7 years as union pipefitter
Cory	Commercial sales	M	50s	5 years and 20+ years in the building trades, recent mathematics course at community college
Buck	Building materials	M	20s	5 years and recent mathematics course at community college
Bill	Building materials	M	50s	7 years with 20+ years in the building trades
Benny	Building materials	M	20s	3 years and recent mathematics course at community college
Oliver	Nursery & outdoors	M	30s	3 years and recent carpentry and construction school with the National Guard
Nancy	Nursery & outdoors	F	40s	7 years and AA degree in landscape botany
Frank	Flooring & walls	M	50s	9 years and previous manager of a picture framing store

The second field day (11 calendar days after the first) consisted primarily of recruitment activities. The store manager introduced me to a large group (approximately 40) of the store's employees. To minimize interference in the store's operations, he did not invite five categories of employees to this meeting: a) cashiers, b) new hires, c) managers and other support staff, d) employees who worked in appliance sales, and e) those he considered "trouble makers." Of the excluded groups, only the "trouble makers" would have contained potential participants, but doubtless few in number.

Given the absence of these five categories of employees, there is no claim in this study that the participating workers represented the entire body of employees since they obviously did not; however, based on four factors, the workers in the study were representative of their departments.

First, excluded cashiers, managers, and support staff did not work in the departments studied, so excluding them had no impact on the selection of employees from the studied departments. Second, according to the store manager, new employees were not typically assigned directly to a department. Rather, based on their experience, they were either initially assigned to loading duties in the parking lot or to directly assisting another experienced employee inside the store. In the first case, parking lot assistants were not part of a department so their exclusion had no impact on the claim that workers in the study were representative of their respective department. In the second case, new employees would have still been learning the layout of the store, procedures, heuristics, etc. Thus, it is doubtful that they would have often taken the lead in assisting a customer or determining a course of action. Additionally, this was not a common event, given that I did not witness any new hires undergoing training at the department level during the study. These aspects suggest that the inclusion of any new hires would have been more detrimental to claims of representation than exclusion. Third, similarly, in respect to frequency, the exclusion of trouble makers did not have an adverse effect on claims of department representation. Fourth, each day when I arrived in the store, the manager asked who I was shadowing. He would always offer a comment indicating the placement of the employee on the mathematical spectrum such as "sharp," "dull," or "adequate." The fact that a range of abilities existed among the participants indicated that the manager did not purposefully exclude employees based on their mathematical abilities, but only on the previously stated five criteria.

The script and topics covered at the meeting conformed with the approval from the Institutional Review Board (IRB) (Appendix A).

Shadowing and clinical interview were the two research methods used to address all four research questions. Typically, there were two parts to each research day: shadowing of a new participant and clinically interviewing a participant who had been shadowed the previous day. That is, each worker was shadowed on one day and interviewed on a second day. I coordinated the days and shift times directly with each worker. At the end of each research day, I reviewed the field notes from shadowing and annotated them as necessary. If a clinical interview had taken place, then I also reviewed its accompanying assessment and field notes. This process of shadow-interview-review continued until all nine workers had been shadowed and interviewed.

The last phase of the study was a debriefing with the store manager and distribution of the gift cards. Due to the workers' schedules, it took three return trips to thank and distribute gift cards to all of the employees who had helped or participated in the study.

The ebb and flow of the store changed with the respective day of the week and the time of day; however, the busiest time for the store was typically from 9:00 AM until 2:00 PM. Store management knew about this busy period and reminded the store's employees of this fact via the reading of a short script over the store's intercom. The announcement cajoled that the period from 8:00 AM to 2:00 PM was for focusing on customers, meaning that regular stocking should not take place during that time and that anything blocking the aisles should be removed. The commercial desk followed the same time pattern of activities, but was closed on the weekends. Customers from the construction industry dominated sales during the early mornings at the commercial desk, whereas after 10:00 AM, the customer mix shifted towards nonprofessional consumers.

Because of my background in the home industry, I knew that the store would have different kinds of customers depending on the time of day and whether it was a weekday or weekend. To capture this variety, I scheduled shadowing across different times and days (Table 3).

Table 3
Day and Time of Shadowing

Name	Department	Shadowing day and time
Caleb	Commercial sales	Tuesday 1:00 PM to 4:00 PM
Cindy	Commercial sales	Wednesday 9:00 AM to 12:00 PM
Cory	Commercial sales	Thursday 6:00 AM to 9:00 AM
Buck	Building materials	Friday 10:00 AM to 1:00 PM
Bill	Building materials	Saturday 10:00 AM to 1:00 PM
Benny	Building materials	Monday 4:00 PM to 7:00 PM
Oliver	Nursery & outdoors	Sunday 9:00 AM to 12:00 PM
Nancy	Nursery & outdoors	Tuesday 11:00 AM to 2:00 PM
Frank	Flooring & walls	Saturday 9:00 AM to 12:00 PM

This was important because different kinds of customers brought different kinds of problems. For example, at 6:00 AM on a weekday, the store primarily had contractors and professional repairmen whose needs and problems greatly differed from the do-it-yourself homeowners who dominated the store on the weekend..

Shadowing, Assessment, and Interview Protocols

The two primary methods used were shadowing and clinical interview. Both methods included contingent questioning, and the clinical interview had a written assessment as its basis. Shadowing primarily addressed research questions 1(a), 2(a), and 2(b), whereas the clinical interview and its accompanying assessment primarily addressed 1(b), 2(a), and 2(b) (Table 4).

Table 4
Research Questions with Methods and Protocols

Research Question	Method	Protocol
1(a) In what ways do workers encounter and utilize the cognates while on the job?	Shadowing (3 hrs)	McDonald (2005); Quinlan (2008); Ericsson & Simon (1993)
1(b) Do workers engage problems they encounter at work differently from similar problems found in a textbook?	Clinical interview (1 hr)	A. H. Rubin & I. Rubin (2005); Ginsburg (1981); Swanson et al.(1981); Ericsson & Simon (1993)
2(a) What mathematical difficulties do workers experience while on the job which involve the cognates?	A. Shadowing (3 hrs) B. Clinical interview (1 hr)	A. McDonald (2005) & Quinlan (2008) B. H. Rubin & I. Rubin (2005); Ginsburg (1981); Swanson et al.(1981); Ericsson & Simon (1993)
2(b) What tools, techniques, and social supports do workers use to augment or supplant their own abilities when confronted with difficulties involving the cognates?	A. Shadowing (3 hrs) B. Clinical interview (1 hr)	A. McDonald (2005) & Quinlan (2008) B. H. Rubin & I. Rubin (2005); Ginsburg (1981); Swanson et al.(1981); Ericsson & Simon (1993)

Shadowing. The shadowing with the contingent questioning method was used to gather data to answer research questions 1(a), 2(a), and 2(b). The process and embedded criteria for gathering data consisted of five steps: 1) identification of a new event, 2) initial evaluation, 3) recording, 4) contingent questioning, and 5) refinement. I identified an event as new whenever a worker began working solo on a new project, interacting with a new customer, or interacting with a fellow employee about a new topic. Examples of solo projects included stocking shelves, sweeping floors, and preparing displays. My initial evaluation of the new event consisted of determining whether mathematics of any type was likely to be involved. If I deemed the use of mathematics unlikely, then I only recorded the time and the appropriate non-mathematics interaction code (see the section *Coding* for details). If the event was likely to involve mathematics, then I recorded snippets of conversation and details such as the type of problem, calculator usage, customer type, and quantities involved. I also recorded my impressions and initial ideas for contingent questioning. I used a three-color pen for recording: black for data and snippets of conversation, blue for my thoughts and impressions, and red for contingent questioning ideas. If customers or other workers were involved, then I tried to ask contingent questions immediately following the completion of the event. Sometimes this was not possible due to more customers appearing or other pressing tasks. During solo events, I was often able to ask contingent questions throughout the project without interrupting the flow of work. I refined my notes during lulls (e.g., a worker sweeping a floor) by adding clarifying notes and thoughts to them. I also collected, copied, or photographed artifacts such as tables, charts, markings, and references during the refinement stage. I continued the refinement process later in the day after leaving the research site. This included: a) reviewing my field notes and annotating them while the events were still clear to me; b) downloading and labeling photos taken that day; and c) converting supporting artifacts into an electronic format by scanning and labeling them.

McDonald (2005, p. 5) listed three appropriate reasons to ask questions during shadowing: a) to clarify, b) to determine purpose, and c) to elicit a running commentary. Questions for all three reasons were used in this study. This is a list of questions asked during the shadowing sessions.

A Clarification

- What did you mean when you said...?
- Where did you just look on that chart?
- When did you say that happened?
- What did the customer say about...?
- Did you say...or...?

B Determination of Purpose

- Why are you doing that?
- What is the reason for this?
- What are you trying to do?

C Eliciting a Running Commentary

- Does that happen often?
- What do you do then?
- Where did you learn that?
- When must that be done?
- Why is that important?

This list is not meant to be exhaustive; rather, it is an illustration of some of the questions asked during the research.

The retrospective prompt, "Report everything you can remember about your thoughts during the last problem" (Ericsson and Simon, 1993, p. 19), was initially tried, but was discarded after the second shadowing session. Ideally, it was supposed to be asked temporally close to the event, but only if it did not interrupt the flow of the workplace; this was seldom the case. I also tried it immediately after shadowing and on the following day prior to the clinical interview. In both of these cases, the responses from the employees were minimal, and further probing was not appropriate due to the danger of made-up responses to pacify my apparent interest (Ericsson and Simon,1987).

I made an earnest attempt to audio record the interactions between the worker and myself during shadowing, but ultimately failed because the recording process was continuously being interrupted (turned off momentarily) due to interactions between the worker and persons who were not part of the study, such as customers or other employees. The dynamics of the store and shadowing did not often allow the conversation to be picked up where it had been interrupted; hence, field notes became the primary recording device during shadowing.

The shadowing protocols from McDonald (2005) and Quinlan (2008) recommend that a researcher strive to rapidly become part of the normal landscape at the study site. To facilitate this, I arrived a minimum of 15 minutes prior to the start of any session and chatted briefly with the manager on duty. Typically, the manager and I walked over and met the participating workers on the sales floor in their assigned departments. I usually had a cup of coffee in my hand and a shopping cart filled with a few random items in order to blend in as a customer. To further achieve the requirement of fitting in, I wore work boots, slightly used blue jeans, and a tucked-in collared shirt.

During the shadowing sessions, it was imperative to not make the employees nervous or apt to change their behaviors, so I

- observed the employee from a working distance,
- recorded notes silently,
- refrained from body language indicating approval or disapproval,
- asked contingent questions only during breaks in the action,
- did not interfere, engage, or help with any tasks or with customers, and
- did not record identifying information of third parties not involved in the study.

After each shadowing session, I confirmed the meeting time for the clinical interview (simply called the 'interview') and asked about any items in the field notes that needed clarifying.

In summary, the focal point of shadowing was to answer the research questions by gathering pertinent data via observation and questioning while recording the data, primarily via field notes. Secondary data collected included copying and photographing artifacts that workers had available to solve the problems they encountered.

A**ssessment.** The assessment initiated the clinical interview and provided the basis for contingent questioning. The combination of assessment and clinical interview was used to answer three research questions (1(b), 2(a), 2(b)) and lasted for one hour (ninety minutes including the orientation). The assessment part consisted of three cognate-based questions. Each question was presented on its own page and was multipart. That is, each question consisted of multiple test items that were related to a product or set of products found in the store. The first two items of each question were based on an authentic scenario which utilized current products and pricing at the research site, whereas the last item had similar mathematical features to the first two items, but was written to resemble proportion problems often found in textbooks. All items were presented in an open response format. Appendix B contains the complete set of assessment questions.

I created a bank of five assessment questions from which I chose the first two to initiate the interview with a worker. I chose the first question based on the worker's current department (e.g., manure topping or grass seed for a worker assigned to nursery & outdoors). Frank was the exception to this practice because I had not piloted (explained later) or even prepared any questions that were based on his department; however, Frank had worked for several years in the building materials department before moving over to flooring & walls, and was still active in the building materials department since he occasionally filled vacancies due to vacations or sickness. I chose the second question to be different in scope from the first question, meaning that if the first question was a question that hinged on the cognates as applied to area, then the second question may have involved the cognates as applied to volume.

The final (third) question presented to each worker was similar to either the first or second question, thus checking competence through the presentation of a similar question. I used two techniques to check workers' competence or "strength of belief" in their answers (Ginsburg, 1981, p. 9): a) counter-suggestions, which required a challenge to the worker's response; and b) presentation of a problem with a high degree of similarity later during the assessment. Ginsburg pointed out that if the counter-suggestion is accepted or if the response to the similar problem is inconsistent, then it may be concluded that the worker's beliefs or strategies are not deeply based. Counter suggestions used included:

- Are you sure this is correct (pointing to a specific number or operation)?
- Is this supposed to be gallons per dollar (questioning the assigned units)?
- Why did you multiply instead of divide?

Table 5 displays the order of the assessment questions given to each worker.

Table 5
Assignment and Order of the Assessment Questions

		Assessment questions				
Name	Department	Joint compound	Manure topping	Concrete	Drywall	Grass seed
Caleb	Commercial sales	1	2		3	
Cindy	Commercial sales			2	1	3
Cory	Commercial sales	1	2	3		
Buck	Building materials	1		2	3	
Bill	Building materials	1	2		3	
Benny	Building materials	1		2	3	
Oliver	Nursery & outdoors		2	3		1
Nancy	Nursery & outdoors	3	2			1
Frank	Flooring & walls	1			3	2

Interview. The interview process was based on protocols described by Swanson, et al. (1981), Ginsburg (1981), Ericsson and Simon (1987, 1993, 1998), and H. Rubin & I. Rubin (2005). All of the interviews were audio taped and later transcribed for coding purposes, with two exceptions. One worker (Cindy) did not want to be audio taped, and another worker's tape (Frank) had poor audio quality, making the transcription incomplete.

Starting in the late 1970s, the clinical interview method was expanded from examining mathematical thinking in children (Ginsburg, 1981) to include adults' mathematical thinking (Sewell, 1981; Ginsburg & Asmussen, 1988; Evans, 2000). During the 1980s, this methodology was further refined by making explicit the contingent basis of the questions to be asked. Swanson et al. (1981) wrote:

> A defining characteristic of clinical interview methodology is its contingent structure. The specific direction an interview takes − the questions that are asked − varies as a function of the subject and the subject's answers to earlier questions. (p. 34)

However, the work of information processing theorists such as Ericsson and Simon (1987, 1993, 1998) demonstrated that the clinical interview method by itself was prone to produce errors if the participant was required to verbally encode while solving the problem. That is, researchers found that they had been inadvertently changing the thoughts of participants by probing them during their thinking. Ericsson and Simon (1993) argued that all similar verbal report methods (e.g., think aloud and explanation) faced the same dilemma. Therefore, this study used the 'talk aloud' method while the workers were actively engaged in solving the assessment questions (Ericsson & Simon, 1987, 1993, 1998). In accordance with the 'talk aloud' method, the workers were asked to simply say aloud their thoughts as they solved the problems rather than try to explain their thoughts or say what they were doing. Participants became accustomed to this method by working practice problems before taking the assessment. Appendix C has a list of practice problems and researcher notes.

The ideal situation was to schedule the clinical interview the day after shadowing, with a start time of 90 minutes prior to the worker's shift. This was not always possible for two reasons. First, the worker might not have been scheduled to work the following day, and second, the worker might have had an unusually early start time (e.g., 4:00 AM), so asking the worker to show up 90 minutes early was unreasonable. Other arrangements were made in these cases.

If necessary, the first 15 minutes were used to finish any issues from the shadowing session, such as clarifying questions about a particular event. The next 15 minutes were used for an orientation that included an explanation of the 'talk aloud' protocol and practice questions. The assessment was only given after the worker demonstrated an adequate 'talk aloud' technique. Throughout the clinical interview, I

- reminded the worker, as needed, to "Please remember to talk aloud;"
- observed the employee from an appropriate distance;
- recorded field notes silently;
- refrained from language which indicated approval or disapproval; and
- did not interfere, engage, or help with any tasks.

With one exception, I never spoke to workers while they were responding to an item. The exception was to say "Please remember to talk aloud" when necessary, and this occurred infrequently.

Workers were allowed to access appropriate tools, charts, or other aids during their interviews, but they were not allowed to interact with other employees. I carefully answered any clarifying questions from the workers, in much the same manner as I would have during the testing of students; however, this rarely occurred.

After the worker being interviewed completed an assessment question, I gave the retrospective prompt of "Report everything you can remember about your thoughts during the last set of problems" (Ericsson & Simon, 1993, p. 19). Once the worker was finished answering, I posed contingent questions based on the field notes just taken from the 'talk aloud' and the retrospective question. Types of contingent questions asked included clarifying, determination of purpose, process, and competence. Explanations of clarifying and determination of purpose questions with examples were given in the *Shadowing* section. Process questions included:

- How did you know to do that?
- What were you thinking when you divided here?
- How did you know to divide this by this instead of vice-versa?
- When did this occur to you?

