# Towards Demographic Information Release in LBS K-Anonymization 

by

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#### Abstract

The increasing number of continually connected mobile persons has created an environment conducive to real time user data gathering for many uses both public and private in nature. Publicly, one can envision no longer requiring a census to determine the demographic composition of the country and its sub regions. The information provided is vastly more up to date than that of a census and allows civil authorities to be more agile and preemptive with planning. Privately, advertisers take advantage of a persons stated opinions, demographics, and contextual (where and when) information in order to formulate and present pertinent offers.

Regardless of its use this information can be sensitive in nature and should therefore be under the control of the user. Currently, a user has little say in the manner that their information is processed once it has been released. An ad-hoc approach is currently in use, where the location based service providers each maintain their own policy over personal information usage.

In order to allow more user control over their personal information while still providing for targeted advertising, a systematic approach to the release of the information is needed. It is for that reason we propose a User-Centric Context Aware Spatiotemporal Anonymization framework. At its core the framework will unify the current spatiotemporal anonymization with that of traditional anonymization so that user specified anonymization requirement is met or exceeded while allowing for more demographic information to be released.


To all those that put up with me

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## Chapter 1

## INTRODUCTION

As of 2013 the number of adults in the United States with mobile phones hit $91 \%$ [15]. The capabilities of many of these devices go beyond phone calls. They are in fact a launching point for many different services including navigation, shopping, and social media. Each of these services requires differing types and amount of information that is as accurate as possible in order to provide a useful result. Navigation for example would require Global Positioning System GPS coordinates. While a social media application requires information such as name, gender, and age. However as the mobile platform has developed, services that require both navigation-esque and social media-esque information have begun to emerge.

Regardless of the information required by the service, privacy remains a concern of the mobile user. In Location-based Services $L B S$ such as navigation the privacy concern is regarding a user's physical location at a given time. Release of this information allows an attacker to determine movement patterns of an user. Possibly leading to a determination of said user's home address, place of employment, establishments frequented, and typical routes taken. In Social media-esque $N L B S$ services the privacy concern is release of demographic information intrinsic to but not directly identifying an user such as age, gender, ethnicity, and country of origin. This information can be combined with other publicly available data sets in a linking attack eroding the user's privacy [8]. Mitigating the risk of releasing information related to but not directly identifying an user while providing accurate information for services has borne the research area of k-Anonymity.

Conceptually k-Anonymity is hiding a user in a crowd of k -1 other individuals that are indistinguishable from said user. It utilizes generalization and suppression of quasi-identifiers to accomplish this end. Generalization decreases the accuracy of quasi-identifiers while suppression removes outliers from a the crowd. Quasi-identifiers are pieces of information which by themselves reveal little to nothing about a user but when coupled with other sources of information may reveal unexpected and/or private data of a user. Due to the nature of their required user information LBS and NLBS differ in their quasi-identifiers. LBS k-Anonymity focuses on spatiotemporal quasi-identifiers. While NLBS k-Anonymity handles demographic quasi-identifiers.

Originally NLBS k-Anonymization was designed for the release of user data from large databases. Later k-Anonymization was adapted for use in LBS by viewing the spatiotemporal point of a user a quasi-identifier. NLBS k-Anonymization is designed to handle relatively static data and produce an absolute minimal generalized data set for the entire initial set. In this paradigm an the initial user data set is considered complete and a minimal generalization is determined using the full information available. On the other hand LBS k-Anonymization is designed to process dynamic data and produce an absolute minimal generalized data set at a given instance. The initial user data set is incomplete and a minimal generalized data set is sought for every new piece of user data. In addition to these differences NLBS and LBS operate on different concepts of generalization; NLBS uses a fixed tree structure for generalizing quasi-identifiers while LBS utilizes a fixed function for generalizing quasi-identifiers. Given these differences between LBS and NLBS k-Anonymization providing user privacy for services which require both spatiotemporal and demographic information is a difficult task.

NLBS k-Anonymization uses a tree structure for anonymization where the leafs represent no generalization and the root is full generalization. With this structure there is only a single path of generalization for each leaf. While LBS uses a fixed search function where a single spatiotemporal point represents no generalization. The full generalization is represented by a maximum sized spatiotemporal region. The search function will yield different results for different initial user data sets. Therefore there exists many possible generalization paths for any given spatiotemporal point. In addition to these different generalization mechanisms NLBS and LBS k-Anonymization also differ in speed goals due to the nature of their use. Many LBS have a tight time limit to produce a result. For example in navigation a user reasonably expects their directions to match their current location which would not be possible if the LBS took longer than a few seconds to operate. NLBS is not expected to operate in a manner of seconds as the datasets they handle are large and timeliness of their resultant does not affect the use of the service. As NLBS was initially formulated to release data to researchers a day or longer of operating time is not unreasonable.

Achieving k-Anonymization for services with both LBS and NBLS user data requires features of both their respective k-Anonymization techniques. Neither LBS nor NBLS k-Anonymization is suitable for handling types of user information that the other handles. NBLS k-Anonymization will generally yield an over generalized spatiotemporal region. While LBS k-Anonymization requires a well-ordered set in order to generalize and user demographic generalization is a tree i.e. partial ordering.

In this thesis we present a k-anonymization approach that is independent from the underlying generalization structure. The approach uses a concept of similarity and difference in order to provide a minimal k-Anonymization for both NLBS and LBS
types of quasi-identifiers. It operates in an LBS environment with time constraints and incomplete a priori knowledge of the user set. The resultant anonymized data set can be used by both advertising services and governmental agencies to best react to the composition of people in a given spatiotemporal area. We have implemented a prototype anonymization service as part of this research as well as simulated various user population compositions, sizes, and rates of movement.

The remainder of the thesis is structured as follows. Chapter 2 covers NLBS and LBS k-Anonymization background information and illustrates the need of a different approach with an example. Chapter 3 overviews the general framework of our approach and describes the anonymization service. Chapter 4 discusses the implementation details and generation of simulated user traces. Chapter 5 covers the evaluation of our approaches performance and discussion of limitations. Chapter 6 concludes the thesis and presents possible future directions of this work.

## Chapter 2

## BACKGROUND

The related works in spatiotemporal k-anonymity have focused on expanding, read generalizing, initial coordinates into a larger and larger spatiotemporal regions until some criteria are met while not degrading the QoS. Please note that the expansion pattern is not fixed in most cases, an initial spatiotemporal point may expand into any number of generalized results. Conversely k anonymity as presented in $[8 ; 5 ; 22 ; 25 ?$ ; 3; 24; 2] uses a fixed anonymization structure.

### 2.1 NBLS k-Anonymization

Releasing truthful information for "circulation or research" [8] is the primary focus of NLBS k-Anonymization techniques. These techniques take in complete user data sets then transforms them into k-anonymous data sets via generalization and/or suppression.

### 2.1.1 Definitions

Definition 1 (k-anonymity). Let $T\left(A_{1}, \ldots, A_{m}\right)$ be a table, and QI be a quasiidentifier associated with it. T is said to satisfy k-anonymity with respect to QI iff each sequence of values in $T[Q I]$ appears at least with $k$ occurrences in $T[Q I]$.

As discussed previously generalization utilizes a tree structure to replace exact values of an quasi-identifier with more general versions. Suppression on the other hand will remove outliers from the original data set. Generalization may or may not be combined with suppression while suppression is normally paired with generalization


Figure 2.1: Gender DGH and VGH

```
        Race \(_{1}=\{\) not_released \(\}\)
    |
Race \(_{0}=\{\) asian,black, \(\ldots\), white \(\}\)
```



Figure 2.2: Race DGH and VGH
as pure suppressive approaches have "limited applicability" [8]. Figures 2.1.1 through 2.1.1 show example generalization hierarchies.

The Domain Generalization Hierarchy (DGH) is well-ordered set representing the levels of generalization possible for a given domain. While the Value Generalization Hierarchy (VGH) is a partially ordered set where all paths from root to a leaf have the same number of intermediate nodes. The VGH contains the actual values a quasiidentifier may assume at any given level. This representation works well for quasiidentifiers with no order amongst ungeneralized values. Take gender for example, though each sex may have their own views of superiority, there is no way to say male comes before female and vice versa. A lattice is used to represent more than one


Origin $_{0}=\{$ afghanistan,$\ldots$, zimbabwe $\}$


Figure 2.3: Origin DGH and VGH
quasi-identifier where every point is a composed of all quasi-identifiers and between adjacent nodes only a single component is changed. I.E. when a node is a single hop away all of its components will match the source node with the exception of a single component and the difference between the source node and destination node component is a single level of said components own DGH.

| Name | Gender | Race | Origin | Content |
| :---: | :---: | :---: | :---: | :---: |
| Mike | male | white | United States | $C_{m}$ |
| France | female | black | Haiti | $C_{f}$ |
| Eusebio | male | white | Mexico | $C_{e}$ |
| Tosh | female | native | United States | $C_{t}$ |
| Nesto | female | asian | Mexico | $C_{n}$ |

Table 2.1: NBLS Raw User Information

| Name | Gender | Race | Origin | Content |
| :---: | :---: | :---: | :---: | :---: |
| Mike | male | not_released | north america | $C_{m}$ |
| France | female | not_released | north america | $C_{f}$ |
| Eusebio | male | not_released | north america | $C_{e}$ |
| Tosh | female | not_released | north america | $C_{t}$ |
| Nesto | female | not_released | north america | $C_{n}$ |

Table 2.2: NLBS k-Anonymized $\mathrm{k}=2$

An example of an NLBS k-Anonymized user data can be seen in tables 2.2 and 2.3 for $\mathrm{k}=2$ and $\mathrm{k}=4$ respectively. Given the user information in table 2.1 an anonymization service begins at the leafs of the VGH for each of quasi-identifiers in table 2.1 and traverses towards the root node until a generalization near the original leafs is located. As these algorithms function on entire data sets the generalization is applied

| Name | Gender | Race | Origin | Content |
| :---: | :---: | :---: | :---: | :---: |
| Mike | not_released | not_released | north america | $C_{m}$ |
| France | not_released | not_released | north america | $C_{f}$ |
| Eusebio | not_released | not_released | north america | $C_{e}$ |
| Tosh | not_released | not_released | north america | $C_{t}$ |
| Nesto | not_released | not_released | north america | $C_{n}$ |

Table 2.3: NLBS k-Anonymized $\mathrm{k}=4$
to all user's regardless of necessity. For $k=2$ a generalized resultant satisfying the k requirement is at Origin $_{1}$, Race $_{1}$, Gender $_{0}$. Increasing the $k$ requirement tends to reduce the amount of useful information that can be released as is shown in the table 2.3 which has a generalization located at Origin $_{1}$, Race $_{1}$, Gender $_{1}$. Table 2.2 releases gender and some origin information while Table 2.3 shows only origin information. Please note that items such as Name are considered direct identifiers, reveal the user directly, and would either be removed or replaced before releasing the k-anonymous table. As discussed above the generalization is applied to the entire data set this results in Mike and Eusebio race being not_released for $k=2$ even though no generalization was required as they shared the same race. The race was withheld due to France, Tosh, and Nesto requiring a generalization of race to meet the k requirement.

### 2.2 LBS k-Anonymization

The general framework of LBS k-anonymization systems is shown in Figure 2.2. The framework states a mobile user has a secure connection with a trusted Anonymization Service that acts as a proxy in communicating over an insecure channel with the semi-trusted Location Based Service.


Figure 2.4: General Framework

An example workflow is also shown in Figure 2.2. The Mobile User sends their coordinates, k requirement, and query to the Anonymization Service over a secure connection. The Anonymization Service then anonymizes the Mobile Users coordinates into a region that encompass at least $\mathrm{k}-1$ other users and forwards anonymized query to the LBS provider. The LBS provider processes the anonymized query and returns the candidate results to the Anonymization Service. At this point the candidate results are either filtered at the Anonymization Service and the actual results are sent over the secure connection to the Mobile User, or the full candidate results are forwarded to the Mobile User and the Mobile Users device is responsible for determining the actual result of the query.

### 2.2.1 Definitions

Definition 2 (Coordinate). Pair of $\mathrm{x}, \mathrm{y}$ values. Normally latitude and longitude values but this is not required as long as the x and y domains are well-ordered.

Definition 3 (Spatial Region). Spatial area with clearly defined edges, i.e. a coordinate is either inside or outside the region

Definition 4 (Degraded Quality of Service (QoS)). Degraded QoS signifies an in-
crease in response time up to no response at all and/or a decrease in accuracy up to completely inaccurate results.

Definition 5 (Maximal Bounding Region (mbr)). Total spatial area a Spatial Region may occupy, area is defined in the geometric sense.

Definition 6 (Spatial LBS k-anonymity). Let R be a Spatial Region. Let U be the set of distinct users within $R$. R is said to satisfy Spatial LBS k-anonymity iff $|U| \geq k$. Definition 7 (Spatiotemporal LBS k-anonymity). Let R be a Spatial Region. Let t 1 and t 2 be instances in time s.t. t 1 t 2 . Let U be the set of distinct users within R during the interval [ $\mathrm{t} 1, \mathrm{t} 2$ ]. R is said to satisfy Spatial-Temporal LBS k-anonymity $|U| \geq k$.

Definition 8 (Location l-diversity [4]). Let R be a Spatial Region. Let L be a set of distinct addressable locations that are within the bounds of $R$. $R$ is said to satisfy Location l-diversity iff $|L| \geq l$.

Definition 9 (LBS (k,T)-Anonymity [19]). Let R be a Spatial Region. Let t be an instant in time s.t. t current time. Let Q be a set of distinct queries released during the interval [ t , current time] that have spatial regions which overlap with $\mathrm{R} . \mathrm{R}$ is said to satisfy LBS $(\mathrm{k}, \mathrm{T})$-Anonymity iff $|Q| \geq k$.

Definition 10 (Extended Spatial-Temporal k-anonymity). Place holder see comments for original definition

Definition 11 (Reciprocity [14]). Let R be a Spatial Region. Let U be the set of distinct users contained within the bounds of $R$. Let I be the user issuing the query. R is said to satisfy Reciprocity iff $I \in U,|U| \geq k$, and every member of U modifies their query region to match $R$.

### 2.2.2 Related LBS K-anonymization

The anonymization algorithm takes a users coordinate as focus and expands into the surrounding spatial area until the users criteria have been met, replace the users identifier with a pseudonym, and replace the original coordinate with the expanded Spatial Region. The region is most often rectangular $[4 ; 13 ; 19 ; 11 ; 10]$ or circular [14] in shape. Other shapes are not explicitly disallowed however they have not been thoroughly explored. The temporal anonymization is normally a side effect of processing the user's spatial generalization, I.E. waiting on additional users to fulfill the k requirement. Please note that this is contrary to the NLBS approach as a single user's anonymization is the focus, once we are able to anonymize that user we will leave the other user's not in k group as they are until another new user pops into existence at which the process repeats with that new user as the focus. While in NLBS the focus is on anonymizing every user. Please note peer-to-peer based anonymization approaches such as Mobihide [12] exist, however we use a trusted third party anonymizer and therefore will not explore p2p approaches.

## Adaptive-Interval Cloaking

Adaptive-Interval Cloaking [13] is based upon Quadtree algorithms and provides both spatial and temporal anonymity based upon the Definition 6 and Definition 7 respectively. Spatial anonymization takes the region surrounding the original query coordinates and sub divides it to the point where the next subdivision would cause the region to no longer satisfy Definition 6. Temporal anonymization takes the region surrounding the original query coordinates and subdivides it to a system specified size at which point the algorithm will hold the query until the region satisfies Definition 7. The k used is a system specified and system wide value. User positions are assumed
known at all times to the system.

