Available online at www.sciencedirect.com

ScienceDirect

Transportation Research Procedia 23 (2017) 827–847

Transportation Research Procedia

www.elsevier.com/locate/procedia

22nd International Symposium on Transportation and Traffic Theory

Network-oriented Household Activity Pattern Problem for System Optimization

Jiangtao Liu [a], Jee Eun Kang [b], Xuesong Zhou [a] *, Ram Pendyala [a]

[a] *School of Sustainable Engineering and the Built Environment, Arizona State University, USA*
[b] *Department of Industrial & Systems Engineering, Institute for Sustainable Transportation & Logistics, University at Buffalo, USA*

Abstract

The recently emerging trend of self-driving vehicles and information sharing technologies, made available by private technology vendors, starts creating a revolutionary paradigm shift in the coming years for traveler mobility applications. By considering a deterministic traveler decision making framework at the household level in congested transportation networks, this paper aims to address the challenges of how to optimally schedule individuals' daily travel patterns under the complex activity constraints and interactions. We reformulate two special cases of household activity pattern problem (HAPP) through a high-dimensional network construct, and offer a systematic comparison with the classical mathematical programming models proposed by Recker (1995). Furthermore, we consider the tight road capacity constraint as another special case of HAPP to model complex interactions between multiple household activity scheduling decisions, and this attempt offers another household-based framework for linking activity-based model (ABM) and dynamic traffic assignment (DTA) tools. Through embedding temporal and spatial relations among household members, vehicles and mandatory/optional activities in an integrated space-time-state network, we develop two 0-1 integer linear programming models that can seamlessly incorporate constraints for a number of key decisions related to vehicle selection, activity performing and ridesharing patterns under congested networks. The well-structured network models can be directly solved by standard optimization solvers, and further converted to a set of time-dependent state-dependent least cost path-finding problems through Lagrangian relaxation, which permit the use of computationally efficient algorithms on large-scale high-fidelity transportation networks.

Peer-review under responsibility of the scientific committee of the 22nd International Symposium on Transportation and Traffic Theory.

Keywords: Household Activity Pattern Problem; Space-time-state network; Activity-based modeling; Dynamic traffic assignment

* Corresponding author. Tel.: +1-480-965-5827.
E-mail address: xzhou74@asu.edu

2352-1465
Peer-review under responsibility of the scientific committee of the 22nd International Symposium on Transportation and Traffic Theory.
10.1016/j.trpro.2017.05.046

1. Introduction

The activity-based modeling approach has been widely studied in the area of transportation planning and operations to better capture various facets of travel behavior and decision making. How to recognize complex resource constraints, multi-agent interactions, and consistency through trip chains of different individuals is an important concern for accurate activity-based modelling and analysis at the household level. Different modeling paradigms have been developed, including deterministic optimization-based models by Recker (1995), and probabilistic micro-simulation-based utility maximization models by Bhat et al. (2004), Pendyala et al. (2005), Pribyl and Goulias (2005), Miller and Roorda (2003), and Arentze and Timmermans (2004).

Currently, the emerging mobile apps with multi-modal traveler information and personal activity schedules enable travelers to intelligently schedule their activities and share their trip requests. In addition, transportation network companies such as Uber and Lyft and the forthcoming autonomous vehicle system would allow and encourage a fully optimized planning process for mapping household activities and travel requests (to be met by personal or shared vehicles). In this paper, we focus on the household activity pattern problem (HAPP) that is first systematically formulated by Recker (1995), which aims to find the optimal path of household members for completing their prescribed activities based on the available number of vehicles, scheduled activity participation, and ride-sharing options within a long period as the unit of analysis.

Typically, based on a conventional mixed integer linear programming model for the pickup and delivery problem with time windows (PDPTW), many typical cases in HAPP, e.g., five cases in a classical paper by Recker (1995), require a very large number of linear and integer constraints to capture the complex rules in real-world household-level activity scheduling progress. Recently, several algorithms had been proposed to address more realistic side constraints and large-sized examples, to name a few, Chow and Recker (2012) and Kang and Recker (2013). In addition, Liao et al. (2013) presented a new set of super-network models for various person-level activity scheduling problems, where the multi-dimensional network construct contains travel links, state transition links and activity transaction links. To formulate HAPP as a mathematically rigorous model, how to fully consider complex coupling constraints among three layers, namely household members, vehicles and mandatory/optional activities, is extremely challenging, especially for large-scale multi-modal transportation network with flexible ride-sharing and household member activity-coordination options.

To consider the traffic congestion and feedback loops associated with complex trip interactions, there are a wide range of studies aiming to combine ABM and DTA to better capture the interplay between human activity-travel decisions and underlying congested networks with tight road capacity constraints. For example, Lin et al. (2008) proposed a conceptual framework and explored the model integration of activity-based model (CEMDAP) and dynamic traffic assignment model (VISTA). Pendyala et al. (2012) further integrated activity-travel demand models (OpenAMOS), DTA tools with the long-term land use modeling layer (UrbanSim). Based on mathematical programs of HAPP, Kang et al. (2013) studied the network design problem considering the interaction between the household-level activity pattern and infrastructure changes. Chow and Djavadian (2015) proposed a new market equilibrium model to capture the interaction of traveler activity schedules in a capacitated system with a macroscopic flow restriction on a link or node facility. In a recent study by Fu et al. (2016), the intra-household interactions are considered through Markov decision processes and the road congestion effect is reflected by the static travel time function. To further study the impacts of dynamic traffic management strategies and real-time traveler information provision, Pendyala et al. (2017) proposed a tightly integrated modeling framework for representing activity-travel demand and traffic dynamics in an on-line environment.

This paper first aims to cast HAPP problems as number of time-dependent and state-dependent path searching problems, which have a class of computationally efficient algorithms available in discretized space-time network and high-dimensional space-time-state networks. To capture the impacts of traffic congestion on activity generation and scheduling, this paper also reformulates two special cases of HAPPs as system-optimal multi-household activity scheduling subject to the tight road capacity constraints. The key is how to prebuild a set of embedded finite state machines (FSM) in a network to precisely represent and translate side constraints from the traditional models, which could eliminate activity time window and vehicle selection constraints in the resulting optimization model.

Specifically, we consider Case A as Multi-vehicle and Multi-person vehicle routing problem with mandatory and discretionary activities. Further, with the given ride-sharing options for each household, we propose one more dimension to represent the activity performing status in each vehicle and model our Case B as Multi-vehicle and Multi-person ridesharing problem with mandatory and discretionary activities. These two problems can be formulated as 0-1 integer linear programming models, with the space-time-state network being indexed through vehicle's location($\boldsymbol{i}$), vehicle's timestamp ($\boldsymbol{t}$) and cumulative activity completion state($\boldsymbol{w}$). Then the road capacity constraint can be directly added to model the network congestion and resulting activity scheduling change. Through dualizing the capacity constraints to the objective function by Lagrangian relaxation, our proposed model can be further solved through time-dependent state-dependent least cost path-finding algorithms, which permits the use of fast computational algorithms on large-scale high-fidelity transportation networks.

Nomenclature

N	Set of nodes in the physical network, including necessary virtual nodes
N_v	Set of vehicle nodes for vehicle selection
L	Set of links in the physical network, including necessary virtual links
P	Set of household members
P_m	Set of household members who have mandatory activities
P_n	Set of household members who chooses one mandatory activity from multiple candidates
P_q	Set of household members who have discretionary activities
V	Set of available vehicles
A	Set of activities
$A_n(p)$	Set of household member p's candidate activities for one kind of mandatory activity
A_v	Set of mandatory activities of vehicle v's driver
R	Set of vertices in the space-time/space-time-state network
E	Set of edges/arcs in the space-time/space-time-state network
W	Set of cumulative vehicle activity-performing state
$E(p, a_m)$	Set of edges/arcs of household member p's mandatory activity a_m
$E(p, a_n)$	Set of edges/arcs of household member p's candidate activity a_n for one kind of mandatory activity
$E(p, a_q)$	Set of edges/arcs of household member p's discretionary activity a_q
$E(v, a_m)$	Set of edges/arcs of mandatory activity a_m of vehicle v's driver
i, j	Index of node set N
(i, j)	Index of link set L
t, s	Index of time intervals in the space-time-state network
w, w'	Index of state in the space-time-state network
(i, t)	Index of vertex in the space-time network
(i, j, t, s)	Index of edges/arcs in the space-time network
(i, t, w)	Index of vertex in the space-time-state network
(i, j, t, s, w, w')	Index of edges/arcs in the space-time-state network
p	Index of household member set P
a	Index of activity set A
$t(i, j)$	Travel time of link (i, j)
$c_{i,j,t,s}^{p}$	Travel cost of arc (i, j, t, s) of person p in the space-time network
$c_{i,j,t,s,w,w'}^{v}$	Travel cost of arc (i, j, t, s, w, w') of vehicle v in the space-time-state network
$[a_k, b_k]$	The time window of event k, such as, activity starting time window, activity ending time window
$TD(p)/TD(v)$	Earliest departure time of household member p/ vehicle v
$O(p)/O(v)$	Origin node of household member p/ vehicle v

$D(p)/D(v)$	Destination node of household member p/ vehicle v
T	The time horizon in the space-time network/space-time-state network
$Cap_{i,j,t,s}$	Capacity of arc (i,j,t,s)
$Cap_{i,j,t,s,w,w'}$	Capacity of arc (i,j,t,s,w,w')
$x^p_{i,j,t,s}$	Binary variable, $=1$, if household member p visits the traveling/waiting arc (i,j,t,s) in the space-time network; $=0$ otherwise
$x^v_{i,j,t,s,w,w'}$	Binary variable, $=1$, if vehicle v visits the traveling/waiting arc (i,j,t,s,w,w') in the space-time-state network; $=0$ otherwise

2. Problem statement

In this paper, our study focuses on three particular cases in HAPP (namely A, B, C) and further extends to one more general case D. The general given input includes the population used for activity generation, a physical transportation network, a set of different types of activities (mandatory, semi-mandatory, optional activities) with specific time windows and utility values, a set of vehicles, as well as the activity/vehicle assignment set to each household member. By adapting the classical assumption/definition from Recker (1995), we present the following problem statements.

