Essays on Cross-Country Inequality and Income Differences

by

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These essays are my attempt to answer a big picture question in economics "why some countries are richer than others?". In the first chapter, I document that for a group of 38 countries ranging from low to high income, managers in richer countries are more skilled, and the relative income of managers to non-managers along with skill premium is lower in richer countries. I use a model of investment in skills and occupational choice in which countries differ in productivity level and size-dependent distortions. I find that exogenous productivity differences alone can produce the above facts qualitatively, but size-dependent distortions are needed to account for these facts quantitatively.

Chapter two\(^1\) accounts for the sources of world productivity growth, using data for more than 36 industries and 40 economies. Productivity growth in advanced economies slowed but emerging markets grew more quickly, which kept global productivity growth relatively constant until 2010. World productivity growth is highly volatile from year to year, which primarily reflects shifts in the reallocation of labor. Deviations from Purchasing Power Parity account for about a third of the shifts. Though markups are large and rise over time, they only modestly affect measured industry-level productivity growth.

In chapter three, I document that the mean and dispersion of pre-tax labor earnings grow faster over the life-cycle in the U.S. than in some European countries and individuals with at least a college degree are key for these facts. I use a life-cycle model of human capital accumulation and elastic labor supply which features non-linear taxation and a college choice and investments during college. The model economy is consistent with earnings distribution among college and non-college individuals in

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\(^1\)This chapter was jointly written with John G. Fernald and Bart Hobijn.
the U.S. Non-linear taxation suppresses pre-tax earnings, reduces college attendance and investments during college. More generous subsidies for college exacerbate labor earning inequality. Differences in taxation and college subsidies account for 94% of the differences in mean earnings, and 80% of the differences in inequality over the life-cycle across the U.S. and European countries.
DEDICATION

To my parents
ACKNOWLEDGMENTS

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Chapter 1

INVESTMENT IN SKILLS, MANAGERIAL QUALITY, AND ECONOMIC DEVELOPMENT

1.1 Introduction

It is well-established that the observed cross-country income differences are large. Development accounting methodology reveals that differences in total factor productivity (TFP) are crucial in understanding why some countries produce more than others.\(^2\) The question then becomes, what determines productivity differences across countries?

There are multiple proposals for the roots of productivity differences such as measurement of the physical capital and its composition, measurement of the quality of human capital, and monopolistic barriers to technology adoption. There is a relatively recent empirical literature that highlights the role of management practices as a root of productivity differences.\(^3\) Management practices vary considerably across countries and across firms within a country and they manifest themselves on the aggregate level by higher total factor productivity and output.

In this paper, I present novel evidence on educational attainment of managers, which I define to be managers based on occupational classification and self-employed, and its relation to output per worker. I first document that the share of managers with more than a high school degree among all managers is higher in richer countries.


\(^3\)See Bloom and Reenen (2011) and Bloom et al. (2011) among others.
I refer to managers with more than a high school degree as skilled. For example, the share of skilled managers in the U.S. is 71%, while it is 17% in Mexico and 14% in Brazil. The correlation between the share of skilled managers and GDP per worker is 0.18. I do this using micro data for a set of low-, middle- and high-income countries.

I subsequently document that the relative income of managers to non-managers, which I call managerial premium, is smaller in richer countries. For example, the managerial premium in the U.S. is 1.4 while it is 1.5 in Mexico and 1.7 in South Africa. The correlation between managerial premium and GDP per worker is -0.4. To my knowledge, this is the first paper to document such a fact about relative income of managers across countries.\textsuperscript{4} I also document that the share of managers among the working age population (age 15-64), which I refer to as managerial rate, is lower in richer countries. For instance, the share of managers in the U.S. is 22% while it is 42% in Mexico and 37% in Greece and the correlation between the managerial rate and GDP per worker is -0.2.

There is the well-established fact that the mean plant size is larger in richer countries. Bento and Restuccia (2017) document this fact using the survey data for the manufacturing sector and find that the correlation between the mean plant size and GDP per worker is 0.33. So richer countries have less managers, more skilled managers and those managers operate larger plants. Finally, I document that the relative income of skilled versus unskilled individuals in the working-age population, which I refer to as skill premium, is lower in richer countries. For example, the skill premium in the U.S. is 1.8 while it is 4.0 in Mexico and 4.3 in Brazil. This finding is not new and Fernandez et al. (2005) documented a similar trend. What is new

\textsuperscript{4}Guner et al. (2018) document that the relative income of managers versus non-managers grow faster over the life-cycle in richer countries. My study abstracts from this growth and document a different fact about the income levels rather than growth.
here is that the correlation between managerial premium and skill premium is 0.6. This shows that the above facts about skill segregation and relative income should be interpreted in a unified framework to better understand the income differences across countries. I develop such a framework in this paper.

I will interpret the differences in management practices, documented by the empirical literature, in my framework as the differences in "managerial quality" coming from differences in selection into management occupation, along the lines of Lucas (1978), and differences in investment in skills. Hence in my framework, the incentives for investment in skills and occupational choice and the resulting endogenous skill distribution are at the core of cross-country income differences.

To study the effects of selection and investment as a source of income differences across countries, I develop a span-of-control model with heterogeneous agents with respect to two generic notions of talent: schooling talent and managerial talent. There is a stand-in household who maximizes the lifetime utility of each household member and every period, a large number of new members are born. The household decides about investment in skills (based on schooling talent), how much to invest in skills, and occupational choice (based on managerial talent), i.e. who becomes a manager within the pool of skilled and unskilled members. The rest of the members become skilled and unskilled workers and their schooling talent is their efficiency units of labor. A key feature of the model is that investment in skills augments both schooling and managerial talents and I will refer to this "augmentation" as becoming skilled. So the initial heterogeneity is amplified after selection for skill investment which will in turn translate into more heterogeneity among managers. Managers use capital, skilled and
unskilled workers along with their managerial talent to operate a plant in order to produce output and collect managerial profits. There is an economy-wide productivity term in the production function which is the same for all managers, but it is a source of variation across countries. The equilibrium sum of the managerial talents of all managers (skilled and unskilled) divided by the number of managers is the average managerial quality in this framework.

Investment in skills in the model indicates the costs (investing resources rather than consuming them) and the benefits (the future reward because of having higher schooling and managerial talents). Since the input for the investment in skills is consumption goods, a lower level of aggregate productivity lowers the incentive for the household to invest in its members’ skills. The lower investment in skills results in lower average managerial quality.

A fundamental element of the model is the complementarity between schooling talent and investment in skills. The members that have higher schooling talent are selected for skill investment, which means an investment in production of both schooling and managerial talent. This selection amplifies and propagates the initial differences in talents and in a stationary equilibrium along a balanced growth path, skilled managers operate larger and more productive production plants.

I calibrate the parameters in the model to match a set of cross-sectional and aggregate facts of the U.S. economy: macroeconomic statistics, composition of working-age population with regard to education (skilled vs unskilled) and occupation (managers and non-managers), and also the composition of managers regarding their educational attainment (skilled vs unskilled managers). Remarkably, the model can reproduce the

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In this paper, I refer to a production unit as a plant. I use the word "establishment" and "plant" interchangeably.
central features of the U.S. working-age population based on education and occupation. The model also does an excellent job of generating the skill premium and managerial premium.

The role of size-dependent distortions as a source of income differences across countries has been emphasized in the literature of misallocation. To study this role, I proceed to introduce these distortions. I model them as progressive taxes on the output of a plant and use a simple parametric function, which was originally proposed by Benabou (2002a). These distortions have two main effects in my model. First, a "rereallocation effect" as the presence of distortions suggests that capital and labor services move from more distorted (large) to less distorted (small) plants. Second, an "incentive effect" as distortions affect the incentive of investment in skills and thus the average plant level productivity. Overall, the model has a natural framework to study how changes in the level of distortions can account for differences in output per worker. It also helps to understand the differences in managerial quality, educational attainment of the working-age population, skill premium and managerial premium. For my purposes, I assume the U.S. economy is free of distortions.

I find that the model can generate the documented facts when the source of variation is productivity differences or size-dependent distortions. In order to find the elasticity of output per worker with respect to productivity, I calibrate the productivity and distortion parameters to match two fact in my sample of countries: GDP per worker and the share of unskilled working-age population. I find that the model can generate almost all of the disparities in managerial rate, share of skilled managers, managerial premium and skill premium. I then calculate the elasticity of output with respect to productivity which is 2.6. This elasticity is higher than most previous

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papers\textsuperscript{7} and the presence of investment in skills coupled with size-dependent distortions are the reasons for higher elasticity.

Consistent with the facts presented above, my model suggests that lower levels of economy-wide productivity result in both lower average quality of managers as well as lower share of skilled managers. I find that a reduction in productivity by 20\% decreases mean plant size by 10\% and the share of managers increases by 5\%. Output per worker declines by 34\% which implies an elasticity of output with respect to productivity of 2.6. This elasticity is higher than the standard neoclassical growth model because by lowering the productivity, the incentives for occupational and educational choices change. The added amplification in the model is due to the investment in talents.

Lower productivity also changes the composition of managers. The share of skilled managers in the workforce declines by 6\%, while the share of skilled managers among the pool of managers decreases by 15\%. This shows a high effect of productivity on the selection into managerial occupation. Lower productivity also means that on average, the quality of managers declines by 46\%. The decline in quality comes from two channels: lower share of skilled manages and lower investment in skill accumulation by the household.

Size-dependent distortions have similar qualitative effects to lower productivity, but the quantitative effects are more dramatic. A level of distortions that generates a similar output per capita to the case of lower productivity, increases the share of managers by 42\%. It also reduces the mean plant size by 84\% and the share of skilled managers drops by 80\%. Moreover, the average quality of managers declines by 80\%.

\textsuperscript{7}To my knowledge, only Manuelli and Seshadri (2014) finds higher elasticity which is 5.7. Their model is a life-cycle model that features investment in skills for childhood, adulthood and training on the job. So productivity differences affect more margins that I abstracted from in this work.
I then fit the model to data using two parameters (productivity and distortions parameter) and assess the performance of the model regarding other statistics. The model successfully generates skill premium, managerial premium, managerial rate and share of skilled managers observed in the data. I take it from this exercise that the important margins of occupational choice and investment in skills along with the presence of size-dependent distortions are necessary to account for the observed differences in the relevant statistics across countries.

1.2 Background

My paper builds on the research on how micro level misallocation of resources can emerge as aggregate income and productivity differences on the macro level. I focus on implicit size distortions as a source of misallocation following Guner et al. (2008) and Restuccia and Rogerson (2008). What is new here, compared to those papers, is that I explicitly model the investment in education and selection into different occupations which results in endogenous distribution of skills.

My emphasis on education and income of managers as well as investment in their skill links my paper to empirical literature on differences in management practices. It also links to the trade and development literature that study how investment in skills and R&D amplify the effects of productivity differences and distortions. Caselli and Gennaioli (2013) emphasizes the importance of management and quality of managers for cross-country income differences. More recent works show how managers and their

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8See Bloom and Reenen (2011) and Bloom et al. (2014)

9See Erosa et al. (2010a), Rubini (2014), Atkeson and Burstein (2011) and Gabler and Poschke (2013) among others.
incentives matter for aggregate productivity and the size distributions of plants.\textsuperscript{10} Guner et al. (2018) consider a life-cycle model of heterogeneous managers based on managerial abilities with investment in skills, and study their wage growth relative to non-managers across countries. In contrast, I provide a tractable model to study multiple facts about skill and occupational segregation of the working-age population along with relative income based on skill level and occupation.

My paper is also connected to work that documents plant and firm-level productivity and size.\textsuperscript{11} Poschke (2017) studies a model of occupational choice where the managerial technology improvement is biased towards the skill level of a manager, i.e. managers with higher skills benefit more from improvements in technology. He uses the skill-biased technology change as the driver of the differences in firm size distribution across countries. Bento and Restuccia (2017) present evidence on manufacturing plant size distribution using country level data and develop a model where distortions affect the investment decision in plant-level productivity. In both their models and mine, distortions are amplified by endogenous investment decisions.

1.3 Data

In this section, I document a set of facts about the number of managers, their skill composition (educational attainment) and relative income across countries using multiple individual level datasets: IPUMS-USA (Ruggles et al. (2017)), IPUMS-International (Minnesota Population Center (2018)) and the European Union Statistics

\textsuperscript{10}See Bhattacharya et al. (2013), Roys and Seshadri (2017) and Akcigit et al. (2016).

on Income and Living Conditions (EU-SILC) (2016). IPUMS-International provides harmonized Census data for a large set of countries. Only few international censuses, however, contain information both on incomes and occupations. The EU-SILC contains both cross-sectional and longitudinal micro-data data for European countries on income, work, poverty, social exclusion and living conditions.

My final sample consists of 38 countries: Austria, Belgium, Bulgaria, Switzerland, Cyprus, Czech Republic, Germany, Denmark, Estonia, Greece, Spain, Finland, France, Croatia, Hungary, Ireland, Italy, Lithuania, Latvia, Luxembourg, Malta, Netherlands, Norway, Poland, Portugal, Romania, Serbia, Sweden, Slovenia, Slovakia, United Kingdom, United States, Canada, South Africa, Brazil, Puerto Rico, Panama and Mexico.

Table (33) in appendix 6 shows the data source for each country as well as the year of the survey and the number of observations. Since I document facts about income of individuals, the sample is limited to countries that has reliable data on income and occupation. I should also emphasize that there are factors other than the initial abilities are important in less developed countries for determining the educational attainment, and occupational choice such as borrowing constraints. I abstracted from these channels and postpone them for future research.

The definition of managers is the individuals working for wage with occupational code 11 to 13 based on ISCO-88 and individuals who identify themselves as self-employed. The occupational codes cover legislators, senior officials and managers. These include occupations whose main tasks consist of determining and formulating government policies, as well as laws and public regulations, overseeing their implementation, representing governments and acting on their behalf, or planning, directing and coordinating the policies and activities of enterprises and organizations, or de-
partments. The reason to consider self-employed as managers is that self-employed have supervisory and coordinating roles in their own business and do a lot of tasks that managers who work for wage do. Individuals are classified as skill if they have strictly more than a high school degree.

Specifically, I run the following regression in each country. The dependent variable is the log of income. This income is either wage income (for workers and managers with occupational code 11-13), or self-employed income or both. I should emphasize that when an individual has more than one occupation (both wage and self-employed income), he/she is classified as self-employed or working for wage based on whichever occupation he/she spends more hours in.

\[
\log(\text{labor income}) = \alpha + \beta X + \delta_1 \mathbb{1}\{\text{skilled manager}\} + \delta_2 \mathbb{1}\{\text{unskilled manager}\} \\
+ \delta_3 \mathbb{1}\{\text{skilled worker}\} + \epsilon.
\]

The vector \( X \) contains individual-specific controls such as log of hours, age, age-squared, gender, race, marital status, veteran status. The inclusion of a variable for years of education is redundant since the skilled vs unskilled classification controls partially for years of education. The coefficients \( \delta_1, \delta_2 \) and \( \delta_3 \) are used to calculate skill premium and managerial premium.

I present three stylized facts about managers based on the sample of countries that I have and two complementary facts that was documented before in the literature, and I intend to explain these facts through the lens of my model.
Stylized Facts

1. The share of skilled managers increases with GDP per worker.

It is a well established fact that in less developed countries, the educational attainment of the workforce is generally lower than developed countries and these differences can potentially explain a large part of the differences in income per capita across countries.\textsuperscript{12} The GEM survey reveals that similar systematic differences are present even when I focus on managers. In other words, as income per capita increases, so does the educational attainment of managers. This shows an interplay between education and selection into managerial occupation.

Figure (35) shows the educational attainment of adults aged 18-64 in each country (dots) along with the educational attainment of managers (stars). Clearly the managers are more educated than the average population. This shows that simple models of education cannot account for the differences in education of managers across countries. If less developed countries have low levels of education, it seems that managers in those countries have less education as well. The figure indicates that this explanation is not enough. The managers are clearly more educated than the general population, so there is a selection for managerial occupation, but there are still systematic differences across countries, though the differences are smaller as indicated by the correlation coefficient.

2. The managerial premium decreases with GDP per worker.

The second fact about the managers is about their income. I define the relative income of managers to non-managers as the managerial premium. Figure (36) illustrates the relationship between managerial premium and GDP per worker. The correlation between the two is -0.33. This is a new finding that relate the relative income of managers to non-managers and economic development. The negative relationship means that in as one moves from poorer to richer countries, the income gap between the managers and non-managers shrinks. To calculate this premium, I control for observable characteristics of individuals such as age, race, marital status and education. These observables cannot account for the income gap between managers and non-managers. In appendix 4, I explain in more detail how the managerial premium in each country is calculated.

3. The managerial rate decreases with GDP per worker.

The third fact is about the relationship between the number of managers and economic development. Figure (37) shows that in richer countries, the share of managers among all the working individuals (age 25-64), which I call the managerial rate, is lower than poorer countries. In other words, as countries become more developed, the number of managers decline and the correlation between managerial rate and GDP per worker is -0.11. This fact is to my knowledge new. I should emphasize that the number of managers come from individual level micro data and it is based on occupation. Due to data limitations, I could not document this fact from
the business side and count the number of managers in establishments. I assume that the same pattern will emerge from analyzing such data.

4. Mean establishment size increases with income per capita.

   The forth fact is about the average plant size and its relation to economic development. There are several datasets that provide information on the size of establishments measured by the number of workers. GEM provides data only on relatively small plants which are basically the left tail of the size distribution of establishments. Bento and Restuccia (2017) provide an internationally comparable dataset for manufacturing plants using country-level data and administrative surveys. Poschke (2017) provides data on establishment size for the right tail of the distribution, namely the large establishments. Since all these data sets show that the mean establishment size increases with income per capita, I replicate the evidence by Bento and Restuccia (2017) since it covers most of the size distribution in an internationally comparable fashion. The correlation between log average plant size and GDP per worker is 0.4.

5. The skill premium decreases with GDP per worker.

   The fifth fact is about the relative income of skilled versus unskilled individuals which I call skill premium. I define a skilled individual as one with strictly more than a high school degree and unskilled otherwise. Figure (39) shows the relationship between skill premium and economic development. As economies become more developed the income gap between skilled and unskilled individuals shrinks and the correlation between skill premium and GDP per worker is -0.56. I should emphasize that this
income gap is only about skill level, regardless of occupation. In other words, I did not take into account the occupation of the individuals to look at the relative income of skilled versus unskilled. The details of calculating the skill premium after controlling for observable characteristics of the individuals are presented in appendix 4.

If I put all the facts together, the following picture emerges: In less developed countries, the share of the working individuals who are skilled is lower, the share of managers is higher while the share of skilled managers is lower. On the income side, the relative income of managers to non-managers is higher and the relative income of skilled versus unskilled individuals is also higher. Finally, managers in less developed countries operate smaller plant. In the next section, I provide a theoretical framework to study all of these facts together and I will also give an intuitive interpretation of managerial quality based on investment in skills. This unifying framework will help us understand economic development better and also works a theoretical background for the empirical literature that connects the managerial quality with economic development.

The above facts tell the following story: in less developed countries, a higher share of the population with low educational attainment start businesses and their plants are small. What is a unifying framework to study the complementarity between initial talent and investment in education? How does investment in education affect the number of plants and the quality of managers running these plants? How does exogenous variations such as economy-wide productivity affect education and occupation choices and how does the complementarity between investment in education and intrinsic talent amplifies these effects? What is the role for size-dependent distortions and how distortionary are them? The theoretical model presented next provides a tractable answer to these questions.
1.4 Theoretical Framework

There is a single representative household in the economy. The household has a continuum of measure $L_t$ members at time $t$, who only value consumption. The size of the household grows at the constant rate of $(g_L)$.\textsuperscript{13} The household is infinitely lived and maximizes

$$\sum_{t=0}^{\infty} \beta^t L_t \log(C_t/L_t)$$

where $\beta \in (0, 1)$ and $C_t$ denotes total household consumption at time $t$.

Endowments

Each household member is born with two innate talents: schooling talent ($a$) and managerial talent ($z$). Talents are distributed with support in $[0, \bar{a}] \times [0, \bar{z}]$, with CDF $F(a, z)$ and density $f(a, z)$. Household members have one unit of time that is supplied inelastically. Each household member can be a skilled manager, unskilled manager, skilled worker or unskilled worker.\textsuperscript{14} I describe the related decisions and income of each type of member below. The household is also endowed with an initial capital stock of $K_0 > 0$.

\textsuperscript{13}I introduce population growth so the model has standard balanced-growth properties, and thus can be better mapped to data.

\textsuperscript{14}When I refer to efficiency units of workers, I refer to them as "labor". The context should eliminate any confusion.
Technology

There are two types of production plants in this economy. One that is operated by a skilled manager which is decreasing returns to scale and requires four inputs: capital \((k)\), two types of labor services: skilled labor \((s)\) and unskilled labor \((u)\) and managerial talent if the manager who is operating this technology \(z\). Output is given by

\[
y = f(k, u, s; A, z) = y = Az^{1-\gamma} \left( k^\alpha (u^{\theta} s^{1-\theta})^{1-\alpha} \right)^\gamma
\]

(1.3)

where \(A\) denotes an exogenous measure of "economy-wide" productivity which I will refer to as TFP from this point forward, \(\gamma \in (0, 1)\) is the span of control parameter and \(\theta, \alpha \in (0, 1)\). The elasticity of substitution between skilled and unskilled labor is one.

An unskilled manager operates the same production technology and the only difference between the two types of managers is that the skilled managers had their managerial talent augmented due to obtaining education. How this augmentation occurs is similar to the skilled workers and I will elaborate on it when discussing the effect of education on the skilled workers in section (1.4).

Both types of managers face competitive prices. The price of one unit of physical capital in period \(t\) is \(R_t\). The price for the use of skilled labor in period \(t\) is \(W_{s,t}\) per efficiency units of skilled labor. \(W_{u,t}\) is a similar price for unskilled labor. From the standpoint of a manager, all efficiency units of skilled labor are perfect substitutes. The same is true for unskilled labor. So a plant is a manager that operates a technology which requires physical capital and efficiency units of skilled and unskilled labor. Capital depreciates at the rate \(\delta\).
Managers

Managers maximize their profit, taking prices of inputs as given. Since their problem is a static one, I omit the subscript \( t \). The problem of a manager with managerial talent \( z \) is:

\[
\max_{\{k,u,s\}} A z^{1-\gamma} \left( k^\alpha \left( u^\theta s^{1-\theta} \right)^{1-\alpha} \right)^\gamma - Rk - W_u u - W_s s
\]  

(1.4)

The solution to the above optimization for an unskilled manager gives the demands for the unskilled labor \( u^u(z, W_u, W_s, R) \), skilled labor \( s^u(z, W_u, W_s, R) \) and capital \( k^u(z, W_u, W_s, R) \). The profits of an unskilled manager is \( \pi^u(z, W_u, W_s, R) \).

Similarly, for a skilled manager, the demands for unskilled labor, skilled labor, capital and profits are \( u^s(z, W_u, W_s, R) \), \( s^s(z, W_u, W_s, R) \), \( k^s(z, W_u, W_s, R) \) and \( \pi^s(z, W_u, W_s, R) \). The following proposition illustrates two characteristics of the above functions:

**Proposition 1** The demand and profit functions of both types of managers satisfy the following conditions:

1. They are strictly increasing in managerial talent and strictly decreasing in prices.
2. They are linear functions of the managerial talent. Specifically, the demands for input factors and profits per managerial talent are equal for both types of managers:

\[
\frac{i^u(z, W_u, W_s, R)}{z} = \frac{i^s(z, W_u, W_s, R)}{z}, \quad i \in \{u, s, k, \pi\} 
\]  

(1.5)

The proof is in (A). I will refer to the demands for factors and profits per managerial talent as \( u(W_u, W_s, R) \), \( s(W_u, W_s, R) \), \( k(W_u, W_s, R) \) and \( \pi(W_u, W_s, R) \).
The Household Problem

The household has to decide which newborn member becomes skilled and within each pool of skilled and unskilled, which occupation each member is assigned to. Specifically, the household observes the schooling talent of each member and assigns the member to one of the two pools of skilled and unskilled. Turning a newborn into skilled is costly; it requires time and goods. After this segregation, the household must decide about the occupation of the newborns in each pool: worker or manager. So the members of the household are categorized into four categories: skilled manager, unskilled manager, skilled worker or unskilled worker.

If a newborn household member is selected for the unskilled labor pool at time $t$, her schooling talent is transformed into efficiency units of unskilled labor in the same period and her income is given by $W_{u,t}$. If she is selected to be an unskilled manager, her schooling talent is foregone and she becomes an unskilled managers and earns profits.

If she instead is selected for the skilled pool, it takes one period to turn her into a skilled member and the household has to forgo her earnings for one period either as an unskilled worker or an unskilled manager. The household invests $x_t$ units of consumption goods to augment her talents. Investing $x_t$ implies that her talents are augmented by the factor $h_{t+1}$, where

$$h_{t+1} = Bx_t^\phi$$

with $\phi \in (0, 1)$ and $B$ is a parameter determining the relative efficiency of consumption goods that are invested in augmenting talents. Her income then is given at $t + 1$ by $W_{s,t+1}a h_{t+1}$ if she becomes a skilled worker. If she is selected to become a skilled manager, her efficiency units as a skilled worker are foregone and she earns profits.
Her managerial talent is augmented by the same factor as her schooling talent, so her managerial talent at time $t + 1$ is $zh_t$. This augmentation of managerial talent is a result of education and the skilled and unskilled managers differ in this regard.

It is worth emphasizing that the segregation of household members is based on the schooling talent $(a)$. The fate of a member is going to be very different when she is selected for one of the skilled or unskilled pools. The members who were selected for the skilled pool obtain education and this investment in education will change their occupational outcomes dramatically. As we will see in section (1.6), the choice of schooling is a dominant margin when the economic environment such as economy-wide productivity changes. Most of the differences across countries can be attributed to the choice of schooling based on the model.

It follows that only members with sufficiently high levels of schooling talent become skilled and within each pool of skilled and unskilled, members with sufficiently high managerial talent become managers. Given rental prices, there exists a unique threshold $\hat{a}_t$ such that newborns with schooling talent below this threshold become unskilled at time $t$, and those with schooling talent above this become skilled from $t + 1$ on. There are also two unique thresholds for becoming a manager. Unskilled members with managerial talent higher than $z^u_t$ become unskilled managers at time $t$ and the ones below this threshold become unskilled workers at time $t$. Skilled members at time $t + 1$ with managerial talent higher than $z^s_{t+1}$ become skilled managers and the ones below this threshold become skilled workers.

Figure (1) shows how the household members are segregated into the four categories based on education and occupation.
The household’s problem is to choose (i) the sequence of consumption, (ii) the fraction of household members who are skilled and unskilled, (iii) the fraction of unskilled managers, (iv) the fraction of skilled managers, (v) the amount of consumption goods invested in augmenting the talents of new skilled members, and (vi) the capital for the next period. Formally, the household’s problem is to choose $\{C_t, I_t, x_t, \hat{a}_t, z^u_t, z^s_t\}_{t=0}^{\infty}$ to maximize (1.2) subject to (1.6) and

$$C_t + I_t + N_t x_t \int_{0}^{\hat{a}_t} \int_{\hat{z}}^{\hat{a}} f(a,z) d\hat{a} d\hat{z} \leq (W_{u,t} U_t + W_{s,t} S_t) + R_t K_t + \Pi^u_t + \Pi^s_t$$

(1.7)

$$K_{t+1} = (1 - \delta)K_t + I_t$$

(1.8)

and

$$N_0, S_0, U_{-1}, \Pi^u_0, \Pi^s_{-1}, K_0 > 0$$

(1.9)
where $U_t$ is the stock of the efficiency units of unskilled labor, $S_t$ is the stock of the efficiency units of skilled labor, $\Pi^u_t$ is the flow of profits of all the unskilled managers and $\Pi^s_t$ is the flow of profits of all the skilled managers at time $t$. The laws of motion for these stocks and flows are as follows.

Each period, the number of newborns is $N_t = g_L L_{t-1}$. Based on the above assignment, the laws of motion for the stocks of the efficiency units of unskilled and skilled labor are:

$$U_t = U_{t-1} + N_t \int_{z_t^u}^{\hat{a}_t} \int_0^{\hat{a}_t} a f(a, z) dz$$

(1.10)

$$S_t = S_{t-1} + N_{t-1} h_t \int_{\hat{a}_{t-1}}^{\hat{a}_t} \int_0^{\hat{a}_t} a f(a, z) dz$$

(1.11)

Note that at any time $t$, $U_t + S_t$ is not equal to $L_t$, since $U_t$ is the total efficiency units of unskilled labor and $S_t$ is the total efficiency units of skilled labor, whereas $L_t$ is the size of the household at time $t$. Household also needs to know about the profits that managers are earning. The laws of motion for profits of unskilled and skilled managers are:

$$\Pi^u_t = \Pi^u_{t-1} + N_t \int_{z_t^u}^{\hat{a}_t} \int_0^{\hat{a}_t} z \pi(W_{u,t}, W_{s,t}, R_t) f(a, z) dz$$

(1.12)

$$\Pi^s_t = \Pi^s_{t-1} + N_{t-1} h_t \int_{\hat{a}_{t-1}}^{\hat{a}_t} \int_0^{\hat{a}_t} z \pi(W_{u,t}, W_{s,t}, R_t) f(a, z) dz$$

(1.13)

where $\Pi^u_{t-1}$ is the flow of profits from unskilled managers who were managers from period $t - 1$ and before. Equation (1.12) states that the flow of profits to household from unskilled managers at time $t$ equals to the flow of profits from all the previously assigned unskilled managers and the flow of profits of the unskilled managers that are
going to be assigned in period \( t \). The law of motion for the flow of profits from skilled managers has a similar interpretation. Note that I used the result of proposition (1) for the profits of both types of managers. They are both linear in managerial talent with the augmentation for skilled managers due to education. The solution to the household’s problem is described by the following first order conditions:

- **Euler Equation**
  \[
  \frac{1}{C_t^L_t} = \beta \left[ \frac{R_{t+1} + (1 - \delta)}{C_{t+1}^{L_{t+1}}} \right]
  \]  
  (1.14)
  Condition (1.14) is the standard Euler equation.

- **Selection for Education** (\( \hat{a} \))
  \[
  \frac{W_{u,t} \int_{z_u}^{z_t} \hat{a}_t f(\hat{a}_t, z)dz + \int_{z_l}^{z_u} z\pi f(\hat{a}_t, z)dz + x_t \int_{0}^{z_t} f(\hat{a}_t, z)dz}{C_t^L_t} = \beta \left( \frac{W_{s,t+1} \int_{\hat{a}_t}^{z_t} \hat{a}_t f(\hat{a}_t, z)dz + \int_{z_l}^{z_t} z\pi f(\hat{a}_t, z)dz}{C_{t+1}^{L_{t+1}}} \right) Bx_t^\phi
  \]  
  (1.15)
  Condition (1.15) states that the optimal decision for education threshold should be in such a way that the compensation of marginal unskilled workers plus profits of marginal unskilled managers plus the cost of skill augmentation on the margin (marginal cost of education) in this period, is equal to the discounted compensation of marginal skilled workers plus the profits of marginal skilled managers (marginal benefits of education) in the next period.

- **Investment in Education** (\( x \))
  \[
  \frac{\int_{\hat{a}_t}^{\hat{a}_t} \int_{0}^{z} f(a, z)dadz}{C_t^L_t} = \beta \left( \frac{W_{s,t+1} \int_{\hat{a}_t}^{z_t} a f(a, z)dadz + \int_{\hat{a}_t}^{z_t} \pi f(a, z)dadz}{C_{t+1}^{L_{t+1}}} \right) Bx_t^\phi^{-1}
  \]  
  (1.16)
  Condition (1.16) states that the marginal cost of investing one unit of consumption good in the talents of marginal skilled members must be equal to the
marginal benefit which is the marginal increase in efficiency units times its rental price times "raw" addition to its pool plus the profits of marginal managers whose managerial talent is augmented.

- Occupation Choice for Unskilled Members \((z^u)\)

\[
W_{u,t} \int_{0}^{\hat{a}_t} af(a, z^u_t)da = \int_{0}^{\hat{a}_t} z^u_t \pi f(a, z^u_t)da
\]  

(1.17)

Condition (1.17) states that the compensation of marginal unskilled workers must be equal to the profits of marginal unskilled managers. I can rewrite this condition to get an expression for the threshold of choosing managerial occupation for the unskilled members:

\[
z^u_t = \frac{W_{u,t} \int_{0}^{\hat{a}_t} af(a, z^u_t)da}{\int_{0}^{\hat{a}_t} f(a, z^u_t)da \pi}
\]  

(1.18)

The numerator is the marginal wage earnings per marginal unskilled worker. The denominator is the profits per unit of managerial talent. The optimality condition dictates that for a chosen threshold of education \((\hat{a})\), if the wage per unskilled efficiency units increases relative to the profits per managerial talent, household decides to choose less unskilled members to become unskilled managers and hence the threshold increases.

- Occupation Choice for Skilled Members \((z^s)\)

\[
W_{s,t} \int_{\hat{a}_{t-1}}^{\hat{a}} af(a, z^s_t)da = \int_{\hat{a}_{t-1}}^{\hat{a}} z^s_t \pi f(a, z^s_t)da
\]  

(1.19)

Condition (1.19) states that the compensation of marginal skilled workers must be equal to the profits of marginal skilled managers. I can rewrite this condition as well:

\[
z^s_t = \frac{W_{s,t} \int_{\hat{a}_{t-1}}^{\hat{a}} af(a, z^s_t)da}{\int_{\hat{a}_{t-1}}^{\hat{a}} f(a, z^s_t)da \pi}
\]  

(1.20)
The numerator is the marginal wage earnings per marginal skilled worker. The denominator is the profits per unit of managerial talent. The optimality condition dictates that for a chosen threshold of education ($\hat{a}$), if the wage per skilled efficiency units increases relative to profits the per managerial talent, household decides to choose less skilled members to become skilled managers and hence the threshold increases.

Equilibrium

A Competitive Equilibrium is a collection of sequences

$$\{C_t^*, K_{t+1}^*, \hat{a}_t^*, \hat{z}_t^{us}, \hat{z}_t^{ss}, x_t^*, W_{u,t}^*, W_{s,t}^*, R_t^*\}_{t=0}^\infty$$

such that given prices, managers maximize their profits, $\{C_t^*, K_{t+1}^*, \hat{a}_t^*, \hat{z}_t^{us}, \hat{z}_t^{ss}, x_t^*\}_{t=0}^\infty$ solves the household’s problem and labor markets for skilled and unskilled workers as well as capital and goods markets clear.

Balanced Growth

Along a balanced growth path, aggregate consumption, investment, production, capital, profits of unskilled and skilled managers, and the pools of skilled and unskilled workers are growing at the rate of population growth. Also, the thresholds for education and occupation choices are fixed as well as the investment in education per member. I omit the subscript $t$ and superscript $*$ since the equilibrium quantities are either constant or growing with constant population growth rate.

