Tolerance Maps For Patterns Of Profiles

by

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ABSTRACT

This thesis contains the applications of the ASU mathematical model (Tolerance Maps, T-Maps) to the construction of T-Maps for patterns of line profiles. Previously, Tolerance Maps were developed for patterns of features such as holes, pins, slots and tabs to control their position. The T-Maps that are developed in this thesis are fully compatible with the ASME Y14.5 Standard. A pattern of square profiles, both linear and 2D, is used throughout this thesis to illustrate the idea of constructing the T-Maps for line profiles. The Standard defines two ways of tolerancing a pattern of profiles – Composite Tolerancing and Multiple Single Segment Tolerancing. Further, in the composite tolerancing scheme, there are two different ways to control the entire pattern – repeating a single datum or two datums in the secondary datum reference frame. T-Maps are constructed for all the different specifications. The Standard also describes a way to control the coplanarity of discontinuous surfaces using a profile tolerance and T-Maps have been developed.

Since verification of manufactured parts relative to the tolerance specifications is crucial, a least squares fit approach, which was developed earlier for line profiles, has been extended to patterns of line profiles. For a pattern, two tolerances are specified, and the manufactured profile needs to lie within the tolerance zones established by both of these tolerances. An *i*-Map representation of the manufactured variation, located within the T-Map is also presented in this thesis.

DEDICATION

This work is dedicated to my parents and my sister for all their support and encouragement.

AKNOWLEDGEMENTS

I would like to thank my advisor and committee chair, Dr. Joseph K. Davidson for his support, guidance and patience without which this work would not have been possible.

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TABLE OF CONTENTS

CHAPTER Page

LIST OF FIGURES

Figure Page

Figure Page

Figure Page

Figure Page

Figure Page

Figure Page

CHAPTER 1

INTRODUCTION

1.1 Tolerance and its types:

Engineering tolerance is a permissible limit of variation allowed in a specified dimension. A tolerance is specified because manufacturing processes inherently induce variations in the finished part. In other words a part cannot be manufactured to perfectly match the specified dimensions. A leeway must be permitted which takes into account the errors that are native to every machine. Hence, it is necessary to specify an allowable range of dimensions and shapes of the desired part to ensure it works as intended.

Tolerances have a direct effect on the ease and cost of manufacturing. Designers like to specify tight tolerances so that the part performs well. This typically requires more expensive and sophisticated manufacturing processes. On the other hand, manufacturers prefer loose tolerances because the parts would then cost less. Therefore, tolerances should be specified keeping in mind both the cost and functionality of the part.

There are two methods to allocate tolerances to a part. They are – Modern tolerancing and Geometric tolerancing.

Modern tolerancing is just a plus or minus variation only on variation of size; all other tolerances are geometric i.e. those identified in the ASME Y14.5 Standard [1] and hence additional tolerances called Geometric tolerances are required.

Geometric tolerances, specified inside a feature control frame, define a tolerance zone of acceptance and may include datums to control position and/or orientations. Geometric tolerances control all the variations except size. The types of control are – Form, Orientation, Location, Profile and Runout [1].

These five forms of control are further divided into sub categories. Form tolerances are comprised of straightness, flatness, circularity, cylindricity tolerances. Orientation tolerances contain parallelism and angularity tolerances. Location tolerances contain position and concentricity tolerances. Profile tolerances include the control of both line and surface profiles, such as the shape of a turbine blade. Runout tolerances control circular and total runouts, often is a means to control imbalance of high-speed rotary components made of nearly homogeneous materials.

The Geometric Dimensioning and Tolerancing (GD&T) Standard [1] contains a description of practical methods for specifying 3-D design dimensions and tolerances on an engineering drawing. Its universally accepted graphic language improves communication between the designers and the manufacturers and improves the quality of the produced part. Some of the advantages of GD&T are:

- Symbols and syntax are universally accepted leaving little room for ambiguity.
- Dimensions and tolerances specified are based on the functionality of the part.
- Datum systems may be used to dimension and tolerance the part.

1.2 Need for a Mathematical Model:

Tolerancing a part often is complicated, and many designers make mistakes while allocating tolerances. There are a few reasons for this:

- The designers are humans and humans tend to make mistakes
- Lack of knowledge of the tolerancing methods
- The guidelines specified in the standard are in the form of examples so designers often need to extrapolate to their applications

- The standard is not based on any mathematical rules but on empirical data collected over the years

Hence, due to these factors validation of the allocated tolerances on a manufactured part is very important. A mathematical model is crucial for the process of validation. Not only this, but also it becomes easier to automate the whole process of tolerance allocation and analysis.

For this very reason, a new mathematical model, the T-Maps model, was proposed by Davidson and Shah [3] for representing the three dimensional geometric variations that could be integrated with a CAD system.

A brief description about the other math models available is presented below along with their limitations.

Models for representing geometric tolerances maybe classified into five major categories - Parametric models, Offset zone models, Variational surfaces models, Kinematic models and DOF models.

- Parametric models have the same concept as parametric CAD where nominal shape and size of the geometry are represented by a set of explicit dimensions and constraints from which simultaneous equations are obtained; these are called the constraint equations. The values of the dependent dimensions are obtained by solving these equations [4, 5, 6, 7]. Parametric models find good usability in 2D but in 3D the geometries are limited due to computational expense and the possibility of multiple solutions to the same problem. Form tolerances, Datum Reference Frames (DRFs) and directed datum-target relationships cannot be modeled.

- In Offset Zone models, the geometry is first offset equally on either side by an amount equal to the tolerance specified and then the tolerance zone is represented by the Boolean subtraction of these volumes (maximal and minimal volumes) [8, 9, 10]. Disadvantages of this model are that important aspects of tolerancing cannot be modeled such as modeling each type of variation separately and representing their interactions, modeling position tolerance on medial planes and axes are not possible.
- Variational surfaces model uses the variational surfaces approach to calculate the surface coefficients independently, in the way of changing the values of the model variables according to the tolerance values [11, 12]. And positions of the vertices and edges are computed from the surface variations. Form tolerances can be handled by using higher-degree surfaces or surface triangulation. However, this model is not compatible with the ASMEY14.5 Standard [1] and also this model has some topological problems, such as maintenance of tangency and incidence conditions among vertices, edges and surfaces of a solid. Variational surfaces cannot model tolerances applied to derived features such as axes or mid-planes.
- Kinematic models use kinematic joints [13] to model different tolerance classes. These combinations are then used to estimate the geometric variations caused by feature tolerances. The developers of kinematic models are yet to show that this approach can be extended to combine the interaction of geometric variations and size dimensions.
- There are a few different models that use the degrees of freedom concept. Clement [14, 15] introduced the concept of Technologically and Topologically Related Surfaces (TTRS). In this model, the elementary surfaces such as planes, cylinders, spheres etc. are used to model the six lower kinematic pairs together with the complete

constraint of a fixed rigid body. A tolerance zone representing each tolerance related to a TTRS is modeled as a displacement torsor (a 6D vector representing six degrees of freedom in 3D space) containing the non-invariant rotations and translations.

A model based on a set inequalities representing tolerance zones has been described by Giordano *et al,* [15]. They are expressed in terms of components of a torsor (called deviation torsor having six components), i.e. a screw, and are mapped to a corresponding geometrical deviation space which is similar to the concept of the T-Map in 3 or 4 dimensions. This model has efficiently dealt with interaction of tolerances in an assembly, which leads to the Minkowski sum of deviation spaces. They have also modeled projected tolerance zones and material conditions.

Finally, the T-Maps model, developed at ASU, is the most general of all the models produced to represent tolerances. It is the only one that has been used to represent all six classes of tolerances that appear in the ASME Standard Y14.5 [1], and it has none of the limitations identified for the alternative models. A detailed description of T-Maps and T-Maps for line profiles is given in chapter 2.

1.3 Problem Statement:

T-Maps for patterns have been developed [17] on the basis of position tolerances allocated to them. This thesis extends the concept of T-Maps to patterns of line profiles. Also, conventional gauges often are not practical and/or economical to verify the tolerances on patterns of profiles. Hence an alternate gauging technique based on Least Square Fit [27] has been developed.

A linear pattern and a 2D pattern of squares and rectangular tabs will be used for most analyses.

1.4 Profile Tolerances:

Since this thesis deals with patterns of profiles, it is important to understand profile tolerances as described in the Standard [1] and their applications.

The Standard [1] describes it as: "Profile tolerances are used to define a tolerance zone to control form or combinations of form, orientation, size and location of a manufactured profile relative to a true profile specified on a drawing of the part."

A true profile is defined by basic radii, basic angular dimensions, basic coordinate dimensions, basic size dimensions, undimensioned drawings, formulas or mathematical data, including design models.

There are two types of profile tolerances, they are – Profile of a Surface and Profile of a Line. They are represented by unique symbols which are presented in the Fig 1.1.

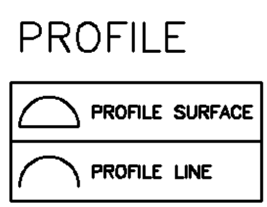
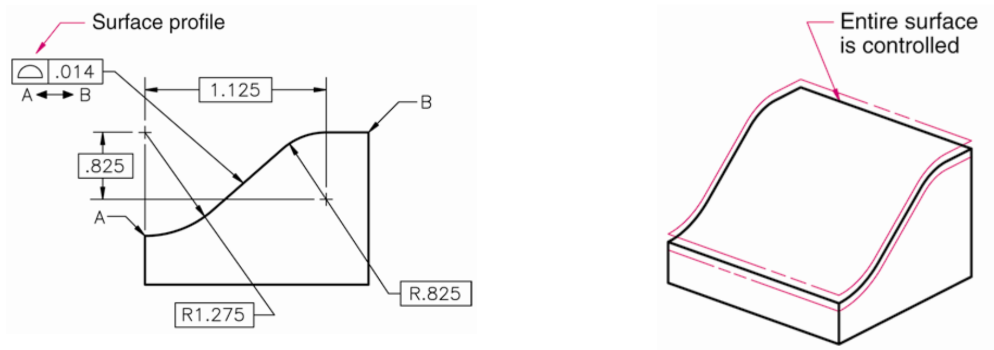


Figure 1.1: The two symbols used to denote the type of profile tolerance specified.

1.4.1 Profile of a Surface:

The tolerance for a surface profile establishes a 3D tolerance zone extending along the full length and width of the surface and requires the entire surface to lie within the limits of the tolerance zone.

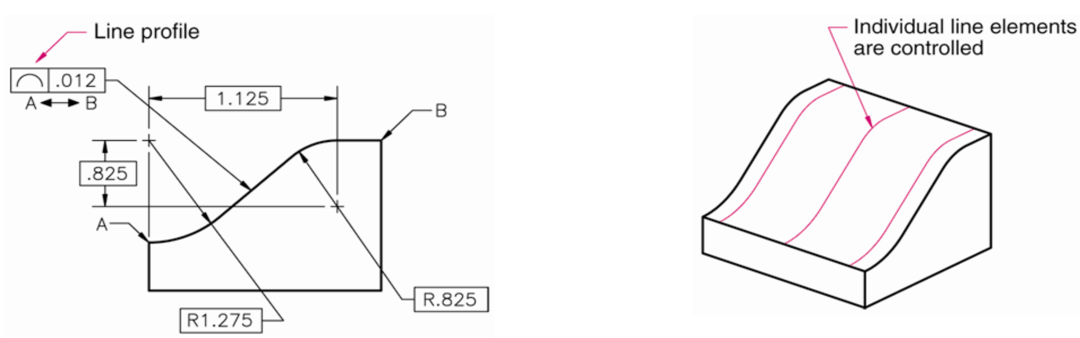


(a) (b)

Figure 1.2 (a) and (b): Surface profile tolerances simultaneously control all points on the surface to which the tolerance is applied.

1.4.2 Profile of a Line:

The tolerance for a line profile establishes a 2D tolerance zone at each plane containing normals to the true profile and requires every line element of the feature to lie within the tolerance zone.

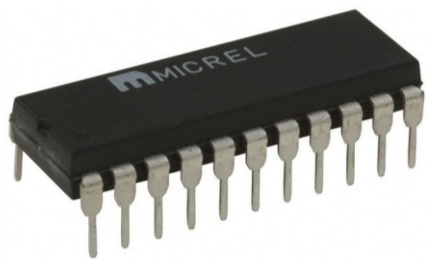


(a) (b)

Figure 1.3 (a) and (b): Line profile tolerances simultaneously control individual elements that are parallel to the view in which the tolerance is specified.

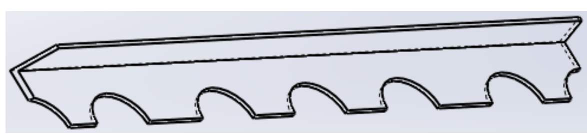
1.5 Patterns of profiles (as explained in the Standard [1]):

A group of similar features in arranged in a particular order is referred to as a pattern. These patterns could either be a linear pattern or a 2D pattern of features. Patterns are extremely common. For instance, patterns of tabs and slots are used to mount brackets, assemble stiffeners etc. in the automotive industry and patterns of pins and holes are used in the electrical industry for semiconductors and serial ports (Fig 1.4).



(a)

(b)



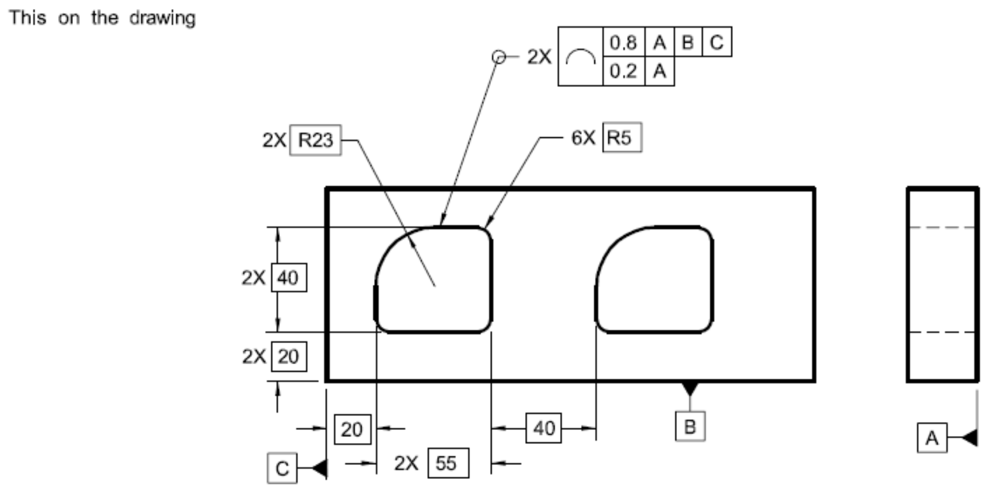
(c)

Figure 1.4 (a) : Transistor with a pattern of pins; (b): Serial port with pattern of holes; (c): Stiffener with tabs.

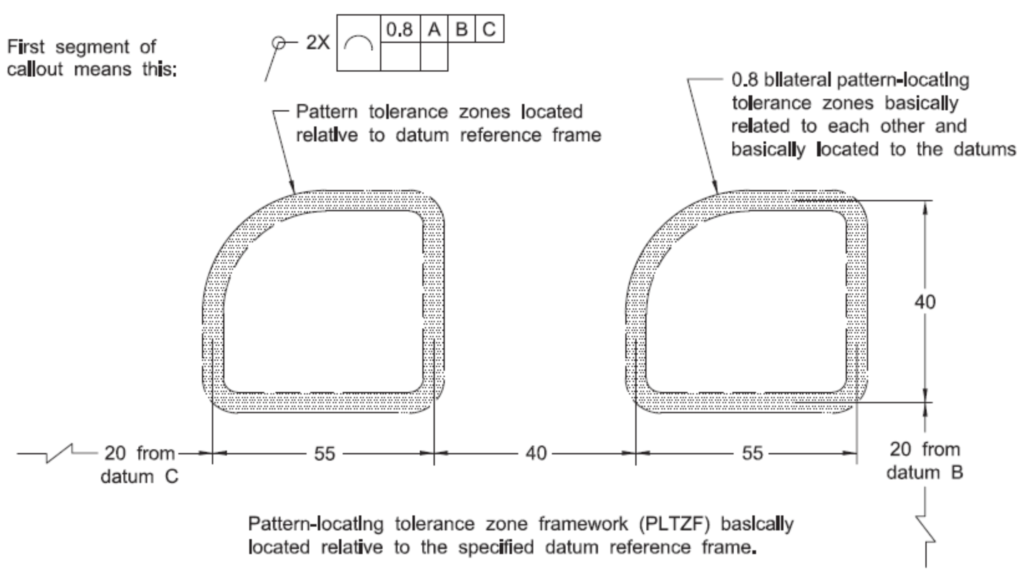
The Standard [1] has defined two ways of tolerancing patterns of profiles - composite tolerancing and multiple single segment tolerancing. For both the methods there are two tolerances specified – a Pattern Locating Tolerance and a Feature Relating Tolerance.

The pattern locating tolerance is the larger of the two and is used to locate the true profile of the features in the pattern with respect to the datums in the upper feature control frame. The boundaries of the tolerance zone are parallel offset curves to the true profiles in the pattern; they are fixed in position with respect to the datums. The feature relating tolerance adds a further refinement that limits the form, orientation and size of the features as a "group". The orientation of this group is restricted by the datums specified along with tolerance value in the lower feature control frame. However, the translation of this group is not restricted and it can move about freely within the larger pattern locating tolerance when the composite scheme is used, whereas this is not the case in the single segment scheme. In the single segment scheme, the true profile is fixed with respect to the datums specified in the FRTZF. This tolerance is specified just below the pattern locating tolerance in a composite frame.

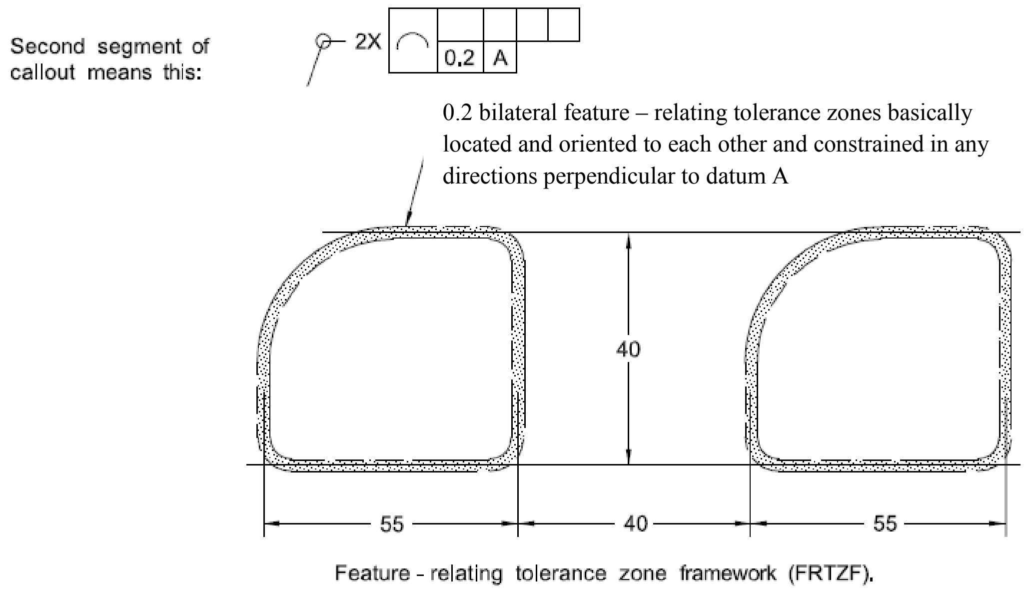
Consider the following example from the ASME Y14.5 Standard which has been slightly modified to show the effect of line profile tolerance instead of the surface profile tolerance. In this example, the Pattern Locating Tolerance (PLT) is 0.8mm and the Feature Relating Tolerance (FRT) is 0.2mm.


This on the drawing
2X
0.8 A B C
0.2 A
2X R23
6X R5
2X 40
2X 20
20
40
2X 55
B
A
C

(a)


First segment of callout means this:
2X
0.8 A B C
Pattern tolerance zones located relative to datum reference frame
0.8 bilateral pattern-locating tolerance zones basically related to each other and basically located to the datums
40
20 from datum C
55
40
55
20 from datum B
Pattern-locating tolerance zone framework (PLTZF) basically located relative to the specified datum reference frame.

(b)



(c)

Figure 1.5 (a) : Example from the Standard [1] to show the effect of composite profile tolerance on a pattern, here modified to apply to a line profile instead of a surface profile; (b): The effect of the pattern locating tolerance (the upper feature control frame); (c): The effect of the feature relating tolerance (the lower feature control frame).

1.6 Outline of this thesis:

Chapter 2 gives a brief description of all the work that has been done on T-Maps by the recent graduates. The work: Tolerance-Maps Applied to Patterns of Features is highlighted as it is very relevant to this thesis and also the first to expand the concept of T-Maps to patterns of line profiles.

Chapter 3 addresses the case of composite tolerancing of a pattern of profiles. It contains an example of a part with a 1D pattern and another with a 2D pattern for which

T-Maps have been constructed and also the Standard's interpretation of the same is presented.

