

Learning-by-doing and the Incidence of the Green Consumption Subsidy

by

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## ABSTRACT

This paper presents a two period general equilibrium model that incorporates firms learning-by-doing under the green subsidies. I use a dynamic version of Dixit-Stiglitz monopolistic competition model to analyze the impact of an introduction of green subsidies in the presence of pre-existing effluent taxes. I first show that introduction of green subsidies promotes the demands of green goods, and consumers are better off each period. I then show that even when the green subsidies directly accrue to consumers, firms in the green sector also benefit via boosted demands of the green goods. Learning-by-doing effect accelerates the speed of expansion of the green sector in the face of green subsidies. On the other hand, even when the demand for the green goods is promoted, and more pollution may involve meeting the increased demand as a whole, environmental quality may still improve if the technology is good enough to sufficiently boost the net positive impact of green consumption on the environment.

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## Chapter 1

### INTRODUCTION

From water pollution to global warming, increasing evidence supports the claim that unless we change our consumption patterns, the world will eventually face unacceptable levels of environmental crisis. In response to an urgent need to tailor patterns of economic growth, many environmental technologies have been developed and adopted for various types of goods. The field of green technology encompasses a wide range of evolving subfields, from energy generation to the creation of recycled or renewable products. The effort to find innovative solutions that meet environmental requirements leads to more effective products. Over the last decade, the federal government has provided massive subsidies to support various emerging activities that push the world toward less dangerous climate change. To improve environmental quality by changing our consumption patterns, the government has an interest in promoting faster adoption of environmentally conscious products by creating economic incentives that ultimately help the environment. Many empirical reports demonstrate that demand-side subsidies encourage the consumption of the targeted good. Green subsidies seem to support targeted industries and restructure the market by letting the consumers choose green goods. Though it sounds plausible, this argument deserves scrutiny. When new technology and new product emerge, I check to see whether the introduction of such a subsidy helps better establish the market by boosting the firm's profit and supporting its effort to push a learning-by-doing effect via promoted demand of goods by individuals.

When the government generates bias in favor of certain goods, incidence analysis

becomes important, because special interest groups often hinder the creation or elimination of such a policy. Since fiscal year 2009, nearly \$54 billion has been granted as demand or supply side energy-related tax subsidies according to the Institute for Energy Research (IER)<sup>1</sup>. There is much debate surrounding the elimination of such subsidies. Moreover, one of the basic issues concerning such an environmental policy is whether it will yield the intended improvement. Green tax credits are known to consumers when they make consumption decisions. Some argue that the reduction of the price of targeted green goods through consumer subsidies only benefits consumers, thereby diverting resources from more efficient uses. When the statutory tax benefit accrues to purchasers of qualifying green goods by way of lump-sum rebate, it is tempting to say that all tax benefits go to the consumers.

To seek true tax credit incidence, I develop a two-period general equilibrium dynamic incidence analysis model that incorporates the supplier's learning-by-doing or technology innovation under the green subsidy. For the simplicity of analysis, I introduce two sectors of production, namely varieties of conventional consumption goods and innovative green consumption goods, such as hybrid cars, energy-efficient kitchen appliances, non-toxic cleaner goods and so on with their conventional counterparts. Because innovative technology is still developing, the cost of production of these goods decreases as more experience is obtained. The growth of technology is modeled by a function of the accumulated outputs that have been sold, capturing the idea of learning-by-doing.

As new initiatives are continuously developed and compete with each other in the green sector, large heterogeneity is introduced via a Dixit and Stiglitz (1977) monop-

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<sup>1</sup><http://data.instituteforenergyresearch.org/tax-subsidies/renewables/>

olistic competition model. Equipped with their own unique features in productivity, firms producing varieties of innovative goods compete with each other's market powers. In this model, given the initial productivity parameter, each firm producing a variety chooses how much to produce today, knowing that it will affect the next period's productivity. The green sector is assumed to face a higher growth rate in technology reflecting its undeniable growth potential. That is, green technology is assumed to evolve faster. It is motivated by the observation that green investment grows fast<sup>2</sup>. In recent years, investor have been supporting innovative research and advanced technologies in much greater size and proportion, which has become a robust source of growth in green sectors. Higher level of accumulated outputs sold can be paired with better ability in learning-by-doing, or a greater chance of attracting such investment.

The firm producing green goods does not necessarily use clean technology. Because pollution is an inevitable byproduct of production, the government levies an effluent tax to limit the amount of carbon that can be emitted from plants. By imposing such taxes, the government urges plants to look into solutions that make production cleaner or more efficient. As long as they are coupled with charges that accurately reflect the environmental damage of each type of goods, the effluent tax provides correct incentive to react. Green subsidies may impose an extra burden on public budgets, however, having prices that reflect true costs or benefits under consideration of the environmental quality would send correct signal to households and firms to use the targeted good more efficiently.

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<sup>2</sup>“Green tech investments growing fast,” The San Francisco Chronicle, January 17, 2008, Green Transition Inflection Point: Green Transition Scoreboard 2013 Report

My model is a dynamic variant of the Dixit and Stiglitz (1977) monopolistic competition model in which a firm's cumulative output of goods determines its constant marginal cost. I study such a model to show that even when a subsidy directly accrues to consumers, a firm also benefits if it is a dynamic entity. The consequence of accumulated output becomes important for a firm's strategy. This is because it makes the firm face a dynamic problem, as firm's output decision in the first period affects the productivity parameter in the second period via cumulative output of goods. Another important feature of my model is that each firm's decision affects the environmental quality, and the government has interest to protect the environment. Even when the demand is boosted as a whole, if the consumption pattern is well tailored and green technology is good enough, the environmental quality may improve.

A firm's process of learning-by-doing follows the idea of Arrow's (1962) seminal paper, "The Economic Implications of Learning by Doing," studying the relationship between cost and cumulative output sold. Further works such as those by Rosen (1972), Spence (1981), and Clarke, Darrough, and Heinecke (1982) showed how learning by doing affects dynamic pricing strategy as the production costs decrease in their cumulative outputs. Fudenberg and Tirole (1982) and Dasgupta and Stiglitz (1988) showed the impact of learning by doing on the industry concentration and imperfect competition. It may have further implication on a firm's exit and entry decision, but in this paper, to focus on how firms behave in the face of particular consumer subsidies, the discussion begins when all firms have already made their decision on entry or exit. This formal exploration of the strategic implications of emerging technologies allows us to study the firms as dynamic entities. The manufacturing sector, especially the innovative industry, displays markedly large magnitudes of learning-

by-doing, economies of scale<sup>3</sup>, or diffusion of innovation. In this way, dynamic aspect is built on Dixit and Stiglitz (1977).

