# Migration and Graduation Measures 

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#### Abstract

The current debate over graduate rate calculations and results has glossed over the relationship between student migration and the accuracy of various graduation rates proposed over the past five years. Three general grade-based graduation rates have been proposed recently, and each has a parallel version that includes an adjustment for migration, whether international, internal to the U.S., or between different school sectors. All of the adjustment factors have a similar form, allowing simulation of estimates from real data, assuming different unmeasured net migration rates. In addition, a new age-based graduation rate, based on mathematical demography, allows the simulation of estimates on a parallel basis using data from Virginia's public schools. Both the direct analysis and simulation demonstrate that graduation rates can only be useful with accurate information about student migration. A discussion of Florida's experiences with longitudinal cohort graduation rates highlights some of the difficulties with the current status of the oldest state databases and the need for both technical confidence and definitional clarity. Meeting the No Child Left Behind mandates for school-level graduation rates requires confirmation of transfers and an audit of any state system for accuracy, and basing graduation rates on age would be a significant improvement over rates calculated using gradebased data.


## Migration and Graduation Measures

There is no agreement on either how to measure graduation from high school or what is the general level of graduation from public high schools. While the Census Bureau and others estimate graduation in the United States as a whole is over $80 \%$ (e.g., Mishel \& Roy, 2006), some researchers estimate that fewer than $75 \%$ of teenagers graduate from high school (e.g., Education Week, 2006; Green \& Winters, 2005, 2006; Seastrom, Hoffman, Chapman, \& Stillwell, 2006). While some of the differences focus on the definition of graduation (do alternative credentials such as the General Educational Development certificate, or GED, count?), most of the debate has focused on technical issues of measurement. Mishel and Roy (2006) emphasize two: the fragility of administrative records and the conflation of ninth-grade enrollment with first-time ninth-grade enrollment. For them, using more rigorous survey methods, such as the Current Population Survey (CPS) or the National Educational Longitudinal Survey (NELS), is more likely to result in accurate national estimates of graduation. In addition, when graduation measures use ninth-grade enrollment, they note, the patterns of ninth-grade retention complicate the measures. Respondents to Mishel and Roy have focused on the selfreported nature of CPS education data and on the limited survey nature of both the CPS and NELS (Greene, Winters, \& Swanson, 2006).

While surveys are limited by their design, administrative record-keeping problems can bias measures. When Education Week (2006) released its estimates of graduation, based on Swanson's (2004) Cumulative Progression Index (CPI), it highlighted Detroit's graduation rate for 2003 as the worst in major cities: $22 \%$. However, the enrollment data that Michigan reported
to the Common Core of Data (CCD) for Detroit in 2002-03 is inconsistent with the preceding and following year, creating an artificial bulge that resulted in an underestimate of graduation for the 2003 CPI measure (see Table 1). Using the data reported to CCD, the CPI plummeted from $74.2 \%$ in 2002 to $21.7 \%$ in 2003. Even using corrected enrollment data released by the Michigan Department of Education (2005), the CPI dropped from $62.2 \%$ to $28.2 \%$. Such a change is an artifact of corrupt data-the CCD is an unaudited database, where states voluntarily provide data. In this case, Michigan began using a new data-collection system in 2002-03, a change that may have been involved in the data anomalies (P. Bielawski, personal correspondence). The transient (and surely incorrect) bulge in enrollment reported for fall 2002 is also analytically equivalent to migration into and out of the school district, artificially inflating and then deflating the CPI (which does not correct for migration). While the primary problem with the Education Week report is the uncritical use of the CCD, it also points to the sensitivity of the measure with regard to changing migration.

Table 1
Enrollment counts reported for grades 9-12, Detroit City Schools, fall 2001-fall 2003

| Grade | 2001 | 2002 | 2003 |
| :--- | ---: | ---: | ---: |
| 9 | 14,494 | 20,025 | 17,837 |
| 10 | 9,291 | 11,275 | 9,899 |
| 11 | 6,355 | 7,795 | 7,421 |
| 12 | 4,618 | 6,020 | 5,244 |

Source: Common Core of Data (http://nces.ed.gov/ccd)
Graduation estimates matter for policy in several ways. First, they shape the general debate over high school reform. Statistics used in public debate imply normative judgments (Starr, 1987). In the past year, would-be high school reformers have framed their calls for change by drawing from the lower range of graduation estimates and discussing the need for a response to the crisis (e.g., National Governors Association, 2005a, 2005b), and other school critics have recently pointed to what they call a graduation rate crisis (Orfield, 2004; Orfield, Losen, Wald, \& Swanson, 2004). In doing so, these reform advocates have begun a new cycle of rhetoric about dropping out, a pattern with a decades-long history (Author). (See Harvey \& Housman, 2004, for a skeptical look at the framing of high school reform in a crisis context.)

In addition to shaping policy debates, graduation measures are directly tied to individual schools' judgments of making Adequate Yearly Progress (AYP). Each state must choose a graduation measure that it applies to individual schools and a standard of progress required to meet for AYP. The first round of definitions by the states were generally created to maximize the measure, leading to criticism by education reform organizations (Hall, 2005) and a proposed common standard by the National Governors Association (NGA) (2005c). The proposed measure by the NGA is longitudinal in nature: Tracking students within a state will allow the calculation of a true cohort measure of graduation, or so the proposal claims.

However, neither the methodological discussion nor the NGA proposal adequately addresses the issue of migration. The purpose of this paper is to analyze the relationship between migration and various measures of graduation. For several reasons, the analysis will emphasize subnational estimates. Because of the ties to AYP, the viability of subnational estimates is an important policy question. Furthermore, subnational estimates involve an additional level of migration (internal migration), which national estimates need not consider. Finally, the simulations in this paper rely on data from a state (Virginia) that has published enrollment both by age and by grade, disaggregation that is not available from the national Common Core of Data.

In this paper, the term migration refers to three different ways in which students move into, out of, and between schools. International migration brings students into schools from outside the United States (and, less often, removes students from schools as they leave the U.S.). As Warren (2005) and Mishel and Roy (2006) point out, estimates of state and national graduation adjusted by the Census Bureau single-year estimates of population are likely to underestimate graduation from U.S. schools because some proportion of international teenage migrants enter the U.S. but never enroll in school. In addition to international migration, internal migration moves students between schools as they move between states, districts, and neighborhoods. A growing number of states (such as Florida, Texas, Michigan, and Virginia) have individual-student tracking databases, allowing them to keep track of inter-school and interdistrict moves, if not interstate moves-assuming that the unique statewide identifier is handled appropriately. Finally, inter-sector migration represents the flows of students among public schools, private schools, and homeschooling. While states with student-level databases can
theoretically track such migration, these records are typically unconfirmed by any follow-up procedure. While these sources of migration have different causes and implications for individual families, students, and schools, they represent the same analytical problem: How does the presence of unmeasured migration affect measures of graduation?

This paper begins with grade-based measures using administrative record sources that have been proposed by other researchers, presents a new measure with roots in mathematical demography, evaluates the sensitivity of all of these measures to migration, and discusses the National Governors Association (2005c) compact on longitudinal measures.

## Grade-Based Measures

The new graduation measures proposed over the past six years generally try to approximate the common-sense notion of a graduation rate (the proportion of teens who graduate) by using some variant of a ratio of the number of graduates to a pool of potential graduates. (See Hauser, 1997, for a description of appropriate characteristics of graduation measures, and Seastrom, Chapman, Stillwell, et al. 2006 for a descriptive analysis of some of the measures discussed below.) With the exception of Swanson's (2004) formula, which is the basis for the recent Education Week (2006) report, the differences revolve around calculating the pool of potential graduates and the nature of any adjustment for migration and mortality. After describing the different formulas, this section will describe a simulation of how three key rates vary with different migration assumptions.