Chapter 4 addresses the case of multiple single segment profile tolerancing. An example has been worked out to show the T-Map construction for a 1D and a 2D pattern of profiles.

Chapter 5 addresses tolerance verification of the patterns. Reduction of CMM data to variables that can be used for verification is discussed.

Chapter 6 deals with development of T-Maps for interrupted plane surfaces using the interpretation provided in the Standard [1].

Chapter 7 contains the conclusion and what could be done in the future to expand the concept of T-Maps and to facilitate its implementation through software.

CHAPTER 2

LITERATURE REVIEW

2.1 Introduction:

This chapter serves as an introduction to T-Maps developed by Davidson and Shah [3] and partly by Mujezinovic [18, 19], Ramaswamy [20], Bhide [21, 22] and Ameta [23]. The model proposed is compatible with the ASME Standard Y14.5 [1] for geometric tolerances. This chapter also contains the results of some of the previous work done on T-Maps by students and hence contains sections taken from published papers that are cited at appropriate places.

2.2 Tolerance Maps:

A T-Map is a hypothetical Euclidean point-space, the shape and size of which represents all allowable variations of a feature within its tolerance-zone. Each point is obtained by one-to-one mapping of all the possible variations of a perfect form feature within its tolerance zone. The T-Map can be a closed area, a volume or a hypervolume of *n*-dimensions depending on the allowable degrees of freedom of the feature within its tolerance zone.

For a line profile, its geometric variations are represented by the true profile, profiles parallel to it, and all of these displaced and within the tolerance zone. Each such variation represents a point in the T-Map for the line profile and tolerance specifications. Areal coordinates [24] are used to build the point space of a T-Map. A brief description about areal coordinates is presented below.

2.2.1 Areal Coordinates:

Areal coordinates [24] of a point are specified by center of mass of masses placed at vertices of a basis simplex, the ratios of which determine the position of the point. Areal coordinates can be formed for *n*-dimensional space with *n*+1 basis points of a simplex. Consider an example of 2-D space, in which the areal coordinates are based on a triangle of reference as shown in Figure 2.1. The basis is set by the three vertices ψ_1, ψ_2 *and* ψ_3 of the triangle. Consider that masses λ_1, λ_2 and λ_3 are located at these vertices, respectively, such that $\lambda_1 + \lambda_2 + \lambda_3 \neq 0$. The values of masses λ_1, λ_2 and λ_3 determine the position of any point ψ such that ψ is the centroid of the masses. The masses λ_1, λ_2 and λ_3 are also known as barycentric coordinates. A negative value for one or more of the coordinates shows that the point ψ is outside of the triangle. The position of the point ψ is uniquely determined by the linear combination

$$(\lambda_1 + \lambda_2 + \lambda_3)\,\psi = \lambda_1 \psi_1 + \lambda_2 \psi_2 + \lambda_3 \psi_3 \tag{2-1}$$

When these barycentric coordinates are normalized i.e. when $\lambda_1 + \lambda_2 + \lambda_3 = 1$, then they are called areal coordinates. The point ψ can also be located by connecting it to the vertices and taking the ratios of the areas of the triangles thus formed (shown in Fig 2.1 below).

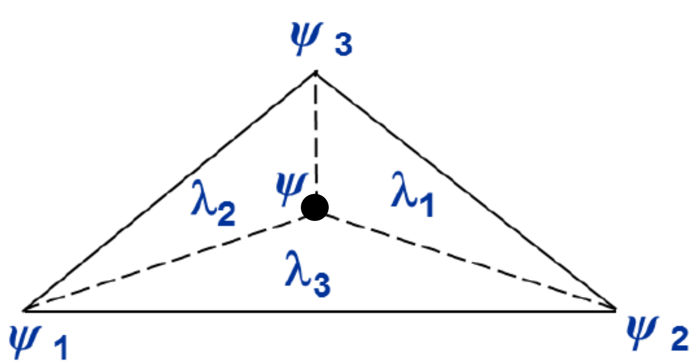


Figure 2.1: Two-simplex for areal coordinates in 2D space.

2.2.2 Tolerance Maps for Square Line Profiles:

This section describes the method for generating a T-Map for square line profiles [25]. Consider a square profile as an example shown in Fig 2.2. It is taken, with little change, from [1]:

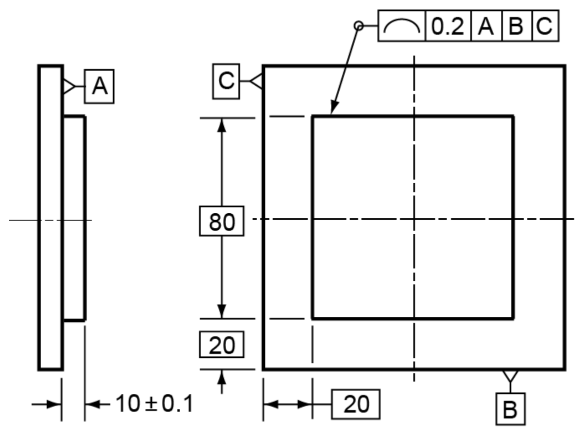


Figure 2.2: Specification of a line profile on a square boss, raised from a plate. Taken from [25].

A Tolerance-Map (T-Map) represents the freedom of a feature in its tolerance-zone. For line-profiles, the manufacturing variations will be represented with the true profile and profiles parallel to it which are allowed by the tolerance specification. These parallel profiles may be rotated or translated with respect to the true profile, as long as they are within the tolerance zone. Each point in the T-Map corresponds to any one of these parallel profiles or to any one of them that is displaced, yet remains within the tolerance-zone. Four degrees of freedom are required to specify the manufacturing variations of a line-profile, such as at any cross-section of the square boss in Figure 2.2. The four degrees of freedom are two translations, a rotation and a size change with respect to its true profile, and these freedoms correspond to the four-dimensional (4-D) T-

Map. Therefore, it becomes necessary to choose five of the parallel and/or displaced profiles as basis profiles and to define the T-Map by placing five corresponding basis points $\psi_1 \ldots \psi_2$ to form the vertices of a basis simplex. Five barycentric coordinates $\lambda_1 \ldots \lambda_5$, each one at its basis point ψ_i, identify any point ψ in the T-Map, and each such point corresponds to one manufacturing variation (one profile) in the tolerance-zone.

Of the five basis-profiles required, two will be: ψ_1, the smallest-sized profile, and ψ_2, the largest-sized profile, i.e. the inner and outer boundaries to the tolerance-zone, respectively (Fig 2.3). These are both locked in place and cannot displace. The remaining basis-profiles are based on displacements of the *middle-sized* square profile, even though the true profile in the design specification may lie at one boundary of the tolerance-zone or be unevenly positioned between both boundaries [1]. Each middle-sized square is represented by its components of basis-profiles displaced to the limits $e_x = t/2$ and $e_y = t/2$ in the *x*- and *y*-directions are labeled ψ_3 and ψ_4, respectively, and the one rotated counterclockwise the maximum amount $\theta = t/2\bar{a}$ is ψ_5 (Figure 2.3 (a)).

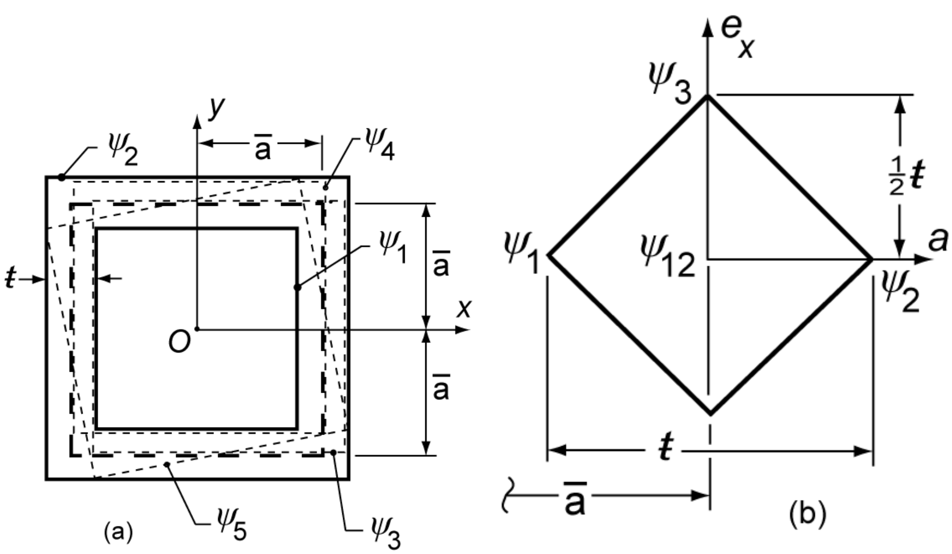


Figure 2.3: (a) The middle-sized profile (dashed-lined square) in the (exaggerated) tolerance-zone that is specified with the profile tolerance t; five variational possibilities are labeled, three with dotted lines; (b) One 2D cross-section of the corresponding T-Map that is confined to all size variations and displacements e_x only in x-direction.

Consider just two geometric variations of the square profile in Figure 2.3(a): its size and displacement e_x in the x-direction. The smallest and largest sized profiles ψ_1 and ψ_2 respectively are regarded as two points on horizontal line in T-Map point space, as shown in Figure 2.3(b). Similarly consider ψ_3 as middle sized profile translated in x-direction to extreme rightward by $t/2$, and then isosceles triangle $\psi_1\ \psi_2\ \psi_3$ in Figure 2.3(b) establishes areal coordinates for 2D T-Map space. Since the dashed square in Figure 2.3(a) can displace leftward by $t/2$ also, the boundary to this 2-D cross-section of the T-Map is a square of side-length $t/\sqrt{2}$. Notice that the middle sized profile shown in dashed line in Figure 2.3(a), corresponds to origin ψ_{12} of T-Map shown as at midpoint of line segment $\psi_1\psi_2$ in Figure 2.3(b). This square profile is shown in Figure 2.3 (a) with the dashed line.

The 3-D T-Map for all the middle-sized square profiles is established with the four basis-points ψ_{12}, ψ_3, ψ_4, and ψ_5 shown in Figure 2.4. Basis-points ψ_3, ψ_4, and ψ_5 are placed at the same distance t /2 from the origin along the three axes of a rectangular Cartesian frame of reference with axes e_x, and e_y, and θ'. Note that the angular limit $\theta = t/2\ \bar{a}$ is multiplied by the length $\bar{a}$, i.e. $\theta' = \bar{a}\ \theta$ so that the units along all axes are the same, i.e. a length [L]. Consistent units on all the axes permit the T-Map to be used for metric computations.

Square profiles that are larger or smaller than the middle-sized one are more limited in their allowable displacements e_x, e_y, and θ, and the limits diminish linearly with change in size.

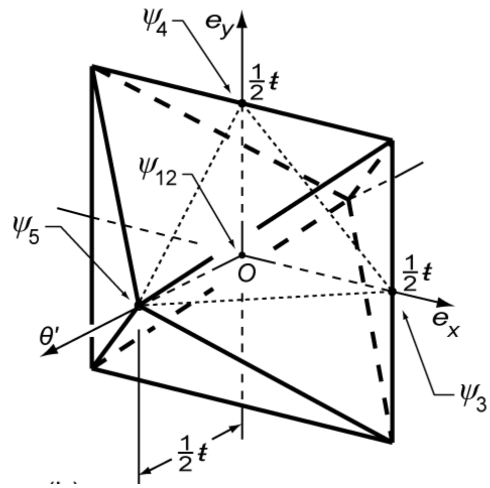


Figure 2.4: 3D T-Map for all middle sized profile shown in Fig 2.3(a). Taken from [25].

Therefore, the full T-Map for the square tolerance-zone in Figure 2.3 (a) is a double hyperpyramid in 4-D that is depicted in Figure 2.4. The base for each single hyperpyramid is the 3-D octahedron from Figure 2.3 (b), and every other section (two are shown) at right angles to the direction of size is a smaller and geometrically similar octahedron. The combined basis-point ψ_{12}, shown in Figure 2.3 (b), has been replaced with the individual basis points ψ_1 and ψ_2.

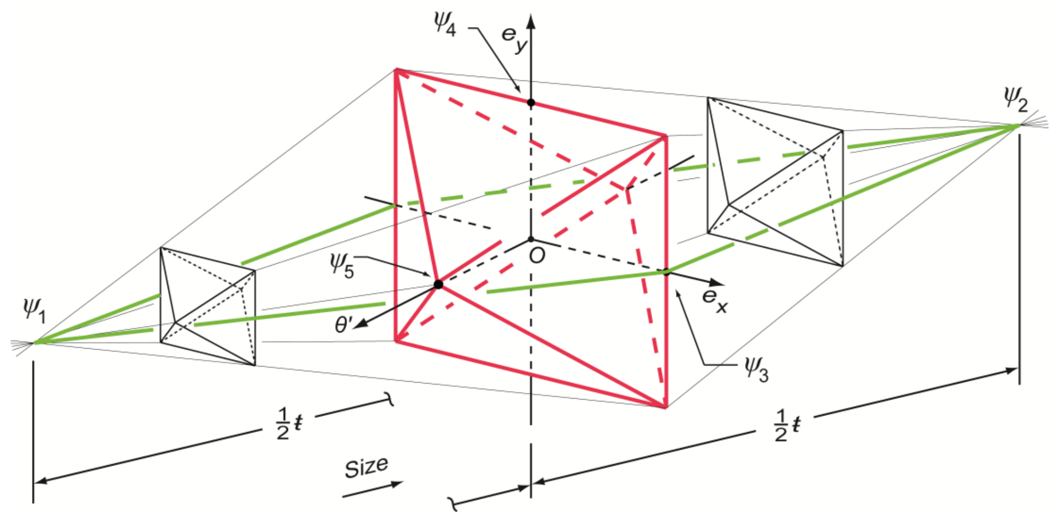


Figure 2.5: 4D T-Map for the tolerance zone in Fig 2.2(a). The scale in the direction of size (ψ_1 ψ_2) is exaggerated for a better understanding.

2.3 Generation of Tolerance Maps for Line Profiles by Primitive T-Map Elements:

Yifei He [26] has developed a way to generate T-Maps for line profiles using primitives. His method is briefly discussed here.

Line-profiles can be divided into three unique types of segments which are straight-line segments, circular-arc segments and freeform-curve segments. T-Maps for line-profiles are generated using the decomposition modeling method wherein the profile is divided into several segments and a primitive T-Map for each of these segments is built. Then, Boolean intersections of these primitive T-Maps give the T-Map for the entire profile.

The steps involved in generating the 3D hypersections of the T-Map are:

1. Decompose the entire line-profile into its line, circular-arc, and/or free-form segments.

2. Define a local reference system for each of the segments, and create the primitive T-Map for each one.

3. Arbitrarily set a temporary global reference frame for the entire profile and represent each primitive T-Map in this reference system.

4. Intersect the transformed primitive T-Maps in the temporary reference frame to get a tentative T-Map for the entire profile.

5. Find the maximum rotation center (pole) of the profile when it is rotated the maximum amount permitted by the tolerance zone boundaries. Reset the origin of the reference frame to the pole and transform the tentative T-Map to its representation in this new frame.

6. Generate the entire 4D of the T-Map by repeating the step 1-4 for several different sizes of the profile.

This method of generating the T-Maps can be used for line-profiles of any shape.

2.4 Tolerance-Maps Applied To Patterns Of Features:

Gagandeep Singh worked on developing T-Maps for a pattern of tabs, slots and cylindrical features [17]. The results from his study are presented in this section. He studied the effect of both composite tolerancing and multiple single segment tolerancing (§ 1.5) on a pattern of features. The two types of patterns studied were Linear pattern of Tabs and slots and 2D patterns of pins and holes.

Since patterns of pins and holes will not be examined in this thesis, the T-Maps for these are not presented in this section.

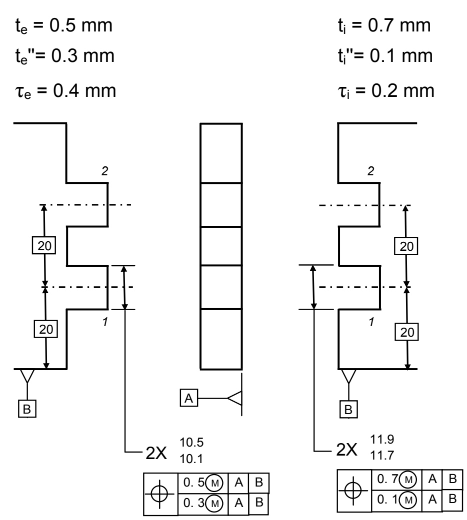
Also, there are two types of T-Maps. One is called the Assembly Level T-Map and the other is called the Part Level T-Map. The assembly level T-Map focuses on the variations in the features within the pattern whereas the part level T-Map takes into account the orientation and the location of the pattern as a whole.

2.4.1 Tolerance-Maps for a linear pattern of slots:

Two tolerances are specified on a tab/slot – one on the size of the tab/slot and the other on its position with respect to the datums. The uppermost frame, called the Pattern Locating Tolerance Zone Frame (PLTZF) is the larger of the two tolerances and controls the location of the pattern as a group. The lower frame is called the Feature Relating Tolerance Zone Frame (FRTZF) and it controls the position of individual features with respect to each other and their orientation with respect to the datums specified.

Consider the following example taken from [17]:

Here t_e is the Pattern Locating Tolerance on the tab, t_e'' is the Feature Relating Tolerance on the tab, τ_e is the size tolerance on the tab, t_i is the Pattern Locating Tolerance on the slot, t_i'' is the Feature Relating Tolerance on the slot, τ_i is the size tolerance on the slot.



Part 1 Part 2

Figure 2.6: Tabs on Part 1 engage with corresponding slots on Part 2. Composite positional tolerancing is used to position the tabs and the slots relative to the common lower face (Datum B) of their respective parts. The center view is common to both parts. Figure is taken from [17].

As shown in Fig 2.6, Part 1 has tabs that assemble with the slots on Part 2. The tabs, external features, slots and internal features are located using position tolerance. Composite tolerance scheme for the pattern is used in this example (Fig 2.6). The pattern locating tolerance is 0.5 for the tab and 0.7 for the slot. The feature relating tolerance is 0.3 for the tab and 0.1 for the slot.

When only the PLTZF is applied, the T-Map is a right-rhombic dipyramid as shown in Fig 2.7.

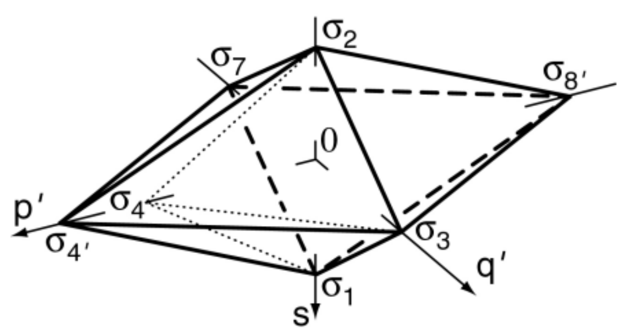


Figure 2.7: Three-dimensional T-Map of the medial plane of a tab in its tolerance zone when only a pattern-locating tolerance is applied. σ_1, σ_2, σ_3 and σ_4 are the four basis points. $\sigma_1\sigma_2 = t_p$.

But when the FRTZF is also applied, there is an additional constraint on the orientation of the tab/slot and hence the T-Map is truncated on its sides.

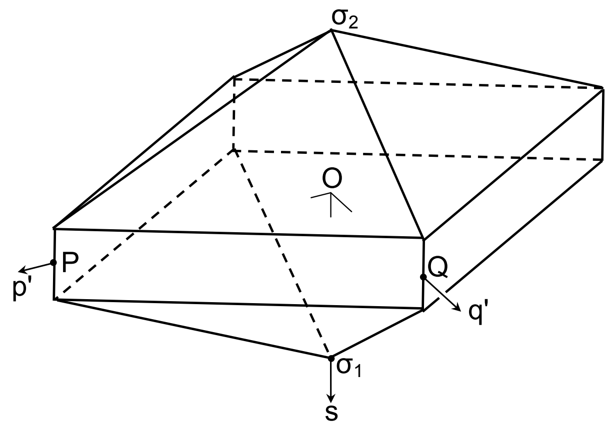


Figure 2.8: The T-Map of the medial plane of a tab when both the pattern-locating tolerance t_p and the feature-relating tolerance t_f are applied. Dimensions OP and OQ are t_f. Taken from [17].

2.5 Least-Squares Fit For Points Measured Along Line-Profiles:

The concept of Least-Squares fit will be used later in this thesis to verify whether the patterns conform to the design specifications or not. A brief review of this method is presented in this section.