These process questions faced the dangers discussed above in the *Shadowing* section concerning made-up thoughts (Ericsson & Simon, 1987); hence, I had to carefully probe the worker to ensure the thoughts originated during the problem solving, and not afterwards as a speculation by the worker.

The two methods (shadowing and clinical interview) in this study had an important connection; the initial shadowing formed the basis for familiarity and trust during the interview. Ginsburg (1981, p. 9) posited that establishing proper participant motivation is important for a successful clinical interview. Ginsburg recommended developing a one-to-one relationship between the interviewer and the participant (i.e., a sense of trust between them). I developed this sort of one-to-one relationship during the shadowing session. Additionally, my familiarity with the subject matter was helpful in placing the worker at ease (Ginsburg, 1981).

Pilot Study

Before the actual study began, I piloted the clinical interview protocol and all of the items from the assessment questions with six people with extensive construction backgrounds. All of them were well known to me and volunteered to take part. The purpose of the pilot study was to check and refine the assessment questions and the clinical interview processes. The pilot study participants were audio taped for the purposes of checking the interview process and reviewing their responses, but the tapes were not transcribed. As much as practicable, I followed all aspects of the clinical interview protocol.

I conducted the pilot study over two days with three participants per day. They arrived at intervals of approximately two hours. This schedule allowed enough time for me to conduct a debriefing and to fix any issues before a next participant arrived. I made several refinements based on the results of the pilot study. For example, I modified two of the assessment items (concrete and joint compound) because two pilot participants interpreted the original verbiage as asking simply which container was cheaper. As another example, I increased the planned number of practice questions for the 'talk aloud' protocol because four of the pilot participants needed more than originally planned. I simply wrote several more questions of the same type to have available as needed during the actual study. I also enlarged the font size used on the questions when one pilot participant complained about his 'old eyes', and I placed my scripts on 5 x 8 cards rather than 8.5 x 11 paper to help me keep them separate from my field notes.

After the completion of the pilot study, I wrote a series of possible questions concerning intensive ratios and units because I had realized during the pilot study that none of the pilot participants had used units to help in solving the questions or items. This was intriguing to me because I think in units and many of the errors my pilot participants committed could have been avoided if they had properly used units. Hence, during the clinical interviews, all nine of the workers were questioned concerning how units could be used when solving problems. Possible questions included:

- What were the units on this number?
- How do you know what the units are here?
- What does that set of units mean?
- How do you know your answer is in dollars per pound?

After all six participants had completed the pilot study, I examined their solution techniques and errors. I did this because Ericsson and Simon (1987, 1993) had argued that understanding the possible solution pathways (order of thoughts heeded) prior to actual data collection was instrumental in analyzing and understanding the 'talk aloud' data, and they were right. For example, a pilot participant used a solution technique during the pilot study that took me substantial effort to understand. He used a product data table to solve the manure question; however, his answer was incorrect. I asked him contingent questions to understand what he had been thinking. His explanations, reasoning, and calculations seemed solid, but the answer was definitely wrong. Though this issue did not reappear during the actual study, I have presented my analysis of what he did, along with a further discussion of it, in the section *Recommendations for Future Research*.

Coding

I began the coding process by developing a set of research codes prior to any collection of data—even before the pilot study. I modified them slightly after the pilot study and then modified them drastically after the first two shadowing sessions. This was not surprising since other researchers (e.g., Constas, 1992; Saldana, 2012, p. 8) have noted that the "*a priori*" (Constas, 1992, p. 261, his italics) codes selected for a study were often changed or discarded as the iterative nature of coding proceeded. I added a new code whenever I noted approximately three or more similar events for which I did not have a code. I discarded a code at the end of the study if it had not been used, or during the study when I replaced it with a set of more descriptive codes. Though the data was constantly being reviewed as it was collected, the final codes were not decided until all of the data had become available, and then only after several iterations of coding.

I designed the assessment and clinical interview to be authentic and to reflect the sorts of problems I would be observing during shadowing. Therefore, most of the codes were universally applicable in the sense that they applied to more than one of the data sources (shadowing, assessment, interview). For example, a mathematics problem that was fundamentally a ratio would have had the event coded PR (problem-ratio) regardless of whether a worker encountered it during shadowing or on the assessment. The following paragraphs and tables explain the final set of research codes used in this study.

I assigned a qualitative competency code to shadowing events that involved the cognates and all of the assessment items. These codes were useful in answering research question 2(a) and took the form of QC-M for qualitative competency-master or QC-A for qualitative competency-average. See Table 6 for the complete list of these codes and their qualitative descriptors. A worker who was 'efficient' solved the problem directly rather than making false starts. I included Poor (minor error) and Fail (major error) to assist me in differentiating between an answer that was primarily correct and an answer that was primarily wrong. For example, I coded QC-P when Frank subtracted incorrectly on a drywall item from the assessment; otherwise, his answer and technique demonstrated competence. In contrast, I coded QC-F when Bill told a customer that it took nearly four smaller sheets (3' x 5') to make one bigger one (4' x 8')—it actually took 2.13 of the smaller sheets to equal one large sheet. Furthermore, during contingent questioning, Bill was not able to cogently explain his process for arriving at four sheets.

Table 6
Coding for Qualitative Competency

Master (QC-M)	Excellent (QC-E)	Average (QC-A)	Struggled (QC-S)	Poor (QC-P)	Fail (QC-F)
Correct Efficient Quick	Correct Efficient	Correct	Eventually correct Not efficient	Minor error	Major error(s)

The codes from Table 7 were used to classify the types of solution techniques and tools used by the workers during shadowing and the assessment. The preconceived codes for difference, additive, and build-up solution strategies were removed from the tables since they were never observed being used during the study. I added an 'avoidance' code on the fourth day of shadowing after observing multiple instances of workers avoiding quantitative engagement with a customer. For example, I retrospectively assigned the avoidance code to an event that had involved Cindy. I had observed her tell a customer to combine ingredients and mix it until it "feels right" in response to a question about the ratio of mortar, sand, and water to use for laying bricks.

I assigned the 'guess & check' code to a family of closely related techniques that I observed being used during the study. Guess & check, in its simplest form, was assigned whenever a worker immediately began performing operations on known quantities, and then thinking about the meaning of the answer. I applied the 'extrinsic data used' code whenever a worker did not utilize the available numerical data, even when encouraged to do so or when the task required it. This follows Capon and Kuhn (1982), who described task extrinsic reasoning as "based on factors extrinsic to the task objective, though task instructions directed the subject's attention explicitly to the criterion of ‘better buy’ and suggested disregarding extraneous factors" (p. 450). If heuristics are defined as practical methods that produce a good-enough answer without a guarantee of optimization, then many of the codes in Table 7 could have been labeled as heuristic; however, I coded only two types of events with the heuristic code: a) during shadowing when workers recommended that a customer buy a customary percentage of extra material to allow for waste, and b) during the assessment when workers used phrases such as “invert & multiply”.

Table 7
Coding for Solution Techniques

Technique or tool used	Code
Calculator	CU
Guess & check	GU
Heuristic	HU
Table or chart	TU
Primitive calculation	PU
Multiplicative (within)	MUW
Multiplicative (between)	MUB
Multiplicative (formal)	MUF
Tape measure (stick)	TMS
Avoidance	AVD
Extrinsic data	EDU
Referred to schooling	RTS
Made an excuse	EXC
Double-checked answer	DBC
Expressed doubt	DBT

I used the codes in Table 8 for classifying the problem types encountered by the workers during shadowing and on the assessment; however, a worker sometimes viewed the problem differently than I did. These sorts of discrepancies were recorded in the field notes. These coding data were used in answering research questions 1(a), 1(b), and 2(a).

Table 8
Coding for Problem Type

Problem type	Code
Part-whole	PP
Quotient	PQ
Execution of division (decimal)	PD
Ratio	PR
Operator	PO
Measurement	PM
Real	RL
Routine	RT
Mathematics-non cognate	NC

Table 9 contains all of the codes that I used for categorizing customer interactions, but several of the codes require an explanation. The term 'mathematical discourse' may have been applied to any set of humans discussing a situation that involved quantity or shape, but it was primarily used to describe the conversation that took place between a worker and a customer that involved quantity or shape. Occasionally, the discourse involved something written, such as a sketch, plan, or product shelf tag. I judged overall discourse as 'good' or 'poor' based on two criteria: a) sufficient mathematical information was exchanged to achieve the apparent goal, and b) the information was understood by the other party. I judged workers' abilities to engage in mathematical discourse based on their skills to: a) extract mathematical information from a customer, b) correctly process the mathematical information, and c) correctly explain its meaning to the customer.

Table 9
Customer Interaction Coding

Interaction types, reason	Code
Customer Type I	CTI
Customer Type II	CTII
Customer Type III	CTIII
Customer Type IV	CTIV
No customer present	NCP
Product information (comparison shop)	IH
Product information (general cost)	IC
Product information (quantity or coverage)	IQ
Good discourse	GD
Poor discourse	PD
Great answer	GA
Loose estimate answer	LEA
Wrong type of answer	WTA
Wrong quantitative answer	WQA
Asked for irrelevant help	HLP
Position of authority	PA

When planning this study, I did not consider the significant impact different types of customers would have on mathematical discourse. This oversight became apparent on the first day of shadowing when I realized that the types of customers and how they impacted mathematical discourse would play a significant role in answering the research questions. Therefore, based on my observations of interactions between customers and workers, I developed a set of four categories and corresponding codes to classify the types of customers being encountered:

- Type I customers arrived already knowing the quantity and type of product they needed (e.g., I need help loading 67 cases of oak laminate flooring—the one right there [pointing]). They had a specific plan (not necessarily written) and had done the calculations themselves. They might have asked if a similar product was on sale, but would have done the comparison (performed calculations) without the assistance of an employee. They assumed mathematical responsibility.
- Type II customers arrived with an idea and the dimensions for a project, but not necessarily a plan (e.g., I want new flooring and the dimensions of the room are...). They needed help with refining their idea and selecting a product. Some Type II customers spoke freely about the dimensions and parameters of their ideas while others had to be asked for the information. They shared mathematical responsibility.
- Type III customers arrived with a complaint or desire (e.g., My flooring looks horrible, or I want new flooring). They did not have dimensions. Some did not know how to take the necessary measurements and a few gave the impression that they did not even realize that measurements were necessary. They were unaware of mathematical responsibility.
- Type IV customers arrived knowing the quantity and type of product they wanted, but were drastically mistaken. Typically, they did not have the dimensions of the project and stated that someone else had done the measuring and calculating. At

times, these customers adamantly claimed that their measurements or calculations were correct. I have recounted an incident involving Cindy and a Type IV customer in section *Question 2(a)* in Chapter 3.

The 'asked for irrelevant help' code means that the customer requested help that was not of a mathematical nature or was otherwise not relevant to this study. Examples include requests for in-store directions and physical help in loading products onto a cart. I added the 'position of authority' code about halfway through the collection of data when I realized that two types of customers (Types II and III) were giving a kind of deference to the workers. This was examined and illustrated in the section *Social supports used to augment and supplant*. An example of an interaction receiving the 'wrong type of answer' code was a worker giving a qualitative comparison of two products when the customer had asked for a quantitative comparison.

Estimation is a necessary part of the home improvement industry, but I added a code for 'loose estimate answer' after noticing that some workers strived to give reasonable or tight estimates, while others gave unreasonably loose estimates. Unreasonably loose estimates differed from reasonable estimates (correct answers) in three ways: a) a second trip to the store to return excess merchandise or to buy more merchandise was assured; b) anyone could have made the loose estimate, meaning that it was not based on experience or special training; and c) the details of drawings, measurements, and calculations were not used. For example, telling a customer to buy four squares (1 square=100 ft^2) of shingles to reshingle a 400 ft^2 shed without consideration of overhangs, roof pitch, style of roof, or waste would be a loose estimate. I initially considered 'loose estimate' a solution technique; however, upon reflection, I realized that is was more closely related to the result (answer form) rather than the process (solution technique).

Analysis

I converted the raw data into useful forms using a four-step iterative process: 1) completion of coding, 2) collating and reducing data, 3) selecting important data, and 4) drawing and verifying conclusions. I began this process as soon as I started collecting data. As the data became more voluminous, I separated it into complete sentences, thoughts, utterances, or events, and typed each onto its own line in an Excel spread sheet. Each line of the spreadsheet received factual codes (e.g., date, person, item number) and applicable research codes. This process allowed me to efficiently sort, view, and count the data using any desired set of codes.

Once all of the data were coded, I selected particular codes based on their frequency and relevancy. The code had to have sufficient frequency (or lack of frequency) to suggest a pattern. For example, workers occasionally used phrases such as 'invert & multiply' during the assessment (n=3), and regardless of whether the phrase was used correctly or not, its use indicated a familiarity with the phrase. In contrast, I never heard a worker use such a phrase when faced with a problem during shadowing, although many of the problems were similar to the ones on the assessment. Hence, in this situation, it was both frequency and lack of frequency that drew my attention as a researcher.

Though frequency (or lack of frequency) of a code was a necessary condition for me to begin analysis, it alone was not sufficient—relevancy was also necessary. For example, I recorded in my field notes all customer-worker interactions, without regard to the presence of mathematics, and began coding them. After the third day of shadowing, I realized that these interactions, which were devoid of mathematics, could be decomposed into sub-codes, such as customers asking for general product information and customers asking for a specific recommendation. Although these events (codes) occurred frequently, by the end of my analysis, I judged them as not relevant to answering the research questions; hence, I did not further analyze these codes or present them in this study.

After a thorough review of the data, I decided that particular bits of data could be relevant in two cases even if they did not directly address the research questions or lend themselves to coding. In the first case, I decided that data was relevant if it helped me mitigate bias I'd brought into the study. For example, Oliver demonstrated ability and joy at doing arithmetic in his head. This was not a frequent event among the workers and did not directly address a research question, but it countered the bias I had brought into the study that assumed workers would not be interested in mathematics just for the challenge or the fun involved; hence, I included an accounting of this event in section *Question 2(a)* in Chapter 3.

In the second case, I chose bits of infrequently occurring data that did not directly address a research question if they added veracity to conclusions I had drawn (Miles, Huberman, & Salanda, 1994). For example, after interacting over several weeks with the manager of the store, I came to realize that he was a mathophile and that his sensibilities about mathematics influenced the social environment in the store. This realization influenced my conclusions in regards to this study; therefore, I described on page 106 some of my interactions with him, as well as my reasons for concluding that he was a mathophile.

Summary. I gathered and analyzed data to answer four specific research questions that, taken together, had not been answered by previous research. The study used shadowing and clinical interview as its primary data collection methods. An assessment consisting of three multipart questions was completed by each participant and formed the basis of the clinical interview. I selected the methods based on their epistemological attributes, meaning their inherent ability to obtain the knowledge necessary to answer the research questions. Each method was implemented by a specific protocol, as explained and cited in the body of this chapter or the appendices, respectively. I conducted the study in a highly contingent manner based on the specifics of its research perspective; however, being contingent did not mean willy-nilly. The methods of this study were based on well-established protocols and similar research paradigms.

CHAPTER 4

RESULTS

Overview

This chapter presents the results of the research, first by the method used (shadowing and interview) and then by research question. A summary for the results of each research question precedes the detailed results.

Overall, I recorded 684 events while shadowing nine workers for three hours each as they performed their regular duties; 106 of these events involved the cognates. I individually interviewed each worker after shadowing for approximately 60 minutes. Six of the interviews were conducted the day after shadowing; two took place two days after shadowing, and one took place seven days after shadowing due to the worker (Frank) being absent because of a minor illness. During the interviews, I collected 981 pertinent data points from tape recorded sentences, utterances, researcher notes, and the written work of the study participants.

Shadowing Results

Of the 684 events recorded during shadowing, 503 involved a worker-customer interaction. Figure 1 displays the following categorization of shadowing events. Mathematics in some form was involved in 108 (≈16%) of the worker-customer interactions. Of these 108 events, 97 (≈90%) involved the cognates; and of these 97 events, 23 (≈21%) received an artifact code and 4 (≈4%) received two artifact codes. The 181 remaining events, which did not directly involve a customer, consisted of events such as stocking shelves and preparing displays. Of these 181 events, 22 (≈12%) involved mathematics in some form. Of these 22 events, 9 involved the cognates.