## PrivacyGrid

PrivacyGrid $[4 ; 9]$ is based upon Grid and utilizes two different approaches, Top-down and Bottom-up algorithms. It provides spatial anonymity based upon the Definition 6, Definition 7, and Definition 8. The Top-down approach begins with the largest possible region permitted by the users mbr and opportunistically erodes the edges of the region until the next erosion iteration would cause the region to no longer satisfy Definition 6. The Bottom-up approach begins with users original cell in the grid and opportunistically expands an edge of the region into the surrounding area until the region satisfies Definition 6. The $\mathrm{k}, \mathrm{l}$, and mbr values used are on a per message basis, user positions are assumed known at all times to the system. Regarding the temporal aspect, the algorithm does not go into detail on the use of the allowed delay, and for that reason it is not included in this description.

## CliqueCloak

CliqueCloak [11] is based upon clique identification within a graph and provides both spatial and temporal anonymity as defined in Definition 6 and Definition 7. Temporal and Spatial anonymization take place concurrently, the original query coordinate is added into a graph data structure with other queries from distinct users within the mbr are checked to see if they form a clique with original query and if the distinct users k values are less than or equal to the original queries k value. If they do not form a clique, or if the clique size does not satisfy Definition 6, the query is maintained in the graph until its allowed delay value has expired at which time it is removed from the graph. The k , mbr, and allowed delay are on a per message basis. User positions are not assumed known to the system, they are gleaned from the users queries and
remain valid until the query is issued or until the allowed delay has expired.

A pitfall with this approach is the formation of the clique, if a message with a large k value arrives prior to a series of messages with smaller k values, the message with the large k value may not be anonymized. For example 1st msg. $\mathrm{k}=3$ and msg.delay=4 arrives at time $\mathrm{T}=0$. Then at $\mathrm{T}=1$ two messages come in with msg. $\mathrm{k}=2$ and msg.delay $=2$. The messages that arrived at $\mathrm{T}=1$ would anonymized with each other, while the message that came in at $\mathrm{T}=0$ would not be anonymized even though there were enough users present at $\mathrm{T}=1$ to anonymize it.

## LBSKT

LBSKT [3] [19] provides both spatial and temporal anonymity as defined by Definition 9. Temporal and Spatial anonymization take place concurrently, the original query coordinates are added into a corresponding cell within a system maintained grid data structure. A region is initialized at this cell and expands until the region satisfies Definition 9. User positions are not assumed known, only their last anonymized querys Spatial Region is known. The k , and T values are designed on a per message basis, however implementation has been done on a system wide k and T value. Another note on implementation is that this system was built upon PrivacyGrid [4].

### 2.3 Unaddressed in Current Techniques

The approaches of the related works in NLBS and LBS k-Anonymization have identified non-ordered quasi-identifiers such as gender, race, and origin, and ordered quasi-identifiers such as coordinates and time, respectively. However, neither is capable of handling the other's quasi-identifiers elegantly.


Figure 2.5: Coordinate DGH and VGH

In the case of NLBS k-Anonymity systems tackling the ordered quasi-identifier of coordinates there will be an excessive loss of data accuracy. This loss is due to the tree based generalization mechanism. For example if we use the x component of the coordinates, the DGH and VGH in Figure 2.3, and the data set in Table 2.4 we will generate the k -anonymous X of $0-3$. This is an unnecessary full generalization while an LBS centric system would produce an X of 2-3. Less accurate location data leads increased processing time for the LBS provider, anonymization service, and increased data transfer overall.

| Name | X | Content |
| :---: | :---: | :---: |
| Mike | 1 | $C_{m}$ |
| France | 2 | $C_{f}$ |

Table 2.4: NBLS Raw User Location Information

The issue of excessive loss of accuracy is not an issue of the chosen DGH or VGH. It results from the tree based structure of the VGH, namely for any VGH on a well ordered set there exists edges between generalization "buckets" and elements falling into said edges will require greater traversal up the VGH. In the above example the edges are 1 and 2 .

|  | 3 |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
|  | 2 |  | $0^{(0) 0}$ | ¢0 $0^{50}$ |  |
|  | 1 | $\mathrm{z}^{\text {cos }}$ | -步 | $x^{\left(20^{0^{0}}\right.}$ |  |
|  | 0 |  |  |  |  |
|  |  | 0 | 1 | 2 | 3 |

Figure 2.6: User locations

Regarding LBS k-Anonymization of NLBS quasi-identifiers, it is not possible. These approaches take advantage of the ordering of their domains to yield k-anonymous data sets. Determining which of the following integers: $\{0,1,5\}$ is closer to 2 is simple, however if the domain was race this task becomes impossible. For example with the given information which of the following races:\{black,white\} is closer to asian. Answering this question is not possible as there is no ordering amongst the elements of the race domain.

If the only quasi-identifier taken into account is the spatiotemporal point of the user as is the case for the current LBS k-anonymization schemes they do not address the possible loss of privacy when additional sensitive attributes are present. Take for example the grid shown in Figure 2.6 that shows the location of the five users presented earlier. The combined data set, location and demographic, is shown in Table 2.5 and the query point is represented by the black dot in Figure 2.6

LBS k -anonymization approaches with $\mathrm{k}=4$ would favor the blue enclosed region shown in Figure 2.6 resulting in the set of queries shown in Table 2.6 to be forwarded to LBS provider. Please note the missing user Nesto, the nature of this type of

| Name | X | Y | Gender | Race | Origin | Content |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Mike | 1 | 1 | male | white | United States | $C_{m}$ |
| France | 2 | 1 | female | black | Haiti | $C_{f}$ |
| Eusebio | 1 | 2 | male | white | Mexico | $C_{e}$ |
| Tosh | 2 | 2 | female | native | United States | $C_{t}$ |
| Nesto | 0 | 1 | female | asian | Mexico | $C_{n}$ |

Table 2.5: Combined Raw User Location Information
anonymization does not necessitate that every element be present in resultant set. It only requires that the initiator user be present in any generated k-anonymous sets. The released table complies with Definition 6 however it does not satisfy Definition 1 , as such the probability of linking the released information to the raw data in Table 2.5 is greater than $\frac{1}{k}$. Please note that row rearrangement has not been preformed to maintain clarity of example.

| Name | $X_{1}$ | $X_{2}$ | $Y_{1}$ | $Y_{2}$ | Gender | Race | Origin | Content |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Pseudonym $_{\text {Mike }}$ | 1 | 2 | 1 | 2 | male | white | United States | $C_{m}$ |
| Pseudonym $_{\text {France }}$ | 1 | 2 | 1 | 2 | female | black | Haiti | $C_{f}$ |
| Pseudonym $_{\text {Eusebio }}$ | 1 | 2 | 1 | 2 | male | white | Mexico | $C_{e}$ |
| Pseudonym $_{\text {Tosh }}$ | 1 | 2 | 1 | 2 | female | native | United States | $C_{t}$ |

Table 2.6: LBS k-anonymous resultant, $\mathrm{k}=4$

In this situation, due to the extra quasi-identifiers being released, the actual probability is $\frac{1}{1}$ that an attacker can link the user to query. In this attack scenario, the attacker is aware of all the users in this section of the grid, and is knowledgeable about their Gender, Race, Origin. However once the attacker obtains the service request data set, Table 2.6, they are able to link the query with the actual user thanks to

| Name | $X_{1}$ | $X_{2}$ | $Y_{1}$ | $Y_{2}$ | Gender | Race | Origin | Content |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Pseudonym Mike | 0 | 3 | 0 | 3 | not_released | not_released | north america | $C_{m}$ |
| Pseudonym France | 0 | 3 | 0 | 3 | not_released | not_released | north america | $C_{f}$ |
| Pseudonym $_{\text {Eusebio }}$ | 0 | 3 | 0 | 3 | not_released | not_released | north america | $C_{e}$ |
| Pseudonym Tosh | 0 | 3 | 0 | 3 | not_released | not_released | north america | $C_{t}$ |
| Pseudonym Nesto | 0 | 3 | 0 | 3 | not_released | not_released | north america | $C_{n}$ |

Table 2.7: NLBS k -anonymous resultant, $\mathrm{k}=4$
the aforementioned attributes, even though the table has had all direct identifiers removed and the spatiotemporal region has been anonymized. Essentially the attacker is able to map the pseudonym'd user to actual user.

NLBS k -anonymization approaches with $\mathrm{k}=4$, the coordinate generalization scheme in Figure 2.3, and the each columns respective scheme presented earlier in this chapter would favor the green enclosed region shown in Figure 2.6 resulting in the set of queries shown in Table 2.7 to be forwarded to LBS provider. The released table complies with both definitions 6 and 1 , as such the probability of linking the released information to the raw data in Table 2.5 is $\frac{1}{k}$. Please note that row rearrangement has not been preformed to maintain clarity of example.

The extra location quasi-identifier did not affect the probability of a linking attack as occurred in the LBS case. However, the extra location information is off little to no use. It is fully generalized, increasing processing time throughout the anonymization service and LBS provider. At this level of accuracy the presence of location data is a moot point at best and a waste of processing power at worst.

## Chapter 3

## FRAMEWORK FOR K-ANONYMOUS M-COMMERCE

The approach taken marries NLBS k-Anonymity and LBS k-Anonymity in the LBS environment that meets or exceeds the cloaked users' anonymization policies. This marriage allows for handling of both location and demographic quasi-identifiers. The necessity to include more quasi-identifier types than the current LBS k-anonymization approaches results from the nature of M-Commerce, Targeted Advertising, and future context sensitive LBS providers. In these domains the service providers require more information than location in order to function. The extra required quasi-identifiers such as gender, age, and ethnicity may used in a linking attack to erode the privacy of a user.

As discussed previously each anonymization domain handles different types of quasi-identifiers. NLBS uses a tree structure for generalization that is geared for quasi-identifier domains with no ordering. While LBS leverages the well-ordered nature of location information for generalization. Essentially NLBS will always follow the same generalization path for a given value while LBS can yield many different generalization paths for a given value. In addition to this the NLBS k-anonymization normally is solving the problem of finding a single generalization scheme that satisfies the k requirement for every member of a data set. LBS on the other hand locates a generalization scheme that satisfies $k$ for a subset of the original data set at a time.


Figure 3.1: Combination Possibilities

### 3.1 Anonymization Service

Minimal generalization is the primary concern of the anonymization service. Rapid processing is a close secondary concern. As the data becomes less accurate the LBS provider is burdened and the amount of post filtering of the LBS providers resultant is increased. The rate of anonymization directly affects the types of services that end-users can utilize. Targeted advertising in M-commerce is highly dependent upon the end-user's current environment. Small changes in location, occurring in a manner of seconds, may greatly affect the ad content.

### 3.1.1 Possible NLBS/LBS k-Anonymization Combinations

Due to the various types of quasi-identifier domains and existence of approaches to handle each class of domain there are four combinations possible as shown in Figure 3.1. We will describe each in the following sections.

## Unified

The first combination merges NLBS and LBS k-anonymization concepts into a single algorithm. The initial data set is transformed into an k-anonymous one by a single unified approach. Further discussion located in Algorithm and Data Structures.

## People First

In this combination the data set is first sent through an NLBS based k -anonymization approach then those results are fed to an LBS based k-anonymization scheme to produce a k-anonymous resultant. However, the intermediate table produced by the NLBS approach is not closed under subset. Without closure under subset there is no guarantee the LBS k-anonymization resultant will still be k -anonymous with respect to the NLBS quasi-identifiers. As the NLBS k-anonymization will not take location into account, it will produce an anonymous data set containing every user present regardless of their proximity to each other. When this data set is processed by the LBS k-anonymization approach k users in the resultant will be selected to minimize the spatial area needed to encompass them. The k users selected are a subset of the NLBS k-anonymous data set. We will prove the non-closure by assuming closure and providing a counter example where the above described process produces a data set that is not k -anonymous.

Take the data set from Table 2.5 and produce a NLBS k-anonymous data set with $\mathrm{k}=2$ and the VGHs from Figures 2.1.1, 2.1.1, and 2.1.1. Please see Table 3.1 for the resulting data set. Now assuming our focus is Nesto when we produce the LBS k-anonymous data set we will group Mike with Nesto as he is closer than any other user. As can be seen in table the resultant is LBS k-anonymous but it is not NLBS k -anonymous as it does not satisfy Definition 1. Namely, they are distinguished by
gender.

| Name | X | Y | Gender | Race | Origin | Content |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Mike | 1 | 1 | male | not_released | north america | $C_{m}$ |
| France | 2 | 1 | female | not_released | north america | $C_{f}$ |
| Eusebio | 1 | 2 | male | not_released | north america | $C_{e}$ |
| Tosh | 2 | 2 | female | not_released | north america | $C_{t}$ |
| Nesto | 0 | 1 | female | not_released | north america | $C_{n}$ |

Table 3.1: NLBS k-Anonymized $\mathrm{k}=2$

| Name | $X_{1}$ | $X_{2}$ | $Y_{1}$ | $Y_{2}$ | Gender | Race | Origin | Content |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Mike | 0 | 1 | 1 | 1 | male | not_released | north america | $C_{m}$ |
| Nesto | 0 | 1 | 1 | 1 | female | not_released | north america | $C_{n}$ |

Table 3.2: LBS k-Anonymized $\mathrm{k}=2$ Unsuccessful

## Places First

Contrasting People First; An LBS k-anonymous data set is generated then input to an LBS k-anonymization algorithm to produce an LBS/NLBS k-anonymous resultant. This approach was pursued initially but due to poor performance exploration was discontinued.

## Separate But Equal

Finally in this combination LBS and NLBS approaches are run in parallel and their results are then joined in the relational algebra sense to produce a k-anonymous data set. Exploration of this approach has been slated for future work.

### 3.1.2 Algorithm and Data Structures

Among the aforementioned combinations we focus on Unified for the reasons presented in the preceding section. As previously stated this is a novel approach unifying the concepts from both LBS and NLBS k-anonymization. Like LBS it operates with a single user focus resulting in a set of anonymized messages containing the original user and at least k-1 other users. The resultant is constructed by gathering a group of users message's with the minimal amount of difference. The group's generalization is applied by looking at the overall messages quasi-identifier extremities and locating a suitable generalization with regards to those two values.

Definition 12 (Difference). Measurement of generalization required to make entities indistinguishable by their quasi-identifiers.

The pseudo-code for this approach is located in Algorithm 1. It is composed of several sub-algorithms, presented in the order they appear in Algorithm 1, and operates on messages structured as below. Where $M$ is set of incoming messages to the anonymization service; $m_{M}$ is a message in $M ; I D$ is a unique identifier; $\{x, y\}$ is the originating point of the message; $Q I_{x}$ is a quasi-identifier; $t$ is the allowed amount of delay; $k$ is required number of indistinguishable users for any resultant containing this message; $C$ is content of message.

$$
m_{M} \in M:\left\langle I D,\{x, y\},\left\{Q I_{0}, Q I_{1}, \ldots, Q I_{n-1}\right\}, t, k, C\right\rangle
$$

The algorithm maintains $Q_{m}$ and $U N A N O N Y M I Z E D . Q_{m}$ is a queue of incoming messages that have yet to be processed. As a queue it is FIFO with the ordering based upon arrival time to the anonymization service. $U N A N O N Y M I Z E D$ is a
fully connected graph structure containing messages which have been unsuccessfully processed. The edges represent the difference between the messages they connect.

The algorithm first checks for a message in $Q_{m}$ then selects the oldest message for processing, lines 1 and 2 respectively. Line 4 ensures the message originated from a location managed this anonymization service. Line 5 generates an interim fully connected graph structure $G_{t}$ that is a clone of $U N A N O N Y M I Z E D$ with the message included. Line 6 is an optimization that prevents processing a message if it's $k$ requirement is unsatisfiable. In this event the message is added to UNANONYMIZED. Lines 6 through 18 repeatedly generate a candidate set of messages from $G_{t}$, unify the $k$ policy of generated set, and remove messages with stricter $k$ requirements than the generated set can satisfy from $G_{t}$ until either a suitable set is found or finding one becomes impossible. A suitable set is found when the generated set's cardinality is $\geq$ the unified $k$ while impossibility condition is triggered when the number of messages in $G_{t}$ falls below the unified $k$ requirement. Line 19 branches on the success of Lines 6 through 18. If it was unsuccessful the message is incorporated into UNANONY MIZED then returns null. If it was successfully the candidate set along with the message are generalized then returned.

