Essays in Organizational Economics: Information Sharing and Organizational

Behavior

by

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ABSTRACT

One theoretical research topic in organizational economics is the information issues raised in different organizations. This has been extensively studied in last three decades. One common feature of these research is focusing on the asymmetric information among different agents within one organization. However, in reality, we usually face the following situation. A group of people within an organization are completely transparent to each other; however, their characters are not known by other organization members who are outside this group. In my dissertation, I try to study how this information sharing would affect the outcome of different organizations. I focus on two organizations: corporate board and political parties. I find that this information sharing may be detrimental for (some of) the members who shared information. This conclusion stands in contrast to the conventional wisdom in both corporate finance and political party literature.

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# Preface

Organizational economics use economic logic and methods to understand the existence, design and performance of organizations. According to Kenneth Arrow (1974), organizations are explained broadly. They include business corporations, unions, legislatures, agencies, churches, and much more.

In last several decades, within Arrow's broad view of organizations, researchers from economics, finance, political science, psychology and many other fields of social science have done important contributions to help us understand a wide range of issues of different organizations. The most influential research includes the following. Coase (1937) raised the question of the boundaries of the firm. Berle and Means (1991) described conflicts of interest arising from the separation of corporate ownership by shareholders from corporate control by top managers. Simon (1951) offered perhaps the first formal model in organizational economics. Williamson (1975) concerned more extensively on the nature and boundaries of modern firms. Marschak and Radner (1972) modeled optimal communication and decision-making processes in an environment with uncertainty. Hurwicz (1973) introduced the concept of mechanism design theory and set up the framework of organizational design. Meanwhile, Mirrlees (1976) and Hölmstrom (1979) introduced formal models of moral hazard, launching a literature that have tremendous influence on organizational economics. A comprehensive survey on these topics is documented in Gibbons and Roberts (2013).

## Outline of the dissertation

As mentioned above, one theoretical research topic in organizational economics is the information issues raised in different organizations. This has been extensively studied in last three decades. Researchers try to understand different questions, including the following. What information is collected, by whom, to whom is communicated and how is it used? How are people rewarded in an organization under uncertainty? What norms exist regarding information asymmetry toward people in the organization, as well as outsiders who are related to the organization?

One common feature of these research is focusing on the asymmetric information among different agents within one organization. However, in reality, we usually face the following situation. A group of people within an organization are completely transparent to each other; however, their characters are not known by other organization members who are outside this group.

For instance, in modern corporate firms, board is a organization. Within the board, they are divided to different groups or factions according to different characters or interests. Board directors usually represent shareholders of different interests. Directors with the same interests usually share information with each other before the board decisions. In the first chapter of my dissertation, I model board directors as two groups of players with different interests and study the effects of market reputation, and information sharing on boardroom collusion and decisions. A new multilateral bargaining model is proposed which incorporates the idea that there may be information sharing among groups of directors with the same interests. I focus on the advisory role of corporate boards and assume that during boardroom decisions, directors provide information (expertise) which is productive but only privately observed. I find that: If a minority director has a good market reputation of having

an expertise in one field and his information contributes more in the current safe project (policy) than the new risky one, there may exist conflicts among the majority directors, leading to a decreased level of private benefits for a certain director, and therefore decrease the probability of collusion among the majority. Then the majority director who can propose the project would collude with a minority director. This result could be independent from the information provided by majority directors and a majority director's market reputation. Using the insights from the model, we also analyze: 1) The interactions between controlling shareholders and minority shareholders; 2) The relation between the project selection and independence of the board; 3) The relation between management selection and market reputation. Some of the results are illustrated with examples from Apple Inc., Microsoft Inc..

In modern political regime, we have the same situations as corporate boards. Politicians are divided to different parties or factions according to their ideologies or interests. Politicians within a party or faction usually engage in information sharing. This information sharing could be forced by some special rules or other norms. However, across parties, information may not be shared. In chapter two of my dissertation, I study the effect of information sharing on distributive politics. I use the bargaining model proposed in chapter one that incorporates the idea that there may be information sharing amongst politicians in the same party. Absent information sharing, the party leadership will provide legislative pork to their party members. However, with information sharing, there may exist conflicts between party members. This may result in a decreased level of legislative pork for certain party members. Thus, information sharing may be detrimental for (some of) the party's own members. This conclusion stands in contrast to the conventional wisdom that parties help their party members achieve legislative goals.

In chapter 3, I switch back to corporate board, but try to study a question which

is different from the previous two chapters. In the literature, we know that firm's performance depends on the "type" of the management—whether it is efficient or non-efficient. Literature also suggests that the board also plays an important role in firm's performance. In particular, this paper views the board as a medium, which transmits the information of the management's type to the shareholder. The shareholder then designs a compensation scheme for both the management and the board conditional on firm's performance. This paper studies how interactions between the board and the management influence the firm's performance and, thereby, influence the shareholder's welfare. It first considers a setting, in which the board can not collude with the management. It observes a noisy signal of the management's type, and reports this signal to the shareholder. The shareholder can only obtain information of the management from the board, and knows how accurate the signal observed by the board. This paper finds that the board can have a positive or negative influence on the firm's performance. Whether the influence is positive or negative depends on the accuracy of the signal and whether the signal infers the management is efficient. This paper then turns to an alternative setting, in which the board is allowed to collude with the management. In particular, the collusion is modeled with a bargaining game, where the board and the management bargain over the payment from the shareholder. With this additional feature, the paper find that: The cost of blocking the collusion depends on the structure of the board—whether the board is dependent or independent. The results have several implications for organization design of corporate board and board regulation.

Chapter 1

# MARKET REPUTATION, INFORMATION SHARING AND BOARDROOM COLLUSION : THEORY

## 1.1   Introduction

From empirical literature, we already know that in many countries, minority share-holders, which usually indicate outside investors and creditors, have a large impact on the economy, see López de Silanes *et al.* (1998) and Porta *et al.* (1999).   [1]

Empirical research in economics, finance and law also document that: in many countries, which is more than 49 countries in the previous research, there are conflicts between minority shareholders and controlling shareholders. When minority shareholders finance firms, they face the risk that the returns on their investment will never be materialized, because the controlling shareholders or managers expropriate them. For instance, the controlling shareholders can manipulate firms' decisions by controlling majority positions in the corporate board; they might sell the output, the assets or the additional securities in the firm they control to another firm they own at a lower price through the firms' decisions. Or the entrepreneurs may extend the size of the firm on the cost of outside investors through project selections, such as merge other firms or set up a new product line. The basic conflict is that the controlling shareholders use the profit of a firm to benefit themselves rather than return the money to the minority shareholders.

This kind of conflict arises in many countries for two reasons: The first reason is due to the incomplete legal system. In some countries, the legal system does not

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[1]In these research, the authors found that the average ratio of stock market capitalization held by minority shareholders to GNP is more than 40% in a sample of 49 countries.

protect minority shareholders because of either poor laws or poor enforcement of laws. And the second is the corporate governance structure of public companies gives controlling shareholders more chances to have more positions in the corporate board, or to install family members in managerial positions, see La Porta *et al.* (2000).

So one question asked in the literature is: why are people willing to be minority shareholders even if they know that the laws or corporate governance mechanisms can not protect them from expropriation by controlling shareholders? This paper tries to answer this question by opening the black box of board room decisions. We want to argue that, despite the fact that a corporate governance mechanism fails in many aspects, the minority shareholders still do well if they can appoint board directors whose expertise could affect the firm's performance.

Under this framework, one key assumption is that board directors represent shareholders of different interests. In some sense, we think that the boardroom conflicts and collusion are inevitable because the board is composed of different interest groups [2] . For example, it is highly unlikely that the interests of inside directors, namely executives within the firm, the outside, independent directors, and the "gray" directors will be congruent. The attitude between the groups ranges from cynical indifference to open defiance. Among outside directors, even if they are in the majority, there are conflicts of interest, in view of the fact that they (1) represent shareholders, debt-holders, and other stakeholders (e.g., Byrd and Mizruchi (2005) on bankers on the board; Baker and Gompers (2003) on Venture capitalists; Faleye *et al.* (2006) on employees); (2) are members of different demographics (e.g., Adams and Ferreira (2009) on gender mix); (3) belong to different social networks (e.g.,Kramarz and

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[2]We have not found any literature in economics or finance which formally document the existence of interest group or factions in the corporate board, even if the stories about this board politics are floating around on newspapers. However, in law literature, we do find some evidences about the interest groups or factions in the corporate board. From the historical view, the board of East Indian Company might be the best example to support our argument here, see Gevurtz (2004).

Thesmar (2006)); (4) possess different strategic views about the business (e.g., Demb *et al.* (1992); (5) were appointed before and after the current CEO took office (e.g., Hermalin and Weisbach (1998)).

Based on anecdote and empirical evidence, this paper attempts to open the black box of the boardroom processes and dynamics by modeling directors as players who represent shareholders of different interests. We try to formalize the interactions within the board and give a rational explanation to the famous anecdote, like Apple's board conflict in 1985. To our best knowledge, it is the first theoretical attempt to study the boardroom collusion.

This model has four features: (1) Information is shared among directors with the same interests; (2) the market has belief in the directors' ability or expertise; (3) production is directly affected by the board directors' advice (information or expertise); and (4) the board is composed of directors with diverse interests. For the first one, information sharing, we assume that in a group with the same interest, directors would share their private information among each other. The motivation is as follows: From the informational perspective, board members are always treated as policy specialists. These policies can be related to the board's monitoring role, its advising role or the directors' visions. The monitoring tasks include the hiring, firing, and assessment of management (e.g., Hirshleifer and Thakor (1994); Hermalin and Weisbach (1998)), while the advising role involves setting of strategy and project selection (e.g., Song and Thakor (2006); Adams and Ferreira (2007)). The vision is a purely personal characteristic. It can help coordinate the firm's activities around a common goal (e.g.,Bolton *et al.* (2010)). While almost all the current theories look at the board as one entity, (see Adams *et al.* (2008) for a review), and neglect this information aspect. But we think that, within board interactions, each of the board members might possess these private, different information about the relationship

between policies (or project) and their consequences, and they might belong to different interest groups [3] . And one important way in which the interest group help its members achieve common goals is by serving as a mechanism for sharing this private information within the group. This feature, information sharing, also has been argued as one key feature in team building (e.g.,Bolton *et al.* (2013)). And one communication mechanism inducing information sharing has been discussed by theoretical literature, see Adams and Ferreira (2007) for one of them.

For the second one, market reputation, the motivation has been argued by the literature for a long time, which may go back to Fama (1980) and Fama and Jensen (1983). They point out that the management and the directors care about their reputation on the market, and the reputation would also affect their compensation.

In this paper, the CEO is not necessarily to be excluded from the board. We explain the directors' market reputation as the shareholders' belief on the directors' "type". For the director who is the management, we follow Milbourn (2003), and explain the "type" as the ability. Their market reputation is the CEO's perceived ability. The ability here is not only about what the CEOs can do, but also relates to the information they get. And the information is assumed to affect the project or policy outcome. Efficient CEOs are assumed to always get more precise news on the outcome of the selected project or policy. For example, John Sculley is an expert on marketing, his network may bring him lots of information on a new market which would benefit the firm. This network would have the same meaning as the ability.

For the non-management directors, the "type" could have very broad explanation. From the view of monitoring, we can think about the directors being vigilant and who

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[3]Numerous empirical works examine board structure and its potential impact on the board's actions and firm performance (e.g., Faleye *et al.* (2006); Linck *et al.* (2008)), but few studies the internal bargaining and decision-making process in the boardroom that links the board's structure and its actions or performance. This missing link is largely due to data limitation.

would monitor management very intensively, or being lax and who would not monitor management intensively. From the view of advice, we can think about the type as whether the directors observe a piece of information on the outcome of the project or if the information they observe is precisely related to the outcome of the project. The similar argument on the directors as experts of policy or project has already been mentioned in the literature, see Adams *et al.* (2008).

For the third one, board director's affect to production, this feature has been documented in both descriptive and empirical literature, e.g., Vafeas (1999). One common argument is that directors provide expertise in board decisions which would further affect the firm's performance. Or a more effective board might give more proper advice or monitor more frequently than others. Then these actions would affect the management, so as the performance of the firm. Following this argument, we would formally model this feature by considering a technology which is directly affected by the board directors' expertise.

The fourth, diversity of board directors, has not been mentioned too much in the literature. But we think it is a crucial feature of the corporate boards. This is because as boards grow, it is possible to accept directors representing different interests and thus induce diversity. Baranchuk and Dybvig (2009) also notices this feature of the board. They try to analyze the decisions in diverse corporate boards. They did not explicitly characterize the interactions among directors, but they focus on some cooperative feature in board decisions. We try to analyze this in another way and focus on the non-cooperative feature in board decisions.

Combining the above four features, in this paper we argue that, by sharing information, controlling shareholders with majority seats in the board could more easily propose and control the selection of new project (policy). This would be preferred by them, in the sense that the director could get an agreement on the proposal with lower

cost and other directors in the group could also get benefits from the new project. Therefore, if we relabel one of the directors as the management, this could be an explanation on a friendly board, see Adams and Ferreira (2007) for other explanations.

Meanwhile, more surprisingly, we also try to argue that: Information sharing may reveal the director's private information regarding his expectation of the size of the project outcome, resulting in possibly a smaller slice received by the directors who share the information. The conflicts induced from information sharing may prevent collusion among directors in the same interest group [4]. And a possible prediction is that the directors in the majority, even if he is in high ability, might be excluded from the collusion due to the information sharing; and the director in the minority would be included in the coalition. This result gives an answer to the question we mentioned above. The minority shareholders can actually protect themselves by appointing an experienced director to a corporate board.

Another point we want to argue is that a director with a good market reputation may not be retained in the position; however, a director with a bad market reputation may be retained in the position. This is because the information sharing induce a non-symmetric expectation on the return of the new project. Meanwhile, the asymmetric information gives the proposer incentives to take risks to gamble on the minority director being non-efficient type and to propose a project which would give hims a high level of residual. These combined two effects induce a non-linear relation between the market reputation and the turn over of management. In other words, this result suggests that the shareholders may not really care about the performance of the firm, what they need is the highest residual to themselves. This is an inefficient result induced by boardroom politics.

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[4]Another trade-off could be that information sharing may intensify mutual monitoring among the peers. See Alchian and Demsetz (1972), Landier *et al.* (2009), and Acharya *et al.* (2011) for theoretical framework and Li (2013) for empirical tests.

The paper is organized as follows. Section 2 previews the assumptions of the model, while Section 3 provide the main results and intuition. Section 4 presents the model. Section 5 analyzes the benchmark model. Section 6 analyzes the main model and gives the main results. Finally, Section 7 gives comparative statics. Section 8 gives implications of our main results. The last section concludes the paper by discussing several extendible assumptions. The appendix contains all the proofs.

## 1.2   Preview of the Approach

We first give a description of the model, with the goal to explain key assumptions. The paper focuses on distribution of net profit in a firm. To simplify analysis but without loss of generality, we assume that there are three board directors divided into two groups. It could be executive and non-executive or insiders and outsiders or any other divisions. We call the one with more directors as a majority group and the other as a minority group. One member of each group is designated as the group leadership. Under this framework, controlling shareholders are represented by the majority group, and the minority shareholders are represented by directors of the minority group. At the beginning, the leadership of the majority group makes a proposal on the new project. The value of the project to each director is the level of benefit each director receives. The "project" and "benefit" here could arise from many specific decisions. For example, the insider directors may propose and vote for a pay raise which is a cost to the outsider directors; the stockholder may propose a new bond issuance while the debt-holder may vote against. In a more general sense, we can understand the "benefit" as a net profit of a project or a pie that all the board members are competing for. The proposal passes if a majority of the board vote to accept. If not, the proposal is rejected and they get their reservation values.

Each board director is associated with a type, which is a private signal obtained

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by themselves. Every signal carries a piece of information about the consequence of a specified project (or policy). When directors in the same group engage in information sharing, the types are commonly known within each group but are still private information across the groups. One way to think of these signals is that each director has expertise in his own field. For instance, some board directors might be experts in marketing, some may be experts on risk control. And we assume that the outcome or the return of the project is composed of consequences of a bundle of the expertise. The types of different directors will influence the return of the project. For instance, if a director is an expert on marketing, then his expertise might affect the firm's sale in the fast east market. This would be part of the return of the firms' new project. In this case, he might know exactly what is the consequence of a new marketing strategy, but this information is difficult to be observed by others. However, when directors with the same interest engage in information sharing, they would credibly communicate with each other. Here we do not model the detail of the communication process. It could be a required rule by the group which is one feature of team build (e.g., Bolton *et al.* (2013)). Or we can think about a communication game between the directors, such as the one described in Adams and Ferreira (2007).

Let us take note of some important features of the model. First, the goal here is to focus on the role of information sharing and market reputation on the directors' types . So we abstract away from other preferences of different groups, such as the ideology or the risk attitude.

Second, directors make proposals about the level of return to be distributed. That is, they bargain over the dollar amount. For instance, the board member will make an offer in the form of a promise to increase the executive compensation, to give money for charity, etc. He offers these concrete objects (or dollar amounts) instead of simply offering a share of the total ex post return.

In a world of complete information, bargaining over the dollar amount is equivalent to bargaining over the share of the pie. So, while typical formulations, e.g., Rubinstein (1982) or Baron and Ferejohn (1989), study models where bargainers negotiate over the share of the pie, their model is equivalent to one in which bargainers negotiate over dollar amounts. But, this need not be the case when there is asymmetric information. In this case, bargainers may have different expectations about the return of the project, i.e., size of the pie, and these different expectations may correspond to different expected shares of the pie. For example, suppose one director expects the return to be 100 million dollars and another expects 200 million dollars. If the first offers the second 50 million dollars, then the first believes she is making an offer of half the pie, while the second believes she received an offer of one quarter of the pie. Assuming that the bargainers negotiate the share of the pie misses an important strategic implication that arises from this mismatch of beliefs.

In practice, the board directors do make offers in terms of dollar amounts and not shares of the pie. The fact that there is uncertainty about the return introduces an important strategic consideration: A director may offer a proposal that turns out to exceed the return. If this happens, we assume that the proposer, think about him as the chairman of the board, can secure funds, through refinance, for the dollar amounts (or projects) that have been promised to other directors but cannot secure the funds for himself. That is, if the chairman promised a return to other board directors, he must deliver. In practice, the board do request an extension of the return to fulfill projects.

Although the model allows for the proposal to go over return, the equilibrium we solve for has the feature that the proposer makes an offer that does not go above return, i.e., for any realization of the directors' types. Thus, the proposer ensures that he also receives positive return.

## 1.3 Summary of Results

We consider three variants of the model. The first two are benchmarks. In the first one, there is information sharing among directors who represents shareholders of the same interest, but the directors' reservation value from no agreement is constant and there is no uncertainty on the reservation values. In the second one, there is no information sharing among directors, even if they represent shareholders of the same interest. But their reservation values from no agreement depend on the outcome, which are also affected by directors' expertise. In the third one, there is information sharing among directors who represent shareholders of the same interest, and their reservation value from no agreement also depends on outcome and each director's information.

First of all, in all these models, majority voting induces that: To reach an agreement, the proposer only needs to collude with one from the other two. The difference among the three models is: who is included and what is proposed?

In the first model, the proposer, one of the majority directors, would always choose the one with the lowest reservation value — smallest share from safe project. That is because a director would always accept any proposal which gives at least as much as his reservation value. Therefore, the proposer would choose the one with lowest share. In this model, new project would always be selected by the board and agreement is reached for sure.

In the second model, for the proposer, he could gamble on the type of the other two directors. Thus, he could give a proposal which is only enough to cover the reservation value from a non-efficient minority director; or he could pay a higher value to get an agreement for sure. The final decision would depend on the comparison between the returns from the safe project and the new proposed risky project. If the benefit from

the safe project is high enough and the cost to buy off the non-efficient director is low enough, the proposer would take the risk to give a proposal which is only enough to cover the reservation value of a non-efficient director. Otherwise, director 1 would pay a higher value to get an agreement for sure. We find that under some conditions, the proposer would collude with the director who has a good market reputation, no matter if he is a majority director or a minority director. In other words, the director with a good reputation would benefit from colluding with the proposer. If directors have the same market reputation, the proposer would be indifferent between buying off any of the other two. However, if we include ideology into the preference, director 1 would strictly prefer to collude with his group member, the majority director. One feature of this model is that the board may not reach an agreement on the selection of new project.

In the third model, the proposer has the same trade off as the second one. However, due to information sharing, if he wants to get an agreement for sure, his group member would never be symmetric to the minority director. We find that, if minority director's information and reputation could induce a higher expected return from no agreement (the safe project) than the new risky project, the majority director who can propose would strictly prefer to collude with minority director. Meanwhile, he would take the risk to give a proposal which is only enough to cover a non-efficient minority director's reservation value. This is because a high expectation from the safe project and the low cost to collude with a non-efficient minority director give incentives to the majority director to collude with the minority directors. A more subtle issue here is that, in this model, the proposal by the majority director would carry information on his group member's type. However, by pooling his proposals on the group member's type, the majority director could manipulate the minority director's belief and lower the expectation on the outcome, and further lower the cost of collusion.

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We also find that if the market believes that with very high probability, say 99%, one of the majority directors is a non-efficient type, then it is possible that a majority director with proposal power will collude with the minority director, even if he knows that his group member is indeed an efficient type.

Let us understand why. On one hand, if the market believes it is almost true that the director in majority is a non-efficient type, then the minority director would put a low weight on the expected return from an efficient type majority director. This would decrease the minority director's expectation on the return. On the other hand, however, if the market believes that the minority director is an efficient type, then the majority director would put a high weight on the expected return from an efficient type minority director. This would increase the majority director's expectation on the return. Therefore, if the majority leadership wants to reach an agreement immediately, he needs to choose the one with low expected pay off from no agreement. Then the director in the minority group would be preferred. This result is also true, even if the leadership knows exactly that his group member is efficient.

The insights of the model also deliver several testable implications:

- For the proposed risky project, if the probability of getting a good project is high, then the management would prefer a less independent (passive or friendly) board, in the sense that the management could share information with the board. Otherwise, the management would prefer a more independent board, in the sense that no information is shared with the board.

- The likelihood of the CEOs' turn over may not have a linear relation to their market reputation.

## 1.4   The Model

### 1.4.1   Prime

Let us consider an environment with three board directors [5] . They are divided into two groups, $I = \{1, 2\}$, which are called majority directors, and $O = \{3\}$, which is called minority director. Under this framework, we can think about group $I$ as directors who represent controlling shareholders, and group $O$ as a director who represents minority shareholders.

Every director has expertise or knowledge in one field. The expertise would affect the firm's performance. Before board decisions, each director privately observes a piece of information or signal in their own fields. The information can be effective or non-effective in the firm's performance. For convenience, we use "type" to indicate the information or expertise obtained by each director. We assume that the type could be either efficient viz. $\theta_i = 1$, or non-efficient [6] viz. $\theta_i = 0$, where $i \in \{1, 2, 3\}$.

After observing signals, director 1's information is assumed to be publicly revealed to all the other two directors. Director 2 and 3's information is still assumed to be privately observed. However, the market has a belief (or priors) on 2 and 3's types, which is represented by probability distributions:

$$\text{Prob}\{\theta_i = 0\} = \pi_i \in [0, 1] \quad \forall i \in \{2, 3\}$$

The types are independently distributed and the probability is common knowledge among all directors.

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[5]Unlike the literature of corporate board which usually uses board as shorthand for the board minus the management, we would include the management as a director of the board.

[6]Here, the word "efficient" refers to situations like: precise information on the return of project, expertise on marking, strong network, or even vigilant in monitoring, i.e., all the characters inducing a hight level return. Meanwhile, the word "non-efficient" refers to situations like: no information on the return of project, amateur on sale, weak network, or lax in monitoring, i.e., all the characters inducing a low level of return.

The market beliefs can be explained as follows. If director $i$ is famous for his expertise in marketing, then the market would believe that with high probability this director would provide precise information about the sale, which would further improve the firm's performance, i.e., $\pi_i \to 0$. Otherwise, the market would have an opposite belief, i.e., $\pi_i \to 1$.

During board decisions, we allow director 1 to have proposal power on the project (policy) selection, i.e., he can given a proposal on projection selections and a division of the outcome. For convenience, we can think about director 1 as the chairman of the board. But our results is not restricted to this explanation.

**Remark** Several remarks on the set up are listed here.

- From the view of advisory role of corporate board, the types could be explained as the expertise or private network of the directors. For example, director 3 might be an expert in risk control, and director 2, the CEO, might be an expert in marketing. Or we can think about the type as a piece of news on the return of the project. For example, one of the directors might be in charge of the marketing department, so it is reasonable to assume that he could get more accurate information about the new market.

- We can also explain types from the view of monitoring, the type could be vigilant or lax. The vigilant directors would always monitor the management very intensively. However, the lax type would not monitor the management too intensively.

- We can also think about director 1's type as the level of investment from some institutional investors and it is publicly revealed to each director. This explanation is not from the view of information, but it would not affect the analysis and main idea of this paper.

## 1.4.2   Technology

We assume that there are two technologies in this environment. One represents a risky project, the other represents a safe project. For any technology, there is an endowment, $e > 0$, which will be used in production.

If a risky project is proposed and finally selected by the board, then the outcome is represented by a linear technology,

$$y = e + \tilde{\mu}(\theta_2, \theta_3|\theta_1)$$

Here, $\tilde{\mu}(\theta_2, \theta_3|\theta_1)$ indicates the return of a new risky project. It is a function of each director's type. We assume that

$$\tilde{\mu}(\theta_2, \theta_3|\theta_1) = \begin{cases} \mu(\theta_2, \theta_3|\theta_1) & \text{If good project} \\ \\ 0 & \text{If bad project} \end{cases}$$

Here, $\mu(\theta_2, \theta_3|\theta_1)$ is return from a good project, and 0 is return from a bad project. And the probability of getting a good project is

$$\text{Prob\{good project\}} = \rho \in (0, 1)$$

Then the expected return from the risky project would be $\rho\mu(\theta_2, \theta_3|\theta_1)$.

If no risky project is finally selected by board, then we assume a safe project is implemented. The outcome of the safe project is represented by another linear technology,

$$y = e + k(\theta_2, \theta_3|\theta_1)$$

Here, $e > 0$ is endowment, $k(\theta_2, \theta_3|\theta_1)$ is the return of the safe project.

The outcome $y$, no matter from a risky or a safe project, is assumed to be not realized during the board decisions. Directors' interactions would be based on their expectation on others' information.

**Remark** One key feature of these technologies is that the directors provide information or expertise in the board decisions, which would further affect the firm's performance through project selection. In the descriptive and empirical literature, people have already documented that directors provide expertise during board decisions, see Adams *et al.* (2008) for a survey on the literature. This set up is also motivated by the fact that: The provision of advice and "monitoring" to management is one of the top functions of board directors in the United States, see Monks and Minow (1996). And this function has been emphasized to be important in the firm's performance, see Adams *et al.* (2008). Therefore, we explicitly put the function of "advice" and "monitoring" in the production function to catch the point that the board's advice would affect the firm's performance.

For the return of the projects, we assume it is true that,

**Assumption 1.4.1** ( Good project > safe project > Bad project)**.**

$$M > \mu(\theta_2, \theta_3 | \theta_1) > k(\theta_2, \theta_3 | \theta_1) > 0 \qquad \forall \theta_1, \theta_2, \theta_3$$

**Assumption 1.4.2** (Monotonicity & Anonymity )**.**

$$\mu(\theta_2, \theta_3 | \boldsymbol{1}) > \mu(\theta_2, \theta_3 | 0) \quad and \quad k(\theta_2, \theta_3 | \boldsymbol{1}) > k(\theta_2, \theta_3 | 0) \qquad \forall \theta_2, \theta_3$$

*For all $\theta_1$*

$$\mu(\boldsymbol{1,1} | \theta_1) > \mu(\boldsymbol{1}, 0 | \theta_1) = \mu(0, \boldsymbol{1} | \theta_1) > \mu(0, 0 | \theta_1) > 0$$

$$k(\boldsymbol{1,1} | \theta_1) > k(\boldsymbol{1}, 0 | \theta_1) = k(0, \boldsymbol{1} | \theta_1) > k(0, 0 | \theta_1) > 0$$

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The idea of the assumption is that:

- In a good state, the risky project always induces a higher return than other projects; a safe project is always better than a bad project.

- Director 2 and 3's information is symmetric in the contribution to the project or only the aggregate information matters. This means that: It does not matter who is efficient in the production, only the number of efficient type matters; or equivalently, only the number of no-efficient type matters.

- Efficient type always induces higher return and the return is increasing with the number of efficient directors. This assumption has been confirmed by some empirical research, e.g., Vafeas (1999). They found that a more efficient board would induce better firm performance.

- The return is bounded and the loss can not be higher than the initial investment, i.e., no bankruptcy is allowed.

Here we assume $\mu(1, 0|\theta_1) = \mu(0, 1|\theta_1)$, but this is not crucial in the analysis. We just want to be consistent with the assumption that directors 2 and 3 are symmetric in the contribution to the firm's performance. All the results are true if we replace the equality with an inequality in any direction.

**Remark** Several remarks on the return are listed here.

- For convenience, we can think about the returns, $\mu(\theta_2, \theta_3|\theta_1)$ or $k(\theta_2, \theta_3|\theta_1)$, as the NVP of the project. But all the results can be explained in a broader way.

- This set up on the returns further implies that each director's expectation on the output would depend on the other directors' type.

- We do not specify the exact form of return. In general, it could be a determined form, which means there is no uncertainty. It could also be a stochastic form, which means there is uncertainty on the return. The only difference is that, if the return has a stochastic form, director 1 needs to take expectation on the residual. However, the main results do not depend on the form of return.

### 1.4.3 Bargaining

For clarity of the sequential decision, we turn to the timing of the bargaining.

1. Each director privately observes an independent signal $\theta_i$ in their own field. Director 1's information is publicly revealed to the other two directors. Market forms a belief on 2 and 3's type.

2. The proposer, director 1, proposes a risky project and a division of the outcome $(p_2(h^0), p_3(h^0))$, which is a mapping from 1's information set to $R_+^2$. Here $h^0$ is the information set, after which director 1 gives a proposal.

3. After seeing the proposal, all three directors vote on the proposal simultaneously. Voting strategy is a mapping from i's information set to {Yes, No} or {Accept, Reject}.

4. Majority rule determines the result, i.e., if at least two choose to accept, then game over, the risky project is launched and proposal is implemented. Otherwise, the safe project is launched, and every director gets their reservation values from the safe project.

To simplify analysis, the reservation value is assumed to be a third of the outcome from the safe project for each director, which is denoted as,

$$r(y|\theta_1, \theta_2, \theta_3) = \frac{1}{3}\left[1 + k(\theta_2, \theta_3|\theta_1)\right] \quad \forall i = 1, 2, 3$$

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Because board structure changes only infrequently, we assume that over the decisions in the model, the composition of the board and each group stay the same.

As mentioned in the preview section, the proposals might be over the return, i.e., larger than the realization of $y$, then we assume that the proposer, director 1, always gets money outside to guarantee that 2 and 3 get what 1 proposed. And the proposer gets a payoff zero. This represents the fact that the chairman could require investment from other investors or financial market.

### 1.4.4 The Preference

We assume that the directors are all risk neutral. Their preferences are defined as follows: for director 1,

$$
u_1 = \begin{cases} \max\left\{\text{Outcome} - p_2 - p_3, 0\right\} & \text{Agreement} \\ \\ \frac{1}{3} \text{ outcome of safe project} & \text{No Agreement} \end{cases}
$$

If an agreement is reached, and the realized outcome of a risky project is less than what is proposed to the other two directors, i.e., $y \geq p_2 + p_3$, director 1 would keep the residual. Otherwise director 1 would guarantee the other two directors get what was proposed, i.e., $p_2, p_3$, and keep 0 for himself. If no agreement is reached, then director 1 gets the reservation value from the safe project. For director $i \in \{2, 3\}$, the preference is,

$$
u_i = p_i
$$

where

$$
p_i = \begin{cases} \text{proposal} & \text{Agreement} \\[2em] \frac{1}{3} \text{ outcome of safe project} & \text{No Agreement} \end{cases}
$$

This means that: if an agreement is reached, $p_i$ is the proposal to director $i$. If no agreement is reached in the end, $p_i$ is $i$'s reservation value.