(1) Case A is a multi-vehicle and multi-person vehicle routing problem with mandatory and discretionary activities, which is similar to Case IV in the paper by Recker (1995). (i) Members of the household share a set of vehicles; a subset of vehicles may be available for use by any member of the household, and the remainder may be reserved for use by certain members; (ii) A subset of activities can be performed by any member of the household, and the remaining activities must be performed by certain members; (iii) Certain members can have specific mandatory activities or optional activities; (iv) Some members may perform no activities; some vehicles may not be used.

(2) Case B is a multi-vehicle and multi-person ridesharing problem with mandatory and discretionary activities, which can be treated as a special sub-problem of Case V in the paper by Recker (1995). The specific definition is: (i) the ride-sharing pattern that which household members will share one vehicle and which one is the driver has been given; (ii) A subset of activities can be performed by any member of the household, and the remaining activities must be performed by certain members; (iii) Certain members can have specific mandatory activities or optional activities.

(3) Case C is an extension of cases A and B, which considers tight road capacity constraints to capture the underlying congestion in physical transportation networks, so that the influence of time-dependent link travel time on household activity patterns can be observed. As a result, this case is a system optimal multi-household activity scheduling problem under time-varying traffic conditions.

(4) Case D is a dynamic household-level equilibrium problem where each household is inclined to choose the optimal activity pattern, which considers vehicle selection, mode choice and ride-sharing options simultaneously. As studied in a recent paper by Liu and Zhou (2016), when there is no link capacity constraint, each agent (e.g., passenger, vehicle, or household) can choose the best/shortest path without affecting each other. Once the limited resource constraint is strictly considered, some agents may have to accept a longer path in order to finish their own travel and this kind of decision mechanism could invoke bounded rationality to those agents.

Specially, Fig.1 (a) and (b) specifically compares the data flow of (i) existing integration of ABM and DTA and (ii) our proposed models. The simulation-based integration in Fig.1 (a) focuses on searching individual activity-travel pattern under dynamic user equilibrium conditions, and the mathematical program oriented modelling framework proposed in this paper aims to optimize the household activity decisions with system optimal goals under household-level activity requirements and network capacity constraints. In our future study, Case D will be further examined to study possible dynamic household-level equilibriums with household activity interactions.

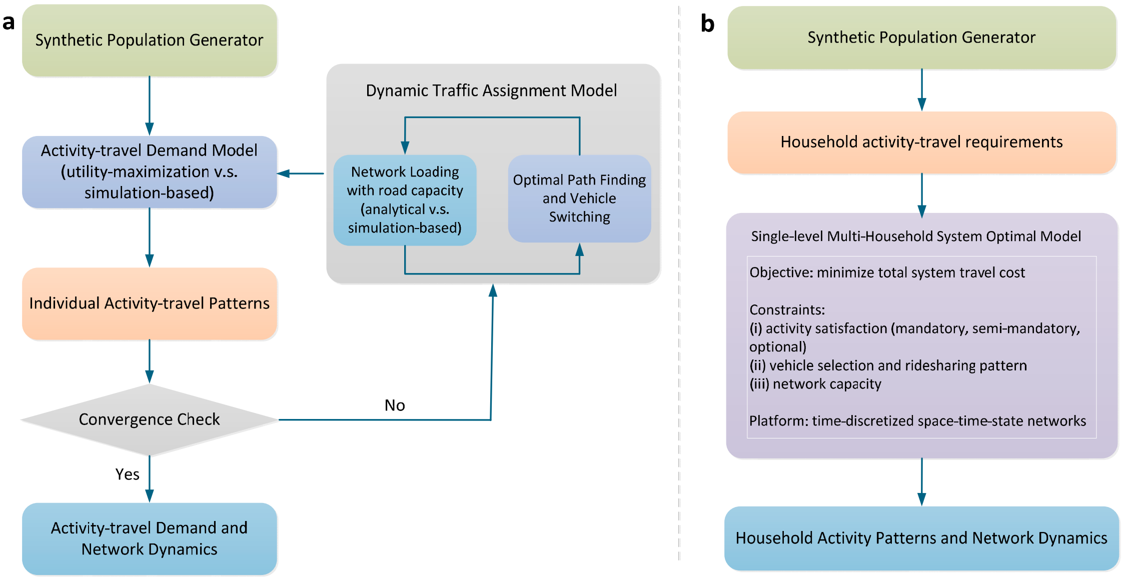

Fig. 1. (a) Existing integration framework of ABM and DTA; (b) Proposed modelling framework of Case C

2.1 Network construction and conceptual illustration of Case A

For illustrative purposes, a hypothetic three-node network shown in Fig. 2(a) is used to explain the problem addressed by Case A. There are two household members (p_1 and p_2), two available vehicles (v_1 and v_2), and two activities (a_1 and a_2). The available vehicle set and activity set of household members p_1 and p_2 is $\{v_1, v_2\}$ and $\{v_2\}$, and $\{a_1, a_2\}$ and $\{a_1\}$, respectively. The two activities belong to the mandatory activity and should be finished finally.

In order to model those requirements above, the physical network is modified as shown in Fig. 2(b), where the previous home node and activity nodes are split as several nodes. The detailed explanations are as follow. First, the home node is extended as six nodes, where (i) each household member has his/her dedicated node as his/her origin node, (ii) two vehicle nodes are created and one can view the links between household member origin node and vehicle nodes as vehicle selection links, while each vehicle node can only be visited less than or equal to once by all passengers, and (iii) node D serves as the super destination node. Moreover, to follow an activity-on-the-link representation scheme, the extended network on the right has activity starting node 1' and ending node 1'' corresponding to the activity node 1 on the left-hand side, and the link between the two nodes can be used to represent the required activity time duration.

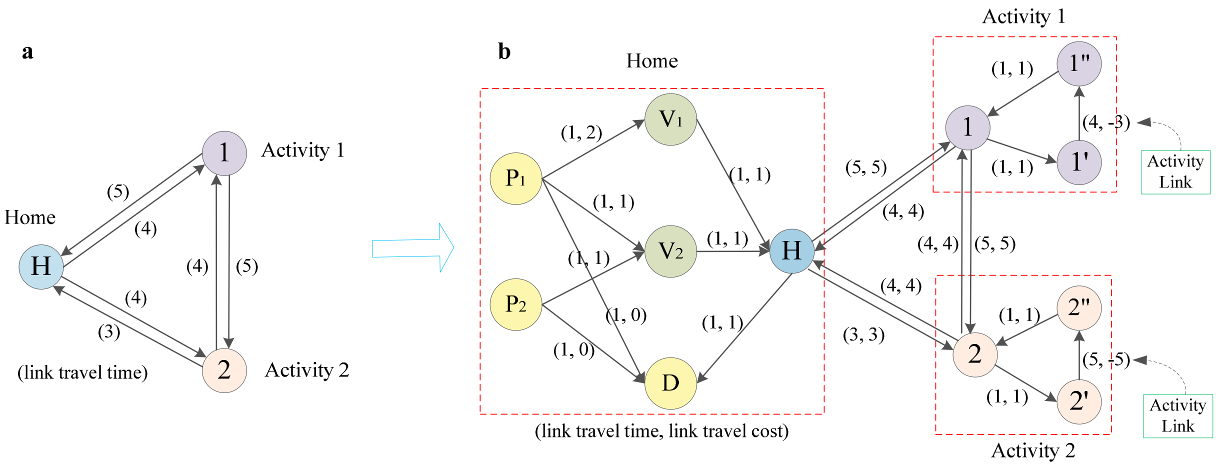

Fig. 2. (a) Physical network; (b) Corresponding modified network

In order to consider passenger-to-vehicle preference, the travel costs on those vehicle selection links can be passenger-specific. For example, the travel cost from passenger p_1's origin to the super destination is 0, which indicates that when passenger p_1 stays at home as one particular vehicle selection, there is no travel cost. In addition, the travel cost on link (p_1, v_1) is higher than that on link (p_1, v_2), indicating passenger p_1's higher preferences toward vehicle 2 compared to vehicle 1.

Each activity in HAPP typically has one specific time window, then we assume that the beginning time windows for (i) passengers p_1 and p_2 and (ii) activities a_1 and a_2 are [1,3], [1,4], [9,10], and [20, 21], respectively, along the total time horizon of 32 time units. Furthermore, the waiting cost of each time interval at origin nodes and destination node is assumed to be 0, and the waiting at activities nodes has a cost of 1 at each time interval. Within a deterministic disutility minimization framework, we assume negative cost values on activity links shown in Fig. 2(b).

A standard time-discretized space-time network can be constructed through the procedure proposed in the papers (Tong et al., 2015; Liu and Zhou, 2016; Lu et al., 2016), and the feasible space-time prism can be greatly reduced as illustrated in Fig. 3. As a result, the problem becomes how to find the best passengers' trajectory satisfying all time windows and activity requirements in the space-time network so as to minimize the total travel cost of all household members.