Calculating the difference between messages is performed by the Calculate Difference Algorithm 2. It is essentially a weighted euclidean distance formula, where each quasi-identifier in has an associated weight $\omega\left(D_{i}\right)$ and difference function $d\left(p_{i}, q_{i}, D_{i}\right)$. Please see Algorithm 3 and Algorithm 4 for the weight and difference function respectively.

The weights given to a quasi-identifier domain depend on type and represent the inverse of the maximal change required for any two values to become indistinguishable.

```
Algorithm 1 LIVE
    if \(Q_{m} \neq \emptyset\) then
        \(m_{0} \leftarrow\) Pop first message of \(Q_{m}\)
        \(m_{0} \leftarrow\) Append \(\mathrm{r}, \mathrm{c}\) to \(m_{0}\) where \(r=\operatorname{row}\left(m_{0} \cdot y\right)\) and \(c=\operatorname{column}\left(m_{0} \cdot x\right)\)
        returnable \(\leftarrow \emptyset\)
        if \(m_{0}\).coordinates \(\in R E G I O N\) then
        Add \(m_{0}\) into \(G_{w}\)
        for all \(m_{i} \in U N A N O N Y M I Z E D\) do
            if \(\left(m_{0} \cdot m b r \geq\left|m_{0} \cdot c-m_{i} \cdot c\right|\right) \&\left(m_{0} \cdot m b r \geq\left|m_{0} \cdot r-m_{i} \cdot r\right|\right)\) then
            if \(\left(m_{i} . m b r \geq\left|m_{0} . c-m_{i} . c\right|\right) \&\left(m_{i} . m b r \geq\left|m_{0} \cdot r-m_{i} \cdot r\right|\right)\) then
            \(d \leftarrow\) calculate_difference \(\left(m_{0}, m_{i}\right)\)
            Add edge \(\left(m_{0}, m_{i}\right)\) with weight \(d\) to \(G_{w}\)
        Add \(m\) into UNANONYMIZED
        if \(G_{w}\left(m_{0}\right) \cdot\) edges \(>=\left(m_{0} \cdot k-1\right)\) then
            clique \(\leftarrow\) get_k_clique \(\left(G_{w}, m_{0}\right)\)
            if clique \(\neq \emptyset\) then
                returnable \(\leftarrow\) apply_generalization(clique, \(k\) )
            for all \(m_{i} \in\) clique do
            Remove \(m_{i}\) from \(G_{w}\)
            Remove \(m_{i}\) from UNANONYMIZED
        return returnable
```

```
Algorithm 2 Calculate Difference
calculate_difference \((p, q)=\sqrt{\sum_{i=1}^{n} \omega\left(D_{i}\right)^{2} d\left(p_{i}, q_{i}, D_{i}\right)^{2}}\)
```

In other words the percentage of accuracy lost for each generalization increase.

As shown in Algorithm 3 a finite well-ordered set such as latitude or age uses $\frac{1}{|D|-1}$. The cardinality of the domain represents the total number of elements that can included while generalizing. We remove one from the total to compensate for the initial value. The inverse is used as it is the percentage of generalization or inaccuracy that each additional element adds when included in the result.

```
Algorithm 3 Weight
    \(\omega(D)= \begin{cases}\frac{1}{|D|-1} & \text { if } \mathrm{D} \text { is finite well-ordered set } \\ \frac{1}{h t(D)} & \text { if }|D|>1 \& \mathrm{D} \text { is finite hierarchical (partially ordered) set } \\ 1 & \text { if } \mathrm{D} \text { is finite unordered set }\end{cases}\)
```

The finite hierarchical set shown in Algorithm 3 has a total number of generalization steps equal to the height of the VGH or DGH. Illustrating this is the Gender domain shown in Figure 2.1.1, there is only a single generalization step for the leaves male and female in the VGH and there is only a single step in the DGH. This results from the from definition of Domain Hierarchies in [8].

Contrary to the hierarchical and well-ordered sets the size of the unordered domain has no bearing on the generalization level. In this case the amount of information lost is completely dependent upon the two users being compared as a user is unable to lose information that they never had in the first place. The unordered set views the percentage of inaccuracy increased per element removed. As it is removal being measured the initial values may be removed, hence no need to deduct one. While
this type of domain has not been explored as of yet it does extend the expression capabilities of k-anonymization. Take for example the multiracial individuals shown in Table 3.3 and the race hierarchies in Figure 2.1.1. Currently the leaves of the VGH are single elements and not sets, therefore there is no specified manner to generalize in these situations. An approach to accommodate these data types is presented later this section.

| Name | Gender | Race | Origin | Content |
| :---: | :---: | :---: | :---: | :---: |
| Mike | male | \{white\} | United States | $C_{m}$ |
| France | female | \{black,white \} | Haiti | $C_{f}$ |
| Eusebio | male | $\{$ white\} | Mexico | $C_{e}$ |
| Tosh | female | \{native,white,asian\} | United States | $C_{t}$ |
| Nesto | female | \{white,asian \} | Mexico | $C_{n}$ |

Table 3.3: Multiracial Modified NBLS Raw User Information

The Difference algorithm 4 determines the number of generalization steps required to make a pair of quasi-identifier elements from the same domain indistinguishable. The inner working of the algorithm are dependent upon the type of domain being operated on.

[^0]In the case of finite well-ordered sets we take the difference of the index or ordinal value of p and q . As the set is well-ordered the amount of generalization required to make p and q indistinguishable is equal to the number of items between p and q. Conceptually we are increasing the interval size until enough users fall within it's boundaries. For example in Figure 2.6 the domain for the quasi-identifier X is the set $\{0,1,2,3\}$. Nesto, Eusebio, and France are located at 0, 1, and 2 respectively. The difference between Nesto and Eusebio is 1 while between Nesto and France it is 2. If we were to want a $k=2$ with respect to the X axis and Nesto as the initiator an anonymized value of $0-1$ or 0-2 would satisfy the requirement as both have at least two users whose x point fall within the boundaries of the anonymized interval.

As described in [8] the minimal number of generalizations needed to be preformed on a pair of leaves p and q in a VGH is the height of the lowest common ancestor of p and q . For example in Figure 2.1.1 the LCA(antigua\&barbuda, unitedstates) is north america with a height of one. Hence antiquaध́barbuda and the united states become indistinguishable at one step of generalization.

The unordered set works in an opposite conceptual manner to both hierarchical and well-ordered sets. The other types view generalization as the inclusion of extra elements in the anonymized resultant. Well-ordered will grow the interval size while traversal up a tree in the hierarchal set will increase the number of leaves. Unordered sets on the other hand will remove elements from their initial sets in order to produce the k-anonymous result. The intersection of the initial sets gives the overlapping values which can be release without distinguishing either original set. For example using Table 3.3 as a source set, $k=2$, ignoring direct identifiers, and the quasiidentifier of race we can produce the NLBS k-anonymous Table 3.4. This table shows
more racial information than that of Table 2.2.

| Name | Gender | Race | Origin | Content |
| :---: | :---: | :---: | :---: | :---: | :---: |
| Mike | male | $\{$ white $\}$ | north america | $C_{m}$ |
| France | female | $\{$ white $\}$ | north america | $C_{f}$ |
| Eusebio | male | $\{$ white $\}$ | north america | $C_{e}$ |
| Tosh | female | $\{$ white $\}$ | north america | $C_{t}$ |
| Nesto | female | $\{$ white $\}$ | north america | $C_{n}$ |

The amount of information lost is equal to one minus the percentage of remaining diversity of the resultant set. The quantity $\frac{|p \cap q|}{|p \cup q|}$ shows the remaining diversity percentage. $|p \cap q|$ is a count of the releasable elements while $|p \cup q|$ is the maximum number of elements available for release. The amount of information loss in unordered domains is respective to the sets being compared and not to the domain itself.

The actual search for the k clique is performed by algorithm an $\mathrm{A}^{*}$ based approach. As $A^{*}$ is geared for path finding in graphs it works on the concept of a source and goal vertex. Though it appears that we have all the components save for a terminal or goal vertex we do not. The former we have as $m$ however we do not have a goal vertex. Therefore we defined goal vertex conditions, namely we know we have arrived at a goal vertex when we have reached our required k size. With both the source vertex and terminating conditions present the algorithm then searches through the possible set of members of the k clique by adding a single member at a time to the initial set containing $m$.

The additional members loosely come from the original graph $G$, however the A* algorithm is not directly searching $G$ it is in fact exploring many derivatives of $G$. At any given step in the process the next possible member to the clique is dependent both upon existence of arcs between all members of the current clique elements and the possible member as well as the policy constraints on the resulting clique.

An initial state is created in lines $5-11$. The states maintain a spatial region defined by lower and upper rows as well as columns of the messages present in the state, this is initialized in line 5 . Policy values of $m b r$ and $k$ are also set in line 5 . As this is an A* algorithm the associated $g, h, f$ values are set in lines 6,7 , and 11 respectively. Each state is also responsible for maintaining the members present in the k clique for the state, as each additional member will signify a new state. The members present set is initialized in line 8 to that of the source message $m$. The state is added to the openlist structure of the A* in line 12 and as we do not have a goal state but rather terminal conditions we will repeatedly loop through lines 14-44 until found which is set to false in line 13 is set to true by the terminal conditions being met. Line 9 sets the handle for the vertex representing the set of nodes in the candidate clique in the states associated $G$.

The inner workings of the loop is an extension the A* algorithm. The lowest $F$ valued state is popped from the openlist, line 15 . Then it is checked to see if it satisfies the terminal conditions in line 16. The state's $h$ being 0 signifies that it is already satisfied with the members present. If it meets the terminal conditions then found is set to true, the nodes present in the clique managed by the state are returned and then removed from the original graph $G$; lines 17 through 19 respectively. If not then additional states are generated and checked.

```
Algorithm get_k_clique: part 1
    function GET_K_CLIQUE (G, m) \(\triangleright\) A* search based approach \(^{*}\)
        returnable \(\leftarrow \emptyset\)
        openlist \(\leftarrow\) Empty priority queue
        closedlist \(\leftarrow \emptyset\)
        initial_state \(\left(\right.\) state \(\left._{0}, m\right)\)
    6: \(\quad\) state \(_{0} . g \leftarrow 0\)
    7: \(\quad\) state \(_{0} . h \leftarrow H\left(G, m\right.\), state \(\left._{0} . k-1\right)\)
    8: \(\quad\) state \(_{0}\). nodes \(\leftarrow\{m\}\)
    9: \(\quad\) state \(e_{0}\).label \(\leftarrow m\)
10: \(\quad\) state \(_{0} . G \leftarrow G\)
11: \(\quad\) state \(_{0} . f \leftarrow\) state \(_{0} . g+\) state \(_{0} . h\)
12: Push state \(_{0}\) to openlist
13: \(\quad\) found \(\leftarrow\) false
14: while ! found do
15: \(\quad\) state \(_{c} \leftarrow\) openlist.pop ()
16: \(\quad\) if state \(_{c} \cdot h==0 \& \mid\) state \(_{c} \cdot\) nodes \(\mid \geq\) state \(_{c} \cdot k\) then
17: \(\quad\) found \(\leftarrow\) true
18: \(\quad\) returnable \(\leftarrow\) state \(_{c}\).nodes
19: \(\quad\) Remove all state \(_{c}\).nodes from \(G\)
20: else
21: \(\quad\) Add state \(_{c}\) to closedlist
22: \(\quad\) for all neighbor \(w\) of state \(_{c} . l a b e l \in \operatorname{state}_{c} \cdot G\) do
23: \(\quad\) state \(_{n}\). nodes \(\leftarrow\) state \(_{c}\).nodes \(\cup w\)
```

```
Algorithm get_k_clique: part 2
24: \(\quad\) valid \(\leftarrow\) enforcers_statisfied \(\left(\right.\) state \(_{c}\), state \(\left._{n}, w\right)\)
25: \(\quad\) if \(!\left(\right.\) state \(_{n} \in\) closedlist \() \&\) valid then
```

As this state does not satisfy the terminating conditions and it has been checked already it is now added to the closed list; line 21. The additional states are generated by looking to label/vertex and graph of this state and enumerating the neighbors of the vertex in the graph; line 22. Each enumerated neighbor will first produce a state shell that is used for comparison and policy enforcement; lines 23 trough 24 . The new state's clique is the union of the preceding state's clique and the enumerated neighbor, line 23. As all messages have associated row and column data the spatial region of the new state is updated in line 24 . The new state's $k$ and $m b r$ policies are updated, checked against the new state, and checks resultant returned in line 24.

Line 25 ensures we are not rechecking a visited state and that new state is valid, if not then the state is added to the closedlist else the new state is promoted from a shell to a full state; lines 26 through 30. This entails generating a graph, label, $h, g$, and $f$ for the newly promoted state. Line 31 ensures only possibly satisfiable states are added to the openlist. If the state is unsatisfiable then it is added to the closedlist; line 41. The lines 32 through 41 update the openlist, by adding a new state, lines 38 - 39, or by replacing a state currently present, lines 34-36.

As get_k_clique algorithm is based upon $A^{*}$ we a need a heuristic or future cost function (h), past path cost function (g), and the composite cost function (f). $f$ is the sum of the cost to our current state and the estimated cost from said point to the goal state thus the equation $f=g+h$. The past path cost is initialized at 0 as when we start we have yet to travel, and sums the edge weights present on the path selected. $g_{n}=g_{n-1}+$ weight $\left(\operatorname{Edge}\left(g_{n-1}\right.\right.$.label,$\left.\left.x\right) \in G_{n-1}\right)$ with $g_{0}=0$ where $G_{n-1}$ is the graph associated with state $_{n-1}$.

```
Algorithm initial_state
    function initial_State(state, m)
    2: \(\quad\) state.row lower \(^{\leftarrow}\) m.row
    3: \(\quad\) state.row \(w_{\text {upper }} \leftarrow\) m.row
    4: \(\quad\) state.col \(l_{\text {lower }} \leftarrow\) m.col
    5: \(\quad\) state.col \(l_{\text {upper }} \leftarrow\) m.col
    6: \(\quad\) if \((m . m b r==\emptyset) \mid(m . m b r<0)\) then
    7: \(\quad m \cdot m b r \leftarrow \infty\)
        state.mbr \(\leftarrow m . m b r\)
        if \((w . k==\emptyset) \mid(w . k<0)\) then
            \(w . k \leftarrow 0\)
        state \(_{0} . k \leftarrow m . k\)
    end function
```

The initial_state Algorithm initializes the state's policy values based upon the message $m$, lines 8 and 11. A sanity check is preformed on the $m b r$, lines 6-7, and $k$, lines $9-10$, policies. As the $m b r$ is a constraint on the spatial region area we create a boundary box around $m$ using $m$ 's row to set bottom and top edges, lines 2 and 3 respectively. Equivalently we set left and right edges using $m$ 's column in lines 4-5.