A feature of the balanced growth equilibrium in the model is that the rental rate of capital is constant and equals $(1/\beta - (1 - \delta))$. This helps with computing the equilibrium. Specifically, starting with a guess for the pair $(W_u, W_s)$, one can
solve for \( \{\hat{a}, z^u, z^s, x\} \) simultaneously using equations (1.15), (1.16), (1.17) and (1.19). Then market clearing for skilled and unskilled labor can determine the new guess for the initial pair. Iterating on this pair, the equilibrium objects can be calculated. Consumption and capital can be calculated using aggregate feasibility and capital market clearing conditions.\(^{15}\)

1.5 Parameter Values

I start by setting the model period to four years to reflect a more realistic time period to become a skilled member. I use the U.S. as a benchmark to calibrate the parameter values and choose these values to match the observed characteristics of the U.S. economy in the steady state equilibrium of the model. Since the model does not have any implications over the life-cycle, I focus on prime age individuals (age 40-54) and years 2014-2016. The details of calculating moment conditions from the data is described in Appendix (A).

Technology

I follow Guner et al. (2018) for choosing the value for the span of control parameter \((\gamma)\) and set it to 0.77. In order to match the capital to output ratio of 0.33 based on the findings of Gollin (2002) on labor shares across countries, I set the share of capital in the production function \((\alpha)\) to 0.43. I choose \((\theta)\) so that in the steady state, the model reproduces the fraction of unskilled labor (worker+managers) in the U.S.

\(^{15}\)See Appendix 2 for more details.
This share is 40% and it is the fraction of the population aged 15 years and older that completed secondary education or less.

I set the depreciation rate to 26.6% (7.4% at the annual rate), so that given the capital-to-output ratio, the model is consistent with the observed investment-to-output ratio. Cubas et al. (2016a) used NIPA data for the period 1960-2010 to calculate an average investment-to-output ratio of 0.27 and a capital-to-output ratio of 0.8 at the four-year frequency (3.2 at the annual rate). The productivity level \(A\) is set to 1.0 as a normalization.

Preferences and Demographics

I set the discount factor so that in the four-year steady-state, the capital-to-output ratio is 0.8. This implies that \(\beta\) equals to 0.869 (0.966 at the annual rate). The value for \(\beta\) means an annual interest rate of 3.6%. Based on the Penn World Tables 7.0, I set the growth rate of population to the annual rate of 0.9%.

Distributions of Talents

I calibrate the distribution of talents to match several moments. I assume exponential distributions for talents with parameters \(\lambda_a\) for schooling talent and \(\lambda_z\) for managerial talent. There is also a level of correlation between talents which I will indicate with a parameter \(\rho\). I construct a joint distribution of talents with CDF \(F(a, z)\) and PDF \(f(a, z)\) using the notion of copula.\(^{16}\) Specifically, I create a bivariate

\[^{16}\text{A copula is a function that produces multivariate distributions out of any set of arbitrary univariate distributions; see Nelsen (2006) for more details.}\]
exponential distribution using the method developed by Gumbel (1960). Suppose the marginal distributions of talent are univariate exponential distributions as follows:

\[
g(a; \lambda_a) = \begin{cases} 
\frac{1}{\lambda_a} e^{-\frac{a}{\lambda_a}} & a \geq 0, \\
0 & a < 0.
\end{cases} \quad G(a; \lambda_a) = \begin{cases} 
1 - e^{-\frac{a}{\lambda_a}} & a \geq 0, \\
0 & a < 0.
\end{cases} \tag{1.21}
\]

and

\[
h(z; \lambda_z) = \begin{cases} 
\frac{1}{\lambda_z} e^{-\frac{z}{\lambda_z}} & z \geq 0, \\
0 & z < 0.
\end{cases} \quad H(z; \lambda_z) = \begin{cases} 
1 - e^{-\frac{z}{\lambda_z}} & z \geq 0, \\
0 & z < 0.
\end{cases} \tag{1.22}
\]

where the upper case letters denote CDF and the lower case letters denote pdf of the distributions. Then the bivariate distribution can be characterized by the following probability distribution function:

\[
f(a, z; \lambda_a, \lambda_z, \rho) = \begin{cases} 
g(a)h(z) \left(1 + \rho(2G(a) - 1)(2H(z) - 1)\right) & a, z \geq 0, \\
0 & a < 0 \text{ or } z < 0.
\end{cases} \tag{1.23}
\]

The parameter \( \rho \) governs the correlation between the random variables with \( \rho \in [-1, 1] \). The correlation coefficient between the two random variables is \( \rho \). Several remarks about the choices of distributions and copula are in order.

First, they allow for richness and flexibility in matching data and at the same time, retaining a parsimonious set of parameters. One dimension of richness is that household members' talents are dependent which a priori seems a reasonable assumption. This allows for the feature that a household member who is talented in one talent is more talented in the other talent as well. Another dimension of richness is that dispersion
in household members’ talent is not the same for each talent. Specifically, schooling
talent is a broad notion for relative advantage in obtaining education which constitutes
a variety of activities related to education such as studying hard, managing personal
finances, balancing between school and social life and so on. Managerial talent is a
different notion for relative advantage in managing a business and performing tasks
such as hiring/firing decisions, forecasting the needs of the business, motivating and
monitoring employees, organizing, negotiating contracts and so on. So these notions
of talent are capturing different aspects of heterogeneity in the household members.
Since managerial talent is a stand-in for arguably more diverse types of activities than
those related to education, one might expect that the dispersion of managerial talent
of the household members is larger than schooling talent. The choice of distributions
allows for this possibility.

The second reason for the above choices is that they allow for closed-form solutions
to double integrations needed to solve the model. Given that the equilibrium objects
consist of threshold levels in integration, the solution is very sensitive to the error level
if I want to compute the integrals numerically. The closed-form solutions eliminate
this difficulty.

The third reason for choosing these distributions is that they allow my theory to
fail. In particular, there is nothing inherent in these distributions that assures that
the model should behave in a certain way to reproduce the empirical targets and
generate the cross country patterns observed in the data. The success of the model in
this regard will be dictated by the data and the calibration procedure.

In order to identify the parameters of the distributions, I calibrate the model to
reproduce three observed moments in U.S. data in equilibrium. The first moment
is managerial rate which is the percentage of prime-age working population who are
either owner-manager of a business or employed to be the manager of a established business. So the managers in the data are those who are employed as managers and can be labeled as managers based on the Census Bureau occupation codes as well as self-employed individuals. Using data from BLS, this rate is about 17%. The second moment is the share of skilled managers in the U.S. where I defined the level of education to be considered "skilled" in section (1.5). Using BLS data, this amounts to 77%. The third moment is the managerial premium which is defined as the relative earnings of a manager to a worker (regardless of skill level).

The curvature ($\phi$) and level parameter ($B$) in the production of skills are set so that the model reproduces two empirical targets in a steady state equilibrium. The first target is the expenditure per tertiary student as a fraction of GDP per worker (at PPP values) in the U.S. According to OECD (2017a) report, this fraction was 25% in U.S. in 2014. The second target is the skill premium. It is defined as the relative earnings of a skilled person to an unskilled one regardless of occupation which is 1.71 in U.S. The details of calculation the skill premium and managerial premium are outlined in Appendix (A).

Table (27) shows the calibrated parameters based on matching the above moments. The interesting result is the parameter values for talent distributions.

As I mentioned before, the notion of managerial talent is capturing a wider variety of activities and skills than the notion of schooling talent. The calibrated values for parameters of the distribution indeed approves this a priori expectation. The schooling distribution has mean and standard deviation of 9.7, while the managerial talent has mean and standard deviation of 10. The correlation between the two talents is positive but small 0.005. It seems that this is reasonable since a negative correlation does not have a good economic meaning and a positive but large correlation means
that individuals who are talented in one dimension are more likely to be talented in the other and it sounds like natural selection.

Table (28) shows the reproduced moments from the model and compares them with the empirical moments. The model does an excellent job of matching the moments in the data. The equilibrium for the benchmark model has an interesting feature about relative quality of managers and their steady state distribution. The share of skilled managers in the U.S. is 77% which model does a good job of reproducing. As a measure of external validity of the model, I reported two additional moments. One is the relative income of skilled managers to unskilled workers, which is 2.2 in the U.S. data. The other is the income of unskilled managers to skilled workers, which is 0.67 in the U.S. data. The model reproduces these two moments fairly well. Note that since there are four types of members in the model, skilled and unskilled managers along with skilled and unskilled workers, I can only match at most three moments related to relative income. The forth moment will be mechanically matched. I chose to match only two income moments and use the other two to test for external validity.

But the important distinction between skilled and unskilled managers in the model is the fact that skilled manager obtained education and augmented their managerial talent which amplifies the starting heterogeneity between skilled and unskilled managers. This feature reveals itself in the relative income of skilled vs unskilled managers. Specifically, the average income of a skilled managers in the model and data is more than twice as much as the average income of an unskilled worker. The selection for schooling along with selecting into higher income occupation based on having a higher managerial talent are the channels contributing to this income gap.

It is worth emphasizing that the mean plant size in the current setup is about 5, whereas in the data, the mean plant size for the U.S. is 17, as calculated by Guner et al.
(2008). The reason being that in the real world, plants can have multiple managers, while in the model, every plant has only one manager. So, unless I deviate from the simple span-of-control setup here to allow for multiple managers per plant, I cannot reproduce simultaneously the share of managers as well as the mean plant size. Guner et al. (2008) focuses on the size distribution to study the aggregate implications of size-dependent distortions without investment in skills, while I focus on the share of managers and the composition of managers with respect to skills. Therefore, the current setup matches the share of managers while failing to match the mean plant size. Since my main concern is not the size distribution per se and the relative change in the mean plant size of skilled vs unskilled managers is a result of the framework under study, I chose to match the share of managers rather than the mean plant size.

Figure (40) shows the equilibrium distribution of managers in the model economy. One can see that the cutoff values for selection into managerial occupation for skilled and unskilled managers are different. Specifically, the cutoff value for unskilled managers is higher. The reason is that the outside options for skilled and unskilled manager are different. The unskilled manger can become an unskilled worker and earn $w_u$ for her efficiency units while the skilled manger gains $w_s$ if she works as a skilled worker. Since in equilibrium, the share of unskilled managers is low, the selection is higher for them in contrast with the skilled managers.

What is important here is the average managerial talent of skilled and unskilled managers in the equilibrium which eliminates the relative share of each type of managers and gives us a better statistic to gauge the relative quality of skilled and unskilled managers. This quality translates one to one into the average plant level productivity run by different types of managers for a fixed level of $A$. It turns out that the average managerial talent of the unskilled managers is 46% lower than the
average managerial talent of the skilled managers. So the investment in education outweighs the higher share of skilled managers and the average plant level productivity for skilled managers is higher than the unskilled managers.

1.6 Model Mechanics

I present a set of results pertaining to economy-wide notion of productivity differences across countries along with an experiment of introducing size-dependent distortions in the model and assessing their effects on the model equilibrium. The two channels, productivity and distortions, have different implications for the model, both qualitatively and quantitatively. The productivity channel has a uniform impact on all firms; the generic productivity term \( A \) reduces by 20%. The reaction of managers to this productivity loss is heterogeneous due to the fact that managers have different levels of managerial talent. The distortion channel has a heterogeneous effect from the beginning since it hits each plant based on its size (value-added in the model). I will elaborate on the exact modeling strategy for size distortions in section (1.6.2).

1.6.1 Productivity Differences

The results from a 20% reduction in productivity is presented in table (29). A reduction in productivity results in a lower level of output per capita,\(^\text{17}\) share of skilled labor, skilled manager, investment in skills and mean size of establishments. It

\(^{17}\)Since in the model, every household member is either a worker or a manager and there is no unemployment, the notion of output per capita which is the total output divided by the size of the household seems more appropriate than output per worker. The latter may confuse the reader into thinking that the total output is divided only by the size of workers in the household.
also results in an increase in the shares of unskilled labor, unskilled manager, share of managers in the population, managerial premium and skill premium.

Before continuing with the intuition behind these results and how they connect to the stylized facts in section (1.3), a note on the notion of size in the model is in order. The size of establishments are usually measured by the number of workers in the establishment. In my theory, managers demand efficiency units and workers supply them. So the size of a plant is not well-defined. But based on the segregation of the population based on occupation and the fact that each manager represents one and only one establishment and all the workers are hired by managers, the notion of the mean size of establishments can be defined as:

\[
\text{mean size} = \frac{\# \text{ of workers}}{\# \text{ of managers}} \quad (1.24)
\]

So the notion of size is a by-product of the model and changes in this variable come from the relative changes in the number of managers and workers. I can also define the mean plant size for skilled and unskilled managers. The trick is to determine the share of skilled and unskilled workers that are employed by each type of manager. I define these shares as follows: the share of efficiency units of skilled workers that is demanded by the skilled managers is the same as the share of the number of skilled workers that is demanded by skilled managers. The rest of the skilled workers are employed by unskilled managers. The share of unskilled workers for each type of manager is defined analogously. Based on the model setup, this is a natural way of mapping the efficiency units demanded to the number of each type of worker that is demanded by managers. When I know the total number of workers demanded by each type of managers, then the mean plant size can be calculated using (1.24).

**Panel A** in table (29) shows how the model conforms to the patterns found in
the data. Fact 1 states that the share of skilled managers and workers increase with income per capita. Fact 2 states that the managerial premium declines with income per capita, while Fact 3 asserts that the managerial rate declines with income per capita. Fact 4 reveals that the mean size of establishments increases with income per capita. Finally, fact 5 states that the skill premium declines with income per capita.

A 20% reduction in productivity in the model results in a decline in the share of skilled labor 34% while the share of skilled managers among managers only decreases by 15% which is almost half of the decline in skilled population share. Based on the calibration of the benchmark economy, the share of skilled managers among managers is still higher than the share of skilled working-age population in the new equilibrium. All of these changes in shares conforms to the fact (1). In line with fact (2), the managerial premium increases by 8%. The managerial rate increases by 5% which is in line with fact (3) and the mean size of establishments declines by 10%, conforming to fact (4). Finally, skill premium increases by 12% which is what fact (5) shows.

**Panel B** gives some additional information about the output and segregation of the household members into different occupations and skill levels. The output per capita falls by 12% which is intuitive based on lower productivity that translates into lower levels of output. The share of unskilled workers increases by 6% while the share of skilled workers declines by 8%. The share of unskilled managers increases by 22%. Putting panels A and B together, the total share of managers in the population increases along with the share of unskilled managers. Within the pool of managers, the share of skilled managers declines. This shows that in the model, an economy with 20% lower productivity exhibits more managers who on average, runs smaller plants and the share of skilled managers among them is lower.

**Panel C** elaborates more on the notion of quality and size among managers.
With 20% lower productivity in the model economy, the investment in skills which provides higher quality declines by 23%, the average quality of all managers drops by 28% and the average quality of skilled managers declines by 12% while the average quality of unskilled managers decreases by 42%. This means that in an economy with lower productivity, the quality of all managers is lower. The relative quality of a skilled manager to an unskilled manager increases by 56%. This shows that the skilled managers in the economy with lower productivity are more selected: they have less investment in them, but the gap between their talents relative to the unskilled managers is higher.

There is an asymmetry between the two types of managers in terms of quality. Specifically, the average quality of the unskilled managers declines more due to two reasons. First, the lower productivity induces the household to select more unskilled members to become managers, which results in lower average productivity among unskilled managers. I call this the "selection effect". The other reason is that the skilled managers have investment in their managerial talents. The investment is lower due to lower productivity, and it is nonexistent for unskilled managers. This investment increases the average quality of skilled managers relative to unskilled managers and I call it the "skill effect". Together, these effects cause a wider gap between the average quality of skilled versus unskilled managers in economies with lower levels of productivity.

The average plant size of skilled managers almost stays the same when productivity is lower while the average plant size of unskilled managers declines by 53%. The relative plant size increases by 114%. This change in plant size is consistent with studies showing that in less developed countries, the distribution of plant size consists of a lot of small plants along with a few big ones and the middle part of the distribution
is "missing".\textsuperscript{18} Although I am not trying to explain the size distribution of plants across countries, it is interesting that even with my notion of size in this paper, the relative size gap is higher in economies with lower productivity.

The intuition behind the results is as follows. As the productivity declines, it is relatively more expensive for the household to invest in skill accumulation of its members, so the household selects fewer members for skill investment and invest in them less. The choice of occupation will change based on three factors: profits per managerial talent, prices per efficiency units of labor (whether skilled or unskilled) and investment in skills. In a simple span-of-control model without skill accumulation such as Lucas (1978), the change in productivity will not affect occupational choice if the technology has a Hicks neutral productivity term.\textsuperscript{19} The profits and wages would adjust in a way that the choice of occupation is unchanged (i.e. the threshold for becoming a manager does not change). The effects of productivity change are more subtle here because of the skill accumulation and selection for skill investment.

For the choice of unskilled managers, the profits per managerial talent declines as a result of the drop in productivity, so it becomes less profitable to be an unskilled manager. On the other hand, the wage per efficiency units of unskilled workers will adjust and it may go up or down depending on the supply and demand for unskilled workers in the new equilibrium. This wage may weaken or strengthen the desire of the household to select unskilled members to become unskilled managers. In the new steady state equilibrium, the share of unskilled managers increases.\textsuperscript{20} The interesting

\textsuperscript{18}See Hsieh and Olken (2014) for a discussion of the literature of the firm size distribution in developing countries.

\textsuperscript{19}This requires a Cobb-Douglas technology which is the case in my model.

\textsuperscript{20}Table (29), Panel B.
feature of the composition of the unskilled managers in the new equilibrium is that the increase is because of the fact that the household selects some members with high initial schooling and managerial talent to become unskilled (i.e. not selecting them for skill investment), but because their managerial talent is high, they become unskilled managers and some of the unskilled managers in the previous equilibrium with low managerial talent are unskilled workers in the new equilibrium.\textsuperscript{21} This means that the choice of the schooling has a very important effect on the choice of occupation.

The effects on the profits per managerial talent for the skilled managers is the same. The other determinant of this choice for the household is the wage per efficiency units of a skilled worker and the investment in skills which augments also managerial talent. With lower investment in skills, lower share of skilled household members and lower profits per managerial talent, one expects that the share of skilled managers declines in the new equilibrium which indeed happens.\textsuperscript{22} Since in the new equilibrium, there are more unskilled managers and less skilled managers, the share of skilled managers among the pool of managers declines.

What happens to the total number of managers? The above explanation shows lower share of skilled managers and higher share of unskilled managers in the equilibrium. The total effect is an increase in the share of managers in the workforce and based on the definition for size of a plant in the model, the mean size declines with a decrease in productivity.

What happens to relative income of based on skills and occupation? The relative income of managers to workers increases since in the new equilibrium the wages decline

\textsuperscript{21}For the mathematically inclined reader, it means that $z^u$ increases, but because $a$ also increases, the net effect on the share of unskilled managers is an increase rather than a decreases. As a visual aid, take a look at figure (1).

\textsuperscript{22}Table (29), Panel A, fact 1.
more than the profits. The fact that there are more managers in the new equilibrium
does not offset the relative income of managers to workers and the managerial premium
increases. The relative income of skilled versus unskilled members increases because of
two effects. The first is an adjustment of wages where in the new equilibrium, wages
are lower. The other effect is the lower share of skilled members in the new equilibrium.
Although the investment in the skills of skilled members decline, the relative income
of skilled members increases which causes the skill premium to increase.

1.6.2 Size Dependent Distortions

I investigate the role of size-dependent distortions in my model here. The environ-
ment is the same as before, but the managers now face distortions when operating
plants and these distortions depend on the size of the operation. My model has a
delicate notion of size; the size of a plant is only defined in a steady-state equilibrium
via the relative share of managers to workers. So, instead of introducing distortions
on the size of the operations, I model them to be output taxes that act based on the
value-added of the plant. In particular, I assume an establishment with output $y$
faces an average tax rate $T(y) = 1 - \nu y^{-\tau}$. This tax function was proposed originally
by Benabou (2002a) and has an intuitive interpretation: if $\tau = 0$, all establishments
face the same constant tax rate $(1 - \nu)$. For $\tau > 0$, the distortions are dependent on
size; larger establishments face higher distortions than smaller ones. Hence, $\tau$ controls
the level of the dependence of distortions on size. In (A), the profit and demand
functions for a manager with managerial talent $z$ and a given level of $\tau$ are presented.
I am going to assume that the economy-wide productivity is constant and normalize
it to $A = 1$. An interesting feature of the distortions is that the demand and profit functions per managerial talent are no longer the same for all managers, specifically:

$$\frac{k(z', W_u, W_s, R, A)}{k(z, W_u, W_s, R, A)} = \frac{u(z', W_u, W_s, R, A)}{u(z, W_u, W_s, R, A)} = \frac{s(z', W_u, W_s, R, A)}{s(z, W_u, W_s, R, A)} = \frac{\pi(z', W_u, W_s, R, A)}{\pi(z, W_u, W_s, R, A)} = \left(\frac{z'}{z}\right)^{\frac{(1-\gamma)(1-\tau)}{1-\gamma(1-\tau)}}$$

(1.25)

where

$$\frac{(1-\gamma)(1-\tau)}{1-\gamma(1-\tau)} < 1$$

(1.26)

as long as $\tau > 0$. This means that for a given distribution of managerial talents, the distribution of plant size becomes more compressed when size-dependent distortions are present.

The Case Without Investment in Skills

It is worth considering a simpler version of the model where the choice of investment in skills is absent. In this case, the dynamics of occupational choice disappears and the choice of occupation between worker and manager is contemporaneous for each cohort of newborns in the same period that they are born. Therefore, the economy contains two types of members: workers and managers. The managers employ capital and efficiency units of workers. This setup is essentially the one studied in Guner et al. (2008).

The distortions reduces the overall demand for labor in the model economy. This reduction in the labor demand drives down the equilibrium wage rate, while the rental rate of capital stays constant. The reduction in the wage rate changes the incentives
of the marginal worker in favor of becoming a manager since the reduction in the wage rate translates one-to-one in reduction in income as a worker, while the reduction in profits for that marginal worker if she was a manager is less than proportional. As a result, the marginal worker becomes manager. The aggregate output declines despite the fact that more members are now managers since the distortion affects the managers with high managerial talent harder than the managers with low managerial talent. The share of output accounted for by the managers with high managerial talent declines and the share of the output accounted for by the bottom managers increases. So, the restrictions on the size increases the number of plants, decreases the mean size of plants, and redistribute production from high talent to low talent managers.

The Case With Investment in Skills

I now study the effects of size-dependent distortions on various variables in the model and present some numerical results. I first set $\nu = 1$ and compare steady state equilibrium of the model under two values for $\tau$: 0 and 0.05. The reason for choosing $\tau = 0.05$ is that the output per worker in the distorted economy is close to an undistorted economy with $A = 0.8$ and so the results in this section can be compared to the results in section (1.6.1).

The results are qualitatively similar to the ones in table (29). Once again the model conforms to the patterns observed in the data as shown in Panel A. The main differences between the two tables is quantitative. Specifically, size-dependent distortions have dramatic impact on some statistics. The number of managers increases by 42%. As a result, the mean size of the establishments drop by 84%. The effect on
share of skilled managers is about 34% and the share of skilled managers among all managers drops by 80%.

**Panel B** shows the decline in output per capita by 30% which is expected since the distortions are modeled as output taxes. The skilled labor share declines by 85%, the managerial premium increases by 49% and the skill premium goes up by 76%. The effects on the quality of managers and the mean size of the establishments managed by different types of managers are summarized in **Panel C**. The investment in skills declines by 44%. As a result, the average quality of all managers goes down by 80% and the average quality of skilled managers drops by 66%. The increase in the number of managers is almost solely due to the addition of unskilled managers which means that the average quality of unskilled managers decreases by 85%. The relative quality of a skilled manager to unskilled manager increases by 175%. The mean size of the plants run by skilled managers declines by 78% while the drop in the mean size is about 92% for the unskilled managers. The relative average establishment size of skilled to unskilled managers increases by 130% which means a very skewed size distribution for the model economy.

To understand the intuition behind these numbers, one should focus on the margins that are affected by the size distortions. Since some fraction of the output is taxed away, the household decides to select fewer members for schooling and invest in them less. This means that the average quality of the skilled managers would decline along with the share of skilled workforce. The choice of occupation would change in favor of household members with lower managerial talent since the distortions for members with higher managerial talents are higher. So the relative advantage of managers with high managerial talent (either intrinsic talent or augmented talent as a result of investment in skills) is diminished because the distortions tax away a higher share
of their output relative to managers with lower levels of managerial talent. This is a reallocation of resources from larger to smaller plants in the model. As a result, the share of unskilled managers increases.

The share of skilled managers decreases since the outside option for them is to work for other managers and with more managers in the economy as a result of an influx of unskilled managers, it is optimal for the household to select some of the previous skilled managers to become skilled workers. This margin turns out to be not so large and the main reason that there are less skilled managers is that, some of them are not selected for investment in skills in the distorted economy and become unskilled, but since their managerial talent is high, the household still selects them to become unskilled managers. This again emphasizes the importance of the choice of schooling for the choice of occupation.

Size distortions can provide an interpretation for the overall productivity in the model economy. Recall that for the results of this section, I set $A = 1$. This means that the managerial talent of a manager can be interpreted as the productivity level of the plant she is running. The productivity of the whole economy in the model can be defined as the average talent (quality) of all the managers, whether skilled or unskilled. This way of looking at the managerial talent provides a new source for cross-country productivity differences. In the model economy in this section, the average quality of managers is 75% lower when distortions are present. This is due to two margins. On the intensive margin, the skilled managers have less investment in them which decreases the average quality of the skilled managers. On the extensive margin, the addition of unskilled managers with lower managerial talent contributes to the overall decline in the average quality of managers. The two margins working
together provide the sharp decline in the average managerial quality in the economy which can be interpreted as the lower productivity in the model economy. The segregation of household members based on skill level and occupation provides more margins of change that is worth noting. In a simple setting without investment in skills such as Guner et al. (2008), the introduction of size-dependent distortions results in a reallocation of resources from large to small plants through addition of more managers with low managerial talent. Therefore, the mean size of the establishments declines and the number of managers go up. The same pattern is present here, but selection and skill effects amplify the effect of distortions through lowering the share of skilled managers and investment in skills.

The results here can be contrasted to Guner et al. (2018) as well. In their model, the population consists of workers and managers. The workers are all the same and managers are heterogeneous based on their managerial talent and they invest in their skills. The effect of distortions in my model is larger than theirs. Specifically, distortions increase the number of managers in both models, but their effects on the incentive of workers and managers are different in each model. The workers and managers in the current setting are heterogeneous based on skill level, so the outside option for a skilled worker, which is a skilled manager, is different from an unskilled worker, which is an unskilled managers. In Guner et al. (2018), the outside option for all managers is becoming a worker and having a fixed level of efficiency units. The fact that my model provides more heterogeneity in terms of skill level and occupation amplifies the effects of size distortion since these distortions are impacting different margins. These margins reveal themselves in selection and skill effects and result in different aggregate implications in my model compared to theirs. For example, the

\[\text{They call it managerial ability.}\]
number of managers increase more in my model with \( \tau = 0.05 \) (it is less than 100 percent increase in their model) and the average quality of managers is lower in mine (80% lower versus 50%).

1.7 Matching Data

In this section, I use the model to replicate two statistics in every country in my sample: the GDP per worker and the share of unskilled working individuals. Then I study the model’s implications for other statistics associated with the facts that I presented in section (1.3), namely managerial rate, share of skilled managers, managerial premium and skill premium. In particular, I calibrate the productivity term \( (A) \) and distortions parameter \( (\tau) \) to match those moments in the data. The model generates other statistics and I can assess how the model performs regarding the unmatched moments.

Table (31) shows the results of this experiment. I compare three sections of the distribution of GDP per worker: top 10% to bottom 10%, top 20% to bottom 20% and top 35% to bottom 35%. The model match GDP per worker and the share of unskilled working individuals exactly and it also produces other statistics related to the facts that I documented, namely managerial rate, share of skilled managers among all managers, managerial premium and skill premium. Since the notion of size in the model is not the same as the data, I refrain from reporting this statistic, but I explained earlier that qualitatively the model generates the fact about mean establishment size and its relation to GDP per worker. I also presented three statistics that the model generates but I do not have reliable data to compare: average managerial quality, investment in skills as a share of GDP per worker and capital per worker.
The model reproduces the other moments in the data quite well. For example, the managerial rate in the model and the data are sufficiently close and the variation in managerial rate coming from the model is less than what I observe in the data. The same pattern is true for the share of skilled managers. The model can also produce the managerial premium quite well, although the variation generated by the model is more than what I find in the data. The most impressive statistic generated from the model is skill premium. This statistic in the model match the data quite well.

I should emphasize that the two moments related to relative income, i.e. managerial premium and skill premium, are quite complicated statistics. They contains all the decision rules about investment in skills and occupational choice as well as equilibrium prices and wages. There is nothing in the model parameters that insure that these statistics can match data closely. In other words, I only calibrate the model parameters to generate skill premium and managerial premium in the U.S. and there is no guarantee that if I match two moments other than these two, the model can match them sufficiently well. In fact, Cubas et al. (2016a) tried to match the skill premium by matching the GDP per worker, the division of labor between skilled and unskilled and distribution of talents matching the PISA distribution in each country. The skill premium from their model is far from the data. This comparison shows that since investment in skills are affecting occupational choice in the current setup, coupled with the presence of size-dependent distortions are necessary to produce the massive variation in the skill premium observed in the data.

I take the results of this exercise as a success story for my current setup. It shows that in order to explain the five facts about economic development, one needs a rich, yet parsimonious setup such as the one I developed. The relevant margins of investment in skills and occupational choice with the presence of size-dependent
distortions are enough to generate the large observed differences in several dimensions of economic development. In this regard, this paper adds a significant understanding to the economic choices that individuals in developing countries are making.

1.8 Discussion

In this section, I provide a discussion about the importance of different margins in the model. One margin is occupational choice and the other is investment in skills.

1.8.1 Importance of Occupational Choice

How important is the occupational choice? For one thing, if I shutdown the margin of occupational choice, I cannot relate to the facts presented about the managerial rate, the skilled managers and managerial premium. I also cannot study the effect of size-dependent distortions. The model is going to reduce to the model presented by Cubas et al. (2016a). Their choice of parameters is different from mine and they identify the distribution of talents, which is a gamma distribution in their setup, by the mean and variance of the PISA test scores in each country. However, the investment in skills and other modeling assumptions are fairly similar. In the case with no occupational choice, the productivity term can be interpreted as total factor productivity (TFP).

Figure (41) illustrates the elasticity of output with respect to productivity term ($A$) when the curvature parameter $\phi$ in the technology of skill investments change from 0 to 0.6. There are three scenarios presented in the figure: no occupational choice, no size-dependent distortions and the model with occupational choice and size-dependent
distortions. In the figure, the elasticity of output with respect to productivity term \((A)\) increases from 1.5 to around 3 which is similar to what Cubas et al. (2016a) found. This case can be contrasted to the scenario with occupational choice and the figure demonstrates that the elasticity is higher for every value of \(\phi\). The reason is that the investment in skills affecting the margin of occupation and each unit of investment in the skills of managers and workers is going to increase output from the production side in two ways: (1) the efficiency units of skilled workers will be higher and (2) the managerial talent of skilled managers will be higher as well. Together, they contribute to more production. Occupational choice has another obvious advantage that it helps me to understand the facts about managers and how the productivity differences can account for observed variations in the data through differences in productivity.

1.8.2 Importance of Investment in Skills

To understand the importance of investment in skills, I did the following experiment. I increased the value of curvature parameter in the production of skills \((\phi)\) from 0 to 0.25 and for each value of \(\phi\) studied the effects of changes in productivity and size-dependent distortions on output per worker, skill premium and managerial premium. Table (32) presents the result of such experiment. When \(\phi = 0\), the channel of skill investment is shutdown and \(\phi = 0.17\) is the benchmark case. In the case of no skill investments, the output drops by 68\%, skill premium by 44\% and managerial premium by 42\% when \(A = 1\) and \(\tau = 0\). This is the case where there is no size-dependent distortions.

Decline in productivity term and increase in size-dependent distortions is going to decrease the output further and increase in skill premium and managerial premium.
But the changes in all these cases are less dramatic than the benchmark case with $\phi = 0.17$. As the value of $\phi$ increases the magnitude of changes in the three statistics increases. This illustrates that fixed changes in productivity and size-dependent distortions are magnified in the model when investment in skills is present and the marginal rate of return on investment in skills ($\phi$) is higher. I previously showed that quantitatively, skill investments are necessary to account for the observed differences in GDP per worker in the data along with the statistics on managerial rate, share of skilled managers, skill premium and managerial premium. I thereby emphasize that to account for the observed differences in these statistics across countries, the channel of investment in skills and a reasonable value for the rate of return of this investment are needed.

I further calculate the elasticity of output with respect to exogenous productivity term $A$ in the model when size-dependent distortions are present. I set the distortion parameter $\tau = 0.015$ since this is the average value for this parameter when I match the data in section (1.7). Figure (41) shows that as the marginal rate of investment in skills increases, the elasticity of output with respect to $A$ increases, when the channel of occupational choice and size-dependent distortions are present. It also shows the output elasticity in case without distortions and without occupational choice. The output elasticity in the model is always higher when distortions are present and diverge from the other two cases as $\phi$ increases. Since the model matched data quite well, I take this evidence as showing the necessary role of investment in skills for accounting for observed differences in GDP per worker in the data.

Finally, I highlight the role that size-dependent distortions are playing in affecting the margins of skill investment and occupational choice. These distortions change the incentives for becoming a manager. As a result, the household does not invest in the
skills of its members enough. So the distortions on the production side ultimately changes the incentive for skill investment. The resulting elasticity of output with respect to productivity shows that a distortion-free environment is not always enough to account for observed differences in income across countries.

1.9 Final Remarks

The importance of investment in skills as a source of variation in the observed GDP per worker across countries was emphasized before in the macro-development literature. In this paper, I developed a parsimonious model to study investment in skills and occupational choice in the presence of size-dependent distortions as the central component of observed differences in managerial rate, share of skilled managers, managerial premium and skill premium. I showed that the model can successfully generate the statistics in the data and emphasized the important role of size-dependent distortions for accounting for the facts in the data. I abstracted from channels such as borrowing constraints, which are very important in developing countries, and organizational considerations in those countries about promotions and occupational choice which are important for the managerial occupations and left them for future research.
Chapter 2


2.1 Introduction

We trace world productivity growth from 1996-2014 to its industry sources, using data on more than 36 industries and 40 countries.\textsuperscript{24} “World productivity” is often discussed in models of economic growth and innovation (e.g., Caselli and Coleman, 2006) in the context of a world technology frontier. But few studies formally account for world productivity growth. In this paper, we use new global growth-accounting techniques and datasets to decompose world GDP growth into parts driven by technology, labor, and capital—importantly, accounting for markups and factor reallocation.

Our results provide a clear narrative regarding global productivity. First, world productivity growth—measured as either Average Labor Productivity (ALP) or Total Factor Productivity (TFP)—is highly volatile from year to year and even over multi-year periods. Second, despite this volatility, the contribution of underlying productivity growth at a country-industry level (that is, the weighted average of productivity growth across the 36 industries in each of the 40 or so countries, for a total of some 1,440 country-industries) is relatively constant until the Great Recession. Since the Great Recession, growth in country-industry productivity (as well as in overall world productivity) has been markedly slower. Third, (net) reallocations of labor across countries are the major source of year-to-year volatility in world productivity growth—reconciling the first two results. Labor reallocation is, on average, a drag of about half

\textsuperscript{24}This chapter was jointly written with John Fernald and Bart Hobijn.
a percentage point per year on world productivity growth, as hours typically grow faster in low-wage/low-productivity countries.