2.5.1 Moore-Penrose Inverse [27]:

Penrose showed that for every rectangular, finite matrix $[\mathbf{K'}]_{mxn}$ where $m>n$, there exists a unique matrix $[\mathbf{K}]^{\#}$ that satisfies the four equations,

$$
\begin{aligned}
&[\mathbf{K}]\,[\mathbf{K}]^{\#}[\mathbf{K}] = [\mathbf{K}],\\
&[\mathbf{K}]^{\#}\,[\mathbf{K}]\,[\mathbf{K}]^{\#} = [\mathbf{K}]^{\#},\\
&([\mathbf{K}]\,[\mathbf{K}]^{\#})^{*} = [\mathbf{K}]\,[\mathbf{K}]^{\#}, \text{ and}\\
&([\mathbf{K}]^{\#}\,[\mathbf{K}])^{*} = [\mathbf{K}]^{\#}\,[\mathbf{K}], \qquad (2\text{-}2)
\end{aligned}
$$

where $[\mathbf{K}]^*$ denotes the conjugate transpose of [**K**]. The matrix $[\mathbf{K}]^\#$ called *Moore-Penrose inverse*, named after Moore and Penrose who presented the conditions (Eq 2-2). Uniquely, the *Moore-Penrose inverse*,

$[\mathbf{K}']^\# = \{([\mathbf{K}']^T [\mathbf{K}'])^{-1} [\mathbf{K}']^T\}$, for an inconsistent set of equations $[\mathbf{b}] = [\mathbf{K}'][\mathbf{x}]$ minimizes $([\mathbf{b}] - [\mathbf{K}'][\mathbf{x}])^2$. The results give fit of a straight line to a set of points arrayed around it. The same property is used to calculate the Least-Squares fits for line profiles.

2.5.2 Least-Squares fit of a square line-profile to measured points:

In Fig. 2.9, the middle-sized square profile (dashed line) and the boundaries of its tolerance-zone are shown drawn on the platform of a planar in-parallel robot that is guided with three linear actuators that lie on the normalized screws $\$'_1$, $\$'_2$, and $\$'_3$. The actuators are attached to the platform at three of the measured points, i.e. at *A*, *B*, and *C*, and the directions of the corresponding $\$'_i$ are the same as for the inward unit normals $\mathbf{n}_i$ from the closest side of the square to the (enlarged) reference envelope for the profile. For what follows in developing formulation for least-squares fit in this section, it is necessary to align coordinate frame of the measured points and coordinate frame of the geometry from which their deviations are measured. Each of the three linear actuators exerts a force of magnitude F_i' and causes a velocity of magnitude v'_i at the measured point where it is attached to the platform. Since speed and time are of no importance in measurement reduction, each v'_i will be replaced with a differential displacement d'_i of the measured point in the direction of $\mathbf{n}_i$. The corresponding deviation torsor for the platform body is represented by $[\$] \equiv (0,\ 0,\ \delta\theta;\ \delta x,\ \delta y,\ 0)$. Since displacements are confined to the *xy*-plane, the three zero-coordinates may be omitted.

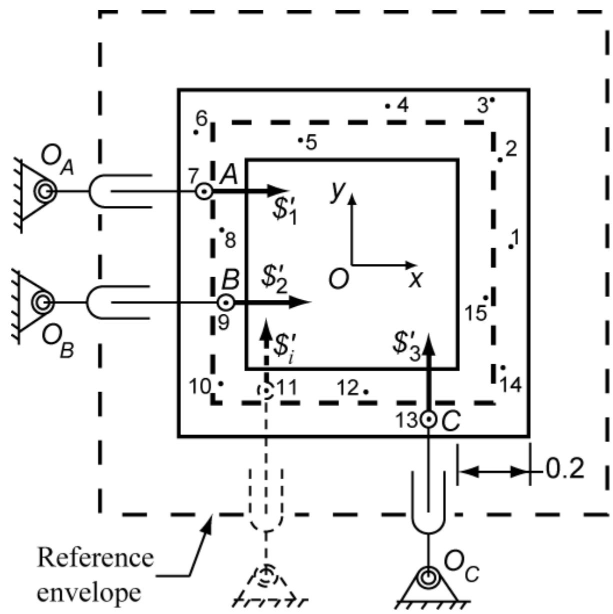


Figure 2.9: The line-profile (dashed line), its tolerance zone boundaries (with an exaggerated scale), and the measured points lying on the platform of a planar in-parallel robot which is guided by three linear actuators lying on screws $\$'_1$, $\$'_2$ and $\$'_3$ at points *A*, *B*, and *C;*

Each of the actuator forces in the *xy*-plane is represented with wrench coordinates, i.e. $F_i' \, \$'_i \equiv (\mathbf{F}'_i; \mathbf{T}'_i) \equiv (\mathrm{L}'_i, \mathrm{M}_i', 0; 0, 0, \mathrm{R}'_i)$, where L'_i and M_i' are the *x*- and *y*-components of actuator-force $\mathbf{F}'_i$ and R'_i is the moment of $\mathbf{F}'_i$ about the origin, i.e. $\mathrm{R}'_i = -y_i\mathrm{L}'_i + x_i\mathrm{M}_i'$. Since all forces will lie in the *xy*-plane, the three zero-coordinates may be omitted, just as for [**$**]. Also, the geometry may be isolated from the statics by normalizing the wrench coordinates, i.e.

$$F'_i\$'_i \equiv (\mathrm{L}'_i, \mathrm{M}'_i ; \mathrm{R}'_i) \equiv F'_i \, (L'_i, M'_i ; R'_i), \tag{2-3}$$

this making $(L'_i)^2 + (M_i')^2 = 1$. The normalized coordinates L'_i, M_i', and R'_i for each $\$'_i$ are the scalar *screw* coordinates for the actuator-wrench $F_i'\$'_i$; they contain only geometry, i.e. direction and location of $F_i'\$'_i$.

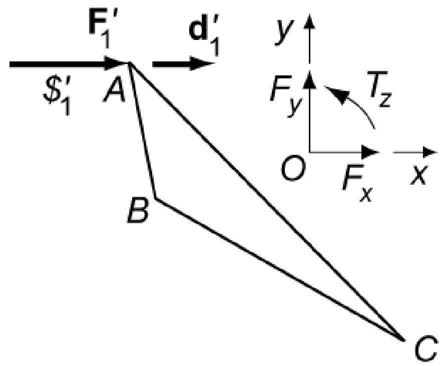


Figure 2.10: The free-body diagram of the platform carrying the profile. The external loads are the force $\mathbf{F}'_1$ acting along the screw $\$'_1$ at point *A* and the equilibrium wrench $(\mathbf{F}_1;\mathbf{T}_1)$ exerted on the platform from the environment and represented with the coordinates (*Fx, Fy; Tz*). Also shown is the differential displacement vector $\mathbf{d}'_1$ that is aligned with $\$'_1$ at *A*. The shape of the platform *ABC*, and the relative location of the *xy*-frame are together congruent to those same features in Fig. 2.9.

A free-body diagram of the platform in Fig 2.9 contains the three forces $\mathbf{F}'_i$ ($i = 1, 2, 3$) and an equilibrium wrench, composed of a force and a couple, exerted on the platform from the environment. The force and couple are represented with the wrench ($\mathbf{F}$; $\mathbf{T}$). Consider now that all of the actuated joints have no force applied and are free to move except one, say $\$'_1$, shown in Fig 2.10. Then, the only additional loads on a free-body diagram of the platform are those *portions* of the equilibrium wrench reacting back on it from the environment which are required to equilibrate $F_1'\$'_1$, i.e. the force and couple ($\mathbf{F}_1$; $\mathbf{T}_1$) shown in Fig 2.10 with the components F_x, F_y, and T_z. Since the virtual work of all forces and moments on the free body must be zero for a kinematically admissible displacement of the platform arising from $\mathbf{d}'_i$, the system of forces and couples for the special case in Fig 2.10 leads to

$$F_1'd'_1 + [T_z \ F_x \ F_y][\delta\theta \ \ \delta x \ \ \delta y]^{\mathrm{T}} = 0, \tag{2-4}$$

in which the order of the coordinates in $(\mathbf{F}_1\,;\mathbf{T}_1)$ has been changed to $(\mathbf{T}_1\,;\mathbf{F}_1)$ and the zero-coordinates again have been omitted. The term $F_1'd'_1$ represents the virtual work of force $\mathbf{F}'_1$ with virtual displacement $\mathbf{d}'_1$, both in the direction of $\$'_1$, at point A on the platform, and the product $[T_z \ \ F_x \ \ F_y][\delta\theta \ \ \delta x \ \ \delta y]^{\mathrm{T}}$ represents the virtual work from the equilibrium-wrench acting on the platform whose deviation torsor is $[\mathbf{\$}] \equiv [\delta\theta \ \ \delta x \ \ \delta y]^{\mathrm{T}}$.

It is helpful to shift attention to the wrench $-(\mathbf{T}_1\,;\mathbf{F}_1)$ exerted on the environment and *produced* at the platform by the force $F_1'\$'_1$ at A. Since the platform in Fig 2.10 is a two-force (two-wrench) member, with each wrench intensity of equal magnitude, $-(\mathbf{T}_1\,;\mathbf{F}_1) \equiv -(T_z\,; F_x\,, F_y) \equiv (\mathrm{R}'_1\,;\mathrm{L}'_1,\mathrm{M}_1') \equiv F_1'(R'_1; L'_1, M_1')$. Making this substitution in Eq. (2-4) gives

$$F'_1d'_1 = F'_1[R'_1\, L'_1,\, M'_1][\delta\theta \ \ \delta x \ \ \delta y]^{\mathrm{T}} \tag{2-5}$$

for the virtual work expression when force is exerted only at $\$'_1$. Two more Eqs. (2-5), with subscripts $_2$ and $_3$, occur when force is applied only at $\$'_2$ and only at $\$'_3$. The force-amplitude at each actuated joint may be removed from each term, and all terms on the right come from the product of a row matrix and a column matrix of three elements each. When the three equations are ordered sequentially, then the rows of screw coordinates, when taken together, comprise a matrix $[\boldsymbol{K}']$ that is formed entirely from the (normalized) coordinates for $\$'_1$, $\$'_2$ and $\$'_3$, and the three equations may be written

$$\begin{bmatrix} d'_1 \\ d'_2 \\ d'_3 \end{bmatrix} = \begin{bmatrix} R'_1 & L'_1 & M'_1 \\ R'_2 & L'_2 & M'_2 \\ R'_3 & L'_3 & M'_3 \end{bmatrix} \begin{bmatrix} \delta\theta \\ \delta x \\ \delta y \end{bmatrix} \quad \text{or} \quad [\mathbf{d'}_i] = [\boldsymbol{K'}][\$]. \tag{2-6}$$

The reader familiar with robotics will recognize $[\boldsymbol{K}']$ as a Jacobian for the actuators of the robot platform in which the normalized coordinates have been rearranged. (For those interested in a more detailed treatment of the principles involved, the notation here has been made nearly consistent with that in Davidson & Hunt [24], §§1.6, 6.11, 8.5, and 9.6.)

So long as the screws $\$'_1$, $\$'_2$ and $\$'_3$ are independent for the three measured deviations d'_1, d'_2, and d'_3 at locations A, B, and C around the profile, the solution to Eq. (2-6) for [**$**], i.e. $[\mathbf{\$}] = [\boldsymbol{K}']^{-1}[\mathbf{d}'_i]$, is unique and all three scalar Eq. (2-6) are satisfied exactly. This solution ensures that d'_1, d'_2, and d'_3 are kinematically consistent with the platform (profile) displacement [**$**]. However, in practical situations, there are many more measured points around a line-profile than three. For instance, in Fig. 2.9 there are 15 points. For every additional point, there would be an added, and redundant, linear actuator with its normalized screw $\$'_i$ exerting a force of amplitude F_i' on the platform. One example is shown with dashed lines at Point 11 in Fig. 2.9. Each of these additional points adds a row to the matrices $[\mathbf{d}'_i]$ and $[\boldsymbol{K}']$ in Eq. (2-6), so that, for all the measured points,

$$[\mathbf{d'}_i] = \begin{bmatrix} d'_1 \\ d'_2 \\ \vdots \\ d'_n \end{bmatrix} = [\boldsymbol{K'}][\$] = \begin{bmatrix} R'_1 & L'_1 & M'_1 \\ R'_2 & L'_2 & M'_2 \\ \vdots & \vdots & \vdots \\ R'_n & L'_n & M'_n \end{bmatrix} [\$]. \tag{2-7}$$

Solution to Eq. (2-7) provides [**$**], rigid body displacements ($\delta\theta, \delta x, \delta y$) of the platform in plan. However, in metrology size is also an important measure of dimensional variation. In next section, Eq. (2-7) are modified to accommodate size variation.

2.5.3 Adding the 4th dimension for metrology

The coordinates ($\delta\theta, \delta x, \delta y$) of [**$**] appear only in a 3-D cross-section of the T-Map (Fig. 2.4), such as in the base of the 4-D double hyperpyramid in Fig. 2.5; they do not represent the *size* of the least-squares envelope, i.e. the fourth dimension of the T-Map. The values for displacements d'_i, then, may all contain a constant value ΔF that represents the change in feature size between that of the middle-sized profile and the least-squares profile, and they *must* contain a value Δs that was introduced artificially (§4.2 in [28]) to establish the correct proximity of a measured point to the profile. For reduction of CMM data, then, each generic Eq. (2-8) must be augmented to

$$d'_i = [R'_i\ L'_i\ M'_i][\delta\theta\ \ \delta x\ \ \delta y]^T + (\Delta s - \Delta F)$$

$$= [R'_i\ L'_i\ M'_i\ 1][\delta\theta\ \ \delta x\ \ \delta y\ \ (\Delta s - \Delta F)\ ]^T \quad (2\text{-}8)$$

(compare to $y_i = mx_i + b$, §3.2 in [28]). The scalar relation in Eq. (2-8) forms the transition between the setting of in-parallel robotics and the setting of reducing CMM data to geometric variables related to Tolerance-Maps. Now the least-squares fit is obtained by minimizing the sum

$$\sum[\ d'_i - \{\ R'_i\delta\theta + L'_i\delta x + M'_i\delta y + (\Delta s - \Delta F)\}]^2 \quad (2\text{-}9)$$

for $i = 1\ldots n$. Matrix [**$**] in Eq. (2-7) is augmented to contain the *four* components $\delta\theta$, δx, δy and $(\Delta s - \Delta F)$, and the matrix $[\boldsymbol{K}']$ in Eq. (2-7) is augmented on the right with a column of ones so that the n Eq. (2-8) (for the n measured points) produce the matrix equation

$$[\mathbf{d}'_i] = \begin{bmatrix} d'_1 \\ d'_2 \\ \vdots \\ d'_n \end{bmatrix} = [\boldsymbol{K}'][\$] = \begin{bmatrix} R'_1 & L'_1 & M'_1 & 1 \\ R'_2 & L'_2 & M'_2 & 1 \\ \vdots & \vdots & \vdots & \vdots \\ R'_n & L'_n & M'_n & 1 \end{bmatrix} [\$] \tag{2-10}$$

The Moore-Penrose solution to Eq. (2-7) for [**$**], i.e. $[\$] = [\boldsymbol{K}']^{\#}[\mathbf{d}'_i]$, produces the least-squares location $(\delta\theta, \delta x, \delta y)$ and size-adjustment $(\Delta s - \Delta F)$ for the profile, i.e. that location and size for a profile which minimizes the sum in Eq. (2-9). Note that coordinates $(\delta\theta, \delta x, \delta y)$ correspond to coordinates (θ, e_x, e_y) in the T-Map of Fig. 2.5.

CHAPTER 3

COMPOSITE TOLERANCING OF A PATTERN OF PROFILES

3.1 Introduction:

The Standard [1] has defined two ways of prescribing tolerances to patterns – composite frame tolerancing and multiple single segment tolerancing (§ 1.5). This chapter focuses on generating T-Maps for a pattern of profiles toleranced using the composite scheme. In the following sections, examples for both one dimensional and two dimensional patterns will be presented along with their respective T-Maps.

3.2 Composite Tolerancing:

Composite tolerancing of a pattern of profiles utilizes a composite frame which contains both the Pattern Locating Tolerance and the Feature Relating Tolerance. The value of the Pattern Locating Tolerance is greater than the Feature Relating Tolerance always. In this section, an example will be presented with the interpretation as per the guidelines in the Standard [1] and also the T-Map for the same will be presented. An example composite frame is presented below in which tolerance t_p is the pattern locating tolerance (PLT) and t_f is the feature relating tolerance (FRT).

⌒	t_p	A	B	C
	t_f	A	B	

Figure 3.1: A sample composite frame which contains both PLT and FRT.

3.2.1 Interpretation as per the Standard:

Consider the following example of two bosses that are raised above the same plate:

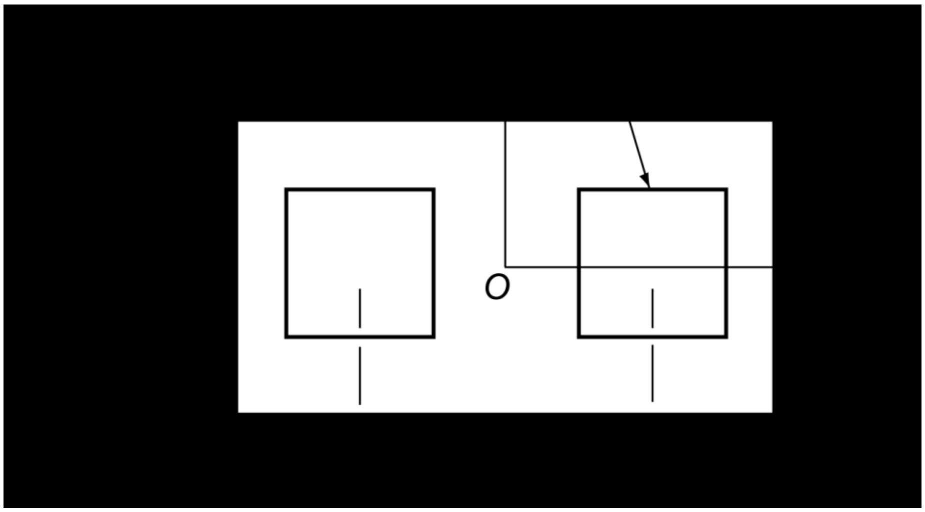


Figure 3.2: A sample part containing two square profiles toleranced using the composite frame scheme. The centers of the squares are separated by a distance of $2L$.

The first part of the callout means that the two square profiles need to lie within the boundaries of the Pattern Locating Tolerance Zone (PLTZ) with a tolerance t_p. The center of the true profiles of the squares need to be separated by $2L$ units. Once the feature is located within the PLTZ an additional constraint is imposed by the Feature Relating Tolerance Zone (FRTZ) with a tolerance t_f. Now the true profiles need to be within the limits of PLTZ and also have their centers separated by $2L$ units. This zone is not tied to any of the datums and can float freely within the PLTZ. The tolerance zones are first presented below in Fig 3.3.

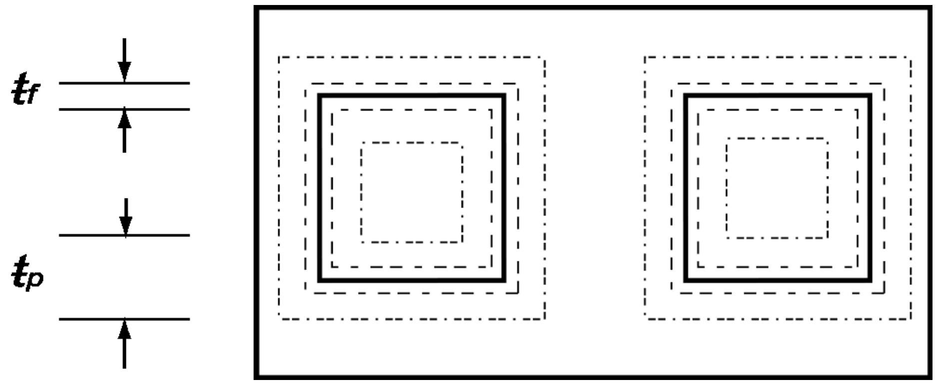


Figure 3.3: The tolerance zones for the part in Fig 3.2 are depicted here. t_p is the pattern locating tolerance and t_f is the feature relating tolerance.

For the tolerance specification in Fig 3.2, the feature relating tolerance zone need not be parallel to any datum. Hence the orientation freedom is larger than what it is for the case of two datums in the lower DRF which is shown later in §3.3. The feature needs to lie within both the PLTZ and the FRTZ. The Fig 3.4 shows the orientation freedom available when just the primary datum A (Fig 3.2) is repeated in the FRTZF. The entire FRTZ can translate and rotate within the PLTZ.

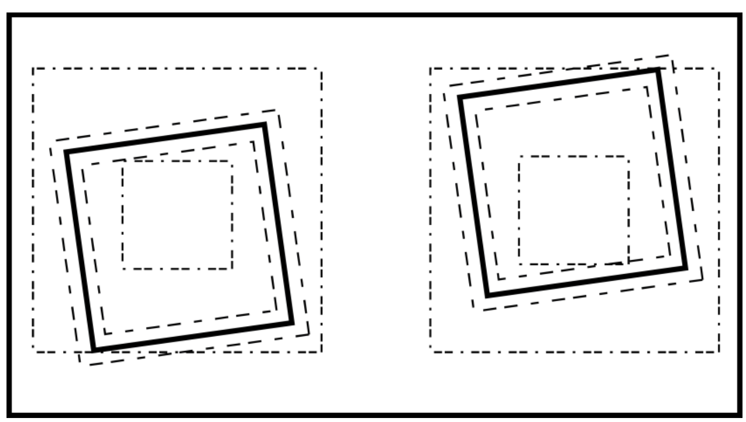


Figure 3.4: Variation within the PLTZ is shown here. The FRTZ has full freedom to move inside the PLTZ for tolerance specification in Fig 3.2.

