

An Optimization Model for Timetabling and Vehicle Assignment
for Urban Bus Systems

by

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ABSTRACT

To guide the timetabling and vehicle assignment of urban bus systems, a group of optimization models were developed for scenarios from simple to complex. The model took the interaction of prospective passengers and bus companies into consideration to achieve the maximum financial benefit as well as social satisfaction. The model was verified by a series of case studies and simulation from which some interesting conclusions were drawn.

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CHAPTER 1 INTRODUCTION

1.1 Background

Bus systems play a significant role in many urban areas even those cities that have well developed subway or light rail systems and widespread use of private vehicles. Compared with tracked transportation, bus systems are much more flexible for redesign or adjustment for emergency. For cities that are not suitable for tracked transportation system due to the population and geographical condition, a bus system could be the only viable choice. Psychologically, a self-driven individual vehicle may provide a greater sense of flexibility, safety and security for passengers. On the other hand taking buses costs less and could be more environment-friendly than driving personally. From the view point of the city, successful bus companies bring revenue to the community as well as employment. Moreover a bus system provides mobility for a wide variety of individuals that may not have access to a private vehicle for health or economic reasons. In order to achieve the potential service, economic and environmental advantages it is important that the city administration and bus companies design routes and schedules considering customers' response. A scientific approach to these decisions can improve the system's societal value.

1.2 Problem Overview

The design of a bus system can be classified hierarchically, in both time and space dimensions. The lowest level contains specific designs to static elements on a bus route like the position and facilities of bus stops. The middle level is for each bus route wherein

the path, speed limit, necessary detours, and drivers are determined. The top level is for the integrated system and contains the macroscopic designs like the boundary of the system scope, budget, and administration rules. Such a large and complex dynamic system with multiple scenarios is not easy to manage. However, system engineers and researchers can concentrate on a specific aspect of the system to reach a local optimality that contributes to the global optimality. This approach may not be able to acquire the exact system optimal solution but with careful consideration of interactions it may provide an acceptable solution that can be implemented for bus companies, particularly with an extra level of input and feedback that links the community and iterates between local optimization and global evaluation.

When the concentration is in a specific aspect, like a single route with fixed start and end depots and stops in the middle, the members in this subsystem can be concluded as buses and prospective passengers. Decision makers of the bus company define parameters of running buses according to the resources they have (e.g. budget) and constraints in the reality (e.g. policies or customs), as well as prospective passengers' requirements and preferences.

1.3 Problem Definition

A global optimization model for route-level service and vehicle assignment best introduces the thesis. In this thesis, we investigate the problem of how buses should be allocated to routes: for a given bus system with known demand, costs and routes, the objective is to determine the headways (dispatching time between two consecutive buses) for each route at each time period that minimizes the cost. The number of buses allocated

to the routes will also be found when the headway is determined, based on a maximum utilization principle (See Chapter 3). It is taken into consideration that the running pattern of buses not only affects cost and revenue of the bus company but also the response of prospective passengers. Such comprehensive viewpoint represents the characteristic of this thesis and the unique modeling approach. The thesis attempts to integrate both the policy makers' goals of serving customers and controlling costs while accounting for the realities of human behavior and traffic network dynamics.

1.4 Resource

Valley Metro of Phoenix serves Maricopa County, AZ with over 100 Metro Bus routes. Annual ridership is over 55 million people. Such a large and complex transportation system is an excellent resource of research and provides a basis for examining bus operations.

Valley Metro actually runs in several cities besides Phoenix including Tempe. Tempe has a smaller geographic scale and population than the entire valley and may be considered as an integral unit of the larger system. It is practical that the model built in this thesis is applied to buses in Tempe at first, or even just part of them. Our intent is to use a part of Tempe as a model for the thesis. If the methodology works then the application can be expended for a larger sample, like the case of Phoenix.

1.5 Thesis Structure

The development of the bus scheduling problem and former research achievements are collected and introduced in Chapter 2 as the literature review and a salute to all pioneers

in this area. In the very last part of Chapter 2 the characteristics and meaning of current research of this thesis is also stated. The optimization models are constructed in Chapter 3. The chapter presents the detail mathematical deviation of the basic model and describes why it is reasonable. In Chapter 4 the models are coded in AMPL and tested by case studies where some meaningful conclusions are drawn from the result of calculation. Simulation models are applied in Chapter 5 to explain and verify the case studies in the former chapter. The last chapter is a conclusion with evaluations to the thesis and suggestions for further research.

CHAPTER 2 LITERATURE REVIEW

2.1 Introduction

Bus timetabling is highly related to bus scheduling. The bus scheduling problem, arising in public transport bus companies, addresses the task of assigning buses to cover a given set of timetabled trips with consideration of practical requirements (Bunte and Kliwer, 2009). Bunte and Kliwer (2009) produced an overview and discussed the modeling approaches for different kinds of vehicle scheduling problems and gave an up-to-date comprehensive overview on the basis of a general problem definition. One example of the standard definition of the bus scheduling problem was given in Bunte and Kliwer's paper in Public Transport in 2009:

Given a set of timetabled trips with fixed travel (departure and arrival) times and start and end locations as well as traveling times between all pairs of end stations, the objective is to find an assignment of trips to vehicles such that

- each trip is covered exactly once,
- each vehicle performs a feasible sequence of trips and
- the overall costs are minimized.

Bunte and Kliwer (2009) defined such a class of problems as the Vehicle Scheduling Problem (VSP).

In the last half century researchers made quite a few achievements on solving this problem. In as early as 1954, J. D. Foulkes, W. Prager, and W. H. Warner from Brown University attempted a rational approach to the sequencing problem in which a

mathematical theorem was developed to generate bus schedule plans and minimize the waiting time (Foulkes et al., 1954). The difficulty of solving such problems mainly stemmed from a large set of complex and conflicting restrictions that must be satisfied by any solution (Foulkes et al., 1954). The size of the scheduling problem itself also contributed to the difficulty of obtaining solutions (Gavish et al., 1978).

2.2 Network Flow Methodology

Under most circumstances, the objective of VSP model was to minimize cost. The cost could be operation cost which was from gas, repairing or setup cost like drivers' salary. Typically the cost could be simplified as the fleet size of a transportation system (Salzborn and Franz, 1972). Actually Salzborn's approach to optimum bus scheduling represented one typical theory that vehicle movement was thought of as continuous time dependent flows on the links of a network (Salzborn and Franz, 1972).

The nodes of such a network defined by Salzborn were called control points since these were the positions from which the vehicle flows could be controlled in order to satisfy the transport demands in an efficient way and a depot which was the source of vehicles was connected with each control point. Vehicles in the depot could be idle until they were needed or used immediately. The fleet size at a control point was defined by the maximum difference between the vehicle capacity that had departed from the control point and that had arrived at the control point before time t (Salzborn and Franz, 1972). Since the departure and passenger arrival rate were regarded as continuous functions of time, Salzborn solved the minimum bus fleet size by mathematical derivation including integration of bus departure rate at time t and a defined load factor which was from the

ratio of passenger arrival rate $c(t)$ and bus departure rate $d(t)$. The theory was tested by data of Adelaide, Australia. The passenger arrival rates were rates per minute over a 6-min interval, and the bus departure rates were the quotients of passenger arrival rates and fleet sizes.

A similar but more intuitive case was described in Kliewer (2006) in which the author discussed the multi-depot, multi-vehicle bus scheduling problem (MDVSP) involving multiple depots for vehicles and different vehicle types for timetabled trips. Kliewer used a time–space-based model instead of connection-based networks for MDVSP modeling. This led to a crucial reduction in the size of the corresponding mathematical models compared to well-known connection-based network flow or set partitioning models. The models use a timetable as shown in Figure.2.1 (Kliewer et al. 2006) as input.

Timetable

Trip	From	To	Dep. time	Arr. time	d1	D2
a	k_1	k_2	1	2	+	+
b	k_2	k_1	3	4	-	+
c	k_2	k_1	5	6	+	+

Figure 2.1 An Example for a Timetable (Kliewer et al. 2006).

The trial of Kliewer was successful. The model size had been substantially reduced through aggregation of incoming and outgoing arcs within each station and there was not any loss of generality. Thus, they were able to solve very large practical instances to the optimality through direct application of standard optimization software since the number of variables in the exact optimization model was reduced considerably (Kliewer et al. 2006).

Naumann and Kramkowski (2011) defined waiting time and delay penalty on arcs in a Time-Space Network (TSN) in a new stochastic programming approach for robust vehicle scheduling for public bus transport. The schedule was represented as a TSN with all connecting arcs to enable independent penalization of every connection between two consecutive service trips. Naumann's method significantly decreased total expected costs compared to simply minimizing planned costs and outperformed a simple approach of adding fixed buffer times between service trips. Despite the increased computational complexity, small and medium-sized real-world instances could be solved.

2.3 Mathematical Programming Methodology

The Mathematical Programming approach to bus scheduling problems can be classified into two classes. Some researchers worked on ideas to improve existing algorithms to make it easier to solve mathematical models. For instance Saha (1970) solved a bus scheduling problem by finding the maximum flow through a bipartite graph instead of using the simplex method.

Most researchers concentrated on coming up with applicable models for specific use then solving the model by programming importing data collected in reality (Fügenschuh and Armin, 2009; Rodrigues et al., 2006; Foulkes et al., 1954; Naumann et al., 2011). The size of the problem varied. The number of variables and constraints of Fügenschuh's cases varied from 18,000 to 240,000, but that of Naumann was only 426.

From the view point of this thesis there is interaction between bus system operators and prospective passengers. Therefore special attention was paid to papers that were related

to game theory and customer response. Besides the overview of mathematical theory of games concluding by Lucas (1972), Su (2007) built a game theory model of urban public traffic networks in which a simplified game theory model with three manipulators was proposed for simulating the evolution of the traffic network. The model was based on an empirically investigated urban public traffic network of Beijing. Statistical properties and simulation results showed a good qualitative agreement with the empirical results (Su et al., 2007). It also reflected that such models were difficult to solve mathematically. However if the game theory could be combined with mathematical programming models there would be better solutions that were closer to the mathematical optimum.

At the beginning of the research, the model built for this thesis appeared to have bilevel characteristics.

Bilevel optimization models include two mathematical programs within a single instance, one of these problems being part of the constraints of the other one. In view of this hierarchical relationship, the program as the constraints is called the lower-level problem while another one corresponds to the upper-level problem (Colson et al., 2007). In the end of Chapter 3 there is a trial on bilevel optimization which can be referred to as an example (see Page 34). Related papers were collected and investigated. Marinakis (2006, 2007) came up with a bilevel formulation for the vehicle routing problem and solved the model using a genetic algorithm. It must be pointed out that the model built by Marinakis was slightly different from the standard bilevel models described by Colson (2007). Since the inner mathematical theories used by all OR researchers were relative there was often some similarities. Typically, integer programming was utilized widely in modeling,

including Marinakis (2006) whose bilevel model had two IP-form models as upper and lower levels. IP programming was also utilized by Rodrigues (2004) as well as Fügenschuh (2007, 2009). Especially Ceder (2011) built a mixed integer model for a specific kind of bus scheduling problem which was heuristic that different types of buses were used for different road situation and he had to solve the model in a heuristic way due to the complexity of such a NP complete problem.

The bi-level modeling was abandoned finally because of two main reasons. Firstly there were very few appropriate algorithms for solving bilevel models. Although some researchers, like Xu (2014), had trials on bilevel algorithm, such algorithms were usually quite limited and lacked flexibility. Therefore another trial was made on modeling for the bus scheduling problem for this thesis and an alternative model in nonlinear form was produced.

Representative nonlinear modeling approaches for scheduling problems tried by former researchers were "Scheduling school buses" by Swersey (1984) and "Optimizing Frequencies in a Transit Network: a Nonlinear Bi - level Programming Approach" by Constantin (1995). Constantin considered optimizing the frequencies of transit lines in an urban transportation network as a nonlinear nonconvex mixed integer programming problem but her objective was only to minimize travel and waiting time and never considered cost. Swersey built a nonlinear mixed-integer model to find the minimum number of school buses but he did not consider stochastic arrival of prospective students.

The different characteristics of Swersey and Constantin's work and the points that they ignored in their papers aroused the fundamental thought of this thesis. The approach described in this thesis combined the deterministic modeling aiming at minimizing cost and stochastic arrival of prospective passengers together. We believe this new model reflects the real situation better although there are a few assumptions. The high level approach adopted does not determine detailed tours for individual buses but does assign the number of buses to each prespecified route in order to meet demand at minimal cost.

The validation of the theory in this thesis requires research results of customer demand and response as well as passengers arrival pattern. Mishalani (2006) generated a series of perceptions on passengers waiting time based on empirical results. This study quantified the relationship between perceived and actual waiting times experienced by passengers awaiting the arrival of a bus at a bus stop, and the results indicated that passengers did perceive time to be greater than the actual amount of time waited. In this thesis an equation was cited at first from Larson and Odoni (1981). Larson and Odoni's textbook was classical for urban operations research. It was based on applied probability theory and rigorous mathematical deduction. The equation calculated the expected waiting time at a bus stop which was related to the length of headway.

Luethi (2007) made direct contribution to the modeling in this thesis. Luethi's research evaluated the influence of headway and other factors on passenger arrival rates at public transport stations based on data collected at 28 stations in Zurich's public transport network. This thesis refers to two significant points in Luethi's paper. Firstly it was admitted that most passengers consulted schedules subjectively to reduce their waiting

time. Second, Luethi developed a logarithmic approximation between waiting time and headway which fitted well with his observation. The logarithmic-form equation was implanted into the model of this thesis with a newly estimated coefficient and it simplified the model as well as coding and solution.