Mechanically, our labor-reallocation term, as in the broader growth-accounting literature, reflects the cross-sectional covariance between hours growth and wage levels. The effect arises because ALP or TFP weight all hours equally—wherever the work takes place. But wages differ substantially across countries. Firm optimization implies that these heterogeneous wages reflect the heterogeneous value of labor’s marginal product. On average, labor hours have grown faster in low-wage/low-marginal-product locations, creating the persistent drag on productivity growth. But there is substantial time series variation in the cross-sectional covariance, creating the volatility.

We discuss several interpretations of labor reallocation. A natural interpretation is as shifts in the global misallocation of labor. Considerable research following Hsieh and Klenow (2009b) argues that resource misallocation can explain TFP differences across countries. The same effect can work in the time series. If factor prices differ across firms, say because of distortionary taxes, then marginal value products won’t be equalized. Global misallocation arguably rises if hours grow faster in low-wage countries, in the sense that global output would have risen by more if those hours had grown in a high-wage (high marginal product) country.

Our results do not hinge on this misallocation interpretation. One alternative is that productivity differences may be embodied in workers themselves. For example, wages may be lower in emerging market economies because educational attainment (and, as a result, productivity) is lower. Labor reallocation would appropriately capture these wage and productivity differences. But moving a worker from a low-wage to a high-wage country would not raise global output. That said, as we discuss in Section 2.5, the weight of the evidence is that moving such a worker would, in fact,
raise his or her marginal product. In addition, even if moving the worker would raise global output, the reallocation might not be Pareto-efficient if each country has its own representative consumer.

Before we can reach the three broad conclusions above, we make three contributions. First, we develop a new growth-accounting decomposition that isolates distortions in product, labor, and capital markets. Second, to implement this decomposition, we use the WIOD as a global growth accounting database. We augment the 2016 vintage of the WIOD database with new data on capital services for industries across countries. Third, to allow for output distortions, we extend recent work by Barkai (2019) and Karabarbounis and Neiman (2018) to the world. Specifically, we estimate (rising) economic profits and (sizeable) markups of price over marginal cost across countries and industries. Interestingly, though profits and markups are quantitatively important—with both labor and capital shares of output falling—the broad narrative about global productivity is robust to whether we control for markups or not.

Our global growth accounting method builds on three strands of literature. The first focuses on cross-country productivity levels using economy-wide data (Conference Board, 2015; Feenstra et al., 2015b). These studies do not include industry-level data, so they do not estimate the industry origins of world productivity growth. Moreover, they also do not formally account for reallocation of resources across countries, which turns out to be quantitatively important in the data.

The second strand of the literature, based on the methodology pioneered by Domar (1962), Hulten (1978), and (especially) Jorgenson et al. (1987), studies productivity growth using industry-level data.25 These studies analyze the industry origins of

25 Among the many studies in this literature are Byrne et al. (2016) and Oliner and Sichel (2000) for the United States, Xu (2011) for China, Das et al. (2016) for India, and Rao and van Ark (2013) for Europe.
productivity growth and the importance of the factor reallocation, but only at the country level or for a few countries. Within countries, factor reallocation typically appears modest (e.g., Jorgenson et al., 2016, Figure 14; Samuels, 2017) and, indeed, is often ignored. In a global setting, in contrast, we find that factor reallocation is of first order importance.26

The growth-accounting in this second strand of literature does not account for markup distortions in output markets. As a result, they show how to aggregate country-industry TFP growth, regardless of whether country-industry TFP growth represents changes in technology. In the presence of markups of price over marginal cost, these TFP changes are not, in general, technology changes.

The third strand of literature corrects country-industry TFP changes for markups. This literature goes back at least to Hall (1986). Most closely, we follow Basu and Fernald (2002) and related literature in aggregating productivity in an economy with distortions in product, capital, and labor markets. Baqae and Farhi (2019b) is an important recent contribution. We develop a novel variant of this accounting that isolates the terms of interest.

Specifically, we start from a decomposition of world GDP growth, measured on the production side, that is similar to that in Jorgenson et al. (1987). We then extend it to the case with markups, along the lines of Basu and Fernald (1997) and Basu and Fernald (2002). Our decomposition isolates terms that represent factor reallocation and the effects of markup distortions.27

26Wu (2016, Table 6.4), finds that factor reallocation in China, measured the same way we do, is large in magnitude and quite variable across subperiods. In our data, the labor reallocation effect is mainly across countries though some countries have sizeable within-country effects.

27The decomposition is also closely related to Hsieh and Klenow (2009b). Our growth accounting requires little structure other than cost-minimization. We are then able to analyze observed shifts and reallocations, taking as given the (potentially) distorted equilibrium. But without additional
The data we use are two vintages (2013 and 2016) of the World Input-Output Database (WIOD), described in Timmer (2012) and Timmer et al. (2015). These data cover input-output and productivity data for more than 40 countries and 36 industries from 1996-2014. These countries cover about 80 percent of World GDP measured in dollars over the years in the sample. Unfortunately, industry capital services are missing from the 2016 vintage of the data. We address this shortcoming by constructing the missing capital services data. We also estimate rates of pure economic profits and (under the assumption of constant returns to scale) markups for all countries and industries.

Our main takeaways—volatile world productivity, relatively smooth country-industry productivity, and a sizeable role for labor reallocation—are robust to the measurement assumptions we make. They hold for ALP, for TFP calculated under the Solow assumption of perfect competition (price equals marginal cost), and for TFP calculated using our estimated markup estimates.

The relative constancy of productivity at a country-industry level until the Great Recession masks a marked change in the regional composition of this part of world productivity growth. Consistent with other evidence, our results reveal a slowdown in growth in ALP and TFP for advanced countries starting in the second half of the 2000s, prior to the Great Recession. At a global level, this slowdown is offset, however, by an acceleration of productivity growth in emerging economies, most notably India and China. After 2007 (for TFP) or 2010 (for labor productivity), the productivity slowdown is more widespread.

structure (e.g., on the demand side of the economy), we cannot do counterfactuals the way Hsieh and Klenow (2009b) can. Fernald and Neiman (2011) also discuss links between growth-accounting approaches and the Hsieh and Klenow (2009b) approach, in a two-sector setting.

28See, for example, Fernald (2015), ECB (2017), and Fernald and Inklaar (2020).
In addition to labor reallocation, our growth accounting method isolates the effect of capital reallocation. When we do not allow for markups of price over marginal cost, reallocations of capital have a substantial effect on growth. The bulk of this reallocation is across industries within countries rather than across countries: Within countries, capital input grows faster in industries with a higher apparent internal rate of return.

However, after accounting for markups, the implied effect of capital reallocation is small, since the high apparent internal rate of return to capital is reapportioned to pure economic profits. Markups also play a quantitatively important role in accounting for world output growth. Interestingly, the inclusion of markups has only a minor effect on the estimated country-industry contribution of technology to global productivity.

In most of our results, nominal values are measured in dollars converted using market exchange rates. Thus, the outsized role we find for labor reallocation hinges on the assumption that relative dollar-denominated wages are equal to relative marginal productivity levels of labor. To drop this assumption, we extend data from Inklaar and Timmer (2014) and construct PPP data at the country-industry level for all countries, industries, and years. We can then measure relative productivity levels directly, rather than inferring them from factor prices.

With this in mind, we generalize the growth accounting methods we use to take into account deviations from Purchasing Power Parity (PPP). This correction for PPP differentials accounts for only a third of the labor reallocation effect that arises using dollar-based measures of world GDP. Even after the PPP correction, labor reallocation is still, on net, a substantial drag on world productivity growth and contributes a lot to its volatility. This suggests that it is important to understand
barriers to factor movements and distortions in labor markets when analyzing global economic performance.

2.2 Global Growth Accounting With Distortions

In this section, we introduce a growth-accounting decomposition of world GDP that separates the parts of GDP growth accounted for by changes in technology, aggregate labor, and aggregate capital from the parts of GDP growth driven by changes in factor reallocation and markups.

Our decomposition draws on a long literature, starting with Hulten (1978), that traces aggregate productivity to its industry sources. Hulten considered the case where the market allocation of resources is efficient. Jorgenson et al. (1987) and Basu and Fernald (2002) extend Hulten’s results to cases with market imperfections, including (in the latter case) imperfect competition. Because of these imperfections, the same factor of production may have a different value of its marginal product, depending on where it is used. Our decomposition builds on this literature.

The growth-accounting decomposition we develop here combines terms that isolate particular distortions. It is important to recognize that, with distortions, there is no unique decomposition and that the one applied depends on the research question. Our aim is to isolate the importance of growth in technology, capital, and labor for world GDP growth as well as the quantitative effects on world GDP growth of distortions in product, capital, and labor markets. The specific decomposition we use here is designed to do so. We discuss how it relates to others in the literature (including a recent contribution by Baqee and Farhi (2019a,b)).
2.2.1 Producer Level

This sub-section discusses the implications of distortions for productivity analysis at the producer level. The next sub-section discusses aggregation in this economy.

We analyze the static cost-minimizing decisions of producers to purchase inputs, and on how those decisions are affected by technology and factor prices. The (world) economy has \( n \) sectors, indexed by \( i = 1 \ldots n \). Each sector reflects a particular country-industry combination. The sector takes technology, \( Z_i \), as given; \( Z_i \), and all variables below, have time subscripts that we suppress for readability. Producers pay \( R_i \) to rent capital, \( W_i \) to hire workers, and \((1 + \tau^j_i)P_j\) to purchase intermediate inputs of product \( j \) (so \( P_j \) is the net price received by the producer of product \( j \)). Any (implicit or explicit) taxes on capital or labor usage are incorporated into the \( W_i \) and \( R_i \). Such taxes would affect the interpretation of some of the effects, but not their derivations.

Producers choose factor inputs, \( \{K_i, L_i, \{M_{i,j}\}_{j=1}^n\} \), to minimize their cost of production

\[
R_iK_i + W_iL_i + \sum_j (1 + \tau^j_i)P_jM_{i,j}, \tag{2.1}
\]

subject to the constraint that they produce a given level of output

\[
Y_i = F_i \left( K_i, L_i, \{M_{i,j}\}_{j=1}^n, Z_i \right). \tag{2.2}
\]

Producers in sector \( i \) charge a price, \( P_i \), that includes a potential net markup, \( \mu_i \), over marginal cost. In other words, if \( MC_i \) is marginal cost, then \((1 + \mu_i) = P_i/MC_i\).

Firms’ cost-minimizing first-order conditions for capital, labor, and intermediate inputs imply
These first-order conditions state that the value of the marginal products are a markup 
\((1 + \mu_i)\) above the nominal cost of the factor to the producer. We can, equivalently, 
express these first-order conditions in terms of factor shares and output elasticities. 
For each input \(J\) in industry \(i\), define \(\tilde{s}^K_i\) as the share of cost of input \(J_i\) in total 
revenue (i.e., in nominal gross output). For example, for \(J_i = L_i\), \(\tilde{s}^K_i\) is labor’s share 
in revenue, \(\frac{W_iL_i}{P_iY_i}\).

It follows that for any factor \(J_i\), the output elasticity is a markup over the factor’s 
revenue share:

\[
\frac{F^J_i J_i}{Y_i} = (1 + \mu_i) \tilde{s}^J_i. 
\]  

As is standard since \(?\), we differentiate the production function to express output 
growth, \(\dot{y}_i\), as the output-elasticity-weighted growth in factor inputs plus the contribu-
tion of technological progress. We follow Hall (1990) and use (2.4) to substitute for 
the output elasticities (normalizing the elasticity with respect to technology to one,
\(F_i^Z Z_i / F_i = 1\)). We find

\[
\dot{y}_i = (1 + \mu_i) \left( \tilde{s}^K_i \dot{k}_i + \tilde{s}^L_i \dot{l}_i + \sum_j \tilde{s}^j_i \dot{m}_{i,j} \right) + \dot{z}_i. 
\]  

If there are zero profits, then payments to factors of production exhaust revenue 
and the factor shares sum to one. The shares sum to less than one if there are pure
economic profits. Although we have suppressed time subscripts, factor shares as well as the markup can vary with time.

Given data on factor shares and growth in inputs and output, any assumed markup $\mu_i$ implies a value for the residual measure of technology growth $\dot{z}_i$. In this sense, equation (2.5) can be viewed as an identity that relates inputs, output, markups, and technology. Of course, $\dot{z}_i$ only measures actual technology growth if the assumptions are correct.

Concretely, consider the Solow residual. If we assume constant returns and perfect competition ($\mu_i = 0$), then the factor shares sum to one and equation (2.5) defines $\dot{z}_i$ as the standard Solow residual. It can be calculated from the data even if markups and pure economic profits are not zero. In that case, of course, $\dot{z}_i$ is no longer (in general) a measure of technology change, so its economic interpretation is less clear.

Aggregate output is a value-added concept, which nets out intermediate-input use. So it is useful to re-express the industry expression (2.5) in terms of value added. The Divisia definition of industry value added is

$$\dot{v}_i = \frac{P_i Y_i}{P_i^Y V_i} \left[ \dot{y}_i - \sum_j s_j^i \hat{m}_{i,j} \right].$$

(2.6)

Value added, as Basu and Fernald (1995) point out, is like a partial Solow residual: It subtracts revenue-share-weighted growth in intermediate inputs from gross-output growth, with no adjustment for markups. It then rescales by the ratio of nominal gross output to nominal value added from the point of view of the producer, where $P_i^Y V_i = P_i Y_i - \sum_j (1 + \tau_j^i) P_j M_{i,j}$ (i.e., nominal gross output less payments to purchase intermediate inputs).

It will also be useful to write output growth identically as

$$\dot{y}_i \equiv \left( \frac{\mu_i}{1 + \mu_i} \right) \dot{y}_i + \left( \frac{1}{1 + \mu_i} \right) \dot{y}_i.$$

(2.7)
Substituting this expression into (2.5), we find

\[
\dot{y}_i = \left( \frac{\mu_i}{1 + \mu_i} \right) \dot{y}_i + \left( s^K_i \dot{k}_i + s^L_i \dot{l}_i + \sum_j \tilde{s}_{i,j} \dot{m}_{i,j} \right) + \left( \frac{1}{1 + \mu_i} \right) \dot{z}_i \quad (2.8)
\]

We can now substitute (2.8) into (2.6) to find

\[
\dot{v}_i = \frac{P_i Y_i}{P_i^V V_i} \left( \frac{\mu_i}{1 + \mu_i} \right) \dot{y}_i + \left( s^K_i \dot{k}_i + s^L_i \dot{l}_i \right) + \left( \frac{1}{1 + \mu_i} \right) \dot{z}_i. \quad (2.9)
\]

In this equation, \( s^K_i \) and \( s^L_i \) are payments to capital and labor, respectively, as shares of nominal value added. For example, \( s^L_i = W_i L_i / (P_i^V V_i) \).

The second and third terms in equation (2.9) show that growth in value added depends on share-weighted growth in capital and labor and technology. With imperfect competition, however, value added-growth is not, in general, simply a function of these factors. Rather, as captured in the first term on the right-hand side, imperfect competition implies that there is an extra effect of inputs (including intermediates) and technology.\(^{29}\)

Note that we have made no assumptions so far about returns to scale (the sum of the output elasticities, \( \sum_j F_j^i J_i / Y_i \)).

\(^{29}\)In the special case in which intermediate inputs and gross output are used in fixed proportions (\( \dot{y}_i = \dot{m}_i = (\sum_j \tilde{s}_{i,j} \dot{m}_{i,j}) / \tilde{s}_M^i \)), then it is straightforward to show that value-added growth can be written so that it does depend just on primary input growth; there is a “value-added” markup multiplying share-weighted primary input growth that exceeds the gross-output markup \( \mu_i \). Otherwise, intermediate inputs also matter (see Basu and Fernald (1997)). Equation (2.9) is agnostic about the production structure.
2.2.2 Aggregate Growth Accounting

Divisia growth in aggregate real GDP is value-added-weighted growth in industry real value added:

\[ \dot{\ln v} = \sum_i s_i^V \dot{v}_i, \text{ where } s_i^V = \frac{P_i^V V_i}{P^V V} \text{ and } P^V V = \sum_i P_i^V V_i. \]  

(2.10)

Substituting for industry value-added growth from equation (2.9) yields

\[ \dot{\ln v} = \sum_i \frac{1}{(1 + \mu_i)} s_i^D \dot{z}_i + \sum_i s_i^V s_i^K \dot{k}_i + \sum_i s_i^V s_i^L \dot{l}_i + \sum_i s_i^P \frac{\mu_i}{(1 + \mu_i)} \dot{y}_i. \]  

(2.11)

In this expression, the Domar (1962) weights of sector \( i \) are given by the ratio of nominal industry gross output to nominal aggregate value added, i.e.,

\[ s_i^D = \frac{P_i^V Y_i}{P^V V}. \]

The first term in equation (2.11) relates aggregate output growth to the contribution of country-industry technology shocks. Dividing the Domar weight by the gross markup, \( (1 + \mu_i) \) removes the effect of the markup on prices from this term, so that it values technology shocks using marginal cost rather than prices. The second and third terms relate aggregate output growth to the contribution of country-industry capital and labor growth. The final term captures the “extra” value added that comes from markups and isn’t already accounted for by primary inputs or by technology.

Of course, aggregate productivity is typically defined in terms of aggregate inputs. (For example, aggregate labor input is given by the sum of hours across country-industries, \( L = \sum_i L_i \).) It will be useful to add and subtract growth in aggregate capital and labor. The resulting decomposition, which we will use for our analysis of
world productivity, is

\[ \dot{v} = \sum_i \frac{1}{(1 + \mu_i)} s_i^D \dot{z}_i + s^K \dot{k} + s^L \dot{l} \]

\[ + \sum_i s_i^D \frac{\mu_i}{(1 + \mu_i)} \dot{y}_i + \sum_i s_i^V s_i^K \left( \dot{k}_i - \dot{k} \right) + \sum_i s_i^V s_i^L \left( \dot{i}_i - \dot{i} \right). \]

Here, the aggregate and sector-specific factor shares in value added equal

\[ s^K = \sum_i s_i^V s_i^K, \text{ where } s_i^K = \frac{R_i K_i}{P_i V_i} \text{ and } s^L = \sum_i s_i^V s_i^L, \text{ where } s_i^L = \frac{W_i L_i}{P_i V_i}. \]

These shares include any implicit or explicit tax wedges in factor costs. For example, for labor they measure employee compensation from the point of view of employers.

Equation (2.12) allows us to account for the sources of growth in real value added in the world economy. The three terms in the first line are the direct effect of technology and the contributions of growth of aggregate capital and labor. The terms in the second line account for how the change in the global allocation of productive resources affects world GDP growth.

### 2.2.3 Interpreting Changes in Resource Allocation

Because the terms in equation (2.12) that measure the effects of markups and changes in resource allocation turn out to be important in our results, we discuss each of them here.

*Markups and product market distortions* The first term on the second line of (2.12) captures the effect of markups. In a direct growth accounting sense, this term captures the fact that, with markups, the revenue-share-weighted growth in primary inputs doesn’t capture the full productive effect of capital, labor, and intermediate input usage.
Clearly, markups are also related to static efficiency and welfare. Markups most obviously lead to static efficiency losses by, for example, distorting the labor-leisure choice; or by distorting producers’s choices about the use of intermediate versus primary inputs. Note also that we quantify the impact of resource changes starting from an already distorted allocation. In that case, output in sectors with high markups is relatively undersupplied. The markup term on the second line of (2.12) captures that output growth in sectors with markups alleviates this distortion.

Of course, the full dynamic general equilibrium effects of markups and the tradeoff between static markup distortions and dynamic Schumpeterian gains from innovation are complicated. We take the path of markups and technological change, $Z_i$, as given and without considerably more structure, which goes beyond the scope of this paper, we cannot quantify the full endogenous effects of markups.\footnote{This paragraph draws on the welfare discussion in Basu and Fernald (2002, p. 981-2).}

**Labor-market distortions** The final term of equation (2.12), $\sum_i s^V_i s^L_i \left( \hat{i}_i - \hat{\dot{i}} \right)$, captures the effect of reallocations of labor. As we explain below, these effects include reallocations that change the magnitude of static labor misallocation.

To better understand the interpretation of this labor-reallocation term, it will be useful to express it a different way. First, define the cross-sectional (across countries and industries) world average gross wage in a given year as $W = (\sum_i W_i L_i) / L$. Second, note that, since world hours are the simple sum across country-industries, growth in world hours is

$$\dot{\dot{i}} = \sum_i \left( \frac{L_i}{L} \right) \dot{i}_i = \sum_i \left( \frac{W L_i}{W L} \right) \dot{i}. \quad (2.14)$$

\footnote{Edmond et al. (2018) discuss the costs of markups in the context of a fully-specified model, and provide references to this literature.}
In the definition of labor reallocation, we use (2.14) to substitute for $\dot{l}$ and note that $s_i^V s_i^L = W_i L_i / PV$; the aggregate labor share is $s^L = W L / PV$. We find:

$$\sum_i s_i^V s_i^L \left( \dot{l}_i - \dot{i} \right) = \sum_i \left( \frac{W_i - \bar{W}}{PV} \right) \dot{l}_i,$$

(2.15)

$$= s^L \sum_i \left( \frac{W_i - \bar{W}}{W} \right) \dot{l}_i.$$

(2.16)

This expression shows that, mechanically, the labor reallocation term entirely reflects the covariance of country-industry (gross) wages and growth in labor input. If wage differences do not co-vary with labor input growth, then labor reallocation is zero. In contrast, if labor grows faster in country-industries where it has a higher-than-average gross wage, then there is a positive reallocation. Other things equal, that reallocation boosts growth in output and aggregate TFP.

Putting an economic interpretation on labor reallocation requires understanding the source of gross wage differences. Suppose that wages differ by country-industry because of differential taxes on labor, $\tau_i^L$. Then $W_i = W \left( 1 + \tau_i^L \right)$ and $\bar{W}(1 + \tau^L) = W \left( 1 + \bar{\tau}^L \right)$. Labor reallocation is then

$$\sum_i s_i^V s_i^L \left( \dot{l}_i - \dot{i} \right) = s^L \sum_i \left( \frac{\tau_i^L - \bar{\tau}^L}{1 + \bar{\tau}^L} \right) \dot{l}_i.$$

(2.17)

Labor reallocation is positive if we shift resources towards the industry with the higher distortionary tax. This is intuitive from the first-order condition (2.3). For given markups, the value of the marginal product (the right-hand side of (2.3)) rises if the gross wage rises (the left-hand side).\footnote{The value of the marginal product also depends on the markup, which we account for in the markup-reallocation term. The labor reallocation term completely accounts for the change in output if there are no net markups ($\mu_i = 0$), as well as no changes in country-industry technology $z_i$, aggregate $L$ or $K$, or in the distribution of $K_i$: $\dot{l} = \dot{k} = 0$, and for all $i$, $\dot{z}_i = \dot{k}_i = 0$. With these assumptions, the only change in the economy is in the distribution of $L_i$ across country-industries. From (2.12), aggregate value added growth is then equal to the labor-reallocation term.}
Given (2.17), a natural interpretation of the labor reallocation term is that it reflects a change in static world misallocation, holding the effects of other distortions fixed. Consider the statically “efficient” allocation, defined as the allocation that maximizes global output from a given flow of labor. It is clear that differential country-industry taxes on labor, $\tau^L_i$, can move the economy away from the allocation that maximizes output. In this situation, if labor “shifts” to where distortions are larger and where—according to the first-order conditions—there is a higher marginal product, then reallocation is positive. The allocation of resources moves closer to the output-maximizing allocation, so misallocation falls.\(^{33}\) Note that this interpretation does not require that the same worker actually move from one location to another, just that labor grows faster in the high-distortion country-industry. The faster growth could reflect faster growth in the working-age population, a business cycle boom that raises the employment rate, or other factors.

Conceptually, this term is akin to changes in spatial misallocation discussed by ?. They argue that, based on productivity differences, there are too few people working in high-productivity San Francisco and New York, and too many working in less productive (and less-densely populated) U.S. regions. If, for any reason, labor input grows faster in high-productivity locations, then this source of misallocation will fall.

Globally, the same force is at work. Productivity in German car manufacturing is much higher than that in Mexico. This means that, from a global perspective, there

\(^{33}\)As noted in footnote 32, this discussion holds the effects of other distortions fixed. In practice, the different distortions that we have identified could interact. For example, consider the stylized example from equation (2.17). Suppose $\tau^L_1 = 0.2, \tau^L_2 = 0$, and that $dL_1 = -dL_2 > 0$. But suppose also that $\mu_1 = 0$, while $\mu_2 = 0.2$. In this case, the value of the marginal product of labor is, in fact, equalized; and shifting the worker from firm 2 to firm 1 would not, in fact, change global output. Our decomposition isolates the distortions coming from markups (holding the distortion from labor taxes fixed) from the distortions coming from labor taxes (holding the distortion from markups fixed). Thus, it would measure this shift as a positive labor reallocation term along with a negative and offsetting markup reallocation term.
is a misallocation of production factors. World GDP would increase if we moved resources, including workers, from Mexican to German car manufacturing (if we could).

Of course, the gross wage differentials observed in the data across countries might not simply reflect (implicit or explicit) distortionary taxes. Several alternatives are possible, including (but not limited to) the following. First, the difference in dollar-based wages could reflect differences in purchasing power across countries. We discuss how we adjust for PPP later in this subsection.

Second, suppose the wage differentials reflect barriers to mobility that prevent the equalization of wages from the point of view of workers. One could straightforwardly interpret this as a higher shadow tax on labor in high-wage countries (with the tax being paid to the worker), as in (2.17). But if different countries have different social welfare functions, it could be that the barriers to mobility are “efficient” from the point of view of the representative consumer in the high-wage country—for example, the representative consumer might not care about the utility of the immigrant who is arbitraging wage differences. This is not something we can assess from data alone.

Finally, note that for understanding productivity dynamics alone, it is not crucial to understand the source of the wage differences across countries. This is because we are measuring productivity using raw hours. An hour is an hour, whoever does the work, and whatever the skills or experience of the worker. The first-order conditions tell us that the wage should be proportional to the marginal product, whether the source of the productivity difference is the technology of the country-industry, or is embodied in the skills and experience of the worker. But of course, if an important source of the wage differences is embodied in the workers themselves—through education, for example—then it would not be the case that moving workers from, say, apparently low-wage Bangkok to high-wage Boston would actually raise global output. This issue
of embodiment also matters for assessing the welfare consequences of labor reallocation. Hence, we return to this issue in Section 2.5.

Capital-market distortions The next-to-last term in (2.12), \( \sum_i s_i^V s_i^K \left( \dot{k}_i - \dot{k} \right) \), captures how changes in the allocation of capital across countries and industries affects world GDP growth. The intuition for this capital-reallocation term is very similar to that of the labor-reallocation term. As with labor, the capital-reallocation term can be written in terms of the covariance of capital rental rates and capital growth across sectors. In a statically efficient allocation, the world capital stock is adjusted in every period to equate the rental rates, that is, the shadow values of capital across all sectors. As Hulten (1978) showed, if these shadow values are equalized, then this term is zero. Capital reallocation is positive if capital grows faster in sectors with high rental rates of capital—which, from the first-order conditions, implies high marginal products of capital. Holding the other terms in equation (2.12) fixed, that reallocation of capital contributes positively to world GDP growth. To the extent the differences in rental rates reflect distortions, such as distortionary capital taxes, output increases because capital misallocation falls.

Distortions in intermediate goods and services demand We explicitly consider distortions in intermediate goods demand in the form of the tax rates, \( \tau_i^j \). Changes in these tax rates, which have only a second-order effects on output growth, do not appear explicitly in our decomposition. Implicitly, these changes show up in the impact of the intermediates’ demand distortions on the marginal products of capital and labor. As a result, these second-order impacts of distortions in intermediates demand show up as part of the factor reallocation terms of capital and labor.

Impact of deviations from PPP In practice, when one considers industries with many different types of output, the units of measurement of the marginal products of capital
and labor differ. That is, in agriculture, the marginal products are measured in terms of agricultural products while in metal manufacturing they are measured in terms of metal.

To compare these marginal products across industries one needs to translate them into common units. This is most naturally done by using relative output prices and that is what is captured by the value-added shares, $s^V_i$. For our global analysis of productivity, we face another choice, namely what unit to express these prices in.

For our baseline results, we use U.S.-dollar-denominated prices. In that case, the reallocation terms in (2.12) measure the degree to which production factors disproportionately grow in industries with high dollar-denominated marginal products. The use of U.S.-dollar-denominated prices makes sense if all goods and services are tradable. In the case of our car manufacturing example, Volkswagen will focus on the dollar-denominated marginal products when it decides on where to produce Beetles that it sells on the global car market.

However, the Balassa-Samuelson-effect (Balassa, 1964; Samuelson, 1964) implies that for non-tradable goods and services, there might be persistent deviations in relative dollar-denominated marginal products from relative physical marginal products. These differences are reflected in deviations from PPP. To take this into account, we also present a set of results in which we use PPP-dollar denominated value-added shares for $s^V_i$. As we discuss in the next section, this requires the use of a newly-constructed dataset with country-industry level PPP price deflators.
2.2.4 Discussion of Alternative Aggregation Equations

The industry-to-aggregate relationships in Hulten (1978) and Jorgenson et al. (1987) are special cases of equation (2.12). Hulten considers the no-markup case (for all $i$, $\mu_i = 0$) and where all purchasers face the same input costs for capital and labor. Jorgenson et al. retain the the no-markup assumption, but allow purchasers to face different input prices.

Basu and Fernald (2002) extend Jorgenson et al. to allow for imperfect competition. Basu and Fernald and note that as the first-order conditions in (2.3) show, markups create a wedge between the “cost” of a factor and the value of its marginal product. Indeed, the social value of the marginal product depends on the markup of the purchasing industry. As a result, if markups differ across industries, then the effect on aggregate output depends on how the extra output is allocated across uses. Basu and Fernald (2002, p.979) chose a benchmark allocation rule for production where intermediate inputs are used in fixed proportions to output. If this assumption is relaxed, then there is an additional aggregation term in the Basu and Fernald (2002) equation for the reallocation of intermediate inputs.

Given this lack of uniqueness in the aggregation, other papers have made different choices about the allocation rule. These include and, more recently, Baqaee and Farhi (2019a,b). Baqaee and Farhi take as their benchmark for measuring aggregate technology the case where, following an industry technology shock, all uses

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34It is the value of the marginal product that matters for aggregate output, not the marginal revenue product. The reason is that aggregate output is valued using prices (marginal rates of substitution).

35This assumption is consistent with typical representative-agent models with imperfect competition, e.g., ?
of industry output (final expenditures and uses as intermediate inputs) expand in equal multiplicative proportions. They argue that this allocation rule is more natural in some settings.

The different decompositions in the literature can all be interpreted as accounting identities. That is, all of them are equally “correct” in an accounting sense, in that all of them describe the data perfectly. But if the benchmark assumptions are not correct, the terms might not necessarily have a clear economic interpretation.\(^\text{36}\)

In this regard, note that the identities include the industry growth-accounting relationship (2.5). As noted, that equation can be considered an identity linking output, inputs, assumed markups, and technology: Given the first three, the fourth (technology) is pinned down as a residual.

Relative to the existing literature, the decomposition in (2.12) does not take an explicit stand on what is being held fixed. Rather, it isolates the effects of markups and differential factor prices across country-industries. Our decomposition is thus well-suited to quantify the effect of shifts in the misallocation of resources on world GDP over time.\(^\text{37}\) It is not suited, however, to do a sources-of-growth accounting that is used to split up world GDP growth in parts due to capital, labor, and technology

\(^{36}\)The Baqae and Farhi (2019b) aggregation equation has very strong data requirements; the authors are not able to estimate all the pieces of their equation directly. In addition, their maintained assumptions include constant returns to scale. Although they argue that some sources of non-constant returns can be accommodated, the interpretation of the terms in their equation in a world with increasing returns remains unclear. In contrast, our equation, and the one in Basu and Fernald (2002) requires no assumptions at all on returns to scale. That said, when we implement the aggregation equation (2.12), we impose constant returns in order to measure markups.

\(^{37}\)As a practical matter, our decomposition has the advantage that we are able to isolate the distortion terms even when we are limited to using data on average labor productivity rather than TFP. Neither the Basu-Fernald nor Baqae-Farhi aggregation equations easily allow this use.
growth. Such an accounting exercise would involve splitting up gross output growth, $\dot{y}_i$ in (2.12) into parts due to capital, labor, technology, and intermediate inputs.\(^{38}\)

### 2.3 WIOD-Data

For the empirical implementation of our global growth accounting method with distortions, we use SEA data from the WIOD. The reason we use these data is that it is the only productivity dataset that covers a broad set of industries across the major world economies.\(^{39}\) Two vintages of the WIOD have been released, one in 2013 and one in 2016. We calculate results using both of them. We merge data from two additional sources with the WIOD: Data from Timmer et al. (2007) for the construction of PPP deflators and data from OECD (2017b) for capital price deflators used for the 2016 vintage of WIOD. (Appendix B.3.2 details how we merge these data.)

For all variables, we approximate continuous-time growth rates in with log-changes. We measure the time-varying factor shares for any given year $t$ as the average share in years $t$ and $t - 1$.

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\(^{38}\)One additional difference between our analysis and that in Baqae and Farhi (2019a,b) is that we do not transform all “distortions” (including differential factor prices) into markups. This turns out to be important, because the effects of markups and differential factor prices yield three separate terms in our decomposition that each coincide with terms already used in other growth accounting decompositions. Hence, our derivation helps show how the decomposition in Baqae and Farhi (2019b) is related to conventional growth accounting results.

\(^{39}\)Other datasets, like Conference Board (2015) and Feenstra et al. (2015b) provide aggregate data only at the country level. The closest alternative dataset is the Organization for Economic Cooperation and Development (OECD)’s STAN database (OECD, 2017b), which covers fewer years and countries than the WIOD data we use.
2.3.1 Comparison Across Vintages and With Other Data Sources

The two vintages differ somewhat in the industries, countries, and years covered. Important for our analysis is that the two vintages contain an overlapping period from 2000-2007. We use this period in the rest of the paper to compare results across vintages to make sure that there are no major qualitative differences in results due to differences in countries and industries covered as well as methodological differences in the construction of variables.

Table 1 compares the two vintages of the WIOD that we use. The top part of the table shows the difference in coverage between the vintages in terms of years, countries, and industries.

The sample of countries is largely comparable across vintages. The 2016 vintage contains three more countries than the 2013, namely Norway, Switzerland, and Croatia. These countries are relatively small, so the average share of world GDP covered is similar in the two vintages. At times, we aggregate our results into regions or country blocks, as shown in Table 44 in Appendix B.3.2.

We also present results for major sectors of the economy (listed in Table 45 in Appendix B.3.2). Each of these sectors comprises ISIC industries for which the WIOD data are reported. Even though the 2016 vintage of the data contains many more industries than the 2013 vintage (see Table 1), the major sectors that we focus on are consistent over time and across vintages.

