# Appendix

## 1 Conditional Strategies without Punishment

The following equations define the respective individual payoff whenever an agent employs a cooperation (defection) strategy:

$$\begin{aligned} X_c &= 1 + b\frac{T_c(n-1)+1}{n} - c \\ X_d &= 1 + b\frac{T_c(n-1)}{n} \end{aligned} \tag{1}$$

.

In this case, the conditional expected payoffs for other group members are:

$$\begin{aligned} \overline{X_c} &= T_c X_c + (1-T_c)\left(X_d + \frac{b}{n}\right) \\ \overline{X_d} &= T_c\left(X_c - \frac{b}{n}\right) + (1-T_c)X_d \end{aligned} \tag{2}$$

Here, note that the following inequalities hold:

$$\begin{aligned} \overline{X_c} &= T_c X_c + (1-T_c)(X_c + c) \\ &= X_c + (1-T_c)c \\ &\geq X_c \end{aligned} \tag{3}$$

$$\begin{aligned} \overline{X_d} &= T_c(X_d - c) + (1-T_c)X_d \\ &= X_d - cT_c \\ &\leq X_d \end{aligned} \tag{4}$$

Thus, the expected utility of cooperation and defection take the form:

$$\begin{aligned} E[U_c] &= \beta\overline{X_c} + (1-\beta)X_c \\ E[U_d] &= \alpha\overline{X_d} + (1-\alpha)X_d \end{aligned}$$

where $\alpha$ and $\beta$ are restricted to $\Omega$:

$$\Omega = \left\{(\alpha,\beta) : -1 \leq \alpha \leq 1,\ -1 \leq \beta \leq \alpha\right\}.$$

The preceding expectations can be re-written as:

$$\begin{aligned}
E[U_c] &= \beta\Bigg[X_c + (1-T_c)c\Bigg] + (1-\beta)X_c \\
&= X_c + \beta(1-T_c)c \\
E[U_d] &= \alpha\Bigg[X_d - cT_c\Bigg] + (1-\alpha)X_d \\
&= X_d - \alpha c T_c
\end{aligned} \tag{5}$$

Thus, cooperation evolves whenever:

$$\begin{aligned}
E[U_c] &> E[U_d] \\
\implies \alpha T_c + \beta(1-T_c) &> 1 - \frac{b}{nc} \\
\implies \beta &> -\left(\frac{T_c}{1-T_c}\right)\alpha + \frac{1-\frac{b}{nc}}{1-T_c}.
\end{aligned} \tag{6}$$

## 2 Conditional Strategies with Punishment

The following equations define the respective individual payoff whenever an agent employs a cooperation (or defection) strategy. Now, groups are also comprised of $(nT_p)$ punishers who reduce the earnings of defectors by $(p)$ at a personal cost $(k)$:

$$\begin{aligned}
X_c &= 1 + b\frac{T_c(n-1)+1}{n} - c \\
X_d &= 1 + b\frac{T_c(n-1)}{n} - pT_pT_c \\
X_{cp} &= 1 + b\frac{T_c(n-1)+1}{n} - c - k(1-T_c).
\end{aligned} \tag{7}$$

The conditional expected payoffs for other group members are:

$$\begin{aligned}
\overline{X_c} &= T_c\Bigg[(1-T_p)X_c + T_pX_{cp}\Bigg] + (1-T_c)\Bigg[X_d + \frac{b}{n}\Bigg] \\
\overline{X_d} &= T_c\Bigg[(1-T_p)(X_c - \frac{b}{n}) + T_p(X_{cp} - \frac{b}{n} - k)\Bigg] + (1-T_c)X_d \\
\overline{X_{cp}} &= T_c\Bigg[(1-T_p)X_c + T_pX_{cp}\Bigg] + (1-T_c)\Bigg[X_d + \frac{b}{n} - p\Bigg].
\end{aligned} \tag{8}$$

We can re-write the equations in (8) as:

$$\begin{aligned}\overline{X_c} &= X_c + T_c\Big[T_p(X_{cp} - X_c)\Big] + (1-T_c)\Big[c - pT_pT_c\Big]\\ &= X_c + (1-T_c)\Big[c - T_pT_c(k+p)\Big]\\ &\geq X_c\end{aligned} \tag{9}$$

where the last inequality holds if: $T_pT_c(k+p) < c$.

$$\begin{aligned}\overline{X_d} &= X_d + T_c\Big[T_p(pT_pT_c - c - k - k(1-T_c)) + (1-T_p)(pT_pT_c - c)\Big]\\ &= X_d + T_c\Big[pT_pT_c - T_pk(2-T_c) - c\Big]\\ &\leq X_d\end{aligned} \tag{10}$$

where the last inequality holds if: $T_p((k+p)T_c - 2k) < c$.

$$\begin{aligned}\overline{X_{cp}} &= X_{cp} + T_c\Big[(1-T_p)k(1-T_c)\Big] + (1-T_c)\Big[c - pT_pT_c - p + k(1-T_c)\Big]\\ &= X_{cp} + (1-T_c)\Big[c + k - p - T_pT_c(k+p)\Big]\\ &\geq X_{cp}\end{aligned} \tag{11}$$

where the last inequality holds if: $T_pT_c(k+p) < c + k - p$.