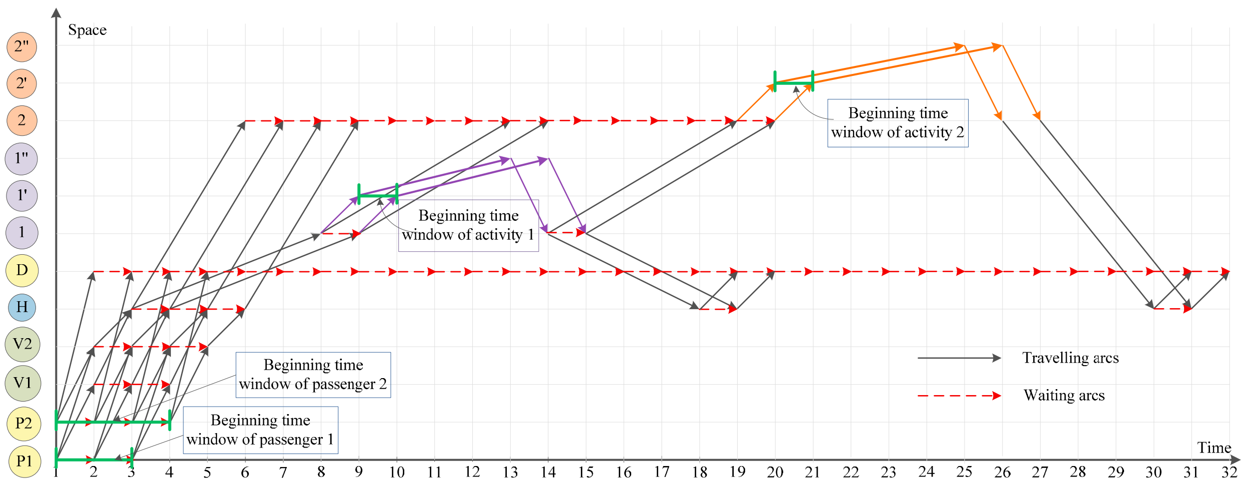

Fig. 3 Feasible searching region in the space-time network

As a remark, the (time-dependent) travel time on each travelling arc could be given in advance to reflect the congestion due to complex travel route choice interactions in the real-world traffic network. However, in the following Case C, we directly consider tight link/arc capacity inside the model to compute the resulting congestion effect explicitly. When the number of inflow vehicles exceeds the capacity of traveling arc, some vehicles have to wait at the waiting arc for available travelling arc capacity at next time interval. The detail about how tight capacity constraint is considered in space-time networks can be found in recent papers by Lu et al. (2016) and Liu and Zhou (2016), and their agent-based approach does not use the traditional flow-based nonlinear link/path cost function.

2.2 Network construction and conceptual illustration of Case B

The most difficult challenge in modeling the household-level ridesharing problem is how to recognize the complex coordination among different household members, pertaining to the following questions such as who is the driver and where/when the driver should drop off and pick up passengers. Considering offline planning applications, our Case B assumes that the set of possible ridesharing patterns is pre-specified with the given drop-off and pick-up locations with time windows to choose.

An illustrative example is given in Fig. 4(a), where there are two household members ($\boldsymbol{p_1}$ and $\boldsymbol{p_2}$), one available vehicle, and three activities ($\boldsymbol{a_1}$, $\boldsymbol{a_2}$ and $\boldsymbol{a_3}$). The given ride-sharing pattern requires that driver $\boldsymbol{p_1}$ needs to drop off the passenger $\boldsymbol{p_2}$ to his/her own activities within given beginning time windows, and then this driver needs to pick up $\boldsymbol{p_2}$ from the activity locations within given activity ending time windows. The driver $\boldsymbol{p_1}$ could accompany passengers to perform their activity, and also can leave to conduct his/her own mandatory activities. In this example with a quite busy household activity agenda, the driver has to perform the mandatory (deriver as D) activity $\boldsymbol{a_1}$ while the passenger needs to finish the mandatory (passenger as P) activities $\boldsymbol{a_2}$ and $\boldsymbol{a_3}$.

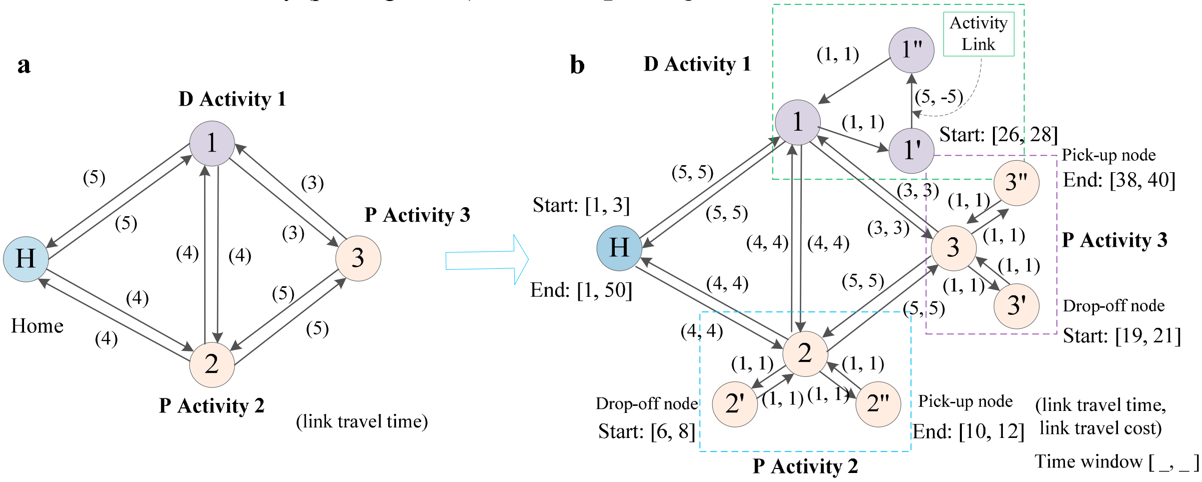

Fig. 4 (a) Physical network; (b) Corresponding modified network

Accordingly, we construct a drop-off node and a pick-up node for each passenger at the activity location in Fig. 4(b). It should be remarked that, in a typical case, one is dropped off and picked up at the same activity location, but our formulation also make it possible that one passenger is dropped off at one activity and then picked up at another location if he/she can take other travel modes (walking, transit or taxi) to the (spatially different) pick-up location. The starting node and ending node of the passenger activity (shown as P activity 3' and 3") is considered as a special drop-off node and pick-up node in drivers' network.

In addition to using the two dimensions (space and time) to depict vehicles' travel trajectory, this section will introduce one more state dimension to model ride-sharing status. More precisely, the state code covers each traveler's service status, including the driver and all passengers. Through adding one more dimension and exogenously listing the possible relation of location, time, and vehicle state, a set of hard activity-performing constraints for the driver and passengers in each vehicle could be embedded in advance in the space-time-state network, which will greatly reduce the set of side constraints and make our proposed mathematical model tractable for network flow optimization algorithms.

To solve the single-vehicle routing problem with pickup and delivery service with time windows (VRPPDTW), Psaraftis (1983) proposed a cumulative service state {1,2,3} to record the service status of each passenger. In this paper, we adopt the cumulative state representation as {0,1,2}: 0 means that the activity has not been performed, 1 means that the activity is being performed or the passenger has been dropped off at the activity location but not been picked up, and 2 means that the activity has been performed or the passenger has been picked up. While Mahmoudi and Zhou (2016) firstly proposed a third dimension as vehicle carrying state to solve the VRPPDTW, our third dimension of household-oriented state (with a rich representation of different household members, driver, passenger and associated activities) and the process of state transition are systematically different with those of Mahmoudi and Zhou (2016).

Since one activity could have 3 different states, if there are n activities for all passengers in one vehicle, it would require 3^n variables to represent all possible states. The total number of states depends on the number of activities. However, if one passenger has multiple activities, the possible states could be reduced because one passenger cannot perform multiple activities simultaneously. Also, the tight time window and transition preference for each activity can greatly reduce the number of possible states reasonable within feasible a space-time prism. In addition, the rapid development of hardware of computers could provide more memory and faster computation speed to address those large number of state search decisions.

We now use the example above to illustrate our cumulative activity-performing state and the state transition at different activity locations and times. There are one vehicle with two household members and three activities, so the vehicle's activity-performing state can be $[a_1, a_2, a_3]$, or more generically denoted as [_, _, _], where the first slot represents the driver's activity-performing state of activity a_1 and the second slot and the third one represent passenger p_2's two activity-performing states of activities a_2 and a_3, respectively. To reduce the number of states in this combinatorial optimization problem, one can also implement the activity-performing requirement as constraints on the activity link $(1' \rightarrow 1'')$for the driver in the mathematical model, so the resulting reduced state vector is $[a_2, a_3]$.

Since activities a_2 and a_3 are mandatory for passenger p_2, all possible vehicle's state could be $[a_2 = 0, a_3 = 0]$, [1,0], [2,0], [0,1], [0,2], [2,1], [1,2], and [2,2] by enumeration. It is noticed that $[a_2 = 1, a_3 = 1]$ is not included because it is impossible that passenger p_2 is dropped off at two locations simultaneously. Fig. 5(a) illustrates a graph of possible state transitions for the example above. In addition, if one of multiple activities with same type can be performed, such as, shopping at location 2 vs. at location 3 for passenger 2, he/she may just need to choose one of the two locations, so the resulting possible state transition will be that shown in Fig. 5(b).