## Simple Ratios

Haney et al. (2004) described a straight ratio of diplomas to a prior eighth- or ninth-grade enrollment, which will be referred to as the Boston College ratios (or BCR) as follows:

$$
\begin{equation*}
\mathrm{BCR}_{9}=\frac{\mathrm{D}_{\mathrm{A}}^{\mathrm{t}}}{\mathrm{~N}_{\mathrm{G} 9}^{\mathrm{t} 9}} \tag{1}
\end{equation*}
$$

where $D_{A}^{t}$ is the number of academic diplomas awarded in academic year $t$ and $N_{G 9}^{t-3}$ is the ninthgrade enrollment in the fall of academic year $t-3$, and

$$
\begin{equation*}
\mathrm{BCR}_{8}=\frac{\mathrm{D}_{\mathrm{A}}^{\mathrm{t}}}{\mathrm{~N}_{\mathrm{G} 8}^{\mathrm{t}}} \tag{2}
\end{equation*}
$$

As Mishel and Roy (2006) note, ninth-grade retention biases all quasi-cohort measures that have ninth grade as a base, including $\mathrm{BCR}_{9}$. In contrast, $\mathrm{BCR}_{8}$ would be more accurate in systems with low eighth-grade retention. As with all quasi-cohort measures, $\mathrm{BCR}_{8}$ and $\mathrm{BCR}_{9}$ assume that the number of diplomas in any academic year is identical to the number of diplomas earned by students in a true cohort. Because some proportion of students earn a diploma later than their cohort, there will be minor fluctuations in diplomas attributable to late degree-earning. This distortion becomes more pronounced if the numerator includes special-education certificates (because some students in special education remain in school until 22). $\mathrm{BCR}_{9}$ and $\mathrm{BCR}_{8}$ include no adjustment for migration or mortality, but Warren's (2005) formula is related to $\mathrm{BCR}_{8}$.

## Adjusted Simple Ratio

Warren (2005) proposed using a variant of $\mathrm{BCR}_{8}$ with an adjustment for migration and mortality. This adjusted Boston College ratio (or $\mathrm{ABCR}_{8}$ ) is calculated as follows:

$$
\begin{equation*}
\mathrm{ABCR}_{8}=\mathrm{BCR}_{8} \cdot \mathrm{~W} \tag{3}
\end{equation*}
$$

where

$$
\begin{equation*}
\mathrm{W}=\frac{\mathrm{N}_{13}^{\mathrm{t}-4}}{\mathrm{~N}_{17}^{\mathrm{t}}} \tag{4}
\end{equation*}
$$

and $\mathrm{N}_{\mathrm{x}}^{\mathrm{t}}$ is the population with last birthday x at the beginning of academic year t . (Readers of Warren's work, as well as that of Greene and Winter $(2002,2005,2006)$, will notice that the adjustment form stated here is a multiplier that is the inverse of what Warren describes. This variant form is used later when comparing adjustment factors.) Warren uses a three-year average of Census Bureau estimates for each state for this adjustment ratio, using the July 1 state population estimate as a substitute for the population at the beginning of the following academic year. Theoretically, an accurate population estimate would incorporate both migration and mortality. Warren's adjustment assumes that the general population ratio is applicable to publicschool enrollment, and there are two threats to that assumption. One is the differential in migration and mortality between public-school and private-school students. The second is the existence of international migration of teens who never enroll in school. In states with a relatively high unenrolled teen immigrant population (such as California), the Census Bureau estimates will deflate the adjustment factor and the overall measure, as Warren (2005) notes.

## Smoothed Ratio

Seastrom, Hoffman, et al. (2006) use a formula that is a variant of Greene and Winter's (2002) earlier, unadjusted measure of graduation, with a smoothing to estimate first-time ninthgrade enrollment. The U.S. Department of Education refers to this as the Averaged Freshman Graduation Rate (or AFGR):

$$
\begin{equation*}
\mathrm{AFGR}=\frac{\mathrm{D}_{\mathrm{A}}^{\mathrm{t}}}{\hat{\mathrm{~N}}_{\mathrm{G} 9}^{\mathrm{t}-3}} \tag{5}
\end{equation*}
$$

where

$$
\begin{equation*}
\hat{\mathrm{N}}_{\mathrm{G} 9}^{\mathrm{t}-3}=\frac{\mathrm{N}_{\mathrm{G} 8}^{\mathrm{t}-4}+\mathrm{N}_{\mathrm{G} 9}^{\mathrm{t}-3}+\mathrm{N}_{\mathrm{G} 10}^{\mathrm{t}-2}}{3} \tag{6}
\end{equation*}
$$

This smoothing attempts to estimate first-time ninth-grade enrollment by averaging the quasicohort enrollment over three grades (and years). There is neither an explicit model nor empirical evidence to justify the use of this average as an estimate of first-time ninth-grade enrollment, though Seastrom, Chapman, et al. (2006) claim that AFGR compares well to a true cohort rates.

## Adjusted Smoothed Ratio

Greene and Winters $(2005,2006)$ make a migration and mortality adjustment to AFGR, which will be referred to here as the adjusted smoothed graduation rate, or ASGR:

$$
\begin{equation*}
\mathrm{ASGR}=\mathrm{AFGR} \cdot \mathrm{G} \tag{7}
\end{equation*}
$$

where

$$
\begin{equation*}
\mathrm{G}=\frac{\mathrm{N}_{14}^{\mathrm{t}-3}}{\mathrm{~N}_{17}^{\mathrm{t}}} \tag{8}
\end{equation*}
$$

One should note that W and G (from equations 4 and 8 ) differ by the number of years of migration and mortality adjusted for (four versus three). In addition, Warren uses an averaged Census Bureau estimate, but Greene and Winters $(2005,2006)$ do not describe any smoothing factors for the migration adjustment. For simulation purposes later, however, we may ignore the smoothing of data sources for population changes.

## Quasi-Period Promotion Index

Swanson (2004) proposed a cumulative promotion index (CPI), a measure that relies on data from adjoining years. This formula is qualitatively different from quasi-cohort rates. As with period calculations of life expectancy and other demographic measures, the concept of the CPI is to capture what might happen with a synthetic ninth-grade cohort if it experienced the conditions documented in a single academic year rather than a cohort. Each factor attempts to model the attrition experienced between two grades in a single year, and between twelfth grade and graduation:

$$
\begin{equation*}
\mathrm{CPI}=\left(\frac{\mathrm{D}_{\mathrm{A}}^{\mathrm{t}}}{\mathrm{~N}_{\mathrm{G} 12}^{\mathrm{t}}}\right) \cdot\left(\frac{\mathrm{N}_{\mathrm{G} 12}^{\mathrm{t}+1}}{\mathrm{~N}_{\mathrm{G} 11}^{\mathrm{t}}}\right) \cdot\left(\frac{\mathrm{N}_{\mathrm{G} 11}^{\mathrm{t} 11}}{\mathrm{~N}_{\mathrm{G} 10}^{\mathrm{t}}}\right) \cdot\left(\frac{\mathrm{N}_{\mathrm{G} 10}^{\mathrm{t}+1}}{\mathrm{~N}_{\mathrm{G} 9}^{\mathrm{t}}}\right) \tag{9}
\end{equation*}
$$

In essence, each factor is a quasi-cohort attrition (or persistence) measure, chained together to create the quasi-period measure. As with the quasi-cohort measures, Swanson's CPI is biased by retention. There is no adjustment for migration and mortality, but one can create one for the purposes here, an adjusted CPI or ACPI. We might imagine some K reflecting the total increment in the population such that

$$
\begin{equation*}
\mathrm{ACPI}=\mathrm{CPI} \cdot \mathrm{~K} \tag{10}
\end{equation*}
$$

where the factor K incorporates the population changes across $3^{2} / 3$ years, between the fall of a synthetic cohort's ninth grade and the synthetic cohort's on-time graduation. The next section begins with the exact form of K and its comparison to W and G .