Two differences between the vintages are important to note for the interpretation of our results. First, there is a discrepancy between the two data vintages in terms of hours growth. In particular, hours growth in the 2001-2004 periods is half as much in the 2016 vintage as in the 2013 vintage. This is largely due to the different ways
hours growth in China and India are constructed in the two vintages. Second, the 2016 vintage does not contain data on capital price deflators. We supplement the available WIOD data and constructed such deflators using data from OECD (2017b).

For the overlapping years, aggregates from the two vintages line up closely, as well as with world-level aggregates from the World Bank (2018). Figure 2 shows that the real GDP growth pattern in the WIOD data mimics that of world GDP. Both show an acceleration in world GDP growth after 2000 up until the Great Recession in 2008. Global economic activity shrunk in 2008, causing a dip in world GDP before accelerating again during the recovery phase of 2009-2014. The main difference is that world real GDP growth is a bit higher from 2002 than in our data because our sample of countries does not include many fast-growing emerging economies. The fact that the WIOD data show the same qualitative patterns as those from the World Bank (2018) makes us confident they capture the main movements at a global level.

So, our sample covers more than three quarters of the global economy and the growth rate of GDP that we decompose in the rest of this paper closely resembles that of the world economy.

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40We discuss these differences in more detail in Appendix B.3.2.

41Value added in World Bank (2018) is measured at purchaser’s prices while WIOD-SEA (Socio-Economic Accounts) value added is reported at basic prices. The difference is taxes on products and imports, i.e. \( \tau_j \) in our theoretical framework. Of course, our data also do not cover all countries in the world.

42See Appendix B.3.1.1 for a comparison of nominal GDP measures.
2.3.2 Implementation of World Productivity Growth Measurement

The WIOD-SEA dataset contains measures that correspond to many of the terms in (2.12): Nominal and real gross output, labor inputs, and compensation. What is not directly reported, for one or both of the vintages, are measures related to capital input and markups.

Gross output and value added: Nominal gross output, $P_iY_i$, nominal value added, $P_i^V V_i$, along with quantity and price indexes are directly reported. The growth in real gross output, $\dot{y}_i$, and real value added $\dot{v}_i$ can be calculated directly.

Labor input and compensation: Hours, i.e., labor input, $L_i$, are included in the data for all industries and countries and the growth rate of hours, $\dot{l}_i$, can thus be directly calculated. In addition, the compensation of labor, i.e. $W_iL_i$ is also directly reported.

Markups and payments to capital: To implement our growth accounting equation, (2.12), we require markups for all 1400 industries (in the 2013 vintage of data, where we have 35 industries in 40 countries). Relatedly, we need capital shares based on required payments to capital, which do not include pure profits. We estimate required payments to capital and infer the level of markups, $\mu_i$, in a similar manner to Barkai (2019) and Karabarbounis and Neiman (2018).

The part of nominal value added that is not paid to labor consists of required payments to capital plus pure economic profits. Denoting profits by $\Pi_i$, we can write

$$ P_i^V V_i - W_i L_i = R_i K_i + \Pi_i. \tag{2.18} $$

We first estimate required payments to capital, $R_i K_i$, as explained below. Second, we impose constant returns to scale and back out a markup consistent with the implied
profit rate.\footnote{Recent literature such as Karabarbounis and Neiman (2018) point out that “profits” potentially include payments for unmeasured capital, notably intangible capital, as well as pure economic profits. Hence, if the accounting identity in (2.18) is applied to data that does not include these and other intangibles, then the right-hand side includes the implicit compensation net of the implicit investment flow. We note that even our measures of standard capital do not include land or inventories. As a result, we are bound to find higher profit estimates than datasets that do include these types of capital.} We follow Hall and Jorgenson (1969) to estimate a required return on capital, $R_i$, in a user-cost framework by assuming that the nominal capital service flows equal the nominal replacement value of the capital stock (reported in the data) times a real user cost of capital. This real user cost consists of a nominal return on capital corrected for depreciation and capital price inflation. We use the 10-yr BBB U.S. nominal corporate bond rate as the nominal rate.\footnote{Our qualitative results are similar when we use the 10-year U.S. treasury yield, e.g. Schmelzing (2017)}

Second, to back out the country-industry-specific markups from the profit estimates, we follow much of the recent literature and assume constant returns to scale at the industry level. With this assumption, profits $\Pi_i = (\mu_i/(1 + \mu_i)) P_i Y_i$.\footnote{One alternative approach, pursued by Baqaee and Farhi (2019b), would be to use direct estimates of firm-level markups, e.g. those by Loecker and Eckhout (2017, 2018). As Traina (2018) discusses, these estimates directly pertain to the wedge between price and marginal cost and their magnitude critically hinges on what is assumed to make up variable costs for firms. In our aggregate growth accounting framework such markups would not be the right measure because they would also be non-zero in the case of fixed operating costs or entry costs in which firms’ individual technology exhibits decreasing returns to scale (increasing marginal cost in variable factors) but aggregate technology exhibits constant returns to scale and the market allocation is efficient, e.g. Hopenhayn and Rogerson (1993). A second alternative approach, following Hall (1990) and Basu and Fernald (1997), estimates industry returns to scale and markups jointly. That approach is more data intensive than is possible with 1400 or more industries in 40 countries. But constant returns is not innocuous here. For example, Ho and Ruzic (2019) find that in U.S. manufacturing, profit rates rose in the 1990s and 2000s despite roughly constant markups, because returns to scale fell (from increasing to approximately constant).}
2.3.3 Calculating Results in Four Steps

The advantage of using the WIOD-SEA data is that they cover a broad set of industries for not only advanced but also for emerging economies. The disadvantage is that some variables in the data are less reliably measured, especially for the latter group of countries.

With these data limitations in mind, we construct the decomposition in (2.12) in four steps. First, we start with a decomposition that uses the most reliably measured components. Namely, we consider ALP growth and ignore markups. This relies only on value-added and hours growth.

To begin, recall that \( \dot{v} = \sum s_i^V \dot{v}_i \) and, trivially, note that world labor growth, \( \dot{l} \), equals \( \sum s_i^V \dot{l}_i \). Using these expressions, and subtracting and adding \( \sum s_i \dot{l}_i \), we can write world ALP growth as

\[
\dot{a}l = \dot{v} - \dot{l} = \sum s_i^V \dot{a}p_i + \sum s_i^V \left( \dot{l}_i - \dot{l} \right)
\]

Here, the first term on the right-hand side is the contribution of country-industry specific ALP growth. The second term reflects shifts in hours growth across country-industries. Some algebraic manipulation shows that the second term can be written as \( \sum_i \left( \frac{L_i}{L} \right) \left( \frac{P_i V_i / V / L}{P_i V_i V / L} - 1 \right) \dot{l}_i \), which will, in general, be nonzero if nominal value added per hour worked differs across country-industries. Nominal value added per hour worked might, in turn, differ across country-industries for efficient reasons (such as differences in factor shares) or because of wedges (such as factor-price wedges or

\[\text{To see this, note that, since } \sum_i s_i^V = \sum_i \frac{P_i V_i}{P_i V / V} = 1 \text{ and } \dot{l} = (L_i / L) \dot{l}_i, \text{ we can write the second term on the right-hand-side of (2.19) as } \sum_i \left( \frac{P_i V_i / L_i}{P_i V / V / L} - 1 \right) \dot{l}_i = \sum_i \left( \frac{L_i}{L} \right) \left( \frac{P_i V_i / L_i}{P_i V / V / L} - 1 \right) \dot{l}_i.\]
Therefore, it is useful to decompose the shift-in-hours term into two pieces:

$$\sum_i s_i^V \left( \hat{i}_i - \hat{i} \right) = \sum_i s_i^V s_i^L \left( \hat{i}_i - \hat{i} \right) + \sum_i s_i^V (1 - s_i^L) \left( \hat{i}_i - \hat{i} \right). \quad (2.20)$$

The first piece is the labor-reallocation term from equation (2.12); as discussed in Section 2.2.3, this term may be non-zero if there are wage differences across country-industries. In case of a statically efficient allocation of resources, this term would be zero. The second piece is a residual, reflecting other differences in factor shares or markups that may affect nominal value-added per hour (which might or might not be efficient).

After presenting these labor-productivity results, we move to the second step, which adds capital to the above decomposition but maintains the assumption of no markups. That is, it considers a version of the full TFP decomposition in (2.12) under the assumption of zero markups ($\mu_i = 0$). This step assumes that $s_i^K = (1 - s_i^L)$; capital’s rental rate in each industry is whatever is needed for this to be true. These results are useful because they directly allow for the comparison with results from other studies that use standard TFP measures calculated under the assumption of constant returns and zero markups, such as those based on Jorgenson et al. (1987).

In the third step, we present the full decomposition (2.12), including non-zero markups. This enables us to quantify the impact of changes in product-market distortions on world GDP growth. By comparing the results from this step with those from step two, we can assess how markups affect global productivity growth estimates.

In the final step of our analysis, we consider the impact of deviations from PPP on the decomposition (2.12). For this we construct PPP value-added measures by country-industry and use them to construct value-added shares, $s_i^V$, in terms of 2005
PPP dollars rather than current U.S. dollars. So, our final set of results implements a PPP value-added share weighted version of (2.12).

2.4 Results

We use the two WIOD vintages to construct annual estimates of each of the components of equations (2.12) and (2.19). The key takeaways from this section are that (i) world productivity growth is volatile from year to year or over multi-year periods, even though (ii) underlying country-industry productivity growth is relatively smooth; and (iii) Reallocation, particularly labor reallocation, explains the bulk of the high-frequency volatility in world productivity.

Before we present the growth-accounting results in the steps described in the previous section, we first discuss the value-added and factor shares that help put the subsequent results in context.

Value-added and factor shares

In some form or another, all our results based on (2.12) are weighted averages of growth rates across industries by country. The weights are the country-industry share in world value-added, either in current U.S. dollars or in 2005 PPP dollars. It is thus important to understand the main properties of these shares.

In terms of current U.S. dollars, the U.S. and Japan are the two largest individual economies, together covering more than 40 percent of world GDP. The share of the U.S.

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47 Appendix B.3.2.4 discusses how we construct these PPP measures. Because we use country-industry level PPP data there can be different degrees of deviation from PPP across industries within a country.
and Japan in world GDP has declined over the 19 years in our sample. This is mainly because of the relatively strong growth performance of China, whose value-added share increased by 10 percentage points.

There are notable differences between value-added shares by country in terms of current U.S. dollars and in terms of PPP dollars. The main difference between the PPP-based and dollar-based valued-added shares is that, due to high PPP prices in the U.S., the U.S. value-added share in U.S. dollars is much higher than in PPP dollars. China and India are the two countries whose value-added shares increase the most when the unit of measurement is changed from current U.S. dollars to 2005 PPP dollars. Both of their shares more than double. This is consistent with the Balassa-Samuelson-effect that more productive economies tend to have “overvalued” currencies.

No matter whether we use dollar-denominated or PPP-denominated value-added shares, manufacturing, trade, and Finance, Insurance, and Real Estate (FIRE) are the sectors with the highest value-added shares. These shares do not fluctuate much across the subperiods we consider. Agriculture and manufacturing have slightly higher PPP-shares than dollar shares, while those in FIRE and business services are slightly lower. This reflects that the latter two sectors are larger in advanced economies, especially the United States.

The other shares that matter for the decomposition in (2.12) are factor shares. Figure 3 plots the global factors shares from 1996-2014 for both vintages of the data. It reveals that the global labor share has declined, as documented by Karabarbounis and Neiman (2014). However, the decline in the labor share pales in comparison to the movements in the factor shares of capital and profits. Just like Barkai (2019) for the United States, we find that the capital share in world GDP has declined substantially, by more than 10 percentage points, since 1996. The joint declines of the labor and
capital shares are absorbed by an increase in the profit share. By the end of the sample, pure profits amount to nearly 20% of world GDP.

These profits are concentrated in manufacturing, trade, and FIRE. Most notably, profit rates in FIRE showed the largest increase over the sample. Markups are particularly high in manufacturing in China and in FIRE in the United States.

Although the estimated profits and markups are high, it is important to note that our main takeaways below are robust to whether or not we account for markups.

Growth-accounting results

We now turn to the growth-accounting results. As discussed, we proceed in four steps: (1) (relatively well measured) labor productivity, (2) conventional TFP, (3) markup-adjusted TFP, and (4) PPP- and markup-adjusted TFP. Each step requires additional, stronger assumptions to construct the data. Nevertheless, the main takeaways remain remarkably consistent throughout this progression, indicating that the data assumptions do not drive the results.

For each step, we group the results by WIOD vintage and, further, into five subperiods: (i) the 1990’s expansion, 1996-2000, (ii) the 2001 recession and recovery, 2001-2004, (iii) the mid-2000’s expansion, 2005-2007, (iv) the Great Recession and early recovery, 2008-2010, and (v) the recovery from the Great Recession, 2011-2014, which is the period of the Euro crisis in many countries in our sample. The 2001-2004 and 2005-2007 periods exist in both WIOD vintages, allowing a direct comparison
of results. We focus primarily on the qualitative results that both vintages have in common, rather than on the precise numbers.\footnote{Section B.3.1 of the Appendix includes the underlying details relevant for the points we make in the main text.}

Step 1: World labor productivity growth

In this step, we implement the world ALP decomposition in equation (2.19). We begin graphically with Figure 4, which illustrates the three key takeaways that apply throughout the four-step analysis that follows. For visual clarity, we show the data only from the 2016 WIOD vintage.

First, the dark line in the figure shows the substantial volatility in world ALP growth, $\dot{v} - \dot{l}$. Second, the light line shows the much smoother contribution of country-industry ALP growth, $\sum_{i} s_{i}^{V} \alpha l p_{i}$. For example, the country-industry growth rate stays relatively constant in the 2003-2007 period; and it drops much less than world ALP growth in 2009 or 2011. Algebraically, equation (2.19) shows that the difference between the two lines reflects shifts in hours across industries with different levels of labor productivity, $\sum_{i} s_{i}^{V} (\dot{l}_{i} - \dot{\bar{l}})$. This effect includes the contribution of labor reallocation, $\sum_{i} s_{i}^{V} s_{i}^{L} (\dot{l}_{i} - \dot{\bar{l}})$. The third takeaway is the year-to-year volatility of this labor reallocation term, which explains much of the difference between the volatile world ALP growth and the smooth country-industry labor productivity growth.

Table 2 shows the detailed subperiod numbers for the two vintages. The rows correspond to components of equation (2.19). Line 1 of the table shows world GDP growth in each period. During the Great Recession period (2008-10, shown in the 2016 vintage), output grows much more slowly than in any previous period; it is followed...
by a sizeable recovery in 2011-14. Line 2 shows growth in world hours. Comparing
the 2001-2004 and 2005-2007 periods across vintages, one can see the discrepancy in
hours growth across vintages that we discussed in Subsection 2.3.1. Specifically, world
growth in hours in the 2016 vintage was about 1-1/4 percent lower from 2001-04 than
in the 2013 vintage, but then was about 1/2 percentage point higher from 2005-07.
These revisions, though large, do not substantially affect the key takeaways from this
section.

Lines 3, 4, and 8 show the key takeaways from implementing equation (2.19). Line 3
shows World ALP growth, which is output growth (line 1) less hours growth (line 2).
Lines 4 and 8 decompose this growth into (line 8) the part that reflects
country-industry ALP growth, \( \sum_{i,s} V_i \dot{a}p_l \); and (line 4) the part that reflects shifts in
hours across country-industries, \( \sum_{i,s} s_i (\dot{i}_i - \dot{i}) \). By construction, line 3 is the sum of
lines 4 and 8.

Line 3 shows the first key takeaway: World ALP growth is volatile across the five
subperiods that we distinguish. During the expansion of the late 1990’s, world ALP
growth was above 2 percent. Growth declined substantially in the early 2000’s and
(in both vintages) rebounded sharply in the mid-2000’s. During the Great Recession
(2008-10), world ALP growth retreated to under 1 percent per year. In the 2011-14
period, world ALP growth got even worse, turning sharply negative.

Line 8 shows the second key takeaway, which is the relatively smooth evolution of
ALP growth at a country-industry level, \( \sum_{i,s} s_i \dot{V} a \dot{p}_l \). Indeed, country-industry ALP
growth was relatively constant at about 2 percent per year—regardless of which
vintage you look at—over the first four of the five subperiods we consider. A sharp
deterioration in country-industry ALP growth is apparent only in the final 2011-14
The third takeaway, from lines 4 and 5, is that the bulk of the variation in world ALP growth arises from substantial volatility in the effects of shifting hours, notably labor reallocation. This follows from the first two takeaways, given that the contribution of shifting hours (line 4) is, as an accounting identity, the difference between the volatile growth rate of world ALP growth and the relatively smooth contribution of country-industry specific ALP.

As discussed in section 2.3.3, this shift-in-hours term reflects the cross-sectional covariance of labor growth and nominal value added per hour. Those differences could be efficient—reflecting, say, technological heterogeneity in factor shares across industries. Or they could be related to wedges, such as markups or labor taxes. For this reason, line 5 of Table 2 breaks out labor reallocation, \( \sum_i s_i^V s_i^L (l_i - \bar{l}) \). This piece, as discussed in Section 2.2.3, reflects the cross-sectional covariance of wages and labor growth. Wage differences are plausibly related to efficiency and welfare (though, as discussed in Section 2.2.3, the efficiency and welfare consequences are not entirely clearcut). This labor-reallocation term in line 5 carries over to the TFP decompositions below.

Within labor reallocation, what turns out to be quantitatively most important is reallocations across countries, reported in line 7 of the table.\(^{49}\) These shifts are, on average, a drag on world GDP growth of between around 0.4 and 0.5 percentage points. This reflects the fact that hours growth in emerging economies, where wages are lower, has typically outpaced hours growth in developed economies. The first-order

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\(^{49}\)See Appendix B.1 for more details on how we split misallocation term into within- and across-country components.
conditions interpret these shifts as a reallocation of labor from high to low marginal-product-of-labor countries, as valued using measured prices. This cross-country term was slightly positive during the expansion in developed economies from 2005-2007. In contrast, the term was more negative in periods when there was a bigger wedge in hours growth between emerging and developed economies, as in 2001-2004, 2008-2010, and 2011-2014. Note also, from line 6, that shifts in the within-country reallocation of labor contribute little to world GDP growth.

Table 3 decomposes the contribution of country-industry ALP growth into its regional composition. It shows that the *composition* of this component across countries has changed notably over time. In terms of the cross-country details, these results are in line with studies that document a broad productivity slowdown in industrialized countries starting in the early 2000’s (e.g., Cette et al., 2016). We find that the contribution of country-industry specific ALP growth of these countries (United States, Japan, and the United Kingdom in particular) declines in the last three periods in our sample that cover 2005-2014. The global productivity impact of this slowdown was largely offset by an increase in the contributions of country-industry specific ALP growth to world GDP growth of Brazil, Russia, India, and China (BRIC countries). The contribution of BRIC countries’ country-industry specific ALP to world productivity growth declined during 2011-2014. This, together with country-industry specific ALP growth in the United States, is the main driver of the decline in world ALP growth during that period.

What this result points out is how important it is to do growth accounting on a global scale to understand shifts in the center of gravity of global productivity growth. This is especially important during the 1996-2014 period that we consider, because of the growth performance of emerging economies in Asia.
Step 2: World TFP growth without markups

In step two, we explicitly account for capital and focus on TFP rather than ALP growth. That is, we implement equation (2.12) assuming net markups are zero everywhere. Table 4 shows the results. Lines 1 (GDP growth) and lines 7 and 8 (hours reallocation within and across countries) repeat lines that were in the ALP results in Table 2. Line 3 (hours growth) is now rescaled by $s^L$. Given this, our discussion here focuses primarily on the contribution of aggregate capital growth (Line 2), world TFP growth (Line 4), capital reallocation (Lines 5 and 6), and country-industry specific TFP growth (Line 10). Line 9 shows shifts in markups, which are assumed to be zero in this step.

Line 2 shows the contribution of aggregate capital growth, $s^K \dot{k}$, to world GDP growth for the subperiods in our data. There is a substantial discrepancy between the two vintages for the overlapping periods 2001-2004 and 2005-2007. This mainly reflects the lower labor share (and, hence, higher residual capital share, $1 - s^L$) in the 2016 vintage, as shown in Figure 3.

Line 4 shows that our first takeaway also holds for TFP: As with world ALP growth, world TFP growth is volatile across the five subperiods that we consider. Line 10 translates the second takeaway to TFP growth: As with ALP growth, the country-industry component of TFP growth, $\sum_i \frac{1}{(1+\mu i)} s^D_i \dot{z}_i$, is much less volatile than world TFP growth. Country-industry TFP growth was relatively strong prior to 2008, and then (looking at the 2016 vintage) stepped down markedly. Country-industry TFP growth was modestly negative from 2008-10 and was only weakly positive from 2011-2014 (both in the 2016 vintage).

Lines 5 and 6 show that, when we do not account for markups, we find sizable
effects of capital reallocation on world GDP growth. Most of this capital-reallocation effect occurs between industries within countries (Line 5) rather than across countries (Line 6). This capital reallocation is largely due to two sectors: Trade, transportation, and utilities as well as business services. The reallocation of capital across countries accounts for a much smaller part of world GDP growth. The reallocation contributions in Lines 5 and 6 of Table 4 are positive, which reflects that capital grows faster in industries and countries for which the implied internal rate of return to capital (i.e., the implied marginal product of capital under the assumption of no markups) is higher.

Finally, we note again that lines 7 and 8 show the third takeaway—volatile labor reallocation. Labor reallocation is somewhat more volatile across subperiods than capital reallocation.

As we showed earlier in this section, our estimates imply that profits make up a substantial, and increasing, fraction of world GDP. The results without markups ignore this evidence. So, in the next step we redo our decomposition, accounting for the role of markups.

Step 3: World TFP growth with markups

Table 5 shows that our main results also hold when we explicitly account for markups. There are some notable differences when we allow for markups. Starting in line 2 with the contribution of world capital to growth, a substantial part of the growth contribution of aggregate capital from Table 4 is attributable to markups in Table 5. The reason is that without markups, capital’s weight was $(1 - s^L)$. With markups and profits, however, this weight is split between capital and profits, $s^K + s^\Pi$. 

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This recharacterization reduces the contribution of capital growth in Line 2 of Table 5 for all subperiods. In fact, accounting for markups reduces the measured contribution of aggregate capital growth to world GDP growth by 0.26 and 0.57 percentage points in the 2013 and 2016 vintages of the data respectively.

Not only is the contribution of capital to world GDP growth lower when we account for markups, it is also remarkably constant, with a mean of 0.78, across subperiods and vintages. Moreover, the large differences across vintages in the contribution of aggregate capital growth for the periods 2001-2004 and 2005-2007 that we found in Line 2 of Table 4 almost disappear.

Compared with Table 4, the lower contribution of aggregate capital growth results in somewhat higher world TFP growth in line 4 of Table 5. That said, world TFP growth remains quite volatile across subperiods and slows substantially after 2007.

A big difference between the results with and without markups is the implied contribution of capital reallocation to World GDP growth, reported in Lines 5 and 6 of the respective tables. After accounting for markups in Table 5, the measured effect of capital reallocation within countries (line 5) is much smaller, particularly in the 2013 vintage. If our markup estimates are accurate, it suggests that we found spurious effects of capital reallocation in Table 4 because we misassessed capital rental rates (and implied marginal products of capital). With or without markups, the effect of changes in the cross-country reallocation of capital (line 6) remains negligible.\(^{50}\)

Line 9 of Table 5 reports the impact of markups on world GDP growth. These shifts add around half a percentage point annually to world GDP growth over the

\(^{50}\)The careful reader might wonder why there is any capital reallocation term left, given we are assuming the same nominal return everywhere. One reason is that there are differences in the levels and growth rates of capital deflators across countries, in part reflecting different capital mixes (which we do not control for). For the same reason, there are differences in average depreciation rates across countries.
period we consider. Our detailed results indicate that the effect of shifts in markups on world GDP growth is mainly due to manufacturing, trade, and FIRE in China and the United States.

Finally, Line 10 of Table 5 lists the part of world GDP growth accounted for by country-industry specific TFP growth. The picture here is very similar as for the contribution of country-industry specific ALP growth in Line 8 of Table 3. Before 2008 the contribution of country-industry specific TFP growth to world productivity was relatively constant at around 1.2 percent. After that, country-industry specific TFP growth declined to near zero during global financial crisis and recovered only modestly afterwards.

It is striking that allowing for markups makes a minimal difference to line 10. Rather, the effect of markups in line 9 largely comes out of a reduced contribution from capital (line 2) and within-country capital reallocation (line 5).

Just like for ALP, the relative constancy of the number reported in Line 10 of Table 5 for before 2008 masks a shift in technology growth from advanced economies to emerging economies, especially from 2005-2007. This can be seen from Table 40, which splits Line 10 up by country.

Step 4: PPP value-added share weighted results

A striking takeaway from the first three steps is that labor reallocation explains much of the volatility in world productivity, as well as being a consistent drag on world growth. These first three steps valued world output using current dollars. A natural question is whether these findings reflect true differences in labor’s marginal productivity across countries, or rather the effects of exchange rates? Table 6 addresses
this question by quantifying the impact of deviations from PPP on the decomposition in equation (2.12). Here, country-industry value-added shares are measured in terms of 2005 PPP dollars rather than current U.S. dollars. Although the specific numbers are quite different, our qualitative results are robust to deviations from PPP.

Line 1 of Table 6 shows that PPP-weighted world GDP grows much faster than current-dollar-weighted GDP growth. The reason is that PPP value-added shares in world GDP tend to be higher than dollar shares for emerging economies; these economies tend to grow faster than average. The growth rate also appears somewhat more volatile. In contrast, comparing lines 2 and 3 with the same lines in Table 5, the contributions of aggregate capital and labor growth are not much changed.\(^{51}\)

World TFP growth, reported in Line 4, is higher for the PPP-weighted case than for the dollar-weighted case. This follows from having faster growth in GDP (line 1) along with roughly similar contributions from capital and labor (lines 2 and 3). World TFP growth remains highly volatile across subperiods as well as slows down after 2007.

Comparing Lines 4 and 10 of Table 6 we find that fluctuations in PPP-deflated world TFP growth are much larger than those in country-industry PPP-deflated TFP growth. This is similar to what we found for dollar-weighted ALP and TFP growth as well (and was our first two takeaways). Moreover, even though level of country-industry TFP growth is higher in the PPP-weighted data, the pattern over time is similar to the dollar-weighted results.

Deviations from PPP do have a marked impact on the contributions of capital and labor reallocation, especially across countries, to world GDP growth. The impact of

\(^{51}\) The numbers do not match exactly since our sample changed slightly due to PPP data availability. See Table 43 in Appendix B.3.2 for more details.
the cross-country capital reallocation in Line 6 of Table 6 is large compared to that in Table 5, in which it was negligible. This potentially reflects that capital flows across the world to equate dollar-denominated returns on investment across country-industry combinations. Equating these dollar-denominated returns is not the same as equating physical marginal products.

For the changes in labor reallocation we find the opposite. Labor reallocation is less important when we consider the PPP-weighted results in Table 6. A portion of cross-country labor reallocation in the dollar-weighted results in Table 5 reflects economic activity shifting to sectors with an international cost advantage. These are industries with low relative wages compared to relative productivity levels—most obviously, manufacturing in China and India.

The labor reallocation results imply that deviations from PPP only account for about a third of the total impact of labor reallocation reported in the earlier tables. Thus, even after adjusting for PPP, labor reallocation remains a drag on world GDP growth as well as being an important source of volatility in world TFP.

Finally, shifts in markups (line 9) contribute slightly more to world GDP growth when PPP-deflated than current-dollar weighted. This is largely due to markups in (Chinese) manufacturing.

2.5 Interpreting the Cross-Country Reallocation of Labor

This section explores sources of wage differences across countries and industries which is important for understanding the labor reallocation term. As discussed in section 2.2.3, labor reallocation reflects the covariance of wage differences with labor
growth. Wage differences between emerging and advanced economies are large, which is what allows this term to be quantitatively significant.

One interpretation of wage differences, besides barriers to movement of workers, is that some or all of the observed wage differences across countries reflect worker productivity differences—most saliently, arising from differences in educational attainment—that are “embodied” in workers. There are large differences in human capital across countries. 52

In our data, we are able to implement a crude human capital adjustment in the 2013 vintage of WIOD (through 2007). This alternative implementation does not change our qualitative results. The 2013 vintage of WIOD provides information on industry labor hours and compensation based on three broad skill groups (low-, medium-, high-skilled). 53 These data allow for a crude accounting of cross-country differences in skill distributions. To do so, we treat the hours worked by each of these skill groups as a separate factor of production, $L^\tau$, where $\tau \in \{L, M, H\}$. The production function from equation (2.2) becomes

$$Y_i = F_i \left( K_i, L^L_i, L^M_i, L^H_i, \{M_{i,j}\}_{j=1}^{n}, Z_i \right). \quad \text{(2.21)}$$

The resulting decomposition of aggregate TFP growth differs from the ones we presented before in three ways. First, aggregate growth of the labor input is measured as a share-weighted average of growth in hours of each skill group. Second, this

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53Labor skill types are classified on the basis of educational attainment levels as defined in the International Standard Classification of Education (ISCED): low-skilled (ISCED categories 1 and 2), medium-skilled (ISCED 3 and 4) and high-skilled (ISCED 5 and 6).
redefinition also affects our measures of aggregate and industry TFP, since each type of labor is effectively treated as a separate input.\footnote{The production function in (2.21) allows for shifts in the contribution of labor “composition,” or “quality.” For example, suppose that total hours are constant, but the low skilled work less while the high skilled work more. Since the high-skilled wage is higher, effective share-weighted labor input increases. For industry TFP, the contribution from hours shifting (at least on average) to the high skilled was previously attributed to technology.} Finally, and most importantly, labor reallocation in this case is a weighted average of labor reallocation across the three types of labor.\footnote{We defer the details of this decomposition to Appendix B.2.}

Table 7 shows the results of the decomposition with three skill types. The earlier findings regarding the importance of cross-country labor reallocations are robust to this extension. Comparing lines 7 and 14 show that, as before, the volatility of world TFP growth is mainly driven by the cross-country labor reallocation term; country-industry TFP growth (line 19) remains very smooth. The cross-country labor reallocation term not only fluctuates a lot, but lines 15 through 17 show that its contribution to world TFP growth is almost always negative for each skill group. Thus, even within skill groups, hours typically grow faster in countries with relatively low wages.

Unfortunately, the three skill groups are crude—capturing only broad buckets of years of schooling, and with no controls for the quality of education. Nevertheless, cross-country analyses of wages of migrants suggest that, even if we could correct for cross-country human capital differences within these skill groups, we would still find the reallocation of hours to be a drag on world GDP. In particular, Hendricks (2002b), Schoellman (2011), and Hendricks and Schoellman (2017) use the wages of immigrants before and after migration to quantify cross-country differences in wages per unit of human capital. These studies show that, after controlling for selection, wage gains
from migrating to the U.S. are large. They are larger for workers who earned lower wages in the country of origin than for workers with high wages in those countries.

Thus, if wages per unit of human capital reflect marginal products of labor measured in constant quality units, then our observation that hours grow faster in countries with lower wages implies that hours grow faster in countries with lower wages per unit of human capital. Hence, correcting for human capital does not overturn our conclusion that the reallocation of labor is a drag on world TFP growth as well as being a substantial source of volatility.56

2.6 Conclusion

We provide new global growth-accounting results from a novel growth decomposition that nests standard decompositions but allows for markups as well as factor "wedges." We implement this decomposition using data on 35 or more industries and 40 or more countries from 1996-2014.

Empirically, we find three main results: (i) world productivity is volatile from year to year and even over multi-year periods, even though (ii) the average rate of productivity growth across country-industries is comparatively smooth; (iii) labor reallocation is the primary source of the volatility in world productivity growth, as well as being a persistent drag on growth. These takeaways apply whether we use labor productivity or TFP, whether or not we control for markups, and whether or not we adjust for PPP.

The quantitative importance of labor reallocation arises from the well-known

56This holds even when we account for PPP data, though the contribution of labor reallocation across countries declines, which is again the same qualitative result we had before.
heterogeneity in wages around the world. Previous research has not examined how this heterogeneity affects productivity measurement. The intuition is straightforward. Cost-minimizing first-order conditions imply that observed differences in (equilibrium) wages correspond to differences in marginal products of labor. Labor input has typically grown faster in low-wage/low marginal-product locations, creating a persistent drag of around 1/2 percent per year for world productivity growth. But over time, the cross-sectional covariance of wages and hours growth varies substantially which, in turn, leads to considerable variability in world productivity.

Our growth-accounting methodology and results extend the insights of the so-called “misallocation” literature (following Hsieh and Klenow (2009b)) to the time-series domain. That literature highlights the importance of the allocation of resources for productivity. Recent critiques of misallocation estimates (e.g., Haltiwanger et al. (2018)) have highlighted the strong assumptions made in the literature. In contrast, our approach should be more robust to these concerns: For growth rates, we show how to account for the effects of changing resource allocation with few structural assumptions beyond cost-minimization.

Importantly, our results do not require a misallocation interpretation. Certainly, it is natural to interpret labor reallocation as capturing shifts in global misallocation, where the global “optimum” is defined as the allocation of labor that would maximize global output. As argue for the United States, output rises if we shift, or “reallocate,” labor input from low-marginal-product to high-marginal-product locations. Nevertheless, our positive results do not hinge on this normative misallocation interpretation. For example, low wages could reflect low skills, so that the marginal product is associated with the worker, not with the location. In that case, shifting a given worker from one country to another would not, in fact, change global output.
That said, the evidence suggests that shifting workers from a low-wage to a high-wage country would, in fact, raise their marginal products. More generally, if each country has its own representative consumer, resource shifts that raise global output might not be Pareto-efficient.

Our results provide new insights into at least two other recent literatures. First, a growing recent literature examines the role of markups and rising profits. We extend Barkai (2019) to emerging markets. We estimate that markups are widespread and that profits rise steadily across a wide range of countries. Indeed, both labor and capital shares fall. Interestingly, although profits and markups are quantitatively important—with both labor and capital shares of output falling—the broad narrative about global productivity is robust to whether we control for markups or not.

Second, a sizeable strand of literature has highlighted the slowdown in recent decades in advanced-economy productivity growth. We provide broader context for this finding: At a global level, the advanced-economy slowdown in country-industry productivity growth in the 2000s is offset until the Great Recession by a rising contribution from emerging markets. World productivity growth (and world country-industry productivity growth) only consistently slows after the Great Recession.

Thus, our analysis shows how important it is to do growth accounting on a global scale to understand shifts in the center of gravity of global productivity growth. With the rise of emerging economies in Asia, this global perspective has become increasingly essential.
Table 1. Comparison of WIOD-SEA vintages

<table>
<thead>
<tr>
<th>Description</th>
<th>2013</th>
<th>2016</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Coverage</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Number of countries</td>
<td>40</td>
<td>43</td>
</tr>
<tr>
<td>Average share of world GDP</td>
<td></td>
<td></td>
</tr>
<tr>
<td>... dollar denominated</td>
<td>80</td>
<td>82</td>
</tr>
<tr>
<td>... PPP deflated</td>
<td>76</td>
<td>77</td>
</tr>
<tr>
<td>Number of industries</td>
<td>35</td>
<td>56</td>
</tr>
<tr>
<td>Industry classification</td>
<td>ISIC v3</td>
<td>ISIC v4</td>
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<td><strong>Factor inputs</strong></td>
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<tr>
<td>Hours</td>
<td>✓</td>
<td>✓</td>
</tr>
<tr>
<td>Capital</td>
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<td>✓</td>
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<tr>
<td>... Nominal current cost</td>
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<td>✓</td>
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<tr>
<td>... Investment</td>
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<tr>
<td>... Capital deflators</td>
<td></td>
<td>✓</td>
</tr>
</tbody>
</table>

Note: Both vintages contain data on value added by country and industry as well as value added deflators and factor prices for inputs for which data is available.