The heuristic function presented in Algorithm 3 under estimates the cost from the vertex in a given graph to the goal state. It sums the edge weights between the vertex and the number of other vertices needed to reach the required k value as having a group of size k is the goal state. The summation is done in ascending order of edge weights, so the minimum weighted edges are included first as in an ideal situation these would be the vertices selected for the group due to the minimal difference from our current group. In addition to the summation the difference between the final or

```
Algorithm 3 Heuristic
    function H (G, vertex, required)
        returnable \(\leftarrow 0\)
        nearest \(\leftarrow\) List sorted in ascending weights order of neighbors of vertex \(\in G\)
        if \(\mid\) nearest \(\mid \geq\) required \& \(\mid\) nearest \(\mid>0\) then
            for all \(n=0 ; n<\) required \(; n++\) do
                returnable \(+=\) weight \((\) Edge \((\) vertex, nearest \([n]))\)
            returnable \(+=\) weight(Edge(vertex, nearest[required - 1])) -
    weight \((E d g e(v e r t e x\), nearest \([0]))\)
        else
            returnable \(\leftarrow-1\)
        return returnable
    end function
```

max edge weight and the initial or min edge weight is also included as we can infer via the triangle inequality that between these two vertices there is an edge of at least this length. In the event that the vertex does not have enough neighbors then -1 is returned as a sentential value signifying that there is not a route from the vertex to a satisfied state.

The enforcers_satisfied function sets and checks the next state's policies. The mbr policy requires the row and column values for the new state to be set, lines $2-5$. If a policy field isn't present or value is below zero, the policy is set it most lenient settings for the user, lines 6-9. A value of infinite for $m b r$ allows the full spatial region to be explored; k of 0 requires no anonymity. The new state's $k$ policy should be the maximum of preceding state's $k$ and enumerated neighbors $k$, line 10 . While the $m b r$ policy should be the minimum of the preceding state's $m b r$ and enumerated

```
Algorithm enforcers_Satisfied
    function ENFORCERS_SATISFIED \(\left(\right.\) state \(_{c}\), state \(\left.{ }_{n}, w\right)\)
    2: \(\quad\) state \(_{n}\). row \(_{\text {lower }} \leftarrow \min \left(\right.\) state \(_{c}\). row \(_{\text {lower }}\), w.row \()\)
    3: \(\quad\) state \(_{n} \cdot\) row \(_{\text {upper }} \leftarrow \max \left(\right.\) state \(_{c} \cdot\) row \(_{\text {upper }}, w\). row \()\)
        state \(_{n}\). col \(_{\text {lower }} \leftarrow \min \left(\right.\) state \(_{c}\). col \(\left._{\text {lower }}, w . c o l\right)\)
        state \(_{n}\). col \(_{\text {upper }} \leftarrow \max \left(\right.\) state \(_{c}\). col \(_{\text {upper }}\), w.col \()\)
        if \((w \cdot m b r==\emptyset) \mid(w \cdot m b r<0)\) then
            \(w . m b r \leftarrow \infty\)
        if \((w \cdot k==\emptyset) \mid(w \cdot k<0)\) then
            \(w . k \leftarrow 0\)
        state \(_{n} . k \leftarrow \max \left(\right.\) state \(\left._{c} . k, w . k\right)\)
        state \(_{n} \cdot m b r \leftarrow \min \left(\right.\) state \(\left._{c} \cdot m b r, w . m b r\right)\)
        valid \(\leftarrow\) state \(_{n}\). row \(_{\text {upper }}-\) state \(_{n}\). row \(_{\text {lower }} \leq\) state \(_{n} . m b r\)
        valid \(\leftarrow\left(\right.\) state \(_{n} \cdot\) col \(_{\text {upper }}-\) state \(_{n} \cdot\) col \(_{\text {lower }} \leq\) state \(\left._{n} \cdot m b r\right) \&\) valid
        return valid
    end function
```

neighbors $m b r$, line 11. The $m b r$ constraint is now checked in lines 12 through 13 . If the difference between the upper and lower rows of the new states spatial region is less than or equal the new state's $m b r$ and the same holds true for the columns then this new state is a valid state.

Algorithm 3 creates a new graph derived from the $G$ with vertex 1 and vertex 2 merged into a single vertex. The set edges of the new vertex are the intersection of the sets of vertex1 neighbors and vertex 2 neighbors. Edge weight is sum of weights from corresponding edge in vertex 1 neighbors and vertex 2 neighbors. Line 4 ensures that the created vertex has a predictable name for access later. Non-mutual adjacent

```
Algorithm 3 get_Merged_Copy
    function GET_MERGED_Copy(G,vertex1,vertex2)
        \(G_{r} \leftarrow\) copy of \(G\)
        Remove vertex 1, vertex 2 , and all associated edges from \(G_{r}\)
        Create \(\min (\) vertex 1, vertex 2\()\) in \(G_{r}\)
        mutual \(\leftarrow\) (neighbors of vertex \(1 \in G) \cap\) (neighbors of vertex \(2 \in G\) )
        mutual \(\leftarrow\) mutual \(-\{\) vertex 1, vertex 2\(\}\)
        for all \(v \in\) mutual do
            \(w \leftarrow\) weight \((E d g e(\) vertex \(1, v) \in G)+\) weight \((E d g e(\) vertex \(2, v) \in G)\)
            Add edge ( \(\min (\) vertex 1, vertex 2\(), v\) ) with weight \(w\) in \(G_{r}\)
        return \(G_{r}\)
10: end function
```

vertices are excluded as neighbors of the merged vertex because their inclusion would violate the $m b r$ policy of the non-mutual neighbor and that of vertex 1 or vertex 2 .

Once the k -clique is found it is generalized via Algorithm which returns an anonymized set mapped to their original messages. First an empty set $V_{i}$ is created for each domain $D_{i}$, lines 3-4. Messages are then iterated through and each of their domain values are added to their corresponding set initialized above, lines 5-7. Next each set $V_{i}$ and its corresponding domain $D_{i}$ are fed to get_generalization which produces a minimal anonymized value $L_{i}$ for each $D_{i}$ given its associated $V_{i}$, lines 8-9. Then the mapped table is constructed via iterating through the original messages and deriving the anonymized version, lines $10-15$. A new message $m_{c}$ is created, it's identifiers set to a suppressed version of the original message $m_{s}$, it's $k$ is set, each domain value $m_{c} . D_{i}$ it's respective anonymized version $L_{i}$, and finally the new message is mapped to the original, lines 11-15 respectively.

```
Algorithm apply_generalization
    function APPLY_GENERALIZATION(C,k)
    2: \(\quad\) returnable \(\leftarrow\) deep copy of C
    3: \(\quad\) for all \(D_{i} \in D\) do
    4: \(\quad V_{i} \leftarrow \emptyset\)
    5: \(\quad\) for all \(m_{c} \in C\) do
    6: \(\quad\) for all \(D_{i} \in D\) do
            \(V_{i} \leftarrow m_{c} . D_{i} \cup V_{i}\)
    for all \(D_{i} \in D\) do
        \(L_{i} \leftarrow\) get_generalization \(\left(V_{i}, D_{i}\right)\)
        for all \(m_{s} \in\) returnable do
        \(m_{c} \leftarrow\) Suppress direct identifiers of \(m_{s}\)
        \(m_{c} . k \leftarrow k\)
        for all \(D_{i} \in D\) do
            \(m_{c} . D_{i} \leftarrow L_{i}\)
            returnable \(\leftarrow\) returnable \(\cup m_{c}\)
        return returnable
    end function
```

The sub algorithm takes in a set of values $V$ and a domain $D$ then produces a single value minimal generalization. As with other algorithms dependent upon the domain it is essentially a piecewise function. Lines 3-12 handle well-ordered domains such as latitude; Lines 13-20 operate on partially-ordered domains such as origin. Currently unordered domains have not been explored, but intuitively it would be the intersection of values in $V$.

```
Algorithm get_generalization
    function GET_GENERALIZATION(V,D)
        returnable \(\leftarrow \emptyset\)
        if \(D\) is well-ordered set then \(e_{\min } \leftarrow V[0] e_{\max } \leftarrow V[0]\)
        for all \(e \in V\) do
            \(e_{\text {min }} \leftarrow \min \left(\right.\) ordinal \(\left.\left(e_{\text {min }}, D\right), \operatorname{ordinal}(e, D)\right)\)
            \(e_{\max } \leftarrow \max \left(\operatorname{ordinal}\left(e_{\max }, D\right), \operatorname{ordinal}(e, D)\right)\)
        if \(\operatorname{ordinal}\left(e_{\min }, D\right) \neq \operatorname{ordinal}\left(e_{\max }, D\right)\) then
        \(\min \leftarrow \operatorname{string}\left(e_{\min }\right)\)
        \(\max \leftarrow \operatorname{string}\left(e_{\max }\right)\)
        returnable \(\leftarrow \min +\operatorname{string}(-)+\max\)
        else
            returnable \(\leftarrow \operatorname{string}\left(e_{\max }\right)\)
        if \(D\) is partially-ordered set then
        returnable \(\leftarrow \emptyset\)
        for all \(e_{0} \in V\) do
        for all \(e_{1} \in V\) do
            if returnable \(==\emptyset\) then
                returnable \(\leftarrow L C A\left(e_{0}, e_{1}, D\right)\)
            returnable \(\leftarrow L C A\left(L C A\left(e_{0}, e_{1}, D\right)\right.\), returnable \()\)
        return returnable
    end function
```

As the well-ordered domains can contain elements of any type, we look at the ordinal position of each value in $V$ to determine the minimum and maximum values, read lowest and highest ordinal, lines 4-6. If the values are different then the generalized resultant should be the interval [minimum-maximum], else it is merely maximum. Please note that interval is inclusive and the choice of maximum over minimum for the else section is arbitrary.

The hierarchal or partially ordered domains are generalized by comparing every value in $V$ to every other value in $V$ and determining the their overall lowest common ancestor (LCA). This is a brute force approach, two values $e_{0}, e_{1}$ are chosen from $V$, $L C A\left(e_{0}, e_{1}\right)$ is calculated and stored $L C A_{p}$, line 18. Then two more values $e_{0}, e_{1}$ are chosen from $V$ and $L C A_{p} \leftarrow L C A\left(L C A\left(e_{0}, e_{1}\right), L C A_{p}\right)$, repeat until $e_{0}$ and $e_{1}$ have assumed all values in $V$.

### 3.2 End-user

Privacy erosion, power consumption, and network data costs are concerns of the end-user. The more information a user yields in exchange for a service the greater the loss of privacy. As the end-user operates a mobile device power consumption of processing large result sets from an LBS provider is detrimental to service adoption. In addition to the power consumption, transferring large results sets incurs carrier charges in most cases.

Each LBS provider maintains a unique policy of manging user data. Policies are malleable to business interests and not user-centric. Monitoring the various terms and
conditions an end-user's data is subject to for changes is a difficult task. Specifying their own policy and anonymizing their data reduces the privacy risk incurred by an end user.

Battery life and data usage are concerns for the typical mobile end-user. Processing and data usage increase with the size of the LBS anonymous query resultant. The size of the query resultant increases as the generalization of query parameters increases. Filtering the resultant at the anonymization service reduces both processing and data usage at the end-user.

### 3.3 LBS Provider

LBS providers feature varied services dependent upon the user's location. Detailing their inner workings is difficult due to the diversity of services offered and their associated operating domains. In general LBS providers take a user's GPS coordinate, process it, and yield a result. The processing may range from querying a database, Location Based Access Control enforcement. The results of processing may be returned to the user, passed to another service, and/or simply stored.

In order for these systems to work with our or any Anonymization Service they must be able to process anonymous queries. In a nut shell, instead of receiving exact latitude longitude coordinates the LBS provider will receive a spatial region containing the user's location. The greater the area given to the provider the larger the result and computer cycles consumed.

Thanks to the advertisement driven business model pushed forward by Google and Android, user information is monetizable. As stated previously the LBS provider may send the query containing user information to a third party service such as advertisers,
over which the end user has no control of data usage, save for those protections offered by impermanent terms and conditions of the LBS provider. Therefore the LBS provider is considered Semi-Trusted.

## Chapter 4

## IMPLEMENTATION

We have implemented our proof of concept Anonymization Service in Java. In order to support the varied quasi-identifier domain types the Anonymization Service is comprised of three packages: Anonymizer, Domains, and Enforcers. Please see Figure 4.1 for the dependencies amongst the components. Currently the service supports both NLBS and LBS quasi-identifier domain types, namely: age, gender, origin, race, latitude, and longitude. In addition to this the policy values of $k$ and mbr are also supported. The components are discussed in further detail below.

The Anonymizer uses Jgrapht [1] for pending message graph manipulation. As messages arrive their difference to all other pending is calculated using the Domains module. While searching for a suitable generalization group the Enforcers component is used to determine next state validity and satisfaction. Once a group has been discovered and the anonymized data determined the Anonymizer produces the anonymous queries, maintains a mapping for to the original queries, and removes the group from the pending messages graph. In addition, messages outside the spatial region of the Anonymizer are filtered out prior to processing.


Figure 4.1: Package Diagram

The Domains handle the quasi-identifiers and use a different underlying data structure for each domain type. Partially ordered NLBS domains are represented by a hierarchy backed by Jgrapht [1]. Well ordered LBS domains are Lists. The unordered domain data would be managed by a SET type structure. This component generates the difference measure for NLBS domains by looking for the LCA of the two corresponding message components. LBS by getting the ordinal value difference between the corresponding components. Unordered domains by comparing the intersection and union of corresponding component. The weights are a function of the number of possible generalization any initial value may yield. Please see algorithms 2 and 3 for further details.

The Enforcers manage the $k$ and mbr policies. $k$ is understood to be required size of anonymized group while $m b r$ is a cap on the spatial area encompassing the group. As new members are incorporated the groups policy values adjust such that the combined group policy will satisfy all individual member policies. Intuitively selecting the largest $k$ and smallest $m b r$ of the group accomplishes this goal.

As this implementation is focused on the Anonymization Service extension we have a wrapper application that reads in user queries from a file and then calls the Anonymization Service to process. The Anonymizer itself is headless and returns the generalized queries mapped to the original to the wrapper application which in turn writes them to a file. As is show in Figure 4 the queries are structured as name, age, gender, race, origin, latitude, longitude, row, col, k, mbr, pseudonym, arrival time, anon start, wait time. The fields of row, col, pseudonym, arrival time, anon time, and wait time are added by the Anonymization Service and are not present in the original query from the user. The Anon start field shows when the message began
anonymization processing.


Figure 4.2: Incoming Queries


Figure 4.3: Outgoing Queries: part 1

Figure 4.4: Outgoing Queries: part 2

The generalized queries presented in Figures 4.3 and 4.4 follow the same message structure as the originals but with different values representing the group instead of the individual with the exception of name and pseudonym. The name receives the originals pseudonym and the pseudonym value get ":ALIAS" appended to it signifying it is the anonymized version of the original message with the pseudonym matching that which immediately precedes ":ALIAS". The fields released to an LBS provider from the anonymized query are only name, age, gender, origin, latitude, and longitude. These fields have had their original values replaced by the k-anonymous equivalents for their grouping. The remaining fields are present for internal use and in the event of a LBS provider returning a result, to link the anonymous query response with the correct user.

Each query contains its own policy values that are parsed from the message by the Anonymizer so there is no need to call the Anonymization Service with different arguments per message. Global values such as spatial region boundaries and row/col

```
Latit̄udel = 36.92940
Latitude2 = 38.30810
Longitude1 = -123.001200
Longitude2 = -121.217000
#NOTE latitude corresponds to rows a.k.a. y and longitude
#How to split this spatial region into quadrants
#if more than one is specified then the order of
#precedence is
# row_count, col_count
# --row_distance, col_distance
# --May want to support quadrant_area, quadrant_count
#Specify the number of rows and cols the grid supports
Row_Count = 358
Col_Count = 270
```

Figure 4.5: Global Configuration
number are set in a configuration file. Figure 4.5 illustrates the mentioned configurations. Changing these values while the Anonymization Service is running will cause indeterminate errors as the user's coming in after the changes will have a different spatial domain than those already present. Privacy erosion may occur from improperly formed k -anonymous groups as result of the old spatial domain users being combined with users from the new spatial domain.