Once both the zones are located, the Feature Relating Tolerance Zone can extend outside the pattern locating tolerance zone to facilitate maximum rotation of the Mid-Sized Profile. Also, additional variations namely, translations in the X and Y directions and rotations about an axis normal to XY plane are possible in the FRTZ.

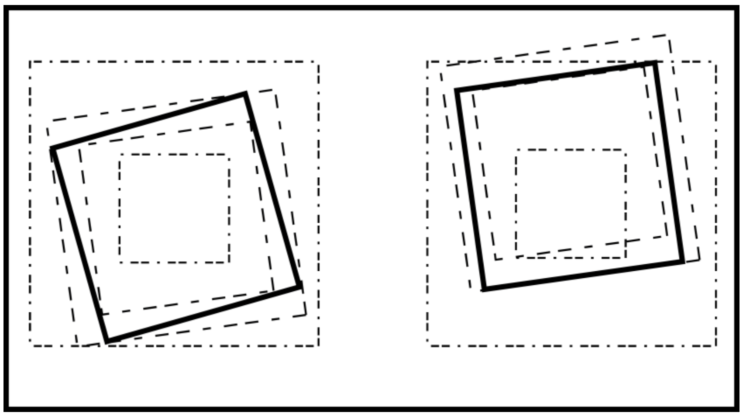


Figure 3.5: The variation within the FRTZ is shown here. One extreme position (orientation) of the left square boss is shown.

3.2.2 Developing the T-Map:

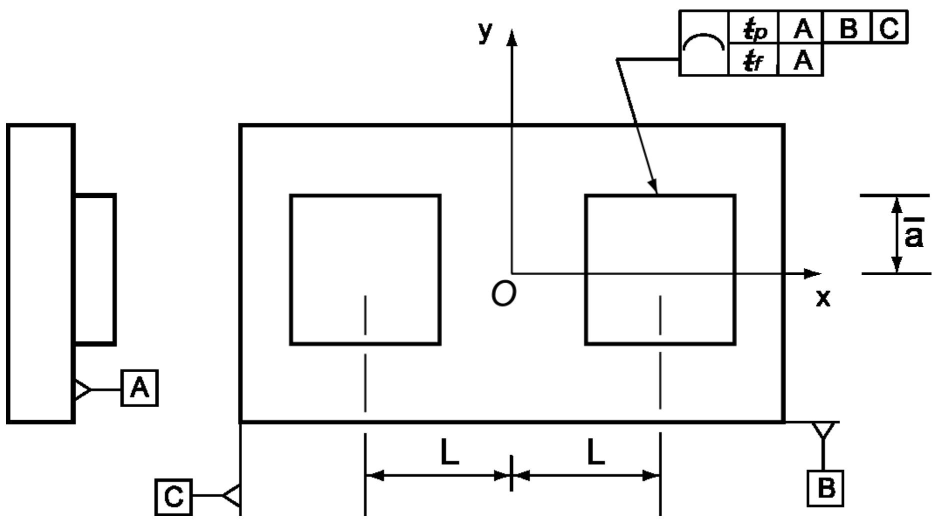


Figure 3.6: The Fig. 3.2 is repeated here for convenience. A line profile tolerance is allocated here since the boss is relatively (compared to its width) shorter.

The T-Map of a square profile has been developed by Davidson and Shah [3]. A detailed review of how the T-Map is constructed has been presented in §§ 2.2.2. This will be used as the starting point to develop the T-Maps for pattern of square profiles. For convenience the T-Map for a single square profile is reproduced below (§ 2.2.2). It represents the T-Map for all the different locations of the Mid-Sized Profile (MSP) of one square profile in a fixed tolerance zone (Fig 2.2).

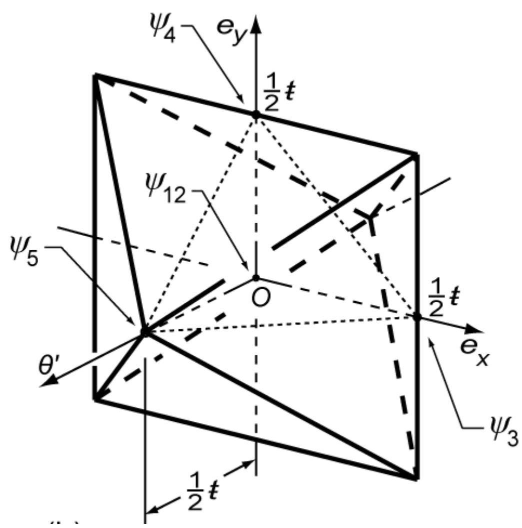


Figure 3.7: Fig 2.4 has been repeated here. The T-Map for all the middle-sized squares in the sharp cornered tolerance-zone of Fig. 2.2; it is one central hypersection of the 4D T-Map that represents the entire tolerance-zone established by a tolerance (as shown in Fig 2.3(a)). The tolerance t shown on the T-Map is equal to the pattern locating tolerance t_p when a pattern is considered.

The first constraint is imposed by the Pattern Locating Tolerance (t_p). This restricts the size, position and orientation of the profile. The maximum allowable angle of rotation for the pattern as a whole is:

$$\theta_{max} = \frac{t_p}{2(L+\bar{a})} \tag{3-1}$$

So that the T-Map may be used for metric computations, the units along all axes should be the same, i.e., a length [L]. For that reason, the scale assigned to the axis for angle θ is made $\theta' = \bar{a}\,\theta$. Hence the equation (3-1) is reduced to:

$$\theta'_{max} = \frac{t_p}{2(L+\bar{a})}\,\bar{a} \tag{3-2}$$

Therefore, the maximum possible variation has been reduced to $\frac{t_p}{2(L+\bar{a})}\,\bar{a}$ from $\frac{t_p}{2}$ (in Fig. 3.7). This results in truncation of the T-Map in the θ' direction.

Further, the possibility of variation within the FRTZ also must be considered. Since the profile can float within the limits of the FRTZ, another T-Map can be developed considering just this. The T-Map obtained will be similar to the one shown in Fig 3.7 because the profile under consideration is a square and is restricted in orientation and size by a tolerance t_f.

The two factors presented above have a combined effect on the final T-Map for the pattern of squares shown in Fig. 3.6. Since the FRTZ can float inside the PLTZ, the e_x and e_y limits of the final T-Map will remain the same ($\frac{t_p}{2}$). The only change is going to occur in the orientation limits. The θ' direction will be stretched to accommodate the small variation that occurs in the FRTZ. Hence, the limits in the θ' direction are now going to be $\pm\left(\frac{t_p}{2(L+\bar{a})}\ \bar{a}\ + \frac{t_f}{2}\right)$.

A series of central sections of the T-Map is presented below to better understand the T-Map construction. Consider the left boss in the Fig. 3.6.

θ' - e_y directions are considered only - the sequence of steps involved in the rotation of the Mid-Sized profile from zero to its maximum position is shown in the Fig. 3.8. First, consider the profile at its least *y* value i.e. pushed all the way down so that it lies on the boundary of the Pattern Locating Tolerance Zone (PLTZ). Now the Feature Relating Tolerance Zone (FRTZ) is pushed vertically upward so that it can accommodate the full rotation of the profile ($\frac{t_f}{2}$) within its boundaries. Next, the entire profile along with the FRTZ can rotate inside the PLTZ by an amount $\frac{t_p}{2(L+\bar{a})}\ \bar{a}$. Using these steps, a 2D central section of the T-Map is constructed and shown in Fig. 3.9.

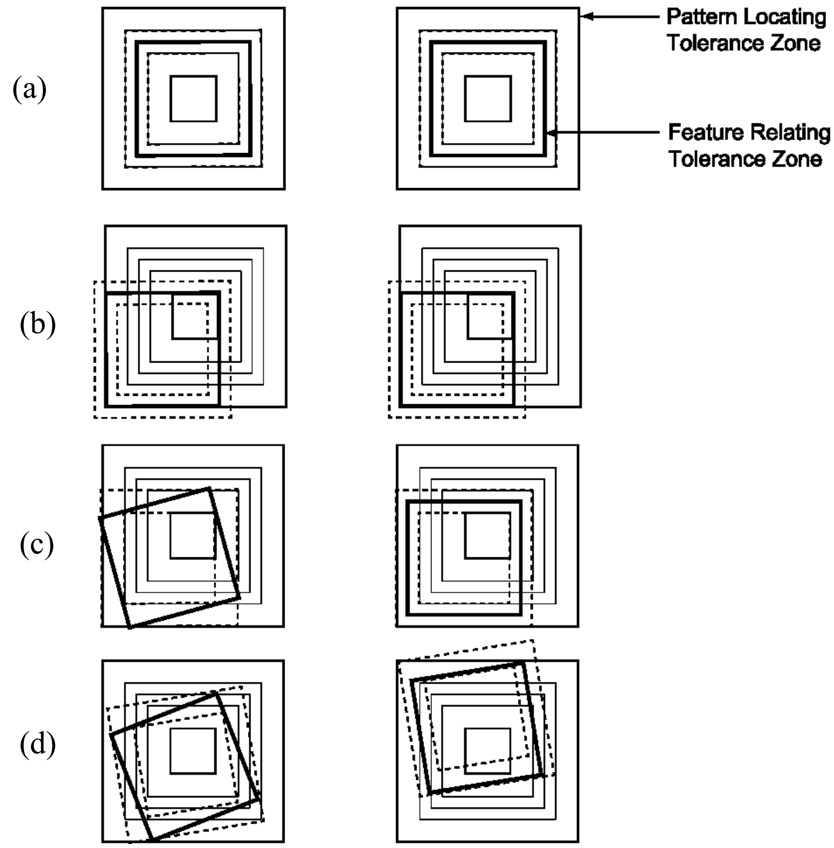


Figure 3.8: A series of steps the profiles in a linear pattern of two squares undergo to rotate from its initial position to its maximum limit is shown.

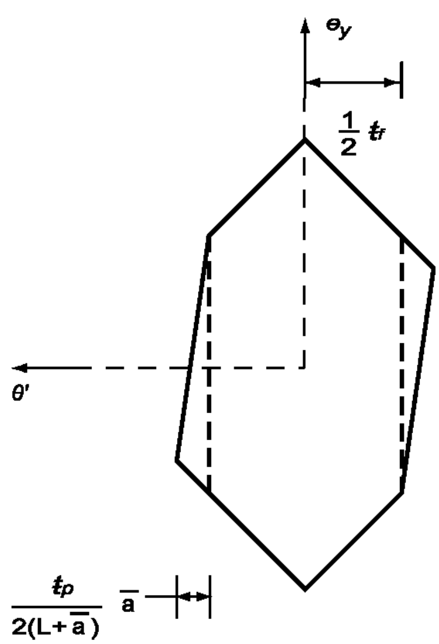
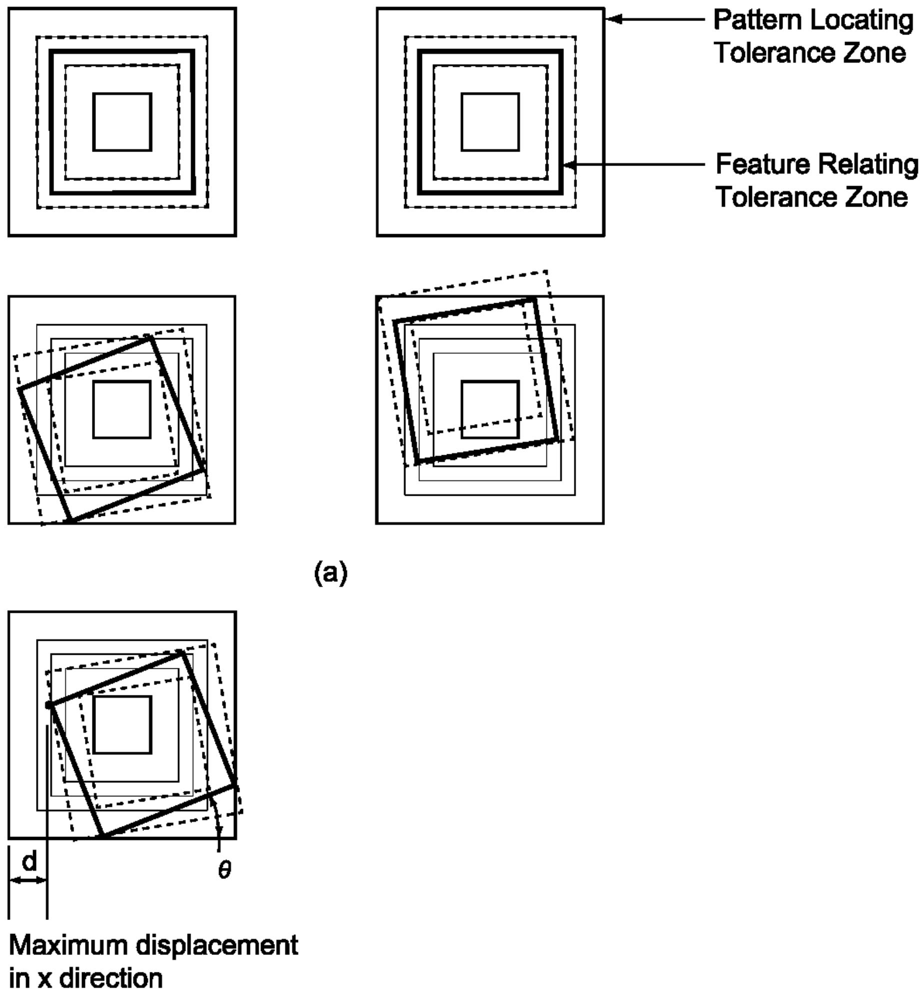


Figure 3.9: A 2D central section through the T-Map for the left tab in Fig 3.2 is shown. The limits of θ' and y are $(\frac{t_p}{2(L+\bar{a})}\ \bar{a}\ + \frac{t_f}{2})$ and $\frac{t_p}{2}$ respectively.

θ' - e_x directions are considered only - the series of steps involved in the translation in the *x* direction of the profile when it is rotated to the maximum angle possible is shown in Fig. 3.10. Consider the profile of the left boss in the Fig. 3.6 to be at its minimum *x* value i.e. at the leftmost position. Now the profile can translate in the *x* direction only till one of its edges touches the Pattern Locating Tolerance Zone boundary. The limit in displacement *d* in the *x* direction is calculated in Eqs (3-3) through (3-7).



(b)

Figure 3.10: (a) Some of the possible variations of the profiles, in a linear pattern of two squares, within the tolerance zone in the *x*-direction when rotated to its limits are shown. (b) The maximum displacement *d* in the *x*-direction is shown.

$$d = \left(2\bar{a} + t_p\right) - 2\bar{a}(\sin\theta + \cos\theta) \quad (3\text{-}3)$$

For small angles, $\sin\theta = \theta$ and $\cos\theta = 1$. Hence equation (3-3) becomes

$$d = \left(2\bar{a} + t_p\right) - 2\bar{a}(\theta + 1) \quad (3\text{-}4)$$

Since the maximum value of θ is to be considered,

$$d = \left(2\bar{a} + t_p\right) - 2\bar{a}\left(\frac{t_p}{2(L+\bar{a})} + \frac{t_f}{2\bar{a}} + 1\right) \quad (3\text{-}5)$$

$$d = \left(2\bar{a} + t_p\right) - 2\bar{a} + \frac{t_p}{2(L+\bar{a})}\, 2\bar{a} + t_f \quad (3\text{-}6)$$

$$d = t_p\left[1 - \frac{\bar{a}}{(L+\bar{a})}\right] - t_f \quad (3\text{-}7)$$

Equation (3-7) represents the maximum possible displacement of the profile in the *x* direction when fully rotated. But when it is rotated just within the Feature Relating Tolerance Zone it can displace by an amount of $(t_p - t_f)$.

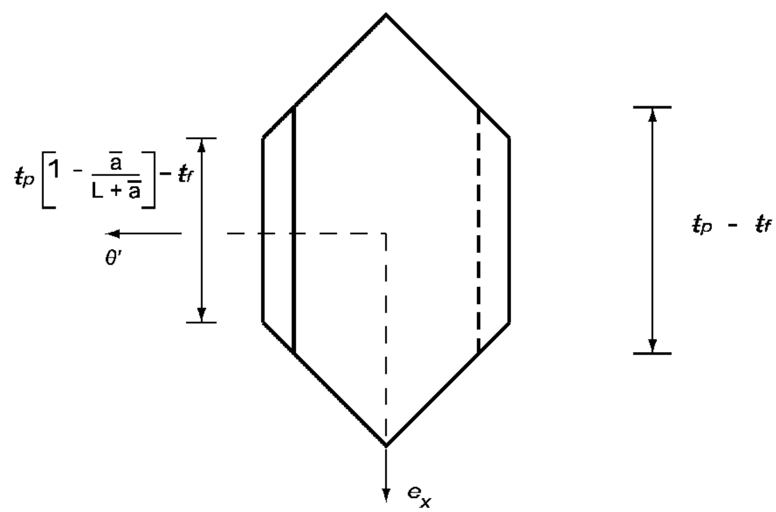


Figure 3.11: A 2D central section (θ' - e_x directions) through the T-Map for the left square tab shown in Fig 3.2.

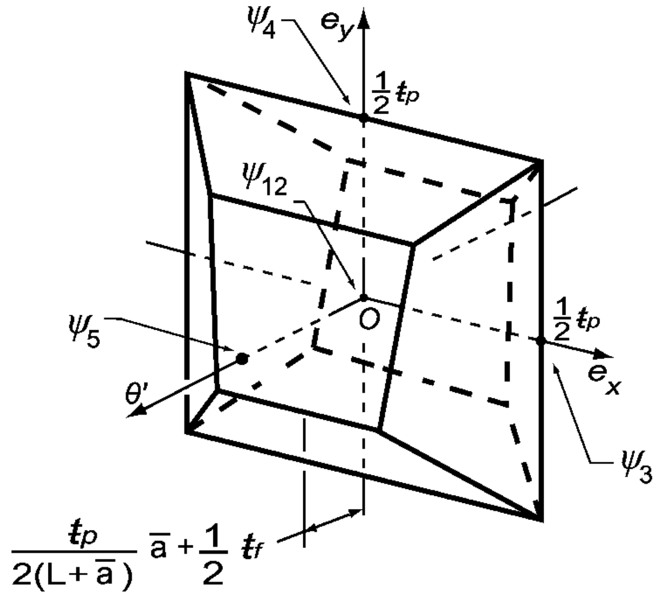


Figure 3.12: The T-Map for all middle-sized squares (left tab in Fig 3.2) in the tolerance zone of Fig 3.6; it is one central hypersection of the 4D T-Map that represents the entire tolerance zone.

The above T-Map is a hypersection of the complete 4D T-Map for the pattern of square profiles. The size variation is governed by the FRTZ (t_f).

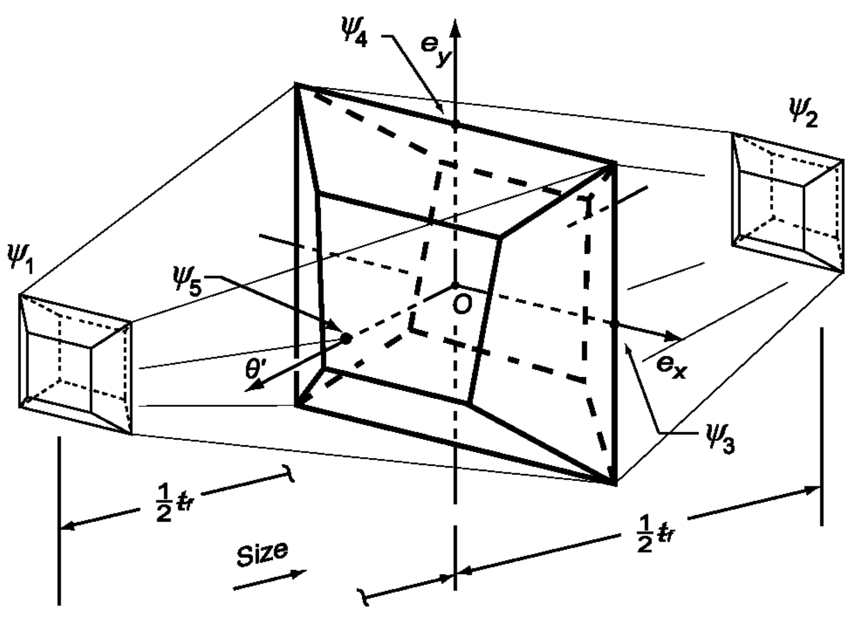


Figure 3.13: The 4D T-Map for the tolerance zone in Fig 3.3 showing all the basis points.