## Analyzing and Simulating Migration Effects

## on Grade-Based Measures

The simulation of migration effects on grade-based measures will use those measures constructed specifically to adjust for migration: $\mathrm{ABCR}_{8}, \mathrm{ASGR}$, and ACPI. Because the model adjustments have identical, two-parameter equivalent forms, they can be discussed and analyzed together.

## Equivalence of Adjustment Forms

For this section, we define the instantaneous net-increment rate of a student population, $\mathrm{ti}(\mathrm{x})$, as the difference between the net in-migration flow $\mathrm{i}(\mathrm{x})$ and the force of mortality $\mathrm{m}(\mathrm{x})$ at age x . For any cohort, the change in population $\mathrm{N}(\mathrm{x})$ between ages $a$ and $b$ is a function of $\mathrm{i}(\mathrm{x})$ and $\mathrm{m}(\mathrm{x})$-or $\mathrm{ti}(\mathrm{x})$, as indicated below.

$$
\begin{equation*}
\frac{N(b)}{N(a)}=e^{\int_{a}^{b}(i(x)-m(x)) d x}=e^{\int_{a}^{b} i(x) d x}=e^{\bar{\pi} \cdot T} \tag{11}
\end{equation*}
$$

where $\bar{t}$ is the average net-increment change rate for the cohort and T is the interval between $a$ and b. In each cohort measure, the adjustment reverses the distortion in the end population "at risk" of graduating, a distortion created by mortality and net migration. Thus, W and G are both of the form $\frac{N(a)}{N(b)}$, and the generic adjustment is of the form $e^{-\bar{t} \cdot T}$, where $\mathrm{T}=3$ for G and $\mathrm{T}=4$ for W .

For ACPI, because each factor is a quasi-cohort measure, we can calculate the synthetic cohort adjustment in the same manner, with $\mathrm{T}=32 / 3$ for K . The relative steepness of the adjustment slopes thus depend on the age interval T in question. Greene and Winters $(2005,2006)$ define the
relative period of risk for cohort population change as three years, Swanson (2004) effectively assumes $3^{2} / 3$ years, and Warren (2005) 4 years (because the base is eighth grade). While they use whole-year data because of their sources (annual state population estimates by single years of age), Greene and Winters and Warren could assume a period of risk eight months longer than their respective definitions assume. (One can transform population data into average rates of change by the inverse of equation 11 and then extend the interval to a non-integer length.)

## Simulation: Virginia Public Schools, 2003

One can simulate the effects of migration by first calculating the unadjusted estimates $\mathrm{BCR}_{8}, \mathrm{AFGR}$, and CPI, and then calculating the adjustments for their respective adjusted rates for a range of $\overline{t i}$. Using published (online) enrollment data from the Virginia Department of Education (n.d.) for the school years 1998-99 through 2003-04 and only academic (standard and advanced-studies) diplomas, the unadjusted rates for 2003 are $80.1 \%\left(\mathrm{BCR}_{8}\right), 76.2 \%$ (AFGR), and $70.6 \%$ (CPI). Table 2 shows the estimates of $\mathrm{ABCR}_{8}, \mathrm{ASGR}$, and ACPI when simulating a mean net-increment rate $(\overline{t i})$ from 0 to 0.05 , and Figure 1 shows the estimates when simulating $\overline{t i}$ from -0.15 to 0.15 . For a large, growing state such as Virginia, $\bar{t}$ is likely to range only between 0 and 0.03 , but even in this narrow range, each graduation estimate still varies by more than $7 \%$, and the whole set of estimates range from $63.2 \%$ to $80.1 \%$, or an almost $17 \%$ difference between the lowest to the highest estimate for plausible values of $\overline{t i}$. Smaller jurisdictions are likely to have a wider range of net migration rates, and Figure 2 demonstrates the corresponding uncertainty in graduation estimates. Figure 2 also shows the intersection of each graduation rate method with the upper limit (100\%) for certain (and unrealistic) negative values of net-increment rates. In real
populations with substantial outmigration, the unadjusted values of each estimate will be lower, allowing for a range of graduation estimates below $100 \%$ for plausible values of net-increment rates.

Table 2
Simulated estimates of graduation rates for Virginia public schools, 2003, $\bar{t}=0$ to 0.05 .

| $\overline{t i}$ | ABCR $_{8}$ | ASGR | ACPI |
| :--- | :---: | :---: | :---: |
| 0 | $80.1 \%$ | $76.2 \%$ | $70.6 \%$ |
| 0.01 | $76.9 \%$ | $74.0 \%$ | $68.1 \%$ |
| 0.02 | $73.9 \%$ | $71.8 \%$ | $65.6 \%$ |
| 0.03 | $71.0 \%$ | $69.7 \%$ | $63.2 \%$ |
| 0.04 | $68.2 \%$ | $67.6 \%$ | $61.0 \%$ |
| 0.05 | $65.5 \%$ | $65.6 \%$ | $58.8 \%$ |

Source: Virginia Department of Education (n.d.).


Figure 1. Simulated estimates of graduation rates for Virginia public schools, 2003, $\bar{t} \bar{i}=-0.15$ to 0.15. Given the data published by the Virginia Department of Education (n.d.), the estimated rates drop as the net-increment rate increases (correcting for the increase in the diplomas corresponding to
net increments). For any jurisdiction, the sloping profiles for $\mathrm{ABCR}_{8}, \mathrm{ASGR}$, and ACPI would be comparable to those in Figure 1, changing by proportion only by adjusting the y-intercept.

## Mathematical Demography and Graduation Measures

This section presents a new method of measuring graduation, one based on mathematical demography and using age-specific rather than grade-specific data. As with the CPI, this measure focuses on a hypothetical cohort, in a fashion comparable to other demographic summary measures such as period life expectancy and period total fertility rate. The advantage of basing graduation rates on age is the elimination of bias from grade retention and other conflations of grade level with student cohort. Birth cohorts age together.

## Stationary Populations

To reiterate from the grade-based measure discussion, the common-sense construct of a graduation rate would be the probability of graduating over one's school career. As described earlier, the administrative definitions of dropout and graduation rates have complicated the task of estimating this proportion. But analyzing graduation and attrition is clearer when compared to other population processes. If one looks at graduation as a way of leaving the non-graduate population, for example, then graduation becomes one of several ways of leaving the population (along with death or migration). Demographic techniques then become tools for analyzing educational experiences. Consider first the simple case of a stationary population closed to migration, with no migration, no population growth, and no changes in the underlying forces of mortality or graduation. Then graduation is one of two paths out of the non-graduation
population. In these simplified circumstances, the proportion of the non-graduate population that eventually leaves through graduation is the number of diplomas earned in a year divided by the number of births, or

$$
\begin{equation*}
\frac{I_{g}}{I_{0}}=\frac{D_{g}}{B} \tag{12}
\end{equation*}
$$

where $l_{g} / l_{0}$ is the proportion of the population that graduates, $\mathrm{D}_{\mathrm{g}}$ is the number of graduates, and $B$ is the number of births in the population. ${ }^{1}$ If real populations met these conditions, then estimating the graduation (and dropout) rate would be simple.