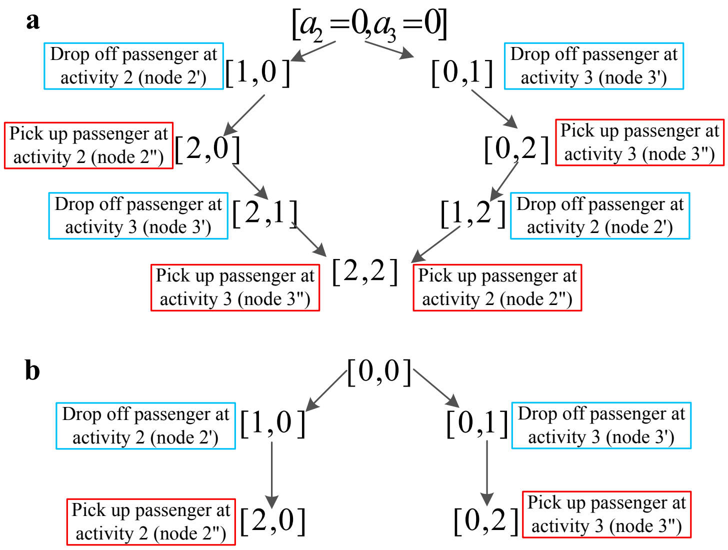

Fig. 5 (a) Both activities need to be performed; (b) Exact one of two activities should be performed

There are three types of mutually exclusive multidimensional arcs in the space-time-state network:

(1) Travelling arcs $(i, j, t, s, w, w' = w)$ with a time-dependent cost on link (i, j) departing at time t, with the same state w as transportation services do not change activity performing states.

(2) Waiting arcs $(i, i, t, t + 1, w, w')$ with a unit of waiting costs at location i from time t to time $t + 1$. A special Case is that, the waiting cost should be zero at the super home origin and destination nodes.

(3) State transition/service arcs (i, i, t, t, w, w') with a utility (i.e. negative travel cost) when performing their activities at the drop off location. As shown in Fig. 6, at node $i = 2'$, time $t = 6$, 7 or 8 within a given time window, we have a number of possible state changes, for example, $w = [0,0]$ with a possible transition to $w' = [1,0]$, or $w = [0,2]$ with a possible transition to $w' = [1,2]$. As the ending state for a_2 must be 2, passenger 2 will be picked up automatically among any feasible solutions and there is no benefit at pick-up nodes to avoid double counting of service utilities.

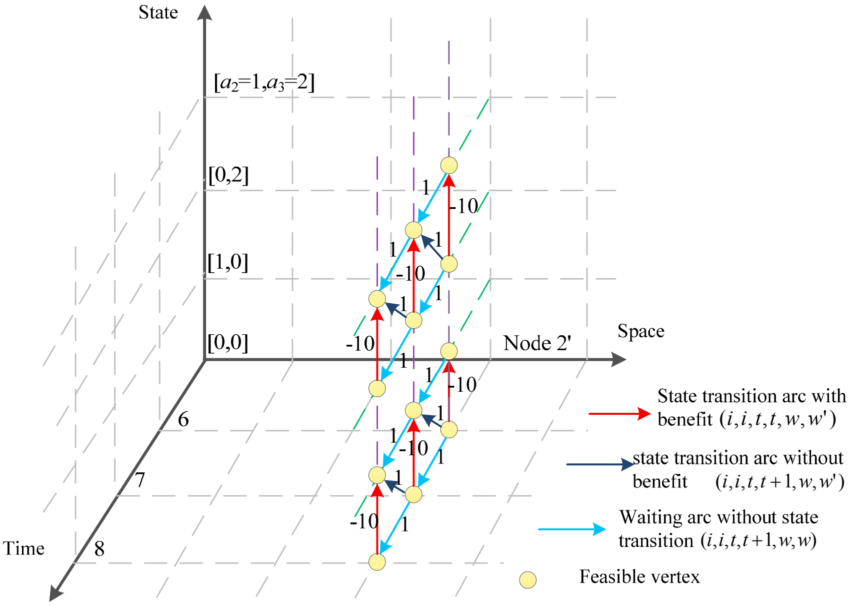

Fig. 6 Feasible arcs at node $2'$ in a space-time-state network

2.3 Conceptual illustration of Case C

As an extension of Cases A and B, Case C strictly honors the travelling arc capacity in the space-time network and space-time-state network, similar to the consideration in the recent papers along this line (Lu et al., 2016; Liu and Zhou, 2016). Compared with the constant link free-flow travel time, the underlying time-varying congestion could dramatically affect the passenger/vehicle's departure time, route choice, mode choice, destination choice, and even activity generation.

Without loss of generality, we adopt a time-invariant network (Liu and Zhou, 2016) shown in Fig. 7 to illustrate the congestion effect for two households with two different activities, where household 1 (household 2) has one member who departs from home node H_1 (H_2) to perform activity A_1 (A_2) then go back home, respectively.

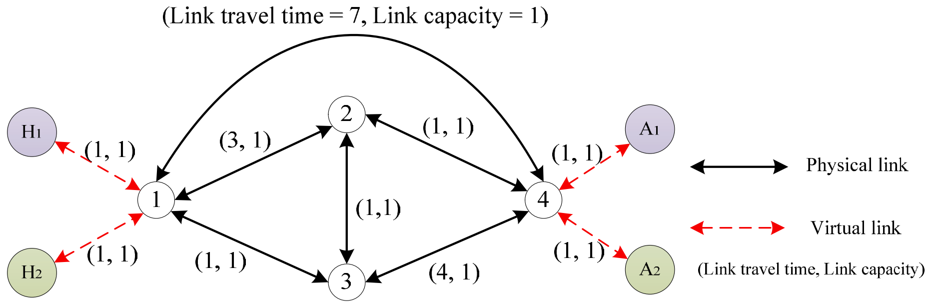

Fig. 7 A simple illustrative network for case C

What can be observed in Table 1 is summarized as follows and those observation can also be applicable to space-time and space-time-state networks.

(1) When the link capacity is not taken into account, the vehicles from both households choose their own shortest path. The physical path node sequences of households 1 and 2 is $1 \rightarrow 3 \rightarrow 2 \rightarrow 4 \rightarrow 2 \rightarrow 3 \rightarrow 1$ with path travel time of 6.

(2) When the link capacity is considered, the system optimal objective of Case C could make household 1 change its path as $1 \rightarrow 2 \rightarrow 4 \rightarrow 2 \rightarrow 1$ with a larger path cost of 8. Meanwhile, household 2 would switch a new path as $1 \rightarrow 3 \rightarrow 4 \rightarrow 3 \rightarrow 1$ with an increased cost of 10.

(3) In observation (1), the travel time of household 2 from 1 to 4 is 3, but now it will increase to 5 due to link capacity constraint in observation (2). If the passenger of household 2 has a strict time window for activity 2, the increased path travel time from 1 to 4 could make passenger depart earlier to satisfy the time window.

(4) If the time budget of household 2 from 1 to 4 is less than 5, the passenger would cancel activity 2 or may change to an alternative by switching to other possible travel modes.

(5) If Case D is considered for possible equilibrium conditions, one household could choose the previous shortest path and the other has to accept the longer path, $1 \rightarrow 4 \rightarrow 1$, with total travel time of 14. It also could lead to changes in departure time, activity cancel or mode choice. In addition, Braess paradox exists in the network above, so blocking links $3 \rightarrow 2$ and $3 \rightarrow 2$ definitely could improve the transportation efficiency and further influence household activity patterns from the perspective of traffic managers.

Table 1. Result analysis of different cases

Different cases		Physical path selection: Path 1	Path 2	Path 3	Path 4	Remarks
Case A/B: without link capacity constraint		×	×	×	√; √	Benchmark
Case C: system optimal with link capacity constraint		√	√	×	×	Compared with Case A/B, household has possible departure time change, route choice change, and possible activity cancel or mode choice change due to link capacity constraints.
Case D: household equilibrium with link capacity constraint	With links 3 ↔ 2	×	×	√	√	Compared with Case C, the total travel time is increased.
	Without links 3 ↔ 2	√	√	×	×	Braess paradox occurs, as the system-wide cost reduces without the link.

√: One household (vehicle) chooses the corresponding path;
×: No household (vehicle) chooses the corresponding path;

3. Mathematical programming models

3.1 space-time network-based optimization model for Case A

Based on the two-dimension space-time network constructed in section 2.2, we formulate our mathematical programing model that satisfies all requirements in *Case A*, which aims to optimize vehicle selection, activity-performing selection and route guidance for each household member so as to minimize the total household travel cost.

Model 1:

Objective function

$$\min \sum_{p} \sum_{(i,j,t,s)\in E} (c^{p}_{i,j,t,s} \times x^{p}_{i,j,t,s}) \tag{1}$$

Subject to

(1) Flow balance constraint for each person:

$$\sum_{i,t:(i,j,t,s)\in E} x^{p}_{i,j,t,s} - \sum_{i,t:(j,i,s,t)\in E} x^{p}_{j,i,s,t} = \begin{cases} -1 & j = O(p), s = DT(p) \\ 1 & j = D(p), s = T \\ 0 & otherwise \end{cases}, \forall p \tag{2}$$

(2) Vehicle selection constraint at vehicle selection node:

$$\sum_{p} \sum_{i,t:(i,j,t,s)\in E} x^{p}_{i,j,t,s} \le 1, \forall j \in N_v \tag{3}$$

(3) Mandatory activity participation for one specific household member:

$$\sum_{i,t:(i,j,t,s)\in E(p,a_m)} x^{p}_{i,j,t,s} = 1, \forall p \in P_m \tag{4}$$

(4) Mandatory activity with multiple candidates for one household member:

$$\sum_{a_n \in A_n(p)} \sum_{i,t:(i,j,t,s)\in E(p,a_n)} x^{p}_{i,j,t,s} = 1, \forall p \in P_n \tag{5}$$

(5) Discretionary activity for each household member

$$\sum_{i,t:(i,j,t,s)\in E(p,a_q)} x^{p}_{i,j,t,s} \le 1, \forall p \in P_q \tag{6}$$

(6) Binary variable: $x^{p}_{(i,j,t,s)} = \{0,1\}$

The objective function is to minimize the total system travel cost of all household members, where the travel cost $c_{i,j,t,s}$ on each arc has been predefined in the space-time network construction stage. Eq. (2) is the standard person-based flow balance constraint. Eq. (3) means that each household member can only choose one vehicle or don't choose any vehicles (a.k.a. staying at home in our example). Eq. (4) represents that the activity duration arc of each mandatory activity of a specific household member should be visit exactly once by that household member. For example, if one household member must go to a company for work, one of working arcs must be visited exactly once by the household member. Eq. (5) ensures that if one household member needs to perform one type of activity with multiple candidate locations/time durations, he/she must choose one candidate to complete one activity instance among all options. For example, if one household member needs to go shopping and there are two candidate shopping malls, finally only one shopping mall should be visited exactly once to mark the completion state of the shopping activity. Inequality (6) represents the flexibility association with those optional activities, as they could be performed or not, depending on the availability of those eligible household members and the required travel cost to reach those locations. In short, the proposed model in this section is a 0-1 integer linear programing model, or more precisely, a multi-commodity flow optimization problem with a limited set of side constraints. This compact formulation enables the use of standard optimization solvers for a real-world transportation network.