Note that the 3D hypersection at the two extreme sizes (maximum and minimum) of the profile have a straight truncation and not an angular one like in Fig. 3.12. This is because at these sizes, there is only one angular variation of the profile and that is within the Pattern Locating Tolerance Zone. The maximum limit of translation in x and y direction is $t_p - t_f$ and the limit in θ' direction is $\frac{t_p}{2(L+\bar{a})}\ \bar{a}$.

The T-Maps for each profile is different in the case of primary datum repeated in the secondary frame. To illustrate this further, a sample part with three square profiles is shown in Fig 3.14.

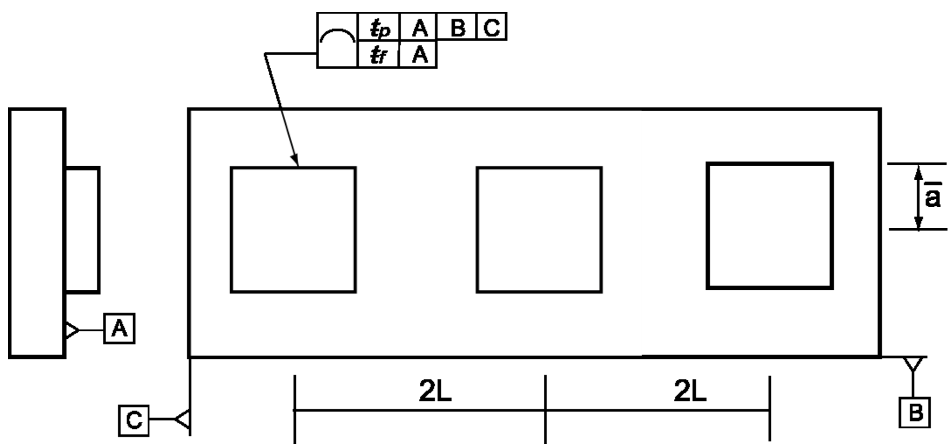


Figure 3.14: A sample part containing two square profiles toleranced using the composite frame scheme. The centers of the squares are separated by a distance of 2*L*.

The tolerance zones established by the two tolerances t_p and t_f are shown in the Fig 3.15. The feature relating tolerance zone can rotate and translate within the larger pattern locating tolerance zone. The profiles within each feature relating tolerance zone can rotate and translate as shown in Fig 3.15.

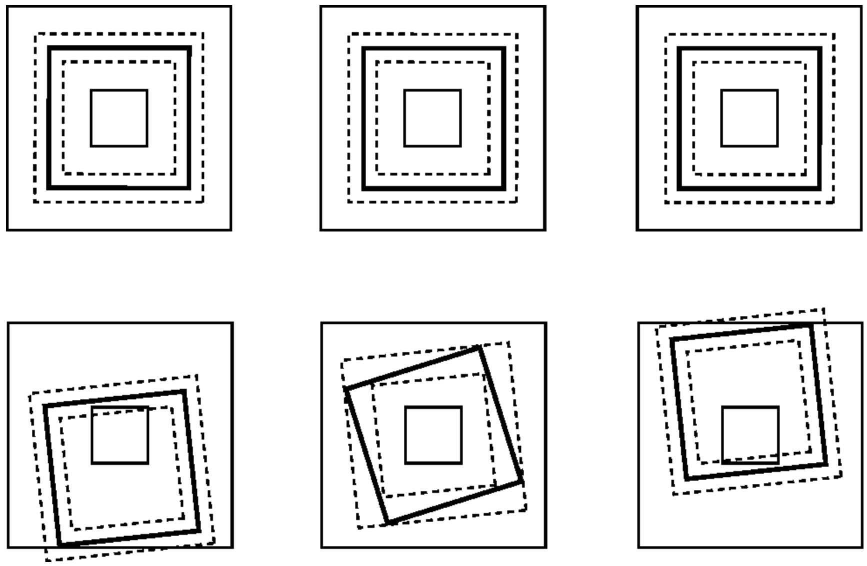


Figure 3.15: The tolerance zone established by tolerance t_p and t_f is shown along with the permitted variations of the profile within the zones. The solid line shows the pattern locating tolerance (t_p) zone and the dashed lines show thefeature relating (t_f) tolerance zone. The thick solid profile is the true profile.

The T-Map for each of the tabs is different. The calculations are similar to those performed in Eqs (3-1) through (3-7) and not shown here. Only the T-Map for the Mid-Sized Profile (MSP) is shown since it is used in stack up analysis. In the Fig 3.16, (a) represents the T-Map for the leftmost square, (b) represents the T-Map for the middle square and (c) represents the T-Map for the rightmost square.

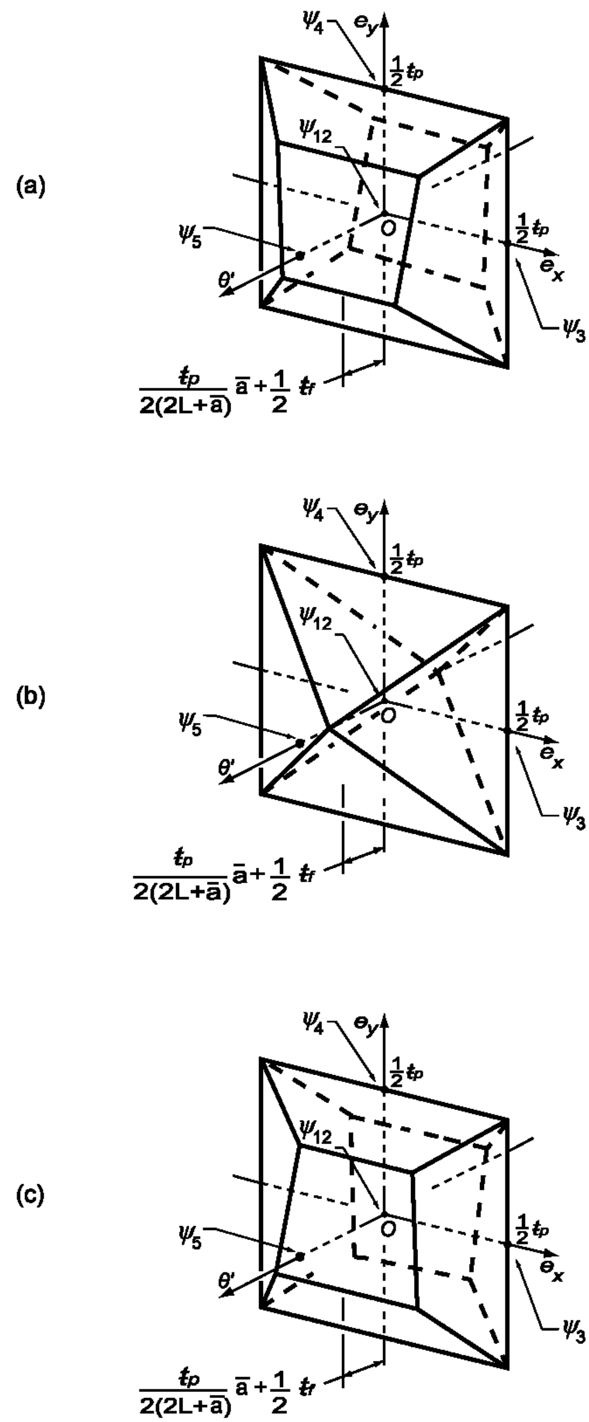


Figure 3.16: The T-Map for all middle-sized squares (left tab in Fig 3.2) in the tolerance zone of Fig 3.6; it is one central hypersection of the 4D T-Map that represents the entire tolerance zone.

3.3 Effect of datums:

There are two ways to tolerance a pattern of profiles using the composite scheme – repeating the primary datum in the second frame (§ 3.2.1) or repeating both the primary and the secondary datums in the FRTZF. Both of these methods have a unique T-Map and need to be studied more carefully. The following section gives a detailed explanation of how the T-Map varies when there aere two datums specified in the FRTZF.

3.3.1 Primary and the Secondary datum in the FRTZ frame:

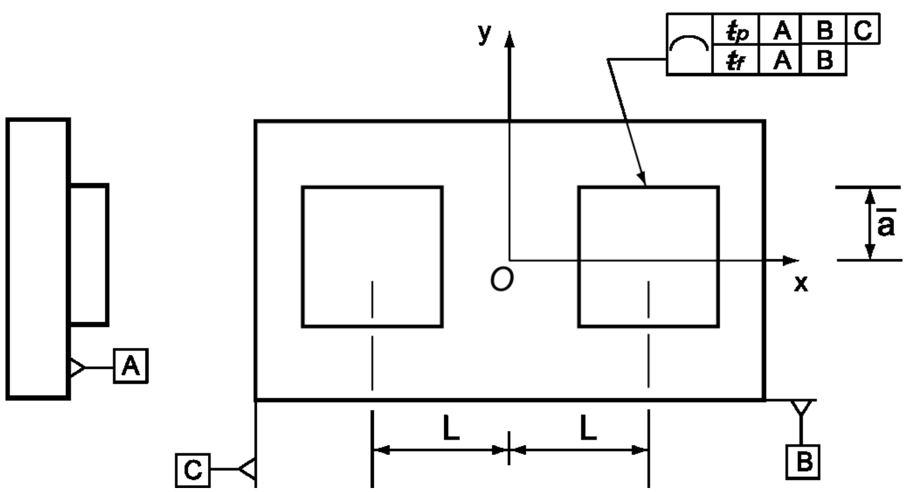


Figure 3.17: A sample part which consists of a pattern of square profiles toleranced with a composite frame where two datums are repeated in the lower frame. The centers of the squares are separated by a distance of $2L$.

When both the primary and the secondary datums are repeated in the lower frame, the normal to Feature Relating Tolerance Zone (FRTZ) needs to be perpendicular to the primary datum and also the zone needs to be parallel to the secondary datum, as required by the Standard [1]. Therefore, the orientation, of both the entire pattern and each of the profiles in the pattern, about an axis normal to the pattern is now limited by the FRTZ which imposes a much tighter control than that described in §3.2.1. The FRTZ can still translate in the larger PLTZ but needs to stay parallel to datum B. Hence the orientation limit is:

$$\theta_{max} = \frac{t_f}{2\bar{a}} \tag{3-5}$$

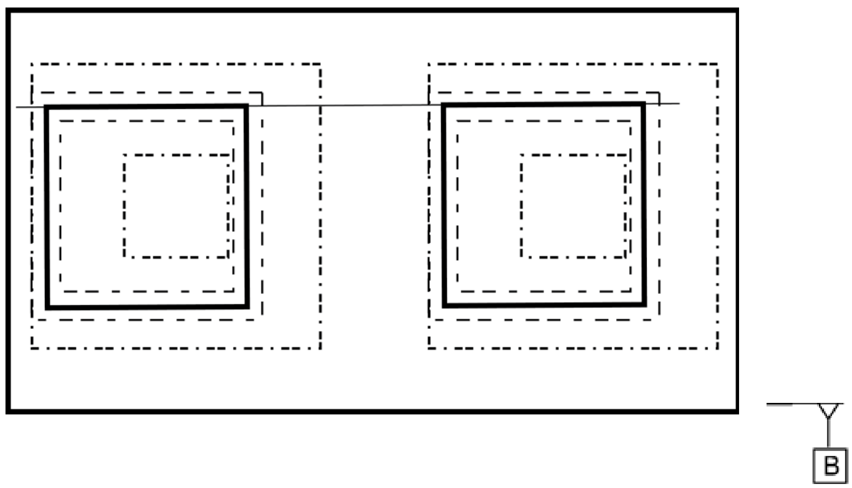


Figure 3.18: Variation within the PLTZ is shown here. The FRTZ has freedom to move inside the PLTZ as long as it stays parallel to datum B.

As in the above case of §3.2.1, the feature needs to lie within both the tolerance zones. In this case, as the Standard [1] states, the FRTZ has to be parallel to the secondary datum and hence the restriction is imposed by the secondary tolerance zone. As a result, the T-Map in the Fig 3.20 will be narrower compared to the case where only a single datum is repeated in the lower frame (§ 3.2.1).

Once the Feature Relating Zones are located, there are additional variations within the FRTZ, like in §3.2.1. The profiles within each of the FRTZs can translate and rotate independently of each other. This is shown below in Fig 3.19:

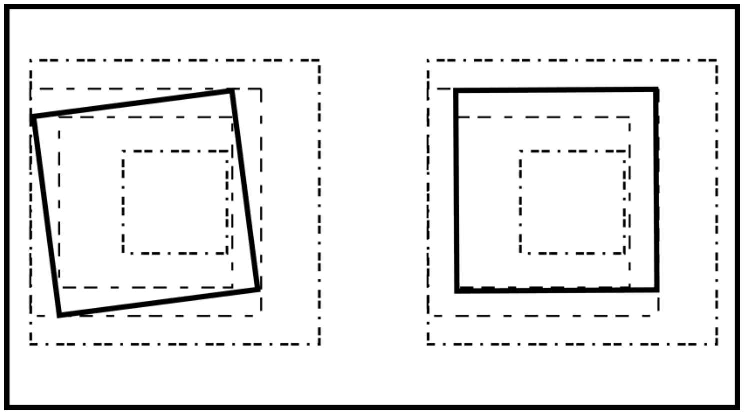


Figure 3.19: The variation within the FRTZ is shown here. One extreme orientation and one extreme translation are shown.

The T-Map for this case is similar to the one presented as Fig 3.12 with a difference in the orientation limit. The translation limit remains the same because the FRTZ can still float inside the larger PLTZ provided it stays parallel to the datum B. Now the maximum orientation possible is only $\frac{t_f}{2\bar{a}}$ as per equation (3-5). Once again, the scale assigned to the axis for angle θ is made $\theta' = \bar{a}\,\theta$. Hence equation (3-5) now becomes:

$$\theta_{max} = \frac{t_f}{2} \qquad (3\text{-}6)$$

Hence the T-Map will be narrower in the θ′ direction compared to the case with just the primary datum repeated in the lower frame (§ 3.2.1). The 3D hypersection for the Mid-Sized Profile (MSP) and the complete 4D T-Map are presented below in Fig. 3.20 and 3.21 respectively.

Figure 3.20: 3D hypersection for the case of pattern of squares shown in Fig 3.17. The rotational limits are truncated whereas the translational limits remain the same as in the Fig 3.12.

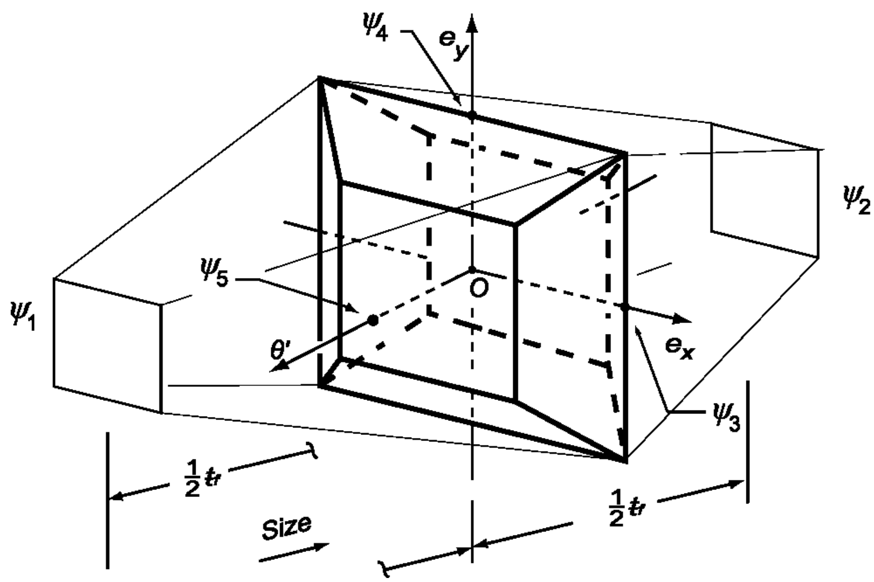


Figure 3.21: The 4D T-Map for the tolerance zone in Fig 3.18 showing all the basis points.

Note that, at the maximum and minimum sizes of the profile, the T-Map is just a 2D square. This is because no angular variation is possible at these sizes since the profiles have to remain parallel to the secondary datum. The squares have a side length of $t_p - t_f$.

3.4 Two Dimensional Patterns of Profiles:

Two dimensional patterns may also be tolerance using composite profile tolerancing so that the Pattern Locating Tolerance Zone can be larger than the Feature Relating Tolerance Zone that controls form and size of the profile [1]. In this section T-Maps will be constructed for two different cases – single datum repeated in the FRTZF (§3.2.1) and two datums repeated in the FRTZF (§3.3.1) – and their respective T-Maps will be constructed. Also, the interpretation of each case according to the Y14.5 Standard [1] will be discussed.

3.4.1 Case 1 – Primary datum only repeated in the FRTZF:

As in the case of a linear pattern, the primary rule is that the profile needs to be located within the boundaries of both the PLTZ and the FRTZ. These zones may overlap as long as every profile is confined to both of its zones. Note that the line-profile specification in Fig. 3.22 requires that these 2D tolerance zones be located parallel to the primary datum A at every cross-section of the feature. The FRTZ has no other orientation limitation with respect to the other datums and hence can rotate and translate within the PLTZ (as long as the profile stays within both the zones). The main difference between a 2D and a 1D pattern lies in the orientation limits. Because of the extra row/column of features in the pattern, there is more restriction imposed on rotation but has the same translation freedom as in the case of 1D patterns. An example is shown in Fig 3.22 below for more clarity on the subject.

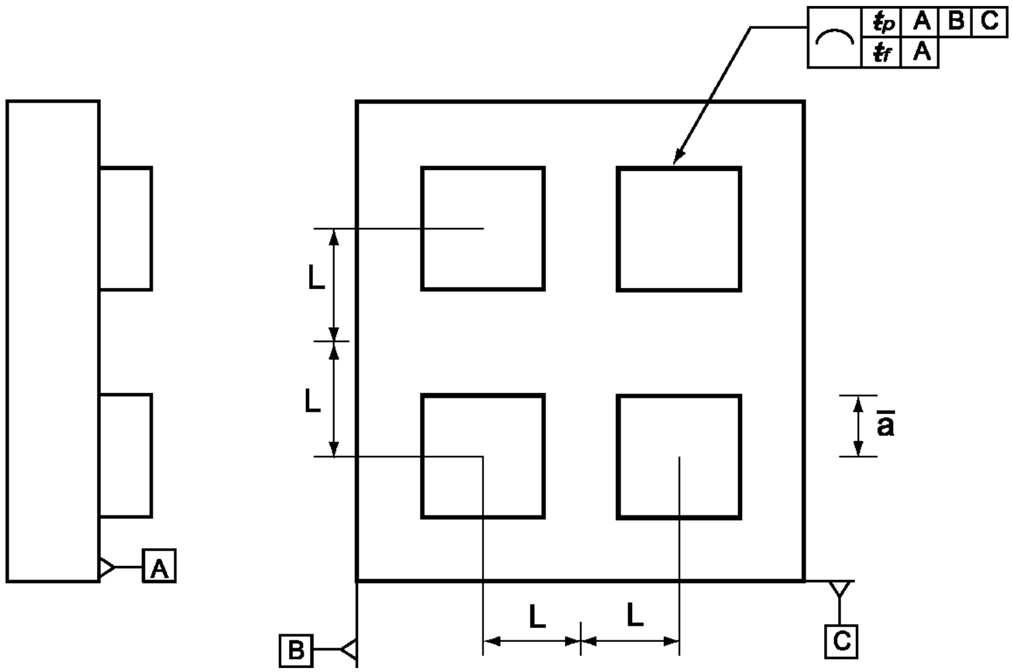


Figure 3.22: A sample part containing a two dimensional pattern of squares, the centers of which are separated by an equal distance 2*L*. The bosses are short enough to have a line profile tolerance allocated to them.

In the Fig 3.22, the Pattern Locating Tolerance is t_p and the Feature Relating Tolerance is t_f. The true profiles are established using basic dimensions and then the PLT Zones for all the profiles of the pattern are established. Once this is located, the FRT Zones are established within the PLTZs. The FRTZ for each of the profiles is separated by basic dimensions specified on the drawing. The FRTZs has the freedom to float within the PLTZs as long as they maintain a distance of 2*L* units between them. The Fig 3.23 (a) below shows one of the many possible orientations of the entire pattern. There could be a case where the FRTZs have simply translated and not rotated at all. There are many possibilities and only one is depicted below. Once the PLTZs and the FRTZs are located there exists an additional possibility of the profiles varying within the FRTZs. This is depicted in the Fig 3.23 (b).