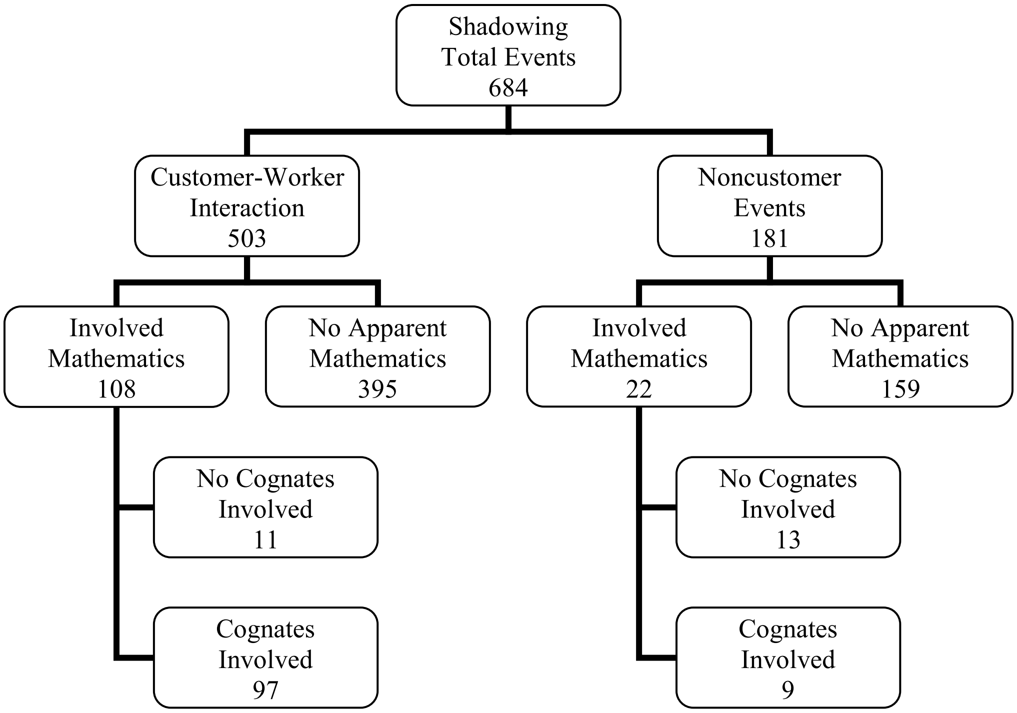


Figure 1. Categorization of shadowing events.

Shadowing events categorized by department, customer presence, and use of mathematics are displayed on Table 10. When total customer interactions were considered, then the workers in the nursery & outdoors department had the highest rate of observed events; however, Table 11, which categorizes events with mathematics present by department, customer presence, and use of cognates, shows that the workers in the flooring & wall department had the highest rate of interactions that involved the cognates. Both tables have a weighted average column. This column displays the average number of interactions by department divided by the average number of all interactions. This partially corrects for the unequal number of workers in each department and allows direct department-to-department and department-to-total comparisons.

Table 10

Frequency and Percentage of Events During Shadowing that Involved Mathematics by Department

	Customer interactions				Noncustomer tasks				Total events			
	Irrelevant help		Involved math		Irrelevant tasks		Involved math					
Department	Total	Weighted ave %	Total	Weighted ave %	Total	Weighted ave %	Total	Weighted ave %	Without math	With math	Percent with math	Events
Commercial	120	91%	44	122%	2	4%	5	68%	122	49	30%	171
Building	64	49%	17	47%	60	113%	10	136%	124	27	10%	151
Nursery	164	187%	26	108%	86	243%	4	82%	250	30	11%	280
Flooring	47	107%	21	175%	11	62%	3	123%	58	24	41%	82
Total	395	Ave=44	108	Ave=12	159	Ave=18	22	Ave=2.4	554	130	19%	684

Note. Weighted average is the average number of interactions by department divided by the average number of all interactions, and the 'Ave=' entry is the average across all departments per worker.

Table 11

Frequency and Percentage of Mathematical Events During Shadowing that Involved the Cognates by Department

	Customer interaction with mathematics				Noncustomer tasks with mathematics				Total events	
	Cognates involved		No cognates		Cognates involved		No cognates			
Department	Total	Weighted ave %	Total	Weighted ave %	Total	Weighted ave %	Total	Weighted ave %	Math with cognates	Math without cognates
Commercial	42	130%	2	55%	4	133%	1	23%	46	3
Building	11	34%	6	164%	3	100%	7	162%	14	13
Nursery	24	111%	2	82%	1	50%	3	104%	25	5
Flooring	20	186%	1	82%	1	100%	2	138%	21	3
Total	97	Ave=11	11	Ave=1.2	9	Ave=1	13	Ave=1.4	106	24

Note. Weighted average is the average number of interactions by department divided by the average number of all intera
and the 'Ave=' entry is the average across all departments per worker.

Table 12 displays the variety and frequency of the techniques and artifacts I saw workers use to solve cognate based problems during shadowing. Workers often used multiple techniques or artifacts to solve a single problem; hence, the total of all the techniques and artifacts used is greater than the total number of problems the workers faced. Note that Table 12 displays the data by customer presence and further breaks it into real and routine problems. Observations indicated that the use of a calculator and the use of a tape measure were nearly mutually exclusive events. That is, problems that required a calculator did not often require a tape measure, and vice-versa. The data in the first column of this table illustrates this observation, since the total of 'calculator used' (n=58) and 'tape measure or stick' used (n=39) sums to slightly more than the total number of events (n=92). An additional observation, not easily extracted form this table, was that multiplicative techniques were always used in conjunction with a calculator and nearly always with guess & check. The use of the additive or build-up techniques were never observed, so they were excluded from this table and any further comment or analysis.

Table 12
Solution Techniques Workers Used to Engage Cognate Problems During Shadowing

	Customer interaction problems		Noncustomer task problems		
Solution Technique	Routine 92 total	Real 5 total	Routine 2 total	Real 7 total	Total
Calculator used	58	5	0	6	69
Guess & check	27	4	0	4	35
Heuristic used	11	1	1	3	16
Table or chart used	6	0	1	0	7
Tape measure or stick	39	3	1	3	46
Avoidance	21	0	0	0	21
Primitive calculation	1	1	0	0	2
Multiplicative (all forms)	14	4	0	4	22

Table 13 shows the frequency of problem types encountered during shadowing. Note that this is how I categorized the problems and that the workers may have treated a particular problem differently than my categorization. The problem type most often encountered by workers in the study was 'measurement' (n=32), with 'ratio' the next most often with 28 events. These 28 events included nine interactions with customers who asked for help with chemicals in the nursery & outdoors department and 14 interactions in which customers asked a quantitative comparison shopping question; hence, 23 of the 28 ratio problems were in actuality two repeated problems. Table 12 does not directly show that workers used 'avoidance' as the solution technique on all nine of the chemical questions and 11 of the 14 quantitative comparison shopping questions; hence, 20 of the 21 uses of the avoidance techniques stemmed from two particular question forms. Also note that workers always solved the 'decimal' problem type with the aid of a calculator, and in seven of the eleven cases, offered a loose estimate answer to the customer.

Table 13
Frequency of Problem Types During Shadowing

Problem type	Frequency
Part-whole	10
Quotient	9
Execution of division (decimal)	11
Ratio	28
Operator	16
Measurement	32

Interview Results

Each employee was interviewed for approximately one hour. The interview always took place after the shadowing session, typically on the following day. Each interview produced three forms of data: interview transcripts, impromptu notes I made, and written work created by the employee. Eight of the nine interviews were audio recorded with the participants' permission, and later transcribed. The exception was Cindy, who opted out of being recorded during the clinical interview. All eight of the recordings were transcribed, but Frank had a style of speaking that made his recording difficult to transcribe accurately. Nonetheless, I was still able to analyze the interview by triangulating his difficult-to-understand transcribed interview with my field notes and his written work All three data forms were examined and cross referenced to find relevant instances, and then coded. A total of 132 relevant instances were identified and assigned a total of 620 codes. The frequency of the assigned codes are in the following tables.

Table 14 shows that a calculator was used by every worker on every question during the interview sessions. Guess & check was an often used technique, and note that the recorded use of heuristics was less than the use of extrinsic data.

Table 14
Frequency of Solution Techniques Used on the Assessment Questions

Solution technique	Assessment questions					
	Joint compound	Manure topping	Concrete	Drywall	Grass seed	Total
Question total*	7	5	5	6	4	27
Calculator	7	5	5	6	4	27
Guess & check	4	4	1	5	3	17
Heuristic	1	0	1	1	1	4
Table or chart	0	5	3	0	0	8
Multiplicative	4	2	1	2	1	10
Extrinsic data	2	0	3	1	0	6

Note. *This row is the total number of workers who were asked a particular question.

Table 15 displays the frequency of competency and other codes assigned to the assessment events. I did not observe any solution that warranted the Master (M) code. I assigned excellent and average codes to the drywall and grass seed questions, although no worker successfully solved the leftover-material part of the questions.

Table 15
Frequency of Competency and Other Codes on the Assessment Questions

	Assessment questions					
Code assigned	Joint compound	Manure topping	Concrete	Drywall	Grass seed	Total
Excellent	1	0	1	1	1	4
Average	1	1	1	2	0	5
Struggled	2	1	1	1	0	5
Poor	0	0	1	0	0	1
Fail	3	3	1	2	3	12
Made an excuse	3	1	1	2	1	8
Expressed doubt	5	4	3	1	1	14
Double checked	1	1	1	1	0	4
Referred to school	1	0	3	0	0	4

Results by Research Question

This section is organized around the original research questions and addresses each in turn. For each research question, answers were triangulated by basing each one on at least three of the five data sources: a) worker-customer interactions observed during shadowing; b) worker solo actions observed during shadowing; c) workers' written responses to the assessment questions; d) workers' talk-aloud utterances during the interview; and e) workers' answers to contingent questions. I presented answers by citing data from this chapter, often accompanied by an illustrative vignette. Summarized answers to the questions are:

1(a). Workers encountered the cognates an average of 3.9 times each hour while at work. The majority of encounters were routine (n=94) rather than real (n= 12) (See page 112 of Smith, 2002, and page 37 of the literature review for an explanation of the differences between routine and real problems).

1(b). Workers engaged in cognate problems at work in a manner vastly different from similar cognate problems found in textbooks. Two stark examples were a reliance on calculators to perform a type of 'guess & check', and the social acceptance of loose estimation calculated using 'quick math'.

2(a). Workers primarily had difficulty in five areas that involved the cognates at work: a) calculating intensive quantities; b) interpreting and interpolating tables, charts, and labels; c) performing calculations by hand; d) understanding and using decimals; and e) conducting mathematical discourse.

2(b). Workers chiefly relied on calculators and product tags as tools to augment, and loose estimation as an answer form, to supplant their own abilities. They used a three-part social phenomenon to mitigate the effects of their mathematical limitations: a) consistent product availability, b) liberal return policy, and c) position of authority. Most of the workers knew a set of common quantities and results that simplified some calculations.

Question 1(a). *In what ways do workers encounter and utilize the cognates while on the job?* Workers encountered the cognates an average of 3.9 times per hour based on observation of 106 pertinent mathematical events in 27 hours of shadowing. The majority of encounters involved routine (n=94) rather than real (n=12) problems. All of the problems stemmed from either customers or departmental duties. I analyzed this research question by utilizing the six subconstructs discussed beginning on page 17 in the literature review.

Part-Whole. Workers occasionally (n=10) encountered the part-whole subconstruct. When it was used, it occurred in two forms: as an adjective or as an adjectival noun.

An example of part-whole being used as an adjective was when a customer stated to Frank, "I need about half a gallon of paint." The customer's use of "half a gallon" rather than "two quarts" was interpreted by Frank as an indication that the customer was a novice paint buyer and was not aware that paint is typically sold in pints, quarts, gallons, and five-gallon containers. Frank countered the customer's 'need' statement by suggesting that a gallon might be a better choice than two single quart containers since a gallon was nearly the same price as two quarts and the customer would then have some left over for touch-up, which was better than having to buy one of the "small cans" for touch-up in the future. Based on this customer interaction and later contingent questioning, Frank, although not assigned to the paint department, showed a competent understanding of the common measures of quarts and gallons. However, he did not understand how pints, the 'small cans', fit into this particular system of part-whole volumes. Additionally, during discourse, he did not query the customer concerning the size of the project in order to verify the volume of paint required.

An example of part-whole being used as an adjectival noun was when Bill remarked to a customer: "I can just cut a whole sheet into 'quarters' for you." In this case, the customer was planning to buy five pieces of plywood that had been precut into 2' x 2' pieces. Bill knew that the common measure of plywood sheets as it arrives in the store is 4'x8', and that a few are cut into smaller 'convenience' pieces. He also knew that an uncut sheet does not fit into a typical car; hence, the purchase of smaller pieces was physically necessary. Bill suggested to the customer that buying an entire sheet and having it cut into 'quarters' was a better option. He argued: a) that these smaller sized pieces would fit into most cars; b) that the price for an entire sheet was less than five 2' x 2' pieces; and c) that the customer would have material left over for another project. However, Bill did not first inquire or measure the car and did not query the customer concerning the final dimensions needed for the project; it may have been better for the customer (and possibly for Bill, if fewer cuts had been needed) to cut the entire sheet into slightly irregular dimensions (e.g., 2' x 3').

Quotient. On average, workers did not often encounter or use the quotient subconstruct (n=9). However, every recorded incident occurred among three workers in two departments (flooring & wall and nursery & outdoors, respectively). Thus, the quotient subconstruct was used approximately once per hour. Examples of its use included Frank helping a customer find a 12-pack of edge tiles to make three frames with four sides each, and Nancy helping a customer determine that a 24-pack of seedling flowers would make four pots of six flowers each.

Execution of Division. Workers regularly used the execution of division subconstruct; however, I was only able to positively identify eleven uses of the subconstruct since the workers frequently used calculators, which tended to hide the process. This is true despite the workers being questioned concerning their thoughts as soon as was practicable after the event. I documented a related occurrence concerning decimals. Typically, when a calculation resulted in a decimal, the worker immediately rounded it up to an integer or converted it to a standard fraction (e.g., , $^1/_2$, $^3/_4$, $^5/_8$, $^7/_{16}$). The conversion to a fraction was not a formal process; rather, it was a loose estimation. I recorded this type of conversion seven times.

Ratio. Workers often encountered problems based on the ratio subconstruct (n=28), but seldom solved them using any 'textbook' paradigm; rather, they solved them using a guess & check method or avoided them altogether. As an example, guess & check was used by Bill when a customer asked him whether he should buy the 3' x 5' sheets or the 4' x 8' sheets of backer board (a cement based fiber board typically used as a substrate for tile on walls and countertops). Bill initially began talking about the benefits of fewer joints if the larger one was purchased versus its unwieldiness in enclosed spaces such as a bathroom. The customer listened, but then directly asked which was "cheaper". Bill immediately replied that the larger ones were normally cheaper, but then took out his calculator and began inputting numbers. He eventually declared that the larger one was "a lot" better of a buy. I later calculated the actual numbers: $0.79 per square foot for the smaller sheets and $0.76 per square foot for the larger sheet. During contingent questioning, Bill explained to me that he had wanted to figure the price per square foot "like over in flooring" but had become confused. So, instead, he estimated by square feet that it took nearly four sheets of the smaller size to make one large one. Actually, it was 2.13 of the smaller sheets to equal one large sheet.

Customers in the nursery & outdoors department asked questions (n=9) concerning how to mix or apply chemicals such as malathion and weed killer. Both of the shadowed workers in the department (Oliver and Nancy) avoided giving specific advice or mixing instructions; instead, they referred the customer to the label instructions, such as those shown in Figure 2.

MIXING INSTRUCTIONS

Tank Sprayer: Use of a Roundup® Brand tank sprayer is recommended. A plastic, fiberglass, plastic-lined steel or stainless steel sprayer may also be used. **Do not mix, store or apply in a galvanized or unlined steel sprayer.**

- Add 6 fl oz (12 Tbs) to 1 gallon of water.
- Spot treat or spray evenly over 300 sq ft.

Hose-End Sprayer: For large areas consider using the Ortho® Dial 'n Spray®

- Set dial to 6 oz.
- To sprayer jar add 6 fl oz (12 Tbs) for each 300 sq ft. DO NOT add water.
- Spray evenly over measured area.
- After spraying, unused product can be poured back into its original container.

1 Tablespoon (Tbs) = 3 teaspoons (tsp)   1 fl oz = 2 Tbs

For easy to kill weeds such as seedlings, add 3 fl oz (6 Tbs) to 1 gallon of water.

Do not apply with a galvanized or unlined steel (except stainless steel) sprayer, or through any irrigation system.

Figure 2. Chemical ratio instructions for weed killer.

Operator. Workers often (n=16) encountered this subconstruct when handling materials such as cinder blocks, pavers, and bags of concrete. Based on contingent questioning, workers viewed stacked material not as full pallets, but rather as an intensive-like operator based on layers (they referred to them as rows) of material such as '15 blocks per row'. An illustrative instance was when a customer needed 55 blocks. Based on contingent questioning immediately following the event, Benny stated: "I need 4 rows because 4 times 15 gives me 60, then I will just take 5 blocks off the top".

