Testing the Mediated Effect in the Pretest-Posttest Control Group Design

by

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ABSTRACT

Methods to test hypotheses of mediated effects in the pretest-posttest control group design are understudied in the behavioral sciences (MacKinnon, 2008). Because many studies aim to answer questions about mediating processes in the pretest-posttest control group design, there is a need to determine which model is most appropriate to test hypotheses about mediating processes and what happens to estimates of the mediated effect when model assumptions are violated in this design. The goal of this project was to outline estimator characteristics of four longitudinal mediation models and the cross-sectional mediation model. Models were compared on type 1 error rates, statistical power, accuracy of confidence interval coverage, and bias of parameter estimates. Four traditional longitudinal models and the cross-sectional model were assessed. The four longitudinal models were analysis of covariance (ANCOVA) using pretest scores as a covariate, path analysis, difference scores, and residualized change scores. A Monte Carlo simulation study was conducted to evaluate the different models across a wide range of sample sizes and effect sizes. All models performed well in terms of type 1 error rates and the ANCOVA and path analysis models performed best in terms of bias and empirical power. The difference score, residualized change score, and cross-sectional models all performed well given certain conditions held about the pretest measures. These conditions and future directions are discussed.

DEDICATION

This document is dedicated to my family and my girlfriend for their continued support.

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I thank my committee chair, David MacKinnon, and the rest of my committee for their help in the creation and execution of this project. Lastly, I thank my friends, family, and girlfriend for their support.

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Introduction

Many research designs consist of two-waves of measurement and aim to test mediation hypotheses. A PsycINFO search of the terms "two-wave or pretest posttest or two time points" and "mediation or mediating or mediator or process variable" during the span 2000 – 2014 resulted in 485 peer-reviewed articles. There are four commonly used models to test mediated effects in the pretest-posttest control group design: Analysis of covariance (ANCOVA), Path analysis, difference score, and residualized change score. Additionally, it is possible to estimate the mediated effect using a cross-sectional model that ignores the pretest information on both the mediator and the outcome variable. Given the wide use of these models in mediation analysis, it is surprising that few studies have evaluated under what conditions these models have don't type 1 error rates above the nominal 0.05 alpha level, produce unbiased estimates of the mediated effect, have confidence interval coverage that is close to 95%, and have high empirical power to detect the mediated effect. This project aims to compare tests of the mediated effect in the pretest-posttest control group design using these four common traditional longitudinal models and the cross-sectional model.

Statistical Mediation

Most research focuses on assessing relations between two variables, with the research question of whether or not there is a *total effect* of an independent variable X on a dependent variable Y (Sobel, 1990). Additional variables can be included to further investigate how or why there is a relation between the two variables. When variables are added to bivariate relations, these additional variables can result in a variety of third

variable effects including confounding, moderating, or mediating effects. Of particular interest in this project are third variables that are defined as mediating variables. A mediating variable is a variable that is both a dependent variable and an independent variable and is intermediate in a causal sequence between two variables (Lazarsfeld, 1955; MacKinnon, 2008; Sobel 1990). Inclusion of a mediating variable in a theoretical model and statistical analyses allows researchers to test *indirect effects* of an independent variable on a dependent variable through the independent variable's effect on the mediating variable (Lazarsfeld, 1955; MacKinnon, 2008; Sobel 1990).

Statistical mediation is important because it allows researchers to investigate *how* two variables are related. Once researchers know two variables are related (e.g., X causes Y) it may be of theoretical interest to investigate through what mechanism X and Y are related. Statistical mediation is a tool by which causal mechanisms can be investigated given assumptions (MacKinnon, 2008; VanderWeele & Vansteelandt, 2009). Statistical mediation is typically conceptualized using a series of three linear regression equations (MacKinnon, 2008). Equation 1 represents the total effect of X on Y (*c* coefficient), Equation 2 represents the effect of X on M (*a* coefficient), and Equation 3 represents the effect of X on Y adjusted for M (*c'* coefficient) and the effect of M on Y adjusted for X (*b* coefficient). Computing the product of *a* and *b* coefficients from Equation 2 and Equation 3, respectively, represents the indirect effect of X on Y through M (*ab*).

$$Y = i_1 + cX + e_1 \quad (1)$$

$$M = i_2 + aX + e_2 \quad (2)$$

$$Y = i_3 + c'X + bM_1 + e_3 \quad (3)$$

These three linear regression equations are used to assess statistical mediation in cross-sectional experimental designs. A cross-sectional experimental design is one which researchers measure variables at a single time point. Longitudinal experimental designs are ones in which researchers measure variables over time or treatment effects at a later time point (Shadish, Cook, & Campbell, 2002). One type of longitudinal design that incorporates a randomized experiment is the pretest-posttest control group design (Bonate, 2002; Shadish, Cook, & Campbell, 2002).

Pretest-Posttest Control Group Design

The pretest-posttest control group design is common in a wide range of research areas. This design consists of randomly assigning units to either a control group or a treatment group, measuring theoretically relevant variables before delivery of a treatment, and then measuring these same variables again at a later point in time after treatment (Bonate, 2002; Shadish, Cook, & Campbell, 2002). Random assignment of units to treatment and initial measurement of variables occur at the *pretest* stage of an experiment. Variables are measured again at the *posttest* stage of an experiment after delivery of a treatment to the treatment group. This document describes the two groups as the treatment and control groups but the control condition may actually be a standard treatment or some other comparison for the treatment investigated.

Pretest-posttest control group designs can assess how much change, or gain, in scores on measured variables has occurred for the treatment and control group between pretest and posttest. Because this design allows researchers to measure variables twice

for each unit, and units are randomly assigned to different treatment groups, researchers can answer questions about within-group changes between pretest and posttest and between-group differences in change between pretest and posttest (Bonate, 2002; Shadish, Cook, & Campbell, 2002).

The timing of posttest measurement is an important aspect of experimental design and should be determined a priori and based on previous research. If timing of posttest measurement does not match timing of a true effect, estimates of this true effect across pretest and posttest will typically underestimate the actual true effect (Cohen, 1991; Collins & Graham, 1991, 2002). In reality, it may be difficult to know exactly when a true effect is going to occur, and it is likely that true effects will diminish with time after the true effect occurs (Collins & Graham, 2002). Because predicting timing effects can be difficult, researchers are often advised to take many repeated measures occurring at short time intervals (Cohen, 1991; Collins & Graham, 1991, 2002). This project assumes the posttest measurement matches the timing of the true effect.

Pretest-Posttest Control Group Design with A Mediating Variable

The pretest-posttest control group design can be extended to research questions regarding mediating variables (as shown in Figure 1). When X is a randomized treatment variable coded zero or one, a common way to assess mediated effects is by estimating a series of linear regression equations similar to those used to assess mediated effects in cross-sectional data. Equation 4 represents the effect of X on the mediator measured at posttest adjusted for pretest mediator (a_{m2x} coefficient) and the pretest outcome, the effect of the mediator measured at pretest (stability) on the mediator measured at posttest (S_{m2m1}

coefficient) adjusted for X and the pretest outcome, and the effect of the outcome measured at pretest on the mediator measured at posttest (b_{m2y1} coefficient) adjusted for X and the pretest mediator. Equation 5 represents the effect of X on the outcome variable measured at posttest (c'_{y2x} coefficient) adjusted for the other variables in the equation, the effect of the outcome variable measured at pretest (stability) on the outcome variable measured at posttest (S_{y2y1} coefficient) adjusted for the other variables in the equation, the effect of the mediator measured at pretest on the outcome variable measured at posttest (b_{y2m1} coefficient) adjusted for the other variables in the equation, and the effect of the mediator measured at posttest on the outcome variable measured at posttest (b_{y2m2} coefficient) adjusted for the other variables in the equation (see Appendices A – B for further explanation of variables and notation).

$$M_2 = i_4 + a_{m2x}X + S_{m2m1}M_1 + b_{m2y1}Y_1 + e_4 \quad (4)$$

$$Y_2 = i_5 + c'_{y2x}X + S_{y2y1}Y_1 + b_{y2m1}M_1 + b_{y2m2}M_2 + e_5 \quad (5)$$

Mediated effects of X on Y_2 through M_2 in a pretest-posttest design can be assessed by taking the product of a_{m2x} coefficient in Equation 4 and b_{y2m2} coefficient in Equation 5 ($a_{m2x}b_{y2m2}$). This computation of mediated effects in a pretest posttest design is similar to the computation of mediated effects in cross-sectional designs based on Equations 1 – 3 except that it includes coefficients from regression equations with pretest measures as predictors of M and Y at posttest. Equations 4 – 5 correspond to the ANCOVA or path analysis model and represent all estimable parameters of the covariance structure in the pretest – posttest control group design with a mediating variable.

The pretest-posttest control group design is the focus of this study because it is widely used for assessing mediation and little is known about the accuracy of different models. The pretest – posttest control group design represents the simplest longitudinal design making it ideal to compare with the commonly-used cross-sectional mediation design. Several different models have been applied to assess mediation in pretest-posttest control group designs such as analysis of covariance (Jang, Kim, & Reeve, 2012; Schmiege, Broaddus, Levin, & Bryan, 2009), path analysis (Cribbie & Jamieson, 2004; MacKinnon, 2001, 2008), difference scores (Hofmann, 2004; Jansen, et al., 2012; MacKinnon et al., 1991), and residualized change scores (Cole, Kemeny, Fahey, Zack, & Naliboff, 2003; Miller, Trost, & Brown, 2002; Reid & Aiken, 2013). The purpose of this project is to evaluate models to assess mediation in the pretest-posttest control group design which consists of two time points (i.e., pretest and posttest) assuming that timing of posttest measurement matches timing of the true effect.

- Insert Figure 1 about here –

Analysis of Change in Mediation Models and Conditions to be Met

Assuming that there is successful randomization of units to the control group and treatment group so that these groups do not differ systematically at pretest, any observed change in a unit from the treatment group from pretest to posttest would not have occurred had that unit been assigned to the control group (Van Breukelen, 2006, 2013).

It is also assumed that all measures of pretest and posttest variables in this study are measured without error. Researchers have expounded on the results of violating these assumptions using ANCOVA and difference score models outside the context of mediation (Jamieson, 1999; Kisbu-Sakarya, MacKinnon, & Aiken, 2013; Van Breukelen, 2006, 2013; Wright, 2006).

Cross-sectional model. The cross-sectional model is the simplest of the models because it does not take into account the pretest measures of the mediator and outcome variable and therefore does not address a question of change across time. The cross-sectional model assumes the stabilities of the mediator and the outcome variable are equal to zero, there is no pretest correlation between the mediator and the outcome, and there are no cross-lagged relations between the mediator and the outcome or the outcome and the mediator. Equation 6 represents the relation between the treatment variable and the posttest mediator (a_{m2x}) and Equation 7 represents the relation between the treatment variable and the posttest outcome (c'_{y2x}) adjusted for the posttest mediator and the relation between the posttest mediator and the posttest outcome (b_{y2m2}) adjusted for the treatment.

$$M_2 = i_6 + a_{m2x}X + e_6 \quad (6)$$

$$Y_2 = i_7 + c'_{y2x}X + b_{y2m2}M_2 + e_7 \quad (7)$$

The cross-sectional mediated effect is estimated by computing the product of a_{m2x} coefficient from Equation 6 and b_{y2m2} coefficient from Equation 7 ($a_{m2x}b_{y2m2}$). The cross-sectional model does not explicitly take into account the pretest measures of the mediator

and the outcome in any way. This model assumes there are no relations between the pretest measures of the mediator and the outcome and no relations between the pretest measures of the mediator and the outcome and the posttest measures of the mediator and the outcome.

Difference score model. Difference scores address the question "On average, how much did each group change across time?" It can be seen that difference scores address a question of change across time that is unconditional on pretest scores (Dwyer, 1983). The difference score model assumes the same measure is used at pretest and at posttest and that the correlation between the prestest and posttest measure (stability) is 1.0 (Bonate, 2002; Campbell & Kenny, 1999; Cronbach & Furby, 1970). Equation 8 represents the difference score that would be calculated for a mediator variable where Δ_M indicates difference in scores on the mediator variable measured at pretest subtracted from scores on the mediator variable measured at posttest. Equation 9 represents the difference scores calculated for the outcome variable where Δ_Y indicates change in scores on the outcome variable measured at pretest subtracted from scores on the outcome variable measured at posttest.

$$\Delta_M = M_2 - M_1 \tag{8}$$

$$\Delta_Y = Y_2 - Y_1 \tag{9}$$

Equations 9 and 10 represent regression equations that are estimated using difference scores for the mediator variable and outcome variable, respectively.

$$\Delta_M = i_8 + a_\Delta X + e_8 \tag{10}$$

$$\Delta_Y = i_9 + c'_\Delta X + b_\Delta \Delta_M + e_9 \quad (11)$$

Mediated effects are estimated by computing the product of a_Δ coefficient from Equation 10 and b_Δ coefficient from Equation 11 ($a_\Delta b_\Delta$). When estimating the mediated effect this way, it becomes clear that the relations between pretest measures of the mediator variable and the outcome variable are not explicitly taken into account and the cross-lagged relations between pretest measures and posttest measures are not explicitly taken into account (pretest mediator to posttest outcome and pretest outcome to posttest mediator). Therefore, the difference score model in the context of mediation, implicitly assumes these relations are equal to zero.

Residualized change score model. Residualized change scores are computed by regressing posttest scores on pretest scores and then computing the difference between observed posttest scores and predicted posttest scores (residual). No treatment group variable is included in the regression of posttest scores on pretest scores, which means posttest scores for units in both treatment groups are adjusted for pretest scores based on an aggregate of pretest scores across both treatment groups. The residualized change score model assumes there are no between group differences in the correlation between the pretest measure and the posttest measure (Cronbach & Furby, 1970). That is, the correlation between the pretest measure and the posttest measure for the control group is equal to the correlation between the pretest measure and the posttest measure for the treatment group.

Residualized change scores answer the question "How different are treatment group posttest scores given equal treatment group pretest scores?" Residualized change

scores address a conditional question of change. That is, given the treatment group and the control group have equal pretest scores, how different are the treatment group and control group posttest scores? Equation 12 represents residualized change scores calculated for the mediator variable, where R_M indicates change in predicted scores on the mediator variable measured at posttest subtracted from observed scores on the mediator variable measured at posttest. Equation 13 represents residualized change scores calculated for the outcome variable, where R_Y indicates change in predicted scores on the outcome variable measured at posttest subtracted from observed scores on the outcome variable at posttest.

$$R_M = Observed\ M_2 - Predicted\ M_2 \quad (12)$$

$$R_Y = Observed\ Y_2 - Predicted\ Y_2 \quad (13)$$

Equations 13 and 14 represent regression equations that are estimated using residualized change scores for the mediator variable and the outcome variable, respectively.

$$R_M = i_{10} + a_R X + e_{10} \quad (14)$$

$$R_Y = i_{11} + c'_R X + b_R R_M + e_{11} \quad (15)$$

Mediated effects are estimated by computing the product of a_R coefficient from Equation 14 and b_R coefficient from Equation 15 ($a_R b_R$). Like the difference score model, the residualized change score model does not explicitly take into account the pretest correlation between the mediator and outcome variables and it does not explicitly take into account the cross-lagged relations. Therefore, this model implicitly assumes these relations are equal to zero.

ANCOVA. ANCOVA is used to assess change by using pretest scores as a covariate when predicting posttest scores (Bonate, 2002; Campbell & Kenny, 2002). ANCOVA removes the influence of pretest scores on posttest scores by computing a within-group regression coefficient of posttest scores on pretest scores for each treatment and control group, separately. Next, these within group regression coefficients are pooled to form a single regression coefficient by which posttest scores are adjusted for pretest scores. ANCOVA specifically addresses the question "On average, how different are the treatment and control groups scores at posttest given that treatment and control groups had equivalent pretest scores?" ANCOVA addresses a conditional question of change. The ANCOVA model assumes that within group regression coefficients are homogenous, there is no interaction of the covariate (e.g., pretest scores) and the treatment group, and that the covariate is measured without error (Huitema, 2011; Maxwell & Delaney, 2004). Equations 15 and 16 represent regression equations that are estimated using ANCOVA to adjust for pretest scores for the mediator and the outcome variable, respectively.

$$M_2 = i_4 + a_{m2x}X + S_{m2m1}M_1 + b_{m2y1}Y_1 + e_4 \quad (16)$$

$$Y_2 = i_5 + c'_{y2x}X + S_{y2y1}Y_1 + b_{y2m1}M_1 + b_{y2m2}M_2 + e_5 \quad (17)$$

S_{m2m1} in Equation 16 represents a pooled regression coefficient relating pretest scores measured on the mediator to posttest scores measured on the mediator within each treatment and control group and then pooled across both groups. S_{y2y1} in Equation 17 represents a pooled regression coefficient relating pretest scores measured on the outcome variable to posttest scores measured on the outcome variable within each treatment and control group and then pooled across both groups. Mediated effects are

estimated by computing the product of a_{m2x} coefficient from Equation 16 and b_{y2m2} coefficient from Equation 17 ($a_{m2x}b_{y2m2}$). Unlike the difference score model and the residualized change score model, the ANCOVA model explicitly takes into account the cross-lagged relations (as long as they are included in the model equations) and takes into account the pretest correlation between the mediator and outcome variables because the predictors in the equations are adjusted for their relations with the other predictors in the same equation (Cohen, Cohen, West, & Aiken, 2003).

Path analysis. An additional approach to analyzing change in the pretest-posttest control group design is to analyze the relations specified by equations 16 and 17 using path analysis. Like the ANCOVA model, path analysis explicitly takes into account the pretest correlation between the mediator and outcome variables and the cross-lagged relations (as long they are specified in the model).

Summary. Overall, the cross-sectional, difference score, and residualized change score model all have specific conditions that need to be met about various model parameters when testing the mediated effect in the pretest-posttest control group design. None of these three models take into account any potential pretest correlation between the mediator and outcome variables and none of the three models directly take into account any potential cross-lagged relations. The difference score model assumes the stabilities for the mediator and outcome variables are 1.0 and the cross-sectional model assumes the stabilities for the mediator and outcome variables are 0.0. The ANCOVA and path analysis models explicitly estimate or take into account the correlation between the mediator and outcome at baseline, cross-lagged relations, and stabilities. Therefore, it is

expected that the mediated effect estimated with the difference score, residualized change score, and the cross-sectional model will be biased across a variety of conditions whereas the mediated effect estimated with the ANCOVA and the path analysis model will not.

The following hypotheses reflect that the performance of the models will be negatively affected when parameters are non-zero in the true model but are not estimated. For example, the residualized change score model does not explicitly take into account any cross-lagged relations between pretest measures and posttest measures therefore when these relations exist and the residualized change score model is used to estimate the mediated effect, it is expected that the mediated effect will be biased.

Hypotheses

The first hypothesis is the difference score, residualized change score, ANCOVA, and path analysis models will perform better than the cross-sectional model in general because they use the pretest information. The second hypothesis is the cross-sectional, difference score, and residualized change score models will be biased when either or both the *by2m1* and *bm2y1*, cross-lagged paths, are non-zero. The third hypothesis is the cross-sectional, difference score, and residualized change score model will have confidence interval coverage lower than 95% when either or both the *by2m1* or *bm2y1* paths are present. The *by2m1* and *bm2y1* paths are expected to bias the results and lead to confidence interval coverage lower than 95% for the cross-sectional, difference score,

and residualized change score models because these models do not directly take into account these paths when estimating the mediated effect. The fourth hypothesis is the cross-sectional, difference score, and residualized change score models will be biased and have confidence interval coverage lower than 95% when there is a pretest correlation between the mediator and outcome variables. The presence of the pretest correlation is expected to bias the results of these models because these models do not take into account this pretest correlation. The fifth hypothesis is that the difference score model will have less power when the stability is low versus high because the difference score model assumes the pretest-posttest correlation (stability) is 1.00. Overall, it is hypothesized all models will not have type 1 error rates that are greater than the nominal 0.05 alpha level and will have increasing power as effect size of the mediated effect increases and sample size increases.

The study hypotheses are important because if researchers use the cross-sectional, difference score, or residualized change score model to estimate mediated effects in the pretest – posttest control group design, there are specific conditions to be met regarding the relations between the mediating and outcome variable at the pretest and the relations of the pretest measures to the posttest measures (i.e., cross-lagged relations). The conditions range from a zero pretest correlation between the mediator and the outcome to zero cross-lagged relations between the pretest and posttest measures. It is unlikely that these conditions are tenable in most research designs and specifically the pretest – posttest control group design involving mediation effects. The proposed simulation is designed to investigate how violating these conditions affects the accuracy of the five models.

Method

Data-Generating Model

The SAS 9.3 programming language was used to conduct a Monte Carlo simulation of a pretest-posttest control group design with a mediating variable. The following equations represent the data-generating model (see Appendix C) and correspond to the ANCOVA/path analysis model in the Monte Carlo simulation where x is an observed value of random variable X and $\tilde{x}$ is the sample median.

$$X \sim N(0,1): x \geq \tilde{x} = 1; x < \tilde{x} = 0 \quad (34)$$

$$M_1 \sim N(0,1) \quad (35)$$

$$Y_1 = b_{y1m1}M_1 + e_1 \quad (36)$$

$$M_2 = a_{m2x}X + b_{m2y1}Y_1 + S_{m2m1}M_1 + e_2 \quad (37)$$

$$Y_2 = c'_{y2x}X + b_{y2m1}M_1 + b_{y2m2}M_2 + S_{y2y1}Y_1 + e_3 \quad (38)$$

$$\sigma_{e1}^2 = 1 \quad (39)$$

$$\sigma_{e2}^2 = 1 \quad (40)$$

$$\sigma_{e3}^2 = 1 \quad (41)$$

$$\sigma_{e1e2} = 0 \quad (42)$$

$$\sigma_{e1e3} = 0 \quad (43)$$

$$\sigma_{e2e3} = 0 \quad (44)$$

The Monte Carlo simulation, varied sample size (N = 50, 100, 200, 500), effect size of the *a* (a_{m2x}) (0 .10, .30, .50), *b*(b_{y2m2}) (0 .10, .30, .50), and *c'*(c'_{y2x}) (0 and .30) paths, effect size of the path from the pretest mediator to the posttest outcome (b_{y2m1}) (0 and .50), effect size of the path from the pretest outcome to the posttest mediator (b_{m2y1}) (0 and .50), stability of the mediator (S_{m2m1}) and the outcome (S_{y2y1}) (.3 and .7), and the correlation between the mediator and outcome at pretest (0 and .5). The b_{y1m1} coefficient in Equation 36 was simulated to be equivalent to a correlation (ρ_{y1m1}) of 0 or .5. The data-generating model diagram (see Appendix D) depicts a causal relation between the mediator and outcome at pretest but this is to make the Equations 34 – 44 match exactly with the diagram. That is, although there was a causal arrow relating the mediator at pretest to the outcome at pretest we will investigate effects of varying the *correlation* between these variables and do not assume a unidirectional causal effect between them.

To summarize, 13 combinations of effect sizes for the *a*, *b*, and *c'* path were studied. The effect sizes of these paths were all in the correlation metric and chosen to reflect small, medium, and large effect sizes (Cohen, 1988). The 13 combinations of effect size were adopted from MacKinnon, Lockwood, and Williams (2004) because they demonstrated that all other combinations of effect sizes had identical results in their study. A caveat should be made that the present study differs from the single mediator model studied in MacKinnon et al. (2004). The combinations are as follows and summarized in Table 1: $a = b = c' = 0$; $a = 0$, $b = .10$, $c' = 0$; $a = 0$, $b = .30$ $c' = 0$; $a = 0$, $b = .50$, $c' = 0$; $a = .10$, $b = .10$, $c' = 0$; $a = .30$, $b = .30$, $c' = 0$; $a = .50$, $b = .50$, $c' = 0$; $a = .10$, $b = .30$, $c' = 0$; $a = .10$, $b = .50$, $c' = 0$; $a = .30$, $b = .50$, $c' = 0$; $a = .10$, $b = .10$, $c' = .30$; $a = .30$, $b = .30$, $c' = .30$; and $a = .50$, $b = .50$, $c' = .30$. There was 832 conditions

defined by 13 effect size combinations, 4 sample sizes, 2 effect sizes of pretest mediator on posttest outcome, 2 effect sizes or pretest outcome on posttest mediator, 2 stabilities of mediator and outcome, and 2 correlations between mediator and outcome at pretest. This resulted in an incomplete factorial design with all factors being fully crossed with one another except for the last three combinations of effect sizes which included a non-zero effect size for the *c'* path. When this path is non-zero, it is known as a direct effect in the mediation literature (MacKinnon, 2008). The presence of this direct effect did not occur for all combinations of effect sizes for the *a* and *b* paths. Therefore, the direct effect was not fully crossed with all the factors in this simulation study. A total of 1,000 replications of each condition was conducted. The focus of this simulation study was to evaluate estimator characteristics of the mediated effect ($a_{m2x}\, b_{y2m2}$) for the cross-sectional single mediator model ignoring the pretest mediator and outcome variables and four longitudinal models (i.e., difference scores, residualized change scores, ANCOVA, and path analysis) for assessing change.

Bias of Parameter Estimates

For each replication in each condition, bias of the parameter estimates of the mediated effect was the parameter estimate minus the true value of the parameter as in Equation 45. All estimates of bias were averaged over all replications within each condition.

$$Bias(\hat{\theta}) = \hat{\theta} - \theta \tag{45}$$

The relative bias of the parameter estimates of the mediated effect was computed by dividing the bias of the parameter estimate from Equation 45 by the true value of the parameter as in Equation 46. All estimates of relative bias were averaged over replications within each condition.

$$RBias(\hat{\theta}) = \frac{(\hat{\theta}-\theta)}{\theta} \quad (46)$$

An estimator was considered acceptable in terms of bias if the absolute value of relative bias was less than .10 (Flora & Curran, 2004; Kaplan, 1988). One drawback of calculating relative bias (i.e., Equation 44) is that it cannot be calculated when the true value of the parameter is equal to zero. To remedy this, the standardized bias (SBias) of the parameter estimates was computed by dividing the bias of the parameter estimate as obtained from Equation 45 by the standard deviation of the parameter estimate across replications (i.e., empirical standard error of the parameter estimate). This measure of relative bias can be calculated when the true value of the parameter is equal to zero (see Equation 47). All estimates of standardized bias were averaged over replications within each condition.

$$SBias(\hat{\theta}) = \frac{(\hat{\theta}-\theta)}{SD\hat{\theta}} \quad (47)$$

Significance Testing

Type 1 error rates were the proportion of times across the 1000 replications per condition a parameter estimate of the mediated effect was statistically significant at the 0.05 alpha level when the true value of the parameter estimate was 0. Bradley's (1978) liberal criterion was used to evaluate the performance of the methods in terms of Type 1

error rates. That is, Type 1 error rates will be deemed acceptable if they fall within the range of [0.025, 0.075]. Power was the proportion of times across the 1000 replications per condition a parameter estimate of the mediated effect was statistically significant at the 0.05 alpha level when the true value of the parameter was not equal to 0. The best performing estimator in terms of statistical power has the highest statistical power given the effect size and sample size generated for a given simulation condition.

Confidence Interval Estimation

Normal theory. Confidence interval coverage will be the proportion of 95% confidence intervals that contain the true value of the parameter estimate of the mediated effect across replications. The width of each arm of the normal theory confidence interval (margin of error; M.O.E) was computed using the following equation:

$$M.O.E.\left(\hat{\theta}\right) = (1.96 * SE_{\hat{\theta}}) \quad (48)$$

The value of 1.96 refers to the critical value of the standard normal distribution (Z-scores) that corresponds to an area above the value equal to 0.025 and $SE_{\hat{\theta}_r}$ refers to the estimated standard error for a given replication. In addition to confidence interval coverage, the proportion of times the true value of the parameter fell above the upper limit of the confidence interval was calculated and the proportion of times the true value of the parameter fell below the lower limit of the confidence interval was calculated across replications.

Percentile bootstrap. Confidence interval coverage was also computed using the percentile bootstrap (Efron & Tibshirani, 1993). For each replication, 1000 bootstrap samples were generated and confidence intervals were computed for each parameter estimate in each bootstrap sample. A 95% percentile bootstrap confidence interval for each replication was computed by rank ordering each bootstrap sample mediated effect and taking the 25^{th} value from the 1000 bootstrapped samples as the lower bound of the confidence interval and the 975^{th} value from the 1000 bootstrapped samples as the upper bound of the confidence interval. Coverage was the proportion of times the true value of the parameter fell within the percentile bootstrap confidence interval. The proportion of times the true value fell below the lower limit of the bootstrapped confidence interval and the proportion of times the true value fell above the upper limit of the bootstrapped confidence interval was calculated.

Distribution of a product. Confidence interval coverage was computed using the PRODCLIN program to create asymmetric confidence intervals based on the non-normal distribution of the product of two regression coefficients (e.g., *ab*; MacKinnon, Fritz, Williams, & Lockwood, 2007). PRODCLIN was used to compute the 95% asymmetric confidence interval for each estimate of the mediated effect for each replication. Coverage was the proportion of times the true value of the mediated effect fell within the asymmetric confidence intervals. The proportion of times the true value fell below the lower limit of the asymmetric confidence interval and the proportion of times the true value fell above the upper limit of the asymmetric confidence interval was calculated.

All else being equal, the best estimator of the mediated effect and the best method for creating confidence intervals (i.e., normal theory, percentile bootstrap, or PRODCLIN) had confidence interval coverage rates that fall within the range of [92.5, 97.5] and equal proportions of true values that fell above the upper limit of the confidence interval within the range of [1.25, 3.75] and true values that fell below the lower limit of the confidence interval within the range of [1.25, 3.75] based on Bradley's (1978) liberal robustness criterion.

The simulation was conducted in two parts. First, the data for each of the 832 conditions was generated with a SAS macro (see Appendix C). Second, the data for each of the 832 conditions was analyzed using separate SAS macros (See Appendices F – K).

Data Analysis Models

Cross-Sectional Single Mediator Model. The following equations estimate the cross-sectional mediated effect and ignore the pretest mediator and outcome variables (see Appendix L).

$$M_2 = i_6 + a_{m2x}X + e_6 \tag{49}$$

$$Y_2 = i_7 + c'_{y2x}X + b_{y2m2}M_2 + e_7 \tag{50}$$

The estimate of the effect of treatment on the posttest mediator scores is $\hat{a}_{m2x}$, the estimate of the effect of treatment on the posttest dependent variable scores adjusted for the effect of posttest mediator scores is $\widehat{c'}_{y2x}$, the estimate of the effect of the posttest

mediator scores on the posttest dependent variable scores adjusted for the effect of the treatment is $\hat{b}_{y2m2}$, and the estimate of the cross-sectional mediated effect is $\hat{a}_{m2x}\hat{b}_{y2m2}$.

Difference Scores. Difference scores for the mediator and the dependent variable will be computed and submitted to regression analyses using the following Equations (see Appendix M for figure).

$$\Delta_M = M_2 - M_1 \quad (51)$$

$$\Delta_Y = Y_2 - Y_1 \quad (52)$$

$$\Delta_M = i_8 + a_\Delta X + e_8 \quad (53)$$

$$\Delta_Y = i_9 + c'_\Delta X + b_\Delta \Delta_M + e_9 \quad (54)$$

The estimate of the effect of treatment on the difference score for the mediator is $\hat{a}_\Delta$, the estimate of the effect of treatment on the difference score for the dependent variable adjusted for the effect of the mediator change score on the difference score of the dependent variable is $\widehat{c'}_\Delta$, the estimate of the effect of the mediator difference score on the difference score for the dependent variable adjusted for the effect of the treatment on the difference score of the dependent variable is $\hat{b}_\Delta$, and the estimate of the mediated effect is $\widehat{a_\Delta b_\Delta}$.

Residualized Change Scores. Residualized change scores for the mediator and the dependent variable will be computed and used in regression analyses using the following Equations (see Appendix N for figure).

$$Predicted\ M_2 = S_{m2m1total} M_1 \quad (55)$$

$$R_M = Observed\ M_2 - Predicted\ M_2 \quad (56)$$

$$Predicted\ Y_2 = S_{y2y1total} Y_1 \quad (57)$$

$$R_Y = Observed\ Y_2 - Predicted\ Y_2 \quad (58)$$

$$R_M = i_{10} + a_R X + e_{10} \quad (59)$$

$$R_Y = i_{11} + c'_R X + b_R R_M + e_{11} \quad (60)$$

The estimate of the effect of treatment on the residualized change score for the mediator is $\hat{a}_R$, the estimate of the effect of treatment on the residualized change score for the dependent variable adjusted for the effect of the mediator residualized change score on the residualized change score of the dependent variable is $\widehat{c'}_R$, the estimate of the effect of the mediator residualized change score on the residualized change score for the dependent variable adjusted for the effect of the treatment on the residualized change score of the dependent variable is $\hat{b}_R$, and the estimate of the mediated effect is $\widehat{a_R b_R}$.

Analysis of Covariance. The following equations represent estimating the effects using ANCOVA with pretest scores as the covariate for the mediator and the dependent variable (see Appendix D).

$$M_2 = i_4 + a_{m2x} X + b_{m2y1} Y_1 + S_{m2m1\ pooled} M_1 + e_4 \quad (61)$$

$$Y_2 = i_5 + c'_{y2x} X + S_{y2y1\ pooled} Y_1 + b_{y2m1} M_1 + b_{y2m2} M_2 + e_5 \quad (62)$$

The estimate of the effect of treatment on the posttest mediator scores adjusted for pretest mediator scores and pretest dependent variable scores is $\hat{a}_{m2x}$, the estimate of the effect

of treatment on the posttest dependent variable scores adjusted for the effect of the pretest mediator scores, posttest mediator scores, and pretest dependent variable scores is $\widehat{c'}_{y2x}$, the estimate of the effect of the posttest mediator scores on the posttest dependent variable scores adjusted for the effect of the treatment, pretest mediator scores, and pretest dependent variable scores is $\hat{b}_{y2m2}$, and the estimate of the mediated effect is $\hat{a}_{m2x}\hat{b}_{y2m2}$.

Path Analysis. The following equations represent estimating the effects using path analysis (see Appendix D). The effects estimated with path analysis will be similar to the effects estimated using ANCOVA except for the estimated standard errors. Standard errors estimated using path analysis model will differ from those from the standard errors estimated using ANCOVA because path analysis uses maximum likelihood estimation as opposed to ANCOVA which uses ordinary least squares estimation. The formulas for standard errors using maximum likelihood estimation differ slightly from those used in ordinary least squares estimation. Thus, the estimated standard errors across the path analysis model results and the ANCOVA model results will differ slightly.

$$M_2 = i_4 + a_{m2x}X + b_{m2y1}Y_1 + S_{m2m1}M_1 + e_4 \quad (63)$$

$$Y_2 = i_5 + c'_{y2x}X + S_{y2y1}Y_1 + b_{y2m1}M_1 + b_{y2m2}M_2 + e_5 \quad (64)$$

$$Cov(X, M_1) = 0 \quad (65)$$

$$Cov(X, Y_1) = 0 \quad (66)$$

$$Cov(M_1, Y_1) = \sigma_{m1y1} \quad (67)$$

The covariance between treatment variable (X) and pretest mediator (M_1) and between treatment variable (X) and pretest outcome (Y_1) will be fixed to zero under the assumption of successful randomization of units to conditions. The covariance between the pretest mediator and pretest outcome variable will be estimated. All covariance terms between residuals will be fixed to zero and all residual variances will be estimated.

The estimate of the effect of treatment on the posttest mediator scores adjusted for pretest mediator scores and pretest dependent variable scores is $\hat{a}_{m2x}$, the estimate of the effect of treatment on the posttest dependent variable scores adjusted for the effect of the pretest mediator scores, posttest mediator scores, and pretest dependent variable scores is $\widehat{c'}_{y2x}$, the estimate of the effect of the posttest mediator scores on the posttest dependent variable scores adjusted for the effect of the treatment, pretest mediator scores, and pretest dependent variable scores is $\hat{b}_{y2m2}$, and the estimate of the mediated effect is $\hat{a}_{m2x}\hat{b}_{y2m2}$(see Appendices O – P for true covariances and correlations).

Results

Organization

The results section was organized in the following way. Type 1 error rates were discussed first followed by bias, confidence interval coverage, and then power results. All type 1error, confidence interval coverage, and power results were reported using the distribution of a product. The distribution of a product results were reported because they perform better than normal theory results and they performed similarly to the percentile

bootstrap results when detecting mediated effects in this study and in prior research (MacKinnon, Lockwood, & Williams, 2004). For each section (except for the type 1 error rates section) the results for a specific model were presented (e.g., ANCOVA) and for each model there was a section of results for when there was no direct effect and a section for when there was a direct effect. The results were analyzed separately for no direct effect versus direct effect because the direct effect was not fully-crossed with the other predictors in this simulation study. For example, there were 10 conditions of different effect sizes of the mediated effect for which there was no direct effect and there were 3 conditions of different effect sizes of the mediated effect for which there was a direct effect. In most situations the patterns of results for different values of the direct effect were identical. When results differed across values of the direct effect, they were reported.

Type 1 Error Rates Regression Analyses

To assess the significant predictors of empirical type 1 error rates logistic regression analyses were conducted with the dependent variable coded as '0' for 'non-significant mediated effect' and '1' for 'significant mediated effect' for all simulation conditions. Sample size was treated as a continuous predictor and was standardized to have a mean of zero and a standard deviation of one prior to the analyses. Pretest correlation was coded '-1' for a pretest correlation of 0.00 and coded '1' for a pretest correlation of 0.50. Stability of the mediator and outcome variables was coded '-1' for stability of 0.30 and coded '1' for stability of 0.70. The *bm2y1* path was coded'-1' for the *bm2y1* path of 0.00 and coded '1' for the *bm2y1* path of 0.50. The *by2m1* path was coded

'-1' for the *by2m1* path of 0.00 and coded '1' for the *by2m1* path of 0.50. Because the coding scheme for the categorical predictors was chosen to be contrast codes (-1 or 1), this resulted in the mean of each categorical variable being equal to zero and the standard deviation being equal to one given equal sample size in each simulation condition (i.e., 1000 replications in each condition).

All possible higher-order interactions were included in these analyses and main and interaction effects that were both statistically significant at $\alpha = 0.05$ and had a standardized beta coefficient (reported as *b*) of at least 0.10 were considered important effects on type 1 error. For interaction terms, predictors were first standardized and then products of the standardized predictors were formed to create the interaction terms. Standardized beta coefficients were computed following the recommendations of Menard (2004) for a fully standardized logistic regression coefficient. The coefficients were standardized in order for all effects to be on a similar metric and 0.10 was chosen as a cut-off point because it corresponds to a 0.20 standard deviation change for a 1-unit change in the predictors that were coded using contrast codes (i.e., pretest correlation, stability, cross-lags). Because the design of this study was an incomplete factorial design with the incomplete factor being the presence of a direct effect, analyses were performed separately for each of the two levels of direct effect.

Type 1 Error Rates

There were no significant interactions or main effects that were both statistically significant and had standardized beta coefficients that were greater than the absolute

value of 0.10 for the ANCOVA, path analysis, difference score, residualized change score, or the cross-sectional model results.

The Type 1 error rates for all methods to assess the mediated effect (i.e., Cross-sectional model, ANCOVA, path analysis, difference scores, and residualized change scores) are not above the nominal cut-off point of 0.05 across any combination of simulation conditions. Therefore, the statistical method that is the least biased and has the most power for detecting the mediated effect given a particular effect size and sample size will be considered the best model for detecting mediated effects in pretest-posttest control group designs.

Insert Tables 2-5 about here

Bias Regression Analyses

To assess which predictors in this study significantly contributed to variation in empirical bias, relative bias, and standardized bias, Ordinary Least Squares (OLS) regression analyses were conducted for each model with bias, relative bias, and standardized bias as the dependent variables for each model. The predictors were coded the same way as for the type 1 error rates analyses and separate analyses were conducted for each level of direct effect. Omega-squared values of 0.01 in combination with a statistically significant main or interaction effect were considered practically significant effects. The analyses were only conducted for standardized bias and relative bias because

these measures of bias are generally more easily interpreted than bias because they are not artificially inflated as the effect size of the mediated effect increases. All bias results are presented in the tables alongside standardized bias and relative bias results.

Standardized Bias Results

ANCOVA. As shown in Table 6, there were no main or interaction effects that were both statistically significant and had omega-squared values of 0.01 or higher and there were no conditions for which the standardized bias exceeded 0.10.

Insert Table 6 about here

Path Analysis. As shown in Table 7, there were no main or interaction effects that were both statistically significant and had omega-squared values of 0.01 or higher and there were no conditions for which the standardized bias exceeded 0.10.

Insert Table 7 about here

Difference Scores. As shown in Table 8, there were no main or interaction effects that were both statistically significant and had omega-squared values of 0.01 or higher and there were no conditions for which the standardized bias exceeded 0.10.

Insert Table 8 about here

Residualized change scores. Because of the number of practically significant interactions of predictors on standardized bias for the residualized change score model, only the results for when the *bm2y1* path and the *by2m1* path were equal to 0.50 are presented. There were no cases when the absolute value of the standardized bias exceeded 0.10 when both of these paths were equal to 0.00. As shown in Tables 9 – 10, there were four-way interactions of effect size, pretest correlation, the *bm2y1* path, and the *by2m1* path (ω^2 = 0.01, F (1, 639,936) = 11,133, p < 0 .05) and effect size, sample size, pretest correlation, and the *bm2y1* path (ω^2 = 0.01, F (1, 639,936) = 9,332.30, p < 0 .05) such that the absolute value of the standardized bias of the mediated effect for the residualized change score model exceeded 0.10 for all sample sizes and effect sizes and when the *bm2y1* path was equal to 0.50 and whether or not the *by2m1* path or pretest correlation were equal to 0.00 or 0.50. There were main effects of pretest correlation (ω^2 = 0.05, F (1, 639,936) = 85,026.90, p < 0 .05), the *bm2y1* path (ω^2 = 0.10, F (1, 639,936) = 191,671, p < 0 .05), the *by2m1* path (ω^2 = 0.02, F (1, 639,936) = 31,284.80, p < 0 .05), effect size (ω^2 = 0.08, F (1, 639,936) = 153,693, p < 0 .05), and sample size (ω^2 = 0.02, F (1, 639,936) = 42,107.50, p < 0 .05) but there were no simple relations between either the pretest correlation, the *bm2y1* path, the *by2m1* path, effect size, or sample size and standardized bias.

Insert Tables 9-10 about here

Cross-Sectional.

Direct effect = 0.00. As shown in Table 11, the absolute value of the standardized bias of the mediated effect with the cross-sectional model exceeded 0.10 for all conditions and became larger when *by2m1* increased from 0.00 to 0.50 and as sample size increased (interaction of effect size, sample size, and the *by2m1* path on the standardized bias, $\omega^2 = 0.01$, F (1, 639,936) = 5,471.52, $p < 0$ .05). The standardized bias was greater for effect sizes 0.01, 0.09, and 0.25 ($\omega^2 = 0.16$, F (1, 639,936) = 144,896, $p <$ 0 .05), as sample size increased ($\omega^2 = 0.03$, F (1, 639,936) = 29,506.80, $p < 0$ .05), and as the *by2m1* path increased from 0.00 to 0.50 ($\omega^2 = 0.03$, F (1, 639,936) = 26,019.70, $p < 0$ .05).

Insert Table 11 about here

Direct effect = 0.30. As shown in Tables 12 – 15 there was no interaction of effect size, sample size, and the *by2m1* path when the direct effect was present. There were, however, interactions of effect size and sample size ($\omega^2 = 0.01$, F (1, 191,936) = 5,036, $p < 0$ .05), effect size and the *by2m1* path ($\omega^2 = 0.02$, F (1, 191,936) = 7,067.87, $p < 0$ .05), sample size and the *by2m1* path ($\omega^2 = 0.02$, F (1, 191,936) = 7,193.63, $p < 0$ .05), pretest correlation and the *bm2y1* path ($\omega^2 = 0.01$, F (1, 191,936) = 1,987.96, $p < 0$

.05), and the *bm2y1* path and the *by2m1* path (ω^2 = 0.01, F (1, 191,936) = 3,990.23, p < 0 .05) such that the absolute value of the standardized bias exceeded 0.10 when the *bm2y1* path increased from 0.00 to 0.50 which increased when the pretest correlation increased from 0.00 to 0.50, the *by2m1* path increased from 0.00 to 0.50 and as effect size and sample size increased. The standardized bias was the highest for N = 500 and large effect sizes when the *bm2y1* path was equal to 0.50, the *by2m1* path was equal to 0.50, and the pretest correlation was equal to 0.50. The only condition for which the absolute value of the standardized bias did not exceed 0.10 was when the pretest correlation was equal to 0.00, and both the *bm2y1* path and the *by2m1* path were equal to 0.00.

Insert Tables 12 – 15 about here

Relative Bias Results

ANCOVA. There were no main or interaction effects that were both statistically significant and had omega-squared values of 0.01 or higher and there were no conditions for which the relative bias exceeded 0.10.

Path Analysis. There were no main or interaction effects that were both statistically significant and had omega-squared values of 0.01 or higher and there were no conditions for which the relative bias exceeded 0.10.

Difference Scores.

Direct effect = 0.00. There were no main or interaction effects that were both statistically significant and had omega-squared values of 0.01 or higher. Subsequently, there were no conditions for which the absolute value of relative bias exceeded 0.10.

Residualized change scores. As shown in Tables 9 – 10, the absolute value of the relative bias of the mediated effect with the residualized change score model exceeded 0.10 for all sample sizes and effect sizes, when the *bm2y1* path was equal to 0.50 and became larger when the pretest correlation increased from 0.00 to 0.50 but was unaffected by the *by2m1* path (interaction of pretest correlation and the *bm2y1* path, ω^2 = 0.01, F (1, 383,936) = 5,571.36, p < 0 .05). Additionally, there were a few other conditions for which the relative bias was greater than 0.10. The relative bias increased when the *bm2y1* path increased from 0.00 to 0.50 (ω^2 = 0.06, F (1, 383,936) = 25,662.40, p < 0 .05). There was a main effect of pretest correlation (ω^2 = 0.01, F (1, 383,936) = 5,587.27, p < 0 .05) but there was no simple interpretation of pretest correlation and relative bias.

Cross-Sectional.

Direct effect = 0.00. As shown in Table 11, the absolute value of the relative bias of the mediated effect with the cross-sectional model exceeded 0.10 for all sample sizes and effect sizes and whether or not the *by2m1* path was equal to 0.00 or 0.50 but was higher for small effect sizes and when the *by2m1* path was equal to 0.50 (interaction of

effect size and the *by2m1* path, ω^2 = 0.01, *F* (1, 383,936) = 2,939.90, *p* < 0 .05). The relative bias was larger when the pretest correlation increased from 0.00 to 0.50 (ω^2 = 0.01, *F* (1, 383,936) = 2,443.24, *p* < 0 .05), when the *by2m1* path increased from 0.00 to 0.50 (ω^2 = 0.04, *F* (1, 383,936) = 17,133.80, *p* < 0 .05), and as effect size decreased (ω^2 = 0.02, *F* (1, 383,936) = 9,206.74, *p* < 0 .05).