Incidence analysis of consumer subsidy has been carried out in several previous empirical works. One of the well-known examples of a government subsidy is the hybrid car subsidy which is provided based on estimated energy efficiencies. In the early part of the 2000s, the government offered a \$2,000 tax deduction for hybrids. The Energy Policy Act of 2005 established a more extensive tax credit program lasting five years that offered to make pricier, but more environmentally friendly vehicles more appealing. The rationale behind all these tax incentives was that many potential buyers required economic incentive to purchase more expensive hybrids over a conventional automobile that runs solely on gasoline. Sallee (2011) used transaction level data on new vehicle purchases and compared transaction prices just before and just after each tax change to show that the actual price consumers paid for Prius (excluding tax credit) moved very little, if at all. Given that Toyota faced a binding production constraint, Sallee (2011) concluded that consumers captured the significant majority of the benefits from tax subsidies for the Toyota Prius. On the other hand, Chupp, Myles and Stephenson (2010) showed that almost half of the hybrid car subsidy is capitalized into car prices using the transaction price. Subsidies on other goods and services, such as higher education (Singell and Stone 2005; Long 2004; Dynarski 2000; Grubb and Oyer 2007) or housing (Susin 2002; Gibbons and Manning 2006) have also been studied.

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<sup>3</sup>Refer to Levitt, List, & Syverson (2013) for learning-by-doing in automobile industry. Refer to Husan (1997), Huang (2002), and Truett and Truett (2003) for the importance of economy of scale in the automobile industry.



My paper contributes to the literature of subsidy incidence by complementing such works constructing a dynamic model of tax credit incidence that captures learning-by-doing across time. Because the incidence analysis in a static model does not consider the firm's dynamic optimization, the true benefit of the tax credit program for the suppliers may have been underestimated. Many emerging firms may have always been focused on the long horizon, determined to bring down their cost through learning-by-doing and economies of scale.

We have been giving attention to environmental issues for decades, and more and more environmentally friendly goods are on the way with developing technology. To address environmental protection, many policies have been proposed (for reviews of this literature, see Alm and Banzhaf 2010), and the promotion of the development and adoption of green technology is one example. As more and more states consider enacting tax credit policies to promote the demand for innovative environmentally conscious goods, it is important to know which parties truly benefit, and how the improvement of environmental quality is achieved. The trend toward green has been and will be persistently driven by an underlying commitment to sustainable development. I therefore believe that my conclusion can provide useful knowledge for legislative decisions to enhance environmental quality via promoted demand of environmentally-friendly products.

## Chapter 2

### TAX INCENTIVES FOR INNOVATIVE GOODS

Green subsidies apply to green counterparts of conventional goods: the Honda Civic hybrid versus a normal Civic, energy efficient kitchen appliances versus conventional appliances, and solar heating systems versus usual heating systems are examples. The government wants to promote green consumption by giving green subsidies on green purchase.

In this section we take a brief view of the tax incentives for hybrid cars as an example. Petroleum consumption in the private transportation sector accounts for 40 percent of gasoline consumption, 60 to 70 percent of total urban air pollution, and 20 percent of the annual U.S. carbon dioxide emissions, according to a report by U.S. Environmental Protection Agency. Dependency of the United States on foreign oil keeps increasing, and in recent years almost 60 percent of total petroleum products have been imported. In the presence the continuous rise of gasoline prices, unstable oil supplies, and growing concern over the increased influence of pollution including global warming, the environment-friendly and fuel-efficient hybrid technology is considered a desirable alternative to conventional technologies. To encourage the purchase of hybrid or alternative fuel vehicles, at the federal level, the Energy Policy Act of 2005 introduced a substantial personal income tax credit for hybrids<sup>1</sup>, while at the state level, thirteen states have passed tax incentives for hybrids, and many others have considered similar actions.

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<sup>1</sup><http://www.irs.gov/Businesses/Corporations/Alternative-Motor-Vehicle-Credit>

From 2003 to 2005, original buyers of hybrid cars were eligible for a \$2,000 income tax deduction, regardless of itemization. The reduction amount depends on his or her marginal tax rate, and relatively small. IRS data indicates that tax liability was reduced by \$300 for those in the 15 percent rate bracket, while those in the 25 percent, 28 percent, and 33 percent tax brackets, received reductions of \$500, \$560, and \$667, respectively. When the Energy Policy Act of 2005 was passed in 2005, from January 1, 2006, the buyer of a qualified car could claim a more extensive tax credit, depending on fuel-efficiency, worth up to \$3,400<sup>2</sup>. For example, the values of these credit for Toyota Prius, Honda Civic Hybrid, and the Honda Accord Hybrid were \$3,150, \$2,100, and \$1,300, respectively. The credit is designed to phase out in steps when an automaker sold 60,000 hybrid vehicles, regardless of the models. Toyota reached the threshold in June, 2006, and Honda in August, 2007. When an automaker hits the bound, the credit stays the same in the corresponding and consecutive quarters, then falls by half for the next two quarters, and falls again by another half for another six months, and then expires completely. When Honda reached accumulated sales of 60,000 in August, 2007, the credit of \$1,300 for the Accord Hybrid decreased to \$650 in half a year, and then Honda stopped selling this model in April, 2008.

When this type of tax policy is enacted, benefits and costs accrue to individuals, namely, individual households, individual firms. Knowledge of how much each party gains can be very useful when the policy is implemented.

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<sup>2</sup><http://www.irs.gov/Businesses/Corporations/Qualified-Hybrid-Vehicles>

## Chapter 3

### THE MODEL

Now, turn to the theoretical framework for assessing the subsidy incidence for innovative green goods. To analyze true incidence of green subsidies, a general equilibrium model is useful to capture the interaction of buyers and sellers in markets. I consider a version of the two period Dixit-Stiglitz monopolistic competition model. Time is discrete and labelled  $t = 1, 2$ . A simple two-sector model is developed to investigate the interactions between tax subsidy and the potential benefits that firms in the innovative sector may face.

#### 3.1 Households

There is a continuum of identical households of measure one. The preferences of a representative consumer are given by a constant elasticity of substitution utility function over two commodities,

$$U = U(c_1, c_2) \tag{3.1}$$

$$= \left( c_1^\rho + c_2^\rho \right)^{\frac{1}{\rho}} \tag{3.2}$$

where  $c_1$  is a conventional consumption commodity, and  $c_2$  is an innovative consumption commodity. Commodities are imperfect substitutes, so  $\rho \in (0, 1)$ <sup>1</sup>. The elasticity of substitution is  $\sigma = \frac{1}{1-\rho} > 1$ . The households have access to technology that uses a composite of differentiated goods  $c_i(z)$ 's to produce each consumption

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<sup>1</sup>If  $\rho = 1$ , then  $c_i$ 's are perfect substitutes, while if  $\rho < 0$ , then they are complements (Brakman et al. 2001).

commodity  $c_i$ . Given the varieties of the conventional or innovative product, the representative household can transform infinitely many types of products into a desirable conventional or innovative composite commodity to use,

$$c_i = \left[ \int_{z \in \Omega_i} c_i(z)^\rho dz \right]^{\frac{1}{\rho}} \quad (3.3)$$

where the measure of the set  $\Omega_i$  represents the mass of available goods. Let  $\Omega_1 = [0, \alpha]$  and  $\Omega_2 = [\alpha, 1]$ , so that there is a unit mass of firms in this economy. The household is endowed with a fixed unit of time, which is normalized to one. The households supply labor to firms for the production of varieties. Since the household does not value leisure, labor is inelastically supplied. The households own the firms and face an intertemporal budget constraint as follows.