## Mathematical Models of Nonstable Populations

But real populations complicate the estimation of graduation. The first complication is population growth: The number of children does not remain constant across cohorts. Thus, crosssectional information about graduation conflates the underlying forces of mortality and graduation with changes in the size of cohorts. A second complication is change in the underlying population characteristics: Mortality and graduation do not remain constant. Crosssectional information not only conflates current conditions with population growth, but the sizes of different cohorts at each age also reflect historical population characteristics from the birth of that cohort through its age at the time of data collection. Standard demographic and survival analysis addresses these problems by finding data to calculate event-exposure rates (how many deaths per person years lived between the 20th and 25th birthdays, for a population, or

[^0]individual-level data from a survey indicating movement from being a non-graduate to a graduate), and from these rates one can create an equivalent synthetic population from the characteristics of a real, observed population. Period life tables describe the characteristics of the synthetic equivalent population, and the period life expectancy at birth is such a synthetic measure that compresses mortality conditions into a summary measure. Period life expectancy at birth is not a prediction of how long babies born that year will live.

But such laborious data collection is not always necessary, especially in circumstances (such as schooling) where some of the data may be difficult to collect. Preston and Coale (1982) develop a population model that reflected changing growth rates by age and parceled out the historical changes in cohort size and mortality conditions from the equivalent synthetic population of an observed population in time. In a population closed to migration, they showed that

$$
\begin{equation*}
\frac{1_{p}}{1_{0}}=\frac{\int_{0}^{\infty} D_{p}(a) e^{\frac{\mathrm{r} r(x) d x}{a}} d a}{B} \tag{13}
\end{equation*}
$$

where $1_{p} l_{0}$ is the proportion of the equivalent synthetic population that would leave through cause $p$ (usually cause-specific mortality), $D_{p}(a)$ is the instantaneous function representing the number of decrements (deaths) at age a through cause $\mathrm{p}, B$ is the number of births, and $\mathrm{r}(\mathrm{x})$ is the instantaneous proportional growth rate for the population. In essence, $e^{\frac{\int_{0} r(x) d x}{i}}$ is a correction factor that adjusts the number of decrements (deaths, or graduations) to factor out the accumulated growth and changing population conditions over the life course up through age x . If a population is growing, for example, or the total forces of decrement in the population are
declining over time, then later cohorts are larger, $\mathrm{r}(\mathrm{x})$ is positive, and $e^{\int_{0}^{j r(x) d x}}$ expands the decrements at older ages to match those changing conditions. Preston (1987) demonstrated how to use the growth corrections to estimate correct mortality rates for cancer. In a population that is open to migration, equation 13 is easily adjusted:

$$
\begin{equation*}
\frac{1_{p}}{1_{0}}=\frac{\int_{0}^{\infty} D_{p}(a) e^{\int_{0}^{a}[r(x)-i(x)] d x} d a}{B} \tag{14}
\end{equation*}
$$

Where $\mathrm{i}(\mathrm{x})$ is the proportional net migration rate for the population. Equation 14 is tautological, true for all populations at all times, in all conditions.

## Application

Applying equation 14 requires transformation from the instantaneous form into an estimate using data gathered with discrete categories (for example, using age last birthday instead of exact age) and often grouped in multi-year age intervals:

$$
\begin{equation*}
\frac{l_{g}}{1_{0}} \cong \frac{\sum_{j} D_{g j} e^{\sum_{x=1}^{\mathrm{i}-1} t_{x}\left(r_{x}-i_{x}\right)+\frac{1}{2} t_{j}\left(r_{j}-i_{j}\right)}}{B} \tag{15}
\end{equation*}
$$

where $D_{g j}$ is the total number of graduations in the $j$ th age interval, $\left(\mathrm{r}_{x}-\mathrm{i}_{x}\right)$ capture the average growth and in-migration rates in the $x$ th age interval (or enrollment-exposure interval), and $\mathrm{t}_{x}$ is the width (in years) of the $x$ th age interval. Equation 15 estimates the growth correction for each successful age interval from birth through the middle of the interval in question. Thus, an adjustment for graduates in the 20-24-year-old age interval would use a correction factor estimated through age 22.5 (halfway through the 5-year interval). For purposes of estimating
graduation, one need not start with birth but any point below the common ages of graduation.
The example below uses 15 as a starting point.

Table 3. Period high school graduation probabilities by sector, United States, 2001.

\begin{tabular}{|c|c|c|c|c|c|c|c|c|c|c|c|c|c|c|c|c|}
\hline (1)
Age \& $$
\begin{gathered}
(2) \\
\mathrm{N}_{\mathrm{j}}^{2001}
\end{gathered}
$$ \& $$
\begin{gathered}
(3) \\
\mathrm{N}_{\mathrm{j}}^{2002}
\end{gathered}
$$ \& (4)

$\mathrm{r}_{\mathrm{j}}$ \& $(5)$
$\mathrm{i}_{\mathrm{j}}$ \& $(6)$
$\mathrm{r}_{\mathrm{j}}-\mathrm{i}_{\mathrm{j}}$ \& (7)
$\mathrm{M}_{\mathrm{j}}$ \& (8)
$\mathrm{e}^{\mathrm{M}_{\mathrm{j}}}$ \& (9)
$\mathrm{D}_{\text {GED }}$ \& $(10)$

$D_{\text {GED }} \cdot \mathrm{e}^{\mathrm{M}_{j}}$ \& $$
\begin{gathered}
(11) \\
\mathrm{N}_{\mathrm{G} 12}^{2001}
\end{gathered}
$$ \& (12)

$D_{\text {Pub }}$ \& (13)

$\mathrm{D}_{\text {Pub }}{ }^{\prime}$ \& $(14)$
$\mathrm{D}_{\text {Pub }}{ }^{\prime} \cdot \mathrm{e}^{\mathrm{M}_{j}}$ \& (15)

$\mathrm{D}_{\text {Pri }}$ \& (16)

$D_{\text {Pri }}$ \& (17)

$$
\mathrm{D}_{\text {Pri }} \cdot \mathrm{e}^{\mathrm{m}_{j}}
$$ <br>

\hline 14 \& 4046 \& \& \& \& \& \& \& \& \& \& \& \& \& \& \& <br>
\hline 15 \& 3984 \& 4033 \& 0.012 \& 0.006 \& 0.007 \& 0.003 \& 1.003 \& \& \& 44 \& 29 \& 10 \& 10 \& 3 \& 1 \& 1 <br>
\hline 16 \& 3942 \& 4036 \& 0.024 \& 0.006 \& 0.018 \& 0.015 \& 1.016 \& \& \& 249 \& 165 \& 74 \& 76 \& 18 \& 8 \& 8 <br>
\hline 17 \& 3807 \& 3798 \& -0.002 \& 0.006 \& -0.008 \& 0.020 \& 1.021 \& \& \& 2,424 \& 1606 \& 645 \& 659 \& 174 \& 70 \& 71 <br>
\hline 18 \& 1659 \& 1609 \& -0.031 \& 0.008 \& -0.039 \& -0.003 \& 0.997 \& 264 \& 263 \& 968 \& 642 \& 1285 \& 1281 \& 70 \& 139 \& 139 <br>
\hline 19 \& 769 \& 701 \& -0.093 \& 0.008 \& -0.101 \& -0.073 \& 0.930 \& \& \& 172 \& 114 \& 466 \& 433 \& 12 \& 51 \& 47 <br>
\hline 20 \& 605 \& 612 \& 0.012 \& 0.013 \& -0.002 \& -0.124 \& 0.883 \& \& \& 59 \& 39 \& 89 \& 79 \& 4 \& 10 \& 9 <br>
\hline 21 \& 576 \& 526 \& -0.091 \& 0.013 \& -0.104 \& -0.177 \& 0.838 \& \& \& 44 \& 29 \& 36 \& 30 \& 3 \& 4 \& 3 <br>
\hline 22 \& 566 \& 555 \& -0.020 \& 0.013 \& -0.033 \& -0.245 \& 0.782 \& 171 \& 134 \& 16 \& 11 \& 23 \& 18 \& 1 \& 2 \& 2 <br>
\hline 23 \& 437 \& 511 \& 0.156 \& 0.013 \& 0.143 \& -0.190 \& 0.827 \& \& \& \& \& \& \& \& \& <br>
\hline 24 \& 516 \& 536 \& 0.038 \& 0.013 \& 0.025 \& -0.106 \& 0.899 \& \& \& \& \& \& \& \& \& <br>
\hline 25-29 \& 2409 \& 2296 \& -0.048 \& 0.013 \& -0.061 \& -0.124 \& 0.883 \& 73 \& 64 \& \& \& \& \& \& \& <br>
\hline 30-34 \& 2338 \& 2492 \& 0.064 \& 0.009 \& 0.055 \& -0.127 \& 0.881 \& 50 \& 44 \& \& \& \& \& \& \& <br>
\hline 35-39 \& 2606 \& 2580 \& -0.010 \& 0.007 \& -0.017 \& -0.108 \& 0.897 \& 90 \& 81 \& \& \& \& \& \& \& <br>
\hline Total \& \& \& \& \& \& \& \& 647 \& 586 \& \& 2635 \& \& 2584 \& \& 286 \& 281 <br>
\hline \multicolumn{8}{|l|}{Proportion earning the degree} \& 16.0\% \& 14.5\% \& \& 65.2\% \& \& 64.0\% \& \& 7.1\% \& 6.9\% <br>
\hline
\end{tabular}