The 2013 vintage includes incomplete data for 2008-2011 that we do not use in our analysis. Share of world GDP reported in percentage of dollar-denominated world value added from World Bank (2018).

The 2016 vintage contains incomplete capital data, especially capital deflators. We construct them by merging data from OECD (2017b) and extrapolating from the 2013 vintage for variables unavailable. See the Appendix for details.
<table>
<thead>
<tr>
<th></th>
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</thead>
<tbody>
<tr>
<td>1)</td>
<td>World GDP growth</td>
<td>$\dot{i}$</td>
<td>3.33</td>
<td>2.51</td>
<td>3.70</td>
<td>3.15</td>
<td>2.31</td>
<td>3.65</td>
<td>0.91</td>
<td>2.56</td>
<td>2.37</td>
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<td>2)</td>
<td>World hours growth</td>
<td>$\dot{l}$</td>
<td>1.18</td>
<td>2.44</td>
<td>0.39</td>
<td>1.40</td>
<td>1.16</td>
<td>0.85</td>
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<td>3.38</td>
<td>1.46</td>
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<tr>
<td>3)</td>
<td>World ALP growth</td>
<td>$\dot{alp}$</td>
<td>2.15</td>
<td>0.07</td>
<td>3.31</td>
<td>1.75</td>
<td>1.15</td>
<td>2.80</td>
<td>0.98</td>
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<td>4)</td>
<td>Relative hours growth</td>
<td>$(\dot{i} - \dot{l})$</td>
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<td>-2.22</td>
<td>0.95</td>
<td>-0.58</td>
<td>-0.79</td>
<td>0.82</td>
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<td>-1.49</td>
<td>-0.62</td>
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<td>5)</td>
<td>Reallocation of hours</td>
<td>$s_L(\dot{i} - \dot{l})$</td>
<td>-0.01</td>
<td>-1.34</td>
<td>0.50</td>
<td>-0.33</td>
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<td>-0.36</td>
<td>-0.97</td>
<td>-0.44</td>
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<td>6)</td>
<td>within countries</td>
<td>-0.07</td>
<td>-0.02</td>
<td>0.15</td>
<td>0.06</td>
<td>0.03</td>
<td>0.08</td>
<td>0.08</td>
<td>0.09</td>
<td>0.07</td>
<td></td>
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<tr>
<td>7)</td>
<td>across countries</td>
<td>-0.08</td>
<td>-1.32</td>
<td>0.35</td>
<td>-0.39</td>
<td>-0.6</td>
<td>0.27</td>
<td>-0.44</td>
<td>-1.07</td>
<td>-0.51</td>
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<tr>
<td>8)</td>
<td>Country-industry ALP growth</td>
<td>$\dot{alp}_i$</td>
<td>2.14</td>
<td>2.11</td>
<td>2.20</td>
<td>2.15</td>
<td>1.94</td>
<td>1.98</td>
<td>1.70</td>
<td>0.67</td>
<td>1.53</td>
<td></td>
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</tbody>
</table>

Note: Lines in this table correspond to parts of equation (2.19). Reported are contributions to average annual growth rates in percentage points over various subperiods.
Table 3. Contribution of country-industry specific ALP growth, by country/region: 1996-2014

<table>
<thead>
<tr>
<th>SEA vintage</th>
<th>2013</th>
<th>2016</th>
</tr>
</thead>
<tbody>
<tr>
<td>Country/region</td>
<td>-</td>
<td>-</td>
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<tr>
<td>Advanced</td>
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<td></td>
</tr>
<tr>
<td>United States</td>
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<td>1.78</td>
</tr>
<tr>
<td>Great Britain</td>
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<td>0.13</td>
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<tr>
<td>Japan</td>
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<td>0.25</td>
</tr>
<tr>
<td>Euro Area</td>
<td>0.33</td>
<td>0.23</td>
</tr>
<tr>
<td>Other Advanced</td>
<td>0.27</td>
<td>0.16</td>
</tr>
<tr>
<td>Emerging</td>
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<td></td>
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<tr>
<td>Brazil</td>
<td>0.04</td>
<td>-0.00</td>
</tr>
<tr>
<td>China</td>
<td>0.30</td>
<td>0.28</td>
</tr>
<tr>
<td>India</td>
<td>0.06</td>
<td>0.02</td>
</tr>
<tr>
<td>Russia</td>
<td>-0.02</td>
<td>0.04</td>
</tr>
<tr>
<td>Other Emerging</td>
<td>-0.00</td>
<td>-0.01</td>
</tr>
<tr>
<td>Total</td>
<td>2.14</td>
<td>2.11</td>
</tr>
</tbody>
</table>

Note: Reported are contributions by country/region to line 8 in Table 2 in percentage points over various subperiods.
Table 4. Summary of global TFP growth accounting without markups: 1996-2014

<table>
<thead>
<tr>
<th>SEA vintage</th>
<th>2013</th>
<th>2016</th>
</tr>
</thead>
<tbody>
<tr>
<td>1) World GDP growth</td>
<td>$\dot{v}$</td>
<td>3.33</td>
</tr>
<tr>
<td>2) World capital growth</td>
<td>$s^K \dot{k}$</td>
<td>0.98</td>
</tr>
<tr>
<td>3) World hours growth</td>
<td>$s^L \dot{l}$</td>
<td>0.71</td>
</tr>
<tr>
<td>4) World TFP growth</td>
<td>$\ddot{tfp}$</td>
<td>1.65</td>
</tr>
<tr>
<td>Misallocation of capital</td>
<td>$s^K_i (\dot{k}_i - \dot{k})$</td>
<td>0.76</td>
</tr>
<tr>
<td>5) ...within countries</td>
<td>0.63</td>
<td>0.20</td>
</tr>
<tr>
<td>6) ...across countries</td>
<td>0.13</td>
<td>0.09</td>
</tr>
<tr>
<td>Misallocation of hours</td>
<td>$s^L_i (\dot{l}_i - \dot{l})$</td>
<td>-0.01</td>
</tr>
<tr>
<td>7) ...within countries</td>
<td>0.07</td>
<td>-0.02</td>
</tr>
<tr>
<td>8) ...across countries</td>
<td>-0.08</td>
<td>-1.32</td>
</tr>
<tr>
<td>9) Shifts in markups</td>
<td>$\frac{\mu}{1+\mu} \dot{y}_i$</td>
<td>0.00</td>
</tr>
<tr>
<td>10) Country-industry TFP growth</td>
<td>0.91</td>
<td>1.18</td>
</tr>
</tbody>
</table>

Note: Lines in this table correspond to parts of equation (2.12). Reported are contributions to average annual growth rates in percentage points over various subperiods. These are results with no markups. Hence line 9 consists of zeros.
Table 5. Summary of global TFP growth accounting with markups: 1996-2014

<table>
<thead>
<tr>
<th>SEA vintage</th>
<th>2013</th>
<th>2016</th>
</tr>
</thead>
<tbody>
<tr>
<td>line</td>
<td></td>
<td></td>
</tr>
<tr>
<td>1) World GDP growth</td>
<td>$\bar{v}$</td>
<td>3.33</td>
</tr>
<tr>
<td>2) World capital growth</td>
<td>$s^K\bar{k}$</td>
<td>0.79</td>
</tr>
<tr>
<td>3) World hours growth</td>
<td>$s^L\bar{l}$</td>
<td>0.71</td>
</tr>
<tr>
<td>4) World TFP growth</td>
<td>$tfp$</td>
<td>1.84</td>
</tr>
<tr>
<td>Misallocation of capital</td>
<td>$s^K_i(\bar{k}_i - \bar{k})$</td>
<td>0.21</td>
</tr>
<tr>
<td>...within countries</td>
<td></td>
<td>0.23</td>
</tr>
<tr>
<td>...across countries</td>
<td></td>
<td>-0.02</td>
</tr>
<tr>
<td>Misallocation of hours</td>
<td>$s^L_i(\bar{l}_i - \bar{l})$</td>
<td>-0.01</td>
</tr>
<tr>
<td>...within countries</td>
<td></td>
<td>0.07</td>
</tr>
<tr>
<td>...across countries</td>
<td></td>
<td>-0.08</td>
</tr>
<tr>
<td>9) Shifts in markups</td>
<td>$\frac{\mu_i}{\bar{y}_i}$</td>
<td>0.51</td>
</tr>
<tr>
<td>10) Country-industry TFP growth</td>
<td></td>
<td>1.13</td>
</tr>
</tbody>
</table>

Note: Lines in this table correspond to parts of equation (2.12). Reported are contributions to average annual growth rates in percentage points over various subperiods. Results with markups.

<table>
<thead>
<tr>
<th>SEA vintage</th>
<th>2013</th>
<th>2016</th>
</tr>
</thead>
<tbody>
<tr>
<td>1) World GDP growth</td>
<td>$\hat{\nu}$</td>
<td>5.42</td>
</tr>
<tr>
<td>2) World capital growth</td>
<td>$s^K \hat{k}$</td>
<td>0.75</td>
</tr>
<tr>
<td>3) World hours growth</td>
<td>$s^L \hat{l}$</td>
<td>0.75</td>
</tr>
<tr>
<td>4) World TFP growth</td>
<td>$\hat{t}_{fp}$</td>
<td>3.92</td>
</tr>
<tr>
<td>Misallocation of capital</td>
<td>$s^K (\hat{k}_i - \hat{k})$</td>
<td>0.32</td>
</tr>
<tr>
<td>5) ...within countries</td>
<td>0.20</td>
<td>0.07</td>
</tr>
<tr>
<td>6) ...across countries</td>
<td>0.11</td>
<td>0.10</td>
</tr>
<tr>
<td>Misallocation of hours</td>
<td>$s^L (\hat{l}_i - \hat{l})$</td>
<td>0.02</td>
</tr>
<tr>
<td>7) ...within countries</td>
<td>0.08</td>
<td>-0.19</td>
</tr>
<tr>
<td>8) ...across countries</td>
<td>-0.06</td>
<td>-0.93</td>
</tr>
<tr>
<td>9) Shifts in markups</td>
<td>$\mu y_i$</td>
<td>0.60</td>
</tr>
<tr>
<td>10) Country-industry TFP growth</td>
<td>2.99</td>
<td>3.59</td>
</tr>
</tbody>
</table>

Note: Lines in this table correspond to parts of equation (2.12). Reported are contributions to average annual growth rates in percentage points over various subperiods. Results with markups.
Table 7. Summary of global TFP growth accounting with markups and labor quality: 1996-2007

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>1)</td>
<td>World GDP growth</td>
<td>$\dot{v}$</td>
<td>3.33</td>
<td>2.51</td>
<td>3.70</td>
<td>3.15</td>
</tr>
<tr>
<td>2)</td>
<td>World capital growth</td>
<td>$s^K \dot{k}$</td>
<td>0.79</td>
<td>0.74</td>
<td>0.80</td>
<td>0.78</td>
</tr>
<tr>
<td>3)</td>
<td>World hours growth across skills</td>
<td>$s^{L\tau} \dot{l}$</td>
<td>1.53</td>
<td>2.17</td>
<td>0.88</td>
<td>1.58</td>
</tr>
<tr>
<td>4)</td>
<td>...low-skilled</td>
<td></td>
<td>-0.02</td>
<td>0.14</td>
<td>-0.04</td>
<td>0.03</td>
</tr>
<tr>
<td>5)</td>
<td>...medium-skilled</td>
<td></td>
<td>0.73</td>
<td>0.82</td>
<td>0.16</td>
<td>0.62</td>
</tr>
<tr>
<td>6)</td>
<td>...high-skilled</td>
<td></td>
<td>0.82</td>
<td>1.21</td>
<td>0.76</td>
<td>0.94</td>
</tr>
<tr>
<td>7)</td>
<td>World TFP growth</td>
<td>$\dot{\text{tfp}}$</td>
<td>1.01</td>
<td>-0.41</td>
<td>2.02</td>
<td>0.79</td>
</tr>
<tr>
<td></td>
<td>Misallocation of capital</td>
<td>$s^K_i (\dot{k}_i - \dot{k})$</td>
<td>0.21</td>
<td>0.03</td>
<td>0.06</td>
<td>0.11</td>
</tr>
<tr>
<td>8)</td>
<td>...within countries</td>
<td></td>
<td>0.23</td>
<td>0.06</td>
<td>0.11</td>
<td>0.14</td>
</tr>
<tr>
<td>9)</td>
<td>...across countries</td>
<td></td>
<td>-0.02</td>
<td>-0.03</td>
<td>-0.05</td>
<td>-0.03</td>
</tr>
<tr>
<td></td>
<td>Misallocation of hours</td>
<td>$s^{L\tau}_i (\dot{l}^\tau_i - \dot{l}^\tau)$</td>
<td>-0.63</td>
<td>-1.88</td>
<td>0.01</td>
<td>-0.89</td>
</tr>
<tr>
<td>10)</td>
<td>within countries</td>
<td></td>
<td>-0.01</td>
<td>-0.11</td>
<td>0.09</td>
<td>-0.02</td>
</tr>
<tr>
<td>11)</td>
<td>...low-skilled</td>
<td></td>
<td>0.00</td>
<td>-0.01</td>
<td>0.03</td>
<td>0.00</td>
</tr>
<tr>
<td>12)</td>
<td>...medium-skilled</td>
<td></td>
<td>-0.01</td>
<td>-0.06</td>
<td>0.04</td>
<td>-0.02</td>
</tr>
<tr>
<td>13)</td>
<td>...high-skilled</td>
<td></td>
<td>-0.00</td>
<td>-0.03</td>
<td>0.02</td>
<td>-0.01</td>
</tr>
<tr>
<td>14)</td>
<td>across countries</td>
<td></td>
<td>-0.62</td>
<td>-1.78</td>
<td>-0.08</td>
<td>-0.87</td>
</tr>
<tr>
<td>15)</td>
<td>...low-skilled</td>
<td></td>
<td>-0.17</td>
<td>-0.26</td>
<td>-0.03</td>
<td>-0.16</td>
</tr>
<tr>
<td>16)</td>
<td>...medium-skilled</td>
<td></td>
<td>-0.35</td>
<td>-0.81</td>
<td>0.09</td>
<td>-0.39</td>
</tr>
<tr>
<td>17)</td>
<td>...high-skilled</td>
<td></td>
<td>-0.11</td>
<td>-0.71</td>
<td>-0.14</td>
<td>-0.32</td>
</tr>
<tr>
<td>18)</td>
<td>Shifts in markups</td>
<td>$\frac{\mu}{1+\mu} \dot{y}_i$</td>
<td>0.51</td>
<td>0.39</td>
<td>0.94</td>
<td>0.58</td>
</tr>
<tr>
<td>19)</td>
<td>Country-industry TFP growth</td>
<td></td>
<td>0.93</td>
<td>1.06</td>
<td>1.01</td>
<td>0.99</td>
</tr>
</tbody>
</table>

Note: Lines in this table correspond to parts of equation (B.6). Reported are contributions to average annual growth rates in percentage points over various subperiods. Results with markups.
Figure 2. Growth in world real GDP in WIOD-SEA and World Development Indicators
Note: World real GDP growth is constructed as dollar-denominated value-added share weighted average of real GDP or real country-industry value-added growth.
Figure 3. World factor shares for both vintages of WIOT
Source: Timmer (2012), OECD (2017b), and authors’ calculations.
Figure 4. ALP growth: World vs. country-industry component, vintage 2016.
Source: Timmer (2012), OECD (2017b), and authors’ calculations.
Chapter 3

INEQUALITY OVER THE LIFE-CYCLE: U.S. VS EUROPE

3.1 Introduction

Inequality in labor earnings is higher in the U.S. cross-section than Europe, as measured by several indicators such as gini coefficient, share of earnings going to the top percentile, etc (Piketty and Saez, 2006; Guvenen et al., 2014). One critical step in understanding earnings inequality in the cross-section is understanding the forces that shape inequality over the life-cycle. Specifically, one can ask, what are the determinants of life-cycle inequality in labor earnings? Are there significant differences in life-cycle inequality across the U.S. and Europe? How do these determinants interact with taxation and education policies? The goal of this paper is a quantitative exploration of these questions by studying life-cycle inequality in (pre-tax) labor earnings for males and the impact of labor market (tax) policies and higher education subsidies/transfers on college attainment and life-cycle earnings, using cross-country data, and focusing on the U.S., U.K., Netherlands, and France.

I document that the mean and dispersion in labor earnings grow fastest in the U.S., followed by the U.K., Netherlands and France (see figure 5). For example, the growth in mean earnings between ages of 25 and 50 is a factor of 2.2 in the U.S., while it is only a factor of 1.5 in France. Differences in mean earnings mask the observed differences in the heterogeneity in earnings growth over the life-cycle across

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57 Guvenen et al. (2014) show similar results for these countries, though they leave out a quantitative exploration of them. Badel et al. (2018) show similar patterns across the U.S., Canada, Sweden, and Denmark. They do not however, make quantitative statements about the profiles.
individuals. For instance, the variance of log earnings grows faster in the U.S. (40 log points), followed by the U.K. (30 log points) and Netherlands and France (both 20 log points). The increase in the variance of earnings indicates that as individuals age in the labor market, the differences in earnings across individuals grow and this growth is faster in the U.S. (see figure 8).

The second new fact that I uncover is that the differences in mean and dispersion of earnings over the life-cycle across the U.S. and Europe are driven by individuals with at least a four-year college degree (or its equivalent). Figure 6 shows that the mean earnings for those individuals without a college degree grow by a factor of 1.5 in “all” countries. Then, by definition, individuals with a college degree drive the differences in mean earnings across these countries. Variance of log earnings over the life-cycle show similar growth across all the countries for non-college individuals (figure 9). This means that the bulk of the growth in dispersion is also due to college graduates.

Differences in life-cycle inequality across these countries are correlated with differences in taxation and education policies. First, I document that labor earnings taxation schedule is steeper in Europe than the U.S., by fitting a tax function to each country’s labor earnings tax code. The steepness comes from the fact that as earnings rise, the marginal tax rate on labor earnings increases and this increase is not uniform across these countries. For instance, moving a person from the mean earnings in the cross-section to three times this level increases her marginal tax rate by 8 percentage points in the U.S., while the increase in marginal rate is 12 pp in France for the same relative change in earnings. The ranking of countries in terms of life-cycle

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58This is consistent with other papers that study the taxation schedule across U.S. and Europe such as Guvenen et al. (2014), Duncan and Sabirianova Peter (2016), and Bick et al. (2019).
inequality coincides with steepness of the tax code, i.e. the steeper the tax schedule is, the smaller is the rise in life-cycle inequality in (pre-tax) labor earnings. A second salient policy difference is the share of public expenditure in overall college expenditure across these countries. The United States devotes the smallest share (39%), while France has the highest (80%). The overall college expenditure share in GDP and college attainment is higher in the U.S. than European countries. The above facts show a correlation between life-cycle inequality and taxation and education policies, but on their own, they fail to provide a quantitative assessment of the importance of each policy and their interactions for college attainment and life-cycle inequality across these countries. For that, I develop a model.

I construct a life-cycle model of human capital accumulation and elastic labor supply, which features uninsurable shocks to human capital and college choice. Individuals enter the model with heterogeneous learning abilities and initial stock of human capital and can accumulate more human capital over the life-cycle in potentially two ways: during college and while working. First, if they choose to go to college, they specialize in human capital accumulation, and invest consumption goods in the production of human capital. Second, if they do not go to college (start working from the beginning of life), or after college graduation, they invest in risky human capital as in Huggett et al. (2011). The heterogeneity in learning ability and initial human capital results in different education choices (whether to attend college and how much to invest during college) and different investments in human capital while working, which translate into different earnings trajectories over the life-cycle across individuals.

The model integrates a college stage into a life-cycle framework to capture intensive and extensive margins of college choice and its effects on life-cycle inequality. Individuals who choose to go to college prior to working, which provides them with a
short period of rapid human capital accumulation, start working with an “endogenous” human capital stock. On the margin, those who did not choose college experience smaller growth in earnings over the life-cycle since rapid human capital accumulation during college “complements” the investments undertaken during the working life.

Steeper taxation schedule provides social insurance against uninsurable idiosyncratic labor earnings risk, while distorting labor supply and human capital investments. Individuals with high learning ability, whose earnings grow faster, face high marginal and average taxes which discourages human capital accumulation. This distortion in human capital accumulation compounds the adverse impact of non-linear taxation through the classic labor supply channel. As a result, earnings grow slower over the life-cycle and the pre-tax earnings distribution becomes less dispersed. Since college investments complement human capital accumulation during the working life, higher non-linearity in labor earnings taxation lowers the value of college attendance. Those high ability individuals who still find college worthwhile to attend invest less during college as a result of steeper taxation. Therefore, the model naturally links tax policies with education choice along the extensive margin (college attendance) as well as an intensive margin (college investments).

In the model, a more generous college subsidies/transfers exacerbate life-cycle inequality. They make college investments “cheaper”, which raise the value of attending college, and induce the marginal individual to attend college. Therefore, more individuals experience the rapid human capital accumulation during college (extensive margin), and while they are in college, they will invest more in human capital

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59 Zhao (2017) finds that more risk averse individuals benefit more from progressive taxation in terms of welfare in a model with human capital, although elastic labor supply dampens the benefits. Cubas and Silos (2020) studies the effects of non-linear taxation and social insurance on occupational mobility in the U.S. and Germany.
accumulation (intensive margin). This is especially true for inframarginal individuals with high learning ability who increase their investments during college significantly. As a result, the composition of the college graduates changes and the overall rate of growth in human capital in the economy increases. Faster growth in human capital accumulation results in higher average earnings growth over the life-cycle. Individuals with higher learning ability benefit the most from college subsidies/transfers and their earnings growth increases the most, which increases the growth in the dispersion in earnings over the life-cycle.

I find that the differences in the steepness of labor earnings taxation account for most of the observed variation in life-cycle inequality in the data. Differences in steepness of the labor earnings tax schedule account for 95% of the differences in the growth in mean earnings between ages 25-50 and 74% of the differences in overall growth in variance of log earnings over the life-cycle.\(^{60}\) When differences in college transfers/subsidies are also considered on top of the differences in taxation, model generates 94% of the growth in mean earnings profiles and 80% of the growth in variance of log earnings. More generous subsidy/transfer from the government for college expenditure exacerbate life-cycle inequality as indicated by larger growth in the variance of log earnings over the life-cycle when differences in these expenditures are considered. On an aggregate level, my model accounts for 91% of the differences in cross-section gini coefficient across U.S., U.K., Netherlands, and France. Finally, model is consistent with the differences in aggregate annual hours worked between U.S. and Europe as emphasized by Prescott (2004).

\(^{60}\) These are averages across European countries, see section 3.7 for more details on each country’s profile.
Current paper provides an integrated framework to study college attainment, human capital differences, and life-cycle inequality in labor earnings across the U.S. and Europe in a heterogeneous-agent model. This essentially relates it to a broad literature that exploit differences in labor market policies across the U.S. and Europe in order to explain differences in various aggregate outcomes such as hours worked, college attainment and quality of human capital, and earning inequality. Prominent examples include Prescott (2004), Ohanian et al. (2008), Rogerson (2008), and Bick et al. (2018) who study labor hours differences, Erosa et al. (2010b), Schoellman (2012), Cubas et al. (2016b), and Hendricks and Schoellman (2018) who focus on college attainment and labor quality differences, and Guvenen et al. (2014) and Zucman (2019) who focus on income inequality.

In terms of empirical analysis, Huggett et al. (2011) present life-cycle inequality facts for the U.S., while Badel et al. (2018) goes beyond the U.S. and include Canada, Sweden, and Denmark. My paper is the first paper to document life-cycle facts across the U.S., U.K., Netherlands, and France systematically and with similar methodology, and uncover the fact that inequality is driven by individuals with a college degree. None of the papers that focus on life-cycle inequality make this point, including the above papers. Although the underlying data sources are different across these papers, they all find that life-cycle inequality varies non-trivially across countries.\textsuperscript{61}

Several papers share similar modeling framework but focus on different questions.

\textsuperscript{61}Most studies focus either on the U.S. or one country in general. See among others, Alvaredo et al. (2013), Piketty and Qian (2009) for China and India, Domeij and Flodén (2010) for Sweden, Blundell and Etheridge (2010) for Britain, Pijoan-Mas and Sánchez-Marcos (2010) for Spain, Fuchs-Schündeln et al. (2010) for Germany, and Jappelli and Pistaferri (2010) for Italy.
These include Heathcote et al. (2010a), Huggett et al. (2011). Heathcote et al. (2010a) study the implications of rising wage inequality in the U.S. over time and find that recent cohorts in the U.S. enjoy welfare gains from higher college premium and a more even division of labor within the household. Huggett et al. (2011) study lifetime inequality in the U.S. and conclude that about 2/3 of the rise in inequality over the life-cycle is due to endogenous human capital accumulation and the rest is due to idiosyncratic shocks to earnings.

Two closely related papers are Guvenen et al. (2014), and Badel et al. (2020). Guvenen et al. (2014) study inequality in the cross-section across U.S. and Europe and its relation to non-linearity of taxation schedule. They conclude that most of the differences in the cross-section gini coefficient can be accounted for by differences in non-linearity of labor earnings taxation. I find similar qualitative result, and go a step further to emphasize life-cycle inequality differences, and how endogenous choices for college and its interaction with labor market (tax) policies can exacerbate inequality. Badel et al. (2020) study optimal top marginal tax rate in the presence of endogenous human capital accumulation in the U.S. and find that the optimal top marginal tax rate is close to the current rate in the U.S. if human capital forces are taken into account. Although some modeling choices are shared with Badel et al. (2020), the focus of this paper on cross-country life-cycle inequality and college attainment and investments, separates it from the rest of the literature.

The paper proceeds as follows. Section 3.2 provides the empirical investigation of life-cycle inequality across the U.S. and Europe. Section 3.3 illustrates a simple life-cycle model of college choice to provide intuition about college selection. Section 3.4 presents the full model, while section 3.5 discusses parametrization. Section 3.6 illustrates the main mechanisms in the model. Section 3.7 presents the main results.
of the paper and discusses various channels through which model generates inequality patterns observed in the data. Section 3.8 provides a discussion about various topics to put the paper in a broader perspective. Section 3.9 concludes.

3.2 Cross-country Inequality Facts

Earnings Inequality Over the Life-Cycle and College Attainment

Studying earnings inequality over the life-cycle requires microdata which includes reliable information on income and its sources, hours worked, and educational attainment. The sample should also be sufficiently large to include a proper number of low and high earners. For the United States, several microdata contain this information. The one that I am using is the Current Population Survey (CPS).\textsuperscript{62}

For the European countries, the underlying microdata which includes all the above information, especially data on income, is the European Union Statistics on Income and Living Conditions (EU-SILC).\textsuperscript{63} Although the sample size is smaller than CPS, this microdata contains sufficient information to study earnings inequality.\textsuperscript{64} The reader can consult Appendix C.1 for further information about EU-SILC.\textsuperscript{65}

Each sample was restricted following the macro literature (Heathcote et al. (2010a); Huggett et al. (2011), among others) by eliminating possible extreme observations.

\textsuperscript{62}Flood et al. (2018).

\textsuperscript{63}EU-SILC (2016).

\textsuperscript{64}See table 17 for information about the sample size in each country.

\textsuperscript{65}Since I do not have access to panel data for Europe, I chose CPS for the U.S. instead of PSID. Also, observations in PSID are selected non-randomly on earnings, see Gouskova (2014).
For example, individuals with positive hours worked but no income reported, and
those who earn half the hourly minimum wage, were dropped. Also, those working in
agriculture and public/government sector are excluded. The age group is limited to
25-60 since I want the working individuals who most likely finished formal education,
while avoiding late-life decisions about the timing of retirement.

The definition of labor earnings is real total wages and/or salaries from individual’s
employer. This means that I am excluding any business income that is a result of
self-employment. The reason is that it is not clear which share of business profits
is labor earnings and which part is capital income. Also, self-employed individuals
consist a small share of the working population (around 10% in the U.S. and less
in Europe). Finally, there is a lot of discrepancies between reported income from
self-employment in the surveys and what is reported to the Internal Revenue System
in the U.S.

For generating the desired statistics of inequality, I focus on the period 2005-2016
in each country and calculate each statistic for each age groups (5-year bins) and
each year. For example, mean_{act} is the mean earnings of individuals in age group a,

66It is not entirely clear how much of the farm income is labor earnings and how much is capital.
Also, most farmers are considered sole proprietors/self-employed which are excluded in my study.
Government/public sector earnings structure is arguably different from the private sector and hence
these individuals are dropped.

67This age group is a good approximation for the United States, but for European countries where
the timing and duration of college is different from the U.S. may pose a problem. I have decided to
stick to this formulation since I want to be consistent in my empirical investigation and there is not
one choice that fits all countries.

68Nominal earnings are deflated by each country’s Consumer Price Index.

69This discrepancy is reported in table 7.14 in NIPA. For example, in 2016, the adjustments
for misreporting on income tax returns on net profit (less loss) of nonfarm proprietorships and
partnerships, plus payments to partners was 660 billion dollars.
which belong to cohort $c$, at time $t$. I then impose that this statistic is driven by age, cohort and time effects, and nothing else. Of course, these three factors are linearly dependent, so I can only control by either time or cohort effects and then focus on the implied age effect using the following regression framework:

$$\text{statistic}_{act} = \beta_a a + \beta_t t + \epsilon_{act}. \quad (3.1)$$

The coefficient of interest is the vector $\beta_a$, which is “age effect”, controlling for time effects. Similar regressions are possible while controlling for “cohort effects”. Which effect is more important is not a settled debate (see Heathcote et al. (2005), Blandin (2018)), but in general, cohort effects are larger.

In what follows, workers are divided into two education groups. Those with at least a 4-year college degree or its equivalent, and those without. The first group is labeled “college” and the later “non-college”. For a detailed treatment of educational levels in Europe and their U.S. equivalents, see Appendix C.1.

Fact 1: Steeper Earnings Profile in the U.S.

The first statistic of interest for inequality over the life-cycle is simply the mean earnings. This statistic shows how earnings evolve over the life-cycle on average and it contains the effect of labor market experience and human capital accumulation over the working life. Figure 5 shows the difference in mean earnings over the life-cycle across the U.S., U.K., Netherlands, and France. Mean earnings profile is steeper in the U.S., followed by the U.K. and Netherlands. France has the flattest mean earnings profile among all four. For example, by the age 50, earnings increase on average by
a factor of 2.2 in the U.S. relative to age 25, while in France, earnings increase by a factor of 1.5.\textsuperscript{70}

Another fact that I uncover is that college graduates on average have steeper mean earnings profiles in all countries relative to non-college individuals. For instance, the earnings of college graduates in the U.S. increases by a factor of 3.1 between ages of 25 to 50, as opposed to a factor of 2.2 for both college and non-college individuals together. This reflects the fact that individuals with different education levels potentially experience different earnings profile over the life-cycle. In fact, as figure 6 shows, non-college individuals experience the same earnings profile over the life-cycle in “all” countries. On the other hand, figure 7 shows that college individuals drive the differences in mean earnings over the life-cycle across countries.

The differences in mean earnings can be a result of several factors. Individuals may select into different occupations with different earnings growth, which can be different across countries. For instance, if doctors are in higher demand in the U.S. than in France for any reason, and the supply of doctors are slower to adjust to this demand, then life-cycle earnings growth of doctors can be larger in the U.S. than France. If most of the professional occupations have a similar situation, then mean earnings profile over the life-cycle in the U.S. will be steeper than France.

Another reason for differences in earnings growth is labor market frictions. Assume that the supply of and demand for doctors are similar across U.S. and France, but longer administrative processes in France make the promotion of doctors in hospitals slower. Then we would observe that the earnings growth of doctors in the U.S. is

\textsuperscript{70}Lagakos et al. (2018) find similar patterns in mean earnings profile over the life-cycle. Their focus is on work experience rather than age. Other papers that emphasize the cross-country differences in mean earnings over the life-cycle include Badel et al. (2018)
mechanically faster. Widespread frictions of this sort can slow down earnings growth across countries.

I argue in this paper that, from a human capital standpoint, steeper labor earnings taxation schedule lowers the incentives for investments in human capital which translates into slower earnings growth over the life-cycle. From the perspective of the model, a doctor in France has less incentive to spend extra hours in the medical school or the lab learning new techniques since any skill acquired will be taxed harder once turned into earnings. Across individuals, differences in college major and career path is interpreted as differences in initial conditions when starting adult life after high school graduation and differential investments decisions during adult life. Similar individuals across countries make different choices given the taxation and higher education policies which results in different mean earnings profiles for the whole cross-section and within education groups. A human capital framework would predict that college graduates are more distorted by steeper taxation schedule. This is backed by the evidence I uncover that the mean earnings of the individuals without a college degree looks almost identical across U.S. and the European countries.

Fact 2: Faster Growth in Variance of Log Earnings in the U.S.

The second statistic of interest is the variation in earnings. Individuals arguably experience different growth rates in life-cycle earnings and just focusing on the mean earnings profile masks this heterogeneity. On way to uncover this heterogeneity is by looking at the variance of log earnings and its evolution over the life-cycle.

Figure 8 shows that the variance of log earnings grows faster in the U.S. than European countries. First, the level of the variance is higher in the U.S. for all ages
and it is followed closely by the U.K. Netherlands and France have smaller variance overall. Second, variance of earnings between ages of 25 to 60 grows by 40 log points in the U.S., followed by 30 log points growth in the U.K. The growth in the Netherlands and France is about 10 log points.\footnote{Guvenen et al. (2014) show qualitatively similar results, though they control for cohort effects and the underlying sample and time-period is different.}

Another notable feature in the above figure is that the main part of the growth is concentrated early on in the life-cycle. U.S. and U.K. both experience faster growth until the age of 35 and this is true in Netherlands and France between the ages of 30 to 45.

Similar to the mean earnings profile, most of the rise in variance of log earnings in concentrated among individuals with a college degree, as shown in figures 9 for non-college, and figure 10 for college individuals. Both college and non-college graduates face increasing variance over the life-cycle in all countries with the U.S. having the largest initial level and overall growth. The profiles are steeper in all countries for college graduates and the main concentration of growth is early on in the life-cycle.

Going back to the example in the previous part, a doctor in the U.S. invests more in her human capital over the life-cycle than a construction worker. This is arguably because of the nature and opportunities available for human capital accumulation in the two occupations. Thus, the difference between the earnings of the two widens as these individuals age. Within individuals without a college degree, variance of earnings can rise. For instance, a construction worker’s earnings may experience less growth than a commercial pilot. But the variance of earnings arguably grows faster among individuals with a college degree (doctor vs art curator). The differences in life-cycle growth in the variance of log earnings across these countries can be interpreted as
different incentives for human capital accumulation across heterogeneous individuals. If individuals with higher potential earnings growth are more distorted in France than in the U.S., then the life-cycle growth in the variance would be slower in France.