**Remark** The reservation value might have different forms. In some situations, if the new project is not selected by the board, the directors may just get the reservation value from an ongoing project, and their values from this project could already be revealed to each other. To simplify analysis, we explain the reservation value as the share from a project with a public observed value on each director's information, and denote the share as $r > 0$, such that $\sum_i r = 1$.

In other situations, if no agreement is reached on the new proposed project, all the directors might work on another new project. This might be proposed by others or just be a safe project. Therefore, the reservation value could also depend on the final outcome from this project. This also induces that the reservation values would depend on all three directors' types. In general, for any $i \in \{1, 2, 3\}$, we denote these reservation values as

$$
r_i(y|\theta_1, \theta_2, \theta_3) \equiv \frac{1}{3}\left[1 + k(\theta_2, \theta_3|\theta_1)\right]
$$

However, in order to simplify the analysis and also make the point, we would assume that the reservation value from this project is a third of the outcome from the new project.

This assumption here is to simplify analysis. In general, what I need is: the share of outcome from the safe project delivered to a minority director is no less than the

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share delivered to director 2, which means a minority director could get more from no agreement than director 2. The main result of this paper is that a majority director will strictly prefer to collude with minority director. In this case, if a minority director gets a smaller share on outcome of the safe project, then it is not surprising for a majority director to collude with him, because he has a smaller reservation value. So to make it interesting, I assume that the minority director and director 2 has the same share from the outcome of the safe project. And in equilibrium, we have: majority director will strictly prefer to collude with a minority director.

If we explain this reservation value as the return from another project (risky or non-risky) which is proposed by another director if the agreement is not reached. Then in this dynamic environment, the reservation value could also be explained as the continuation value from no agreement of that project, see Wu (2013b) for an environment in which this reservation value is endogenously determined as an equilibrium result from continuation bargaining.

### 1.4.5   Information Sharing in the Majority Directors

In order to model information sharing in a group, we assume that after observing signals but before board decisions, directors in the same group would share information with each other. This means that director 2 would honestly report his signal to director 1. We can think about this information sharing as an equilibrium outcome of any communication game between director 2 and director 1. To support this assumption, refer to Adams and Ferreira (2007) for a mechanism about information transmission between board directors. In fact, this information sharing could be supported by any direct mechanism which induces truth telling. However, across the two groups, there is no information sharing, i.e., there is bilateral asymmetric information between director 3 and 2, but there is only one side asymmetric information between

director 3 and 1. To be more specific, we assume director 3 only knows that with probability $\pi_2$, director 2 is a non-efficient type. For director 1 and 2, they know each others' type, but only know that with probability $\pi_3$, 3 is a non-efficient type in his field.

We use information sharing among the group of inside directors as an example because they, as "agents", have very different interests than the outsiders. And the tension between insiders and outsiders is the most important one in most boards (e.g., Fama and Jensen (1983)). It might be easier for majority directors to share information with each other. Empirically, this kind of information sharing among them appears more prevalent and detrimental to the firm. However, this model can possibly apply to any kind of division in the board.

All these are assumed to be common knowledge among all the directors. At the beginning of the game, the above priors on the types are realized. After that, the sequence of bargaining is the same as the one in the timing section.

### 1.4.6    How to Think of Proposals

In the previous section, we described the proposal in a very abstract way. This abstraction could be mapped to different proposals in real board decisions. To demonstrate the relation between the abstract and real world situations, we give a few examples here.

In the first example, we can think about a proposal on the extension of a product line. It is a very common board decision. In reality, the proposal might look like this,

> In the next year, we need to set up a new research department and support some new projects, e.g., iWatch and iCar. To fulfill this project, we need to use \$100,000,000 from our last year's net profit to finance this new risk project. Therefore, this year's return to our shareholders may be limited.

But our management needs more compensation to finish the risky project.

In this example, we can map this new research department plus the new projects to the risky project in the proposal. The return to the shareholders and the compensations can be mapped to the division of outcome from the risky project. Board directors' choice on this project can be mapped to the action of Acceptance and Rejection on this proposal.

The merge policy has the same features. When Microsoft merged with Nokia, the stock price of Microsoft went down, but the stock price of Nokia went up. Then the controlling shareholders who have both firms' stock get positive benefit, but the shareholders who only had Microsoft's stock lost money. We can think about this merge policy as the risky project, the return from stock market as the division of outcome.

Another example is appointment of management. One famous example could go back to Apple's board decision in 1985. John Sculley, who was a board director and CEO of Apple, proposed to the board:

> . . . remove Jobs from his position as Apple Vice President and General Manager of the Mac department. . .

Under this circumstance, the proposal is to remove Jobs from management position. If the board directors had an agreement on this proposal, which is what happened in 1985, Jobs would be removed from the position. This induces that the new risky projects planned by Jobs would not be launched. Then shareholders would receive return from another safe project. In 1985, after the decision of the board, Apple kept on working with IBM to produce more products which were compatible with PC. We can map this to the safe project in our model. Jobs chose to resign and quit Apple and start up a new firm, NeXT. Several new projects launched in NeXT where the

ones planned to launch in Apple, if Jobs has not been removed by the board. This could be mapped to the risky project in our model.

The proposals in the real world could be more complicated than what is described above. The bottom line is that proposals carry directors' information; this information would relate to the outcome of projects and also a division of outcomes. Some directors may benefit from these proposals, some may not. And this raises the conflicts among board directors.

## 1.5 Benchmark

### 1.5.1 Benchmark I: Constant Reservation Value

In the benchmark, we first consider the case in which there is information sharing among the directors in the same group, and all directors would get a constant value from the safe project. The main result is:

**Proposition 1.** *If the directors get constant value from no agreement, then for any beliefs $\pi_i \in (0,1)$, $i = 2,3$ and any $\theta_1$, director 1 would form a coalition with the directors with the lowest share from the safe project.*

The intuition is that: First, by the majority rule, for the proposer, director 1, he only needs to give a proposal to convince one of another two directors. Second, there is only one dimension of uncertainty to director 1, the type of director 3. This uncertainty only affects director 1's expectation on the outcome, but nothing to his cost of buying off any of the other two directors. Third, for director 2 and 3, they would accept any offer which is at least as high as their reservation constant value; and reject otherwise. Therefore, independent from the market belief, director 1 would collude with the one with the lowest reservation value. This result is still true even if there is no information sharing in the group. The key point is that the type

independent reservation value would cancel out all the trade off caused by asymmetric information.

### 1.5.2  Benchmark II: No Information Sharing

In this benchmark, we would consider the case in which the directors in the same group would not share information with each other. The information structure is defined as follows, i.e.,

$$\text{Prob}\{\theta_i = 0\} = \pi_i \in [0,1] \qquad \forall i \in \{1,2,3\}$$

Other than this, the game is exactly the same as the one described in the model section. The main result is as follows.

**Proposition 2.** *Given that there is no information sharing among the directors in the same group,*

- *If return from the safe project is larger than the proposed risky project, and director 2 has a better reputation of being efficient than director 3, i.e.,$\pi_2 \leq \pi_3$, director 1 would collude with director 2 by proposing in favor of him; or director 1 would collude with director 3 if director 2 has a relatively bad reputation of being an efficient director, i.e., $\pi_3 < \pi_2$.*

- *If director 2 and 3 have the same market reputation, director 1 would be indifferent between colluding with director 2 or director 3.*

The intuition is that, by the majority rule, director 1 only needs to convince one of the directors in 2 or 3. Since director 2 and 3's types are private information, director 1 has two choices. He could either gamble on the type of each director by proposing a project which is only enough to cover the reservation value from a non-efficient director; or he can pay a high cost to get a sure agreement. Obviously, the

comparison between the two cases depends on the return from the safe project and the benefit from the new proposed project. If the benefit from the safe project is high enough and the cost to buy off the non-efficient director is low enough, director 1 would take the risk to give a proposal which is only enough to cover the reservation value from the non-efficient director. Otherwise, director 1 would pay a higher cost to get a sure agreement.

In the former case, director 2 and director 3 are not symmetric to director 1. This is because their expectation from no agreement would depend on the other's reputation of being efficient. For instance, if director 3 has a relatively good reputation of being efficient, i.e., $\pi_3 < \pi_2$, director 2's expectation from no agreement would be higher than the one from director 3. Thus, director 1 would collude with director 3. In the latter case, the proposer will be indifferent between buying off any of the other two. This is because the cost of buying off any of them is the same. This induces that it would be indifferent for director 1 to buy off the other two. However, if we include ideology into the preference, director 1 would strictly prefer to collude with his group member, director 2. The idea is that, if the directors also value the ideology, then given the same expected payoff from collusion, the proposers will prefer to buy off the one with the similar ideology preference. Therefore director 1 will buy off his group member in this case. If we reexplain director 2 as the CEO of the firm, and model the turn over of CEO by the exclusion from the coalition or zero benefit from the new proposed project, this result is consistent with current research on market reputation and CEO turn over, see Milbourn (2003). But the intrinsic mechanisms are quite different. In those papers, they emphasize the relation between CEO's compensation and market reputation. A low reputation induces a low wage compensation, so as the probability of being retained in the position. However, our argument purely depends on the internal interactions in the board and the conflicts among the board directors.

## 1.6  Analysis of the Game: Information Sharing and Type Depend Reservation Value

In benchmark I, we assumed that the safe project is independent from the directors' types. In reality this may not be true. If the proposal of a new project is rejected by the board, the board usually runs another project; it could be the one proposed by other directors or another running project which would also be affected by directors' information. Thus, directors may not strictly prefer to collude with the one with the lower share. It would depend on the real amount from the safe project.

In this section, we would show that if the directors' reservation value depends on directors' types and directors in the same group share their private information, then even if they have the same share, majority director 1 would strictly prefer to collude with minority director 3.

Given the belief system,

$$\text{Prob}\{\theta_i = 0\} = \pi_i \in [0, 1] \quad \forall i \in \{2, 3\}$$

The main results are as follows:

**Main Results.** *There exist equilibria in which the majority director who can propose would collude with the minority director who does not share information with him.*

This result is supported by several other propositions. These propositions are shown to be true by construction. The solution concept we are using is Perfect Bayesian Equilibrium and weakly dominance. The construction follows the logic of backward induction. We first go through the interactions in the voting stage. Then we go back to analyze majority director 1's best response in the proposing stage.

### 1.6.1 Analysis of Voting

We first go to the information set of voting game among all three directors. In general, there might be many strategy profiles which could be Bayesian Nash equilibrium in the sub-game of voting. This might complicate our analysis. In order to simplify analysis in a reasonable way, we use the solution concept: weakly dominance to restrict each director's equilibrium strategies in the voting stage. A basic requirement is that a rational director would never choose weakly dominated actions in the voting stage. Formally, we introduce the following definition:

**Definition** An action $a_i \in A_i \equiv \{Yes, No\}$ is weakly dominant for director $i \in \{1, 2, 3\}$ in the voting, if for any $a_i' \in A_i$ $(a_i \neq a_i')$, $a_{-i} \in A_{-i}$ and $\vec{\theta} = (\theta_1, \theta_2, \theta_3)$

$$u_i(a_i, a_{-i}, \vec{\theta}|h) \geq u_i(a_i', a_{-i}, \vec{\theta}|h)$$

with strict inequality for some $a_{-i} \in A_{-i}$, $h = (p_2, p_3)$.

In other words, the "weakly dominant" requires that: given a proposal $h = (p_2, p_3)$ an action $a_i$ in the voting is optimal independently of what others know, which is type/information $\vec{\theta}$ observed by other directors, and what others do in voting $a_{-i}$.

Follow this definition, we have our first prediction as follows,

### Prediction 1:

In the voting stage, each director will

- accept any proposal which is larger than the reservation value given that the uncertainty is realized as efficient type, i.e.,

  For majority directors, they will accept any proposer which is at least as large as $r(y|\theta_1, \theta_2, \theta_3 = 1)$

  For minority, he will accept any proposer which is at least as large as $r(y|\theta_1, \theta_2 = 1, \theta_3)$

- reject any proposal which is smaller than the reservation value given that the uncertainty is realized as non-efficient type, i.e.,

  For majority directors, they will reject any proposer which is no more than $r(y|\theta_1, \theta_2, \theta_3 = 0)$

  For minority director, he will reject any proposer which is no more than $r(y|\theta_1, \theta_2 = 0, \theta_3)$

The proposition supporting this prediction is as follow:

**Claim 1.** *For any belief system $\{\pi_i\}_{i \in \{2,3\}}$ in the sub-game of voting process, and for any history $h = (p_2, p_3)$,*

1. *Given a type profile $(\theta_1, \theta_2)$ it is a weakly dominant action for the majority group members, $i \in \{1, 2\}$, to accept any offer $p_i \geq r(y|\theta_1, \theta_2, 1)$. And it is weakly a dominant action for each of them to reject any offer $p_i < r(y|\theta_1, \theta_2, 0)$.*

2. *Given a $\theta_3$, it is a weakly dominant action for the minority director to accept any offer $p_3 \geq r(y|\theta_1, 1, \theta_3)$. And it is a weakly dominant action for each of them to reject any offer $p_3 < r(y|\theta_1, 0, \theta_3)$. Here, $p_i$ is the proposal to director $i$ and $r$ is $i$'s reservation value from no agreement.*

The intuition of this result is as follows. Majority rule induces that director i's action matters if and only if the other two's actions are split, which means the other two directors have different actions in the voting. Therefore, given a proposal, if director i could get more utility from acceptance, then it is weakly dominant for him to accept. Meanwhile, if director i could get more utility from rejection, then it is weakly dominant for him to reject.

To illustrate the idea of this proof, let us take majority director 2 as an example. Figure 1 demonstrates director 2's utility from different voting strategy profiles. We first define $\bar{r} \equiv r(y|\theta_1, \theta_2, 1)$, which is reservation value from no agreement given that
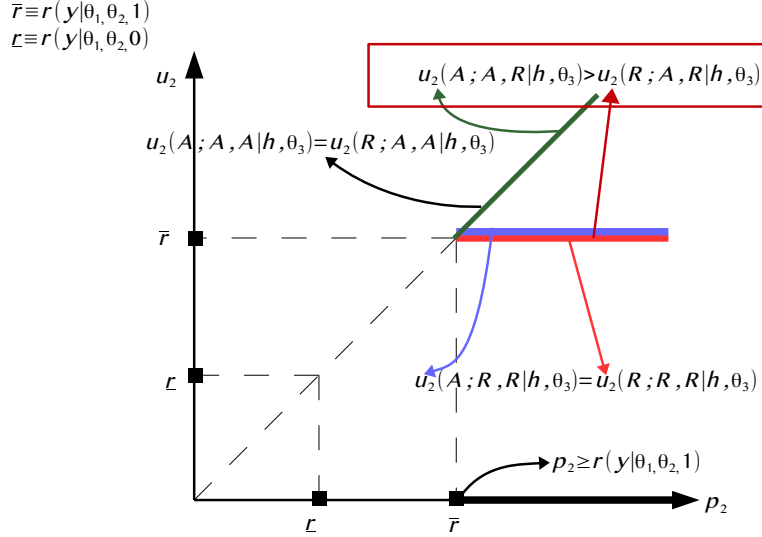
**Figure 1.1:** Weakly dominant of Acceptance

minority director is efficient; and define $\underline{r} \equiv r(y|\theta_1, \theta_2, 0)$, which is reservation value from no agreement given that minority director is non-efficient. The horizontal line, $p_2$, is the proposal to director 2. For any proposal which is larger than $\bar{r}$, if the other two directors choose to reject, then given a proposal $h$ and any information obtained by each director, $(\theta_1, \theta_2, \theta_3)$, director 2's action will not change the voting result. Majority rule induces that no agreement will be reached. For director 2, if he is efficient type, he would get $\underline{r}$.

For any proposal which is larger than $\bar{r}$, if the other two directors choose to accept, then given a proposal $h$ and any information obtained by each director, $(\theta_1, \theta_2, \theta_3)$, director 2's action will not change the voting result. Majority rule induces that: agreement will always be reached. Then director 2 would always get what was proposed, $p_2$.
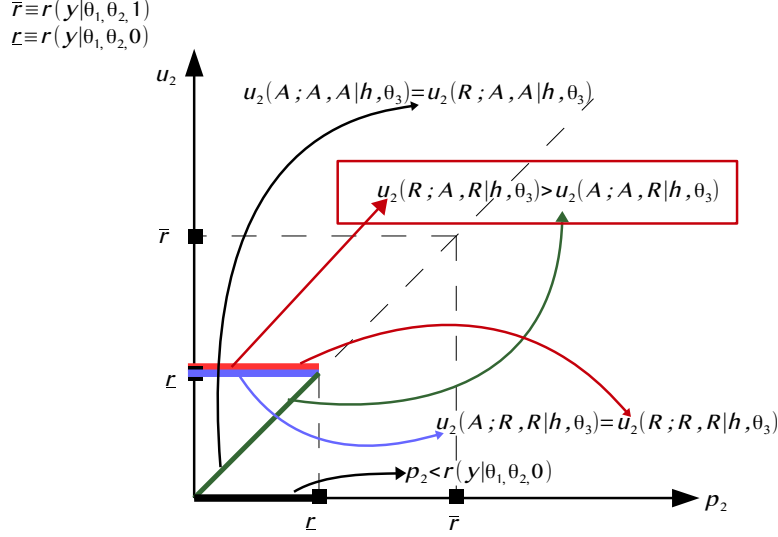
**Figure 1.2:** Weakly dominant of Rejection

However, if one of the other two directors chooses to reject and the other of the two chooses to accept, then director 2's action would affect the outcome of voting. If director 2 chooses to accept, then majority rule induces an agreement, director 2 gets $p_2$. If he chooses to reject, then majority rule induces no agreement in the voting, and director 2 gets reservation value $\bar{r}$ or $\underline{r}$. From figure 1, it is easy to check that, for any proposer less than $\underline{r}$, we always have

$$u_2(A; A, R|h, \vec{\theta}) \geq u_2(R; A, R|h, \vec{\theta})$$

Then follow the definition, we would say: it is a weakly dominant action for the majority director 2 to accept any proposal which is larger than $r(y|\theta_1, \theta_2, 1)$.

For the rejection part, let us refer to figure 2. The coordinates and notations are the same as the previous analysis. For any proposal which is less than $\underline{r}$, if the other two directors choose to reject, then given a proposal $h$ and any information

obtained by each director, $(\theta_1, \theta_2, \theta_3)$, director 2's action will not change the voting result. Majority rule induces that no agreement will be reached. For director 2, if he is non-efficient type, he would get $\underline{r}$. If he is an efficient type, he would get $\bar{r}$.

For any proposal which is less than $\underline{r}$, if the other two directors choose to accept, then given a proposal $h$ and any information obtained by each director, $(\theta_1, \theta_2, \theta_3)$, director 2's action will not change the voting result neither. Majority rule induces that agreement will always be reached. Then director 2 would always get what was proposed, $p_2$.

However, if one of the other two director chooses to reject and the other of the two choose to accept, then director 2's action would affect the outcome of voting. If director 2 choose to accept, then majority rule induces an agreement, director 2 gets $p_2$. If he chooses to reject, then majority rule induces no agreement in the voting, and director 2 gets reservation value $\bar{r}$ or $\underline{r}$. From figure 2, it is easy to check that, for any proposer less than $\underline{r}$, we always have

$$u_2(R; A, R|h, \vec{\theta}) \geq u_2(A; A, R|h, \vec{\theta})$$

Then follow the definition, we would say: it is weakly dominant action for the majority director 2 to reject any proposal which is less than $r(y|\theta_1, \theta_2, 0)$.

For the other two directors' optimal decisions in the voting stage, the same logic will be applied. This proposition is equivalent to the following expression:

**Corollary 1.** *For any belief system $\{\pi_i\}_{i \in \{2,3\}}$ in the sub-game of voting process, and for any history $h = (p_2, p_3)$,*

1. *Given a type profile $(\theta_1, \theta_2)$ it is a weakly dominated action for the majority group members, $i \in \{1, 2\}$, to reject any offer $p_i > r(y|\theta_1, \theta_2, 1)$. And it is a weakly dominated action for each of them to accept any offer $p_i \leq r(y|\theta_1, \theta_2, 0)$.*

2. *Given a $\theta_3$, it is weakly dominated action for the minority director to reject any offer $p_3 > r(y|\theta_1, 1, \theta_3)$. And it is weakly dominated action for each of them to accept any offer $p_3 \leq r(y|\theta_1, 0, \theta_3)$. Here, $p_i$ is the proposal to director $i$ and $r$ is $i$'s reservation value from no agreement.*

### 1.6.2  Analysis of Proposing

Now let us go to the proposing stage. We are going to analyze majority director 1's best response at his information set. We use Perfect Bayesian Equilibrium as the solution concept. Formally we introduce the following definition:

**Definition** A proposal $(p_2^*, p_3^*)$ and voting strategies form a **Perfect Bayesian Equilibrium**, if there exists a belief system such that,

- Director 1's proposal is optimal for each type, given the strategies in the voting.

- Given the belief, all directors' voting strategies respond optimally to each other and every proposal.

- In equilibrium, weakly dominated actions are not allowed in the voting stage.

- Beliefs associated with equilibrium strategy satisfy Bayes rule.

One additional requirement here is that, in equilibrium, given proposals from the proposing stage, each director's equilibrium actions in voting stage can not be weakly dominated. This simply requires that, for each director, any proposal less than $\underline{r}$ should be rejected, and any proposal larger than $\bar{r}$ should be accepted, no matter what happened, what others know, and what others do in voting.

Before going to the decision problem of majority director 1, let us first show the following argument is true.

**Lemma 1.** *The majority director 1's expected payoff is decreasing with the total proposal, $p_2 + p_3$, given to the other two directors.*

The intuition is that when the total proposal increases, on one hand, the probability of getting a positive residual will decrease; on the other hand, the residual kept by the majority director who can propose will also decrease. Then with the assumption [7] in the set up, the expected payoff will decrease too.

The above result can be proved to be true in a more general case, if we relax the assumption as follows,

> *When the total proposal is over the return, the proposer always get a non-positive payoff.*

The intuition is that, when the proposal increases, other than what is mentioned above is true, we also have the following result: the probability of getting a non-positive payoff will be increasing, and because the payoff is non-positive, this induces that the expected payoff will decrease more.

Now, let us go to the information set of director 1, we are going to characterize equilibria which maximizes his expected payoff. Our prediction is:

**Prediction 2:**

> If a minority director has a good reputation of being expertise, the majority director who can propose will collude with a minority director by proposing in favor of him. Majority directors' information may not be revealed and agreement may not be reached.

This is the most surprising result of this paper. Two equilibria would support this prediction. In these results, the majority director 1 would strictly prefer to collude with the minority director who does not share information with him.

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[7] When the total proposal is over the return, the proposer always get a zero payoff.

For the majority director who has proposal power, director 1, the basic trade-off is:

1. He can give a proposal in favor of other majority directors who shared information with him. In this case, agreement will be reached and a new policy/project will be launched. However, he can give a proposal in favor of the minority director, but he might face the risk of no agreement.

2. If he proposes in favor of the minority director, another trade-off is to reveal their information to the minority director or manipulate the minority director's belief through proposal. For instance, if a majority director is an efficient type, he might have incentives to conceal information to lower the minority director's expectation from no agreement. This would induce a low cost of collusion. If the minority director's information could induce high reservation value, he would also take the risk of no agreement and propose a low offer to the minority director.

The next result will support the argument I made in the introduction: The majority director would collude with a minority director by proposing in favor of him.

**Proposition 3.** *If an efficient minority director contributes more in a safe project than in a risky project, s.t., for all $\theta_1, \theta_2$*

$$\underbrace{k(\theta_2, 1|\theta_1) - \rho\mu(\theta_2, 1|\theta_1)}_{\Delta \text{ between safe and risky return}} \geq \frac{1}{2}\left[e + \rho\mu(\theta_2, 1|\theta_1)\right]$$

*then for all $\theta_2$ $\pi_2 \in (0,1)$, $\pi_3 \in (0, \pi^*)$, it is the best response for director 1 to propose*

$$(p_2^*, p_3^*) = \left(0, w(\pi_2, 0|\theta_2)\right)$$

*where*

$$w(\pi_2, 0|\theta_2) = \pi_2 r(y|\theta_1, 0, \theta_3 = 0) + (1 - \pi_2)r(y|\theta_1, 1, \theta_3 = 0)$$

First it is easy to prove that the decision problem of the majority director who can propose can be transferred to the comparison among the costs of different proposals. If he chooses to collude with the other majority director, director 2, he only needs to pay director 2's reservation value from no agreement. Since the minority director's information is not realized, director 2's expectation would be based on the minority director's market reputation, $\pi_3$. And the cost of colluding with director 2 would be, for all $\theta_1, \theta_2$

$$C_2(\pi_3|\theta_1, \theta_2) = \pi_3 r(y|\theta_1, \theta_2, 0) + (1 - \pi_3) r(y|\theta_1, \theta_2, 1)$$

This reads as follows: with probability $\pi_3$, the minority director is a non-efficient type, and because there is no private information between majority directors. So, in order to reach an agreement, the proposer only needs to pay $r(y|\theta_1, \theta_2, 0)$. With probability $1 - \pi_3$, the minority director is an efficient type, then the cost would be $r(y|\theta_1, \theta_2, 1)$.

If the majority director chooses to collude with a minority director, one choice is to give a proposal, which supposes that the minority director is an efficient type. Since director 2's information is not revealed to the minority director, the minority director's expectation from no agreement would be based on the other majority director's market reputation $\pi_2$. This offer would induce an agreement for sure. This cost would be, for all $\theta_1$

$$C_3(\pi_2|\theta_1, \theta_3 = 1) = \pi_2 r(y|\theta_1, 0, 1) + (1 - \pi_2) r(y|\theta_1, 1, 1)$$

Another choice is to give a proposal, which supposes the minority director is a non-efficient type. If it is lucky, i.e., with probability $\pi_3$ he faces a non-efficient minority director, then the majority director only needs to pay

$$\pi_2 r(y|\theta_1, 0, 0) + (1 - \pi_2) r(y|\theta_1, 1, 0)$$

and agreement will be reached. Otherwise, with probability $1 - \pi_3$, there would be no agreement. This is because the minority director is an efficient type, and he has a higher reservation value. This cost of no agreement can be divided to two parts, the first part is the return from agreement, which is

$$e + \rho\mu(\theta_2, 1|\theta_1)$$

It is just the outcome from a risky project, endowment plus return. The second part is the return from no agreement, which is $r(y|\theta_1, \theta_2, 1)$. This is just the outcome from a safe project, or the continuation value from no agreement. By giving this lower level offer, the majority director would fail to get the first part but, would get the second one. Combine these two, we have the cost of no agreement as,

$$C_3(\pi_2, \pi_3|\theta_2, \theta_3 = 0) = \pi_3 \overbrace{\left[ \pi_2 r(y|\theta_1, 0, 0) + (1 - \pi_2) r(y|\theta_1, 1, 0) \right]}^{\text{Cost of agreement}}$$
$$+ (1 - \pi_3) \underbrace{\left[ e + \rho\mu(\theta_2, 1|\theta_1) - r(y|\theta_1, \theta_2, 1) \right]}_{\text{Cost of No agreement}}$$

For convenience, we use

$$L(\theta_2) \equiv e + \rho\mu(\theta_2, 1|\theta_1) - r(y|\theta_1, \theta_2, 1)$$

to denote the cost of no agreement.

Now, I am going to use some figures to demonstrate why the argument is true. The horizontal line is the market belief on the minority director's type, which is $\pi_3$. The vertical line is cost of collusion. Here, $r(y|\theta_1, 1, 1)$ is reservation value for an efficient director 2, given that the minority director is also an efficient type. $r(y|\theta_1, 1, 0)$ is reservation value for an efficient director 2, given that the minority director is non-efficient. $r(y|\theta_1, 0, 0)$ is reservation value for a non-efficient director 2, given that the minority director is also non-efficient.
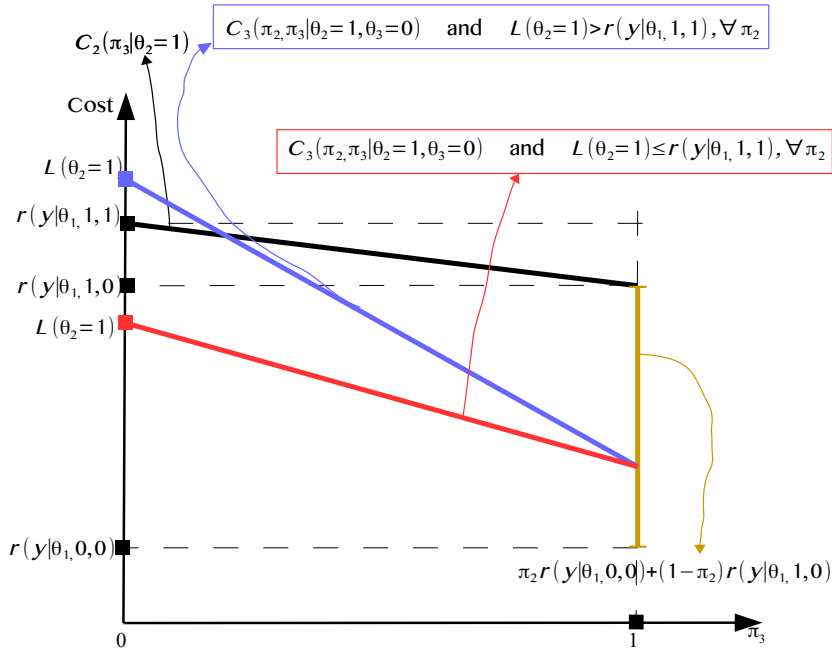
Figure 1.3: Cost for $\theta_2 = 1$

From the previous analysis, we know that the cost depends on a majority director's type, so we will go through the two types. The equilibrium I just described has the feature of polling information, which means that majority director 1 gives the same proposal for director 2 of a different type. Therefore, if we find a common choice for director 2 in a different type, then this choice could be the candidate of equilibrium. Given this candidate, we only need to check if we have a belief system to support it as a Perfect Bayesian Equilibrium.

If majority director 2 is an efficient type, i.e., $\theta_2 = 1$, then the black line is the cost to collude with the other majority director, given that he is efficient. It is the expectation across the minority director's market belief, given that director 2 is efficient. If the majority director tries to gamble on the minority director's type, then the location of cost line would depend on the cost of no agreement, which is denoted by $L(\theta_2)$ and also the market belief of director 2, $\pi_2$. The points on the yellow line
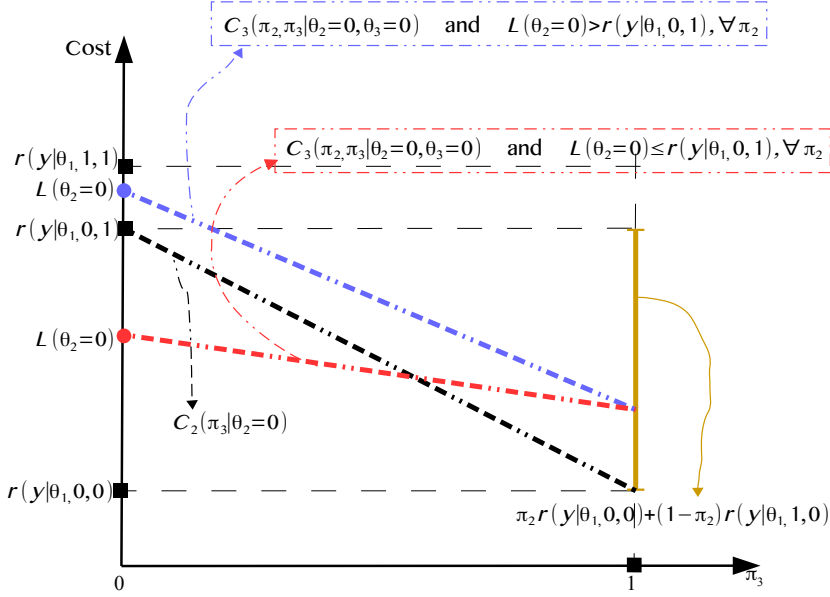
**Figure 1.4:** Cost for $\theta_2 = 0$

indicate the cost of colluding with a non-efficient minority director. This means, if director 3 is non-efficient, i.e., $\pi_3 = 1$, the cost to get an agreement is the expectation on director 2's market belief.