## Chapter 5

## EVALUATION

We evaluated the performance of the Anonymization Service on simulated user data. The data is based upon the Mill Avenue demographic data provided by the City of Tempe in $[20 ; 21]$ as well as movement traces generated by Thomas Brinkhoff's simulator $[7 ; 6]$. The box is a 64 -bit Windows 7 Professional SP 1 Intel Core i7-2620M @ 2.70 GHz with 4 GB of RAM.

A single large set of movement traces $(100,000)$ was generated with Brinkhoff's simulator in sections of about 10,000 that were combined. Demographic data of Mill Avenue district was then incorporated into the traces, yielding a large set of user movement and demographic information. The policy values were normally distributed with a standard deviation of one about three for $k$ and 36 for $m b r$. The policy values are modified at runtime by the addition of an test specific offset to the initial value. A total of 49 tests were performed such that every value of $k_{\mu} \in\{3,4,5,6,7,10\}$ was paired with every value of $m b r_{\mu} \in\{36,72,108,144,180,216,252\}$. The mean and standard deviations are derived from the union of above stated test resultants. From the combined data set we evaluate the affect of various policy pairings on time,Tables A. 5 and A.6, as well as information loss, Tables A.1, A.3, A.2, and A.4.

### 5.1 Affects of Policy on Time

The nature of LBS necessitates a rapid turnaround time from query to result, we have settled on five seconds an allowable delay threshold. Figure 5.1 shows the mean turnaround time for queries with the given $k, m b r$ pair on a linear scale while


Figure 5.1: Average Turnaround Time (sec)


Figure 5.2: Log Average Turnaround Time (sec)

mbr
Figure 5.3: Average Processing Time (ms)

Figure 5.2 is on a logarithmic scale. We observe in Figure 5.1 that turnaround time grows exponentially longer as the $m b r$ approaches its minimum value of 36 . As the $k$ value increases we again see an exponential growth in turnaround time. Figure 5.2 illustrates that $k$ values up to seven with the largest $m b r$ and $k$ up to five for the smallest mbr meet the turnaround time QoS threshold of five seconds. We evaluate the components, processing and waiting time, of turnaround time next to determine the influence of each on the overall time taken.

### 5.1.1 Affects of Policy on Processing Time

Processing time, generating a generalization once enough users are present, is dependent upon algorithm used for generalization. Figure 5.3 illustrates that $k$ values up to 11 with the largest $m b r$ and $k$ up to seven for the smallest $m b r$ meet the turnaround time QoS threshold of five seconds. The $k$ value has greater influence over the processing time than $m b r$; processing time is exponentially related to $k$. We can also see that on average in the worst case the processing time was under 40


Figure 5.4: Average Wait Time (sec)
seconds while the turnaround time in an equivalent scenario is around 25,000 seconds. Overall the contribution of processing time to turnaround is minuscule.

### 5.1.2 Affects of Policy on Waiting Time

Waiting time, waiting on enough users to be present for generalization, is dependent upon the Anonymization Service's environment, specially the rate at which users fall into each other mbr. Figure 5.4 shows the mean wait time for queries with the given $k, m b r$ pair on a linear scale while Figure 5.5 is on a logarithmic scale. We observe in Figure 5.4 that turnaround time grows exponentially longer as the mbr approaches its minimum value of 36 . As the $k$ value increases we again see an exponential growth in wait time. Figure 5.5 illustrates that $k$ values up to seven with the largest $m b r$ and $k$ up to five for the smallest $m b r$ meet the turnaround time QoS threshold of five seconds. We can also see that on average in the worst case the waiting time was around 25,000 seconds while the turnaround time in an equivalent scenario is around 25,000 seconds. Waiting time has a major effect on the overall


Figure 5.5: Log Average Wait Time (sec)
turnaround time.

### 5.2 Affects of Policy on Information Loss

In current LBS k-anonymous systems the information released to the LBS provider is quasi-identifier of location. The proof of concept is capable of releasing location as well as other demographic information. The graphs presented below show the amount of information lost by generalization for the given $k, m b r$ pairs. If all quasi-identifier information except location is suppressed then information loss is .66 . We observe in Figure 5.6 that information loss grows to an asymptote exponentially as $k$ increases linearly. While information loss grows linearly as mbr grows linearly. As the user is composition of differing quasi-identifiers types we will next examine the effects of policy on NLBS and LBS.


Figure 5.6: Average Overall Percent Generalized


Figure 5.7: Average NLBS Percent Generalized

### 5.2.1 Effects of Policy on NLBS Information Loss

The NLBS information of the user is composed of age, gender, origin, and race, Figures 5.8 through 5.11 respectively. These graphs all grow exponentially to an asymptote as $k$ grows linearly and show no change as the $m b r$ varies. The combined effect is presented in Figure 5.7 that also follows this trend. This signifies that demographic information preservation is not dependent upon the $m b r$ policy.


Figure 5.8: Average Age Percent Generalized


Figure 5.9: Average Gender Percent Generalized


Figure 5.10: Average Origin Percent Generalized


Figure 5.11: Average Race Percent Generalized


Figure 5.12: Average LBS Percent Generalized

### 5.2.2 Effects of Policy on LBS Information Loss

The LBS information of the user is composed of latitude and longitude Figures 5.13 through 5.14 respectively. These graphs all grow exponentially to an asymptote as $k$ grows linearly and shows linear growth as the $m b r$ grows. The combined effect is presented in Figure 5.12 that also follows this trend. This signifies that location information preservation is dependent upon both policy values. The mbr policy field limits the maximum spatial area of a group hence the asymptotic nature with respect to $k$, limiting the $m b r$ value has a greater effect on the location information loss than changes $k$.

### 5.3 Discussion

The approach suffers a performance hit as the number of waiting users increases; primarily due to the underlying graph data structure. As the number of users waiting increases the nodes present in the graph also increasing leading to processing overhead

k
Figure 5.13: Average Latitude Percent Generalized


Figure 5.14: Average Longitude Percent Generalized
for the graph dependent operations throughout our algorithm. Implementing an expire time for a message would help to solve this problem as well as communicating the anonymization fail to the user. Along the performance limitations vein is the use of $A^{*}$, as in the worst case it will end up brute-forcing the solution.

We implicitly trust the user and as such the Anonymization Service is vulnerable to crafted message attacks possibly resulting in erosion of privacy and or denial of service. If an attacker was to flood the Anonymization Service with messages with relatively small $k$ and large $m b r$ they will deny the anonymization of legitimate request because the Anonymization Service. If an attacker was to flood the service with messages with demographic $D$ and message Content $C$ any resultant of the Anonymizer can be used to infer more information about legitimate traffic than expected. This is possible as the attacker can compare their input for a given time period with the anonymized output of the Anonymization Service remove all those with $C$ and infer what would be required to make $D$ match the demographic of the output of the Anonymization Service.

Regardless of the pitfalls of the approach it is able to handle the combined nature of M-Commerce Anonymization meeting time QoS while allowing for up to $80 \%$ privacy. We attempted to use open source k-anonymization package Incognito described in [16] but it failed to satisfactorily solve the problem. Failure due to processing time or inability to handle quasi-identifiers with more than a single generalization path.

## Chapter 6

## CONCLUSION

We have designed and implemented a proof concept Anonymization Service that unifies traditionally disparate approaches for LBS and NLBS information in the LBS domain. The Anonymization Service utilizes a generic approach towards generalization that is not dependent upon the hierarchical structure of NLBS quasi-identifier domains nor the well-ordered structure of LBS quasi-identifier domains. We used a difference measure and single source clique generation to attain k -anonymity. Currently the Anonymization Service supports the NLBS quasi-identifiers of race, age, gender, and origin; the LBS quasi-identifiers of longitude and latitude; the policy values of $k$ and $m b r$. The increased expressive power of generic view of PII allows for domains such race to be represented more accurately, as an individual may have more than a single race in their immediate heritage. It also allows for the quasi-identifiers regardless of expression; hierarchy, line, or set, to be processed simultaneously.

The generic view of personally identifiable information (PII) as quasi-identifiers as well as the Anonymization Service required to process this view are our contributions. Namely we have designed the following:

1. User-Centric Anonymization algorithm separated from the quasi-identifier domains it handles
2. Difference measure as comparator of people
3. Set based quasi-identifier domains
4. Methodologies for working with Set based quasi-identifier domains
5. Quasi-identifier Domain abstraction/interface
6. User policy unification technique

The implementation of some of the aforementioned designs requires submodules so the implementation list below does not directly reflect the design list above.

1. User-Centric Anonymization algorithm separated from the quasi-identifier domains it handles (Anonymizer)
2. Difference measure as comparator of people (Anonymizer)
3. Quasi-identifier Domain abstraction/interface (Domains)
4. User policy unification technique for $k$ and $m b r$ (Enforcers)
5. Jgrapht based Hierarchical or partially order quasi-identifier domain module. (Domains)
6. List based well-ordered quasi-identifier domain module. (Domains)

### 6.1 Future work

Regarding future work, the unordered set can be implemented into the Anonymization Service. A Framework for the handling for domains may be developed to support the Anonymization Service in generalizing the varied domains and the LBS provider process the anonymous queries composed of the varied domains. The current implemented Anonymization Service can be extended to include LBS provider policy constraints and $l$-diversity [18] and $t$-closeness [17] of query content. As the $m b r$
policy has little influence on the amount of NLBS information lost a different timewindow approach may be taken that is presented in [19]. The approach increases the diversity of users available by retaining a sliding history of users in a given region, the increased diversity should give Anonymization Service more choices within the same region and yield k-anonymous groups with less NLBS information loss.

Beyond performance improvements a distribution and lookup service for the various Generalization Domains needs to implemented. Information required tends to be service type specific so the number of Generalization domains is expected to grow quite large. Aside from the Anonymization Service research needs to continue on the how to best present the policy management to users, educating them on the expected degradation of QoS given their selected policy values.

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## APPENDIX A

DATA COLLECTED FROM EXPERIMENTATION ON SIMULATED USER BASE
mobihide [12]
Table A.1: Average NLBS Generalization Percentage

| k | mbr | Gender | Origin | Race | Age | NLBS |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 1 | 36 | 0 | 0 | 0 | 0 | 0 |
| 1 | 72 | 0 | 0 | 0 | 0 | 0 |
| 1 | 108 | 0 | 0 | 0 | 0 | 0 |
| 1 | 144 | 0 | 0 | 0 | 0 | 0 |
| 1 | 180 | 0 | 0 | 0 | 0 | 0 |
| 1 | 216 | 0 | 0 | 0 | 0 | 0 |
| 1 | 252 | 0 | 0 | 0 | 0 | 0 |
| 2 | 36 | 0.493888996 | 0.602123812 | 0.510543971 | 0.083670074 | 0.422556713 |
| 2 | 72 | 0.496751733 | 0.603997764 | 0.511900635 | 0.083332129 | 0.423995565 |
| 2 | 108 | 0.504160878 | 0.607513322 | 0.514776281 | 0.084087181 | 0.427634415 |
| 2 | 144 | 0.495794393 | 0.609064532 | 0.512699757 | 0.084445505 | 0.425501047 |
| 2 | 180 | 0.501270498 | 0.611254 | 0.51289956 | 0.083613209 | 0.427259317 |
| 2 | 216 | 0.49970548 | 0.607994152 | 0.507301377 | 0.08363392 | 0.424658732 |
| 2 | 252 | 0.505938109 | 0.606273366 | 0.511334336 | 0.084112286 | 0.426914524 |
| 3 | 36 | 0.747693902 | 0.780715011 | 0.704879337 | 0.12487615 | 0.5895411 |
| 3 | 72 | 0.75149294 | 0.781563756 | 0.702294848 | 0.127406732 | 0.590689569 |
| 3 | 108 | 0.754621299 | 0.782584416 | 0.703353885 | 0.125354512 | 0.591478528 |
| 3 | 144 | 0.745835035 | 0.780147589 | 0.703348344 | 0.127073885 | 0.589101213 |
| 3 | 180 | 0.755825821 | 0.78419798 | 0.710146375 | 0.127419016 | 0.594397298 |
| 3 | 216 | 0.758615367 | 0.782143351 | 0.697633992 | 0.12633935 | 0.591183015 |
| 3 | 252 | 0.763678992 | 0.783166614 | 0.703915973 | 0.125930404 | 0.594172996 |
| 4 | 36 | 0.874408651 | 0.863855001 | 0.819491176 | 0.153787061 | 0.677885472 |
| 4 | 72 | 0.881070115 | 0.86130582 | 0.825193105 | 0.156256351 | 0.680956348 |
| 4 | 108 | 0.879739283 | 0.853532104 | 0.815824338 | 0.154541256 | 0.675909245 |
| 4 | 144 | 0.872680124 | 0.862702713 | 0.812250212 | 0.153683398 | 0.675329112 |
| 4 | 180 | 0.887461665 | 0.862284643 | 0.806430018 | 0.153247547 | 0.677355968 |
| 4 | 216 | 0.886194714 | 0.864255622 | 0.802000968 | 0.154449627 | 0.676725233 |
| 4 | 252 | 0.884609457 | 0.856342684 | 0.800011233 | 0.15352642 | 0.673622448 |
| 5 | 36 | 0.935055305 | 0.913516062 | 0.891950581 | 0.181861923 | 0.730595968 |
| 5 | 72 | 0.926387728 | 0.916399222 | 0.870275835 | 0.178914559 | 0.722994336 |
| 5 | 108 | 0.938130363 | 0.91135122 | 0.868398775 | 0.18263147 | 0.725127957 |
| 5 | 144 | 0.927611259 | 0.901643433 | 0.862902254 | 0.17483903 | 0.716748994 |
| 5 | 180 | 0.926928375 | 0.902709232 | 0.863695907 | 0.178806291 | 0.718034951 |
| 5 | 216 | 0.937969431 | 0.898626251 | 0.873641687 | 0.179278807 | 0.722379044 |
| 5 | 252 | 0.941698484 | 0.905740109 | 0.875890194 | 0.179876228 | 0.725801254 |
| 6 | 36 | 0.971963544 | 0.935874205 | 0.906523942 | 0.195815245 | 0.752544234 |
| 6 | 72 | 0.963284441 | 0.936720405 | 0.916721209 | 0.20067008 | 0.754198266 |
| 6 | 108 | 0.962212062 | 0.938200719 | 0.922278728 | 0.191245876 | 0.753484346 |
| 6 | 144 | 0.973938851 | 0.943944397 | 0.930931031 | 0.228160491 | 0.769243693 |
| 6 | 180 | 0.974223326 | 0.948361963 | 0.918386032 | 0.226329878 | 0.7668253 |
|  |  |  | Continued | on next page | 0 |  |
|  |  |  |  |  |  |  |
| 2 |  |  |  |  |  |  |