Fact 3: Larger Growth in the Gini Coefficient in the U.S.

Other statistics of inequality over the life-cycle show similar qualitative patterns; the growth in the U.S. is higher than Europe and it is concentrated among college graduates. For example, the gini coefficient, which has been used extensively for studying cross-section inequality, is another indicator of the variation in earnings over the life-cycle. Huggett et al. (2011) document the gini coefficient over the life-cycle in the U.S. This is the first paper that goes beyond the U.S. for this statistic.

Figure 11 shows the gini coefficient over the life-cycle in all countries. The initial value is normalized to zero to emphasize the growth. It shows that similar to the variance of log earnings, gini coefficient grows faster in the U.S. over the life-cycle followed by the U.K., Netherlands, and France. The ranking of the countries do not change, whether I use the variance of log earnings or the gini coefficient. Most of the growth in all countries happens before age 40, and most of the life-cycle growth is concentrated among individuals with a college degree.72

Together, the above three facts show that earnings inequality over the life-cycle grows and this growth is faster in the U.S. than in Europe. Earnings grow faster on average in the U.S. over the life-cycle, but this growth is masking different growth rates across individuals. In other word, individuals do not experience similar growth rates, and so both variance of log earnings and the gini coefficient grow over the

72 See figures 20 and 21 for non-college and college gini coefficient respectively.
life-cycle. This pattern of rising inequality through life is present in all countries, but both levels and growth rates are larger in the U.S. Education subgroups follow the same qualitative patterns, but life-cycle inequality is mainly driven by college graduates.

Fact 4: College Attainment and Investments Are Higher in the U.S.

The previous facts show that the segregation based on college is important for earnings inequality patterns across these countries. It should be noted that there are critical differences in college attainment rate and expenditure between U.S. and European countries. Table 8 shows summary statistics about college across these countries. U.S. has the highest share of college attainment among working individuals (35%), while France has the lowest share (25%). Total expenditure for college (tuition, research, amenities, etc) from both public and private sources as a share of GDP is the highest in the U.S. (2.6%), while the share of this expenditure that comes from public sources (taxes) is the lowest in the U.S. (39%).

What the differences in the college attainment and investments across these countries show is that (1) the composition of individuals who choose college is different, and (2) college has a differential effect on human capital formation across these countries. The goal of the paper is to quantify to what extent different policies (taxation, higher education subsidies/transfers) are contributing to these differential selection and investment patterns which lead to differences in life-cycle inequality across these countries.
Fact 5: Labor Earnings Taxation Is Less Steep in the U.S.

In general, European countries have steeper labor earnings tax schedule (Guvenen et al., 2014; Duncan and Sabirianova Peter, 2016). I document the same fact within the model using a tax function first introduced by Benabou (2002b), and estimate the relevant parameters.

As an illustration, consider figure 12 which shows the marginal tax rates for labor earnings based on the multiples of mean earnings in each country. For instance, an individual who earns the mean earnings level in the U.S. faces a marginal tax rate of 30%. To understand the differences in the steepness of the tax schedule, note that moving an individual who earns the mean earnings level, to three times the mean earnings level increases her marginal tax rate by 8 percentage points in the U.S. Similar relative change in the level of earnings in France results in 12 percentage points increase in the marginal tax rate.

The differences in the steepness of the labor earnings schedule creates different distortions in these countries. Individuals with a potentially high earnings growth (e.g. college graduates) face the high marginal tax rates relatively early in the life-cycle. As a result, they are more distorted and reduce their human capital investments more. College choice amplifies this effect since steeper taxation lowers the value of attending college and the incentives to invest in human capital during college and all subsequent periods.

All of these empirical facts point to a role for differential growth rates in earnings over the life-cycle within each country, but with different magnitude across them. For understanding the underlying mechanisms for differential growth rates within and

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73See OECD (2011) for more details.
across countries, I develop a human capital model of earnings growth over the life-cycle and exploit policy differences in taxation and higher education. Before presenting the full model, I show how college selection depends on individual characteristics that shape human capital accumulation over the life-cycle and how policy changes can impact this margin.

3.3 A Simple Model of College Choice

In this section, I present a simple two-period model of college choice in order to show how changes in taxes can shift college selection and investments during college. This model is helpful for building intuition about different margins at play for college selection and human capital investments. The basic argument is the same in the full model in section (3.4). What is important here is how the individual makes college choice. The details of the derivations are in Appendix C.3.

Consider an individual who lives for two periods. The individual can potentially work in both periods, or she can go to college in period one and accumulate human capital while only work in the second period. There is no channel to increase human capital besides college and there is no human capital depreciation. I assume for simplicity that the only taxation system is a flat rate labor tax ($\tau$). Individuals discount future at rate ($r$), which equals the real interest rate. This implies a discount factor $\beta = \frac{1}{1+r}$. There is a rental rate per unit of human capital so that the labor earnings of an individual with human capital ($h$) equals ($wh$). There are no shocks to earnings.

The individual has three state variables at the start of period one: learning ability ($a$), initial human capital ($h_0$) and disutility for college ($\eta$). Higher learning ability
means higher marginal return on investment in human capital during college. The utility cost of attending college captures the psychic cost of exerting effort during college. The individual compares the present discounted value of going to college $V^C(a, h_0, \eta)$ and not going to college $V^{NC}(a, h_0, \eta)$ and if $V^C > V^{NC}$, she chooses college. Otherwise, she does not choose college.

If the individual goes to college, she will have access to a technology to increase her human capital. This technology uses her initial human capital, and investments in terms of consumption goods to produce new human capital. There is complementarity between initial human capital and investments in this technology as follows:

$$h_1 = h_0 + ah_0^\phi d^\nu, \quad \phi, \nu > 0, \quad \phi + \nu < 1,$$

where $(d)$ is the amount of investments in terms of consumption goods. The individual has to borrow this amount in order to fund her college education. There is a government that subsidizes college expenditure at rate $g_d$ and provides a fixed transfer during college in the form of a grant, denoted by $\bar{d}$.

Solution to the problem of the individual consists of a cutoff value for disutility for college $(\eta^*)$, so that if $\eta < \eta^*$, individual chooses to go to college in the first period, and above that cutoff, she works in both periods. The cutoff value for the disutility for college is given by:

$$\eta^* = 2 \log \left( \frac{1}{\frac{1}{2} + \nu} \left[ 1 + \left( \frac{aw\nu}{(1 + r)(1 - g_d)} \right) \frac{\phi + \nu - 1}{\nu} h_0^\frac{\nu}{1 - \nu} (1 - \tau)^\frac{\nu}{1 - \nu} + \frac{\bar{d}(1 + r)}{(2 + r)wh_0(1 - \tau)} \right] \right).$$

Also, the optimal investments during college is given by:

$$d = \left( \frac{aw\nu h_0^\phi}{(1 + r)(1 - g_d)} \right)^\frac{1}{\nu} (1 - \tau)^\frac{1}{\nu}. $$
Equation (3.3) can be used as the basis of analyzing college choice and the impact of taxation in this simple setup. First, higher ability individuals have higher cutoffs and are more likely to go to college. This is intuitive since ability increases the marginal return of investments during college. This means that higher ability individuals also invest more if they choose to go to college as is evident from equation (3.4).

Second, higher initial human capital decreases the cutoff point which means that these individuals are less likely to choose college. The reason is that when the initial human capital is high, it is very costly to forgo the earnings in the first period and attend college, given that the marginal return to investment in college is diminishing. These two forces together decreases the likelihood of attending college when initial human capital increases. However, if the individual makes it to college, she will invest more since there are complementarities between initial human capital and investments during college.

Third, a more generous college subsidy rate \( (g_d) \) and grants \( (\bar{d}) \) will result in larger cutoff which means individuals are more likely to go to college. The reason is that the investment in human capital during college is “cheaper” since the subsidies are more generous and there is also a fixed transfer which can be used for college expenditure or consumption. Working in the first period means that the individual does not get this transfer. As a result of more generous subsidy rate and transfers, individuals are more likely to go to college on the extensive margin, and college expenditure increases on the intensive margin, as shown in equation (3.4).

Finally, an increase in the tax rate \( (\tau) \), decreases the likelihood of choosing college. The reason is that higher taxes decreases the marginal benefit of investments in college since a higher share of the new stock of human capital is now taxed. This means that if the individual attends college, she invests less during college (see equation (3.4)),
which results in lower level of new stock of human capital \((h_1)\). This stock is now taxed at a higher rate, which reduces the incentive to attend college.

The above model illustrates that higher taxes results in lower college attendance, lower expenditure during college, and higher average ability for those who attend college. I explore the above forces in a more realistic setup with college, work, and retirement stages of life along with progressive taxation and college subsidies in the next section.

3.4 A Model of Human Capital Accumulation and College Choice

The economy is populated by a measure one of individuals. Each individual starts her economic life at the end of high school at age \(j_h\). The individual decides to go to college or start working right away. If the individual goes to college, she has to stay there for 4 periods, and after that enters the labor market at age \(j_w\). All individuals retire at age \(j_r\) and die after 20 periods of retirement at age \(j_d\). All of these ages are exogenously fixed.

Individuals have an innate learning ability \((a)\) which is fixed over the life-cycle. Their stock of human capital at the end of high school that encompasses all the childhood investments by parents until the end of high school is denoted by \((h_0)\). Individuals accumulate human capital \((h)\) and borrow/save using a risk-free asset \((x)\) at a real interest rate \((r)\). Human capital encompasses both general human capital such as health, and specific human capital such as sector specific skills and on-the-job training. The earnings of an individual during the working life is determined by her labor supply, her human capital, which is subject to a stochastic shock, and a rental rate \((w)\). There is a natural borrowing constraint, namely expected future earnings.
Individuals can accrue debt so long as they can pay back the debt with interest using their future earnings.

The asset level of all individuals upon high school graduation is assumed to be zero. This assumption rules out parental savings for college and/or general transfers from parents to children.\footnote{This assumption is for the sake of simplicity since finding the distribution of assets at the start of adult life is difficult in the data, and even if I find this distribution from other sources for the U.S., it is generally not available for the European countries.} In a human capital model similar to mine, Ionescu (2009) showed that leaning ability and initial human capital are much more important for college choice than parental wealth.

The human capital accumulation is determined by the choices of the individual. It depreciates at the rate ($\delta_h$) every period until death, so the individual should make investment decisions for her human capital. The individual has access to two types of technologies that allow for human capital accumulation. During working life, individual can invest time and accumulate human capital according to a Ben-Porath technology\footnote{See Mincer (1997) for a review of the history and evolution of this technology specification.} as follows:

$$h_{t+1} = (1 - \delta_h) h_t + a(h_t s_t)\phi.$$

Here, $s$ is the amount of time that the individual invests in the production of new human capital. This technology uses the learning ability, ($a$), and the current stock of human capital to produce new human capital. The economic interpretation is that during the working life, individual accumulates human capital in the form of gaining experience and/or learning on the job. The learning ability of the individual is an important factor in this technology. An individual with higher learning ability has a
larger marginal gain from investing time in production of human capital. Therefore, she can accumulate more human capital than an individual with a lower ability, holding the current stock of human capital and time investment fixed.

Another technology that is available to the individual for human capital accumulation is during college. This technology differs from the previous one in that the individual needs to invest time and consumption goods in order to produce new human capital during college. I assume that the individual does not work during college and all of her time endowment (normalized to one) is used for human capital production. The law of motion of human capital during college is

\[ h_{t+1} = h_t + a h_t^\phi d_t^\nu. \]

Here, \((d)\) is the amount of goods that the individual invests during college periods. The learning ability of individuals plays an important role here as well. Individuals with high learning ability have higher marginal benefit of investments during college. Therefore, they value college more and one in college, they are going to invest more for human capital production. This means that the gains from college attendance is heterogeneous across individuals. The only way to invest goods during college is to borrow from future earnings. Therefore, individuals with higher learning ability and higher initial human capital (after high school graduation), who have enough future earnings and can borrow against that, enter college.

Individuals are subjected to taxation of consumption, labor earnings (non-linear schedule) and returns on assets (capital income tax).\(^76\) The tax rates and functional

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\(^76\) These taxes differ across countries and I study their quantitative importance later on.
forms are presented in section 3.5. All individuals pay flat rate consumption ($\tau_c$) and capital income taxes ($\tau_k$), while working individuals pay labor earnings taxes as well.\footnote{If the individual is borrowing, she will not pay capital income tax. This tax is only relevant if she is saving and has positive capital income.}

At the beginning of economic life, the individual decides whether to go to college or not. So the life-cycle path is either of the form college-work-retirement for college graduates or work-retirement for individuals with no college education. Figure 13 summarizes the structure of the life-cycle in the model. I present the problem of the individual in a recursive format starting from the retirement period and moving backwards to the high school graduation. In what follows, subscript $j$ represents the individual’s age.

Retirement

The state variable of the individual aged ($j$) is her assets ($x$). The individual makes consumption ($c$) and savings ($x'$) decisions during the retirement periods. There is no human capital accumulation and her human capital depreciates each period. The terminal value for the individual states that at the last period of life, the individual consumes all of her assets since there is no bequest motive. Learning ability and the level of human capital are not relevant for the retirement periods. Government-funded social security transfers are denoted by ($ss$). These transfers are not dependent on the earnings history of the individual for simplicity.

$$V_j(x) = \max_{c,x'} \left[ u(c) + \beta V_{j+1}(x') \right]$$

$$(1 + \tau_c)c + x' = (1 + r(1 - \tau_k))x + ss,$$
\[ V_{j_d}(x) = u(x), \]

\[ j \in \{j_r, \ldots, j_d\}. \]

Working

The state variables of the individual ages \((j)\) are her learning ability \((a)\), human capital stock \((h)\), asset holdings \((x)\), and the shock to the stock of human capital \((z)\). The individual makes consumption \((c)\), savings \((x')\), and labor supply \((l)\) decisions. She also makes a choice about time investment \((s)\) for production of human capital, which results in the next period’s human capital \((h')\).

\[
V_j(h, x, z; a) = \max_{c,l,s,x',h'} \left[ u(c, l, s) + \beta \mathbb{E} V_{j+1}(h', x', z'; a|z) \right] \tag{3.6}
\]

\[
I = wzhl
\]

\[
(1 + \tau_c)c + x' = (1 + r(1 - \tau_k))x + I - T(I),
\]

\[
h' = (1 - \delta_h)h + a(hs)\phi,
\]

\[
V_{j_r}(h', x', z'; a) = V_{j_r}(h', x'; a), \quad \forall z,
\]

\[ j \in \{j_w, \ldots, j_r - 1\}. \]

The terminal value states that at the last period of working life, the individual starts the next period in retirement for all possible future values of the shock. In the above formulation, labor earnings is defined as \(I = wzhl\), where \((w)\) is the rental rate of human capital in the labor market, \((z)\) is an idiosyncratic, age-independent, and \(iid\) shock to the stock of human capital, \((h)\) is the stock of human capital, and \((l)\) is labor supply.
The idiosyncratic shock represents uninsurable labor earnings risk. A negative human capital shock can occur when a worker loses firm- or sector-specific human capital following job termination. A decline in health (disability) is another example of negative human capital shock. In this case, both general and specific human capital might be lost. Internal promotions, bonuses, and upward movements in the labor market are examples of positive shocks to human capital.

Human capital investments in this formulation represent on the job training. They are also the vehicle for propagating the iid shocks to the stock of human capital in the model. A large literature that estimate the statistical models of earnings posit that there is a stochastic and persistent shock component to earnings that is important for various policy analysis. In a human capital model, iid shocks will result in a similar stochastic component of earnings. In other words, even though the shocks are iid, they propagate in the model in a way that earnings shocks look like an auto-regressive process or a more sophisticated stochastic process to an econometrician. This point was emphasized in Huggett et al. (2011) where a similar shock process in a human capital model generates income processes estimated in other empirical work such as Guvenen (2007).

While working, individuals are subject to a non-linear labor earnings tax, \( T(I) \). The non-linear taxation means that as the earnings of the individuals increases, the marginal tax rate on labor earnings goes up. The non-linear schedule distorts the individual’s incentives beyond labor supply decision and affects human capital investments. Since the marginal cost of time investment is not changing under non-linear taxation (it is a direct utility cost), but the marginal benefit of an increase in future earnings decreases with higher marginal tax rates, non-linear taxation depresses
human capital investments. On the other hand, non-linear taxation provide valuable social insurance by lowering the marginal tax rate when individual experiences a negative shock.

Whether the benefits from non-linear taxation dominates the distortionary effects of this taxation schedule is a quantitative question that I will address in section 3.7.

College

The state variables for an individual age \((j)\) in college is her learning ability \((a)\), her human capital stock \((h)\), and her asset holdings \((x)\), and disutility for college \((\eta)\). During college, the individual makes consumption \((c)\) and savings \((x')\) decisions, along with goods investments \((d)\) for production of human capital. She invests all of her available time for full specialization in human capital production. This means that the individual cannot work, and there is no depreciation of human capital during college.

The individual incurs a utility cost \((\eta)\) of college attendance. This cost reflects the idiosyncratic psychic cost of studying, learning new skills, etc, during college. In order to smooth consumption, individual has to borrow while in college against her future earnings. Thus, individuals with sufficiently high future earnings and low disutility for college will choose to attend college. For simplicity, the individual

\[\text{Other papers that emphasize the distortionary effects of non-linear taxation for human capital accumulation include Huggett et al. (2011), Guvenen et al. (2014), and Badel et al. (2020), among others.}\]

\[\text{In order to better match some observable moments regarding college choice and investments, the inclusion of a utility cost of college attendance is very desirable. See for example, MacDonald (1981), Card (2001), and Hai and Heckman (2017) among others. In my setup, the distribution of this parameter helps with matching the college share in the data.}\]
borrows at the same capital market interest rate. So, there is no student loan program with a different interest rate and repayment schedule.\(^{80}\)

Government contributes to college expenditure through two channels. The first is a subsidy program which subsidizes private expenditure in college at the rate \(g_d\). This subsidy reflects scholarships and tuition subsidies for college students. The second is a fixed transfer, \(\bar{d}\), to individuals in college in the form of a grant. This grant is capturing expenditures on stipends, health insurance, housing allowances, bus passes, etc, that can be used for consumption, saving, or spent directly on human capital production. The subsidy program and grants constitute public expenditure in college. In reality, part of the public expenditure may not directly benefit college students, but indirectly influence their human capital accumulation through better facilities and higher quality of classes/labs. Finally, there are no uncertainty about college graduation, which means that every individual who enters college will graduate.

The problem of an individual during college is given by:

\[
V_j(h, x; a, \eta) = \max_{c, d, x'} \left[ u(c) - \eta + \beta V_{j+1}(h', x'; a) \right]
\]

\[
(1 + \tau_c) c + (1 - g_d) d + x' = (1 + r(1 - \tau_k)) x + \bar{d}
\]

\[
h' = h + ah''d''
\]

\[
V_{j_w}(h', x'; a, \eta) = \mathbb{E}V_{j_w}(h', x', z'; a),
\]

\[
j \in \{j_h, \ldots, j_w - 1\}.
\]

The production of new human capital during college means that the individuals who attend college start working with an “endogenous” initial condition, rather than

\(^{80}\)For a treatment of student loan programs and their implication for college choice, graduation rates, and so on see Ionescu (2009), Ionescu (2011), Chatterjee and Ionescu (2012), and Ionescu and Simpson (2016), among others.
the exogenous draw they had when graduated from high school. This points to complementarities between college investments and human capital accumulation while working. Individuals who attend college can accumulate human capital much faster than those who do not, since college requires investments in the form of consumption goods. Therefore, a high ability individual who goes to college, starts working with a much higher stock of human capital than she otherwise would have if she did not attend college. She enjoys higher earnings after college and potentially a higher growth in earnings since returns on human capital investments while working are increasing in the level of human capital.

Individuals who go to college forgo earnings during the time in college and accrue debt, but they will have potentially high earnings in the future, which makes college worthwhile. Therefore, if there is a change in the environment that shrinks future earnings, the composition of college graduates and their investments during college will change.

It should also be noted that human capital in college is not risky. The reason, aside from simplicity, is not theoretical, but rather empirical. Any strategy to identify shocks to earnings relies on observed earnings, wages, hours, etc. It is true that students in the real world work during college, but given the type of jobs they can get while in college (mostly minimum-wage jobs), their human capital is not arguably as relevant for their earnings as later in life when they are not in college anymore, and on a career path. Therefore, shocks to human capital during college are ruled out theoretically. Graduation risk is also ignored to keep the model simple, though it is possible to add that to the current setup. Given that the facts are driven by individuals with a college degree, the model only allows for college completion.
Educational Choice

The life-cycle choice for an individual with a high school degree is to choose whether to go to college or not by comparing the life-cycle value of college path \( V^C \) and non-college path \( V^{NC} \):

\[
V(a, h_0, \eta) = \max\{V^C, V^{NC}\} \quad (3.8)
\]

Technology

There is a stand-in firm that demands capital and labor for production of the consumption good using a constant returns to scale technology. Physical capital depreciates at rate \( \delta \). The capital is the total assets in the economy and the labor is the total human capital supplied to the labor market.

Stationary Competitive Equilibrium

The definition of a steady-state stationary competitive equilibrium is standard. It consists of decision rules, population measures, and factor prices such that, given factor prices, decision rules are optimal, factor prices are competitive, and total taxes equals social security transfers, college subsidies/transfers, and government expenditure, and population measures are consistent with optimal choices. The formal definition of the equilibrium can be found in Appendix C.3.
3.5 Parametrization

Calibration consists of two steps. In the first step, I borrow some common parameters usually used in the literature and set them exogenously. The parameters governing demographics, technology, and the tax rates for consumption and capital income, are set in this way. The parameter governing the standard deviation of the shock to human capital is set to the estimate by Huggett et al. (2011). The depreciation of human capital is set to match the average decline in the hourly wage in the data towards the end of working life. The model implies that at the end of working life, human capital investment is essentially zero. This means that any decline in the hourly wage is due to depreciation of human capital. Given this set of parameters, I will set the rest in the second step to match some moments in the data. All the parameters except for the labor earnings tax schedule can be found in table (15).

Benchmark Model Functional Forms

Utility function:

\[ u(c, s, l) = \begin{cases} 
\log(c) - \eta & \text{college} \\
\log(c) - B \frac{(l + s)^{1+\frac{1}{\gamma}}}{1 + \frac{1}{\gamma}} & \text{working} \\
\log(c) & \text{retirement}
\end{cases} \]  

Production function:

\[ Y = AK^\alpha L^{1-\alpha} \]
Initial conditions: Marginal distributions for learning ability and initial human capital are both Pareto-Log-Normal (PLN) distributions.\footnote{The way initial human capital is constructed from the learning ability results in a Pareto tail for its marginal distribution. For a formal proof, see Badel et al. (2020).}

\begin{align*}
a &\sim PLN(\mu_a, \sigma_a^2, \lambda_a) \\
\log h_0 &= \beta_0 + \beta_1 \log a + \log \epsilon, \quad \epsilon \sim LN(0, \sigma_\epsilon^2) \\
\eta &\sim \exp(\lambda_\eta)
\end{align*}

The random variable \(\epsilon\) used in the above construction is independent of learning ability. The distribution for disutility of college is exponential. The distributions for learning ability and initial human capital are from the power law family, which feature fatter tails than other distributions. This is important for generating enough top earners in the model so that the earnings distribution matches sufficiently with the data. Given that the top earners are arguably more distorted by taxation, it is important quantitatively to generate enough top earners in the model.

Demographics

An individual enters the model at a real-life age of \(j_h = 20\), graduates from college at age \(j_w = 24\) if she goes to college, retires at age \(j_r = 60\) and lives up to age \(j_d = 80\). The population growth rate \(n = 0.01\) is set to the geometric average growth rate of the U.S. population over the period 1960-2015. Population fractions \(\mu_j\) sum to 1 and decline with age by the factor \((1 + n)\).
Technology and human capital

U.S. national accounts data imply that capital’s share, the investment-output ratio and capital-output ratio averaged \((0.4, 0.2, 3.2)\) over the period 1960-2015. I set \(\alpha\) to match capital’s share, and \(\delta\) to be consistent with the investment-output ratio and the capital-output ratio, given \(n\). I normalize \(A\) to one.

Finally, the value of the curvature parameter for the human capital production function is set to value \(\phi = 0.6\), following Badel et al. (2020). This value is within the range of the estimates for this parameter in the literature.\(^{82}\)

Taxation

There are different sources of taxes in the model. Consumption and capital income are taxed at a flat rate \((\tau_c)\), and \((\tau_k)\), respectively. There is also a non-linear tax system for labor earnings such that the marginal tax rate increases with earnings. The functional form for the non-linear taxation is the one used in numerous papers, (e.g. Benabou (2002b); Storesletten et al. (2004); Guner et al. (2014)), and reasonably approximates the U.S. tax code:

\[
I = wzhl,
\]

\[
T_f(I) = I(1 - \lambda(\frac{I}{\tilde{I}})^{-\tau}),
\]

where \(\tilde{I}\) is the mean cross-sectional income. Dividing by the mean is consistent with a balanced growth path since the average taxes will be unchanged if the earnings of all individuals are multiplied by a constant factor. The parameter \((\lambda)\) governs the

\(^{82}\)For a detailed discussion about the different estimates of this parameter, see Blandin (2018).
level of taxation, while the parameter \((\tau)\) governs the steepness of the tax schedule. This means that as \(\tau\) increases, the marginal tax rate increases faster as earnings grow.

I need three pieces of information in order to estimate the parameters of the above tax function: (1) mean cross-sectional income, (2) individual’s income, and (3) tax liability of each individual. For each country, I observe individual’s income and so, I can calculate the mean cross-sectional income directly from the data. The part that cannot be calculated straightforwardly from the data is the individual’s tax liability. In order to calculate that, I need to look at the tax code for each country and year and calculate labor earnings tax liabilities (federal, state, local, social security, etc) for each individual. I will rely on the programs provided by Bick et al. (2019) for calculating tax liability for each individual, which are basically simulating the tax code for the U.S. and some European countries.

In many countries, the tax code considers household as the unit of observation. Therefore, tax liabilities are calculated at the household level.\(^{83}\) This is true for the U.S., France, and the Netherlands.\(^{84}\) U.K. is the only country in my sample that is truly individual-based. This pose a challenge since the empirical facts and the model are based on only male earnings and countries differ in their respective tax unit.

I treat the taxes at the household level for all countries.\(^{85}\) This means that all the required information for estimating the tax function are at the household level. In

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\(^{83}\)Social security taxes are individual-based.

\(^{84}\)Taxes in the Netherlands are individual-based. But part of tax code indicates a tax credit which is calculated at the household level. This nuance among others make the tax code in the Netherlands effectively household-based. For detailed information see OECD (2011).

\(^{85}\)The interpretation of household-based tax parameters in the model is that each individual is part of a household and faces tax liabilities according to a household-based system. Of course, this affects the individual’s decision if the other members of the household are also making similar human
this way, I know the household income as well as the mean cross-sectional household income directly from the data. Bick et al. (2019) then gives tax liabilities for each household. Once I know these, I can look for parameters of the tax function so that the difference between what the tax function is producing and tax liabilities in the data is minimized.\footnote{See Appendix C.1 for more information about these estimations.}

The country-specific parameters are presented in table 11.\footnote{As a robustness check, I estimated the tax function for the U.K. based only on male earnings and their tax liabilities. Compared to the household estimates, males face higher average tax than households as well as a slightly more steepness.} The average taxes are smallest in the U.S. and the U.K., with the Netherlands having higher average taxes and France the highest. In terms of steepness, U.S. and U.K. are similar, with the Netherlands more progressive than them. France has the most progressive tax system among these four countries.

Preference Parameters

The parameters in the utility function are set to the values presented by Badel et al. (2020). The parameter governing the level of disutility of work is set to $B = 12.4$, while the parameter governing the labor supply elasticity is set to $\gamma = 0.6$. They calibrate these parameters to match the patterns of hours over the life-cycle in the data. Essentially, they regress the log of hours worked on the log of hourly wage (similar to MaCurdy (1981)) and match the coefficients of this regression in the model. Given the similar structure of the model for the working stage of individuals' capital investment and labor supply decisions. For simplicity, I am abstracting from those and remain agnostic about household structure in the model.
life-cycle (endogenous and risky human capital with elastic labor supply), I set these parameters based on their calculations. The amount of social security transfers during retirement is 40% of the mean cross-sectional income, which pertains to a replacement rate reported by Social Security Administration.

Remaining Parameters

All the remaining parameters are chosen to match a set of data moments. These parameters are

1. mean of the distribution of disutility for college $\lambda_\eta$; 1 parameter,
2. distribution of learning ability and initial human capital; a total of 7 parameters,
3. human capital production parameter for college $\nu$; 1 parameter,
4. subsidy rate for college, $g_d$; 1 parameter,
5. grant level, $\bar{d}$; 1 parameter,
6. discount factor $\beta$; 1 parameter.

There is a total of 12 parameters to be jointly calibrated. The mean of the disutility for college is governing the percentage of college graduates in the model, which is 35% in the U.S. over the period 2004-2016. The parameters for the initial distribution of ability and human capital are set to reproduce the mean and variance of log earnings

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88 The value for the Frisch elasticity is in the range estimated for macro models previously documented in the literature which feature endogenous human capital. For a more extensive discussion on Frisch elasticity see Blundell and Macurdy (1999), Domeij and Flodén (2006), and Keane (2011), among others.

89 See Biggs and Springstead (2008) for alternative measures of replacement rates.
for each education group. The human capital production parameter is governing the expenditure during college in the model. It is set to reproduce the total expenditure during college as a percentage of GDP (both public and private). Based on OECD (2016), total expenditure on college education is 2.6 percent of the GDP in the U.S.

The subsidy rate for college and the grant level are calibrated to produce two moments: (1) the share of public expenditure in total college expenditure, which is 38.6%, and (2) the share of public expenditure in the government revenue (2.9%). Total government expenditure in 4-year colleges (excluding community colleges, vocational schools, etc) is available on the Department of Education website. These expenditures include all expenditures by federal, state, and local governments. Moreover, total government tax receipts are reported in National Income and Product Accounts (NIPA) table 3.2 for the federal government and table 3.3 for the local and state governments. Finally, the discount factor is set so that the model produces a capital to output ratio of 3.2.

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90 This constitutes 6 moments for mean earnings profile for each group (the first moment is a normalization), and 7 moments for each group for variance of log earnings. Together, there are $6 + 6 + 7 + 7 = 26$ moments to match.

91 The value 2.6 is the average over 2005-2016. It excludes expenditure for community colleges and vocational/training schools.

92 See table 8 for summary statistic for the U.S. higher education.
3.5.1 Benchmark Economy

The results of the above calibration are summarized, for mean earnings, in figure 14 (all individuals), and for variance of log earnings in figure 15 (all individuals).\textsuperscript{93} The model does an excellent job at matching the targeted moments for each education group. The mean earnings profiles are matched almost perfectly, while the profiles of variance of log earnings are matched with lower precision. The mean and variance profiles for all individuals are matched very well, even though they were not directly targeted (the mean and variance profile of each education group were targeted for calibration).\textsuperscript{94} The match will never be perfect since the number of parameters is less than the number of moments.

The benchmark economy features all the relevant statistics related to college perfectly relative to the data. Model generates a 35\% share for college individuals, while the total expenditure (both public and private) for college in GDP is 2.6\%. The share of public expenditure for college in total college expenditure is 38\%, and the share of government revenues dedicated to college subsidies/transfers is 2.9\%.

Figure (16) shows non-leisure time in the model and hours worked over the life-cycle from the data (annual hours), which was not targeted for calibration.\textsuperscript{95} Both profiles

\textsuperscript{93}For mean earning within education groups, see figures 22 (non-college individuals), and 23 (college individuals). For variance of log earnings within education groups, see figures 24 (non-college individuals), and 25 (college individuals).

\textsuperscript{94}The mean earnings profile for all is a weighted average of the profiles for each education group, with the weights being the share of each education group. Since the college share is matched perfectly, the mean earnings profile for all is unsurprisingly matched. The variance of log earnings profile however, is a nonlinear combination of the two education groups. The model does a good job since college graduates are driving the overall profile.

\textsuperscript{95}Figures (26) and (27) show the hours profile for non-college and college individuals, respectively.
are normalized to 100 at age 40. Hours worked over the life-cycle in the data follows a hump-shaped patterns whereas in the model, both labor supply and time investment for human capital accumulation are monotonically decreasing.\footnote{The reason is that, in order to have a realistic capital to output ratio in a life-cycle setup where the only borrowing constraint is the expected future earnings, the discount factor has to be close to one ($\beta > 1/(1 + r)$). Euler equation then implies that consumption is growing over the life-cycle. This means that the optimal labor supply has to decline if the growth in consumption is faster than earnings, which is the case here. Time investment in human capital declines monotonically over the life-cycle since individuals reap the benefits of human capital accumulation over shorter periods as they age. Therefore, they optimally invest less time over the life-cycle.}

Subsequently, there are additional moments that the model generates and are not targeted for calibration. One important moment is the relative earnings of college to non-college individuals (skill premium) over the life-cycle. Figure (17) shows the skill premium. Producing the skill premium is important since it incorporates the college decision, earnings levels and heterogeneous earnings growth over the working life across individuals. It basically summarizes all the relevant decision margins for generating a sensible earnings distribution. Even though none of these data moments are targeted by the calibration, the model still matches these moments closely.

Looking closer at the earnings distribution in the U.S., I check whether the model can generate the percentage of earnings that flow to different quintiles of the earnings distribution, the cross-section gini coefficient, and the concentration of the earnings at the top. Panel A in table 9 shows these statistics for the U.S. which are calculated from the CPS. Model generates closely all sections of the earnings distribution, even though none of the statistics in this table are targeted by the calibration procedure. What is also important for my analysis is that the model generates a reasonable distribution of earnings for college and non-college individuals as well. Table 10 shows that the earnings distribution for each education subgroup follows the data closely. All the above gives external validity to the model that it can reproduce relevant statistics.
of the U.S. earnings distribution both at an aggregate level and for each education subgroups.

The last piece of evidence for the performance of the model that I present is related to the wealth distribution. Specifically, I define the wealth of an individual at any age as her asset holdings. I then look at certain quintiles of the wealth distribution and ask, how much of total wealth in the economy is held by each quintile? This shows how skewed the wealth distribution is in the model.