Direct effect = 0.30. As shown in Tables 12 – 15, the same pattern of results held for when there was a direct effect as compared to when there was not a direct effect. The absolute value of relative bias of the mediated effect exceeded 0.10 for the cross-sectional model when the *by2m1* path increased from 0.00 to 0.50 and as effect size decreased (interaction of effect size and *by2m1* path, ω^2 = 0.01, *F* (1, 191,936) = 3,265, *p* < 0 .05). The relative bias increased as effect size decreased (ω^2 = 0.05, *F* (1, 191,936) = 10,570.10, *p* < 0 .05), when the pretest correlation increased from 0.00 to 0.50 (ω^2 = 0.01, *F* (1, 191,936) = 1,342.46, *p* < 0 .05), when stability increased from 0.30 to 0.70 (ω^2 = 0.01, *F* (1, 191,936) = 1,166.56, *p* < 0 .05), when the *bm2y1* path increased from 0.00 to 0.50 (ω^2 = 0.01, *F* (1, 191,936) = 1,138.91, *p* < 0 .05), and when the *by2m1* path increased from 0.00 to 0.50 (ω^2 = 0.04, *F* (1, 191,936) = 9,089.92, *p* < 0 .05).

Summary.

In summary, the mediated effect was not biased when estimated with the ANCOVA, path analysis, or difference score model. The mediated effect estimated with the residualized change score model was biased when the pretest correlation was equal to 0.00 or 0.50 and the *bm2y1* and *by2m1* paths were equal to 0.50. When the *bm2y1* and *by2m1* paths were equal to 0.00, the mediated effect was not biased and therefore

comparable to the mediated effect estimated with the ANCOVA and path analysis models. The mediated effect was biased with the cross-sectional model when there was no direct effect present (0.00) and became more biased as effect size increased and as the *by2m1* path increased from 0.00 to 0.50. When a direct effect was present, the only time the mediated effect was not biased was when the pretest correlation was equal to 0.00 and the *by2m1* path was equal to 0.00. For this combination of conditions, the mediated effect estimated with the cross-sectional model was comparable to the mediated effect estimated with the ANCOVA and path analysis models.

Confidence Interval Coverage and Power Regression Analyses

Logistic regression analyses were conducted in the same way for confidence interval coverage and power as for type 1 error rates with the addition of effect size of the mediated effect as a standardized predictor in the analyses of confidence interval coverage and power. For confidence interval coverage and power, separate analyses were conducted for the conditions with no direct effect and for the conditions with a direct effect because the direct effect manipulation was not completely crossed for all levels of the other factors in this simulation study.

When assessing the performance of confidence interval coverage, coverage values below 92.5% were considered low and were highlighted in red in the results tables. Coverage values above 97.5% were considered high and were highlighted in green in the results tables. These cutoff points correspond to Bradley's (1978) robustness criterion. The percent of cases falling above the upper limit and the percent of cases falling below the lower limit did not differ across the models so those results were not reported here.

Confidence Interval Coverage.

ANCOVA. As shown in Table 16, the confidence interval coverage for the ANCOVA model never fell below 92.5% for any combination of sample size and effect size. There were a few instances when the coverage exceeded 97.5% which generally happened when the effect was small (0.01) or zero and sample size was small (50 to 200). There were no significant predictors of the confidence interval coverage of the mediated effect with the ANCOVA model. That is, there were no significant main or interaction effects that were both statistically significant and had a standardized beta coefficient with an absolute value greater than 0.10.

Insert Table 16 about here

Path analysis. As shown in Table 17, the confidence interval coverage for the path analysis model never fell below 92.5% for any combination of sample size and effect size. There were a few instances when the coverage exceeded 97.5% which generally happened when the effect size (0.01) and sample size was small (50 to 100). There were no significant predictors of the confidence interval coverage of the mediated effect with the path analysis model. That is, there were no significant main or interaction effects that were both statistically significant and had a standardized beta coefficient with an absolute value greater than 0.10.

Insert Table 17 about here

Difference scores. As shown in Table 18, the confidence interval coverage for the path analysis model never fell below 92.5% for any combination of sample size and effect size. There were a few instances when the coverage exceeded 97.5% which generally happened when the effect size (0.01) and sample size was small (50 to 100). There were no significant predictors of the confidence interval coverage of the mediated effect with the path analysis model. That is, there were no significant main or interaction effects that were both statistically significant and had a standardized beta coefficient with an absolute value greater than 0.10.

Insert Table 18 about here

Residualized change scores.

Direct effect = 0.00. As shown in Tables 19 – 20 confidence interval coverage fell below 92.5% as sample size, pretest correlation, and the *bm2y1* path increased (interaction of sample size, pretest correlation, and the *bm2y1* path, b = -0.11, χ^2 (1, N = 384,000) = 1,822.28, $p < 0.05$). Coverage was low for large sample sizes when the pretest correlation the *bm2y1* path both increased from 0.00 to 0.50. There was also an interaction of pretest correlation, the *bm2y1* path, and the *by2m1* path (b = -0.11, χ^2 (1, N = 384,000) = 2,043.63, $p < 0.05$) such that as pretest correlation, the *bm2y1* path, and the

by2m1 path all increased, coverage decreased. That is, coverage decreased when the pretest correlation increased from 0.00 to 0.50 and when both the *bm2y1* path and the *by2m1* path increased from 0.00 to 0.50. There were also main effects of the *bm2y1* path (b = -0.31, χ^2 (1, N = 384,000) = 16,827.79, $p < 0.05$), sample size (b = -0.17, χ^2 (1, N = 384,000) = 4,303.29, $p < 0.05$), pretest correlation (b = -0.21, χ^2 (1, N = 384,000) = 8,020.70, $p < 0.05$), and effect size (b = -0.13, χ^2 (1, N = 384,000) = 2,529.66, $p < 0.05$) but there were no simple interpretation of either the *bm2y1* path, sample size, or effect size and coverage.

The confidence interval coverage fell below 92.5% for the residualized change score model for N = 50, when the effect size was 0.15 and 0.25 and when the *bm2y1* path was equal to 0.50 but the *by2m1* path and the pretest correlation were equal to 0.00. The confidence interval coverage was also low for N = 50, effect sizes 0.05, 0.09, 0.15, and 0.25 when the *bm2y1* path and the pretest correlation were equal to 0.50 but the *by2m1* path was equal to 0.00. When the *by2m1* path increased from 0.00 to 0.50, the coverage dropped to 92.5% for an additional effect size of 0.03. For N = 100, the confidence interval coverage dropped below 92.5% for effect sizes 0.09, 0.15, and 0.25 when the *bm2y1* path, the *by2m1* path, and the pretest correlation decreased from 0.50 to 0.00. When the *by2m1* path increased from 0.00 to 0.50, the coverage decreased for effect sizes 0.15 and 0.25. When the pretest correlation increased from 0.00 to 0.50, the coverage decreased for effect sizes 0.03, 0.05, 0.09, 0.15, and 0.25 when the *bm2y1* path increased from 0.00 to 0.50 and whether or not the *by2m1* path increased from 0.00 to 0.50.

For N = 200, the coverage was low for effect sizes 0.09, 0.15, and 0.25 when the *bm2y1* path increased from 0.00 to 0.50 but the *by2m1* path and the pretest correlation decreased from 0.50 to 0.00. When the *by2m1* path increased from 0.00 to 0.50, the coverage decreased for effect sizes 0.15 and 0.25. When the pretest correlation increased from 0.00 to 0.50 the coverage decreased for effect sizes 0.03, 0.05, 0.09, 0.15, and 0.25 when the *bm2y1* path increased from 0.00 to 0.50 but the *by2m1* path decreased from 0.50 to 0.00. When the *by2m1* path increased from 0.00 to 0.50, the coverage also decreased for effect size 0.01 with the coverage decreasing to 0% in some cases. For N = 500, coverage was low for effect sizes, 0.03, 0.05, 0.09, 0.15, and 0.25 when the *bm2y1* path was equal to 0.50, the pretest correlation was equal to 0.00 and whether or not the *by2m1* path increased was equal to 0.00 or 0.50. When the pretest correlation increased from 0.00 to 0.50, coverage decreased for all effect sizes when the *bm2y1* path increased from 0.00 to 0.50 and whether or not the *by2m1* path increased from 0.00 to 0.50. In some cases, the coverage went to 0%. The confidence interval coverage exceeded 97.5% for N = 50 and N = 100 when there was no effect or the effect size was small (0.01) and occurred across both values of pretest correlation, the *bm2y1* path, and the *by2m1* path.

Insert Tables 19-20 about here

Direct effect = 0.30. As shown in Tables 21 – 22 there was an additional interaction of sample size, pretest correlation and the *by2m1* path (b = -0.10, χ^2 (1, N = 192,000) = 416.01, $p < 0.05$) on the confidence interval coverage for the residualized change score model when there was a direct effect present. As sample size, pretest correlation, and the *by2m1* path increased, the coverage decreased.

For N = 50 and effect size 0.25 the confidence interval coverage dropped below 92.5% when the *bm2y1* path and the pretest correlation both increased from 0.00 to 0.50, and when the *by2m1* path decreased from 0.50 to 0.00. For N =100 and effect size 0.09 and 0.25, the confidence interval coverage dropped to or below 92.5% when the *bm2y1* path increased from 0.00 to 0.50, and when the pretest correlation and the *by2m1* path both decreased from 0.50 to 0.00. It was also low for effect size 0.25 when the *bm2y1* path and the pretest correlation both increased from 0.00 to 0.50 and when the *by2m1* path decreased from 0.50 to 0.00. The coverage was also low for effect size 0.25 when the *bm2y1* path and the *by2m1* path both increased from 0.00 to 0.50, and when the pretest correlation decreased from 0.50 to 0.00. When the pretest correlation increased from 0.00 to 0.50, the coverage decreased for effect sizes 0.01 and 0.25. A similar pattern emerged for N = 200 and N = 500. The coverage exceeded 97.5% when the *by2m1* path increased from 0.00 to 0.50 and the *bm2y1* path decreased from 0.50 to 0.00 for a range of effect sizes and exceeded 97.5% when the effect size was small (0.01).

Insert Tables 21-22 about here

Cross Sectional.

Direct effect = 0.00. As shown in Table 23, as sample size and effect size increased, the confidence interval coverage of the cross-sectional model decreased (interaction of sample size and effect size, b = -0.10, χ^2 (1, N = 384,000) = 2,976, p < 0.05). The coverage decreased as a function of effect size more for large sample sizes than small sample sizes. There was also an interaction of effect size and the *by2m1* path (b = -0.11, χ^2 (1, N = 384,000) = 3,418.07, p < 0.05) such that as effect size and the *by2m1* path increased, coverage decreased. Coverage decreased as a function of effect size more when the *by2m1* path was equal to 0.50 than when the *by2m1* path was equal 0.00 with coverage being as low as 0.02 in one case. The coverage was the lowest for large effect sizes, large sample sizes, and when the *by2m1* path was equal to 0.50. There were also main effects of sample size (b = -0.23, χ^2 (1, N = 384,000) = 16,296.86, p < 0.05), effect size (b = -0.23, χ^2 (1, N = 384,000) = 16,855.86, p < 0.05), and the *by2m1* path (b = -0.21, χ^2 (1, N = 384,000) = 12,807.41, p < 0.05). In this case, there was no simple relation between either sample size, effect size, or the *by2m1* path and coverage.

For N = 50, confidence interval coverage fell below 92.5% for effect sizes 0.09, 0.15, and 0.25 when the *by2m1* path increased from 0.00 to 0.50. For N = 100, coverage decreased for effect size 0.25 when the *by2m1* path decreased from 0.50 to 0.00 and for effect sizes 0.09, 0.15, and 0.25 when the *by2m1* path increased from 0.00 to 0.50. For N = 200, coverage decreased for effect sizes 0.09, 0.15, and 0.25 when the *by2m1* path increased from 0.00 to 0.50 and decreased for effect sizes 0.01, 0.03, 0.09, 0.15, and 0.25 when the *by2m1* path increased from 0.00 to 0.50. For N = 500, coverage decreased for

effect sizes 0.01, 0.09, 0.15, and 0.25 when the *by2m1* path decreased from 0.50 to 0.00 and decreased for effect sizes 0.01 and higher when the *by2m1* path increased from 0.00 to 0.50. Confidence interval coverage never exceeded 97.5%.

Insert Table 23 about here

Direct effect = 0.30. As shown in Table 24, there was an additional interaction of sample size, effect size, and the *by2m1* path (b = -0.12, χ^2 (1, N = 192,000) = 2,243.01, p < 0.05) on the confidence interval coverage of the cross-sectional model when there was a direct effect present. That is, as sample size, effect size, and the *by2m1* path increased, the coverage decreased.

For N = 50 and N = 100 and effect sizes 0.09 and 0.25, the confidence interval coverage dropped below 92.5% when the *by2m1* path increased from 0.00 to 0.50. For N = 200 and N=500 and effect sizes 0.01, 0.09, and 0.25, the coverage was low when the *by2m1* path increased from 0.00 to 0.50. There was a significant three-way interaction of effect size, sample size, and *by2m1* path (b = -0.12, χ^2 (1, N = 192,000) = 2,243.01, p < 0.05) on the confidence interval coverage of the mediated effect for the cross-sectional model. The confidence interval coverage never exceeded 97.5%.

Insert Table 24 about here

Summary.

In summary, the confidence interval coverage for the ANCOVA, path analysis, and difference score model were similar and generally fell within the range of 92.5% - 97.5% coverage. The confidence interval coverage for the residualized change score model was low when the pretest correlation and both the *bm2y1* path and the *by2m1* path were equal to 0.50 and as sample size and effect size increased. The confidence interval coverage for the residualized change score model was comparable to the confidence interval coverage for the path analysis model when the *bm2y1* path was equal to 0.00 and whether or not the *by2m1* or the pretest correlation was equal to 0.00 or 0.50. The results for the confidence interval coverage for the cross-sectional model were low when the *by2m1* path was equal to 0.50 and as effect size and sample size increased. The confidence interval coverage for the cross-sectional model was comparable to the confidence interval coverage for the path analysis model when the *by2m1* path was equal to 0.00, when the sample size was small (N = 50), and when the effect size was less than 0.25.

Power Results

ANCOVA. As shown in Figure 2, as sample size and effect size increased simultaneously, the power to detect the mediated effect increased substantially (interaction of sample size and effect size, $b = 0.25$, χ^2 (1, $N = 384{,}000$) = 9,748.88, $p < 0.05$). Power increased as a function of effect size faster for large sample sizes (N = 500) compared to small sample sizes (N = 50). Power to detect the mediated effect using the ANCOVA model increased as sample size ($b = 0.36$, χ^2 (1, $N = 384{,}000$) = 27,071.26, $p <$

0.05) and effect size (b = 0.55, χ^2 (1, N = 384,000) = 61,739.35, p < 0.05) increased. Overall, power reached 0.80 or higher for N = 50 when the effect size was 0.25, for N = 100 when the effect size was 0.15 or higher, for N = 200 when the effect size was 0.09 or higher, and for N = 500 when the effect size was 0.09 or higher.

- Insert Figure 2 about here –

Path analysis. As shown in Figure 3, as sample size and effect size increased simultaneously, the power to detect the mediated effect increased substantially (interaction of sample size and effect size, b = 0.24, χ^2 (1, N = 384,000) = 8,738.57, p < 0.05). Power increased as a function of effect size faster for large sample sizes (N = 500) compared to small sample sizes (N = 50). Power to detect the mediated effect using the path analysis model increased as sample size (b = 0.35, χ^2 (1, N = 384,000) = 25,179.87, p < 0.05) and effect size (b = 0.56, χ^2 (1, N = 384,000) = 61,661.44, p <0.05) increased. Overall, power reached 0.80 or higher for N = 50 when the effect size was 0.25, for N = 100 when the effect size was 0.15 or higher, for N = 200 when the effect size was 0.09 or higher, and for N = 500 when the effect size was 0.09 or higher.

- Insert Figure 3 about here –

Difference scores. As shown in Figure 4, as sample size, effect size, and stability increased simultaneously, the power to detect the mediated effect increased substantially (interaction of sample size, effect size, and stability, b = 0.12, χ^2 (1, N = 384,000) = 3,220.13, p < 0.05). Power increased as sample size and effect size increased but power increased faster when stability was equal to 0.70 compared to when stability was equal to 0.30. Power to detect the mediated effect with the difference score model increased as

sample size increased (b = 0.31, χ^2 (1, N = 384,000) = 31,624.90, $p < 0.05$), effect size increased (b = 0.41, χ^2 (1, N = 384,000) = 46,292.63, $p < 0.05$), and stability increased (b = 0.19, χ^2 (1, N = 384,000) = 17,024.69, $p < 0.05$). Overall, power reached 0.80 or higher for N = 50 and stability = 0.70 when the effect size was 0.25, for N = 100 and stability = 0.70 when the effect size was 0.15 or higher, for N = 200 and stability = 0.70 for effect sizes 0.09 or higher, and for N = 500 and stability = 0.70 for effect sizes 0.09 or higher. Power never reached 0.80 when stability was equal to 0.30 for any combination of effect size and sample size.

- Insert Figure 4 about here –

Residualized change scores.

Direct effect = 0.00. A caveat should be mentioned. Because the residualized change score model resulted in biased estimates of the mediated effect, power results should be interpreted with caution. As shown in Figure 5, as sample size and effect size increased simultaneously the power to detect the mediated effect with the residualized change score model increased substantially (interaction of sample size and effect size, b = 0.29, χ^2 (1, N = 384,000) = 17,289.46, $p < 0.05$). Power increased as effect size increased but power increased faster for large sample sizes (N = 500) than small sample sizes (N = 50). Power to detect the mediated effect with the residualized change score model increased as sample size increased (b = 0.36, χ^2 (1, N = 384,000) = 38724.90, $p < 0.05$)

and effect size increased (b = 0.47, χ^2 (1, N = 384,000) = 58207.76, $p < 0.05$). Power decreased when the *bm2y1* path increased from 0.00 to 0.50 (b = -0.11, χ^2 (1, N = 384,000) = 1901.00, $p < 0.05$).

Overall, power reached 0.80 or higher for N = 50 and *bm2y1* path equal to 0.00 for effect size 0.25, for N = 100 and *bm2y1* path equal to 0.00 for effect size 0.15 or higher, for N = 200 and *bm2y1* path equal to 0.00 for effect size 0.09 or higher, for N = 500 and *bm2y1* path equal to 0.00 for effect size 0.09 or higher. Power also reached 0.80 or higher when the *bm2y1* path was equal to 0.50 for N = 200 and N = 500 and for effect sizes 0.15 or higher.

- Insert Figure 5 about here -

Direct effect = 0.30. As shown in Figure 6 and Figure 7, power decreased when either the *by2m1* or *bm2y1* paths increased from 0.00 to 0.50 for small sample sizes more than large sample sizes (interaction of sample size and *by2m1* path, b = -0.10, χ^2 (1, N = 192,000) = 705.35, $p < 0.05$, and interaction of sample size and the *bm2y1* path, b = -0.10, χ^2 (1, N = 192,000) = 739.36, $p < 0.05$). The decrease in power was more pronounced for N = 50, 100, and 200 but power was still low for N = 500 when either or both the *by2m1* path and *bm2y1* path increased from 0.00 to 0.50.

Power increased less as effect size increased when either the *by2m1* path or the *bm2y1* path was equal to 0.50 than when either *by2m1* path or the *bm2y1* path was equal to 0.00 (interactions of effect size and *by2m1* path, b = -0.10, χ^2 (1, N = 192,000) = 1,068.48, $p < 0.05$) and effect size and *bm2y1* path, b = -0.10, χ^2 (1, N = 192,000) = 1,047.38, $p < 0.05$). In general, power decreased when either the *by2m1* path was equal

to 0.50 (b = -0.13, χ^2 (1, N = 192,000) = 1,828.92, $p < 0.05$) or the *bm2y1* path was equal to 0.50 (b = -0.13, χ^2 (1, N = 192,000) = 1,976.48, $p < 0.05$).

Overall, power reached 0.80 or higher for N = 50 when the *bm2y1* path and the *by2m1* path were equal to 0.00 for effect size 0.25, for N = 100 when either the *bm2y1* path or the *by2m1* path were equal to 0.50 for effect size 0.25, for N = 200 when either the *bm2y1* path or the *by2m1* path was equal to 0.50 for effect size 0.09 or higher, for N = 500 when either the *bm2y1* path or the *by2m1* path was equal to 0.50 for effect size 0.09. The only case when power reached 0.80 or higher and when both the *bm2y1* path and the *by2m1* path were equal to 0.50 was for N = 500 and for effect size 0.25.

- Insert Figures 6 - 7 about here -

Cross-sectional.

Direct effect = 0.00. Because the cross-sectional model resulted in biased estimates of the mediated effect, power results should be interpreted with caution. As shown in Figure 8, as sample size and effect size increased simultaneously, the power to detect the mediated increased substantially (interaction of sample size and effect size, b = 0.24, χ^2 (1, N = 384,000) = 10,063.63, $p < 0.05$). Power increased as effect size increased faster for large sample sizes than small sample sizes. Power to detect the mediated effect with the cross-sectional model increased as sample size increased (b = 0.32, χ^2 (1, N = 384,000) = 28,117.66, $p < 0.05$) and effect size increased (b = 0.51, χ^2 (1, N = 384,000) = 62,557.86, $p < 0.05$). Overall, power reached 0.80 or higher for N = 50 when the effect size was 0.25, for N = 100 when the effect size was 0.25, for N = 200 when the effect size was 0.09 or higher, and for N = 500 when effect size was 0.09 or higher.

- Insert Figure 8 about here -

Direct effect = 0.30. As shown in Figure 9, power decreased as stability increased from 0.30 to 0.70 more for small sample sizes than large sample sizes (interaction of sample size and stability, b = -0.10, χ^2 (1, N = 192,000) = 553.42, p < 0.05) and power decreased as stability increased from 0.30 to 0.70 more for small effect sizes than large effect sizes (interaction of effect size and stability, b = -0.10, χ^2 (1, N = 192,000) = 897.22, p < 0.05). Power decreased as stability increased from 0.30 to 0.70 (b = -0.11, χ^2 (1, N = 192,000) = 1,108.92, p < 0.05). Overall, power reached 0.80 or higher for N = 50 and stability equal to 0.30 for effect size 0.25, for N = 100 when effect size was 0.25 regardless of stability (0.30 vs. 0.70), for N = 200 when effect size was 0.09 and higher regardless of stability (0.30 vs. 0.70), and for N = 500 for effect size 0.09 or higher regardless of stability (0.30 vs. 0.70).

- Insert Figure 9 about here -

Summary.

In summary, the power results for the ANCOVA, path analysis, and difference score models were similar for direct effect equal to 0.00 and direct effect equal to 0.30 conditions. It was expected the ANCOVA and path analysis model results would be similar because they are identical models with the exception that ANCOVA uses ordinary least squares (OLS) to estimate model parameters whereas path analysis uses maximum likelihood (ML) to estimate model parameters.

All models achieved power of 0.80 or higher for some combinations of effect size of the mediated effect and sample size. The ANCOVA and path analysis models achieved power of 0.80 or higher with the lowest combination of effect size for the mediated effect and sample size. The difference score model readily achieved power of 0.80 or higher as long as stability was equal to 0.70. The difference score model did not achieve power of 0.80 or higher for any combination of effect size and sample size when stability was equal to 0.30. The residualized change score model achieved power of 0.80 or higher unless both the *bm2y1* path and the *by2m1* path were equal to 0.50. When the *bm2y1* path and the *by2m1* path were equal to 0.50, the residualized change score model did not achieve power of 0.80 or higher regardless of effect size or sample size. The cross-sectional model performed similarly to the ANCOVA and path analysis models.

ANCOVA vs. Cross-sectional. Because the cross-sectional model resulted in biased estimates of the mediated effect, power results should be interpreted with caution. When comparing the significance across models for some power analyses (e.g., ANCOVA vs. cross-sectional), the dependent variable was created by taking the absolute value of the difference in conclusions each model reached regarding statistical significance of the mediated effect. For example, for each model, the dependent variable was '0' for 'non-significant mediated effect' and '1' for 'significant mediated effect. When comparing the performance of one model to another (e.g., ANCOVA vs. cross-sectional) the absolute value of the difference of this value (0 or 1) for each model was taken as the dependent variable to use for model comparisons of statistical significance of the mediated effect. The coding scheme resulted in a variable that was coded '1' if, for that particular observation, there was a difference in the statistical conclusion regarding

the significance of the mediated effect across the two models and coded '0' if, for that particular observation, there was no difference in the statistical conclusion regarding the significance of the mediated effect. The same procedure was followed for all comparisons across models. These new variables were analyzed with logistic regression analyses previously described in the type 1 error rates section.

As shown in Figure 10, there was an interaction of sample size and effect size on the differences in power across the ANCOVA and cross-sectional model (b = -0.19, χ^2 (1, N = 384,000) = 10,497.68, $p < 0.05$). The cross-sectional model had more power than the ANCOVA model for effect size of 0.01 for N = 50 to 500 and for effect sizes 0.03 and 0.05 for N = 50. The ANCOVA model had more power than the cross-sectional model across all other combinations of sample size and effect size. There were also main effects of sample size (b = -0.11, χ^2 (1, N = 384,000) = 4,357.81, $p < 0.05$) and effect size (b = -0.14, χ^2 (1, N = 384,000) = 7,308.63, $p < 0.05$) but these main effects were not interpreted because there was no simple relation between either sample size or effect size and differences in power across the two models.

- Insert Figure 10 about here -

Path Analysis vs. Cross-sectional. As shown in Figure 11, the cross-sectional model had more power than the path analysis model for effect size of 0.01 for N = 50 to 500 and for effect sizes 0.03 for N = 50 (interaction of sample size and effect size, b = -0.19, χ^2 (1, N = 384,000) = 10,481.10, $p < 0.05$). The path analysis model had more power than the cross-sectional model across all combinations of sample size and effect size. There were also main effects of sample size (b = -0.11, χ^2 (1, N = 384,000) =

4,461.01, $p < 0.05$) and effect size (b = -0.14, χ^2 (1, N = 384,000) = 7,442.22, $p < 0.05$) but there was no simple relation between either sample size or effect size and the differences in power across the two models.

- Insert Figure 11 about here -

Difference Scores vs. Cross-sectional.

Direct effect = 0.00. As shown in Figure 12 and Figure 13, there was an interaction of effect size, stability, and *by2m1* path (b = 0.10, χ^2 (1, N = 384,000) = 3,099.08, $p < 0.05$) such that the cross-sectional model had more power than the difference score model for all effect sizes and values of the *by2m1* path when the stability was 0.30. When stability increased to 0.70, the power for the cross-sectional model and difference score model became more similar except for a discrepancy at effect size 0.15 and 0.25 for N =50, when the *by2m1* path was equal to 0.00, and for effect size 0.15 for N = 100.

As sample size (b = -0.10, χ^2 (1, N = 384,000) = 2,709.765, $p < 0.05$) and stability (b = -0.13, χ^2 (1, N = 384,000) = 6,778.41, $p < 0.05$) increased, the differences in power across the difference score model and the cross-sectional model became less pronounced. Overall, the differences in power between the two models was affected less by the inclusion of the *by2m1* path and became less discrepant as sample size and stability increased.

- Insert Figures 12 - 13 about here -

Direct effect = 0.30. As shown in Figure 14, when the direct effect was present there was no longer an interaction of effect size, stability and the *by2m1* path. Power across the difference score model and the cross-sectional model did become less discrepant for medium to large effect sizes as sample size increased (interaction of sample size and effect size, b = -0.23, χ^2 (1, N = 192,000) = 5,551.16, $p < 0.05$). The difference in power across these models was almost zero for a small effect size (0.01).

The cross-sectional model had more power than the difference score model for all effect sizes when stability was 0.30 but as stability increased to 0.70, the power for the cross-sectional model and difference score model was almost identical (interaction of effect size and stability, b = -0.13, χ^2 (1, N = 192,000) = 2,308.15, $p < 0.05$). In this case, the difference score model had more power than the cross-sectional model for effect size 0.25 for N = 50. Overall, the difference in power between the difference score model and the cross-sectional model became less discrepant for medium to large effect sizes as sample size and stability increased.

- Insert Figure 14 about here -

Residualized Change scores vs. Cross-sectional. As shown in Figure 15, the residualized change score model had more power than the cross-sectional model for effect sizes 0.10 and 0.15 for N = 100, effect sizes 0.03, 0.05, and 0.15 for N = 200, and for effect sizes 0.03 and 0.05 for N = 500 (interaction of sample size and effect size, b = 0.22, χ^2 (1, N = 384,000) = 11,984.99, $p < 0.05$). The cross-sectional model had more power than the residualized change score model for all other combinations of sample size and effect size. There were also main effects of sample size (b = -0.13, χ^2 (1, N =

384,000) = 5,150.72, $p < 0.05$) and effect size (b = -0.11, χ^2 (1, N = 384,000) = 4,330.22, $p < 0.05$) but there was no simple relation between either sample size or effect size and the differences in power across the two models.

- Insert Figure 15 about here –

Path analysis vs. Difference score.

Because the path analysis model and the ANCOVA model are identical models, only comparisons between the path analysis model and the difference score model and the residualized change score model were presented. The path analysis model was treated as the "ideal" model to use as a benchmark for the other models because it estimates all the parameters of the covariance structure in the pretest – posttest control group design with a mediating variable.

Direct effect = 0.00. As shown in Figures 16 – 17, there was more discrepancy in the power across the path analysis and difference score models when sample sizes were small and stability was equal to 0.30 (interaction of sample size and stability, b = -0.10, χ^2 (1, N = 384,000) = 1,069.97, $p < 0.05$), more discrepancy in the power results for medium to large effect sizes for small sample sizes (interaction of sample size and effect size, b = -0.24, χ^2 (1, N = 384,000) = 5,326.53, $p < 0.05$), and more discrepancy in the power results for medium to large effect sizes when the stability was equal to 0.30 (interaction of effect size and stability, b = -0.15, χ^2 (1, N = 384,000) = 2,857.50, $p < 0.05$).

The discrepancy between the models decreased as both sample size (b = -0.14, χ^2 (1, N = 384,000) = 1,935.26, $p < 0.05$) and stability increased (b = -0.22, χ^2 (1, N =

384,000) = 7,394.77, $p < 0.05$). The discrepancy between the model became more extreme when the *by2m1* path increased from 0.00 to 0.50 ($b = 0.10$, χ^2 (1, N = 384,000) = 1,539.44, $p < 0.05$). Overall, there is a large discrepancy in power across the path analysis and difference score model for N = 50 – 100, when stability was equal to 0.30 and when the *by2m1* path was equal to 0.50 for effect size 0.09 and higher. The discrepancy between the models became less extreme as stability increased to 0.70 and there was less discrepancy for N = 200 – 500 regardless of the value of the *by2m1* path and effect size.

- Insert Figures 16 – 17 about here –

Direct effect = 0.30. As shown in Figures 18 – 21, when the direct effect was present, there was no longer an interaction between sample size and stability. All effects that were present when the direct effect equaled 0.00 were present with the addition of an interaction of the *bm2y1* path and the *by2m1* path ($b = -0.10$, χ^2 (1, N = 192,000) = 406.04, $p < 0.05$) such that the discrepancy between the models was more pronounced when the *bm2y1* path was equal to 0.50 and the *by2m1* path was equal to 0.50. There was an interaction of effect size and the *by2m1* path ($b = 0.10$, χ^2 (1, N = 192,000) = 493.51, $p < 0.05$) such that the discrepancy was more pronounced for medium to large effect sizes when the *by2m1* path was equal to 0.50. There was a main effect of the *by2m1* path ($b = 0.13$, χ^2 (1, N = 192,000) = 690.86, $p < 0.05$) but there was not a simple relation between the *by2m1* path and the differences in power across the two models. Overall, there was a large discrepancy in power across the path analysis model and the difference score model when stability was equal to 0.30 and either the *bm2y1* path or the *by2m1* path were equal

to 0.50. As stability increased to 0.70, the discrepancy between the models virtually disappeared for all combinations of effect size, sample size, and values of the *bm2y1* path and the *by2m1* path.

- Insert Figures 18 – 21 about here –

Path analysis vs Residualized change score. Because the residualized change score model resulted in biased estimates of the mediated effect, power results should be interpreted with caution. As shown in Figures 22 – 23, the discrepancy between the path analysis model and the residualized change score model was more pronounced for medium to large effect sizes for small sample sizes than large sample sizes (interaction of sample size and effect size, b = -0.24, χ^2 (1, N = 384,000) = 3,513.36, $p < 0.05$). The discrepancy between the models became less pronounced as sample size increased (b = -0.19, χ^2 (1, N = 384,000) = 2,417.01, $p < 0.05$), as effect size increased (b = -0.12, χ^2 (1, N = 384,000) = 1,095.76, $p < 0.05$), when the *bm2y1* path decreased from 0.50 to 0.00 (b = 0.20, χ^2 (1, N = 384,000) = 3761.27, $p < 0.05$) and when the *by2m1* path decreased from 0.50 to 0.00 (b = 0.09, χ^2 (1, N = 384,000) = 813.10, $p < 0.05$). Overall, there was a large discrepancy in power across the path analysis model and the residualized change score model when either the *bm2y1* path or the *by2m1* path were equal to 0.50 for medium to large effect sizes for N = 50 to 100. When the sample size increased to N = 200 and higher, there was only a discrepancy between the models when both the *bm2y1* path and the *by2m1* path were equal to 0.50 for medium to large effect sizes.

- Insert Figures 22 – 23 about here –

Summary.

In summary, the power results across the ANCOVA and path analysis models compared to the cross-sectional model were generally not discrepant except when the effect size of the mediated effect was small (0.01). The power results across the difference score model compared to the cross-sectional model were discrepant when stability was equal to 0.30 and the *by2m1* path was equal to 0.50. Any discrepancies between these two models went away when stability increased to 0.70 and sample size increased. The power results across the residualized change score model compared to the cross-sectional model were discrepant for numerous values of effect size of the mediated effect and across all sample sizes. Despite the discrepancy for numerous conditions, the two models had similar power curves across all combinations of sample size and effect size.

The power results across the path analysis model compared to the difference score model were discrepant when stability was equal to 0.30 and either the *bm2y1* path or the *by2m1* path was equal to 0.50. Any discrepancies between these models generally went away as stability increased to 0.70 and sample size increased regardless of the value of the *bm2y1* path or the *by2m1* path. The power results across the path analysis model compared to the residualized change score model were discrepant when the *bm2y1* path or the *by2m1* path was equal to 0.50. The discrepancies between these models went away as sample size increased and as either the *bm2y1* or the *by2m1* path decreased to 0.00.

Ancillary Analyses

A comparison was made between the sample size needed to detect the mediated effect of a given effect size to achieve power of 0.80 or greater from this simulation study

and the sample size needed to detect a mediated effect to achieve power of 0.80 from Fritz and MacKinnon (2007). As shown in Table 25, there was a general consensus with the sample size and effect size combination that was needed to detect the mediated effect with power of 0.80 or higher in this simulation study compared to the sample size and effect size combination that was needed to detect a mediated effect with 0.80 power from Fritz and MacKinnon (2007). One discrepancy occurs when the effect size of the mediated effect was small (0.02 – 0.03). In this simulation study, when the effect size was small (0.03) and sample size was equal to 500, the power to detect a significant mediated effect with the cross-sectional model was 0.46. Fritz and MacKinnon (2007) demonstrated that when the effect size of the mediated effect was small (0.02) a sample size of 533 was required to detect a significant mediated effect. Therefore, the cross-sectional estimate of the mediated effect is under-powered for small effect sizes when the underlying model is a longitudinal mediation model.

A summary index was created that represented the percentage of times a mediated effect was detected for each of the five models as a way to gauge the consensus regarding the significance of the mediated effect across all five models. This index was created as a five digit number with each digit representing one of the five models' decision of detecting a significant mediated effect (i.e., labeled '1') or not detecting a significant mediated effect (i.e., labeled '0'). The first digit in this index represented the path analysis model, the second represented the ANCOVA model, the third represented the residualized change score model, the fourth represented the difference score model, and the fifth represented the cross-sectional model. For example, the first row of Table 26 is the five digit number '00000' with percent equal to 36.18% which indicates that all five

models reached the same conclusion of not detecting the mediated effect 36.18% of time. The second row is the five digit number '00001' with percent of occurrence of 3.79% which indicates that less than 4% of the time the only model that indicated a significant mediated effect was the cross-sectional model.

Insert Table 25 about here

For all models, the same conclusion of not detecting a mediated effect (36.18%) and the same conclusion of detecting the mediated effect (29.29%) was observed. The difference score model was the only model that did not detect the mediated effect 10.84% of the time. The residualized change score model was the only model that did not detect the mediated effect 4.14% of the time. The cross-sectional model was the only model that did not detect the mediated effect 3.81% of the time and it was the only model to detect the mediated effect 3.79% of the time. Further, 3.47% of the time the residualized change score and difference score models did not detect the mediated effect while the other models did. Lastly, 1.09% of the time the cross-sectional and difference score models did not detect the mediated effect while the other models did and 1.17% of the time the cross-sectional and residualized change score models did not detect the mediated effect while the other models did. Overall, there is a high percentage of occurrence of all models reaching the same conclusion (65.47%) regarding detecting either a significant mediated effect or a non-significant mediated effect.

Insert Table 26 about here

A two-dimensional plot was created using tetrachoric correlations between the power of each of the five models. The dimension on the y-axis represents whether or not the models estimated the stability of the mediator and outcome variables across time and how they estimated the stability and the dimension on the x-axis represents how the models that did not measure the stability were differentially affected by the stability. The ANCOVA, path analysis, and residualized change score models all estimated the stability of the mediator and outcome variables but they did so differently therefore they are clustered close together and separated from the cross-sectional and difference score models. The cross-sectional and difference score models were affected differently by stability because neither the cross-sectional nor difference score model estimated the stability and they each made different assumptions regarding the stability. For example, the difference score model assumes that the stability is equal to 1.0 and we can think of the cross-sectional model as assuming the stability is equal to 0.0 which explains why each of these models are opposite each other on the dimension 'Effect of stability'.

- Insert Figure 24 about here –

Summary

Overall, the best performing models were the ANCOVA and the path analysis model in terms of type 1 error, bias, confidence interval coverage, and power. The

difference score model performed well in terms of type 1 error, bias, confidence interval coverage, and power but it never outperformed the ANCOVA or path analysis models. The performance of the residualized change score and cross-sectional models were negatively affected when the pretest correlation was non-zero between the mediator and the outcome and when there were non-zero cross-lagged paths (*by2m1* and *bm2y1*). When the pretest correlation was zero and the cross-lagged paths were zero, the performance of the residualized change score and cross-sectional models were comparable to the ANCOVA and path analysis models in this simulation but these models (residualized change score and cross-sectional) did not consistently outperform the ANCOVA and path analysis models.

Discussion

The aim of this project was to compare four common longitudinal models and the cross-sectional model for assessing the mediated effect in the pretest – posttest control group design. The main conclusion of this project is that the only models that were not biased, did not have inflated Type 1 error rates, and had high empirical power were the ANCOVA and path analysis models. Recall the effect size of the mediated effect in this project was the product of the effect size of the coefficient relating the treatment to the mediator at posttest and the coefficient relating the mediator at posttest to the outcome at posttest adjusting for all other effects in the model. The effect sizes for each of the coefficients were picked to reflect small, medium, and large effects in the correlation metric with the effect size of the mediated effect being the product of these effect sizes. When the sample size was small (N = 50), the ANCOVA and path analysis models reached 0.80 or higher when the effect size of the mediated effect was 0.25 (which

corresponds to a large effect for both the *am2x* and *by2m2* paths). When the sample size was slightly larger (N = 100), the ANCOVA and path analysis models reached 0.80 when the effect size of the mediated effect was 0.15 (which corresponds to a medium and large effect size for either the *am2x* or *by2m2* path). Finally, when the sample size was large (N = 200 to N = 500), the ANCOVA and path analysis models reached 0.80 power when the effect size of the mediated effect was 0.09 (which corresponds to medium effects for both the *am2x* and *by2m2* paths) or higher.

The ANCOVA and path analysis models performed the best because they estimated every parameter of the covariance structure for mediation in the pretest – posttest control group design. That is, these models took into account the pretest correlation between the mediator and the outcome, the stability of the mediator and the outcome, and any cross-lagged relations between the pretest measures and the posttest measures. Further, the ANCOVA and path analysis models performed the best because they most closely matched the data-generating model of this simulation study. Researchers should use either the ANCOVA or path analysis models when assessing the mediated effect in the pretest – posttest control group design.

The difference score, residualized change score, and the cross-sectional model all produced biased estimates of the mediated effect or had low empirical power for some conditions of this simulation study. The difference score model was generally not biased even when there was a pretest correlation and cross-lagged relations. Despite not producing a biased estimate of the mediated effect, the difference score model did not have as high empirical power as did the ANCOVA/path analysis model and the empirical

power of the difference score model decreased as stability of the mediator and outcome decreased. The residualized change score model and the cross-sectional model produced biased estimates of the mediated effect when either of the cross-lagged paths were present. The estimate of the mediated effect also became more biased when there was a pretest correlation compared to when the pretest correlation was not present for the residualized change and cross-sectional model. However, when the cross-lagged paths and the pretest correlation was zero in the population model, the residualized change score and cross-sectional models did not produce biased estimates of the mediated effect. The residualized change score model had comparable power to the ANCOVA/path analysis model when the pretest correlation and cross-lagged paths were zero. The cross-sectional model had lower power than the ANCOVA/path analysis model except in a few instances when the sample size was small and the effect size of the mediated effect was small. The empirical power to detect the mediated effect with the cross-sectional model decreased as stability of the mediator and outcome increased.

Because the difference score model did not produce a biased estimate of the mediated effect and because type 1 error rates were not above the nominal 0.05 alpha level, a researcher could estimate the mediated effect with the difference score model and have empirical power similar to the ANCOVA or path analysis models as long as the stability of the mediator and outcome is high. The residualized change score model requires that more conditions are met in order for unbiased estimation of the mediated effect and empirical power similar to the ANCOVA and path analysis models. If there is no pretest correlation and no cross-lagged relations, a researcher could estimate the mediated effect with the residualized change score model and produce an unbiased

estimate of the mediated effect with comparable albeit lower power than the ANCOVA/path analysis model. In order for the cross-sectional model to unbiasedly estimate the mediated effect in the pretest – posttest control group design, similar conditions are needed. The mediated effect will not be biased using the cross-sectional model when there are no cross-lagged paths and when there is no pretest correlation between the mediator and the outcome. Additionally, the power to detect the mediated effect with the cross-sectional model will be comparable to the power to detect the mediated effect using the ANCOVA/path analysis model if the pretest correlation and cross-lags are zero and if stability of the mediator and outcome is low. Given a pretest – posttest control group design with low stability of the mediator and outcome, no pretest correlation, and no cross-lagged relations, a researcher could estimate the mediated effect with the cross-sectional model and have empirical power comparable to the ANCOVA or path analysis models and have an unbiased estimate of the mediated effect.

Although there exist conditions for which a researcher could estimate the mediated effect for all the models mentioned, it is not recommended for researchers to use the difference score, residualized change score, or cross-sectional models when estimating mediated effects in the pretest – posttest control group design because there were no conditions for which these models outperform the ANCOVA/path analysis model. Also, it may be difficult to find evidence of these conditions in sample data (especially with small samples) so either the ANCOVA or the path analysis model would be the best model for this research design.

The results provide practical insight about the conditions needed when using the cross-sectional, difference score, and residualized change score models. That is, when researchers use the cross-sectional, difference score, or residualized change score model to estimate mediated effects in the pretest – posttest control group design, there are additional conditions that need to be met regarding the relation between the pretest measures of the mediator and outcome variable and conditions that need to be met regarding the relations between the pretest measures and the posttest measures of the mediator and outcome variables. These additional conditions may be untenable but if they are satisfied, then the estimation of mediated effects can be unbiased and have empirical power similar to the ANCOVA or path analysis models.

Implications

Researchers can apply several different models when assessing mediated effects in the pretest – posttest control group design including the cross-sectional model, the difference score model, the residualized change score model, ANCOVA, and path analysis. Given the findings of this study, anytime researchers use the cross-sectional model (Jouriles et al., 2010), the difference score model (Hofmann, 2004; Jansen et al., 2012; MacKinnon et al., 1991), or the residualized change score model (Cole, et al., 2003; Miller, et al., 2002; Reid & Aiken, 2013) they are making assumptions about the conditions that need to be met regarding the relations between the pretest variables and any cross-lagged relations across time. In general, the assumption is there are no relations between the pretest variables or cross-lagged relations. When these conditions are met the cross-sectional model and residualized change score model result in biased

mediated effect estimates and low empirical power to detect the mediated effect even when these conditions are met. The difference score model never resulted in biased estimates of the mediated effect but it did have low empirical power in some case. If researchers use the cross-sectional, change score, or residualized change score model, they may be inadvertently missing true mediated effects that are present and they may be reporting biased estimates of true mediated effects.

Previous research has expounded on the limitations of using cross-sectional data to estimate mediated effects (Gollob & Reichardt, 1991; Maxwell & Cole, 2007; Maxwell, Cole, & Mitchell, 2011). Cross-sectional estimates of mediated effects will often be biased because they do not allow for mediating variables (M) to exert their influence on outcome variables (Y), which presumably occurs over a specific period of time. As time interval varies, so do estimates of mediated effects because estimates of mediated effects depend on the time interval during which they are assessed. This study confirms the general findings of previous literature regarding the bias of the cross-sectional mediated effect as an estimate of a longitudinal mediated effect. The implications of the findings of this study is that there are some conditions for which the cross-sectional estimate of the mediated effect is unbiased (i.e., when there is no pretest correlation or cross-lagged relations) and that there were no cases for which the cross-sectional model ever resulted in type 1 error rates above the nominal 0.05 alpha level. These implications build on previous research and provide a more detailed picture of when the cross-sectional model will result in biased estimates of longitudinal mediated effects and how this affects empirical power and type 1 error rates.

Limitations

This project compared longitudinal models and the cross-sectional model in the pretest – posttest control group design. Recall, the pretest – posttest control group design involves the random assignment of units to either a treatment or a control group. Successful randomization of units to groups in this experimental design ensures that any pre-existing differences between the units in the treatment and control groups are due to chance and do not reflect systematic differences. Therefore, the results of this project do not necessarily extend to situations for which there are systematic pre-existing differences between the units in groups that are to be compared.

Previous research demonstrated the effect of the stability of X and M was an important predictor of bias of the cross-sectional estimate of the mediated effect when assessing longitudinal mediation processes such that bias of the mediated effect was different when X was more stable than M compared to when M was more stable than X (Maxwell & Cole, 2007; Maxwell, Cole, & Mitchell, 2011). This study only had one measure of X because it corresponded to an experimental manipulation but the stability of M and Y was varied in this project. The stability of M and Y was manipulated simultaneously to the same value, so there were no conditions for which the stability of M and the stability of Y were different from each other. Because stability of M and Y was not varied separately, this project was unable to confirm similar findings regarding the bias of the cross-sectional estimate of the mediated effect when variables in the model have different stability values from one another.

This project assumed that the mediator variable and outcome variable were measured without any measurement error. That is, it assumed the measures of the mediator and outcome variable were perfectly reliable. This project also assumed that the units randomized to the treatment and control groups fully adhered to the assigned treatment condition. That is, this project assumed that there was complete treatment compliance when assessing the mediated effect.