$r_{j}$ is calculated as the natural $\log$ of the ratio of the second year's enrollment to the first.
$M_{j}=\sum_{x=1}^{i-1} t_{x}\left(r_{x}-i_{x}\right)+\frac{1}{2} t_{j}\left(r_{j}-i_{j}\right)$.
$\mathrm{D}_{\text {GED }}$ represents the GED certificates (in thousands).
$D_{\text {Pub represents standard academic public-school diplomas (in thousands). }}$
$\mathrm{D}_{\text {Pri }}$ represents private-school diplomas (in thousands).
Estimate of inter-survey $15^{\text {th }}$ birthdays: $\left(\mathrm{N}_{14}^{2001}+\mathrm{N}_{15}^{2002}\right) / 2=4040$.
See text for source information.

Table 3 applies equation 15 to three sources of graduation for the United States population for 2001-02: GED alternative credentials, public-school diplomas, and private-school diplomas. Columns (1) through (8) illustrate the calculation of the correction factors for each age, from the enrollments in fall 2001 and 2002 (columns 2 and 3, enrollment in thousands from the Current Population Reports; U.S. Census Bureau, n.d. a), the age-specific growth rates (column 4), the immigration rates (column 5, from the Current Population Reports; calculated from U.S. Census Bureau, n.d. b), the age-specific $\mathrm{r}_{j}-\mathrm{i}_{j}$ (column 6), the cumulative growth to the middle of each age interval (column 7, where $M_{j}=\sum_{x=1}^{i-1} t_{x}\left(r_{x}-i_{x}\right)+\frac{1}{2} t_{j}\left(r_{\mathrm{r}}-i_{j}\right)$ ), and the correction factor (column 8). Columns (9) and (10) finish the calculations for lifetime GED receipt and columns (11) through (14) and (15) through (17) show the estimates of public-school and private-school graduation in the equivalent synthetic population, respectively.

The U.S. Department of Education (2004) provides data for GED and regular high school diploma recipients. GED recipients (in thousands in column 9 ) are grouped in age intervals of below 20 years, 20-24, 25-29, 30-34, and 35+. Here, GED recipients below 20 years old are assumed to be 15-19 and placed for the purposes of estimates at the midpoint of the age interval (18 at last birthday), and those 35 and older are likewise assumed mostly to be recipients 35-39 for the purpose of estimating the equivalent synthetic population's probability of
receiving a GED. (In both cases, because of the age-specific growth rates, moving the average age of recipients downward inappropriately may slightly bias the estimate upward, moreso for teenagers than for GED recipients ages 35 and up.) Column (9) shows the adjusted GED recipients, and the bottom of the column compares the sum to 15 th birthdays. The estimate of 15 th birthdays in the year between the fall of 2001 and fall of 2002 is simply the average of 14 -year-olds in 2001 and 15-year-olds in 2002.

The estimation of public- and private-school graduation is more complicated, because states do not provide age-specific data on graduation. Columns (11) through (13) therefore apportion the 2,635,000 non-adult-program graduates of public schools by the 12th-grade distribution below age 23 (column 11 , in thousands $)^{2}$ to create an estimate of the distribution of graduates according to October 2001 ages (column 12, also in thousands) and then aged forward 8 months to the end of the spring (column 13). Column 14 shows the growth-and-migration-adjusted diplomas and the comparison with 15th birthdays. Columns (15) through (17) parallel columns (12) through (14), except apportioning private-

[^1]school (not public-school) diplomas by the total 12th-grade enrollment the prior spring and calculating the adjusted counts and probability.

In each case, adjusting the raw proportions for age-specific growth and migration rates lowers the estimates of graduation through each route. Stationarypopulation assumptions would lead to estimates that $16.0 \%$ of the equivalent synthetic population would receive a GED, $65.2 \%$ would receive a public-school diploma, and $7.1 \%$ a private-school diploma. With corrections, the estimates are $14.5 \%, 64.0 \%$, and $6.9 \%$, respectively. The GED estimate moves further with correction because GED recipients are on average older than regular graduates, and the growth correction spans a wider age range. The sum of all routes to graduate, $85.4 \%$, is close to the U.S. Census Bureau (2003) estimate that $84.5 \%$ of 20-24 year olds reported some high school credential in March 2002.

## Limitations

Several assumptions limit the accuracy of these estimates. The estimates assume that high-school graduates in the spring of 2002 were distributed proportionately to the 12 th grade population in the fall of 2001 ; while $91.5 \%$ of the public-school twelfth graders in fall 2001 graduated the next spring, they may not have done so in ages proportionate to the fall population. Public-school and private-school age distributions of graduates are almost certainly not identical. In addition, formal graduation credentials omit the informal attainment of
homeschoolers. However, these flaws are not likely to appreciably change estimates. Shifting the school-graduate age distribution older or younger would not appreciably change the estimate, given the near-zero cumulative growth and migration rates at the ages of greatest graduation (between 16 and 19). While homeschooling may include up to 1 million children nationwide, homeschooled students are disproportionately elementary-age children (Bauman, 2001). A greater limitation of this approach is the need for age-specific data on the nongraduate population and credentials. As more states create student-level databases, however, more states will have the capacity to report data by age.