$$\int_{z \in \Omega_1} \hat{p}_1(z) c_1(z) dz + \int_{z \in \Omega_2} \hat{p}_2(z) c_2(z) dz \leq E \quad (3.4)$$

$$\hat{p}_i(z) = p_i(z) + \tau_i(z) \quad (3.5)$$

$$T^g = \int_z \gamma_z c_2(z) dz \quad (3.6)$$

where  $p_i(z)$  is the price of the differentiated good indexed by  $z$  in the sector  $i = 1, 2$  and  $\tau_i(z)$  is the effluent tax for the firm indexed by  $z$  in sector  $i$ ,  $E$  is the total disposable income given by  $E \equiv w + \Pi_1 + \Pi_2 + T + T^g - \tilde{T}$ ,  $w$  is the equilibrium wage rate,  $\Pi_i$  is the aggregated profit earned due to the ownership of the firms in each sector  $i$ , that is,  $\Pi_i = \int_{z \in \Omega_i} \pi_i(z) dz$  where  $\pi_i(z)$  is the profit of the firm indexed by  $z$ ,  $T$ ,  $T^g$ ,  $\tilde{T}$  and  $\gamma_z$  are government policies,  $T$  is lump-sum transfer,  $T^g$  is lump-sum green subsidy,  $\tilde{T}$  is the lump-sum tax, and  $\gamma_z$  is the green subsidy rate for the differentiated good indexed by  $z$  in the green sector 2. When consumers make their decisions over consumption, they know that the green subsidy,  $T^g$  is calculated by (5) given  $\{\gamma_z\}$  and is given back in the lump-sum manner. Consumption commodities,  $c_i$ 's, directly enter

the utility function. As was shown in Dixit and Stiglitz (1977), consumer behavior can be modeled with consumption commodities,  $c_i$ 's, with associated aggregate prices,  $p_i$ 's.

$$\hat{p}_1 c_1 + \hat{p}_2 c_2 \leq E \quad (3.7)$$

$$\begin{aligned} \hat{p}_1 &= \left[ \int_{z \in \Omega_1} \hat{p}_1(z)^{\frac{\rho}{\rho-1}} dz \right]^{\frac{\rho-1}{\rho}} \\ \hat{p}_2 &= \left[ \int_{z \in \Omega_2} \{\hat{p}_2(z) - \gamma_z\}^{\frac{\rho}{\rho-1}} dz \right]^{\frac{\rho-1}{\rho}} \end{aligned} \quad (3.8)$$

Using these aggregates, the two-stage budgeting procedure with homothetic utility function gives us the demands,  $c_i(z)$ 's, and the optimal expenditures,  $r_i(z) = p_i(z)c_i(z)$ ,

$$\begin{aligned} c_1(z) &= \left[ \frac{\hat{p}_1(z)}{\hat{p}_1} \right]^{-\sigma} c_1 \\ c_2(z) &= \left[ \frac{\hat{p}_2(z) - \gamma_z}{\hat{p}_2} \right]^{-\sigma} c_2 \end{aligned} \quad (3.9)$$

$$\begin{aligned} r_1(z) &= \left[ \frac{\hat{p}_1(z)}{\hat{p}_1} \right]^{1-\sigma} R_1 \\ r_2(z) &= \left[ \frac{\hat{p}_2(z) - \gamma_z}{\hat{p}_2} \right]^{1-\sigma} R_2 \end{aligned} \quad (3.10)$$

where the aggregate expenditure  $R_i$  is given by  $R_i \equiv \hat{p}_i c_i = \int_{z \in \Omega_i} r_i(z) dz$ . Note that in the absence of green subsidy, the inter-sector elasticity of substitution and intra-sector elasticity of substitution are assumed to be identical, as they are both consumption goods. In the presence of green subsidy, the demand for green goods becomes more elastic, and intra-sector elasticity is larger than inter-sector elasticity,

$$\epsilon_1^D = \frac{1}{1-\rho} = \sigma$$

$$\epsilon_2^D = \frac{1}{1 - \rho} \frac{\hat{p}_2(z)}{\hat{p}_2(z) - \gamma_z} = \sigma \frac{\hat{p}_2(z)}{\hat{p}_2(z) - \gamma_z} \equiv \tilde{\sigma} > \sigma$$

### 3.2 Producers

The production structure consists of two consumption goods sectors: conventional goods ( $\{c_1(z)\}$ ) and innovative goods ( $\{c_2(z)\}$ ). Each sector contains a continuum of firms, each of which chooses to produce a different variety,  $z$ , in the corresponding sector. The innovative sector is the green sector, which produces green counterparts to conventional consumption goods. Green goods are more eco-friendly, characterized by their positive (net) impact on environmental quality. For example, the Honda Civic hybrid is the green counterpart to the normal Civic. The Civic hybrid is classified as a green good because of its use of alternative energy, which has a relatively positive impact on the environment. As another example, an energy-efficient kitchen appliance is a green counterpart to a conventional kitchen appliance. A green appliance is characterized by its improved energy efficiency.

Production requires only labor input. Firm technology is represented by a cost function that exhibits a constant marginal cost, that is, labor demand,  $n_{i,t}(z)$ , is given by a linear function of output,  $y_{i,t}(z) = A(q_{i,t-1}(z))n_{i,t}(z)$ , where  $q_{i,t-1}$  is the accumulated output level up to time  $t - 1$ , evolving by  $q_{i,t}(z) = q_{i,t-1}(z) + y_{i,t}(z)$ , so it is a fixed number at  $t$ . In this two-period model, the initial productivity parameter,  $q_{i,0}$ , is given at the first period, and  $q_{i,1}$  is determined endogeneously according to the firm's optimal decision at  $t = 1$ . In this way, each firm indexed by  $z$  incurs production cost according to its productivity parameter,  $q_{i,t}(z)$ . The firm's productivity parameter is given by its accumulated number of units sold so far. Because firm's output decision in the first period affects the productivity parameter in the

second period, the firm faces a dynamic problem. A firm at  $t$  with productivity index  $q_{i,t-1}(z)$  has productivity of  $A(q_{i,t-1}(z)) = q_{i,t-1}(z)^{\psi_i}$  with  $\psi_2 > \psi_1$ , reflecting the undeniable growth potential of the green sector. Higher productivity is modeled by lower marginal cost, which captures the concept of learning-by-doing, or innovation. A firm that has sold more units demonstrates higher productivity based on its past experience, that is, productivity at the firm level evolves over time, depending on the level of production decision by the firm. At the beginning of period 1, a firm with a productivity parameter  $q_{i,0}$  makes its output decision  $y_{i,1}$ , knowing that it contributes the next period's productivity via the productivity parameter  $q_{i,1}$ .