Customer demand and preference for urban public transportation have also been investigated by many researchers. Microscopically Strathman (2003) did research on the effects of headway deviation on bus passenger loads based on data of Tri-Met, the public transit system in the Portland area. Strathman stated an important conclusion that in peak hours a delay for 1 minute of a bus led to an increase of 2.6 to the load of a bus, which means in the situation that Strathman investigated, the passenger's arrival rate was 2.6 and this is the most direct reference on passenger's deterministic arrival rate. Macroscopically Frankena (1978) empirically estimated the demand functions for urban bus services in Canada. Based on this he concluded that the quantity of bus service demanded per capita in an urban area depended upon the money and time costs of travel by bus and by average income and other socioeconomic characteristics of the population and geographical characteristics of the urban area (Frankena and Mark 1978). Paulley (2006) also investigated the factors affecting the demand for public transport. While a wide range of factors were examined in the study, the paper concentrated on the findings regarding the influence of fares, quality of service and income and car ownership. Paulley drew a series of interesting conclusions like public transport use was remarkably sensitive to car costs but car use was much less dependent on public transport costs, and the effect of service quality was much less than that of fares. Besides, income and car ownership

growth were fundamental to the underlying demand for public transport and there had been almost continual decline in the demand for bus travel over the past 25 years. A conclusion of Paulley (2006) was introduced and utilized in this thesis. Golob (1972) discussed the structure of a market research study to design an evolutionary public transportation system. Frankena, Paulley and Golob's work helped more on macroscopic research on public transportation planning issues.

Comprehensively, the bus scheduling problem was usually included as a part of urban transportation planning research and analysis, along with vehicle routing problem (VRP) (Fügenschuh et al., 2009). A typical example was the book written by Meyerand Miller (2001) that held the opinion that the major purpose of transportation planning was to inform decision making. Besides transportation issues, the demand for urban transportation also contributed to customer behavioral analysis. The book of Domencich and McFadden (1975) developed a theory of demand, for populations of individual economic consumers.

2.4 Conclusion

The vehicle scheduling problem has been studied by researchers for more than half a century and abundant research results had been produced to assist public transportation administration including planning and operation departments. Besides macroscopic works that were closely related to social science instead of operations research, approaches to this problem were usually based on mathematical optimization models with similar objectives and various constraints.

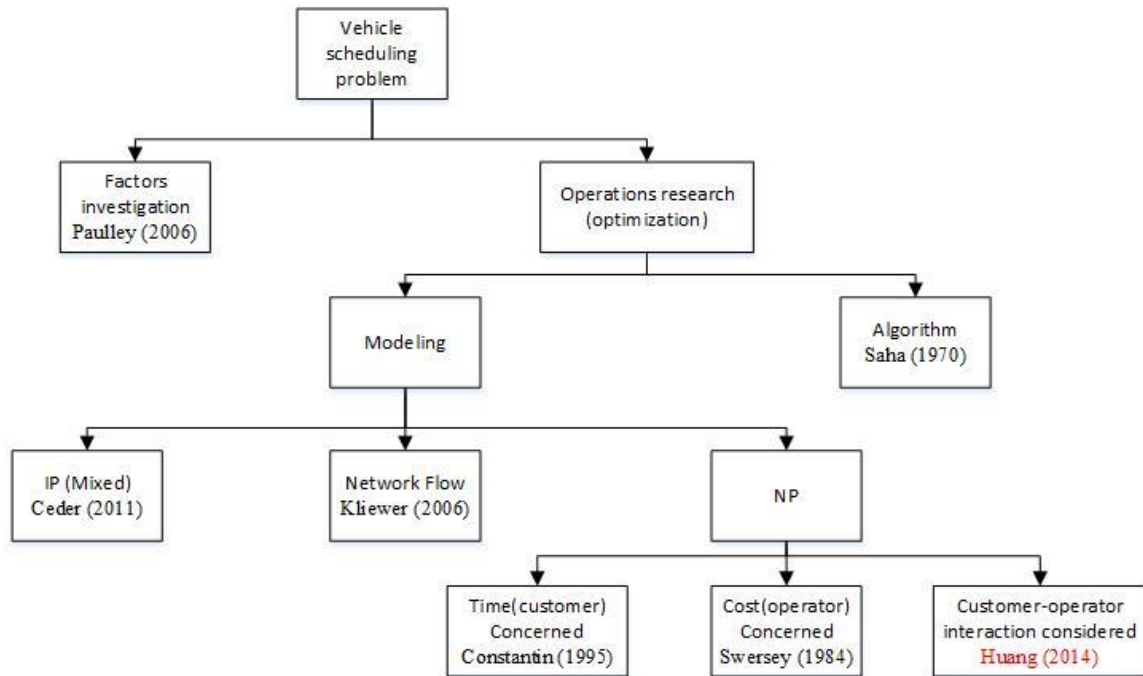


Figure 2.2 Approaches to Vehicle Scheduling Problem

Figure 2.2 is an overview of general classification of approaches to vehicle scheduling. Researchers used to regard passengers and bus companies as isolated factors and they analyzed and made decisions unilaterally ignoring the impact of their decisions on the other agent. Hardly any researchers considered prospective passengers and decision makers of bus companies as interactive factors, however the world is connected and any change may lead to unexpected feedback. In this thesis, the intent is to reflect this characteristic in a simple and computationally feasible way that will allow investigation of alternatives.

CHAPTER 3 PROBLEM DESCRIPTION AND MODELING

Unlike traditional approaches for bus scheduling where the decision makers of bus companies and prospective passengers were regarded as separate factors in the system, the model in this thesis took the interaction of these two factors into consideration. From the bus company's standpoint, besides internal conditions including finance and time limitations, the passengers' satisfaction affects potential customer-preference and revenue. The more frequent the buses run, the more likely someone is to use such public transportation method other than driving. It has been investigated by several former researchers that customers' preference for public transportation is not decided by a single factor like bus frequency but by a comprehensive set of factors including population density, natural conditions and economic level. However the relationship between customer preference and bus frequency cannot be erased. For instance, some students of ASU take the bus Orbit Mars to the main campus in downtown Tempe every day. Since the expected time between two consecutive Mars is 15 minutes, once they miss a bus, they have to take the risk of being late therefore they may turn to cars or bikes. However, if the headway is 5 minutes, they may keep waiting for the next bus. Thus whereas the approach in prior bus scheduling research has focused on allocating specific buses to depots and connecting physical bus moves to cover predefined timetables, the focus in this thesis is on determining the frequency of buses for each predefined route by time of day. Estimations of service time are used to link route frequency to total bus resource needs. Likewise, customer satisfaction and ridership are included as functions of headway.

3.1 Problem Description

For any single bus route, the frequency of buses affects the time that passengers wait at bus stations in a logarithmic rate (Luethi, 2007). Furthermore it affects the tendency that passengers will take a bus to their destination to some degree. Ben-Akiva (2002) made comparisons between customer choice on buses and railways and revealed that lower frequency did harm the interest of taking any public transportation. Similarly, Beirão (2007) compared private vehicles and public transportation and the car users admitted that one of the reasons why they did not take public transport was its low frequency. Therefore it was supposed that the demand for a bus route was a function of its headway.

Such a deterministic function is deduced in Section 3.6. The logarithmic relationship between headway and waiting time was concluded from real data investigation by Luethi (2007) but he did not give out any theoretic proof. It can be deduced assisted by another theory in his paper which will be explained in Section 3.5.

For the bus company, higher route frequency brings more passengers, however, higher cost as well. The tradeoff between losing passengers and reducing cost could reach a balance and they all can be formed into an optimization problem. In addition, since such decisions are made for the common good, social welfare, environmental concerns and other externalities may be important to consider.

Specifically, for a single route, the decision makers of the bus company have to decide the headway--the time between buses dispatched running on the route. The decision is made for each scenario, such as weekdays or weekends and rush hour or non-rush hour.

Furthermore, to consider the problem in a global view, the model can be expanded to multiple routes.

The modeling process below was based on an abstract bus route that was idealized, but followed by a generalization.

3.2 Parameter Definition

Scenario: Bus demand varies with human activity. The 24 hours in a day can be divided into several segments, as well as the 7 days in a week. Bus companies can treat the scenarios differently. Empirically the scenarios can be regarded as separate time periods and these are denoted by subscript t .

Decision variable:

T_{rt} : the headway of buses on route r in time t

Parameters:

r : routes

t : scenarios.

L_r : length of route r

M_{rt} : the total number of buses on route r at time period t

v_{rt} : average bus travel speed at route r in time t

P_t : Length of time period of scenario t

c_r : Unit cost for one trip of buses of route r , for all t . There is not any subscript t for this parameter since it is assumed that for a given route the unit cost remains the same including driver's salary and fuel cost.

s_r : Number of seats of a bus on route r

N_{rt} : Ridership (potential demand) of route r in scenario t

b : Average fare of one bus rider

3.3 Maximum Utilization Principle

In this section an important equation is generated based on and aims at the maximum utilization of buses. It is deduced from a simple scenario and then broadened to generality.

We begin by establishing some basic concepts that are then gradually applied to models of increasing scope. Consider Figure 3.1 showing a basic route consisting of a sequence of bus stops. A bus starts every T_{rt} time. We assume spacing remains constant. The route length is L_r and the travel speed is v_{rt} . Initially assume the bus retraces its steps "out of service" after each run and travels at the same speed. To reach the most utilization of a fixed number of buses so as to minimize rider wait time and maximize route rider capacity, when the backward traveling bus arrives at the start station this bus should be dispatched immediately to begin the next trip. This maintains the bus separation as T_{rt} in time or L_r/v_{rt} in distance. In this deterministic model arrival time between consecutive buses at a stop is always T_{rt} time units.

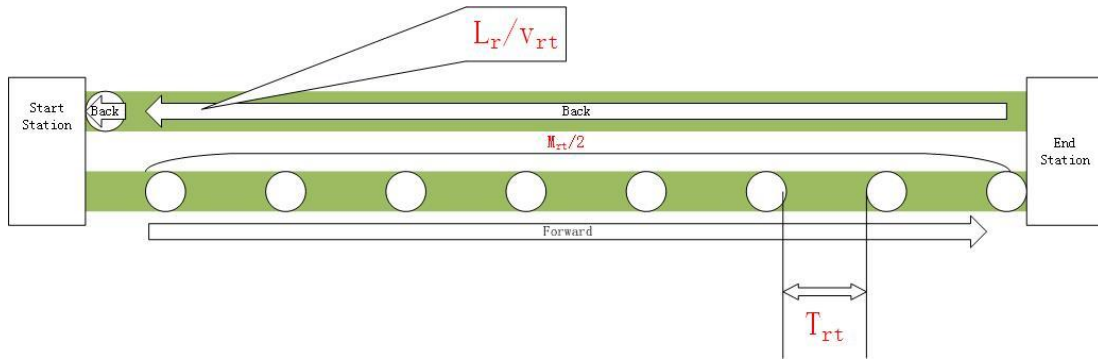


Figure 3.1 Optimal Utilization of Buses on A Single Route

Therefore with no delay at the start and end stations, since the number of buses is equal to the time for a tour ($2L_r/v_{rt}$) times the tour completion rate ($1/T_{rt}$), we obtain Equation (3-1)

$$M_{rt} = \frac{2L_r}{v_{rt}T_{rt}} \quad (3-1)$$

To explain the situation in a more intuitive angle, the time that a bus spends on the service portion of the route equals the summation of the spacings between half of the buses, which leads to Equation (3-2), the basic equation of the Maximum Utilization Principle in this case.

$$\frac{L_r}{v_{rt}} = \frac{M_{rt}T_{rt}}{2} \quad (3-2)$$

Actually Equation (3-1) is the transformed Little's Law for this case. Little's Law which is commonly expressed as $L = \lambda W$, defines the mean relationship between parameters in steady-state queuing systems. For a stable system, the average number of entities L equals to the average entity arrival rate, λ , times the average time an entity spends in the

system, W . Referring to Equation (3-1), L is substituted by M_{rt} , λ is substituted by $(1/T_{rt})$, and W is substituted by $(2L_r/v_{rt})$.

Furthermore the Maximum Utilization Principle can be released to inequality since the idealized case cannot and is not necessary to achieve. Then the principle can be generalized to enable the model to reflect any bus route cases besides the simplest case like Figure 3.1 or Figure 3.2.

3.3.1 Generalization

In reality there always are time delays which can be either short or long for a bus to get ready for next trip. For instance drivers may need time to get prepared and the bus should be refueled and cleaned. Besides, buses may run directly from the end station to the start station without any passengers or stoppings. Assume that the distance for such an empty return is L_r' , then the bus routes may have different shapes as well as L_r 's like Figure 3.2 to 3.4.

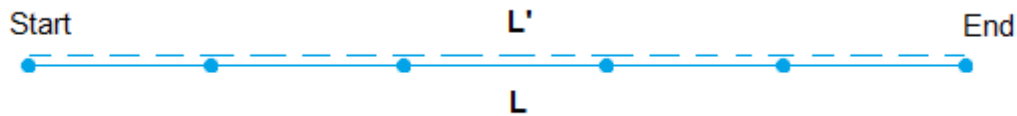


Figure 3.2 Length of Empty Return $L_r' = L_r$

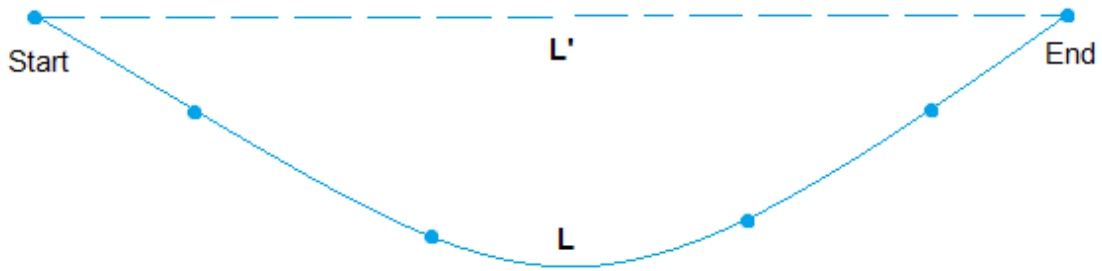


Figure 3.3 Length of Empty Return $L_r' < L_r$

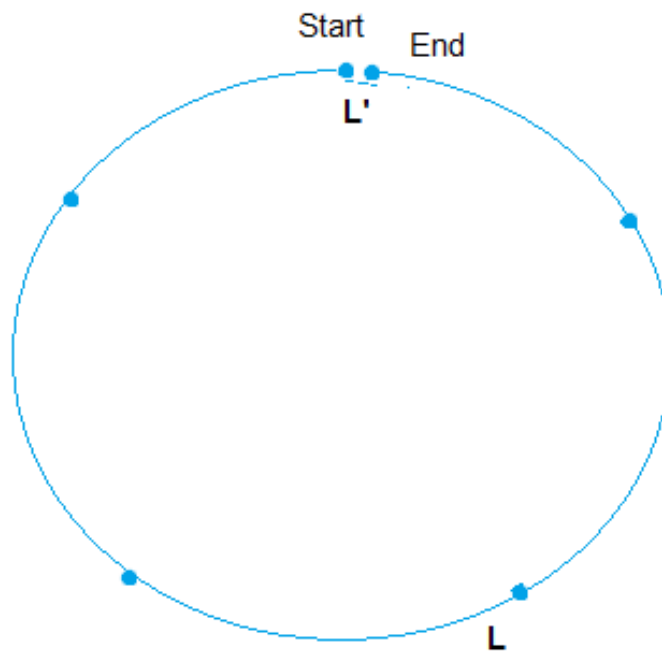


Figure 3.4 Length of Empty Return $L_r' \rightarrow 0$

Furthermore, the travel on L_r' can be characterized by a separate travel time. For each bus therefore there is a ζ_{rt} between completing a trip and the start of the next trip on the same route. Factor ζ_{rt} is for the drivers' preparation and for empty return. Assume that the average speed for trip L_r' is v_{rt}' , then $\xi_{rt} = \frac{L_r'}{v_{rt}'}$.