Table A. 1 - Continued from previous page

| k | mbr | Gender | Origin | Race | Age | NLBS |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 6 | 216 | 0.974677076 | 0.846983136 | 0.920152913 | 0.221673032 | 0.740871539 |
| 6 | 252 | 0.974919942 | 0.947068641 | 0.925416601 | 0.189126137 | 0.75913283 |
| 7 | 36 | 0.98624807 | 0.959078935 | 0.950426094 | 0.211227202 | 0.776745075 |
| 7 | 72 | 0.990130077 | 0.961961816 | 0.939647714 | 0.214700235 | 0.776609961 |
| 7 | 108 | 0.981181796 | 0.954894789 | 0.943771269 | 0.206414587 | 0.77156561 |
| 7 | 144 | 0.978286071 | 0.969164716 | 0.953713712 | 0.200253367 | 0.775354467 |
| 7 | 180 | 0.988707352 | 0.966339046 | 0.946567765 | 0.201762445 | 0.775844152 |
| 7 | 216 | 0.987200805 | 0.847266194 | 0.952533922 | 0.251402713 | 0.759600909 |
| 7 | 252 | 0.988179096 | 0.966912716 | 0.954485896 | 0.212697176 | 0.780568721 |
| 8 | 36 | 0.989310237 | 0.967420006 | 0.97500961 | 0.226941804 | 0.789670414 |
| 8 | 72 | 0.972440244 | 0.97900065 | 0.951234623 | 0.22669132 | 0.782341709 |
| 8 | 108 | 0.993114189 | 0.965959539 | 0.94305225 | 0.215730854 | 0.779464208 |
| 8 | 144 | 0.992521685 | 0.981723015 | 0.963122174 | 0.263360858 | 0.800181933 |
| 8 | 180 | 0.99242818 | 0.975227981 | 0.959362597 | 0.214308173 | 0.785331733 |
| 8 | 216 | 0.998763906 | 0.973062746 | 0.945481684 | 0.193980023 | 0.77782209 |
| 8 | 252 | 0.998148148 | 0.97549264 | 0.960600665 | 0.21850623 | 0.788186921 |
| 9 | 36 | 0.997470489 | 0.977146571 | 0.98777403 | 0.238346648 | 0.800184435 |
| 9 | 72 | 0.986918336 | 0.991525424 | 0.963836672 | 0.239382428 | 0.795415715 |
| 9 | 108 | 0.996551724 | 0.979177719 | 0.958222812 | 0.238256935 | 0.793052297 |
| 9 | 144 | 0.999533147 | 0.992063492 | 0.981064426 | 0.303834579 | 0.819123911 |
| 9 | 180 | 0.992927284 | 0.977803037 | 0.985412522 | 0.311755963 | 0.816974702 |
| 9 | 216 | 0.998756219 | 0.99063644 | 0.979336807 | 0.231170579 | 0.799975011 |
| 9 | 252 | 0.998347107 | 0.98953168 | 0.982691408 | 0.215140937 | 0.796427783 |
| 10 | 36 | 0.997356828 | 0.983700441 | 0.985903084 | 0.246182454 | 0.803285702 |
| 10 | 72 | 0.998392283 | 0.986334405 | 0.985530547 | 0.247604582 | 0.804465454 |
| 10 | 108 | 0.996363636 | 0.98 | 0.987272727 | 0.242256738 | 0.801473275 |
| 10 | 144 | 0.999271137 | 0.991618076 | 0.994169096 | 0.306602253 | 0.822915141 |
| 10 | 180 | 0.999174917 | 0.994636964 | 0.992574257 | 0.240078279 | 0.806616104 |
| 10 | 216 | 0.999019608 | 0.990196078 | 0.989215686 | 0.315435018 | 0.823466598 |
| 10 | 252 | 0.996726678 | 0.989770867 | 0.990180033 | 0.227680958 | 0.801089634 |
| 11 | 36 | 1 | 0.981632653 | 1 | 0.250969131 | 0.808150446 |
| 11 | 72 | 1 | 0.984962406 | 0.984962406 | 0.251445196 | 0.805342502 |
| 11 | 108 | 1 | 0.995867769 | 0.991735537 | 0.258565555 | 0.811542215 |
| 11 | 144 | 1 | 0.996774194 | 1 | 0.252034365 | 0.81220214 |
| 11 | 180 | 0.992125984 | 0.992125984 | 0.992125984 | 0.258576346 | 0.808738575 |
| 11 | 216 | 0.991150442 | 0.982300885 | 0.991150442 | 0.265486726 | 0.807522124 |
| 11 | 252 | 1 | 0.996268657 | 0.992537313 | 0.251856296 | 0.810165567 |
| 12 | 36 | 1 | 1 | 1 | 0.2860134 | 0.82150335 |
| 12 | 72 | 1 | 1 | 1 | 0.252261307 | 0.813065327 |
| 12 | 108 | 1 | 1 | 1 | 0.298157454 | 0.824539363 |
| 12 | 144 | 1 | 0.9375 | 1 | 0.239949749 | 0.794362437 |
| 12 | 180 | 1 | 1 | 1 | 0.257717157 | 0.814429289 |
| Continued on next page |  |  |  |  |  |  |

Table A. 1 - Continued from previous page

| k | mbr | Gender | Origin | Race | Age | NLBS |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 12 | 216 | 1 | 1 | 1 | 0.24321608 | 0.81080402 |
| 12 | 252 | 1 | 1 | 1 | 0.270638909 | 0.817659727 |

Table A.2: Average STD of NLBS Generalization Percentage

| k | mbr | Gender | Origin | Race | Age | NLBS |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 1 | 36 | 0 | 0 | 0 | 0 | $6.29 \mathrm{E}-16$ |
| 1 | 72 | 0 | 0 | 0 | 0 | $6.30 \mathrm{E}-16$ |
| 1 | 108 | 0 | 0 | 0 | 0 | $6.30 \mathrm{E}-16$ |
| 1 | 144 | 0 | 0 | 0 | 0 | $6.29 \mathrm{E}-16$ |
| 1 | 180 | 0 | 0 | 0 | 0 | $6.30 \mathrm{E}-16$ |
| 1 | 216 | 0 | 0 | 0 | 0 | $6.31 \mathrm{E}-16$ |
| 1 | 252 | 0 | 0 | 0 | 0 | $6.31 \mathrm{E}-16$ |
| 2 | 36 | 0.499959537 | 0.392863972 | 0.499867678 | 0.078404718 | 0.144227903 |
| 2 | 72 | 0.499987785 | 0.393471838 | 0.499804963 | 0.077463929 | 0.143790331 |
| 2 | 108 | 0.499933982 | 0.392226635 | 0.499722908 | 0.077588807 | 0.144970911 |
| 2 | 144 | 0.499938882 | 0.391223936 | 0.499755859 | 0.07894809 | 0.146128812 |
| 2 | 180 | 0.499997522 | 0.39179699 | 0.499780379 | 0.078393062 | 0.146798634 |
| 2 | 216 | 0.4999698 | 0.391913579 | 0.499945247 | 0.07774391 | 0.148687942 |
| 2 | 252 | 0.499945812 | 0.391803053 | 0.499867387 | 0.078739465 | 0.150740119 |
| 3 | 36 | 0.434231024 | 0.29957524 | 0.45585525 | 0.087840234 | 0.118121697 |
| 3 | 72 | 0.432081899 | 0.300410314 | 0.457245346 | 0.08727228 | 0.118869844 |
| 3 | 108 | 0.430231986 | 0.302068188 | 0.456776561 | 0.08505019 | 0.119136515 |
| 3 | 144 | 0.435363313 | 0.305510494 | 0.456751332 | 0.086672851 | 0.123455048 |
| 3 | 180 | 0.429503585 | 0.300625776 | 0.453593266 | 0.086173058 | 0.123119112 |
| 3 | 216 | 0.427656383 | 0.301408911 | 0.459113374 | 0.08659315 | 0.125758227 |
| 3 | 252 | 0.423769889 | 0.302394918 | 0.456405595 | 0.086378171 | 0.126523495 |
| 4 | 36 | 0.331357916 | 0.238352696 | 0.384041066 | 0.089542341 | 0.096239235 |
| 4 | 72 | 0.322022646 | 0.23761377 | 0.378607507 | 0.089805839 | 0.092856598 |
| 4 | 108 | 0.324930887 | 0.246488493 | 0.38734144 | 0.088332348 | 0.097325386 |
| 4 | 144 | 0.333316153 | 0.241547278 | 0.390293564 | 0.08929902 | 0.098256831 |
| 4 | 180 | 0.314159655 | 0.239613699 | 0.395071837 | 0.088555564 | 0.097508958 |
| 4 | 216 | 0.315451007 | 0.239386821 | 0.397742782 | 0.08774691 | 0.101356739 |
| 4 | 252 | 0.318727036 | 0.246027065 | 0.39962549 | 0.088440369 | 0.104799312 |
| 5 | 36 | 0.246318863 | 0.192828476 | 0.307419983 | 0.092852646 | 0.075126193 |
| 5 | 72 | 0.258396246 | 0.18890261 | 0.334404697 | 0.094039465 | 0.07982684 |
| 5 | 108 | 0.240835548 | 0.19511258 | 0.337669374 | 0.092555036 | 0.077855439 |
| 5 | 144 | 0.255972051 | 0.207868413 | 0.34150829 | 0.086662698 | 0.084131746 |
| 5 | 180 | 0.257396993 | 0.20155078 | 0.342119897 | 0.090815846 | 0.084633976 |
| 5 | 216 | 0.240690785 | 0.210061444 | 0.332137427 | 0.088674044 | 0.084340724 |
| 5 | 252 | 0.234020692 | 0.205856378 | 0.328890313 | 0.089094666 | 0.084221529 |
|  |  |  | $C 0 n t i n u e d$ | $0 n$ | $n e x t$ | $p a g e$ |
|  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |
|  |  | 0 | 0 | 0 |  |  |

Table A. 2 - Continued from previous page

| k | mbr | Gender | Origin | Race | Age | NLBS |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 6 | 36 | 0.164264163 | 0.168831346 | 0.289948425 | 0.093023341 | 0.062324932 |
| 6 | 72 | 0.185634923 | 0.168667871 | 0.275774739 | 0.092165754 | 0.063758592 |
| 6 | 108 | 0.188637268 | 0.166016848 | 0.266643131 | 0.089718732 | 0.064474732 |
| 6 | 144 | 0.141770014 | 0.144772862 | 0.224432173 | 0.07156474 | 0.05666156 |
| 6 | 180 | 0.140936711 | 0.135019263 | 0.238852447 | 0.071161645 | 0.057031818 |
| 6 | 216 | 0.139665781 | 0.136589578 | 0.236873268 | 0.071172293 | 0.055601981 |
| 6 | 252 | 0.138976738 | 0.136909479 | 0.231849567 | 0.071418766 | 0.057767597 |
| 7 | 36 | 0.114231724 | 0.136426364 | 0.216390885 | 0.090107704 | 0.051031576 |
| 7 | 72 | 0.08043713 | 0.132218463 | 0.237022653 | 0.090568085 | 0.05193208 |
| 7 | 108 | 0.135808735 | 0.143704004 | 0.230306275 | 0.087840671 | 0.054189224 |
| 7 | 144 | 0.11759354 | 0.102478827 | 0.179524591 | 0.068732772 | 0.045358296 |
| 7 | 180 | 0.090965555 | 0.107133228 | 0.191283543 | 0.066173256 | 0.043139484 |
| 7 | 216 | 0.096998671 | 0.107234463 | 0.182109834 | 0.071681622 | 0.048572944 |
| 7 | 252 | 0.092964731 | 0.106241442 | 0.178034214 | 0.066524035 | 0.044312567 |
| 8 | 36 | 0.082742913 | 0.123250215 | 0.155745408 | 0.093596827 | 0.041803316 |
| 8 | 72 | 0.159312328 | 0.099451175 | 0.214569609 | 0.090047916 | 0.054489066 |
| 8 | 108 | 0.06328785 | 0.125410695 | 0.224525156 | 0.08758222 | 0.047844186 |
| 8 | 144 | 0.059767653 | 0.080112018 | 0.160859621 | 0.065904665 | 0.036912009 |
| 8 | 180 | 0.060475549 | 0.092707675 | 0.167442871 | 0.064918896 | 0.037581206 |
| 8 | 216 | 0.017535551 | 0.096285522 | 0.188225583 | 0.064883776 | 0.039982574 |
| 8 | 252 | 0.021436735 | 0.092262404 | 0.163622608 | 0.06437068 | 0.036543825 |
| 9 | 36 | 0.035473329 | 0.104312052 | 0.077223769 | 0.085588906 | 0.030675744 |
| 9 | 72 | 0.109132281 | 0.045241857 | 0.173912955 | 0.096594439 | 0.040152052 |
| 9 | 108 | 0.04137931 | 0.099808244 | 0.195942208 | 0.095676792 | 0.04131766 |
| 9 | 144 | 0.012465948 | 0.035493143 | 0.10816863 | 0.065732807 | 0.023351211 |
| 9 | 180 | 0.059581561 | 0.082723601 | 0.097392453 | 0.060989429 | 0.027389251 |
| 9 | 216 | 0.020323551 | 0.054264545 | 0.114797536 | 0.059619423 | 0.028759724 |
| 9 | 252 | 0.023414357 | 0.040436793 | 0.104948735 | 0.057830998 | 0.028720768 |
| 10 | 36 | 0.051343797 | 0.088792478 | 0.117890599 | 0.086122006 | 0.031087411 |
| 10 | 72 | 0.040064102 | 0.081523303 | 0.119415612 | 0.082676277 | 0.029216422 |
| 10 | 108 | 0.060192529 | 0.09797959 | 0.112095001 | 0.08210199 | 0.032823601 |
| 10 | 144 | 0.01907617 | 0.045002493 | 0.053679162 | 0.043406695 | 0.016642994 |
| 10 | 180 | 0.020294346 | 0.036221498 | 0.060479167 | 0.04444942 | 0.015821227 |
| 10 | 216 | 0.022118655 | 0.048526936 | 0.072635084 | 0.042806572 | 0.02018784 |
| 10 | 252 | 0.040323028 | 0.049524216 | 0.069379766 | 0.044545739 | 0.0203557 |
| 11 | 36 | 0 | 0.094054846 | 0 | 0.093070062 | 0.023192918 |
| 11 | 72 | 0 | 0.085397118 | 0.121702361 | 0.088066517 | 0.028230332 |
| 11 | 108 | 0 | 0.045266327 | 0.090532654 | 0.089994272 | 0.026132452 |
| 11 | 144 | 0 | 0.040031205 | 0 | 0.082608999 | 0.020866786 |
| 11 | 180 | 0.088385608 | 0.06224956 | 0.088385608 | 0.082385024 | 0.031485405 |
| 11 | 216 | 0.093654914 | 0.092392093 | 0.093654914 | 0.088205552 | 0.036853117 |
| 11 | 252 | 0 | 0.04303195 | 0.0860639 | 0.080680546 | 0.032165321 |
| Continued on next page |  |  |  |  |  |  |

Table A. 2 - Continued from previous page

| k | mbr | Gender | Origin | Race | Age | NLBS |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 12 | 36 | 0 | 0 | 0 | 0.092669147 | 0.021479627 |
| 12 | 72 | 0 | 0 | 0 | 0.102856846 | 0.020953404 |
| 12 | 108 | 0 | 0 | 0 | 0.103866104 | 0.025485034 |
| 12 | 144 | 0 | 0.165359457 | 0 | 0.068152543 | 0.04148346 |
| 12 | 180 | 0 | 0 | 0 | 0.096012942 | 0.033783807 |
| 12 | 216 | 0 | 0 | 0 | 0.06456731 | 0.019583789 |
| 12 | 252 | 0 | 0 | 0 | 0.100014197 | 0.026502434 |