The firms producing the varieties in each sector are monopolistically competitive, so demands are assumed to be known to the firms. With this evolution of firm productivity, the expected discounted present value of profits for the firm is given as follows. Taking  $w_t$ ,  $\tau_{i,t}(z)$ , demand, cost function,  $R$ , and the initial productivity parameter  $q_{i,0}$  as given, the firm producing the type  $z$  chooses  $p_{i,t}(z)$ ,  $y_{i,t}(z)$ ,  $n_{i,t}(z)$  to solve



$$\begin{aligned}
v_i(z) &= \max \left[ \pi_{i,1}(z) \right] + \frac{1}{R} \left[ \pi_{i,2}(z) \right] & (3.11) \\
s.t. \quad \pi_{i,t}(z) &= \{p_{i,t}(z)y_{i,t}(z) - w_t n_{i,t}\} \\
y_{i,t}(z) &= q_{i,t}(z)^{\psi_i} n_{i,t}(z) \quad (\text{cost function}) \\
q_{i,1}(z) &= q_{i,0}(z) + y_{i,1}(z) \\
y_{1,t}(z) &= \left[ \frac{\hat{p}_{1,t}}{\hat{p}_{1,t}(z)} \right]^{\frac{1}{1-\rho}} y_{1,t} \quad (\text{demand}) \\
y_{2,t}(z) &= \left[ \frac{\hat{p}_{1,t}}{\hat{p}_{1,t}(z) - \gamma_z} \right]^{\frac{1}{1-\rho}} y_{2,t} \\
\hat{p}_{i,t}(z) &= p_{i,t}(z) + \tau_{i,t}(z) \\
&\text{given } q_{i,0}.
\end{aligned}$$

where  $\tau_{i,t}(z)$  is the effluent tax rate for each firm, and  $R$  is the exogeneously given gross interest rate. It can be solved by backward induction from the terminal period.

The marginal firm is defined as the one that yields zero profit, that is,  $v_i(q_{i,0}) = 0$ . To focus the analysis on the effect of the green subsidy, all firms in each sector are assumed to make  $v_i \geq 0$ . That is, the firm with lowest productivity parameter in each sector is the marginal firm, and no firm enters or exit. It can be viewed that time in this economy starts after the firm's entry or exit decision. Some firms may make non-positive profit at  $t = 1$ , but overall profit must be  $v_i \geq 0$ .

### 3.3 Environment and Government Regulation

Environmental (air) quality is affected by the production of the economy. The production process contributes to air pollution. Denote  $\xi$  as the air quality.

$$\xi_t = H_t(y_{1,t}, y_{2,t}, c_{2,t}) \quad (3.12)$$

where  $y_{i,t}$  is a vector of  $y_{i,t}(z)$ 's, and  $c_{2,t}$  is a vector of  $c_{2,t}(z)$ 's.  $H_t$  is decreasing in  $y_{i,t}(z)$ 's, reflecting the fact that pollution is an inevitable byproduct of production, and increasing in  $c_{2,t}(z)$  for all  $z$ , since they are green goods. Assume that  $H_t(\cdot)$  is strictly convex in  $y_{i,t}$ 's. When each firm makes an optimal decision, it does not internalize its impact on the environmental quality.

Pollution is an inevitable byproduct generated in the process of production. The government cares about environment quality, and thus takes command and control environmental regulation, setting a Pigouvian effluent tax on the production side, and providing a consumption subsidy on the green demand side. We assume that the government knows the marginal impact of each firm's output on the environment. Environmental regulation by the government is given as follows. Based on its estimation of the environmental effect of the arguments, it sets per unit emission taxes  $\tau_{i,t}(z)$  by

$$\tau_{i,t}(z) = -\frac{\partial H_t}{\partial y_{i,t}(z)} \quad (3.13)$$

where not necessarily  $\tau_{1,t}(z) > \tau_{2,t}(z)$ , reflecting the fact that a green sector may not be a clean sector. Similarly, we also assume that the government knows the net marginal impact of each green good demand on the environment. For example, when the consumer chooses to purchase an energy efficient refrigerator instead of a conventional one, the government knows how much energy will be saved due to this decision. Based on its correct estimation of environmental effect of the green demand, it provides green subsidy  $c_{2,t}(z)$ 's by

$$\gamma_{z,t} = \frac{\partial H_t}{\partial c_{2,t}(z)}. \quad (3.14)$$

Following Harberger (1964), the government rebates all tax revenues to the representative agent in a lump-sum manner, hence:

$$T_t = \Sigma_i \int_z \tau_{i,t}(z) y_{i,t}(z)$$

$$T_t^g = \int_z \gamma_z y_{2,t}(z)$$

### 3.4 Equilibrium

Assume that we have an interior solution. In equilibrium, all markets are cleared at the equilibrium price. Hence, feasibility requires for the consumption goods:

$$c_{i,t}(z) = y_{i,t}(z) \tag{3.15}$$

The feasibility constraint on labor is given by

$$\int n_{1,t}(z) dz + \int n_{2,t}(z) dz = 1 \tag{3.16}$$

An equilibrium in this economy is a collection of sequences of aggregate prices and wages  $\{R, p_{1,t}, p_{2,t}, w_t\}$ , monopolistic competitors' prices  $\{p_{i,t}(z)\}$ , a collection of sequences of marginal firms' prices  $\{p_{i,t}^*(z)\}$ , a collection of sequences of aggregate quantities  $\{y_{i,t}, c_{i,t}, n_{i,t}\}$ , and quantities of monopolistically competitive goods  $\{c_{i,t}(z), n_{i,t}(z)\}$ , a collection of sequences of marginal firms' quantities  $\{q_{i,t}^*(z)\}$ , a collection of sequences of the profit of the firms  $\{\pi_{i,t}(z)\}$ , and a collection of government policies  $\{\tau_{i,t}(z), \gamma_z, T_t, T_t^g, \tilde{T}\}$  such that

1. Taking prices, wage rate, government policies as given,  $\{c_{i,t}(z)\}$  solves the household problem each period;

2. Taking prices, wage rate, demand, government polices as given,  $\{p_{i,t}(z)\}$ ,  $\{y_{i,t}(z)\}$ ,  $\{n_{i,t}(z)\}$  solve the firm's problem;

3. The marginal firm earns zero profit: marginal firm with  $q_{i,0}^*$  is such that  $v_{i,1}(q_{i,0}^*) = 0$ ;

4. The markets are cleared;

$$y_{i,t}(z) = c_{i,t}(z)$$
$$\sum_i \int_z n_{i,t}(z) dz = 1$$

5. The government's budget constraint holds each period.

$$T_t = \sum_i \int_z \tau_{i,t}(z) y_{i,t}(z)$$

$$T_t^g = \int_z \gamma_z y_{2,t}(z)$$

## Chapter 4

### THE INCIDENCE OF THE GREEN CONSUMPTION SUBSIDY

Green subsidies are known to consumers when they make decisions. The statutory tax benefit accrues to purchasers of qualifying innovative goods by way of a lump sum rebate. I analyze the reaction of each firm in the innovative sector in the presence of the green subsidy, and check how its profit varies accordingly. Because producers are assumed to focus solely on profit, the change in profit is the relevant monetary measure of the change in welfare. A bigger increase in profit represents a bigger benefit from a tax incentive on innovative goods.

In real-world examples of consumer subsidies, the government evaluates the positive impact of targeted activity on the environment and returns its estimated monetary contribution. Hybrid or pure electric car subsidies, home energy efficiency subsidies, and solar financing subsidies are examples. Regarding the tax credit for the hybrid car, Beresteanu and Li (2011) used the dataset of new vehicle registrations in 22 U.S. metropolitan statistical areas from 1999 to 2006 to find that the federal hybrid tax credit accounts for 27 percent of the diffusion of hybrid vehicles sales in 2006. Especially for the Toyota Prius, Sallee (2011) finds the surprising result that consumers are the only party that fully captures the hybrid tax incentive. Here, I show that some benefits of the subsidies also pass on to producers to the extent that productivity rises in response to subsidies. That is, the green good supplier may receive some positive amount of the tax preferences.