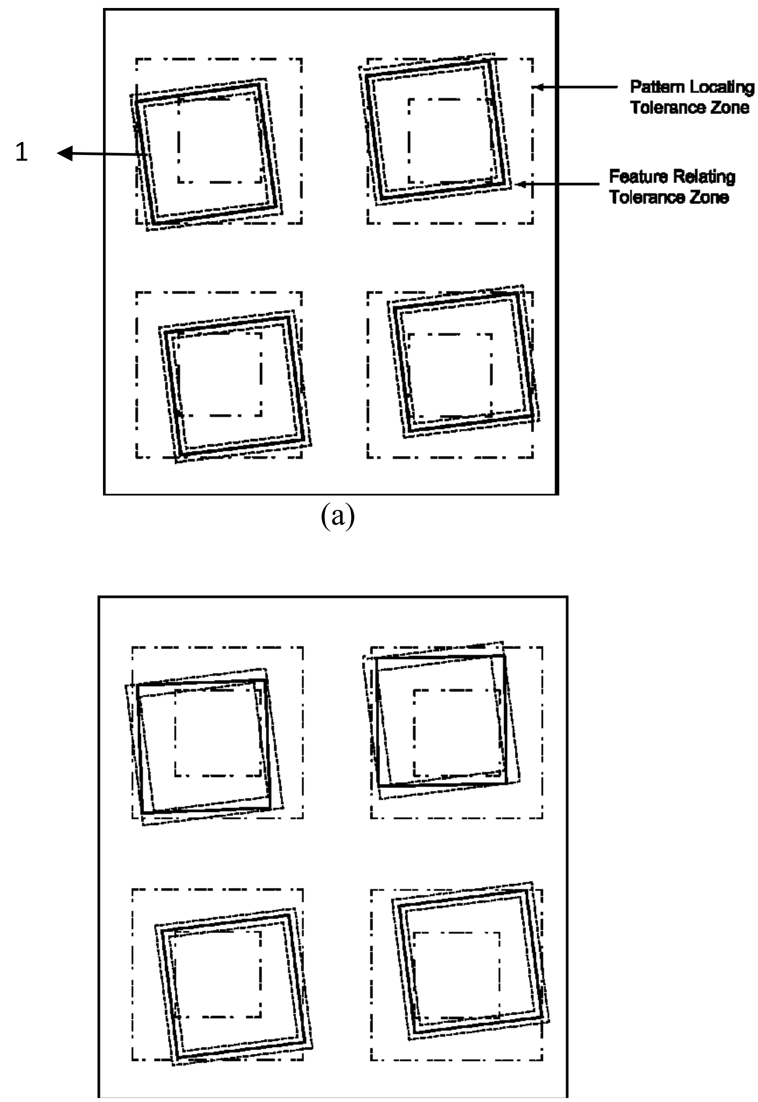
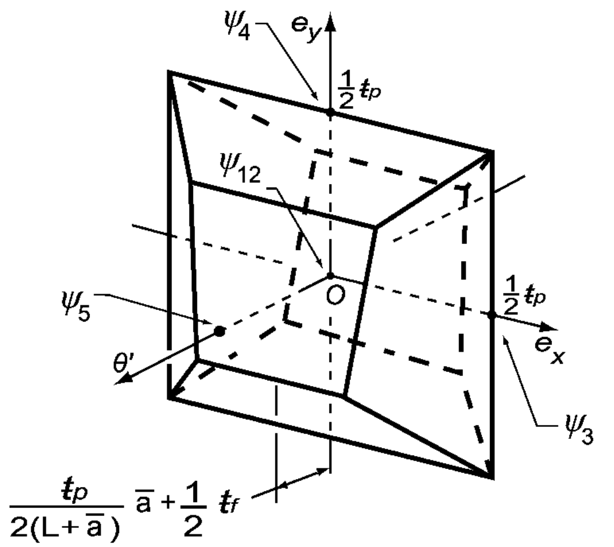


Figure 3.23: (a) Maximum possible orientation of the FRTZ within PLTZ is shown. (b) Additional variation (orientation) within the FRTZ is shown.

The T-Maps for 2D patterns of square bosses are exactly the same as the T-Maps for linear pattern of squares bosses. Each of the boss will have a unique T-Map and only the T-Map of square profile #1 is presented in this section. The entire procedure is the same as that presented in §3.2.2 and is not repeated here.



(a)

(b)

Figure 3.24: (a) The T-Map for all middle-sized squares in the tolerance zone of Fig 3.22; it is one central hypersection of the 4D T-Map that represents the entire tolerance zone.

(b) The 4D T-Map for the tolerance zone in Fig 3.23 (a) & (b) showing all the basis points.

3.4.2 Case 2 – Primary and the Secondary datums repeated in the FRTZF:

The case of Primary and Secondary datums repeated in the Feature Relating Tolerance Zone Frame for a 2D pattern of profiles is similar to linear patterns. The profile has to be located within the Pattern Locating Tolerance Zone and the Feature Relating Tolerance Zone. The true profile needs to stay parallel to the secondary datum and the normal to the profiles needs to stay perpendicular to the primary datum. Because of the parallelism constraint imposed by the secondary datum, orientation freedom is limited. The translational freedom, however, is not affected by the secondary datum. Hence, the entire FRTZ can float inside the PLTZ as long as it stays parallel to the secondary datum. An illustration is shown below with an interpretation as per Y14.5 Standard [1].

Consider a plate with two square bosses as shown in Fig. 3.25. A line profile tolerance is allocated to them as the bosses are short in length. The centers of the squares are separated by a distance *L*.

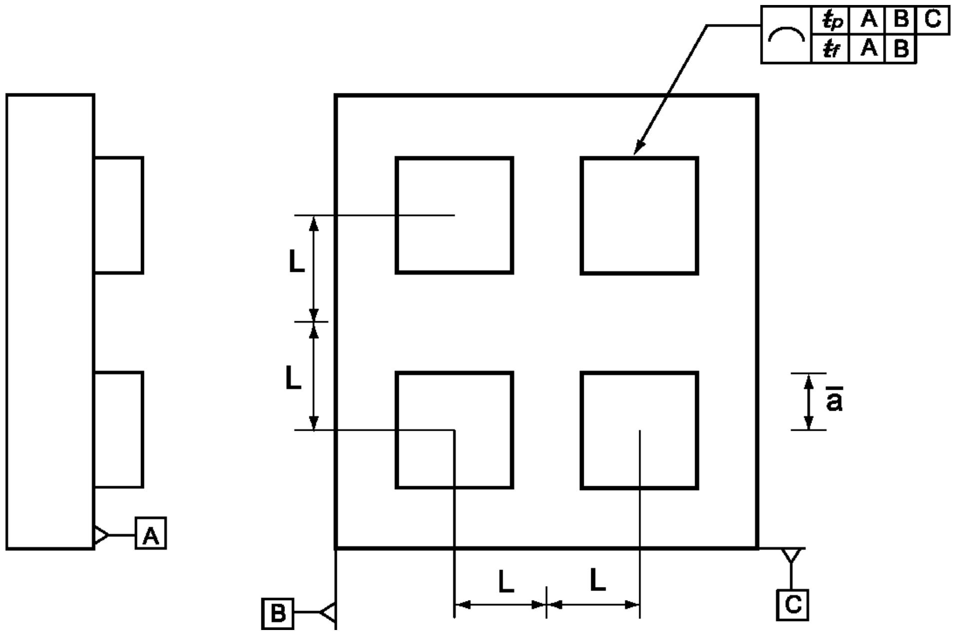


Figure 3.25: A sample part containing a two dimensional pattern of squares, the centers of which are separated by an equal distance *L*. The secondary datum B is repeated in the FRTZF.

In the Fig 3.25, the Pattern Locating Tolerance is t_p and the Feature Relating Tolerance is t_f. The true profiles are established using basic dimensions and then the PLT Zones for all the profiles of the pattern are established. Once this is located, the FRT Zones is established within the PLT Zones. The FRT Zones for each of the profiles is separated by basic dimensions specified in the drawing. The FRT Zones can float inside the PLT Zones as long as they stay parallel to the datum B and maintain a distance of *L* units between them. The profiles within the FRT Zones have full freedom to translate. The Fig 3.26 shows two possible variations of the profiles, two of them (1 & 3) are fully translated and the other two (2 & 4) depict the extreme orientations possible.

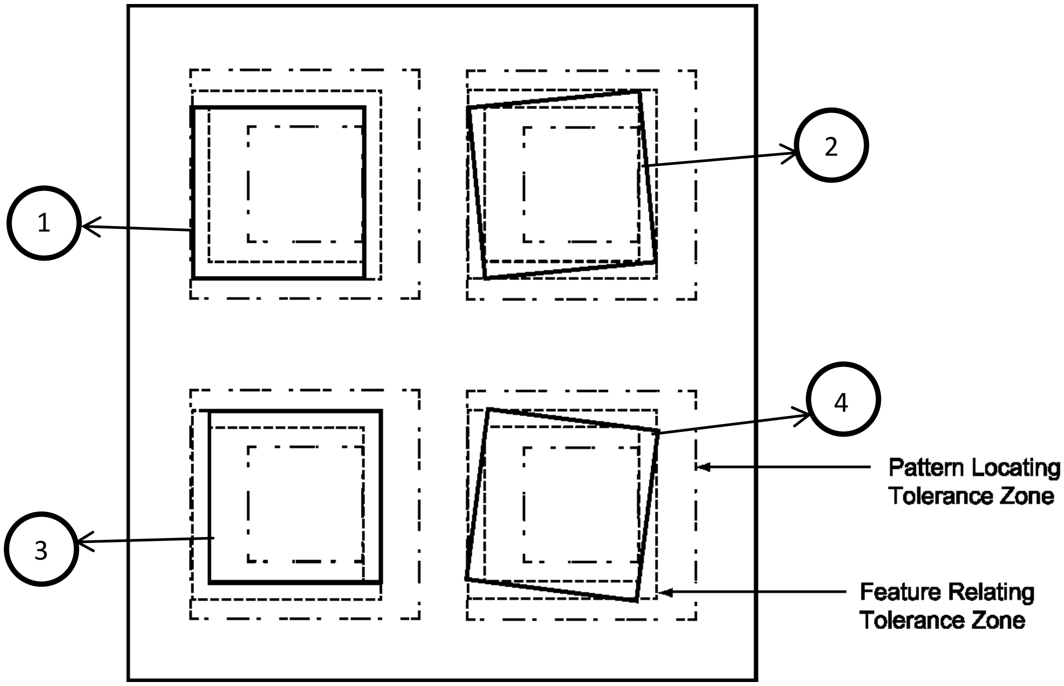
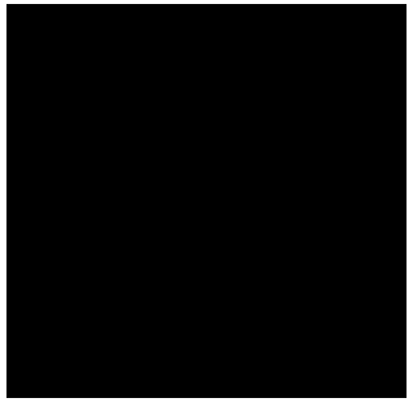
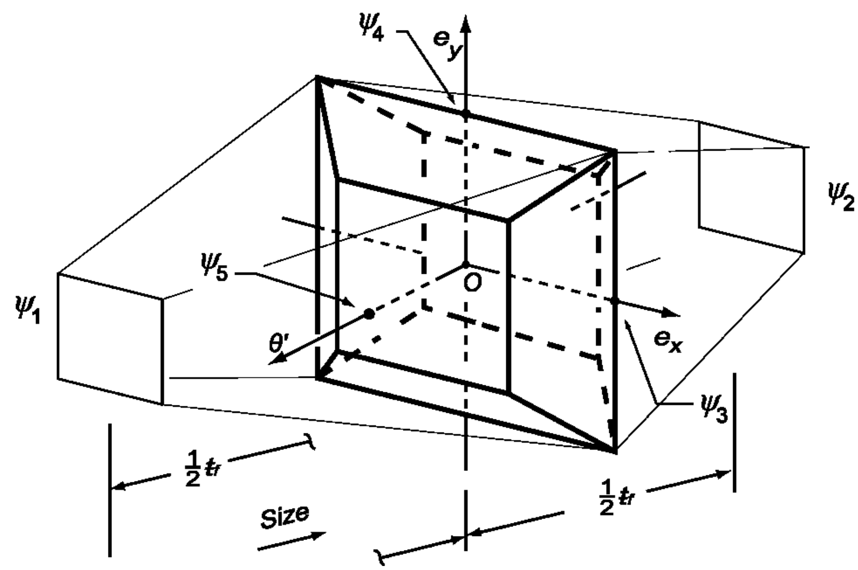


Figure 3.26: Two extreme possibilities of the Mid – Sized Profile with respect to orientation and translation.

The T-Map for this case is similar to that of the linear patterns with two datums repeated in the FRTZF (§ 3.3.1). The orientation freedom is governed by the FRT Zones and its limits are $\frac{t_f}{2}$ (Eqn. (3-6)). The translation freedom is governed by the PRT Zones and its limits are $\frac{t_p}{2}$. The 3D hypersection and the 4D T-Maps are presented below in the Fig 3.27(a) & (b). Note that the T-Maps are represent the variations of line profile #1 only.



(a)



(b)

Figure 3.27: (a) 3D hyper section for the case of pattern of squares shown in Fig 3.25. (b) The 4D T-Map for the tolerance zone in Fig 3.26 showing all the basis points.

CHAPTER 4

SINGLE – SEGMENT TOLERANCING OF A PATTERN OF PROFILES

4.1 Introduction:

As per the Y14.5 Standard [1], single-segment feature control frames are used where it is desired to invoke basic dimensions along with the datum references. In this chapter, the single-segment tolerancing on linear and two dimensional patterns will be studied. An illustration will be provided to better understand the interpretation as per the Standard [1] and also then respective T-Maps will be developed.

Unlike composite tolerancing (§3.1, §3.2), single-segment tolerancing has just one case – primary and secondary datum repeated in the Feature Relating Tolerance Zone Framework. If only the primary datum is repeated, it becomes equivalent to composite tolerancing, and, if all three datums are repeated, the purpose of specifying the Pattern Locating Tolerance (PLT) becomes void as it equivalent to a single profile tolerance with a value equal to that of the Feature Relating Tolerance (FRT).

4.2 Single-Segment Tolerancing:

A pattern can be toleranced using two single-segment frames. This offers a tighter control over the entire pattern when compared to composite tolerancing. An example of a multiple single segment frame is shown below in the Fig 4.1.

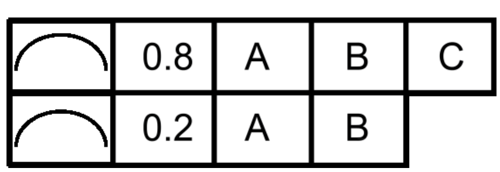


Figure 4.1: A sample multiple single-segment frame.

In the above frame, the PLT or t_p is assigned at the top and the FRT or t_f is assigned at the bottom. Note that the value of PLT is always greater than that of the FRT. Also, both the primary and the secondary datums are repeated.

4.3 Single-Segment Tolerancing on Linear Patterns of Profiles:

When a multiple single-segment tolerancing is applied to a linear pattern of profiles, there is a greater control over the position of the individual features of the pattern. Consider the following example:

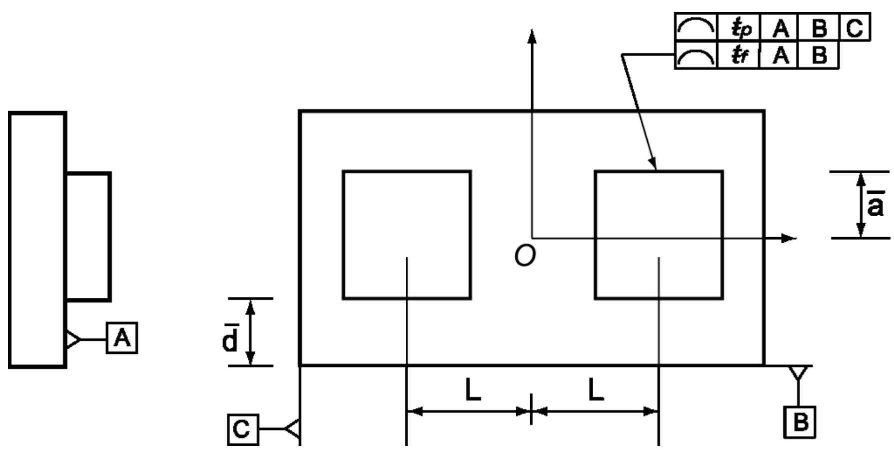
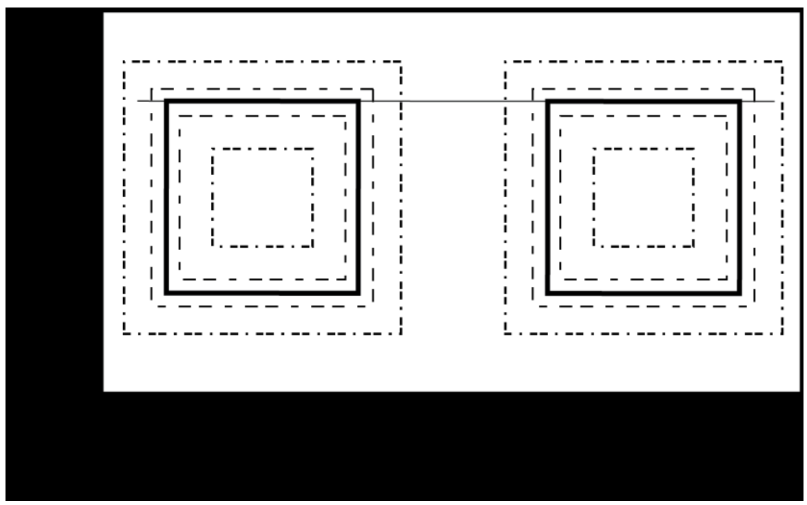


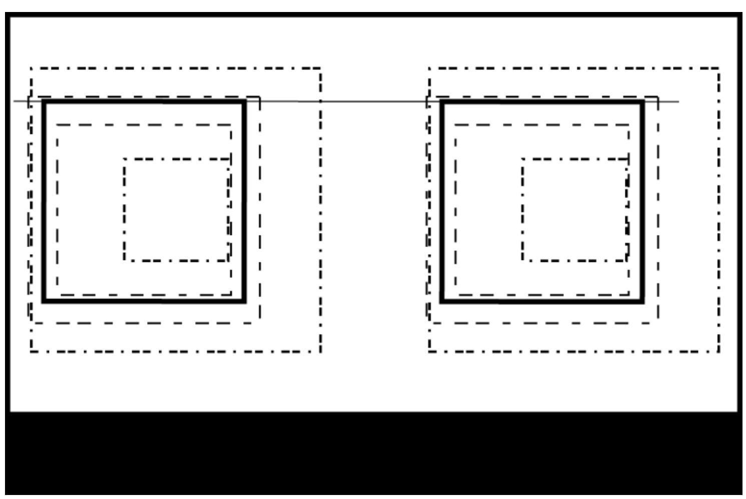
Figure 4.2: A sample part with a pattern of square profiles toleranced using the multiple single segment scheme. The centers of squares are $2L$ units apart.

According to the Standard [1], true profiles are first drawn using the basic dimensions given on the drawing. Once the true profiles are drawn, Pattern Locating Tolerance Zones (PLTZ) are established around them. Now, in case of multiple single-segment tolerancing, the Feature Relating Tolerance Zones (FRTZ) are also aligned with the datums, i.e. the true profile used to establish the FRTZ of the leftmost square is $\bar{d}$

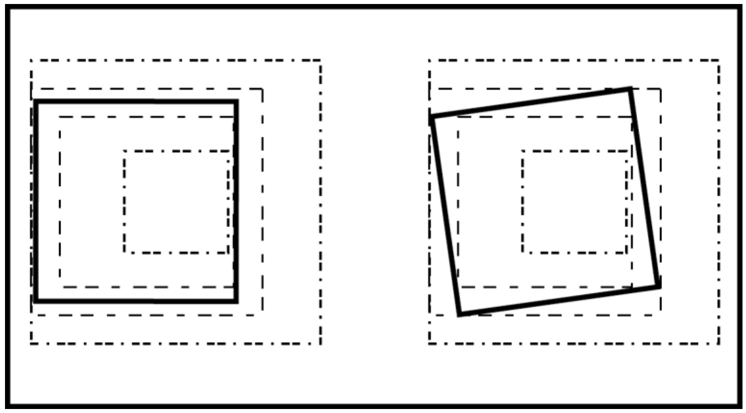
units away from the datum B and parallel to it. The FRTZs are separated by basic (theoretical) distance 2*L*. The FRTZs have the freedom to slide parallel to the datum B as long as the profiles stays within both the zones and the FRTZs maintain the basic distance between them. Two possible positions of the FRTZs are shown below in the Fig 4.3 (a) & (b). Once the zones are located, the profiles can still vary within the boundaries of the FRTZ. There can be a translation or rotation of each profile within the boundaries of the FRTZ. This is shown in the Fig 4.3 (c).



(a)



(b)



(c)

Figure 4.3: (a) The two tolerance zones are located by the true profile which is tied to the datum B. (b) One of the possible locations of the Mid-Sized Profile after the FRTZ has translated parallel to the datum B. (c) An additional variation of individual profiles within the FRTZ – both translation and rotation of the Mid-Sized Profile is shown.