If the cost of no agreement $L(\theta_2 = 1)$ is larger than $r(y|\theta_1, 1, 1)$, which is the cost of colluding with the other majority director, given that the minority is efficient, then we have this blue cost line. It is easy to see that for any $\pi_2$, which means we can move the blue line along this yellow interval, with some value of $\pi_3$, the cost of colluding with a minority director is less than the cost of colluding with the other majority director. However, if the cost of no agreement, $L(\theta_2 = 1)$, is less than $r(y|\theta_1, 1, 1)$, then we have this red cost line. We can see, for any $\pi_2 \in (0, 1)$ and $\pi_3 \in (0, 1)$, the cost of colluding with a minority director is less than the cost of colluding with the other majority director. Now let us go to the case in which majority director 2 is a non-efficient type, i.e., $\theta_2 = 0$. We use the dashed line to indicate the cost. We use

the same coordinate as before. This black dashed line is the cost to collude with the other majority director, given that he is non-efficient. It is the expectation across the minority director's market belief, given director 2 is non-efficient.

If the majority director tries to gamble on the minority director's type, then the location of the cost line would also depend on the cost of no agreement, $L(\theta_2 = 0)$, and also the market belief of director 2, $\pi_2$. It moves on the yellow line with different market belief $\pi_2$. As the previous case, the points on the yellow line indicate the cost of colluding with a non-efficient minority director. This means, if director 3 is non-efficient, i.e., $\pi_3 = 1$, then the cost to get an agreement is the expe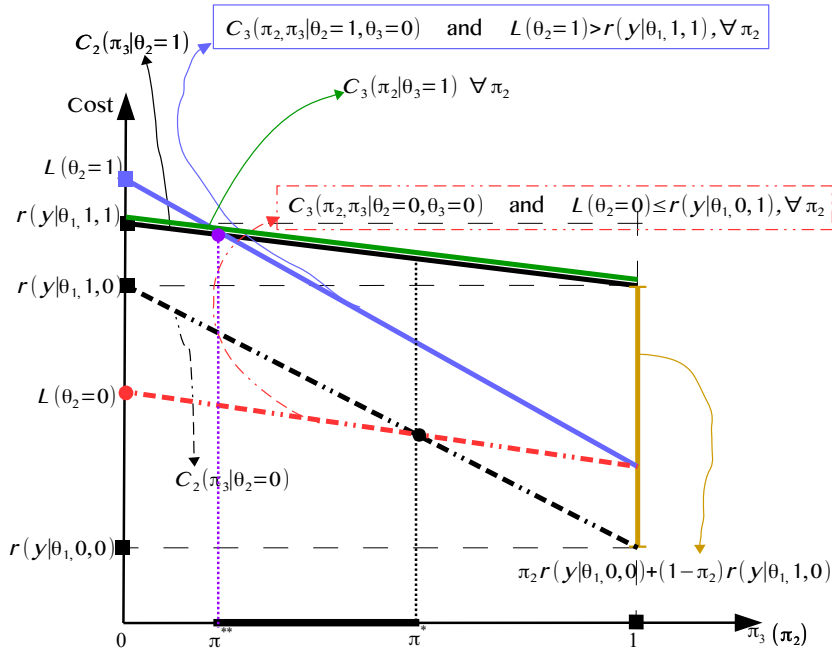ctation on director 2's market belief. If the cost of no agreement $L(\theta_2 = 0)$ is larger than $r(y|\theta_1, 0, 1)$, which is the cost of colluding with a non efficient majority director 2, given that the minority is efficient, then we have this dashed blue cost line. It is easy to see that for any $\pi_2$, which means we can move the blue line along this yellow interval, and any $\pi_3$, the cost of colluding with the minority director is larger than the cost of colluding with the other majority director. If the cost of no agreement, the $L(\theta_2)$, is less than $r(y|\theta_1, 0, 1)$, i.e., the cost of colluding with the other majority director, given that the minority is efficient. Then we have this dashed red cost line. We can see, for any $\pi_2 \in [0, 1]$, there exist some $\pi_3$ under which the cost of colluding with the minority director is less than the cost of colluding with the other majority director. Recall the previous case of $\theta_2 = 1$, it is easy to check that in order to support the argument, we need the cost of no agreement, $L(\theta_2)$, to be less than the cost of colluding with the other majority director, given that the minority is efficient. This is demonstrated by the black and red lines. The continuous line is for $\theta_2 = 1$, the dashed line is for $\theta_2 = 0$. The next step is to check the location of cost of colluding with minority director given that he is efficient. It is just the expectation across director 2's market reputation. We use the green line to indicate this cost. It is easy to check that this

**Figure 1.5:** Cost for all $\theta_2$

cost is above all the red lines for any $\pi_2 \in [0,1]$ and any $\pi_3 \in [0,1]$. Given all these cost lines, we get that, for any $\pi_2 \in [0,1]$, and any $\pi_3$ less than $\pi^*$, it would be the best response for the majority director who can propose to collude with the minority director by gambling on him as a non efficient type. It is because this proposal always induce the lowest cost of collusion.

We can further prove that there exists a belief system to support the above proposal as an equilibrium which satisfies the equilibrium requirement we defined before.

From above analysis, we also observe that the prediction can also be supported by another equilibrium. Recall that if majority director 2 is an efficient type, and director 1 chooses to gamble on the minority director being non-efficient, then the cost of collusion is this continuous blue line. It is easy to check that, for any $\pi_3$ larger than $\pi^{**}$, the cost would be less than other choices.

**Figure 1.6:** Cost for all $\theta_2$

If majority director 2 is non efficient, and the cost of no agreement $L(\theta_2 = 0)$ is less than $r(y|\theta_1, 1, 0)$, then the cost of colluding with the minority director by gambling on him as non-efficient is this dashed red line. And for any $\pi_2 \in [0, 1]$ and $\pi_3$ less than $\pi^*$, this proposal would induce the lowest cost. Put the two cases together. It is easy to check that, under some conditions, for any $\pi_2 \in [0, 1]$, there exist $\pi^{**} < \pi^*$, such that it would be the best response for the majority director who can propose to collude with the minority director by gambling on him as a non efficient type, which means give a proposal equal to the minority director's expectation from no agreement. This intuition is summarized by the following proposition:

**Proposition 4.** *If, for all $\theta_1$, condition $Z(\pi_2, e, \rho)$ is true, and*

$$k(1, 1|\theta_1) - \rho\mu(1, 1|\theta_1) < \frac{1}{2}\left[e + \rho\mu(1, 1|\theta_1)\right]$$

$$k(0, 1|\theta_1) - \rho\mu(0, 1|\theta_1) \geq \frac{1}{2}\left[e + \rho\mu(0, 1|\theta_1)\right]$$

*then there exists $0 < \pi^{**} < \pi^* < 1$, s.t., for all $\theta_2$, $\pi_3 \in (\pi^{**}, \pi^*)$, it is best response for director 1 to propose*

$$(p_2^*, p_3^*) = \left(0, w(\pi_2, 0|\theta_2)\right)$$

*where*

$$w(\pi_2, 0|\theta_2) = \pi_2 r(y|\theta_1, 0, \theta_3 = 0) + (1 - \pi_2)r(y|\theta_1, 1, \theta_3 = 0)$$

$Z(\pi_2, e, \rho)$ *is a condition to guarantee $\pi^{**} < \pi^*$.*

We want to emphasize some features of this result. First, the information sharing might hurt directors in the majority group, in the sense that the majority director who can propose would collude with a minority director by proposing in favor of him. Second, this result indicates that the bargaining may not be efficient in the sense that there might be no agreement between the board directors. Third, majority directors' information will not be revealed. Fourth, from the view of production, a less productive project might be finally selected.

## Prediction 3:

> There is no other equilibrium inducing majority director to collude with a minority director.

And this prediction is supported by next proposition.
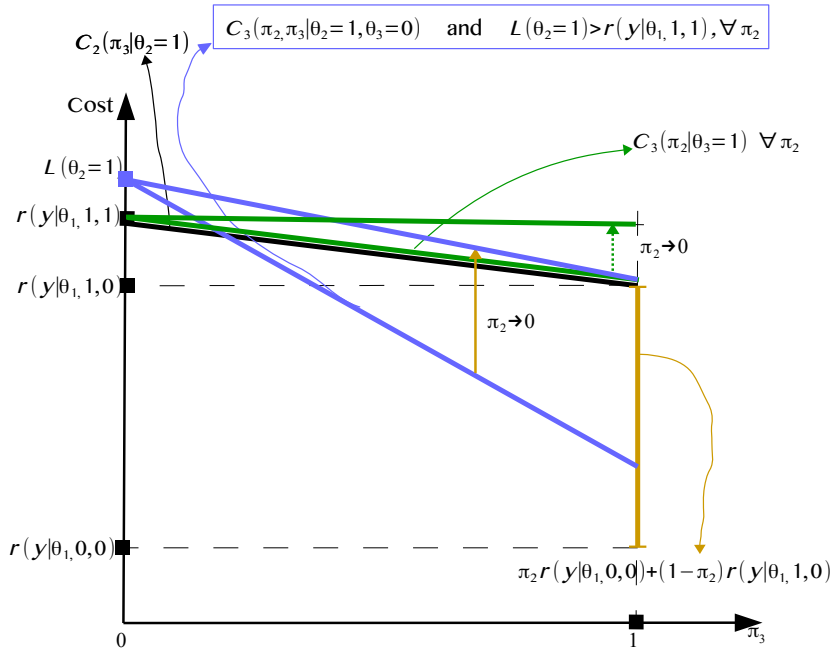
**Figure 1.7:** Cost for all $\theta_2$

**Proposition 5.** *If for all $\theta_1, \theta_2$*

$$\underbrace{k(\theta_2, 1|\theta_1) - \rho\mu(\theta_2, 1|\theta_1)}_{\Delta \text{ between safe and risky return}} < \frac{1}{2}\left[e + \rho\mu(\theta_2, 1|\theta_1)\right]$$

*then for all $\pi_2, \pi_3 \in (0, 1)$, there is no equilibrium, s.t., the proposer will collude with a minority director*

The idea of the proof is as follows. We use the same coordinates as the previous analysis to support this result. If this condition is satisfied, it is equivalent that the cost of no agreement, $L(\theta_2)$ for all $\theta_2 \in \{0, 1\}$, is larger than the cost of colluding with the other majority director. This is true for any $\pi_2 \in [0, 1]$, and $\pi_3 \in [0, 1]$. And the cost of colluding with a minority director, given that he is an efficient type, is also larger than the cost of colluding with the other majority director. The green line is above the dashed black line. If director 2 is efficient, it is possible the cost of

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colluding with a minority director is smaller than other cases. However, in equilibrium, majority director will not collude with a minority director. First, an efficient majority director can not conceal info by proposing in favor of the minority director. This is because, a non-efficient majority director will never collude with a minority director. The dashed blue line is always above the dashed black line. Second, if only the efficient majority collude with a minority director, then in equilibrium the type will be perfectly revealed, which means a minority director knows that with probabilty $1$ $\theta_2 = 1$, if he gets a positive proposal. This induces a high level cost for director 1. Both the blue line and the green line will move above to the black line. Therefore, majority director 1 would only prefer to collude with the other majority director.

The more subtle issue here is that a different expectation on the outcome would perfectly reveal the true type of director 2. This would change minority director 3's belief on director 2's type. Therefore, it would induce a higher cost to collude with the minority director. However, the cost of colluding with his group member, director 2, would always be the same. Thus, it would always be the best response to collude with his group member.

As previous analysis, we can also prove there exists a belief system to support this as an equilibrium.

## 1.7   Comparative Statics

### 1.7.1   Market Reputation

The above analysis is true for a wide range of beliefs, i.e., $\pi_i \in (0, 1)$. However, if we move the value of belief to extreme case, in which $\pi_3 \rightarrow 0$ and $\pi_2 \rightarrow 1$, the equilibrium result would be different. This extreme belief system reads as: director 3 has a reputation of being an efficient type and director 2 has a reputation of being

a bad type. Given this belief system, director 2's expectation would put high weight on the high level outcome, but director 3 would put high weight on the low level outcome. This induces that the cost to include the another majority director to the coalition is higher than the cost of including the minority directors in the coalition. To summarize this intuition, we have the following result:

**Claim 2.** *For all $\theta_2$, if $\pi_3 \to 0$ and $\pi_2 \to 1$, then there exist an equilibrium in which it is the best response for director 1 to include director 3, the minority, in the coalition.*

This proposition is another point to support the argument that the information sharing might hurt directors in the same group. If we think about director 2 as the current management, such as the CEO, and director 3 as a candidate for the replacement of current management, then one message delivered in this proposition is that: Market reputation on directors' type might induce an inefficient outcome of production in the sense that an efficient current management might be replaced by an non-efficient candidate.

One immediate implication from this result relates to CEO turn over in a modern firm. Recall the famous case of Apple in 1985. The board fired the CEO, Steve Jobs, who was director of the board at that time, and the new management, John Sculley, was supported by the board as the new CEO. The above result explains this as follows: the previous performance of Steve Jobs lowered the market evaluation of his ability. This induces that with a high probability Steve might be a non-efficient type, even though he is in fact an efficient type. But the asymmetric information blocks our eyes from seeing the truth. Meanwhile, John Sculley's previous performance in Pepsi gave him a reputation of being an efficient type, even though in fact he was not. According to a previous result, their market reputation changes the board's expectation on the final outcome. Steve Job's reputation induces a relative low cost of making John

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Sculley a coalition member.

### 1.7.2 Risks of the Project

From the previous analysis, we know that the equilibria results depend on the probability that the proposed project is good, $\rho$. It is easy to see that when $\rho \to 1$, i.e., probability of getting a good project is high, then the condition

$$k(\theta_2, 1|\theta_1) - \mu(\theta_2, 1|\theta_1) \geq \frac{1}{2}\left[1 + \mu(\theta_2, 1|\theta_1)\right]$$

would never be true; however, condition

$$k(\theta_2, 1|\theta_1) - \mu(\theta_2, 1|\theta_1) < 0 < \frac{1}{2}\left[1 + \mu(\theta_2, 1|\theta_1)\right]$$

would always be satisfied. Therefore, if we explain director 2 as the management, it is obvious that the information sharing is good to both the management and the aligned board directors, in the sense that they all get their preferred project. Rational directors anticipate this benefit, then they would both prefer a less independent board, which would allow them to share information.

Now, if we let $\rho \to 0$, i.e., the probability of getting a good project is low, then the return from the current safe project is not high enough. Then it might only be true that

$$k(\theta_2, 1|\theta_1) < \frac{1}{2} \quad \text{for some } \theta_1, \theta_2$$

then, the board would not propose a project in favor of the management. Now we introduce the following assumption

$$k(0, 1|1) < \frac{1}{2}, \quad k(\theta_1, 1|0) < \frac{1}{2} \quad \forall \theta_1 \text{ and } k(1, 1|1) > \frac{1}{2}$$

If we explain majority director 1's type as an investment he financed, then this condition reads as: A safe project with low investment or a high investment with lazy

inside directors, $\theta_2 = 0$, all induce a low return from the safe project. However a high level investment with efficient supervision and efficient management would induce a relatively high return.

Under this assumption, our results imply that, given a low level investment, the management may not prefer a less independent board. Meanwhile, a board with a non-efficient inside director 2, $\theta_2 = 0$, may not prefer a less independent board. However, if the investment is high, the management is efficient and the inside director also provides accurate information then the management would prefer a less independent board. This is because, based on our theory, the management would always get the preferred project.

To summarize the above analysis, we have the following result,

**Claim 3.** *If the probability of the good project is high, then the management would prefer a less independent board. However, if the probability of the good project is low, then*

- *given the return or the NVP from the current safe project is low, then the management would not prefer a less independent board.*

- *or given the return or the NVP from the current safe project is high, then the management would also prefer a less independent board.*

To our knowledge, this implication has not been discussed in the literature. A related result is from Song and Thakor (2006) which shows that when the probability of good projects is low, then the board will be biased toward underinvestment. Our results try to connect the board structure to the project selection. For the empirical test, it might be a good idea to check the data of board structure during economic booms and economic downturns. To support our results, we hope the data shows us

that during economic booms, a less independent board would be selected, however, during the economic downturns, another board structure would be selected.

## 1.8    Discussion and Implications

In this part, we want to do comparisons between the results of three models. In the first benchmark model, the proposer's optimal decision would depend on the share delivered to each director, and the one with the lower share would always be preferred. However, this result may not be true if we allow the directors to affect the production. This feature is especially true when we focus on the advisory rule of the board directors. To support this argument, we assume that the share to each director in the safe project is the same, but the results indicate that the proposer would never be indifferent between the other two directors. The key point here is that information sharing induces different expectations of return to each director. Then, even if the directors have the same share on the safe project, they still might have different expectations on the real outcome from a safe project. Therefore, the proposer would strictly prefer to collude with one of them, i.e., propose a project in favor of one of them.

In the second benchmark model, the proposer would either prefer to collude with his group member or to collude with the one with a good reputation of being an efficient director. However, this result may not be true after considering information sharing within a group. Our results indicate that the proposer may also collude with the director who does not have a good reputation of being efficient. Meanwhile, after information sharing, it may never be the best choice for the director to get an agreement for sure, i.e., it would be optimal for him to take a risk to gamble on the minority director's type and collude with him with a low cost. This also induces that after information sharing, for the proposer, it may never be indifferent

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between colluding with his group member and the minority. Our results also have implications to current research in corporate finance. The most related two topics are friendly board and CEO turn over.

### 1.8.1 Directors Information and Board Structure

Fist of all, we can think of the minority director as an outside director. The main result says: If the outside directors can bring information/knowledge to board, and the information can affect the firm's performance, then the shareholders will be protected, even if they only have minority position in board.

If we focus on the advisory role of the corporate board, this model implies that the number of the outside directors may not be the key. What really matters is the information provided by the outside directors. If the information provided by a minority director is more productive in a safe or the safe project. The outside investors or minority shareholders will be protected. At the same time, the asymmetric information may actually protect the minority director and the represented shareholders.

Think about one of the majority directors who is a gray director and he is aligned with management. Then this model tells us that: Boards still can protect shareholders if they include some gray directors who have a conflict of interest, but also bring some information to the board, especially if the boards also have outside directors whose information is more productive/influential. A similar implication was also presented in Baranchuk and Dybvig (2009), even though it comes from the cooperative view.

If we think about the new risk project as a firm's R&D, it implies that the conflicts in the board may actually block R&D. Meanwhile, the project/policy proposed may not be the one inducing the highest expected outcome.

### 1.8.2 Friendly Board —information Sharing Between the Management and Board

In this model, we do not exactly specify each directors' role in the firm. If we explain the proposer, director 1, as the one who represents the largest portion of the shareholders and director 2 as the CEO of the firm, our results imply that friendly (or passive) boards may be preferred by the shareholders and the majority of the board. A similar question has been studied recently in the theoretical literature, e.g., Adams and Ferreira (2007). However, the underground mechanisms are quite different. Our theory explains as follows, a passive board might prefer to select a project or policy in the favor of the CEO. Therefore, the CEO would have incentive to share information with the board directors to benefit himself in the board decisions. Meanwhile, this selection would also be preferred by the shareholder, because this would also maximize the shareholders' expected payoff from the selection of the new project or policy.

Another more surprising message delivered here is that the information sharing may not necessarily benefit the management. Under some conditions this would only be beneficial to the largest proportion of the shareholders. In some sense, the information sharing might hurt the management. This could be a non-efficient result induced by information sharing.

### 1.8.3 Market Reputation and CEO Turn over

In the current literature, research already argue that: On the one hand, for higher evaluation of CEO ability, there is a greater likelihood that the incumbent CEO will stay in position. On the other hand, if the initial evaluation of the incumbent CEO's ability is low, then the likelihood of being retained in the position is also low, see Milbourn (2003).

However, our results want to argue that, after considering the interactions in

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the board, this may not be the case. If we explain director 2 as the current CEO, director 3 as the candidate of the new CEO, and director 1 as the representative of the largest shareholders who also controls the CEO turn over by project (policy) selection, then our results predict that: First, the CEO turn over might both depend on the reputation of the incumbent and the entrant. Our results say that: if the market has low evaluation on the incumbent CEO's ability, but it does not have high evaluation on the candidate CEO's ability either, then the incumbent CEO might be retained in the position. Second, the board might not keep the CEO with high market evaluation on ability. Meanwhile, the board might also keep the CEO even if the market has low evaluation of his ability.

## 1.9    Conclusion

The research status quo has left the working of the board as a black box [8] . What they do has been extensively studied, but not how they do it. How do boards function? What are the mechanics by which they do their jobs? These are the questions raised by the survey papers such as John and Senbet (1998), Hermalin and Weisbach (2001), Adams *et al.* (2008). Every time, however, there were no satisfactory answers. This paper steps into the black box, though a small step, by modeling complex, economic tensions inside the board. In this section, let us summarize our main findings and point out some extendible assumptions.

The main message delivered in this paper is as follows. Unlike previous research on corporate governance which emphasizes majority positions for the outside directors in the board, our main result says: if directors who represent minority shareholders could provide expertise or information in production, especially if these directors

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[8]Only a few theory papers touch on the topic of tensions within the board (e.g., Harris and Raviv (2008) on board control; Galai and Wiener (2008) on power sharing).

also have a good reputation of expertise, then minority shareholders can actually be protected from expropriation by controlling shareholders, even if they only have minority position in board.

Some of our modeling assumptions deserve further discussion. First, we assume that the members representing shareholders of the same interest only share information before board room decisions. However, sometimes, directors with the same interest also help members to coordinate actions. It would be interesting to investigate how this function would affect directors' actions and boardroom decisions. A plausible conjecture is that the coordination might give some benefits to the directors who shared information. To support this result, there might be some inside transfers or some fiduciary responsibility to force the majority director to fulfill this job. However this does not necessarily contradict the results in our model. Because the leaderships' benefit still might be hurt in this case.

Second, in this model, we only consider one shot negotiation among the directors. However, in real board decisions, they might negotiate more than one round. Therefore, a model with multi-round negotiation might be a better description of the board decisions. But this might induce a very complicated signaling process, see Wu (2013b) on an attempt of two rounds negotiation.

Some points in this paper also need further support. One key feature of our model is that the directors with the same interest would share information. From newspaper and anecdotes, we have numerous stories to support this assumption. For instance, in the Walmart's Mexican Bribery Scandal, the so called "gray" directors and the management share information with each other, and get a new project passed and built in Mexico. However, all these are descriptive, we need more precise and detailed evidence about the information they shared and how they formed the coalition. This could help us to better understand the interactions among the board directors. Based

on this situation, methods from experiment economics might be useful, see Gillette *et al.* (2008) for an attempt on the laboratory study on the boardroom communication.

## Appendix

In this part, we give the complete proof of main propositions. The notations are the same as the setup of the model.

**Claim 4.** *For any belief system $\{\pi_i\}_{i \in \{2,3\}}$ in the sub-game of voting process, and for any history $h = (p_2, p_3)$,*

1. *Given a type profile $(\theta_1, \theta_2)$ it is weakly dominant action for the majority group members, $i \in \{1, 2\}$, to accept any offer $p_i \geq r(y|\theta_1, \theta_2, 1)$. And it is weakly dominant action for each of them to reject any offer $p_i < r(y|\theta_1, \theta_2, 0)$.*

2. *Given a $\theta_3$, it is weakly dominant action for the minority director to accept any offer $p_3 \geq r(y|\theta_1, 1, \theta_3)$. And it is weakly dominant action for each of them to reject any offer $p_3 < r(y|\theta_1, 0, \theta_3)$. Here, $p_i$ is the proposal to director $i$ and $r$ is $i$'s reserved value from no agreement.*

*Proof.* To simplify analysis, we first introduce the following notations:

$$\bar{r} \equiv r(y|\theta_1, 1, \theta_3) = r(y|\theta_1, \theta_2, 1)$$

$$\underline{r} \equiv r(y|\theta_1, 0, \theta_3) = r(y|\theta_1, \theta_2, 0)$$

The two equalities are induced by the anonymity assumption.

For any proposal $h = (p_2, p_3)$, if the other two directors choose the same actions in the voting, i.e., either both choose "Yes" or both choose "No", then for director $i \in \{2, 3\}$, we have

$$u_i(a_i; \hat{a}, \hat{a}, \vec{\theta}|h) = u_i(a_i'; \hat{a}, \hat{a}, \vec{\theta}|h)$$

where $a_i, a'_i, \hat{a} \in \{Yes, No\}$ and $a_i \neq a'_i$. This is becuase majority rule induces that director $i$'s action will not affect the result of voting.

However, if the other two directors choose different actions in the voting, i.e., one chooses "Yes", the other chooses "No", then majority rule induces that director $i \in \{2, 3\}$ will be pivotal in the voting. If director $i$ gets a proposal $p_i \geq \bar{r}$, and he chooses "Yes", then we have

$$u_i(Yes; \hat{a}, \hat{a}', \vec{\theta}|h) \geq u_i(No; \hat{a}, \hat{a}', \vec{\theta}|h)$$

where $a_i, a'_i \in \{Yes, No\}$ and $a_i \neq a'_i$. Strictly inequality holds for any proposal $p_i > \bar{r}$. Recall the definition of weakly dominance, then we have: it is weakly dominant for director $i \in \{2, 3\}$ to accept any proposal which is no less than $\bar{r}$.

If director $i \in \{2, 3\}$ gets a proposal $p_i < \bar{r}$, and he chooses "No", then we have

$$u_i(No; \hat{a}, \hat{a}', \vec{\theta}|h) > u_i(Yes; \hat{a}, \hat{a}', \vec{\theta}|h)$$

where $a_i, a'_i \in \{Yes, No\}$ and $a_i \neq a'_i$. Recall the definition of weakly dominance, then we have: it is weakly dominant for director $i \in \{2, 3\}$ to reject any proposal which is less than $\underline{r}$.

For director 1, the same argument will be applied. The only difference is that: if the return has a stochastic form, there is uncertainty on director 1's residual. Therefore, director 1 would take expectation on the proposal. From the set up, we know that, with some probability, the realized outcome is less than the proposal given to the other two director, $p_2 + p_3$, director 1 would get 0. However, if the realized outcome is large than $p_2 + p_3$, director 1 would take the residual. $\square$

**Lemma 2.** *The majority director 1's expected payoff is decreasing with the total proposal, $p_2 + p_3$, given to the other two directors.*

*Proof.* We define the total proposal given to directors 2 and 3 as $z$, i.e., $z = p_2 + p_3$.

Let $0 \leq z' \leq z$, then for any $\theta_1$, director 1's expectation on the residual is

$$g(z'|\theta_2, \theta_3) = \int \text{Prob}\{z' < y|\theta_2, \theta_3\}\left(y - z'\right)dF(y|\theta_2, \theta_3)$$

$$\geq \int \text{Prob}\{z < y|\theta_2, \theta_3\}\left(y - z\right)dF(y|\theta_2, \theta_3)$$

$$= g(z|\theta_2, \theta_3)$$

The first inequality is determined by the following facts. For any $z' \leq z$, we always have $y - z \leq y - z'$ and

$$\text{Prob}\{z < y|\theta_2, \theta_3\} \leq \text{Prob}\{z' < y|\theta_2, \theta_3\} \quad \forall \theta_2, \theta_3$$

Since for all $\theta_2, \theta_3$ and $z \geq 0$, we always have

$$\text{Prob}\{z < y|\theta_2, \theta_3\} \leq 0$$

Then we have above result. $\qquad\square$

**Proposition 6.** *If directors get constant reservation value from no agreement, then for any beliefs $\pi_i \in (0,1)$, $i = 2, 3$ and any $\theta_1$, director 1 would form a coalition with the directors with the lowest share from safe project.*

*Proof.* First, we can prove that: it is weakly dominate action for each director to accept any proposal which is no less than his reservation value, and it is weakly dominate action to reject any proposal which is less than his reservation value. Here we denote these reservation values as $r_i \in (0,1)$, such that $\sum_i r_i = 1$. The proof is a special case of Claim 1, such that, for each director $i$, $\bar{r}_i = \underline{r}_i$. [9] Therefore, the equilibrium proposal would be one of the $r_i$.

---

[9] Under this set up, each director could have different $\bar{r}$ and $\underline{r}$, so we use subscript $i$ to denote the difference.

There is no information sharing between directors in the same group, director 1's information is publicly revealed, and each director's reservation value is fixed, therefore director 1's decision problem is simply do comparison among all the reservation values. The lowest reservation value would induce the highest expected payoff to director 1. It is obvious that this decision would be independent from each director's market reputation.

□

**Proposition 7.** *Given that there is no information sharing among the directors in the same party,*

- *If return from safe project is larger than the proposed risky project, and director 2 has better reputation of being efficient than director 3, i.e.,$\pi_2 \leq \pi_3$, director 1 would collude with director 2 by proposing in favor of him; or director 1 would collude with director 3 if director 2 is in relative bad reputation of being an efficient director, i.e., $\pi_3 < \pi_2$.*

- *If director 2 and 3 has the same market reputation, director 1 would be indifferent between collude with director 2 or director 3.*

*Proof.* First of all, the analysis of voting game is similar to Claim 1. We have it is weakly dominate action for each director to accept any proposal which is at least as large as $\bar{r}$, and it is weakly dominate action for each director to reject any proposal which is no more than $\underline{r}$.

Now, we go to director 1's information set. In this set up, director 2 and 3's reservation values depends on each other's type which is private information. Therefore, in order to get an agreement. Director 1 first needs to choose a director to collude with. Second, he needs to decide the amount given to that director. He has two choices. The first is to give a proposal given that this director is efficient type. Under

the proposal, agreement will be reached for sure. The second choice is to gamble on director 2 or 3's type, and give a proposal which is the same as the reservation value of a non-efficient type.

One trivial case is that director 2 and 3 have the same market belief, then they are symmetric to director 1. Therefore it would be indifferent for director 1 to choose either of them. He only need to decide the amount of the offer.

Now, we go to the non-trivial case in which director 2 and 3 have different market reputation. If director 1 gives a proposal

$$\pi_j r(y|\theta_1, 0, 1) + (1 - \pi_j) r(y|\theta_1, 1, 1)$$

to collude with director $i \in \{2,3\}$, where $j \neq i$ and $j \in \{2,3\}$, which is director $i$'s reservation value given that he is efficient type. Then director 1's expected utility is

$$V - \left( \pi_j r(y|\theta_1, 0, 1) + (1 - \pi_j) r(y|\theta_1, 1, 1) \right)$$

where

$$V \equiv \pi_2 \pi_3 \left( e + \rho\mu(0, 0|\theta_1) \right) + \pi_2(1 - \pi_3) \left( e + \rho\mu(0, 1|\theta_1) \right)$$
$$+ (1 - \pi_2)\pi_3 \left( e + \rho\mu(1, 0|\theta_1) \right) + (1 - \pi_2)(1 - \pi_3) \left( e + \rho\mu(1, 1|\theta_1) \right)$$

which is the expectation on the outcome of a risky project.