Table 2 offers a systematic comparison for detailed modelling techniques between our proposed model and classical model proposed by Recker (1995), specifically between our Case A and Case IV of Recker.

Table 2. Comparison between Case IV (Recker, 1995) and our Case A

Modelling constraints	Model R4: Case IV (Recker, 1995)	Model 1 for our Case A	Remarks
(1) Time representation	Continuous	Discretized	
(2) Network representation	Abstract physical traffic network	Time-discretized space-time network	
(3) Objective function	Eqns (1a)-(1f) with multiple goals	Eqn (1) with travel cost only	
(4) Coupling constraints for vehicle selection of household member	Constraints (40a)-(40b)	Embedded in the modified physical network	
(5) Vehicle spatial connectivity constraints	Constraints (2), (3), (4’), (5’) and (6)	Constraints (2)-(6) in the space-time network for modelling constraints (5)-(9)	Model 1 needs to build one specific activity duration link for each activity to represent the activity process
(6) Vehicle temporal constraints	Constraints (7)-(10)		
(7) Household spatial constraints	Constraints (26)-(30)		
(8) Household temporal constraints	Constraints (31)-(33)		
(9) Illogical activity constraints	Constraints (21)-(24) and (36)-(39)		
(10) Vehicle capacity constraints	Constraints (14)-(17)	Always satisfied (solo driving pattern)	
(11) Activity time window constraints	Constraints (11)-(13) and (34)-(35)	Embedded in the space-time network	Model R4 provides a starting time window and return-home window for each activity, but in Model 1 each activity only has a starting time window and does not have the return-home window. Instead, each household member has a return-home window for his/her arrival at home
(12) Travel cost/time budget constraint	Constraints (18)-(19)	Not considered but can be easily added	
(13) Variable definitional constraints	Binary and continuous variables	Binary variables only	Model R4 is a mixed integer linear programming model. Model 1 is a 0-1 integer linear programming model.

3.2 space-time-state network-based optimization model for Case B

Before presenting the model for case B, it should be emphasized that the space-time-state network needs be pre-built and satisfies the given time windows of each activity and the predefined arc attributes, such as, the location of each node, the travel time or travel cost of each arc, and the logically feasible state transition in the three-dimension network. More importantly, the slate of passengers’ activity-performing states in the final solution for each vehicle exactly depends on the type of different activities, mandatory activity vs. discretionary activity. As shown in Fig. 5 (a) in section 2.3, when the two activities are mandatory for passenger 2, the super starting state at the origin and super ending state at the destination are [0,0] and [2,2], respectively. On the other hand, when only one of two activities needs to be executed in a daily schedule in Fig. 5(b), the final arrival state could be [2,0] or [0,2], with a virtual ending state shown in Fig. 5(a).

Similarly, if the two activities are optional, the final state could be one of four possible alternatives [0,0], [2,0], [0,2] or [2,2], while the final selection of the optimal activity states is highly depending on the vehicle and time resources it consumed along the daily activity chain as well as the corresponding objective function in terms of benefit and travel costs. To satisfy the flow balance constraint for a network flow programming model, we need to build a virtual super ending state, as shown in Fig. 8(b), with connections from those possible ending states at the physical destination, e.g., four states in the above example, [0,0], [2,0], [0,2] and [2,2]. This state-transition based modeling paradigm could systematically capture the complicated possible interactions between multiple household members in a daily scheduling process.

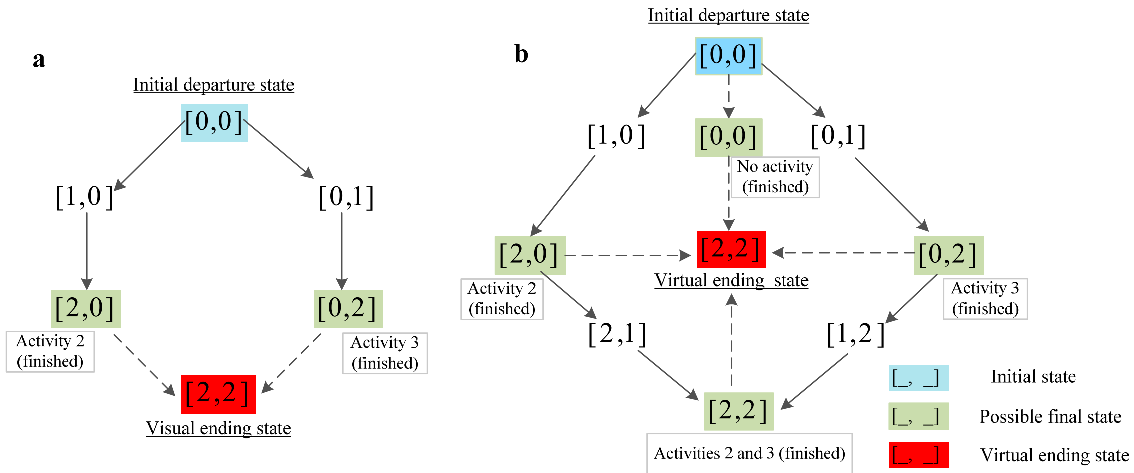

Fig. 8 State transit graph (a) one of two activities should be performed; (b) Two activities are optional

Based on the prebuilt 3D space-time-state network and given ride-sharing patterns, we now present our optimization model that satisfies all requirements in Case B that provides the optimal vehicle route guidance to the driver(s) to enable the scheduling of everyone's activities.

Model 2:

Objective function

$$\min \sum_{v} \sum_{(i,j,t,s,w,w')\in E} (c^{v}_{i,j,t,s,w,w'} \times x^{v}_{i,j,t,s,w,w'}) \tag{7}$$

Subject to,

(1) Flow balance constraint for each vehicle:

$$\sum_{i,t,w:(i,j,t,s,w,w')\in E} x^{v}_{i,j,t,s,w,w'} - \sum_{i,t,w:(j,i,s,t,w',w)\in E} x^{v}_{i,j,t,s,w,w'} = \begin{cases} -1 & j=O(v), s=DT(v), w=[0,0,\ldots,0] \\ 1 & j=D(v), s=T, w=[2,\ldots,2] \\ 0 & otherwise \end{cases}, \forall v \tag{8}$$

(2) Mandatory activity performing constraint for the driver on the activity arcs (including ride-sharing):

$$\sum_{i,t,w:(i,j,t,s,w,w')\in E(v,a_m)} x^{v}_{i,j,t,s,w,w'} = 1, \ \forall v, \forall a \in A(v) \tag{9}$$

(3) Binary variable: $x^{v}_{i,j,t,s,w,w'} = \{0,1\}$

The objective function in Eq. (7) aims to minimize the total travel cost of the household, including the travel cost of vehicles and the benefit from everyone's performed activities. Eq. (8) is the standard vehicle-based flow balance constraint. With the given initial departure state [0,0, … ,0] and virtual ending states for each vehicle, the given activity requirements of each passenger have been embedded in the space-time-state network. Similar to Eq. (4), Eq. (9) ensures that the household driver can finish his/her mandatory activity with given time windows and time duration, which means that the activity duration arc of each mandatory activity should be visit exactly once by the driver/vehicle. The decision variable $x^{v}_{i,j,t,s,w,w'}$ is a binary variable that indicates whether or not the arc (i,j,t,s,w,w') will be chosen in the space-time-activity path of vehicle v. Finally, the model we proposed is also a 0-1 integer linear programing model, which has one more dimension compared to Case A but still can be directly solved in GAMS in a reasonable-size network.

In our Case B, the ridesharing pattern is prescribed, so the case can be viewed as a sub-problem of Case V in the paper (Recker, 1995). In Case V of Recker (1995), it requires to build drop-off and pick-up nodes at each activity location and the set of available vehicles is expanded by designating driver seat and passenger seat(s) for each vehicle. The corresponding model has six categories of constraints, including vehicle temporal constraints, household member temporal constraints, vehicle spatial constraints, household member spatial constraints, vehicle capacity and budget constraints, and vehicle and household member coupling constraints. In our Case B, we also build drop-off and pick-up nodes for each activity with specific time windows. Since the ridesharing pattern is given a priori and modelled as a pair of drop-off-first then pick-up actions, we do not need identify the specific driver seat and passenger seat(s), and the coupling constraints for vehicle and household member is automatically coded through the state transition graph or explicitly taken as activity-performing constraints in Eq. (9). The temporal and spatial constraints of vehicle and household member are all embedded in the well-structured space-time-state network where the state transition graph defines the possible activity visit sequences of passengers/vehicles.

It should be reminded that if we treat the start node and the end node of the activity duration link for the driver as a drop-off node and pick-up node, respectively, the driver's activities can also be added into the cumulative activity-performing state. As a result, side constraints (9) can also be embedded in the space-time-state network, and the mathematical model above is reduced to a time-dependent state-dependent least cost path-finding problem, which could be efficiently solved by dynamic programming with parallel computing technology on large-scale networks.