Measurement. This was the most common manner (n = 32) in which the cognates were encountered. It was ubiquitous in all of the departments that were shadowed, including even nursery & outdoors: pot size, mature plant diameter, paving stone area, and drip line perimeter. All of the participants in the study were able to use a tape measure and discuss, add, and subtract common measurements in inches (e.g., , $^1/_2$, $^3/_4$, $^5/_8$, $^7/_{16}$). My observations during shadowing and contingent questioning suggested that six of the participants were highly skilled in this area. Buck illustrated this skill while he was being shadowed. He had been tasked to build a display for the store that required him to cut a series of rectangles from a 4' x 8' sheet of $^{1}/_{8}$" thick paneling. I queried what he was thinking. He replied,

> *These boxes need 16 by 24 bottoms cut out, but not really because I have to leave a thirty-second* [$^{1}/_{32}$ of an inch] *all the way around for a reveal. So that is a sixteenth* [$^{1}/_{16}$ of an inch] *but the kerf is an eighth* [the saw blade consumes $^{1}/_{8}$" of material]. *So* ...[pause for a few seconds as he did the math mentally], *I need to mark it out as 16 $^{1}/_{16}$ by 24 $^{1}/_{16}$ on the factory edges* [the original side of the material] *and add an eighth for the other cuts because there will be two kerfs.*

Asked if he would explain it again, he repeated nearly verbatim the same line of reasoning and came to the same final cut measurements.

Question 1(b). *Do workers engage cognate problems they encounter at work differently than similar cognate problems found in a textbook?* I observed and documented several differences between how workplace problems are engaged as compared to textbook problems. Perhaps the simplest difference was absence of solution paradigms first learned in school, such as 'invert and multiply' and 'extremes times the means'. I never observed their use during shadowing, but at least some of the workers knew of them (n=3) since they used them on the assessment questions during the clinical interview.

Solutions were not written down in the manner taught in school. That is, there was an absence of neat and numbered work with clearly defined steps that ended with an answer that included units. Instead, workers may have scribbled a few numbers on a small note pad taken from their aprons but only if needed.

Mathematics and science courses often teach unit analysis as a technique to solve problems with intensive quantities. Cory used unit analysis whenever he faced a problem with intensive quantities (n=4), but none of the other workers used units or even seemed aware of them until the end of a problem, when they would attach them as an afterthought such as, 'Oh, yeah, that's square feet.'

Likewise, Cory was the only worker during the entire study to record an equation with a variable. He set up a proportion with an unknown represented by a variable (x); nonetheless, he incorrectly solved the problem. The proportion was set up in response to an item posed during the clinical interview. Cory wanted pounds per gallon but set up gallons per pound.

Another readily apparent difference between school mathematics and mathematics at this workplace was the manner in which mathematical information was obtained. In most observed cases, the relevant information to solve the problem was not given. This meant that, unlike textbook problems, workers had to extract the necessary information via discourse with a customer or by examining the situation (e.g., examining a sketch made by the customer). My observations suggested that workers' success at acquiring sufficient information to solve problems depended on their own abilities and on the customer type.

All of the workers in the study had a calculator in their apron and often (n=69) used it. There were two exceptions to ubiquitous calculator usage. First, Oliver enjoyed challenging himself by "doing math" (the four basic operations) in his head. He only used his calculator occasionally as a check on his mental work or for a particularly challenging problem. Second, workers did not use calculators for addition and subtraction of standard fractions used in measurement (e.g., , $^{1}/_{2}$, $^{3}/_{4}$, $^{5}/_{8}$, $^{7}/_{16}$); moreover, workers did not write these standard fractions down and convert them to have common denominators before performing addition or subtraction; instead, they did the math mentally. I asked Buck how he worked with fractions in his head. He replied that most of them were memorized, but that "a lot them are just two times the other." Upon further questioning, he replied that "half was just two quarters and two eighths was just a quarter."

Workers avoided the use of decimals, meaning that if a calculator displayed an answer with a nonzero decimal, they would immediately round up to the nearest quarter of an inch. Only when it was necessary to input a value expressed as a fraction into a calculator did workers convert fractions to decimals by rounding up to the nearest quarter of an inch. They would immediately convert the decimal back into a standard fraction once an answer had been calculated. Through contingent questioning, it was not possible to determine precisely where or when the workers had learned to round up, but their reasoning always expressed a sense of wanting the customer to have too much material available while completing a project, rather than too little.

Loose estimation was universally used by all workers during the study. Furthermore, its use was reinforced and taught by management under the label of 'quick math.' Loose estimation based on quick math did not mean rounding or estimating as taught in school textbooks (e.g*., Pre Algebra* by McGraw Hill, 2007; *Math: Grade 8* by Harcourt, 2003). It was not even a formal system; rather, it was a loose system of arithmetic estimation with a heavy emphasis on answering the question, "Does it make sense?" In nearly all of the observed instances, the workers in the study understood the question or problem at hand and had the ability to offer a reasonable answer. These two attributes, understanding and ability, were developed well beyond anything I have witnessed in 9 years of teaching high-school mathematics. During shadowing and the interviews, the workers had good number sense concerning the approximate answer—they knew if their calculator was displaying a nonsensical answer. The study protocol did not allow me to interact with customers, so I was not often privy to the sketches and drawings that customers brought with them. Because of this, I was not able to meaningfully estimate an average order of magnitude error of the loose estimation technique; however, I did not witness any of the workers make a mistake that could have led to a significant negative outcome (e.g., death, major property damage, or large financial loss).

Some textbooks have included guess & check as a solution technique, though it has not usually been emphasized. Conversely, the main solution technique used by workers was some form of guess & check (n=36). Of the various forms they used, the most often used (n=17) followed a loose 4-step paradigm: 1) when first confronted with a problem, the worker immediately began entering numbers into a calculator and performing operations; 2) when the calculator displayed an answer, the worker thought momentarily about the meaning of the answer and whether its magnitude (numerical size) made sense; 3) if the displayed number was either substantially greater or much less than expected, then a "No, that's not it" or similar utterance was muttered, followed by more calculator manipulations; and 4) the worker repeated steps two and three until reaching an acceptable answer, and failing that, the worker switched solution techniques. A difference, sometimes observed, was that if a displayed number was somehow satisfactory, then it was written down and the process of calculator inputting began again, but this time using the newly found, satisfactory number. These behaviors suggested that finding an answer consisted of a series of piecemeal steps with no overall plan, even if the goal was well understood. The term guess & check was used because it was the only textbook strategy that remotely resembled the process being used by the workers.

Double-checking has been a mainstay of school mathematics, but I never observed double-checking during shadowing, and only observed it four times during the assessment. I was not able to determine a reason for the lack of double-checking. When I asked workers contingent questions concerning this behavior, the typical response was: "Oh, I probably should have."

A minor but distinct and interesting difference between workplace and school mathematics was the use of adjectival nouns in a mathematical setting, meaning that numbers, particularly fractions, would take the place of nouns. This was seen in comments such as, "Do you want me to cut it into quarters?" and "Do you need another half?"

Question 2(a). *What mathematical difficulties involving the cognates do workers experience while on the job?* A substantial number of the workers in the study displayed difficulties or committed errors when faced with: a) conducting mathematical discourse; b) understanding labels and interpolating tables; and c) solving problems involving intensive quantities. Small subsets of workers had difficulties with other topics, such as performing calculations by hand and understanding decimals.

Conducting mathematical discourse. I documented ineffective mathematical discourse in approximately 19% (n=22 out of 97) of the worker-customer interactions, and noted that it arose from two primary factors along with several minor ones. Customers, especially Type III and Type IV, were one of the primary factors. For example, a Type IV customer stated at the commercial desk that he was pouring a "14 yard" concrete pad and that he needed help loading 43 bags of 80 lb concrete to do the job. Cindy, sensing that the numbers were not correct, took out her construction-estimating calculator and asked the customer if it was going to be a 4-inch thick slab (a fairly standard thickness). He replied, "Yeah, 14 yard, and 4 inches thick." Cindy, showing the customer the calculator, calculated that he would need 45 bags for every yard and not 43 bags for the entire project. The customer insisted that "his contractor buddy" was good at this and had done the calculations correctly. Cindy then asked, while signaling area (square feet) with a hand gesture, "How big is your slab going to be?" He replied with an incredulous look, "14 yards." She then explained that she was wondering about the perimeter dimensions. He was becoming flustered and asked pointedly if he could just get some help loading the 43 bags. Cindy phoned for the lot attendants to give the man a hand, but as he left, she calculated that even if the slab was 14 square yards of 4-inch deep concrete, he would have needed 63 bags. As a final note, all 43 bags—3,440 pounds—were placed, at the customer's insistence, into the back of a standard half-ton pickup truck. It was severely overloaded.

The second primary factor affecting discourse was the difference between the individual workers in their observed abilities to engage in mathematical discourse with customers. Having shadowed nine employees for 3 hours each, I considered Cindy, from the above concrete example, as moderately effective based on the criteria explained on page 62. In particular, she was not able to extract all of the necessary mathematical information from the customer, but was able to process what she did receive, and attempted to explain its meaning to the customer. By contrast, Nancy, from the nursery & outdoors department, was ineffective, as illustrated by the following vignette.

Customer: *I am interested in these pavers for my patio* (pointing to a type of irregularly shaped paver stones shown in Figure 3).

Nancy: *OK* (taking out her calculator and inputting some numbers).

Nancy: *A patio using these pavers will cost about $300, not including sand or any other necessary prep work.*

Customer: *OK, I will have to think about it—thanks* (customer leaves the area).

Researcher: *How did you do that estimate?*

Nancy: *I used the cost per square foot from the tag* (pointing at the product shelf tag shown in Figure 4).

Researcher: *Did you multiply by 100?*

Nancy: *Yes*.

Researcher: *Why 100?*

Nancy: *Because you have to multiply the cost per square foot times the size of the patio*.

Researcher: *Understood. How did you know his patio was 100 square feet?*

Nancy: *That's a standard size*.

Researcher: *Don't patios vary in size?*

Nancy: *Not by much. It's the prep work that will blow a budget*.

In particular, note that Nancy began to work on calculating an answer before fully understanding the parameters of the problem, meaning that she did not successfully extract mathematical information from the customer. She provided an estimate based on the idea that a common patio size was 100 square feet, but she did not explain this to the customer. Another example of her ineffective mathematical discourse can be found in the section, *Understanding and interpolating tables and labels*.

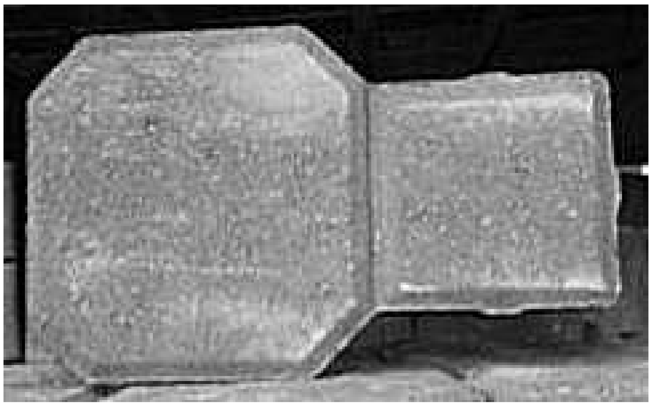


Figure 3. Irregularly shaped paver from the nursery & outdoor department.

Figure 4. Product shelf tag for irregularly shaped pavers.

Once again, in contrast to Cindy and Nancy, I considered Frank, who was the lone participant from the flooring & wall department, excellent at engaging in mathematical discourse. The following example illustrates his interaction with a set (man and woman) of Type III customers.

Frank: *Hello, can I help you with some carpet this morning?*

Customer (man): *Well, we are thinking about it.*

Frank: *Is it for inside your house?* (Frank had noticed that the customers were looking at outdoor carpet.)

Customer (man): *No, it's for the porch.*

Frank: *Does the porch have steps?* (During contingent questioning after this event, Frank stated that this question helps to establish if it is a mobile home.)

Customer (both): *Yes.*

Frank: *Are they going to be carpeted?*

Customer (man): *Yes.* (hesitatingly, and looking at the woman for approval)

Frank: *How are they carpeted?*

Customer (both): Silence

Frank: *Are just the tops of the steps carpeted or does the carpet flow down the steps like a waterfall?* (using his hands for emphasis)

Customer (both): *Like a waterfall*.

Frank: *OK. Do you have the measurements of the porch?*

Customer (man): *Yes.* (taking a piece of paper out of pocket)

Frank: *Good. How about for the steps?*

Customer (man): *Oh...No, didn't think about that*.

Frank: *No problem. Are the steps wider than this?* (pointing to the 6 foot wide outdoor rolls of carpet)

Customer (male): *No, the two of us can barely pass each other on them.*

Frank: *OK, let me take a look at your measurements*.

After briefly examining the measurements, Frank continued the conversation with an explanation and recommendation for the direction in which the carpet should be installed since the direction has an impact on the amount needed. He went over a few facts concerning the durability of the various carpets for sale, and questioned the couple concerning whether the porch received direct sunlight and for how long they intended to live in the home. He used this information as a basis to discuss the pros and cons of the various carpets, using a loose lifetime cost estimate. Here is a snippet from that conversation.

Frank: *See, this carpet for 73 cents a square foot* (pointing to a display) *? It will only last about three years in the sun. So that costs you about 25 cents a year to own it* (calculation done without a calculator). *This one* (pointing to a different display) *lasts at least five years. See, it's guaranteed* (pointing to a small placard) *and it's only 90 cents, or about* (getting his calculator out and punching in the numbers) *18 cents a year. So, even though it is more expensive now, it's cheaper over the five years, and I think it's a nicer looking carpet anyway.*

Frank finally stated the amount of carpet needed and used his calculator to determine the total cost, excluding taxes (which he specifically stated), of three different qualities of carpet. Overall, Frank displayed three well developed attributes that made his mathematical discourse highly effective: he a) explained mathematical principles (geometric and lifetime cost) as they related to the specific situation; b) extracted mathematical information without causing any apparent anxiety; and c) correctly processed the mathematical information and explained its meaning to the customer.

Minor causes of ineffective discourse were different vocabulary (e.g., “Visqueen” instead of “Tyvek,” house wrap, or plastic sheeting), label misinterpretations, and product specifications changing. See the next sections, *Understanding and interpolating tables and labels* and *Understanding and using decimals,* for examples of these causes in action.

Understanding labels and interpolating tables. I found evidence that workers were sometimes confused by product labels (n=12). For example, customers asked Oliver three questions concerning chemicals found in the nursery & outdoors department during his shadowing session. Likewise, Nancy was asked six questions during her shadowing session. During each interaction, Oliver and Nancy immediately referred their customer to the chemical's label. This was done without querying the customer further or actually opening the label and explaining it to the customer. Thinking that it might be store policy due to chemicals being involved, I asked Oliver about this rapid reference to the label. He said it was not policy. He further explained that the labels were always changing and were difficult to understand. As proof, he took me to a container of 'weed stopper' on the shelf that "doesn't even contain ounces on the label" (Figure 5).

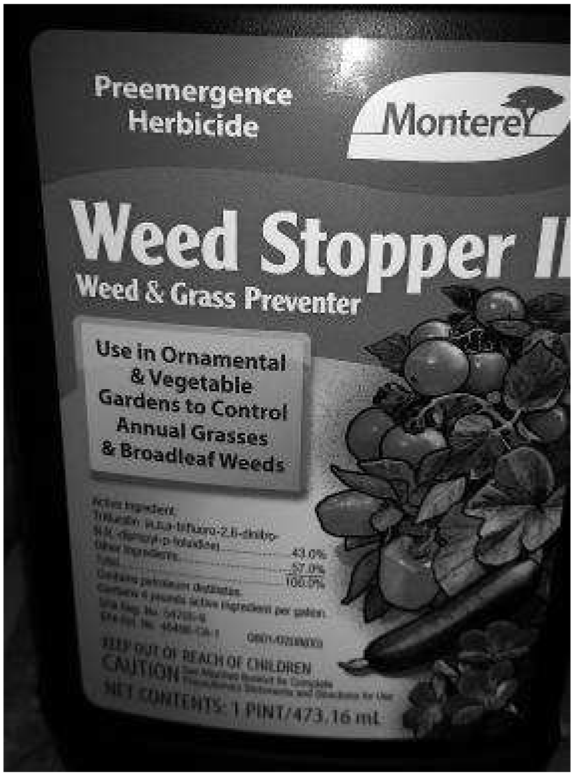


Figure 5. Chemical in the nursery & outdoor department without ounces on the label.

I also queried Nancy. She offered remarks such as "I really couldn't help him [customer] because I didn't know how big his yard is" and "I didn't know what type of weed she [customer] was trying to kill." I did not ask Nancy why she did not ask the customer for the yard size or weed type. It seemed to me that asking her such a question might have indicated that she ought to be doing so, and hence changed the behavior I was observing. Nancy's lack of questioning customers and her responses to my questions provided evidence that she had difficulty thinking about mathematical discourse. My observations of Nancy indicated that she was a conscientious employee, so I doubt that she had ill reasons (e.g., spite, laziness, apathy) for not asking mathematical questions; rather, it seemed to never occur to her to engage in mathematical discourse.

During shadowing, I noted that workers twice needed to interpolate tables, but in neither case did the worker succeed. For example, Cindy was asked by a customer for the ratio of sand to mortar to make a mixture suitable for repairing the mortar bed of a few bricks. Cindy initially told the customer that the directions were on the bag, but the customer persisted, saying that the table on the bag was only for a full bag, and that "I need to know how to mix it by the pound," since he only needed a small batch for some repairs on a barbecue grill. Cindy walked with him over to the product area and read the label. She was unable to interpolate the data, and said to the customer that he should mix it until it 'feels right'.