Kuhn and Rios-Rull (2016) document the distributional properties of wealth for U.S. households, based on the Survey of Consumer Finances (SCF).\footnote{Other papers that document wealth distribution in the U.S. and other countries include Piketty and Saez (2006), Saez and Zucman (2016), Piketty et al. (2017) and Zucman (2019). Most of the data used in these papers come from tax records in order to measure wealth inequality over time. For a more detailed account of wealth and income inequality in the U.S. since the 1950s, see Kuhn et al. (2020).} I use their data to compare the model generated wealth distribution with the SCF data. Panel B in table (9) shows the comparison. The model does an excellent job capturing the distribution of wealth for each quintile as well as the wealth gini coefficient. However, the model fails to generate the concentration of wealth at the top. This is a known feature of life-cycle models that focus on labor earnings and goes back to Huggett (1996). The reason is that creating a realistic wealth distribution in the model requires more savings motive beyond consumption smoothing. One such motive can be saving for business operation and entrepreneurship.\footnote{See Quadrini (2000), Quadrini (2005), and Cagetti and Nardi (2006), among others, for models of entrepreneurship and wealth inequality.}
3.6 Model Mechanisms

One of the differences between the countries in section 3.2 is the difference in the steepness of the taxation system. Specifically, U.S. and the U.K. have similar steepness, while the Netherlands is a bit more steep. The steepest of all is France. Given that steepness in taxation schedule decreases the marginal benefit of human capital accumulation, but leave the marginal cost unchanged, it depresses investment in human capital as well as labor supply. Together, they cause smaller earnings growth and less variation in earnings at the top. The degree to which non-linear taxation can depress human capital accumulation in two hypothetical scenarios is studied in this section.

In the model, the parameter $\tau$ governs the steepness of the taxation system. Specifically, increasing this parameter means that the marginal taxes grow faster as the earnings increase. The benchmark calibrated value is $\tau = 0.13$. I increase $\tau$ to 0.15 and 0.17, while keeping other parameters constant.

Figures 18 shows the effect of increasing the steepness parameter on mean earnings of all individuals. Not surprisingly, the steepness of the earnings profile declines. This is true for all individuals as well as within each education group. The main difference within each education group is that the college educated ones are affected more since they experience higher earnings growth rates on average and they hit the higher marginal tax rates more often than non-college graduates. Hence, the earnings profile for college graduates becomes less steep.

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\[^{99}\)See figures 28 and 29 for the mean earnings profiles for non-college and college individuals, respectively.
Figures 19 shows what happens to the variance of log earnings for all individuals.\textsuperscript{100} The slower growth rate in earnings on average means that there is less variation in the earnings over the life-cycle. As a result, variance of log earnings grows slower as the tax system become more steep. The qualitative pattern is true for both education subgroups, although the college graduates are affected more as the high marginal taxes kicks in for these individuals more frequently relative to the non-college ones.

Given the above effects on earnings, it is not surprising that college selection and expenditure for college changes under different tax system. Table 12 shows the share of college graduates as well as total expenditure for college as a percentage of GDP. Higher progressiveness of the taxation system lowers the benefit of human capital accumulation and hence, discourages college attainment. Once in college, the same effect depresses expenditure for human capital accumulation.

An interesting observation, which is consistent with the stylized facts in section 3.2, is that the variance of log earnings drops for the first age group. This happens through three channels: (1) the fact that less individuals go to college means smaller variance of human capital at age 25, which is the first year of working after college, (2) less investments during college implies that college graduates have smaller growth in human capital during college, and (3) higher marginal taxes because of the more progressive system depresses hours for all. Together, these three channels contribute to smaller variance of earnings in logs at age 25.

\textsuperscript{100}\textit{See figures 31 and 30 represent the variance profiles for non-college and college individuals, respectively.}
3.7 Policy Analysis

In this section, I use the model to study life-cycle inequality patterns documented in section 3.2 to see how much of the differences in the data is generated by the model under different policies. I consider U.S. and the European countries to be different from each other in two dimensions: labor earnings taxation, and college transfers/subsidies.

I move from the benchmark model that is calibrated to the U.S. in both dimensions to the European countries in two steps. In the first step, I change the relevant parameters for each dimension individually. In the second step, I combine them together and study their interactions. The first step shows that taxation differences go a long way for accounting for inequality differences. It also illustrates that increasing college transfers exacerbates inequality. The second step reveals that gains from more generous college transfers in terms of earnings growth are dampened by steep taxation schedule.

3.7.1 Only Taxation Differences

I change the taxation system by applying the pair \((\lambda, \tau)\) that was estimated in section 3.5 for each country, while keeping other parameters constant. I then solve and simulate the model and compare model produced statistics with data. In doing so, I keep the interest rate fixed, given that the European countries can be viewed as small
open economies. This means that based on the production function and normalization of the TFP parameter, the rental rate for human capital is also constant.\textsuperscript{101}

In the model, the parameter $\tau$ governs the degree of steepness of the taxation system. Specifically, increasing this parameter means that the marginal taxes grow faster as earnings increase. The benchmark model is calibrated with $(\lambda = 0.83, \tau = 0.13)$.

First row of table 13 summarizes the model generated profiles of mean and variance, respectively, after allowing for labor earnings taxes to differ across countries. The numbers for mean earnings are the share of overall growth in mean earnings between age 25-50 in the model relative to the data. The numbers for variance are the share of overall growth in variance between age 25-60 in the model relative to the data.

The table shows that considering only the differences in labor earnings taxation account for the bulk of the variations in the data. Model generates on average, 95% of the mean earnings growth, and 74% of the growth in the variance of log earnings across European countries.

Model does a good job of generating the mean and variance profiles for the U.K. which is not surprising given that the steepness of the tax system in the U.K. is very similar to the U.S. For France and Netherlands, however, model generates less growth in both mean and variance profiles. This means that the disincentives from higher marginal tax rates for potentially top earners are so large that the accumulation of human capital slows down a lot. As a result, inequality profiles become flatter than the ones observed in the data.

The other reason for the flatter profiles is related to college selection and change in

\textsuperscript{101}European countries are considered small open economies. Therefore, I always fix the interest rate and TFP level for all of them at the benchmark calibration. This means that unless TFP level changes, the rental rate for human capital is fixed as well since a fixed interest rate and TFP level implies a fixed capital to labor ratio: $r + \delta = A(K/L)^{\alpha-1}$. In section 3.8, I discuss how changing the TFP level which translates into different rental rate for human capital affect inequality patterns.
composition of college graduates, which is presented in the first row of table 14. The numbers in this table are the share of college graduates and total college expenditure in GDP relative to these statistics in the data. Table shows that college choice is severely distorted under steeper taxation. On average, model generates 78% of the differences in college graduation and 86% of the total expenditure in GDP across European countries.

In all European countries, both the share of individuals going to college and the share of total expenditure for human capital formation during college is smaller than the data. The exception is U.K. which is to be expected since taxation structure in the U.K. is almost identical to the U.S.

It should be noted here that U.S. and Europe differ in terms of the average duration of college degree. While in the U.S. it is customary to finish college in 4 years, college takes about 3 years in the U.K. and 4 to 5 years in the Netherlands and France. Some occupations also require degrees that take more than 4 years fulfill. For instance, medical and law school students spend more years to obtain proper college credentials for a potential future career. The model has a fixed number of periods for college which is 4. This means that a potential margin about the duration of college is missing. The distortionary effects of progressive taxation is more pronounced for individuals who spend more time in college accumulating human capital since their potential earnings growth is much faster after graduation. This means that model is generating a lower bound for the distortionary effects of non-linear taxation.

Given that college graduates in the data have steeper profiles and are important to reconcile the model with the data, it is of first order importance to consider differences in college subsidies/transfers across these countries. The next section focuses on this dimension.
3.7.2 Only College Subsidies/Transfers

In this section, only the parameters related to college stage is matched to each country and I find that increasing college subsidies/transfers increases earnings inequality. Specifically, I calibrate the subsidy rate $g_d$ and grant level $\bar{d}$ to match the share of total college expenditure in GDP and the share of public expenditure in total college expenditure, while leaving the labor earnings taxation unchanged.\footnote{For a brief treatment of higher education finances in the U.S. and Europe, see Appendix C.1.}

The second row of Table 13 shows that country-specific college subsidies and transfers alone are not enough to reproduce the patterns observed in the data. The only country that is a bit closer to the data is the U.K. Even there, model fit becomes worse since U.K. spends less on college, both publicly and privately. The mean and variance profiles for the Netherlands and France are steeper than the data. The reason is that as college subsidies/transfers become more generous, the marginal individual finds it worthwhile to attend college since attending college is “cheaper”. Those potentially top earners who were already in college increase their human capital investments in college much more that the rest of the college individuals. As a result, the earnings of the top earners grow faster after graduation which increases the growth in mean and especially variance of log earnings over the life-cycle for the whole cross-section. As a result, college subsidies/transfers exacerbate earnings inequality.

3.7.3 Taxation + College Subsidies/Transfers

In this section, I combine taxation and higher education subsidies/transfers and study their joint effects for earnings inequality and show that the fit of the model is
improved relative to previous sections. I already showed that the model can generate most of the variation in the data by country-specific labor earnings taxation alone. Given that inequality patterns in the data are driven mostly by workers with a college degree, selection for college is an important margin to consider. However, matching proper aggregate and public expenditure for college “alone” did not improve the fit of the model much.

The third row of table 13 shows the interaction of taxation differences and higher education subsidies/transfers. It is most informative if this row is contrasted with the first row of the same table. Comparing the rows of this table indicates that a more generous college subsidies/transfers provide more incentive for individuals to select for college, despite steeper taxation. Specifically, the marginal individual values college more since she can invest in college in a “cheaper” way. This cheaper way comes from the fact that investment in human capital production, $d$, is subsidized at a higher rate, and the amount of college grant, $d$, is higher. Together, they mean that the individual does not need to incur as much debt and can benefit from higher human capital during the working life even though the taxation system is more progressive. This is certainly true for the Netherlands and France, but in the case of U.K. lower overall college expenditure relative to the U.S. makes the fit of the model worse.

College subsidies/transfers improved the fit of the model for European countries in the presence of non-linear taxation since they induce more individuals to select for college. This is evident when comparing the first and third rows of table 14. Model

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103 When taxes become steeper, the private expenditure during college is depressed a lot. This means that in order to match the total college expenditure in GDP, the level of grants ($d$) should increase. In the absence of an increase in steepness, the grant level goes down since more generous subsidy rate implies a lot of expenditure during college and the share of college expenditure in GDP would be too high. This means that if one only changes the college subsidies/transfers across these countries, she would find that college transfers in the Netherlands are smaller than the U.S. which seems counterfactual. Taxation differences correct this prediction of the model as well.
now accounts for 94% of the growth in mean earnings and 80% of the growth in the variance of log earnings across countries.

3.8 Discussion

In this section, I discuss some results that are important in order to put the paper in a broader perspective. I first briefly discuss how the focus on life-cycle inequality relates to the cross-section inequality which has been the focus of many empirical papers. The model does a good job at generating cross-section gini in earnings across countries. I then study the role of TFP for accounting for life-cycle inequality across countries. Changes in TFP allows for full general equilibrium effects in the model and worth further analysis. I find that TFP differences only matter when taxation differences are present.

3.8.1 Life-Cycle vs Cross-Section Inequality

I showed in section 3.7 that the model does a good job of generating the differences in life-cycle inequality when differences in taxation and college subsidies/transfers are taken into account. What does this mean for cross-section inequality in labor earnings? In other words, can the model generate a realistic cross-section inequality? To answer this question, I look at the cross-section gini coefficient in labor earnings across U.S. and Europe from the data and compare them to model generated ones.

Table 18 shows that the model generates almost all the variation in cross-section gini coefficient across these countries. For instance, the gini coefficient is 0.28 in the Netherlands in the data, and 0.25 in the model. This means that accounting for
life-cycle inequality ultimately accounts for the observed differences in inequality at the aggregate level.

3.8.2 The Role of TFP

There is a large literature in macro development that emphasizes the role of TFP differences as the main factor for income differences across countries. In this section, I explore this fact in the context of life-cycle inequality. Given that the emphasis of this paper is on human capital formation, which is also emphasized in the macro development literature, it is worthwhile to think about how TFP differences is going to shape life-cycle inequality on the micro level as well as aggregate variables such as GDP and hours worked per worker at the macro.

Changing Only The TFP

In order to understand the role of TFP differences for life-cycle inequality, I calibrate the TFP parameter, $A$, for each country to match the GDP per adult relative to the U.S. in the data, which are documented in table 19 from Penn World Tables. I then proceed by comparing the model generated earnings profiles with the data.

The first row of table 21 show that TFP alone is not of first order importance for

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104 See Caselli (2005a) for a review of this literature and the references therein.


106 The second column in table 20 shows the TFP levels in this calibration. The U.K. has the lowest TFP level, while the Netherlands and France have higher TFP than the U.K. The reason TFP level in the U.K. is smaller than the other countries is that the GDP per adult in the U.K. is smaller relative to the hours worked. In other words, it looks like the aggregate hours worked is large in
generating life-cycle patterns in the data. Lower TFP will result in a lower rental rate for human capital in the market. This leads to lower return on human capital accumulation and lower value of college investments. Therefore, individuals will invest less in human capital overall which leads to flatter profiles for mean and variance of earnings. In a model with flat rate labor earnings taxation, changes in the rental rate would not change human capital investment decisions since earnings over the life-cycle would change by the same factor \( w \). This is not true when non-linear taxation is present. Share of individuals with a college degree as well as overall expenditure in college also goes down as reported in table 22.

\[
\text{Taxation + College Subsidies/Transfers + TFP}
\]

For each country, I keep the taxation country-specific and calibrate the TFP level to match GDP per worker relative to benchmark (U.S.) as the data. College subsidy rate and grant level are recalibrated to match college expenditure in GDP and share of public expenditure in college.\(^{107}\)

Second row in table 21 shows how the model performs in producing inequality profiles. Lower level of TFP with fixed interest rate means a lower rental rate for human capital. As a result, investment in human capital has a lower present discounted value and individuals are less inclined for this investment. This is apparent specifically in the case of U.K. where a low level of TFP and rental price for human capital results in a flatter earnings profile and less growth in the variance of log earning.

\(^{107}\)See table 20 for the calibrated TFP levels.
These changes in inequality is driven by a smaller share of college graduation in the U.K. as reported in the last row of panel B in table 22. Similar thing happens in the Netherlands and France.

Accounting for differences in aggregate TFP alone is not of first order importance to reconcile the model with data in terms of GDP and average annual hours worked per adult. When TFP levels are country-specific and model matches the observed differences in GDP per adult across the U.S. and Europe, model cannot generate the observed inequality patterns. Only when differences in taxation and higher education subsidies/transfers are considered, TFP differences are amplified enough. This means that although TFP plays an important role for aggregate variables (GDP, hours worked, etc), it is not of first order importance for micro-level differences (life-cycle earnings inequality) across these countries.

Gdp per Adult and Aggregate Hours Worked

Lastly, I document the performance of the model (with country-specific taxation and college subsidies/transfers) regarding GDP per worker and average annual hours worker per worker relative to the data.

Table 19 shows GDP and hours worked per adult in 2005 in European countries relative to the U.S. from Penn World Table.\textsuperscript{108} Netherlands has the highest GDP per adult, while France has the lowest annual hours worked among European countries. In the model, GDP per adult is simply output since the population size is one. Hours

\textsuperscript{108}The reason I chose GDP per adult in the data not GDP per worker is that I do not want to pick up the effect of differences in labor force participation to be in the data. When I do a robustness check and choose GDP per worker in the data, the qualitative conclusion remains the same, and quantitative conclusion changes very slightly. See Appendix C.2 for more details.
worked is non-leisure time which includes both labor supply and time investment in human capital production.

Figure 32 shows that the model with fixed TFP but different taxation and college subsidies/transfers is doing a good job producing differences in GDP per worker relative to the data except for the U.K. With fixed TFP level and prices across these countries, U.K. looks somehow similar to the U.S. while the model generates less output in the case of the Netherlands and France.

A similar story about the hours worked in figure 33 is true, with the U.K. looking similar to the U.S. in the model and hours worked in the model for the Netherlands and France are less than data. The two figures show that even though a combination of taxation differences and college subsidies/transfers are important to generate micro level inequality in labor earnings, the aggregate variables are still a bit far from the data.

Finally, in terms of aggregate hours, lower TFP level in the U.K. reduces aggregate hours worked, which deteriorates the fit of the model for the U.K. a bit. This is depicted in figure 34. Average hours worked in the Netherlands and France goes down as well. Overall, the model on average can account for 85% of the differences in hours across U.S. and Europe. This version of the model accounts for all the differences in GDP per worker across U.S. and Europe as well as most of the differences in aggregate labor hours. These results are in line with other studies that connect aggregate hours worked with differences in tax rates such as Prescott (2004); Ohanian et al. (2008); Rogerson (2008); Bick et al. (2018).
3.9 Final Remarks

Overall the model account for 94% of the mean earnings and 80% of the variance profiles observed in the data. There are other differences in terms of taxation across these countries, namely consumption and capital income (wealth taxes). For future work, I will consider them as well so that a more comprehensive taxation differences across these countries is explored. These additional taxation differences may matter especially the consumption taxes which are high in Europe and matter for labor supply decision.

There are potentially other factors that can differ across countries that are left unchanged. One is the distribution of shocks. The shock process is assumed to be similar across individuals. This may not be true in the data for the U.S. and deserve more treatment. The propagation of shocks in the model is through human capital decisions, which are foundational for earnings inequality. Different shock processes change the decisions for college attendance and life-cycle human capital growth if the variance of the innovation to the human capital stock differs across education groups. This channel will be important to account for the cross-country facts since the shock process across countries may differ as well.

Another channel is the distribution of initial conditions, namely learning ability, initial human capital and disutility for college. For future work, I will calibrate these distributions to match each country’s mean earnings and variance profiles in the presence of taxation and college subsidies/transfers differences. I then study how these distributions differ from one another and what they imply about human capital formation earlier in the life-cycle, and other labor market conditions that are summarized in these distributions. For instance, if France has more unions
which reduces the dispersion of earnings over the life-cycle, then I may find that the
distribution of learning ability has a lower mean and less dispersion in France. This
does not say anything about the innate ability of French people relative to Americans.
Learning ability is a model object in a specific setup. It potentially summarizes other
differences in the labor market besides taxation and college subsidies/transfers that
are important for life-cycle earnings inequality.

Table 8. Summary statistics for college graduation and expenditure

<table>
<thead>
<tr>
<th>Country</th>
<th>(1)</th>
<th>(2)</th>
<th>(3)</th>
<th>(4)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>College share</td>
<td>College expenditure in GDP</td>
<td>Share of public expenditure in college expenditure</td>
<td>Share of college expenditure in government revenues</td>
</tr>
<tr>
<td>United States</td>
<td>35</td>
<td>2.6</td>
<td>38.6</td>
<td>2.9</td>
</tr>
<tr>
<td>United Kingdom</td>
<td>33</td>
<td>1.7</td>
<td>40.5</td>
<td>2.0</td>
</tr>
<tr>
<td>Netherlands</td>
<td>28</td>
<td>1.2</td>
<td>69.2</td>
<td>2.9</td>
</tr>
<tr>
<td>France</td>
<td>25</td>
<td>1.2</td>
<td>79.7</td>
<td>2.3</td>
</tr>
</tbody>
</table>

Note: All the numbers are in percentage. Government revenues include federal, local, and state government revenues. Any expenditure in vocational and training schools as well as community colleges that do not lead to a bachelor’s degree are excluded from college expenditure. Source: OECD (2016).

Table 9. Earnings and wealth distribution

Panel A: Earnings

<table>
<thead>
<tr>
<th></th>
<th>1st</th>
<th>2nd</th>
<th>3rd</th>
<th>4th</th>
<th>5th</th>
<th>Gini coef.</th>
<th>Top 5%</th>
<th>Top 1%</th>
</tr>
</thead>
<tbody>
<tr>
<td>Data</td>
<td>2.6</td>
<td>7.0</td>
<td>12.6</td>
<td>21.3</td>
<td>56.5</td>
<td>0.4</td>
<td>27.2</td>
<td>10.7</td>
</tr>
<tr>
<td>Model</td>
<td>2.4</td>
<td>6.8</td>
<td>11.0</td>
<td>21.0</td>
<td>58.8</td>
<td>0.4</td>
<td>28.4</td>
<td>11.5</td>
</tr>
</tbody>
</table>

Panel B: Wealth

<table>
<thead>
<tr>
<th></th>
<th>1st</th>
<th>2nd</th>
<th>3rd</th>
<th>4th</th>
<th>5th</th>
<th>Gini coef.</th>
<th>Top 5%</th>
<th>Top 1%</th>
</tr>
</thead>
<tbody>
<tr>
<td>Data</td>
<td>-0.5</td>
<td>0.9</td>
<td>3.9</td>
<td>10.7</td>
<td>85</td>
<td>0.8</td>
<td>60.4</td>
<td>34.1</td>
</tr>
<tr>
<td>Model</td>
<td>-0.8</td>
<td>0.6</td>
<td>3.2</td>
<td>8.2</td>
<td>88.8</td>
<td>0.8</td>
<td>47.0</td>
<td>11.7</td>
</tr>
</tbody>
</table>

Note: Quintiles of wealth and earnings distribution are reported. Data for earnings distribution are from the CPS and author’s calculations, averaged over 2004-2016. Data for wealth distribution are from Kuhn and Rios-Rull (2016) for the same period.
Table 10. Earnings distribution for college and non-college individuals

Panel A: Non-college

<table>
<thead>
<tr>
<th></th>
<th>1st</th>
<th>2nd</th>
<th>3rd</th>
<th>4th</th>
<th>5th</th>
<th>Gini coef.</th>
<th>Top 5%</th>
<th>Top 1%</th>
</tr>
</thead>
<tbody>
<tr>
<td>Data</td>
<td>3.1</td>
<td>8.1</td>
<td>14.3</td>
<td>23.1</td>
<td>51.4</td>
<td>0.42</td>
<td>22.0</td>
<td>8.1</td>
</tr>
<tr>
<td>Model</td>
<td>2.9</td>
<td>7.9</td>
<td>13.5</td>
<td>24.2</td>
<td>51.5</td>
<td>0.41</td>
<td>21.1</td>
<td>8.8</td>
</tr>
</tbody>
</table>

Panel B: College

<table>
<thead>
<tr>
<th></th>
<th>1st</th>
<th>2nd</th>
<th>3rd</th>
<th>4th</th>
<th>5th</th>
<th>Gini coef.</th>
<th>Top 5%</th>
<th>Top 1%</th>
</tr>
</thead>
<tbody>
<tr>
<td>Data</td>
<td>2.4</td>
<td>7.0</td>
<td>13.0</td>
<td>21.5</td>
<td>56.1</td>
<td>0.38</td>
<td>27.1</td>
<td>9.0</td>
</tr>
<tr>
<td>Model</td>
<td>1.8</td>
<td>6.8</td>
<td>12.4</td>
<td>23.3</td>
<td>55.7</td>
<td>0.38</td>
<td>26.8</td>
<td>9.4</td>
</tr>
</tbody>
</table>

Note: Quintiles of earnings distribution are reported. Data are from the CPS and author’s calculations, averaged over 2004-2016.

Table 11. Country-specific labor earnings tax parameters

<table>
<thead>
<tr>
<th></th>
<th>United States</th>
<th>United Kingdom</th>
<th>Netherlands</th>
<th>France</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\lambda$</td>
<td>0.83</td>
<td>0.83</td>
<td>0.78</td>
<td>0.76</td>
</tr>
<tr>
<td></td>
<td>(0.0003)</td>
<td>(0.0015)</td>
<td>(0.0032)</td>
<td>(0.0010)</td>
</tr>
<tr>
<td>$\tau$</td>
<td>0.11</td>
<td>0.13</td>
<td>0.16</td>
<td>0.20</td>
</tr>
<tr>
<td></td>
<td>(0.0002)</td>
<td>(0.0012)</td>
<td>(0.0019)</td>
<td>(0.0004)</td>
</tr>
</tbody>
</table>

Note: The standard errors are bootstrapped by 10% resampling for 1000 times, and are reported in parentheses.

Table 12. Selection for college for different progressive tax systems

<table>
<thead>
<tr>
<th></th>
<th>$\tau = 0.13$</th>
<th>$\tau = 0.15$</th>
<th>$\tau = 0.17$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Share of college (%)</td>
<td>35</td>
<td>31</td>
<td>27</td>
</tr>
<tr>
<td>Expend. for college in GDP (%)</td>
<td>2.6</td>
<td>2.0</td>
<td>1.3</td>
</tr>
</tbody>
</table>
Table 13. Inequality profiles under different policies

<table>
<thead>
<tr>
<th>Policy</th>
<th>Mean Earnings</th>
<th>Variance of Log Earnings</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>U.K.</td>
<td>Netherlands</td>
</tr>
<tr>
<td>taxation</td>
<td>0.98</td>
<td>0.93</td>
</tr>
<tr>
<td>college sub/tran</td>
<td>0.95</td>
<td>1.21</td>
</tr>
<tr>
<td>taxation + college sub/tran</td>
<td>0.91</td>
<td>0.96</td>
</tr>
</tbody>
</table>

Note: The figures for mean earnings show the share of growth in earnings between age 25 to 50 that is accounted by the model relative to data. For example, 0.93 in the first row for the Netherlands mean that 93% of the growth in mean earnings between ages 25 and 50 that is observed in the data is generated by the model. The figures for the variance show the share of overall growth in variance of log earnings that is generated within the model relative to data.

Table 14. College selection under different policies

<table>
<thead>
<tr>
<th>Policy</th>
<th>Share of College Graduates</th>
<th>Expenditure in GDP</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>U.K.</td>
<td>Netherlands</td>
</tr>
<tr>
<td>taxation</td>
<td>0.98</td>
<td>0.75</td>
</tr>
<tr>
<td>college sub/tran</td>
<td>0.95</td>
<td>1.45</td>
</tr>
<tr>
<td>taxation + college sub/tran</td>
<td>0.92</td>
<td>0.89</td>
</tr>
</tbody>
</table>

Note: The figures for both the share of college graduates and college expenditure in GDP are those statistics generated in the model relative to data. The data is reported in Table 8.
Figure 5. Mean earnings, individuals with and without a college degree, earnings are normalized to 1 at age 25.

Figure 6. Mean earnings, individuals without a college degree, earnings are normalized to 1 at age 25.
Figure 7. Mean earnings, individuals with a college degree, earnings are normalized to 1 at age 25.

Figure 8. Variance of log earnings, individuals with and without a college degree.
Figure 9. Variance of log earnings, individuals without a college degree.

Figure 10. Variance of log earnings, individuals with a college degree.
Figure 11. Gini coefficient, individuals with and without a college degree.

Figure 12. Marginal labor earnings tax rates based on multiples of mean cross-sectional income.
Figure 13. The life-cycle path of an individual in the model
Notes: $j_h$ is the age of high school graduation, $j_w$ is the age when individual start working, $j_r$ is retirement age, and $j_d$ is age of death.

Figure 14. Mean earnings, individuals with and without a college degree, not targeted for calibration
Figure 15. Variance of log earnings, individuals with and without a college degree, not targeted for calibration

Figure 16. Hours worked over the life-cycle; data vs model (all individuals). Hours worked in the model is non-leisure time which includes labor supply and time investment for human capital accumulation.
Figure 17. Skill premium, which is defined as the mean earnings of individuals with a college degree to those without one, not targeted for calibration.

Figure 18. Mean earnings, individuals with and without a college degree, different progressiveness of the tax system.
Figure 19. Variance of log earnings, individuals with and without a college degree, different progressiveness of the tax system

3.10 Additional Figures

Figure 20. gini coefficient, individuals without a college degree.
Figure 21. Gini coefficient, individuals with a college degree.

Figure 22. Mean earnings, individuals without a college degree, targeted for calibration.
Figure 23. Mean earnings, individuals with a college degree, targeted for calibration

Figure 24. Variance of log earnings, individuals without a college degree, targeted for calibration
Figure 25. Variance of log earnings, individuals with a college degree, targeted for calibration

Figure 26. Hours worked over the life-cycle; data vs model (non-college individuals). Hours worked in the model is non-leisure time which includes labor supply and time investment for human capital accumulation.
Figure 27. Hours worked over the life-cycle; data vs model (college individuals). Hours worked in the model is non-leisure time which includes labor supply and time investment for human capital accumulation.

Figure 28. Mean earnings, individuals without a college degree, different progressiveness of the tax system
Figure 29. Mean earnings, individuals with a college degree, different progressiveness of the tax system

Figure 30. Variance of log earnings, individuals without a college degree, different progressiveness of the tax system
Figure 31. Variance of log earnings, individuals with a college degree, different progressiveness of the tax system

Figure 32. GDP per worker (output) in the model (taxation + college subs/trans) vs. GDP per adult in the data.

Model and data for the U.S. are normalized to one. Differences in taxation and college subsidies/transfers are present. TFP level is normalized to 1 in all countries. Source: Feenstra et al. (2015a).
Figure 33. Hours (non-leisure time) per worker in the model (taxation + college subs/trans) vs. average annual hours worked per adult in the data.

Model and data for the U.S. are normalized to one. Differences in taxation and college subsidies/transfers are present. TFP level is normalized to 1 in all countries. Source: Feenstra et al. (2015a).

Figure 34. Hours (non-leisure time) per worker in the model (taxation + college subs/trans + TFP) vs. average annual hours worked per adult in the data.

Model and data for the U.S. are normalized to one. TFP levels are country-specific. Source: Feenstra et al. (2015a).
### Table 15. All parameter values

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Technology</strong></td>
<td></td>
</tr>
<tr>
<td>$\alpha$</td>
<td>$1/3$</td>
</tr>
<tr>
<td>$A$</td>
<td>$1.00$</td>
</tr>
<tr>
<td><strong>Human capital</strong></td>
<td></td>
</tr>
<tr>
<td>$\phi$</td>
<td>$0.55$</td>
</tr>
<tr>
<td>$\nu$</td>
<td>$0.16$</td>
</tr>
<tr>
<td>$\delta_h$</td>
<td>$0.01$</td>
</tr>
<tr>
<td>$g_d$</td>
<td>$0.29$</td>
</tr>
<tr>
<td>$\bar{d}$</td>
<td>$2358.4$</td>
</tr>
<tr>
<td><strong>Utility</strong></td>
<td></td>
</tr>
<tr>
<td>$\beta$</td>
<td>$0.985$</td>
</tr>
<tr>
<td>$B$</td>
<td>$12.40$</td>
</tr>
<tr>
<td>$\gamma$</td>
<td>$0.61$</td>
</tr>
<tr>
<td><strong>Shocks</strong></td>
<td></td>
</tr>
<tr>
<td>$z \sim N(\mu_z, \sigma_z^2)$</td>
<td>$(0,0.111)$</td>
</tr>
<tr>
<td><strong>Initial conditions</strong></td>
<td></td>
</tr>
<tr>
<td>$\lambda$</td>
<td>$160$</td>
</tr>
<tr>
<td>$(\mu_a, \sigma_a^2, \lambda_a)$</td>
<td>$(0.80,0.20,10.00)$</td>
</tr>
<tr>
<td>$(\beta_0, \beta_1, \sigma_\epsilon^2)$</td>
<td>$(5.20,0.15,0.22)$</td>
</tr>
</tbody>
</table>

Note: The consumption and capital income tax rates come from McDaniel (2011). The parameters for the progressive system are estimated.

### Table 16. Parameters for the tax system

<table>
<thead>
<tr>
<th>Tax source</th>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Consumption</td>
<td>$\tau_c$</td>
<td>$0.10$</td>
</tr>
<tr>
<td>Assets</td>
<td>$\tau_k$</td>
<td>$0.20$</td>
</tr>
<tr>
<td>Progressive</td>
<td>$(\lambda, \tau)$</td>
<td>$(0.83,0.11)$</td>
</tr>
</tbody>
</table>

Note: The consumption and capital income tax rates come from McDaniel (2011). The parameters for the progressive system are estimated.
Table 17. Average sample size of 5-year age bins for each country over the period 2004-2016

<table>
<thead>
<tr>
<th>Country</th>
<th>Size</th>
</tr>
</thead>
<tbody>
<tr>
<td>United States</td>
<td>4425</td>
</tr>
<tr>
<td>United Kingdom</td>
<td>505</td>
</tr>
<tr>
<td>Netherlands</td>
<td>458</td>
</tr>
<tr>
<td>France</td>
<td>460</td>
</tr>
</tbody>
</table>

Table 18. Gini coefficient of labor earnings for the whole cross-section, model vs data

<table>
<thead>
<tr>
<th>Country</th>
<th>Data</th>
<th>Model</th>
</tr>
</thead>
<tbody>
<tr>
<td>United States</td>
<td>0.39</td>
<td>0.39</td>
</tr>
<tr>
<td>United Kingdom</td>
<td>0.35</td>
<td>0.33</td>
</tr>
<tr>
<td>Netherlands</td>
<td>0.28</td>
<td>0.25</td>
</tr>
<tr>
<td>France</td>
<td>0.28</td>
<td>0.23</td>
</tr>
</tbody>
</table>

Note: Gini coefficients in the data are calculated for each year and averaged over the period 2005-2016. Model generated data are generated when differences in taxation and college subsidies/transfers are both considered.

Table 19. Cross-country aggregate variables

<table>
<thead>
<tr>
<th>Country</th>
<th>GDP per adult</th>
<th>Hours worked per adult</th>
</tr>
</thead>
<tbody>
<tr>
<td>United States</td>
<td>1.00</td>
<td>1.00</td>
</tr>
<tr>
<td>United Kingdom</td>
<td>0.75</td>
<td>0.97</td>
</tr>
<tr>
<td>Netherlands</td>
<td>0.86</td>
<td>0.83</td>
</tr>
<tr>
<td>France</td>
<td>0.68</td>
<td>0.77</td>
</tr>
</tbody>
</table>

Note: All figures are for 2005, and the U.S. is normalized to one. Source: Feenstra et al. (2015a).
Table 20. Country-specific TFP level

<table>
<thead>
<tr>
<th>Country</th>
<th>TFP only counterfactual</th>
<th>TFP with taxation and college subs/trnas</th>
</tr>
</thead>
<tbody>
<tr>
<td>United States</td>
<td>1.00</td>
<td>1.00</td>
</tr>
<tr>
<td>United Kingdom</td>
<td>0.71</td>
<td>0.85</td>
</tr>
<tr>
<td>Netherlands</td>
<td>0.81</td>
<td>0.94</td>
</tr>
<tr>
<td>France</td>
<td>0.77</td>
<td>0.92</td>
</tr>
</tbody>
</table>

Note: U.S. is normalized to one.

Table 21. The role of TFP for inequality profiles

<table>
<thead>
<tr>
<th>Policy</th>
<th>Mean Earnings</th>
<th>Variance of Log Earnings</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>U.K.</td>
<td>Netherlands</td>
</tr>
<tr>
<td>TFP</td>
<td>0.77</td>
<td>1.13</td>
</tr>
<tr>
<td>taxation + college sub/tran + TFP</td>
<td>0.84</td>
<td>0.95</td>
</tr>
</tbody>
</table>

Note: The figures for mean earnings show the share of growth in earnings between age 25 to 50 that is accounted by the model relative to data. The figures for the variance show the share of overall growth in variance of log earnings that is generated within the model relative to data.

Table 22. The role of TFP for college selection

<table>
<thead>
<tr>
<th>Policy</th>
<th>Share of College Graduates</th>
<th>Expenditure in GDP</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>U.K.</td>
<td>Netherlands</td>
</tr>
<tr>
<td>TFP</td>
<td>0.72</td>
<td>0.92</td>
</tr>
<tr>
<td>taxation + college sub/tran + TFP</td>
<td>0.82</td>
<td>0.81</td>
</tr>
</tbody>
</table>

Note: The figures for both the share of college graduates and college expenditure in GDP are model generated relative to data. The data is reported in Table 8.