As a remark, it is also possible to define another state instead of cumulative activity-performing state to model Case B. Based on the specific requirements in one problem, different state definitions could lead to different model formulations (less or more side constraints), different network structure and computation complexity. One specific example can be found in recent papers by Mahmoudi and Zhou (2016) and Mahmoudi et al., (2016) where they applied vehicle carrying state {0,1} and vehicle cumulative service state {0,1,2} to solve the VRPPDTWs, respectively, with different model formulation, networks, and algorithms. Therefore, our proposed formulation for Cases A and B is not the only possible modelling choice, and one should examine the size of state variables and nature of complex constraints to reformulate the problem based on the preferred network structure and available space and time complexity requirements.

3.3 Link capacity constraints of Case C

Since Case A considers a solo-driving pattern, one vehicle can only carry one person. In the mathematical model of Section 3.1, the person-based formulation is equivalent to the vehicle-based model. After converting the hourly road capacity into "specific time interval-based travelling arc capacity in the space-time network, the tight arc capacity constraint can be formulated as,

$$\sum_{p} x^{p}_{i,j,t,s} \le cap_{i,j,t,s}, \forall (i,j,t,s) \in E \tag{10}$$

To consider the "queue spillback" phenomenon, additional inequality needs be added to represent the link storage capacity constraint by using cumulative arrival counts and cumulative departure counts on that link. The detailed formulation can be found in the paper by Li et al. (2015).

Similarly, since the mathematical model of Section 3.2 is vehicle-based formulation, the tight capacity constraint can be formulated as,

$$\sum_{v} x^{v}_{i,j,t,s,w,w'} \le cap_{i,j,t,s,w,w'}, \forall (i,j,t,s,w,w') \in E \tag{11}$$

Regarding the queue spillback and congestion propagation property from Newell's simplified Kinematic wave model, the specific formulation is similar to the constraints in the paper by Li et al. (2015), but with one more dimension *w*.

As stated at the end of Section 3.2, the driver's activity participation constraint can also be embedded in the space-time-state networks so that case B becomes a time-dependent state-dependent least cost path-finding problem. When the road resource capacity constraint (11) is recognized, there are two research directions to solve our proposed system optimal problem for large-scale real-world applications:

(1) Lagrangian relaxation: the link/arc capacity constraints can be dualized to objective function (7), so a new time-dependent state-dependent least cost path problem is transformed in the Lagrangian relaxation framework to obtain a lower bound. Since the optimal sub-gradient in binary integer programming model is hard to be obtained, the gap between the lower bound and the optimal solution cannot be well analytically proved. Meanwhile, when more side constraints from queue spillback consideration are taken into account, dualizing those constraints might not be a suitable approach.

(2) Queue-based simulation: Since our proposed model is a system optimal problem considering complex traffic dynamics, we can apply event-based simulation to solve the large-scale problem where (i) event-based simulation process is consistent with the time-discretized space-time-state network, (ii) different travel flow models can be handled, and (iii) the marginal cost analysis by Ghali and Smith (1995) can be used to calculate the least marginal cost path for system optimal solutions. The specific algorithm design can refer to the paper by Lu et al. (2016), which proposed a simulation framework to solve agent-based eco-system optimal traffic assignment in congested networks.

4. Numerical experiments

4.1 Small-scale experiment for Case A

The proposed model for Case A in Section 3.1 will be tested in the following network shown in Fig. 9(a), where there are two household members $\boldsymbol{p_1}$ and $\boldsymbol{p_2}$, two available vehicles $\boldsymbol{v_1}$ and $\boldsymbol{v_2}$, and four candidate activities $\boldsymbol{a_1}$, $\boldsymbol{a_2}$, $\boldsymbol{a_3}$ and $\boldsymbol{a_4}$. Household member $\boldsymbol{p_1}$ can choose any one of the two vehicles, and has one mandatory activity $\boldsymbol{a_1}$ to meet with others and one optional activity to swim. Household member $\boldsymbol{p_2}$ can only choose $\boldsymbol{v_2}$ and will go to one of the two shopping malls. The corresponding modified network is constructed in Fig. 9(b) where nodes 1 and 2 are origin nodes, nodes 3 and 4 are vehicle nodes, and node 5 is the final destination node. It is observed from the activity links that the time durations and costs for performing activities 1 to 4 are (60, -20), (30, -10), (30, -15), and (20, -20), respectively. The specific time windows are listed in Table 3. The waiting cost at each time interval is 0 at origin and destination nodes and 1 at activity nodes.

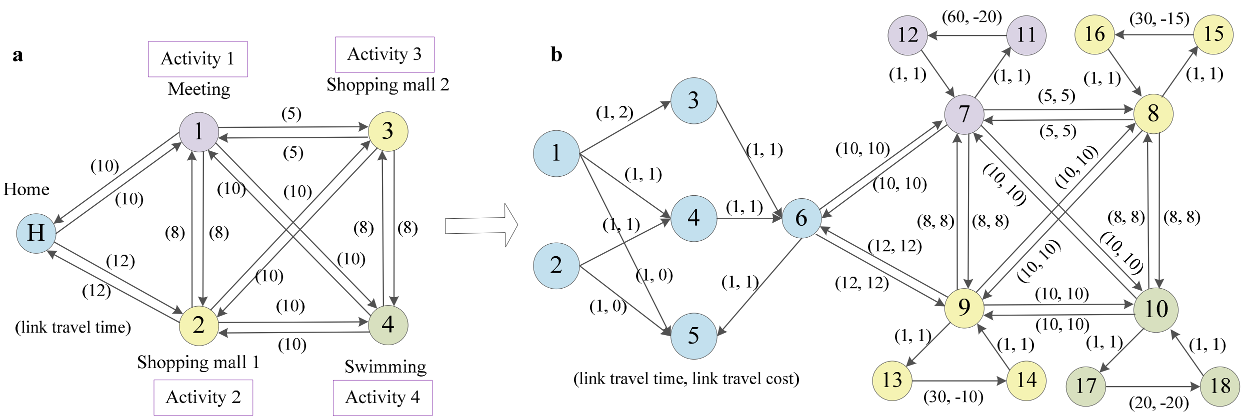

Fig. 9 (a) Physical network; (b) Corresponding modified network

Table 3. Specific time window for each event

Location (node)	1	2	5	11	13	15	17
Time window	[1, 3]	[1, 3]	[1, 130]	[15, 18]	[15, 18]	[18, 20]	[86, 90]

Our proposed 0-1 integer linear programming model for this example is solved in GAMS. The related source code can be downloaded at the website: https://www.researchgate.net/publication/306459026_Experiment_1_1. Finally, the total travel cost of this household is 24. The specific optimal solution is listed in Table 4, and can be also illustrated in Fig. 10.

Table 4. Optimal solution for each household member

Household member p_1: $x^1_{i,j,t,s} = 1$				Household member p_2: $x^2_{i,j,t,s} = 1$			
i	j	t	s	i	j	t	s
1	3	3	4	2	4	1	2
3	6	4	5	4	6	2	3
6	7	5	15	6	9	3	15
7	11	15	16	9	13	15	16
11	12	16	76	13	14	16	46
12	7	76	77	14	9	46	47
7	6	77	87	9	6	47	59
6	5	87	88	6	5	59	60

It is observed that p_1 should not go to activity 4 (swimming) and p_2 does not need to go to activity 3 (shopping mall 2) due to the trade-off between the required travel costs and corresponding activity benefits. Therefore, if we increase the benefits of activities 3 and 4 to 17 and 23, respectively, the optimal solution will be that (i) the total cost is 22, (ii) household member p_1 will visit activities 1 and 4 sequentially and then go back home, and (iii) household member p_2 will visit activity 3 (shopping mall 2) rather than activity 2. Meanwhile, if we assume that the link travel time increases due to tight link capacity constraints when more other household activity trips are considered, the activity pattern of this household is expected to change again. In short, the final activity selection and route guidance are comprehensively evaluated and selected based on the possible time-varying travel cost in the physical network, available time windows, and the benefits of performing individual available activities.

4.2 Small-scale experiment for Case B

This section will test our proposed model for Case B in Section 3.2 based on the network shown in Fig. 10(a), where there are three household members with one driver and two passengers. They will share one vehicle to perform their daily activities. The driver $\boldsymbol{p_1}$ has one mandatory activity $\boldsymbol{a_1}$ and needs to drop off and pick up two passengers to conduct their activities. Passenger $\boldsymbol{p_2}$ has one mandatory activity $\boldsymbol{a_2}$ and one optional activity $\boldsymbol{a_3}$, and passenger $\boldsymbol{p_3}$ has one mandatory activity $\boldsymbol{a_4}$. The corresponding modified network is plotted in Fig. 10(b) where the activity of the driver is represented by one specific activity link and each activity node of passengers is added with two additional nodes as drop-off node and pick-up node.