First rearrange Equation (3-2) into Equation(3-3).

$$\frac{2L_r}{v_{rt}} = M_{rt}T_{rt} \quad (3-3)$$

Noting that the equality requires full activation of the bus, relax Equation (3-3) to an inequality and take the trip L_r' into consideration. This yield:

$$\frac{L_r}{v_{rt}} + \frac{L_r'}{v_{rt}'} \leq M_{rt}T_{rt} \quad (3-4)$$

In the default case, $L_r=L_r'$ and $v_{rt}=v_{rt}'$ therefore $\frac{L_r}{v_{rt}} + \frac{L_r'}{v_{rt}'} = \frac{2L_r}{v_{rt}}$.

When parameter ξ_{rt} is inserted, Equation (3-4) turns into

$$\frac{L_r}{v_{rt}} + \xi_{rt} \leq M_{rt}T_{rt} \quad (3-5)$$

It is not realistic or necessary to reach such a maximum utilization. However, to ensure the continuation of operation, there must be such a constraint that

$$\frac{1}{T_{rt}} \left(\frac{L_r}{v_{rt}} + \xi_{rt} \right) \leq M_{rt} \quad (3-6)$$

for all shapes of bus routes.

3.4 Cost Definition

Then total direct cost of the bus company is defined by the unit cost and total number of trips, which is formulated as

$$Cost = \sum_r \sum_t \frac{P_t}{T_{rt}} c_{rt} \quad (3-7)$$

This is part of the objective function of this model.

3.5 Waiting time

According to Luethi (2007), the median lasting time that passengers wait at the bus station is logarithmically related to bus headway. See Figure 3.5.

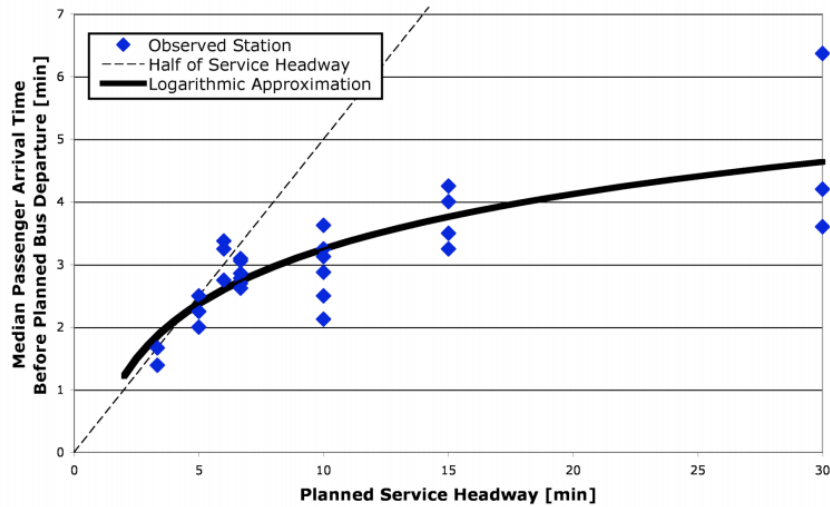


Figure 3.5 Median Passenger Waiting Time versus Headway for Peak Periods in Zurich (Luethi, 2007)

Luethi (2007) did not have any explanation on the logarithmic shape of this relationship curve. In his paper he fitted a curve of the probability density function of timetable-dependent passengers arriving at the bus stop over headway of the bus according to

collected data. Typically when they headway is 10 minutes, the histogram and fitted curve are given as Figure 3.6 and Figure 3.7.

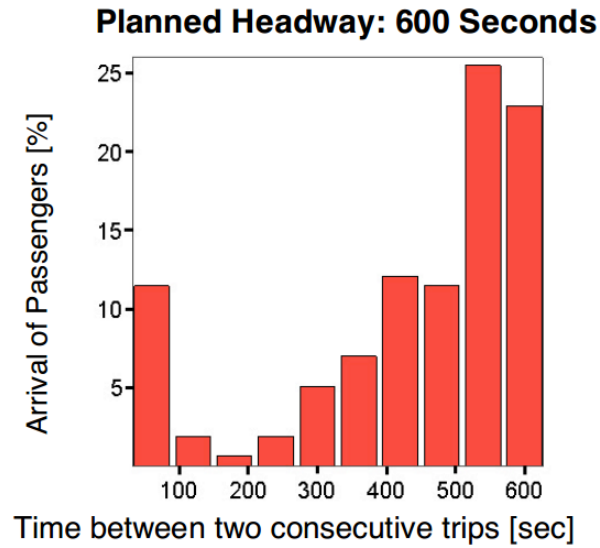


Figure 3.6 Density of Passenger Arrivals at Stops over A Headway Time Period

(Luethi, 2007)

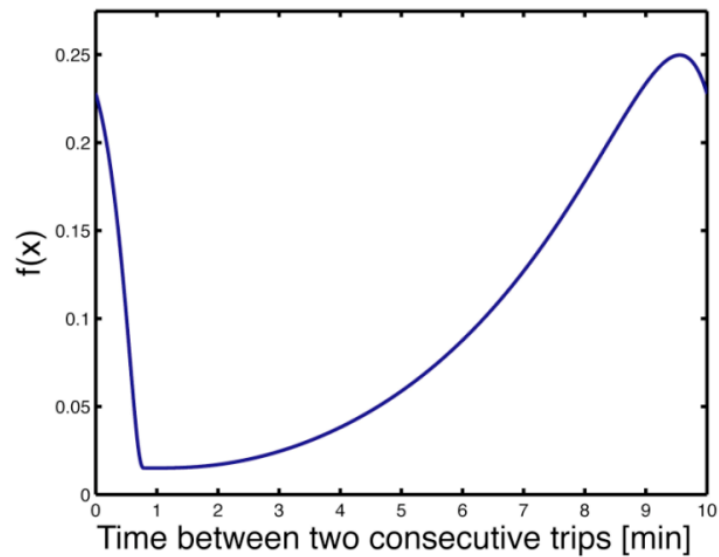


Figure 3.7 The John SB Curve Fitted for Histogram Figure 3.6 (Luethi, 2007)

It is obvious that most people who are dependent on the timetable avoided arriving at the bus stop right after the last bus left although some unlucky ones just missed it. Most customers prefer arriving at the bus stop as close to the arriving time of the bus they planned to take as possible to shorten their waiting time. The longer the headway is, the more obvious this effect is since nobody likes waiting for too long. This explained why the rate of waiting time increasing is slower when the headway is longer. Such an effect resulted in the logarithmic curve like Figure 3.5.

Luethi (2007) did not give out the exact mathematical equation of this logarithmic relation. It is assumed that the average waiting time

$$WT(T) = a \cdot \ln(T) \tag{3-8}$$

Where a is a coefficient to be determined.

By testing the coefficient a , the equation that best fit the graph by Luethi (2007) was found and $a=1.45$.

Table 3.1 shows the fits by points, and it can be observed that Equation (3-8) is quite accurate.

Table 3.1

The Fit of Equation (3-8) to Figure 3.5

Headway (T)	Waiting time, estimate in Figure 3.2 by ruler	1.45 · log(T)
3	1.6	1.6
5	2.4	2.3

7	2.8	2.8
10	3.1	3.3
12	3.7	3.6
15	3.8	3.9
20	4.1	4.3
30	4.8	4.9

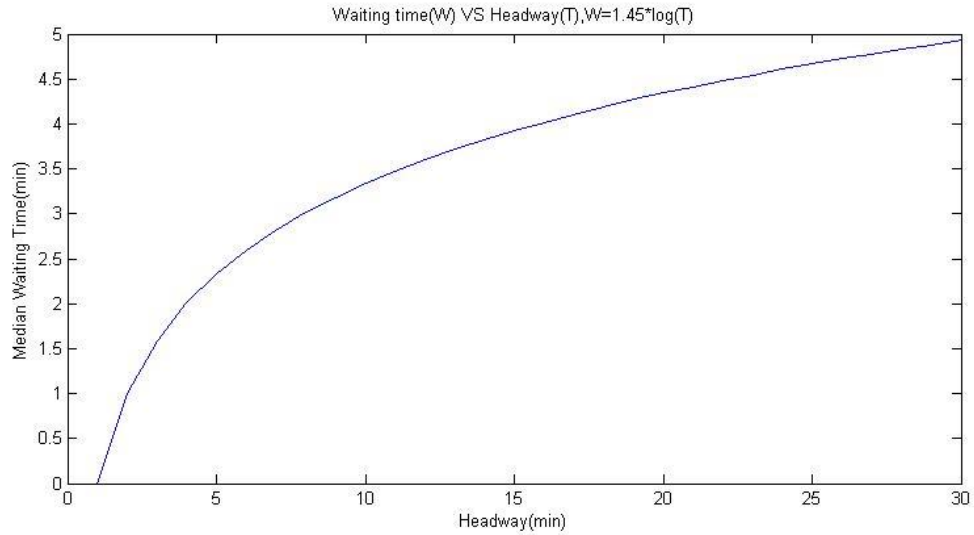


Figure 3.8 Fit The Equation of Luethi (2007)

In line with the desire to ensure adequate service, a constraint is generated to limit the waiting time in our model as follows:

$$WT(T) = 1.45 \ln(T) \leq \varepsilon \quad (3-9)$$

where ε is the limitation reflecting that the bus company does not want their customers to wait for too long. In this case, it is assumed that nominally $\varepsilon = 5$ (minutes). In reality ε should be determined by large scale of social investigation. .

3.6 Ridership and Demand Fulfillment

When service frequency is determined by a bus company, the total capacity, or conveying ability, is determined. From the Annual Ridership Report (ARP) of Valley Metro of Phoenix, it can be known that in every month how many people come to ride the buses. To fulfill the demand, the total capacity should not be less than the total boarding population given by ARP. That is to say,

$$\frac{P_t}{T_{rt}} \cdot \gamma \cdot s_r \geq \sigma \quad (3-10)$$

where $\gamma \cdot s_r$ is the unit capacity of one bus. Setting $\gamma > 1$ considers that there is space for people standing on buses while values less than one would indicate planned empty seats. The parameter σ is an index indicating that how the bus company wishes to fulfill the passenger's requirement. To exactly meet the boarding demand, $\sigma = 1$, to go further, $\sigma > 1$ so that more conveying ability is provided by the bus company.

If the conclusion of Luethi (2007) is combined with that of Paulley et al. (2006), the relationship of ridership and headway can be found. Paulley et al. (2006) did a series of research on the public transportation of England and used the concept of 'elasticity' to represent the relationship between bus demand and the impact factors. For the factor of

average waiting time, the elasticity appeared to be -0.64, which gives the ratio of the proportional change in bus demand to the proportional change in average waiting time.

The real ridership ($N_0=4900$) and headway ($T_0=20$) of Valley Metro can be applied as the basis to build the function of expected ridership, for Route 72, as an example. According to Paulley (2006), there is

$$\frac{\frac{N - N_0}{N_0}}{\frac{WT(T) - WT(T_0)}{WT(T_0)}} = -0.64 \quad (3-11)$$

Since $WT(T) = 1.45 \ln(T)$, Equation (3-11) can be deduced and re-written into equation(3-12), using value of N_0 and T_0

$$N_r = 4900(1.64 - 0.21 \ln(T_r)) \quad (3-12)$$

Equation(3-12) reveals that the real demand of the bus route 72 is actually the function of its headway. Figure 3.9 is the graph of the function.

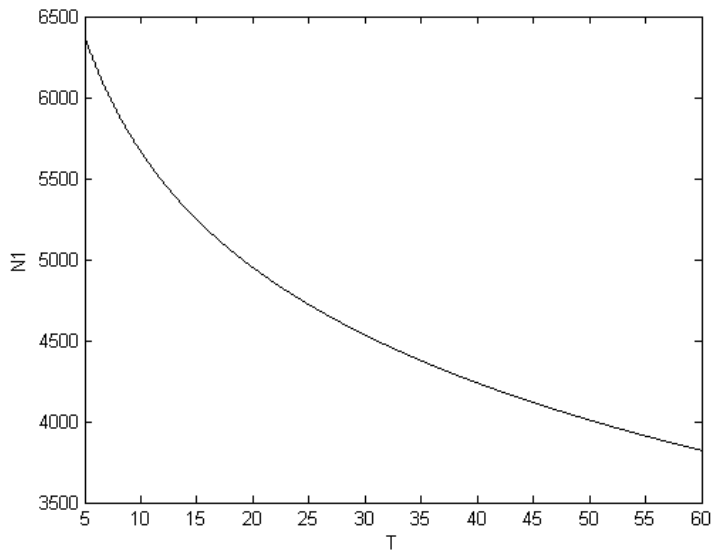


Figure 3.9 Graph of Function (3-12)

The part of $T < 5$ minutes was truncated since it was not realistic for a bus system. It can be observed that the demand decreases when the headway keeps increasing. If the headway is less than 20 minutes, such a high frequency brings more prospective passengers. For headways longer than 20 minutes, the effect becomes weaker and it gets closer to a linear function, like shown in Figure 3.10.