Before starting analysis, note that the consumer's problem is a standard concave

optimization program with a strictly concave and continuous objective function over a convex constraint. Therefore, we know that there exists a unique solution. The firm's problem is a bit tricky. Each firm's problem can be solved by backward induction, its objective function can be reduced to one-variable problem in the first period's price with given constraints. The second period's profit decreases as the first period's price increases (see Appendix). I find the first period's optimal price, which leads to some loss today, expecting further gain tomorrow.

Proposition 1 : Green subsidies promote the purchase of green goods. That is, the demands of green goods and expenditure on them increase, as do the demand for aggregate good, and its budget share.

Proof. First we solve the consumer's problem to derive demands function for green goods. The consumer's problem can be solved in two-steps. The Lagrangian  $L = U(c_1, c_2) + \lambda[E - \hat{p}_1 c_1 - \hat{p}_2 c_2]$  yields the first-order conditions:

$$\begin{aligned}\frac{\partial L}{\partial c_i} &= U_{c_i} - \lambda \hat{p}_i \\ \frac{\partial L}{\partial \lambda} &= \hat{p}_1 c_1 + \hat{p}_2 c_2 - w\end{aligned}$$

which implies that

$$\frac{c_1}{c_2} = \left( \frac{\hat{p}_1}{\hat{p}_2} \right)^{-\sigma}.$$

Denote  $s(c_2)$  is the budget share for  $c_2$ . Then, since  $U$  is homothetic, the commodity demands are:

$$\begin{aligned}c_1 &= (1 - s(c_2))w/\hat{p}_1 \\ c_2 &= s(c_2)w/\hat{p}_2.\end{aligned}$$

For the next step, from the Lagrangian  $L = \left[ \int_z c_2(z)^\rho \right]^{\frac{1}{\rho}} + \lambda [s(c_2)E - \int_z \{\hat{p}_2(z) - \gamma_z\} c_2(z)]$ , we obtain the first-order conditions:

$$\begin{aligned} \frac{\partial L}{\partial c_2(z)} &= c_2(z)^{\rho-1} c_2^{\frac{1}{\rho}} - \lambda \{\hat{p}_2(z) - \gamma_z\} = 0 \\ \frac{\partial L}{\partial \lambda} &= \int_z \{\hat{p}_2(z) - \gamma_z\} c_2(z) - s(c_2)E = 0 \end{aligned}$$

which implies that

$$c_2(z) = \left[ \frac{\hat{p}_2}{\hat{p}_2(z) - \gamma_z} \right]^{\frac{1}{1-\rho}} c_2$$

where the price index  $\hat{p}_2$  is given by  $\hat{p}_2 = [\int_z \hat{p}(z)^{\frac{\rho}{\rho-1}}]^{\frac{\rho-1}{\rho}}$ . From (3.8)  $\partial \left( c_{2,t}(z) = \left[ \frac{\hat{p}_{2,t}(z)}{\hat{p}_{2,t}} \right]^{-\sigma} c_{2,t} \right) / \partial p_{2,t}(z) = -\sigma \left[ \frac{\hat{p}_{2,t}(z)}{\hat{p}_{2,t}} \right]^{-\sigma-1} c_{2,t} < 0$ , we know that  $c_{2,t}(z)$  increases for all  $z$ , and so does  $c_{2,t}$ . From (3.9),  $\partial \left( r_{2,t}(z) = \left[ \frac{\hat{p}_{2,t}(z)}{\hat{p}_{2,t}} \right]^{1-\sigma} R_{2,t} \right) / \partial p_{2,t}(z) = (1-\sigma) \left[ \frac{\hat{p}_{2,t}(z)}{\hat{p}_{2,t}} \right]^{-\sigma} R_{2,t} < 0$ , since  $\sigma > 1$ , we know that  $r_{2,t}(z)$  increases, and so do  $R_{2,t} = \hat{p}_{2,t} c_{2,t} = s(c_{2,t})E$  and  $s(c_{2,t})$ . Apparently, as green subsidy increases, disposable income  $E$  increases, and the consumers get better off at each period.  $\square$

**Proposition 2 :** The green sector benefits under green subsidies, which accrue directly to consumers. The learning-by-doing effect amplifies this benefit.

**Proof.** Solving by backward induction, in the terminal period, each firm in sector 1 chooses the profit maximizing markup equals to  $\sigma/(\sigma-1) = 1/\rho$ , and those in the sector 2 choose  $\tilde{\sigma}/(\tilde{\sigma}-1) = 1/\tilde{\rho}$ . The optimal pricing rules in the second period become:

$$\begin{aligned}
\hat{p}_{1,2}(z) &= \frac{w_2}{\rho} \left( \frac{1}{q_{1,t-1}(z)} \right)^{\psi_1} \\
\hat{p}_{2,2}(z) - \gamma_z &= \frac{w_2}{\tilde{\rho}} \left( \frac{1}{q_{2,t-1}(z)} \right)^{\psi_2}
\end{aligned} \tag{4.1}$$

where  $\frac{1}{\tilde{\rho}} \equiv \sigma \left( \frac{\hat{p}_{2,2}(z)}{\hat{p}_{2,2}(z) - \gamma_z} \right) / \left[ \sigma \left( \frac{\hat{p}_{2,2}(z)}{\hat{p}_{2,2}(z) - \gamma_z} \right) - 1 \right] > \frac{1}{\rho}$  with  $\tilde{\rho} < \rho$ ,  $w_t$  is wage rate hereafter normalized to one, so  $R$  is expressed in terms of labor unit. In the presence of  $\gamma_z > 0$ , the effective markup gets higher. This is because the green subsidy makes the demand more elastic. Given this optimal pricing rule, optimal output level is given by (3.9) and (3.15), and the labor demand is given by cost function. Note that all  $z, z'$  with  $z \neq z'$ ,  $q_{i,t-1}(z) = q_{i,t-1}(z')$  solve the same problem. Rewriting (4.1) yields,

$$\begin{aligned}
\hat{p}_{1,2}(q_{i,t-1}) &= \frac{w_2}{\rho} \left( \frac{1}{q_{1,t-1}} \right)^{\psi_1} \\
\hat{p}_{2,2}(q_{i,t-1}) - \gamma_z &= \frac{w_2}{\tilde{\rho}} \left( \frac{1}{q_{2,t-1}} \right)^{\psi_2}
\end{aligned} \tag{4.2}$$