## Sensitivity to Migration

One can theoretically apply the same approach used for Table 2 and Figure 1 to the public-school enrollment of any jurisdiction (whether a state, school district, or school). With the algorithm described in equation 15 and shown in Table 3, the relationship between estimates of mortality and migration, on the one hand, and graduation $\left(\frac{l_{g}}{l_{0}}\right)$ on the other, involves corrections to graduation counts at each age and is not easily amenable to algebraic analysis. But simulations are still possible. In the public-school context, the population is the enrollment membership and migration includes internal migration and inter-sector migration, as well as international migration. Virginia published official public
enrollment on line by age and grade from 1996-97 through 2003-04 (Virginia Department of Education, n.d.). ${ }^{3}$ Distributing the reported diplomas in the same manner as in the prior section (aging the twelfth-grade student enrollment by twelve months and then apportioning the diplomas in the same distribution) allows one to estimate graduation rates for academic diplomas, special-education diplomas, and other exit documents, assuming no net migration. To simplify the simulation of different migration rates, $i_{\mathrm{x}}$ is replaced by a constant migration level $i$ and varied from -0.15 to 0.15 at increments of 0.01 . (This $i$ does not incorporate mortality, but teen mortality is largely ignorable for the U.S. in recent decades. In most cases, $i$ and $\overline{t i}$ are comparable.) As mentioned earlier, large jurisdictions such as states are unlikely to have migration that is lower than -0.03 or greater than 0.03 , but smaller jurisdictions have greater variations in migration. Table 4 shows the changing estimate of academic graduation in Virginia between 1997 and 2003, for values of $i$ between 0 and 0.05 , with Figure 2 showing the estimates as values of $i$ range between -0.15 and 0.15 . (As with CPI and CPI, this estimate uses enrollment data from both academic years that include each year in question. Thus, the 2003 estimates use 2002-03 enrollment and graduation data and 200304 enrollment.)

[^2]Table 4
Simulated estimates of graduation rates for Virginia public schools, 1997-2003, i $=0$ to 0.05 .

| $i$ | 1997 | 1998 | 1999 | 2000 | 2001 | 2002 | 2003 |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 0 | $78.1 \%$ | $74.0 \%$ | $77.3 \%$ | $78.6 \%$ | $84.6 \%$ | $70.1 \%$ | $83.5 \%$ |
| 0.01 | $74.6 \%$ | $70.6 \%$ | $73.8 \%$ | $75.1 \%$ | $80.8 \%$ | $67.0 \%$ | $79.7 \%$ |
| 0.02 | $71.3 \%$ | $67.5 \%$ | $70.5 \%$ | $71.7 \%$ | $77.1 \%$ | $64.0 \%$ | $76.2 \%$ |
| 0.03 | $68.1 \%$ | $64.5 \%$ | $67.4 \%$ | $68.5 \%$ | $73.6 \%$ | $61.1 \%$ | $72.8 \%$ |
| 0.04 | $65.1 \%$ | $61.6 \%$ | $64.4 \%$ | $65.4 \%$ | $70.3 \%$ | $58.4 \%$ | $69.5 \%$ |
| 0.05 | $62.2 \%$ | $58.9 \%$ | $61.5 \%$ | $62.5 \%$ | $67.1 \%$ | $55.8 \%$ | $66.4 \%$ |

Source: Virginia Department of Education (n.d.).


Figure 2. Regular-diploma graduation proportions, $\frac{l_{g}}{l_{0}}$, by migration by year Virginia public schools, 1997-2003. Source: Virginia Department of Education (n.d.).

Before discussing the relationship between migration and graduation, it is important to note that the estimates of $\frac{l_{g}}{l_{0}}$ are not stable from year to year. For example, the lowest curve on Figure 2, corresponding to the lowest graduation estimate for any chosen constant migration rate, is from 2002, immediately after and before the highest curves from 2001 and 2003. In the case of 2002, the number of graduates reported is lower than for the immediately preceding and following year, and there does not appear to be any obvious pattern of unreported diplomas (such as a single large school system with anomalous data). As with the enrollment data from Detroit in 2002-03, this instability shows the inherent difficulties of relying on administrative records.

As with the grade-based measures, assumptions about migration have significant consequences for estimates of graduation. The curves for $\frac{l_{g}}{l_{0}}$ are steeper than for any of the grade-based measures, so that a change from an assumption of $i=0.01$ to $i=0.02$ results in a drop in the estimate of $\frac{l_{g}}{l_{0}}$ between $3.0 \%$ and $3.7 \%$ for the years in question. For 2003, a change in $i$ or $\bar{t}$ between 0.01 and 0.02 results in a $3.6 \%$ drop in $\frac{l_{g}}{l_{0}}$, a $2.5 \%$ drop in ACPI, a $3.0 \%$ drop in $\mathrm{ABCR}_{8}$, and a $2.2 \%$ drop in ASGR. Figure 3 compares the Virginia 2003 migration-estimate curves for all four measures.


Figure 3. Regular-diploma graduation estimates by migration, Virginia public schools, 2003.

## National Governors Association Compact:

## A Solution?

The NGA (2005c) compact to develop longitudinal student databases and use a true cohort measure of graduation as the states' official graduation rates promises to address some of the problems discussed here and elsewhere, such as the conflation of ninth-grade retention with first-time enrollment in high school. At least in theory, the compact defines graduation, declares a cohort as the group
of all students who enter high school at a similar time except for transfers, allows for separate calculations of graduation for cohorts of individuals in special circumstances, and calls for an "audit" of current data collection methods, including the codes used to classify student exits from enrollment. The graduation-rate compact parallels the National Institute of Statistical Sciences and Education Statistics Services Institute (2004) recommendations for cohort-based measures adjusted by migration. However, there is nothing in the compact that calls for regular steps to confirm the accuracy of each exit code, especially transfers. In addition, definitional problems will still cause problems unless there is a standard agreement on what constitutes a transfer that removes a student from the cohort or the correct cohort for a student transferring into the jurisdiction. The experience of Florida, which has had an individual-level student database since the early 1990s, is a cautionary tale of these problems and of the need for finetuning longitudinal graduation rate data collection and analysis.

## Florida's Graduation-Rate Definition

Florida's official graduation rate (e.g., Florida Department of Education, 2005) attempts to follow individual cohorts from their first enrollment in ninth grade through exiting the state's public schools. The following definition is from the Florida Department of Education (2006) guide for calculating the rate:

Determining the denominator for the formula involves the following steps: determine the cohort of students who enrolled as first-time ninth-graders four years prior to the year for which the graduation rate is to be measured; add to this group any subsequent incoming transfer students who are on the same schedule to graduate; and subtract students who transfer out for various reasons, or who are deceased. The numerator simply consists of the number of graduates from this group (diploma recipients). (p. 5)

Theoretically, this meets the requirements of the NGA compact. However, the details are critical. Removed from the cohort are students "who left to enroll in an adult education program" (Florida Department of Education 2006, p. 3)—in other words, dropouts who immediately enroll in a GED preparation program and who are coded W26 in the state database. In addition, Florida counts as graduates all those who receive GED certificates and special-education certificates as well as academic diplomas.

## Failure To Confirm or Audit Exit Codes

Florida's database of students is one of the oldest in the country and has a number of steps counties take to clean data before it is uploaded to the state department of education. However, there is no auditing of the withdrawal codes. If a student or a student's parent claims that a student is leaving to move to
another state, to enter a private school, or to be homeschooled, there is nothing in law or written rule to prevent the data processing clerk from recording that as reported. There is no guarantee that the recorded code is an accurate reflection of what happens when the student leaves the school building, as there is no public record of any follow-up procedure. There is sufficient experience nationwide of reporting flaws that even the recording of codes can be inaccurate without auditing (Lewin \& Medina, 2003; Schemo, 2003).

## The Effect of Florida's Definition on Graduation Rates

Because the Florida Department of Education provides the counts of dropout-to-GED (W26) attrition by cohort, it is a relatively simple calculation to include the W26 withdrawals in the denominator, as shown in Table 5. The difference between the official rate and the rate corrected for the W26 exclusions ranges between $3.7 \%$ (for the 1999 cohort) and 6.2 (for 2002) and averages 5.3\%.