Table A.3: Average LBS Generalization Percentage

| k | mbr | Latitude | Longitude | LBS | Overall |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 1 | 36 | 0.002793296 | 0.003703704 | 0.0032485 | 0.001082833 |
| 1 | 72 | 0.002793296 | 0.003703704 | 0.0032485 | 0.001082833 |
| 1 | 108 | 0.002793296 | 0.003703704 | 0.0032485 | 0.001082833 |
| 1 | 144 | 0.002793296 | 0.003703704 | 0.0032485 | 0.001082833 |
| 1 | 180 | 0.002793296 | 0.003703704 | 0.0032485 | 0.001082833 |
| 1 | 216 | 0.002793296 | 0.003703704 | 0.0032485 | 0.001082833 |
| 1 | 252 | 0.002793296 | 0.003703704 | 0.0032485 | 0.001082833 |
| 2 | 36 | 0.044179411 | 0.051990198 | 0.048084805 | 0.297732744 |
| 2 | 72 | 0.085197454 | 0.107779185 | 0.096488319 | 0.314826483 |
| 2 | 108 | 0.128144811 | 0.155699124 | 0.141921968 | 0.332396933 |
| 2 | 144 | 0.167093188 | 0.216943497 | 0.192018342 | 0.347673478 |
| 2 | 180 | 0.199975641 | 0.259064179 | 0.22951991 | 0.361346181 |
| 2 | 216 | 0.243500424 | 0.277357023 | 0.260428724 | 0.369915396 |
| 2 | 252 | 0.281044047 | 0.279808209 | 0.280426128 | 0.378085059 |
| 3 | 36 | 0.060047472 | 0.070309353 | 0.065178413 | 0.414753538 |
| 3 | 72 | 0.116374541 | 0.150497975 | 0.133436258 | 0.438271798 |
| 3 | 108 | 0.178994412 | 0.218487653 | 0.198741032 | 0.460566029 |
| 3 | 144 | 0.229686846 | 0.309795796 | 0.269741321 | 0.482647916 |
| 3 | 180 | 0.277741323 | 0.382503617 | 0.33012247 | 0.506305689 |
| 3 | 216 | 0.347074843 | 0.411635493 | 0.379355168 | 0.520573733 |
| 3 | 252 | 0.418302121 | 0.416983088 | 0.417642604 | 0.535329532 |
| 4 | 36 | 0.068176977 | 0.079346798 | 0.073761888 | 0.476510944 |
| 4 | 72 | 0.133872889 | 0.171199608 | 0.152536248 | 0.504816315 |
| 4 | 108 | 0.208305226 | 0.255083913 | 0.231694569 | 0.527837687 |
| 4 | 144 | 0.266379127 | 0.362541082 | 0.314460105 | 0.555039443 |
| 4 | 180 | 0.323770585 | 0.459986624 | 0.391878605 | 0.582196847 |
| 4 | 216 | 0.407398871 | 0.499396759 | 0.453397815 | 0.60228276 |
| 4 | 252 | 0.495901162 | 0.502186517 | 0.49904384 | 0.615429579 |
| 5 | 36 | 0.072683008 | 0.086224564 | 0.079453786 | 0.513548574 |
| 5 | 72 | 0.144316073 | 0.191185788 | 0.167750931 | 0.537913201 |
| 5 | 108 | 0.225975636 | 0.27991296 | 0.252944298 | 0.567733404 |
|  |  |  | Continued on next page |  |  |

Table A. 3 - Continued from previous page

| k | mbr | Latitude | Longitude | LBS | Overall |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 5 | 144 | 0.282534554 | 0.398098577 | 0.340316566 | 0.591271518 |
| 5 | 180 | 0.350403751 | 0.50247821 | 0.42644098 | 0.620836961 |
| 5 | 216 | 0.439773074 | 0.541407073 | 0.490590074 | 0.645116054 |
| 5 | 252 | 0.550022095 | 0.544650834 | 0.547336464 | 0.666312991 |
| 6 | 36 | 0.076574167 | 0.09000662 | 0.083290393 | 0.52945962 |
| 6 | 72 | 0.153441808 | 0.201062606 | 0.177252207 | 0.561882913 |
| 6 | 108 | 0.236745346 | 0.299676383 | 0.268210865 | 0.591726519 |
| 6 | 144 | 0.266389494 | 0.437538419 | 0.351963956 | 0.630150447 |
| 6 | 180 | 0.397987613 | 0.531673535 | 0.464830574 | 0.666160391 |
| 6 | 216 | 0.495090093 | 0.616008134 | 0.555549113 | 0.679097397 |
| 6 | 252 | 0.591442805 | 0.516434267 | 0.553938536 | 0.690734732 |
| 7 | 36 | 0.078805957 | 0.093481253 | 0.086143605 | 0.546544585 |
| 7 | 72 | 0.158782179 | 0.214482871 | 0.186632525 | 0.579950816 |
| 7 | 108 | 0.243727231 | 0.305184641 | 0.274455936 | 0.605862386 |
| 7 | 144 | 0.267708025 | 0.451801852 | 0.359754938 | 0.636821291 |
| 7 | 180 | 0.360566928 | 0.574081392 | 0.46732416 | 0.673004155 |
| 7 | 216 | 0.51443255 | 0.588970479 | 0.551701514 | 0.690301111 |
| 7 | 252 | 0.614737456 | 0.593654049 | 0.604195752 | 0.721777732 |
| 8 | 36 | 0.080890307 | 0.097297714 | 0.089094011 | 0.556144947 |
| 8 | 72 | 0.164212153 | 0.215501539 | 0.189856846 | 0.584846755 |
| 8 | 108 | 0.253596979 | 0.313274541 | 0.28343576 | 0.614121392 |
| 8 | 144 | 0.289192893 | 0.461465873 | 0.375329383 | 0.658564416 |
| 8 | 180 | 0.368984108 | 0.586513734 | 0.477748921 | 0.682804129 |
| 8 | 216 | 0.533516671 | 0.602100976 | 0.567808823 | 0.707817668 |
| 8 | 252 | 0.641328878 | 0.646261827 | 0.643795352 | 0.740056398 |
| 9 | 36 | 0.082156359 | 0.097858829 | 0.090007594 | 0.563458821 |
| 9 | 72 | 0.169279596 | 0.223956229 | 0.196617913 | 0.595816448 |
| 9 | 108 | 0.255178526 | 0.332841144 | 0.294009835 | 0.62670481 |
| 9 | 144 | 0.295852568 | 0.478713836 | 0.387283202 | 0.675177008 |
| 9 | 180 | 0.431370344 | 0.561018785 | 0.496194564 | 0.710047989 |
| 9 | 216 | 0.546412294 | 0.553997309 | 0.550204802 | 0.716718275 |
| 9 | 252 | 0.64521568 | 0.532429575 | 0.588822628 | 0.727226065 |
| 10 | 36 | 0.084180346 | 0.102264643 | 0.093222495 | 0.566597966 |
| 10 | 72 | 0.171203902 | 0.223490532 | 0.197347217 | 0.602092709 |
| 10 | 108 | 0.26111224 | 0.330572391 | 0.295842315 | 0.632929622 |
| 10 | 144 | 0.292829861 | 0.487112623 | 0.389971242 | 0.678600508 |
| 10 | 180 | 0.4521383 | 0.623288718 | 0.537713509 | 0.716981906 |
| 10 | 216 | 0.5533374 | 0.592509078 | 0.572926409 | 0.739953201 |
| 10 | 252 | 0.645335973 | 0.678008123 | 0.661672048 | 0.754617105 |
| 11 | 36 | 0.083741877 | 0.099168556 | 0.091455216 | 0.569252036 |
| 11 | 72 | 0.17244928 | 0.225814536 | 0.199131908 | 0.603272304 |
| 11 | 108 | 0.265363128 | 0.339087848 | 0.302225488 | 0.641769973 |
|  |  | Continued on next | $p a g e$ |  |  |
|  |  |  |  |  |  |

Table A. 3 - Continued from previous page

| k | mbr | Latitude | Longitude | LBS | Overall |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 11 | 144 | 0.341070463 | 0.455937873 | 0.398504168 | 0.674302816 |
| 11 | 180 | 0.426516518 | 0.594225722 | 0.51037112 | 0.709282756 |
| 11 | 216 | 0.539476937 | 0.655293346 | 0.597385142 | 0.737476463 |
| 11 | 252 | 0.639769032 | 0.681260365 | 0.660514698 | 0.760281944 |
| 12 | 36 | 0.081936685 | 0.088580247 | 0.085258466 | 0.576088389 |
| 12 | 72 | 0.174301676 | 0.239259259 | 0.206780468 | 0.610970374 |
| 12 | 108 | 0.255586592 | 0.328395062 | 0.291990827 | 0.647023185 |
| 12 | 144 | 0.333798883 | 0.450925926 | 0.392362404 | 0.60362426 |
| 12 | 180 | 0.464884278 | 0.552910053 | 0.508897165 | 0.712585248 |
| 12 | 216 | 0.530167598 | 0.705185185 | 0.617676391 | 0.746428144 |
| 12 | 252 | 0.640462889 | 0.705820106 | 0.673141497 | 0.769486984 |

Table A.4: Average STD of LBS Generalization Percentage

| k | mbr | Latitude | Longitude | LBS |
| :---: | :---: | :---: | :---: | :---: |
| 1 | 36 | $2.36 \mathrm{E}-15$ | $3.18 \mathrm{E}-15$ | 0 |
| 1 | 72 | $2.37 \mathrm{E}-15$ | $3.17 \mathrm{E}-15$ | 0 |
| 1 | 108 | $2.36 \mathrm{E}-15$ | $3.18 \mathrm{E}-15$ | 0 |
| 1 | 144 | $2.36 \mathrm{E}-15$ | $3.17 \mathrm{E}-15$ | 0 |
| 1 | 180 | $2.37 \mathrm{E}-15$ | $3.17 \mathrm{E}-15$ | 0 |
| 1 | 216 | $2.36 \mathrm{E}-15$ | $3.18 \mathrm{E}-15$ | 0 |
| 1 | 252 | $2.37 \mathrm{E}-15$ | $3.18 \mathrm{E}-15$ | 0 |
| 2 | 36 | 0.029872769 | 0.040380919 | 0.216126451 |
| 2 | 72 | 0.058154164 | 0.080425915 | 0.214374919 |
| 2 | 108 | 0.087647776 | 0.114967493 | 0.213776471 |
| 2 | 144 | 0.115435818 | 0.156385702 | 0.213435266 |
| 2 | 180 | 0.143352764 | 0.184449644 | 0.212402757 |
| 2 | 216 | 0.173555566 | 0.196750373 | 0.213172638 |
| 2 | 252 | 0.199611685 | 0.199586712 | 0.214204675 |
| 3 | 36 | 0.026946938 | 0.039592989 | 0.176971476 |
| 3 | 72 | 0.053966169 | 0.077682756 | 0.176954218 |
| 3 | 108 | 0.080549079 | 0.110526731 | 0.175727251 |
| 3 | 144 | 0.105157284 | 0.148006958 | 0.179603015 |
| 3 | 180 | 0.135197137 | 0.170958679 | 0.17613253 |
| 3 | 216 | 0.163713361 | 0.18523251 | 0.178986108 |
| 3 | 252 | 0.185536445 | 0.189428563 | 0.176600442 |
| 4 | 36 | 0.023953953 | 0.037744439 | 0.144152269 |
| 4 | 72 | 0.048366956 | 0.071935502 | 0.137078878 |
| 4 | 108 | 0.070187114 | 0.100531525 | 0.142612197 |
| 4 | 144 | 0.092715396 | 0.130307802 | 0.141738341 |
| 4 | 180 | 0.122491008 | 0.143995236 | 0.139735294 |
| Continued on next page |  |  |  |  |

Table A. 4 - Continued from previous page

| k | mbr | Latitude | Longitude | LBS |
| :---: | :---: | :---: | :---: | :---: |
| 4 | 216 | 0.146561895 | 0.159903158 | 0.140012176 |
| 4 | 252 | 0.15784674 | 0.169058763 | 0.140792071 |
| 5 | 36 | 0.022082317 | 0.036702061 | 0.112178297 |
| 5 | 72 | 0.045595212 | 0.065600938 | 0.118272143 |
| 5 | 108 | 0.061255343 | 0.090356038 | 0.113459886 |
| 5 | 144 | 0.086068468 | 0.11304168 | 0.119903177 |
| 5 | 180 | 0.110282355 | 0.122973127 | 0.119163981 |
| 5 | 216 | 0.134295819 | 0.14337753 | 0.11634864 |
| 5 | 252 | 0.131150079 | 0.150881978 | 0.113700952 |
| 6 | 36 | 0.020232234 | 0.036108148 | 0.093197121 |
| 6 | 72 | 0.039038468 | 0.061394942 | 0.093897253 |
| 6 | 108 | 0.056660048 | 0.082478233 | 0.09450942 |
| 6 | 144 | 0.061048345 | 0.080000233 | 0.08036765 |
| 6 | 180 | 0.083926459 | 0.082118275 | 0.079201595 |
| 6 | 216 | 0.093495259 | 0.105401565 | 0.078268292 |
| 6 | 252 | 0.095400047 | 0.111453511 | 0.079213956 |
| 7 | 36 | 0.018954167 | 0.03371069 | 0.07640427 |
| 7 | 72 | 0.037823208 | 0.052475019 | 0.076462317 |
| 7 | 108 | 0.05209244 | 0.077304396 | 0.078869383 |
| 7 | 144 | 0.050467437 | 0.070701091 | 0.061779236 |
| 7 | 180 | 0.072973475 | 0.066376278 | 0.060558915 |
| 7 | 216 | 0.080158177 | 0.087584662 | 0.063644927 |
| 7 | 252 | 0.07313282 | 0.098590345 | 0.058140728 |
| 8 | 36 | 0.016963659 | 0.033235401 | 0.061455718 |
| 8 | 72 | 0.03265702 | 0.050935363 | 0.080065356 |
| 8 | 108 | 0.044223635 | 0.071633105 | 0.070794724 |
| 8 | 144 | 0.047750323 | 0.064442111 | 0.051051205 |
| 8 | 180 | 0.063098999 | 0.058942651 | 0.050910856 |
| 8 | 216 | 0.069705147 | 0.083132787 | 0.056543583 |
| 8 | 252 | 0.061580501 | 0.089622825 | 0.049735502 |
| 9 | 36 | 0.016010949 | 0.032332932 | 0.043856421 |
| 9 | 72 | 0.031618382 | 0.040419278 | 0.058644247 |
| 9 | 108 | 0.040700071 | 0.059781609 | 0.059094547 |
| 9 | 144 | 0.044477086 | 0.053391154 | 0.03202526 |
| 9 | 180 | 0.054863592 | 0.053195501 | 0.036634255 |
| 9 | 216 | 0.053230993 | 0.0661019 | 0.035833067 |
| 9 | 252 | 0.04226616 | 0.073033853 | 0.035603904 |
| 10 | 36 | 0.014607595 | 0.028280949 | 0.04586925 |
| 10 | 72 | 0.02816648 | 0.044627679 | 0.041308571 |
| 10 | 108 | 0.039530678 | 0.059659499 | 0.046008508 |
| 10 | 144 | 0.026959654 | 0.042268527 | 0.021480541 |
| 10 | 180 | 0.035403479 | 0.033531714 | 0.020972049 |
|  |  | $C 0 n t i n u e d$ | on next page |  |
|  |  |  |  |  |
| 5 |  |  |  |  |

Table A. 4 - Continued from previous page

| k | mbr | Latitude | Longitude | LBS |
| :---: | :---: | :---: | :---: | :---: |
| 10 | 216 | 0.043485771 | 0.047432957 | 0.024991763 |
| 10 | 252 | 0.037342133 | 0.055789477 | 0.02634867 |
| 11 | 36 | 0.014940282 | 0.029444483 | 0.03387408 |
| 11 | 72 | 0.027840176 | 0.043017423 | 0.042520415 |
| 11 | 108 | 0.033730503 | 0.054157389 | 0.034607713 |
| 11 | 144 | 0.05182558 | 0.067260299 | 0.023651753 |
| 11 | 180 | 0.068500035 | 0.06989317 | 0.041058177 |
| 11 | 216 | 0.062987855 | 0.095494629 | 0.046124053 |
| 11 | 252 | 0.065988077 | 0.112996498 | 0.031087909 |
| 12 | 36 | 0.016656 | 0.038622199 | 0.023167287 |
| 12 | 72 | 0.031730663 | 0.010102357 | 0.025714212 |
| 12 | 108 | 0.025230026 | 0.046929822 | 0.025965526 |
| 12 | 144 | 0.062911205 | 0.074114572 | 0.047170123 |
| 12 | 180 | 0.020518691 | 0.127989845 | 0.024003235 |
| 12 | 216 | 0.045365055 | 0.068916761 | 0.016139183 |
| 12 | 252 | 0.071896305 | 0.079727567 | 0.025003549 |