The assessment revealed a similar finding; that is, only one worker, Cory, successfully interpolated two of the tables found in the store. An item from the concrete question (Appendix B) asked the worker to determine the number of bags of concrete that would be needed to pour a slab with specified dimensions. The item asked that the answer be given in both 60 pound and 80 pound sized bags. Two tables were included that showed the number of bags needed for standard size slabs. These tables were ubiquitous, as they were printed on every concrete bag, and posted, in a blown-up size, in three places throughout the store for easy referral. Based on the results of the pilot study, I expected that workers assigned to the concrete question (n=5) would try either a straight volume calculation or use the table with interpolation techniques. Cindy used a volume calculation on a specialized calculator, Buck used primitive reasoning, and three workers attempted to solve the problem using tables. Two of them, Cory and Oliver, were successful at using the tables. Cory interpolated the data in the tables, whereas Oliver used the information in the tables for addition and estimation. I assigned the concrete question to both of them as a competency check after they had each successfully completed the manure topping question.

The manure topping question was similar to the concrete question. It also asked the worker to determine the number of bags of product that would be necessary to accomplish a task and provided a table from the product's packaging. Five workers attempted to solve this question, but only Cory and Oliver succeeded. Cory used interpolation, whereas Oliver used a system of addition. The other three understood how to read the table, but did not know how to use the information via interpolation or another method to find a value not explicitly in the table.

Solving problems involving intensive quantities. Eight of the nine workers (Cory being the exception) in the study did not use units to help them set up or solve the items presented to them during the interview. On the contrary, the study results suggested that the workers were unaware of how to use units, unit analysis, dimensional analysis, intensive ratios, extensive ratios, and rates. Besides the one exception, units did not appear in any way (e.g., written or uttered during talk-aloud) during actual problem solving. Workers often added units to the answer at the end of the solution process. I interpreted uttered phrases such as "Oh, yeah, that's feet cubed" and "Oops, just about forgot, that's dollars per pound" as indicators that units were an afterthought rather than an integral part of the solution process. I first noticed the lack of unit usage by participants during the pilot study, so I developed a set of pertinent contingent questions focused on units to be used during the interview. Cory gave cogent explanations, but none of the others were able to explain or conjecture about how units could be used.

For example, I asked Oliver about his answer to the concrete question.

Researcher: *You divided \$3.45 by 80 lbs, and you're saying that's about 4.3 cents per pound, right?* (pointing to Oliver's written work)

Oliver: *Yeah, I divided three forty-five by eighty and got 4.3 cents a pound.*

Researcher: *OK. What if you had done it the other way? What would it mean if you divided it the other way?*

Oliver: *So, like 80 divided by 345...there's sort of some significance in my head. I'm thinking like...I just can't see it. It's blocked because I'm used to thinking the other way, my mind is blocking that thought...like why would you do that?*

Researcher: *All right, but sort of just forget about why you would do it. Just tell me what it would mean? What would the answer tell you or indicate?*

Oliver: (using the calculator) *23.18. All right 23.2 let's do that again...be sure, yeah 23.2, right. Yeah...that's....I honestly don't know what that would represent. So 80 divided by 345. I'm not doing the decimal thing, I will just add that in later, 80 divided by 345, what would that, that would be, it would have no significance in my mind to anything.*

In this same line of questioning, I asked five of the workers about the '2' in feet squared. Frank gave a cogent answer, whereas Caleb gave a response typical of the other four workers.

Researcher: *When someone writes squared feet like this* (pointing to 'ft^2'), *what does the '2' mean?*

Caleb: *It means squared. You know, for area.*

Researcher: *How do you know that?*

Caleb: *I don't know, it's what it means.*

Researcher: *Why a '2'? Why not a '3' for example?*

Caleb: *Because '3' means cubed—that's not area.*

Researcher: *Sure, but does the '2' have anything to do with area?*

Caleb: *I guess a square has two sides sort of. We have to multiply the two sides—length and width—together to get it.*

Researcher: *Where did you learn about square feet and area?*

Caleb: *I don't know. Thirty years ago? In school? I don't know.*

Caleb's responses were not unique; the written work, talk aloud protocol, planned questions, and contingent questioning all supported this nearly universal lack of knowledge or skill in using units to solve problems. Moreover, the talk-aloud protocol indicated that numbers and operations—but not units—were the primary thoughts occurring during problem solving. In a sense, their thoughts appeared to be "unitless."

Further examination of the data revealed several more instances of faulty answers that could be attributed to unitless thoughts. For example, Benny stated during (joint compound) contingent questioning that he wanted to "find out how much each pound costs." To do this, he had incorrectly divided 61.7 pounds by $13.47 (which gave him the number of pounds per dollar rather than dollars per pound). Additional contingent questioning determined that he had divided the "bigger" number by the "smaller" number "because that's how you have to" and to divide the "smaller number by the larger number doesn't make any sense." When asked what the units on his answer were, he replied after some thought, "dollars per pound." When asked how he knew those were the units, he replied, "Well, that's what I wanted." This sort of inversion of division occurred four times during the interviews, and each time a similar round of contingent questioning ensued with similar results.

During shadowing, I noted that workers did not always view comparison shopping questions from customers as ratios or even as mathematical questions. Instead, the workers most often (11 times out of 14 situations) treated the question as an opportunity to explain the qualitative differences between the two products or packages. The three exceptions all had the common characteristic of product shelf tags that allowed for easy cost comparison. The data from the assessment showed a similar weakness in comparison shopping.

Comparison shopping items accounted for a major part of the questions asked during the assessment; 22 of the 27 questions asked had comparison shopping items. Every worker received at least two questions with comparison shopping items. Three workers did not successfully solve any of the comparison shopping items. The other six workers answered 45% of the items correctly (10 correct out of 22 possible); however, three of these workers did not successfully answer follow-up competency items.

Results suggested that understanding and successfully calculating area problems were routine events, but the data also suggested that several of the studied workers had difficulty understanding volume. The question about concrete, which was asked five times, elicited two telling remarks. In the first case, Benny initially wrote that there were three cubic feet in a cubic yard. During contingent questioning he became unsure of his answer, but also knew "it can't be nine cubic feet because that's area." In the second case, Caleb stated, when already deep in the manure question, "I should have the chart." When alerted to the fact that he had the chart from the bag to help solve the question, he replied that there was a different chart he liked to use. I asked him if he wanted to get the chart, but he said he was OK without it. Afterwards, he explained that a chart at the commercial desk converts cubic feet into square feet no matter the application (e.g., concrete, manure, and joint compound). The following day, I saw Steve at the commercial desk and asked if I could see the chart we had been discussing; he did not find it, but once again remarked how useful it was when solving area and volume problems. He asked two other employees (not in the study) about the location of the chart. They appeared to know what he was talking about but also did not find it.

Another example that suggested volume confusion occurred during shadowing. I had just started shadowing Oliver, and during a slow moment (no customers present), he was explaining to me how and why he liked to do calculations in his head. Another worker in the study (Nancy) walked up and verified that Oliver was "quick with math in his head." Oliver wanted to do an example:

> Oliver: *Let's say you have pi-r-squared* (πr^2) *and 'r' is equal to 4. Then 4 squared is 16 and you multiply that by 3.14 which is...*(thinking while doing minor multiplication movements with his fingers in the air)*...50.24.*
>
> Nancy: *That's amazing*.
>
> Researcher: *What is πr^2 all about? What's it mean?*
>
> Oliver: *It's the volume of a cone.*
>
> Researcher: *Explain it to me.*
>
> Oliver: *Well, it's just r is the radius—I used 4—and pi is 3.14.*
>
> Researcher: *Four what? I mean, what are the units on the four in your example?*

Oliver: *Oh, like inches; could be feet, whatever.*

Researcher: *What about pi?*

Oliver: (thinking) *It doesn't have any*.

Researcher: *OK, so when you multiplied the r^2 out, what units did you end up with?*

Oliver: *Well, 4 times 4 is 16. Wait, what do you mean?*

Researcher: *It was 4* ***inches*** *times 4* ***inches*** *wasn't it?* (emphasized 'inches' while speaking)

Oliver: *Oh, yeah. Well, you get volume, because it, πr^2, is the volume of a cone.*

A customer appeared and the conversation ended. Later contingent questioning (at the end of shadowing) continued to find that Oliver did not consider units an integral part of the problem. I asked Oliver directly during the clinical interview to explain the units on his volume formula while stating that it only seemed to have squared units. His answer was not cogent. In the past, he had not thought about units and formulas, but continued to insist that πr^2 was the volume for a cone.

Trouble with other topics. The following difficulties and errors appeared in only a few instances. However, they may be more prevalent and important than initially thought, given the small sample size of the study and that some of these issues also appeared during the pilot study.

Multiplication and division. As part of each clinical interview, I began the demonstration of the talk-aloud protocol by hand multiplying a two digit number by a one digit number while talking aloud (Appendix C). I would then ask the worker to try it. Cindy and Caleb stated that they had forgotten how to do multiplication by hand and that they relied exclusively on the calculator. However, both said that watching me do the one as an example had reminded them of the process, and they were willing to try. Both were successful, albeit Caleb took a few tries (without my help) to fully recall the standard algorithm.

Eventually, as workers continued to practiced the talk-aloud protocol, I presented a long division problem (single digit divisor, three digit dividend). This time three workers (Cindy, Caleb, and Benny) stated that they had forgotten how to do long division by hand. I asked all three workers to attempt it if they remembered anything at all about the process, and in all three cases, they talked their way through the problem. Caleb's first attempt was reversed—he divided the divisor into the one's place and then the ten's place and hundred's place digits. He realized his answer was nowhere near correct and then resolved the problem correctly by reversing the order, thus using the standard long division algorithm.

Algorithmic phrases. The misuse or misunderstanding of algorithmic phrases (e.g., invert and multiply) first appeared during the pilot study and was also present during the clinical interviews. Nancy and Benny incorrectly referenced the paradigm "invert and multiply" when faced with what they believed was a fraction, but in both cases, they had set up a ratio with intensive quantities. Nancy spent several minutes repeating that she knew she needed to "invert and multiply" but did not decide by what she should multiply the inverted fraction [ratio]. Benny stated, "OK, now I need to, um... invert and multiply to get this right." Just like Nancy, Benny had difficulty deciding the other factor. During contingent questioning, both Nancy and Benny indicated a belief that, if a fraction is involved in a problem, then one must invert and multiply sometime during the process in order to get the correct answer. A third worker, Cory, used "multiply the extremes by the means." In this case, he incorrectly identified the extremes and means, but had also set up the proportion incorrectly. These two errors led to a number (decimal) that was in the form of x^{-1} from the correct answer. Cory recognized that the answer displayed on his calculator was wrong and tried inverting it by using his calculator to divide one (1) by the displayed answer. He checked the result and was satisfied that it was correct (it was). During contingent questioning, I asked him to talk me through his solution technique. As he explained his process, he pointed out that he knew "the answer was too small" and that he suspected that he had "flipped" something, so he tried flipping the answer, and when he "plugged" it back into the proportion, it worked.

Understanding and using decimals. As previously noted, when a calculator displayed an answer involving decimals, workers immediately rounded or converted the answer to the closest standard fraction. Contingent questioning led to phrases such as: "Decimals and fractions don't measure the same thing" and "Inches in decimal is metric." One event combined the misunderstanding of labels, decimals, and the metric system. Furthermore, it led to the rejection of an entire lift (100 sheets) of paneling.

A customer wanted a sheet of $^1/_8{}^{th}$ inch paneling (Figure 6), but the shelf contained a new bundle of 100 sheets that had not yet been cut open. Benny cut open the bundle and helped load the sheet onto the customer's cart. As it was being loaded, the customer complained that the sheet felt too thin. Benny examined the product shelf tag and pointed to the '0.106IN', and stated that this was the correct place for $^1/_8{}^{th}$ paneling, but that it did feel thin (fact: $^1/_8$" = 0.125"). He then took out a tape measure and determined that the paneling was less than $^{1}/_{8}{}^{th}$ inch. Benny verbally called over Caleb from the commercial desk, which was about 100 feet away. Caleb verified the 'error' and removed the tag, complaining aloud that they (the supplier) had sent metric paneling rather than $^{1}/_{8}{}^{th}$ inch paneling.

Figure 6. Product shelf tag with "metric numbers."

Caleb then phoned the ordering department in the store to arrange for a return of the entire pallet to the vendor. The sheeting was in fact the correct thickness because, typically, lumber is nominally dimensioned (i.e., in name only), meaning that it is not actually the dimension listed in its description. Some common examples are that 2"x4"s are actually $1^{1}/_{2}$"x$3^{1}/_{2}$", and half-inch thick plywood is actually $^{15}/_{32}$" thick. I noted that most of the other lumber tags also had decimal equivalents, and asked both Benny and Caleb (I had shadowed Caleb two days earlier) about the tags. They commented that the tags had begun to have the "metric numbers" about a month ago.

Question 2(b). *What tools, techniques, and social supports do workers use to augment or supplant their own abilities when confronted with difficulties involving the cognates?* Workers chiefly relied on calculators, tape measures, and various forms of product markings as tools to augment, and in some circumstances supplant, their abilities. The two techniques primarily used included: a) avoidance, and b) guess & check, used in tandem with loose estimation. They used a three-part social phenomenon to mitigate the effects of their mathematical limitations: a) consistent product availability, b) liberal return policy, and c) position of authority. Occasionally, other tools, techniques, and methods were used. In particular, most of the workers had a set of factoids that they called upon to simplify calculations.

Tools used to augment and supplant. Calculator use was ubiquitous when solving routine and real problems (n=58 out of 94 and n=11 out of 12, respectively). Calculators were so heavily relied upon that three workers, Caleb, Cindy, and Benny, claimed to have forgotten how to do multiplication and division by hand. Additionally, even a seemingly simple problem, approximating 100 x $3.04, was solved with the aid of a calculator by Nancy.

Measuring devices, meaning tape measures and measuring sticks, were the second most often used tool. During shadowing, a measuring device was used in 46 of the 106 recorded events involving the cognates.

Product data sheets and labels were regularly used as references by the workers, but they played a significant role only five times in solving a problem. On occasion, the product shelf tags contained useful information, but that information was not used. For example, a customer asked Oliver how many of the irregularly shaped pavers were needed to make an 80 square foot pad (Figure 3). Oliver referred to the product shelf tag (Figure 4) and stated that the tag did not appear to have the information. Oliver continued to examine the tag and pointed out that the tag had all sorts of other information (e.g., pallet weight and stone weight), but that the number of pavers per square foot did not appear.

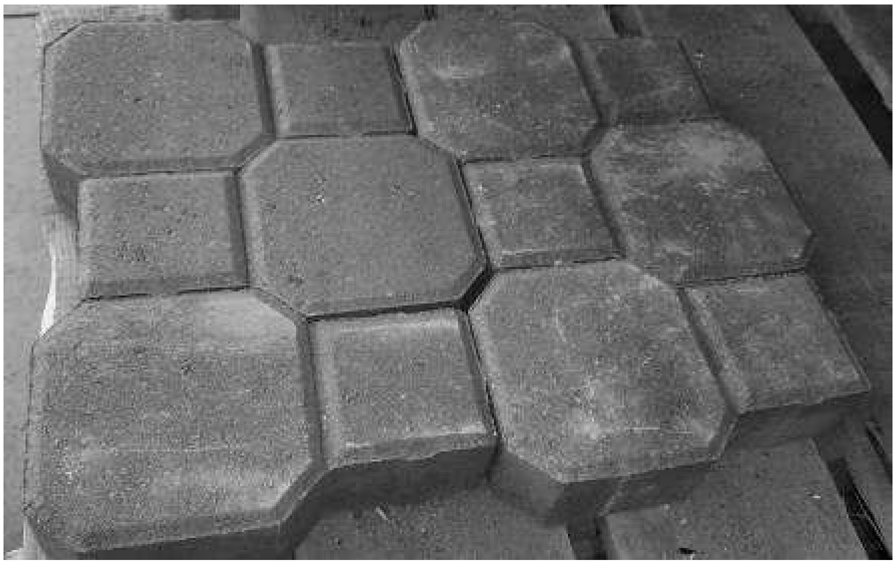


Figure 7. Pavers laid out to determine the quantity in a square foot.

To assist the customer, Oliver had an idea and laid out a sufficient number of pavers on a pallet (Figure 7) to estimate the number of pavers the customer would need. It did not occur to Oliver (or the customer) that the tag contained sufficient information to calculate the number of pavers per square foot.

End stripes on sheet goods were another type of product marking that was regularly used, as shown in Figure 8. The number of stripes indicated the thickness of the material in eighths of an inch with no stripe indicating $^1/_8$th inch (e.g., three stripes means $^1/_2$ inch and five stripes means $^3/_4$ inch ($^6/_8$)). I noticed that these marks were not often used by the workers in building materials; they already knew their products. Instead, the end stripes seemed to be primarily used by cashiers to speed up checkout and decrease errors. Note, however, that cashiers were not included in this study.

