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Reduced Order Level Modeling of Structure-Based Uncertainty on Fluid Forces for the Dynamics of Nearly-Straight Pipes

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Abstract

This investigation is focused on the consideration of structural uncertainties in nearly-straight pipes conveying fluid and on the effects of these uncertainties on the dynamic behavior of the pipes. Of interest more specifically are the structural uncertainties which affect directly the fluid flow and its feedback on the structural response, i.e., uncertainties on/variations of the inner cross-section and curvature of the pipe. A finite element-based discovery effort is first carried out on randomly tapered straight pipes to understand how the uncertainty in inner cross-section affects the behavior of the pipes. It is found that the dominant effect originates from the variations of the exit flow speed, induced by the change in inner cross-section at the pipe end, with the uncertainty on the cross-section at other locations playing a secondary role. The development of a *generic* model of the uncertainty in fluid forces is next considered by proceeding directly at the level of modal models by randomizing simultaneously the appropriate mass, stiffness, and damping matrices. The maximum entropy framework is adopted to carry out the stochastic modeling of these matrices with appropriate symmetry constraints guaranteeing that the nature, e.g., divergence or flutter, of the bifurcation is preserved when introducing uncertainty. To achieve this property, it is proposed that the fluid related mass, damping, and stiffness matrices of the stochastic reduced order model (ROM) all be determined from a single random matrix and a random variable. The predictions from this stochastic ROM are found to closely match the corresponding results obtained with the randomized finite element model.

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Keywords: pipes conveying fluid; uncertainty; reduced order model; stochastic model; divergence; flutter

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1. Introduction

The structural dynamic response of pipes conveying a fluid has been the subject of a large series of investigations summarized in the landmark text by Paidoussis [1]. This vast body of work has focused on the stability and response of pipes of various configurations, boundary conditions, and with different flow properties but always under the assumption that the fluid-structure system considered has well defined properties. In many other applications of structural dynamics, e.g., see the review in [2], the effects of uncertainty on the geometric and/or material properties of the structure have been quite thoroughly investigated. Somewhat surprisingly given the mature state of both uncertainty modeling methods and knowledge on the dynamic behavior of pipes conveying fluid, there appears to be only a few prior investigations of the effects of uncertainty on the response of such pipes [3-5]. In fact, references [3] and [4] considered only a handful of slightly curved pipes focusing primarily on the nonlinear response vs. a systematic study of the effects of uncertainty as considered here and in [5]. In this regard, three main sources of uncertainty in the pipe-fluid system can be identified. First are the uncertainties associated with the structural problem that have no effect on the fluid flow. They include the density of the pipe, its Young's modulus, the external shape and diameter (assuming only internal flow), etc., and can be addressed by now well known methods, e.g., see [2]. At the opposite end are the fluid only uncertainties such as density of the fluid, non-uniformity of the flow speed, and fluid modeling which are typically convected along the pipe creating time dependent variations of the system. Finally are the uncertainties on the structure-fluid coupling terms which include variations of the internal shape and diameter of the pipe and of its curvature. Changes, e.g., taper, of the internal area of the pipe modifies the local flow speed and in turn the overall magnitude of the local fluid effects on the pipe. A curvature of the pipe also modifies the fluid flow effects as demonstrated in [1,3,4].

The objectives of the present effort are thus to propose and assess a global modeling of the uncertainty in the structure-fluid coupling terms (different than the one proposed in [5]). To support this modeling, carried out at a reduced order model (ROM) level, a "computational experiment" is first performed in which the internal diameter of the pipe is randomly varied along its length and a dedicated finite element code is used to predict the behavior of the pipe as a function of the flow speed. The phenomenological understanding gathered from that effort is then relied upon in the construction of a stochastic ROM representing the desired global model of uncertain pipes.

2. Computational Experiment

Owing to the paucity of data on the effects of structural uncertainty on the dynamic behavior of pipes, a discovery effort was carried out for tapered straight pipes with randomly varying inner radius. This study was performed through a finite element implementation (see [6] for details) of the governing equation for straight tapered pipes, i.e., [1]