If director 1 gives a proposal

$$\pi_j r(y|\theta_1, 0, 0) + (1 - \pi_j) r(y|\theta_1, 1, 0)$$

to collude with director $i \in \{2,3\}$, where $j \neq i$ and $j \in \{2,3\}$, which is director $i$'s reservation value given that he is non-efficient type. Then director 1's expected utility is

$$V - \pi_i \left( \pi_j r(y|\theta_1, 0, 0) + (1 - \pi_j) r(y|\theta_1, 1, 0) \right)$$
$$- (1 - \pi_i) \left[ e + \pi_j \left( \rho\mu(0, 1|\theta_1) - r(y|\theta_1, 0, 1) \right) + (1 - \pi_j) \left( \rho\mu(1, 1|\theta_1) - r(y|\theta_1, 1, 1) \right) \right]$$

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Manipulate the algebra, we find that if it is true that

$$\rho\mu(\theta_2, \theta_3|\theta_1) < r(y|\theta_1, \theta_2, \theta_3)$$

then the proposal which is equal to the reservation value of non-efficient director would induce a higher collusion cost. This further induces that director 1 would give director $i$ a proposal equal to

$$\pi_j r(y|\theta_1, 0, 1) + (1 - \pi_j)r(y|\theta_1, 1, 1)$$

This is also the cost of colluding with director $i$, we can rewrite it as

$$r(y|\theta_1, 1, 1) - \pi_j\left[r(y|\theta_1, 1, 1) - r(y|\theta_1, 0, 1)\right]$$

Given this expression, it is easy to check that: this cost is increasing with $\pi_j$. This means that the cost of colluding with director $i$ is lower than the cost of colluding with director $j$, if $\pi_i \leq \pi_j$. Now we have proved this proposition.

□

**Proposition 8.** *If efficient minority director contributes more in reserved project than risky project, s.t., for all $\theta_1, \theta_2$*

$$\underbrace{k(\theta_2, 1|\theta_1) - \rho\mu(\theta_2, 1|\theta_1)}_{\Delta \text{ between safe and risky return}} \geq \frac{1}{2}\left[e + \rho\mu(\theta_2, 1|\theta_1)\right]$$

*then for all $\theta_2$, $\pi_2 \in (0, 1)$, $\pi_3 \in (0, \pi^*)$, it is best response for director 1 to propose*

$$(p_2^*, p_3^*) = \left(0, w(\pi_2, 0|\theta_2)\right)$$

*where*

$$w(\pi_2, 0|\theta_2) = \pi_2 r(y|\theta_1, 0, \theta_3 = 0) + (1 - \pi_2)r(y|\theta_1, 1, \theta_3 = 0)$$

*Proof.* Now, we go to director 1's information set. First of all, director 1's decision problem can be transferred to the comparison among the cost of colluding with different directors. Director 1 has three choices and the basic trade-off is: he can give a

proposal in favor of other majority directors who shared information with him. In this case, agreement will be reached for sure and a new policy/project will be launched. However, he can also give a proposal in favor of the minority director, but he might face the risk of no agreement in this case. If he proposes in favor of the minority director, another trade-off is: to reveal their information to the minority director or manipulate the minority director's belief through proposals. For instance, if director 2 is efficient type, director 1 might have incentives to conceal this information, and pretend to be non efficient type, this could lower minority director's expectation from no agreement. And also lower the cost of collusion.

If he chooses to collude with the other majority director, director 2, he only needs to pay director 2's reservation value from no agreement. And agreement will be reached for sure. Since minority director's information is not realized, director 2s expectation would based on minority director's market reputation, $\pi_3$. Director 1's expected utility is

$$
\pi_3\bigg(e + \rho\mu(\theta_2, \theta_3|0)\bigg) + (1 - \pi_3)\bigg(e + \rho\mu(\theta_2, \theta_3|1)\bigg)
$$
$$
- \bigg(\pi_3 r(y|\theta_1, \theta_2, 0) + (1 - \pi_3)r(y|\theta_1, \theta_2, 1)\bigg)
$$

If director 1 chooses to collude with minority director, one choice is to give a proposal, which supposes that the minority director is efficient type. Since director 2's information is not revealed to minority director, minority director's expectation from no agreement would be based on the other majority director's market reputation $\pi_2$. This offer would also induce an agreement for sure. Director 1's expected utility is

$$
\pi_3\bigg(e + \rho\mu(\theta_2, \theta_3|0)\bigg) + (1 - \pi_3)\bigg(e + \rho\mu(\theta_2, \theta_3|1)\bigg)
$$
$$
- \bigg(\pi_2 r(y|\theta_1, 0, 1) + (1 - \pi_3)r(y|\theta_1, 1, 1)\bigg)
$$

Another choice is to give a proposal, which supposes the minority director is non-efficient type. If it is lucky, i.e., with probability $\pi_3$ he faces a non-efficient minority director, agreement will be reached with offer which is at least

$$\pi_2 r(y|\theta_1, 0, 0) + (1 - \pi_3) r(y|\theta_1, 1, 0)$$

then he gets

$$e + \rho\mu(\theta_2, \theta_3|0) - \left( \pi_2 r(y|\theta_1, 0, 0) + (1 - \pi_3) r(y|\theta_1, 1, 0) \right)$$

Otherwise, no agreement will be reached, director 1 gets his reservation value, which is

$$r(y|\theta_1, \theta_2, 1)$$

which depends on director 2's type. Combine these two cases, we have director 1's expected utility as

$$\pi_3 \left[ e + \rho\mu(\theta_2, \theta_3|0) - \left( \pi_2 r(y|\theta_1, 0, 0) + (1 - \pi_3) r(y|\theta_1, 1, 0) \right) \right] + (1 - \pi_3) \left[ r(y|\theta_1, \theta_2, 1) \right]$$

We can rewrite this expression as follows,

$$\pi_3 \left( e + \rho\mu(\theta_2, \theta_3|0) \right) + (1 - \pi_3) \left( e + \rho\mu(\theta_2, \theta_3|1) \right)$$
$$- \left( \pi_3 \left[ \pi_2 r(y|\theta_1, 0, 0) + (1 - \pi_2) r(y|\theta_1, 1, 0) \right] \right.$$
$$+ (1 - \pi_3) \left[ e + \rho\mu(\theta_2, 1|\theta_1) - r(y|\theta_1, \theta_2, 1) \right] \bigg)$$

It is easy to check that, the parts before the minus symbol are the same for all the three cases. The only differences are on the parts after the minus symbol. We call these are cost of collusion, and introduce the following notations:

$$C_2(\pi_3|\theta_1, \theta_2) = \pi_3 r(y|\theta_1, \theta_2, 0) + (1 - \pi_3) r(y|\theta_1, \theta_2, 1)$$

which is the cost of colluding with director 2 would be, for all $\theta_1, \theta_2$.

$$C_3(\pi_2|\theta_1, \theta_3 = 1) = \pi_2 r(y|\theta_1, 0, 1) + (1 - \pi_2) r(y|\theta_1, 1, 1)$$

which is the cost of colluding with director 3, given that he is efficient, for all $\theta_1$.

$$C_3(\pi_2, \pi_3|\theta_2, \theta_3 = 0) = \pi_3 \overbrace{\left[ \pi_2 r(y|\theta_1, 0, 0) + (1 - \pi_2) r(y|\theta_1, 1, 0) \right]}^{\text{Cost of agreement}}$$
$$+ (1 - \pi_3) \underbrace{\left[ e + \rho\mu(\theta_2, 1|\theta_1) - r(y|\theta_1, \theta_2, 1) \right]}_{\text{Cost of No agreement}}$$

which is the cost of colluding with director 2, given that he is non-efficient, for all $\theta_1$. The explanation of this third cost is as follows: If it is lucky, i.e., with probability $\pi_3$ director 1 faces a non-efficient minority director, then the cost to director 1 is

$$\pi_2 r(y|\theta_1, 0, 0) + (1 - \pi_2) r(y|\theta_1, 1, 0)$$

and agreement will be reached. Otherwise, with probability $1 - \pi_3$, there would be no agreement. This cost of no agreement can be divided to two parts, the first part is the return from agreement, which is

$$e + \rho\mu(\theta_2, 1|\theta_1)$$

It is just the outcome from risky project, endowment plus return. The second part is the return from no agreement, which is $r(y|\theta_1, \theta_2, 1)$. This is just the outcome from safe project, or the continuation value from no agreement. By giving this lower level offer, the majority director would fail to get the first part but get the second one instead.

To support the argument, we need to find a condition to guarantee that

$$C_3(\pi_2, \pi_3|\theta_2, \theta_3 = 0)$$

is the smallest among all the three costs, and it should be true independently from director 2's type. For convenience, we use

$$L(\theta_2) \equiv e + \rho\mu(\theta_2, 1|\theta_1) - r(y|\theta_1, \theta_2, 1)$$

to denote the cost of no agreement.

1. First we want to argue that: for any $\theta_1, \theta_2, \theta_3$, the relation between $C_2(\pi_3|\theta_1, \theta_2)$ and $C_3(\pi_2, \pi_3|\theta_2, \theta_3 = 0)$ depends on the value of cost of no agreement, $L(\theta_2)$. This is supported by the following arguments.

   From expression of each cost, we know that, for any $\theta_2$, $C_2(\pi_3|\theta_1, \theta_2)$ is decreasing with minority director's market belief, $\pi_3$. This is because $r(y|\theta_1, \theta_2, ) - r(y|\theta_1, \theta_2, 1) < 0$. And it is also true that:

   $$r(y|\theta_1, \theta_2, 0) \le C_2(\pi_3|\theta_1, \theta_2) \le r(y|\theta_1, \theta_2, 1)$$

   For any $\theta_2$, $C_3(\pi_2, \pi_3|\theta_2, \theta_3 = 0)$ could be either decreasing or increasing with minority director's market belief, $\pi_3$ . It depends on the values of $L(\theta_2)$ and director 2's market belief $\pi_2$. If it is increasing with $\pi_3$, this cost reaches the maximal value when $\pi_3 = 1$, and we have, $\forall \pi_3 \in [0, 1]$, , and given $\theta_2 = 1$,

   $$C_3(\pi_2, \pi_3 = 1|\theta_2 = 1, \theta_3 = 0) = \pi_2 r(y|\theta_1, 0, 0) + (1 - \pi_2)r(y|\theta_1, 0, 1)$$
   $$\le r(y|\theta_1, 0, 1) = r(y|\theta_1, 1, 0)$$
   $$\le C_2(\pi_3|\theta_1, \theta_2 = 1)$$

   However, if it is decreasing with respect to $\pi_3$, and $L(\theta_2 = 1) \le r(y|\theta_1, 1, 1)$, then we also have $\forall \pi_3 \in [0, 1]$

   $$C_3(\pi_2, \pi_3 = 1|\theta_2 = 1, \theta_3 = 0) \le C_2(\pi_3|\theta_1, \theta_2 = 1)$$

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This is because, $\forall \pi_2$,

$$\max_{\pi_3} C_3(\pi_2, \pi_3 | \theta_2 = 1, \theta_3 = 0) \leq \max_{\pi_3} C_2(\pi_3 | \theta_1, \theta_2 = 1)$$

and

$$\min_{\pi_3} C_3(\pi_2, \pi_3 | \theta_2 = 1, \theta_3 = 0) \leq \min_{\pi_3} C_2(\pi_3 | \theta_1, \theta_2 = 1)$$

and both are linear in $\pi_3$.

If $L(\theta_2 = 1) > r(y | \theta_1, 1, 1)$, then there exists a $\pi^*$, such that, for any $\pi_3 \in [0, \pi^*]$

$$C_3(\pi_2, \pi_3 | \theta_2 = 1, \theta_3 = 0) \geq C_2(\pi_3 | \theta_1, \theta_2 = 1)$$

and for any $\pi_3 \in [\pi^*, 1]$

$$C_3(\pi_2, \pi_3 | \theta_2 = 1, \theta_3 = 0) \leq C_2(\pi_3 | \theta_1, \theta_2 = 1)$$

This is because, when $L(\theta_2 = 1) > r(y | \theta_1, 1, 1)$, we have, $\forall \pi_2$,

$$\max_{\pi_3} C_3(\pi_2, \pi_3 | \theta_2 = 1, \theta_3 = 0) \geq \max_{\pi_3} C_2(\pi_3 | \theta_1, \theta_2 = 1)$$

and

$$\min_{\pi_3} C_3(\pi_2, \pi_3 | \theta_2 = 1, \theta_3 = 0) \leq \min_{\pi_3} C_2(\pi_3 | \theta_1, \theta_2 = 1)$$

These induce that there exist a point $\pi^{**}$, such that, for any value left to $\pi^{**}$, colluding with minority director is more costly than colluding with majority director 2. However, for any value right to $\pi^{**}$, colluding with majority director 2 is more costly than colluding with minority director. Above analysis is for the case of $\theta_2 = 1$. In order to support the result, we will go through the case of $\theta_2 = 0$.

The same as the case of $\theta_2 = 1$, $C_3(\pi_2, \pi_3 | \theta_2 = 0, \theta_3 = 0)$ could be either decreasing or increasing with minority director's market belief, $\pi_3$. It depends

on the values of $L(\theta_2)$ and director 2's market belief $\pi_2$. If $L(\theta_2 = 0) \geq r(y|\theta_1, 0, 1) = r(y|\theta_1, 1, 0)$, it is easy to check that $C_3(\pi_2, \pi_3|\theta_2 = 0, \theta_3 = 0)$ is decreasing with $\pi_3$, and this cost reaches the maximal value when $\pi_3 = 0$, and we have, $\forall \pi_3 \in [0, 1]$, , and given $\theta_2 = 0$,

$$C_3(\pi_2, \pi_3|\theta_2 = 0, \theta_3 = 0) \leq C_2(\pi_3|\theta_1, \theta_2 = 0)$$

This is because, $\forall \pi_2$,

$$\max_{\pi_3} C_3(\pi_2, \pi_3|\theta_2 = 0, \theta_3 = 0) \geq \max_{\pi_3} C_2(\pi_3|\theta_1, \theta_2 = 0)$$

and

$$\min_{\pi_3} C_3(\pi_2, \pi_3|\theta_2 = 0, \theta_3 = 0) \geq \min_{\pi_3} C_2(\pi_3|\theta_1, \theta_2 = 0)$$

and both are linear in $\pi_3$.

However, if If $L(\theta_2 = 0) < r(y|\theta_1, 0, 1) = r(y|\theta_1, 1, 0)$, then we have that: there exists a $\pi^*$, such that, for any $\pi_3 \in [0, \pi^*]$

$$C_3(\pi_2, \pi_3|\theta_2 = 0, \theta_3 = 0) \leq C_2(\pi_3|\theta_1, \theta_2 = 0)$$

and for any $\pi_3 \in [\pi^*, 1]$

$$C_3(\pi_2, \pi_3|\theta_2 = 0, \theta_3 = 0) \geq C_2(\pi_3|\theta_1, \theta_2 = 0)$$

This is because, when $L(\theta_2 = 1) < r(y|\theta_1, 0, 1) = r(y|\theta_1, 1, 0)$, we have, $\forall \pi_2$,

$$C_3(\pi_2, \pi_3 = 0|\theta_2 = 1, \theta_3 = 0) \geq C_2(\pi_3 = 0|\theta_1, \theta_2 = 1)$$

and

$$C_3(\pi_2, \pi_3 = 1|\theta_2 = 1, \theta_3 = 0) \leq C_2(\pi_3 = 1|\theta_1, \theta_2 = 1)$$

These induce that there exist a point $\pi^*$, such that, for any value right to $\pi^*$, colluding with minority director is more costly than colluding with majority

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director 2. However, for any value left to $\pi^*$, colluding with majority director 2 is more costly than colluding with minority director.

Combine above analysis, one conjecture is that: as long as it is true that, for any $\theta_2$,

$$L(\theta_2) \leq r(y|\theta_1, \theta_2, 1)$$

then there exists a $\pi^* \in (0,1)$, such that, for any $\pi_3 \in [0, \pi^*]$, director 1 maximizes his expected utility by colluding with minority director and give a proposal which is the same as the minority director's reservation value from no agreement, given that minority director is non-efficient.

To check the conjecture, we only need to check: for any $\pi_2$ and given $L(\theta_2) \leq r(y|\theta_1, \theta_2, 1)$, it is true that

$$C_3(\pi_2, \pi_3|\theta_2 = 1, \theta_3 = 0) < C_3(\pi_2|\theta_3 = 1)$$

It is easy to check as long as $L(\theta_2) \leq r(y|\theta_1, \theta_2, 1)$, we always have, for any $\theta_2, \theta_3$, it is true that,

$$C_3(\pi_2, \pi_3 = 0|\theta_2, \theta_3 = 0) = L(\theta_2) \leq r(y|\theta_1, \theta_2, 1) = C_3(\pi_3 = 0|\pi_2|\theta_3 = 1)$$

and

$$C_3(\pi_2, \pi_3 = 1|\theta_2, \theta_3 = 0) \leq r(y|\theta_1, 0, 1) = r(y|\theta_1, 1, 0) = C_3(\pi_3 = 1|\pi_2|\theta_3 = 1)$$

Because these costs are linear in $\pi_3$, so the conjecture is true.

2. Above analysis is on the equilibrium path, to complete the proof, we also need to check off equilibrium path. If director 1 deviates to any other proposals $(p_2', p_3')$ which are different from

$$\left(0, w(\pi_2, 0|\theta_2)\right)$$

where $w(\pi_2, 0|\theta_2) \equiv \pi_2 r(y|\theta_1, 0, 0) + (1 - \pi_2) r(y|\theta_1, 1, 0)$.

If director 1 deviates to $(p'_2, p'_3) = (\hat{p}, 0)$, where $\hat{p} > 0$, in order to get an agreement, the $\hat{p}$ needs to be at least as large as director 2's reservation value. The cost of doing this is $C_2(\pi_3|\theta_1, \theta_2)$. Previous analysis already shows that this cost is higher than the equilibrium proposal.

If director 1 deviates to $(p'_2, p'_3) = (0, \hat{p})$, where $\hat{p} \neq w(\pi_2, 0|\theta_2)$, follow the definition of Perfect Bayesian Equilibrium defined in the paper, we need to specify the posterior belief of minority director. This is because, director 1's proposal might change minority director's belief about director 2's type. To support the equilibrium, we assume that for any proposal $(p'_2, p'_3) = (0, \hat{p})$ where $\hat{p} \neq w(\pi_2, 0|\theta_2)$, minority director's belief on director 2's type is

$$\text{Prob}\{\theta_2|(p'_2, p'_3)\} = \pi \in (\pi_2, 1]$$

This means that if minority director gets an offer which is positive but is different from the equilibrium proposal, then minority director believes that the probability of facing a non-efficient director 2 will increase, i.e., $\pi > \pi_2$.

Now, we are going to prove that, given above off equilibrium path belief, director 1 has no incentive to deviate. If he deviates to any proposal no less than $\pi_2 r(y|\theta_1, 0, 1) + (1 - \pi_2) r(y|\theta_1, 1, 1)$, which is minority director's reservation value from no agreement given that he is efficient type, minority director would accept the proposal for sure. From previous analysis, we know that, for any belief $\pi_2$, this proposal would induce higher cost than the equilibrium proposal.

Now let us go to deviations between 0 and $\pi_2 r(y|\theta_1, 0, 1) + (1 - \pi_2) r(y|\theta_1, 1, 1)$, but different from $w(\pi_2, 0|\theta_2)$. For any proposals satisfying these condition, the

cost of colluding with minority director is

$$C_3(\pi, \pi_3 | \theta_2, \theta_3 = 0) = \pi_3 \left[ \pi r(y|\theta_1, 0, 0) + (1 - \pi) r(y|\theta_1, 1, 0) \right]$$
$$+ (1 - \pi_3) \left[ e + \rho \mu(\theta_2, 1|\theta_1) - r(y|\theta_1, \theta_2, 1) \right]$$

It is easy to check that, for any $\pi > \pi_2$, it is true that,

$$C_3(\pi, \pi_3 | \theta_2, \theta_3 = 0) = \pi_3 \left[ \pi r(y|\theta_1, 0, 0) + (1 - \pi) r(y|\theta_1, 1, 0) \right]$$
$$+ (1 - \pi_3) \left[ e + \rho \mu(\theta_2, 1|\theta_1) - r(y|\theta_1, \theta_2, 1) \right]$$
$$> \pi_3 \left[ \pi_2 r(y|\theta_1, 0, 0) + (1 - \pi_2) r(y|\theta_1, 1, 0) \right]$$
$$+ (1 - \pi_3) \left[ e + \rho \mu(\theta_2, 1|\theta_1) - r(y|\theta_1, \theta_2, 1) \right]$$
$$= C_3(\pi_2, \pi_3 | \theta_2, \theta_3 = 0)$$

Therefore director 1 has no incentive to deviate. One key condition inducing this equilibrium is

$$L(\theta_2) \leq r(y|\theta_1, \theta_2, 1) \quad \forall \theta_2$$

which is equivalent to

$$k(\theta_2, 1|\theta_1) - \rho \mu(\theta_2, 1|\theta_1) \geq \frac{1}{2} \left[ e + \rho \mu(\theta_2, 1|\theta_1) \right] \quad \forall \theta_2$$

Now we have proved this proposition.

$\square$

**Proposition 9.** *If, for all $\theta_1$, condition $Z(\pi_2, e, \rho)$ is true, and*

$$k(1, 1|\theta_1) - \rho \mu(1, 1|\theta_1) < \frac{1}{2} \left[ e + \rho \mu(1, 1|\theta_1) \right]$$
$$k(0, 1|\theta_1) - \rho \mu(0, 1|\theta_1) \geq \frac{1}{2} \left[ e + \rho \mu(0, 1|\theta_1) \right]$$

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*then there exists $0 < \pi^{**} < \pi^* < 1$, s.t., for all $\theta_2, \pi_3 \in (\pi^{**}, \pi^*)$, it is best response for director 1 to propose*

$$(p_2^*, p_3^*) = \left( 0, w(\pi_2, 0|\theta_2) \right)$$

*where*

$$w(\pi_2, 0|\theta_2) = \pi_2 r(y|\theta_1, 0, \theta_3 = 0) + (1 - \pi_2) r(y|\theta_1, 1, \theta_3 = 0)$$

$Z(\pi_2, e, \rho)$ *is a condition to guarantee* $\pi^{**} < \pi^*$.

*Proof.* One key condition to support the previous equilibrium is

$$L(\theta_2) \le r(y|\theta_1, \theta_2, 1) \quad \forall \theta_2$$

From previous analysis, we know that, fix all other conditions, but change one condition to

$$L(\theta_2 = 1) > r(y|\theta_1, \theta_2 = 1, 1)$$

then there exist a point $\pi^{**}$, such that, for any value left to $\pi^{**}$, colluding with minority director is more costly than colluding with majority director 2. However, for any value right to $\pi^{**}$, colluding with majority director 2 is more costly than colluding with minority director.

Recall previous analysis, one conjecture is that: as long as $\pi^{**} < \pi^*$, there is one equilibrium under which director 1 will propose in favor of minority director. Next step is to find if there exists such condition.

First, $\pi^*$ can be determined by

$$C_3(\pi, \pi_3|\theta_2 = 0, \theta_3 = 0) = C_2(\pi_3|\theta_2 = 0)$$

and $\pi^{**}$ can be determined by

$$C_3(\pi, \pi_3|\theta_2 = 1, \theta_3 = 0) = C_2(\pi_3|\theta_2 = 1)$$

Manipulate the algebra, we get

$$\pi^* = \frac{r(y|\theta_1, 0, 1) - L(\theta_2 = 0)}{r(y|\theta_1, 0, 1) - L(\theta_2 = 0) + (1 - \pi_2)\left[r(y|\theta_1, 0, 1) - r(y|\theta_1, 0, 0)\right]}$$

and

$$\pi^{**} = \frac{L(\theta_2 = 1) - r(y|\theta_1, 0, 1)}{L(\theta_2 = 1) - r(y|\theta_1, 0, 1) + (1 - \pi_2)\left[r(y|\theta_1, 0, 1) - r(y|\theta_1, 0, 0)\right]}$$

It is easy to check, as long as

$$L(\theta_2 = 1) - r(y|\theta_1, 0, 1) \le r(y|\theta_1, 0, 1) - L(\theta_2 = 0)$$

we have $\pi^{**} < \pi^*$. We denote above condition as condition $Z(\pi_2, e, \rho)$.

The analysis off equilibrium path is the same as previous proposition, we use the same belief to support this as an equilibrium.

$\square$

**Proposition 10.** *If for all $\theta_1, \theta_2$*

$$\underbrace{k(\theta_2, 1|\theta_1) - \rho\mu(\theta_2, 1|\theta_1)}_{\Delta \text{ between safe and risky return}} < \frac{1}{2}\left[e + \rho\mu(\theta_2, 1|\theta_1)\right]$$

*then for all $\pi_2, \pi_3 \in (0, 1)$, there is no equilibrium, s.t., the proposer will collude with minority director*

*Proof.* From previous analysis, we know that if it is true that

$$L(\theta_2) > r(y|\theta_1, \theta_2, 1) \quad \forall \theta_2$$

then director 1 will not collude with minority director if $\theta_2 = 0$. This is because the cost of colluding with minority director is the lowest one among the three possible cases. If director 2 is efficient, it is possible that for some $\pi_3$, the cost of colluding with

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minority director is less than other cases. However, in equilibrium efficient director 2 will not collude with minority director. This is because, if director 2 is efficient, then director 1 will never collude with minority director. Therefore, if director 1 proposes in favor of minority director, the minority will infer that director 2 is efficient, then $\pi_2 = 0$. This would further induce a higher expectation on reservation value. The cost of colluding wit minority director changes to be

$$
\begin{aligned}
C_3(\pi_2 = 0, \pi_3 | \theta_2 = 0, \theta_3 = 0) =& \pi_3 r(y|\theta_1, 1, 0) + (1 - \pi_3)\Big[e + \rho\mu(\theta_2, 1|\theta_1) - r(y|\theta_1, 1, 1)\Big] \\
\geq& \pi_3 r(y|\theta_1, 0, 1) + (1 - \pi_3) r(y|\theta_1, 1, 1) \\
=& C_2(\pi_3 | \theta_1, \theta_2 = 1)
\end{aligned}
$$

if director 1 tries to gamble on minority director being a non-efficient type. Or, the cost changes to be

$$
C_3(\pi_2 = 0, \pi_3 | \theta_3) = r(y|\theta_1, 1, 1) \geq \pi_3 r(y|\theta_1, 0, 1) + (1 - \pi_3) r(y|\theta_1, 1, 1)
$$
$$
= C_2(\pi_3 | \theta_1, \theta_2 = 1)
$$

if director 1 tries to give a proposal given that minority director is an efficient type. Therefore, colluding with director 2 will be the best choice for director 1. Now, we have proved this proposition. □

Chapter 2

INFORMATION HURTS: DO PARTIES REALLY BENEFIT THEIR MEMBERS?

## 2.1 Introduction

Political parties are an instrumental aspect of legislative politics. They help politicians achieve their goals, whether these goals are to hold office or whether they are legislative goals. A vast literature has explored how parties help politicians achieve each of these goals.

In the formal literature, the idea that political parties help politicians win office goes back, at least, to Downs (1957). Downs viewed political parties as providing a 'brand name' for politicians. This idea was formalized by Alesina and Spear (1987) Alesina (1988), Cox and McCubbins (1993), Snyder Jr and Ting (2002), and Ashworth and de Mesquita (2008). Alesina and Spear (1987) and Alesina (1988) point to a different mechanism by which parties can help politicians win reelection: Because parties are long-lived organizations, they can help short lived politicians commit to implementing electorally attractive policies.

The literature has pointed to two mechanisms by which political parties help politicians achieve their legislative goals. One idea, that goes back to Schwartz (1986) and Aldrich (2011) is that parties help members form winning coalitions and, thereby, pass legislation. ( Jackson and Moselle (2002) can be viewed as a formalization of this idea.) A second idea, formalized by Levy (2004), is that parties help party members commit to particular policies and thereby implement legislation desired by the party members.

From the informational perspective, politicians are always treated as policy expertise. Each of them might possess some information which is about the relationship between policies and their consequences Krehbiel (1992). And one important way in which parties help politicians achieve their legislative goals is by serving as a mechanism for sharing these information among party members. For instance, in the United States, Whips helps the party leadership collect information about party members. Smith *et al.* (2011) report:

> ... [Party] leaders can sometimes pry information from members, lobbyists, and others who want something from them. The whip system and party task forces are often activated to gather and disseminate information. ... By exercising care in granting access to their information, party leaders can affect the strategies of other important players. (Page 157-158)

The idea that information sharing between politicians may help them achieve their legislative goals is also applicable to a more general class of political factions. For instance, in many Communist parties, such as in The Soviet Union or in China, party members are forced to make reports to the party on a monthly basis. The leadership also encourages members to report information of other party members. For instance, the Constitution of the Communist Party of China requires party members to reveal their personal characteristics publicly. Article 5 requires that:

> Party members who recommend an applicant must make genuine efforts to acquaint themselves with the applicant's ideology, character, personal record and work performance ...

In practice, the informational constraints are met by members reporting, not to the party leadership, but to their political factions within the party; see Huang (2006).

The fact that parties (or, more generally, political factions) serve as a mechanism to share information amongst party members is documented in the qualitative literature, e.g., Smith *et al.* (2011). The formal theory literature has not investigated the impact of information sharing on legislative outcomes. At first glance, the effect of information sharing on legislative outcomes may appear positive: By sharing information, party members can more easily propose and pass legislative policies that achieve the legislative goals of the party and, in particular, are preferred by all party members. But, in this paper, I show that there is a second—and potentially detrimental—effect. Sharing information may induce conflicts between politicians of the same party and thereby hurt certain party members.

The main result of this paper establishes that this detrimental effect of information sharing can indeed occur. To establish this, I study the effect of information sharing on distributive politics. I propose a new legislative bargaining model that incorporates the idea that there may be information sharing amongst party members. Absent information sharing, the party leadership will provide legislative pork to their party members. However, with information sharing, there may exist conflicts between party members. This may result in a decreased level of legislative pork for certain party members. Thus, information sharing may be detrimental for (some of) the party's own members. This conclusion stands in contrast to the conventional wisdom that parties help their party members achieve legislative goals.

The paper is organized as follows. Section 2.2 provides a preview of the main assumptions of the model and provides an intuition for the main result. Section 2.3 presents the model. Section 2.4 analyzes the benchmark model. Section 2.5 analyzes the main model and gives the main result. Finally, I conclude the paper by discussing several extendable assumptions. The appendix contains all the proofs.

## 2.2  Preview of the Approach

This section gives a description of the model. The goal is to explain key assumptions and to give an intuition for the main result.

The paper focuses on distributive politics. There are three politicians divided into two parties: A majority party and a minority party. One member of each party is designated as the party leader. In the first period, the leader of the majority party makes a proposal of the level of pork each politician receives. The politicians vote to accept or reject the proposal. The proposal passes if a majority of the politicians vote to accept. If not, the proposal is rejected and a party is randomly recognized to make a new proposal. The leader of the party is the one to actually make the proposal. Again, the politicians vote to accept or reject the new proposal. If the new proposal is also rejected, then the game is over, and they divide the pie equally.

Each politician is associated with a *type*, which is a private signal obtained by the politician. Every signal carries a piece of information about the consequence of a spcified policy. When parties engage in information sharing, the types are commonly known within the parties, but are private information across parties. One way to think of these signals is that each politician is a policy expertise, and the budget is composed of consequences of a bundle of policies. The signals from different policies will influence the expected size of the budget. For instance, one politician might be an experter on the trade policy with China or he might get some private news from some lobbyists. In this case, he might know exactly what is the consequence of that trade policy, but this information is difficult to be observed by others. However, when parties engage in information sharing, the politicians within a party would credibly communicate their information only to other party members.