Table 5
Florida's graduation rate, including dropout-to-GED attrition in each cohort

| Graduation <br> year | Official <br> denominator | Corrected <br> denominator | Dropouts- <br> to-GEDs $^{\text {b }}$ | Official <br> rate | Corrected rate |
| :--- | :---: | :---: | :---: | :---: | :---: |
| 1999 | 166,736 | 177,525 | 10,789 | $60.2 \%$ | $56.6 \%$ |
| 2000 | 167,723 | 179,352 | 11,629 | $62.3 \%$ | $58.2 \%$ |
| 2001 | 171,301 | 186,940 | 15,639 | $63.8 \%$ | $58.5 \%$ |
| 2002 | 174,203 | 191,682 | 17,479 | $67.9 \%$ | $61.7 \%$ |
| 2003 | 180,578 | 198,012 | 17,434 | $69.0 \%$ | $62.9 \%$ |
| 2004 | 174,732 | 190,461 | 15,729 | $71.6 \%$ | $65.7 \%$ |
| 2005 | 182,969 | 199,080 | 16,111 | $71.9 \%$ | $66.1 \%$ |

Source: Florida Department of Education, 2005; and data conveyed in correspondence with Florida Department of Education.

Calculating the longitudinal cohort graduation rate based only on academic diplomas is difficult given the inconsistencies in the data available from the Florida Department of Education. In personal correspondence with the author, as well as in official publications, the department thus far has not made available the distribution of cohort-specific diplomas as academic, special-education, and GED. The best available option is to extrapolate the proportion of academic diplomas given the reporting of total diplomas in a year from the state Department of Education. But reporting is inconsistent. For example, for the 2002-03 school year, the department reported on its website 120,905 academic diplomas, 6,160 special diplomas, 6,225 standard certificates of completion, and 115 special certificates of completion (Sims, 2003). To the U.S. Department of Education, the state reported 127,484 academic diplomas, 14,161 diploma equivalents, and 6,326 other types of completion documents (Common Core of Data). In its cohort calculations, the state used a final figure of 124,577 total diplomas of all types. Some of the variations come from different definitions of categories-the federal request for reporting of all other exit documents presumably includes certificates of completion (e.g., for regular-curriculum students who do not pass the graduation tests in Florida), a number not included in the state's official graduation-rate numerator. But the state's own reporting does not include an explicit count of GEDs. From the federal figures, one calculates that $86.2 \%$ of all
reported diplomas were academic diplomas. The state's reporting implies a potential $90.6 \%$ of exit documents as standard academic diplomas. A generous estimate that $91 \%$ of reported cohort diplomas are standard academic diplomas would correspond to an additional 5-6\% inflation of Florida's official graduation rates.

## Improving the NGA Compact

There are several steps that states can take to maximize the accuracy and transparency of longitudinal graduation rates. First, states can clearly define which students are excluded from a cohort by transferring, and this definition should eliminate the possibility that a dropout will be counted as a transfer, as happens currently in Florida. Second, states should take steps to ensure the accuracy of a transfer code by requiring a transcript request or other confirmation step at the local level. Third, states should design an audit of the assignment of exit codes on an annual basis to ensure accuracy of the system as a whole, in addition to other editing and audit mechanisms. Fourth, states should group cohorts by birth year rather than the year in which they entered high school. There are several reasons for this last recommendation. Reporting graduation rates by birth cohort will eliminate the bias of differential retention rates. In addition, reporting graduation rates by birth cohort will eliminate any bias from differential placement of students transferring into a state's public high schools. With student-
level databases, there is no significant cost to reporting graduation rates by birth cohort.

## Conclusion

Recently proposed grade-based graduation measures and a new age-based measure are all subject to bias from misestimating student migration, whether international, internal, or inter-sector. For one case, Virginia public schools in 2003, moving from an assumption of zero net migration or net-increment rates to 0.03 rates corresponds to changes in the graduation estimate between $6.6 \%$ and $10.7 \%$, depending on the measure. In absolute terms, the various measures ranged from $63.2 \%$ to $83.5 \%$ given plausible net in-migration or net-increment rates between 0 and 0.03 . Even relatively small changes in the assumed in-migration or net-increment rate, between 0.01 and 0.02 , resulted in measurable drops of the graduation estimate between $2.2 \%$ and $3.6 \%$, depending on the measure chosen. For Virginia 2003, the ideal age-based measure $\frac{l_{g}}{l_{0}}$ had both the highest estimate of graduation for plausible estimates of migration and also the greatest sensitivity to migration.

In addition to the relationship between migration and graduation rates, this analysis demonstrates the fragility of estimates based on unaudited administrative record-keeping. The variation in graduation estimates for Virginia using published
data varied significantly, beyond what one would expect for normal year-to-year changes. In addition, the estimates for 1998 and 1999 had to be recalculated because the official enrollment reported for 15-year-olds in 1998-99, 77,707, was implausibly low (and was replaced by an average of nearby values). The Education Week (2006) estimate of Detroit's CPI, which relies on clearly faulty data in the Common Core of Data, is only the most obvious example of corrupted data at the foundation of many graduation estimates. Finally, as Mishel and Roy (2006) explain, grade-based estimates are also biased by retention, especially when they use ninth-grade enrollment.

Florida's experience with longitudinal cohort graduation rates shows both the promise of the NGA compact on graduation rates and also the need for appropriate operationalization of definitions and steps to improve the technical adequacy of the information. Florida's rates are inflated because the graduation rate simultaneously eliminates responsibility for students who drop out and then immediately enroll in a GED program - and then credits public schools for the students who eventually earn a GED. Florida's database is also one with no confirmation or auditing of transfer codes.

Finally, serious consideration needs to focus on the question of whether grade-based or age-based graduation rates are better. Most current school statistics report information by grade or grade cohort, including several recentlyproposed graduation-rate formulas and also the NGA compact and its progenitors
(including Florida's official graduation rate). Yet grade-based graduation rates conflate grade level with cohort. Quasi-cohort methods that use ninth-grade enrollment statistics cannot distinguish first-time ninth-graders from repeaters. Longitudinal student databases such as Florida's cannot always determine the cohort to which a student transferring into the public schools truly belongs. Age is less vulnerable to such conflation problems, and any state with an accurate student database can report information by birth cohort (for longitudinal cohort rates) or by age (for period rates).

Given the requirements in No Child Left Behind to calculate a graduation rate for every high school, it appears from the analysis here that there is no broadly-used measure currently able to estimate graduation with degree of precision at a state level, let alone at the school level. While the National Governors Association (2005c) compact on a longitudinal cohort rate is appropriate, at least in theory, in practice states that already have a longitudinal rate show some evidence of inflating graduation rates. The No Child Left Behind requirement is desirable but currently impossible to meet. Meeting the law requires a well-operated student registration system, a system where records of diplomas, enrollments, and transfers are all audited regularly to raise confidence in the accuracy of transfer and migration data.

## References

Bauman, Kurt J. (2001). Home schooling in the United States: Trends and characteristics. U.S. Census Bureau Population Division Working Paper No. 53. Retrieved December 21, 2005, from http://www.census.gov/population/www/documentation/twps0053.html.

Education Week (2006, June). Diplomas count: An essential guide to graduation policy and rates. Bethesda, MD: Editorial Projects in Education, Inc. Retrieved June 26, 2006, from http://www.edweek.org/ew/toc/2006/06/22/index.html.

Florida Department of Education. (2005). 1989-99-2004-05 graduates.
Tallahassee, FL: Author. Retrieved June 19, 2006, from
http://www.firn.edu/doe/eias/eiaspubs/grad.htm.
Florida Department of Education. (2006). Technical guide for the 2005-06 Florida high school graduation rate. Tallahassee, FL: Author. Retrieved June 26, 2006, from http://www.firn.edu/doe/eias/eiaspubs/pdf/gradgde.pdf.