The T-Maps of Single-Segment tolerancing is different from all the other cases (§3.1, 3.2) discussed previously in Chapter 3. Due to the basic dimensions, the true profile, used to locate the PLTZ and the FRTZ, is fixed in its position with respect to datum B. The effect of this is that it constrains the translation along the direction perpendicular to datum B. But, since datum C is omitted in the second feature control frame, there is no additional constraint horizontally, and there is still freedom for the profile to translate within the boundaries of the FRTZ. The translation freedom in a direction parallel to the datum B is still controlled by the PLTZ much like the other cases (§3.1, 3.2) discussed until now. So the maximum translations in the *y*-direction which now becomes half the value of the FRT

$$e_y = \frac{t_f}{2}, \tag{4-1}$$

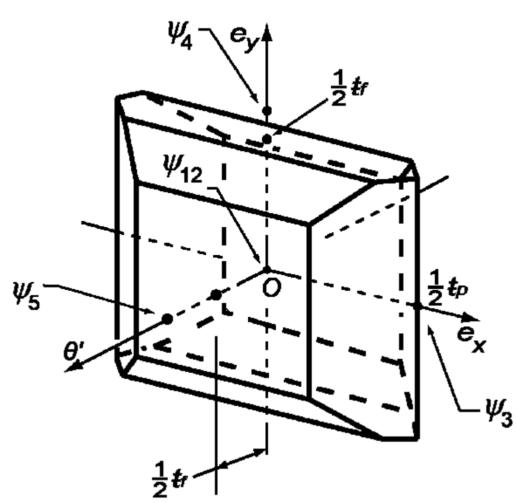
As shown by the small dot in center of the face at the top of the T-Map in Fig 4.4(a). The orientation freedom is also limited to just the FRTZ. This is because the FRTZ cannot rotate about an axis perpendicular to the datum A. Hence the maximum angle of rotation for each individual profile in the pattern is:

$$\theta_{max} = \frac{t_f}{2\,\bar{a}}\,. \tag{4-2}$$

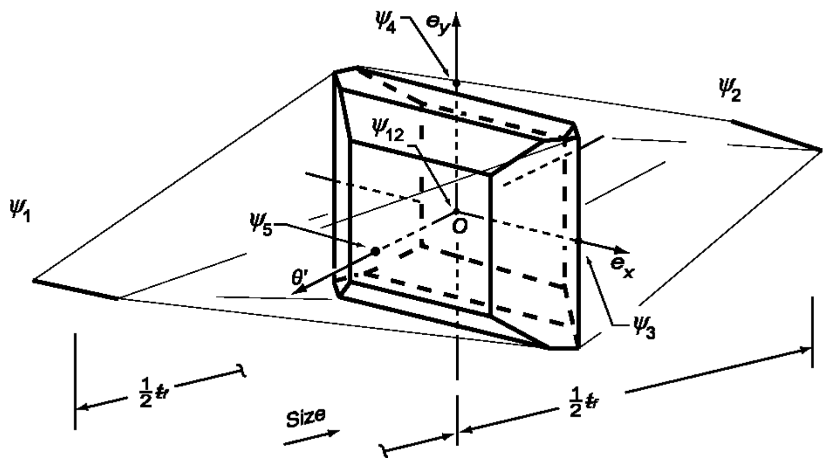
So that the T-Map may be used for metric computations, the units along all axes should be the same, i.e., a length [L]. For that reason, the scale assigned to the axis for angle θ is made $\theta' = \bar{a}\,\theta$. Hence the equation (4-2) is reduced to:

$$\theta'_{max} = \frac{t_f}{2} \tag{4-3}$$

Hence the T-Map is clipped in both the e_y and the θ' directions, and the boundaries in these directions are is dictated by the FRTZ. In Fig 4.4 (a) & (b) a 3D hypersection and the 4D T-Maps are presented.



(a)



(b)

Figure 4.4: (a) A 3D hypersection of T-Map for all mid-sized squares in the tolerance zone of the part shown in Figure 4.2. (b) The 4D T-Map of the tolerance zone shown in Figure 4.3 with all the basis points.

Note that at the extreme sizes (minimum and maximum) the T-Map is a horizontal line. This is because the tolerance specification limits displacement of the FRTZs in the *y*-direction i.e. is constrained and displace can occur only along the x. The length of one of the lines is $t_p - t_f$.

4.4 Single-Segment Tolerancing on Two Dimensional Patterns of Profiles:

This section addresses the case of two dimensional patterns with multiple single-segment tolerances allocated to them. First, an interpretation according to the Standard [1] will be provided and then, a T-Map will be constructed based on the interpretation.

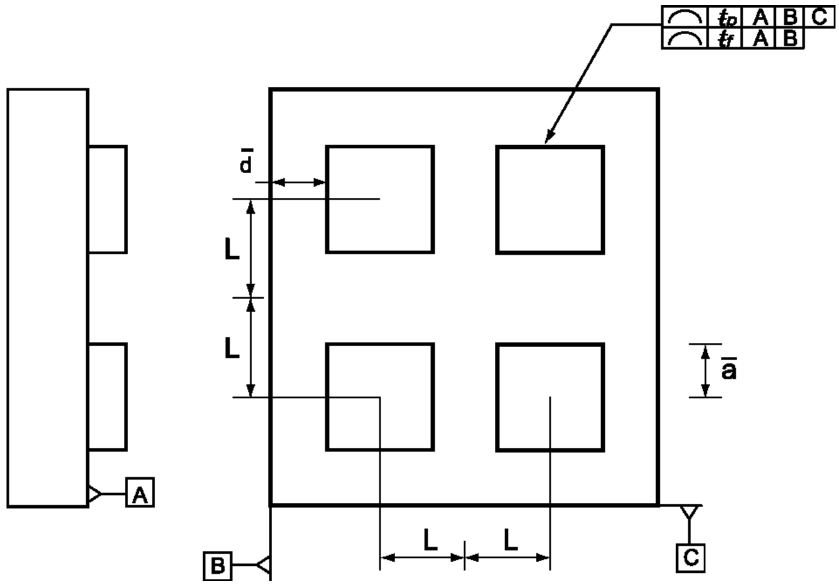
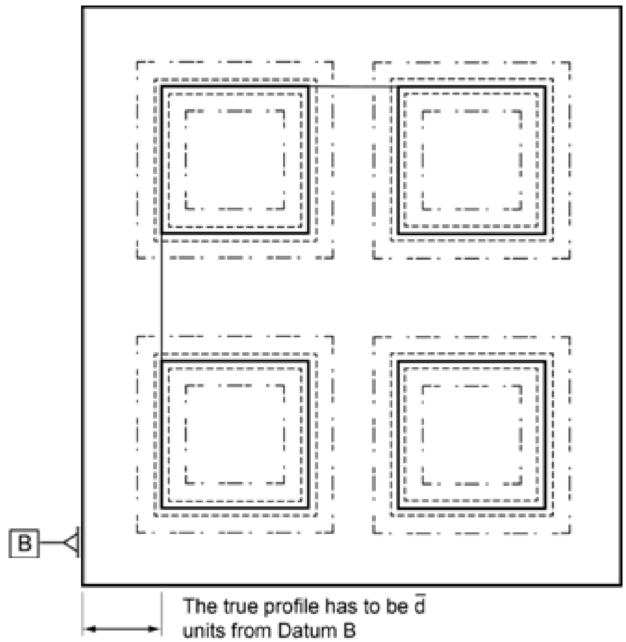
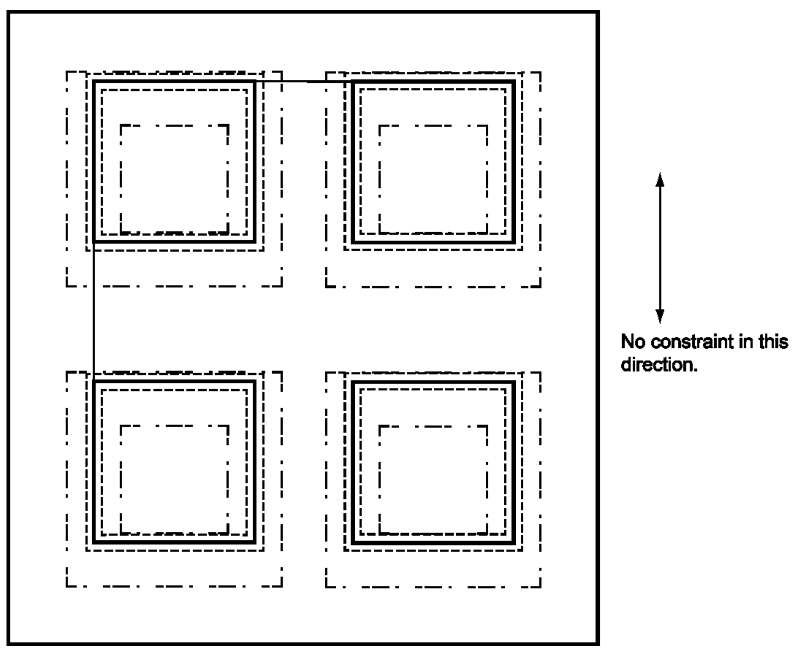


Figure 4.5: A sample part with a 2D pattern of square profiles, showing multiple single segment tolerancing scheme.

A part containing a 2D pattern of square bosses is shown in the Fig 4.5. The centers of these bosses are separated by a distance of $2L$. The true profile for each of the squares is first established using the basic dimensions and then the respective tolerance zones are established according to the specifications in Fig 4.5. The true profile which establishes the PLTZ and FRTZ is always at a fixed distance of $\bar{d}$ from datum B. Note that the FRTZ can move parallel to the datum B as long as it maintains the fixed distance (true profile being $\bar{d}$ units) from it. Also, the true profiles are spaced with basic dimensions and must maintain this distance. This effect is shown in Fig 4.6 (a) and (b) below.


B
The true profile has to be $\bar{d}$
units from Datum B

(a)


No constraint in this
direction.

(b)

Figure 4.6: (a) The two different zones established around the true profile which stays fixed with respect to the datum B always. (b) The direction in which the FRTZs can translate (parallel to datum B).

Apart from these variations, there can be translation and rotation of the profile within the FRTZ just as in the other cases (§3.1, 3.2). These variations are limited by the boundaries of the FRTZ. This is depicted in the Fig 4.7 below.

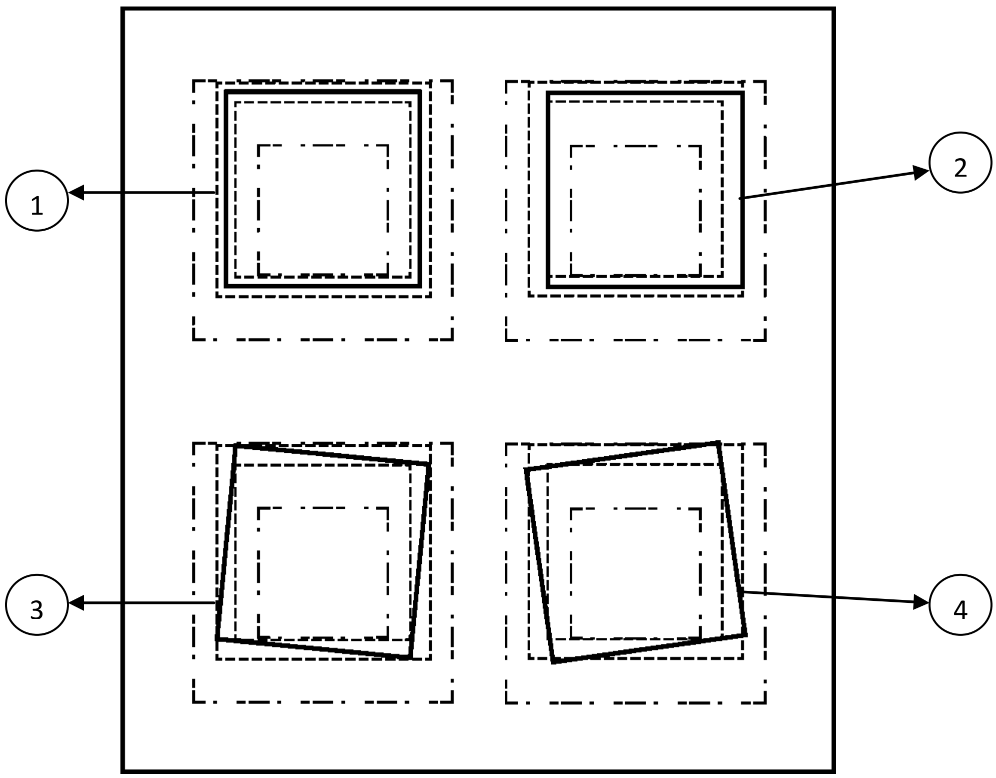


Figure 4.7: The variations within the FRTZ are shown. The MSPs of the two squares at the bottom (3 & 4) are oriented at extreme positions while the MSP of the square at the top right (2) is translated to an extreme position.

Since there is no difference between the interpretation of two dimensional patterns and linear patterns in the case of multiple single segment tolerancing, the T-

Maps remain the same. Hence the calculations and the figures of the 3D hypersection of the T-Map (Fig 4.4 (a)) and the 4D T-Map (Fig 4.4 (b)) are not repeated in this section.

CHAPTER 5

GAGING OF PATTERNS AND LEAST SQUARES FIT

5.1 Introduction:

Once a pattern of profiles is manufactured, it is important to verify if the features are within the tolerances specified by the designer. There are two approaches to this – (a) physical (hard) gages and (b) fitting of features to CMM data to identify geometric variations that can be used for verification. A detailed description of both the methods is presented in the following sections.

5.2 Physical Gaging Of Patterns:

To inspect patterns with gages, two types of gages required are - Pattern locating gages to check the position of the pattern with respect to the datums and Feature relating gages to check the relative position of the pins with respect to each other. A worst case analysis performed on the part to be manufactured will give the dimensions required for the gages.

Since gages work only with a Maximum Material Condition specification, they are not used in verification of profiles. However, the worst case analysis will give us dimensions of the gage that can be used to check if the profile is within the outer boundary of the tolerance zone.

5.3 Least Squares fit [27]:

In §2.5, the concept of Least-Squares was introduced for the fit of points around a single manufactured profile. The least-Squares fit for a profile also can be used to verify both tolerances t_p and t_f for a pattern of profiles after it has been manufactured. This is done by using a CMM to measure points along the machined profiles and then using Moore-Penrose inverse (§2.5.1) to fit a profile to these set of points. The necessary equations are reproduced in this section.

The *Moore-Penrose inverse* for any finite rectangular matrix $[\mathbf{K'}]_{mxn}$ is [29]:

$$[\mathbf{K'}]^{\#} = \{([\mathbf{K'}]^{T}[\mathbf{K'}])^{-1}[\mathbf{K'}]^{T}\} \tag{5-7}$$

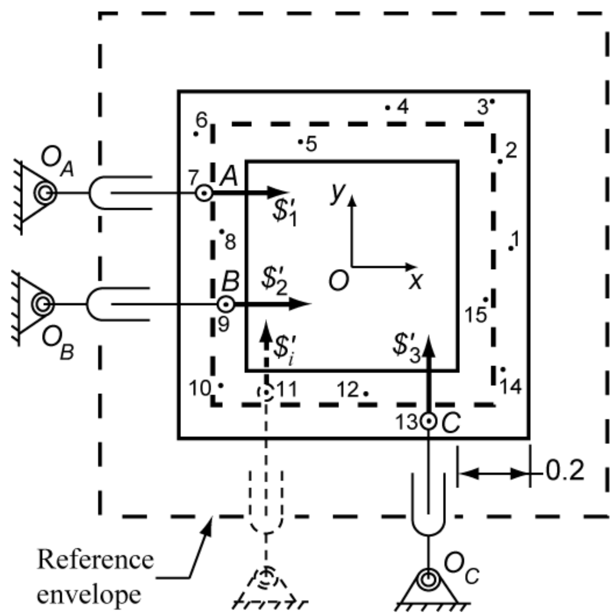


Figure 5.1: Figure 2.9 is reproduced here. The line-profile (dashed line), its tolerance zone boundaries (with an exaggerated scale), and the measured points lying on the platform of a planar in-parallel robot which is guided by three linear actuators lying on screws $\$'_1$, $\$'_2$ and $\$'_3$ at points *A*, *B*, and *C;*

The minimum distances to the points measured are all computed from a *reference-envelope*. This is because computing minimum distances at the corners of a profile requires a decision about association of a point with a particular segment; the decision can be made by identifying the closest envelope tangent-line to the point. The *reference-envelope* is a parallel curve larger than the MSP (Middle-Sized Profile). Now distance for every point measured is:

$$[\mathbf{d'}_i] = \begin{bmatrix} d'_1 \\ d'_2 \\ \vdots \\ d'_n \end{bmatrix} = [\boldsymbol{K'}][\$] = \begin{bmatrix} R'_1 & L'_1 & M'_1 \\ R'_2 & L'_2 & M'_2 \\ \vdots & \vdots & \vdots \\ R'_n & L'_n & M'_n \end{bmatrix} [\$]. \tag{5-8}$$

where $d'_1 \ldots d'_n$ are distances to points 1...n respectively from the *reference-envelope* and **[\$]** is a matrix of the rigid body displacements ($\delta\theta, \delta x, \delta y$) of the platform in plan. And when the size is included, equation (5-8) becomes:

$$[\mathbf{d'}_i] = \begin{bmatrix} d'_1 \\ d'_2 \\ \vdots \\ d'_n \end{bmatrix} = [\boldsymbol{K'}][\$] = \begin{bmatrix} R'_1 & L'_1 & M'_1 & 1 \\ R'_2 & L'_2 & M'_2 & 1 \\ \vdots & \vdots & \vdots & \vdots \\ R'_n & L'_n & M'_n & 1 \end{bmatrix} [\$] \tag{5-9}$$

where **[\$]** is now ($\delta\theta \;\; \delta x \;\; \delta y \;\; (\Delta s - \Delta F)$). ΔF represents the change in feature size between the mid-sized profile and the least-squares profile and Δs is the distance between the mid-sized profile and the *reference-envelope*. The values of ($\bar{a}\, \delta\theta, \delta x, \delta y$) correspond to coordinates (θ', e_x, e_y) in the T-Map (Fig. 2.4).

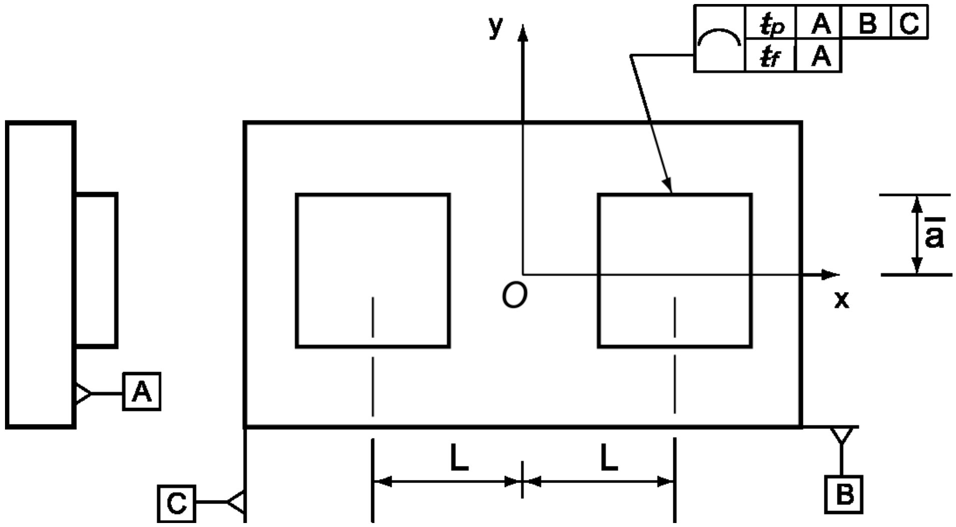


Figure 5.2: A sample part showing two square bosses. Here the t_p is the pattern locating tolerance and t_f is the feature relating tolerance.

The least squares fit is done in two steps as. First, a profile is fitted to the points around the entire pattern – a rectangle if the pattern is linear and contains squares as shown in Fig 5.2. This will aid in checking if the pattern as a whole is within the pattern locating tolerance zone. In the next step, a profile is fitted for points around each profile – square in case of Fig 5.2. This will aid in checking if each profile is within the feature relating tolerance zone.

5.3.1 Least Squares fit for the pattern:

When equation (5-9) is solved for **[$]**, the rigid body displacements and the change in size from the Mid-Sized Profile is obtained. The values of δx δy denote the displacement of the center (with respect to the center of the true profile) of the least-squares profile in the x and y directions respectively. It is also rotated by an amount $\delta\theta$ with respect to the Mid-Sized Profile.

To start the least squares fitting process, a CMM is used to measure points around each of the profiles in the pattern (Fig. 5.7(b)). To first verify whether the features are within the pattern locating tolerance zone, only the points on the outside of the pattern are considered, for example, the points shown in Fig 5.7(b) only points 3-13, 16-21 and 26-30 would be used. A rectangular least squares profile is then fitted to these points. The values of δx δy $\delta\theta$ and ΔF from Eqn. (5-9) must all lie within the limits of a T-Map constructed for a Rectangular Profile of height $2\bar{a}$ and width $(2\bar{a}+2*n*L)$,where n is the number of features in the pattern, and $t = t_p$ i.e. the pattern locating tolerance (Fig. 5.3). Although the size is controlled by the feature relating tolerance, it is taken into consideration in this section as well.