Let us take note of some important features of the model. First, ideological politics

are absent from this model. This is striking given that ideology is often seen as a key feature of party politics. But, in fact, as highlighted in the Introduction, parties serve many other roles. The goal here is to focus on the role of information sharing. So I abstract away from ideological politics. That said, it is, of course, important to ensure that the analysis is robust to introducing policy preference. Later, I will discuss some choices in the formal analysis that are meant to ensure this form of robustness.

Second, politicians make proposals about the level of pork to be distributed. That is, politicians bargain over the dollar amount. For instance, the proposer will make an offer of legislative pork in the form of a promise to build a bridge, to give money for education, etc. He offers these concrete objects (or dollar amounts) instead of simply offering a share of the total *ex post* budget.

In a world of complete information, bargaining over the dollar amount is equivalent to bargaining over the share of the pie. So, while typical formulations, e.g., Rubinstein (1982) or Baron and Ferejohn (1989), study models where bargainers negotiate over the share of the pie, their model is equivalent to one in which bargainers negotiate over dollar amounts. But, this need not be the case when there is asymmetric information. In this case, bargainers may have different expectations about the size of the pie, and these different expectations may correspond to different expected shares of the pie. For example, suppose one politician expects the budget (i.e., the level of legislative pork) to be 100 million dollars and another politician expects the budget to be 200 million dollars. If the first politician offers the second politician 50 million dollars, then the first believes she is making an offer of half the pie, while the second believes she received an offer of one quarter of the pie. Assuming that the bargainers negotiate the share of the pie misses an important strategic implication that arises from this mismatch of beliefs.

In practice, politicians do make offers in terms of dollar amounts and not shares of the pie. Many Appropriations Bills exceeds the President's budget request or the previous year's funding. [1] This should not happen if politicians bargain over the share of the pie. But, if they bargain over the dollar amount, this is certainly a possibility.

The fact that there is uncertainty about the budget introduces an important strategic consideration: A politician may offer a proposal that turns out to exceed the budget. If this happens, I assume that the proposer can secure funds for the dollar amounts (or projects) that have been promised to other politicians but cannot secure the funds for his own projects. That is, if the proposer promised a bridge to a legislator, he must deliver on that bridge. In practice, committees do request an extension of the budget to fulfill projects. Likewise, Congress does try to request an extension of the budget from the President.

Although the model allows for the proposal to go over budget, the equilibrium I solve for has the feature that the proposer makes an offer that does not go above budget, i.e., for any realization of the politicians' types. Thus, the proposer ensures that he also receives legislative pork.

### 2.2.1   Summary of Results

I consider two variants of the model: One where parties do not engage in information sharing and a second where they do engage in information sharing.

In the benchmark model, there is no information sharing, and politicians' types

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[1] As evidence to support this fact, I give two examples: First, in the fiscal 2004 Labor/HHS/Education Appropriations Bill,The Labor/HHS Bill contained 1,951 projects, an 8 percent increase over last year's 1,805 projects. The projects cost $943 million, 16 percent less than the $1.1 billion in 2003. In addition, 100 percent of the 1,951 earmarks lacked a budget request, and 99.9 percent or 1,950 earmarks were added in conference. Second, In 2006, Yazoo Basin projects are receiving 63.3 percent more than the state of Mississippi received from the entire Energy and Water bill in fiscal year 2005 and have exceeded the President's fiscal 2006 budget request of 28,920,00 by 188 percent.

are private information. Thus, each politician has the same expectation about the level of pork available to distribute. There is an equilibrium where the leadership of the majority party 'buys off' his own party member and they reach immediate agreement. In one such equilibrium, the leadership offers his own party member one-third of the expectation of the pie, and this is exactly what his party member expects to get in the second period, independent of who makes the second-period proposal. There is another such equilibrium where the leadership offers his own party member only one-sixth of the expectation of the pie, as this is exactly what his party member expects to get in the second period.

Importantly, there is also an equilibrium of this game where the leadership of the majority party 'buys off' the member of the minority party and they reach immediate agreement. But, in any such equilibrium, the leadership must be indifferent between between buying off his party member vs. the non-party member. For instance, in any second-period sub-game, there is an equilibrium where each politician—other than the leadership of the majority party—expects to get one-third of the pie. This makes it equally costly for the leadership of the majority party to 'buy off' his own party member vs. non-party member in the first-period. But, in a slight perturbation of the game where the leadership has partisan preferences, he would strictly prefer to buy off his own party member.

Now add the feature of information sharing. The result is quite different: Within the party, the party members have the same information about the expected level of the budget. But, this information is not shared across parties. Thus, politicians from different parties will have different expectations about the level of pork to distribute and this mismatch of expectations can make it cheaper for the leadership to buy off the minority party member in the first period.

The key is that because the members of the majority party have the same expec-

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tations about the size of the pie, it is strictly cheaper for the leadership to buy off his own party member in the second period. As such, if the leadership of the majority party is to reach immediate agreement, it is cheaper to buy off the minority party member earlier on, as the minority party member's expected future benefit is lower.

Let us understand why, in the second period, it is cheaper for the leadership to buy off his own party member. First, consider the case where at least one member of the majority party received bad news about the size of the budget. In this case, each majority member has a lower expectation about the outside option, i.e., disagreement, than the member of the minority party. Thus, it is more expensive to buy off the minority party member. The more subtle case is when both members of the majority party receive good news about the size of the budget. Suppose, contra hypothesis, that, in this case, it is cheaper to buy off the minority party member vs. a majority party member. Then, in this case, the minority party member will infer, from the fact that she was offered a proposal, that both the majority party members received good news. Thus, her expectation of the outside option cannot be lower than the expectation of the majority party members, contradicting the hypothesis that it is cheaper to buy off the minority party member vs. a majority party member.

Note, carefully, this equilibrium is robust with respect to a slight perturbation of the game where the leadership has partisan preferences: When the majority party leader makes a first-period offer to the minority party member, the offer is a unique best response (for the majority leader). As a consequence, even if we perturb the game to give the majority party member small partisan incentives to make offers to his own party member, the equilibrium would still obtain.

### 2.2.2   Related Literature

This paper connects to a large existing literature. I now turn to discuss some connections.

*Internal Organization of Political Parties:* The bulk of the literature on political parties treats the parties as a black-box; that is, they do not delve into the internal workings of the party. This paper moves away from that standpoint. It looks at a feature of how parties operate—namely, by information sharing. Recently, some researchers have explored a different aspect of the internal workings of the party—namely, internal party competition. Caillaud and Tirole (2002) focus on how party competition influences the 'image' of the party and the general election. Persico *et al.* (2011) focus on how factional competition within the party affects the persistence of policy and public spending.

*Bargaining in Stochastic Environments:* A number of papers focus on bargaining in political environments where the size of the pie is stochastic. See Merlo and Wilson (1995), Eraslan and Merlo (2002), and Diermeier *et al.* (2003). In that literature, the size of the pie is determined anew in each period by a sequence of random shocks. That is, the size of the pie tomorrow may differ from the size of the pie today. In each period, all politicians have the same expectation about the size. In my paper, the size of the pie is fixed. Nonetheless, the expected size of the pie may differ from one period to another, in so far as offers made early on influence the beliefs about the size of the pie in later periods. This fact raises a conflict between the politicians' expectations about the size of the pie.

*Information Sharing in Oligopoly Markets:* There is related literature on "information sharing" in oligopoly markets. The literature was pioneered by Novshek and Sonnenschein (1982), Clarke (1983) and Vives (1984). The main theoretical results are summarized in Vives (2001). The idea is that industry-wide agreements allow firms to exchange information about costs, demand conditions etc. They ask whether information sharing increases or reduces expected profits. This paper shares some common features with that literature, but the voting process here is part and parcel of the political application.

## 2.3   The Model

There are three politicians, which are denoted by a set $\{1, 2, 3\}$. They are divided into two parties, $L = \{1, 2\}$, which is called the majority party, and $R = \{3\}$, which is called the minority party. We refer to politician 1 as the leadership of the majority party. Each politician $i \in \{1, 2, 3\}$ either gets a piece of good news viz. $\bar{\theta}$, or a bad news, viz. $\underline{\theta}$, where $\bar{\theta} > \underline{\theta}$. We assume that the type $\theta_i$ is private information to each politician. The prior on the type is

$$Prob\{\theta_i = \bar{\theta}\} = \frac{1}{2} \qquad \forall i \in \{1, 2, 3\}$$

This probability is commonly known by all politicians and the types are independently distributed.

In each period, one politician gives a proposal. And after that all the politicians vote to Accept or Reject the proposal. Majority rule determines the voting results. Here, the proposal is the level of pie which will be distributed to the other two politicians. The pie is an output of a linear technology,

$$y = x + \epsilon$$

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where $x > 0$ , $\epsilon \in [-\frac{x}{2}, \frac{x}{2}]$ is a random variable. The size of pie $y$ is not realized until the end of the game.

Politician's type is the signal about the size of the pie. Good news is assumed to indicate a large size of pie and the bad news is assumed to indicate a small size of pie. The signals only affect the expectation on the size of pie through the random variable $\epsilon$, in the sense that the type profile $(\theta_1, \theta_2, \theta_3)$ is the parameters of the distribution of $\epsilon$. Specifically, I am going to assume $\epsilon$ is a random drawn from a continuous CDF which is conditional on type profiles $(\theta_1, \theta_2, \theta_3)$:

$$F(\epsilon|(\theta_1, \theta_2, \theta_3)) = \int f(\epsilon|(\theta_1, \theta_2, \theta_3))d\epsilon \qquad \forall \theta_i \in \{\underline{\theta}, \bar{\theta}\}$$

We further assume that the politicians' information are anonymous to the probability distribution. In other word, it does not matter who has the good news on what policy, only the number of good news matters; or equivalently, only the number of bad news in the type profiles matters. For instance, if both politicians 1 and 2 have good news on their specialized policies and 3 has bad news on his specialized policy, then we have the same distribution on $\epsilon$ as the case in which politician 1 and 3 have good news on their specified policies and 2 has bad news on his specified policy. From this assumption, we can simplify the eight possible type profiles to four, which are

$$(\underline{\theta}, \underline{\theta}, \underline{\theta});$$
$$(\underline{\theta}, \underline{\theta}, \bar{\theta}) = (\underline{\theta}, \bar{\theta}, \underline{\theta}) = (\bar{\theta}, \underline{\theta}, \underline{\theta});$$
$$(\underline{\theta}, \bar{\theta}, \bar{\theta}) = (\bar{\theta}, \underline{\theta}, \bar{\theta}) = (\bar{\theta}, \bar{\theta}, \underline{\theta});$$
$$(\bar{\theta}, \bar{\theta}, \bar{\theta})$$

Another assumption I want to introduce is that, the expectation on the $\epsilon$ is bounded and decreasing with number of high type in the type profiles, to be more specific, we

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have, [2]

$$\frac{x}{4} \geq \mathbb{E}_\epsilon(\epsilon|\bar{\theta},\bar{\theta},\bar{\theta}) \geq \mathbb{E}_\epsilon(\epsilon|\underline{\theta},\bar{\theta},\bar{\theta}) \geq 0 \geq \mathbb{E}_\epsilon(\epsilon|\underline{\theta},\underline{\theta},\bar{\theta}) \geq \mathbb{E}_\epsilon(\epsilon|\underline{\theta},\underline{\theta},\underline{\theta}) \geq -\frac{x}{4}$$

The idea here is that, high type always induce high expectation on the size of pie.

### 2.3.1  Timing

For clarity of the sequential decision, we turn to the timing of the game.

**In the first period , t=0,**

1. Nature determines the realization of each politician's type and distribution of the random variable $\epsilon$. These realizations are privately observed by the politicians.

2. The majority party leadership, politician 1, gives a proposal $(p_2(h^0), p_3(h^0))$, which is a mapping from 1's information set to

$$[0, \frac{3x}{2}] \times [0, \frac{3x}{2}]$$

s.t.

$$p_2(h^0) + p_3(h^0) \leq \frac{3x}{2}$$

where $h^0$ is the information set, after which politician 1 gives a proposal. $3x/2$ is the largest realization of the pie.

3. After seeing the proposal, $i \in \{1,2,3\}$ votes on the proposal simultaneously. Voting strategy is a mapping from i's information set to {Accept, Reject}.

4. Majority rule determines the result. If at least two choose to accept, then game over, the proposal is implemented. Otherwise, they go to the second period.

---

[2]From now on, we are going to use $\mathbb{E}_X(X|\theta_1,\theta_2,\theta_3)$ to denote the expectation of random variable $X$ conditional on type profile $(\theta_1,\theta_2,\theta_3)$.

**In the second period , t=1,**

1. With probability $1/2$, each party is recognized as the new proposer in the second period. If the majority party is recognized, we assume only the leadership has the right to give proposals.

2. Proposer $i \in \{1, 3\}$ gives a proposal, $(p_j(h^1), p_k(h^1))$: a mapping from i's information set to

$$[0, \frac{3x}{2}] \times [0, \frac{3x}{2}]$$

   s.t.

$$p_j(h^1) + p_k(h^1) \leq \frac{3x}{2}$$

   where $j, k \in \{1, 2, 3\}$, but $i \neq j \neq k$. $h^1$ is the information set, after which politician i gives a proposal in the second period.

3. After seeing the proposal, $i \in \{1, 2, 3\}$ votes on the proposal simultaneously. Voting strategy is a mapping from i's information set to {Accept, Reject}.

4. Majority rule determines the result. If at least two choose to accept, then game over, the proposal is implemented. Otherwise, they all get a third of the realized pie.

As mentioned in the preview section, the proposals might be over the budget, i.e., larger than the realization of $y$, then we assume that proposer $i$ always gets money outside to guarantee that $j$ and $k$ get what $i$ proposed. And the proposer $i$ gets a payoff zero.

## 2.3.2 The Preferences

We assume the politicians are all risk neutral. The preference are defined as follows,

$$u_i = \delta^t p_i^t$$

where the discount factors $\delta \in (0, 1)$, $t \in \{0, 1\}$, $i \in \{1, 2, 3\}$. If an agreement is reached, $p_i^t$ is the proposal to i at time t. If no agreement is reached in the end, $p_i^t$ is a third of the realized $y$. If $i$ is proposer and proposal is over the budget, $p_i^t = 0$.

## 2.3.3 Information Sharing in a Party

In order to model information sharing in a party, we are going to assume politician 1 and 2's types are public information within the majority party, but it is private information across parties. And also, 3' type is private information across parties. To be more specific, I am going to assume: 3 only knows

$$Prob\{\theta_1 = \bar{\theta}\} = \frac{1}{2} \quad \text{and} \quad Prob\{\theta_2 = \bar{\theta}\} = \frac{1}{2}$$

1 and 2 know

$$Prob\{\theta_3 = \bar{\theta}\} = \frac{1}{2}$$

and all these are assumed to be common knowledge among all the politicians. At the beginning of the first period, the above priors on the types and the distribution on the $\epsilon$ are realized. After that the sequence of bargaining is the same as the one in the timing section.

## 2.4 Benchmark

As a benchmark, we assume that no party engages in the information sharing, i.e., there is no information sharing between politicians in the same party. The information

structure is the same as the one defined in the model set up, i.e.,

$$Prob\{\theta_i = \bar{\theta}\} = \frac{1}{2} \qquad \forall i \in \{1, 2, 3\}$$

Other than this, the game is exactly the same as the one described in the model section.

The main result of the benchmark model is as follows. There exists one equilibrium, such that, if the majority party has the proposal power, then the majority party leadership is going to buy off his party member. Thereby an agreement is reached immediately by the majority rule.

The intuition is that, given the set up in the benchmark model, all the other two politicians are symmetric to the proposer. In this case the proposer will be indifferent between buying off any of the other two. This induces the possibility of multiple equilibria. But we focus on the equilibrium in which leadership is going to buy off his party member. This is because the specified equilibrium is robust if we include policy into the preference. The idea is that, if the parties also value the policy, then given the same expected payoff from distributive politics, the proposers will prefer to buy off the one with the similar policy preference. Therefore the leadership will buy off party member in this case.

## 2.5   Results

The main result of the model with information sharing is as follows. There exists one equilibrium, such that, if the majority party has the proposal power in the first period, then the majority party leadership is going to buy off the minority party member. Thereby an agreement is reached immediately by the majority rule. If delay happens (off equilibrium path), leadership of the majority party will strictly prefer to include the party member in the winning coalition.

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Two points in this result support the argument of the paper — dynamics and information sharing hurts party members' benefits.

The first one is what happens off equilibrium path. Key difference between this model and the benchmark model is that the information structure of two parties are not symmetric anymore. This induces that, in any period, from the view of the proposer, the other two politicians are not symmetric anymore. If no agreement is reached in the first period, then in the second period, the leadership of the majority party strictly prefers to buy off his party member. This is because, in the equilibrium I constructed, the politician in the minority party has a very high expected payoff on the size of the pie, this will raise the cost of buying off the politician in the minority party. So the leadership of the majority will strictly prefer to buy off his party members.

However, the expectation of the politician in minority party will be relative lower in this case than the benchmark model. This is because, the information sharing makes the politicians in the majority party know each other's type, which might induce a high expectation of the size of the pie, therefore a high reserved value. This induces a high cost of buying of one of the politicians of the majority party. But the minority party politician's expectation on the the size of the pie is still the same as the benchmark model. Therefore, if the minority party is recognized as the proposer, he will get a lower expected pay off than the benchmark case.

The second one is the equilibrium proposal in the first period. the majority leadership will exclude party members from winning coalition, no matter what the types are. The intuition of this result is as follows. As analyzed before, if delay happens, the information sharing, on the one hand, will raise the majority party members' expectation from delay. On the other hand, this will lower other minority pary members' expectation from delay. In this case if the majority leadership has the

proposal power in the first period, it will be more beneficial for him to sacrifice his party member by including the politician from the minority party.

The proof of this result is shown to be true by construction. The solution concept we are using is Perfect Bayesian Equilibrium and weakly dominance. The construction follows the logic of backward induction. In next part, we go through the idea of the main results. The details of the proof are in the appendix.

### 2.5.1  Analysis in the Second Period

As described in the timing section, in each period, one politician first gives a proposal, then all three vote to accept and reject on the proposal. Therefore, we first analyze what happens in the voting process in the second period. We have the following propositions:

**Proposition 11.** *In t=1, in the sub-game of voting process, for any history*

$$h^2 = (h^1, p_k^{t=1}, p_l^{t=1})$$

*where $k, l \in \{1, 2, 3\}, j \neq k$*

$$h^1 = (p_2^{t=0}, p_3^{t=0}; \text{At least two choose Rejection})$$

*and for any belief system,*

1. *Given a type profile $(\theta_1, \theta_2)$, it is weakly dominant action for the majority party members, $i \in \{1, 2\}$, to accept any offer*

$$p_i^{t=1} \geq \frac{1}{3} \mathbb{E}_y(y | \theta_1, \theta_2, \bar{\theta})$$

   *And it is weakly dominant action for each of them to reject any offer*

$$p_i^{t=1} < \frac{1}{3} \mathbb{E}_y(y | \theta_1, \theta_2, \underline{\theta})$$

2. *Given a $\theta_3$, it is weakly dominant action for the minority party member 3 to accept any offer*

$$p_3^{t=1} \geq \frac{1}{3}\mathbb{E}_y(y|\bar{\theta}, \bar{\theta}, \theta_3)$$

*And it is weakly dominant action for each of them to reject any offer*

$$p_3^{t=1} < \frac{1}{3}\mathbb{E}_y(y|\underline{\theta}, \underline{\theta}, \theta_3)$$

Here, $p_i^{t=1}$ is the proposal to politician $i$ in $t=1$ and $\frac{1}{3}\mathbb{E}_y(y|\theta_1, \theta_2, \theta_3)$ is the expectation from no agreement, given a type profile $(\theta_1, \theta_2, \theta_3)$.

The intuition of this result is that, no matter what actions are chosen by each politicians in the sub-game of voting, we only have two possible results from the voting — agreement or no agreement. Given this fact, we know that with some probability there is an agreement, then the politicians from majority party will get what proposed, say $p_i^{t=1}$, $i \in \{1, 2\}$. However, with some probability there is no agreement. If no agreement is reached, and suppose he faces a minority member with low type, then his expectation from no agreement will be a third of the realized outcome, give that 3 is low type, which is $\frac{1}{3}\mathbb{E}_y(y|\theta_1, \theta_2, \underline{\theta})$. If the politician in minority party is hight type, then his expectation from no agreement will be a third of the realized come , give that 3 is hight type, which is $\frac{1}{3}\mathbb{E}_y(y|\theta_1, \theta_2, \bar{\theta})$. This induces that, the expected payoff of the politicians in the majority party is a convex combination of any two from above three values,

$$p_i^{t=1}, \quad \frac{1}{3}\mathbb{E}_y(y|\theta_1, \theta_2, \underline{\theta}) \quad \text{and} \quad \frac{1}{3}\mathbb{E}_y(y|\theta_1, \theta_2, \bar{\theta})$$

Therefore, if the politicians in the majority party get a proposal $p_i^{t=1}$ which is larger than the maximal element of the three, he will accept. Meanwhile, if the proposal is smaller than the minimal element of the three, he will reject.

For the politicians in minority party, the intuition is very similar to above argument. The difference is that, if no agreement is reached, with some probability 3 is facing two politicians in low types, one in high type one in low type, or both in high type. This induces that, 3's expected payoff from the sub-game of voting is a convex combination of any three of the following four values,

$$p_3^{t=1}, \quad \frac{1}{3}\mathbb{E}_y(y|\underline{\theta}, \underline{\theta}, \theta_3), \quad \frac{1}{3}\mathbb{E}_y(y|\underline{\theta}, \bar{\theta}, \theta_3) \quad \text{and} \quad \frac{1}{3}\mathbb{E}_y(y|\theta_1, \theta_2, \bar{\theta})$$

Therefore, if the politicians in the majority party get a proposal $p_3^{t=1}$ which is larger than the maximal element of the three, he will accept. Meanwhile, if the proposal is smaller than the minimal element of the three, he will reject.

Now let us go back to the information set of the proposer in the second period. Before going to the best response of the proposer, let us first show the following argument is true.

**Lemma 3.** *In any period, the proposer's expected payoff is decreasing with the total proposal given to the other two politicians.*

The intuition is that when the total proposal increase, on one hand, the probability of getting a positive residual will be decreasing; on the other hand, the residual kept by the proposer will also decrease. Then give the assumption [3] in the set up, the expected pay off will be decreasing.

Above result can be proved to be true in a more general case. If we relax the assumption as follows,

> When the total proposal is over the budget, the proposer always get a non positive pay off.

---

[3] When the proposal is over the budget, the proposer always gets a zero payoff.

The intuition is that, when the proposal increase, other than what mentioned above is true, we also have the following result: the probability of getting a non positive pay off will be increasing, and because the pay off is non positive, this induce that the expected pay off will decrease more.

Now let us go to the proposers' information set in the second period. Because the random selection rule will give all the politicians a chance to be the proposer in the second period. We will first go to the case in which the politicians in the majority have the proposal power, then we will go to the case in which the minority party has the proposal power.

In the equilibrium we constructed, agreement is reached immediately in the first period. Therefore all the analysis in the second period is on the off equilibrium path. Follow the most popular definition of Perfection Bayesian Equilibrium (Furderberg and Tirole 1991), we could give any belief here. However, we are going to assume that the beliefs will be the same as the prior, which is 3 only knows

$$Prob\{\theta_1 = \bar{\theta}\} = \frac{1}{2} \quad \text{and} \quad Prob\{\theta_2 = \bar{\theta}\} = \frac{1}{2}$$

1 and 2 knows

$$Prob\{\theta_3 = \bar{\theta}\} = \frac{1}{2}$$

and all these are assumed to be common knowledge among all the politicians. The reason here is that, there is no information revealed at the beginning of the second period. Thus one reasonable belief should be the same as the prior. However, the main result is robust to the change of this belief. The main result still holds for many other beliefs. The only difference is the equilibrium proposal given in the second period.

**Proposition 12.** *In t=1, given that the posterior beliefs are the same as the prior, and given the subsequent strategies in the voting process, if the majority party has the*

*proposal power, then for any history*

$$h^1 = (p_2^{t=0}, p_3^{t=0}; At\ least\ two\ choose\ Rejection)$$

*For any $(\theta_1, \theta_2)$, it is best response for the majority party leadership, politician 1, is to propose*

$$(p_2^{t=1}, p_3^{t=1}) = (w(\theta_1, \theta_2), 0)$$

*where*

$$w(\theta_1, \theta_2) = \frac{1}{3}\mathbb{E}_{y,\theta_3}(y|\theta_1, \theta_2, \theta_3)$$
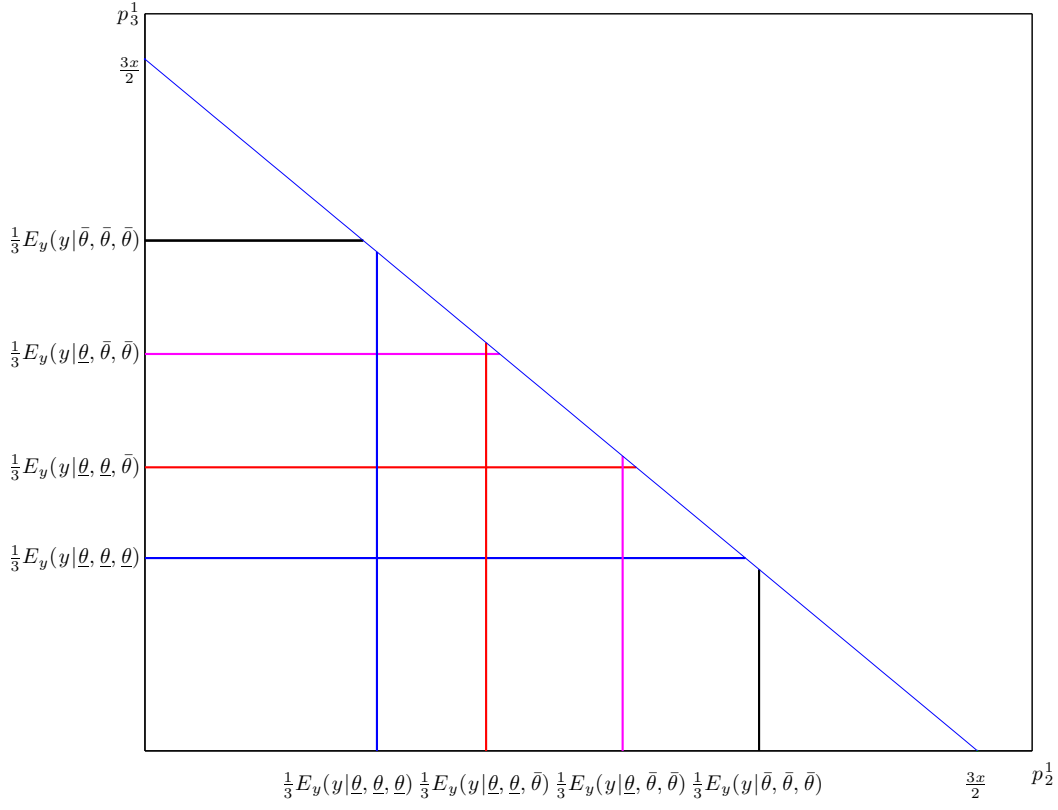
The intuition of this result is simple, but the construction needs some space. So, we first give the intuition, then give the idea of the proof. All the detail on the constriction can be find in the appendix. The intuition is as follows. First, given that the beliefs are the same as the prior, then, for the politician in the minority party, he is going to take expectation across all the four possible type profiles to get the expected pay off from no agreement. Second, for the politicians from majority party, they only need to take expectation on the two possible types of the politician from the minority party. However, different type profile in the majority party will give different expectation. For example, if both the politicians in the majority party are high types, then their expectation on the size of the pie will be relative larger than the case in which one of the politician in the majority party is low type. Therefore, some type profiles of the majority party might induce a high expectation then the politician in minority party. However, for the proposer, his first objective is to maximize his expected pay off. By majority rule, he only needs to buy off the one who has lower expected pay off from no agreement. In this result, we can prove that if both the politicians in the majority are high types, then their expectation on the size of the pie will be larger than the expectation from minority party. If at least one of the

politician in the majority party is low type, then their expectation on the size of the pie will be smaller than the minority party's expectation on the size of the pie.

However, in this case, the type profiles of the majority party will be perfectly reviewed. Then the minority party member is not going to take the expectation across the all the possible type profiles, but realized that both the politicians in the majority party are high type. This will raise the minority party' expectation of the size of the pie, so as the reserved value from delay. Therefore, the leadership of the majority party will switch to buy off his party member.

Now, we are going to give the idea of the construction which will support above intuition. First, there might be multiple equilibria in the sub-game of voting. We are going to apply Proposition 1 to restrict our attention to the non-dominated actions. Under this restriction, we still might face multiple equilibria. Then we try to focus on the following equilibria, in which, the equilibria will maximize the expected pay off of the proposer. To get this job done, we are going to get the maximal value of proposer's expected pay off in the sub-game of voting. Then we go back to check if there is a proposal or proposals to implement this maximal value. If we can find the proposals to support this maximal value, then the proposal will be one best response of of the proposer.