$$\frac{\partial^2}{\partial x^2} \left[EI \frac{\partial^2 w}{\partial x^2} \right] - \frac{\rho_i A_i}{2\pi} \frac{dA_i}{dx} \frac{\partial}{\partial x} \left\{ \frac{\partial^2 w}{\partial t^2} + 2U_i \frac{\partial^2 w}{\partial x \partial t} + U_i^2 \frac{\partial^2 w}{\partial x^2} \right\} + \rho_i A_i \left\{ \frac{\partial^2 w}{\partial t^2} + 2U_i \frac{\partial^2 w}{\partial x \partial t} + U_i U_i(L) \frac{\partial^2 w}{\partial x^2} \right\} + m \frac{\partial^2 w}{\partial t^2} = 0 \quad (1)$$

in which gravity and pressurization effects, and any externally imposed tension have been neglected. In the above equation, m is the mass per unit length of pipe, E and I are its Young's modulus and cross-section moment of inertia, and $A_i(x)$ is the inner cross-sectional area at location x along the pipe. Moreover, ρ_i is the mass per unit length of fluid, $U_i(x)$ is the flow speed at location x satisfying the conservation of mass $A_i(x)U_i(x) = \text{constant} = VFR$ (the volumetric flow rate). Finally, $w(x,t)$ is the transverse deflection of the pipe at axial location x and time t . Note in Eq. (1) that $U_i(L)$, the flow speed at the end of the pipe, appears explicitly.

A series of "computational experiments" were conducted [6] with the tapered finite element code in which the inner radius of the pipe varied linearly between nodes with the values at these points randomly perturbed from the mean model value except at the inlet. The inner radius at that node was kept constant for convenience so that the various samples simulated could be compared based on the same flow speed at the inlet as opposed to matching the volumetric flow rate. The inner radii at the finite element nodes were modeled as independent random numbers following either a uniform or a truncated Gaussian distribution both of mean equal to the mean model and specified

standard deviations. The simulated Gaussian values were limited to ± 4 standard deviations to avoid the occurrence of unphysical, very large changes of the inner radius. Since the present investigation is focused solely on the effects of uncertainty on the fluid forces, the purely structural properties, i.e., EI and m , were kept constant along the beam, even though variations in inner radius would affect them.

Since the exit flow speed $U_i(L) = VFR / A_i(L)$ appears explicitly in Eq. (1) and depends solely on the inner radius at that point, three options were considered in the computational experiment. In the first one, only this speed was varied by randomly changing the outlet inner radius. In the second option that radius was fixed while all others were varied. Finally, in the last option, the inner radius at all nodes (except the first as discussed above) were varied. The results of this computational experiment for all three options with both normal and uniform distributions of inner radii and for both simply supported and cantilevered pipes are presented in [6] for the pipe-fluid system of [7], i.e., with masses per unit length $m = 2330 \text{ kg/m}^3$ and $\rho_i = 1000 \text{ kg/m}^3$, length $L = 200 \text{ nm}$, and a circular cross section of inner and outer radii equal to 8 nm and 10 nm. A representative sample of these results, for the cantilevered beam with options 1 and 2 and a truncated Gaussian distribution of inner radii, is shown in Fig. 1. It is observed from these figures that the uncertainty of the exit flow speed alone leads to much larger uncertainty of the pipe dynamic behavior (option 1) than the uncertainty on the inner radius at all other locations combined (option 2). Moreover, the distribution of the critical (flutter here) flow speed obtained with the truncated normal and uniform distributions (see [6]) are nearly identical for option 2 but somewhat different for option 1, as expected since many random values of the inner radius are involved in the former case vs. only one in the latter case.