Using this pricing rule, denoting  $v_i = v_{i,1} + \frac{1}{R}v_{i,2}$ , firm profits in the second period are then (see Appendix for derivations)

$$\begin{aligned}
v_{i,2}(q_{i,1}) &= r_{i,2}(q_{i,1}) - n_{i,2}(q_{i,1}) \\
v_{1,2}(q_{1,1}) &= \frac{1 + \sigma}{\sigma} r_{1,2}(q_{1,1}) - \frac{\tau_{1,2}(z)}{q_{1,1}(z)^{\psi_1}} \\
v_{2,2}(q_{2,1}) &= \frac{1 + \tilde{\sigma}}{\tilde{\sigma}} r_{2,2}(q_{2,1}) - \frac{\tau_{2,2}(z)}{q_{2,1}(z)^{\psi_1}}
\end{aligned} \tag{4.3}$$

where  $\frac{1+\tilde{\sigma}}{\tilde{\sigma}} > \frac{1+\sigma}{\sigma}$  and the revenues are given by



$$\begin{aligned}
r_{1,2}(q_{1,1}) &= R_{1,2} \left( \frac{\hat{p}_{1,2} \rho q_{1,1}^{\psi_1}}{1 + \tau_{1,2}(q_{1,1}) \rho q_{1,1}^{\psi_1}} \right)^{\sigma-1} \\
r_{2,2}(q_{2,1}) &= R_{2,2} \left( \frac{\hat{p}_{2,2} \tilde{\rho} q_{2,1}^{\psi_1}}{1 + \tau_{2,2}(q_{2,1}) \tilde{\rho} q_{2,1}^{\psi_1}} \right)^{\sigma-1}
\end{aligned} \tag{4.4}$$

so that the profits expressed in terms of aggregate price index and aggregate revenue are

$$\begin{aligned}
v_{1,2}(q_{1,1}) &= \frac{1 + \tilde{\sigma}}{\tilde{\sigma}} R_{1,2} \left( \frac{\hat{p}_{1,2} \rho q_{1,1}^{\psi_1}}{1 + \tau_{1,2}(q_{1,1}) \rho q_{1,1}^{\psi_1}} \right)^{\sigma-1} - \frac{\tau_{1,2}(z)}{q_{1,1}(z)^{\psi_1}} \\
v_{2,2}(q_{2,1}) &= \frac{1 + \tilde{\sigma}}{\tilde{\sigma}} R_{2,2} \left( \frac{\hat{p}_{2,2} \tilde{\rho} q_{2,1}^{\psi_1}}{1 + \tau_{2,2}(q_{2,1}) \tilde{\rho} q_{2,1}^{\psi_1}} \right)^{\sigma-1} - \frac{\tau_{2,2}(z)}{q_{2,1}(z)^{\psi_1}}
\end{aligned} \tag{4.5}$$

From (3.9), (4.2),

$$\frac{y_{i,2}(q_{i,1})}{y_{i,2}(q'_{i,1})} = \left( \frac{q_{i,1}}{q'_{i,1}} \right)^{\sigma+\psi_i}; \quad \frac{r_{i,2}(q_{i,1})}{r_{i,2}(q'_{i,1})} = \left( \frac{q_{i,1}}{q'_{i,1}} \right)^{\sigma-1+\psi_i} \tag{4.6}$$

Therefore, in the second period, a more productive firm yields larger output and revenue, charges lower prices, and earns higher profit. If the green sector had not admitted the learning-by-doing effect and its productivity had stayed unchanged at its initial level, then it would have earned lower profit. Since  $\partial v_{i,2}(z)/\partial p_{i,1}(z) < 0$  (see Appendix), if the firm sets the first period price according to its markup as in the terminal period, it would not be optimal. The firm would rather choose lower price in the first period to maximize the discounted sum of the profits according to  $\partial v_i/\partial p_{i,1}(z) = \partial v_{i,1}(z)/\partial p_{i,1}(z) + \partial v_{i,2}(z)/\partial p_{i,1}(z) = 0$ . Since  $\partial^2 v_{i,2}(z)/\partial p_{i,1}(z) \partial \gamma_z < 0$ , in the presence of green subsidy, we know that the firm's discounted present value  $v_i(z)$  satisfies  $\partial v_i/\partial \gamma_z = (\partial v_{i,1}(z)/\partial p_{i,1}(z))(\partial p_{i,1}(z)/\partial \gamma_z) + (\partial v_{i,2}(z)/\partial p_{i,2}(z))(\partial p_{i,2}(z)/\partial \gamma_z) > 0$ .

Therefore, the firms in the green sector benefit in the presence of green subsidies.  $\square$

Note that in the presence of pre-existing emission taxes, introduction of a green subsidy may improve air quality if the technology is sufficiently green. Since the firm does not take into account the impact of its decision on environmental quality, it generates negative externality in production, namely pollution, while the demand on a green good similarly generates positive externality. Even when the demand for the green goods is promoted, and more pollution may involve meeting the increased demand as a whole, environmental quality may still improve if the technology is good enough to sufficiently boost the net positive impact of green consumption on the environment. For example, when consumers purchase electric vehicles instead of conventional cars, the amount of contributing factors to pollution will decrease, which is described by the net positive impact of the electric vehicles consumption. In turn, air quality will improve, which will decrease the effluent tax rate. By substitution effect, it may increase the total number of cars in the economy, and increase the emission in the production process accordingly. If the positive impact of green consumption, that is, the reduction in the greenhouse gas intensity of green goods that occurs is large enough to offset the negative impact from production, the changed allocation among conventional and innovative green goods will improve the environmental quality.

CONCLUDING REMARK

This paper presents a two-period general equilibrium model that incorporates the firm's learning-by-doing under the green subsidies. I use a dynamic version of the Dixit-Stiglitz monopolistic competition model to analyze the impact of the introduction of green subsidies in the presence of pre-existing effluent taxes. I first show that the introduction of green subsidies promotes the demand for green goods, and consumers are better off each period. I then show that even when the green subsidies directly accrue to consumers, firms in the green sector also benefit via boosted demand for green goods. The learning-by-doing effect accelerates the speed of expansion of the green sector in the face of green subsidies. On the other hand, even when the demand for the green goods increases, and greater pollution may result from meeting the increased demand as a whole, environmental quality may still improve if the technology is good enough to sufficiently boost the net positive impact of green consumption on the environment.

People seem to agree that tax credits on innovative environment-friendly goods works well for environmental problems, and also make it easier for people to adopt clean and efficient green technologies. In spite of efforts to spare the environment, some people raise concern about this type of promotion, especially when it seems to impose an extra burden on public budgets. That is, the double dividend issue arises. For example, when comes to the renewable energy, because it causes gasoline tax revenue to drop, which are used to pay for local highway and bridge maintenance.

Almost 40 percent of state revenue for highways comes from fuel taxes revenue<sup>1</sup>. Three states enacted additional fees on the cars and at least five others states have considered similar actions to make up for losses in tax revenues, under the assumption that people who use roads must pay for them. Washington state introduced a \$100 annual fee for electric-car owners,; Virginia imposed \$64 for hybrid and electric cars, and; New Jersey considers \$50 for electric and compressed natural-gas cars<sup>2</sup>.

Regarding the decreased amount of tax revenue, we may take the view that if the government were to transfer the exact amount it would need to abate the increased amount of pollution emitted from conventional vehicles in the absence of the tax policy, the decreased tax revenue would be less of a concern. But, if the transfer exceeds that amount, then net tax revenue decreases, and may worsen the distortion effect of the pre-existing taxes. We leave this question for our future work. The focus of this paper remains the question of incidence—which parties truly benefit from hybrid tax incentives.