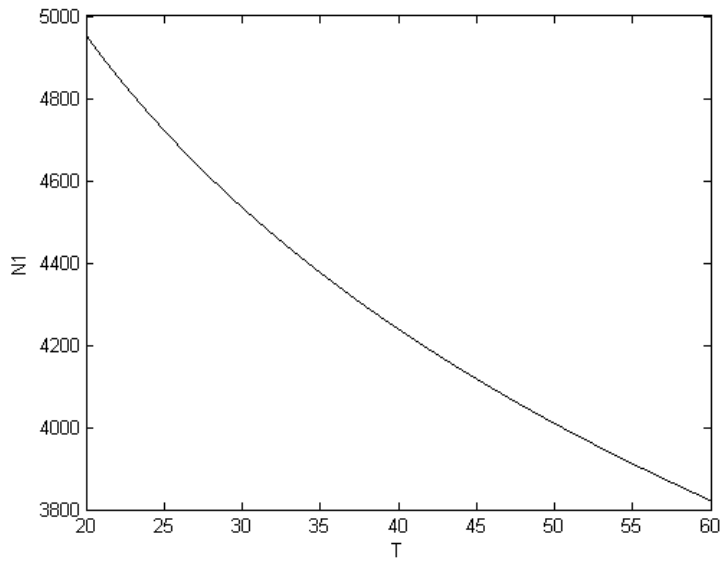


Figure 3.10 Graph of Function (3-12), When $T > 20$ min

To predict the ridership of any routes, the Equation(3-12) can be modified into equation (3-13)

$$N_r = (1.64 - 0.21 \ln(T_r)) N_{r0} \quad (3-13)$$

N_{r0} is the current ridership in the Ridership Report. Equation (3-13) will be used in the modeling for multiple routes.

3.7 Revenue and Profit

If the average price of one bus rider (written as b) is known, the revenue can be acquired combined with the predicted ridership in Equation (3-13), yielding Equation (3-14)

$$Revenue = bN_r \quad (3-14)$$

And

$$Profit = Revenue - Cost = bN_r - \sum_r \sum_t \frac{P_r}{T_r} c_r \quad (3-15)$$

3.8 Integrated Model for Single Route

Initially a simple model was built for a single bus route that headways were to be determined on 7 days in a week.

$$MaxProfit = b \sum_t N_t - \sum_t \frac{P_r}{T_t} c \quad (3-16)$$

Subject to:

$$\frac{L + \xi_t v_t}{v_t T_t} \leq M \quad \text{for all } t \quad (3-17)$$

$$1.45 \log(T_t) \leq \varepsilon \quad \text{for all } t \quad (3-18)$$

$$\frac{\frac{P_r}{T_t} \cdot \gamma \cdot s}{N_t} \geq \sigma \quad \text{for all } t \quad (3-19)$$

Where

$$N_r = (1.64 - 0.21 \ln(T_r)) N_{r0} \quad (3-20)$$

The subscript r is omitted since so far there is only one single route.

3.9 Model Extending—Single Route with Accurate Scenario Definition

Furthermore the operation time of a single bus route can be divided into more scenarios such that the optimization can be pushed forward. Theoretically the bus company can divide the operation time period into hours or even half-hours to control the system most accurately. However this scheme has to be based on extreme accurate statistics (like big data methodologies) on ridership that is of same scale of time dividing thus the problem might even become dynamic, that is to say, the bus service rate will become a continuous function of time like Salzborn (1972) . On the other hand, even if such accurate statistic data can be acquired, it is not that necessary to reach such accuracy. Empirically during weekdays the investigation only in rush hours and non-rush hours are of great value, and even Valley Metro is aware of it thus some routes run with longer headways at night Furthermore rush hours can be divided into morning rush hours and afternoon rush hours. For a city like Tempe which has a considerable population of university students, operation time of related routes can have more intervals to meet the demand at peak hours when students go to classes and back home.

An existing reference is the bus system CyRide of Ames, IA, which has three different headways in a day, 15, 20 and 40 minutes.

Visually an extended model of the characteristics above is the same with the model in section 3.7. The difference is the additional alternatives of parameter t .

Take route No.72 as an example again. Operations time can be divided as:

$t=1$: 5:00am to 7:00am;

$t=2$: 7:00am to 9:00am (Morning Rush Hour);

$t=3$: 9:00am to 2:00pm;

$t=4$: 2:00pm to 3:00pm (Students Peak);

$t=5$: 3:00pm to 5:00pm;

$t=6$: 5:00pm to 7:00pm (Afternoon Rush Hour);

$t=7$: 7:00pm to 11:00pm;

3.10 Model Extension—Multiple Routes

The model can be extended to multiple routes in a district or even all routes operated by Valley Metro of Phoenix or other entity. Parameter r representing routes was implemented into the model in section 3.7. Additionally, special requirements could be reflected in this extended model, such as the limitation on the total number of available buses of several routes M_0 .

$$Max Profit = b \sum_t \sum_r N_{tr} - \sum_r \sum_t \left(\frac{P_{rt}}{T_{rt}} c_r \right) \quad (3-21)$$

where

$$N_{rt} = (1.64 - 0.21 \ln(T_{rt})) N_{rt0} \quad (3-22)$$

Subject to:

$$\frac{L_r + \xi v_{rt}}{v_{rt} T_{rt}} \leq M_{rt} \quad \text{for all } r, t \quad (3-23)$$

$$\sum_r M_r \leq M_0 \quad \text{for all } t \quad (3-24)$$

$$1.45 \log(T_{rt}) \leq \varepsilon \quad \text{for all } r, t \quad (3-25)$$

$$\frac{\frac{P_{rt}}{T_{rt}} \cdot \gamma \cdot S_r}{N_{rt}} \geq \sigma \quad \text{for all } r, t \quad (3-26)$$

$$f\left(\frac{L_r + \xi v_{rt}}{v_{rt} T_{rt}}\right) \geq \delta \quad \text{for all } r, t \quad (3-27)$$

The last constraint (3-27) was for special requirements. For example sometimes it happened that the city regarded one bus route as more important than another due to the population distribution or other reasons. Therefore it was ordered that the number of buses on one route must be more than another route. Thus constraint (3-27) was designed for these constraints but it was not used in the case studies in Chapter 4 since no such information was collected from Valley Metro.

In the model M_0 is the total number of buses distributed to the selected routes by the operating bus company. Buses can be shared between neighbor routes. To provide a better scheduling plan, the number of buses dispatched to each route in each time period M_{rt} was also made into decision variables, which will be coded as a variable in Chapter 4.

As before the constraints ensure all routes have sufficient buses in all periods to achieve their selected headway, the total limit on buses is respected, maximum allowable average waiting time is achieved and buses have sufficient capacity to serve nominal demand.

3.11 Further Expansion

The model could be modified furthermore to include wait time as an objective.

Initially, for a single route, there should be

$$1.45 \log(T_{rt}) \leq \varepsilon \quad (3-28)$$

Then transferred to objective-form:

$$\text{Minimize } \sum_r \sum_t 1.45 \log(T_{rt}) \cdot N_{rt} \quad (3-29)$$

If this objective is implemented to the model, the model becomes a bi-level optimization model. This describes the problem more accurately however it also becomes much more difficult to solve.

Bi-level model:

$$\text{Max Profit} = b \sum_t \sum_r N_{rt} - \sum_r \sum_t \left(\frac{P_{rt}}{T_{rt}} c_r \right) \quad (3-30)$$

where

$$N_{rt} = (1.64 - 0.21 \ln(T_{rt})) N_{rt0} \quad (3-31)$$

Subject to:

$$\text{Minimize } \sum_r \sum_t 1.45 \log(T_{rt}) \cdot N_{rt} \quad (3-32)$$

$$\frac{L_r + \xi v_{rt}}{v_{rt} T_{rt}} \leq M_{rt} \quad \text{for all } r, t \quad (3-33)$$

$$\sum_r M_r \leq M_0 \quad \text{for all } r, t \quad (3-34)$$

$$\frac{\frac{P_{rt}}{T_{rt}} \cdot \gamma \cdot s_r}{N_{rt}} \geq \sigma \quad \text{for all } r, t \quad (3-35)$$

$$f\left(\frac{L_r + \xi v_{rt}}{v_{rt} T_{rt}}\right) \geq \delta \quad \text{for all } r, t \quad (3-36)$$

CHAPTER 4 SOLVER INTRODUCTION AND CASE STUDY

4.1 Solver Introduction

The model was built in AMPL and used the MINOS solver, a linear and nonlinear mathematical optimization solver.

AMPL is the abbreviation of “A Mathematical Programming Language”, which is a practical tool for comprehensive mathematical programming. AMPL was developed by Bell Laboratories of Lucent Technologies. Compared to other programming tools the most significant characteristic of AMPL is that it enables users to describe complex mathematical models by simple algebraic symbols that match with the describing and thinking method of modelers.

AMPL supports mathematical programming models in a mechanism like Figure 4.1. Users edit .txt files for model and input data then save them as model (.mod) and data (.dat) files. After loading the files in AMPL, solvers will be called to solve the model.

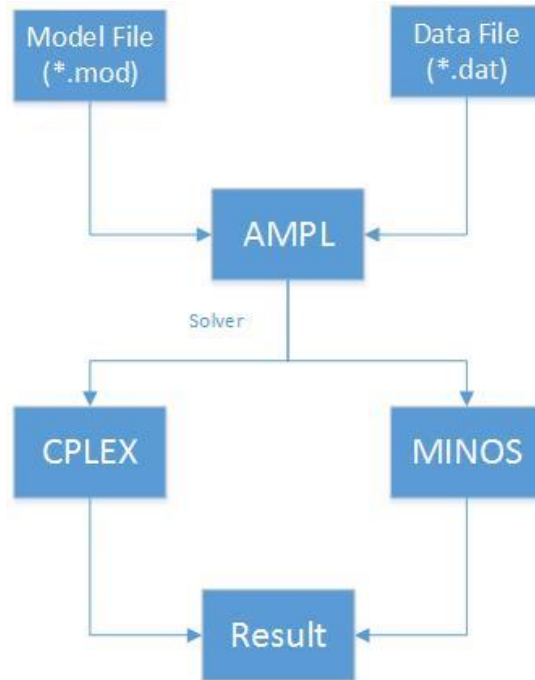


Figure 4.1 AMPL Working Flow Chart

4.2 Assumptions

Before presenting the case study there are a series of assumptions that have to be announced.

Firstly in the case study there is no case of empty returns. Buses run in both directions and always have the same headway.

Secondly, the relaxation of preparing time before bus dispatching remains the same for all buses and drivers.

For more complex cases, the two assumptions above can be made into detail by editing the matrix of ξ instead of using a constant value. See Chapter 3.

Thirdly, the passenger arrival rates during rush hours are twice their arrival rates during non-rush hours. This setting is due to the lack of information on the difference of the two arrival rates therefore they were artificial.

4.3 Single Route Case Study

To verify the mathematical programming model described in Chapter 3, a series of examples were evaluated.

The model in section 3.7 for a single bus route was tested by data of Valley Metro Bus No.72. The scenarios were set and separated as Weekdays and Weekends. The data of the 2- day weekends was from those of Sunday to simplify the testing. The number of buses and cost were artificial and assumed before formal investigation, partially according to the ridership and financial report publicized by Valley Metro. The objective was to minimize the weekly cost of a single bus route.

The model to be solved is in a modified form of the Single Route Model in Chapter 3.

The model can be easily read in Appendix A.

Table 4.1

List of Inputs of Case Study for Single Route No.72

Parameters	Value	Explanation
Route Length	$L=28.1$ (mile);	The operation length of Route 72 from Chandler Fashion Center to Scottsdale Healthcare Thompson Peak.

Scenario 1 Operation Time	$P_1=1140$ (min);	The bus runs for 19 hours on a weekday.
Scenario 2 Operation Time	$P_2= 1020$ (min);	The bud runs for 17 hours on a weekend day.
Operation Cost per Trip	$c=\$30$;	The estimated operation cost per trip includes the cost of salary, fuel and maintaining.
Available Number of Buses	$M=10$;	It is assumed that there are 10 buses distributed to Route 72 independently.
Average Velocity in Scenario 1	$v_1=0.23$ (miles per minute);	The current average speed of the buses during weekdays (Route 72 runs 28.1 miles in 2 hours).
Average Velocity in Scenario 2	$v_2=0.28$ (miles per minute);	The current average speed of the buses during weekends (Route 72 runs 28.1 miles in 1.5 hours).
Seats on the Bus	$s=40$;	The bus type is New Flyer MiDi which has 40 seats.
Capacity Coefficient	$\gamma=2.5$;	The artificial parameter for the bus capacity, considering standing spaces in the bus and the released space when passengers get off the bus.
Satisfaction Coefficient	$\sigma=0.95$;	95% of prospective passengers are served by bus.
Ridership in Scenario 1	$N_1=24000$;	The total ridership in weekdays, since the ridership is 4800 in a single day, according to Ridership Report.
Ridership in Scenario 2	$N_2=4400$;	The total ridership in weekends.
Pre-Operation time	$\zeta=5$ (min);	For a straight bus line ξ only represents the time between each trip, including drivers hand-over, cleaning and examining.
Maximum Waiting Time	$\varepsilon=5$ (min);	The expected maximum waiting time to achieve.
Average Fare per Rider	$b=\$3.00$	The artificial average fare considering all ticket types.

Note: Since the bus routes under investigation were not circles or arcs, there was not any empty return and ζ was treated as a constant parameter instead of a matrix.

The model generated a satisfying result as shown in Figure 4.2.

```

ampl: reset;
ampl: model H:/Thesis/code/SingleRoute.mod;
ampl: data H:/Thesis/code/SingleRouteD.dat;
ampl: solve;
MINOS 5.51: optimal solution found.
3 iterations, objective 65341.37873
Nonlin evals: obj = 30, grad = 29, constrs = 30, Jac = 29.
ampl: display T,Y1,Y2;
T [*] :=
1 25.4348
2 31.4461
;

Y1 = 11428.9
Y2 = 4098.5

```

Figure 4.2 The Calculation Result of The Single Route Model.