To be more specific on above argument, let us consider the following case. We assume politician 1 is the proposer, he gives proposals to 2 and 3. First, from Proposition 1, we can divide the feasible set of proposals to several different ranges, which is shown in Figure 2.5.1. In this graph, the horizontal line is the proposals to 2, the vertical line is the proposals to 3. Let us focus on range $X$ in Figure 2.5.1. From Proposition 1 we know that, for any proposals in range $X$, politician 2 is going to reject no matter what the type profiles are. For politician 3, he is going to accept any proposals in the range if he is low type; but the action is not determined if 3 is

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$p_3^1$

$\frac{3x}{2}$

$\frac{1}{3}E_y(y|\bar\theta,\bar\theta,\bar\theta)$

$\frac{1}{3}E_y(y|\underline\theta,\bar\theta,\bar\theta)$

$\frac{1}{3}E_y(y|\underline\theta,\underline\theta,\bar\theta)$

$\frac{1}{3}E_y(y|\underline\theta,\underline\theta,\underline\theta)$

$\frac{1}{3}E_y(y|\underline\theta,\underline\theta,\underline\theta)\ \ \frac{1}{3}E_y(y|\underline\theta,\underline\theta,\bar\theta)\ \ \frac{1}{3}E_y(y|\underline\theta,\bar\theta,\bar\theta)\ \ \frac{1}{3}E_y(y|\bar\theta,\bar\theta,\bar\theta)$ $\qquad\qquad \frac{3x}{2}\quad p_2^1$

**Figure 2.1:**

hight type. However, as the argument in Proposition 1, we do know that there are two possible results — agreement is reached or no agreement is reached. If there is an agreement reached by some proposals in range $X$, then the proposer's expected pay off will be

$$g(p_1^{t=1} + p_2^{t=1}|\theta_1,\theta_2,\theta_3)$$
$$\equiv \int Pr\{p_2^{t=1} + p_3^{t=1} \le y|\theta_1,\theta_2,\theta_3\}(y - p_2^{t=1} - p_3^{t=1})dF(y|\theta_1,\theta_2,\theta_3)$$

$$s.t. \qquad (p_2^{t=1}, p_3^{t=1}) \in X$$

If there is no agreement, the proposer's expected pay off will be

$$\frac{1}{3}\mathbb{E}_y(y|\theta_1,\theta_2,\theta_3)$$

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The figure contains the following labels:

- $p_3^1$
- $\frac{3x}{2}$
- $\frac{1}{3}E_y(y|\bar{\theta}, \bar{\theta}, \bar{\theta})$
- $\frac{1}{3}E_y(y|\underline{\theta}, \bar{\theta}, \bar{\theta})$
- In $X$, 2 rejects $\forall(\theta_1, \theta_2)$. 3 accepts if $\theta_3 = \underline{\theta}$. Action is not determined for $\theta_3 = \bar{\theta}$
- $X$
- $\frac{1}{3}E_y(y|\underline{\theta}, \underline{\theta}, \underline{\theta})$
- $\frac{3x}{2}$
- $p_2^1$
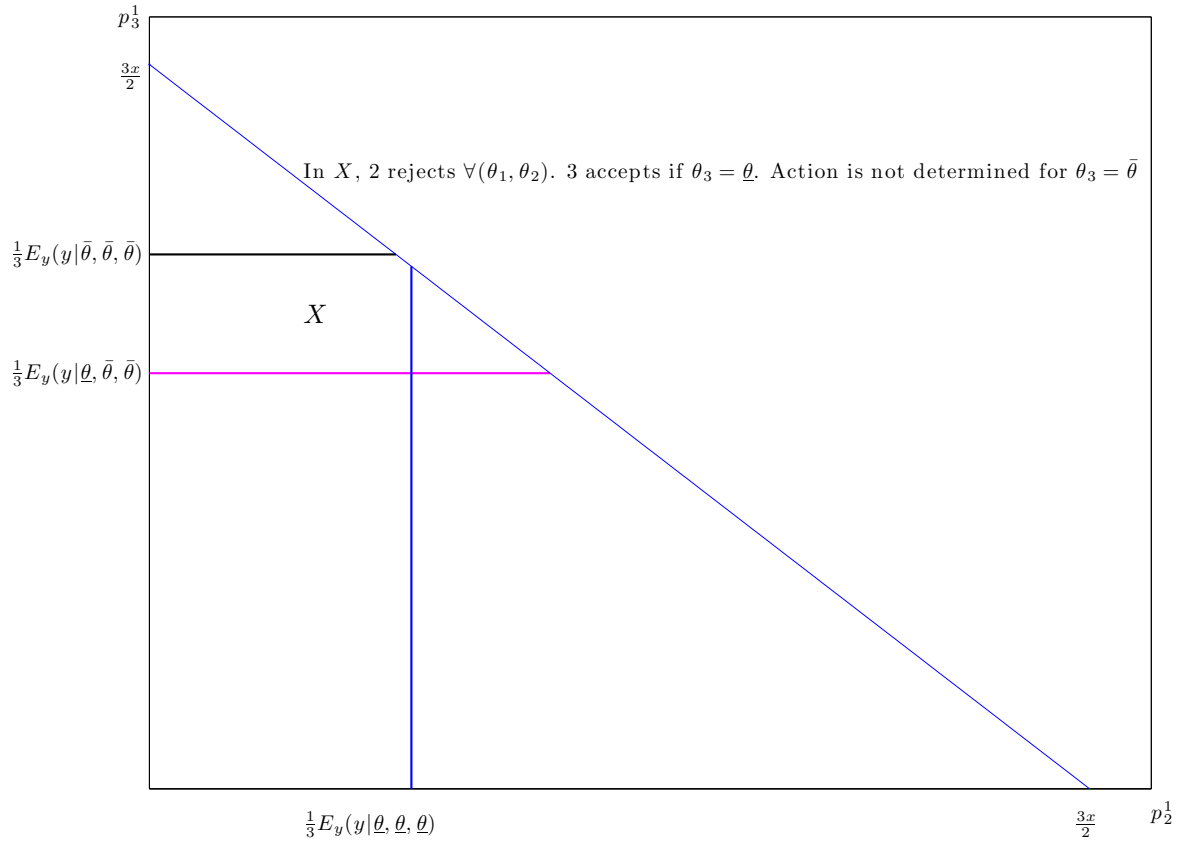
**Figure 2.2:**

Then fixed a type profile $(\theta_1, \theta_2, \theta_3)$, we are going to get the maximal among above two expected pay off, which is denoted by $m(\theta_3)$,

$$m(\theta_3) = \max\left\{ g(p_2^{t=1} + p_3^{t=1}|\theta_1, \theta_2, \theta_3), \quad \frac{1}{3}\mathbb{E}_y(y|\theta_1, \theta_2, \theta_3)\right\}$$

Now, we are going to calculate the proposer's expected pay off in the sub-game of voting. This will depend on the proposer's actions in the sub-game of voting. If the proposer chooses to reject the proposal in range $X$, then no agreement will be reached, therefore the proposer's expected pay off will be,

$$U_1(R|X, h^2) = \frac{1}{3}\mathbb{E}_{y,\theta_3}(y|\theta_1, \theta_2, \theta_3)$$

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1's expected payoff of being the proposer in t=1

Upper bound of i's expected payoff in the subgame of voting, given $(\underline{\theta},\underline{\theta})$

Upper bound of i's expected payoff in the subgame of voting, given $(\underline{\theta},\underline{\theta})$

Upper bound of i's expected payoff in the subgame of voting, given $(\underline{\theta},\bar{\theta})$

Expected payoff to 1 if no agreement is reached

$k(\bar{\theta})$

$1/3\ E_y(y|\underline{\theta}\underline{\theta}\underline{\theta})$    $1/3\ E_y(y|\underline{\theta}\underline{\theta}\bar{\theta})$    $1/3\ E_y(y|\underline{\theta}\bar{\theta}\bar{\theta})$    $1/3\ E_y(y|\bar{\theta}\bar{\theta}\bar{\theta})$    Total proposal

$w(\underline{\theta}\underline{\theta})$    $w(\underline{\theta}\bar{\theta})$    $w(\bar{\theta}\bar{\theta})$
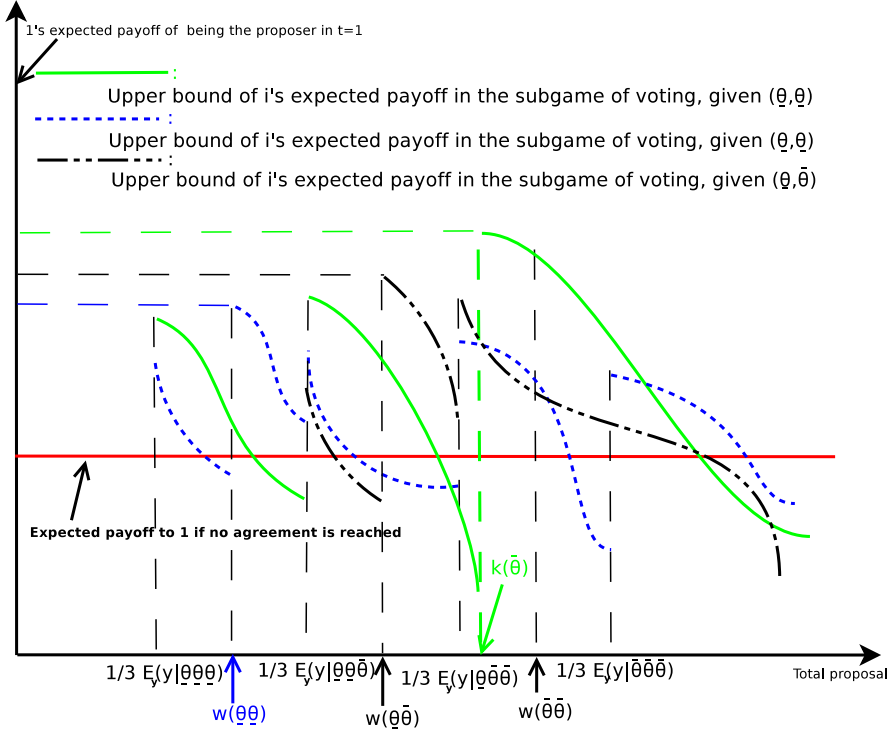
**Figure 2.3:**

Here, $U_1(R|X,h^2)$ denotes politician 1's expected pay off from choosing to reject the proposal after history $h^2$. $h^2$ is the history after which all the three politicians vote on the proposal, which is defined in Proposition 1. If the proposer chooses to accept the proposal in range $X$, then the proposer's expected pay off will be,

$$U_1(A|X,h^2) \leq \frac{1}{2} \times g(p_2^{t=1} + p_3^{t=1}|\theta_1, \theta_2, \underline{\theta}) + \frac{1}{4} \times m(\bar{\theta})$$

Here, $U_1(A|X,h^2)$ denotes politician 1's expected pay off from choosing to accept the proposal after history $h^2$. The right hand side of the above expression is one upper bound of the $U_1(A|X,h^2)$. Then we do the same analysis across all the ranges described in Figure 2.5.1. After that, we find one upper bound for all the possible proposals. To illustrate the point, we show a graph in Figure 2.5.1, which describes the upper bound of proposer's expected pay off for all the possible proposals. Here, the horizontal line is the total proposal given to politicians 2 and 3, the vertical line

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is the upper bound of the proposer's expected pay off. As shown in Lemma 1, the proposer's expected pay off will be decreasing when the total proposal is increasing. However, after increasing to certain values, the probability of getting an agreement will be increasing. This will make the proposer's expected pay off jump to a higher value, but after that jump, proposer's expected pay off will decrease again with the increasing of the proposals. As shown in Figure 2.5.1, if the politicians in the majority party are both hight type, then the proposer get the maximal expect pay off at the the proposal $k(\bar{\theta})$. This value is the politician 3's expected pay off from no agreement. For all the other type profiles, the proposer reach the maximal expected pay off at his party member's expected pay off from no agreement.

Now, we turn to the case in which the politician in minority party is the proposer.

**Proposition 13.** *In t=1, given that the posterior beliefs are the same as the prior, and given the subsequent strategies in the voting process, if the minority party has the proposal power, then for any history*

$$h^1 = (p_2^{t=0}, p_3^{t=0}; At \ least \ two \ choose \ Rejection)$$

*it is best response for 3 to propose*

$$(p_1^{t=1}, p_2^{t=1}) = (0, w(\bar{\theta}, \bar{\theta}))$$

*where*

$$w(\bar{\theta}, \bar{\theta}) = \frac{1}{3} \mathbb{E}_{y, \theta_3}(y|\bar{\theta}, \bar{\theta}, \theta_3)$$

If politician 3 has the proposal power, then he will be indifferent between buying off 1 or 2. The problem here is how much to propose to get an agreement. The trade off here is that, the politician in minority party could give a low offer to one of the politicians in the majority party, but he need to take a risk of getting no agreement, then gets the expected pay off from no agreement. This result shows that, facing this

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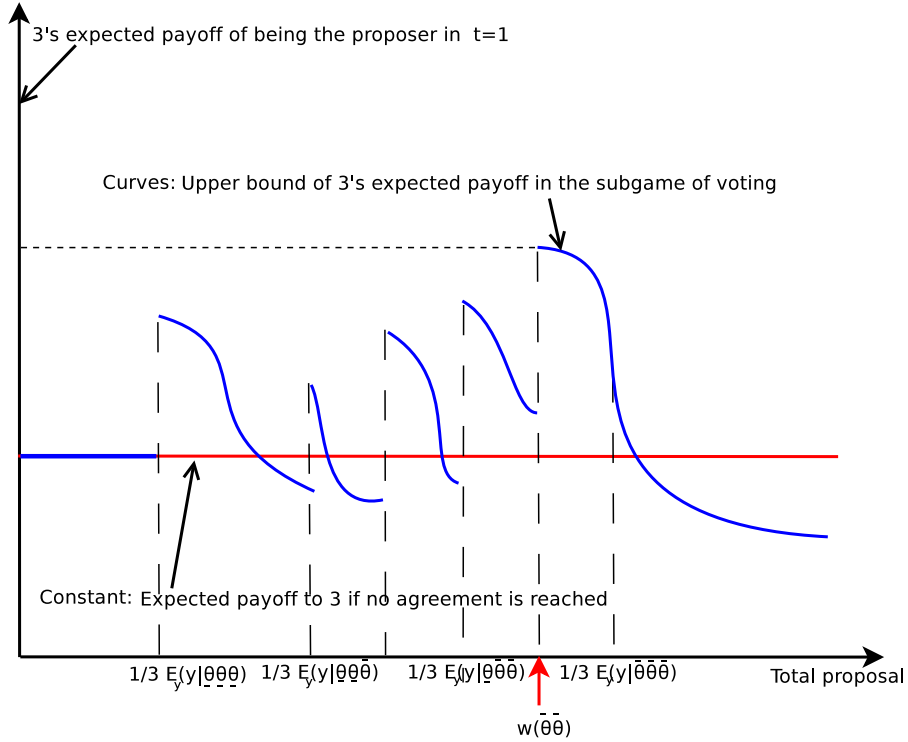
**Figure 2.4:**

trade off, it is best for the proposer to give a relative high proposal, which will be accepted by the politician in the majority party, no matter what the type profiles are.

The logic of the proof is the same as Proposition 2. We go through all the possible proposals to find one upper bound of the expected payoff to the proposer, then go back to check if there is a proposal or proposals to support this upper bound. The upper bound of the expected pay off for every proposal is shown in Figure 2.5.1. In this picture, we only show the the case in which the proposer is low type. The case of high type looks very similar to this picture. Both get the maximal value at $w(\bar{\theta}, \bar{\theta})$.

Now we have finished the construction of the equilibrium strategies in the second period. In the next step, we will go back to the first period. Before that, given the best response proposed in Proposition 2 and Proposition 3, we are going to calculate the expected payoff to each legislator. We are going to denote the expected value from delay as $V_i(\theta_1, \theta_2)$, for any $\theta_1, \theta_2$; and $V_3(\theta_3)$, for any $\theta_3$.

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For legislator 1, if he is selected to be the proposer, he is going to give an offer $(p_2^{t=1}, p_3^{t=1}) = (w(\theta_1, \theta_2), 0)$, to his party member. If the minority party member, politician 3, is selected to be the proposer, then 1 also get 0. Therefore, we have, for all $(\theta_1, \theta_2)$,

$$V_1(\theta_1, \theta_2) = \frac{1}{2} \times [\mathbb{E}_{y, \theta_3}(y|\theta_1, \theta_2, \theta_3) - w(\theta_1, \theta_2)]$$

For the member of the majority party, legislator 2, he has no chance to be the proposer. With probability half, he gets $w(\theta_1, \theta_2)$ from politician 1; and with probability half, he gets $w(\bar{\theta}, \bar{\theta})$ from politician 3. Therefore, we have, for all $(\theta_1, \theta_2)$,

$$V_2(\theta_1, \theta_2) = \frac{1}{2} \times w(\theta_1, \theta_2) + \frac{1}{2} \times w(\bar{\theta}, \bar{\theta})$$

For legislator 3, if he is selected to be the proposer, then he proposes $(p_1^{t=1}, p_2^{t=1}) = (0, w(\bar{\theta}, \bar{\theta}))$. If the leadership of majority party is recognized to be the proposer, then he gets zero, therefore, we have, for any $\theta_3$,

$$V_3(\theta_3) = \frac{1}{2} \times [\mathbb{E}_{y, \theta_1, \theta_2}(y|\theta_1, \theta_2, \theta_3) - w(\bar{\theta}, \bar{\theta})]$$

By simple algebra and boundary assumption of $\mathbb{E}_\epsilon(\epsilon|\theta_1, \theta_2, \theta_3)$, we get the following result,

$$V_2(\bar{\theta}, \bar{\theta}) > V_2(\underline{\theta}, \bar{\theta}) = V_2(\bar{\theta}, \underline{\theta}) > V_3(\bar{\theta}) > V_3(\underline{\theta}) > 0$$

$$V_2(\bar{\theta}, \bar{\theta}) > V_2(\underline{\theta}, \bar{\theta}) = V_2(\bar{\theta}, \underline{\theta}) > V_2(\underline{\theta}, \underline{\theta}) > V_3(\underline{\theta}) > 0$$

$$\frac{x}{2} > V_3(\bar{\theta}) > V_3(\underline{\theta})$$

### 2.5.2 Analysis in the First Period

In the first period, we are going to assume that all the politicians will take the value $V_i$ as the expected pay off from no agreement in the first period. This means, if no agreement is reached in the first period, all the politicians think they will get $V_i$ from the second period.

**Proposition 14.** *In t=0, in the sub-game of voting, for any history $h^0 = (p_2^{t=0}, p_3^{t=0})$ and any belief system, given the best responses described in Proposition 2 and Proposition 3, and fix the expected payoff from t=1, we have: for a given $(\theta_1, \theta_2)$ it is weakly dominant action for $i \in \{1, 2\}$ to accept any proposal*

$$p_i^{t=0} \geq \delta V_i(\theta_1, \theta_2)$$

*And for a given $\theta_3$, it is weakly dominant action for 3 to accept any proposal*

$$p_3^{t=0} \geq \delta V_3(\theta_3)$$

*Here discount factor $\delta \in (0, 1)$, $V_i$ are the expected payoff from the give best response described in Proposition 2 and Proposition 3.*

The intuition of this proposition is as follows. No matter what actions are chosen by the politicians in the sub-game of voting, as in the Proposition 1, we only have two possible results, an agreement is reached or no agreement is reached. For any proposal, with some probability an agreement is reached, then the politician $i$ gets the proposal, say $p_i^{t=0}$, where $i \in \{1, 2, 3\}$. With some probability, no agreement is reached, then the politician $i$ gets the expected payoff, say $V_i$, from the second period. However, unlike the Proposition 1, the $V_i$ is independent of $i$'s type, therefore, politician $i$'s expected payoff in the sub-game of voting in the first period is the convex combination of the following two elements,

$$p_i^{t=0} \quad \text{and} \quad V_i$$

Thus, if the proposal is larger than the $V_i$, then politician $i$ will accept it, no matter what actions is chosen by the other two. Meanwhile, if the proposal is less than $V_i$, then politician $i$ will reject it, no matter what actions is chosen by others.

Given the subsequent strategies described above, let us go to the information set after which politician 1 gives a proposal. The next proposition describes the best response of politician 1.

**Proposition 15.** *In t=0, given the prior, and the subsequent strategies in the voting process and t=1, for any $(\theta_1, \theta_2)$, it is best response for the majority party member, legislator 1, to propose*

$$(p_2^{t=0}, p_3^{t=0}) = (0, \delta V_3(\bar{\theta}))$$

This is the proposition supporting the main result of this paper — the information sharing in a party will hurt the party members. The intuition is that, the information sharing makes the leadership of the majority party buy off the politicians in the same party. On the one hand, this will raise the majority party politicians' expected payoff from delay. This is supported by the following two reason. First if both the majority politicians are in high type, then they have a high expectation on the size of the pie. This will induce a high reserved value. Secondly, the minority party member will give a hight proposal to buy off the member of the majority party. These two effect will increase the majority party member's expected payoff from the second period.

On the other hand, the information sharing will lower the minority party politician's expected payoff. This is because, information sharing only increase the the minority party politician's cost of buying off one of the majority politicians, but still keep the same expectation on the size of the pie. This will induce a lower expectation payoff from the second period. As a result, this gives incentive to the majority party leadership to buy off the politician who has a lower expected payoff from delay. Given this fact, the majority party leadership still needs to decide which proposal should be offered. One trade-off here is that, the leadership of the majority party could give a lower offer which is just the low type minority party politician's expected payoff from delay, say $V_3(\underline{\theta})$. With some probability this will give the majority leaderships a high residual; but, with some probability, there will be no agreement, therefore no agreement will be reached, and politician 1 will get the expected payoff from delay, which is $V_1(\theta_1, \theta_2)$, for any $(\theta_1, \theta_2)$. However, it turns out that this is not the best

for politician 1. The best response for him is to give a offer which is the high type minority party politician's expected payoff from delay. Therefore, no delay happens under this offer, and it will give the proposer, politician 1, the largest expected payoff, give the equilibrium strategies constructed in the second period.

The logic of the proof is very similar to the Proposition 2 and 3. We first apply Proposition 4 to divide the set of feasible proposals to several ranges, then go through every range to get an upper bound of politician 1's expected payoff. After that we go back to find one proposal implement the upper bound. The detail of the proof is shown in the appendix.

## 2.6   Conclusion

In this part, let us summarize our main finding and point out some extendable assumptions. The main point of this paper — party might hurt its member's benefit— hinges on the following arguments.

Information sharing makes the politicians in the same party know each other better than the politicians from other parties. This might induce a low expectation on the size of the budget to the party members. However, after considering the random selection rule of the proposer, the member might have a higher expected payoff than the politician in the other party. In a dynamic environment, when the majority leadership wants to get an agreement immediately, he needs to choose the politician with low expected payoff from future. This would be the politician from the other party.

Some of our modeling assumptions deserve further discussions. First, we assume that the party only help members to share information in the legislation. However, sometimes, parties also help members to coordinate actions in the legislation. For example, the whips in the U.S political system also try to coordinate members' actions

in the legislation. It would be interesting to investigate how this function of party will affect party members' benefits. A reasonable conjecture is that, the coordination might give some benefits to the party members. To support this result, there might be some inside transfers or some party doctrine to force the leadership to fulfill this job. However this does not necessary contradict the results in my model. Because the leaderships' benefit still might be hurt in this case.

Second, in this model, the leadership of the majority is predetermined to be politician 1 and fixed across the whole game. But this is not true in general, in almost all the democratic parties, the leaderships is not predetermined. The leaderships also try to hold the office, this gives them incentives to benefit their members so as to get supports in the re-election. If we consider this fact, some of the results in this model might change. One reasonable conjecture is that, in order to maintain the reputation so as to get re-elected. The leaderships might sacrifice his own benefits to include his party members in the winning coalition. These and other interesting questions must await further discussions.

Chapter 3

## ON BOARD AS A MEDIUM OF INFORMATION TRANSMITTER

### 3.1   Introduction

Conventional wisdom describes boards as ineffective rubber-stamper controlled by management. The typical complaints about the indolent behavior of boards might date back to Mace (1971). Recent research, most empirically, argue that the board of directors performs the critical function on firm performance. And empirically it is true that if the board of directors could perform in a great level of independence on effective monitoring, the firms' performance would be improved, see Weisbach (1988), Brickley *et al.* (1994), just name a few here.

In the theoretical study on the interactions between the boards and the management, most researches focus on questions, such as why boards may not monitor too intensively, see Warther (1998), Almazan and Suarez (2003), Adams and Ferreira (2007); and they argue that the passive (or weak) boards may be optimal in the view of shareholders. Only a few researches try to consider the possible collusion between the board and the management, see Bourjade and Germain (2012). Li and Zhenhua (2013) studies the information sharing and the collusion between the directors and management.

However, one common assumption in most of these researches [1] , theoretical or empirical, is that, in the monitoring process, the board of directors always perfectly represent the shareholders to maximize the welfare of them. But in the real

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[1]Li and Zhenhua (2013) divides the shareholders to different interest groups and study the collusion between the management and the directors who represents shareholders in different interests. They open a quite new channel to study the collusion between the boards directors and the management.

interactions between the board directors and management, they are more privately connected. The personal relationship might be beyond the control of the shareholders. Meanwhile, the dual board system [2] also pushes the board of directors to collude with the management. So one conjecture is that there might be underground deals between the directors and the management. It also has been argued that the lack of shareholder power gives rise to situations in which the management and the board of directors might mutually protect each other, see Beetsma *et al.* (2000). Most recent corporate scandals in the US and Europe have also emphasized the collusion between the board of directors and the management. For instance, as reported by the New York Times (Jan 2013), in Walmart Bribery scandal, the directors collude with the management to bribe Mexican officials and share part of the additional benefits. [3]

Therefore, the purpose of this paper is to theoretically study how the interactions between the board directors and the management would affect the firm performance and the relation (contract) between the shareholder and the management. Our model has four key features. First, board of directors do not perfectly aligned with shareholders, and the management has private information on his "type", e.g., ability, marginal cost. This ability is not observed by the shareholders. Second, the board is working as a medium to monitor and transfer the management's information to the shareholder. In the monitoring process, the board can only observe an imperfect (noisy) signal about the management's "type". Third, there is a market of directorships to evaluate directors' actions in the monitoring. Fourth, the interaction between the directors and the management are explicitly modeled as a bargaining process.

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[2]In this system, the board of directors are separated from the management. This is very common in the organization of modern firms.

[3]Newspapers also document many other evidences. For instance, in Netherland, there is a case of Vie d'Or in which the directors and managements were involved jointly in outright fraud, see Beetsma *et al.* (2000). It is obvious that only the cases end badly would get attention from the media.

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The first feature is motivated by the facts that the directors might be more privately connected to the management. It is also supported by the fact that the directors might represent shareholders in different interests. For example, among outside directors, they represent shareholders, debt-holders, and other stakeholders in different interests, (e.g., Byrd and Mizruchi (2005) on bankers on the board; Baker and Gompers (2003) on Venture capitalists; Faleye *et al.* (2006) on employees). This induce that the directors may not be perfectly aligned with the majority of shareholders. Meanwhile, the CEO's ability may not be observed by the shareholders, (e.g., Milbourn (2003)). This feature has been supported by evidences, such as the Sabarnes-Oxley Act, the NYSE and the NASDAQ regulations in the US request that the board directors need to supervise the firms' management. The scandals of Enron, Worldcom and others also tell us that the accountants who are supposed to help reveal information may actually help the management conceal the truth. These actions might induce more transaction cost in the monitoring process and make it much harder for the board to see the real face of the management. The third feature could go back to Fama and Jensen (1983), who notices that the important incentives for directors to monitor comes from the reputation effect in market of directorships. This argument is also supported by empirical research on the reputation effects in the market for directorships, e.g., Yermack (2004) and Fich and Shivdasani (2007). Recently, Knyazeva *et al.* (2013) studies the relationship between board structures and the market of directorships. They empirically suggest that firm's performance, board independence are significantly related to market of directorships; and the shareholders' welfare is related to directors' reputation. But they did not explain the intrinsic mechanism. To this end, we try to formalize this point and discuss this mechanism. For the fourth point, we believe that the bargaining is the most popular communication channel in business interactions, see Raiffa (1982).

In our setting, we focus on the function of monitoring of the boards directors, so the final production would only be affected by the ability of the management. The ability is assumed to be private information which is not observed by the shareholder. In this case, from the classical principal-agent model, we know that in order to maximize the welfare of the shareholders, they need to pay additional information rent to the management who tries to mimic, see Laffont and Martimort (2009). For the shareholders, in order to lower the information rent and also monitor the true ability of the management, we introduce the board of directors who might observe the true ability of the management. We also assume that there is a market of directorships. This market would respond to the board directors' action in the monitoring process. The directors care the evaluation (their reputation)from the market. An honest reporting of signals on the management's ability would always induce a potential positive benefit from this market. However, any dishonest behaviors would induce a potential loss from this market.

In the monitoring process, we assume that the type of the management may not be perfectly observed by the directors, they can only observe a noisy signal related to the true ability of the management. Meanwhile, the directors could privately communicate with the management to make a deal and conceal the signal on the management's true ability. We explicitly model the communication process as an infinitely alternative bargaining between the directors and the management. In practice, this collusion may result a decreased level of production and an increased level of transaction cost between the shareholders and the management. However, we argue that, a proper designed institution (board structure) and incentive to the directors might block the collusion. Thus it would also improve the shareholders' benefit.

To our knowledge, our paper is the first theoretical study on the collusion between the board and the management by explicitly modeling the interaction between

the management and the board of directors as a bargaining process. Collusion has been studied in the mechanism design literature. The seminal paper of Tirole (1986) studies a three level organization with a principal, a supervisor and an agent in the setting with moral hazard. However, in this literature, the interaction between the supervisor and the agent are essentially a black box. They notice that there is a underground bargaining between the supervisor and the agent, but they assume that the outcome of the bargaining always reach the Pareto efficient allocation. In general this is not true. The equilibrium outcomes of the bargaining could depend on many features of the bargaining, such as the players' risk attitudes, their outside options and information structure, see Muthoo (1999) and Ausubel *et al.* (2002). Meanwhile, from the existing research, both empirical and theoretical, we already notice that the boards directors do care their reputation from the market of directorships, see Adams *et al.* (2008) for the survey on this topic. Therefore, this paper tries to open the black box by considering the reputation effects in the market of directorships and also the bargaining between the boards directors and the management.

This framework allows us to derive many results and implications which may not be delivered by current researches. More precisely, our results are the following. First, after introducing a board of directors who do not communicate with the management, we show that, whether the boards' influence is positive or negative depends on the accuracy of the signal and whether the signal infers the management is efficient, i.e., the acctual signal observed by the board. Here, the signal is explained as the indicator of the management's "type". Given that the board could observe the management's true "type" with high probability, and the board reports that a non-efficient management is observed, then the shareholders' posterior on the management being an efficient type would be decreased. This would reduce the shareholders' fear of giving up an information rent, but increase the production level of the non-efficient man-

agement. In other words, the board's would improve the firm's performance, so as the welfare of the shareholders, in the sense that a high lever of outcome would be produced, and a low expectation of giving up information rent. But we would never hope the board helps the shareholders to reach the first best in which there is no asymmetric information between the shareholder and the management. The same argument would apply for the case with low accuracy signal plus a report of facing an efficient management. However, if the board could accurately observe the true signal of the management, and he actually observes that the management is an efficient type, then it would increase the shareholders' fear of giving up the information rent. Meanwhile, a low lever of outcome would be produced. This induce that, the presence of the board would actually worse the performance of the firm, so as the welfare of the shareholders.

Second, if we allow communication between the board and the management, then the optimal compensation to block the collusion between the board and the management would decrease the production level and the information rent payed to the management would also be decreased.

Third, after introducing a market which could evaluate the board directors' reputation, and a bargaining between the boards and the management, we find that: If the market is sensitive to board of director's reports (more dependent directors), a high rewards to the truth reporting or a serious punishment to the fake reporting from the market may not really encourage the board to report truth. This is because, high rewards or serious punishment from the market of directorships only gives the board more bargain chips to raise his share of the information rent from the management. Meanwhile this would increase the shareholders' cost of blocking the collusion between the board and the management. Furthermore, as before, this would lower the production level.

However, if the market of directorships does not sensitively respond to board's actions (more independent directors), the shareholder would pay less to block the collusion, and the level of production would increase. This is because, a non-sensitive market would lower the board's bargain chips, therefore lower his share of the information rent from the management. Then, from the view of shareholders, this board structure would lower the shareholder's cost of blocking the collusion. Furthermore, this would increase the production level. Meanwhile, from the view of the management, he would also prefer a more independent board. This is because more information rent could be kept by the management, if the board is structured with more independent directors.

These results theoretically confirm the empirical argument that firm's performance might be better with more independent directors in the board, see Adams *et al.* (2008) for a survey on the empirical results. Two other theoretical paper also deliver the similar message as our model here, see Adams and Ferreira (2007), Bourjade and Germain (2012). However, the intrinsic mechanism driving our result is quite different from theirs.

If we only focus on the interactions between the board and the mangement, above resluts also tell us why the management prefers a more independent board than a less independent one. This is because, the management could keep more information rent from the more independent board.

Fourth, if the size of board is large, it might be more expensive for the management to make all the board directors conceal his private information. Knowing this potential deal between the directors and the management, the shareholders could properly design a compensation rule to block the deal by increasing the transaction cost between them. We expect, therefore, that a proper designed compensation rule plus a proper market institution would lower the transaction cost between the share-

holder and management. Thus also improve the performance of the firm and the welfare of the shareholder. However, if the size of the board is not large enough to block the collusion, then the large the size of the board, the higher the cost for the shareholders to block the collusion.

More interestingly, we also find that, if each directors' compensation from the market of directorships depends on the per capita contribution to the firm's performance, i.e., link the evluation from the market of directorships to the size of the board, then the rent to block the collusion would be independent from the size of the board. This result is not addressed in the literature about optimal boar size, such as Raheja (2005) and Coles *et al.* (2008).