Figure 8. End stripes on sheet goods indicating the thickness of the material.

Frank in the floor & wall department used a chart (Figure 9) to convert lineal feet of 12-foot wide carpet into square yards. Buying carpet was complicated because: a) the checkout computers often priced carpet by the square yard but sometimes by the lineal foot; b) the shelf tags had varied pricing schemes, including by the square foot, the square yard, and the lineal foot; c) carpet came on 12-foot or 6-foot wide rolls; and d) carpet was cut by the lineal foot but seldom priced by the lineal foot.

measures 48'3" in length, how many square yards are there in that piece of flooring?

48' equals 64.00 sq. yd.
3" equals .33 sq. yd.

64.33 sq. yd. total

Length Inches	Square-Yard Equivalents	Length in Feet	Square-Yard Equivalents	Length in Feet	Square-Yard Equivalents	Length in Feet	Square-Yard Equivalents
1	.11	1	1.33	34	45.33	67	89.33
2	.22	2	2.67	35	46.67	68	90.67
3	.33	3	4.00	36	48.00	69	92.00
4	.44	4	5.33	37	49.33	70	93.33
5	.55	5	6.67	38	50.67	71	94.67
6	.67	6	8.00	39	52.00	72	96.00
7	.78	7	9.33	40	53.33	73	97.33
8	.89	8	10.67	41	54.67	74	98.67
9	1.00	9	12.00	42	56.00	75	100.00
10	1.11	10	13.33	43	57.33	76	101.33
11	1.22	11	14.67	44	58.67	77	102.67
		12	16.00	45	60.00	78	104.00
		13	17.33	46	61.33	79	105.33
		14	18.67	47	62.67	80	106.67
		15	20.00	48	64.00	81	108.00
		16	21.33	49	65.33	82	109.33
		17	22.67	50	66.67	83	110.67
		18	24.00	51	68.00	84	112.00
		19	25.33	52	69.33	85	113.33
		20	26.67	53	70.67	86	114.67
		21	28.00	54	72.00	87	116.00
		22	29.33	55	73.33	88	117.33
		23	30.67	56	74.67	89	118.67
		24	32.00	57	76.00	90	120.00
		25	33.33	58	77.33	91	121.33
		26	34.67	59	78.67	92	122.67
		27	36.00	60	80.00	93	124.00
		28	37.33	61	81.33	94	125.33
		29	38.67	62	82.67	95	126.67
		30	40.00	63	84.00	96	128.00
		31	41.33	64	85.33	97	129.33
		32	42.67	65	86.67	98	130.67
		33	44.00	66	88.00	99	132.00

Figure 9. Table to convert the length of flooring on 12-foot wide rolls into square yards.

As an example, a customer wanting 75 yards of carpet would require the worker to select and use a table to determine the number of lineal feet to cut off of the roll. The worker would have to determine the lineal footage that was closest, but over, what the customer wanted. By contrast, a customer who wanted 48' 3" (lineal) of carpet cut off of the roll would require the worker to use the chart in a different manner, as shown by the example on the top part of Figure 9. In both example cases, the worker would have to write a tag with the type and square yardage of the carpet, with the exception of those 6-foot wide carpets that were priced by the lineal foot. Then the cashier would have to interpret and input the data from the tag correctly. According to Frank, errors were not common, but were made.

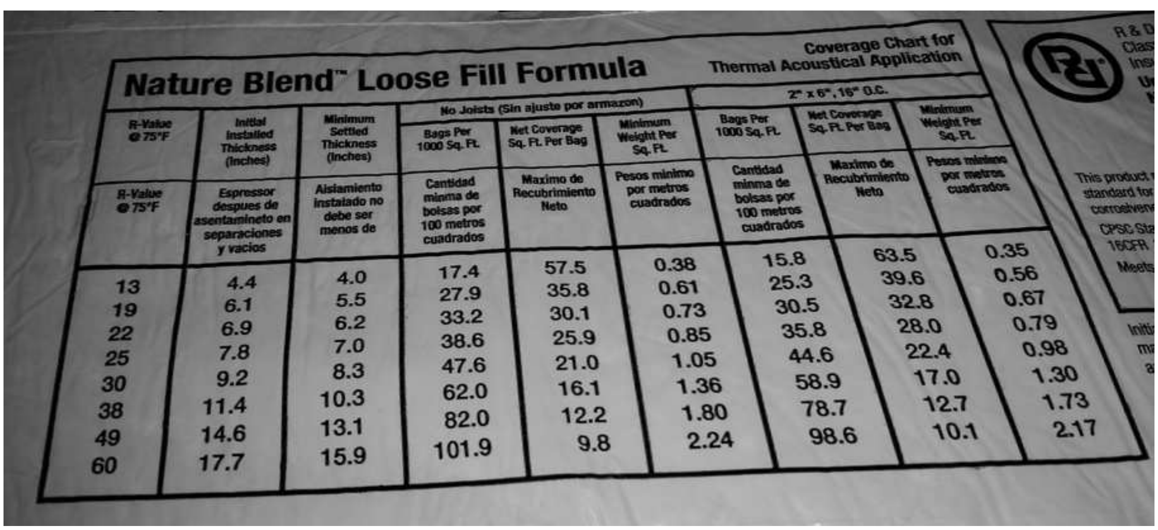
Techniques used to augment and supplant. Workers used 'guess & check' in tandem with loose estimation as the primary pair of techniques to augment their abilities. Workers in the nursery & outdoors department (Oliver and Nancy) used avoidance in every observed instance when confronted with cognate questions involving chemicals.

Social supports used to augment and supplant. The head manager enthusiastically supported this study and was a bit of a mathophile. This piece of his personality displayed itself in the social atmosphere of the store. He called finding a correct loose estimation 'quick math', and being good at quick math in the form of getting a "good enough" answer was socially prevalent and supported. Regularly, he provided on the spot training to employees in the form of a verbal quiz, followed by any needed coaching (I saw it happen 3 times to employees not in the study while I was shadowing). He called the coaching 'cardboard talks' because he used the back of the nearest piece of cardboard for a chalk board. He reported that he regularly provided formal mathematics training classes to help develop his assistant managers and department heads. Though I never witnessed one of his classes, all of the workers knew of them and spoke fondly of having to go through the manager's quick math quizzes. His background also adds plausibility to the existence of an environment that supports mathematics: a) he took calculus in high school and still has the book; and b) he majored in a STEM field while in college, but dropped out when he started working part-time for the company that allowed this study to take place. His comments indicated that he still yearns to learn higher mathematics.

The workers relied heavily on three social phenomenon to mitigate their mathematical foibles. This means that the store, in its totality, including the physical structure of the store, the arrangement of the cashiers, and the policies affecting worker-customer interactions, was built through social arrangements, and that these social arrangements (though perhaps 'agreements' is a better word) gave rise to three phenomenon that played a pivotal role in augmenting or supplanting the workers' abilities to provide building materials and services to their customers. They were:

a. consistent product availability mitigated any damages that stemmed from a worker advising a customer to buy too little of a product; ‘just come back and get some more as it will certainly still be available’;
b. a liberal return policy with at least one dedicated cashier (and sometimes two) mitigated any damages that stemmed from a worker advising a customer to buy too much of a product; ‘just come back and return the item with no questions asked’; and
c. the general deference given the workers by Type II and Type III customers, meaning that customers blamed themselves for not following directions, measuring wrong, buying too little, buying too much, wasting too much material, or even wasting too little.

I conceptualized the last point as workers holding a position of authority, but did not begin to suspect the existence of this position until about halfway through the study. A set of incidents, worker comments, and customer comments began to paint a picture of behavioral self blame on the part of the customers. In one instance, a Type III customer told Bill that it was the third time that week that he was returning to the store to get attic insulation for his home. He also indicated that it was going to take a total of 50 bags, and not the 20 he had initially purchased. Bill simply said "Oh", and was beginning to help the customer put the bulky bags onto the cart when the customer shared that he was probably putting it on too thickly. Bill began discussing the situation and asked about the size of the house and what R-value he wanted. The man replied that it was 2200 square feet and that an employee who had helped him earlier in the week had told him to install the insulation nine inches thick to achieve R-30. Bill looked at the table on a bag of insulation (Figure 11) and replied, "Yeah, that's right." The customer looked over Bill's shoulder, and Bill showed him the part of the table. The customer, once again, stated that he was probably just installing it too thickly. The customer did not indicate any prior familiarity with the table.



Nature Blend™ Loose Fill Formula

Coverage Chart for Thermal Acoustical Application

R-Value @ 75°F	Initial Installed Thickness (Inches)	Minimum Settled Thickness (Inches)	No Joists (Sin ajuste por armazon) Bags Per 1000 Sq. Ft.	Net Coverage Sq. Ft. Per Bag	Minimum Weight Per Sq. Ft.	2" x 6", 16" O.C. Bags Per 1000 Sq. Ft.	Net Coverage Sq. Ft. Per Bag	Minimum Weight Per Sq. Ft.
R-Value @ 75°F	Espressor despues de asentamineto en separaciones y vacios	Aislamiento instalado no debe ser menos de	Cantidad minma de bolsas por 100 metros cuadrados	Maximo de Recubrimiento Neto	Pesos minimo por metros cuadrados	Cantidad minma de bolsas por 100 metros cuadrados	Maximo de Recubrimiento Neto	Pesos minimo por metros cuadrados
13	4.4	4.0	17.4	57.5	0.38	15.8	63.5	0.35
19	6.1	5.5	27.9	35.8	0.61	25.3	39.6	0.56
22	6.9	6.2	33.2	30.1	0.73	30.5	32.8	0.67
25	7.8	7.0	38.6	25.9	0.85	35.8	28.0	0.79
30	9.2	8.3	47.6	21.0	1.05	44.6	22.4	0.98
38	11.4	10.3	62.0	16.1	1.36	58.9	17.0	1.30
49	14.6	13.1	82.0	12.2	1.80	78.7	12.7	1.73
60	17.7	15.9	101.9	9.8	2.24	98.6	10.1	2.17

Figure 10. Table on package of insulation.

During contingent questioning, I asked Bill how many total bags the customer should have initially bought. Bill read the table and responded that he needed "at least 47 twice". When Bill was asked how he knew the man's attic did not have joists, he looked at the table again and changed his answer to "at least 100 bags." Bill explained the table indicated about 45 bags per 1000 square feet with joists present, but because the house was bigger than 2000 square feet, the customer should have gotten 10 extra bags. When asked why the man would have bought only 20 bags initially, Bill responded, "Probably, someone read the table wrong."

Since this customer did not seem familiar with the table printed on the bag, it is doubtful that he had initially read the table incorrectly. It seems more likely that the first employee, who had helped this customer, made a mistake in reading the table. However, this possibility did not seem to occur to the customer; rather, the customer blamed himself.

Minor tools, techniques and methods. On two occasions during the study, workers reported that they had had to teach themselves how to do something involving the cognates: a) Buck taught himself how to use the measuring tools embedded in the panel saw and how to account for kerfs when cutting; and b) Oliver, when he had worked in building materials prior to the study, had realized that certain items such as concrete bags and cinder blocks were not stacked in a standard fashion (Figure 10). This meant that multiplying base by width by height would not determine how many items were in a stack. Therefore, Oliver studied the various ways that the materials arrived and memorized the counts per layer of material.

Figure 11. Top and side views of irregularly stacked material.

Related to Oliver memorizing counts per layer of material, workers often either assumed or had memorized a set of quantities that allowed them to simplify their thinking or calculations. Examples include that Frank knew some of the U.S. customary liquid volume measurements, Bill knew that sheet goods are 4' x 8', Nancy used 100 square feet for all patio estimates, Cindy knew that most concrete patios were poured 4 inches thick, and all of the workers were aware that lumber was dimensionally sized, meaning that the size listed on the product tag was not the actual size of the lumber.

CHAPTER 5

DISSCUSSION

Overall, the results of this study suggest that the workers knew their jobs and were effective in assisting customers. At times, however, a person reading this study may get the opposite impression based solely upon the viewpoint of the research. For example, two of the research questions specifically addressed difficulties that workers faced with the cognates. The answers to 'difficulty' questions are inherently negative sounding. To answer these two questions, I had to emphasize that 19% of customer-worker interactions (22 out of 97) involved ineffective mathematical discourse, rather than that 81% of the customer-worker interactions involved effective mathematical discourse. The data certainly showed that some workers were more skilled at certain aspects of their jobs than others, yet all of them successfully filled the roles to which they were assigned on the day of shadowing. Throughout this study, I witnessed workers who knew their jobs, and based on the review of the literature (pp. 29-32), the cognate abilities of the shadowed workers were at least on par with what other researchers had found among their participants.

In this chapter, I make and support several claims based on the data from the study. Chief among these claims are that: a) the entire environment and sales process acted as a heuristic, b) mathematical discourse was a vitally important activity, and c) routine problems encountered during this study did not match other research findings. Additionally, this chapter contains a) an analysis of two hypotheses that were originally presented in the literature review, b) a discussion of the limitations of this study, c) recommendations for future research, and d) a summary of the study.

A Heuristic

It is possible to argue that the workers and store in this study should be viewed as a case of distributed cognition, not unlike the cockpit examined by Hutchins and Klausen (1996). They argued that the expertise found in the cockpit of an aircraft resided in the "knowledge and skills of the human actors" and in the "organization of the tools in the work environment" (p. 34). These two aspects, to some degree, were also found at the study site; however, they can also be found wherever humans use tools to accomplish a task. This is not to say that they would be found to the same degree as inside a cockpit. I am sure Hutchins and Klausen purposefully chose the word 'expertise' as opposed to a pedestrian word such as 'ability' because they saw expertise and not just ability. I did not see expertise at the study site; rather, I saw 'good enough'—the hint of a large scale heuristic, certainly on a store-wide basis, and perhaps even franchise-wide. I proffer that the system of human actors, tools, and social structures that I studied were an arrangement to give quick and satisfactory solutions, but not necessarily optimal solutions—the entire system was the heuristic. The system did not distribute cognition; rather, it arose to mitigate cognitive load for both the employees and customers. Nor was cognition distributed; instead, it was ignored. The system eliminated any need for high-level cognition in the form of precise estimates or accurate price comparisons, and instead generated answers that were good enough for the customers involved.

Mathematics in the Workplace

I summarized the reviewed literature in Chapter 3 into three principal findings and a theory relevant to this study. The principal findings were that: a) mathematics at work is different than mathematics at school; b) mathematics used at work is mostly routine, and tricks, heuristics, stand-ins, or other tools are often used; and c) adults in a workplace setting solve problems using vastly more effective techniques, as compared to adults in a formal test setting. The theory concerned the developmental flow of heuristics and stand-ins.

Work versus school. Mathematics at this worksite was most certainly different than mathematics at school, as described in the answer to research question 1(b). Some findings of this study mirror those of Harris (1991), such as that data are noisy, accuracy is defined by the situation, work is collaborative, correctness is negotiable, and language is imprecise (Table 1). However, Harris' study observed and examined workplace mathematics from a close-in viewpoint. This led to the small grain size descriptions just listed (e.g., data are noisy). The viewpoint used by Harris, while effective in listing the characteristics of workplace mathematics, did not acknowledge that these characteristics are really descriptors of atomized mathematical discourse. That is, by zooming out, it is apparent that each of the above characteristics is actually a component of mathematical discourse, and that effective mathematical discourse is the key to, for example, dealing with noise, defining accuracy, and negotiating correctness. None of the research reviewed for this study, including Harris', emphasized the significant difference in mathematical discourse between work and school.

Other researchers of adults found significance in mathematical discourse, but called it something else and did not link it back to school. Pozzi, Noss, and Hoyles (1998) found that nurses had to negotiate the meaning of numbers in various forms with other health practitioners to avoid medical errors (e.g., determining proper drug doses and interpreting fluid pump output volumes). The authors described, in detail, two nurses discussing the charting and meaning of numbers during a fluid balance monitoring breakdown episode (p. 111). They never used the words 'mathematical discourse' in their analysis of the event, but I will—these two medical professionals discussing the methods of recording numbers and their meanings were engaged in mathematical discourse. Further, Ancker and Kaufman (2007, p. 714), while studying health numeracy, wrote that even people with advanced mathematical skills were susceptible to poorly managing their conditions if numerical information had been explained poorly. On the other hand, they claimed that excellent communications could offset the weak mathematical skills of individuals. Once again, these researchers did not use the term 'mathematical discourse', but instead wrote: "Both the patient and the information provider must be able to manipulate and interpret quantitative information, as well as communicate about it" (p. 714). 'Being able to manipulate and interpret quantitative information and communicate about it' could pass as a definition of mathematical discourse.

The pedagogical results of ignoring mathematical discourse may be profound. Imagine a student who is highly skilled at performing calculations and extracting numerical information from the environment (e.g., charts, spreadsheets, lab experiments), but is unwilling or unable to extract information directly from another person, in much the same way that Nancy avoided mathematical discourse by failing to ask customers about their yard size or the type of weed they wanted to kill.