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<sup>1</sup>2012 report by the Denver-based National Conference of State Legislatures

<sup>2</sup><http://www.bloomberg.com/news/2013-06-04/losing-revenue-from-hybrids-prompts-states-to-hit-owners.html>

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APPENDIX A

APPENDIX

Derivation of (4.1):

$$\begin{aligned}
 v_{i,2}(z) &= \max\{p_{i,2}(z)y_{i,2}(z) - w_2 n_{i,t}(z)\} \\
 &= \max\{p_{i,2}(z)y_{i,2}(z) - w_2 [\{1/A(q_{i,1}(z))\}y_{i,2}(z)]\} \\
 [p_{i,2}(z)] &: y_{i,2}(z) + p_{i,2}(z) \frac{\partial y_{i,2}(z)}{\partial p_{i,2}(z)} - \{w_2/A(q_{i,1}(z))\} \frac{\partial y_{i,2}(z)}{\partial p_{i,2}(z)} = 0 \\
 &1 + \frac{\partial y_{i,2}(z)}{\partial \hat{p}_{i,2}(z)} \frac{\hat{p}_{i,2}(z) p_{i,2}(z)}{y_{i,2}(z) \hat{p}_{i,2}(z)} - \{w_2/A(q_{i,1}(z))\} \frac{\partial y_{i,2}(z)}{\partial \hat{p}_{i,2}(z)} \frac{\hat{p}_{i,2}(z)}{y_{i,2}(z) \hat{p}_{i,2}(z)} \frac{1}{\hat{p}_{i,2}(z)} = 0 \\
 [p_{1,2}(z)] &: 1 - \sigma = -\frac{w_2/A(q_{1,1}(z))}{p_{1,2}(z)} \sigma \\
 Markup_1 &= \frac{p_{1,2}(z)}{w_2/A(q_{1,1}(z))} = \frac{\sigma}{\sigma - 1} = \frac{1}{\rho} > 1 \\
 [p_{2,2}(z)] &: Markup_2 = \frac{\sigma \left( \frac{\hat{p}_{2,2}(z)}{\hat{p}_{2,2}(z) - \gamma_z} \right)}{\sigma \left( \frac{\hat{p}_{2,2}(z)}{\hat{p}_{2,2}(z) - \gamma_z} \right) - 1} = \frac{1}{\tilde{\rho}} > \frac{1}{\rho}
 \end{aligned}$$

Derivation of (4.3):

$$\begin{aligned}
 y_{1,2}(q_{1,1}) &= \left( \frac{\hat{p}_{1,2}(z)}{\hat{p}_{1,2}} \right)^{-\sigma} c_{1,2} = \left( \frac{r_{1,2}(z)}{R_{1,2}} \right) \left( \frac{\hat{p}_{1,2}(z)}{\hat{p}_{1,2}} \right)^{-1} c_{1,2} \\
 &= r_{1,2}(z) \{ \rho q_{1,1}(z)^{\psi_1} + \tau_{1,2}(z) \} \\
 \{1/A(q_{1,1}(z))\} y_{1,2}(z) &= r_{1,2}(z) \rho + \tau_{1,t}(z) / q_{1,1}(z)^{\psi_1} \\
 v_{1,2}(q_{1,1}) &= \frac{1 + \sigma}{\sigma} r_{1,2}(q_{1,1}) - \frac{\tau_{1,2}(z)}{q_{1,1}(z)^{\psi_1}}
 \end{aligned}$$

Similarly,

$$\begin{aligned}
 y_{2,2}(q_{2,1}) &= \left( \frac{\hat{p}_{2,2}(z) - \gamma_z}{\hat{p}_{2,2}} \right)^{-\sigma} c_{2,2} = \left( \frac{r_{2,2}(z)}{R_{2,2}} \right) \left( \frac{\hat{p}_{2,2}(z) - \gamma_z}{\hat{p}_{2,2}} \right)^{-1} c_{2,2} \\
 &= r_{2,2}(z) \{ \tilde{\rho} q_{2,1}(z)^{\psi_2} + \tau_{2,t}(z) \} \\
 \{1/A(q_{2,1}(z))\} y_{2,2}(z) &= r_{2,2}(z) \tilde{\rho} + \tau_{2,t}(z) / q_{2,1}(z)^{\psi_2} \\
 v_{2,2}(q_{2,1}) &= \frac{1 + \tilde{\sigma}}{\tilde{\sigma}} r_{2,2}(q_{2,1}) - \frac{\tau_{2,2}(z)}{q_{2,1}(z)^{\psi_1}}
 \end{aligned}$$

Note.  $1 - \rho = \frac{1}{\sigma}$ ;  $\tilde{\rho} < \rho$ ;  $\tilde{\sigma} < \sigma$ , so  $\frac{1+\tilde{\sigma}}{\tilde{\sigma}} > \frac{1+\sigma}{\sigma}$ .

Derivation of (4.4):

$$\begin{aligned} r_{1,2}(q_{1,1}) &= R_{1,2} \left( \frac{\hat{p}_{1,2}(q_{1,1})}{\hat{p}_{1,2}} \right)^{1-\sigma} = R_{1,2} \left( \frac{1/A(q_{1,1})}{\rho \hat{p}_{1,2}} + \frac{\tau_{1,2}(q_{1,1})}{\hat{p}_{1,2}} \right)^{1-\sigma} \\ &= R_{1,2} \left( \frac{\hat{p}_{1,2}}{1/\rho q_{1,1}^{\psi_1} + \tau_{1,2}(q_{1,1})} \right)^{\sigma-1} = R_{1,2} \left( \frac{\hat{p}_{1,2} \rho q_{1,1}^{\psi_1}}{1 + \tau_{1,2}(q_{1,1}) \rho q_{1,1}^{\psi_1}} \right)^{\sigma-1} \end{aligned}$$

Similarly,

$$\begin{aligned} r_{2,2}(q_{2,1}) &= R_{2,2} \left( \frac{\hat{p}_{2,2}(q_{2,1}) - \gamma_z}{\hat{p}_{2,2}} \right)^{1-\sigma} = R_{2,2} \left( \frac{1/A(q_{2,1})}{\tilde{\rho} \hat{p}_{2,2}} + \frac{\tau_{2,2}(q_{2,1})}{\hat{p}_{2,2}} \right)^{1-\sigma} \\ &= R_{2,2} \left( \frac{\hat{p}_{2,2} \tilde{\rho} q_{2,1}^{\psi_1}}{1 + \tau_{2,2}(q_{2,1}) \tilde{\rho} q_{2,1}^{\psi_1}} \right)^{\sigma-1} \end{aligned}$$

Proposition 2:

$$\begin{aligned} v_{1,2}(z) &= \frac{1}{\rho} [w_2/A(q_{1,1}(z))] y_{1,2}^*(z) - w_2 [1/A(q_{1,1}(z))] y_{1,2}^*(z) \\ &= \left[ \frac{1}{\rho} - 1 \right] \{1/A(q_{1,1}(z))\} y_{1,2}^*(z) \quad (\because w_2 = 1) \\ &= \left[ \frac{1-\rho}{\rho} \right] \{1/A(q_{1,1}(z))\} \left[ \frac{\rho \hat{p}_{1,2}}{1/A(q_{1,1}(z))} \right]^{\frac{1}{1-\rho}} y_{1,2} \\ &= \left[ \frac{1-\rho}{\rho} \right] (\rho \hat{p}_{1,2})^{\frac{1}{1-\rho}} \{1/A(q_{1,1}(z))\}^{-\frac{\rho}{1-\rho}} y_{1,2} \\ &= \left[ \frac{1-\rho}{\rho} \right] (\rho \hat{p}_{1,2})^{\frac{1}{1-\rho}} q_{1,1}(z)^{\frac{\rho \psi_1}{1-\rho}} y_{1,2} \\ &= \left[ \frac{1-\rho}{\rho} \right] (\rho \hat{p}_{1,2})^{\frac{1}{1-\rho}} \left\{ q_{1,0}(z) + \left[ \frac{\hat{p}_{1,1}}{\hat{p}_{1,1}(z)} \right]^{\frac{1}{1-\rho}} y_{1,2} \right\}^{\frac{\rho \psi_1}{1-\rho}} y_{1,2} \\ \frac{\partial v_{1,2}(z)}{\partial p_{1,1}(z)} &= \left[ \frac{1-\rho}{\rho} \right] (\rho \hat{p}_{1,2})^{\frac{1}{1-\rho}} \hat{p}_{1,1}^{\frac{\rho \psi_1}{(1-\rho)^2}} \left( -\frac{\rho \psi_1}{(1-\rho)^2} \right) \hat{p}_{1,1}(z)^{\frac{\rho \psi_1}{(1-\rho)^2}-1} y_{1,2}^{\frac{\rho \psi_1}{(1-\rho)^2}} < 0 \end{aligned}$$