The model results showed that under the given condition the best scheduling solution for the scheduling of the bus No.72 was setting the headway as 25.4 minutes on weekdays and 31.4 minutes on weekends. The total weekly profit would be \$65,431 under such settings which was the optimal financial result. The daily profit on a weekday is \$11,429 and on a weekend day it is \$4,099. These results will be verified in Chapter 5 by simulation. Besides, once the headway was determined, other criterions like customer waiting time could also be acquired. They were not included in the AMPL result but if they were needed just one more line of code could display them in Figure 4.2.

4.4 Scenario and Routes Expand

The models for multiple scenarios or routes described in section 3.8 and 3.9 were mainly a combination of parallel modes for single route, except for the potential interactions between routes in the overall model of section 3.9. The mechanism of solving also

remained the same. The parameter setting in this expanding section is slightly different with Section 4.3 since it is from an optimization perspective. Especially, the average speed of buses is increased ($v_1=0.45$, $v_2=0.59$) to reduce the necessary number of buses. The speed remains below the speed limit of Phoenix Metropolitan.

4.4.1 Multiple Routes Case

Now consider a more realistic scenario. Instead of dividing one day into multiple periods like what is described in section 4.3.1, the case could be simplified to dividing the day into two scenarios according to whether it was busy hours or not. Furthermore, it was common that several bus routes shared a number of buses which were dispatched to meet their respective demand. In this case, Route 65 and Route 62 were taken into consideration along with Route 72 since they shared the Tempe Transportation Center as a starting or ending station and it was possible to make the bus dispatching flexible. A model was built to come up with a comprehensive optimal solution for the chosen routes.

As with the model described in section 4.2 and 4.3.1, the ridership data was collected from the Valley Metro Ridership Report and the route length was tested by Google Map. The artificial parameters were shared and remained the same with the former model in Section 4.4.1. The total number of buses was limited to 20.

To provide the bus dispatching solution, a new variable M was inserted into the model which represented the number of buses that was needed by the route according to the scheduling pattern. Correspondingly the constraint for available buses was also modified to match the case of three routes.

The calculation result was shown in Figure 4.3.

```

ampl: reset;
ampl: model H:/Thesis/code/MR.mod;
ampl: data H:/Thesis/code/MRD.dat;
ampl: solve;
MINOS 5.51: optimal solution found.
154 iterations, objective 20951.07973
Nonlin evals: obj = 276, grad = 275, constrs = 276, Jac = 275.
ampl: display T,M,utilization,operation_cost,revenue,financial_profit;
:
  T      M      :=
1 1  19.0035  8
1 2   8.40642  6
1 3  12.5123  6
2 1  21.7083  5
2 2  10.5101  4
2 3  15.7652  4
;

utilization = 0.825
operation_cost = 3990.35
revenue = 24941.4
financial_profit = 20951.1

```

Figure 4.3 Calculation Result of The Multiple Route Model.

The new scheduling scheme maximized the daily profit of this three-route system to \$20,951. The scheduling method was explained by Table 4.2 as below.

Table 4.2

Bus Scheduling Solution for Multiple Route Case

Period	Busy Hours		Non-Busy Hours	
	Headway	Number of Buses	Headway	Number of Buses
72	19.0 min	8	21.7 min	5

65	8.4 min	6	10.5 min	4
62	12.5 min	6	15.7 min	4

Unfortunately the situation failed to reflect the flexibility of vehicle assignment between three routes, therefore the number of available buses was set to 10 and the updated model was run again, like Figure 4.4.

```

ampl: reset;
ampl: model H:/Thesis/code/MR.mod;
ampl: data H:/Thesis/code/MRD.dat;
ampl: solve;
MINOS 5.51: optimal solution found.
150 iterations, objective 19824.82057
Nonlin evals: obj = 241, grad = 240, constrs = 241, Jac = 240.
ampl: display T,M,utilization,operation_cost,revenue,financial_profit;
:      T      M      :=
1 1    28.1866    5
1 2    24.6014    2
1 3    31.4461    3
2 1    28.9101    4
2 2    14.4012    3
2 3    21.3744    3
;

utilization = 1
operation_cost = 2599.62
revenue = 22424.4
financial_profit = 19824.8

```

Figure 4.4 Calculation Result of The Multiple Route Model, $M=10$.

It can be observed in Figure 4.4 that during rush hours one bus of Route 65 was adjusted to Route 72 to achieve the global optimality. In reality the number of available vehicles should be more than only 10. To make the case study more applicable, a sensitivity analysis for the number of available buses M is carried out in section 4.4.2.

4.4.2 Sensitivity Analysis

In the sensitivity analysis the number of buses ranges from 3 to 30, in a step of 3. Table 4.3 displays how the model performs under different value of M . The weighted average waiting time \overline{WT}_r is calculated by Equation (4-1)

$$\overline{WT}_r = \frac{WT(T_{r1}) \cdot N(T_{r1}) + WT(T_{r2}) \cdot N(T_{r2})}{N(T_{r1}) + N(T_{r2})} \quad (4-1)$$

Table 4.3

Sensitivity Analysis of M

M	Profit	Routes	Headway (min) T_{r1}, T_{r2}	Total Ridership $N(T_{r1})+N(T_{r2})$	Weighted Average Waiting time $WT_w(\text{min})$
3	Infeasible				
6	Infeasible				
9	Infeasible				
12	\$20,155	#72	28.2, 25.6	2984	4.2
		#65	17.8, 12.6	2451	4.7
		#62	26.3, 18.8	2263	4.2
15	\$20,565	#72	26.9, 21.7	3059	4.6
		#65	12.0, 10.5	2588	3.5
		#62	17.8, 15.8	2396	4.1
18	\$20,824	#72	21.6, 21.7	3124	4.5
		#65	9.6, 10.5	2640	3.3
		#62	14.3, 15.8	2448	3.9
21	\$20995	#72	18.1, 21.7	3179	4.3

		#65	8.0, 10.5	2684	3.2
		#62	11.9, 15.8	2491	3.8
24	\$21,092	#72	15.7, 21.7	3221	4.2
		#65	6.9, 10.5	2718	3.1
		#62	10.3, 15.8	2525	1.7
27	\$21,152	#72	13.8, 21.7	3260	4.1
		#65	6.0, 10.5	2751	3.0
		#62	8.9, 15.8	2557	3.6
30	\$21,174	#72	12.3, 21.7	3294	4.1
		#65	5.3, 10.5	2779	2.9
		#62	7.9, 15.8	2585	3.5

A series of graphs can be generated and visualize how the profit and expected passengers waiting time are influenced by the number of available buses. See Figure 4.5, 4.6 and 4.7.

In Figure 4.6 the AWT is the average waiting time of all three bus routes.

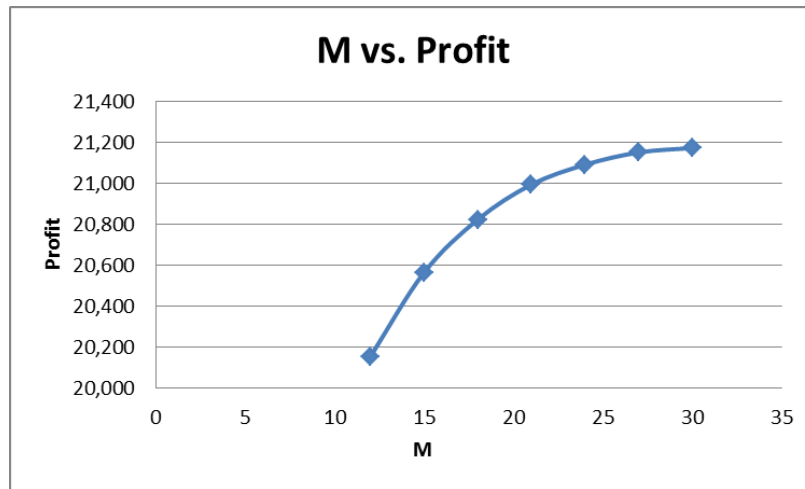


Figure 4.5 M vs. Profit.

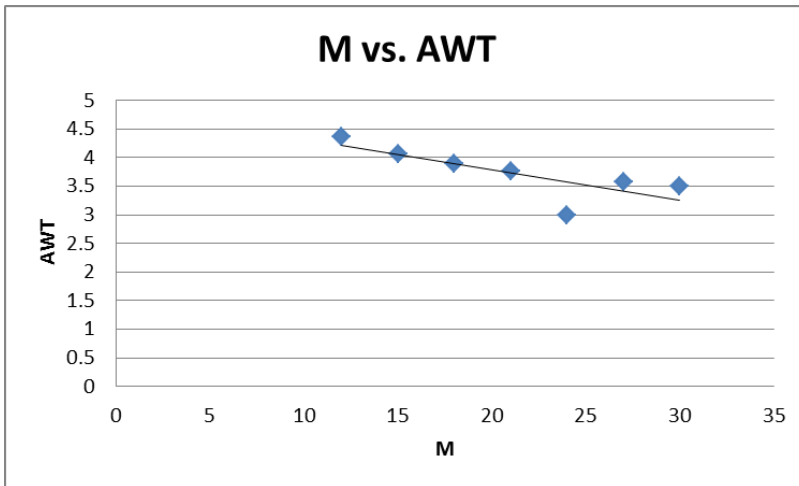


Figure 4.6 M vs. Average Waiting Time.

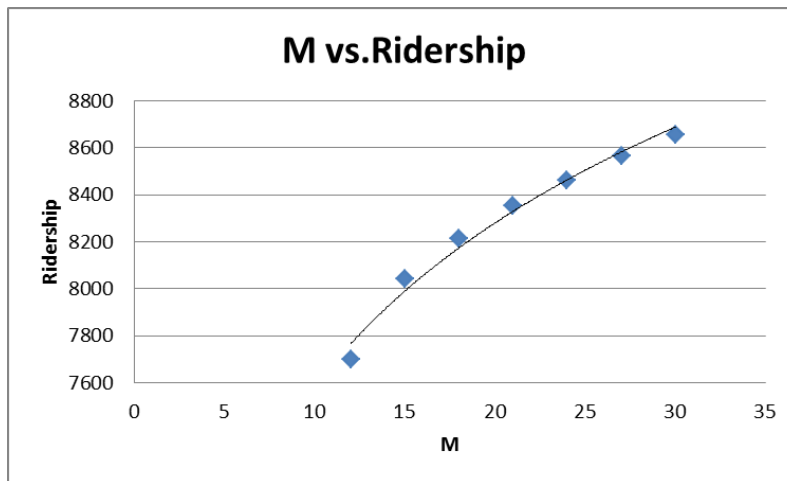


Figure 4.7 M vs. Ridership.

It can be observed from the sensitivity analysis that M provides a significant condition for the model as well as operations in reality and increasing of M brings more benefit to the system. On the other hand Figure 4.5 and 4.7 reveals that there should be an upper limit for M and beyond which the ridership and profit can hardly be improved further.

Moreover, increasing M may also bring new financial burden and such decision requires more consideration and discussion.

4.5 Limitation and Concerns

The only concern is that AMPL lacks the ability to utilize well prepared data saved in database, which means that if there is a large data set, the matrix of inputs including all the variables and parameters have to be typed into the data file manually. Valley Metro of Phoenix runs over 60 bus routes. If the buses were organized according to two scenarios, the input matrix for the whole system would take a long time to edit. Once the condition was changed, re-editing and debugging also called for a large amount of labor.

CHAPTER 5 SIMULATION VERIFICATION

Simulation enables the mathematical optimal solution to be verified with regard to stochastic elements before configured into the real system. A simulation model is built in this chapter to reflect the customer- response to the timetabling of buses. The simulation was completed by SIMUL8©.

5.1 Bus Stop Simulation Overview

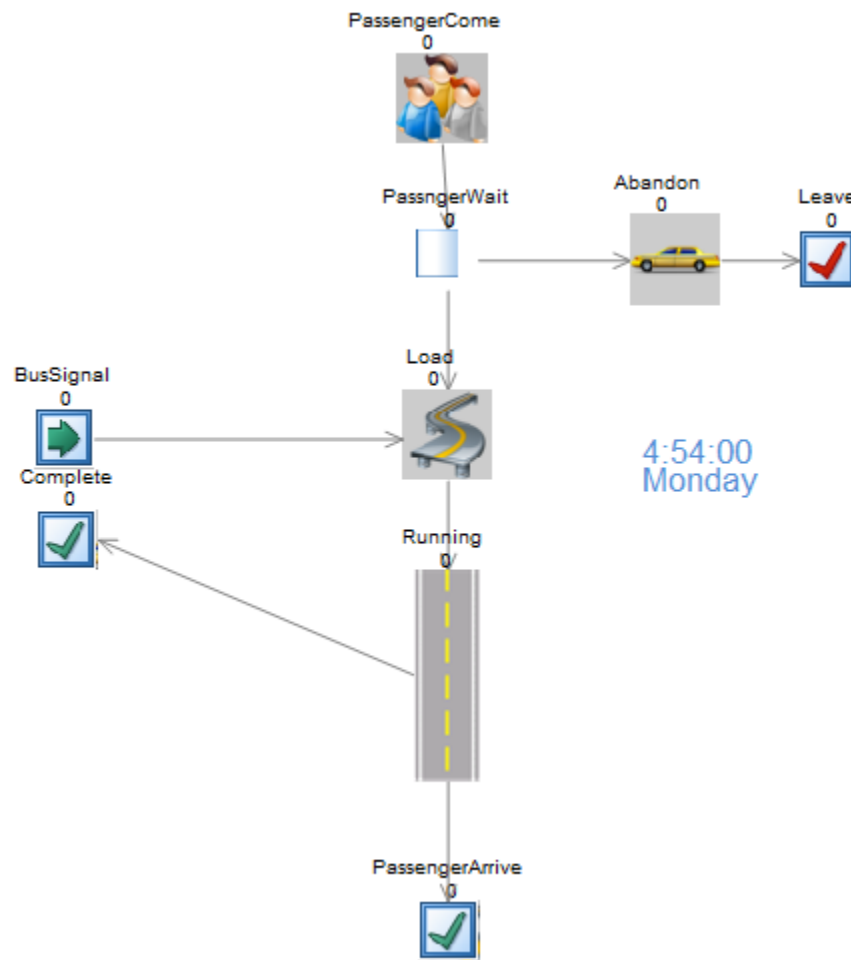


Figure 5.1 Bus Stop Simulation

Passengers are generated from the start point PassengerCome and wait in the queue PassengerWait. If they wait for too long they will leave the queue and abandon taking a bus. When a bus comes, all passengers waiting in the queue get on the bus immediately and arrive to their destination after some time. The main object of the simulation model in Figure 5.1 was the response of passengers in the queue to the changing of bus headway.