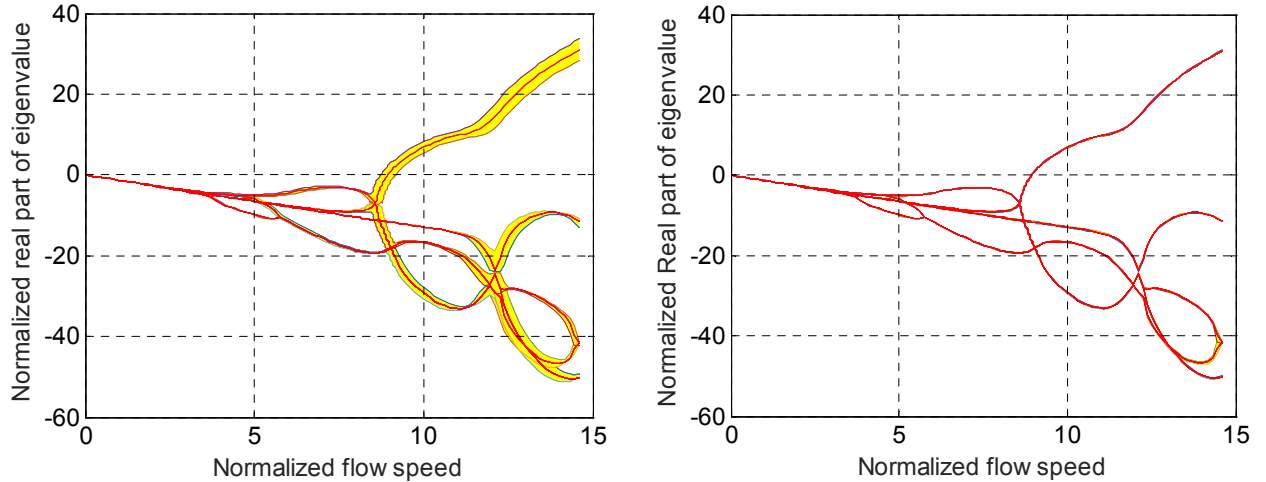


Figure 1. Normalized real part of the eigenvalues vs. normalized flow speed obtained from the tapered finite element model. (a) Option 1, only the inner radius at the exit is varied, (b) Option 2, all inner radii, except the one at the exit are varied. Truncated Gaussian distribution of inner radii of coefficient of variation 0.0075.

3. Stochastic Reduced Order Modeling

The application of the finite element formulation of the previous section requires a detailed characterization of the geometry, e.g., the inner radius as a stochastic field but also the variations in shape of the cross-section, its possible curvature, etc., all of which would be painstaking to measure and time consuming to implement. Instead, the present effort focuses on the construction of a generic stochastic model of the uncertainty on the fluid forces induced by a similar uncertainty of the pipe inner geometry. This task will be achieved by (i) proceeding at the reduced order model level and (ii) relying on maximum entropy [2] concepts. Central to the construction of this stochastic ROM is an understanding of the properties that the ROM matrices must satisfy. To initiate this analysis, consider the governing equation of an undamped straight pipe of constant cross section with gravity and pressurization effects, and any externally imposed tension have been neglected, e.g., Eq. (1) with $A_i(x)$ constant

$$\frac{\partial^2}{\partial x^2} \left[EI \frac{\partial^2 w}{\partial x^2} \right] + \rho_i A_i \left\{ \frac{\partial^2 w}{\partial t^2} + 2U_i \frac{\partial^2 w}{\partial x \partial t} + U_i^2 \frac{\partial^2 w}{\partial x^2} \right\} + m \frac{\partial^2 w}{\partial t^2} = 0 \quad (2)$$

Next, assume a solution in the expansion form below

$$w(x, t) = \sum_{j=1}^N q_j(t) \varphi_j(x) \quad (3)$$

where $\varphi_j(x)$ are basis functions satisfying the geometric boundary conditions, $q_j(t)$ are the corresponding generalized coordinates, and N is the number of such terms. Then, proceeding in a Galerkin format, leads to the set of ordinary differential equations for the generalized coordinates stacked in the vector q

$$(M_p + M_f) \ddot{q} + C_f \dot{q} + (K_p + K_f) q = \underline{0} \quad (4)$$

where M_p , M_f , C_f , K_p , K_f are, respectively, the pipe and fluid-induced mass matrices, the fluid-induced damping matrix, and the pipe and fluid-induced stiffness matrices. In fact, proceeding similarly with Eq. (1) would give a set of equations similar to Eq. (4) but with different matrices which would all involve the variations of the pipe's inner geometry in different manner. This observation would suggest that M_p , M_f , K_p , K_f and C_f should all be randomized together to model uncertainty in the internal cross-section properties. Of course, independent uncertainty on the density of the pipe, its outer diameter, Young's modulus, etc. would also come in inducing further uncertainty on the pipe's own matrices M_p and K_p and decreasing the dependence between the elements of these matrices and those of the fluid-induced ones. On this basis, it is suggested here that M_p and K_p can be modeled independently of the remaining three matrices M_f , C_f , and K_f which however ought to be modeled together since they originate from the same source, the fluid flow, and involve at least one common uncertain factor, i.e., the fluid mass per length which depends on the inner cross-section area. In considering the modeling of K_f , it should further be recognized that this matrix would be the only one affected by the exit flow velocity $U_i(L)$, see Eq. (1), and mostly proportionally to this variable when the pipe taper is small.