Hence, the expected utility of cooperation, defection, and punishment take the form:

$$\begin{aligned}E[U_c] &= \beta\overline{X_c} + (1-\beta)X_c; \quad && \overline{X_c} > X_c\\ E[U_d] &= \alpha\overline{X_d} + (1-\alpha)X_d \quad && \overline{X_d} < X_d\\ E[U_p] &= \beta\overline{X_{cp}} + (1-\beta)X_{cp}; \quad && \overline{X_{cp}} > X_{cp}\end{aligned} \tag{12}$$

where $(\alpha, \beta) \in \Omega$. The preceding expectations may written as:

$$\begin{aligned}E[U_c] &= X_c + \beta(1-T_c)\Big[c - T_pT_c(k+p)\Big]\\ E[U_d] &= X_d + \alpha T_c\Big[(pT_pT_c - T_pk(2-T_c) - c\Big]\\ E[U_p] &= X_{cp} + \beta(1-T_c)\Big[c + k - p - T_pT_c(k+p)\Big]\end{aligned}$$

.

**Notes:**

* $\overline{X_c} \geq X_c \implies \overline{X_d} \leq X_d$ whenever $T_c > \frac{2k}{k+p}$. The converse is not necessarily true.

* $X_{cp} - X_c = -k(1-T_c) < 0$. Also:

$$X_c - X_d = \frac{b}{n} - c + pT_pT_c$$

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## 2.1 Evolution of Cooperation: $E[U_c] > E[U_d]$

**Case 1:** $\overline{X_c} > X_c$; $\overline{X_d} < X_d$

$$\begin{aligned} & E[U_c] > E[U_d] \\ \implies & X_c - X_d + \beta(1-T_c)\Big[c - T_pT_c(k+p)\Big] > \alpha T_c\Big[pT_pT_c - T_pk(2-T_c) - c\Big] \\ \implies & \beta > \frac{T_c\left[pT_pT_c - T_pk(2-T_c) - c\right]\alpha + c - \frac{b}{n} - pT_pT_c}{(1-T_c)[c - T_pT_c(k+p)]} \end{aligned} \tag{13}$$

.

Note here that setting $T_p = 0$ in condition (13) recovers condition (7).

**Case 2:** $\overline{X_c} < X_c$; $\overline{X_d} < X_d$

$$\begin{aligned} & E[U_c] > E[U_d] \\ \implies & X_c - X_d + \alpha(1-T_c)\Big[c - T_pT_c(k+p)\Big] > \alpha T_c\Big[pT_pT_c - T_pk(2-T_c) - c\Big] \\ \implies & \alpha > \frac{c - \frac{b}{n} - pT_pT_c}{(1-T_c)\left[c - T_pT_c(k+p)\right] - T_c\left[pT_pT_c - T_pk(2-T_c) - c\right]} \end{aligned} \tag{14}$$

provided $(1-T_c)(c - T_pT_c(k+p)) > T_c(pT_pT_c - T_pk(2-T_c) - c)$.

**Case 3:** $\overline{X_c} < X_c$; $\overline{X_d} > X_d$

$$\begin{aligned} & E[U_c] > E[U_d] \\ \implies & \beta < \frac{(1-T_c)\left[c - T_pT_c(k+p)\right]\alpha - c + \frac{b}{n} + pT_pT_c(n-1)}{T_c[pT_pT_c - T_pk(2-T_c) - c]}. \end{aligned} \tag{15}$$

**Case 4:** $\overline{X_c} > X_c$; $\overline{X_d} > X_d$

$$E[U_c] > E[U_d]$$

$$\implies \beta > \frac{c - \frac{b}{n} - pT_pT_c}{(1-T_c)\left[c - T_pT_c(k+p)\right] - T_c\left[pT_pT_c - T_pk(2-T_c) - c\right]}. \tag{16}$$

## 2.2 Evolution of Punishment: $E[U_p] > E[U_c]$

Costly punishment evolves whenever it yields a larger expected utility than cooperation alone. This expectation is influenced by within-group strategy distribution and the social welfare preferences of agents.

**Case 1:** $\overline{X_c} > X_c$; $\overline{X_{cp}} > X_{cp}$

$$E[U_p] > E[U_c]$$

$$\implies X_{cp} + \beta(1-T_c)\Big[c + k - p - T_pT_c(k+p)\Big] > X_c + \beta(1-T_c)\Big[c - T_pT_c(k+p)\Big]$$

$$X_{cp} - X_c + \beta(1-T_c)(k-p) > 0$$

$$\implies \beta > \frac{k}{k-p} \tag{17}$$

provided $k > p$.

**Case 2:** $\overline{X_c} < X_c$; $\overline{X_{cp}} < X_{cp}$

$$E[U_p] > E[U_c]$$

$$\implies \alpha > \frac{k}{k-p}. \tag{18}$$

**Case 3:** $\overline{X_c} > X_c$; $\overline{X_{cp}} < X_{cp}$

$$E[U_p] > E[U_c]$$

$$\implies X_{cp} + \alpha(1-T_c)\Big[c + k - p - T_pT_c(k+p)\Big] > X_c + \beta(1-T_c)\Big[c - T_pT_c(k+p)\Big]$$

$$\implies \beta < \frac{\left[c + k - p - T_pT_c(k+p)\right]\alpha - k}{c - T_pT_c(k+p)} \tag{19}$$

**Case 4:** $\overline{X_c} < X_c$; $\overline{X_{cp}} > X_{cp}$

$$E[U_p] > E[U_c]$$

$$\implies \beta > \frac{[c - T_p T_c (k+p)]\,\alpha + k}{c + k - p - T_p T_c (k+p)} \tag{20}$$