Since SIMUL8© was developed for discrete event simulation, the determined pattern of vehicles was difficult to realize. However by defining the entities in this simulation model from two start points, the problem could be solved.

5.2 Passenger Arrival

The start point PassengerCome produced entities representing prospective passengers. The arrival rate of passengers was in a similar pattern with Figure 3.6 which was introduced in Chapter 3, section 3.5.

SIMUL8© enables the modeler to define arrival rates by histograms but the rates can only be represented by inter-arrival time. For instance, if the required passenger arrival rate has a Poisson distribution, then the modeler has to define an exponential distribution for the inter-arrival time as the input of the simulation model. That is to say, there is no way to define the time-dependent passenger arrival pattern like what was shown in Figure 3.6 directly in SIMUL8© since the shape of the histogram of the inter-arrival time is unknown .

Fortunately, SIMUL8© enables to modelers define the arrival of every passenger externally in a CSV file and load it into the simulation model. To generate such a

stochastic pattern that does make sense, a Monte Carlo simulation is executed in Excel. The scenario of 20-minute headway which is currently used by Valley Metro for Route 72 in weekdays is used in the example below. Firstly the number of arriving passengers at each minute is generated, and then it is transplanted to the CSV file in which the passengers are recorded one by one according to their arriving time.

5.2.1 Arrival Rates in Each Minute

The proportion of arrival rates at a bus stop in each minute λ_t obeys the distribution shown in Figure 5.2. According to the ridership report, the number of passengers at one bus stop for one trip of the bus is estimated as 4. Therefore there is

$$\sum_{t=1}^{20} \lambda_t = 4 \quad (5-1)$$

$$\lambda_1 : \lambda_2 : \lambda_3 : \dots : \lambda_{19} : \lambda_{20} = 8 : 7 : 2 : \dots : 13 : 13 \quad (5-2)$$

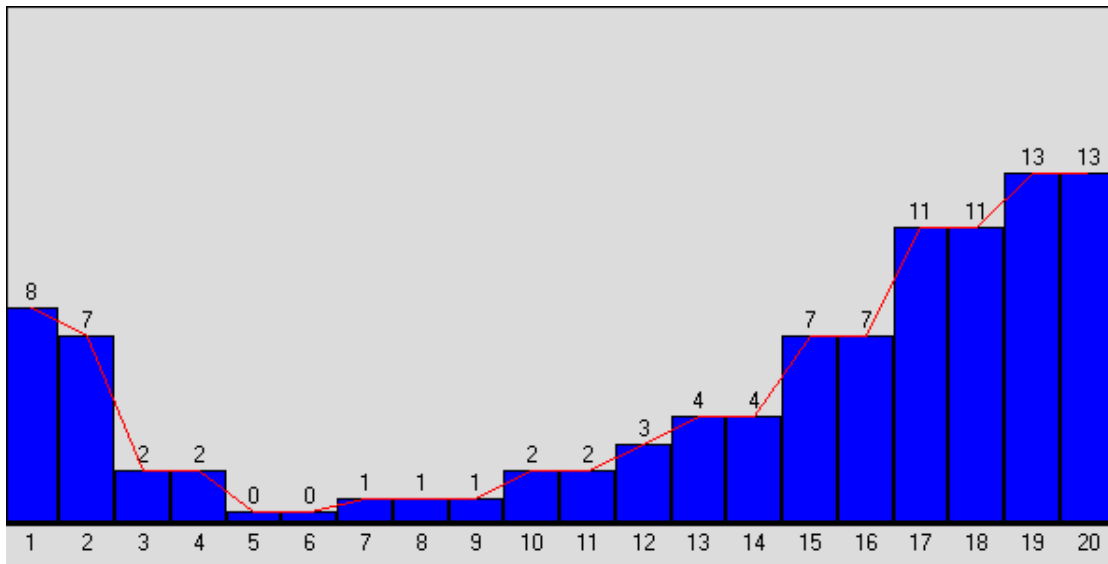


Figure 5.2 Passenger Arrival Rate Proportion (Headway =20 min)
 $\lambda_5 = \lambda_6 = 0.5$, though displayed as 0 in the figure.

From Equation (5-1) and (5-2), the arrival rates in each minute can be calculated. See Table 5.1

Table 5.1

The Arrival Rates

t	1	2	3	4	5	6	7	8	9	10
λ_t	0.29	0.26	0.11	0.11	0.065	0.065	0.08	0.08	0.08	0.11
t	11	12	13	14	15	16	17	18	19	20
λ_t	0.11	0.14	0.17	0.17	0.26	0.26	0.38	0.38	0.44	0.44

According to the Queuing Theory, assuming that N denotes the number of arrival customers, the probability of N equals to x

$$P(N=x) = Poisson(x, \lambda t) \quad (5-3)$$

Here $t=1s$, therefore $\lambda t = \lambda_t$.

The Monte Carlo Simulation is executed by Excel. A column of uniform random numbers is generated and by judging the random numbers against the value of equation(5-3), there will be stochastic results of the number of arriving passengers in each minute. See Figure 5.3

=IF(F20<J20, 0, IF(F20<=J20+K20, 1, IF(F20<=J20+K20+L20, 2, IF(F20<=SUM(J20:M20), 3, 4))))

E	F	GH	I	J	K	L	M	N	O
Minutes	RN		Lambda	N=0	N=1	N=2	N=3	N	
1	0.625971		0.29	0.748264	0.216996	0.031464	0.00304157	0	
2	0.137052		0.26	0.771052	0.200473	0.026062	0.00225867	0	
3	0.363515		0.11	0.895834	0.098542	0.00542	0.00019873	0	
4	0.209887		0.11	0.895834	0.098542	0.00542	0.00019873	0	
5	0.280909		0.065	0.937067	0.060909	0.00198	4.289E-05	0	
6	0.033279		0.065	0.937067	0.060909	0.00198	4.289E-05	0	
7	0.336213		0.08	0.923116	0.073849	0.002954	7.8773E-05	0	
8	0.889349		0.08	0.923116	0.073849	0.002954	7.8773E-05	0	
9	0.833348		0.08	0.923116	0.073849	0.002954	7.8773E-05	0	
10	0.582906		0.11	0.895834	0.098542	0.00542	0.00019873	0	
11	0.489112		0.11	0.895834	0.098542	0.00542	0.00019873	0	
12	0.752279		0.14	0.869358	0.12171	0.00852	0.00039759	0	
13	0.390137		0.17	0.843665	0.143423	0.012191	0.00069082	0	
14	0.133089		0.17	0.843665	0.143423	0.012191	0.00069082	0	
15	0.634151		0.26	0.771052	0.200473	0.026062	0.00225867	0	
16	0.909185		0.26	0.771052	0.200473	0.026062	0.00225867	1	
17	0.029517		0.38	0.683861	0.259867	0.049375	0.00625414	0	
18	0.724953		0.38	0.683861	0.259867	0.049375	0.00625414	1	
19	0.97719		0.44	0.644036	0.283376	0.062343	0.0091436	2	
20	0.455011		0.44	0.644036	0.283376	0.062343	0.0091436	0	

Figure 5.3 A Snapshot of The Monte Carlo Simulation for Passenger Arrival per Minute
(Headway =20 min)

Figure 5.3 is a snapshot of one typical experiment run of the simulation. In this experiment, there are three passengers that come to the bus stop at the 16st, and 18th minute respectively and two passengers come to the stop at the 19th minute. All passengers come independently.

5.2.2 Arrival Time of Passengers and CSV File

The results of the simulation in section 5.2.1 are transplanted into another Excel table to form a formal CSV file that SMUL8© can read. The original table is like Figure 5.4

PassengerArrival		
Passenger	ArrivalTime	Bus Service Number
1	2	1
2	18	1
3	2	2
4	11	2
5	15	2
6	13	3
7	10	4
8	15	4
9	15	4
10	17	4
11	17	4
12	19	4

Figure 5.4 Passenger Arrival Time (Partial Snapshot)

Like what is shown in Figure 5.4, Passenger 1 and 2 come to take the 1st bus of the day and they arrive at the bus stop 18 minutes and 2 minutes before the bus arriving respectively. Then Passenger 3, 4 and 5 come to take the second bus and arrive 18, 9 and 5 minutes earlier. The number of passengers can be generated as many as possible until it exceed the maximum number in the simulation time.

The absolute arriving time in the system that is required in CSV file can be calculated by Equation (5-4)

$$AbsoluteArrivingTime = ArrivalTime + Headway \times (BusServiceNumber - 1) \quad (5-4)$$

The calculation can be done automatically.

The snapshot of the CSV file is shown in Figure 5.5

PassengerArrival			
Passenger	ArrivalTime	Time	TypeB1P2
1	2	Passenger	2
2	18	Passenger	2
3	22	Passenger	2
4	31	Passenger	2
5	35	Passenger	2
6	53	Passenger	2
7	70	Passenger	2
8	75	Passenger	2
9	75	Passenger	2
10	77	Passenger	2
11	77	Passenger	2
12	79	Passenger	2
13	94	Passenger	2
14	99	Passenger	2
15	100	Passenger	2
16	102	Passenger	2
17	111	Passenger	2
18	134	Passenger	2
19	141	Passenger	2
20	154	Passenger	2
21	156	Passenger	2

Figure 5.5 Partial Snapshot of CSV File

In the CSV file the columns of Time and TypeB1P2 are the values of labels used in SIMUL8©, which have to be attached. In the simulation SIMUL8© reads the data and passengers are generated in the exactly defined pattern in the CSV file.

5.3 Bus Dispatching and Passenger Loading

The buses dispatched and passengers are both abstracted as discrete event entities in the simulation model, and this lead to interactions that have to be dealt with in the construction of simulation model.

5.3.1 Reality Introduction

Take Route 72 as an example. There are 101 stops along the whole route but the adjacent stops can merge into one abstract stop to represent a research objective. According to the bus schedule provided by Valley Metro, there are 11 of such stops.

Passengers come to a bus stop to take a bus. When the bus arrives, all passengers can be loaded. The time between a passenger's arrival and the bus's arrival becomes the waiting time of this passenger. Passengers refer to the public bus schedule (by texting or online schedule) to decide their arrival time and buses keep their arrival just on time as much as possible. If a bus arrives a bus stop earlier than the schedule, it will wait for a while, correspondingly if it arrives late it will leave as soon as possible after all passengers are loaded. Some passengers may abandon taking a bus if they wait for too long.

The stopping services at each stop and the travel between them form a whole trip of the bus. The daily total number of trips is determined by a designed headway and the daily operation time. If the headway is 20 minutes and the bus system operates for 19 hours, which is the current scheme of Valley Metro, the daily number of services will be 57.

5.3.2 Simulation Implementation

There are two start points in the simulation model. Start Point PassengerCome generates the entities representing every passengers coming to the bus stop. To guide the dispatching of buses, another Start Point Signal was implemented into the model. When one of the entities generated by Signal (name it "signal") enters the Activity Load, it will stay in the activity and the activity Load becomes blocked. The activity is defined as a

single sever therefor the block disables the entities representing passengers to be loaded and they must wait. The duration of the block is in the same length of headway and when the block is over, passengers in the queue can be loaded immediately.

The block actually simulates the effect of headway that makes passengers wait. Figure 5.6 explains this in graph. The red periods represent the blocks, in other words, the time of headway.

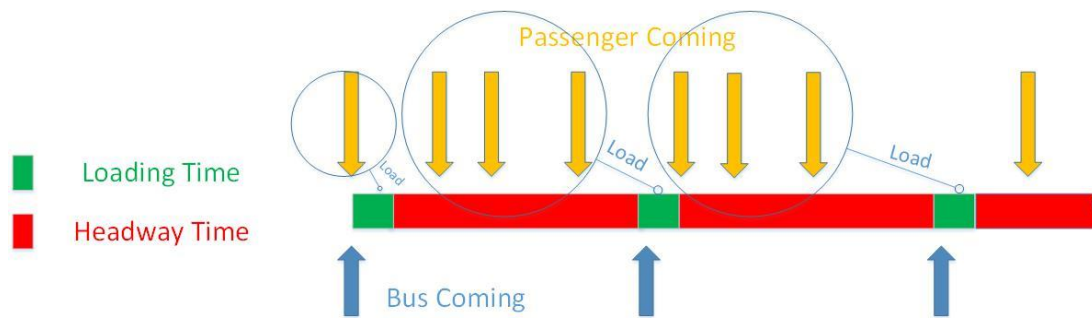


Figure 5.6 Interaction between Coming Buses and Passengers

To ensure the continuity of bus services, it is required that when a batch of passengers waiting at the bus stop is loaded by one bus, there should be another block immediately as long as headway that makes upcoming passengers wait for the next bus. Therefore the frequency of Start Point Signal generating “signals” is set much higher than passengers’ arrival rate; meanwhile the time of loading is set to 0. Moreover for Activity Load passengers are given priority upon “signals”. So far such a process is realized: Block (Last bus leaves)—Passengers come and wait—Block remove (Next bus comes)—Passengers being loaded—Block again (Bus leaves). It is a successful simulation to what happens at bus stops. The time on activity “Load” of entity Passenger has a distribution

that is different from entity Signal. This is realized by defining label based distribution in SIMUL8. Since passengers' response to various headways of buses is the concentration, their time spent on activity Load and Running can be simply set as 0.

Passengers' tolerance to waiting is defined by shelf life of the queue. Passengers that wait for some time more than shelf life will be removed from the queue and left the system. The activity "Abandon" is set as "expired only" to accept the expired entities. In reality, those who abandon to take a bus might choose to drive or by other transportation methods to their destination.