Another implication of our theoretical prediction is that: The payment to the CEO would be affected by board independence. The more independent of the board, the less might be payed to the CEO. This result is in contrary to the empirical findings which argues that total CEO pay is not affected by board independence, see, Knyazeva *et al.* (2013).

The paper is structured as follows. Section 2 presents the model. Section 3 sets up the benchmark without the board of directors. In section 4, we introduce the board as a medium to transfer manager's information to the shareholders. In section 5, we analyze the equilibrium result of the bargaining between the board and the management. In section 6, we discuss the reputation effects of market of directorships on the boards' supervision. In section 7, we extend the model to discuss the board size and the supervision of the board. Last two sections highlight the policy implications of our analysis and concludes. We present proofs in the Appendix.

## 3.2 Model

Let us consider an environment with three players which are called shareholder, management and board of directors [4]. The shareholders authorizes the management to operate a firm. The production is affected by the "type" of the management which is measure by the marginal cost of the management, $S$. The marginal cost of the management is not observed by the shareholders which can take one of two values $\{\theta_g, \theta_b\}$ with respective probabilities $\pi \in (0, 1)$ and $1 - \pi$. We let $\Delta\theta = \theta_b - \theta_g > 0$. This probability is common knowledge, but only the management knows the true value of $\theta$.

The "type" here could be explained in many different ways. One popular explanation is ability. managements with high ability are more easily to catch the prospective market. This would save lots of cost for the firm. However, the low ability ones may spends lots of budget to explore the market. Another explanation is personal network. managements with strong network are more easily to extend the market of the firm; however, the ones with weak network may need lots of resource to extend the market.

### 3.2.1  Technology of Supervision

The main questions in this paper is to study how to give incentives to the board to make them truthfully report the signal, and how to prevent the possible collusion between the board directors and the management. So we are going to assume that the board always chooses to monitor the management and gets information on management's type.

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[4]In this paper, we do not model the difference between outside director and inside directors. In reality, directors in these two groups would behave quite differently. In a companion paper, we try to study the boardroom collusion with heterogeneous directors, see Wu (2013a) .

Let us consider an environment in which the directors receives a signal $s \in \{g, b\}$ about the true type, $S \in \{\theta_g, \theta_b\}$, of the management. We assume that the signal is imperfect but informative and distributed according to

$$\text{Prob}(g|\theta_g) = \text{Prob}(b|\theta_b) = \mu \in (0, 1)$$

This reads as: With a positive probability $\mu$ [5] , the director would observe a signal which truthfully indicates the real type of the management, i.e., observe $s$ when the true state is $S = \theta_s$. We assume that shareholders place prior probability $\pi \in (0, 1)$ on the management's type being efficient, $\theta_g$. After seeing the signal, we assume that the directors would truthfully report the signal they have got.

Here, we assume that the information observed by the board is "hard" or verifiable in the sense that: When the board observes the state of manager's type (marginal cost), the board can convey this information to the shareholders in a credible way in which, the shareholders can look at the evidence and be convinced that the board has announce the true state of the marginal cost. However, the board could lie and announce nothing or announce that they observed the other type. In reality, the board usually presents the evidence from auditor or other source to show the manager's ability or effort in the management. The evidence is verifiable but could be concealed or affect by the board. A board could affect the evidence through the choice of auditor, the oversight over reporting requirements, and the control over accounting practices.

**Remark** In general, when the directors give a report, $\hat{s} \in \{\hat{g}, \hat{b}\}$, to the shareholders, the directors are free to report either $\hat{b}$ or $\hat{g}$, and we write the directors' strategy

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[5]The value of $\mu$ could be endogenized as a function of the board's efforts in monitoring, the transaction cost in board decision, the size of the board, or any other characters of the boards. We denote these characters as $x$. If $x$ is explained as the effort putting in monitoring, then $\mu(x)$ would be increasing with the effort. This means that a high level effort leads to a high level monitoring. This further induce a high level of probability of getting the real type of the management.

conditional on its signal by $\sigma_s(\hat{s}) = \Pr(\hat{s}|s) \in [0, 1]$. Without loss of generality, we will restrict attention to strategies with $\sigma_b(\hat{b}) \geq \sigma_g(\hat{b})$ and $\sigma_g(\hat{g}) \geq \sigma_b(\hat{g})$. However, this strategic reporting would not affect the main results of the paper. So we simply assume that the directors would truthfully report the signal to the shareholders.

### 3.2.2 Technology

There is a technology which is denoted by

$$
y = \begin{cases} q_1 & \text{if } \theta = \theta_g \\ q_2 & \text{if } \theta = \theta_b \end{cases}
$$

In this paper, we want to focus on the monitoring from the board, not the provision of advice, so in this technology, only the management's type affects the outcome.

In this paper we are going to consider two different board institutions. In the first institution, there is no communication between the board and the management, i.e., there is no chance for the board to make deal with the management. In the second institution, the board could communicate with the management and make a deal on the share of the information rent. But in both institutions, the board will transfer the information on management's type to shareholders. Only difference is whether board would collude with management.

### 3.2.3 Market of Directorships

After the report of the management's type, we assume that there is a market which can evaluate the precision of the report. The board could get utility from this evaluation which is denoted by $k(\varepsilon, \eta, y) \in \mathbb{R}$. This utility is determined by the precision of the report, which is measured by $\varepsilon \in \{0, 1\}$, the board's relation to this

market which is measured by $\eta \in [0, 1]$ [6] and the outcome from the technology which is described by $y$. We assume $k(\varepsilon, \eta, y)$ is differentiable with respect to $\eta$ and $y$, such that [7]

$$k_y(\varepsilon, \eta, y) \geq 0 \text{ and } k_\eta(\varepsilon, \eta, y) \geq 0$$

We further assume that with probability $\rho \in (0, 1)$, the true state $\theta$ will be perfectly revealed. This also induce that with probability $\rho$ the market and the shareholders can tell if the board tells the truth. If $\hat{\theta} = \theta$, the board gets $k(\varepsilon = 1, \eta, y)$ from the market; otherwise, the board gets $k(\varepsilon = 0, \eta, y)$ such that,

$$k(\varepsilon = 1, \eta, y) > k(\varepsilon = 0, \eta, y) \geq 0 \quad \forall \eta, y$$

Here $k(\varepsilon = 1, \eta, y)$ could be explained as the ego rent to the board directors. Or just think about it as a reward from the market for the precise report. $k(\varepsilon = 0, \eta, y)$ is the punishment [8] of concealing information. We can think about $k(\varepsilon = 0, \eta, y)$ as a present discount value from the market. The story behind this could be explained as follows, if the market knows that the board director is bribing with the management, then the director would not get any benefit from the market. For instance, if the board directors are CEO from other firms, and they are revealed to concealing information or bribing, the scandal might affect their normal income, their firm's stock price and so on, see Fich and Shivdasani (2007) on a study about the relationship between the financial fraud and directors' compensation from the market of directorships.

The key point we need is that the directors' benefit from market of directorships is increasing with the firm's performance and the directors' actions in the monitoring

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[6]The explanation of $\varepsilon$ here shares the same feature as the $\mu$ in Raheja (2005) which is a measure of sensitivity of directors' payoffs to firm value.

[7]Here, we use the subscript to indicate the differentiation on the subscripted element.

[8]In reality the punishment could a negative value, for instance, in the scandals of Enron and Worldcom, the directors of each board had to pay investor plaintiffs, which are out of their pocket. Our results will not change if we allow this value to be negative. The key point here is that the market compensation to the honest behaviors is always higher than the one to dishonest behaviors.

process. In order to simplify analysis, we assume the market evaluation has the following form,

$$k(\varepsilon, \eta, y) \equiv k(\varepsilon, \eta)y$$

where $k(\varepsilon, \eta) > 0$ is continuous with respect to $\mu \in [0, 1]$, $\forall \varepsilon$. We also assume that,

$$\lim_{\eta \to 1} k(\varepsilon = 1, \eta, y) = \bar{k} > 0 \quad \forall \eta, y$$

$$\lim_{\eta \to 1} k(\varepsilon = 0, \eta, y) = 0 \quad \forall \eta, y$$

and

$$\lim_{\eta \to 0} k(\varepsilon, \eta, y) = 0 \quad \forall \varepsilon, \eta, y$$

Here, $\varepsilon = 1$ means perfectly revealing of the true state (the type of the management), i.e., the board tells the truth, otherwise no information on the state is revealed. $\eta \to 0$ means that the boards directors does not value the response from market of directorships. Then the value from the market does not matter to the directors. One explanation is that, the board directors might be from a non-finance market. For instance, board directors could be a professor of college, and the value of precision on his report in the market does not affect what he earns from the college or academia. However, when $\eta \to 1$, which means that the director is perfectly related to the finance market. Then the market value would matter to the boards directors. The precision of the report and firm's performance would affect the directors' utility a lot. For example, the CEO of Google, Eric Schmidt, used to be a board member of Apple, in this case, Google CEO's any action as a board member of Apple might affect his benefit from Google's shareholders. Any negative information about his action in Apple's board monitoring could lower his benefit from the market of directorships. Meanwhile, any positive information about his actions as a director could increase his value from the market of directorship. Meanwhile, the high level production or

performance also could induce a high level payment to the board. This could be cash reward or compensations in other form.

If the true state is not revealed, then the board gets $r \geq 0$, to simplify analysis, we normalize $r = 0$. The main results of this paper would not change, if we allow $r > 0$

To simplify analysis, and guarantee that the board always tells the truth when no deal is made between him and the management, we further assume that,

$$\rho k(\varepsilon = 1, \eta, y) \geq c$$

This means that the expected payoff of truth telling from the market is larger than the cost of monitoring the management [9] . This induces that, the board always has incentive to monitor the management. Then the only problem for the shareholders is how to give incentives to the board to make them report the true information and not to collude with the management.

### 3.2.4    The Bargaining Between the Board and the Management

In this section, we would explicitly model the communication between the board and management. We assume that they sign a contract to deliver $\alpha \in [0, 1]$ share of the information rent, $Q > 0$, to the board. In this subsection, we would model the communication between the two as a bargaining process. The side contract [10] between the management and the board is explained as an agreement in the bargaining.

To make it more clear, we formally describe the bargaining process in this section.

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[9]The goal of the paper is to study how to give incentives to the board to report the truth and how to lower the chance of collusion between the board and the management. So we will simplify the board's decision on whether to monitor the management.

[10]We do not give any restriction on the form of the contract. In many situations, the contract may not be a formal contract which could be justified by the court. It could be any kind of agreement between the board of director and the management with an asset could be measure by dollar amount. It could be liquid asset, such as cash or other asset, such as a jet plane.

The board and the management bargain over the partition of the potential rent according to an alternating offer procedure. One of the player $i \in \{S, A\}$, where $S$ indicates board and $A$ indicates the management, would have chance to give the first proposal $x_i > 0$. It is the amount kept by player $i$. After seeing the offer, the responder has three choices, say: i) accept the offer, ii) reject the offer and make a counter offer $x_{-i}$, and iii) reject the offer and quit the bargaining, in which case the true state would be reported to the shareholders. In case i), the bargaining would end with the given the proposal. In case ii), player $-i$ would given another offer and player $i$ would respond the same choice as player $-i$. This bargaining keeps going until an agreement is reached, or one of them want to quit the bargaining.

The payoffs are as follows. If the board and the management reaches agreement at time $t$, where $t = 0, 1, 2, \ldots$, on a partition that gives player $i \in \{S, A\}$ $x_i(t)$, $0 \le x_i(t) < Q$, then his payoff would be

$$x_i(t)e^{-\gamma_i t}$$

To save some space for notation, we define $\delta_i \equiv e^{-\gamma_i}$. If no agreement is reached at $t$, and the responder chooses to quit, then the players would take up their outside option. For the board, he would report the management's type truthfully, and get the expected value from the market

$$\delta_S^t w_S \quad \text{where } w_S = \rho k(\varepsilon = 1, \eta, y)$$

For the management, he would get no information rent.We denote it as $w_A = 0$. If the board and the management can not reach an agreement perpetually, then each of them gets zero [11] .

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[11]Think about this payoff as the case in which the discount factor converge to zero, the rent to be divided would be vanished.

**Remark** Here, $\delta_i$ could be explained as the discount factor in the normal sense. We can also explain it as the measure of risk aversion. Under this explanation, $\delta_i$ is a function of the risk aversion index $r_i$. Therefore, large $\delta_i$ indicates low level of risk aversion, and small $\delta_i$ indicates high level of risk aversion. Because the board and the management signs the contract underground and both are afraid to be found; therefore, if the agreement is not reached, the risk of getting exposed would be higher. Thus their value from future would be discounted.

### 3.2.5  Preference

The role of shareholders here is to maximize his own utility

$$V = v(q) - t_a - t_s$$

by choosing a compensation rule $t_a > 0$ to the management and $t_s > 0$ to the board. We assume $v' > 0, v'' < 0$. The management's preference is

$$U = t_a - \theta q$$

For the board, if there is no communication between the board and the management, then he is going to maximize the expected value from the market. Otherwise, he is going to maximize the expected payoff from the market and the collusion from the management.

If the management or the board do not participate the game described above, then both of them get zero utility from outside. Therefore the production can not happen, then the shareholders also get zero.

### 3.3 Benchmark

#### 3.3.1 Complete Information Without Board

Under complete information about the management's type $\theta$, the shareholders would equal the marginal utility of the production to the marginal cost, i.e.,

$$v'(q_1^*) = \theta_g \text{ and } v'(q_2^*) = \theta_b \tag{3.1}$$

and would give no rent to management, so that

$$t_1^* = \theta_g q_1^* \text{ and } t_2 = \theta_b q_2^* \tag{3.2}$$

#### 3.3.2 Asymmetric Information Without Board

In this benchmark, we only consider the interaction between the shareholders and the management. And the board is omitted from the model. In order to make the management participate the production, the benefit from the participating the production should be no less than the value form not participating, i.e., for all $\theta$

$$U = t_a - \theta q \geq 0 \tag{3.3}$$

Given above constraint, the task of the shareholders is to specify the appropriate compensation rule to the management in order to maximize expected utility from production. From revelation principle, we know that this can be obtained from the optimal revelation mechanism, which is a pair of compensation rule $(q_1, t_1), (q_2, t_2)$ and satisfy the following incentive compatible conditions.

$$t_1 - \theta_g q_1 \geq t_2 - \theta_g q_2 \tag{3.4}$$

$$t_2 - \theta_b q_2 \geq t_1 - \theta_b q_1 \tag{3.5}$$

Under these conditions, the optimal compensation rule can be characterized by the following condition,

$$v'(q_1^*) = \theta_g \tag{3.6}$$

$$v'(q_2^*) = \theta_b + \frac{\pi}{1-\pi}\Delta\theta \tag{3.7}$$

### 3.4    Asymmetric Information with Board

#### 3.4.1    Inferring Type from Board

As a first step to analyzing the game, we consider how the shareholders' posterior belief on manager's type depends on the board's reports, the accuracy of the signal, $\mu$, and the shareholders' prior, $\pi$.

Given above set up, we are going to determine the shareholders' posterior on the management's type of being efficient, $\theta_g$, after seeing a report $\hat{s} = \hat{g}$, i.e.,

$$\Pr(S = \theta_g | \hat{s} = \hat{g}) = \frac{\Pr(\hat{s} = \hat{g} | S = \theta_g) \times \Pr(S = \theta_g)}{\Pr(\hat{s} = \hat{g} | S = \theta_g) \times \Pr(S = \theta_g) + \Pr(\hat{s} = \hat{g} | S = \theta_b) \times \Pr(S = \theta_b)}$$

On the one hand, we know that with probability $\pi$ the true type of the management is $\theta_g$, i.e., $\Pr(S = \theta_g) = \pi$.  However, the directors may not perfectly observe this. With probability $\mu$ directors observes $g$, and they report $\hat{g}$ with probability $\sigma_g(\hat{g})$, the mixed strategy of reporting $\hat{g}$ after seeing g. Meanwhile, with probability $1 - \mu$, the directors would also report $\hat{g}$ with probability $\sigma_b(\hat{g})$ , even if they observe a signal $b$.

On the other hand, we know that with probability $1 - \pi$ the true type of the management is $\theta_b$, i.e., $\Pr(S = \theta_b) = 1 - \pi$. In this case, after seeing a report $\hat{s} = \hat{g}$, the shareholders know the follows is happening.  With probability $1 - \mu$ directors observe a signal $g$ and report $\hat{g}$ with probability $\sigma_g(\hat{g})$. Meanwhile, with probability $\mu$ directors observes a signal $b$ but they choose to report as $\hat{g}$ with probability $\sigma_b(\hat{g})$.

Therefore, we have the following result,

$$\Pr(S = \theta_g|\hat{s} = \hat{g}) = \frac{\pi[\sigma_g(\hat{g}) \times \mu + \sigma_b(\hat{g}) \times (1 - \mu)]}{\pi[\sigma_g(\hat{g}) \times \mu + \sigma_b(\hat{g}) \times (1 - \mu)] + (1 - \pi)[\sigma_g(\hat{g}) \times (1 - \mu) + \sigma_b(\hat{g}) \times \mu]}$$

Similarly, we can determine the shareholders' posterior belief on the management's type of being efficient, $\theta_g$, after seeing a report $\hat{s} = \hat{b}$:

$$\Pr(S = \theta_g|\hat{s} = \hat{b}) = \frac{\Pr(\hat{s} = \hat{b}|S = \theta_g) \times \Pr(S = \theta_g)}{\Pr(\hat{s} = \hat{b}|S = \theta_g) \times \Pr(S = \theta_g) + \Pr(\hat{s} = \hat{g}|S = \theta_b) \times \Pr(S = \theta_b)}$$

$$= \frac{\pi[\sigma_g(\hat{b}) \times \mu + \sigma_b(\hat{b}) \times (1 - \mu)]}{\pi[\sigma_g(\hat{b}) \times \mu + \sigma_b(\hat{b}) \times (1 - \mu)] + (1 - \pi)[\sigma_g(\hat{b}) \times (1 - \mu) + \sigma_b(\hat{b}) \times \mu]}$$

If the directors always truthfully report the signal, i.e.,

$$\sigma_s(\hat{s}) = \Pr(\hat{s}|s) = 1 \quad \text{and} \quad \sigma_{s'}(\hat{s}) = \Pr(\hat{s}|s') = 0$$

where, $s' \neq s$, then we put them into $\Pr(S = \theta_g|\hat{s} = \hat{g})$ and $\Pr(S = \theta_g|\hat{s} = \hat{b})$ defined above. We have the following expression,

$$\Pr(S = \theta_g|\hat{s} = \hat{g}) = \frac{\pi\mu}{\pi\mu + (1 - \pi)(1 - \mu)}$$

and

$$\Pr(S = \theta_g|\hat{s} = \hat{b}) = \frac{\pi(1 - \mu)}{\pi(1 - \mu) + (1 - \pi)\mu}$$

From these expressions, we can immediately have the following statement:

**Lemma 4.** *Suppose that the board truthfully reports the signal with positive probability, then the posterior belief of the management being an efficient type would depend on the report from the board.*

- *If the board reports that the management is efficient type, i.e., $\hat{s} = \hat{g}$, then the posterior belief of the management's type, $Pr(S = \theta_g|\hat{s} = \hat{g})$, is increasing with the prior, $\pi$, and the accuracy of the signal, $\mu$.*

- *If the board reports that the management is non-efficient type, i.e., $\hat{s} = \hat{b}$, then the posterior belief of the management's type, $Pr(S = \theta_g | \hat{s} = \hat{b})$, is increasing with the prior, $\pi$, but is decreasing with the accuracy of the signal, $\mu$.*

These results are very intuitive. They tell us that the more the shareholders' prior beliefs favor the efficient type, $\theta_g$, the more likely the shareholders will believe that the management is efficient, no matter what report they get from the board. Meanwhile, the more accurate the signal of being an efficient management is received by the board, i.e., $\mu$ is large, the more likely the shareholders believe that the management is efficient if a message of being efficient is reported. However, if a message of being non-efficient is reported, and given that a high accuracy of a non-efficient signal might be received by the board, then it would be less likely for the shareholders to believe that the management is efficient.

Meanwhile, it is easy to check that, in general

$$\Pr(S = \theta_g | \hat{s} = \hat{g}) \neq \Pr(S = \theta_g | \hat{s} = \hat{b})$$

unless we have $\mu = \frac{1}{2}$. This tells us: In general, the shareholders would have different posterior on the management's type after seeing different reports from the board.

### 3.4.2  No Communication Between the Board and the Management

In this section, we first consider a board institution in which the board directors or board could not communicate with the management. And the technology of supervision is explained as above.

**Proposition 16.** *Given that there is no communication between the board and the management, if the following conditions are true:*

$$\mu \in (\frac{1}{2}, 1) \ and \ \hat{s} = \hat{b} \ is \ reported.$$

*or*

$$\mu \in (0, \frac{1}{2}) \ and \ \hat{s} = \hat{g} \ is \ reported.$$

*then the presence of the board would improve shareholders' benefit, in the sense that,*

- *The chance of giving up an information rent is lower than the case of no board monitoring;*

- *The production of the non-efficient type would be higher than the case of no board monitoring.*

Let us see why it is true. Given that the board can not communicate with the management, the directors would not share any information rent with the management. Then given the posterior belief, the shareholders choose the contract to maximize the expected utility. The optimal contract needs to satisfy the following condition,

$$v'(\hat{q}_1^{0*}) = \theta_g$$

$$v'(\hat{q}_2^{0*}) = \theta_b + \frac{\hat{\pi}(\hat{s})}{1 - \hat{\pi}(\hat{s})}\Delta\theta$$

where $\hat{\pi}(\hat{s}) \equiv \Pr(S = \theta_g|\hat{s})$ and $\hat{s} \in \left\{\hat{b}, \hat{g}\right\}$. By manipulating algebras, we get: $\hat{\pi}(\hat{g}) < \pi$, if $\mu \in (0, \frac{1}{2})$; and $\hat{\pi}(\hat{b}) < \pi$, if $\mu \in (\frac{1}{2}, 1)$.

We can also check that if these conditions are not satisfied, then the posterior of the management being efficient would be higher than the prior. This means the chance of giving up an information rent would be higher. From above optimization conditions, we know that the non-efficient type management would induce a lower level production. All these would worsen the shareholder's benefit. The key point here is that, after getting the null report from the board, the shareholders knows that he is facing a high ability management with a lower probability $\hat{\pi}$. Accordingly, the information rent give to the low ability management will also be lower.

### 3.4.3 A Simplified Supervision Technology

From now on, we are going to focus on the cases in which the board would improve the efficiency when there is no collusion between the board and the management. Therefore, we can study how the collusion between the board and the management would distort the equilibrium outcomes. In order to fulfill this job in a simple way, we introduce the following simplified supervision technology.

We assume that the board's monitoring can only inform about the efficient type $\theta_g$. That is, if $\theta = \theta_g$, then with probability $\mu \in (0,1)$, the board perfectly observes the efficient type, i.e.,

$$\text{Prob}(g|\theta_g) = \mu$$

and with probability $1 - \mu$, he observes nothing, i.e., if $S = \theta_b$, then $\hat{s} = 0$. It is easy to check this supervision technology is a special case of the one defined above. The detail is in the Appendix.

Give this simplified supervision technology, if the shareholders report nothing, then the shareholders revises his belief based on the Bayesian rule. The posterior of the hight ability is

$$
\begin{aligned}
&\text{Prob}\{\theta = \theta_g|\hat{s} = 0\} \\
&= \frac{\text{Prob}\{\theta_g\} \times \text{Prob}\{\hat{s} = 0|\theta_g\}}{\text{Prob}\{\theta_b\} \times \text{Prob}\{\hat{s} = 0|\theta_b\} + \text{Prob}\{\theta_g\} \times \text{Prob}\{\hat{s} = 0|\theta_1\}} \\
&= \frac{(1 - \mu)\pi}{(1 - \mu)\pi + (1 - \pi)} = \frac{(1 - \mu)\pi}{1 - \mu\pi} \equiv \hat{\pi}
\end{aligned}
$$

It is easy to check that $\hat{\pi} < \pi$.

Given this belief, the shareholders choose the contract to maximize the expected

utility. Then the optimal contract needs to satisfy the following condition,

$$v'(\hat{q}_1^{0*}) = \theta_1$$

$$v'(\hat{q}_2^{0*}) = \theta_2 + \frac{(1-\mu)\pi}{1-\pi}\Delta\theta$$

One immediate implication from above result is that,

**Proposition 17.** *1. Strong level of monitoring by the board would give a more accurate information about the type of the management. This means that, if a signal of efficient type is reported, then it is sure that the management is efficient. Furthermore, no information rent would be payed to the management.*

*2. Low level of monitoring by the board would give no information to the shareholders, then no type is revealed, thus the shareholders still need to pay the information rent to the management.*

The proof of this proposition is simple. For the first part, since we explain the probability $\mu$ as a function of the board's effort. High level of monitoring could induces a high probability of a signal which indicates the efficient management, i.e., $\mu$ would converge to probability 1, For the second part, it is easy to show that when the board lower the effort in the monitoring, the probability $\mu$ would converge to probability 0. Then the would be the same as the case of no board.

The intuition is as follows: With a high level of monitoring, an efficient type management would more easily be revealed to the board. Then a piece of more accurate information is reported to the shareholders. After perfectly revealing the information on the management, the contract between the shareholders and the management would be the same as the case of complete information. Therefore, there would be no information rent payed to the management, the production level of non-efficient management would increase. However, with a low level of monitoring, the board

may get nothing. Then the situation would be the same as the case of incomplete information with no board, i.e., the board would be useless.

### 3.4.4 Communication Between the Board and the Management

In the previous analysis, the board and the management has no way to communicate and no deal would be made between them.

However, in practice, and also as criticized by corporate finance literature, the board members may belong to the same social network as the management. In this case, the board members and the management might sign a side contract, or get a underground deal to share the information rent.

If the board could communicate and make deal with the management, then after seeing a signal which indicate a efficient management, the board has incentive to conceal his signal and share the information rent $\Delta\theta\hat{q}_2$ with the management. Meanwhile, a rational board would also consider the potential loss from the labor market if he misreports the signal. Then without considering the detail of the agreement between the board and the management, we know that, the maximum amount of money the board could get from the management is $\Delta\theta\hat{q}_2$, however, the board would face a potential loss of $\rho k(\varepsilon = 0, \eta, y)$ in this case. Then the board's maximum expected benefit from collusion would be

$$\Delta\theta\hat{q}_2 + \rho k(\varepsilon = 0, \eta, y)$$

Then we have the following result:

**Proposition 18.** *If there is communication between the board and the management, then*

1. *There exist an optimal compensation to the board which would block the collusion between the board and the management.*

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2. *The optimal production level of the efficient management would be decreased.*

3. *The expected information rent to the management would be decreased.*

*Proof.* We first assume that in the communication between the board and the management, they have an agreement, a contract, on the division of the rent. In this agreement, the board would get share $\alpha \in [0,1]$ of the rent and the management keeps $1 - \alpha$ of the rent. This also includes the case of no side contract between the management and board. It is capture by $\alpha = 0$.

The detail of the communication would be modeled later. And we are going to prove that there is a unique equilibrium which would support the share $\alpha$. But for now we assume that in equilibrium the board would get $\alpha$ from the management. In general, there might be multiple equilibrium in the communication between the board and management. But no matter what agreement is reached between them, we can always rewrite the rent delivered to the board as $\alpha \Delta \theta \hat{q}_2$. Here $\hat{q}_i$, $i = 1, 2$, is the outcome of production if a $\theta_i$ management and the board are presented.

Therefore, in order to block the possible agreement between the management and the board, the shareholders only needs to provide the same amount to the board, then it would be weakly dominate strategy for the board to reject the offer from the management, but to accept the offer from the shareholders and truthfully report the management's type to the shareholders. And this would be happening with probability $\pi \mu$. This is explained as follows. First, with probability $\pi$ the board would meet a $\theta_g$ type management, and this would give the board a chance to conceal the signal. However, it is not certain for the board to observe the signal of type, he can only observe $\theta_g$ with probability $\mu$, given that the true state is $\theta_g$.

If the board reports that the management is efficient, then the shareholders only need to pay the first best compensation to the management which is $\theta_g q_1^*$. This

happens with probability $\pi\mu + (1-\pi)(1-\mu)$, i.e., with probability $\pi\mu$, an efficient management is observed to be efficient. Meanwhile, with probability $(1-\pi)(1-\mu)$, a non-efficient management is observed to be efficient. This induces that, with probability $\pi\mu + (1-\pi)(1-\mu)$, the shareholders would get utility

$$v(q_1^*) - \theta_g q_1^*$$

Then with probability $1 - \pi\mu - (1-\pi)(1-\mu)$, a non-efficient type is reported.

For other events which happen with probability $1 - \pi\mu$. But after the null signal, the shareholders would update the prior to posterior described before. That is, the shareholders face a management in efficient type, $\theta_g$, with probability $\hat{\pi}(\hat{s}) < \pi$. Since in this event, the board also observes nothing, there is no way to get the real type of management reported. Thus the shareholders has to pay information rent $\theta_g \hat{q}_1 + \Delta\theta \hat{q}_2$ to the $\theta_g$ type management. Here $\hat{q}_1$ and $\hat{q}_2$ are the outcomes of production from $\theta_g$ and $\theta_b$ management respectively, given that no information is revealed to the board and the shareholders. Therefore, with probability $\hat{\pi}(1 - \pi\mu)$, the shareholders would get utility

$$v(\hat{q}_1) - \theta_g \hat{q}_1 - \Delta\theta \hat{q}_2$$

On the other hand, given no information is reported, with probability $1 - \hat{\pi}$ the shareholders would face a non-efficient type management, and the shareholders would give $\theta_b \hat{q}_2$. Therefore, with probability $(1 - \hat{\pi})(1 - \pi\mu)$, the shareholders would get utility

$$v(\hat{q}_2) - \theta_b \hat{q}_2$$

Summarize above analysis, we get the no collusion expected utility for the sharehold-

ers as

$$\pi\mu\left[v(q_1^*) - \theta_1 q_1^* - \alpha\Delta\theta\hat{q}_2\right]$$
$$+ (1 - \pi\mu)\left[\hat{\pi}\left(v(\hat{q}_1) - \theta_1\hat{q}_1 - \Delta\theta\hat{q}_2\right) + (1 - \hat{\pi})\left(v(\hat{q}_2) - \theta_2\hat{q}_2\right)\right]$$

The shareholders optimize above problem with respect to $\hat{q}_1(\hat{s})$ and $\hat{q}_2(\hat{s})$. Then we get

$$v'(\hat{q}_1^*) = \theta_g \tag{3.8}$$

$$v'(\hat{q}_2^*) = \theta_b + \Delta\theta\frac{\pi}{1 - \pi}[1 - \mu(1 - \alpha)] \tag{3.9}$$

From above expressions, it is easy to check that the compensation to the $\theta_g$ management would be the first best which is the same as the case of complete information and the case of no communication between board and management. However, for the $\theta_b$ management, the board may not improve the welfare of the shareholders if $\alpha \to 1$. It is easy to check that, as $\alpha \to 1$, the optimal compensation to $\theta_b$ management would be the same as the case of asymmetric information without board. This implies that, if there is communication between the board and the management, then the board might be less useful then the case of no communication between the management and the board. However, when $\alpha \to 0$, the optimal compensation to $\theta_b$ management would be converging to the case of no communication between the board and management. This implies that if the rent delivered to the board is small enough, then even if communication is allowed between the management and the board. Then the board would be useful in the sense that he would improve the welfare the shareholders.