The need for a combined modeling of the flow-induced matrices was also stated in [3]. The authors argued additionally that the fluid-induced mass matrix could be lumped with the pipe's own since they exhibit a similar symmetry. That left the fluid-induced stiffness and damping matrices which were ingeniously combined into a complex impedance modeled according to the extension of the nonparametric method applicable to general matrices [2].

It is wondered here whether additional properties of M_f , C_f , and K_f should be imposed in the modeling. To this end, consider first the case of simply supported pipes. Then, it can be shown [6] that M_f is symmetric positive definite, C_f is skew-symmetric, and K_f is symmetric and negative definite. In fact, these properties are critical to prove that the pipe will exhibit divergence as opposed to flutter. Moreover, it can also be shown [6] that the symmetric matrix F defined as

$$F = \begin{bmatrix} M_f & 0.5C_f \\ 0.5C_f^T & K_f \end{bmatrix}, \quad (5)$$

where T denotes matrix transposition, is positive definite.

The above properties may not hold after variations in geometry are introduced. Yet, for appropriately small variations, it is expected that:

- (i) the pipe becomes unstable at flow speeds close to the divergence speed of the perfectly straight pipe, and
- (ii) the symmetric part of F is positive definite.

The property (i) is in fact much more stringent that could be imagined at first. Indeed, since the perfectly straight pipe diverges, the real part of all its eigenvalues is exactly zero for all of them until divergence takes place. Then, small arbitrary variations of M_f , C_f , and K_f typically lead to the occurrence of eigenvalues with small negative real part (indicative of flutter) at much lower speeds than the divergence speed, sometimes even as $U \rightarrow 0$ which is clearly unphysical.

To avoid this issue, it is necessary to constrain the modeling of the random matrices \hat{M}_f , \hat{C}_f , and \hat{K}_f of the

uncertain pipes. More specifically, it will be *assumed* here that they satisfy the same properties as the perfectly straight pipe. In the particular case of the simply supported pipe, this constraint is satisfied by selecting

$$\hat{M}_f = P^T M_f P \quad \hat{C}_f = P^T C_f P \quad \hat{K}_f = P^T K_f P \quad (6a), (6b), (6c)$$

where P is a $N \times N$ random matrix to be selected. While the above discussion strictly holds only for the simply supported pipe, it is desirable that the construction of the stochastic reduced order model be independent of the boundary conditions. Thus, Eqs (6) will also be applied for all boundary conditions, e.g., also for cantilevered pipes.

It remains finally to address the explicit dependence of \hat{K}_f on the exit flow velocity $U_i(L)$. Since this speed scales the largest component of \hat{K}_f , it is proposed here to modify Eq. (6c) to read

$$\hat{K}_f = R P^T K_f P \quad (7)$$

where the positive random variable R is introduced to model the effects of the exit flow velocity $U_i(L)$.

To complete the stochastic ROM construction, it then remains to specify the distribution of the random variable R and of the random matrix P . It is proposed here to rely on the maximum entropy approach [2]. More specifically, the application of this methodology to the mass matrix M_f alone would lead to

$$\hat{M}_f = \bar{L} H_N H_N^T \bar{L}^T \quad \text{where} \quad M_f = \bar{L} \bar{L}^T \quad (8)$$

where H_N is a $N \times N$ lower triangular matrix the elements of which are all independent of each other with the off diagonal ones identically distributed as zero mean Gaussian random variables. Moreover, the diagonal elements are distributed as square root of Gamma random variables, see [2] for details. The single hyperparameter of this distribution is the overall measure of deviation, δ_N . Equating the random mass matrices \hat{M}_f of Eqs (6a) and (8) yields the desired expression for P , i.e.,

$$P = \bar{L}^{-T} H_N^T \bar{L}^T \quad (9)$$

Finally, recognizing R as a 1×1 positive definite random matrix leads directly to its modeling as $R = H_1^2$ where H_1 is similar to H_N but with a matrix size of 1 and appropriate dispersion δ_1 .

4. Application

The reduced order model of the mean model, i.e., the straight pipe with constant inner radius of [7], was achieved by using the modes of the pipe without fluid. An excellent match of the finite element predictions of the real and imaginary part of the eigenvalues vs. flow speed was obtained for both mean model and uniformly tapered pipes with 6 modes in the simply supported case and 8 for the cantilevered one, see [6] for further discussion.