A critique of Monte Carlo simulation studies is that the results of the simulation typically favor the model that was used to generate the simulated data. This is not necessarily a limitation of this study. The data-generating model for this study was the path analysis model because it was the most general model that could have been used to generate data for this study. The cross-sectional, difference score, and residualized change score models are each special cases of the path analysis model (when there is no pretest correlation or cross-lagged relations present) which can each be estimated with specific constraints put in place. Further, it would be impossible to generate data from a purely cross-sectional model (i.e., only one time point of data generated) and be able to estimate the parameters of the longitudinal models in this study (i.e., ANCOVA/path analysis, difference score, and residualized change score). For example, it would be impossible to estimate a pretest correlation between the mediator and outcome variable if data were not generated for these pretest variables.

Recall, a fully-standardized logistic regression coefficient was used in the type 1 error, confidence interval coverage, and power analyses as a rough proxy for an effect size measure. Standardized regression coefficients are generally not used as standardized

measures of effect size but do allow all the predictors in a given analysis to be on the same metric. Because standardized regression coefficients are not true standardized effect size measures, it is possible the standardized regression coefficients did not detect all of the practically significant effects in this study. To further explore this possibility, a subset of the analyses were conducted using alternate measures of effect size including: reduction in pseudo R^2 (Demaris, 2002), odds ratios (Fleiss & Berlin, 2009), and omega-squared measure of effect size from an Ordinary Least Squares analysis. All these methods reached the same conclusions regarding the practically significant effects in this study. Although standardized regression coefficients are not generally used as standardized measures of effect size, they seemed to perform similarly to more typical standardized measures of effect size. Ideally, a more traditional measure of effect size for binary outcomes (e.g., reduction in pseudo R^2 or odds ratios) would have been used at the outset of this study instead of fully-standardized regression coefficients to determine the practically significant effects but it does not seem that the results of this study would have changed.

Future Directions

Future directions for this research directly extend from the limitations of this project. The first future direction would be to compare the performance of the models investigated in this project to the case for which there exist systematic pre-existing differences between the groups of interest being compared. It is known from previous research that ANCOVA and difference score models can lead to very different results regarding change across two-waves of data (Jamieson, 1999; Kisbu-Sakarya,

MacKinnon, & Aiken, 2013; Lord, 1967; Pearl, 2014; Wright, 2006) but further work is needed in order to examine which of the four longitudinal models discussed in this project would perform the best for estimating mediated effects when systematic pre-existing differences exist.

Another direction of interest is the performance of these models for assessing the mediated effect in the pretest – posttest control group design when the mediator and outcome variable are not measured reliably. Unreliable measures of the mediator and outcome variables can substantially bias estimates of the mediated effect in most cases but the pattern of results can be complicated and even counter-intuitive in some cases (Baraldi, Valente, & MacKinnon, 2014; Fritz, MacKinnon, & Kenny, 2014; Hoyle & Kenny, 1999).

Another potential future direction would be to compare the longitudinal models discussed in this project when there is treatment noncompliance in the pretest – posttest control group design. Treatment noncompliance can lead to biased estimates of treatment effects when using traditional statistical methods in experimental designs (Angrist, Imbens, & Rubin, 1996; Efron & Feldman, 1991; Sagarin, et al., 2014). It is important to see how the longitudinal models presented in this project perform with causal estimators of the mediated effect in the pretest – posttest control group design when there is not complete treatment compliance.

It is possible to extend the pretest – posttest control group design to more than two waves of data (e.g., 3 or more waves of data) and have mediation effects across all waves. The addition of more waves of data may complicate the estimation of mediation

effects (Cole & Maxwell, 2003; MacKinnon, 1994; 2008). That is, all the assumptions and effects regarding stability, timing of effects, and cross-lagged relations across two waves of data will now apply across three or more waves of data. For example, if there are three waves of data, there will be stability between the mediator and outcome at wave 1 and wave 2 and wave 2 and wave 3. The cross-lagged relations between wave 1 and wave 2 will be extended to include cross-lagged relations between wave 2 and wave 3 and the estimate of the mediated effect will vary depending on the time at which it is estimated. For example, when there are three waves of data including a pretest measure prior to a treatment exposure, a researcher could estimate a cross-sectional mediated effect at wave 2 and wave 3 separately and a researcher could estimate longitudinal mediation effects from treatment exposure to mediator at wave 2 and outcome at wave 2, mediator at wave 2 and outcome at wave 3, and mediator at wave 3 and outcome at wave 3. Also, similar to how the difference score and residualized change score models were used in the pretest – posttest control group design to reduce the number of waves from two to one, these models could be used in a design consisting of three waves of data to reduce the number of waves from 3 to 2.

Generally, there are three autoregressive longitudinal models that a researcher could consider when estimated longitudinal mediation effects which deserve further investigation as outlined by MacKinnon (2008) and experimental designs that vary time lags between units which allow for the estimation of the effects of varying time lags to address not only how variables are related but *when* variables are related (Selig, Preacher, Little, 2012).

Finally, the longitudinal models discussed in this project all handled the pretest information on the mediator and outcome variable in different ways (e.g., condition on it or remove it via difference scores). There are additional ways derived from the potential outcomes model of causal inference that would handle the pretest information in yet different ways than the models discussed in this project. It is possible to remove the effect of the pretest measures on the outcome variable at posttest through a series of regression equations using Sequential G-estimation (Vansteelandt, 2009) or to inversely weight observations based on pretest scores on the mediator and outcome using inverse propensity weighting analyses (IPW; Coffman, 2011; Robins, Hernan, & Brumback, 2000). Both of these estimation techniques could provide new ways to unbiasedly estimate the mediated effect in the pretest – posttest control group design.

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Table 1
All combinations of effect size adopted from MacKinnon, Lockwood, and Williams (2004)

a	*b*	*c'*
0	0	0
0	.10	0
0	.30	0
0	.50	0
.10	.10	0
.30	.30	0
.50	.50	0
.10	.30	0
.10	.50	0
.30	.50	0
.10	.10	.30
.30	.30	.30
.50	.50	.30

Table 2

PRODCLIN Type 1 error rates of the mediated effect for N = 50 averaged across all combinations of a and b = 0

	Stability									
	0.3					0.7				
	ANCOVA	Path	Diff	Res	Cross	ANCOVA	Path	Diff	Res	Cross
Pretest Correlation										
0	0.02	0.02	0.01	0.01	0.04	0.02	0.02	0.01	0.01	0.04
0.5	0.02	0.02	0.01	0.01	0.04	0.02	0.02	0.02	0.01	0.05

Table 3

PRODCLIN Type 1 error rates of the mediated effect for N = 100 averaged across all combinations of a and b = 0

	Stability									
	0.3					0.7				
	ANCOVA	Path	Diff	Res	Cross	ANCOVA	Path	Diff	Res	Cross
Pretest Correlation										
0	0.02	0.03	0.02	0.02	0.04	0.03	0.03	0.02	0.02	0.04
0.5	0.03	0.03	0.02	0.02	0.04	0.02	0.03	0.02	0.01	0.05

Table 4

PRODCLIN Type 1 error rates of the mediated effect for N = 200 averaged across all combinations of a and b = 0

	Stability									
	0.3					0.7				
	ANCOVA	Path	Diff	Res	Cross	ANCOVA	Path	Diff	Res	Cross
Pretest Correlation										
0	0.03	0.03	0.03	0.02	0.04	0.03	0.03	0.02	0.02	0.04
0.5	0.03	0.03	0.03	0.02	0.05	0.03	0.03	0.03	0.02	0.05

Table 5

PRODCLIN Type 1 error rates of the mediated effect for N = 500 averaged across all combinations of a and b = 0

	Stability									
	0.3					0.7				
	ANCOVA	Path	Diff	Res	Cross	ANCOVA	Path	Diff	Res	Cross
Pretest Correlation										
0	0.03	0.03	0.04	0.03	0.05	0.03	0.03	0.03	0.03	0.05
0.5	0.03	0.03	0.04	0.03	0.05	0.03	0.03	0.03	0.03	0.05

Table 6

Bias, Relative bias, and Standardized bias of the mediated effect for the ANCOVA model

	N											
	50			100			200			500		
	Bias	Rel Bias	Stand Bias	Bias	Rel Bias	Stand Bias	Bias	Rel Bias	Stand Bias	Bias	Rel Bias	Stand Bias
True Value												
0	0.00	N/A	0.00	0.00	N/A	0.00	0.00	N/A	0.01	0.00	N/A	0.00
0.01	0.00	0.03	0.01	0.00	-0.02	-0.01	0.00	-0.01	-0.01	0.00	0.00	0.00
0.03	0.00	-0.02	-0.01	0.00	0.01	0.00	0.00	0.00	0.01	0.00	-0.01	-0.02
0.05	0.00	-0.01	-0.01	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.01	0.01
0.09	0.00	0.00	-0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
0.15	0.00	0.00	0.01	0.00	0.00	0.00	0.00	0.00	0.01	0.00	0.00	0.00
0.25	0.00	0.00	0.01	0.00	0.00	-0.01	0.00	0.00	-0.01	0.00	0.00	0.01

Table 7

Bias, Relative bias, and Standardized bias of the mediated effect for the Path analysis model

	N											
	50			100			200			500		
	Bias	Rel Bias	Stand Bias	Bias	Rel Bias	Stand Bias	Bias	Rel Bias	Stand Bias	Bias	Rel Bias	Stand Bias
True Value												
0	0.00	N/A	0.00	0.00	N/A	0.00	0.00	N/A	0.01	0.00	N/A	0.00
0.01	0.00	0.03	0.01	0.00	-0.02	-0.01	0.00	-0.01	-0.01	0.00	0.00	0.00
0.03	0.00	-0.02	-0.01	0.00	0.01	0.00	0.00	0.00	0.01	0.00	-0.01	-0.02
0.05	0.00	-0.01	-0.01	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.01	0.01
0.09	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
0.15	0.00	0.00	0.01	0.00	0.00	0.00	0.00	0.00	0.01	0.00	0.00	0.00
0.25	0.00	0.00	0.01	0.00	0.00	-0.01	0.00	0.00	-0.01	0.00	0.00	0.01

Table 8

Bias, Relative bias, and Standardized bias of the mediated effect for the Difference score model

	N											
	50			100			200			500		
	Bias	Rel Bias	Stand Bias	Bias	Rel Bias	Stand Bias	Bias	Rel Bias	Stand Bias	Bias	Rel Bias	Stand Bias
True Value												
0	0.00	N/A	0.00	0.00	N/A	0.01	0.00	N/A	0.00	0.00	N/A	0.00
0.01	0.00	0.01	-0.01	0.00	0.00	-0.01	0.00	0.01	0.01	0.00	0.00	0.00
0.03	0.00	0.07	-0.02	0.00	0.06	0.00	0.00	-0.02	0.00	0.00	0.00	-0.01
0.05	0.00	-0.00	-0.01	0.00	-0.00	-0.01	0.00	0.00	0.01	0.00	0.01	0.01
0.09	0.00	-0.08	0.00	0.00	0.02	-0.01	0.00	-0.01	0.01	0.00	0.00	0.00
0.15	0.00	0.01	0.01	0.00	0.01	0.00	0.00	0.00	0.01	0.00	0.00	0.00
0.25	0.00	-0.01	0.00	0.00	0.00	0.00	0.00	0.00	-0.01	0.00	0.00	0.00

Table 9

Bias, relative bias, and standardized bias of the mediated effect for the residualized change score model when pretest correlation = 0.00

	N											
	50			100			200			500		
	Bias	Rel Bias	Stand Bias	Bias	Rel Bias	Stand Bias	Bias	Rel Bias	Stand Bias	Bias	Rel Bias	Stand Bias
True Value												
0	0.00	N/A	0.00	0.00	N/A	-0.01	0.00	N/A	0.00	0.00	N/A	0.01
0.01	-0.01	-0.30	-0.11	-0.01	-0.34	-0.22	-0.01	-0.29	-0.30	-0.01	-0.29	-0.52
0.03	-0.02	-0.31	-0.21	-0.02	-0.24	-0.24	-0.02	-0.29	-0.45	-0.02	-0.28	-0.68
0.05	-0.04	-0.31	-0.25	-0.03	-0.27	-0.30	-0.03	-0.29	-0.46	-0.03	-0.26	-0.68
0.09	-0.06	-0.31	-0.50	-0.05	-0.27	-0.66	-0.05	-0.27	-0.99	-0.05	-0.27	-1.49
0.15	-0.11	-0.29	-0.60	-0.10	-0.29	-0.87	-0.10	-0.27	-1.19	-0.10	-0.26	-1.82
0.25	-0.19	-0.29	-0.78	-0.19	-0.29	-1.13	-0.18	-0.27	-1.52	-0.18	-0.27	-2.38

Note. bm2y1 and *by2m1* paths = 0.50

Table 10

Bias, relative bias, and standardized bias of the mediated effect for the residualized change score model when pretest correlation = 0.50

	N											
	50			100			200			500		
	Bias	Rel Bias	Stand Bias	Bias	Rel Bias	Stand Bias	Bias	Rel Bias	Stand Bias	Bias	Rel Bias	Stand Bias
True Value												
0	0.00	N/A	0.00	0.00	N/A	-0.01	0.00	N/A	0.01	0.00	N/A	0.01
0.01	-0.08	-1.30	-1.42	-0.09	-1.34	-2.40	-0.09	-1.33	-3.50	-0.09	-1.33	-6.14
0.03	-0.11	-0.96	-1.97	-0.11	-0.97	-3.43	-0.11	-0.96	-6.18	-0.11	-0.96	-10.84
0.05	-0.14	-0.81	-1.70	-0.14	-0.82	-2.81	-0.14	-0.79	-3.97	-0.14	-0.79	-6.47
0.09	-0.35	-0.96	-3.30	-0.35	-0.97	-5.10	-0.35	-0.97	-7.79	-0.35	-0.97	-12.46
0.15	-0.45	-0.81	-3.41	-0.45	-0.80	-5.14	-0.45	-0.80	-7.41	-0.45	-0.80	-12.25
0.25	-0.85	-0.82	-3.77	-0.86	-0.82	-5.74	-0.85	-0.82	-8.10	-0.86	-0.82	-12.91

Note. *bm2y1* and *by2m1* paths = 0.50

Table 11

Bias, Relative bias, and Standardized bias of the mediated effect for Cross-sectional model when direct effect = 0.00

		N											
		50			100			200			500		
by2m1	True Value	Bias	Rel Bias	Stand Bias	Bias	Rel Bias	Stand Bias	Bias	Rel Bias	Stand Bias	Bias	Rel Bias	Stand Bias
0	0	0.00	N/A	0.00	0.00	N/A	0.00	0.00	N/A	0.00	0.00	N/A	0.00
	0.01	0.03	1.43	0.21	0.03	1.38	0.32	0.03	1.39	0.45	0.03	1.40	0.72
	0.03	0.03	0.44	0.13	0.03	0.47	0.19	0.03	0.46	0.27	0.03	0.45	0.43
	0.05	0.03	0.24	0.08	0.03	0.27	0.14	0.03	0.29	0.20	0.03	0.28	0.32
	0.09	0.09	0.47	0.39	0.09	0.47	0.55	0.09	0.48	0.81	0.09	0.47	1.28
	0.15	0.10	0.28	0.29	0.10	0.27	0.42	0.10	0.27	0.58	0.10	0.27	0.91
	0.25	0.19	0.28	0.49	0.19	0.28	0.69	0.18	0.28	0.96	0.19	0.28	1.57
0.5	0	0.00	N/A	0.00	0.00	N/A	0.00	0.00	N/A	0.00	0.00	N/A	-0.01
	0.01	0.10	4.89	0.41	0.10	4.84	0.58	0.10	4.92	0.86	0.10	4.88	1.36
	0.03	0.10	1.63	0.31	0.11	1.69	0.47	0.10	1.59	0.64	0.10	1.56	0.98
	0.05	0.10	0.85	0.23	0.10	0.88	0.34	0.10	0.86	0.47	0.10	0.89	0.77
	0.09	0.32	1.59	0.89	0.32	1.62	1.32	0.32	1.61	1.87	0.32	1.61	2.99
	0.15	0.33	0.90	0.71	0.32	0.88	1.01	0.32	0.89	1.45	0.32	0.89	2.29
	0.25	0.61	0.91	1.20	0.61	0.92	1.74	0.62	0.92	2.46	0.62	0.92	3.89

Table 12

Bias, Relative bias, and Standardized bias of the mediated effect for the Cross-sectional model when direct effect = 0.30 and N = 50

		bm2y1											
		0						0.5					
		by2m1						*by2m1*					
		0			0.5			0			0.5		
		Bias	Rel Bias	Stand Bias	Bias	Rel Bias	Stand Bias	Bias	Rel Bias	Stand Bias	Bias	Rel Bias	Stand Bias
Pretest Correlation	True Value												
0	0.01	0.00	0.06	0.02	0.07	3.66	0.42	0.04	1.88	0.31	0.09	4.61	0.40
	0.09	0.00	0.00	0.00	0.24	1.22	0.86	0.11	0.57	0.52	0.29	1.49	0.87
	0.25	0.00	0.01	0.01	0.46	0.69	1.07	0.19	0.28	0.54	0.58	0.87	1.22
0.5	0.01	0.02	1.21	0.22	0.11	5.19	0.43	0.05	2.56	0.31	0.13	6.23	0.39
	0.09	0.09	0.47	0.43	0.33	1.69	0.94	0.18	0.91	0.63	0.40	2.03	0.91
	0.25	0.20	0.31	0.58	0.64	0.96	1.28	0.35	0.53	0.80	0.76	1.13	1.25

Table 13

Bias, Relative bias, and Standardized bias of the mediated effect for the Cross-sectional model when direct effect = 0.30 and N = 100

		bm2y1											
		0						0.5					
		by2m1						*by2m1*					
		0			0.5			0			0.5		
		Bias	Rel Bias	Stand Bias	Bias	Rel Bias	Stand Bias	Bias	Rel Bias	Stand Bias	Bias	Rel Bias	Stand Bias
Pretest Correlation	True Value												
0	0.01	0.00	0.02	0.01	0.07	3.58	0.58	0.03	1.70	0.43	0.09	4.53	0.55
	0.09	0.00	-0.02	-0.04	0.24	1.21	1.25	0.11	0.56	0.75	0.32	1.60	1.32
	0.25	0.00	-0.00	-0.01	0.45	0.68	1.56	0.21	0.32	0.90	0.60	0.89	1.75
0.5	0.01	0.03	1.34	0.37	0.10	4.97	0.62	0.05	2.56	0.45	0.12	5.97	0.54
	0.09	0.10	0.48	0.63	0.33	1.65	1.39	0.17	0.86	0.82	0.41	2.07	1.35
	0.25	0.20	0.30	0.80	0.68	1.01	1.92	0.35	0.52	1.09	0.80	1.19	1.88

Table 14

Bias, Relative bias, and Standardized bias of the mediated effect for the Cross-sectional model when direct effect = 0.30 and N = 200

		bm2y1											
		0						0.5					
		by2m1						*by2m1*					
		0			0.5			0			0.5		
		Bias	Rel Bias	Stand Bias	Bias	Rel Bias	Stand Bias	Bias	Rel Bias	Stand Bias	Bias	Rel Bias	Stand Bias
Pretest Correlation	True Value												
0	0.01	0.00	-0.04	-0.04	0.07	3.65	0.88	0.04	1.73	0.64	0.09	4.54	0.81
	0.09	0.00	0.01	0.01	0.24	1.20	1.70	0.11	0.56	1.07	0.30	1.51	1.85
	0.25	0.00	-0.01	-0.03	0.45	0.68	2.15	0.21	0.31	1.21	0.59	0.89	2.37
0.5	0.01	0.03	1.43	0.56	0.11	5.19	0.95	0.05	2.51	0.60	0.13	6.22	0.79
	0.09	0.10	0.50	0.93	0.34	1.71	2.04	0.17	0.87	1.26	0.41	2.09	1.90
	0.25	0.20	0.30	1.12	0.66	0.99	2.66	0.35	0.53	1.60	0.77	1.16	2.59

Table 15

Bias, Relative bias, and Standardized bias of the mediated effect for the Cross-sectional model when direct effect = 0.30 and N = 500

		bm2y1											
		0						0.5					
		by2m1						*by2m1*					
		0			0.5			0			0.5		
		Bias	Rel Bias	Stand Bias	Bias	Rel Bias	Stand Bias	Bias	Rel Bias	Stand Bias	Bias	Rel Bias	Stand Bias
Pretest Correlation	True Value												
0	0.01	0.00	0.00	0.00	0.08	3.76	1.40	0.03	1.64	0.97	0.10	4.67	1.36
	0.09	0.00	0.00	0.00	0.24	1.20	2.81	0.11	0.55	1.70	0.31	1.55	2.98
	0.25	0.00	0.00	0.02	0.45	0.67	3.42	0.22	0.32	1.99	0.58	0.88	3.74
0.5	0.01	0.03	1.44	0.91	0.10	5.12	1.45	0.05	2.56	0.98	0.12	6.07	1.24
	0.09	0.10	0.49	1.51	0.33	1.69	3.26	0.17	0.87	1.98	0.41	2.06	2.99
	0.25	0.20	0.31	1.84	0.66	0.99	4.11	0.35	0.53	2.55	0.79	1.19	4.25

Table 16

PRODCLIN 95% Confidence interval coverage of the mediated effect for the ANCOVA model

	N			
	50	100	200	500
	95% C.I. Coverage	95% C.I. Coverage	95% C.I. Coverage	95% C.I. Coverage
True Value				
0	0.980	0.975	0.973	0.968
0.01	0.994	0.992	0.974	0.948
0.03	0.971	0.948	0.950	0.949
0.05	0.945	0.946	0.948	0.951
0.09	0.943	0.946	0.952	0.950
0.15	0.945	0.948	0.949	0.951
0.25	0.946	0.950	0.949	0.950

Table 17

PRODCLIN 95% confidence interval coverage of the mediated effect for the Path analysis model

	N			
	50	100	200	500
	95% C.I. Coverage	95% C.I. Coverage	95% C.I. Coverage	95% C.I. Coverage
True Value				
0	0.975	0.973	0.972	0.955
0.01	0.992	0.990	0.971	0.947
0.03	0.962	0.945	0.949	0.948
0.05	0.937	0.942	0.947	0.950
0.09	0.933	0.942	0.936	0.949
0.15	0.937	0.945	0.947	0.950
0.25	0.937	0.946	0.931	0.949

Table 18

PRODCLIN 95% confidence interval coverage of the mediated effect for the Difference score model

	N			
	50	100	200	500
	95% C.I. Coverage	95% C.I. Coverage	95% C.I. Coverage	95% C.I. Coverage
True Value				
0	0.986	0.980	0.974	0.966
0.01	0.991	0.983	0.971	0.954
0.03	0.983	0.971	0.963	0.953
0.05	0.970	0.962	0.957	0.950
0.09	0.955	0.950	0.951	0.948
0.15	0.956	0.952	0.950	0.950
0.25	0.948	0.948	0.950	0.950

Table 19

PRODCLIN 95% confidence interval coverage of the mediated effect for the residualized change score model when Direct effect = 0.00, N = 50, and N = 100

		N							
		50				100			
		bm2y1				*bm2y1*			
		0		0.5		0		0.5	
		by2m1		*by2m1*		*by2m1*		*by2m1*	
		0	0.5	0	0.5	0	0.5	0	0.5
		95% C.I. Coverage	95% C.I. Coverage	95% C.I. Coverage	95% C.I. Coverage	95% C.I. Coverage	95% C.I. Coverage	95% C.I. Coverage	95% C.I. Coverage
Pretest Corr.	True Value								
0	0	0.980	0.992	0.985	0.993	0.978	0.981	0.979	0.988
	0.01	0.995	0.999	0.997	0.998	0.991	1.000	0.995	0.998
	0.03	0.970	0.989	0.977	0.990	0.943	0.965	0.937	0.962
	0.05	0.950	0.972	0.932	0.959	0.944	0.960	0.936	0.932
	0.09	0.939	0.976	0.917	0.939	0.949	0.982	0.885	0.937
	0.15	0.953	0.972	0.897	0.918	0.957	0.966	0.865	0.895
	0.25	0.943	0.975	0.854	0.915	0.951	0.979	0.769	0.865
0.5	0	0.981	0.989	0.989	0.998	0.977	0.982	0.984	0.993
	0.01	0.995	0.999	0.996	0.964	0.994	0.998	0.994	0.486
	0.03	0.965	0.982	0.950	0.896	0.941	0.962	0.865	0.355
	0.05	0.939	0.955	0.884	0.732	0.951	0.956	0.851	0.475
	0.09	0.939	0.962	0.777	0.296	0.944	0.966	0.660	0.059
	0.15	0.940	0.949	0.717	0.329	0.945	0.966	0.543	0.086
	0.25	0.947	0.968	0.554	0.165	0.944	0.971	0.313	0.015

Table 20

PRODCLIN 95% confidence interval coverage of the mediated effect for the residualized change score model when Direct effect = 0.0, N = 200, and N = 500

		N							
		200				500			
		bm2y1				*bm2y1*			
		0		0.5		0		0.5	
		by2m1		*by2m1*		*by2m1*		*by2m1*	
		0	0.5	0	0.5	0	0.5	0	0.5
		95% C.I. Coverage	95% C.I. Coverage	95% C.I. Coverage	95% C.I. Coverage	95% C.I. Coverage	95% C.I. Coverage	95% C.I. Coverage	95% C.I. Coverage
Pretest Corr.	True Value								
0	0	0.973	0.980	0.975	0.979	0.967	0.971	0.970	0.973
	0.01	0.975	0.993	0.978	0.997	0.940	0.971	0.922	0.936
	0.03	0.958	0.968	0.927	0.925	0.955	0.965	0.895	0.895
	0.05	0.940	0.955	0.921	0.923	0.952	0.962	0.894	0.902
	0.09	0.959	0.983	0.839	0.918	0.958	0.988	0.688	0.819
	0.15	0.958	0.979	0.798	0.842	0.954	0.984	0.569	0.674
	0.25	0.956	0.978	0.633	0.769	0.963	0.979	0.279	0.497
0.5	0	0.975	0.977	0.976	0.986	0.971	0.972	0.972	0.975
	0.01	0.973	0.992	0.912	0.081	0.951	0.968	0.812	0.000
	0.03	0.955	0.955	0.812	0.091	0.953	0.946	0.630	0.002
	0.05	0.951	0.953	0.787	0.237	0.946	0.949	0.574	0.018
	0.09	0.952	0.969	0.497	0.002	0.953	0.973	0.181	0.000
	0.15	0.948	0.970	0.306	0.005	0.955	0.968	0.071	0.000
	0.25	0.957	0.966	0.122	0.000	0.959	0.972	0.005	0.000

Table 21

PRODCLIN 95% confidence interval coverage of the mediated effect for the residualized change score model when Direct effect = 0.30, N = 50, and N = 100

		N							
		50				100			
		bm2y1				*bm2y1*			
		0		0.5		0		0.5	
		by2m1		*by2m1*		*by2m1*		*by2m1*	
		0	0.5	0	0.5	0	0.5	0	0.5
		95% C.I. Coverage	95% C.I. Coverage	95% C.I. Coverage	95% C.I. Coverage	95% C.I. Coverage	95% C.I. Coverage	95% C.I. Coverage	95% C.I. Coverage
Pretest Corr.	True Value								
0	0.01	0.996	0.997	1.000	1.000	0.991	0.999	0.996	1.000
	0.09	0.949	0.974	0.912	0.944	0.941	0.980	0.896	0.940
	0.25	0.940	0.976	0.834	0.916	0.959	0.982	0.810	0.880
0.5	0.01	0.993	0.998	0.998	0.962	0.990	0.996	0.992	0.464
	0.09	0.948	0.961	0.802	0.307	0.938	0.978	0.679	0.064
	0.25	0.947	0.961	0.556	0.167	0.955	0.972	0.314	0.013

Table 22

PRODCLIN 95% confidence interval coverage of the mediated effect for the residualized change score model when Direct Effect = 0.30, N = 200 and N = 500

		N							
		200				500			
		bm2y1				*bm2y1*			
		0		0.5		0		0.5	
		by2m1		*by2m1*		*by2m1*		*by2m1*	
		0	0.5	0	0.5	0	0.5	0	0.5
		95% C.I. Coverage	95% C.I. Coverage	95% C.I. Coverage	95% C.I. Coverage	95% C.I. Coverage	95% C.I. Coverage	95% C.I. Coverage	95% C.I. Coverage
Pretest Corr.	True Value								
0	0.01	0.972	0.999	0.973	0.995	0.949	0.978	0.906	0.954
	0.09	0.950	0.984	0.848	0.919	0.954	0.982	0.692	0.816
	0.25	0.952	0.982	0.624	0.782	0.958	0.984	0.276	0.488
0.5	0.01	0.970	0.991	0.909	0.068	0.941	0.965	0.802	0.001
	0.09	0.947	0.973	0.489	0.001	0.953	0.970	0.176	0.000
	0.25	0.956	0.973	0.116	0.000	0.946	0.970	0.006	0.000

Table 23

PRODCLIN 95% confidence interval coverage of the mediated effect for the cross-sectional model when Direct effect = 0.00

	N							
	50		100		200		500	
	by2m1		*by2m1*		*by2m1*		*by2m1*	
	0	0.5	0	0.5	0	0.5	0	0.5
	95% C.I. Coverage	95% C.I. Coverage	95% C.I. Coverage	95% C.I. Coverage	95% C.I. Coverage	95% C.I. Coverage	95% C.I. Coverage	95% C.I. Coverage
True Value								
0	0.968	0.952	0.963	0.949	0.958	0.951	0.957	0.949
0.01	0.974	0.936	0.958	0.910	0.937	0.861	0.865	0.719
0.03	0.952	0.931	0.943	0.924	0.935	0.902	0.924	0.829
0.05	0.944	0.938	0.944	0.933	0.942	0.924	0.929	0.882
0.09	0.934	0.851	0.909	0.737	0.843	0.522	0.688	0.142
0.15	0.935	0.889	0.930	0.826	0.902	0.689	0.809	0.357
0.25	0.915	0.774	0.875	0.589	0.799	0.281	0.579	0.020

Table 24

PRODCLIN 95% confidence interval coverage of the mediated effect for the cross-sectional model when Direct effect = 0.30

	N							
	50		100		200		500	
	by2m1		*by2m1*		*by2m1*		*by2m1*	
	0	0.5	0	0.5	0	0.5	0	0.5
	95% C.I. Coverage	95% C.I. Coverage	95% C.I. Coverage	95% C.I. Coverage	95% C.I. Coverage	95% C.I. Coverage	95% C.I. Coverage	95% C.I. Coverage
True Value								
0.01	0.973	0.941	0.959	0.907	0.931	0.864	0.869	0.709
0.09	0.927	0.858	0.903	0.735	0.846	0.518	0.681	0.137
0.25	0.919	0.772	0.878	0.564	0.777	0.276	0.556	0.020

Table 25

Comparison of sample size requirement for 0.80 power to detect the cross-sectional mediated effect across current study and Fritz & MacKinnon (2007)

Study				
Fritz & MacKinnon (2007)		Current Simulation Study		
N	Effect size	N	Effect Size	Observed Empirical Power
57.5	0.23	50	0.25	0.84
105.0	0.15	100	0.15	0.72
124.5	0.10	100 – 200	0.09	0.61 – 0.92
402.5	0.05	200 – 500	0.05	0.21 – 0.44
403.0	0.08	200 – 500	0.09	0.92 – 0.99
533.0	0.02	500	0.03	0.46

Table 26

Significance test summary indicator of the mediated effect for the five models

Power	Percent	Cumulative Percent
00000	36.18	36.18
00001	3.79	39.97
00010	0.94	40.90
00011	0.99	41.89
00100	0.14	42.03
00101	0.14	42.17
00110	0.19	42.36
00111	0.14	42.50
01001	0.00	42.50
01010	0.00	42.50
01011	0.00	42.50
01100	0.00	42.50
01101	0.00	42.51
01110	0.00	42.51
01111	0.01	42.52
10000	0.29	42.82
10001	0.17	42.98
10010	0.05	43.03
10011	0.05	43.08
10100	0.03	43.11
10101	0.03	43.14
10110	0.03	43.17
10111	0.01	43.18
11000	3.01	46.19
11001	3.47	49.66
11010	1.17	50.83
11011	4.14	54.97
11100	1.09	56.06
11101	10.84	66.90
11110	3.81	70.71
11111	29.29	100.00

Note: Order of indicator is Path analysis, ANCOVA, res. change, diff. score, and cross-sectional

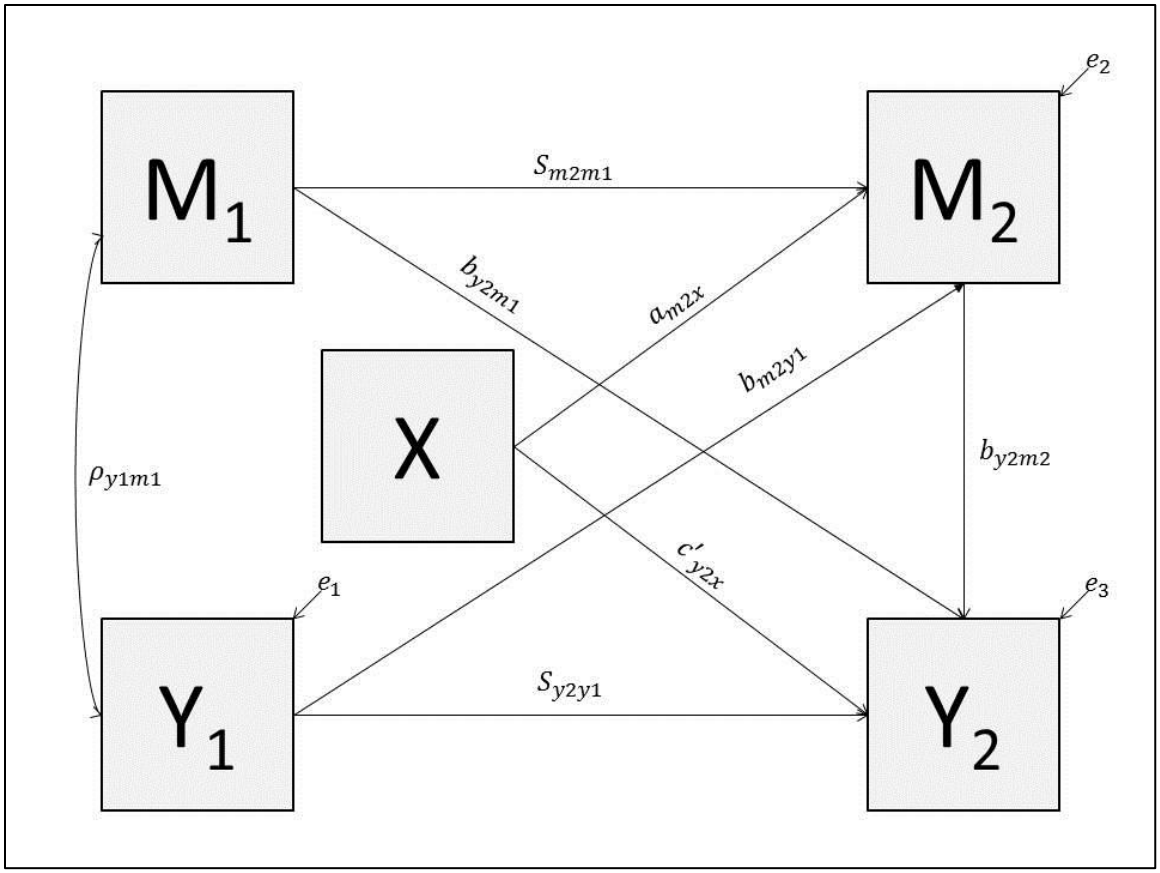


Figure 1. Path diagram of pretest posttest control group design with mediating variable.

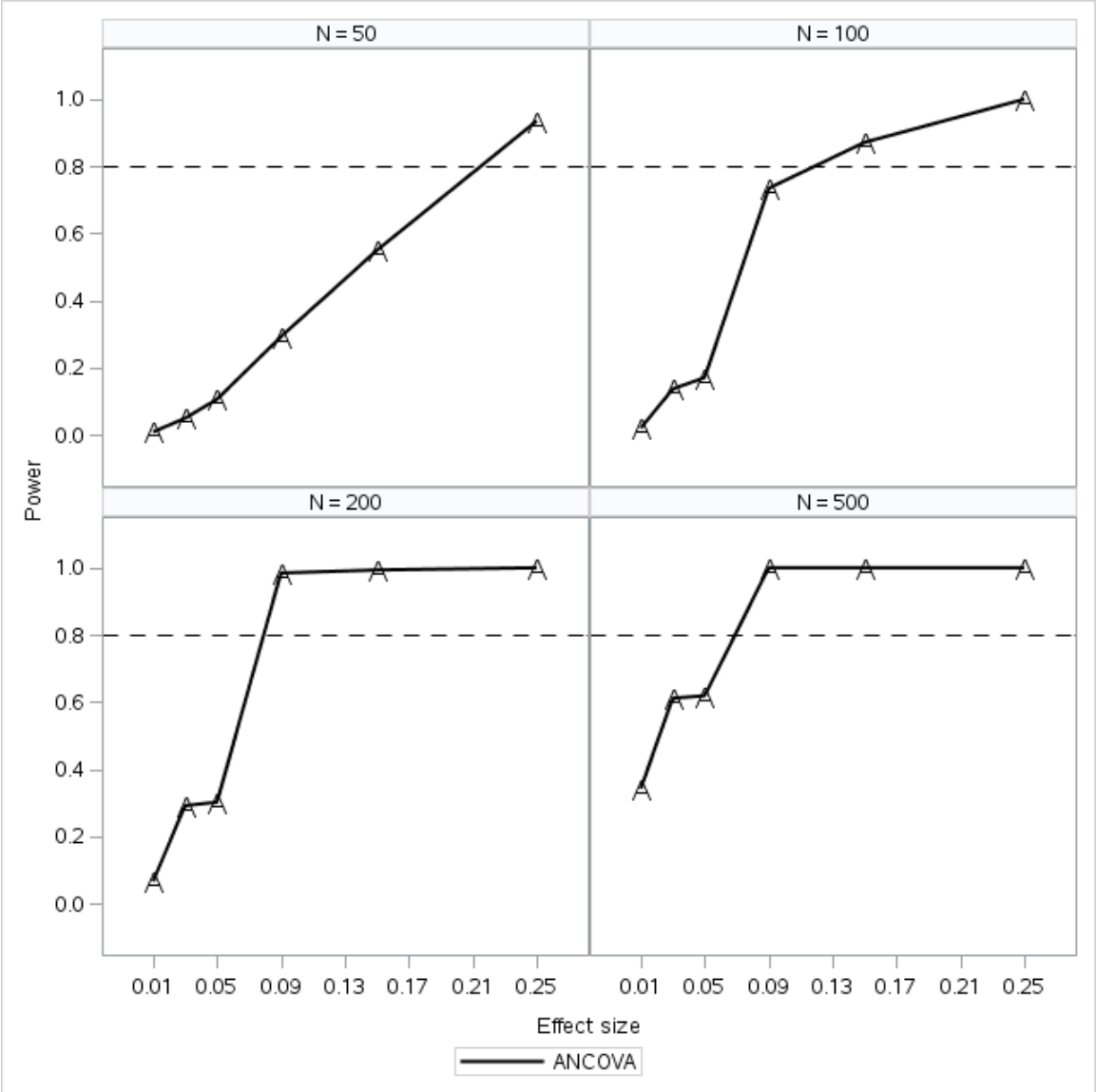


Figure 2. Power plot of ANCOVA model mediated effect results by sample size and effect size collapsed across all simulation conditions.

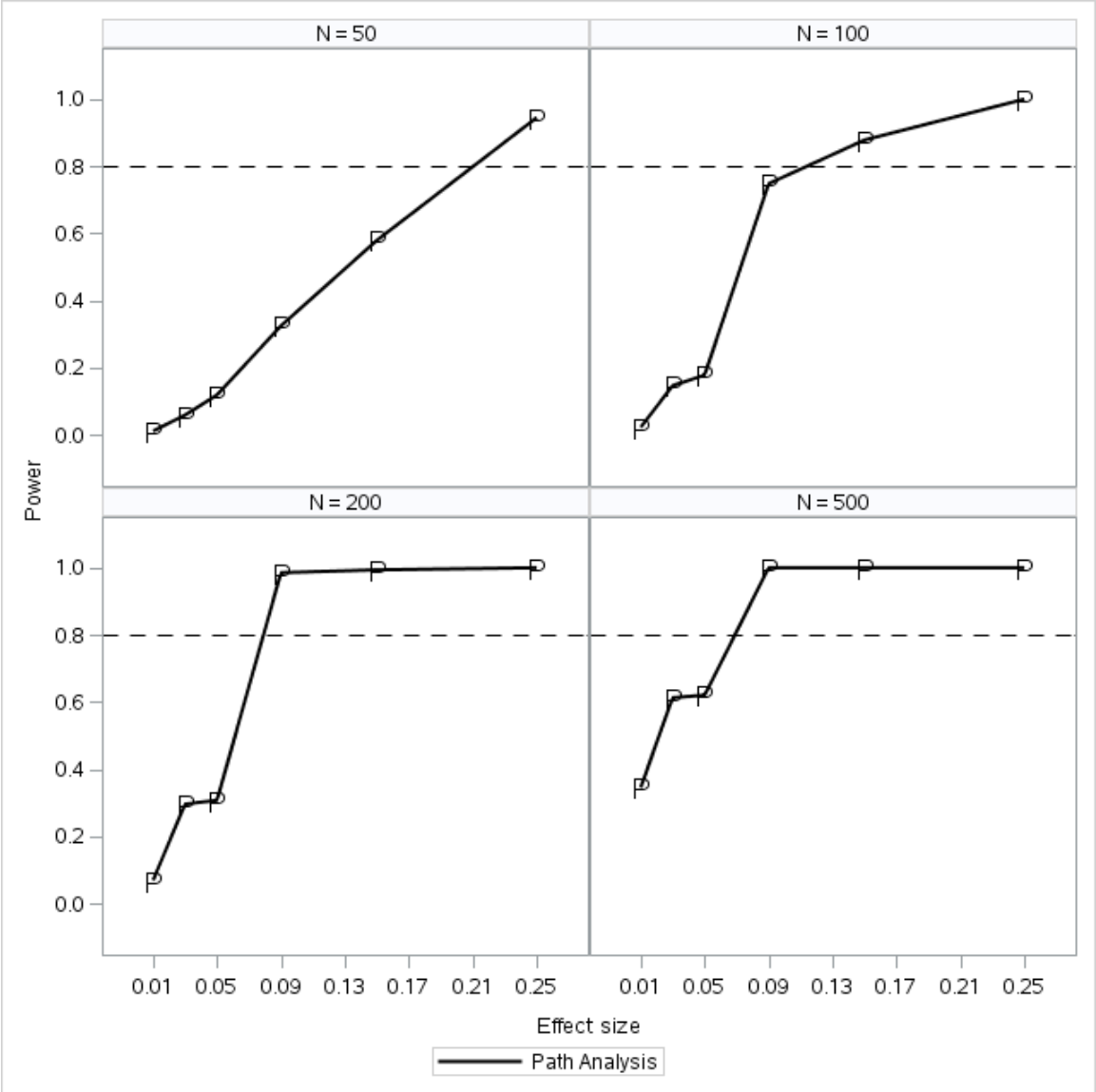


Figure 3. Power plot of Path analysis model mediated effect results by sample size and effect size collapsed across all simulation conditions.

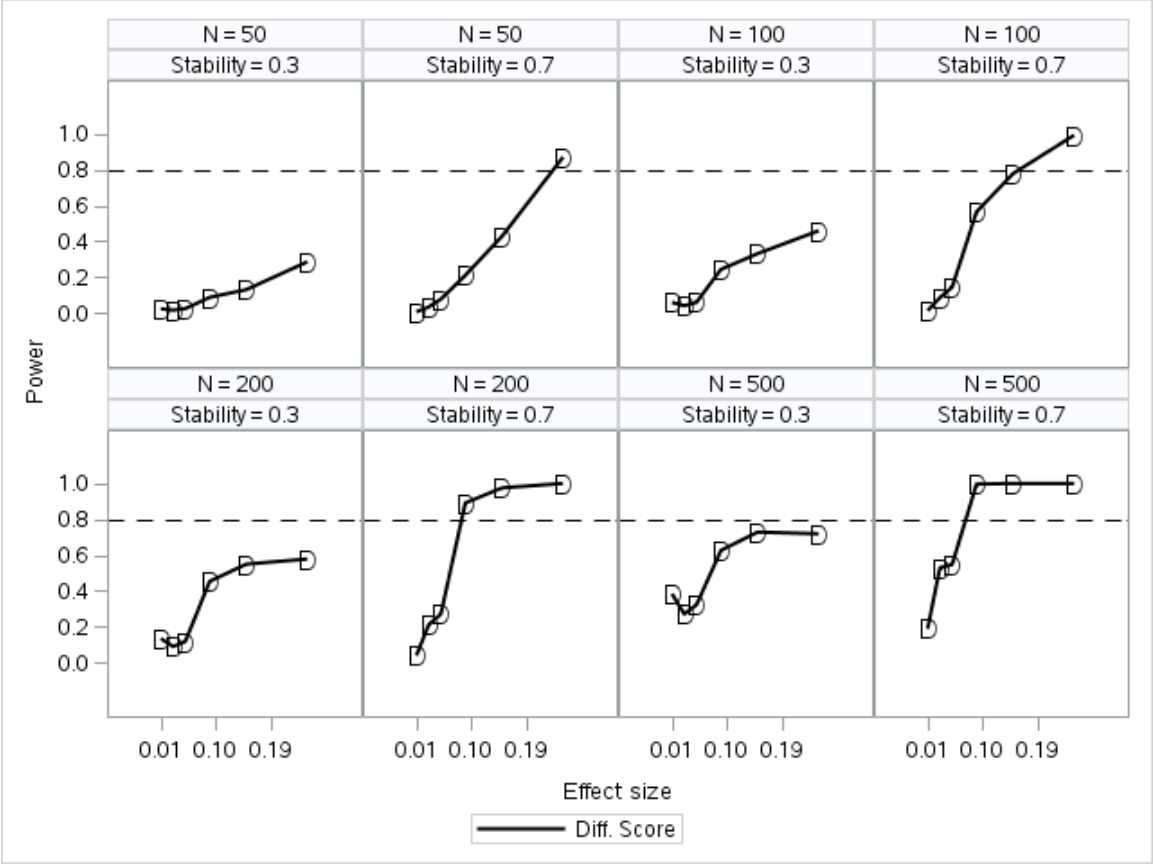


Figure 4. Power plot of difference score model mediated effect results by sample size, effect size, and stability collapsed across all simulation conditions.