The values of ($\delta\theta$ δx δy ΔF) need to be stored since they will be used in the following sections. A subscript p is attached to each of the variables to distinguish it from the other results. Hence the deviation of the entire pattern is ($\delta\theta_p$ δx_p δy_p ΔF_p).

5.3.1.1 *i*-Map Representation of the Solution:

The solution for the equation (5-9) will give four deviations ($\delta\theta_p$ δx_p δy_p ΔF_p) that can be modelled in the T-Map for a rectangular profile with tolerance t_p. The co-ordinates of the point would be

$$(\theta', e_x, e_y, \Delta F) = (\bar{a}\,\delta\theta_p, \delta x_p, \delta y_p, \Delta F_p) \quad (5\text{-}10)$$

Fig. 5.3: i-Map for the solution obtained in §5.3.1. The tolerance shown in the figure is the pattern locating tolerance. ψ_3, ψ_4 and ψ_5 each intersect an axis at $t_p/2$.

5.3.2 Least Squares fit for individual profiles:

For each individual profile, only the points around it are considered, including those that were omitted in the previous section (§5.3.1). A Least Squares Profile (squares in this case) is then fitted to the points measured around each of the profiles. The deviations that are obtained by computing the pseudo inverse are given a subscript f followed by the feature number. Hence, the values of ($\delta\theta_{fn}$ δx_{fn} δy_{fn} ΔF_{fn}) that are obtained from (5-9) for these set of points are then compared with the values of ($\delta\theta_p$ δx_p δy_p ΔF_p) obtained in §5.3.1. This will determine how much of a deviation each feature has from the pattern as a whole.

5.3.2.1 *i*-Map Representation of the Solution:

The solution from § 5.3.1 ($\delta\theta_p \;\; \delta x_p \;\; \delta y_p \;\; \Delta F_p$) needs to be subtracted from the solution from § 5.3.2 ($\delta\theta_{fn} \;\; \delta x_{fn} \;\; \delta y_{fn} \;\; \Delta F_{fn}$) since the relative position of the feature with respect to the entire pattern is to be calculated. This solution can then be modelled into a T-Map with tolerance equal to t_f for the given square profile.

$$(\delta\theta_n \;\; \delta x_n \;\; \delta y_n \;\; \Delta F_n) = (\delta\theta_{fn} \;\; \delta x_{fn} \;\; \delta y_{fn} \;\; \Delta F_{fn}) - (\delta\theta_p \;\; \delta x_p \;\; \delta y_p \;\; \Delta F_p) \qquad (5\text{-}11)$$

where n is the feature number in the pattern. The coordinates for the *i*-Map are:

$$(\theta', e_x, e_y, \Delta F) = (\bar{a}\,\delta\theta_n\, ,\, \delta x_n\, ,\, \delta y_n\, ,\, \Delta F_n) \qquad (5\text{-}12)$$

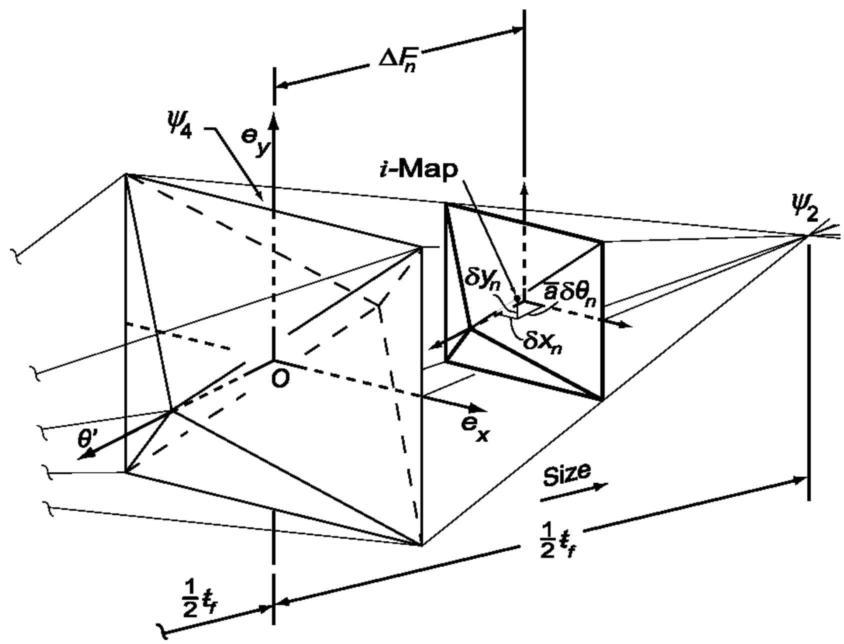


Figure 5.4: *i*-Map for the solution obtained §5.3.2. Taken from [28]. The tolerance shown here is the feature relating tolerance.

An alternative to the validation by separation of the tolerances t_p and t_f, presented in §5.3.1 and §5.3.2, is a combined result. This approach would provide a single point for the *i*-Map of each individual feature. Also, the T-Map into which it would be placed would in general be different for each profile in the pattern because the T-Maps are different (Fig 3.16).

As an example of the individual evalutions, consider a sample part with two square bosses whose centers are 2*L* units apart. These bosses have a line profile tolerance associated with them.

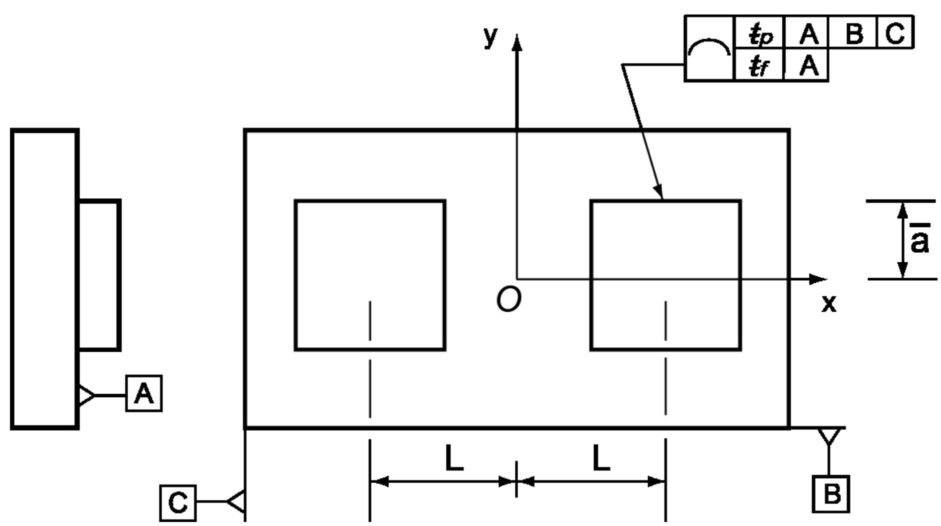


Figure 5.5 (a): Example part with a pattern of two square bosses is shown. The centers are 2*L* units apart.

A CMM (Coordinate Measuring Machine) is used to measure points around the manufactured profile (Fig 5.8). The points shown around the profile in Fig 5.8 are only for illustration purposes and have not been measured by a CMM. First, the profile will be checked for compliance with the Pattern Locating Tolerance and later with the Feature Relating Tolerance.

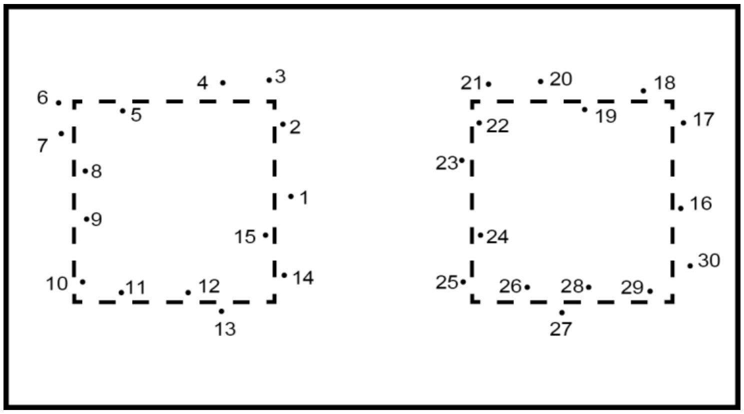


Figure 5.5 (b): The points that are measured around the square profiles using a CMM.

The next step in verification is to fit a least squares profile only to the exterior points, i.e. everything except 1, 2, 14, 15, 22, 23, 24 and 25. The least squares profile will be rectangular in shape. This rectangle needs to be within the Pattern Locating Tolerance Zone. When the equation (5-9) is solved, ($\delta\theta_p \quad \delta x_p \quad \delta y_p \quad \Delta F_p$) values are obtained. Note that these values will be a representation of the deviation of the rectangular least squares profile from its true position. The values depend on the points and can vary if more or different points are considered.

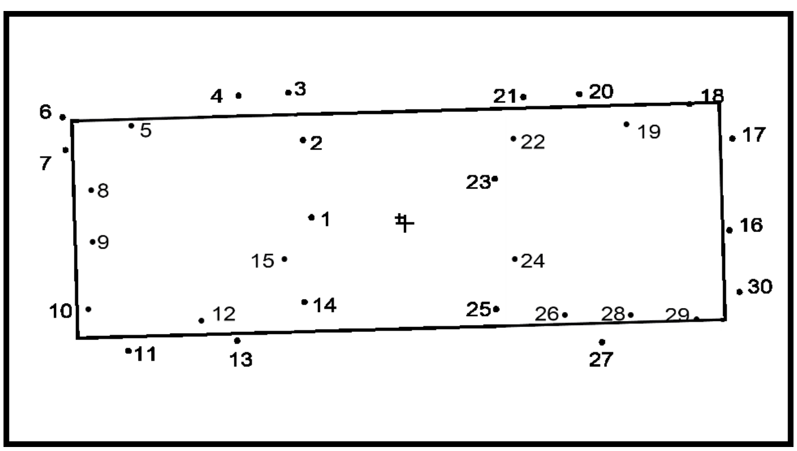


Figure 5.6: The least squares rectangle that is fit to the exterior points is shown. Note that the centers are slightly offset from each other and this is due to the variations in the manufactured profile.

Once the pattern is checked for compliance with the PLT, least square profiles are fitted to the measured points, including the interior points. This results in a square shaped least squares profile. These profiles need to be within the limits of the FRTZ. So, when the equation (5-9) is again solved for these set of points, the results obtained are ($\delta\theta_{fn}$ δx_{fn} δy_{fn} ΔF_{fn}), where n is the feature number. The values ($\delta\theta_p$ δx_p δy_p ΔF_p) obtained previously are now subtracted from ($\delta\theta_{fn}$ δx_{fn} δy_{fn} ΔF_{fn}). This is because the relative position of each of the feature with respect to the entire pattern needs to be determined. This solution will give the variation of each of the profiles and an *i*-Map representation can be used to determine whether the solution is in the T-Map space or not.

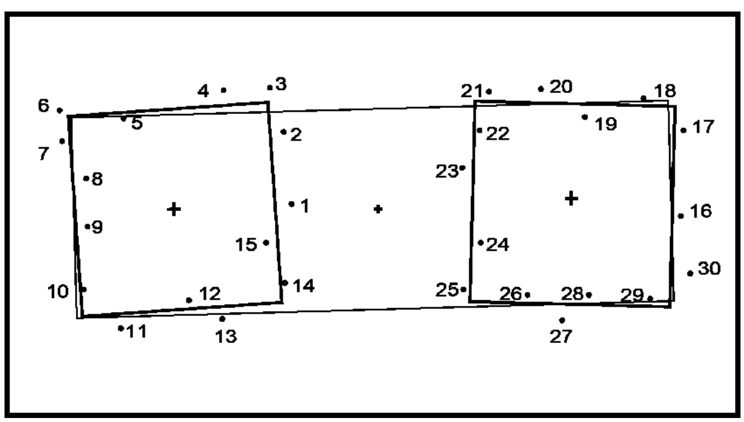


Figure 5.7: Least squares fits is used to fit square profiles to the measured points. Note that the centers are offset with respect to each other and the center of the rectangle.

CHAPTER 6

COPLANARITY USING PROFILE TOLERANCES

6.1 Introduction:

Profile tolerancing may be used to control relative orientation, form and location of plane surfaces. This finds application where two surfaces that are not continuous need to be treated as one uninterrupted surface. This is especially useful when those surfaces are required to be treated as a single datum. The Standard [1] has a brief description on this subject and it is presented later in this chapter, and an example tolerance specification is shown in Fig. 6.1.

6.2 Coplanarity:

Coplanarity is the condition of two or more surfaces having all elements in one plane [1]. A line profile tolerance can be used to when it is desired to treat two or more surfaces as a single continuous, non-interrupted surface. To achieve this, as in Fig. 6.1, two tolerances are specified. One is a size tolerance that locates the surfaces that are intended to be a single datum, and the other is a profile tolerance specified on the remaining surfaces with respect to the established datum.

In the following sections, an example is presented along with the interpretation as per the Standard [1].

6.3 Interpretation:

Consider the part shown in Fig. 6.1. It is similar to Fig. 1.4 (c) which is a stiffener with multiple tabs. The two surfaces A and B are identified as datums and have been assigned a line profile tolerance (similar to a straightness tolerance) and another line profile tolerance is assigned to the remaining two surfaces with respect to datum A-B. Surface #1 is a square with sides $2\bar{a}$ units.

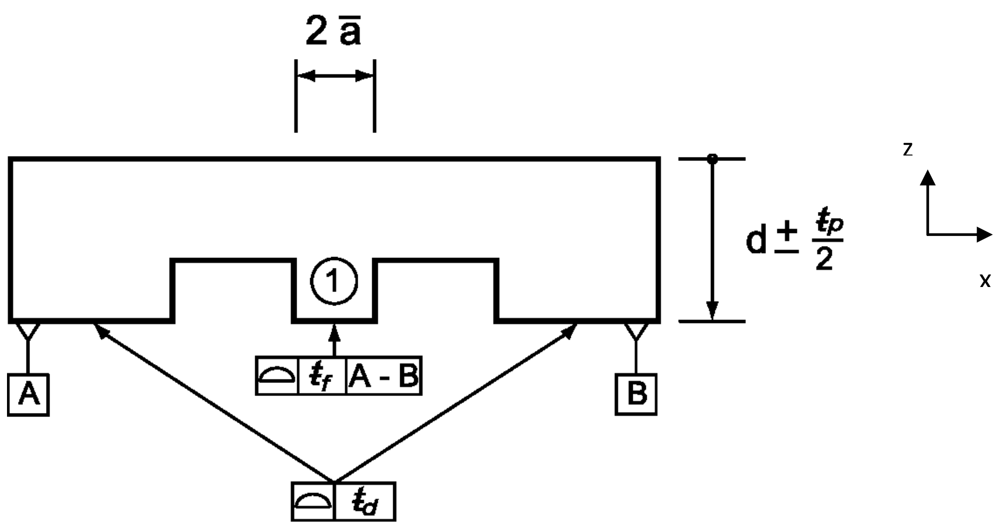


Figure 6.1: A sample part where coplanarity of the surface #1 with respect to datum A and B needs to be controlled is shown. A size tolerance t_p is assigned to the datums along with a profile tolerance t_d and a profile tolerance of t_f to the other surface (#1).

The tolerance specified on the datums (t_d) has to be smaller compared to the tolerance on the other surface (t_f) because its role is to control a unified form of both datums A and B. The surfaces A and B must be within the limits of tolerance zone defined by t_d. This zone is established by two common parallel planes (to the true profile of the surface A and surface B) separated by a distance of t_d. Each individual portion of the surface A-B can translate and rotate freely within its zone. The lower plane that defines this zone acts as a simulated datum A-B.

The second tolerance t_f controls the remainder of the surfaces (surface #1 in this case). It establishes a tolerance zone, two common parallel planes that are equally offset from the simulated datum A-B, which is t_s units wide. The surface 1is free to translate and rotate within the limits of this zone.

Both these zones float within the larger zone established by the size tolerance t_p. Also, the smaller zones need to remain parallel to the simulated datum.

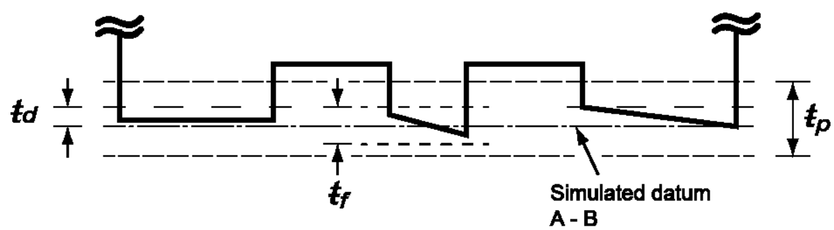


Figure 6.2: Some possible variations within the tolerance zones are shown. Note that the tolerance zone established by t_f is equally disposed about the simulated datum A-B.

6.3.1 Construction of Tolerance Map:

The specific case of the leftmost tab from Fig. 6.1 (similar to the example in the Standard [1]) and the surface #1 is repeated here for a detailed explanation of the construction of T-Map.

Figure 6.3: (a) The left most tab from Fig. 6.1 is shown. (b) The surface #1 and some of its possible variations are shown.

The limits of the possible variations need to be calculated. The translational limit in the *s*-direction is:

$$s_{max} = \frac{t_p}{2} \tag{6-1}$$

The rotational limit in the *p'* and *q'* is equal to

$$p'_{max} = q'_{max} = \frac{t_f}{2\bar{a}} \tag{6-2}$$

So that the T-Map may be used for metric computations, the units along all axes should be the same, i.e., a length [L]. For that reason, the scale assigned to the axis for angle is multiplied by the characteristic length $\bar{a}$. Hence the equation (6-2) is reduced to:

$$p'_{max} = q'_{max} = \frac{t_f}{2} \tag{6-3}$$

The resulting 3D T-Map is shown in Fig 6.4. It is truncated along axes *p'* and *q'*.

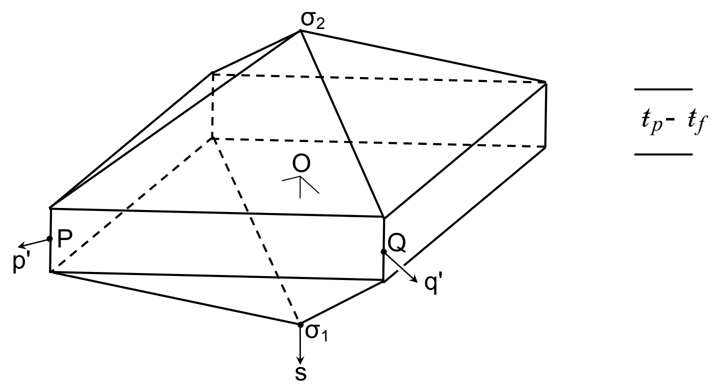


Figure 6.4: The 3D T-Map for the surface #1 shown in Fig. 6.1. The dimension for *OQ'* and *OP'* is t_f. Taken from [17].

CHAPTER 7

CONCLUSION

7.1 Summary

This thesis expands the domain of Tolerance Maps into patterns of profiles. T-Maps have been constructed for both a linear pattern and two-dimensional patterns of square bosses have been addressed. Since a profile tolerance may be used to control interrupted surfaces, T-Maps for this tolerance specification have been generated. Also, the least squares approach to tolerance verification was employed to verify tolerance specification for a pattern of profiles.

Some of the key accomplishments from this research are:

- Tolerance maps were constructed for patterns of line profile tolerances using square bosses in patterns as primary examples. These square bosses are short enough to use a line profile tolerance to control form, orientation, position and size both for each profile and for an entire pattern.
- The Standard [1] describes two ways of controlling a pattern. One is to use a composite frame and the other is to use multiple single segment frames. Two methods for specifying tolerance with composite frames are allowed by the Standard [1]. These are a single datum repeated in the lower frame and two datums repeated in the lower frame. The interpretation of these methods of tolerancing was presented and the Tolerance Maps for each of the above mentioned cases were constructed (Chapters 3 and 4).

- The Standard [1] suggests a way to control the coplanarity of two surfaces. A slight variation to this method of controlling the coplanarity has been suggested in this thesis. Interpretation and the procedure to construct the respective Tolerance Maps were presented (Chapter 6).
- A method which employs a least squares [27] to fit both an individual profile and an entire pattern to measured points. The variations in these profiles are used to determine whether the manufactured part is acceptable or not (Chapter 5).

7.2 Future Work:

Although the Tolerance Maps were constructed for patterns of profiles, it has not been automated for more general shapes. The next step would be to use the code that generates T-Maps for any line profile [26] and build on it to generate T-Maps for any patterns of line profiles. The results from this thesis can serve as a theoretical foundation for automating the whole process.

The least squares method to verify the tolerances has not been implemented in code, although it is theoretically sound. If a code can be developed using the method suggested and run on a few test cases, the method can be validated and used widely.

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