Discourse, per se, was not the focus of this study; however, the data from the study supports the significant role mathematical discourse plays in effective customer interactions. This is illustrated by the different levels of effectiveness I found when observing workers and customers interacting: Nancy, who tended to avoid certain forms of mathematical discourse with customers; Cindy, who knew the customer's answer was wrong but was not able to convince him to reconsider, though she tried; and Frank, who ascertained the needs of two customers, educated them, and provided helpful decision-making comparisons.

Given the significant role that mathematical discourse plays in the workplace, its development in K-12 classrooms should be reevaluated. Several minor changes may be sufficient to enhance the mathematical discourse skills of students: a) asking students questions without regard to their hands being raised, b) writing questions that have more than just the pertinent data (i.e., introducing noise), and c) writing questions that do not have all of the pertinent information. In regards to this third suggestion, the students would have to either: a) ask another person, b) search the web or other sources for written data, or c) take measurements from actual items, drawings (plans), or models.

Another difference between mathematics at work and school was the framework in which the person solving the problem viewed the problem. For example, ratio problems (defined and discussed in the literature review in the section *What is Known*) occurred 28 times during the shadowing, and were solved by guess & check or were avoided. Researchers (e.g., Kieren, 1976) have recorded several methods by which students solve ratio problems, but neither guess & check or avoidance were methods featured in the reviewed literature. In any particular case, if the student solved the problem using an established 'taught-at-school' method, then it was considered a successfully solved problem. If the student failed to solve the problem or used an unacceptable method, then the attempt was typically labeled as a construction or experience failure. Moreover, Kieren (1976) claimed that understanding rational numbers required knowing the meaning of the subconstructs and how they interrelated. The menagerie of concepts and research just listed all view ratio problems as initially selecting the correct abstract notation or form, and then working towards a quantitative answer. This means that researchers have typically conceived of 'solution' as consisting of several ordered steps, including: 1) understanding the problem, 2) selecting a correct form, and 3) calculating a numeric answer. By contrast, when faced with a ratio problem, workers immediately, informally, and entirely within their minds, estimated an answer. This was often done without fully understanding the problem's parameters. Moreover, the solution was not a process undertaken to arrive at some quantity—that had already been estimated—but rather was a process of verification by guess & check. This paradigm also applied to measurement problems. Often, workers would say something like "That's $^3/_4$ inch" as they took out their tape measure for verification, meaning that they had already estimated an answer based on experience, and were using the tool to verify their estimate. This is the opposite of school, where students are taught to first find an answer and then ask if the answer makes sense.

Using this sort of reverse technique (estimation and then verification) on cognate problems, workers were often able to provide answers in the form of loose estimates to customers. Exceptions included problems that involved chemicals, and comparison shopping problems that did not have readily available information on the product tag.

Identifying routine problems. Workers encountered 94 problems categorized as routine and 12 problems categorized as real. The expected tools, such as calculators, charts, tables, and product tags, were used as described in the answer to research question

2(b). However, several unexpected issues and techniques were noted that differed from previous research.

I found it difficult at times to distinguish between real and routine problems as described by Smith (2002). For example, when a customer asked Oliver how many pavers he would need for his patio, it was difficult to categorize the problem because determining the number of pavers needed, on its face, seemed to be a routine problem, both by definition and given that the store is in the business of providing such things. Routine problems are supposed to be recognizable by their set of protocols, tools, stand-ins, or other techniques available to solve them rapidly and efficiently (e.g., Millroy, 1992; de la Rocha, 1981; Carraher & Schliemann, 2002a). Oliver used nothing from this set; rather, he reverted to a primitive method to solve it. Note that I do not consider laying out multiple pavers and grossly measuring an irregular shape to loosely estimate the number of pavers needed as a rapid and effective technique or protocol. This situation was also not readily classified as a real problem, because it was not unique; on the contrary, it had, no doubt, been faced many times before for varying styles of pavers and many other similar products.

Other situations involving real and routine problems also caused consternation. For example, prior to the study, I expected to categorize comparison shopping questions posed by customers as routine. Shadowing found that they were moderately frequent (n=14), meaning that, on average, customers asked a worker one comparison shopping question every two hours, or four comparison shopping questions during every eight-hour shift. Based on prior research (e.g., Millroy, 1992), I considered this common enough to evoke routine paradigms being developed; however, workers did not develop them.

Workers gave quantitative answers to 3 of the 14 comparison shopping questions asked of them. The three situations that evoked quantitative answers all had product tags that lent themselves to easy comparison shopping (e.g., carpeting with the price per square yard clearly indicated). In these three events, it may be argued that workers used the tags as heuristics. This is true, but the workers did not develop them, and the other 11 events (79% of them) are still left unexplained.

The results from the interview questions supported the lack of development of routine techniques for comparison shopping situations. Workers calculated correct solutions 45% of the time (10 out of 22 items) on assessment items that asked for comparison shopping solutions, and none of the workers engaged an item in a manner that evoked a sense of 'routine' problem solving as described in previous research (e.g., Smith, 2002). It is possible that workers avoided numerically answering comparison shopping questions because those sorts of questions cannot be effectively answered by guess & check, and its attendant loose estimation. However true this may be, it does not explain why a solution method did not arise, given the frequency of comparison shopping questions.

Solving routine problems. Workers in this study often solved routine problems that varied in at least one of four ways from the existing research. First, the existing research has identified a set of stand-alone solution techniques (e.g., heuristics, stand-ins, tricks) that adults commonly use in specific workplace settings to solve routine problems (e.g., Millroy, 1992; de la Rocha, 1981; Carraher & Schliemann, 2002a). Stand-alone means that the technique is specific to the problem type at hand, and has limited use in other situations that are not nearly identical. Second, past research (e.g., Hoyles, et al., 2001; Kaushal, 2001; Smith 2002) found that adults solved problems at work using more effective techniques than adults in a formal test setting. Third, the goal when solving routine problems was centered on correctness and accuracy. For example, Smith (2002, p. 121) wrote, concerning automotive workers: "First, when workers needed to compute, the most important feature of those computations was their accuracy. The total elimination of error was the overall goal." Fourth, workers had economic reasons for getting the problem right. Smith (2002) wrote, "...in all cases, computational errors mattered. Mistakes had human and economic consequences beyond the simple issue of getting a correct or incorrect answer" (p. 121). In summation, a set of techniques exist to solve routine problems at work that are more effective, more accurate, and avoid more negative economic consequences than techniques used in formal testing environments. I found little of this to be true at my study site.

A set of stand-alone solution techniques did not exist at the study site; rather, a three-part solution system existed. The first part of the solution system provided a loose-estimate answer via a worker using a calculator in an intensive guess & check process. Inherently, accuracy and "total elimination of error" were not goals. Interestingly, the first part of the solution system was not influenced by the situation. This means that workers used the same process for routine problems, real problems, and assessment items; moreover, the effectiveness of the first part was approximately the same for all three of these problem situations. The second part of the solution system was used whenever a worker failed to find an acceptable loose estimate via guess & check. In these cases, the workers switched to qualitatively describing quantity, such as when describing the quantities of water and mortar to mix together as "until it feels right." The third part of the solution system mitigated any negative human and economic consequences that arose due to the inherent inaccuracies (looseness) of the first two parts of the solution system. It had three components: a) management supported the loose-estimate technique so there was no risk of losing one's job; b) certain customers gave a deference to the workers and blamed themselves for material or process errors; and c) shortages or overages in materials because of loose estimates were not seen as problems due to consistent product availability and an easy return policy.

After the collection and analysis of data, I interviewed the manager of the store to get details of merchandise return policies. I wanted to verify that store employees did not face negative consequences for overselling product (i.e., too much quantity) that then led to merchandise returns. The unequivocal answer from the manager was, "No." I then asked the manager if he could "imagine a situation" when a worker would get into trouble for selling too much of a product. The manager replied that an employee could be "written up" if he or she wrote a customer order promising material on a certain date that the employee knew was impossible to fill. However, the manager had never actually written up an employee for such an infraction. The manager further explained that net sales were tracked by departments but that there was no record of individual sales except for the commercial sales department. At the store level, the manager explained that net sales were tracked and discussed at "district meetings" (meetings of several store managers with a district manager), but since all stores had to accept all returns, regardless of where the original sale took place, net sales were not the emphasized metric. Moreover, he explained that any returns done without a receipt, which accounted for slightly over 40% of all returns, could not be tracked back to the original store. Put together, this meant that a store could directly influence gross sales but had little control over returns. In summary, it was apparent that gross sales were the primary metric upon which success was measured.

Overall, this solution system cannot be the most effective system for building things. However, this solution system appeared to be an effective method for making a sale and moving on to the next customer without any attachment of responsibility for the answer. Oliver's solution to the paver problem provided an answer, but was it a correct answer? Perhaps equally important to ask: correct for whom—Oliver, the customer, the store, or society?

Theory of heuristic and stand-in development. I offered a theory in the summary of the literature review that suggested the existence of two possible flows for the construction of heuristics: one for developers and the other for daily users:

1) The flow for workers who develop heuristics and stand-ins may look like this:

School mathematics → Workplace/Everyday mathematics → Heuristics/Stand-ins

2) The flow for the daily users of heuristics and stand-ins may look like this:

School mathematics → Heuristics/Stand-ins → Workplace/Everyday tasks

Observations during shadowing support the first flow and contraindicate the second flow. Three examples illustrate the first flow, in that:

- Oliver experienced irregularly stacked cinder blocks (Figure 10), so he developed a personal heuristic. He wrote down and eventually memorized the number of blocks per layer, and he repeated this process for other similarly stacked materials.

- Cindy had difficulty calculating concrete amounts using tables or 4-function calculators, so she sought out and purchased a construction calculator that hides all of the mathematics, with the exception of inputting measurements.
- Buck experienced cutting material that was coming out too short. He developed the rule-of-thumb to allow an extra $^{1}/_{8}{}^{th}$ inch for the kerf, or width of the blade.

In each of the above cases, all of the workers experienced the issue before they developed or were taught the heuristic or stand-in. Contraindicating the second flow are the tables, charts, product tags, and labels found throughout the store. These heuristics and stand-ins were developed by others and were available to the workers before they ever experienced a need for them. This meant that the workers did not participate in their development and did not fully understand them. This was supported by the assessment questions that involved tables, and by shadowing observations, such as: a) avoidance of chemical labels, b) Caleb's misunderstanding of product tags and paneling widths, and c) Oliver's understanding of product tags and pavers per square foot. These examples demonstrate that heuristics and stand-ins 'given' without meaning to workers tend to be ignored. These results suggest that the second flow (without modification to include training) was not a viable model at this store, and that usable heuristics and stand-ins were sometimes developed on an as needed basis by individual workers.

Shades of Correctness

Results of this study align closely with results from the study by Capon and Kuhn (1982) in two ways. First, Capon and Kuhn (1982) found that 19% of their comparison shoppers did not use mathematical data that was available; rather, they used extraneous cues even though "task instructions directed the subject's attention explicitly to the criterion of 'better buy' and suggested disregarding extraneous factors" (p. 450). Although I used similar instructions, 44% (four out of nine) of the workers in the present study used extraneous cues, at least once, to answer comparison items. One item was particularly notorious: three out of five workers (60%) used extraneous cues and ignored the numerical data to answer the first item from the concrete question. In each case, the worker was reminded that the situation was only concerned with which concrete bag represented a better buy based on price alone. None of the workers changed or modified their responses.

A second alignment was in the percentage of adults who were able to determine the best buy by using some form of proportional reasoning; lucky guesses or non-cogent arguments did not count as correct. Capon and Kuhn found that approximately 40% of their shoppers did not determine the best buy; whereas, in the present study, 33% (three out of nine) did not correctly solve any of the price comparison items included in the questions. Three other workers solved one of the price comparison items, but did not pass a second competency testing item. This result is significant because Inhelder and Piaget (1958) claimed that the ability to reason proportionally appears during the formal operations stage of development, and that this stage is not obtained by all people; however, they did not determine if 'needing to know' could somehow affect development. The work of Capon and Kuhn (1982) began to address the need-to-know issue when they studied shoppers who ostensibly would benefit from knowing how to calculate a best buy at the grocery store, but perhaps buying garlic was just not that important. Medical researchers (e.g., Noss, 2002) found results similar to Capon and Kuhn, and a person's health is important, but perhaps the need to reason using the cognates to make a medical decision was too infrequent to affect one's development. In their study of nurses in the workplace, Hoyles, Noss, and Pozzi (2001) included all three components: importance, frequent need, and use over an extended period of time. This makes the nurses study similar to the current study; however, the participants in the nurses study differed substantially from the workers in the current study. In particular, the nurses had received formal and apprenticeship-like training in the use of cognate based methods, and which was focused on the task of correctly calculating medications, followed by extensive written and hands-on assessments. This means that those who did not learn to use the cognates would not have become nurses and would not have been included in the Hoyles, Noss, and Pozzi study. None of this was true of the workers in the current study.

If individuals at my research site were going to universally and spontaneously develop proportional reasoning skills due to important and frequent need over time, then there should have been some evidence of it; however, none was found. Rather, I witnessed a few workers who used the cognates, a few workers who used a variety of work-around methods that alleviated their need to use the cognates, a few workers who avoided using the cognates all together; and a social environment that had arisen to support all three types. I argue that my study, with frequency, importance, and a longitudinal aspect, along with many other studies, suggests that a substantial number of adults cannot reason using the cognates, even if their jobs, health, and daily actions would benefit from being able to reason with them. The big question: Why can't they?

It could be argued that the workers in the present study did not develop skilled cognate use because the environment suppressed it, and even supported a different sort of solution system not based on the cognates, as, for example, quick math. The research site did have three necessary features for a solution system based on quick math: a) an unofficial proscription on taking too much time or care in calculating an answer, as seen by the manager's 'cardboard talks' and frequent quizzes emphasizing quick calculations; b) an easily learned arithmetic replacement system in the form of guess & check with a socially accepted resulting loose estimate; and c) a waiver of responsibility through an arrangement of readily available product and easy returns. However, this argument is temporally backwards. The store did not develop a liberal return policy to replace skilled mathematical thinking. On the contrary, the store developed the policy in response to 'good enough' mathematical thinking.

This study extended the work of others (e.g., Capon & Kuhn, 1982) who examined proportional reasoning in adults as it applied to comparison shopping. The extension was asking for the value of leftover material. That is, construction activities generate waste, so material is left over or wasted. Two of the assessment questions (see the grass seed and drywall questions in Appendix B) asked for the value of the material leftover or wasted after the initial best buy was determined. This type of item was asked ten times. No worker was able to successfully calculate the value of the leftover material. I conjecture that this situation became a real problem because there were too many quantities to successfully use guess & check. That is, the number of numbers available to 'guessingly' input into the calculator followed by 'guessingly' selecting a basic operation was simply too great to manage using guess & check. Additionally, the workers' well developed sense of 'materials-needed' estimation did not extend to 'materials-leftover' estimation.

A Conjecture About a Possible Problem

During the proposal phase of this study, I conjectured several possible scenarios regarding the mathematics knowledge of employees who would be in the study, and possible responses by the business. The conjectures tended to be mutually exclusive. For example, regarding new employees' mathematical knowledge, the conjecture stated that either 'There is no problem' or 'There is a problem.' The results from the study have suggested that both were true to some extent. The original conjectures and findings based on this study are shown in Table 16. Note that the original conjectures referred to all employees, whereas the findings only applied to the workers in the study.

The manager allowed me, during the course of the study, to take three employment tests. The first test was typically given to entry level employees. The next two were normally given to experienced tradesmen seeking employment. This testing protocol indicated that the corporate hiring process categorized job applicants into two pools. The first pool consisted of applicants who had no prior experience in the field. The screening of these applicants focused on how they would respond to particular job situations, such as witnessing another employee stealing while on the job. Though I did not find any mathematical testing at this level, it may have existed for particular entry-level positions (such as cashier). The second pool was for applicants who had prior experience in the building trades. I found, in addition to the entry-level questions just described, that these applicants had to answer three mathematical questions. These questions focused on situations of the sort that I classified as routine during the study, such as finding the perimeter of a fence, calculating the square footage of a small four-sided building, and determining a number of stacked boxes.

Table 16
Pre-study Conjectures Versus Study Findings

Conjecture	Findings
There is no problem because	There is no problem because
the vast majority of employees arrive prepared to handle the mathematics found in this business, or	the workers I studied arrived at the store with at least the mathematics necessary to start as a lot attendant or equivalent; even the current head manager started as a lot attendant, and
the company tests before hiring, and any applicant who does not pass a mathematics test is not hired.	if one already has experience, then online tests containing routine problems are given as part of the application process.
There is a problem and	There is a problem and
OJT allows an employee to succeed one department at a time, or	some informal OJT takes place on a daily basis without regard to length of employment; however, most of the information and knowledge transfer did not involve the cognates or any form of mathematics, and
formal training with single event application is required before an employee is allowed to work in a department, or	formal online training with multiple event applications was required before an employee could be promoted; however, the training did not involve the cognates or even mathematics generally, and
general mathematics classes are offered or required.	the head manager conducted 'cardboard talks' and formal general mathematics classes for his assistant managers and department heads, and
no viable solution exists; errors are common.	the business is situated in a set of social phenomenon that mitigate the effects of common errors.