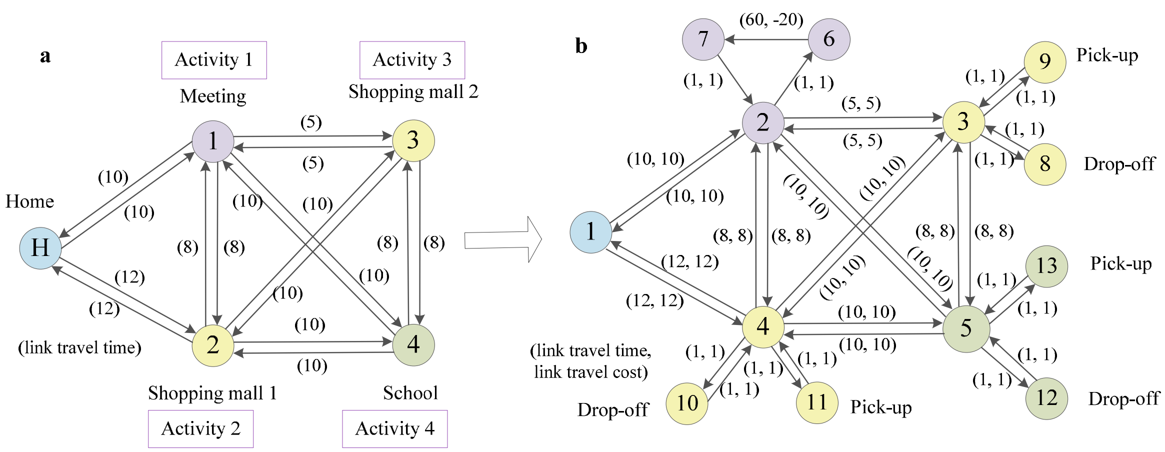

Fig. 10 (a) Physical network; (b) Corresponding modified network

Based on the procedure explained in Section 3.2, the state has three slots as [_, _, _], of which the first two slots are for activities 2 and 3 of passenger p_2 and the last slot is for activity 4 of passenger p_3. We still use cumulative activity-performing state of $\{0,1,2\}$ as before. The time window for each event is listed in Table 5.

Table 5. Specific time window for each event

Location (node number)	Node 1 (departure)	Node 1 (arrival)	Node 10	Node 11	Node 12	Node 13	Node 8	Node 9	Node 6
Time window	[1, 3]	[1, 170]	[15, 16]	[114, 115]	[28, 30]	[127, 139]	[127, 129]	[137, 139]	[41, 43]

Based on the time window information, it is impossible that (i) activity 3 happens before activity 2, (ii) the drop-off event and pick-up event of activity 4 happens before those of activity 2, respectively, and (iii) the drop-off event and pick-up event of activity 3 happens before those of activity 4. Therefore, the remaining possible states will be [0, 0, 0], [1, 0, 0], [1, 0, 1], [2, 0, 1], [2, 0, 2], [2, 1, 1], [2, 1, 2], and [2, 2, 2]. For the convenience of implementation in algorithms, we can label each state with one corresponding ID, such as, using 1 to 8 to represent the eight states above sequentially. The final possible state transition is demonstrated in Fig. 12, where virtual arcs with virtual ending state are also built for developing a single-origin-to-single-destination problem. In addition, the benefit or negative cost of performing activity for all passengers is assumed to occur during the state transition at drop-off nodes, as illustrated in Section 2.3. The negative travel costs for activities 1,2, 3, and 4 are given as -20, -10, -15, and -15.

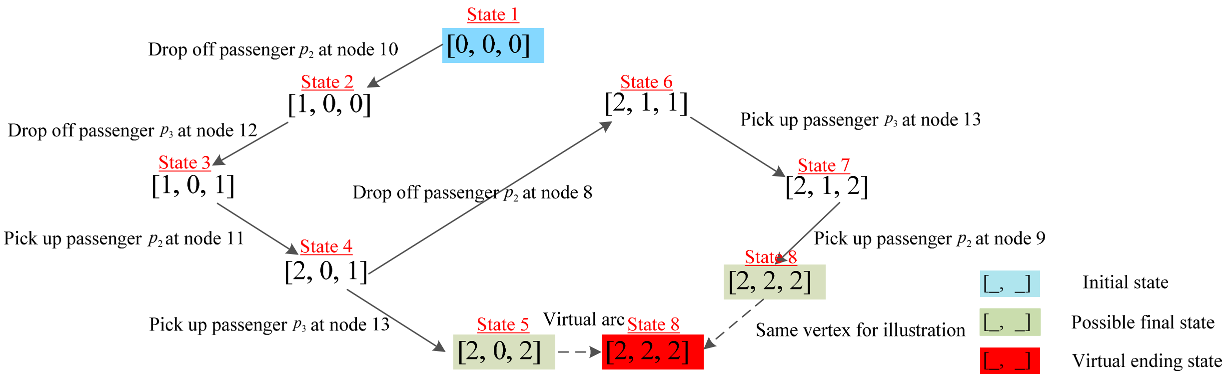

Fig. 11 State transition graph

Based on the constructed space-time-state network, our proposed 0-1 integer linear programming model for this example is solved in GAMS. The related source code can be downloaded at the website: https://www.researchgate.net/publication/306458887_Experiment_2_1. Finally, the total travel cost of this household is 40. The specific optimal solution is listed in Table 6.

Table 6. Optimal solution for the household

The only vehicle: $x^1_{i,j,t,s,w,w'} = 1$													
i	j	t	s	w	w'	Remarks	i	j	t	s	w	w'	Remarks
1	4	3	15	1	1	Depart at home at time 3	7	2	103	104	3	3	
4	10	15	16	1	1		2	4	104	112	3	3	
10	10	16	16	1	2	State transition (passenger p_2 is dropped off at node 10 for activity 2)	4	4	112	113	3	3	
10	4	16	17	2	2		4	11	113	114	3	3	
4	5	17	27	2	2		11	11	114	114	3	4	State transition (passenger p_2 is picked up at node 11 for activity 2)
5	12	27	28	2	2		11	4	114	115	4	4	
12	12	28	28	2	3	State transition (passenger p_3 is dropped off at node 12 for activity 4)	4	5	115	125	4	4	

12	5	28	29	3	3		5	5	125	126	4	4	
5	2	29	39	3	3		5	13	126	127	4	4	
2	2	39	40	3	3		13	13	127	127	4	5	State transition (passenger p_3 is picked up at node 13 for activity 4)
2	2	40	41	3	3		13	5	127	128	5	5	
2	6	41	42	3	3		5	2	128	138	5	5	
6	6	42	43	3	3		2	1	138	148	5	5	Arrive at home at time 148
6	7	43	103	3	3	The driver p_1 performs activity 1	1	1	148	149	5	8	State transition (from final state to assumed final state, the virtual arc cost is 0)

It is observed that passenger p_2 will not perform activity 3 due to the trade-off between the required travel costs and corresponding activity benefits. If we increase the benefit of activity 3 from 15 to 20, the optimal solution will change to be that (i) the total cost is 37, and (ii) activity 3 will be performed by passenger p_2. Moreover, when the link travel time is modelled as a time-dependent attribute due to road congestion effect, the final household activity pattern is expected to change accordingly.

4.3 Medium-scale experiment within a Lagrangian relaxation framework using cumulative activity-performing state

This section aims to examine the computation efficiency of using cumulative activity-performing state for a general HAPP in a medium-scale transportation network. We choose a subarea of Phoenix regional network as our study case with 1186 nodes, 3164 links and 387 activity locations, shown in Fig. 12. The given input data for this experiment are listed in Table 7.

Table 7. The input data of this experiment

agent_id	agent_type	from_node_id	to_node_id	departure_time_start	departure_time_window	arrival_time_start	arrival_time_window	base_profit	optional
1	0	23	23	30	5	40	5	150	0
2	0	24	24	10	20	70	10	133.33	0
3	0	26	26	40	10	60	5	83.33	0
4	0	25	25	20	20	80	5	150	0
5	0	39	39	70	5	90	5	133.33	0
6	0	35	35	20	5	110	5	183.33	0
7	0	38	38	35	10	120	5	133.33	1
Veh 1	1	13	13	1	1	120	1		
Veh 2	1	13	13	1	1	120	1		

"agent_id" could be activity id or vehicle id, "agent_type" = 1 for vehicles, and 0 means activities. Field "from_node_id" and "to_node_id" are the same and define (i) the activity performing location or (ii) vehicle's origin/destination (home). "departure_time_start" defines the start time of activity or vehicle departure, and "depature_time_window" is the feasible time window duration. "arrival_time_start", and "arrival_time_window" defines the activity/vehicle end time window. "base_profit" is the benefit/utilities of performing the corresponding activity. The "optional" flag indicates that if an activity is optional, its value is 1, otherwise it is mandatory as 0. As a result, the problem becomes that two vehicles at home (node 13) plans to perform 6 mandatory activities and 1optional activity.

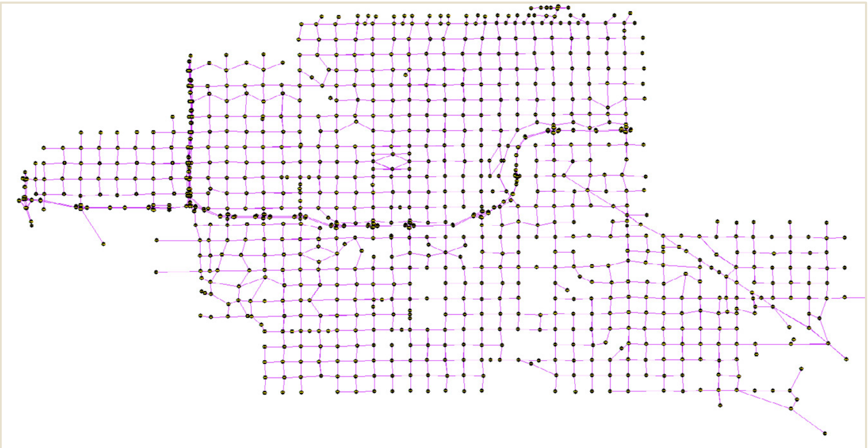

Fig. 12 One subarea of Phoenix regional transportation network

To solve this problem, we use cumulative activity-performing state {0,1,2} to record the activity completion process. It is reminded that the maximum number of possible states could be 3^7 for 7 activities. In order to model the competition for one activity by two vehicles simultaneously, we dualize that constraint to our objective function and adopt the forward dynamic programming algorithm within a Lagrangin relaxation framework, which can refer to the process of solving the VRPPDTW for multiple vehicles by Mahmoudi and Zhou (2016). The related C++ source code and data set can be downloaded at the website: https://github.com/xzhou99/Agent-Plus/tree/master/HAPP. Table 8 lists the impact of different number of activities on the CPU computation time of 5 Lagrangian iterations (for distributing different tasks to two vehices) and computer memory usage. In the above case, the vehicle/activity preference for household members is not considered. If a pre-specified vehicle-to-activity mapping is given, the search space in the space-time-state network could be further reduced.