$$\begin{aligned}
v_{2,2}(z) &= \left( \frac{1}{\tilde{\rho}} [w_2/A(q_{2,1}(z))] + \gamma_z \right) y_{2,2}^*(z) - w_2[A(q_{2,1}(z))] y_{2,2}^*(z) \\
&= \left[ \left( \frac{1-\tilde{\rho}}{\tilde{\rho}} \right) / A(q_{2,1}(z)) + \gamma_z \right] \left[ \frac{\hat{p}_{2,2}}{\hat{p}_{2,2}(z) - \gamma_z} \right]^{\frac{1}{1-\rho}} y_{2,2} \\
&= \left[ \frac{1-\tilde{\rho}}{\tilde{\rho}} \right] \{ 1/A(q_{2,1}(z)) \} \left( \frac{\tilde{\rho}\hat{p}_{2,2}}{1/A(q_{2,1}(z))} \right)^{\frac{1}{1-\rho}} y_{2,2} + \gamma_z \left( \frac{\tilde{\rho}\hat{p}_{2,2}}{1/A(q_{2,1}(z))} \right)^{\frac{1}{1-\rho}} y_{2,2} \\
&= \left[ \frac{1-\tilde{\rho}}{\tilde{\rho}} \right] \left( \tilde{\rho}\hat{p}_{2,2} \right)^{\frac{1}{1-\rho}} q_{2,1}^{\frac{\rho\psi_2}{1-\rho}} y_{2,2} + \gamma_z \left( \tilde{\rho}\hat{p}_{2,2} \right)^{\frac{1}{1-\rho}} q_{2,1}(z)^{\frac{\psi_2}{1-\rho}} y_{2,2} \\
&= \left( \tilde{\rho}\hat{p}_{2,2} \right)^{\frac{1}{1-\rho}} y_{2,2} \left( \left[ \frac{1-\tilde{\rho}}{\tilde{\rho}} \right] q_{2,1}^{\frac{\rho\psi_2}{1-\rho}} + \gamma_z q_{2,1}(z)^{\frac{\psi_2}{1-\rho}} \right) \\
&= \left( \tilde{\rho}\hat{p}_{2,2} \right)^{\frac{1}{1-\rho}} y_{2,2} \left( \left[ \frac{1-\tilde{\rho}}{\tilde{\rho}} \right] \left\{ q_{2,0}(z) + \left[ \frac{\hat{p}_{2,1}}{\hat{p}_{2,1}(z) - \gamma_z} \right]^{\frac{1}{1-\rho}} y_{2,2} \right\}^{\frac{\rho\psi_2}{1-\rho}} \right. \\
&\quad \left. + \gamma_z \left\{ q_{2,0}(z) + \left[ \frac{\hat{p}_{2,1}}{\hat{p}_{2,1}(z) - \gamma_z} \right]^{\frac{1}{1-\rho}} y_{2,2} \right\}^{\frac{\psi_2}{1-\rho}} \right) \}
\end{aligned}$$

$$\begin{aligned}
\frac{\partial v_{2,2}(z)}{\partial p_{2,1}(z)} &= \left( \tilde{\rho}\hat{p}_{2,2} \right)^{\frac{1}{1-\rho}} y_{2,2} \left( \left[ \frac{1-\tilde{\rho}}{\tilde{\rho}} \right] \left\{ \hat{p}_{2,1}^{\frac{\rho\psi_2}{(1-\rho)^2}} \left( -\frac{1}{1-\rho} \frac{\rho\psi_2}{1-\rho} \right) \right. \right. \\
&\quad \times \left[ \hat{p}_{2,1}(z) - \gamma_z \right]^{\frac{-\rho\psi_2}{(1-\rho)^2}} \frac{\rho\psi_2}{y_{2,2}^{\frac{1-\rho}{1-\rho}}} \\
&\quad \left. \left. + \gamma_z \hat{p}_{2,1}^{\frac{\psi_2}{(1-\rho)^2}} \left( -\frac{1}{1-\rho} \frac{\psi_2}{1-\rho} \right) \left[ \hat{p}_{2,1}(z) - \gamma_z \right]^{\frac{-\psi_2}{(1-\rho)^2}} \frac{\psi_2}{y_{2,2}^{\frac{1-\rho}{1-\rho}}} \right\} \right) < 0
\end{aligned}$$

$$\begin{aligned}
\frac{\partial v_{2,2}^2(z)}{\partial p_{2,1}(z) \partial \gamma_z} &= \left( \tilde{\rho}\hat{p}_{2,2} \right)^{\frac{1}{1-\rho}} y_{2,2} \left( \left[ \frac{1-\tilde{\rho}}{\tilde{\rho}} \right] \left\{ \hat{p}_{2,1}^{\frac{\rho\psi_2}{(1-\rho)^2}} \left( -\frac{1}{1-\rho} \frac{\rho\psi_2}{1-\rho} \right) \right. \right. \\
&\quad \times \frac{\rho\psi_2}{(1-\rho)^2} \left[ \hat{p}_{2,1}(z) - \gamma_z \right]^{\frac{-\rho\psi_2}{(1-\rho)^2} - 1} \frac{\rho\psi_2}{y_{2,2}^{\frac{1-\rho}{1-\rho}}} \\
&\quad + \hat{p}_{2,1}^{\frac{\psi_2}{(1-\rho)^2}} \left( -\frac{1}{1-\rho} \frac{\psi_2}{1-\rho} \right) \left[ \hat{p}_{2,1}(z) - \gamma_z \right]^{\frac{-\psi_2}{(1-\rho)^2}} \frac{\psi_2}{y_{2,2}^{\frac{1-\rho}{1-\rho}}} \\
&\quad \left. \left. + \gamma_z \hat{p}_{2,1}^{\frac{\psi_2}{(1-\rho)^2}} \left( -\frac{1}{1-\rho} \frac{\psi_2}{1-\rho} \right) \frac{\psi_2}{(1-\rho)^2} \left[ \hat{p}_{2,1}(z) - \gamma_z \right]^{\frac{-\psi_2}{(1-\rho)^2} - 1} \frac{\psi_2}{y_{2,2}^{\frac{1-\rho}{1-\rho}}} \right\} \right) < 0
\end{aligned}$$