Limitations of the Study

Results of this study were limited by several factors.

- I am biased, as are all researchers, but I brought specific biases to this study due to my background in the construction industry. These biases may have caused me to be more critical than others may have been in regard to the mathematics and techniques that the workers were using.
- I was the only researcher during the study; hence, there was no way to check my notes or coding against those of an independent observer.
- The scope of the study did not allow any meaningful demographic information or previous educational experiences to be analyzed.
- Available time was a limitation for this study. Three hours of shadowing did not seem to produce a sufficient number of real problems for analysis.
- The number of employees studied was a limitation. A different nine employees may have given different results.
- The employees selected for the study were probably not representative of all the employees in the store since certain groups were specifically not invited to join the study.
- The size of the study was a limitation, meaning that it only included a single site controlled and influenced by a single manager; hence, the data collected during this study should not be used to infer that other similar sites use quick math, loose estimation, or guess & check.
- The results of this study cannot be generalized. Any attempt to apply these findings to a broader scope is folly.

Recommendations for Future Research

Perhaps all research raises more questions than it answers, and that was certainly the case for this study. Based on the results of the current study, three salient categories of future research exist: a) acquiring additional information from the current site; b) replicating the study at a similar site; and c) adding to the set of questions. Note that, in all categories, a team of researchers should be employed to validate: a) the assignment of codes, b) note taking, and c) researchers' interpretations of events.

I have developed a level of trust with the workers at the current site, so it may be possible to conduct further research at the site. For example, the assessment questions were not nuanced enough to determine the location of workers on the Piagetian spectrum of stages. The questions determined that certain workers did not use the cognates to solve workplace problems, and a little information about how they struggled with some mathematical situations. Ideally, I would have the same workers complete a set of problems similar to the original Noelting juice problems. This would be a relatively easy study and would help determine if certain workers had not reached the formal operations stage or if something else had prevented them from solving the original assessment questions.

Another possibility is to focus on the same workers and their knowledge of units. I am intrigued as to why certain workers lacked what seems to be fundamental knowledge of units and how to apply them. Why did they not develop a working knowledge of them? Did workers who have had chemistry, physics, or other classes where units are emphasized understand units better? Cory had taken a physics course within the last five years at a community college, and had also taken chemistry in high school many years prior. Did these courses change how Cory approached problem solving? Had other workers also taken these sorts of classes, but not changed how they approached problem solving?

Closely replicating this study at a different site would help to determine if the manager's focus on quick math had moved the workers away from thoughtful and relatively slow reasoning (using the cognates) towards calculator intensive loose estimation. From a pragmatic standpoint, is the way in which different managers balance the need for quickness of calculation with the need for accuracy of calculation related to the volume of store returns? Knowing this could produce a huge economic benefit, given that I noted at times three registers selling and two registers processing returns. I also noted during shadowing that restocking was inefficient and labor intensive.

This study also generated a set of questions well beyond itself. Foremost is the issue of inverse proportionality which arose during the pilot study. One of the participants during the pilot study used the provided table on the manure question, but failed to calculate a correct answer. I asked extensive contingent questioning, but at that moment, I was not able to figure out what had gone wrong; his reasoning and explanations seemed solid. After he had left, and after an extensive review of my notes and whiteboard tinkering, I determined that he had framed the question as an inverse proportionality question, and that trying to interpolate the table in such a manner had been the culprit. This study and others have provided a fairly robust estimate of the number of adults who do not or cannot use proportional reasoning; however, of the ones who demonstrate proportional reasoning, how many can recognize and solve problems that involve inverse proportionality, and why?

Summary of the Study

I focused the research on four questions: a) in what ways do workers encounter and utilize the cognates while on the job; b) do workers engage cognate problems they encounter at work differently than similar cognate problems found in a textbook; c) what mathematical difficulties involving the cognates do workers experience while on the job; and d) what tools, techniques, and social supports do workers use to augment or supplant their own abilities when confronted with difficulties involving the cognates? To answer these questions, I gathered data via shadowing with contingent questioning and clinical interviews driven by a written assessment, followed up with more contingent questioning.

Results suggest several unremarkable findings, such as that workers did not set up equations, used unknowns (variables), or solved problems in a formal manner on paper (i.e., no neat columns with arrows and explanations) while at work. An unexpected result was that workers in this study, on average, displayed an ability to reason proportionally, approximately the same as participants in earlier studies with widely differing populations. The finding was unexpected because these workers faced authentic problems on a frequent basis, with consequences (albeit not extreme), and a paper and pencil assessment containing additional authentic problems; whereas, many other studies lacked at least one of these facets: authentic, frequent, consequences, or written.

Results also suggest four noteworthy findings.

- Workers encountered mathematical problems primarily through discourse with others and not through written or electronic means. Furthermore, contrary to accepted lore, asking and telling seemed as important as listening.
- Workers infrequently used units to help solve a problem, even when confronted with intensive ratios. Contingent questioning revealed that this was not an instance of minor sloth; rather, with one exception, none of the workers understood the role units could play during calculation.
- Workers often solved problems using a calculator and a method of guess & check to produce a loose estimate.
- Workers relied on the social structure of the store to mitigate the impact and defuse the responsibility for any errors they made.

Based on the totality of the evidence, I proffered three hypotheses for further examination. First, I rejected a binomial conjecture that stated employees were hired either with sufficient mathematical skills or with deficient skills. I proffered that both were true. Second, I proffered that heuristics, tables, stand-ins, etc., were maximally effective only if workers individually developed them after a need was recognized. Third, I argued that the store was not best described using a framework of distributed cognition, but instead proffered that the studied workers and their environment formed a system that was itself a heuristic on a grand scale, in fact being a heuristic on a scale that possibly encompassed the entire franchise.

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APPENDIX A

IRB APPROVAL

ASU Knowledge Enterprise Development

EXEMPTION GRANTED

James Middleton
EMTE: Engineering of Matter, Transport and Energy, School for
480/965-9644
JAMES.MIDDLETON@asu.edu

Dear James Middleton:

On 12/3/2014 the ASU IRB reviewed the following protocol:

Type of Review:	Initial Study
Title:	The Use of Proportional Reasoning and Rational Number Concepts by Adults in the Workplace
Investigator:	James Middleton
IRB ID:	STUDY00001910
Funding:	None
Grant Title:	None
Grant ID:	None
Documents Reviewed:	• Consent for Proportional Reasoning by Adults.pdf, Category: Consent Form; • HRP 20503a Proportional Reasoning in Adults.docx, Category: IRB Protocol; • Proportional Reasoning by Adults Recruitment.pdf, Category: Recruitment Materials;

The IRB determined that the protocol is considered exempt pursuant to Federal Regulations 45CFR46 (2) Tests, surveys, interviews, or observation on 12/3/2014.

In conducting this protocol you are required to follow the requirements listed in the INVESTIGATOR MANUAL (HRP-103).

Sincerely,

IRB Administrator

cc: Darryl Orletsky
Darryl Orletsky
James Middleton
Carole Greenes
Eugene Judson

APPENDIX B

COMPLETE SET OF ASSESSMENT QUESTIONS

Assessment Problems

Question identifier	Best price	Missing value	Table based	Volume present	Value of waste
Drywall	X				X
Joint Compound	X	X		X	
Manure Topper		X	X	X	
Grass Seed	X	X			
Concrete	X		X	X	X

Building Materials—Drywall Problem

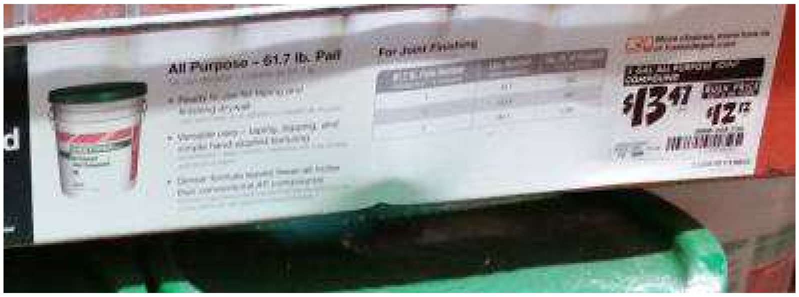
The price for ultralight drywall is shown in the above pictures.

1. If a customer needs 100 square feet of this drywall and was only worried about the price of the drywall, then what would you recommend?

2. If your recommendation produces any leftover material, then what was the dollar cost of the leftover material?

3. Which is larger? $\frac{23}{11}\times 7$ or $\frac{19}{12}\times 9$

Building Materials—Joint Compound Problem



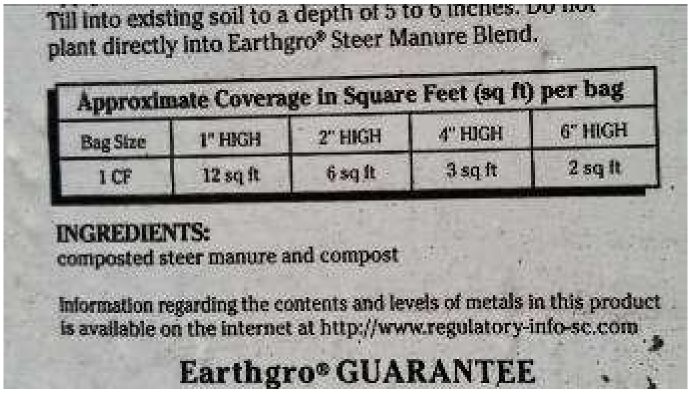
The price for joint compound is shown in the above pictures. The 5-gallon pail sells for \$13.47 and weighs 61.7 pounds. The 3.5 gallon carton (cube) sells for \$8.47. Its label does not state its weight.

1. Assume that the joint compound mixtures of the two brands are chemically identical and that the container weights are negligible. What does the 3.5 gallon carton (cube) weigh?

2. Based only on price, which is the better buy for non-bulk quantities?

3. Find the value of x given $\dfrac{7}{13} = \dfrac{x}{22}$

Nursery—Manure Topping Problem



This is part of the label from a bag of Earthgro manure which costs \$1.09 for 1 cubic foot.

1. A customer needs enough material to cover 100 square feet to a depth of 3" high. How many bags of Earthgro manure will you recommend the customer buy?

2. How high would one bag cover an 8 square foot area?

3. Find the value of x given $\frac{9}{13} = \frac{11}{x}$

Nursery—Grass Seed Problem

The label on the 50 pound bag of annual rye grass states that it will cover up to 10,000 square feet, and it sells for $39.98.

↓

The label on the 10 pound bag of annual rye grass states that it will cover up to 2,000 square feet, and it sells for $17.98.

1. A customer is only concerned about initial cost and needs to seed 4,750 square feet. What would you recommend to the customer?

2. If your recommendation produces any leftover material, then what is the dollar value of the leftover material?

3. Which is larger? $\frac{14}{11}\times 7$ or $\frac{19}{12}\times 9$

Commercial—Concrete Problem

Use the non-bulk price for concrete mix as shown in the above pictures. The 60 pound bag sells for \$2.80. The 80 pound bag sells for \$3.45. Assume that the concrete mixture in both bags is identical. If you need them, mixing tables from bags of concrete are on the attached page.

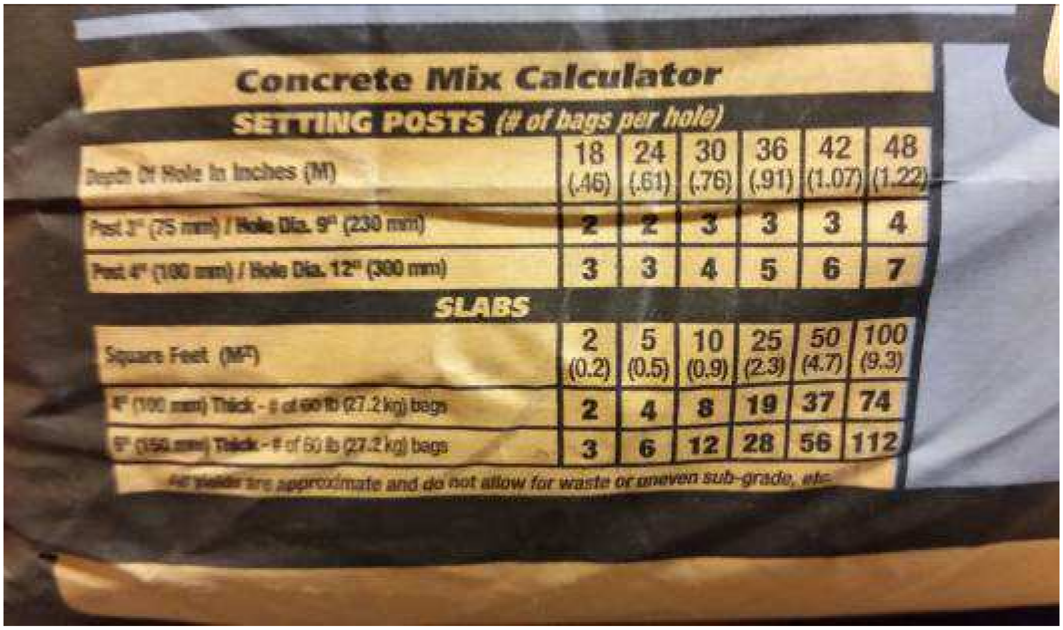
1. A customer is only worried about the final price. Which bag would you recommend to the customer?

2. Given the following table (next page) of 'bags needed' information, how many bags would you recommend that the customer buy to build a 107 square foot slab 5" thick? Calculate your answer in both 80 pound and 60 pound bags.

3. Thinking about your answer from #2, do you still agree with your recommendation from answer #1?

4. Which is larger? $\frac{14.5}{11.3}$ or $\frac{17}{12}$

Concrete Problem Continued



Concrete Mix Calculator

SETTING POSTS (# of bags per hole)

Depth Of Hole In Inches (M)	18 (.46)	24 (.61)	30 (.76)	36 (.91)	42 (1.07)	48 (1.22)
Post 3" (75 mm) / Hole Dia. 9" (230 mm)	2	2	3	3	3	4
Post 4" (100 mm) / Hole Dia. 12" (300 mm)	3	3	4	5	6	7

SLABS

Square Feet (M²)	2 (0.2)	5 (0.5)	10 (0.9)	25 (2.3)	50 (4.7)	100 (9.3)
4" (100 mm) Thick - # of 60 lb (27.2 kg) bags	2	4	8	19	37	74
6" (150 mm) Thick - # of 60 lb (27.2 kg) bags	3	6	12	28	56	112

approximate and do not allow for waste or uneven sub-grade, etc.

Number of Bags Needed

Número de bolsas necesarias

Thickness Grosor	Area Área			
	4 Sq. Ft. 4 pies²	9 Sq. Ft. 9 pies²	16 Sq. Ft. 16 pies²	25 Sq. Ft. 25 pies²
4"	(3) 80 lb. bags 3 bolsas de 80 lb	(5) 80 lb. bags 5 bolsas de 80 lb	(9) 80 lb. bags 9 bolsas de 80 lb	(14) 80 lb. bags 14 bolsas de 80 lb
4"	(3) 60 lb. bags 3 bolsas de 60 lb	(7) 60 lb. bags 7 bolsas de 60 lb	(12) 60 lb. bags 12 bolsas de 60 lb	(19) 60 lb. bags 19 bolsas de 60 lb

All yields are approximate and do not allow for waste or uneven sub-grade, etc.

APPENDIX C

CLINICAL INTERVIEW NOTES AND TALK ALOUD PRACTICE

1. Welcome
    a. Reminder of privacy
    b. Not judging, just gathering data, but
    c. Do the best you can and be honest so the data will be useful.
    d. bathroom or other interruptions
2. Schedule
    a. Follow-up from shadowing
    b. Orient to the room
        1. Worker's seat and table
        2. Researcher's seat
        3. Audio device
        4. Tools (e.g., calculator and tape measure)
    c. Practice talk-aloud problem(s) (see below)
    d. Presentation of the questions.
    e. Report everything you can remember about your thoughts during the last problem.
    f. Contingent and competency questioning.

Talk Aloud Instructions

Detailed initial instructions are required for successful implementation of the talk aloud method.

- Distinguish between talk aloud, explanation and thinking aloud for the participant. See (Boren & Ramey, 2000) if the distinctions are not clear.
- Request that the participant speak constantly as if alone in the room without regard for coherency.
- Inform the participant that reminders will be given every 20 seconds if silent, but that no apology from the participant is wanted and no interruption is intended.
- Reminders should not encourage a sense of personal contact, so do not use the worker's name. A successful reminder should not cause a pause for reflection or retrospection by the participant; hence "Please remember to think aloud" is all that should be said.
- All other interactions during the assessment should be avoided.

Assessment 'Talk Aloud' Practice Questions

37	46	19
x2	x3	x16

Convert 7/5 to a decimal.
Convert 13/7 to a decimal.
Using long division, divide 723 by 4 without a calculator.
Add similar if more practice is needed.