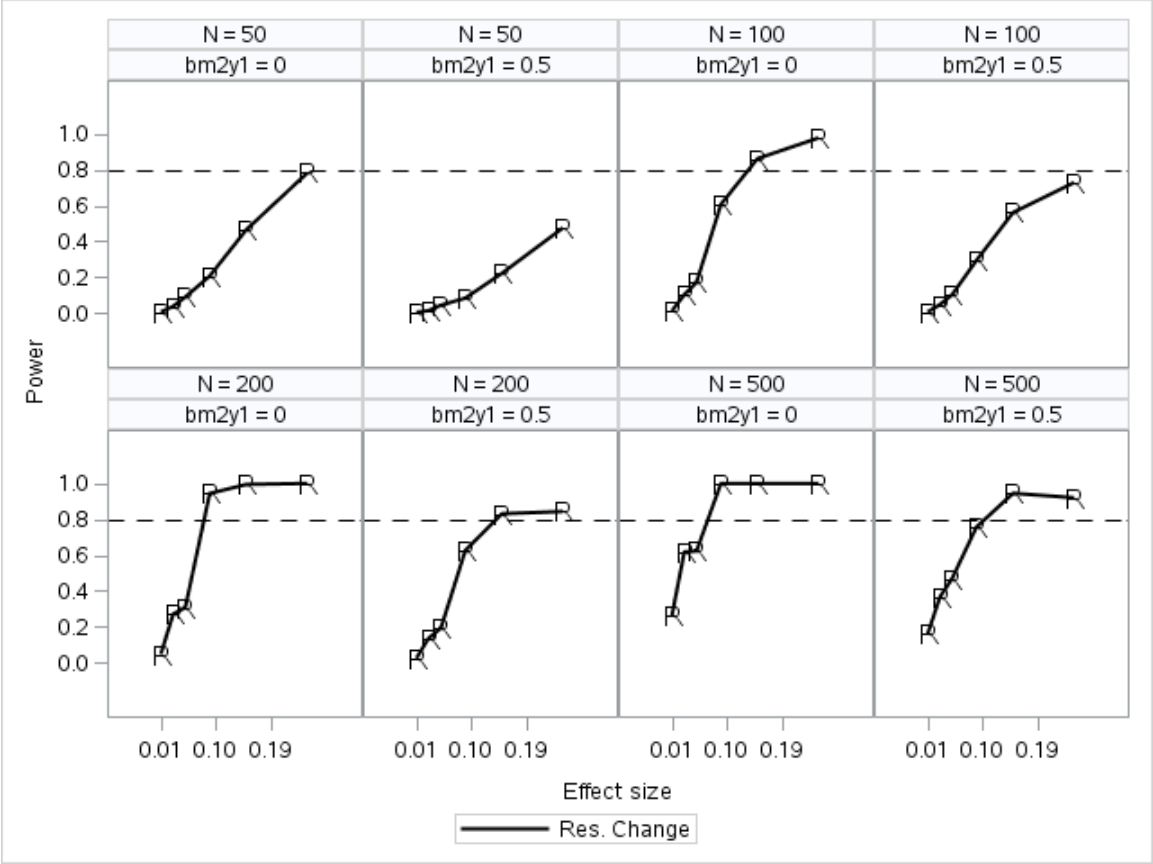


Figure 5. Power plot of residualized change score model mediated effect results by sample size, effect size, and *bm2y1* path and direct effect = 0.00 collapsed across all simulation conditions.

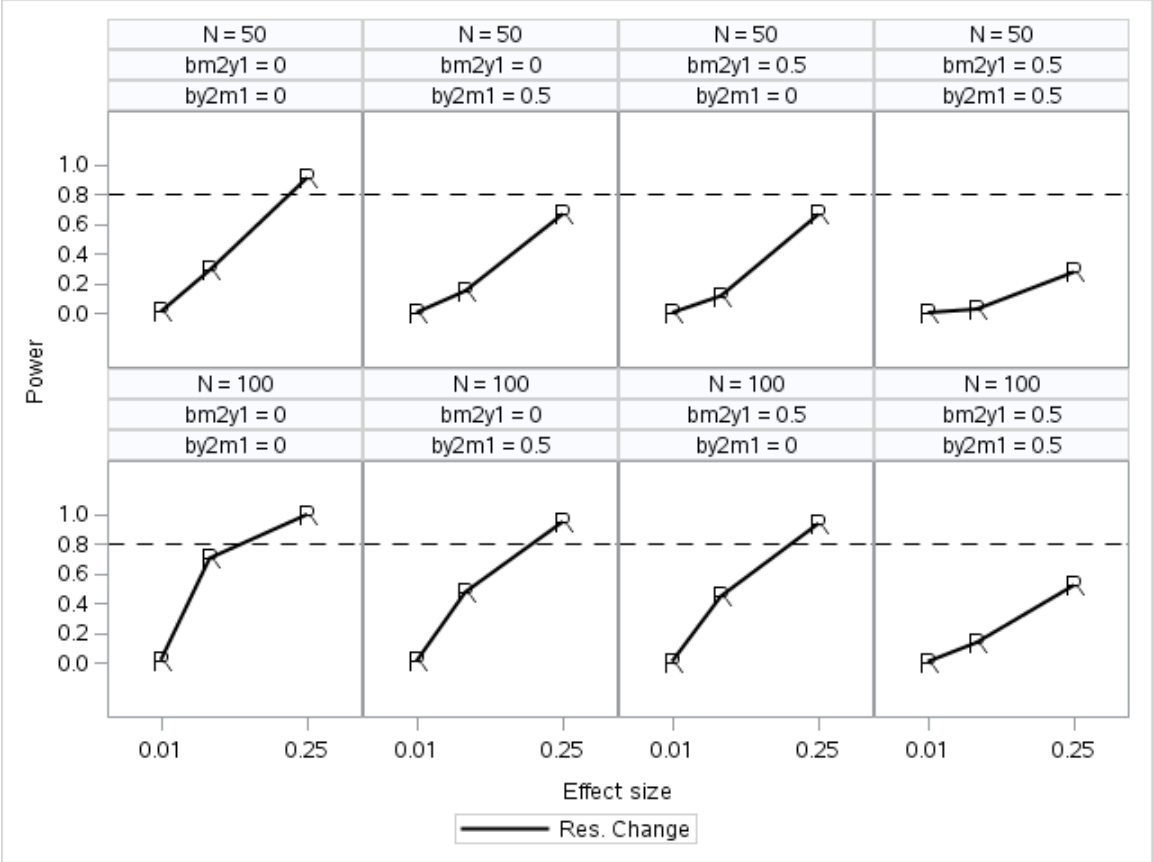


Figure 6. Power plot of residualized change score model mediated effect results by effect size, *bm2y1* path, and *by2m1* path for sample size N= 50 and N = 100 and direct effect = 0.30 collapsed across all simulation conditions.

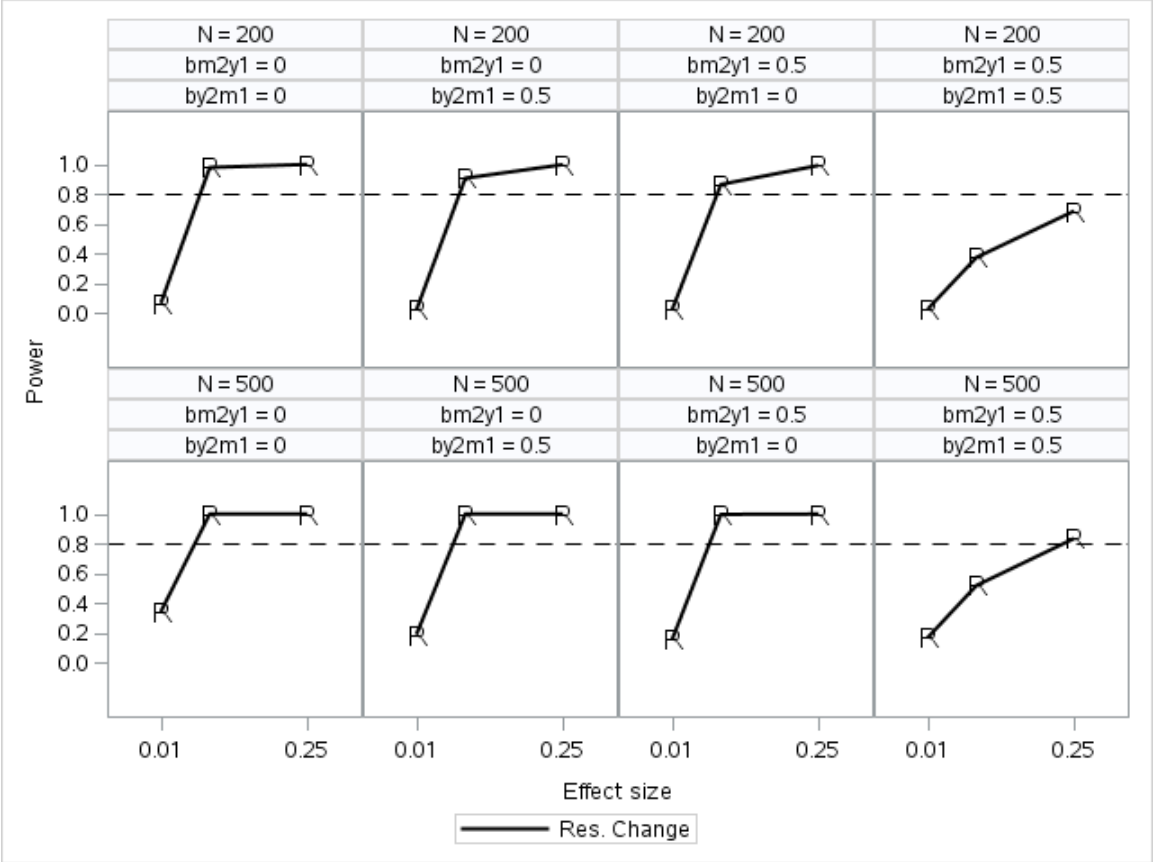


Figure 7. Power plot of residualized change score model mediated effect results by effect size, *bm2y1* path, and *by2m1* path for sample size N= 200 and N = 500 and direct effect = 0.30 collapsed across all simulation conditions.

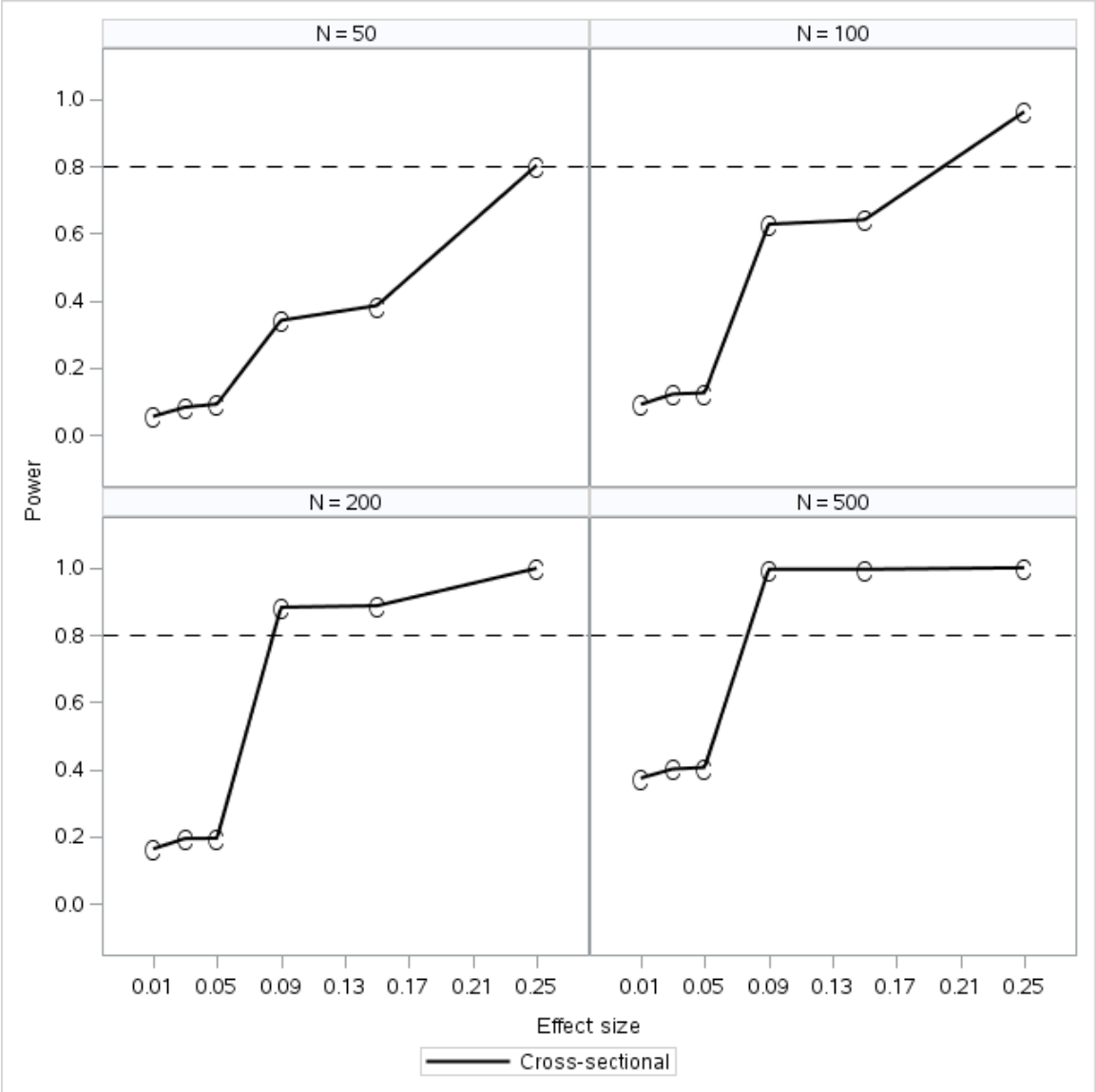


Figure 8. Power plot of cross-sectional model mediated effect results by sample size and effect size for direct effect = 0.00 collapsed across all simulation conditions.

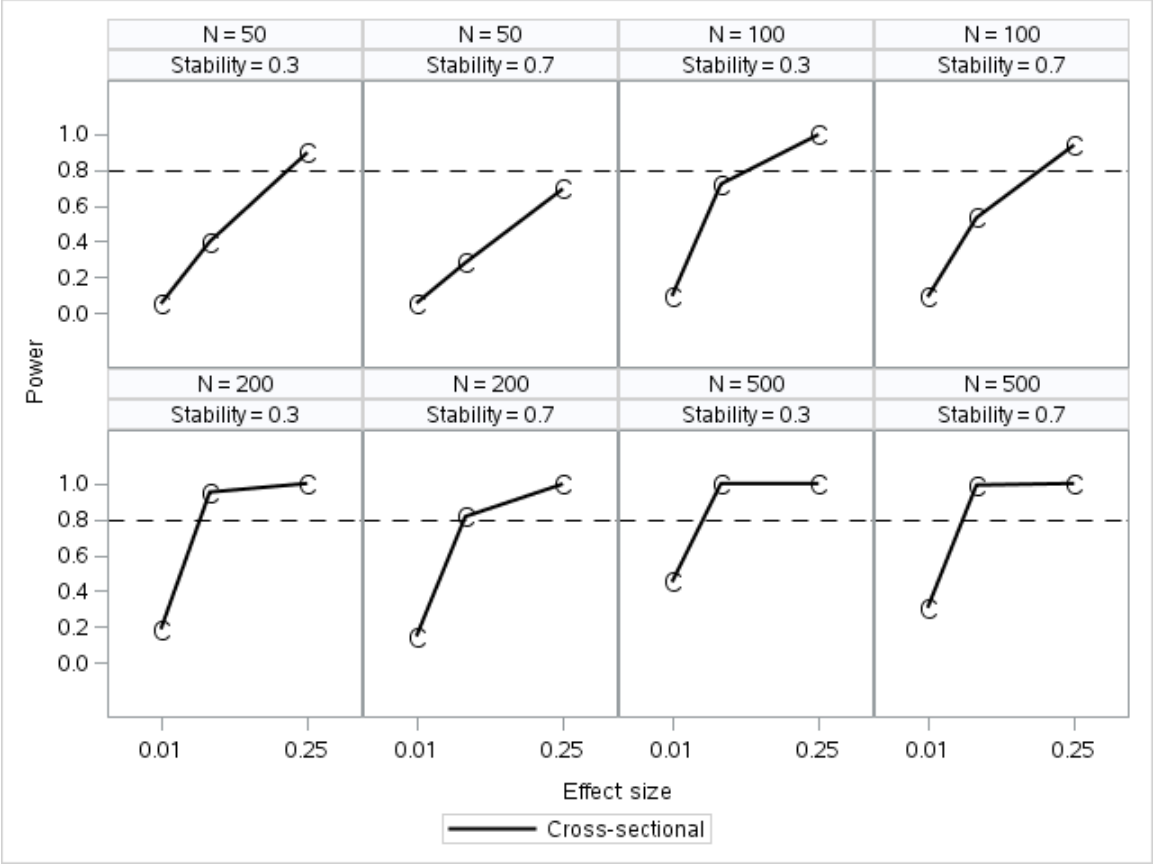


Figure 9. Power plot of cross-sectional model mediated effect results by sample size, effect size, and stability for direct effect = 0.30 collapsed across all simulation conditions.

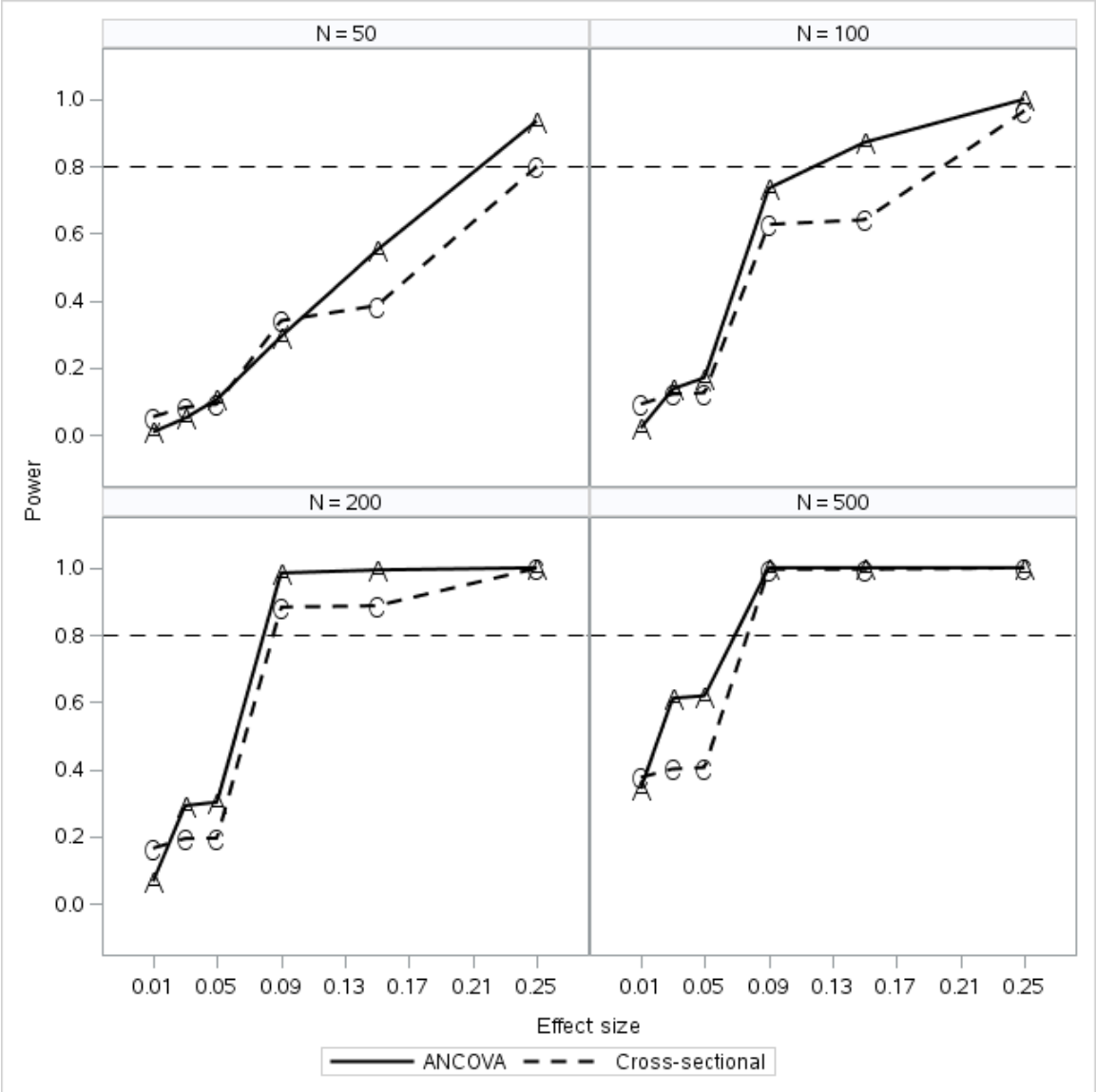


Figure 10. Power plot of ANCOVA vs. cross-sectional model mediated effect results by sample size and effect size collapsed across all simulation conditions.

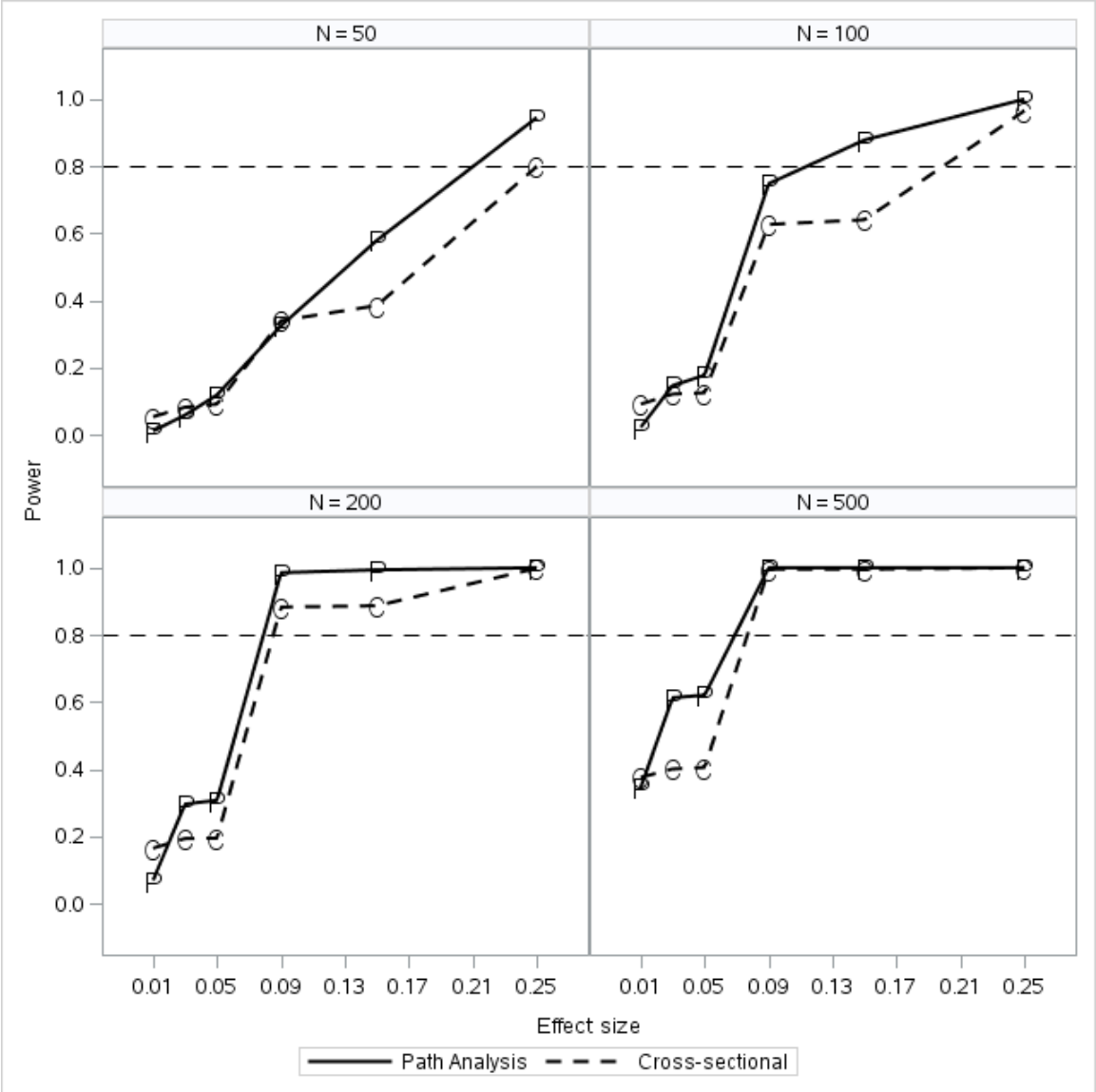


Figure 11. Power plot of path analysis vs. cross-sectional model mediated effect results by sample size and effect size collapsed across all simulation conditions.

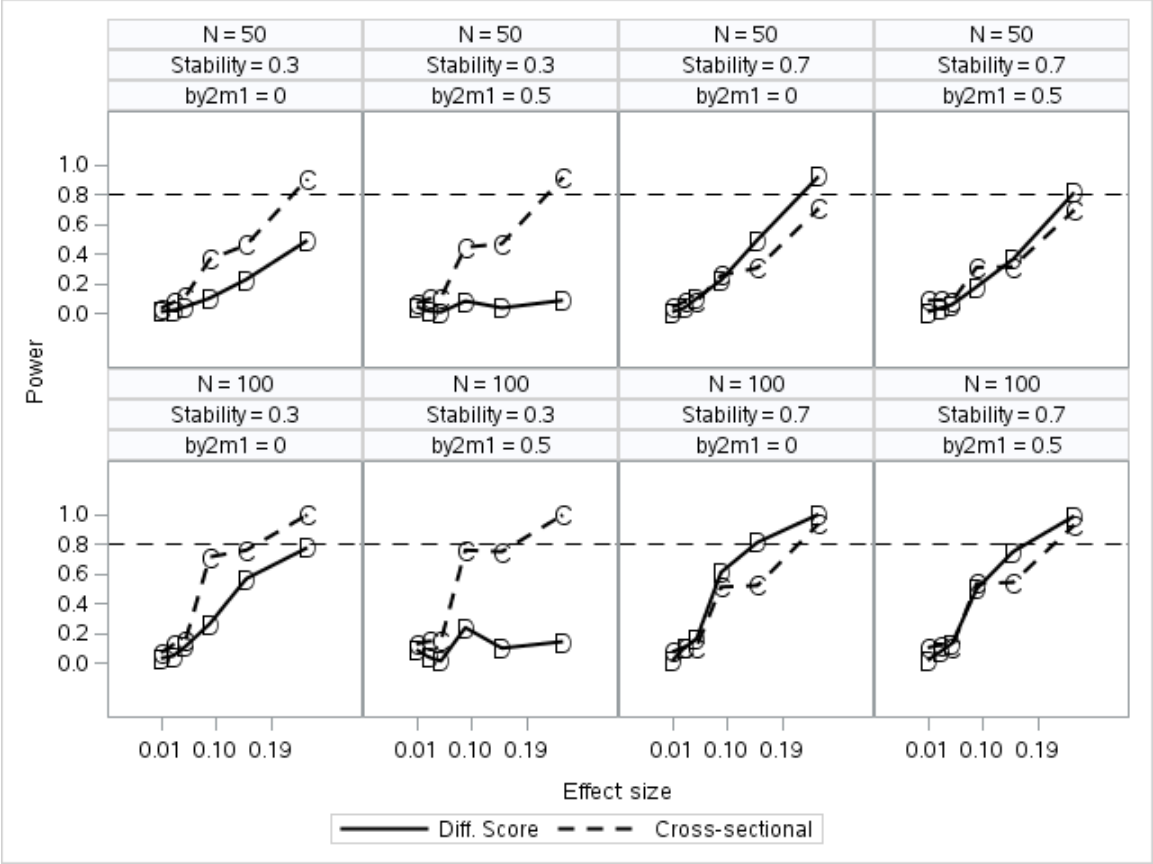


Figure 12. Power plot of difference score model mediated effect results by effect size, stability, and *by2m1* path for sample size N= 50 and N = 100 and direct effect = 0.00 collapsed across all simulation conditions.

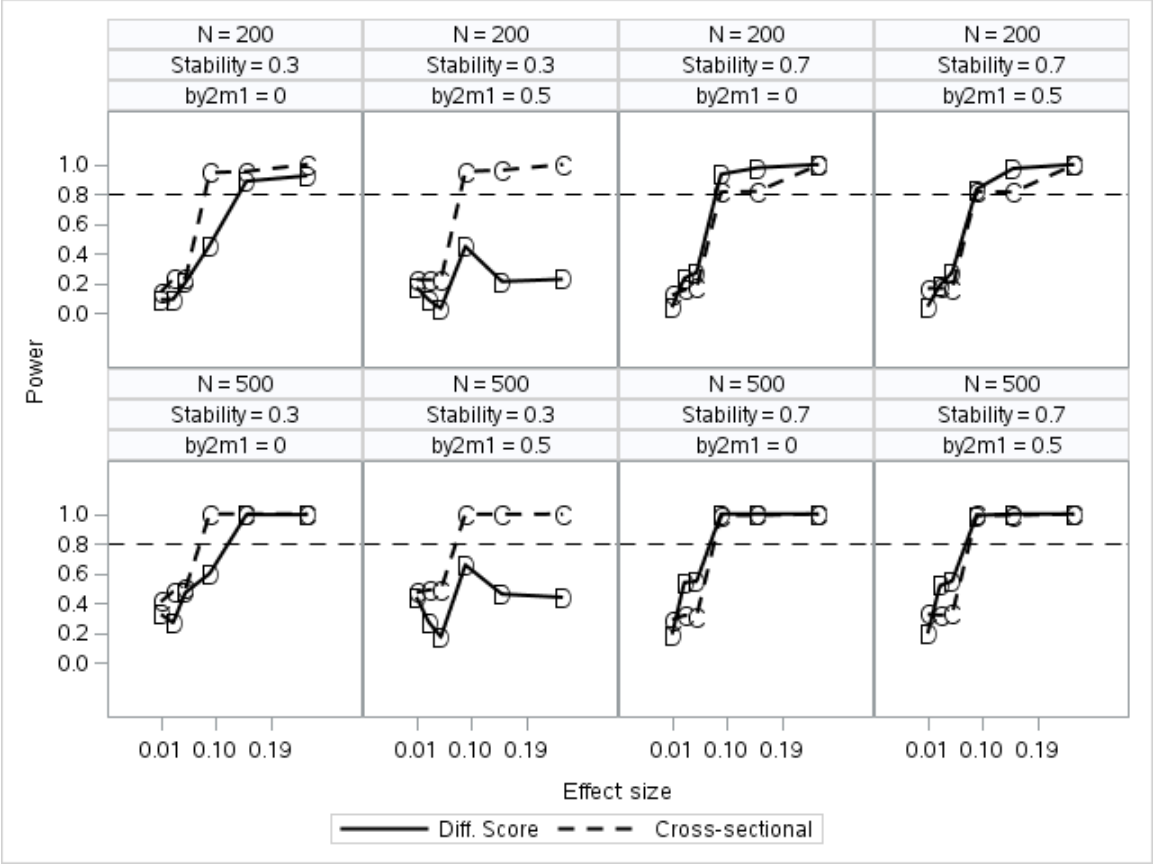


Figure 13. Power plot of difference score model mediated effect results by effect size, stability, and *by2m1* path for sample size N= 200 and N = 500 and direct effect = 0.00 collapsed across all simulation conditions.

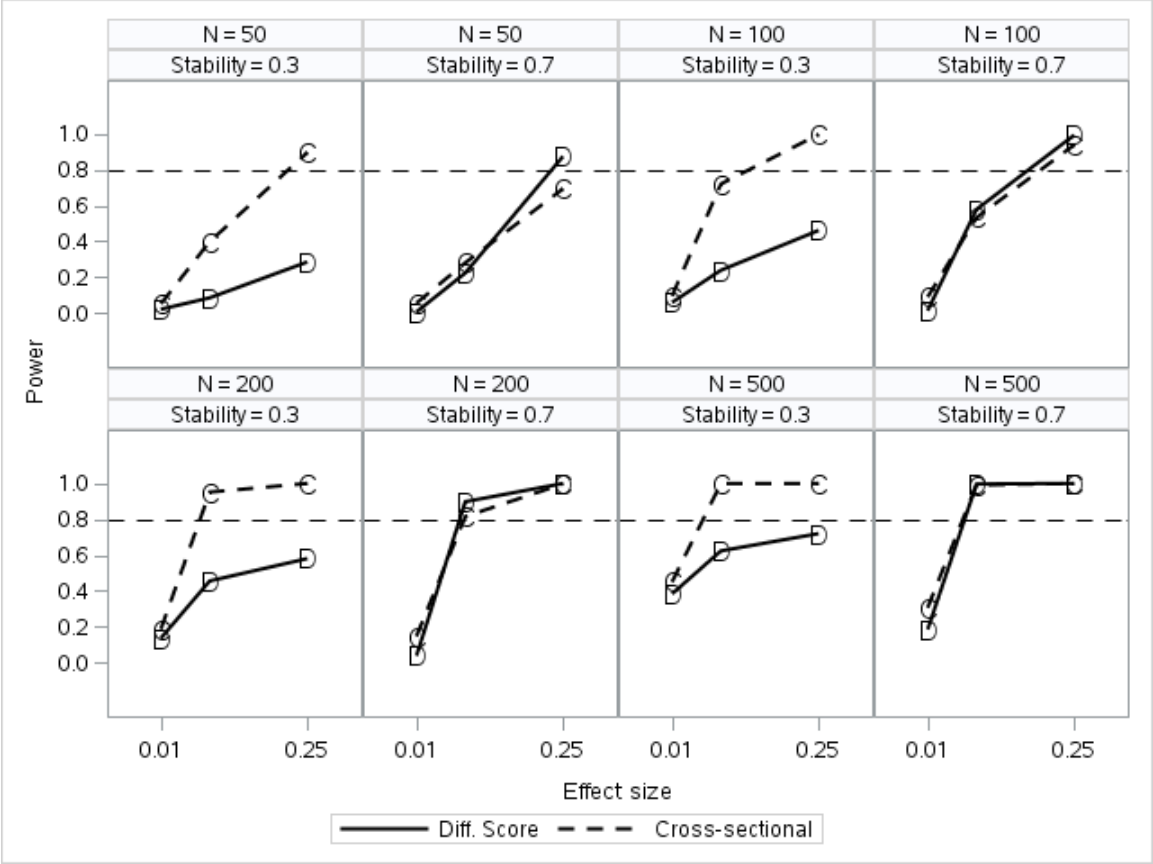


Figure 14. Power plot of difference score vs. cross-sectional model mediated effect results by effect size, sample size, and stability for direct effect = 0.30 collapsed across all simulation conditions.

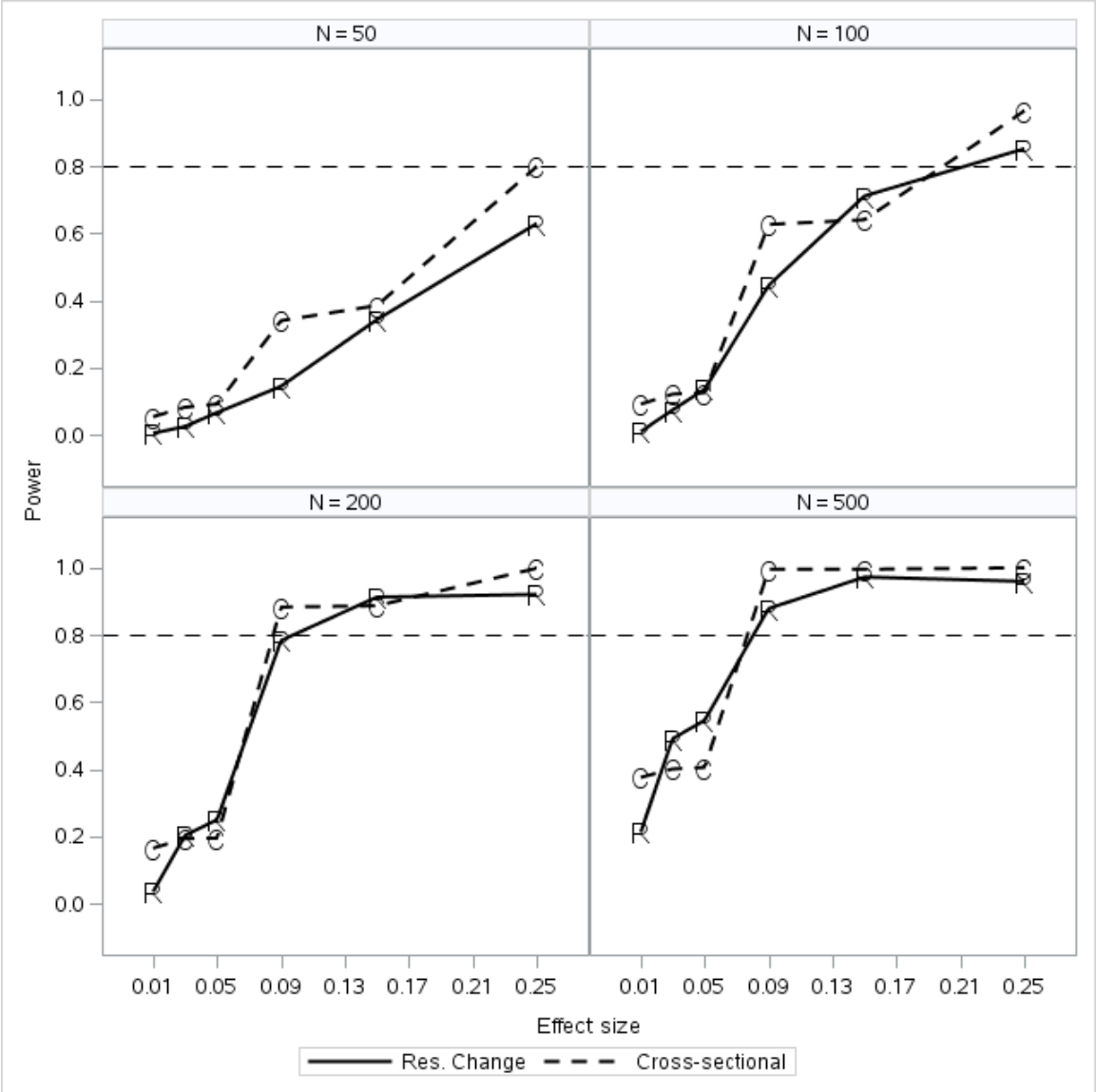


Figure 15. Power plot of residualized change score vs. cross-sectional model mediated effect results by sample size and effect size collapsed across all simulation conditions.

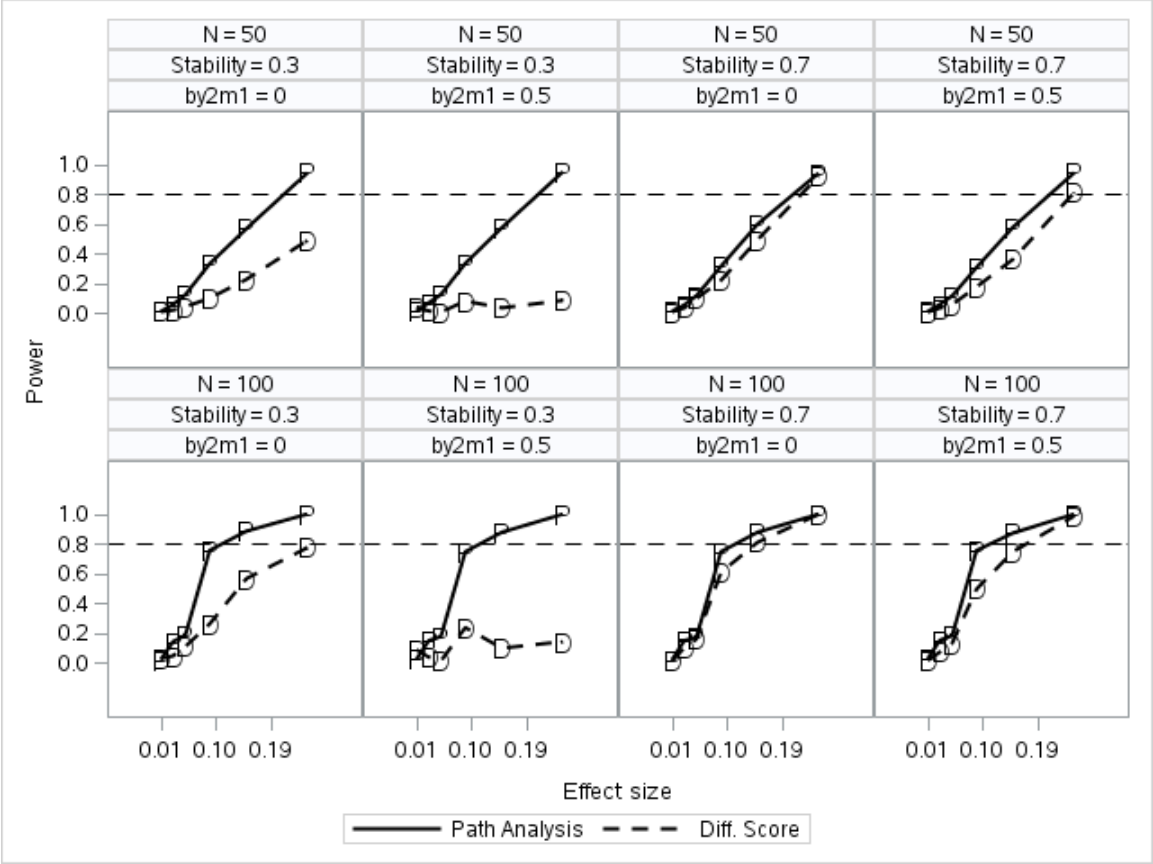


Figure 16. Power plot of path analysis vs. difference score model mediated effect results by sample size, effect size, stability and *by2m1* path for N = 50 – 100 and direct effect = 0.00 collapsed across all simulation conditions.

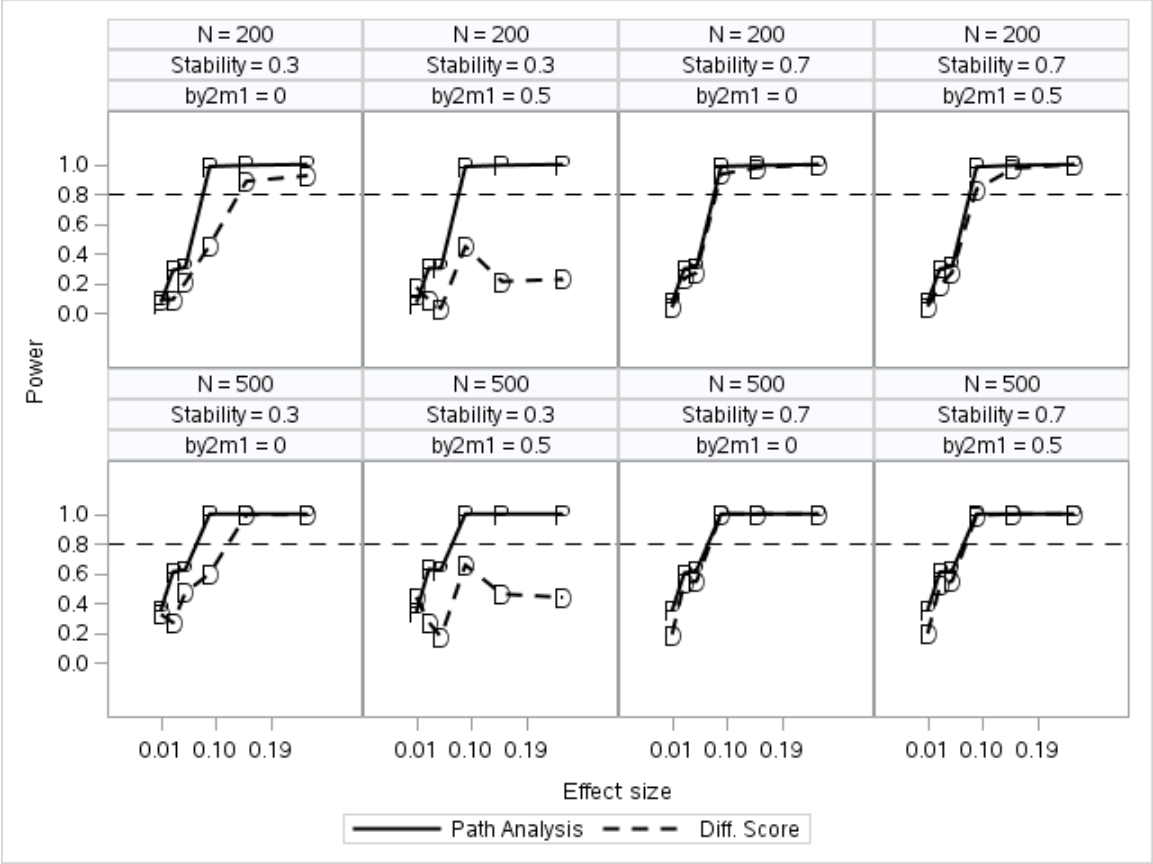


Figure 17. Power plot of path analysis vs. difference score model mediated effect results by sample size, effect size, stability and *by2m1* path for N = 200 – 500 and direct effect = 0.00 collapsed across all simulation conditions.

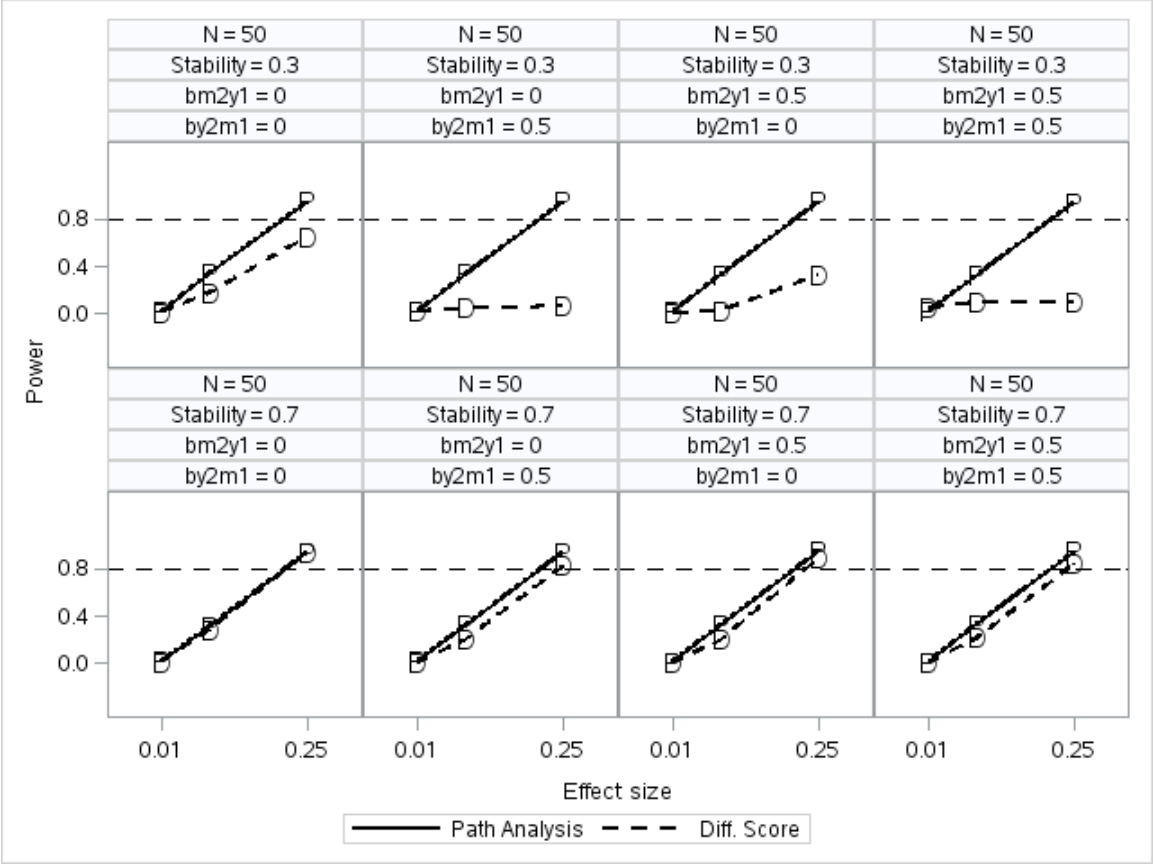


Figure 18. Power plot of path analysis vs. difference score model mediated effect results by sample size, effect size, stability, *bm2y1* path, and *by2m1* path for N = 50 and direct effect = 0.30 collapsed across all simulation conditions.