Table A.5: Average Generalization Time

| k | mbr | Processing (ms) | Wait (ms) | Turnaround (ms) |
| :---: | :---: | :---: | :---: | :---: |
| 1 | 36 | 0.253107961 | 0 | 0.253107961 |
| 1 | 72 | 0.385436745 | 0 | 0.385436745 |
| 1 | 108 | 0.436895769 | 0 | 0.436895769 |
| 1 | 144 | 0.384693978 | 0 | 0.384693978 |
| 1 | 180 | 0.457208173 | 0 | 0.457208173 |
| 1 | 216 | 0.414558428 | 0 | 0.414558428 |
| 1 | 252 | 0.363879184 | 0 | 0.363879184 |
| 2 | 36 | 1.034204001 | 71.10724862 | 73.17565662 |
| 2 | 72 | 0.889716226 | 17.65871035 | 19.4381428 |
| 2 | 108 | 0.808591556 | 8.136275565 | 9.753458678 |
| 2 | 144 | 0.817483352 | 5.249483993 | 6.884450698 |
| 2 | 180 | 0.766131847 | 3.166394478 | 4.698658173 |
| 2 | 216 | 0.668815354 | 2.005116006 | 3.342746714 |
| 2 | 252 | 0.600261504 | 1.503890122 | 2.70441313 |
| 3 | 36 | 2.622752842 | 491.2808451 | 499.1491037 |
| 3 | 72 | 1.777575999 | 101.591578 | 106.924306 |
| 3 | 108 | 1.640686197 | 44.02348792 | 48.94554651 |
| 3 | 144 | 1.388802097 | 29.65557278 | 33.82197908 |
| 3 | 180 | 1.25650545 | 16.1323912 | 19.90190755 |
| 3 | 216 | 1.129552899 | 8.883940829 | 12.27259953 |
| 3 | 252 | 0.942624115 | 6.311566594 | 9.139438941 |
| 4 | 36 | 7.861772195 | 2620.642722 | 2652.089811 |

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Table A. 5 - Continued from previous page

| k | mbr | Processing (ms) | Wait (ms) | Turnaround (ms) |
| :---: | :---: | :---: | :---: | :---: |
| 4 | 72 | 3.848735484 | 516.6523182 | 532.0472601 |
| 4 | 108 | 3.0873596 | 208.3178708 | 220.6673092 |
| 4 | 144 | 2.960779338 | 136.6210309 | 148.4641482 |
| 4 | 180 | 2.594377712 | 76.3349346 | 86.71244544 |
| 4 | 216 | 2.170910678 | 39.49565474 | 48.17929745 |
| 4 | 252 | 1.847057008 | 27.85817903 | 35.24640706 |
| 5 | 36 | 16.0008502 | 4975.206082 | 5055.210333 |
| 5 | 72 | 7.907684226 | 1277.363616 | 1316.902037 |
| 5 | 108 | 6.257659387 | 650.3863978 | 681.6746947 |
| 5 | 144 | 5.825741153 | 412.5298517 | 441.6585575 |
| 5 | 180 | 5.352661694 | 302.6469925 | 329.410301 |
| 5 | 216 | 3.803288726 | 240.6451942 | 259.6616378 |
| 5 | 252 | 3.44090965 | 165.691438 | 182.8959862 |
| 6 | 36 | 39.18439624 | 11154.63231 | 11389.73869 |
| 6 | 72 | 19.20963306 | 2784.659013 | 2899.916811 |
| 6 | 108 | 15.33742996 | 1309.012925 | 1401.037505 |
| 6 | 144 | 14.63355644 | 5876.529555 | 5964.330893 |
| 6 | 180 | 9.176394143 | 6799.997287 | 6855.055652 |
| 6 | 216 | 7.227602195 | 2913.260082 | 2956.625695 |
| 6 | 252 | 5.770132762 | 2749.907005 | 2784.527802 |
| 7 | 36 | 96.74495333 | 32772.83138 | 33450.04606 |
| 7 | 72 | 49.91560124 | 7349.667159 | 7699.076367 |
| 7 | 108 | 40.8062244 | 3455.635102 | 3741.278673 |
| 7 | 144 | 33.33202057 | 14442.40119 | 14675.72533 |
| 7 | 180 | 27.59577649 | 12782.33535 | 12975.50578 |
| 7 | 216 | 18.92090747 | 4463.778594 | 4596.224947 |
| 7 | 252 | 19.45608758 | 5422.949601 | 5559.142214 |
| 8 | 36 | 1679.925852 | 978787.1481 | 992226.5549 |
| 8 | 72 | 1075.761539 | 178784.0603 | 187390.1526 |
| 8 | 108 | 415.786874 | 51961.94497 | 55288.23996 |
| 8 | 144 | 353.4881488 | 52767.78538 | 55595.69057 |
| 8 | 180 | 175.6996125 | 30451.07756 | 31856.67446 |
| 8 | 216 | 172.4398466 | 14989.92252 | 16369.44129 |
| 8 | 252 | 109.9881292 | 12303.38375 | 13183.28878 |
| 9 | 36 | 4817.977849 | 1738493.364 | 1781855.165 |
| 9 | 72 | 2144.710324 | 294776.8116 | 314079.2045 |
| 9 | 108 | 1602.823674 | 98003.32918 | 112428.7422 |
| 9 | 144 | 1077.050626 | 139474.5241 | 149167.9797 |
| 9 | 180 | 669.4562534 | 87417.98251 | 93443.08879 |
| 9 | 216 | 572.4226274 | 47013.51342 | 52165.31707 |
| 9 | 252 | 317.8748272 | 33705.08005 | 36565.95349 |
| 10 | 36 | 18897.58767 | 3793792.293 | 3982768.17 |
|  |  | Continued $0 n$ |  |  |
|  |  | $n e x t ~ p a g e$ |  |  |

Table A. 5 - Continued from previous page

| k | mbr | Processing $(\mathrm{ms})$ | Wait $(\mathrm{ms})$ | Turnaround $(\mathrm{ms})$ |
| :---: | :---: | :---: | :---: | :---: |
| 10 | 72 | 10043.95016 | 636542.3666 | 736981.8682 |
| 10 | 108 | 4996.109091 | 198419.7745 | 248380.8655 |
| 10 | 144 | 3661.336735 | 506316.7507 | 542930.1181 |
| 10 | 180 | 2422.682343 | 339446.67 | 363673.4934 |
| 10 | 216 | 2026.002941 | 114146.6961 | 134406.7255 |
| 10 | 252 | 1087.738134 | 95157.38789 | 106034.7692 |
| 11 | 36 | 25365.55102 | 7763448.502 | 8042469.563 |
| 11 | 72 | 22049.70677 | 1350612.286 | 1593159.06 |
| 11 | 108 | 14481.07438 | 386818.3719 | 546110.1901 |
| 11 | 144 | 13103.8129 | 299888.5097 | 444030.4516 |
| 11 | 180 | 8890.023622 | 129689.3228 | 227479.5827 |
| 11 | 216 | 8025.814159 | 87438.47788 | 175722.4336 |
| 11 | 252 | 4897.716418 | 43525.90299 | 97400.78358 |
| 12 | 36 | 30238.75 | 24700343.67 | 25063208.67 |
| 12 | 72 | 36885.2 | 5420809.4 | 5863431.8 |
| 12 | 108 | 34520.33333 | 2439758.167 | 2854002.167 |
| 12 | 144 | 26495.25 | 2649958.625 | 2967901.625 |
| 12 | 180 | 18687.14286 | 744434.1429 | 968679.8571 |
| 12 | 216 | 23167.2 | 1146606.2 | 1424612.6 |
| 12 | 252 | 10038 | 455945.7143 | 576401.7143 |

Table A.6: Average STD of Generalization Time

| k | mbr | Processing (ms) | Wait (ms) | Turnaround (ms) |
| :---: | :---: | :---: | :---: | :---: |
| 1 | 36 | 1.96358292 | 0 | 1.96358292 |
| 1 | 72 | 2.463672966 | 0 | 2.463672966 |
| 1 | 108 | 6.96407595 | 0 | 6.96407595 |
| 1 | 144 | 2.421065747 | 0 | 2.421065747 |
| 1 | 180 | 7.108534799 | 0 | 7.108534799 |
| 1 | 216 | 7.268984025 | 0 | 7.268984025 |
| 1 | 252 | 2.406638721 | 0 | 2.406638721 |
| 2 | 36 | 3.719487347 | 106.436722 | 106.7482519 |
| 2 | 72 | 3.617670752 | 38.30682441 | 38.94572032 |
| 2 | 108 | 3.515517126 | 18.12312065 | 19.27781484 |
| 2 | 144 | 3.497862692 | 24.22019289 | 25.17625774 |
| 2 | 180 | 3.359863544 | 7.323607987 | 9.666249746 |
| 2 | 216 | 3.188972511 | 14.06001368 | 15.72182836 |
| 2 | 252 | 3.000066184 | 10.36110726 | 12.51691764 |
| 3 | 36 | 5.46219861 | 486.1441511 | 486.8002156 |
| 3 | 72 | 4.91253134 | 129.1824712 | 129.9679495 |
| 3 | 108 | 10.81349658 | 65.21637617 | 80.74220577 |
| 3 | 144 | 4.426726486 | 59.85512829 | 61.6190609 |
| Continued on next page |  |  |  |  |

Table A. 6 - Continued from previous page

| k | mbr | Processing (ms) | Wait (ms) | Turnaround (ms) |
| :---: | :---: | :---: | :---: | :---: |
| 3 | 180 | 4.216029104 | 32.45819875 | 34.9785983 |
| 3 | 216 | 4.018292871 | 28.36477192 | 30.99786347 |
| 3 | 252 | 3.771688262 | 15.54458977 | 19.02016037 |
| 4 | 36 | 11.7555436 | 2327.472278 | 2332.448005 |
| 4 | 72 | 6.555696393 | 599.4091888 | 601.5902091 |
| 4 | 108 | 6.308084955 | 225.6348567 | 227.2933404 |
| 4 | 144 | 5.873725463 | 203.3397137 | 205.9155451 |
| 4 | 180 | 5.756364907 | 134.9288273 | 137.4534199 |
| 4 | 216 | 11.81120295 | 69.06033019 | 95.54402731 |
| 4 | 252 | 4.20078346 | 72.68672218 | 76.33237478 |
| 5 | 36 | 12.31947936 | 4315.015242 | 4324.069789 |
| 5 | 72 | 8.898819406 | 1079.287748 | 1086.781291 |
| 5 | 108 | 8.137525521 | 690.7118658 | 696.4748134 |
| 5 | 144 | 8.111839097 | 472.8618721 | 479.2947992 |
| 5 | 180 | 8.083011642 | 326.1629259 | 331.752818 |
| 5 | 216 | 6.302196817 | 327.2067377 | 333.594623 |
| 5 | 252 | 15.06562553 | 229.7100048 | 266.0554967 |
| 6 | 36 | 31.09592858 | 8984.498733 | 9009.984668 |
| 6 | 72 | 21.07827864 | 2262.475375 | 2286.991319 |
| 6 | 108 | 15.27196347 | 1171.855929 | 1194.29176 |
| 6 | 144 | 11.99042157 | 656.9965292 | 675.6293582 |
| 6 | 180 | 9.705807591 | 499.625851 | 519.853453 |
| 6 | 216 | 8.132141232 | 362.2641002 | 380.5120251 |
| 6 | 252 | 6.417586185 | 295.7670516 | 310.1288778 |
| 7 | 36 | 60.31352736 | 27180.85195 | 27243.94007 |
| 7 | 72 | 51.05633094 | 6101.114642 | 6183.678841 |
| 7 | 108 | 35.97479528 | 3146.282521 | 3225.490972 |
| 7 | 144 | 30.34159308 | 1361.950305 | 1438.86859 |
| 7 | 180 | 23.62277033 | 1069.752156 | 1142.724437 |
| 7 | 216 | 17.64579192 | 696.7556449 | 759.9480621 |
| 7 | 252 | 12.66920347 | 604.081029 | 646.98553 |
| 8 | 36 | 2520.530606 | 835532.1999 | 840801.0853 |
| 8 | 72 | 2312.331917 | 194882.4664 | 201478.2165 |
| 8 | 108 | 550.5228342 | 77121.66644 | 77440.79628 |
| 8 | 144 | 497.0280835 | 31968.31423 | 32575.5229 |
| 8 | 180 | 177.8764969 | 8664.327803 | 9172.366665 |
| 8 | 216 | 220.1135743 | 10028.40771 | 10757.50193 |
| 8 | 252 | 86.75150511 | 4429.54602 | 4667.988856 |
| 9 | 36 | 9633.880533 | 1912085.452 | 1933059.044 |
| 9 | 72 | 3699.209305 | 346174.2384 | 354408.1009 |
| 9 | 108 | 3025.122322 | 114117.0254 | 121594.4488 |
| 9 | 144 | 2093.647536 | 62072.29003 | 66146.8071 |
|  |  | Continued $0 n$ |  |  |
|  |  | next page |  |  |

Table A. 6 - Continued from previous page

| k | mbr | Processing $(\mathrm{ms})$ | Wait $(\mathrm{ms})$ | Turnaround $(\mathrm{ms})$ |
| :---: | :---: | :---: | :---: | :---: |
| 9 | 180 | 919.4456501 | 24488.50235 | 27501.70007 |
| 9 | 216 | 721.7572108 | 19921.40098 | 22771.01697 |
| 9 | 252 | 345.8375576 | 8202.144088 | 10080.12948 |
| 10 | 36 | 123923.0019 | 3863270.83 | 4102400.798 |
| 10 | 72 | 22467.55516 | 663791.1865 | 745115.1621 |
| 10 | 108 | 8522.548716 | 246403.4048 | 270368.713 |
| 10 | 144 | 9148.616417 | 99927.8812 | 145813.5876 |
| 10 | 180 | 3544.093001 | 36047.32814 | 54862.67139 |
| 10 | 216 | 3363.052701 | 31102.2559 | 55598.35468 |
| 10 | 252 | 1347.924471 | 13173.60608 | 23868.37198 |
| 11 | 36 | 33546.34673 | 7731496.702 | 7806682.297 |
| 11 | 72 | 44791.72897 | 1369108.128 | 1619202.015 |
| 11 | 108 | 24451.48312 | 282780.2884 | 454708.8244 |
| 11 | 144 | 17737.90622 | 281910.0593 | 375078.964 |
| 11 | 180 | 16105.68095 | 197803.2417 | 295600.0617 |
| 11 | 216 | 11673.35685 | 96380.48262 | 199055.6448 |
| 11 | 252 | 6197.944965 | 53055.36202 | 101682.5357 |
| 12 | 36 | 26437.16622 | 11979934.03 | 12084523 |
| 12 | 72 | 29831.88041 | 2993636.811 | 3276795.37 |
| 12 | 108 | 40987.3182 | 825480.8954 | 1134758.627 |
| 12 | 144 | 25538.98752 | 1507622.305 | 1527478.561 |
| 12 | 180 | 24749.616 | 481461.4804 | 759188.3068 |
| 12 | 216 | 22273.11256 | 642202.1161 | 880196.8399 |
| 12 | 252 | 5972.794584 | 273960.1507 | 285853.991 |


[^0]:    Algorithm 4 Difference
    $d(p, q, D)= \begin{cases}\operatorname{index}(p)-\operatorname{index}(q) & \text { if } \mathrm{D} \text { is finite well-ordered set } \& \mathrm{p}, \mathrm{q} \text { non-set elements } \\ h t(L C A(p, q)) & \text { if }|D|>1 \& \mathrm{D} \text { is finite hierarchical } \\ & \text { (partially ordered) set \& p,q non-set elements } \\ 1-\frac{|p \cap q|}{|p \cup q|} & \text { if D is finite unordered set }\end{cases}$