From the property of the management's utility function, it is easy to check that the communication between the board and the management would increase the production of $\theta_b$ management. To prove this, we only need to check the optimal condition

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Since $\alpha \in [0, 1]$, then it is true that the right hand side of the optimal condition is larger than the case of no communication which is

$$v'(\hat{q}_2^*) = \theta_b + \Delta\theta \frac{\pi}{1-\pi}[1 - \mu(1-\alpha)]$$
$$\geq \theta_b + \Delta\theta \frac{\pi}{1-\pi}[1 - \mu]$$
$$= v'(\hat{q}_2^{0*})$$

Recall that $v''(y) < 0$, then it induces $\hat{q}_2^* \leq \hat{q}_2^{0*}$. Therefore the expected rent kept by the management is also decreased, which would be

$$\pi(1-\mu)(\theta_b - \theta_g)\hat{q}_2^* \leq \pi(1-\mu)(\theta_b - \theta_g)\hat{q}_2^{0*}$$

The left hand side expression reads as follows. With probability $\pi$, the management would be an efficient type $\theta_g$, and with probability $1 - \mu$ no signal would be revealed to the board, so no reports to the shareholders, then the management could keep all the rent.

The intuition is that: To prevent the potential collusion between the board and the management, the shareholders needs to pay a rent to the board. This is costly to the shareholders, then the production level will be decreased. Meanwhile, the potential collusion benefit to the management also decreases. Now we have proved Proposition 18.

$\square$

We want to emphasize one point of Proposition 18. In the Proposition, we only focus on the reward from the shareholders. But in reality, there are many other possible channels of rewards. One possible channel is the reward from the market of directorships which is already described in the model section. Intuitively, the reward from the market of directorships should affect the communication between the board

and management, so as the optimal decisions of the shareholders. In next section, we are going to show that the value from the market of directorships might largely affect the shareholders' decision and the collusion between the board and the management.

One immediate implication of above Proposition is that:

**Proposition 19.** *When the share of rent divided to the board is increased,*

1. *The expected payoff to the shareholders would be decreased;*

2. *The production level of non-efficient management would be distorted downward;*

3. *The expected rent to the management would be decreased also.*

*Proof.* To prove these properties, we only need to check the shareholders's optimization problem. It is obvious that the expected payoff of the shareholders is decreasing as $\alpha \to 1$. The last two points have been shown in Proposition 18. $\square$

From above analysis, we know that, except for the board, all the player's expected payoff are affected (downward) from the possible collusion between the board and the management. In particular, the management's benefit would also be affected. Recall that, the production level of the non-efficient type management would decrease. This may further imply that: The management might have incentive to make his information revealed in exchange for reasonable transfer from the shareholders and hope the shareholders not to hire a board to monitor him. This is because, with the presence of the board, the management might get less than the reasonable compensation from the shareholders. Meanwhile, for the shareholders, he might get a lower production level then the case of no supervision. Thus both the shareholders and the management would have incentives to make the exchange.

To make the transaction happen, the shareholders only needs to set the reasonable payment between the amount of expected rent given to the corrupted board and the

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amount of expected information rent given to the management in the case of no board.

## 3.5    Analysis of the Bargaining

In the set up of the bargaining, the proposal power is not specified to any players. But we are going to analyze two cases later. In the first one, the board has the proposal power. This means that the board would first give the offer. The motivation is as follows. When the board notices that the management is efficient and could get an information rent, then both know that this rent is completely controlled by the board. Since the board's reports would determine the management's rent. Given this situation, it is reasonable to assume that the board has the proposal.

In the second case, we would assume that the management has the proposal power. The motivation is as follows. Think about the case in which the information rent is large, and both the board and management value the rent. Then it is reasonable to assume that the management might use the rent as a bait to capture the board and make him conceal the information.

However, there do exist cases in which the bargaining, so as the potential collusion, between the board and the management would not happen. This would depend on the benefit from the market of reputation. Next proposition formally describe this argument.

**Proposition 20.** *For the board, if the expected difference between telling the truth and lying is larger than the information rent payed to the management, then the board would always truthfully report the signals to the shareholders. There would be no collusion between the board and the management.*

*Proof.* If the board meets a efficient management, then he has chance to get at most $\Delta\theta\hat{q}_2$ from the bargaining. However, all of the two's decisions in the bargaining would

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depend on their outside options. For the board, he would get

$$w_S = \rho k(\varepsilon = 1, \eta, y)$$

from quiting the bargaining. This reads as: The board observes that the management is efficient. He truthfully reports the type of the management to the shareholder, but does not engage in the collusion. Then with probability $\rho > 0$, he gets reward $k(\varepsilon = 1, \eta, y)$ from the market of directorships. However, with probability $1 - \rho$, he might get nothing from the market of directorships. Thus if the board's outside option is larger than the most he could get from collusion, i.e.,

$$w_S = \rho k(\varepsilon = 1, \eta, y) \geq \Delta\theta\hat{q}_2 + \rho k(\varepsilon = 0, \eta, y)$$

the bargaining between the board and the management would not take place. Above condition is equivalent to

$$\rho\left[k(\varepsilon = 1, \eta, y) - k(\varepsilon = 0, \eta, y)\right] \geq \Delta\theta\hat{q}_2 \tag{3.10}$$

$\square$

Above result also induce that,

**Proposition 21.** *If the total outside options of the board and the management is larger than the information rent, i.e.,*

$$w_A + w_S \geq \Delta\theta\hat{q}_2$$

*there would be no collusion between the board and the management.*

One immediate implication of above proposition is that, if the reward from the market of directorships of telling truth is large enough or the punishment from lying is series enough too, then the board would have no incentive to bargaining with

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the management to divide the potential information rent. Meanwhile, neither does the management can pay for the bait to capture the board. Thus the type of the management would be perfectly reported to the shareholders, then the shareholders would implement the first best contract to compensate the efficient type management. This would be the best result to the shareholders.

Now let us go to the more interesting case in which the two have common stake, i.e.,

$$w_S + w_A = \rho k(\varepsilon = 1, \eta, y) < \Delta\theta\hat{q}_2 + \rho k(\varepsilon = 0, \eta, y)$$

If this is the case, then both of them would have incentives to engage in the bargaining process so as to get a share of the rent.

The next result is about the equilibrium proposal in the bargaining between the board and the management, given the above common stake. And this result would not depend on the proposal power.

**Proposition 22.** *In the bargaining between the board and the management, there exists a unique sub-game perfect equilibrium, such that*

- *board always offers $x_S^*$, always accepts an offer $x_A$ if and only if $x_A \leq x_A^*$, and always chooses to quit after receiving an offer $x_A > x_A^*$ if and only if $\delta_S x_S^* \leq w_S$*

- *management always offers $x_A^*$, always accepts an offer $x_S$ if and only if $x_S \leq x_S^*$, and never choose to quit the bargaining, where* [12]

$$x_S^* = \begin{cases} \beta\left(Q[1 - \delta_A] - \rho\delta_A k(0)\right) & \text{if } \rho k(1) \leq \beta\delta_S\left(Q[1 - \delta_A] - \rho\delta_A k(0)\right) \\ Q[1 - \delta_A] + \rho\delta_A\Delta k & \text{if } \rho k(1) > \delta_S Q[1 - \delta_A] + \rho\delta_A\delta_S\Delta k \end{cases}$$

---

[12]To save some place for notation but without confusion, we let

$$k(0) \equiv k(\varepsilon = 0, \eta, y) \text{ and } k(1) \equiv k(\varepsilon = 1, \eta, y) \quad \forall \eta, y$$

*and*

$$
x_A^* = \begin{cases} \beta\Big(Q[1 - \delta_S] + \rho k(0)\Big) & \text{if } \rho k(1) \leq \beta\delta_S\Big(Q[1 - \delta_A] - \rho\delta_A k(0)\Big) \\ Q + \rho\Delta k & \text{if } \rho k(1) > \delta_S Q[1 - \delta_A] + \rho\delta_A\delta_S\Delta k \end{cases}
$$

*where $\Delta k = k(1) - k(0)$ and $\beta = 1/(1 - \delta_A\delta_S)$.*

*Proof.* The proof of this equilibrium strategy in the bargaining needs some space. But the idea is straightforward. In any SPE of the bargaining, the responder $i$ would be indifferent between accepting and not accepting (quit or give a counter offer) the proposer $j$'s ($j \neq i$) equilibrium offer. That is

$$
Q - x_S^* = \max\{\delta_A x_A^*, w_A\}
$$

$$
Q - x_A^* + \rho k(\varepsilon = 0, \eta, y) = \max\{\delta_S x_S^*, w_S\}
$$

Here, $Q$ is the information rent which would be divided by the board and the management. Since we always have $x_A \geq 0$ and $w_A = 0$, then above conditions coud be written as

$$
Q - x_S^* = \delta_A x_A^*
$$

$$
Q - x_A^* + \rho k(\varepsilon = 0, \eta, y) = \max\{\delta_S x_S^*, w_S\}
$$

It is easy to check there is only one solution to above equations which is the one stated in the proposition. We can further prove that this strategy profile is the unique subgame perfect equilibrium of the bargaining.

**Claim 5.** *The strategy profile described above is a subgame perfect equilibrium in the bargaining between the board and the management.*

*Proof.* See Appendix. $\square$

**Claim 6.** *This is the unique subgame perfect equilibrium of the bargaining between the board and the management.*

*Proof.* See Appendix. □

□

Now, we have characterized the unique SPE of the bargaining between the board and the management. It is easy to check that agreement would be reached immediately at $t = 0$ no matter who gets the proposal power; and the board would not quit the bargaining process.

## 3.6    Market of Directorships and Supervision

Thus far we have taken the bargaining process separately to the shareholders's decision. In this section, we explore the affection from the market of directorships to the shareholders in more detail. We discuss how the market of reputation would affect the equilibrium proposal so as the shareholders's optimal decisions/contract and the welfare to shareholders.

### 3.6.1    Sensitive Market

In the first case, we consider the following market mechanism. The market of directorships are sensitive to the board's behavior. In other word, the board cares very much about the value on the market of directorships. To catch this point, we study the shareholders' optimal decisions when $\eta \to 1$. If the board is observed to misreporting the true type of the management, then the market would give a serious punishment, $k(0, \eta \to 1)$, to the management. If the punishment is serious enough, then the condition

$$\rho k(1, \eta \to 1) > \delta_S Q[1 - \delta_A] + \rho \delta_A \delta_S \Delta k$$

may not be true anymore. This is because, the more serious the punishment from lying, the higher value on the right side of the condition. But the left side is only affected by the reward of telling the truth. Serious punishment would induce a violation of this condition. However, for the similar explanation, the other condition

$$\rho k(1, \eta \to 1) \le \beta \delta_S \Big( Q[1 - \delta_A] - \rho \delta_A k(0, \eta \to 1) \Big)$$

would be more easily satisfied.

So if the punishment increase too much then quitting the bargaining process would not be a credible threat to any of the players.

Then the outcome of bargaining would only depends on the board's expected punishment from helping management to conceal the signal of his type, which is $k(0, \eta \to 1)$ and, of course, also depends on the total information rent $Q$. This analysis is true, no matter who has the proposer power. In both case, the share of rent delivered to the board would be increasing with the punishment from the market.

Thus far, the analysis is only about the equilibrium proposal in the bargaining. Let us go further to discuss how the equilibrium outcome in the bargaining would affect the shareholders's decision. Recall the analysis in the shareholders's optimization problem. In that analysis, we use $\alpha Q$ to represent the equilibrium proposal to the board. If the board has the proposal power, then it is $x_S^*$, otherwise it is $1 - x_A^*$. Since the share of rent delivered to the board is increasing with the absolute value the punishment from the market of directorships. Then an increase on the punishment from lying would increase the share $\alpha$ delivered to the board. This would further reduce the outcome of the production. To prove it, we only need to check the optimal condition (3.9). For convenience, we represent it here and rewrite $\alpha$ as a function of the punishment from lying,

$$v'(\hat{q}_2^{**}) = \theta_b + \Delta \theta \frac{\pi}{1 - \pi} [1 - \mu(1 - \alpha(k(\varepsilon = 0, \eta \to 1)))]$$

When $k(0, \eta \to 1)$ increases, the right hand side of above expression also increases, since $v'' < 0$, then we have $\hat{q}_2^{**}$ decreased. Now we have

$$\hat{q}_2^{**} \leq \hat{q}_2^{*} \leq \hat{q}_2^{0*}$$

This also induce that the expected information rent kept by the management would be decreased. An immediate implication of above result is as follows. The punishment of lying from market of directorships may not really encourage the board to report the truth to the shareholders. However, a higher punishment would make the board get more share from the information rent to the efficient management. This would induce a higher cost to the shareholders to block the potential collusion between the board and the management. Furthermore, as it is more costly for the shareholders to block the collusion, the level of production would decrease more.

In the previous analysis, we only allow the market to give serious response only if the board chooses to lie about the efficient management's type. Now we are going to fix the punishment from the market, but only allow the market to give serious rewards to the board, if he chooses to report the true type of the management. Therefore, the condition

$$\rho k(1, \eta \to 1) \leq \beta \delta_S \left( Q[1 - \delta_A] - \rho \delta_A k(0, \eta \to 1) \right)$$

may not be satisfied easily. This is because, the reward to truth reporting would only increase the left side of the condition. Then it might be violated easily. However, the other condition

$$\rho k(1, \eta \to 1) > \delta_S Q[1 - \delta_A] + \rho \delta_A \delta_S \Delta k$$

would be easily satisfied in this case. This is because, the high rewards would increase on both sides of the condition. This induce that, the high reward from the market of directorships would make the threat of quiting the barging more credible. Therefore,

both player would consider the opposite might choose to quit in the bargaining. Unlike the previous case, the two players need to consider both the punishment and reward from the market of directorships. What matters here would be the absolute value of the punishment plus the reward and the output. The intuition is as follows, since the reward of truth reporting is high, so in order to get the board involved, the management need to give a share of the rent which would be at least as high as the compensation from the market. However, this would not be enough to make a deal with the board. Since after they have an agreement, the board also face the risk of getting exposed. This would make the board get the punishment from the market which would be a loss to the board. Thus, in order to get the board make a deal with the management. The management needs to pay as least the rewards from the market plus the possible loss to the management, which is $\Delta k$.

Above argument is true, no matter who has the proposer power. If the rational management has the proposal power, then he would think as above. If the board has the proposal power, he would know that a rational management would think as above, and would know that the board also knows this. Then proposal from the board also related to $\Delta k$, the only difference would be on other parameters.

As described in the equilibrium outcome of the bargaining, no matter who gets the proposal power, the rent delivered to the board would depend on $\Delta k$, the absolute value of punishment plus the reward, and the total information rent $Q$. It is easy to check that this rent would be increasing with $\Delta k$.

As previous analysis, we can now discuss how the equilibrium outcome of bargaining affect the shareholders's decision. Follow above analysis, we know that the share $\alpha$ here would be increasing with the rewards to the board, meanwhile we can write

the shareholders's optimal condition as,

$$v'(\hat{q}_2^{**}) = \theta_b + \Delta\theta \frac{\pi}{1-\pi}[1 - \mu(1 - \alpha(\Delta k))]$$

Then it is easy to check that: When $\Delta k$ increases, the right hand side of above expression increases, since $v'' < 0$, then we also have $\hat{q}_2^{**}$) decreased, and also, as previous case,

$$\hat{q}_2^{**} \leq \hat{q}_2^{*} \leq \hat{q}_2^{0*}$$

The expected information rent kept by the management would also be decreased in this case. We now have another implication. The rewards from market of directorships may not really encourage the board to report truth neither. High rewards from the market of directorships only gives the board more bargain chips to raise his share of the information rent from the management. Meanwhile this would increase the shareholders's cost of blocking the collusion between the board and the management. Furthermore, as before, this would lower the production level.

Now, let us go to the case in which both the punishment and rewards would increase. This represents a market mechanism in which give large rewards to the honest behavior by the board and give a heavy punishment to the dishonest behavior. No matter which condition is satisfied this time. The share of information rent delivered to the board in the bargaining process would always be increasing with the punishment $k(0, \eta \to 1)$ or the reward $k(1, \eta \to 1)$. So all the previous analysis would be applied here.

We can thus summarize above analysis as the following result.

**Proposition 23.** *If the market of directorships sensitively responds to board's misreporting, i.e., $\eta \to 1$ then, compare to the case of no communication,*

- *the shareholders would pay more to the board to block the collusion;*

- *the level of production would decrease;*

- *the information rent kept by the management also decreases.*

### 3.6.2   Non-sensitive Market

Thus far we have assume that, the market of directorships are sensitive to the board's behavior. The result we get right now is, if the market of directorships are sensitive to the board's behaviors. Then the more sensitive of the market, the more costly for the shareholders to block the collusion and the lower production level would be achieved. Now we are going to do the opposite analysis. We study the case in which the market of directorships are not sensitive to the board's behavior. In other word, the board does not care too much about the value on the market of directorships.

Above situations comes up very often in the board members. In many cases, the board members are from some other industries or professions which have no direct relations to the financial industry. One profession is the professor from college. Their reputation on the board market has nothing to do with their professions. This non-sensitive market is similar to the non-financial market introduced in the current research, such as Knyazeva *et al.* (2013).

To formalize the analysis, we study the shareholders's optimal decisions when $\eta \to 0$. Follow the previous procedure, we first see what happens if the market does not response to much when the board truthfully report the management's type, i.e., $k(1, \eta \to 0) \to 0$. If this is the case, then the condition

$$\rho k(1, \eta \to 0) > \delta_S Q[1 - \delta_A] + \rho \delta_A \delta_S \Delta k$$

may not be true any more. The is because the board would not value the reward from truthfully reporting too much, then quiting the bargaining to report the management's type becomes a incredible threat to both players. Then what matters

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would only be the potential loss from concealing the management's type. Thus the equilibrium rent divided to the board would only depend on the possible loss from the market of directorships, which is $k(0, \eta \to 0)$. This has been described in the equilibrium strategy profiles of the bargaining.

Next, observe that given the equilibrium proposal in the bargaining process, decreasing the punishment of lying, i.e., $k(0, \eta \to 0) \to 0$, would lower the rent delivered to the board, i.e., $\alpha$ would decrease. Then the right side of the of condition (3.9)

$$v'(\hat{q}_2^{***}) = \theta_b + \Delta\theta \frac{\pi}{1 - \pi}[1 - \mu(1 - \alpha(k(\varepsilon = 0, \eta \to 0)))]$$

would decrease. Recall that $v'' < 0$, then the production level would increase, such that

$$\hat{q}_2^{***} \geq \hat{q}_2^{**} \text{ and } \hat{q}_2^{***} < \hat{q}_2^{0*}$$

Meanwhile, the expected information rent kept by the management would increase.

Now let us assume that the market does not response to much to the lying behavior of the board, then we have $k(0, \eta \to 0) \to 0$, this induce that the condition

$$\rho k(1, \eta \to 0) \leq \beta\delta_S\left(Q[1 - \delta_A] - \rho\delta_A k(0, \eta \to 0)\right)$$

may not be satisfied easily. This is because if the punishment is not serious, then the expected cost of colluding with the management would not be high, this means the board may not get a high share of the information rent from the management. This would induce the board to truthfully report the type of the management to the shareholders, and get a higher expected rewards from the market of directorships. Therefore, quiting the bargaining becomes a credible threat to the management. In order to make the board conceal the type, the management needs to propose based on both the reward and the punishment. This is exactly what we described in the equilibrium strategy profile.

Next, given the equilibrium bargaining proposal, decreasing the rewards to reporting truthfully would also decrease the share of rent delivered to the board, i.e., $\alpha$ would decrease. Recall condition (3.9) and $v'' < 0$, then we have a similar the production level increased, i.e.,

$$\hat{q}_2^{***} \geq \hat{q}_2^{**} \text{ and } \hat{q}_2^{***} < \hat{q}_2^{0*}$$

and a decreased level of the management's expected information rent.

Now, let us go to the case in which both the punishment and rewards would decrease. This represents a market mechanism in which give no or very small reward to the honest behavior by the board and give a light or no punishment to the dishonest behavior. No matter which condition is satisfied this time. The share of information rent delivered to the board in the bargaining process would always be decreasing with the punishment $k(0, \eta \to 0)$ or the reward $k(1, \eta \to 0)$. Then we have that the production level would increase.

Summarize above analysis, we have the following result.

**Proposition 24.** *If the market of directorships does not sensitively respond to board's actions , i.e., $\eta \to 0$, then, compare to the case of sensitive market,*

- *the shareholders would pay less to the board to block the collusion;*

- *the level of production would increase;*

- *the information rent kept by the management also increases.*

A strong implication from above analysis is that, in order to make the board truthfully report the type of the efficient management and lower the cost of blocking the collusion between the board and the management, a good choice for the shareholders is to find a board who has the ability to monitor the type of the management but

also has no direct relation to the financial market. This implication is consistent with empirical studies which support the positive relation between board independence and firm performance (see Weisbach 1988, Borokhovich et.al 1996, just name a few here).

## 3.7    Board Size and Supervision

In previous analysis, we trade the board of directors as one player. But in reality, the board is composed by many directors, outside or inside. Therefore the board room itself is a complicated organization but one assumption here is that there is a representative board representing the directors.

Now we relax this assumption simply by assuming that there are $N$ identical directors in the board. The management could bargain with each directors privately on the rent. In order to make boards conceal the signal of the type, the management needs to capture all the $N$ directors. The bargaining process is the same as the one director one management case. We assume that the management can bargain with each director only once, i.e., no matter if there is an agreement or not between the management and director $i$ in the bargaining, they will not bargain again.

The analysis of above game is a $N$ duplication of the one director one management bargaining. The difference is that each director would get a share from $Q/N$. Here $Q/N$ is the highest possible rent delivered to director $i$.

As the analysis in the one management one director case, it is possible that the bargain between directors and the management may not happen. The reason is as follows. Think about the $N$ directors as a board with an outside option $\hat{w}_S \equiv N w_S$. Recall the analysis in Proposition 20, the condition (3.10) would change to

$$N\rho\left[k(\varepsilon = 1, \eta, y) - k(\varepsilon = 0, \eta, y)\right] \geq \Delta\theta\hat{q}_2$$

It is easy to see that this condition would be more easily to be satisfied when $N$ increases. This induces that no bargaining would happen among the management and the directors if the size of the board is large enough. Therefore the chance of of collusion among them would be decreased.

If the above condition is not satisfied, then the bargaining among management and all directors would take place. Then in each bargaining between director $i$ and management, they would reach the unique equilibrium outcome which is similar to the one described in Proposition 22. The unique equilibrium strategy profile is described as below,

**Proposition 25.** *In the bargaining between each director $i$ and the management, there exists a unique sub-game perfect equilibrium, such that*

- *For any director $i$, he always offers $\hat{x}_S^*$, always accepts an offer $\hat{x}_A$ if and only if $\hat{x}_A \leq \hat{x}_A^*$, and always chooses to quit after receiving an offer $\hat{x}_A > \hat{x}_A^*$ if and only if $\delta_S \hat{x}_S^* \leq w_S$*

- *management always offers $\hat{x}_A^*$, always accepts an offer $\hat{x}_S$ if and only if $\hat{x}_S \leq \hat{x}_S^*$, and never choose to quit the bargaining, where*

$$\hat{x}_S^* = \begin{cases} \beta\left(\frac{Q}{N}[1 - \delta_A] - \rho\delta_A k(0)\right) & \text{if } \rho k(1) \leq \beta\delta_S\left(\frac{Q}{N}[1 - \delta_A] - \rho\delta_A k(0)\right) \\ \frac{Q}{N}[1 - \delta_A] + \rho\delta_A \Delta k & \text{if } \rho k(1) > \delta_S \frac{Q}{N}[1 - \delta_A] + \rho\delta_A\delta_S \Delta k \end{cases}$$

*and*

$$\hat{x}_A^* = \begin{cases} \beta\left(\frac{Q}{N}[1 - \delta_S] + \rho k(0)\right) & \text{if } \rho k(1) \leq \beta\delta_S\left(\frac{Q}{N}[1 - \delta_A] - \rho\delta_A k(0)\right) \\ \frac{Q}{N} + \rho\Delta k & \text{if } \rho k(1) > \delta_S \frac{Q}{N}[1 - \delta_A] + \rho\delta_A\delta_S \Delta k \end{cases}$$

*where $\Delta k = k(1) - k(0)$ and $\beta = 1/(1 - \delta_A \delta_S)$.*

The proof of this proposition is the same as Proposition 22. Follow the analysis of one management one director case, we discuss how the size of board would affect the performanc of the firm. In the first case, we consider $\eta \to 0$ which corresponds to outside directors or non-sensitive market. No matter who gets the proposal power, the share delivered to director $i$, $\alpha$, is not only a function of $k(0)$, but also a function of $1/N$. Recall the optimization problem of the shareholders, then we have

$$v'(\hat{q}_2^*) = \theta_b + \Delta\theta \frac{\pi}{1 - \pi} \left[ 1 - \mu \left( 1 - N \left[ \alpha(\frac{1}{N}, k(0)) \right] \right) \right]$$

It is easy to check that: When $N$ increases, the right hand side of above expression increases, since $v'' < 0$, then we have $\hat{q}_2^*$ decreased.

Now, let us go to the case of $\eta \to 1$, which corresponds to the inside directors or sensitive market. No matter who gets the proposal power, the share delivered to director $i$, $\alpha$ would be a function of $\Delta k$ and $1/N$. Recall the optimization problme of the shareholders, then we have

$$v'(\hat{q}_2^{**}) = \theta_b + \Delta\theta \frac{\pi}{1 - \pi} \left[ 1 - \mu \left( 1 - N \left[ \alpha(\frac{1}{N}, \Delta k) \right] \right) \right]$$

For the same reason as the previous case, we get: When $N$ increases, the right hand side of above expression increases, since $v'' < 0$, then we also have $\hat{q}_2^{**}$ decreased.

Above results tell us that, as long as the bargainings take place, the larger the size of the board, the higher the cost for the management to capture the directors, and the lower performance of the firm. The intuition of these results is as follow, to get an agreement with one director, the management needs to pay a fixed rent to director $i$, and this rent will not depend on the size of the board. Therefore, the increas of the size of the baord, $N$, would only increase the total cost of of the management.

Summarize above analysis, we have the following implication:

**Proposition 26.** *The larger the size of the board, the higher the cost for the management to capture the directors, thus the lower chance for the management and the directors to collude. However, if the collusion could happen, then the cost for the shareholder to block the collusion among the management and directors would increase. And the larger the size of the board, the lower the performance of the firm. Therefore, the lower expected welfare to the shareholder.*

**Remark** Thus far we have assumed that the outside value of each director i.e., the directors' value from the market of directorships only depends on the firm's performance which is measured by the outcome of the production. One result from this assumption is that the bargaining chip of each director will increase with the performance of the firm. Therefore this would induce a high cost for the management to collude with the directors. Thus, the cost of blocking the collusion would also increase. This would seriously affect the firm's performance especially when the size of board is large. However, if the outside value of each director depends on the average performance of each director, $y/N$, then the performance of the firm would be independent from the size of the board, i.e., the size of the board would not affect the performance of the firm. Let us see why. If directors' outside options depend on the size of the board, then the share of the information rent delivered to each director would either be

$$N\alpha(\frac{1}{N}, \frac{k(0)}{N}) \text{ or } N\alpha(\frac{1}{N}, \frac{\Delta k}{N})$$

Recall the equilibrium strategy specified in Proposition 25, it is easy to check that these share would not depend on the size of the board.

Above argument is summarized as follows,

**Proposition 27.** *If each directors' compensation (penalty) from the market of directorships is related to the size of the board, such as the one defined above, then the rent*

*to block the collusion would be independent to the size of the board.*

We have not found any results in the existing research which share the same feature as this proposition. All the exiting research try to find the optimal size of the board, theoretical and empirical, see Raheja (2005), Coles *et al.* (2008), and Adams *et al.* (2008). However, our result induces that, from the view of anti-collusion, the board size could be independent to the cost of blocking the collusion, if we choose a proper market mechanism, i.e., correspond the directors' per capita contribution to the firm's performance. One advantage of doing this is that, this can at least isolate the influence from collusion between the directors and the management when we try to find the optimal size of board. But this result still needs more discussion, such as the case with heterogeneous directors.

### 3.8    Implications and Policy Suggestion

Our analysis has numerous implications on the board structure and policy suggestion of board regulations. First, in a very general way, if we focus on the function of monitoring, the board could either improve or worsen the performance of the firm. The final effects would depend on the accuracy of signal the board gets. In reality the accuracy of the signal could be related to the efforts of the board directors putting in the monitoring. Or we can relate it to the transaction cost of board decisions. Therefore, one policy suggestion is that: If the shareholders want to improve the efficiency, e.g., high output, they could encourage the board to monitor the management in a more frequent way. This has been confirmed by some empirical research. And our results give a rational explanation to these empirical predictions. Meanwhile, to improve the accuracy of the signal, the board could hire specialized account to get more accurate information on the type of the management.

Second, given that the board could obtain a accurate signal on the type of the

management, the collusion between the board and the management could also worsen the performance of the firm, so as the welfare of the shareholders. The magnitudes of this negative affections would depend on the structure of the board. The more independent directors in the board, the better the performance of the firm. Meanwhile, the more dependent directors in the board, the worse the performance of the firm. Therefore, from the view of shareholders or the regulators, fix the position of the board, then increase the proportion of independent directors could improve the firm's performance, e.g., the production level of the firm.

Third, from our last result, we know that, the size of the board could also increase the shareholders' cost of anti-collusion, in the sense that, the larger the size of the board, the higher the cost of blocking the collusion between the board and management. However, through a proper designed compensation rules, the cost of blocking collusion would be independent from the size of the board. This has very important policy implications. Empirically, people find evidences that both the small size and large size of board could improve the firm's performance. The final affection from the size of board might depend on the features of firms, see Coles *et al.* (2008). So it may not be proper to suggest the optimal size of board in general. However, based on our result, if we link the directors' market evaluations to the size of the board, then it is possible that the cost of blocking collusion would be independent from the size of the board. Therefore, we can focus on other aspects of the board other than the anti-collusion.

Forth, if we isolate the shareholders from the model, and only focus on the interaction between the board and the management, i.e., the bargaining process between the board and the management, we know that, to get a deal with the board, the management would pay less rent to the board, if the directors are independent. However, more rent would be payed if the directors are dependent. These results explain why

the management might prefer a more independent board other than a more dependent board. And our explanation is that the formal one might help the management get a deal with the board with a relative low cost. But the later one would induce a high cost to the management.

## 3.9    Conclusion

The model in this paper tries to help us understand how the board's supervision would affect the performance of the firm and how the collusion between the board and the management would downward the firm's performance.

The first message delivered in this paper is that: In general, the board can have a positive or negative influence on the firm's performance. Whether the influence is positive or negative depends on the accuracy of the signal and whether the signal infers the management is efficient. Under some conditions, if there is no communication between the board and the management, the shareholder's welfare would be largely improved.

However, if the board and the management could communicate to collude, which is implicitly modeled as a bargaining process, then the shareholders need to transform part of the information rent, which was supposed to be payed to the efficient management, to the board. More importantly, this model tells us that board members who are independent to the financial market, would lower the shareholder's cost of blocking collusion. Meanwhile, this also would better the firm's performance than the case of board members who are not independent to the financial market. The last result has been already confirmed by some empirical research. And this models gives an explanation of the underground mechanism to the empirical research.

If we isolate the affections from the shareholders, then the bargaining outcomes from the collusion tell us that: The management may prefer a less dependent board to

a more dependent one. This is because the management could keep more information rent from the former one than the later.

A simple extension of the model also indicate that larger the size of the board the higher cost to block the collusion. However, a proper designed market mechanism, e.g., link the directors' market evaluation to the size of the board, could potential induce an independent result between the board size and the cost of anti-collusion.

One critical restriction of this paper is that, the board is composed of homogeneous directors, and we can alway find a representative to do decisions for the board. However, if the directors represents shareholders in different interest, such as the one discussed in Li and Zhenhua (2013), then directors representing different shareholders might have conflict in colluding with the management. Then one possible result is that the check balance among the directors might block the collusion between the board and the management. This might be a Pareto improvement to the shareholders.

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