Table 8. CPU computation time and memory use under different number of activities

# of activities	Maximum numbers of activity performing states	CPU time (seconds)	RAM (GB)
4	81	15.5	0.3
5	243	38.2	1.3
6	729	112.3	3.6
7	2187	337.4	11.3

4.4 Large-scale experiment within a simulation-based framework with simplified activity representation and road capacity constraints

This section aims to present the initial test result of the simulation-based approach for system optimal dynamic vehicle routing under road capacity constraints. The Salt Lake City regional traffic network is selected and it has 13,923 nodes, 26,768 links and 2,302 zones. The total number of simulated vehicles is about 1.35 million from 15:00 to18:00. The traffic flow model chooses point queue model, which just considers the tight road capacity constraints. The details of implementing spatial queue model and Newell's simplified kinematic wave model by simulation can be found in the paper (Zhou and Taylor, 2014).

This experiment can be treated as a special version of Case A. Each origin zone is analogous to one household and those destination zones can be viewed as those mandatory activity locations. The process that vehicles depart from origin to destination is like that household members complete their mandatory activities with flexible time windows. The simulated average trip time index (mean trip simulated travel time/trip free-flow travel time) of 100 iterations is depicted in Fig. 13 and finally shows a convergence pattern. A parallel computing technique (Qu and Zhou, 2017) is embedded in the simulation process, and the search process for single activity is extremely simple compared to the full scale space-time-state search presented in the medium-scale example. The computational time for one iteration is just 1 min 25sec in our workstation with 40 available CPU threads and 192G memory. As stated in the paper (Lu et al., 2016), this simulation algorithm still needs further improvements on path marginal travel time calculation and step size optimization of each iteration.

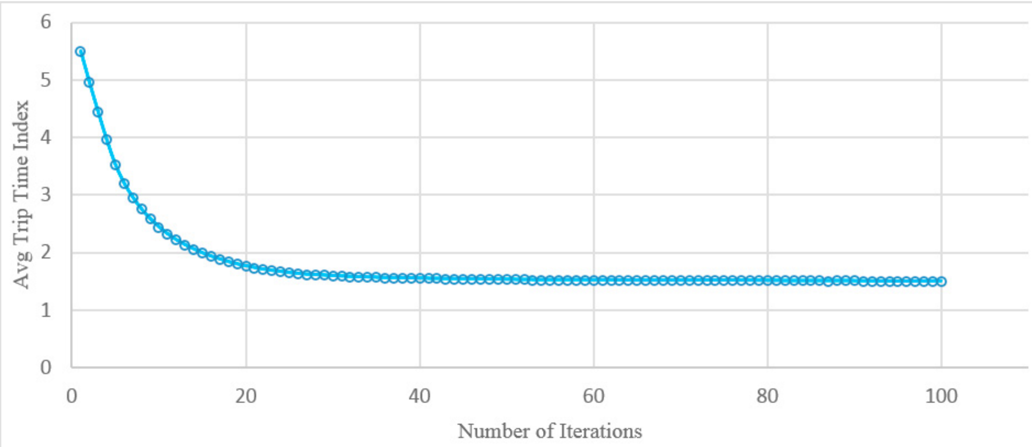

Fig. 13 Average trip time index of each iteration

5. Conclusion and future research

By embedding a set of hard constraints into a well-structured (space-time and space-time-state) network structure, we reformulate two difficult cases in HAAP using a person-based or vehicle-based network flow programming model with very few side constraints, which could be directly solved by some standard optimization solvers, or directly further solved by time-dependent state-dependent shortest path algorithms. Meanwhile, the tight road capacity is highly considered for capturing the underlying congestion effect for the cases above. The numerical experiments demonstrate our proposed methodology and analyze the impacts of different activity benefits on the final vehicle routing and household member activity selection.

It is important to further consider vehicle selection (mode choice) and ride-sharing simultaneously so that an optimal travel pattern, such as driving alone or ride-sharing, can be found for each household. Then the household optimal principle can be applied to the whole household system and finally reaches a dynamic household-level equilibrium. We are also currently conducting additional numerical experiments on large-scale networks with realistic household activity data to examine the sensitivity and possible improvements of the space-time-state routing algorithms.

Acknowledgements

We would like to thank Dr. Yunchao Qu at Beijing Jiaotong University in China for the insightful discussion. This paper is partially supported by National Science Foundation – United States under Grant No. CMMI 1538105 “Collaborative Research: Improving Spatial Observability of Dynamic Traffic Systems through Active Mobile Sensor Networks and Crowdsourced Data”. The work presented in this paper remains the sole responsibility of the authors.

References

Arentze, T. A., Timmermans, H. J., 2004. A learning-based transportation oriented simulation system. Transportation Research Part B *38*(7), 613–633

Bhat, C. R., Guo, J. Y., Srinivasan, S., Sivakumar, A., 2004. Comprehensive econometric micro-simulator for daily activity-travel patterns.

Transportation Research Record: Journal of the Transportation Research Board 1894(1), 57–66.
Chow, J.Y., Djavadian, S., 2015. Activity-based market equilibrium for capacitated multimodal transport systems. Transportation Research Part C 59, 2-18.
Chow, J. Y., Recker, W. W., 2012. Inverse optimization with endogenous arrival time constraints to calibrate the household activity pattern problem. Transportation Research Part B 46(3), 463–479.
Fu, X., Lam, W.H., Xiong, Y., 2016. Modelling intra-household interactions in household's activity-travel scheduling behaviour. Transportmetrica A: Transport Science 12(7), 1-21.
Ghali, M.O., Smith, M.J., 1995. A model for the dynamic system optimum traffic assignment problem. Transportation Research Part B 29 (3), 155–170.
Kang, J.E., Chow, J.Y., Recker, W.W., 2013. On activity-based network design problems. Transportation Research Part B 57:398-418
Kang, J.E., Recker, W.W., 2013. The location selection problem for the household activity pattern problem. Transportation Research Part B 55, 75-97.
Li, P., Mirchandani, P., Zhou, X., 2015. Solving simultaneous route guidance and traffic signal optimization problem using space-phase-time hypernetwork. Transportation Research Part B 81(1), 103–130.
Liao, F., Arentze, T., Timmermans, H., 2013. Incorporating space–time constraints and activity-travel time profiles in a multi-state supernetwork approach to individual activity-travel scheduling. Transportation Research Part B 55, 41–58.
Lin, D.Y., Eluru, N., Waller, S., Bhat, C., 2008. Integration of activity-based modeling and dynamic traffic assignment. Transportation Research Record: Journal of the Transportation Research Board 2076, 52-61.
Liu, J., Zhou, X., 2016. Capacitated transit service network design with boundedly rational agents. Transportation Research Part B 93, 225-250.
Lu, C.C., Liu, J., Qu, Y., Peeta, S., Rouphail, N.M., Zhou, X., 2016. Eco-system optimal time-dependent flow assignment in a congested network. Transportation Research Part B 94, 217–239.
Mahmoudi, M., Zhou, X., 2016. Finding Optimal Solutions for Vehicle Routing Problem with Pickup and Delivery Services with Time Windows: A Dynamic Programming Approach Based on State-space-time Network Representations. Transportation Research Part B 89, 19-42.
Mahmoudi, M., Chen, J., Zhou, X., 2016. Embedding Assignment Routing Constraints through Multi-Dimensional Network Construction for Solving the Multi Vehicle Routing Problem with Pickup and Delivery with Time Windows. arXiv:1607.01728.
Miller, E. J., Roorda, M. J., 2003. Prototype model of household activity-travel scheduling. Transportation Research Record: Journal of the Transportation Research Board 1831 (1), 114–121.
Pendyala, R., Konduri, K., Chiu, Y.C., Hickman, M., Noh, H., Waddell, P., Wang, L., You, D., Gardner, B., 2012. Integrated land use-transport model system with dynamic time-dependent activity-travel microsimulation. Transportation Research Record: Journal of the Transportation Research Board 2303, 19-27.
Pendyala, R., Kitamura, R., Kikuchi, A., Yamamoto, T., Fujii, S., 2005. Florida activity mobility simulator: overview and preliminary validation results. Transportation Research Record: Journal of the Transportation Research Board 1921(1), 123–130.
Pendyala, R., You, D., Garikapati, V., Konduri, K., Zhou, X., 2017. Paradigms for integrated modeling of activity-travel demand and network dynamics in an era of dynamic mobility management. The 96th Annual Meeting of the Transportation Research Board, accepted for presentation only.
Pribyl, O., Goulias, K.G., 2005. Simulation of daily activity patterns incorporating interactions within households: Algorithm overview and performance. Transportation Research Record: Journal of the Transportation Research Board 1926, 135-141.
Psaraftis, H.N., 1983. An exact algorithm for the single-vehicle many-to-many dial-a-ride problem with time windows. Transportation Science 17 (3), 351–357.
Qu, Y., Zhou, X., 2017. Large-scale dynamic transportation network simulation: a space-time-event parallel computing approach. Transportation Research Part C 75, 1-16.
Recker, W.W., 1995. The household activity pattern problem: general formulation and solution. Transportation Research B 29 (1), 61-77.
Tong, L., Zhou, X., Miller, H.J., 2015. Transportation network design for maximizing space–time accessibility. Transportation Research Part B 81(2), 555–576.
Zhou, X., Taylor, J., 2014. DTALite: a queue-based mesoscopic traffic simulator for fast model evaluation and calibration. Cogent Engineering 1 (1), 961345.