To assess the potential of the stochastic ROM to match the physical behavior of the randomly tapered pipes shown in Fig. 1, the dispersion parameters δ_N and δ_1 were selected so that the standard deviations of the flutter speed obtained by the stochastic ROM match their counterparts for options 1 (when $\delta_N = 0$; $\delta_1 \neq 0$) and 2 (when $\delta_N \neq 0$; $\delta_1 = 0$). Then, Shown in Fig. 2 are the corresponding plots of the real parts of the eigenvalues vs. flow velocity predicted for options 1 and 2 by the stochastic ROM. Comparing these figures to those of Fig. 1, it is seen that a close match is indeed accomplished through the entire range of flow speed. Additional comparisons of the imaginary parts of the eigenvalue and of the probability density functions of the critical speeds in both cantilevered and simply supported cases, not shown here for brevity, see [6] for full details, confirm the match of Figs 1 and 2. This ensemble of results provides a strong confirmation of the applicability of the proposed stochastic ROM not only to model small uncertainties in the inner radius of the pipe but also of its curvature, see [6].

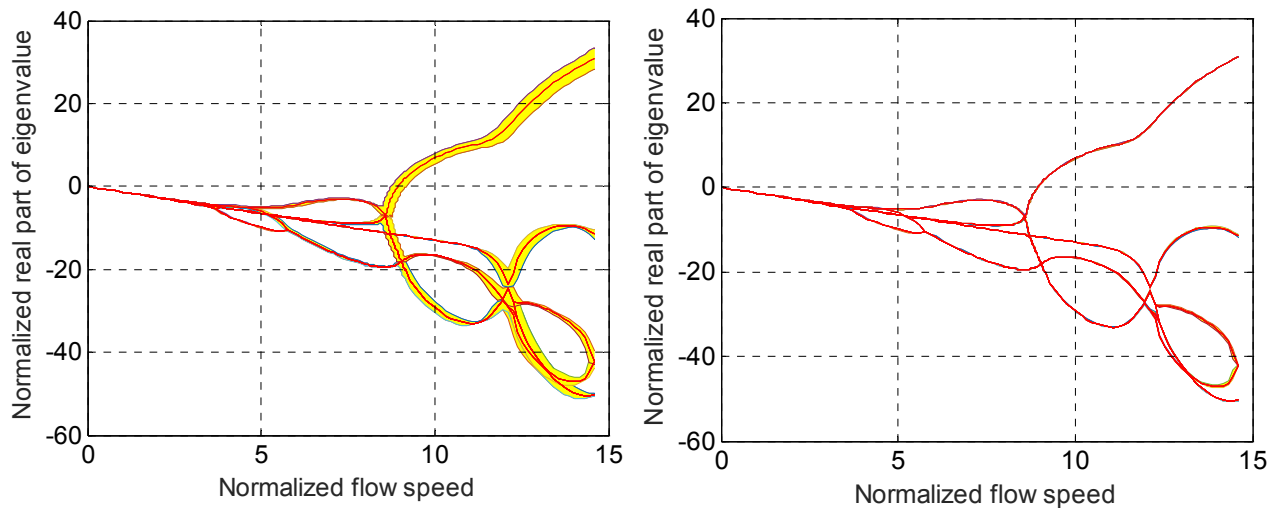


Figure 2. Normalized real part of the eigenvalues vs. normalized flow speed obtained from the stochastic reduced order model. (a) Option 1, only the inner radius at the exit is varied, (b) Option 2, all inner radii, except the one at the exit are varied. Truncated Gaussian distribution of inner radii of coefficient of variation 0.0075.

5. Summary

This investigation focused on the modeling of structural uncertainties in nearly-straight pipes conveying fluid and on the effects of these uncertainties on the dynamic response and stability of those pipes. The structural uncertainties of interest here are those which affect directly the fluid flow and its feedback on the structural response, e.g., uncertainties on/variations of the inner cross-section and curvature of the pipe. A stochastic reduced order model (ROM) was developed to capture the effects of these uncertainties, it is found that the fluid related mass, damping, and stiffness matrices of this stochastic ROM are all determined from a single random matrix and a random variable.

A series of finite element computations were carried out for pipes with straight centerline but inner radius varying randomly along the pipe to (i) understand the effects of this uncertainty on the pipe dynamic behavior and (ii) provide “experimental” data to assess the applicability of the stochastic ROM. With regard to (i), it was in particular found that the uncertainty in the exit flow speed induced by the variation of the pipe inner radius at that location has the most significant effect on the pipe dynamic behavior. Moreover, comparisons of the results obtained with the random finite element model and the stochastic ROM appropriately calibrated were found in close agreement providing strong support for the appropriateness and applicability of the stochastic ROM.

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