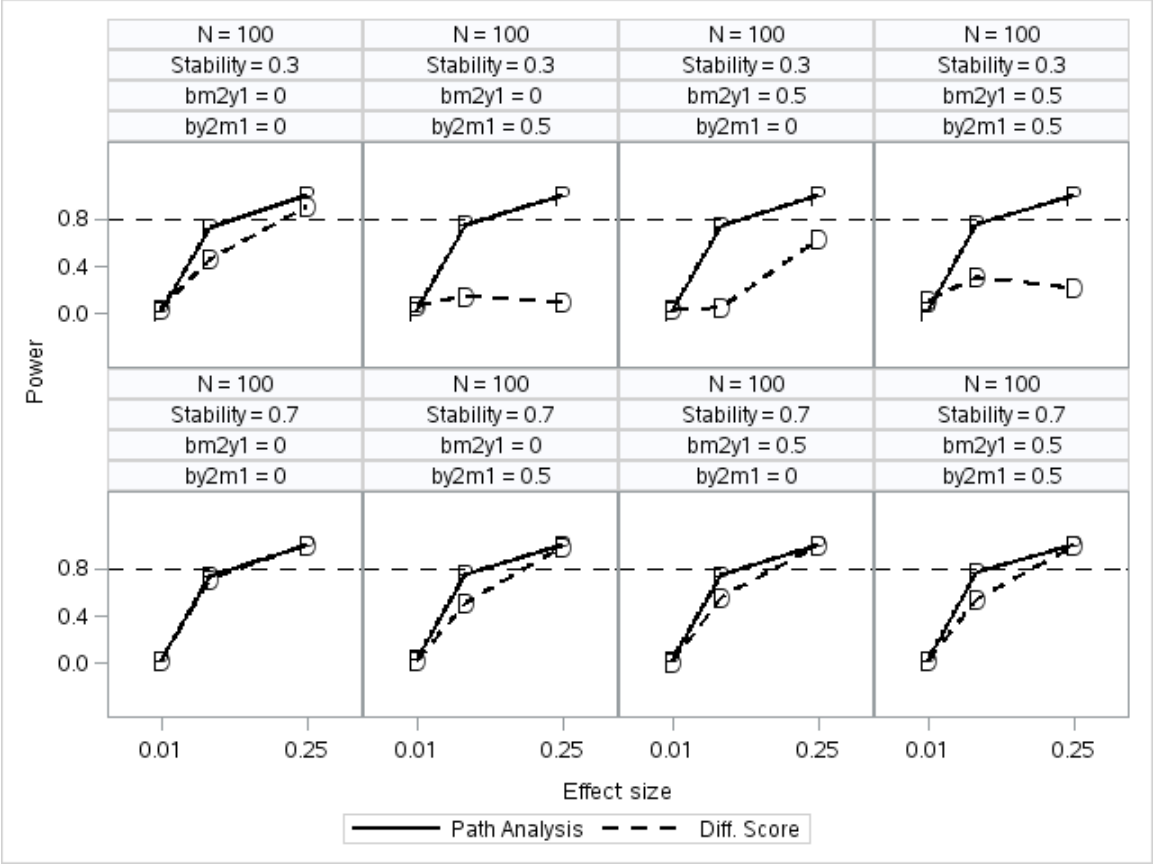


Figure 19. Power plot of path analysis vs. difference score model mediated effect results by sample size, effect size, stability, *bm2y1* path, and *by2m1* path for N = 100 and direct effect = 0.30 collapsed across all simulation conditions.

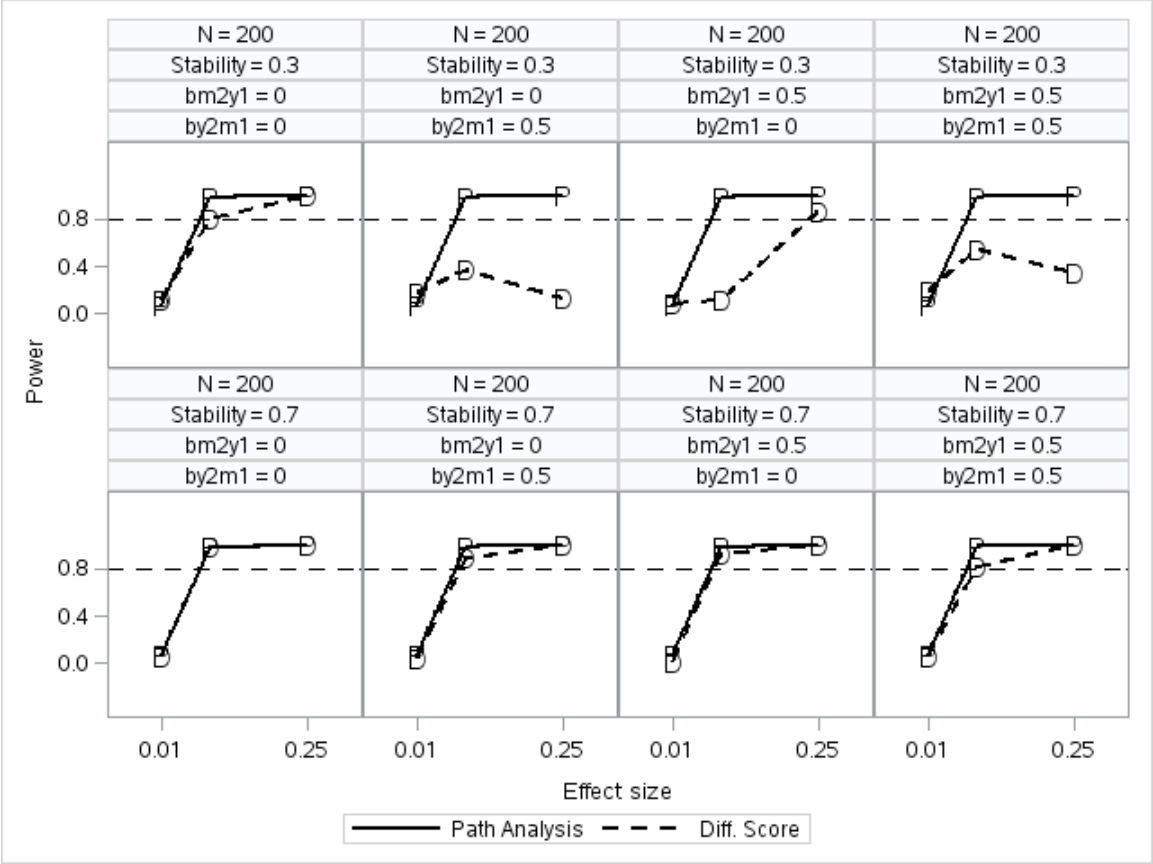


Figure 20. Power plot of path analysis vs. difference score model mediated effect results by sample size, effect size, stability, *bm2y1* path, and *by2m1* path for N = 200 and direct effect = 0.30 collapsed across all simulation conditions.

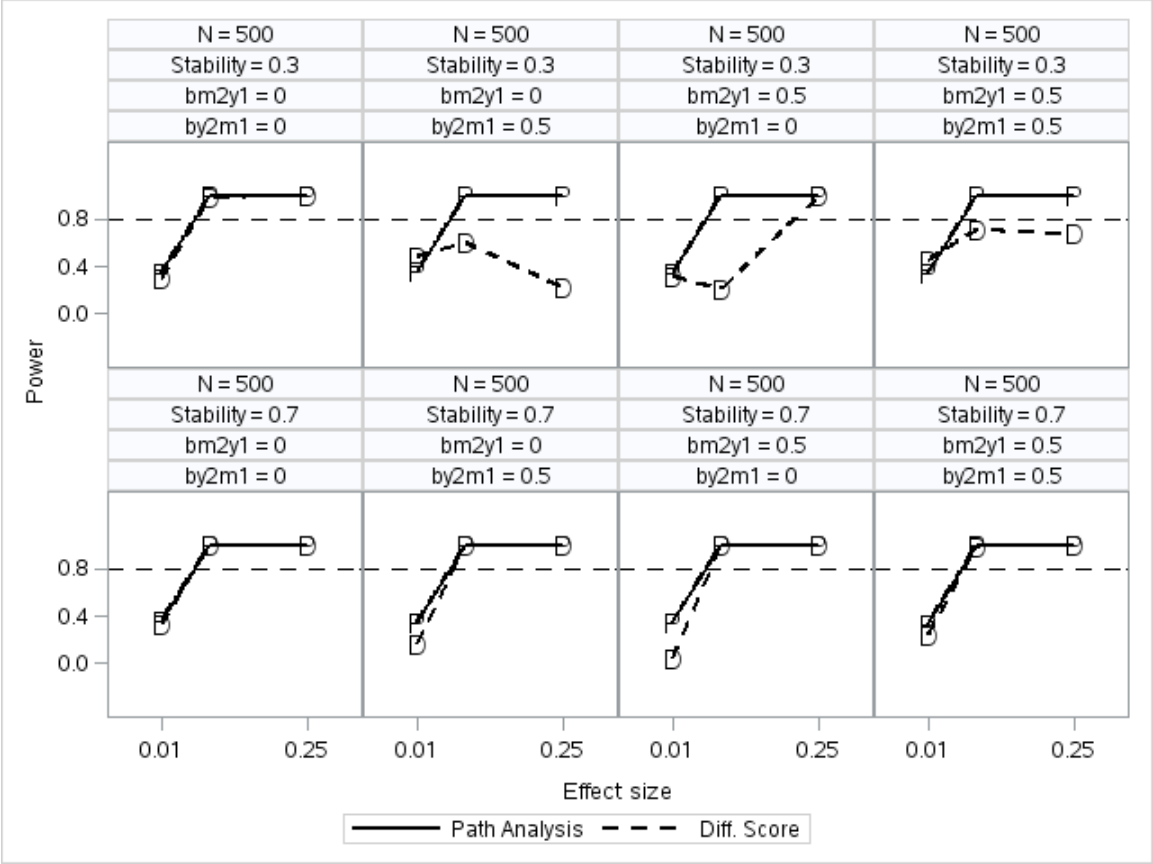


Figure 21. Power plot of path analysis vs. difference score model mediated effect results by sample size, effect size, stability, *bm2y1* path, and *by2m1* path for N = 500 and direct effect = 0.30 collapsed across all simulation conditions.

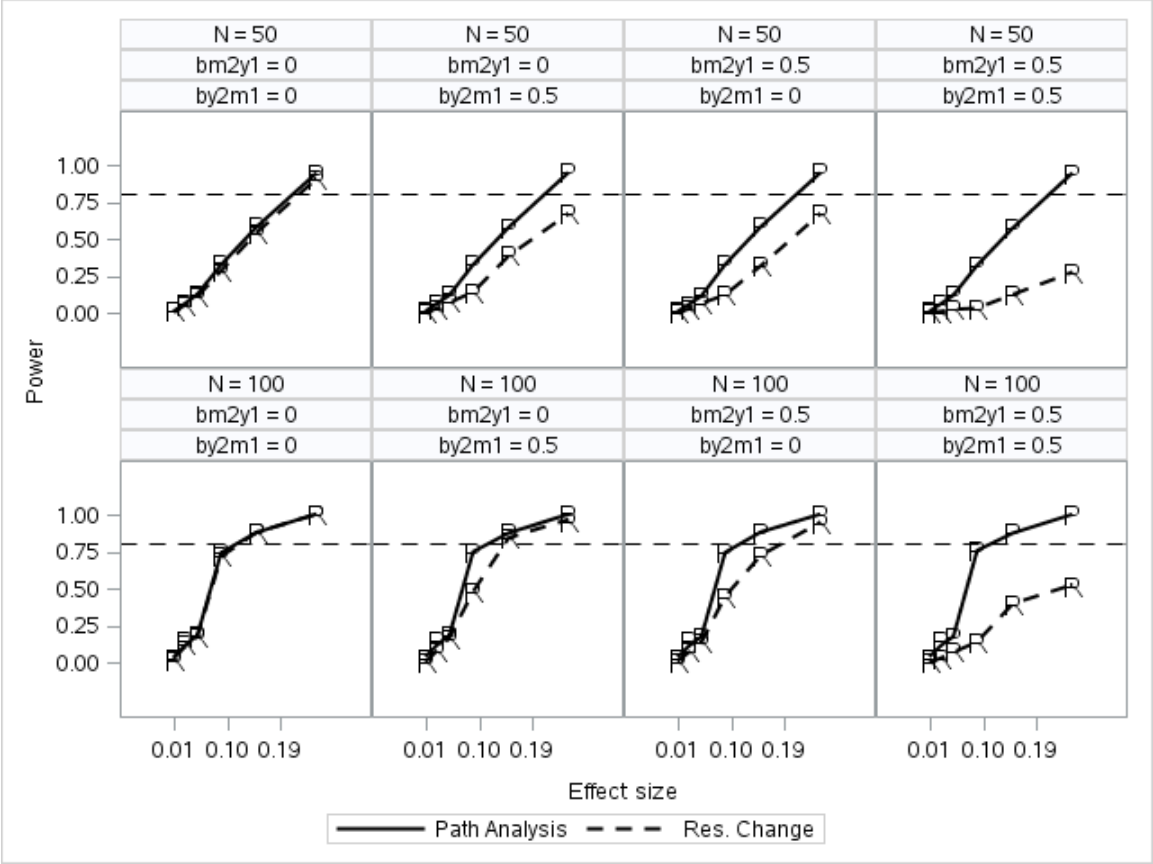


Figure 22. Power plot of path analysis vs. residualized change score model mediated effect results by sample size, effect size, *bm2y1* path, and *by2m1* path for N = 50 – 100 collapsed across all simulation conditions.

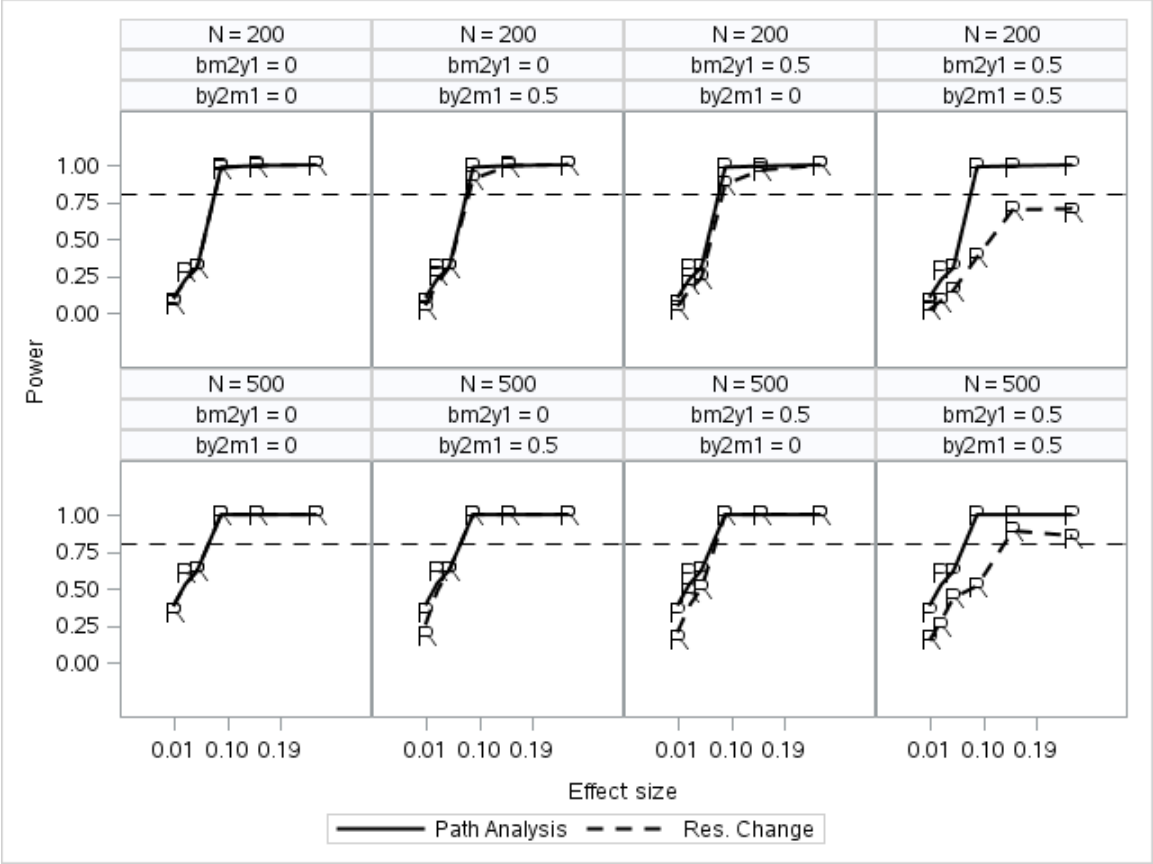


Figure 23. Power plot of path analysis vs. residualized change score model mediated effect results by sample size, effect size, *bm2y1* path, and *by2m1* path for N = 200 – 500 collapsed across all simulation conditions.

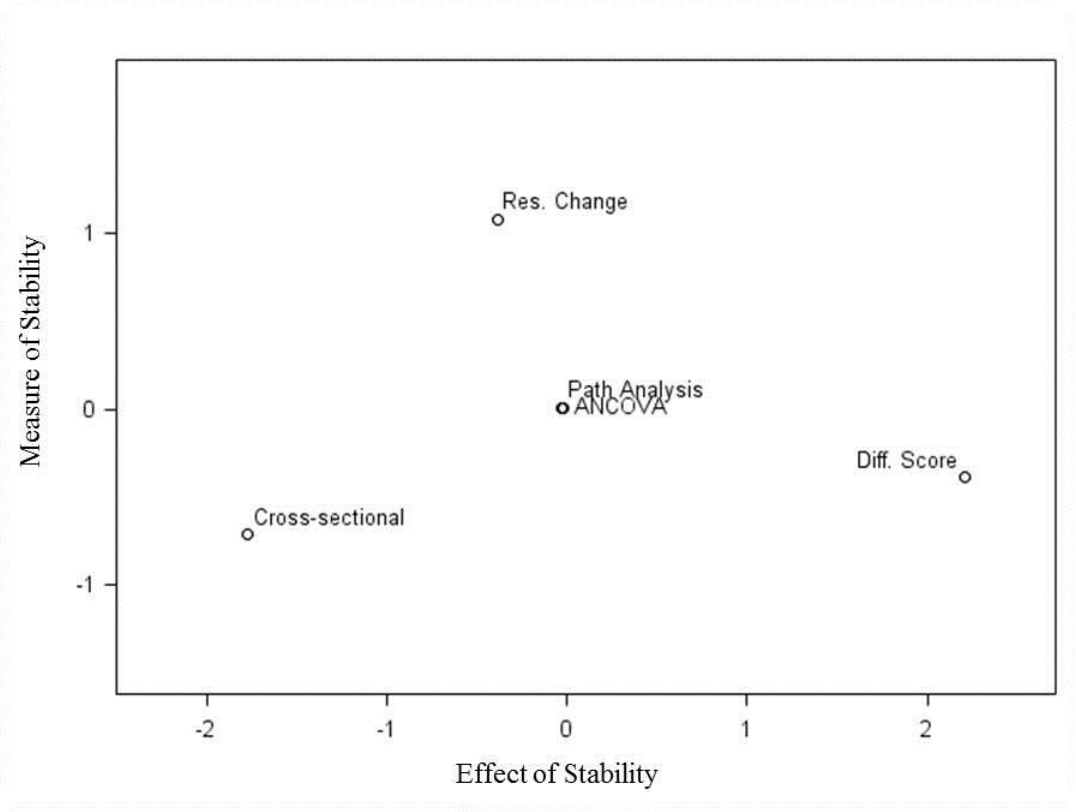


Figure 24. Multidimensional scaling using tetrachoric correlations to categorize the power to detect the mediated effect for the five models on two dimensions.

APPENDIX A

LIST OF VARIABLE NAMES AND DESCRIPTIONS

Variable	Description
X	Treatment
M_1	Pretest Mediator
Y_1	Pretest Outcome
M_2	Posttest Mediator
Y_2	Posttest Outcome
Δ_M	Mediator difference score
Δ_Y	Outcome difference score
R_M	Mediator residualized change score
R_Y	Outcome residualized change score

APPENDIX B

LIST OF MODEL COEFFICIENTS AND DESCRIPTIONS

Coefficient	Description
a_{m2x}	Effect of treatment X on posttest mediator M_2 adjusting for M_1 and Y_1
S_{m2m1}	Pooled within-group stability of mediator
$S_{m2m1total}$	Stability of mediator ignoring group information
b_{m2y1}	Effect of pretest outcome Y_1 on posttest mediator M_2 adjusting for X and M_1
c'_{y2x}	Effect of treatment X on posttest outcome Y_2 adjusting for Y_1, Y_2, and M_2
S_{y2y1}	Pooled within-group stability of outcome
$S_{y2y1total}$	Stability of outcome ignoring group information
b_{y2m1}	Effect of pretest mediator M_1 on the posttest outcome Y_2 adjusting for X, Y_1, and M_2
b_{y2m2}	Effect of posttest mediator M_2 on posttest outcome Y_2 adjusting for X, Y_1,and M_1
b_{y1m1}	Effect of pretest mediator M_1 on pretest outcome Y_1
a_{Δ}	Effect of treatment X on the mediator difference score
b_{Δ}	Effect of the mediator difference score on the outcome difference score adjusting for X
c'_{Δ}	Effect of the treatment X on the outcome difference score adjusting for mediator difference score
a_R	Effect of treatment X on the mediator residualized change score
b_R	Effect of the mediator residualized change score on the outcome residualized change score adjusting for X
c'_R	Effect of the treatment X on the outcome residualized change score adjusting for mediator residualized change score

Note. b_{y1m1} appears in the data generating model figure but was simulated to be equivalent to a pretest correlation between the mediator and the outcome.

APPENDIX C

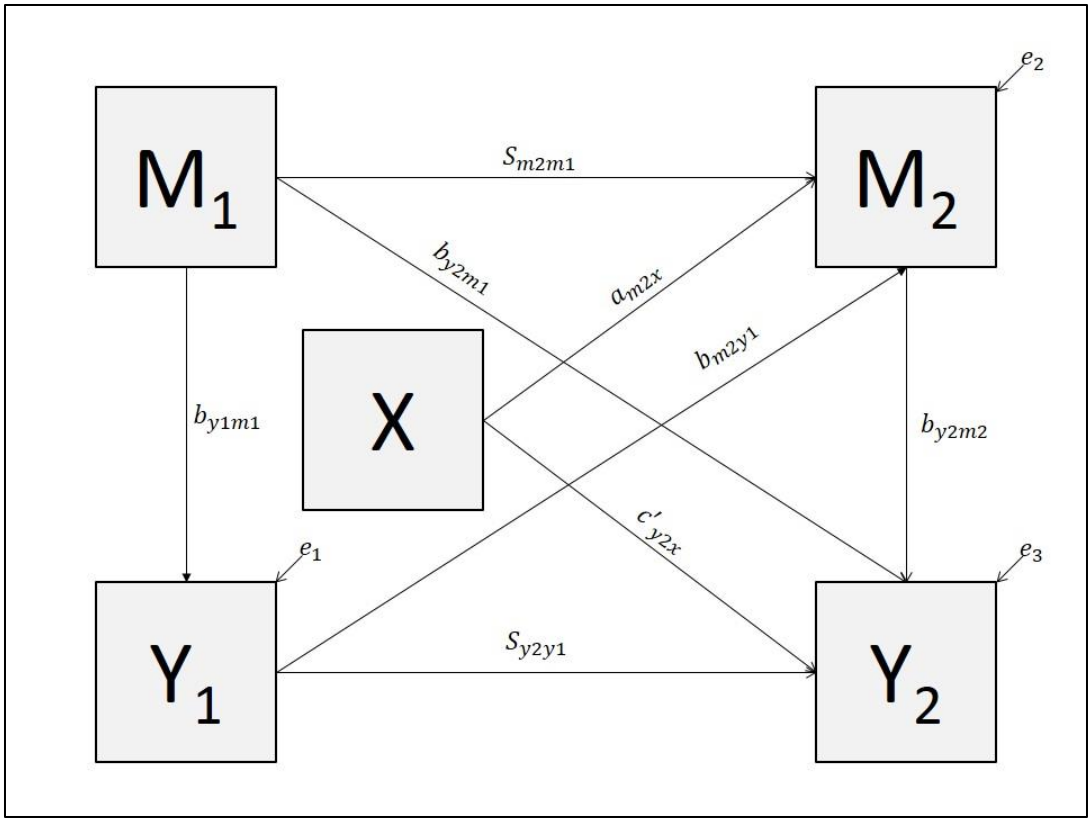
SAS MACRO FOR DATA-GENERATION

```
libname DATAGEN "F:\ASU\Sims\";

TITLE 'DATA GENERATION OF TWO-WAVE MEDIATION';

OPTIONS PS=59 LS=80 REPLACE NONOTES;

*The following two lines of code stop the SAS log
from being saved over iterations of the macro program;

FILENAME NULLOG DUMMY 'C:\NULL';
PROC PRINTTO LOG=NULLOG;

PROC DATASETS LIBRARY=WORK KILL NOLIST; RUN;
%MACRO simulate(nsim,nobs,BMX,BYX,BYM,
aM1X,sm2m1,am2x,bm2y1,
by1m1,cy1x,cy2x,sy2y1,
by2m1,by2m2,varx,varm1,
varm2,vary1,vary2,RELM1,
RELM2, RELY1, RELY2,file,
TYPE,ERROR);

TITLE 'SIMULATION OF TWO-WAVE MEDIATION';
DATA DATAGEN.&FILE;
totaln=&NSIM*&NOBS;
DO I=1 TO totaln;
/*variances and covariances for relability - Reliability will not be
varied in this simulation!*/
VarX=0.25;
VarM1=(&aM1X**2)*VarX+(&VarM1)**2;
CovXM1=&aM1X*VarX;
CovXY1=&cY1X*VarX+&bY1M1*CovXM1;
CovXM2=&aM2X*VarX+&sM2M1*CovXM1+&bM2Y1*CovXY1;
CovXY2=&cY2X*VarX+&SY2Y1*CovXY1+&bY2M1*CovXM1+&bY2M2*CovXM2;
CovM1Y1=&aM1X*&cY1X*VarX+&aM1X*&bY1M1*CovXM1+&bY1M1*&VarM1**2;
CovM1M2=&aM1X*&SM2M1*CovXM1+&aM1X*&bM2Y1*CovXY1+&aM1X*&aM2X*VarX+&SM2M1
*&VarM1+&bM2Y1*&bY1M1*&VarM1**2;
CovM1Y2=&aM1X*&cY2X*VarX+&aM1X*&SY2Y1*CovXY1+&aM1X*&bY2M1*CovXM1+&aM1X*
&bY2M2*CovXM2+&SY2Y1*&bY1M1*&VarM1**2+&bY2M1*&VarM1**2+&bY2M2*&SM2M1*&V
arM1**2+&bY2M2*&bM2Y1*&bY1M1*&VarM1**2;
CovM2Y1=&aM2X*&bY1M1*CovXM1+&aM2X*&cY1X*VarX+&SM2M1*&bY1M1*VarM1+&SM2M1
*&cY1X*CovXM1+&bM2Y1*&bY1M1*CovM1Y1+&bM2Y1*&cY1X*CovXY1+&bM2Y1*(&VarY1)
**2;
VarY1=(&cY1X**2)*VarX+2*&cY1X*&bY1M1*CovXM1+(&bY1M1**2)*VarM1+(&VarY1)*
*2;
VarM2=(&SM2M1**2)*VarM1+2*&SM2M1*&bM2Y1*CovM1Y1+2*&SM2M1*&aM2X*CovXM1+(
&bM2Y1**2)*VarY1+2*&bM2Y1*&aM2X*CovXY1+(&aM2X**2)*VarX+(&VarM2)**2;
VarY2=(&cY2X**2)*VarX+2*&cY2X*&SY2Y1*CovXY1+2*&cY2X*&bY2M1*CovXM1+2*&cY
2X*&bY2M2*CovXM2+(&SY2Y1**2)*VarY1+2*&SY2Y1*&bY2M1*CovM1Y1+2*&SY2Y1*&bY
2M2*CovM2Y1+

(&bY2M1**2)*VarM1+2*&bY2M1*&bY2M2*CovM1M2+(&bY2M2**2)*VarM2+(&VarY2)**2
;
```

```
CovM2Y2=&SM2M1*&cY2X*CovXM1+&SM2M1*&SY2Y1*CovM1Y1+&SM2M1*&bY2M1*VarM1+&
SM2M1*&bY2M2*CovM1M2+&bM2Y1*&cY2X*CovXY1+&bM2Y1*&SY2Y1*VarY1+&bM2Y1*&bY
2M1*CovM1Y1+

&bM2Y1*&bY2M2*CovM2Y1+&aM2X*&cY2X*VarX+&aM2X*&SY2Y1*CovXY1+&aM2X*&bY2M1
*CovXM1+&aM2X*&bY2M2*CovXM2+&bY2M2*(&VarM2)**2;
CovY1Y2=&cY1X*&SY2Y1*CovXY1+&cY1X*&bY2M1*CovXM1+&cY1X*&bY2M2*CovXM2+&cY
1X*&cY2X*VarX+&bY1M1*&SY2Y1*CovM1Y1+&bY1M1*&bY2M1*VarM1+&bY2M2*&bY1M1*C
ovM1M2+
      &bY1M1*&cY2X*CovXM1+&bY2M2*&bM2Y1*(&VarY1)**2+&SY2Y1*(&VarY1)**2;


/*model variables*/
X=&varx*rannor(0); IF X LT 0 THEN X=0; IF X GT 0 THEN X=1;
M1t=&varm1*rannor(0);
Varm1e=(Varm1/&RELM1)-Varm1;
M1e=sqrt(varm1e)*rannor(0);
M1=m1t+m1e;
M2T=&bm2y1*y1+&sm2m1*m1+&am2x*x+&varm2*rannor(0);
VarM2e=(VarM2/&RELM2)-VarM2;
M2e=sqrt(varm2e)*rannor(0);
M2=M2T+M2e;
Y1T=&by1m1*m1+&vary1*rannor(0);
Vary1e=(vary1/&RELY1)-vary1;
Y1e=sqrt(vary1e)*rannor(0);
Y1=Y1T+Y1e;
Y2T=&cy2x*x+&sy2y1*y1+&by2m1*m1+&by2m2*m2+&vary2*rannor(0);
Vary2e=(Vary2/&RELY2)-Vary2;
Y2e=sqrt(vary2e)*rannor(0);
Y2=Y2T+y2e;
Mdiff=M2-M1;
Ydiff=Y2-Y1;
OUTPUT;
END;
drop y1e y1t m1e m1t y2e y2t m2e m2t vary1e varm1e vary2e varm2e;
run;
%END;


%MEND;
run;

PROC DATASETS LIB=WORK NOLIST;
%simulate(nsim=10,nobs=200,BMX=0,BYX=0,BYM=0,
aM1X=0, sm2m1=.981,am2x=0,bm2y1=0, by1m1=.578,
cy1x=0, cy2x=0,sy2y1=.686, by2m1=.876,by2m2=0,
varx=1,varm1=1,varm2=1,vary1=1,vary2=1,
RELM1=1, RELM2=1, RELY1=1, RELY2=1,
FILE=n200cond1,TYPE='CCC',ERROR=1);
run; quit;
```

APPENDIX D

FIGURE OF DATA-GENERATING MODEL


M_1
M_2
X
Y_1
Y_2
e_1
e_2
e_3
S_{m2m1}
b_{y2m1}
a_{m2x}
b_{m2y1}
b_{y1m1}
b_{y2m2}
c'_{y2x}
S_{y2y1}

APPENDIX E

ESTIMATED QUANTITIES AND MODEL CONDITIONS

Model	Quantity Estimated	Conditions			
		Stability	M – Y Cross-lag	Y – M Cross-lag	Pretest correlation
ANCOVA	$a_{m2x} * b_{y2m2}$	Equal across groups	None	None	None
Path analysis	$a_{m2x} * b_{y2m2}$	Equal across groups	None	None	None
Difference Score	$a_{\Delta} * b_{\Delta}$	Equal to 1	None	None	None
Residualized Change score	$a_{R} * b_{R}$	Equal across groups	Equals 0	Equals 0	Equals 0
Cross-sectional	$a_{m2x} * b_{y2m2}$	Equals 0	Equals 0	Equals 0	Equals 0

Note. The mediated effect estimated with the cross-sectional model consists of the same path coefficients as the ANCOVA/path analysis model but ignores all pretest information on M and Y when estimating these quantities.