Greene, J. P., \& Winters, M. A. (2002). High school graduation rates in the United States. New York: Center for Civic Innovation, Manhattan Institute. Retrieved June 26, 2006, from http://www.manhattaninstitute. $\mathrm{org} / \mathrm{html} / \mathrm{cr}$ 31.htm.

Greene, J. P., \& Winters, M. A. (2005). Public high school graduation and college-readiness rates: 1991-2002. New York: Center for Civic Innovation, Manhattan Institute. Retrieved June 26, 2006, from http://www.manhattaninstitute.org/html/ewp 08.htm.

Greene, J. P., \& Winters, M. A. (2006). Leaving boys behind: Public high school graduation rates. New York: Center for Civic Innovation, Manhattan Institute. Retrieved June 26, 2006, from http://www.manhattaninstitute.org/html/cr_48.htm.

Greene, J. P., Winters, M. A., \& Swanson, C. (2006, March 29). Missing the mark on graduation rates. Education Week. Retrieved June 26, 2006, from http://www.edweek.org/ew/articles/2006/03/29/29greene.h25.html.

Hall, D. (2005, June). Getting honest about grad rates: How states play the numbers and students lose. Washington, D.C.: Education Trust. Retrieved June 26, 2006, from http://www2.edtrust.org/NR/rdonlyres/C5A6974D-6C04-4FB1-A9FC-05938CB0744D/0/GettingHonest.pdf.

Haney, W., Madaus, G., Abrams, L., Wheelock, A., Miao, J., \& Gruia, I. (2004). The education pipeline in the United States, 1970-2000. Chestnut Hill, MA: National Board on Educational Testing and Public Policy, Boston College. Retrieved June 26, 2006, from
http://www.bc.edu/research/nbetpp/statements/nbr3.pdf.

Harvey, J., \& Housman, N. (2004). Crisis or possibility? Conversations about the American high school. Washington: National High School Alliance. Retrieved June 26, 2006, from http://www.hsalliance.org/resources/docs/Crisis\ or\ Possibility.pdf.

Hauser, R. M. (1997). Indicators of high school completion and dropout.
In R. M. Hauser, B. V. Brown \& W. R. Prosser (Eds.), Indicators of children's well-being (pp. 152-184). New York: Russell Sage Foundation.

Lewin, T., \& Medina, J. (2003, July 31). To cut failure rate, schools shed students. New York Times.

Miao, J. \& Haney, W. (2004, October 15). High school graduation rates: Alternative methods and implications. Education Policy Analysis Archives, 12(55). Retrieved June 26, 2006, from http://epaa.asu.edu/epaa/v12n55/.

Mishel, L. \& Roy, R. (2006). Rethinking high school graduation rates and trends. Washington, D.C.: Economic Policy Institute. Retrieved June 26, 2006, from
http://www.epi.org/books/rethinking_hs grad_rates/rethinking_hs_grad_ratesFULL_TEXT.pdf.

National Governors Association. (2005a). An action agenda for improving America's high schools. Washington, D.C.: National Governors Association. Retrieved June 26, 2006, from
http://www.nga.org/Files/pdf/0502ACTIONAGENDA.pdf.

National Governors Association. (2005b). American high school crisis and state policy solutions. Washington, D.C.: National Governors Association. Retrieved June 26, 2006, from http://www.nga.org/Files/pdf/1003SCHOOLCRISIS.pdf.

National Governors Association. (2005c). Graduation counts: A compact on state high school graduation data. Washington, D.C.: National Governors Association. Retrieved June 26, 2006, from http://www.nga.org/Files/pdf/0507GRADCOMPACT.PDF.

National Institute of Statistical Sciences and Education Statistics Services Institute. (2004). National Institute of Statistical Sciences/Education Statistics Services Institute Task Force on Graduation, Completion, and Dropout Indicators. Washington, DC: National Center for Education Statistics.

Orfield, G., (Ed.). (2004). Dropouts in America: Confronting the graduation rate crisis. Cambridge, MA: Harvard Education Publishing Group.

Orfield, G., Losen, D., Wald, J., \& Swanson, C. B. (2004). Losing our future: How minority youth are being left behind by the graduation rate crisis. Washington: Urban Institute. Retrieved June 26, 2006, from http://www.urban.org/UploadedPDF/410936_LosingOurFuture.pdf.

Preston, S. H. (1987). Relations among standard epidemiologic measures in a population. American Journal of Epidemiology, 162, 336-345.

Preston, S. H., \& Coale, A. J. (1982). Age structure, growth, attrition and accession: A new synthesis. Population Index, 48, 217-259.

Schemo, D. J. (2003, July 11). Questions on data cloud luster of Houston schools. New York Times.

Seastrom, M. M., Chapman, C., Stillwell, R., et al. (2006). User's guide to computing high school graduation rates. Washington, DC: National Center for Education Statistics.

Seastrom, M., Hoffman, L., Chapman, C., \& Stillwell, R. (2006). Graduation rate for public high schools from the Common Core of Data: School years 2002-03 and 2003-04. Washington, DC: U.S. Department of Education. Retrieved June 26, 2006, from http://nces.ed.gov/pubs2006/2006606.pdf.

Sims, R. (2003). Untitled spreadsheet of data on 2002-03 graduates from Florida public schools. Tallahassee, FL: Florida Department of Education. Retrieved June 26, 2006, from http://www.firn.edu/doe/eias/eiaspubs/xls/grad0203.xls.

Starr, Paul. 1987. The sociology of official statistics. In P. Starr and W. Alonso, eds., The politics of numbers (pp. 7-57). New York: Russell Sage Foundation.

Swanson, C. B. (2004). Who graduates? Who doesn't? A statistical portrait of public high school graduation, class of 2001. Washington, D.C.: The

Urban Institute. Retrieved June 26, 2006, from
http://www.urban.org/publications/410934.html.
U.S. Census Bureau (n.d. a). School enrollment tables, retrieved December 21, 2005, from
http://www.census.gov/population/www/socdemo/school.html
U.S. Census Bureau (n.d. b). Geographic mobility/migration tables, retrieved December 21, 2005, from http://www.census.gov/population/www/socdemo/migrate.html.
U.S. Census Bureau. (2003). Educational attainment in the United States: March 2002. Detailed Tables (PPL-169), retrieved December 21, 2005, from http://www.census.gov/population/www/socdemo/education/ppl-169.html.
U.S. Department of Education. (2004). Digest of education statistics 2004. Washington, DC: Author. Tables retrieved December 21, 2005, from http://nces.ed.gov/programs/digest/d04 tf.asp.

Virginia Department of Education. (n.d.). Superintendent's annual report [various years]. Richmond, VA: Author. Retrieved June 26, 2006, from http://www.pen.k12.va.us/VDOE/Publications/rep page.htm.

Warren, J. R. (2005). State-level high school completion rates: Concepts, measures, and trends. Education Policy Analysis Archives, 13(51). Retrieved December 24, 2005, from http://epaa.asu.edu/epaa/v13n51/.


[^0]:    ${ }^{1}$ In standard demographic analysis, $l_{0}$ represents the hypothetical birth cohort in a period life table. This paper keeps demographic notation throughout, to be consistent and to develop the notion of an equivalent hypothetical population for any real population whose characteristics are known.

[^1]:    ${ }^{2}$ Most public-school programs conclude for students with disabilities at the end of the school year in which students turn 22. Regardless of how respondents on the Current Population Survey reported their attending grade, Table 3 does not use selfreported 12th graders who are 23 years old and older.

[^2]:    ${ }^{3}$ Because the enrollment for 15-year-olds reported for 1998-99 was implausibly low, the average of 15 -year-olds in surrounding years, 14-year-olds in 1997-98, and 16-year-olds in 1999-2000 was used as a substitute for estimate purposes.