5.4 Bus Stop Simulation

A microscopic simulation model for a single bus stop is built and run, to investigate the impact of different headways on customers' response especially the tolerance to waiting time. There are three groups of inputs based on the calculation result of the single route model in Chapter 3 and Chapter 4, the current 20-minute-headway case and the optimized case with a 25-minute-headway and a 30-minute-headway.

5.4.1 Original Case (20-Minute-Headway)

Currently Route 72 of Valley Metro has headway of 20 minutes in weekdays. Therefore 20 minutes is set in the simulation model as the time on activity of bus signals and the passenger's arrival pattern was from the CSV file generated under 20-minute headway condition.

Firstly the model ran and generated series of output data for evaluation of current bus timetabling scheme. Initially the shelf life of the queue is long enough to assume that no one abandoning taking a bus.

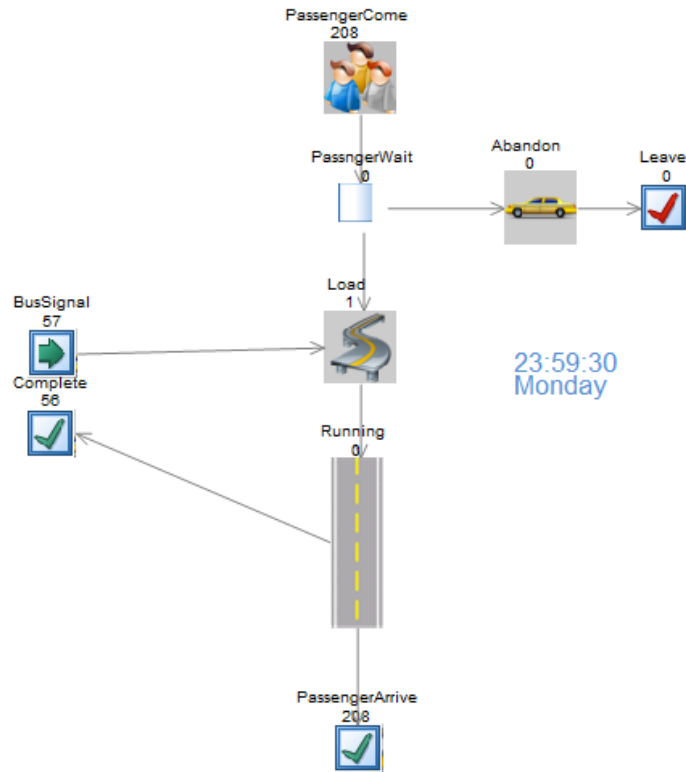


Figure 5.7 Result Overview (Simulated for 19 hours on a weekday, no shelf life)

SIMUL8 - Income Statement	
Costs	20.00
Revenue	710,368.00
PassengerArrive	713,728.00
Complete	7-3,360.00
Profit	710,368.00

Figure 5.8 Financial Report of Simulation Model (Simulated for 19 hours on a weekday,
Headway =20 min)

The financial report displayed that the simulated daily profit of Route 72 is \$10,368. According to the Ridership Report, the average daily ridership of Route 72 is 4900 therefore theoretically the estimated daily profit should be \$12,990. Since the simulated ridership at one bus stop is 208 which is a bit less than reality and leads to a difference on daily ridership of whole route of about 330, the accuracy of the simulation model can be accepted. (Error = $(\$10,268 + \$3.00 \times 330 - \$12,990) / \$12,990 = 12\%$)

If a waiting tolerance is set, like the maximum waiting time of prospective passengers is 5 minutes, the situation will be quite different.

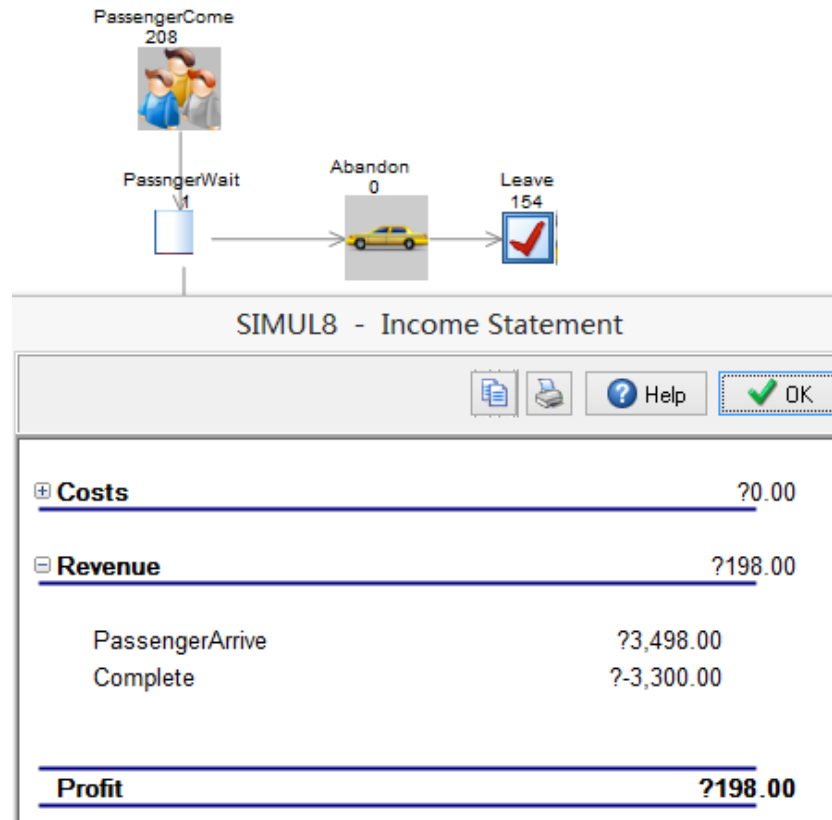


Figure 5.9 Passenger Leaving and Lost Revenue (Shelf life = 5min)

About $\frac{3}{4}$ of potential passengers give up taking a bus if they have a waiting tolerance of 5 minutes. Meanwhile the profit declines to just offsetting the operation cost. It indicates that the tolerance level may be much higher than our settings if they do exist, and it motivates that further optimization can be carry on, like the Single Route Model in Chapter 3 and 4 that increase the headway. If the shelf life is removed, the reason why so many people abandon taking a bus can be found. The data of passengers' waiting are recorded, see Figure 5.10.

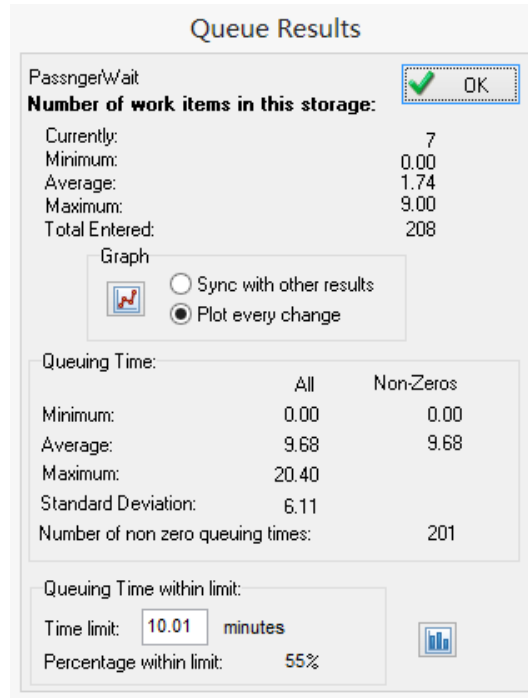


Figure 5.10 Queuing Result with No Shelf Life (Shelf life \geq Headway)

It can be observed that the average waiting time (9.68 minutes) is slightly shorter than the natural case, that if people come to the bus stop uniformly, the average waiting time should be half of the headway (10 minutes).

Furthermore, the simulation displayed the histogram of waiting time, see Figure 5.11.

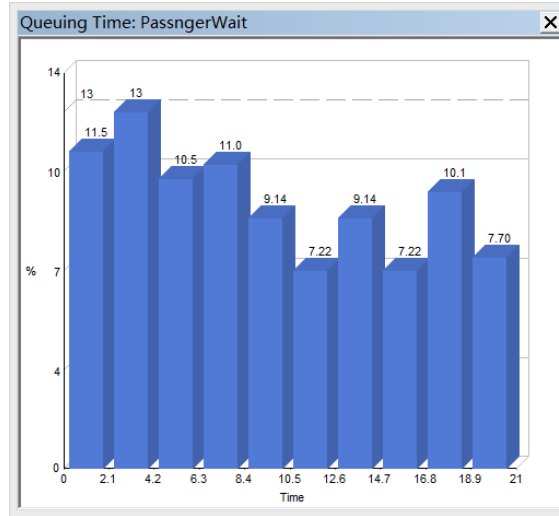


Figure 5.11 Queuing Time Histogram (Shelf life \geq Headway)

It is expected that the histograms at the tail is short, which means the percentage of passengers waiting for long time is small. Unfortunately the effect is not as obvious as expected. If the experiment does not last for such a long time, like just 5 hours, the effect becomes much more obvious. The only explanation is that the large number of people weakens the difference on arrival time choices.

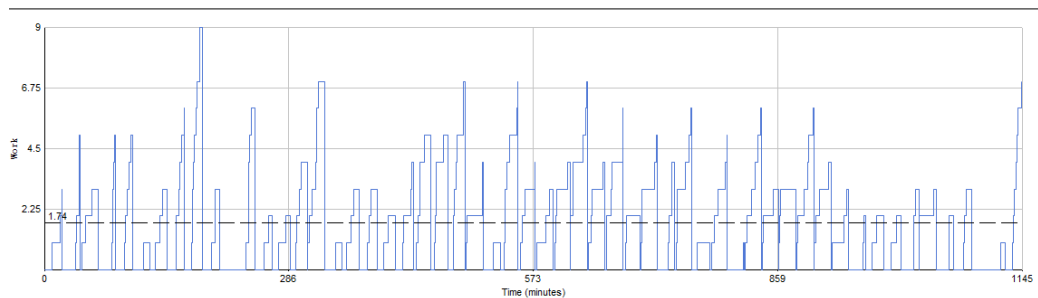


Figure 5.12 Queuing Number of Passengers (Shelf life \geq Headway)

The sharp discrete drops in Figure 5.12 means that at this time a bus comes and takes all passengers at the stop. Those passengers arriving at the bus stop at the same time with the bus can all get on the bus.

5.4.2 25-Minute-Headway Case

The first case study generates an optimal solution of 25-minute-headway on weekdays; therefore it is implemented into the simulation model for verification. The overview is in Figure 5.13.

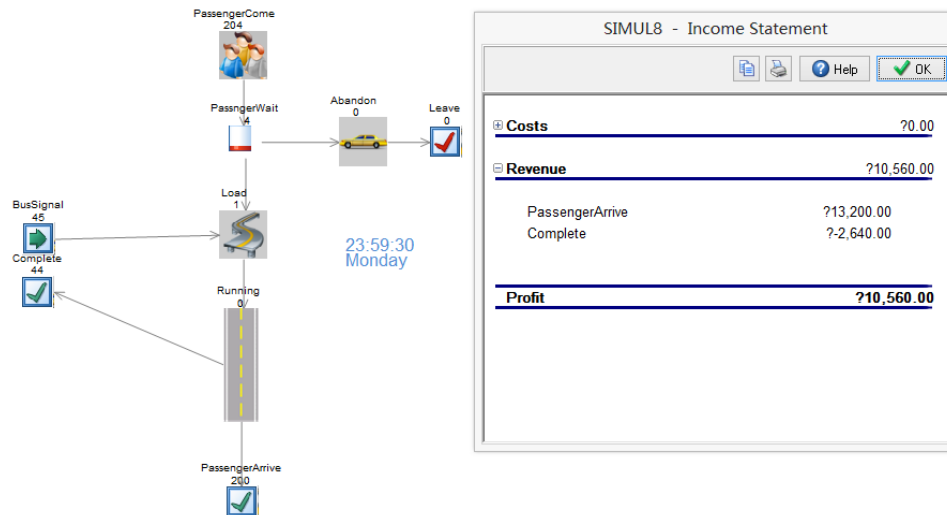


Figure 5.13 Overview of 26-minute-headway Model

The ridership is almost the same with the scenario of 20-minute-headway, and so as the revenue and profit.

5.4.3 30-Minute-Headway Case

Next the headway was changed to the optimal headway given by the case study in Chapter 4. The calculation result shows that there is a 31-minute headway in the group of

optimal solution however since the 30-minute is utilized by Valley Metro in weekends, no changes are made therefore the implementation can be easier but the impact is not too big.

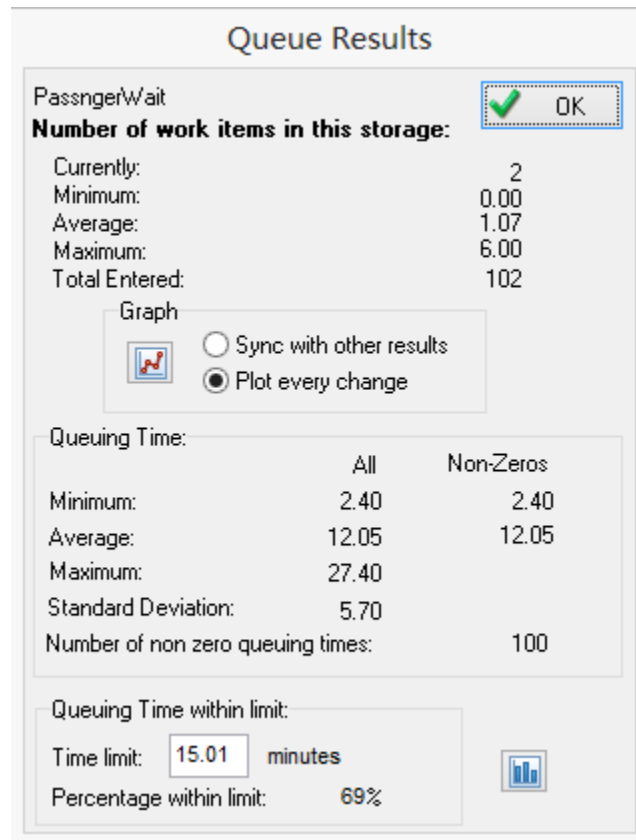


Figure 5.14 Queue Result when Headway=30 min

In Figure 5.14, refer to Figure 5.10, the average waiting time was shortened to a larger degree, comparing with half of headway. It could be observed that stretching of headway enhances the effect of catching a bus on time. The histogram of queuing time Figure 5.15 can lead to similar conclusion since less passengers wait for long time. The curve of the changing in the length of the queue in Figure 5.16 is also displayed and it can be observed that this curve looks more sparse than Figure 5.12.