APPENDIX F

SAS MACRO FOR ANALYSIS OF ALL MODELS

```
libname DATAGEN "E:\ASU\Sims\Two-wave model\Valente Masters
Sim\DATAGEN\";
libname DATAOUT "E:\ASU\Sims\Two-wave model\Valente Masters
Sim\DATAOUT";

FILENAME NULLOG DUMMY 'D:\NULL';
PROC PRINTTO LOG=NULLOG;
PROC DATASETS LIBRARY=WORK KILL NOLIST; RUN;
%MACRO ANALYZE (nsim,nobs,BMX,BYX,BYM,
aM1X,sm2m1,am2x,bm2y1,
by1m1,cy1x,cy2x,sy2y1,
by2m1,by2m2,varx,varm1,
varm2,vary1,vary2,RELM1,
RELM2, RELY1, RELY2,file,
TYPE,ERROR);


DATA SIM; SET DATAGEN.&FILE;
J=&nobs;
DO J=0 to totaln by &nobs;
IF 1+J<=I<=&NOBS+J then rep=1+(J/&nobs);
end;
run;

/*************************************************************/
/*                                                           */
/*                        PATH MODEL                         */
/*                                                           */
/*************************************************************/
PROC CALIS DATA=SIM METHOD=ML NORPINT PLC OUTEST=OUT1 outstat=out2;
by rep;
LINEQS
M2=sm2m1_ M1 + am2x_ X + bm2y1_ Y1+ E2,
y2=cy2x_ X +  sy2y1_ y1 + by2m1_ M1 + by2m2_ M2 + E3;

Cov
X M1,
X Y1,
M1 Y1;

STD
   E2 = EE2,
   E3 = EE3;

RUN;
*SAVING THE PARAMETER VALUES OF FROM THE PATH MODEL OUTPUT. PARAMETERS
ARE DENOTED WITH _ AT THE END OF THE NAME;
DATA CALPARMS; SET OUT1;
IF _TYPE_="PARMS";
KEEP sM2M1_ aM2X_ cY2X_ sY2Y1_ bY2M1_ bY2M2_;
run;
*SAVING THE STANDARD ERRORS OF THE PARAMETERS FROM THE PATH MODEL
OUTPUT;
DATA CALSTDERR1; SET OUT1;
```

```
IF _TYPE_="STDERR"; SEsM2M1_= sM2M1_;
KEEP SEsM2M1_;
DATA CALSTDERR2; SET OUT1;
IF _TYPE_="STDERR"; SEaM2X_= aM2X_;
KEEP  SEaM2X_;
DATA CALSTDERR3; SET OUT1;
IF _TYPE_="STDERR"; SEcY2X_= cY2X_;
KEEP SEcY2X_;
DATA CALSTDERR4; SET OUT1;
IF _TYPE_="STDERR"; SEsY2Y1_= sY2Y1_;
KEEP SEsY2Y1_;
DATA CALSTDERR5; SET OUT1;
IF _TYPE_="STDERR"; SEbY2M1_= bY2M1_;
KEEP SEbY2M1_;
DATA CALSTDERR6; SET OUT1;
IF _TYPE_="STDERR"; SEbY2M2_= bY2M2_;
KEEP SEbY2M2_;

*MERGING THE STANDARD ERROR DATA SETS FOR EACH PARAMETER VALUE INTO ONE
DATASET;
DATA CALSTDERR; MERGE CALSTDERR1 CALSTDERR2 CALSTDERR3 CALSTDERR4
CALSTDERR5 CALSTDERR6;
RUN;

*SAVING THE COVARIANCES AND VARIANCES OF THE VARIABLES FROM THE PATH
MODEL OUTPUT. COVARIANCES DENOTED CV AND VARIANCES DENOTED VARI;
DATA CALISAA; SET OUT2;
IF _TYPE_="COV";IF _NAME_= "M1";CVXM1= X;
KEEP CVXM1;
DATA CALISAB; SET OUT2;
IF _TYPE_="COV";IF _NAME_= "M2";CVXM2= X;
KEEP CVXM2;
DATA CALISAC; SET OUT2;
IF _TYPE_="COV";IF _NAME_= "Y1";CVXy1= X;
KEEP CVXy1;
DATA CALISAD; SET OUT2;
IF _TYPE_="COV";IF _NAME_= "Y2";CVXY2= X;
KEEP CVXY2;
DATA CALISAE; SET OUT2;
IF _TYPE_="COV"; IF _NAME_='M1'; CVM1M2=M2;
KEEP CVM1M2;
DATA CALISAF; SET OUT2;
IF _TYPE_="COV"; IF _NAME_='M1'; CVM1Y1=y1;
KEEP CVM1Y1;
DATA CALISAG; SET OUT2;
IF _TYPE_="COV"; IF _NAME_='M1'; CVM1Y2=Y2;
KEEP CVM1Y2;
DATA CALISAH; SET OUT2;
IF _TYPE_="COV"; IF _NAME_='M2'; CVM2Y1=y1;
KEEP CVM2Y1;
DATA CALISAI; SET OUT2;
IF _TYPE_="COV"; IF _NAME_='M2'; CVM2Y2=Y2;
KEEP CVM2Y2;
DATA CALISAJ; SET OUT2;
```

```
IF _TYPE_="COV"; IF _NAME_='Y1'; CVY1Y2=Y2;
KEEP CVY1Y2;
DATA CALISAK; SET OUT2;
IF _TYPE_='COV'; IF _NAME_="X"; VARIX=X;
KEEP VARIX;
DATA CALISAL; SET OUT2;
IF _TYPE_='COV'; IF _NAME_="M1"; VARIM1=M1;
KEEP VARIM1;
DATA CALISAM; SET OUT2;
IF _TYPE_='COV'; IF _NAME_="M2"; VARIM2=M2;
KEEP VARIM2;
DATA CALISAN; SET OUT2;
IF _TYPE_='COV'; IF _NAME_="Y1"; VARIY1=y1;
KEEP VARIY1;
DATA CALISAO; SET OUT2;
IF _TYPE_='COV'; IF _NAME_="Y2"; VARIY2=Y2;
KEEP VARIY2;

*MERGING THE COVARIANCES AND VARIANCES OF THE VARIABLES FROM THE PATH
MODEL OUTPUT INTO ONE DATASET;
DATA CALCOV; MERGE CALISAA CALISAB CALISAC CALISAD CALISAE CALISAF
CALISAG CALISAH CALISAI CALISAJ
CALISAK CALISAL CALISAM CALISAN CALISAO;
RUN;

*COMPUTES CORRELATIONS BASED ON PATH MODEL OUTPUT;
DATA CALCORR; SET CALCOV;
CRXM1=CVXM1/(SQRT(VARIX)*SQRT(VARIM1));
CRXM2=CVXM2/(SQRT(VARIX)*SQRT(VARIM2));
CRXY1=CVXY1/(SQRT(VARIX)*SQRT(VARIY1));
CRXY2=CVXY2/(SQRT(VARIX)*SQRT(VARIY2));
CRM1M2=CVM1M2/(SQRT(VARIM1)*SQRT(VARIM2));
CRM1Y1=CVM1Y1/(SQRT(VARIM1)*SQRT(VARIY1));
CRM1Y2=CVM1Y2/(SQRT(VARIM1)*SQRT(VARIY2));
CRM2Y1=CVM2Y1/(SQRT(VARIM2)*SQRT(VARIY1));
CRM2Y2=CVM2Y2/(SQRT(VARIM2)*SQRT(VARIY2));
CRY1Y2=CVY1Y2/(SQRT(VARIY1)*SQRT(VARIY2));
RUN;

*THIS MERGES ALL THE OUTPUT FROM THE PATH MODEL OUTPUT INTO ONE
DATASET;
DATA CALIS; MERGE CALPARMS CALSTDERR CALCORR;

/*************************************************************/
/*                                                           */
/*                      CHANGE SCORES                        */
/*                                                           */
/*************************************************************/
/*ESTIMATING (DIFFM=X Y1) REGRESSION AND SAVING THE VALUES OF THE
COEFFICIENTS AND THEIR STANDARD ERRORS*/
PROC REG DATA=SIM OUTEST=DiffFILE1 COVOUT noprint; by rep; MODEL Mdiff=
X/;
DATA DiffA; SET DiffFILE1; IF _TYPE_='PARMS'; aM2XDiff=X;
MSEAM2XDiff=_RMSE_*_RMSE_;
```

```
DROP _MODEL_ _NAME_ _TYPE_ _DEPVAR_ _RMSE_ INTERCEP X;
KEEP aM2XDiff MSEAM2XDiff;
DATA DiffB; SET DiffFILE1;IF _NAME_='X'; SEAM2XDiff=SQRT(X);
DROP _MODEL_ _NAME_ _TYPE_ _DEPVAR_ _RMSE_ INTERCEP X;
KEEP SEAM2XDiff;
DATA DiffMODEL1; MERGE DiffA DIffB;


*Estimating the (DiffY=X DiffM) REGRESSION AND SAVING THE VALUES OF THE
COEFFICIENTS AND THEIR STANDARD ERRORS;
PROC REG DATA=SIM OUTEST=DiffFILE2 COVOUT noprint; by rep; MODEL
Ydiff=X Mdiff/;
DATA DiffE; SET DiffFILE2;IF _TYPE_='PARMS'; cY2XDiff=X;
MSECY2XDiff=_RMSE_*_RMSE_;
DROP _MODEL_ _NAME_ _TYPE_ _DEPVAR_ _RMSE_ INTERCEP X MDiff;
KEEP cY2Xdiff MSECY2Xdiff;
DATA DiffF; SET DiffFILE2;IF _NAME_='X'; SECY2XDiff=SQRT(X);
DROP _MODEL_ _NAME_ _TYPE_ _DEPVAR_ _RMSE_ INTERCEP X MDiff;
KEEP SECY2XDiff;
DATA DiffG; SET DiffFILE2;IF _TYPE_='PARMS'; bY2M2Diff=Mdiff;
MSEBY2M2DIff=_RMSE_*_RMSE_;
DROP _MODEL_ _NAME_ _TYPE_ _DEPVAR_ _RMSE_ INTERCEP X MDiff;
KEEP bY2M2Diff MSEBY2M2Diff;
DATA DiffH; SET DiffFILE2;IF _NAME_='Mdiff'; SEBY2M2Diff=SQRT(Mdiff);
DROP _MODEL_ _NAME_ _TYPE_ _DEPVAR_ _RMSE_ INTERCEP X MDiff;
KEEP SEBY2M2Diff;
DATA DiffMODEL2; MERGE DiffE DiffF DiffG DiffH;

DATA DiffMODELS; MERGE DiffMODEL1 DiffMODEL2;

/*************************************************************/
/*                                                           */
/*                RESIDUALIZED CHANGE SCORES                 */
/*                                                           */
/*************************************************************/
/*MODEL COMPUTING AND SAVING RESIDUALIZED DIFFERENCE SCORE OF M1 AND
M2*/
PROC REG DATA=SIM noprint; by rep; MODEL M2=M1/;
output out=resid1 r=residm;
data rchange1;set resid1;
keep residm x;
/*MODEL COMPUTING AND SAVING RESIDUALIZED DIFFERENCE SCORE OF Y1 AND
Y2*/
PROC REG DATA=SIM noprint; by rep; MODEL Y2=y1/;
output out=resid2 r=residy;
data rchange2;set resid2;
keep residy;

proc sort; by I;
data rchange; merge resid1 resid2; by I;

/*ESTIMATING (RESIDM2=X Y1) REGRESSION AND SAVING THE VALUES OF THE
COEFFICIENTS AND THEIR STANDARD ERRORS*/
```

```
PROC REG DATA=rchange OUTEST=RFILE1 COVOUT noprint; by rep; MODEL
residm= X/;
DATA RA; SET RFILE1; IF _TYPE_='PARMS'; aM2XRES=X;
MSEAM2XRES=_RMSE_*_RMSE_;
DROP _MODEL_ _NAME_ _TYPE_ _DEPVAR_ _RMSE_ INTERCEP X;
KEEP aM2XRES MSEAM2XRES;
DATA RB; SET RFILE1;IF _NAME_='X'; SEAM2XRES=SQRT(X);
DROP _MODEL_ _NAME_ _TYPE_ _DEPVAR_ _RMSE_ INTERCEP X;
KEEP SEAM2XRES;
DATA RESMODEL1; MERGE RA RB;


*Estimating the (RESIDY2=X RESIDM2) REGRESSION AND SAVING THE VALUES OF
THE COEFFICIENTS AND THEIR STANDARD ERRORS;
PROC REG DATA=rchange OUTEST=RFILE2 COVOUT noprint; by rep; MODEL
RESIDY=X RESIDM/;
DATA RE; SET RFILE2;IF _TYPE_='PARMS'; cY2XRES=X;
MSECY2XRES=_RMSE_*_RMSE_;
DROP _MODEL_ _NAME_ _TYPE_ _DEPVAR_ _RMSE_ INTERCEP X RESIDM;
KEEP cY2XRES MSECY2XRES;
DATA RF; SET RFILE2;IF _NAME_='X'; SECY2XRES=SQRT(X);
DROP _MODEL_ _NAME_ _TYPE_ _DEPVAR_ _RMSE_ INTERCEP X RESIDM;
KEEP SECY2XRES;
DATA RG; SET RFILE2;IF _TYPE_='PARMS';
bY2M2RES=RESIDM;MSEBY2M2RES=_RMSE_*_RMSE_;
DROP _MODEL_ _NAME_ _TYPE_ _DEPVAR_ _RMSE_ INTERCEP X RESIDM;
KEEP bY2M2RES MSEBY2M2RES;
DATA RH; SET RFILE2;IF _NAME_='residm'; SEBY2M2RES=SQRT(residm);
DROP _MODEL_ _NAME_ _TYPE_ _DEPVAR_ _RMSE_ INTERCEP X RESIDM;
KEEP SEBY2M2RES;
DATA RESMODEL2; MERGE RE RF RG RH;

DATA RESMODELS; MERGE RESMODEL1 RESMODEL2;
/*************************************************************/
/*                                                           */
/*                LINEAR REGRESSION (ANCOVA)                 */
/*                                                           */
/*************************************************************/

*Estimating the (M1=X) regression and saving the value of aM1X and its
standard error;
/*PROC REG DATA=SIM OUTEST=FILE1 COVOUT noprint; MODEL M1=X/;
DATA B; SET FILE1;IF _TYPE_='PARMS'; aM1X=X; MSEM1X=_RMSE_*_RMSE_;
DROP _MODEL_ _NAME_ _TYPE_ _DEPVAR_ _RMSE_ INTERCEP X M1;
KEEP aM1X MSEM1X;
DATA C; SET FILE1;IF _NAME_='X'; SEAM1X=SQRT(X);
DROP _MODEL_ _NAME_ _TYPE_ _DEPVAR_ _RMSE_ INTERCEP X M1;
KEEP SEAM1X;
DATA MODEL1; MERGE B C; */

*Estimating the (M2=M1 X) regression and saving the value of sM2M1,
aM2X, and their standard errors;
PROC REG DATA=SIM OUTEST=FILE2 COVOUT noprint; by rep; MODEL M2=M1 Y1
X/;
```

```
DATA D; SET FILE2;IF _TYPE_='PARMS'; sM2M1=M1; MSEM2M1=_RMSE_*_RMSE_;
DROP _MODEL_ _NAME_ _TYPE_ _DEPVAR_ _RMSE_ INTERCEP X M1 M2;
KEEP sM2M1 MSEM2M1;
DATA E; SET FILE2;IF _NAME_='M1'; SESM2M1=SQRT(M1);
DROP _MODEL_ _NAME_ _TYPE_ _DEPVAR_ _RMSE_ INTERCEP X M1 M2;
KEEP SESM2M1;
DATA F; SET FILE2; IF _TYPE_='PARMS'; aM2X=X;MSEAM2X=_RMSE_*_RMSE_;
DROP _MODEL_ _NAME_ _TYPE_ _DEPVAR_ _RMSE_ INTERCEP X M1 M2;
KEEP aM2X MSEAM2X;
DATA G; SET FILE2;IF _NAME_='X'; SEAM2X=SQRT(X);
DROP _MODEL_ _NAME_ _TYPE_ _DEPVAR_ _RMSE_ INTERCEP X M1 M2;
KEEP SEAM2X;
DATA H; SET FILE2; IF _TYPE_='PARMS'; bM2Y1=Y1;MSEBM2Y1=_RMSE_*_RMSE_;
DROP _MODEL_ _NAME_ _TYPE_ _DEPVAR_ _RMSE_ INTERCEP X Y1 M1 M2;
KEEP bM2Y1 MSEBM2Y1;
DATA I; SET FILE2; IF _NAME_='y1'; SEBM2Y1=SQRT(y1);
DROP _MODEL_ _NAME_ _TYPE_ _DEPVAR_ _RMSE_ INTERCEP X Y1 M1 M2;
KEEP SEBM2Y1;
DATA MODEL2; MERGE D E F G H I;

*Estimating the (Y1=M1) regression and saving the value of bY1M1 and
its standard errors;
PROC REG DATA=SIM OUTEST=FILE3 COVOUT noprint; by rep; MODEL Y1=M1/;
DATA J; SET FILE3;IF _TYPE_='PARMS'; bY1M1=M1;MSEBY1M1=_RMSE_*_RMSE_;
DROP _MODEL_ _NAME_ _TYPE_ _DEPVAR_ _RMSE_ INTERCEP M1 Y1;
KEEP bY1M1 MSEBY1M1;
DATA K; SET FILE3;IF _NAME_='M1'; SEBY1M1=SQRT(M1);
DROP _MODEL_ _NAME_ _TYPE_ _DEPVAR_ _RMSE_ INTERCEP M1 Y1;
KEEP SEBY1M1;
/*DATA L; SET FILE3;IF _TYPE_='PARMS'; cY1X=X;MSECY1X=_RMSE_*_RMSE_;
DROP _MODEL_ _NAME_ _TYPE_ _DEPVAR_ _RMSE_ INTERCEP X M1 Y1;
KEEP cY1X MSECY1X;
DATA M; SET FILE3;IF _NAME_='X'; SECY1X=SQRT(X);
DROP _MODEL_ _NAME_ _TYPE_ _DEPVAR_ _RMSE_ INTERCEP X M1 Y1;
KEEP SECY1X;*/
DATA MODEL3; MERGE J K;

*Estimating the (Y2=X Y1 M1 M2) regression and saving the value of
sM2M1, aM2X, bM2Y1, and their standard errors;
PROC REG DATA=SIM OUTEST=FILE4 COVOUT noprint; by rep; MODEL Y2=X Y1 M1
M2/;
DATA N; SET FILE4;IF _TYPE_='PARMS'; cY2X=X;MSECY2X=_RMSE_*_RMSE_;
DROP _MODEL_ _NAME_ _TYPE_ _DEPVAR_ _RMSE_ INTERCEP X Y1 Y2 M1 M2;
KEEP cY2X MSECY2X;
DATA O; SET FILE4;IF _NAME_='X'; SECY2X=SQRT(X);
DROP _MODEL_ _NAME_ _TYPE_ _DEPVAR_ _RMSE_ INTERCEP X Y1 Y2 M1 M2;
KEEP SECY2X;
DATA P; SET FILE4;IF _TYPE_='PARMS'; sY2Y1=Y1;MSESY2Y1=_RMSE_*_RMSE_;
DROP _MODEL_ _NAME_ _TYPE_ _DEPVAR_ _RMSE_ INTERCEP X Y1 M1 M2;
KEEP sY2Y1 MSESY2Y1;
DATA Q; SET FILE4;IF _NAME_='Y1'; SESY2Y1=SQRT(Y1);
DROP _MODEL_ _NAME_ _TYPE_ _DEPVAR_ _RMSE_ INTERCEP X Y1 M1 M2;
KEEP SESY2Y1;
DATA R; SET FILE4;IF _TYPE_='PARMS'; bY2M1=M1;MSEBY2M1=_RMSE_*_RMSE_;
```

```
DROP _MODEL_ _NAME_ _TYPE_ _DEPVAR_ _RMSE_ INTERCEP X Y1 M1 M2;
KEEP bY2M1 MSEBY2M1;
DATA S; SET FILE4;IF _NAME_='M1'; SEBY2M1=SQRT(M1);
DROP _MODEL_ _NAME_ _TYPE_ _DEPVAR_ _RMSE_ INTERCEP X Y1 M1 M2;
KEEP SEBY2M1;
DATA T; SET FILE4;IF _TYPE_='PARMS'; bY2M2=M2;MSEBY2M2=_RMSE_*_RMSE_;
DROP _MODEL_ _NAME_ _TYPE_ _DEPVAR_ _RMSE_ INTERCEP X Y1 M1 M2;
KEEP bY2M2 MSEBY2M2;
DATA U; SET FILE4;IF _NAME_='M2'; SEBY2M2=SQRT(M2);
DROP _MODEL_ _NAME_ _TYPE_ _DEPVAR_ _RMSE_ INTERCEP X Y1 M1 M2;
KEEP SEBY2M2;
DATA MODEL4; MERGE N O P Q R S T U;

/*Estimating various total effect of X on Y2 IN PROGRESS
PROC REG DATA=SIM OUTEST-FILE5 COVOUT noprint; MODEL Y2=X/;
DATA V; SET FILE5L IF _TYPE_='PARMS'; CX=X; MSECX=_RMSE_*_RMSE_;
DROP _MODEL_ _NAME_ _TYPE_ _DEPVAR_ _RMSE_ INTERCEP X Y2;
KEEP CX MSECX;
PROC REG DATA=SIM OUTEST-FILE5 COVOUT noprint; MODEL Y2=X/;
DATA W; SET FILE5L IF _NAME_='X'; SECX=SQRT(X);
DROP _MODEL_ _NAME_ _TYPE_ _DEPVAR_ _RMSE_ INTERCEP X Y2;
KEEP SECX;
MODEL5; MERGE V W;*/

/*************************************************************/
/*                                                           */
/*                CROSS SECTIONAL MEDIATOR MODEL             */
/*                                                           */
/*************************************************************/

*Estimating the (M2=X) regression and saving the value of a and its
standard error;
PROC REG DATA=SIM OUTEST=FILE5 COVOUT noprint; by rep; MODEL M2= X/;
DATA V; SET FILE5; IF _TYPE_='PARMS'; a=X;MSEA=_RMSE_*_RMSE_;
DROP _MODEL_ _NAME_ _TYPE_ _DEPVAR_ _RMSE_ INTERCEP X M2;
KEEP a MSEA;
DATA W; SET FILE5;IF _NAME_='X'; SEA=SQRT(X);
DROP _MODEL_ _NAME_ _TYPE_ _DEPVAR_ _RMSE_ INTERCEP X M2;
KEEP SEA;
DATA MODEL5; MERGE V W;

*Estimating the (Y2=X M2) regression and saving the value of c b and
their standard errors;
PROC REG DATA=SIM OUTEST=FILE6 COVOUT noprint; by rep; MODEL Y2=X M2/;
DATA X; SET FILE6;IF _TYPE_='PARMS'; c=X; MSEC=_RMSE_*_RMSE_;
DROP _MODEL_ _NAME_ _TYPE_ _DEPVAR_ _RMSE_ INTERCEP X Y2 M2;
KEEP c MSEC;
DATA Y; SET FILE6;IF _NAME_='X'; SEC=SQRT(X);
DROP _MODEL_ _NAME_ _TYPE_ _DEPVAR_ _RMSE_ INTERCEP X Y2 M2;
KEEP SEC;
DATA Z; SET FILE6;IF _TYPE_='PARMS'; b=M2;MSEB=_RMSE_*_RMSE_;
DROP _MODEL_ _NAME_ _TYPE_ _DEPVAR_ _RMSE_ INTERCEP X Y2 M2;
KEEP b MSEB;
DATA AA; SET FILE6;IF _NAME_='M2'; SEB=SQRT(M2);
```

```
DROP _MODEL_ _NAME_ _TYPE_ _DEPVAR_ _RMSE_ INTERCEP X Y2 M2;
KEEP SEB;
DATA MODEL6; MERGE X Y Z AA;


*THIS MERGES ALL THE PREVIOUS REGRESSION, CHANGE SCORE, RES CHANGE, AND
PATH MODEL OUTPUT;
DATA ALL; MERGE MODEL2 MODEL3 MODEL4 MODEL5 MODEL6 DIFFMODELS RESMODELS
CALIS;
RUN;


*THIS COMPUTES COVARIANCES OF VARIABLES VIA PROC CORR;
PROC CORR DATA=SIM cov nocorr OUTPUT=COV noprint; by rep;
var x m1 m2 y1 y2 YDIFF MDIFF;
DATA SA; SET COV;
IF _NAME_='M1'; COVXM1=X;
KEEP COVXM1;
DATA SB; SET COV;
IF _NAME_='M2'; COVXM2=X;
KEEP COVXM2;
DATA SC; SET COV;
IF _NAME_='Y1'; COVXY1=X;
KEEP COVXY1;
DATA SD; SET COV;
IF _NAME_='Y2'; COVXY2=X;
KEEP COVXY2;
DATA SE; SET COV;
IF _NAME_='M1'; COVM1M2=M2;
KEEP COVM1M2;
DATA SF; SET COV;
IF _NAME_='M1'; COVM1Y1=y1;
KEEP COVM1Y1;
DATA SG; SET COV;
IF _NAME_='M1'; COVM1Y2=Y2;
KEEP COVM1Y2;
DATA SH; SET COV;
IF _NAME_='M2'; COVM2Y1=y1;
KEEP COVM2Y1;
DATA SI; SET COV;
IF _NAME_='M2'; COVM2Y2=Y2;
KEEP COVM2Y2;
DATA SJ; SET COV;
IF _NAME_='Y1'; COVY1Y2=Y2;
KEEP COVY1Y2;
DATA SK; SET COV;
IF _TYPE_='STD'; VAX=X*X;
KEEP VAX;
DATA SL; SET COV;
IF _TYPE_='STD'; VAM1=M1*M1;
KEEP VAM1;
DATA SM; SET COV;
IF _TYPE_='STD'; VAM2=M2*M2;
```

```
KEEP VAM2;
DATA SN; SET COV;
IF _TYPE_='STD'; VAY1=y1*y1;
KEEP VAY1;
DATA SO; SET COV;
IF _TYPE_='STD'; VAY2=Y2*Y2;
KEEP VAY2;
DATA SP; SET COV;
IF _TYPE_='STD'; VAYDIFF=YDIFF*YDIFF;
KEEP VAYDIFF;
DATA SQ; SET COV;
IF _TYPE_='STD'; VAMDIFF=MDIFF*MDIFF;
KEEP VAMDIFF;
DATA SR; SET COV;
IF _NAME_='Mdiff'; COVXMDIFF=X;
KEEP COVXMDIFF;
DATA SS; SET COV;
IF _NAME_='Ydiff'; COVXYDIFF=X;
KEEP COVXYDIFF;
DATA ST; SET COV;
IF _NAME_='Ydiff'; COVMDIFFYDIFF=Mdiff;
KEEP COVMDIFFYDIFF;
RUN;

PROC CORR DATA=rchange cov nocorr OUTPUT=COV2 noprint; by rep;
var X RESIDM RESIDY;
DATA SU; SET COV2;
IF _NAME_='residm'; COVXRESM=x;
KEEP COVXRESM;
DATA SV; SET COV2;
IF _NAME_='residy'; COVXRESY=x;
KEEP COVXRESY;
DATA SW; SET COV2;
IF _NAME_='residy'; COVRESMRESY=residm;
KEEP COVRESMRESY;
DATA SX; SET COV2;
IF _NAME_='residy'; VARESY=residy;
KEEP VARESY;
DATA SY; SET COV2;
IF _NAME_='residm'; VARESM=residm;
KEEP VARESM;
RUN;


*THIS MERGES ALL OF THE INDIVIDUAL COVARIANCE DATASETS INTO ONE
DATASET;
DATA SCOVS; MERGE SA SB SC SD SE SF SG SH SI SJ SK SL SM SN SO SP SQ SR
SS ST SU SV SW SX SY;
RUN;

*THIS COMPUTES THE CORRELATIONS FOR ALL THE VARIABLES VIA PROC CORR;
PROC CORR DATA=SIM OUTPUT=CORR noprint; by rep;
var x m1 m2 y1 y2;
DATA SAR; SET CORR;
```

```
IF _NAME_='M1'; CORRXM1=X;
KEEP CORRXM1;
DATA SBR; SET CORR;
IF _NAME_='M2'; CORRXM2=X;
KEEP CORRXM2;
DATA SCR; SET CORR;
IF _NAME_='Y1'; CORRXY1=x;
KEEP CORRXY1;
DATA SDR; SET CORR;
IF _NAME_='Y2'; CORRXY2=X;
KEEP CORRXY2;
DATA SER; SET CORR;
IF _NAME_='M1'; CORRM1M2=M2;
KEEP CORRM1M2;
DATA SFR; SET CORR;
IF _NAME_='M1'; CORRM1Y1=y1;
KEEP CORRM1Y1;
DATA SGR; SET CORR;
IF _NAME_='M1'; CORRM1Y2=Y2;
KEEP CORRM1Y2;
DATA SHR; SET CORR;
IF _NAME_='M2'; CORRM2Y1=y1;
KEEP CORRM2Y1;
DATA SIR; SET CORR;
IF _NAME_='M2'; CORRM2Y2=Y2;
KEEP CORRM2Y2;
DATA SJR; SET CORR;
IF _NAME_='Y1'; CORRY1Y2=y2;
KEEP CORRY1Y2;
RUN;

*THIS MERGES ALL THE INDIVIDUAL CORRELATION DATASETS INTO ONE DATASET;
DATA SCORRS; MERGE SAR SBR SCR SDR SER SFR SGR SHR SIR SJR;
RUN;

*P DENOTES POPULATION VALUE OF PARAMETER;
DATA TEST;SET ALL;
NSIM=&NSIM;
NOBS=&NOBS;
PaM1X=&aM1X;
PsM2M1=&sM2M1;
PaM2X=&aM2X;
PbM2Y1=&bM2Y1;
PbY1M1=&bY1M1;
PcY1X=&cY1X;
PcY2X=&cY2X;
PsY2Y1=&sY2Y1;
PbY2M1=&bY2M1;
PbY2M2=&bY2M2;
VARX=&VARX;
VARM1=&VARM1;
VARM2=&VARM2;
VARY1=&VARY1;
VARY2=&VARY2;
```

```
FILE=&FILE;
TYPE=&TYPE;
ERROR=&ERROR;

*The code below calculates the product of coefficients for all four
methods;
TRUEAB=PaM2X*PbY2M2;
AB=aM2X*bY2M2;
AB_=aM2X_*bY2M2_;
ABDIFF=aM2XDiff*bY2M2Diff;
ABRES=aM2XRES*bY2M2RES;
ABX=A*B;

*The code below computes the standard error for the product of the
coefficients for all four methods;
SEAB=SQRT((aM2X**2)*(SEbY2M2**2)+(bY2M2**2)*(SEaM2X**2));
SEAB_=SQRT((aM2X_**2)*(SEbY2M2_**2)+(bY2M2_**2)*(SEaM2X_**2));
SEABDIFF=SQRT((aM2XDiff**2)*(SEbY2M2Diff**2)+(bY2M2Diff**2)*(SEaM2XDiff
**2));
SEABRES=SQRT((aM2XRES**2)*(SEbY2M2RES**2)+(bY2M2RES**2)*(SEaM2XRES**2))
;

*The code below calculates empirical power of paths a, b, and c;
ZA=AM2X/SEAM2X;
PZA=1-PROBNORM(ABS(ZA));
SZA=0; IF PZA<=0.025 THEN SZA=1;

ZB=BY2M2/SEBY2M2;
PZB=1-PROBNORM(ABS(ZB));
SZB=0; IF PZB<=0.025 THEN SZB=1;

ZC=CY2X/SECY2X;
PZC=1-PROBNORM(ABS(ZC));
SZC=0; IF PZC<=0.025 THEN SZC=1;

ZADIFF=AM2XDIFF/SEAM2XDIFF;
PZADIFF=1-PROBNORM(ABS(ZADIFF));
SZADIFF=0; IF PZADIFF<=0.025 THEN SZADIFF=1;

ZBDIFF=BY2M2DIFF/SEBY2M2DIFF;
PZBDIFF=1-PROBNORM(ABS(ZBDIFF));
SZBDIFF=0; IF PZBDIFF<=0.025 THEN SZBDIFF=1;

ZCDIFF=CY2XDIFF/SECY2XDIFF;
PZCDIFF=1-PROBNORM(ABS(ZCDIFF));
SZCDIFF=0; IF PZCDIFF<=0.025 THEN SZCDIFF=1;

ZARES=AM2XRES/SEAM2XRES;
PZARES=1-PROBNORM(ABS(ZARES));
SZARES=0; IF PZARES<=0.025 THEN SZARES=1;

ZBRES=BY2M2RES/SEBY2M2RES;
PZBRES=1-PROBNORM(ABS(ZBRES));
SZBRES=0; IF PZBRES<=0.025 THEN SZBRES=1;
```

```
ZCRES=CY2XRES/SECY2XRES;
PZCRES=1-PROBNORM(ABS(ZCRES));
SZCRES=0; IF PZCRES<=0.025 THEN SZCRES=1;

ZA_=AM2X_/SEAM2X_;
PZA_=1-PROBNORM(ABS(ZA_));
SZA_=0; IF PZA_<=0.025 THEN SZA_=1;

ZB_=BY2M2_/SEBY2M2_;
PZB_=1-PROBNORM(ABS(ZB_));
SZB_=0; IF PZB_<=0.025 THEN SZB_=1;

ZC_=CY2X_/SECY2X_;
PZC_=1-PROBNORM(ABS(ZC_));
SZC_=0; IF PZC_<=0.025 THEN SZC_=1;

*The following code computes bias and relative bias for the four
methods (PATH MODEL, RES. CHANGE, CHANGE, ANCOVA(REGRESSION), AND CROSS
SECTIONAL MODEL);
BA=AM2X-PAM2X;
BAR=BA/PAM2X;
B2A=BA*BA;
BADIFF=AM2XDIFF-PAM2X;
BADIFFR=BADIFF/PAM2X;
B2ADIFF=BADIFF*BADIFF;
BARES=AM2XRES-PAM2X;
BARESR=BARES/PAM2X;
B2ARES=BARES*BARES;
BA_=AM2X_-PAM2X;
BA_R=BA_/PAM2X;
B2A_=BA_*BA_;
BAX=A-PAM2X;
BAXR=BAX/PAM2X;
B2AX=BAX*BAX;
BB=BY2M2-PBY2M2;
BBR=BB/PBY2M2;
B2B=BB*BB;
BBDIFF=BY2M2DIFF-PBY2M2;
BBDIFFR=BBDIFF/PBY2M2;
B2BDIFF=BBDIFF*BBDIFF;
BBRES=BY2M2RES-PBY2M2;
BBRESR=BBRES/PBY2M2;
B2BRES=BBRES*BBRES;
BB_=BY2M2_-PBY2M2;
BB_R=BB_/PBY2M2;
B2B_=BB_*BB_;
BBX=B-PBY2M2;
BBXR=BBX/PBY2M2;
B2BX=BBX*BBX;
BC=CY2X-PCY2X;
BCR=BC/PCY2X;
B2C=BC*BC;
SDBC=BC/SECY2X;
```

```
BCDIFF=CY2XDIFF-PCY2X;
BCDIFFR=BCDIFF/PCY2X;
B2CDIFF=BCDIFF*BCDIFF;
BCRES=CY2XRES-PCY2X;
BCRESR=BCRES/PCY2X;
B2CRES=BCRES*BCRES;
BC_=CY2X_-PCY2X;
BC_R=BC_/PCY2X;
B2C_=BC_*BC_;
BCX=C-PCY2X;
BCXR=BCX/PCY2X;
B2CX=BCX*BCX;
BAB=AB-TRUEAB;
BABR=BAB/TRUEAB;
B2AB=BAB**2;
BAB_=AB_-TRUEAB;
BAB_R=BAB_/TRUEAB;
B2AB_=BAB_**2;
BABX=ABX-TRUEAB;
BABXR=BABX/TRUEAB;
B2ABX=BABX*BABX;


*The following code computes confidence 95% C.I. coverage;
LaM2X=aM2X-1.96*SEaM2X; UaM2X=aM2X+1.96*SEaM2X;
LaM2Xdiff=aM2XDiff-1.96*SEaM2XDiff; UaM2XDiff=aM2XDiff+1.96*SEaM2XDiff;
LaM2XRes=aM2XRes-1.96*SEaM2XRes; UaM2XRes=aM2XRes+1.96*SEaM2XRes;
LaM2X_=aM2X_-1.96*SEaM2X_; UaM2X_=aM2X_+1.96*SEaM2X_;

LbY2M2=bY2M2-1.96*SEbY2M2; UbY2M2=bY2M2+1.96*SEbY2M2;
LbY2M2diff=bY2M2Diff-1.96*SEbY2M2Diff;
UbY2M2Diff=bY2M2Diff+1.96*SEbY2M2Diff;
LbY2M2Res=bY2M2Res-1.96*SEbY2M2Res; UbY2M2Res=bY2M2Res+1.96*SEbY2M2Res;
LbY2M2_=bY2M2_-1.96*SEbY2M2_; UbY2M2_=bY2M2_+1.96*SEbY2M2_;

LcY2X=cY2X-1.96*SEcY2X; UcY2X=cY2X+1.96*SEcY2X;
LcY2Xdiff=cY2XDiff-1.96*SEcY2XDiff; UcY2XDiff=cY2XDiff+1.96*SEcY2XDiff;
LcY2XRes=cY2XRes-1.96*SEcY2XRes; UcY2XRes=cY2XRes+1.96*SEcY2XRes;
LcY2X_=cY2X_-1.96*SEcY2X_; UcY2X_=cY2X_+1.96*SEcY2X_;

RGaM2X=0; LFaM2X=0; RGaM2XDiff=0; LFaM2XDiff=0; RGaM2XRes=0;
LFaM2XRes=0; RGaM2X_=0; LFaM2X_=0;
RGbY2M2=0; LFbY2M2=0; RGbY2M2Diff=0; LFbY2M2Diff=0; RGbY2M2Res=0;
LFbY2M2Res=0; RGbY2M2_=0; LFbY2M2_=0;
RGcY2X=0; LFcY2X=0; RGcY2XDiff=0; LFcY2XDiff=0; RGcY2XRes=0;
LFcY2XRes=0; RGcY2X_=0; LFcY2X_=0;

If PaM2X GT UaM2X then RGaM2X=1;
If PaM2X LT LaM2X then LFaM2X=1;
If PaM2X GT UaM2XDiff then RGaM2XDiff=1;
If PaM2X LT LaM2XDiff then LFaM2XDiff=1;
If PaM2X GT UaM2XRes then RGaM2XRes=1;
If PaM2X LT LaM2XRes then LFaM2XRes=1;
If PaM2X GT UaM2X_ then RGaM2X_=1;
```

```
If PaM2X LT LaM2X_ then LFaM2X_=1;

If PbY2M2 GT UbY2M2 then RGbY2M2=1;
If PbY2M2 LT LbY2M2 then LFbY2M2=1;
If PbY2M2 GT UbY2M2Diff then RGbY2M2Diff=1;
If PbY2M2 LT LbY2M2Diff then LFbY2M2Diff=1;
If PbY2M2 GT UbY2M2Res then RGbY2M2Res=1;
If PbY2M2 LT LbY2M2Res then LFbY2M2Res=1;
If PbY2M2 GT UbY2M2_ then RGbY2M2_=1;
If PbY2M2 LT LbY2M2_ then LFbY2M2_=1;

If PcY2X GT UcY2X then RGcY2X=1;
If PcY2X LT LcY2X then LFcY2X=1;
If PcY2X GT UcY2XDiff then RGcY2XDiff=1;
If PcY2X LT LcY2XDiff then LFcY2XDiff=1;
If PcY2X GT UcY2XRes then RGcY2XRes=1;
If PcY2X LT LcY2XRes then LFcY2XRes=1;
If PcY2X GT UcY2X_ then RGcY2X_=1;
If PcY2X LT LcY2X_ then LFcY2X_=1;

CVGaM2X=1-(RGaM2X+LFaM2X);
CVGaM2XDiff=1-(RGaM2XDiff+LFaM2XDiff);
CVGaM2XRes=1-(RGaM2XRes+LFaM2XRes);
CVGaM2X_=1-(RGaM2X_+LFaM2X_);

CVGbY2M2=1-(RGbY2M2+LFbY2M2);
CVGbY2M2Diff=1-(RGbY2M2Diff+LFbY2M2Diff);
CVGbY2M2Res=1-(RGbY2M2Res+LFbY2M2Res);
CVGbY2M2_=1-(RGbY2M2_+LFbY2M2_);

CVGcY2X=1-(RGcY2X+LFcY2X);
CVGcY2XDiff=1-(RGcY2XDiff+LFcY2XDiff);
CVGcY2XRes=1-(RGcY2XRes+LFcY2XRes);
CVGcY2X_=1-(RGcY2X_+LFcY2X_);
RUN;


/* This section computes the true variances and covariances for the
two-wave mediation model. X is group assigment and is
measured once, M1 is the mediator measured at Time1, M2 is the mediator
measured at Time2, Y1 is the outcome
measured at Time1, Y2 is the outcome measured at Time2. */

DATA TEST2; SET TEST;
VX=0.25;
VM1=(PaM1X**2)*VX+(&VarM1)**2;
VM1pred=(PaM2X**2)*VX;
CXM1=PaM1X*VX;
CXY1=PcY1X*VX+PbY1M1*CXM1;
CXM2=PaM2X*VX+PsM2M1*CXM1+PbM2Y1*CXY1;
CXY2=PcY2X*VX+PSY2Y1*CXY1+PbY2M1*CXM1+PbY2M2*CXM2;
CM1Y1=PaM1X*PcY1X*VX+PaM1X*PbY1M1*CXM1+PbY1M1*&VarM1**2;
CM1Y1pred=PaM1X*PcY1X*VX+PaM1X*PbY1M1*CXM1+PbY1M1;
```

```
CM1M2=PaM1X*PSM2M1*CXM1+PaM1X*PbM2Y1*CXY1+PaM1X*PaM2X*VX+PSM2M1*&VarM1+
PbM2Y1*PbY1M1*&VarM1**2;
CM1M2pred=PaM1X*PSM2M1*CXM1+PaM1X*PbM2Y1*CXY1+PaM1X*PaM2X*VX+PSM2M1*&Va
rM1+PbM2Y1*PbY1M1;
CM1Y2=PaM1X*PcY2X*VX+PaM1X*PSY2Y1*CXY1+PaM1X*PbY2M1*CXM1+PaM1X*PbY2M2*C
XM2+PSY2Y1*PbY1M1*&VarM1**2+PbY2M1*&VarM1**2+PbY2M2*PSM2M1*&VarM1**2+Pb
Y2M2*PbM2Y1*PbY1M1*&VarM1**2;
CM2Y1=PaM2X*PbY1M1*CXM1+PaM2X*PcY1X*VX+PSM2M1*PbY1M1*VM1+PSM2M1*PcY1X*C
XM1+PbM2Y1*PbY1M1*CM1Y1+PbM2Y1*PcY1X*CXY1+PbM2Y1*(&VarY1)**2;
CM2Y1pred=PaM2X*PbY1M1*CXM1+PaM2X*PcY1X*VX+PSM2M1*PbY1M1*VM1+PSM2M1*PcY
1X*CXM1+PbM2Y1*PbY1M1*CM1Y1+PbM2Y1*PcY1X*CXY1+PbM2Y1;

RUN;

DATA TEST3; SET TEST2;
VY1=(PcY1X**2)*VX+2*PcY1X*PbY1M1*CXM1+(PbY1M1**2)*VM1+(&VarY1)**2;
VY1pred=(PcY1X**2)*VX+2*PcY1X*PbY1M1*CXM1+(PbY1M1**2)*VM1;
VM2=(PSM2M1**2)*VM1+2*PSM2M1*PbM2Y1*CM1Y1+2*PSM2M1*PaM2X*CXM1+(PbM2Y1**
2)*VY1+2*PbM2Y1*PaM2X*CXY1+(PaM2X**2)*VX+(&VarM2)**2;
VY2=(PcY2X**2)*VX+2*PcY2X*PSY2Y1*CXY1+2*PcY2X*PbY2M1*CXM1+2*PcY2X*PbY2M
2*CXM2+(PSY2Y1**2)*VY1+2*PSY2Y1*PbY2M1*CM1Y1+2*PSY2Y1*PbY2M2*CM2Y1+
    (PbY2M1**2)*VM1+2*PbY2M1*PbY2M2*CM1M2+(PbY2M2**2)*VM2+(&VarY2)**2;
CM2Y2=PSM2M1*PcY2X*CXM1+PSM2M1*PSY2Y1*CM1Y1+PSM2M1*PbY2M1*VM1+PSM2M1*Pb
Y2M2*CM1M2+PbM2Y1*PcY2X*CXY1+PbM2Y1*PSY2Y1*VY1+PbM2Y1*PbY2M1*CM1Y1+

PbM2Y1*PbY2M2*CM2Y1+PaM2X*PcY2X*VX+PaM2X*PSY2Y1*CXY1+PaM2X*PbY2M1*CXM1+
PaM2X*PbY2M2*CXM2+PbY2M2*(&VarM2)**2;
CY1Y2=PcY1X*PSY2Y1*CXY1+PcY1X*PbY2M1*CXM1+PcY1X*PbY2M2*CXM2+PcY1X*PcY2X
*VX+PbY1M1*PSY2Y1*CM1Y1+PbY1M1*PbY2M1*VM1+PbY2M2*PbY1M1*CM1M2+
      PbY1M1*PcY2X*CXM1+PbY2M2*PbM2Y1*(&VarY1)**2+PSY2Y1*(&VarY1)**2;
VYDIFF=(VY2+VY1)-2*CY1Y2;
VMDIFF=(VM2+VM1)-2*CM1M2;
CXYDIFF=(PcY2X-PcY1X)*VX+PSY2Y1*CXY1+(PbY2M1-PbY1M1)*CXM1+PbY2M2*CXM2;
CXMDIFF=(PaM2X-PaM1X)*VX+PSM2M1*CXM1+PbM2Y1*CXY1;
CMDIFFYDIFF=(PaM2X-PaM1X)*(PcY2X-PcY1X)*VX+(PaM2X-
PaM1X)*PSY2Y1*CXY1+(PaM2X-PaM1X)*(PbY2M1-PbY1M1)*CXM1+(PaM2X-
PaM1X)*PbY2M2*CXM2+
             PSM2M1*(PcY2X-
PcY1X)*CXM1+PSM2M1*PSY2Y1*CM1Y1+PSM2M1*(PbY2M1-
PbY1M1)*VM1+PSM2M1*PbY2M2*CM1M2+PbM2Y1*(PcY2X-
PcY1X)*CXY1+PbM2Y1*PSY2Y1*VY1+
             PbM2Y1*(PbY2M1-PbY1M1)*CM1Y1+PbM2Y1*PbY2M2*CM2Y1-
PbM2Y1*(&VARY1)**2+PbY2M2*(&VARM2)**2-PSY2Y1*PbY1M1*(&VARM1)**2-
(PbY2M1-PbY1M1)*(&VARM1)**2-
PbY2M2*(PSM2M1*(&VARM1)**2+PbM2Y1*PbY1M1*(&VARM1)**2);
CXRESM=(PaM2X*VX)+PbM2Y1*CXY1;
CRESMRESY=PaM2X*PcY2X*VX+PaM2X*PbY2M1*CXM1+PaM2X*PbY2M2*CXM2+PbM2Y1*PcY
2X*CXY1+PbM2Y1*PbY2M1*CM1Y1+PbM2Y1*PbY2M2*CM2Y1+PbY2M2*(&VarM2)**2;
CXRESY=PcY2X*VX+PbY2M1*CXM1+PbY2M2*CXM2;
VRESM=PaM2X**2*VX+2*PaM2X*PbM2Y1*CXY1+PbM2Y1**2*VY1+(&VARY2**2);
VRESY=PcY2X**2*VX+PcY2X*PbY2M1*CXM1+PcY2X*PbY2M2*CXM2+PbY2M1*PcY2X*CXM1
+PbY2M1**2*VM1+PbY2M1*PbY2M2*CM1M2+PbY2M2*PcY2X*CXM2+PbY2M2*PbY2M1*CM1M
2+PbY2M2**2*VM2+(&VarY2)**2;
```

```
/*USED TO CALCULATE MSE FOR M2 EQUATION AND Y2 EQUATION CORRESPONDING
TO TWO-WAVE MODEL*/

VM2PRED=(PaM2X**2)*VX+2*PaM2X*PSM2M1*CXM1+(PSM2M1**2)*VM1;
VY2PRED=(PcY2X**2)*VX+2*PcY2X*PSY2Y1*CXY1+2*PcY2X*PbY2M1*CXM1+2*PcY2X*P
bY2M2*CXM2+(PSY2Y1**2)*VY1pred+2*PSY2Y1*PbY2M1*CM1Y1pred+2*PSY2Y1*PbY2M
2*CM2Y1pred+
    (PbY2M1**2)*VM1+2*PbY2M1*PbY2M2*CM1M2pred+(PbY2M2**2)*VM2pred;
RUN;

DATA TEST4; SET TEST3;
STDX=SQRT(VX);
STDM1=SQRT(VM1);
STDY1=SQRT(VY1);
STDM2=SQRT(VM2);
STDY2=SQRT(VY2);
STDMDIFF=SQRT(VMDIFF);
STDYDIFF=SQRT(VYDIFF);
STDMres=SQRT(VRESM);
STDYres=SQRT(VRESY);
RUN;

/*Zero-order correlations*/
DATA TEST5; SET TEST4;
RXM1=CXM1/(STDX*STDM1);
RXY1=CXY1/(STDX*STDY1);
RXM2=CXM2/(STDX*STDM2);
RXY2=CXY2/(STDX*STDY2);
RM1Y1=CM1Y1/(STDM1*STDY1);
RM1M2=CM1M2/(STDM1*STDM2);
RM1Y2=CM1Y2/(STDM1*STDY2);
RM2Y1=CM2Y1/(STDM2*STDY1);
RM2Y2=CM2Y2/(STDM2*STDY2);
RY1Y2=CY1Y2/(STDY1*STDY2);
RXMDIFF=cxmdiff/(STDX*STDMDIFF);
RXYDIFF=cxydiff/(STDX*STDYDIFF);
RMYDIFF=cmdiffydiff/(STDMDIFF*STDYDIFF);
RXMres=cxresm/(STDX*STDMres);
RXYres=cxresy/(STDX*STDYres);
RMYres=cresmresy/(STDMres*STDYres);
RUN;
/*First-Order partial correlations. Variable partialed out is at the
beginning of the variable name*/
DATA TEST6; SET TEST5;
m1RXY1=(RXY1-(RM1Y1*RXM1))/(SQRT(1-RM1Y1**2)*SQRT(1-RXM1**2));
xRM1Y1=(RM1Y1-(RXM1*RXY1))/(SQRT(1-RXM1**2)*SQRT(1-RXY1**2));
m1RXM2=(RXM2-(RXM1*RM1M2))/(SQRT(1-RXM1**2)*SQRT(1-RM1M2**2));
y1RXM2=(RXM2-(RXY1*RM2Y1))/(SQRT(1-RXY1**2)*SQRT(1-RM2Y1**2));
y1RXY2=(RXY2-(RXY1*RY1Y2))/(SQRT(1-RXY1**2)*SQRT(1-RY1Y2**2));
m2RXY2=(RXY2-(RXM2*RM2Y2))/(SQRT(1-RXM2**2)*SQRT(1-RM2Y2**2));
m1RY1M2=(RM2Y1-(RM1Y1*RM1M2))/(SQRT(1-RM1Y1**2)*SQRT(1-RM1M2**2));
xRY1M2=(RM2Y1-(RXY1*RXM2))/(SQRT(1-RXY1**2)*SQRT(1-RXM2**2));
```

```
xRM1M2=(RM1M2-(RXM1*RXM2))/(SQRT(1-RXM1**2)*SQRT(1-RXM2**2));
y1RM1M2=(RM1M2-(RM1Y1*RM2Y1))/(SQRT(1-RM1Y1**2)*SQRT(1-RM2Y1**2));
xRM1Y2=(RM1Y2-(RXM1*RXY2))/(SQRT(1-RXM1**2)*SQRT(1-RXY2**2));
y1RM1Y2=(RM1Y2-(RM1Y1*RY1Y2))/(SQRT(1-RM1Y1**2)*SQRT(1-RY1Y2**2));
m2RM1Y2=(RM1Y2-(RM1M2*RM2Y2))/(SQRT(1-RM1M2**2)*SQRT(1-RM2Y2**2));
xRM2Y2=(RM2Y2-(RXM2*RXY2))/(SQRT(1-RXM2**2)*SQRT(1-RXY2**2));
y1RM2Y2=(RM2Y2-(RM2Y1*RY1Y2))/(SQRT(1-RM2Y1**2)*SQRT(1-RY1Y2**2));
m1RM2Y2=(RM2Y2-(RM1M2*RM1Y2))/(SQRT(1-RM1M2**2)*SQRT(1-RM1Y2**2));
xRY1Y2=(RY1Y2-(RXY1*RXY2))/(SQRT(1-RXY1**2)*SQRT(1-RXY2**2));
m1RY1Y2=(RY1Y2-(RM1Y1*RM1Y2))/(SQRT(1-RM1Y1**2)*SQRT(1-RM1Y2**2));
m2RY1Y2=(RY1Y2-(RM2Y1*RM2Y2))/(SQRT(1-RM2Y1**2)*SQRT(1-RM2Y2**2));
m1RXY2=(RXY2-(RXM1*RM1Y2))/(SQRT(1-RXM1**2)*SQRT(1-RM1Y2**2));
y1RXM1=(RXM2-(RXY1*RM1Y1))/(SQRT(1-RXY1**2)*SQRT(1-RM1Y1**2));
xRMYDiff=(RMYdiff-(RXMdiff*RXYdiff))/(SQRT(1-RXMdiff**2)*SQRT(1-
RXYdiff**2));


/*Second-order partial correlations. Variables partialed out are at the
beginning of the variable name*/

m1y1RXM2=(m1RXM2-(m1RY1M2*m1RXY1))/(SQRT(1-m1RY1M2**2)*SQRT(1-
m1RXY1**2));
m1y1RXY2=(m1RXY2-(m1RXY1*m1RY1Y2))/(SQRT(1-m1RXY1**2)*SQRT(1-
m1RY1Y2**2));
y1m2RXY2=(y1RXY2-(y1RXM2*y1RM2Y2))/(SQRT(1-y1RXM2**2)*SQRT(1-
y1RM2Y2**2));
m1m2RXY2=(m1RXY2-(m1RXM2*m1RM2Y2))/(SQRT(1-m1RXM2**2)*SQRT(1-
m1RM2Y2**2));
xy1RM1M2=(xRM1M2-(xRM1Y1*xRY1M2))/(SQRT(1-xRM1Y1**2)*SQRT(1-
xRY1M2**2));
xy1RM1Y2=(xRM1Y2-(xRM1Y1*xRY1Y2))/(SQRT(1-xRM1Y1**2)*SQRT(1-
xRY1Y2**2));
y1m2RM1Y2=(y1RM1Y2-(y1RM1M2*y1RM2Y2))/(SQRT(1-y1RM1M2**2)*SQRT(1-
y1RM2Y2**2));
xm2RM1Y2=(xRM1Y2-(xRM1M2*xRM2Y2))/(SQRT(1-xRM1M2**2)*SQRT(1-
xRM2Y2**2));
xm1RY1M2=(xRY1M2-(xRM1Y1*xRM1M2))/(SQRT(1-xRM1Y1**2)*SQRT(1-
xRM1M2**2));
xy1RM2Y2=(xRM2Y2-(xRY1M2*xRY1Y2))/(SQRT(1-xRY1M2**2)*SQRT(1-
xRY1Y2**2));
xm1RM2Y2=(xRM2Y2-(xRM1M2*xRM1Y2))/(SQRT(1-xRM1M2**2)*SQRT(1-
xRM1Y2**2));
y1m1RM2Y2=(y1RM2Y2-(y1RM1M2*y1RM1Y2))/(SQRT(1-y1RM1M2**2)*SQRT(1-
y1RM1Y2**2));
xm1RY1Y2=(xRY1Y2-(xRM1Y1*xRM1Y2))/(SQRT(1-xRM1Y1**2)*SQRT(1-
xRM1Y2**2));
xm2RY1Y2=(xRY1Y2-(xRY1M2*xRM2Y2))/(SQRT(1-xRY1M2**2)*SQRT(1-
xRM2Y2**2));
m1m2RY1Y2=(m1RY1Y2-(m1RY1M2*m1RM2Y2))/(SQRT(1-m1RY1M2**2)*SQRT(1-
m1RM2Y2**2));
y1m2RXM1=(y1RXM1-(y1RXM2*y1RM1M2))/(SQRT(1-y1RXM2**2)*SQRT(1-
y1RM1M2**2));
m1m2RXY1=(m1RXY1-(m1RXM2*m1RY1M2))/(SQRT(1-m1RXY1**2)*SQRT(1-
m1RY1M2**2));
```

```
xm2RM1Y1=(xRM1Y1-(xRM1M2*xRY1M2))/(SQRT(1-xRM1Y1**2)*SQRT(1-
xRY1M2**2));
/*Third-order partial correlations. Variables partialed out are at the
beginning of the variable name*/

y1m1m2RXY2=(m1y1RXY2-(m1y1RXM2*y1m1RM2Y2))/(SQRT(1-m1y1RXM2**2)*SQRT(1-
y1m1RM2Y2**2));
y1xm2RM1Y2=(xy1RM1Y2-(xy1RM1M2*xy1RM2Y2))/(SQRT(1-xy1RM1M2**2)*SQRT(1-
xy1RM2Y2**2));
xm1m2RY1Y2=(xm1RY1Y2-(xm1RY1M2*xm1RM2Y2))/(SQRT(1-xm1RY1M2**2)*SQRT(1-
xm1RM2Y2**2));
xm1y1RM2Y2=(xm1RM2Y2-(xm1RY1M2*xm1RY1Y2))/(SQRT(1-xm1RY1M2**2)*SQRT(1-
xm1RY1Y2**2));


/*True value of bY2M2 coefficient and product of AB under the
difference score model and residualized change score model*/
TrueBYMDiff=((RMYdiff-(RXMdiff*RXYdiff))/(1-
RXMdiff**2))*(stdydiff/stdmdiff);
TrueBYMRES=((RMYRES-(RXMRES*RXYRES))/(1-RXMRES**2))*(stdyres/stdmres);
TrueABDIFF=Pam2X*TrueBYMDiff;
TrueABRes=Pam2X*TrueBYMRES;

/*Bias of AB under the difference score and residualized change score
model*/
BBDiffalt=bY2M2diff-trueBymdiff;
BBDiffaltR=BBDiffalt/trueBymdiff;
B2BDiffalt=BBDiffalt**2;
LbY2M2diffalt=bY2M2Diff-1.96*SEbY2M2Diff;
UbY2M2Diffalt=bY2M2Diff+1.96*SEbY2M2Diff;
RGbY2M2Diffalt=0; LFbY2M2Diffalt=0;
If trueBYMDiff GT UbY2M2Diffalt then RGbY2M2Diffalt=1;
If trueBYMDiff LT LbY2M2Diffalt then LFbY2M2Diffalt=1;
CVGbY2M2Diffalt=1-(RGbY2M2Diffalt+LFbY2M2Diffalt);
BABDIFF=ABDIFF-TRUEABDiff;
BABDIFFR=BABDIFF/TRUEABDiff;
B2ABDIFF=BABDIFF**2;
BABRES=ABRES-TRUEABRES;
BABRESR=BABRES/TRUEABRES;
B2ABRES=BABRES**2;
RUN;

/*Estimated partial correlations to test the analytic formulas*/
/*First-order partial correlations*/
PROC CORR DATA=SIM OUTPUT=CORRTEST NOPRINT; by rep;
VAR X Y1;
PARTIAL M1;
DATA PC1; SET CORRTEST; IF _NAME_= 'Y1'; m1CORRXY1=X;
KEEP m1CORRXY1;

PROC CORR DATA=SIM OUTPUT=CORRTEST NOPRINT; by rep;
VAR M1 Y1;
PARTIAL X;
DATA PC2; SET CORRTEST; IF _NAME_= 'Y1'; xCORRM1Y1=M1;
```

```
KEEP xCORRM1Y1;

PROC CORR DATA=SIM OUTPUT=CORRTEST NOPRINT; by rep;
VAR X M2;
PARTIAL M1;
DATA PC3; SET CORRTEST; IF _NAME_= 'M2'; m1CORRXM2=X;
KEEP m1CORRXM2;

PROC CORR DATA=SIM OUTPUT=CORRTEST NOPRINT; by rep;
VAR X M2;
PARTIAL y1;
DATA PC4; SET CORRTEST; IF _NAME_= 'M2'; y1CORRXM2=X;
KEEP y1CORRXM2;

PROC CORR DATA=SIM OUTPUT=CORRTEST NOPRINT; by rep;
VAR X Y2;
PARTIAL y1;
DATA PC5; SET CORRTEST; IF _NAME_= 'Y2'; y1CORRXY2=X;
KEEP y1CORRXY2;

PROC CORR DATA=SIM OUTPUT=CORRTEST NOPRINT; by rep;
VAR X Y2;
PARTIAL M2;
DATA PC6; SET CORRTEST; IF _NAME_= 'Y2'; m2CORRXY2=X;
KEEP m2CORRXY2;

PROC CORR DATA=SIM OUTPUT=CORRTEST NOPRINT; by rep;
VAR y1 M2;
PARTIAL M1;
DATA PC7; SET CORRTEST; IF _NAME_= 'M2'; m1CORRY1M2=y1;
KEEP m1CORRY1M2;

PROC CORR DATA=SIM OUTPUT=CORRTEST NOPRINT; by rep;
VAR y1 M2;
PARTIAL X;
DATA PC8; SET CORRTEST; IF _NAME_= 'M2'; xCORRY1M2=y1;
KEEP xCORRY1M2;

PROC CORR DATA=SIM OUTPUT=CORRTEST NOPRINT; by rep;
VAR M1 M2;
PARTIAL X;
DATA PC9; SET CORRTEST; IF _NAME_= 'M2'; xCORRM1M2=M1;
KEEP xCORRM1M2;

PROC CORR DATA=SIM OUTPUT=CORRTEST NOPRINT; by rep;
VAR M1 M2;
PARTIAL y1;
DATA PC10; SET CORRTEST; IF _NAME_= 'M2'; y1CORRM1M2=M1;
KEEP y1CORRM1M2;

PROC CORR DATA=SIM OUTPUT=CORRTEST NOPRINT; by rep;
VAR M1 Y2;
PARTIAL X;
DATA PC11; SET CORRTEST; IF _NAME_= 'Y2'; xCORRM1Y2=M1;
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```
KEEP xCORRM1Y2;

PROC CORR DATA=SIM OUTPUT=CORRTEST NOPRINT; by rep;
VAR M1 Y2;
PARTIAL y1;
DATA PC12; SET CORRTEST; IF _NAME_= 'Y2'; y1CORRM1Y2=M1;
KEEP y1CORRM1Y2;

PROC CORR DATA=SIM OUTPUT=CORRTEST NOPRINT; by rep;
VAR M1 Y2;
PARTIAL M2;
DATA PC13; SET CORRTEST; IF _NAME_= 'Y2'; m2CORRM1Y2=M1;
KEEP m2CORRM1Y2;

PROC CORR DATA=SIM OUTPUT=CORRTEST NOPRINT; by rep;
VAR M2 Y2;
PARTIAL X;
DATA PC14; SET CORRTEST; IF _NAME_= 'Y2'; xCORRM2Y2=M2;
KEEP xCORRM2Y2;

PROC CORR DATA=SIM OUTPUT=CORRTEST NOPRINT; by rep;
VAR M2 Y2;
PARTIAL y1;
DATA PC15; SET CORRTEST; IF _NAME_= 'Y2'; y1CORRM2Y2=M2;
KEEP y1CORRM2Y2;

PROC CORR DATA=SIM OUTPUT=CORRTEST NOPRINT; by rep;
VAR M2 Y2;
PARTIAL M1;
DATA PC16; SET CORRTEST; IF _NAME_= 'Y2'; m1CORRM2Y2=M2;
KEEP m1CORRM2Y2;

PROC CORR DATA=SIM OUTPUT=CORRTEST NOPRINT; by rep;
VAR y1 Y2;
PARTIAL X;
DATA PC17; SET CORRTEST; IF _NAME_= 'Y2'; xCORRY1Y2=y1;
KEEP xCORRY1Y2;

PROC CORR DATA=SIM OUTPUT=CORRTEST NOPRINT; by rep;
VAR y1 Y2;
PARTIAL M1;
DATA PC18; SET CORRTEST; IF _NAME_= 'Y2'; m1CORRY1Y2=y1;
KEEP m1CORRY1Y2;

PROC CORR DATA=SIM OUTPUT=CORRTEST NOPRINT; by rep;
VAR y1 Y2;
PARTIAL M2;
DATA PC19; SET CORRTEST; IF _NAME_= 'Y2'; m2CORRY1Y2=y1;
KEEP m2CORRY1Y2;

PROC CORR DATA=SIM OUTPUT=CORRTEST NOPRINT; by rep;
VAR X Y2;
PARTIAL M1;
DATA PC20; SET CORRTEST; IF _NAME_= 'Y2'; m1CORRXY2=X;
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KEEP m1CORRXY2;

PROC CORR DATA=SIM OUTPUT=CORRTEST NOPRINT; by rep;
VAR X M1;
PARTIAL M2;
DATA PC21; SET CORRTEST; IF _NAME_= 'M1'; m2CORRXM1=X;
KEEP m2CORRXM1;

PROC CORR DATA=SIM OUTPUT=CORRTEST NOPRINT; by rep;
VAR X Y1;
PARTIAL M2;
DATA PC22; SET CORRTEST; IF _NAME_= 'Y1'; m2CORRXY1=X;
KEEP m2CORRXY1;


/*Second-Order partial correlations*/
PROC CORR DATA=SIM OUTPUT=CORRTEST NOPRINT; by rep;
VAR X M2;
PARTIAL M1 y1;
DATA PC23; SET CORRTEST; IF _NAME_= 'M2'; m1y1CORRXM2=X;
KEEP m1y1CORRXM2;

PROC CORR DATA=SIM OUTPUT=CORRTEST NOPRINT; by rep;
VAR X Y2;
PARTIAL M1 y1;
DATA PC24; SET CORRTEST; IF _NAME_= 'Y2'; m1y1CORRXY2=X;
KEEP m1y1CORRXY2;

PROC CORR DATA=SIM OUTPUT=CORRTEST NOPRINT; by rep;
VAR X Y2;
PARTIAL y1 M2;
DATA PC25; SET CORRTEST; IF _NAME_= 'Y2'; y1m2CORRXY2=X;
KEEP y1m2CORRXY2;

PROC CORR DATA=SIM OUTPUT=CORRTEST NOPRINT; by rep;
VAR X Y2;
PARTIAL M1 M2;
DATA PC26; SET CORRTEST; IF _NAME_= 'Y2'; m1m2CORRXY2=X;
KEEP m1m2CORRXY2;

PROC CORR DATA=SIM OUTPUT=CORRTEST NOPRINT; by rep;
VAR M1 M2;
PARTIAL X y1;
DATA PC27; SET CORRTEST; IF _NAME_= 'M2'; xy1CORRM1M2=M1;
KEEP xy1CORRM1M2;

PROC CORR DATA=SIM OUTPUT=CORRTEST NOPRINT; by rep;
VAR M1 Y2;
PARTIAL X y1;
DATA PC28; SET CORRTEST; IF _NAME_= 'Y2'; xy1CORRM1Y2=M1;
KEEP xy1CORRM1Y2;

PROC CORR DATA=SIM OUTPUT=CORRTEST NOPRINT; by rep;
VAR M1 Y2;
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```
PARTIAL y1 M2;
DATA PC29; SET CORRTEST; IF _NAME_= 'Y2'; y1m2CORRM1Y2=M1;
KEEP y1m2CORRM1Y2;

PROC CORR DATA=SIM OUTPUT=CORRTEST NOPRINT; by rep;
VAR M1 Y2;
PARTIAL X M2;
DATA PC30; SET CORRTEST; IF _NAME_= 'Y2'; xm2CORRM1Y2=M1;
KEEP xm2CORRM1Y2;

PROC CORR DATA=SIM OUTPUT=CORRTEST NOPRINT; by rep;
VAR y1 M2;
PARTIAL X M1;
DATA PC31; SET CORRTEST; IF _NAME_= 'M2'; xm1CORRY1M2=y1;
KEEP xm1CORRY1M2;

PROC CORR DATA=SIM OUTPUT=CORRTEST NOPRINT; by rep;
VAR M2 Y2;
PARTIAL X y1;
DATA PC32; SET CORRTEST; IF _NAME_= 'Y2'; xy1CORRM2Y2=M2;
KEEP xy1CORRM2Y2;

PROC CORR DATA=SIM OUTPUT=CORRTEST NOPRINT; by rep;
VAR M2 Y2;
PARTIAL X M1;
DATA PC33; SET CORRTEST; IF _NAME_= 'Y2'; xm1CORRM2Y2=M2;
KEEP xm1CORRM2Y2;

PROC CORR DATA=SIM OUTPUT=CORRTEST NOPRINT; by rep;
VAR M2 Y2;
PARTIAL Y1 M1;
DATA PC34; SET CORRTEST; IF _NAME_= 'Y2'; y1m1CORRM2Y2=M2;
KEEP y1m1CORRM2Y2;

PROC CORR DATA=SIM OUTPUT=CORRTEST NOPRINT; by rep;
VAR y1 Y2;
PARTIAL X M1;
DATA PC35; SET CORRTEST; IF _NAME_= 'Y2'; xm1CORRY1Y2=y1;
KEEP xm1CORRY1Y2;

PROC CORR DATA=SIM OUTPUT=CORRTEST NOPRINT; by rep;
VAR y1 Y2;
PARTIAL X M2;
DATA PC36; SET CORRTEST; IF _NAME_= 'Y2'; xm2CORRY1Y2=y1;
KEEP xm2CORRY1Y2;

PROC CORR DATA=SIM OUTPUT=CORRTEST NOPRINT; by rep;
VAR y1 Y2;
PARTIAL M1 M2;
DATA PC37; SET CORRTEST; IF _NAME_= 'Y2';m1m2CORRY1Y2=y1;
KEEP m1m2CORRY1Y2;

/*Third-Order partial correlations*/
PROC CORR DATA=SIM OUTPUT=CORRTEST NOPRINT; by rep;
```

```
VAR X Y2;
PARTIAL y1 M1 M2;
DATA PC38; SET CORRTEST; IF _NAME_= 'Y2';y1m1m2CORRXY2=X;
KEEP y1m1m2CORRXY2;

PROC CORR DATA=SIM OUTPUT=CORRTEST NOPRINT; by rep;
VAR M1 Y2;
PARTIAL y1 X M2;
DATA PC39; SET CORRTEST; IF _NAME_= 'Y2';y1xm2CORRM1Y2=M1;
KEEP y1xm2CORRM1Y2;

PROC CORR DATA=SIM OUTPUT=CORRTEST NOPRINT; by rep;
VAR Y1 Y2;
PARTIAL X M1 M2;
DATA PC40; SET CORRTEST; IF _NAME_= 'Y2';xm1m2CORRY1Y2=Y1;
KEEP xm1m2CORRY1Y2;

PROC CORR DATA=SIM OUTPUT=CORRTEST NOPRINT; by rep;
VAR M2 Y2;
PARTIAL X M1 y1;
DATA PC41; SET CORRTEST; IF _NAME_= 'Y2';xm1y1CORRM2Y2=M2;
KEEP xm1y1CORRM2Y2;
RUN;

DATA PARTCORR; MERGE PC1 PC2 PC3 PC4 PC5 PC6 PC7 PC8 PC9 PC10 PC11 PC12
PC13 PC14 PC15 PC16 PC17 PC18 PC19 PC20 PC21 PC22 PC23 PC24
PC25 PC26 PC27 PC28 PC29 PC30 PC31 PC32 PC33 PC34 PC35 PC36 PC37 PC38
PC39 PC40 PC41;
RUN;

DATA ALLCORR; MERGE SCOVS SCORRS PARTCORR;

/*True Standard errors*/
DATA TEST7; SET TEST6;
TRUEMSEM2=VM2-VM2PRED;
TRUESEA=sqrt((TRUEMSEM2/(NOBS-1))*((1/VX)/(1-m1RXM2**2)));
TRUEMSEY2=VY2-VY2PRED;
TRUESEB=SQRT((TRUEMSEY2)/((NOBS-1))*((1/VM2)/(1-xm1y1RM2Y2**2)));
TRUESEC=SQRT((TRUEMSEY2)/((NOBS-1))*((1/VX)/(1-y1m1m2RXY2**2)));
TRUESEAB=SQRT((PaM2X**2)*(TRUESEB**2)+(PbY2M2**2)*(TRUESEA**2));
*The following computes the Bias, Bias squared, and relative Bias for
the standard errors;
BSEA=SEaM2X-TrueSEA;
BSEAR=BSEA/TrueSEA;
B2SEA=BSEA**2;
BSEB=SEbY2M2-TrueSEB;
BSEBR=BSEB/TrueSEB;
B2SEB=BSEB**2;
BSEC=SEcY2X-TrueSEC;
BSECR=BSEC/TrueSEC;
B2SEC=BSEC**2;
BSEADIFF=SEaM2Xdiff-TrueSEA;
BSEADIFFR=BSEADIFF/TrueSEA;
B2SEADIFF=BSEADIFF**2;
```

```
BSEBDIFF=SEbY2M2Diff-TrueSEB;
BSEBDIFFR=BSEBDIFF/TrueSEB;
B2SEBDIFF=BSEBDIFF**2;
BSECDIFF=SEcY2XDIFF-TrueSEC;
BSECDIFFR=BSECDIFF/TrueSEC;
B2SECDIFF=BSECDIFF**2;
BSEARES=SEaM2XRES-TrueSEA;
BSEARESR=BSEARES/TrueSEA;
B2SEARES=BSEARES**2;
BSEBRES=SEbY2M2RES-TrueSEB;
BSEBRESR=BSEBRES/TrueSEB;
B2SEBRES=BSEBRES**2;
BSECRES=SEcY2XRES-TrueSEC;
BSECRESR=BSECRES/TrueSEC;
B2SECRES=BSECRES**2;
BSEA_=SEaM2X_-TrueSEA;
BSEA_R=BSEA_/TrueSEA;
B2SEA_=BSEA_**2;
BSEB_=SEbY2M2_-TrueSEB;
BSEB_R=BSEB_/TrueSEB;
B2SEB_=BSEB_**2;
BSEC_=SEcY2X_-TrueSEC;
BSEC_R=BSEC_/TrueSEC;
B2SEC_=BSEC_**2;
BSEAB=SEAB-TRUESEAB;
BSEABR=BSEAB/TRUESEAB;
B2SEAB=BSEAB**2;
BSEABDIFF=SEABDIFF-TRUESEAB;
BSEABDIFFR=BSEABDIFF/TRUESEAB;
B2SEABDIFF=BSEABDIFF**2;
BSEABRES=SEABRES-TRUESEAB;
BSEABRESR=BSEABRES/TRUESEAB;
B2SEABRES=BSEABRES**2;
BSEAB_=SEAB_-TRUESEAB;
BSEAB_R=BSEAB_/TRUESEAB;
B2SEAB_=BSEAB_**2;

RUN;

DATA ALLDAT; MERGE TEST7 ALLCORR;
run;

DATA DATAOUT.&FILE.outalt; SET ALLDAT;
RUN;
%MEND;
run;

PROC DATASETS LIB=WORK NOLIST;
%ANALYZE(nsim=1000,nobs=50,BMX=0,BYX=0,BYM=0,
aM1X=0, sm2m1=.981,am2x=0,bm2y1=0, by1m1=.578,
cy1x=0, cy2x=0,sy2y1=.686, by2m1=0,by2m2=0,
varx=1,varm1=1,varm2=1,vary1=1,vary2=1,
RELM1=1, RELM2=1, RELY1=1, RELY2=1,
FILE=n50cond1,TYPE='CCC',ERROR=1); run; quit;
```

APPENDIX G

SAS MACRO CALLING PRODCLIN TO ESTIMATE CONFIDENCE INTERVALS FOR THE MEDIATED EFFECT FOR THE ANCOVA MODEL

```
*Using PRODCLIN to estimate asymmetric confidence intervals for the
mediated effect for ANCOVA;
FILENAME NULLOG DUMMY 'C:\NULL';
PROC PRINTTO LOG=NULLOG;

libname DATAOUT "D:\Valente Masters Sim\DATAOUT\";
libname PRODOUT "D:\Valente Masters Sim\PRODOUT\";


*proc printto log=dum;
*options nosource nonotes;

%MACRO CONFLIM(FILE);

PROC DATASETS LIBRARY=WORK KILL NOLIST; RUN;
DATA PRODCLIN; set DATAOUT.&file;
run;

data test; length ii $8; set PRODCLIN;
i+1;
ii=left(put(i, 8.));
call symput ('b'||ii, by2m2);
call symput ('a'||ii, am2x);
call symput ('seb'||ii,seby2m2);
call symput ('sea'||ii,seam2x);
call symput ('nobs', _n_);
run;



options noxwait ;
*Designate location of prdclinforSAS.sas and prodclinsas2.exe;


Data summary;

%macro prodclin(a, sea, b, seb, rho, alpha);

data data1;
*Change file address to match the location of the file prodclin.exe;
file "D:\Valente Masters Sim\raw.txt";
a=&a; sea=&sea; b=&b; seb=&seb; rho=&rho; alpha=&alpha;
put a @; put sea @; put b @; put seb @; put rho @; put alpha @;
run;

*Change file address to match the location of the file prodclin.exe;
X cd    D:\Valente Masters Sim\;
*Change file address to match the location of the file prodclin.exe;
X call "D:\Valente Masters Sim\ProdClin2_Sas.exe";


data data2;
do;
```

```
rc=system("D:\Valente Masters Sim\ProdClin2_Sas.exe");
end;
run;

data data2;
infile "D:\Valente Masters Sim\critval.txt";
input lowz highz;
a=&a; sea=&sea; b=&b; seb=&seb; rho=&rho; alpha=&alpha;
r=rho;
da=a/sea;
db=b/seb;
sedadb=sqrt(da*da+db*db+1);
dadb=da*db;
ab=a*b;
sobelse=sqrt(a*a*seb*seb+b*b*sea*sea);
se_ab=sobelse;
MVDSE = sqrt(a*a*seb*seb+b*b*sea*sea);
prodlow=lowz;
produp=highz;
nl=probit(alpha/2);
normlow=ab+nl*se_ab;
normup=ab-nl*se_ab;
TesT_SE = sqrt(A*a*seb*seb+b*b*sea*sea+2*a*b*r*sea*seb-
(r*sea*seb)*(r*sea*seb)+sea*sea*seb*seb);
run;

*proc print data=data2 noobs;
       *var a sea b seb ab rho alpha prodlow produp Test_SE;
       *run;

data summary; set summary data2;
keep a b seb sea ab prodlow produp normlow normup;
run;

%mend prodclin;

%macro prodclinbootstrap;

%do i=1 %to &nobs;
%prodclin (a=&&a&i, sea=&&sea&i, b=&&b&i, seb=&&seb&i, rho=0, alpha =
.05);
%end;

%mend prodclinbootstrap;
%prodclinbootstrap;
run;

data prodout.&file.prodANCOVA; set summary;
if _n_=1 then delete;

run;

%mend CONFLIM;
%conflim(FILE=n50cond1out);
```

APPENDIX H

SAS MACRO CREATING PERCENTILE BOOTSTRAPPED CONFIDENCE INTERVALS FOR ALL MODELS

```
*Creating Percentile Bootstrapped CIs for the mediated effect for all
models;
libname DATAGEN "C:\Users\psyripl\Desktop\";
libname DATABOOT "D:\Valente Masters Sim\DATABOOT\";
FILENAME NULLOG DUMMY 'C:\NULL';
PROC PRINTTO LOG=NULLOG; run;
%Macro Bootstrap (nsim,nobs,BMX,BYX,BYM,
aM1X, sm2m1,am2x,bm2y1,by1m1,
cy1x,cy2x,sy2y1, by2m1,by2m2,
varx,varm1,varm2,vary1,vary2,
RELM1, RELM2, RELY1, RELY2,
FILE,TYPE,ERROR);

DATA SIM; SET DATAGEN.&file;
J=&nobs;
DO J=0 to totaln by &nobs;
IF 1+J<=I<=&nobs+J then Key=1+(J/&nobs);
end;
keep I X M1 M2 Y1 Y2 MDIFF YDIFF Key file;
run;

*Resampling;
%let nboot=1000;
proc surveyselect data=SIM noprint out=outtemp method=urs
sampsize=&nobs rep=&nboot outhits;
by Key;
run;
quit;

/*************************************************************/
/*                                                           */
/*                        PATH MODEL                         */
/*                                                           */
/*************************************************************/
PROC CALIS DATA=outtemp METHOD=ML NORPINT PLC OUTEST=OUT1 outstat=out2;
by key replicate;
LINEQS
M2=sm2m1_ M1 + am2x_ X + E2,
y2=cy2x_ X +  sy2y1_ y1 + by2m1_ M1 + by2m2_ M2 + E3;

Cov
X M1,
X Y1,
M1 Y1;

STD
   E2 = EE2,
   E3 = EE3;

RUN;
*SAVING THE PARAMETER VALUES OF FROM THE PATH MODEL OUTPUT. PARAMETERS
ARE DENOTED WITH _ AT THE END OF THE NAME;
DATA CALPARMS; SET OUT1;
IF _TYPE_="PARMS";
```

```
KEEP aM2X_ bY2M2_ key replicate;
run;

/*************************************************************/
/*                                                           */
/*                      CHANGE SCORES                        */
/*                                                           */
/*************************************************************/
/*ESTIMATING (DIFFM=X Y1) REGRESSION AND SAVING THE VALUES OF THE
COEFFICIENTS AND THEIR STANDARD ERRORS*/
PROC REG DATA=outtemp OUTEST=DiffFILE1 COVOUT noprint; by key
replicate; MODEL Mdiff= X/;
DATA DiffA; SET DiffFILE1; IF _TYPE_='PARMS'; aM2XDiff=X;
DROP _MODEL_ _NAME_ _TYPE_ _DEPVAR_ _RMSE_ INTERCEP X;
KEEP aM2XDiff key replicate;
run;


*Estimating the (DiffY=X DiffM) REGRESSION AND SAVING THE VALUES OF THE
COEFFICIENTS AND THEIR STANDARD ERRORS;
PROC REG DATA=outtemp OUTEST=DiffFILE2 COVOUT noprint; by key
replicate; MODEL Ydiff=X Mdiff/;
DATA DiffG; SET DiffFILE2;IF _TYPE_='PARMS'; bY2M2Diff=Mdiff;
DROP _MODEL_ _NAME_ _TYPE_ _DEPVAR_ _RMSE_ INTERCEP X MDiff;
KEEP bY2M2Diff key replicate;
run;

DATA DiffMODELS; MERGE Diffa Diffg;
run;
/*************************************************************/
/*                                                           */
/*               RESIDUALIZED CHANGE SCORES                  */
/*                                                           */
/*************************************************************/
/*MODEL COMPUTING AND SAVING RESIDUALIZED DIFFERENCE SCORE OF M1 AND
M2*/
PROC REG DATA=outtemp noprint; by key replicate; MODEL M2=M1/;
output out=resid1 r=residm;
run;

/*MODEL COMPUTING AND SAVING RESIDUALIZED DIFFERENCE SCORE OF Y1 AND
Y2*/
PROC REG DATA=outtemp noprint; by key replicate; MODEL Y2=y1/;
output out=resid2 r=residy;
run;

data resid1; set resid1;
J=_n_;
run;

data resid2; set resid2;
J=_n_;
run;
```

```
data rchange; merge resid1 resid2; by J;
run;


/*ESTIMATING (RESIDM2=X Y1) REGRESSION AND SAVING THE VALUES OF THE
COEFFICIENTS AND THEIR STANDARD ERRORS*/
PROC REG DATA=rchange OUTEST=RFILE1 COVOUT noprint; by key replicate;
MODEL residm= X/;
DATA RA; SET RFILE1; IF _TYPE_='PARMS'; aM2XRES=X;
DROP _MODEL_ _NAME_ _TYPE_ _DEPVAR_ _RMSE_ INTERCEP X;
KEEP aM2XRES key replicate;
run;


*Estimating the (RESIDY2=X RESIDM2) REGRESSION AND SAVING THE VALUES OF
THE COEFFICIENTS AND THEIR STANDARD ERRORS;
PROC REG DATA=rchange OUTEST=RFILE2 COVOUT noprint; by key replicate;
MODEL RESIDY=X RESIDM/;
DATA RG; SET RFILE2;IF _TYPE_='PARMS'; bY2M2RES=RESIDM;
DROP _MODEL_ _NAME_ _TYPE_ _DEPVAR_ _RMSE_ INTERCEP X RESIDM;
KEEP bY2M2RES key replicate;
run;

DATA RESMODELS; MERGE RA RG;
/*************************************************************/
/*                                                           */
/*                LINEAR REGRESSION (ANCOVA)                 */
/*                                                           */
/*************************************************************/


*Estimating the (M2=M1 X) regression and saving the value of sM2M1,
aM2X, and their standard errors;
PROC REG DATA=outtemp OUTEST=FILE2 COVOUT noprint; by key replicate;
MODEL M2=M1 X/;
DATA F; SET FILE2; IF _TYPE_='PARMS'; aM2X=X;
DROP _MODEL_ _NAME_ _TYPE_ _DEPVAR_ _RMSE_ INTERCEP X M1 M2;
KEEP aM2X key replicate;
run;



*Estimating the (Y2=X Y1 M1 M2) regression and saving the value of
sM2M1, aM2X, bM2Y1, and their standard errors;
PROC REG DATA=outtemp OUTEST=FILE4 COVOUT noprint; by key replicate;
MODEL Y2=X Y1 M1 M2/;
DATA T; SET FILE4;IF _TYPE_='PARMS'; bY2M2=M2;
DROP _MODEL_ _NAME_ _TYPE_ _DEPVAR_ _RMSE_ INTERCEP X Y1 M1 M2;
KEEP bY2M2 key replicate;
run;

DATA MODEL4; MERGE F T;
```

```
/*************************************************************/
/*                                                           */
/*                CROSS SECTIONAL MEDIATOR MODEL             */
/*                                                           */
/*************************************************************/

*Estimating the (M2=X) regression and saving the value of a and its
standard error;
PROC REG DATA=outtemp OUTEST=FILE5 COVOUT noprint; by key replicate;
MODEL M2= X/;
DATA V; SET FILE5; IF _TYPE_='PARMS'; a=X;
DROP _MODEL_ _NAME_ _TYPE_ _DEPVAR_ _RMSE_ INTERCEP X M2;
KEEP a key replicate;
run;

*Estimating the (Y2=X M2) regression and saving the value of c b and
their standard errors;
PROC REG DATA=outtemp OUTEST=FILE6 COVOUT noprint; by key replicate;
MODEL Y2=X M2/;
DATA Z; SET FILE6;IF _TYPE_='PARMS'; b=M2;
DROP _MODEL_ _NAME_ _TYPE_ _DEPVAR_ _RMSE_ INTERCEP X Y2 M2;
KEEP b key replicate;
run;

DATA MODEL6; MERGE V Z;



*THIS MERGES ALL THE PREVIOUS REGRESSION, CHANGE SCORE, RES CHANGE, AND
PATH MODEL OUTPUT;
DATA ALL; MERGE  MODEL4 MODEL6 DIFFMODELS RESMODELS CALPARMS;
run;
DATA ALL; set ALL;
ABancovaboot=am2x*by2m2;
ABdiffboot=am2xdiff*by2m2diff;
ABresboot=am2xres*by2m2res;
ABpathboot=am2x_*by2m2_;
ABboot=a*b;
run;


proc sort data=all; by key ABancovaboot; run;
data ancova; set all;
boot+1;
by key;
if first.key then boot = 1;
run;

data ancovaout; set ancova;
if boot = 25 then ancovaLCL=ABancovaboot;
if boot = 975 then ancovaUCL=ABancovaboot;
keep key ancovaLCL ancovaUCL;
```

```
run;quit;

proc sort data=all; by key ABdiffboot; run;
data diff; set all;
boot+1;
by key;
if first.key then boot = 1;
run;

data diffout; set diff;
if boot = 25 then diffLCL=ABdiffboot;
if boot = 975 then diffUCL=ABdiffboot;
keep key diffLCL diffUCL;
run;quit;

proc sort data=all; by key ABresboot; run;
data res; set all;
boot+1;
by key;
if first.key then boot = 1;
run;

data resout; set res;
if boot = 25 then resLCL=ABresboot;
if boot = 975 then resUCL=ABresboot;
keep key resLCL resUCL;
run;quit;

proc sort data=all; by key ABpathboot; run;
data path; set all;
boot+1;
by key;
if first.key then boot = 1;
run;

data pathout; set path;
if boot = 25 then pathLCL=ABpathboot;
if boot = 975 then pathUCL=ABpathboot;
keep key pathLCL pathUCL;
run;quit;

proc sort data=all; by key ABboot; run;
data cross; set all;
boot+1;
by key;
if first.key then boot = 1;
run;

data crossout; set cross;
if boot = 25 then crossLCL=ABboot;
if boot = 975 then crossUCL=ABboot;
keep key crossLCL crossUCL;
run;quit;
```

```
DATA prodboot; merge ancovaout diffout resout pathout crossout; by key;
run;

data UCL; set prodboot;
keep key ancovaUCL diffUCL resUCL pathUCL crossUCL;
if ancovaUCL=. then delete;
if diffUCL=. then delete;
if resUCL=. then delete;
if pathUCL=. then delete;
if crossUCL=. then delete;
run;

data LCL; set prodboot;
keep key ancovaLCL diffLCL resLCL pathLCL crossLCL;
if ancovaLCL=. then delete;
if diffLCL=. then delete;
if resLCL=. then delete;
if pathLCL=. then delete;
if crossLCL=. then delete;
run;

DATA DATABOOT.&file.boot; merge UCL LCL; by key;
run;



%mend bootstrap;
run;
quit;

PROC DATASETS LIB=WORK NOLIST;
*Effect size test condition for stability of M and Y at .7 and pretest
corr at .5 and M1 on Y2 at 0;
%Bootstrap(nsim=1000,nobs=50,BMX=0,BYX=0,BYM=0,
aM1X=0, sm2m1=.981,am2x=0,bm2y1=0, by1m1=.578,
cy1x=0, cy2x=0,sy2y1=.686, by2m1=0,by2m2=0,
varx=1,varm1=1,varm2=1,vary1=1,vary2=1,
RELM1=1, RELM2=1, RELY1=1, RELY2=1,
FILE=n50cond1,TYPE='CCC',ERROR=1);
```

APPENDIX I

SAS MACRO FOR SUMMARIZING RESULTS ACROSS ALL MODELS FOR USE IN ANOVA FOR RESULTS SECTION

```
*Summarizing Bootstrapped results across all models for use in ANOVA
for results section;
FILENAME NULLOG DUMMY 'C:\NULL';
PROC PRINTTO LOG=NULLOG;
libname DATAOUT "F:\ASU\Sims\Two-wave model\Valente Masters
Sim\DATAOUT\";
libname DATABOOT "F:\ASU\Sims\Two-wave model\Valente Masters
Sim\DATABOOT\";
libname BOOT "F:\ASU\Sims\Two-wave model\Valente Masters Sim\BOOT\";

DATA SUMMARY;
%MACRO CONFLIM(FILE1, FILE2, COND, NOBS);

DATA TRUE; set DATAOUT.&file2;
true=Pam2x*Pby2m2;
cond=&cond;
nobs=&nobs;
keep cond nobs true trueabdiff trueabres;
run;


DATA BOOT; set DATABOOT.&file1;
run;

DATA boot; merge boot true;
run;

data boot; set boot;
if true ge ancovaucl then numancovaup=1; else numancovaup=0;
if true le ancovalcl then numancovalow=1; else numancovalow=0;
if 0 ge ancovaucl then powancovaup=1; else powancovaup=0;
if 0 le ancovalcl then powancovalow=1; else powancovalow=0;
powbootancova=powancovalow+powancovaup;
SUMbootancova=numancovaup+numancovalow;
cvgbootancova=1-(numancovaup+numancovalow);

if true ge pathucl then numpathup=1; else numpathup=0;
if true le pathlcl then numpathlow=1; else numpathlow=0;
if 0 ge pathucl then powpathup=1; else powpathup=0;
if 0 le pathlcl then powpathlow=1; else powpathlow=0;
powbootpath=powpathlow+powpathup;
SUMbootpath=numpathup+numpathlow;
cvgbootpath=1-(numpathup+numpathlow);

if trueabdiff ge diffucl then numdiffup=1; else numdiffup=0;
if trueabdiff le difflcl then numdifflow=1; else numdifflow=0;
if 0 ge diffucl then powdiffup=1; else powdiffup=0;
if 0 le difflcl then powdifflow=1; else powdifflow=0;
powbootdiff=powdifflow+powdiffup;
SUMbootdiff=numdiffup+numdifflow;
cvgbootdiff=1-(numdiffup+numdifflow);

if trueabres ge resucl then numresup=1; else numresup=0;
if trueabres le reslcl then numreslow=1; else numreslow=0;
```

```
if 0 ge resucl then powresup=1; else powresup=0;
if 0 le reslcl then powreslow=1; else powreslow=0;
powbootres=powreslow+powresup;
SUMbootres=numresup+numreslow;
cvgbootres=1-(numresup+numreslow);

if true ge crossucl then numcrossup=1; else numcrossup=0;
if true le crosslcl then numcrosslow=1; else numcrosslow=0;
if 0 ge crossucl then powcrossup=1; else powcrossup=0;
if 0 le crosslcl then powcrosslow=1; else powcrosslow=0;
powbootcross=powcrosslow+powcrossup;
SUMbootcross=numcrossup+numcrosslow;
cvgbootcross=1-(numcrossup+numcrosslow);

run;

DATA SUMMARY; SET SUMMARY boot;
run;

%mend CONFLIM;
run;
PROC DATASETS LIB=WORK NOLIST;
%Macro loop;
%do i=1 %to 208;
%conflim(FILE1=n50cond&i.boot,FILE2=n50cond&i.out,COND=&i,NOBS=50);
%end;

%do i=1 %to 208;
%conflim(FILE1=n100cond&i.boot,FILE2=n100cond&i.out,COND=&i,NOBS=100);
%end;

%do i=1 %to 208;
%conflim(FILE1=n200cond&i.boot,FILE2=n200cond&i.out,COND=&i,NOBS=200);
%end;

%do i=1 %to 208;
%conflim(FILE1=n500cond&i.boot,FILE2=n500cond&i.out,COND=&i,NOBS=500);
%end;

%mend loop;
%loop;
run;


DATA boot.anovasummary; Set SUMMARY;
if _n_=1 then delete;
if cond=1 then do; true=0; direct=0; Y2lag=0; M2lag=0; PretestCorr=.5;
Stability=.7; end;
```

APPENDIX J

SAS MACRO FOR SUMMARIZING PRODCLIN RESULTS FOR USE IN ANOVA FOR RESULTS SECTION

```
*Summarizing PRODCLIN results for use in ANOVA for results section;
FILENAME NULLOG DUMMY 'C:\NULL';
PROC PRINTTO LOG=NULLOG;
libname DATAOUT "C:\Users\mvalent4\Desktop\DATAOUT\";
libname PRODOUT "C:\Users\mvalent4\Desktop\PRODOUT\";
libname PRODCL "C:\Users\mvalent4\Desktop\PRODCL\";


*proc printto log=dum;
*options nosource nonotes;
PROC DATASETS LIBRARY=WORK KILL NOLIST; RUN;
DATA SUMMARY;
%MACRO CONFLIM(FILE1,FILE2,COND,NOBS,METHOD);


DATA TRUE; set DATAOUT.&file2;
if &method=1 then do; true=Pam2x*Pby2m2; end;
if &method=2 then do; true=Pam2x*Pby2m2; end;
if &method=3 then do; true=trueabdiff; end;
if &method=4 then do; true=trueabres; end;
if &method=5 then do;true=Pam2x*Pby2m2; end;
cond=&cond;
i=&i;
nobs=&nobs;
keep true cond nobs;
run;

DATA PRODBOOT; set prodout.&file1;
run;

DATA prodboot; merge prodboot true;
run;

data prodboot; set prodboot;
if true ge normup then numnormup=1; else numnormup=0;
if true le normlow then numnormlow=1; else numnormlow=0;
if true ge produp then numprodup=1; else numprodup=0;
if true le prodlow then numprodlow=1; else numprodlow=0;
if 0 ge normup then pownormup=1; else pownormup=0;
if 0 le normlow then pownormlow=1; else pownormlow=0;
pownorm=pownormlow+pownormup;
if 0 ge produp then powprodup=1; else powprodup=0;
if 0 le prodlow then powprodlow=1; else powprodlow=0;
powprod=powprodlow+powprodup;
SUMnorm=numnormup+numnormlow;
SUMprod=numprodup+numprodlow;
cvgnorm=1-(numnormup+numnormlow);
cvgprod=1-(numprodup+numprodlow);
run;
```

```
/*
proc means data=prodboot N mean median std min max sum noprint;
var normlow normup prodlow produp numnormlow numnormup numprodlow
numprodup
pownormlow pownormup pownorm powprodlow powprodup powprod cvgnorm
cvgprod SUMnorm SUMprod;
output out=CLs
mean=normlow normup prodlow produp numnormlow numnormup numprodlow
numprodup
pownormlow pownormup pownorm powprodlow powprodup powprod cvgnorm
cvgprod SUMnorm SUMprod;
run;
*/

DATA SUMMARY; SET SUMMARY PRODBOOT;
drop a sea b seb ab true;
run;

%mend CONFLIM;
run;
PROC DATASETS LIB=WORK NOLIST;
%Macro loop;


%do i=1 %to 208;
%conflim(FILE1=n50cond&i.outprodancova,FILE2=n50cond&i.out,COND=&i,nobs
=50,method=1);
%end;

%do i=1 %to 208;
%conflim(FILE1=n100cond&i.outprodancova,FILE2=n100cond&i.out,COND=&i,no
bs=100,method=1);
%end;

%do i=1 %to 208;
%conflim(FILE1=n200cond&i.outprodancova,FILE2=n200cond&i.out,COND=&i,no
bs=200,method=1);
%end;

%do i=1 %to 208;
%conflim(FILE1=n500cond&i.outprodancova,FILE2=n500cond&i.out,COND=&i,no
bs=500,method=1);
%end;

DATA SUMMARYANCOVA; SET SUMMARY;
merger=1;
if _n_=1 then delete;
pownormancova=pownorm;
powprodancova=powprod;
cvgnormancova=cvgnorm;
cvgprodancova=cvgprod;
run;
```

```
DATA SUMMARY; SET SUMMARY;
merger=1;
if merger=1 then delete;
run;


%do i=1 %to 208;
%conflim(FILE1=n50cond&i.outprodpath,FILE2=n50cond&i.out,COND=&i,nobs=5
0,method=2);
%end;

%do i=1 %to 208;
%conflim(FILE1=n100cond&i.outprodpath,FILE2=n100cond&i.out,COND=&i,nobs
=100,method=2);
%end;

%do i=1 %to 208;
%conflim(FILE1=n200cond&i.outprodpath,FILE2=n200cond&i.out,COND=&i,nobs
=200,method=2);
%end;

%do i=1 %to 208;
%conflim(FILE1=n500cond&i.outprodpath,FILE2=n500cond&i.out,COND=&i,nobs
=500,method=2);
%end;


DATA SUMMARYPATH; SET SUMMARY;
merger=1;
pownormpath=pownorm;
powprodpath=powprod;
cvgnormpath=cvgnorm;
cvgprodpath=cvgprod;
run;

DATA SUMMARY; SET SUMMARY;
merger=1;
if merger=1 then delete;
run;


%do i=1 %to 208;
%conflim(FILE1=n50cond&i.outproddiff,FILE2=n50cond&i.out,COND=&i,nobs=5
0,method=3);
%end;

%do i=1 %to 208;
%conflim(FILE1=n100cond&i.outproddiff,FILE2=n100cond&i.out,COND=&i,nobs
=100,method=3);
%end;

%do i=1 %to 208;
%conflim(FILE1=n200cond&i.outproddiff,FILE2=n200cond&i.out,COND=&i,nobs
=200,method=3);
```

```
%end;

%do i=1 %to 208;
%conflim(FILE1=n500cond&i.outproddiff,FILE2=n500cond&i.out,COND=&i,nobs
=500,method=3);
%end;


DATA SUMMARYDIFF; SET SUMMARY;
merger=1;
pownormdiff=pownorm;
powproddiff=powprod;
cvgnormdiff=cvgnorm;
cvgproddiff=cvgprod;
run;

DATA SUMMARY; SET SUMMARY;
merger=1;
if merger=1 then delete;
run;

%do i=1 %to 208;
%conflim(FILE1=n50cond&i.outprodres,FILE2=n50cond&i.out,COND=&i,nobs=50
,method=4);
%end;

%do i=1 %to 208;
%conflim(FILE1=n100cond&i.outprodres,FILE2=n100cond&i.out,COND=&i,nobs=
100,method=4);
%end;

%do i=1 %to 208;
%conflim(FILE1=n200cond&i.outprodres,FILE2=n200cond&i.out,COND=&i,nobs=
200,method=4);
%end;

%do i=1 %to 208;
%conflim(FILE1=n500cond&i.outprodres,FILE2=n500cond&i.out,COND=&i,nobs=
500,method=4);
%end;


DATA SUMMARYRES; SET SUMMARY;
merger=1;
pownormres=pownorm;
powprodres=powprod;
cvgnormres=cvgnorm;
cvgprodres=cvgprod;
run;

DATA SUMMARY; SET SUMMARY;
merger=1;
if merger=1 then delete;
run;
```

```
%do i=1 %to 208;
%conflim(FILE1=n50cond&i.outprodcross,FILE2=n50cond&i.out,COND=&i,nobs=
50,method=5);
%end;

%do i=1 %to 208;
%conflim(FILE1=n100cond&i.outprodcross,FILE2=n100cond&i.out,COND=&i,nob
s=100,method=5);
%end;

%do i=1 %to 208;
%conflim(FILE1=n200cond&i.outprodcross,FILE2=n200cond&i.out,COND=&i,nob
s=200,method=5);
%end;

%do i=1 %to 208;
%conflim(FILE1=n500cond&i.outprodcross,FILE2=n500cond&i.out,COND=&i,nob
s=500,method=5);
%end;


DATA SUMMARYCROSS; SET SUMMARY;
merger=1;
pownormcross=pownorm;
powprodcross=powprod;
cvgnormcross=cvgnorm;
cvgprodcross=cvgprod;
run;

DATA SUMMARY; SET SUMMARY;
merger=1;
if merger=1 then delete;
run;

%mend loop;
%loop;
run;
QUIT;

DATA PRODCL.ANOVASUMMARY; merge SUMMARYANCOVA SUMMARYPATH SUMMARYDIFF
SUMMARYRES SUMMARYCROSS; BY MERGER;
if cond=1 then do; true=0; direct=0; Y2lag=0; M2lag=0; PretestCorr=.5;
Stability=.7; end;
```

APPENDIX K

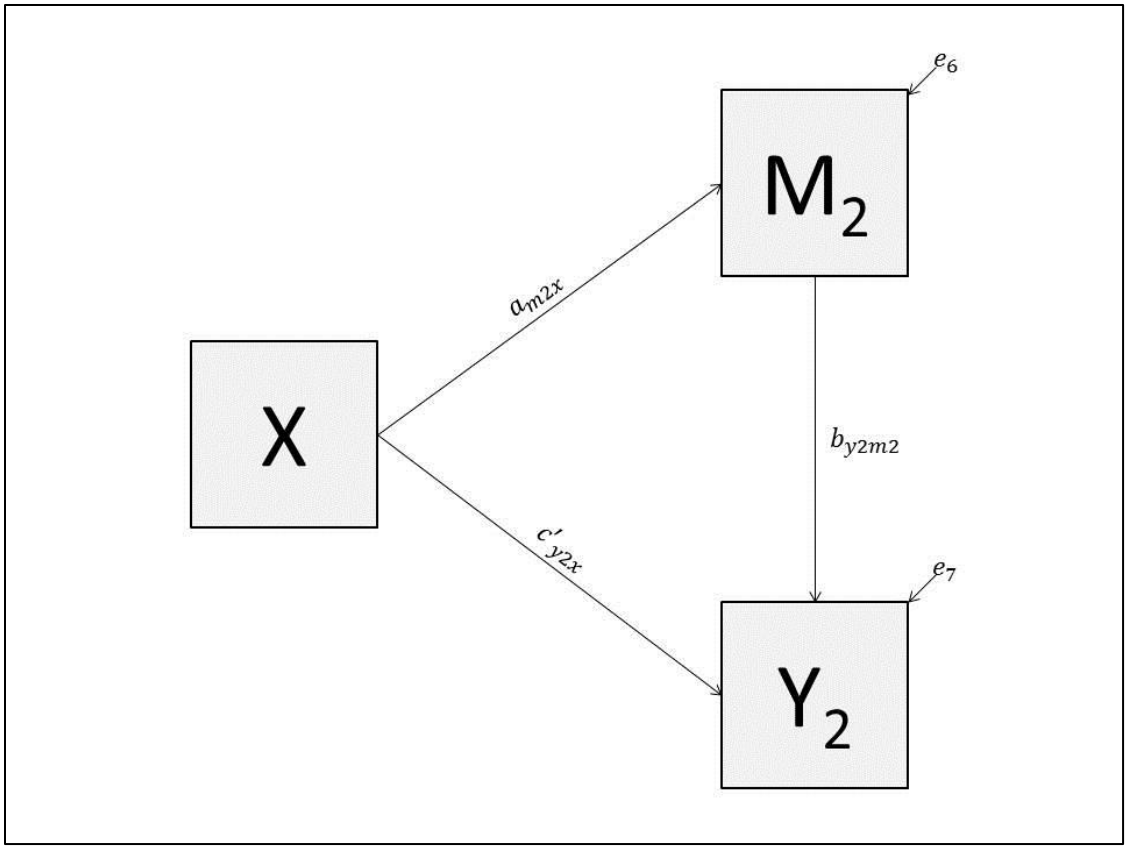
SAS MACRO FOR SUMMARIZING RESULTS BY AVERAGING ACROSS REPLICATIONS FOR USE IN TABLES FOR RESULTS SECTION

```
*Summarizing results by averaging across replications for use in tables
for results section;
libname DATAOUT "C:\Users\mvalent4\Desktop\DATAOUT";
libname DATASUM "C:\Users\mvalent4\Desktop\DATASUMMARY";

FILENAME NULLOG DUMMY 'C:\NULL';
PROC PRINTTO LOG=NULLOG;

DATA SUMMARY;
%MACRO SUMMARIZE (nsim,nobs,BMX,BYX,BYM,
aM1X,sm2m1,am2x,bm2y1,
by1m1,cy1x,cy2x,sy2y1,
by2m1,by2m2,varx,varm1,
varm2,vary1,vary2,RELM1,
RELM2, RELY1, RELY2,file,
TYPE,ERROR,COND);

Data sim; set DATAOUT.&file.out;
merger=1;
cond=&cond;
nobs=&nobs;
keep BAB BAB_ BABDIFF BABRES BABX BABR BAB_R BABDIFFR BABRESR BABXR AB
AB_ ABDIFF ABRES ABX cond nobs merger;
run;

Proc means data=sim noprint;
var BAB BAB_ BABDIFF BABRES BABX BABR BAB_R BABDIFFR BABRESR BABXR AB
AB_ ABDIFF ABRES ABX;
output out=bias
mean = BAB BAB_ BABDIFF BABRES BABX BABR BAB_R BABDIFFR BABRESR BABXR
AB AB_ ABDIFF ABRES ABX
STD = STDBAB STDBAB_ STDBABDIFF STDBABRES STDBABX STDBABR STDBAB_R
STDBABDIFFR STDBABRESR STDBABXR
STDAB STDAB_ STDABDIFF STDABRES STDABX;
run;

Data bias; set bias;
merger=1;
keep STDAB STDAB_ STDABdiff STDABRes STDABX merger;
run;

data sim2; merge sim bias; by merger;
SAB=BAB/STDAB;
SAB_=BAB_/STDAB_;
SABDIFF=BABDIFF/STDABdiff;
SABRES=BABRES/STDABRES;
SABX=BABX/STDABX;
run;


Data SUMMARY; SET SUMMARY sim2;
run;
```

```
%MEND;
run;

PROC DATASETS LIB=WORK NOLIST;
*Effect size test condition for stability of M and Y at .7 and pretest
corr at .5 and M1 on Y2 at 0;
%SUMMARIZE(nsim=1000,nobs=50,BMX=0,BYX=0,BYM=0,
aM1X=0, sm2m1=.981,am2x=0,bm2y1=0, by1m1=.578,
cy1x=0, cy2x=0,sy2y1=.686, by2m1=0,by2m2=0,
varx=1,varm1=1,varm2=1,vary1=1,vary2=1,
RELM1=1, RELM2=1, RELY1=1, RELY2=1,
FILE=n50cond1,TYPE='CCC',ERROR=1,COND=1); run; quit;
```