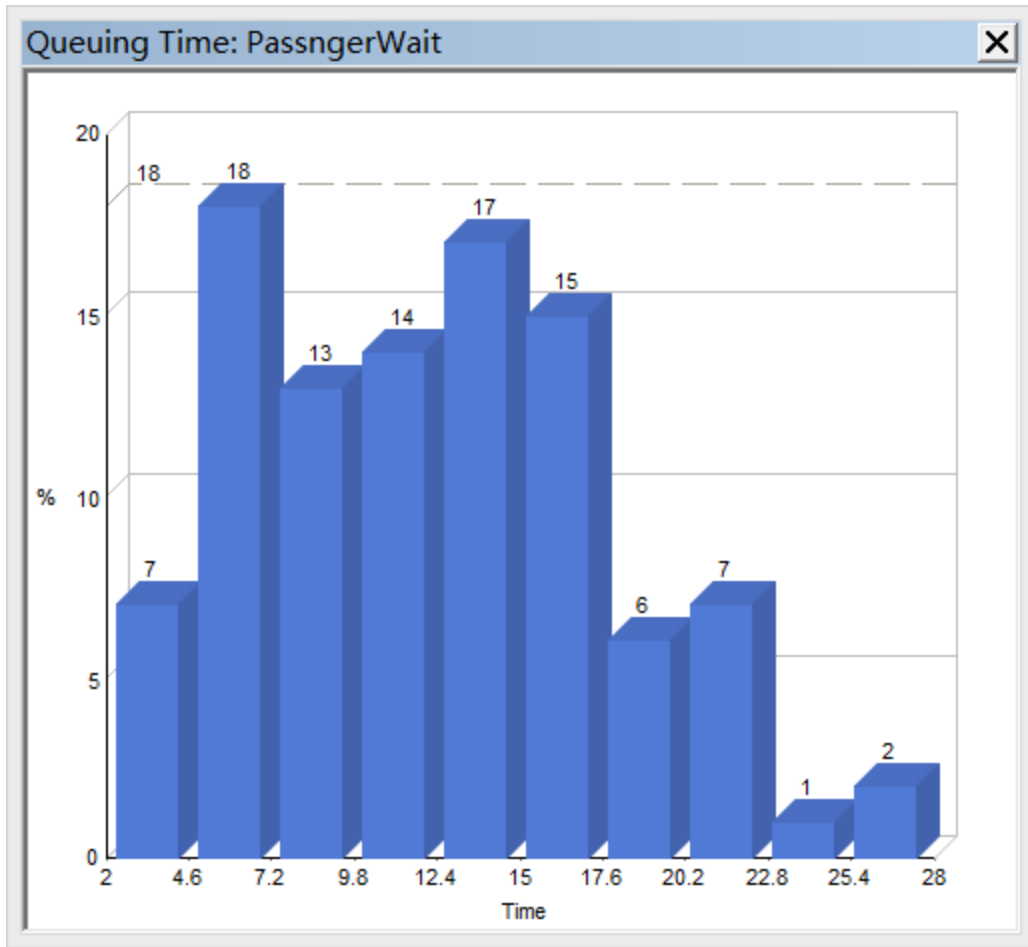


Figure 5.15 Queuing Time Histogram (Headway=30 min)

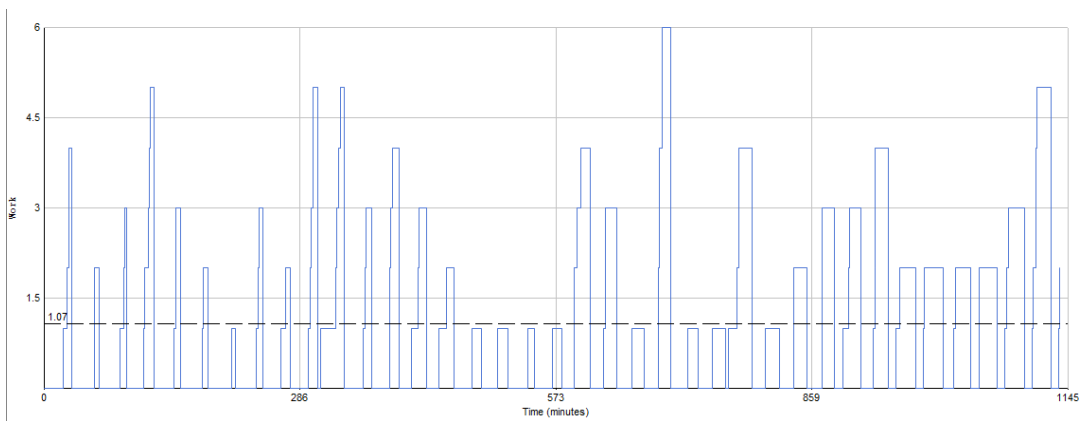


Figure 5.16 Queue Length Changing (Headway=30 min)

5.5 Comparison and Conclusion

Obviously the former managers of Valley Metro did a lot of their homework in that their schedule is very close to the theoretic calculation result in this thesis. Table 5.2 reveals the customer-response to the changing headway in weekdays and weekends.

Table 5.2

Comparison of Headways

25-Minute-Headway	Criteria	30-Minute-Headway
12.22 min	Average waiting time (<i>WT</i>)	12.05 min
7.54	Standard Deviation of <i>WT</i>	5.70
<i>WT</i> shortened for 2.24%	Compare to Average Case	<i>WT</i> shortened for 19.6%
24%	Percentage of <i>WT</i> <5 min	10%
44%	Percentage of <i>WT</i> <10 min	39%
60%	Percentage of <i>WT</i> <15 min	69%
80%	Percentage of <i>WT</i> <20 min	90%

Table 5.3 and 5.4 verify the effect of optimized headway by comparing the profits under different waiting tolerance (Shelf Life) generated by simulation model (SimM) and the calculation result of the Single Route optimization model (SRM) in Chapter 4.

Table 5.3

Profit Comparison (SRM vs. SimM, Headway=25min)

	5min	10min	15min	20min	None
SRM Weekday Daily Profit	\$11,429	\$11,429	\$11,429	\$11,429	\$11,429
SimM Weekday Daily Profit	\$462	\$3,036	\$5,148	\$7,920	\$10,560
Potential Profit Loss	96%	73%	55%	31%	NA

Table 5.4

Profit Comparison SRM vs. SimM (Headway=30min)

	5min	10min	15min	20min	None
SRM Weekend Daily Profit	\$4,099	\$4,099	\$4,099	\$4,099	\$4,099
SimM Weekend Daily Profit	-\$1,560	\$354	\$2,334	\$3,720	\$4,380
Potential Profit Loss	138%	91%	43%	9%	NA

Suggestion for Bus Companies: Questionnaires about how passengers are willing to wait for a bus may be cast to public. If passengers' feelings re always ignored, maybe someday a decision by mistake would lead to severe financial loss.

Suggestion for passengers: if you want to catch a bus, get to the bus stop early. Murphy's Law tells us that if you might miss a bus, then you will miss it.

CHAPTER 6 EVALUATION AND RECOMMENDATION

This thesis developed a new optimization model for a bus assignment problem that took the interaction between customers and system cost into consideration. By providing advice on bus headway, the model may be viewed as an integrated, strategic input to the more widely researched vehicle scheduling problem that requires the bus timetable as input. In addition, a simulation model was constructed to explain and verify the model utilizing discrete event simulation.

6.1 Highlights

The optimization model generated satisfying results for situations from a simple scenario that only one bus route was concerned to the complex one that multiple routes and time periods were investigated.

Since the calculation result was quite reasonable, and also verified by simulation, several highlights could be concluded.

Conciseness

The model has an objective function requiring only minimal data – bus costs per route trip and set of decision periods. Only three constraints were always necessary. The total number of constraints depends on the complexity of scenarios. In real operation process the operators could divide the system to maintain the complexity of subsystems in a relatively low degree and apply the model to help scheduling. In general the model is concise enough that it presented little difficulty in solving with standard optimization engines, at least for small city examples.

Comprehensiveness

The theme of this thesis was to treat the decision makers of bus companies and passengers as interacting agents and to tradeoff system costs with service for a strategic/tactical planning tool. The model provides a means to reconcile prospective passenger's service expectations and bus company finances. Such a model reflects a harmonious relationship between the service provider and the consumer and this approach could be applied to other areas where similar interactions exist.

Applicability

Currently the local buses of Phoenix Metropolitan Area operate with headway between 20 to 30 minutes. The suggested headway given by the model in this thesis varies from over 10 minutes to over 30 minutes. Literally the difference between current operating pattern and newly suggested one was not so large that adjustment on bus schedule would be relatively difficult to execute. Besides the calculation result of the optimization model could also help on bus dispatching that could promote the utilization of the resources that the bus company was able to govern.

Innovative viewpoint

In Chapter 4 the achievements of two former researchers were combined and the bus headways and passengers' interest was given a deterministic relationship. In Chapter 5 besides the verification, the construction of the microscopic model itself is a unique application of Discrete Event Simulation (DES).

6.2 Limitations

The thesis addressed bus system design at a high level making several assumptions such as fixed routes and known demand functions. One limitation of this thesis was that the critical parameters, including the unit cost of one trip, lacked real data support from Valley Metro or any other similar bus operators. The initiation of the parameters was based on empirical estimation of a low precision, therefore although the model could run well but the result was difficult to evaluate referring to the total cost given in the Annual Financial Reports of Valley Metro.

6.3 Future Research

This model is a first step in providing decision support. A more complete model would include service at individual stops (or neighborhoods) and a thorough financial study to determine actual costs. In addition, consumer choice investigations would be helpful to more clearly define the rider demand as a function of fare and headway. The model could also be integrated with a more detailed bus routing algorithm to specifically plan bus trips at a more detailed level.

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APPENDIX A

AMPL CODE FOR SINGLE ROUTE MODEL

MODEL FILE:

```
##SingleRouteCostModel
param period;#Time length of the scenario
param L;# route length
param P{t in 1..period};# period length
param c;#cost per trip
param M;#number of buses for one route
param v{t in 1..period};#average speed under scenario(period t)
param s;#seats of a bus on the route
param sigma;#the ratio of more passengers can be conveyed
param gamma;#include standing
param N{t in 1..period};#total boardings
param kes;
var T{t in 1..period}>=0.001;
var N1{t in 1..period}=(1.64-0.21*log(T[t]))*N[t];
var Y1=3*N1[1]-2*c*(P[1]/T[1]);#daily profit of weekdays
var Y2=3*N1[2]-2*c*(P[2]/T[2]);#daily profit of weekends
maximize revenue:5*(3*N1[1]-2*c*(P[1]/T[1]))+2*(3*N1[2]-2*c*(P[2]/T[2]));
subject to st1{t in 1..period}: 2*(L+kes*v[t])/(v[t]*T[t])<=M;
subject to st2{t in 1..period}: (P[t]*gamma*s)/(N1[t]*T[t])>=sigma;
subject to st3{t in 1..period}:1.45*log(T[t])<=5;
```

DATA FILE:

```
##SingleRouteCostModelData
param period:=2;
param L:=28.1;# route length
param P:=1 1140
       :=2 1020;# period length
param c:=30;
param M:=10;
param v:=1 0.23
       2 0.31;
param s:=40;
param sigma:=0.95;
param gamma:=2.5;
param N:=1 4900
       2 2200;
param kes:=5;
```


APPENDIX B

AMPL CODE FOR MULTIPLE ROUTES MODEL

MODEL FILE:

```
##MultiRouteModel
param period;#Time length of the scenario
param route;#Number of routes
param L{r in 1..route};# route length
param P{t in 1..period,r in 1..route};# period length
param c{r in 1.. route};#cost per trip
param M0;#Total number of buses available
param v{t in 1..period,r in 1..route};#average speed under scenario(period t)
param s;#seats of a bus
param sigma;#the ratio of more passengers can be conveyed
param gamma;#include standing
param N{t in 1..period,r in 1..route};#total boardings
param kes;
param e;#penalty coefficient
var T{t in 1..period,r in 1..route}>=0.001;
var M{t in 1..period,r in 1..route}=ceil(2*(L[r]+kes*v[t,r])/(v[t,r]*T[t,r]));
var N1{t in 1..period,r in 1..route}=(1.64-0.21*log(T[t,r]))*N[t,r];
var utilization=(sum{t in 1..period,r in 1..route}M[t,r])/(M0*period);
var operation_cost=sum{t in 1..period,r in 1..route}(c[r]*(P[t,r]/T[t,r]));
var revenue=3*sum{t in 1..period,r in 1..route}N1[t,r];
var financial_profit=revenue-operation_cost;
maximize profit: 3*sum{t in 1..period,r in 1..route}N1[t,r]
                -sum{t in 1..period,r in 1..route}(c[r]*(P[t,r]/T[t,r]));
subject to st1{t in 1..period,r in 1..route}: (P[t,r]*gamma*s)/(N1[t,r]*T[t,r])>=sigma;
subject to st2{t in 1..period}:sum{r in 1..route}2*(L[r]+kes*v[t,r])/(v[t,r]*T[t,r])<=M0-1;
subject to st3{t in 1..period,r in 1..route}:1.45*log(T[t,r])<=5;
```

DATA FILE:

```
##MultiRouteData
param period:=2;
param route:=3;
param L:=1 28.1 #R72
          2 8.6 #R65
          3 13.7; #R62
param P:1 2 3:=
          1 360 360 360
          2 780 780 780;# period length
param c:=1 30
```

```
2 10
3 15;
param M0:=10;
param v:1 2 3:=
1 0.45 0.45 0.45
2 0.59 0.59 0.59;
param s:=40;
param sigma:=0.95;
param gamma:=2.5;
param N: 1 2 3:=
1 1432 1108 1108
2 1711 1178 1178;
param kes:=5;
```