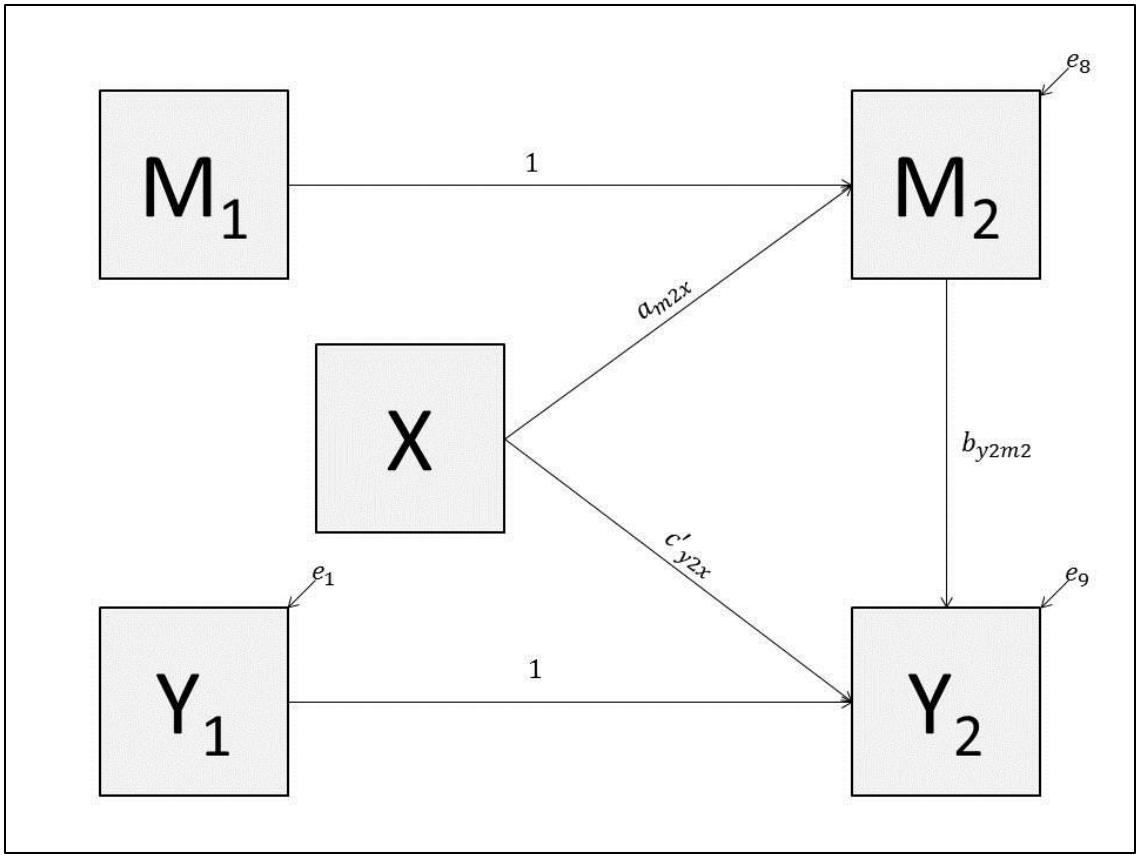
APPENDIX L

FIGURE OF CROSS-SECTIONAL MODEL


e_6
M_2
a_{m2x}
X
b_{y2m2}
c'_{y2x}
e_7
Y_2

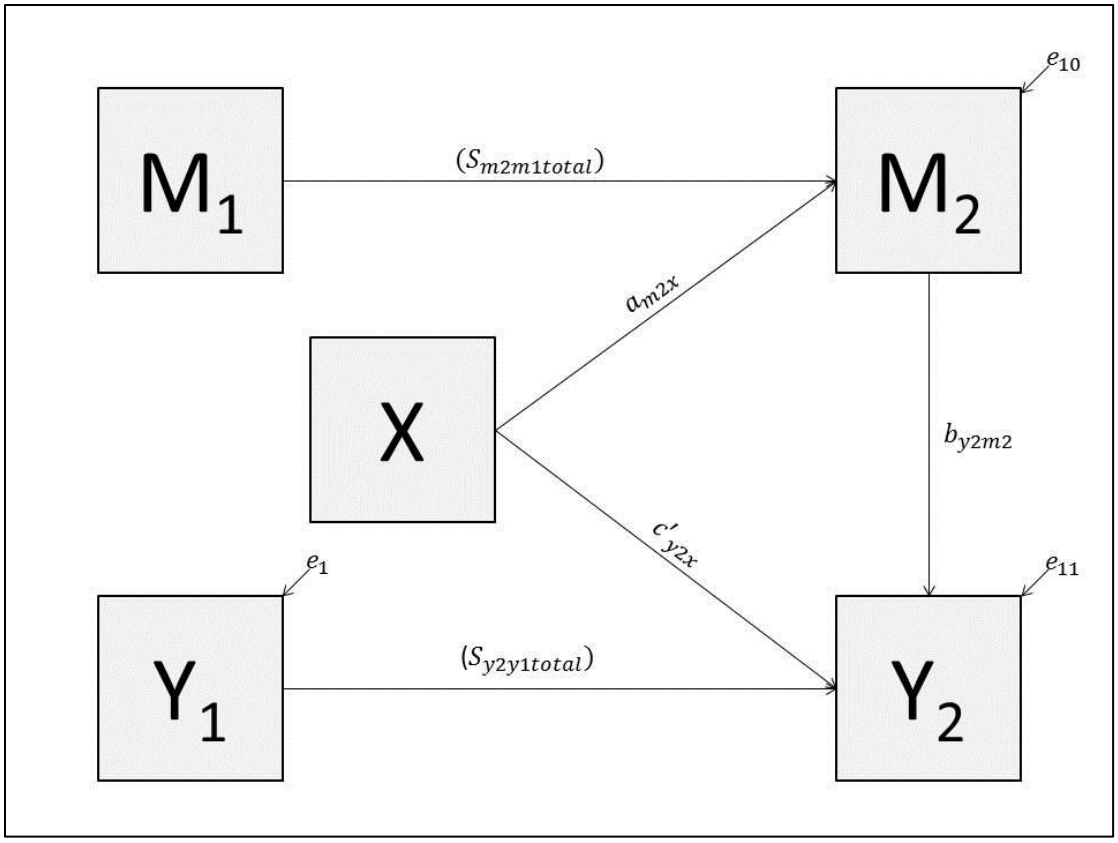
APPENDIX M

FIGURE OF DIFFERENCE SCORE MODEL


e_8
M_1
1
M_2
a_{m2x}
X
b_{y2m2}
c'_{y2x}
e_1
e_9
Y_1
1
Y_2

APPENDIX N

FIGURE OF RESIDUALIZED CHANGE SCORE MODEL


M_1
$(S_{m2m1total})$
M_2
e_{10}
a_{m2x}
X
b_{y2m2}
c'_{y2x}
e_1
e_{11}
Y_1
$(S_{y2y1total})$
Y_2

APPENDIX O

COVARIANCE ALGEBRA

$X = 0{,}1$

$M1 = aM1X * X + e1$

$Y1 = cY1X * X + bY1M1 * M1 + e2$

$M2 = aM2X * X + sM2M1 * M1 + bM2Y1 * Y1 + e3$

$Y2 = cY2X * X + sY2Y1 * Y1 + bY2M1 * M1 + bY2M2 * M2 + e4$

$True\ total\ effect\ of\ X\ on\ Y2 = (aM2X * X) * (bY2M2 * M2) + (aM1X * X) * (sM2M1 * M1) * (bY2M2 * M2) + (aM1X * X) + (bY2M1 * M1) + (aM1X * X) * (bY1M1 * M1) * (bM2Y1 * Y1) * (bY2M2 * M2) + (cY1X * X) * (bM2Y1 * Y1) * (bY2M2 * M2) + (cY1X * X) * (sY2Y1 * Y1) + (cY2X * X)$

$\boldsymbol{Cov(X, M1)} = Cov(X, aM1X * X + e1)$

$= Cov(X, aM1X * X) + Cov(X, e1)$

$= aM1X * \sigma_X^2$

$\boldsymbol{Cov(X, Y1)} = Cov(X, cY1X * X + bY1M1 * M1 + e2)$

$= Cov(X, cY1X * X) + Cov(X, bY1M1 * M1) + Cov(X, e2)$

$= cY1 * \sigma_X^2 + bY1M1 * Cov(X, M1)$

$\boldsymbol{Cov(X, M2)} = Cov(X, aM2X * X + sM2M1 * M1 + bM2Y1 * Y1 + e3)$

$= Cov(X, aM2X * X) + Cov(X, sM2M1 * M1) + Cov(X, bM2Y1 * Y1) + Cov(X, e3)$

$= aM2X * \sigma_X^2 + sM2M1 * Cov(X, M1) + bM2Y1 * Cov(X, Y1)$

$\boldsymbol{Cov(X, Y2)} = Cov(X, cY2X * X + sY2Y1 * Y1 + bY2M1 * M1 + bY2M2 * M2 + e4)$

$= Cov(X, cY2X * X) + Cov(X, sY2Y1 * Y1) + Cov(X, bY2M1 * M1) + Cov(X, bY2M2 * M2) + Cov(X, e4)$

$= cY2X * \sigma_X^2 + sY2Y1 * Cov(X, Y1) + bY2M1 * Cov(X, M1) + bY2M2 * Cov(X, M2)$

$$\boldsymbol{Cov}(\boldsymbol{M1},\boldsymbol{Y1}) = Cov(aM1X*X + e1, cY1X*X + bY1M1*M1 + e2)$$

$$= Cov(aM1X*X, cY1X*X) + Cov(aM1X*X, bY1M1*M1) + Cov(aM1X*X, e2)\, Cov(e1, cY1X*X) + Cov(e1, bY1M1*M1) + Cov(e1, e2)$$

$$= aM1X*(cY1X*\sigma_X^2) + aM1X*[bY1M1*Cov(X, M1)] + bY1M1*\sigma_{e1}^2$$

$$\boldsymbol{Cov}(\boldsymbol{M1},\boldsymbol{M2}) = Cov(aM1X*X + e1, sM2M1*M1 + bM2Y1*Y1 + aM2X*X + e3)$$

$$= Cov(aM1X*X, sM2M1*M1) + Cov(aM1X*X, bM2Y1*Y1) + Cov(aM1X*X, aM2X*X) + Cov(aM1X*X, e3) + + Cov(e1, sM2M1*M1) + Cov(e1, bM2Y1*Y1) + Cov(e1, aM2X*X) + Cov(e1, e3)$$

$$= aM1X*[sM2M1*Cov(X, M1)] + aM1X*[bM2Y1*Cov(X, Y1)] + aM1X*(aM2X*\sigma_X^2) + sM2M1*\sigma_{e1}^2 + bM2Y1*(bY1M1*\sigma_{e1}^2)$$

$$\boldsymbol{Cov}(\boldsymbol{M1},\boldsymbol{Y2}) = Cov(aM1X*X + e1, cY2X*X + sY2Y1*Y1 + bY2M1*M1 + bY2M2*M2 + e4)$$

$$= Cov(aM1X*X, cY2X*X) + Cov(aM1X*X, sY2Y1*Y1) + Cov(aM1X*X, bY2M1*M1) + Cov(aM1X*X, bY2M2*M2) + Cov(aM1X*X, e4) + Cov(e1, cY2X*X) + Cov(e1, sY2Y1*Y1) + Cov(e1, bY2M1*M1) + Cov(e1, bY2M2*M2) + Cov(e1, e4)$$

$$= aM1X*(cY2X*\sigma_X^2) + aM1X*[sY2Y1*Cov(X, Y1)] + aM1X*[bY2M1*Cov(X, M1)] + aM1X*[bY2M2*Cov(X, M2)] + sY2Y1*(bY1M1*\sigma_{e1}^2) + bY2M1*\sigma_{e1}^2 + bY2M2*[(sM2M1*\sigma_{e1}^2) + (bM2Y1*bY1M1*\sigma_{e1}^2)]$$

$$\boldsymbol{Cov}\,(\boldsymbol{M2},\boldsymbol{Y1}) = Cov(aM2X*X + sM2M1*M1 + bM2Y1*Y1 + e3, bY1M1*M1 + cY1X*X + e2)$$

$$= Cov(aM2X*X, bY1M1*M1) + Cov(aM2X*X, cY1X*X) + Cov(aM2X*X, e2) + Cov(sM2M1*M1, bY1M1*M1) + Cov(sM2M1*M1, bY1M1*M1) + Cov(sM2M1*M1, cY1X*X) + Cov(sM2M1*M1, e2) + Cov(bM2Y1*Y1, bY1M1*M1) + Cov(bM2Y1*Y1, cY1X*X) + Cov(bM2Y1*Y1, e2) + Cov(e3, bY1M1*M1) + +Cov(e3, cY1X*X) + Cov(e3, e2)$$

$$= aM2X * [bY1M1 * Cov(X, M1)] + aM2X * (cY1X * \sigma_X^2) + sM2M1 * (bY1M1 * \sigma_{M1}^2) + sM2M1 * [cY1X * Cov(X, M1)] + bM2Y1 * [bY1M1 * Cov(Y1, M1)] + bM2Y1 * [cY1X * Cov(X, Y1)] + bM2Y1 * \sigma_{e2}^2$$

$$\boldsymbol{Cov(Y1, Y2)} = Cov(cY1X * X + bY1M1 * M1 + e2, sY2Y1 * Y1 + bY2M1 * M1 + bY2M2 * M2 + cY2X * X + e4)$$

$$= Cov(cY1X * X, sY2Y1 * Y1) + Cov(cY1X * X, bY2M1 * M1) + Cov(cY1X * X, bY2M2 * M2) + Cov(cY1X * X, cY2X * X) + Cov(cY1X * X, e4) + Cov(bY1M1 * M1, sY2Y1 * Y1) + Cov(bY1M1 * M1, bY2M1 * M1) + Cov(bY1M1 * M1, bY2M2 * M2) + Cov(bY1M1 * M1, cY2X * X) + Cov(bY1M1 * M1, e4) + Cov(e2, sY2Y1 * Y1) + +Cov(e2, bY2M1 * M1) + Cov(e2, bY2M2 * M2) + Cov(e2, cY2X * X) + Cov(e2, e4)$$

$$= cY1X * [sY2Y1 * Cov(X, Y1)] + cY1X * [bY2M1 * Cov(X, M1)] + cY1X * [bY2M2 * Cov(X, M2)] + cY1X * (cY2X * \sigma_X^2) + bY1M1 * [sY2Y1 * Cov(M1, Y1)] + bY1M1 * (bY2M1 * \sigma_{M1}^2) + bY1M1 * [bY2M2 * Cov(M1, M2)] + bY1M1 * cY2X * Cov(X, M1) + bY2M2 * (bM2Y1 * \sigma_{e2}^2) + sY2Y1 * \sigma_{e2}^2$$

$$\boldsymbol{Cov(M2, Y2)} = Cov(sM2M1 * M1 + bM2Y1 * Y1 + aM2X * X + e3, cY2X * X + sY2Y1 * Y1 + bY2M1 * M1 + +bY2M2 * M2 + e4)$$

$$= Cov(sM2M1 * M1, cY2X * X) + Cov(sM2M1 * M1, sY2Y1 * Y1) + Cov(sM2M1 * M1, bY2M1 * M1) + +Cov(sM2M1 * M1, bY2M2 * M2) + Cov(sM2M1 * M1, e4) + Cov(bM2Y1 * Y1, cY2X * X) + Cov(bM2Y1 * Y1, sY2Y1 * Y1) + Cov(bM2Y1 * Y1, bY2M1 * M1) + Cov(bM2Y1 * Y1, bY2M2 * M2) + +Cov(bM2Y1 * Y1, e4) + Cov(aM2X * X, cY2X * X) + Cov(aM2X * X, sY2Y1 * Y1) + Cov(aM2X * X, bY2M1 * M1) + +Cov(aM2X * X, bY2M2 * M2) + Cov(e3, cY2X * X) + Cov(e3, sY2Y1 * Y1) + Cov(e3, bY2M1 * M1) + Cov(e3, bY2M2 * M2) + Cov(e3, e4)$$

$$sM2M1 * sY2Y1 * [Cov(M1, Y1)] + sM2M1 * (bY2M1 * \sigma_{M1}^2) + sM2M1 * [bY2M2 * Cov(M1, M2)] + bM2Y1 * (sY2Y1 * \sigma_{Y1}^2) + bM2Y1 * [bY2M1 * Cov(Y1, M1)] + bM2Y1 * [bY2M2 * Cov(Y1, M2)] + aM2X * (cY2X * \sigma_X^2) + aM2X * [bY2M2 * Cov(X, M2)] + bY2M2 * \sigma_{e3}^2$$

$$\boldsymbol{Cov(M1, M1)} = Cov(aM1X * X + e1, aM1X * X + e1)$$

$$= Cov(aM1X * X, aM1X * X) + 2 * Cov(aM1X * X, e1) + Cov(e1, e1)$$

$$= aM1X^2 * \sigma_X^2 + \sigma_{e1}^2$$

$$\boldsymbol{Cov}(\boldsymbol{M2}, \boldsymbol{M2}) = Cov(sM2M1 * M1 + bM2Y1 * Y1 + aM2X * X + e3, sM2M1 * M1 + bM2Y1 * Y1 + aM2X * X + e3)$$

$$= Cov(sM2M1 * M1, sM2M1 * M1) + Cov(sM2M1 * M1, bM2Y1 * Y1) + Cov(sM2M1 * M1, aM2X * X) + +Cov(sM2M1 * M1, e3) + Cov(bM2Y1 * Y1, sM2M1 * M1) + Cov(bM2Y1 * Y1, bM2Y1 * Y1) + Cov(bM2Y1 * Y1, aM2X * X) + Cov(bM2Y1 * Y1, e3) + Cov(aM2X * X, sM2M1 * M1) + Cov(aM2X * X, bM2Y1 * Y1) + +Cov(aM2X * X, bM2Y1 * Y1) + Cov(aM2X * X, aM2X * X) + Cov(aM2X * X, e3) + +Cov(e3, sM2M1 * M1) + +Cov(e3, bM2Y1 * Y1) + Cov(e3, aM2X * X) + Cov(e3, e3)$$

$$= sM2M1 * \sigma_{M1}^2 + 2 * sM2M1 * [bM2Y1 * Cov(M1, Y1)] + 2 * sM2M1 * [aM2X * Cov(X, M1)] + bM2Y1^2 * \sigma_{Y1}^2 + 2 * bM2Y1 * [aM2X * Cov(X, Y1)] + aM2X^2 * \sigma_X^2 + \sigma_{e3}^2$$

$$\boldsymbol{Cov}(\boldsymbol{Y1}, \boldsymbol{Y1}) = Cov(cY1X * X + bY1M1 * M1 + e2, cY1X * X + bY1M1 * M1 + e2)$$

$$= Cov(cY1X * X, cY1X * X) + Cov(cY1X * X, bY1M1 * M1) + Cov(cY1X * X, e2) + Cov(bY1M1 * M1, cY1X * X) + +Cov(bY1M1 * M1, bY1M1 * M1) + Cov(bY1M1 * M1, e2) + Cov(e2, cY1X * X) + Cov(e2, bY1M1 * M1) + +Cov(e2, e2)$$

$$= cY1X^2 * \sigma_X^2 + 2 * cY1X * [bY1M1 * Cov(X, M1)] + bY1M1^2 * \sigma_{M1}^2 + \sigma_{e2}^2$$

$$\boldsymbol{Cov}(\boldsymbol{Y2}, \boldsymbol{Y2}) = Cov(cY2X * X + sY2Y1 * Y1 + bY2M1 * M1 + bY2M2 * M2 + e4, cY2X * X + sY2Y1 * Y1 + bY2M1 * M1 + bY2M2 * M2 + e4)$$

$$= Cov(cY2X * X, cY2X * X) + Cov(cY2X * X, sY2Y1 * Y1) + Cov(cY2X * X, bY2M1 * M1) + Cov(cY2X * X, bY2M2 * M2) + Cov(cY2X * X, e4) + Cov(sY2Y1 * Y1, cY2X * X) + Cov(sY2Y1 * Y1, sY2Y1 * Y1) + Cov(sY2Y1 * Y1, bY2M1 * M1) + +Cov(sY2Y1 * Y1, bY2M2 * M2) + Cov(sY2Y1 * Y1, e4) + Cov(bY2M1 * M1, cY2X * X) + Cov(bY2M1 * M1, sY2Y1 * Y1) + Cov(bY2M1 * M1, bY2M1 * M1) + Cov(bY2M1 * M1, bY2M2 * M2) + +Cov(bY2M1 * M1, e4) + Cov(bY2M2 * M2, cY2X * X) + Cov(bY2M2 * M2, sY2Y1 * Y1) + Cov(bY2M2 * M2, bY2M1 * M1) + Cov(bY2M2 * M2, bY2M2 * M2) + Cov(bY2M2 * M2, e4) + Cov(e4, cY2X * X) + Cov(e4, sY2Y1 * Y1) + Cov(e4, bY2M1 * M1) + Cov(e4, bY2M2 * M2) + Cov(e4, e4)$$

$$= cY2X^2 * \sigma_X^2 + 2 * cY2X * [sY2Y1 * Cov(X, Y1)] + 2 * cY2X * [bY2M1 * Cov(X, M1)] + 2 * cY2X * [bY2M2 * Cov(X, M2)] + sY2Y1 * \sigma_{Y1}^2 + 2 * sY2Y1 * [bY2M1 * Cov(Y1, M1)] + 2 * sY2Y1 * [bY2M2 * Cov(Y1, M2)] + bY2M1 * \sigma_{M1}^2 + 2 * bY2M1 * [bY2M2 * Cov(M1, M2)] + bY2M2 * \sigma_{M2}^2 + \sigma_{e4}^2$$

$$\boldsymbol{Cov}[(\boldsymbol{M2} - \boldsymbol{M1}), (\boldsymbol{M2} - \boldsymbol{M1})] = VAR(M2) + VAR(M1) - 2 * Cov(M1, M2)$$

$$\boldsymbol{Cov}[(\boldsymbol{Y2} - \boldsymbol{Y1}), (\boldsymbol{Y2} - \boldsymbol{Y1})] = VAR(Y2) + VAR(Y1) - 2 * Cov(Y1, Y2)$$

$$\boldsymbol{Cov}[\boldsymbol{X}, (\boldsymbol{Y2} - \boldsymbol{Y1})] = Cov[X, (cY2X * X + SY2Y1 * Y1 + bY2M1 * M1 + bY2M2 * M2 + e4) - (cY1X * X + bY1M1 * M1 + e2)]$$

$$= Cov[X, (cY2X - cY1X) * X + SY2Y1 * Y1 + (bY2M1 - bY1M1) * M1 + bY2M2 * M2 + e4 - e2]$$

$$= Cov[X, (cY2X - cY1X) * X] + Cov[X, SY2Y1 * Y1] + Cov[X, (bY2M1 - bY1M1) * M1] + Cov[X, bY2M2 * M2] + Cov[X, e4] + Cov[X, -e2]$$

$$= (cY2X - cY1X) * \sigma_X^2 + SY2Y1 * Cov(X, Y1) + (bY2M1 - bY1M1) * Cov(X, M1) + bY2M2 * Cov(X, M2)$$

$$\boldsymbol{Cov}[\boldsymbol{X}, (\boldsymbol{M2} - \boldsymbol{M1})] = Cov[X, (aM2X * X + SM2M1 * M1 + bM2Y1 * Y1 + e3) - (aM1X * X + e1)]$$

$$= Cov[X, (aM2X - aM1X) * X + SM2M1 * M1 + bM2Y1 * Y1 + e3 - e1]$$

$$= Cov[X, (aM2X - aM1X) * X] + Cov[X, SM2M1 * M1] + Cov[X, bM2Y1 * Y1] + Cov[X, e3] + Cov[X, -e1]$$

$$= (aM2X - aM1X) * \sigma_X^2 + SM2M1 * Cov(X, M1) + bM2Y1 * Cov(X, Y1)$$

$$\boldsymbol{Cov}[(\boldsymbol{M2 - M1}), (\boldsymbol{Y2 - Y1})] = Cov[(aM2X - aM1X) * X + SM2M1 * M1 + bM2Y1 * Y1 + e3 - e1, (cY2X - cY1X) * X + SY2Y1 * Y1 + (bY2M1 - bY1M1) * M1 + bY2M2 * M2 + e4 - e2]$$

$$= Cov[(aM2X - aM1X) * X, (cY2X - cY1X) * X] + Cov[(aM2X - aM1X) * X, SY2Y1 * Y1] + Cov[(aM2X - aM1X) * X, (bY2M1 - bY1M1) * M1] + Cov[(aM2X - aM1X) * X, bY2M2 * M2] + Cov[(aM2X - aM1X) * X, e4] + Cov[(aM2X - aM1X) * X, -e2] + Cov[SM2M1 * M1, (cY2X - cY1X) * X] + Cov[SM2M1 * M1, SY2Y1 * Y1] + Cov[SM2M1 * M1, (bY2M1 - bY1M1) * M1] + Cov[SM2M1 * M1, bY2M2 * M2] + Cov[SM2M1 * M1, e4] + Cov[SM2M1 * M1, -e2] + Cov[bM2Y1 * Y1, (cY2X - cY1X) * X] + Cov[bM2Y1 * Y1, SY2Y1 * Y1] + Cov[bM2Y1 * Y1, (bY2M1 - bY1M1) * M1] + Cov[bM2Y1 * Y1, bY2M2 * M2] + Cov[bM2Y1 * Y1, e4] + Cov[bM2Y1 * Y1, -e2] + Cov[e3, (cY2X - cY1X) * X] + Cov[e3, (cY2X - cY1X) * X] + Cov[e3, SY2Y1 * Y1] + Cov[e3, (bY2M1 - bY1M1) * M1] + Cov[e3, bY2M2 * M2] + Cov[e3, e4] + Cov[e3, -e2] + Cov[-e1, (cY2X - cY1X) * X] + Cov[-e1, SY2Y1 * Y1] + Cov[-e1, (bY2M1 - bY1M1) * M1] + Cov[-e1, bY2M2 * M2] + Cov[-e1, e4] + Cov[-e1, -e2]$$

$$= (aM2X - aM1X) * (cY2X - cY1X) * \sigma_X^2 + (aM2X - aM1X) * Cov(X, Y1) + (aM2X - aM1X) * (bY2M1 - bY1M1) * Cov(X, M1) + (aM2X - aM1X) * bY2M2 * Cov(X, M2) + SM2M1 * (cY2X - cY1X) * Cov(X, M1) + SM2M1 * SY2Y1 * Cov(M1, Y1) + SM2M1 * (bY2M1 - bY1M1) * \sigma_{M1}^2 + SM2M1 * bY2M2 * Cov(M1, M2) + bM2Y1 * (cY2X - cY1X) * Cov(X, Y1) + bM2Y1 * SY2Y1 * \sigma_{Y1}^2 + bM2Y1 * (bY2M1 - bY1M1) * Cov(M1, Y1) + bM2Y1 * bY2M2 * Cov(Y1, M2) - bM2Y1 * Cov(Y1, e2) + bY2M2 * Cov(M2, e3) - SY2Y1 * Cov(Y1, e1) - (bY2M1 - bY1M1) * Cov(M1, e1) - bY2M2 * Cov(M2, e1)$$

$$\boldsymbol{Cov}(\boldsymbol{Y1}, \boldsymbol{e2}) = Cov(cY1X * X + bY1M1 * M1 + e2, e2)$$

$$= Cov(cY1X * X, e2) + Cov(bY1M1 * m1, e2) + Cov(e2, e2)$$

$$= \sigma_{e2}^2$$

$$\boldsymbol{Cov}(\boldsymbol{M2}, \boldsymbol{e3}) = Cov(aM2X * X + SM2M1 * M1 + SM2Y1 * Y1 + e3, e3)$$

$$= Cov(aM2X * X, e3) + Cov(SM2M1 * M1, e3) + Cov(bM2Y1 * Y1, e3) + Cov(e3, e3)$$

$$= \sigma_{e3}^2$$

$$\boldsymbol{Cov}(\boldsymbol{Y1}, \boldsymbol{e1}) = Cov(cY1X * X + bY1M1 * M1 + e2, e1)$$

$$= Cov(cY1X * X, e1) + Cov(bY1M1 * M1, e1) + Cov(e2, e1)$$

$$= bY1M1 * Cov(M1, e1)$$

$$\boldsymbol{Cov}(\boldsymbol{M1}, \boldsymbol{e1}) = Cov(aM1X * X + e1, e1)$$

$$= Cov(aM1X * X, e1) + Cov(e1, e1)$$

$$= \sigma_{e1}^2$$

$$\boldsymbol{Cov}(\boldsymbol{M2}, \boldsymbol{e1}) = Cov(aM2X * X + SM2M1 * M1 + bM2Y1 * Y1 + e3, e1)$$

$$= Cov(aM2X * X, e1) = Cov(SM2M1 * M1, e1) + Cov(bM2Y1 * Y1, e1) + Cov(e3, e1)$$

$$= SM2M1 * Cov(M1, e1) + bM2Y1 * Cov(Y1, e1)$$

$$= SM2M1 * \sigma_{e1}^2 + bM2Y1 * bY1M1 * \sigma_{e1}^2$$

$\widehat{\boldsymbol{Y2}} = \text{SY2Y1} * \text{Y1}$

$\boldsymbol{Res\ \Delta Y} = Y2 - \widehat{Y2}$

$= cY2X * X + SY2Y1 * Y1 + bY2M1 * M1 + bY2M2 * M2 + e4 - SY2Y1 * Y1$

$= cY2X * X + bY2M1 * M1 + bY2M2 * M2 + e4$

$\widehat{\boldsymbol{M2}} = \text{SM2M1} * \text{M1}$

$\boldsymbol{Res\ \Delta M} = M2 - \widehat{M2}$

$= aM2X * X + SM2M1 * M1 + bM2Y1 * Y1 + e3 - SM2M1 * M1$

$= aM2X * X + bM2Y1 * Y1 + e3$

$\boldsymbol{Cov}(\boldsymbol{ResΔM}, \boldsymbol{ResΔM})$

$= Cov(aM2X * X + bM2Y1 * Y1 + e3, aM2X * X + bM2Y1 * Y1 + e3)$

$= Cov(aM2X * X, aM2X * X) + Cov(aM2X * X, bM2Y1 * Y1) + Cov(aM2X * X, e3)$
$+ Cov(bM2Y1 * Y1, aM2X * X) + Cov(bM2Y1 * Y1, bM2Y1 * Y1)$
$+ Cov(bM2Y1 * Y1, e3) + Cov(e3, aM2X * X) + Cov(e3, bM2Y1 * Y1)$
$+ Cov(e3, e3)$

$= aM2X^2 * \sigma_X^2 + 2 * aM2X * bM2Y1 * Cov(X, Y1) + bM2Y1^2 * \sigma_{Y1}^2 + \sigma_{e3}^2$

$\boldsymbol{Cov}(\boldsymbol{ResΔY}, \boldsymbol{ResΔY})$

$= Cov(cY2X * X + bY2M1 * M1 + bY2M2 * M2 + e4, cY2X * X + bY2M1$
$* M1 + bY2M2 * M2 + e4)$

$= Cov(cY2X * X, cY2X * X) + Cov(cY2X * X, bY2M1 * M1) + Cov(cY2X * X, bY2M2 * M2)$
$+ Cov(cY2X * X, e4) + Cov(bY2M1 * M1, cY2X * X)$
$+ Cov(bY2M1 * M1, bY2M1 * M1) + Cov(bY2M1 * M1, bY2M2 * M2)$
$+ Cov(bY2M1 * M1, e4) + Cov(bY2M2 * M2, cY2X * X)$
$+ Cov(bY2M2 * M2, bY2M1 * M1) + Cov(bY2M2 * M2, bY2M2 * M2)$
$+ Cov(bY2M2 * M2, e4) + Cov(e4, cY2X * X) + Cov(e4, bY2M1 * M1)$
$+ Cov(e4, bY2M2 * M2) + Cov(e4, e4)$

$= cY2X^2 * \sigma_X^2 + cY2X * bY2M1 * CXM1 + cY2X * bY2M2 * CXM2 + bY2M1 * cY2X * CXM1$
$+ bY2M1^2 * \sigma_{M1}^2 + bY2M1 * bY2M2 * CM1M2 + bY2M2 * cY2X * CXM2$
$+ bY2M2 * bY2M1 * CM1M2 + bY2M2^2 * \sigma_{M2}^2 + \sigma_{e4}^2$

$\boldsymbol{Cov}(\boldsymbol{X}, \boldsymbol{ResΔM}) = Cov(aM2X * X + bM2Y1 * Y1 + e3)$

$= aM2X * \sigma_X^2 + bM2Y1 * Cov(X, Y1)$

$\boldsymbol{Cov(ResΔM, ResΔY)}$
$= Cov(aM2X * X + bM2Y1 * Y1 + e3, cY2X * X + bY2M1 * M1 + bY2M2 * M2 + e4)$

$= Cov(aM2X * X, cY2X * X) + Cov(aM2X * X, bY2M1 * M1)$
$+ Cov(aM2X * X, bY2M2 * M2) + Cov(aM2X * X, e4)$
$+ Cov(bM2Y1 * Y1, cY2X * X) + Cov(bM2Y1 * Y1, bY2M1 * M1)$
$+ Cov(bM2Y1 * Y1, bY2M2 * M2) + Cov(bM2Y1 * Y1, e4)$
$+ Cov(e3, cY2X * X) + Cov(e3, bY2M1 * M1) + Cov(e3, bY2M2 * M2)$
$+ Cov(e3, e4)$

$= aM2X * cY2X * \sigma_X^2 + aM2X * bY2M1 * Cov(X, M1) + aM2X * bY2M2 * Cov(X, M2)$
$+ bM2Y1 * cY2X * Cov(X, Y1) + bM2Y1 * bY2M1 * Cov(M1, Y1) + bM2Y1 * bY2M2 * Cov(Y1, M2) + bY2M2 * \sigma_{e3}^2$

$\boldsymbol{Cov(X, ResΔY)} = Cov(X, cY2X * X + bY2M1 * M1 + bY2M2 * M2 + e4)$

$= Cov(X, cY2X * X) + Cov(X, bY2M1 * M1) + Cov(X, bY2M2 * M2) + Cov(X, e4)$

$= cY2X * \sigma_X^2 + bY2M1 * Cov(X, M1) + bY2M2 * Cov(X, M2)$

APPENDIX P

TRUE CORRELATIONS

$X = 0{,}1$

$M1 = aM1X * X + e1$

$Y1 = cY1X * X + bY1M1 * M1 + e2$

$M2 = aM2X * X + sM2M1 * M1 + bM2Y1 * Y1 + e3$

$Y2 = cY2X * X + sY2Y1 * Y1 + bY2M1 * M1 + bY2M2 * M2 + e4$

Zero-Order Correlations

$\rho_{xm1} = \frac{CXM1}{STDX*STDM1}$

$\rho_{xy1} = \frac{CXY1}{STDX*STDY1}$

$\rho_{xm2} = \frac{CXM2}{STDX*STDM2}$

$\rho_{xy2} = \frac{CXY2}{STDX*STDY2}$

$\rho_{m1y1} = \frac{CM1Y1}{STDM1*STDY1}$

$\rho_{m1m2} = \frac{CM1M2}{STDM1*STDM2}$

$\rho_{m1y2} = \frac{CM1Y2}{STDM1*STDY2}$

$\rho_{m2y1} = \frac{CM2Y1}{STDM2*STDY1}$

$\rho_{m2y2} = \frac{CM2Y2}{STDM2*STDY2}$

$\rho_{y1y2} = \frac{CY1Y2}{STDY1*STDY2}$

First-order partial correlations

$\rho_{xy1.m1} = \frac{\rho_{xy1}-(\rho_{m1y1}*\rho_{xm1})}{\sqrt{1-\rho_{m1y1}{}^2}*\sqrt{1-\rho_{xm1}{}^2}}$

$\rho_{xm2.m1} = \frac{\rho_{xm2}-(\rho_{xm1}*\rho_{m1m2})}{\sqrt{1-\rho_{xm1}{}^2}*\sqrt{1-\rho_{m1m2}{}^2}}$

$\rho_{xy2.y1} = \frac{\rho_{xy2}-(\rho_{xy1}*\rho_{y1y2})}{\sqrt{1-\rho_{xy1}{}^2}*\sqrt{1-\rho_{y1y2}{}^2}}$

$\rho_{y1m2.m1} = \frac{\rho_{y1m2}-(\rho_{m1y1}*\rho_{m1m2})}{\sqrt{1-\rho_{m1y1}{}^2}*\sqrt{1-\rho_{m1m2}{}^2}}$

$\rho_{m1m2.x} = \frac{\rho_{m1m2}-(\rho_{xm1}*\rho_{xm2})}{\sqrt{1-\rho_{xm1}{}^2}*\sqrt{1-\rho_{xm2}{}^2}}$

$\rho_{m1y1.x} = \frac{\rho_{m1y1}-(\rho_{xm1}*\rho_{xy1})}{\sqrt{1-\rho_{xm1}{}^2}*\sqrt{1-\rho_{xy1}{}^2}}$

$\rho_{xm2.y1} = \frac{\rho_{xm2}-(\rho_{xy1}*\rho_{m2y1})}{\sqrt{1-\rho_{xy1}{}^2}*\sqrt{1-\rho_{m2y1}{}^2}}$

$\rho_{xy2.m2} = \frac{\rho_{xy2}-(\rho_{xm2}*\rho_{m2y2})}{\sqrt{1-\rho_{xm2}{}^2}*\sqrt{1-\rho_{m2y2}{}^2}}$

$\rho_{y1m2.x} = \frac{\rho_{xy2}-(\rho_{xm2}*\rho_{m2y2})}{\sqrt{1-\rho_{xm2}{}^2}*\sqrt{1-\rho_{m2y2}{}^2}}$

$\rho_{m1m2.y1} = \frac{\rho_{m1m2}-(\rho_{m1y1}*\rho_{m2y1})}{\sqrt{1-\rho_{m1y1}{}^2}*\sqrt{1-\rho_{m2y1}{}^2}}$

$$\rho_{m1y2.x} = \frac{\rho_{m1y2} - (\rho_{xm1} * \rho_{xy2})}{\sqrt{1-\rho_{xm1}{}^2} * \sqrt{1-\rho_{xy2}{}^2}}$$

$$\rho_{m1y2.y1} = \frac{\rho_{m1y2} - (\rho_{m1y1} * \rho_{y1y2})}{\sqrt{1-\rho_{m1y1}{}^2} * \sqrt{1-\rho_{y1y2}{}^2}}$$

$$\rho_{m1y2.m2} = \frac{\rho_{m1y2} - (\rho_{m1m2} * \rho_{m2y2})}{\sqrt{1-\rho_{m1m2}{}^2} * \sqrt{1-\rho_{m2y2}{}^2}}$$

$$\rho_{m2y2.x} = \frac{\rho_{m2y2} - (\rho_{xm2} * \rho_{xy2})}{\sqrt{1-\rho_{xm1}{}^2} * \sqrt{1-\rho_{xy2}{}^2}}$$

$$\rho_{m2y2.y1} = \frac{\rho_{m2y2} - (\rho_{m2y1} * \rho_{y1y2})}{\sqrt{1-\rho_{m2y1}{}^2} * \sqrt{1-\rho_{y1y2}{}^2}}$$

$$\rho_{m2y2.m1} = \frac{\rho_{m2y2} - (\rho_{m1m2} * \rho_{m1y2})}{\sqrt{1-\rho_{m1m2}{}^2} * \sqrt{1-\rho_{m1y2}{}^2}}$$

$$\rho_{y1y2.x} = \frac{\rho_{y1y2} - (\rho_{xy1} * \rho_{xy2})}{\sqrt{1-\rho_{xy1}{}^2} * \sqrt{1-\rho_{xy2}{}^2}}$$

$$\rho_{y1y2.m1} = \frac{\rho_{y1y2} - (\rho_{m1y1} * \rho_{m1y2})}{\sqrt{1-\rho_{m1y1}{}^2} * \sqrt{1-\rho_{m1y2}{}^2}}$$

$$\rho_{y1y2.m2} = \frac{\rho_{y1y2} - (\rho_{m2y1} * \rho_{m2y2})}{\sqrt{1-\rho_{m2y1}{}^2} * \sqrt{1-\rho_{m2y2}{}^2}}$$

$$\rho_{xy2.m1} = \frac{\rho_{xy2} - (\rho_{xm1} * \rho_{m1y2})}{\sqrt{1-\rho_{xm1}{}^2} * \sqrt{1-\rho_{m1y2}{}^2}}$$

Second-order partial correlations

$$\rho_{xm2.m1y1} = \frac{\rho_{xm2.m1} - (\rho_{m2y1.m1} * \rho_{xy1.m1})}{\sqrt{1-\rho_{m2y1.m1}{}^2} * \sqrt{1-\rho_{xy1.m1}{}^2}}$$

$$\rho_{y1m2.xm1} = \frac{\rho_{y1m2.x} - (\rho_{m1y1.x} * \rho_{m1m2.x})}{\sqrt{1-\rho_{m1y1.x}{}^2} * \sqrt{1-\rho_{m1m2.x}{}^2}}$$

$$\rho_{xy2.m1y1} = \frac{\rho_{xy2.m1} - (\rho_{xy1.m1} * \rho_{y1y2.m1})}{\sqrt{1-\rho_{xy1.m1}{}^2} * \sqrt{1-\rho_{y1y2.m1}{}^2}}$$

$$\rho_{m2y2.xy1} = \frac{\rho_{m2y2.x} - (\rho_{y1m2.x} * \rho_{y1y2.x})}{\sqrt{1-\rho_{y1m2.x}{}^2} * \sqrt{1-\rho_{y1y2.x}{}^2}}$$

$$\rho_{xy2.y1m2} = \frac{\rho_{xy2.y1} - (\rho_{xm2.y1} * \rho_{y2m2.y1})}{\sqrt{1-\rho_{xm2.y1}{}^2} * \sqrt{1-\rho_{y2m2.y1}{}^2}}$$

$$\rho_{m2y2.xm1} = \frac{\rho_{m2y2.x} - (\rho_{m1m2.x} * \rho_{m1y2.x})}{\sqrt{1-\rho_{m1m2.x}{}^2} * \sqrt{1-\rho_{m1y2.x}{}^2}}$$

$$\rho_{xy2.m1m2} = \frac{\rho_{xy2.m1} - (\rho_{xm2.m1} * \rho_{m2y2.m1})}{\sqrt{1-\rho_{xm2.m1}{}^2} * \sqrt{1-\rho_{m2y2.m1}{}^2}}$$

$$\rho_{m2y2.y1m1} = \frac{\rho_{m2y2.y1} - (\rho_{m1m2.y1} * \rho_{m1y2.y1})}{\sqrt{1-\rho_{m1m2.y1}{}^2} * \sqrt{1-\rho_{m1y2.y1}{}^2}}$$

$$\rho_{m1m2.xy1} = \frac{\rho_{m1m2.x} - (\rho_{m1y1.x} * \rho_{y1m2.x})}{\sqrt{1-\rho_{m1y1.x}{}^2} * \sqrt{1-\rho_{y1m2.x}{}^2}}$$

$$\rho_{y1y2.xm1} = \frac{\rho_{y1y2.x} - (\rho_{m1y1.x} * \rho_{m1y2.x})}{\sqrt{1-\rho_{m1y1.x}{}^2} * \sqrt{1-\rho_{m1y2.x}{}^2}}$$

$$\rho_{m1y2.xy1} = \frac{\rho_{m1y2.x} - (\rho_{m1y1.x} * \rho_{y1y2.x})}{\sqrt{1-\rho_{m1y1.x}{}^2} * \sqrt{1-\rho_{y1y2.x}{}^2}}$$

$$\rho_{y1y2.xm2} = \frac{\rho_{y1y2.x} - (\rho_{y1m2.x} * \rho_{m2y2.x})}{\sqrt{1-\rho_{y1m2.x}{}^2} * \sqrt{1-\rho_{m2y2.x}{}^2}}$$

$$\rho_{m1y2.y1m2} = \frac{\rho_{m1y2.y1} - (\rho_{m1m2.y1} * \rho_{m2y2.y1})}{\sqrt{1-\rho_{m1m2.y1}{}^2} * \sqrt{1-\rho_{m2y2.y1}{}^2}}$$

$$\rho_{y1y2.m1m2} = \frac{\rho_{y1y2.m1} - (\rho_{y1m2.m1} * \rho_{m2y2.m1})}{\sqrt{1-\rho_{y1m2.m1}{}^2} * \sqrt{1-\rho_{m2y2.m1}{}^2}}$$

$$\rho_{m1y2.xm2} = \frac{\rho_{m1y2.x} - (\rho_{m1m2.x} * \rho_{m2y2.x})}{\sqrt{1-\rho_{m1m2.x}{}^2} * \sqrt{1-\rho_{m2y2.x}{}^2}}$$

Third-order partial correlations

$$\rho_{xy2.y1m1m2} = \frac{\rho_{xy2.m1y1} - (\rho_{xm2.m1y1} * \rho_{m2y2.y1m1})}{\sqrt{1-\rho_{xm2.m1y1}{}^2} * \sqrt{1-\rho_{m2y2.y1m1}{}^2}}$$

$$\rho_{m1y2.xy1m2} = \frac{\rho_{m1y2.xy1} - (\rho_{m1m2.xy1} * \rho_{m2y2.xy1})}{\sqrt{1-\rho_{m1m2.xy1}{}^2} * \sqrt{1-\rho_{m2y2.xm1}{}^2}}$$

$$\rho_{y1y2.xm1m2} = \frac{\rho_{y1y2.xm1} - (\rho_{y1m2.xm1} * \rho_{m2y2.xm1})}{\sqrt{1-\rho_{y1m2.xm1}{}^2} * \sqrt{1-\rho_{m2y2.xm1}{}^2}}$$

$$\rho_{m2y2.xm1y1} = \frac{\rho_{m2y2.xm1} - (\rho_{y1m2.xm1} * \rho_{y1y2.xm1})}{\sqrt{1-\rho_{y1m2.xm1}{}^2} * \sqrt{1-\rho_{y1y2.xm1}{}^2}}$$