Byrne, D., J. Fernald, and M. B. Reinsdorf (2016). Does the united states have a productivity slowdown or a measurement problem? *Brookings Papers on Economic Activity* 47(1 (Spring)), 109–182.


NCES (2020). National center for education statistics (nces) home page, part of the u.s. department of education.


APPENDIX A

INVESTMENT IN SKILLS, MANAGERIAL QUALITY, AND ECONOMIC DEVELOPMENT
Appendix 1

**Proof of Proposition 1.** First, I solve the maximization problem of a manager with managerial talent $z$.

$$\max_{\{k,u,s\}} A^z A^{z-\gamma} k^{\alpha} (u^{\theta} s^{1-\theta})^{1-\alpha} - Rk - W_u u - W_s s \tag{A.1}$$

The optimality conditions for input factors are:

- **Capital**
  
  $$(\alpha \gamma) A^z A^{z-\gamma} k^{\alpha \gamma - 1} u^{\theta(1-\alpha)\gamma} s^{(1-\theta)(1-\alpha)\gamma} = R$$

- **Unskilled labor**
  
  $$\theta(1-\alpha) \gamma A^z A^{z-\gamma} k^{\alpha \gamma} u^{\theta(1-\alpha)\gamma - 1} s^{(1-\theta)(1-\alpha)\gamma} = W_u \tag{A.2}$$

- **Skilled labor**
  
  $$(1-\theta)(1-\alpha) \gamma A^z A^{z-\gamma} k^{\alpha \gamma} u^{\theta(1-\alpha)\gamma - 1} s^{(1-\theta)(1-\alpha)\gamma - 1} = W_s$$

Solving the above system of equations gives the following factor demand and profit functions:

- **Capital**
  
  $$k(z,W_u,W_s,R) = z A^{\frac{1}{1-\gamma}} \left( \frac{\alpha \gamma}{R} \right)^{\frac{1-\gamma}{1-\gamma}} \left( \frac{(1-\alpha)\theta \gamma}{W_u} \right)^{\frac{(1-\alpha)(1-\theta)\gamma}{W_s}} \tag{A.3}$$

- **Unskilled labor**
  
  $$u(z,W_u,W_s,R) = z A^{\frac{1}{1-\gamma}} \left( \frac{\alpha \gamma}{R} \right)^{\frac{1-\gamma}{1-\gamma}} \left( \frac{\theta(1-\alpha)\gamma}{W_u} \right)^{\frac{(1-\gamma)(1-\theta)(1-\alpha)\gamma}{W_s}} \tag{A.4}$$

- **Skilled labor**
  
  $$s(z,W_u,W_s,R) = z A^{\frac{1}{1-\gamma}} \left( \frac{\alpha \gamma}{R} \right)^{\frac{1-\gamma}{1-\gamma}} \left( \frac{\theta(1-\alpha)\gamma}{W_u} \right)^{\frac{(1-\alpha)(1-\theta)\gamma}{W_s}} \tag{A.5}$$

- **Profit**
  
  $$\pi(z,W_u,W_s,R) = (1-\gamma) g(z,W_u,W_s,R) \tag{A.6}$$

Based on the assumptions about the production technology parameters, all the demand and profit functions are strictly increasing in managerial talent and strictly decreasing in factor prices. The second part of the proposition is straightforward given that the above functions are linear in managerial talent $z$. ■
The Lagrangian for the household problem is:

\[
\mathcal{L} = \sum_{t=0}^{\infty} \beta^t \left\{ L_t \ln \left( \frac{C_t}{L_t} \right) + \mu_t \left[ w_{u,t} \left( U_{t-1} + N_t \int_0^{a_t} \int_0^{z_t} a f(a, z) \text{d}a \text{d}z \right) + \\
 w_{s,t} \left( S_{t-1} + N_{t-1} h_t \int_0^{a_t} \int_0^{z_t} a f(a, z) \text{d}a \text{d}z \right) + \\
 \Pi^u_{t-1} + N_t \int_{z_t}^{z_t} \int_0^{a_t} z \pi(W_{u,t}, W_{s,t}, R_t) f(a, z) \text{d}a \text{d}z + \right. \\
 \left. \Pi^s_{t-1} + N_{t-1} h_t \int_{z_t}^{z_t} \int_0^{a_t} z \pi(W_{u,t}, W_{s,t}, R_t) f(a, z) \text{d}a \text{d}z + \\
 r_t K_t + (1 - \delta) K_{t+1} - C_t - N_t x_t \int_0^{a_t} \int_0^{z_t} a f(a, z) \text{d}a \text{d}z \right\}
\]

\[\text{(A.7)}\]

The first order conditions with respect to \( \{C_t, K_{t+1}, x_t, \hat{a}_t, z^u_t, z^s_t\} \) are equations (1.14), (1.15), (1.16), (1.17) and (1.19) with \( \mu_t = \frac{1}{L_t} \). The market clearing conditions and aggregate feasibility are\(^{109}\):

- **Unskilled labor market**
  \[
  \frac{U}{L} = \int_0^{a_t} \int_{z^u_t}^{z_t} z u(W_{u}, W_{s}, R) f(a, z) \text{d}a \text{d}z + (Bx^0) \int_{a_t}^{a_t} \int_{z^u_t}^{z_t} z u(W_{u}, W_{s}, R) f(a, z) \text{d}a \text{d}z
  \]
  \[\text{(A.8)}\]

- **Skilled labor market**
  \[
  \frac{S}{L} = \int_0^{a_t} \int_{z^u_t}^{z_t} z s(W_{u}, W_{s}, R) f(a, z) \text{d}a \text{d}z + (Bx^0) \int_{a_t}^{a_t} \int_{z^u_t}^{z_t} z s(W_{u}, W_{s}, R) f(a, z) \text{d}a \text{d}z
  \]
  \[\text{(A.9)}\]

- **Capital Market**
  \[
  \frac{K}{L} = \int_0^{a_t} \int_{z^u_t}^{z_t} z k(W_{u}, W_{s}, R) f(a, z) \text{d}a \text{d}z + (Bx^0) \int_{a_t}^{a_t} \int_{z^u_t}^{z_t} z k(W_{u}, W_{s}, R) f(a, z) \text{d}a \text{d}z
  \]
  \[\text{(A.10)}\]

- **Aggregate feasibility**
  \[
  \frac{C}{L} + \delta \frac{K}{L} = \frac{g_L}{1 + g_L} \int_{a_t}^{a_t} \int_0^{z_t} f(a, z) \text{d}a \text{d}z = \frac{Y}{L}
  \]
  \[\text{(A.11)}\]

\(^{109}\)I omitted the market clearing condition for the goods market since it will clear by Walras’ law.
From the laws of motion for the stocks of unskilled and skilled labor, we can further simplify the market clearing conditions in the labor market:

\[
\frac{U}{L} \equiv \bar{u} = \int_0^\hat{a} \int_{z_u}^{z_u} af(a, z) \, da \, dz = \\
\int_0^\hat{a} \int_{z_u}^{z_u} zu(W_u, W_s, R) f(a, z) \, da \, dz + (Bx^\phi) \int_\hat{a}^\hat{a} \int_{z_s}^{z_s} zu(W_u, W_s, R) f(a, z) \, da \, dz
\]

(A.12)

\[
\frac{S}{L} \equiv \bar{s} = \frac{Bx^\phi}{1 + g_L} \int_0^\hat{a} \int_{z_s}^{z_s} af(a, z) \, da \, dz = \\
\int_0^\hat{a} \int_{z_u}^{z_u} zs(W_u, W_s, R) f(a, z) \, da \, dz + (Bx^\phi) \int_\hat{a}^\hat{a} \int_{z_s}^{z_s} zs(W_u, W_s, R) f(a, z) \, da \, dz
\]

(A.13)

Equations (A.12) and (A.13) can be used to refine the initial guess for \((W_u, W_s)\) pair. Equation (A.10) gives the capital per worker while equation (A.11) can be used to calculate consumption per worker. Aggregate production is calculated based on the equilibrium production of managers:

\[
\frac{Y}{L} = \int_0^\hat{a} \int_{z_u}^{z_u} y(W_u, W_s, R) f(a, z) \, da \, dz + (Bx^\phi) \int_\hat{a}^\hat{a} \int_{z_s}^{z_s} y(W_u, W_s, R) f(a, z) \, da \, dz
\]

(A.14)

where \(y(W_u, W_s, R)\) is the production per managerial talent of a manager.
Appendix 3

The problem of a manager with managerial talent \( z \) facing size distortions is as follows:

\[
\max_{\{k,u,s\}} \nu A^{1-\tau} (1-\gamma)(1-\tau) \left( k^{1-\theta} s^{1-\gamma} \right)^{1/(1-\tau)} - Rk - W_u u - W_s s \tag{A.15}
\]

The optimality conditions for input factors are:

- **Capital**
  \[
  (\alpha \gamma (1 - \tau)) \nu A^{1-\tau} z^{(1-\gamma)(1-\tau)} k^{(\alpha \gamma)(1-\tau)} (1-\gamma)(1-\tau) - R = 0
  \]

- **Unskilled labor**
  \[
  \theta (1 - \alpha) \gamma (1 - \tau) \nu A^{1-\tau} z^{(1-\gamma)(1-\tau)} k^{(\alpha \gamma)(1-\tau)} u^{\theta(1-\alpha)\gamma(1-\tau)} s^{(1-\theta)(1-\alpha)\gamma(1-\tau)} = W_u
  \tag{A.16}
  \]

- **Skilled labor**
  \[
  (1-\theta)(1-\alpha) \gamma (1-\tau) \nu A^{1-\tau} z^{(1-\gamma)(1-\tau)} k^{\alpha \gamma(1-\tau)} u^{\theta(1-\alpha)\gamma(1-\tau)} s^{(1-\theta)(1-\alpha)\gamma(1-\tau) - 1} = W_s
  \]

Solving the above system of equations gives the following factor demand and profit functions:

- **Capital**
  \[
  k(z, W_u, W_s, R) = \left( (1 - \tau) \nu A^{1-\tau} z^{(1-\gamma)(1-\tau)} \right)^{\frac{1}{1-\gamma(1-\tau)}}
  \]
  \[
  \left( \frac{(1 - \theta)(1 - \alpha) \gamma (1 - \tau)}{W_s} \right)^{\frac{(1-\theta)(1-\alpha)\gamma(1-\tau)}{1-\gamma(1-\tau)}}
  \]

- **Unskilled labor**
  \[
  u(z, W_u, W_s, R) = \left( (1 - \tau) \nu A^{1-\tau} z^{(1-\gamma)(1-\tau)} \right)^{\frac{1}{1-\gamma(1-\tau)}}
  \]
  \[
  \left( \frac{(1 - \theta)(1 - \alpha) \gamma (1 - \tau)}{W_u} \right)^{\frac{(1-\theta)(1-\alpha)\gamma(1-\tau)}{1-\gamma(1-\tau)}}
  \]

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• Skilled labor

\[ s(z, W_u, W_s, R) = \left( (1 - \tau)A^{1-\tau}z^{(1-\gamma)(1-\tau)} \right)^{1-\gamma(1-\tau)} \]

\[
\left( \frac{\alpha \gamma}{R} \right)^{\frac{\alpha \gamma (1-\tau)}{1-\gamma(1-\tau)}} \left( \frac{\theta (1-\alpha) \gamma}{W_u} \right)^{\frac{1-\gamma(1-\tau)(1-\theta(1-\alpha))}{1-\gamma(1-\tau)}} \\
\left( \frac{(1-\theta)(1-\alpha) \gamma (1-\tau)}{W_s} \right)^{\frac{1-\gamma(1-\tau)(1-\theta(1-\alpha))}{1-\gamma(1-\tau)}}
\]

• Profit

\[ \pi(z, W_u, W_s, R) = (1 - \gamma)g(z, W_u, W_s, R) \]  (A.20)
Appendix 4

In this appendix, I explain in more details the data source and methods used to identify moments for calibration.

American Community Survey

The American Community Survey (ACS) is an ongoing survey that provides vital information on a yearly basis about our nation and its people. Information from the survey generates data that help determine how more than 675 billion dollars in federal and state funds are distributed each year. Through the ACS, we know more about jobs and occupations, educational attainment, veterans, whether people own or rent their home, and other topics. Public officials, planners, and entrepreneurs use this information to assess the past and plan the future. When you respond to the ACS, you are doing your part to help your community plan hospitals and schools, support school lunch programs, improve emergency services, build bridges, and inform businesses looking to add jobs and expand to new markets, and more.

One important fact to remember about the ACS is that the forms are not mailed to specific people, but rather to specific addresses. The sample is designed to ensure good geographic coverage and does not target individuals. By focusing on quality geographic coverage, the ACS can produce a good picture of the community’s people and housing by surveying a representative sample of the population. The Census Bureau selects a random sample of addresses to be included in the ACS. Each address has about a 1-in-480 chance of being selected in a month, and no address should be selected more than once every 5 years. The Census Bureau mails questionnaires to approximately 295,000 addresses a month across the United States. This is a small number of households considering there are more than 180 million addresses in the United States and an address that receives ACS instructions will not likely find a neighbor or friend who has also received them.

The Census Bureau informs people living at an address that they have been selected to participate in the ACS. Shortly thereafter (for most U.S. addresses), instructions for completing the survey online are mailed. In Puerto Rico and some hard to reach areas in the U.S., only a paper questionnaire is mailed. Households are asked to complete the survey online or to mail the completed paper questionnaire back to the Census Bureau’s National Processing Center in Jeffersonville, Indiana. If the Census Bureau does not receive a completed survey within a few weeks, it will mail an additional paper survey questionnaire.
Data Cleaning and Variable Selection

The process of cleaning the data and variables that are used in the regressions are outlined in this section.

- **Year**: The sample years are 2014-16. The reason for the small selection of years is that I do want to pick up as little as possible any employment effect resulting from the Great Recession. Hours of work, occupational standings, wages and worker composition may differ very much form year to year close to the recession and although I control the year effects with year-dummies, anything related to the business cycle is not relevant for my purposes here. I do a robustness check and run the regression just for the year 2015 and results are basically the same.

- **Hours**: The workers report their hours worked in the reference week in the survey. Part-time workers are those who work less than 35 hours per week and full-time workers are those who work 35 hours and more. I exclude all the part-time workers.

- **Age**: I focus on prime age, namely from 40 to 54. The reason is that my model does not have any implications for the changes in earnings, hours and occupation over the life-cycle and most of these transitions have died out for the prime age workers.

- **Class of Worker**: The workers are categorized by the ACS based on the nature of their job as "works for wages" and "self employed". This category based on a survey question that asks about the type of job with the following categories: a private for-profit company or business, or of an individual, for wages, salary, or commissions, a private non-for-profit, tax-exempt, or charitable organization, a local government employee (city, county, etc.), a state government employee, a Federal government employee, self-employed in own not incorporated business, professional practice, or farm, self-employed in own incorporated business, professional practice, or farm, working without pay in family business or farm. The workers who are not in the last two categories (self-employed in own incorporated business and working without pay in family) are listed as "works for wages". The self-employed workers who work in their own incorporated business are categorized as "self employed".

- **Labor Earnings**: The labor earnings in the ACS consists of wages, salary, commission, tips, pay-in-kind, or piece rates for a private, for-profit employer or a private not-for-profit, tax-exempt or charitable organization. These include private and salary workers and government employees. Self-employed people whose business was incorporated are included with private wage and salary workers because they are paid employees of their own companies. For the "self-employed" workers, roughly two-third of their profit can be attributed to wage earnings based on a simple aggregate neoclassical production function. This adjustment excludes the earning from capital. None of the earnings for workers include asset earnings of any kind such as housing.
bonds and securities. All the earnings data were deflated by Consumer Price Index (CPI) from Bureau of Labor Statistics (BLS).\textsuperscript{110}

I will also follow a procedure to eliminate badly incomplete or highly implausible observations from the sample following Heathcote et al. (2010b) and Heathcote et al. (2014). Based on the hours worked and wage earnings, I can impute the hourly wage for the sample. I drop observations with positive income but zero annual hours. Also, those with hourly wage income less than half of the federal minimum wage.\textsuperscript{111} Finally I drop observations with more than $7 \times 12 = 84$ hours of work per week.

**Occupation:** I categorize my sample into workers and managers as follows. First, all the self employed people are considered managers. Also, the people who work in broad categories of "Management Occupations", which corresponds to the 2010 Census Occupation codes of 0010-0430, are considered managers as well. The rest are categorized as workers.

**Education:** The people in the sample are categorized based on educational level into two groups of skilled and unskilled. The unskilled people are those who finished at most the high school (grade 12). Any education more than high school is considered skilled. These include vocational and trainings, college dropout, college graduates and graduate level degrees among other forms of postsecondary education.

Regression Framework

I use a simple regression framework to estimate the relative earnings of skilled manager, unskilled managers, skilled workers and unskilled workers. Since the ACS data is not a panel, I cannot use the panel specifications. However, I can pool the ACS data for a couple of years, as repeated cross sections, and use year dummies to control for year fixed effects.

\[
\log(\text{labor earnings}) = \alpha + \beta X_{it} + \xi_t + \delta_1 \mathbb{1}\{\text{skilled manager}\} + \delta_2 \mathbb{1}\{\text{unskilled manager}\} + \delta_3 \mathbb{1}\{\text{skilled worker}\} + \epsilon_{it}.
\]

(A.21)

The vector $X_{it}$ contains individual-specific controls such as log of hours, age, age-squared, gender, race, marital status, veteran status and detailed occupational codes. The inclusion of a variable for years of education is redundant since the skilled vs unskilled classification controls partially for years of education. The variables $\xi_t$ are year fixed effects. I run the above regression under two specifications: One that

\textsuperscript{110}https://www.bls.gov/cpi/data.htm.

\textsuperscript{111}The federal hourly minimum wage is 7.5 $ in 2014 to 2016.
includes self employed people in the sample and one that excludes them and only focus on managers with the 2010 Census codes classification. My preferred specification is the one that includes self-employed people. As a robustness check, I run the regression for the 2015 cross section and the whole sample from 2014 to 2016. The results stays basically the same.

2015 Cross Section

The result of the above regression without the year dummy for the cross-section of 2015 is summarized in table (23).

Table 23. Relative Earnings of managers and skilled workers to unskilled workers (2015 cross section)

<table>
<thead>
<tr>
<th></th>
<th>skilled manager</th>
<th>unskilled manager</th>
<th>skilled worker</th>
</tr>
</thead>
<tbody>
<tr>
<td>with self-employed</td>
<td>1.91</td>
<td>1.07</td>
<td>1.61</td>
</tr>
<tr>
<td>without self-employed</td>
<td>2.22</td>
<td>1.40</td>
<td>1.55</td>
</tr>
</tbody>
</table>

The skilled workers and all managers have a premium over the unskilled workers in terms of earnings, no matter how I define earnings and occupation. Based on these premia, the other moments in the model are summarized in table (24). Given that the inclusion of self-employed people reduces the premia of all managers, it is reasonable that the skill premium and managerial premium increase in the absence of self-employed people as is the case in the first two columns of table (24). The share of managers is higher with the inclusion of self-employed people which is expected. The share of skilled managers among the pool of managers increases with the exclusion of self-employed people, suggesting that the majority of self-employed people are unskilled. Finally, the share of unskilled labor, which is the sum of unskilled worker and managers, changes very little.

Table 24. Moments for calibration under different specifications (2015 cross section)

<table>
<thead>
<tr>
<th></th>
<th>skill premium</th>
<th>managerial premium</th>
<th>manager share</th>
<th>skilled manager share</th>
<th>unskilled labor</th>
</tr>
</thead>
<tbody>
<tr>
<td>with self employed</td>
<td>1.70</td>
<td>1.14</td>
<td>0.17</td>
<td>0.77</td>
<td>0.40</td>
</tr>
<tr>
<td>without self employed</td>
<td>1.57</td>
<td>1.42</td>
<td>0.13</td>
<td>0.81</td>
<td>0.40</td>
</tr>
</tbody>
</table>
The above regression correctly gives the required moments. The only concern is that given the nature of a cross section, I cannot separate the effect of year fixed effect from the effect of age. Specifically, since I am only looking at prime age individuals, the effect of cohort is eliminated. What remains is the effect of the age of individuals, which I control for, and the effect of years. In order to see whether there is something special about the year 2015, I have to pool the ACS data from several years and run the regression (A.21) with year fixed effects and access the sensitivity of the results to the particular year of 2015. I present these results in the following section.

2014-2016 Pooled Cross Section

The following tables illustrate the results of running the regression (A.21) for the pooled cross section of ACS from 2014 to 2016 with year fixed effects. Comparing table (25) with table (23) reveals that the cross section and pooled data regressions result in fairly similar calculations. In other words, there is not much change in the cross section during 2014-2016. The same comparison is also true for tables (26) and (24).

Table 25. Relative Earnings of managers and skilled workers to unskilled workers (pooled data)

<table>
<thead>
<tr>
<th></th>
<th>skilled manager</th>
<th>unskilled manager</th>
<th>skilled worker</th>
</tr>
</thead>
<tbody>
<tr>
<td>with self-employed</td>
<td>1.92</td>
<td>1.07</td>
<td>1.62</td>
</tr>
<tr>
<td>without self-employed</td>
<td>2.33</td>
<td>1.44</td>
<td>1.62</td>
</tr>
</tbody>
</table>

Table 26. Moments for calibration under different specifications (pooled data)

<table>
<thead>
<tr>
<th></th>
<th>skill premium</th>
<th>managerial premium</th>
<th>manager share</th>
<th>skilled manager share</th>
<th>unskilled labor</th>
</tr>
</thead>
<tbody>
<tr>
<td>with self employed</td>
<td>1.71</td>
<td>1.14</td>
<td>0.17</td>
<td>0.77</td>
<td>0.40</td>
</tr>
<tr>
<td>without self employed</td>
<td>1.62</td>
<td>1.44</td>
<td>0.12</td>
<td>0.81</td>
<td>0.40</td>
</tr>
</tbody>
</table>
## Appendix 5

### Table 27. Parameter values

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Notation</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Discount factor</td>
<td>$\beta$</td>
<td>0.966</td>
</tr>
<tr>
<td>Population growth rate</td>
<td>$g_L$</td>
<td>0.009</td>
</tr>
<tr>
<td>Span-of-control parameter</td>
<td>$\gamma$</td>
<td>0.770</td>
</tr>
<tr>
<td>Capital Share</td>
<td>$\alpha$</td>
<td>0.43</td>
</tr>
<tr>
<td>Depreciation Rate</td>
<td>$\delta$</td>
<td>0.074</td>
</tr>
<tr>
<td>Unskilled labor share</td>
<td>$\theta$</td>
<td>0.287</td>
</tr>
<tr>
<td>Skill efficiency parameter</td>
<td>$B$</td>
<td>0.998</td>
</tr>
<tr>
<td>Skill curvature parameter</td>
<td>$\phi$</td>
<td>0.180</td>
</tr>
<tr>
<td>Schooling talent distribution</td>
<td>$\lambda_a$</td>
<td>9.724</td>
</tr>
<tr>
<td>Managerial talent distribution</td>
<td>$\lambda_z$</td>
<td>10.022</td>
</tr>
<tr>
<td>Correlation between talents</td>
<td>$\rho$</td>
<td>0.021</td>
</tr>
</tbody>
</table>

Note: The numbers are the values of the calibrated parameters for the benchmark economy. Discount factor and the depreciation rate are reported at the annual rate.

### Table 28. Empirical targets: model and data (U.S.)

<table>
<thead>
<tr>
<th>Statistic</th>
<th>Model</th>
<th>Data</th>
</tr>
</thead>
<tbody>
<tr>
<td>Capital-to-output ratio</td>
<td>0.80</td>
<td>0.80</td>
</tr>
<tr>
<td>Investment rate</td>
<td>0.27</td>
<td>0.27</td>
</tr>
<tr>
<td>Fraction of unskilled labor</td>
<td>0.40</td>
<td>0.40</td>
</tr>
<tr>
<td>Expenditure per tertiary student (% GDP per worker)</td>
<td>25</td>
<td>25</td>
</tr>
<tr>
<td>Managerial rate</td>
<td>0.17</td>
<td>0.17</td>
</tr>
<tr>
<td>Share of skilled managers</td>
<td>0.77</td>
<td>0.77</td>
</tr>
<tr>
<td>Skill premium</td>
<td>1.71</td>
<td>1.71</td>
</tr>
<tr>
<td>Managerial premium</td>
<td>1.14</td>
<td>1.14</td>
</tr>
<tr>
<td>Income of skilled manager to unskilled worker</td>
<td>2.20</td>
<td>2.10</td>
</tr>
<tr>
<td>Income of unskilled manager to skilled worker</td>
<td>0.67</td>
<td>0.69</td>
</tr>
</tbody>
</table>

Note: The numbers are the values for the U.S. statistics used as moment targets as the benchmark economy. Capital-to-output ratio and the investment rate are reported at the annual rate.
Table 29. Results of a 20% reduction in exogenous productivity on steady state equilibrium in the model

<table>
<thead>
<tr>
<th>Panel A</th>
<th>Model versus Facts in the Data</th>
<th>Productivity</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>A = 1</td>
</tr>
<tr>
<td>Fact 1</td>
<td>Skilled labor Share</td>
<td>100</td>
</tr>
<tr>
<td>Fact 1</td>
<td>Share of skilled managers (among managers)</td>
<td>100</td>
</tr>
<tr>
<td>Fact 2</td>
<td>Managerial premium</td>
<td>100</td>
</tr>
<tr>
<td>Fact 3</td>
<td>Managerial rate</td>
<td>100</td>
</tr>
<tr>
<td>Fact 4</td>
<td>Mean Size</td>
<td>100</td>
</tr>
<tr>
<td>Fact 5</td>
<td>Skill premium</td>
<td>100</td>
</tr>
</tbody>
</table>

Panel B  Output and Shares

<p>| | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Output per Capita</td>
<td>100</td>
<td>88</td>
</tr>
<tr>
<td>Unskilled Labor Share</td>
<td>100</td>
<td>106</td>
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<tr>
<td>Unskilled Manager Share</td>
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<tr>
<td>Skilled Labor Share</td>
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<td>92</td>
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</table>

Panel C  Quality and Size

<p>| | | |</p>
<table>
<thead>
<tr>
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<tbody>
<tr>
<td>Investment in skills</td>
<td>100</td>
<td>89</td>
</tr>
<tr>
<td>Average Quality of All Managers</td>
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<td>72</td>
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<tr>
<td>Average Quality of Skilled Managers</td>
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<td>88</td>
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<tr>
<td>Average Quality of Unskilled Managers</td>
<td>100</td>
<td>58</td>
</tr>
<tr>
<td>Relative Average Quality of Skilled to Unskilled Managers</td>
<td>100</td>
<td>156</td>
</tr>
<tr>
<td>Average Plant Size of Skilled Managers</td>
<td>100</td>
<td>99</td>
</tr>
<tr>
<td>Average Plant Size of Unskilled Managers</td>
<td>100</td>
<td>47</td>
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<tr>
<td>Relative Average Plant Size of Skilled to Unskilled Managers</td>
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</tr>
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</table>

**Note:** All the shares are reported as the relevant shares in the workforce, i.e. managers plus workers. I also indicate the shares that are calculated only among managers.
Table 30. Effects of size-dependent distortions on steady state equilibrium in the model

<table>
<thead>
<tr>
<th>Panel A</th>
<th>Model versus Facts in the Data</th>
<th>Distortions</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
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<td>$\tau = 0$</td>
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<tr>
<td>Fact 1</td>
<td>Skilled labor share</td>
<td>100</td>
</tr>
<tr>
<td>Fact 1</td>
<td>Share of Skilled Managers (among managers)</td>
<td>100</td>
</tr>
<tr>
<td>Fact 2</td>
<td>Managerial premium</td>
<td>100</td>
</tr>
<tr>
<td>Fact 3</td>
<td>Managerial rate</td>
<td>100</td>
</tr>
<tr>
<td>Fact 4</td>
<td>Mean Size</td>
<td>100</td>
</tr>
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<td>Fact 5</td>
<td>Skill premium</td>
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</table>

<table>
<thead>
<tr>
<th>Panel B</th>
<th>Output and Shares</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Output per Capita</td>
</tr>
<tr>
<td></td>
<td>Unskilled Manager Share</td>
</tr>
<tr>
<td></td>
<td>Skilled Labor Share</td>
</tr>
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</table>

<table>
<thead>
<tr>
<th>Panel C</th>
<th>Quality and Size</th>
</tr>
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<tbody>
<tr>
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<td>Investment in skills</td>
</tr>
<tr>
<td></td>
<td>Average Quality of All Managers</td>
</tr>
<tr>
<td></td>
<td>Average Quality of Skilled Managers</td>
</tr>
<tr>
<td></td>
<td>Average Quality of Unskilled Managers</td>
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<td></td>
<td>Relative Average Quality of Skilled to Unskilled Managers</td>
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<td>Average Plant Size of Skilled Managers</td>
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<tr>
<td></td>
<td>Relative Average Plant Size of Skilled to Unskilled Managers</td>
</tr>
</tbody>
</table>

Note: All the shares are reported as the relevant shares in the workforce, i.e. managers plus workers. I also indicate the shares that are calculated only among managers.
Table 31. Model implications for cross-country comparison

<table>
<thead>
<tr>
<th></th>
<th>Top 10% Bottom 10%</th>
<th>Top 20% Bottom 20%</th>
<th>Top 35% Bottom 35%</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Output</strong></td>
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<td></td>
</tr>
<tr>
<td>Data &amp; Model</td>
<td>10.50</td>
<td>6.65</td>
<td>4.99</td>
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<tr>
<td><strong>Share of Unskilled</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Data &amp; Model</td>
<td>0.80</td>
<td>0.84</td>
<td>0.88</td>
</tr>
<tr>
<td><strong>Managerial rate</strong></td>
<td></td>
<td></td>
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</tr>
<tr>
<td>Data</td>
<td>0.42</td>
<td>0.66</td>
<td>0.88</td>
</tr>
<tr>
<td>Model</td>
<td>0.40</td>
<td>0.58</td>
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<tr>
<td><strong>Skilled managers</strong></td>
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<td></td>
<td></td>
</tr>
<tr>
<td>Data</td>
<td>5.05</td>
<td>2.98</td>
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<tr>
<td>Model</td>
<td>4.89</td>
<td>2.71</td>
<td>1.94</td>
</tr>
<tr>
<td><strong>Managerial Premium</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Data</td>
<td>0.48</td>
<td>0.53</td>
<td>0.61</td>
</tr>
<tr>
<td>Model</td>
<td>0.55</td>
<td>0.68</td>
<td>0.81</td>
</tr>
<tr>
<td><strong>Skill premium</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Data</td>
<td>0.66</td>
<td>0.75</td>
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</tr>
<tr>
<td>Model</td>
<td>0.63</td>
<td>0.71</td>
<td>0.79</td>
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<tr>
<td>Average managerial quality</td>
<td>3.05</td>
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<td>2.04</td>
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<tr>
<td>Investment in skills</td>
<td>1.86</td>
<td>1.61</td>
<td>1.42</td>
</tr>
<tr>
<td>Capital per worker</td>
<td>8.55</td>
<td>6.47</td>
<td>3.16</td>
</tr>
</tbody>
</table>

**Note:** Two parameters, productivity \((A)\) and distortions \((\tau)\), are calibrated to match two moments in each country: GDP per worker and the share of unskilled. I compare each statistic in the data for three sections of the distribution of GDP per worker: top 10% to bottom 10%, top 20% to bottom 20% and top 35% to bottom 35%. I report three more statistics, average managerial quality, investment in skills and capital per worker. I do not have data on these statistics and I report what the model generates.
Table 32. The importance of investment in skills and complementarities between productivity and size-dependent distortions

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<td>$\phi = 0$</td>
<td>Y/L</td>
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<td>31</td>
<td>28</td>
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<td></td>
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<td>56</td>
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<td>Managerial Premium</td>
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<td>$\phi = 0.05$</td>
<td>Y/L</td>
<td>69</td>
<td>64</td>
<td>59</td>
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<tr>
<td></td>
<td>Skill premium</td>
<td>73</td>
<td>62</td>
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<td>Managerial Premium</td>
<td>76</td>
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<td>$\phi = 0.1$</td>
<td>Y/L</td>
<td>75</td>
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<td></td>
<td>Skill premium</td>
<td>86</td>
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</tr>
<tr>
<td>Country</td>
<td>Source</td>
<td>Sample size</td>
<td>Year</td>
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<td>2016</td>
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<td>Mexico</td>
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<td></td>
</tr>
</tbody>
</table>

Figure 35. Educational Attainment of Managers and Working Individuals

Notes: The y-axis is the percentage of managers and working individuals (age 25-64) who has at least a high school degree. The stars represent managers and the dots represent population. The correlation coefficient is 0.59 for managers and 0.67 for working individuals.

Source: Author’s calculation based on individual-level survey data.
Figure 36. Managerial Premium

\[ \rho = -0.36 \]

Notes: The y-axis is the relative income of managers to non-managers.
Source: Author’s calculation based on individual-level survey data.
Notes: The y-axis is the share of managers in working individuals (age 25-64). Source: Author’s calculation based on individual-level survey data.
Figure 38. Mean Establishment Size

Notes: The y-axis is the mean establishment size of the manufacturing plants in logs. Source: Bento and Restuccia (2017).
Figure 39. Skill Premium

Notes: The y-axis is the relative income of skilled to unskilled working individuals. 
Source: Author’s calculation based on individual-level survey data.
Figure 40. Steady State Equilibrium Distribution of Managers in the Benchmark Economy

Notes: The average managerial talent of unskilled managers is 46% lower than the average managerial talent of skilled managers.
Figure 41. Amplification from skill investment and occupational choice

Notes: The x-axis shows different values for the returns to investment in skills. The y-axis is elasticity of output with respect to productivity term ($A$). The distortion parameter $\tau$ in the case with size-dependent distortions is set to 0.015.
APPENDIX B

WORLD PRODUCTIVITY: 1996-2014
B.1 Accounting for within- and across-country contributions

As mentioned in the main text, we split up the contribution of shifts in misallocation into with-country component and across-country one. We elaborate here how we do this. We focus on equation (2.12), but the same calculations can be applied to shifts-in-hours term in (2.19) and (2.20) as well.

Remember that the index $i$ in equation (2.12) represents a country-industry pair. We rewrite this equation again with a new indexation: $i$ for industry and $c$ for country:

\[
\dot{v} = \sum_c \sum_i \frac{1}{(1 + \mu_{ci})} s^D_{ci} \dot{z}_{ci} + s^K \dot{k} + s^L \dot{l} 
\]

\[
+ \sum_c \sum_i s^D_{ci} \frac{\mu_{ci}}{(1 + \mu_{ci})} \dot{y}_{ci} + \sum_c \sum_i s^V_{ci} s^K_{ci} \left( \dot{k}_{ci} - \dot{k} \right) + \sum_c \sum_i s^V_{ci} s^L_{ci} \left( \dot{i}_{ci} - \dot{i} \right). \tag{B.1}
\]

We can now split up the capital and labor misallocation terms into within- and across-country component. For example, labor misallocation term can be written as

\[
\sum_c \sum_i s^V_{ci} s^L_{ci} \left( \dot{i}_{ci} - \dot{i} \right) = \sum_c s^V_c \sum_i \frac{s^V_{ci} s^L_{ci}}{s^V_c} \left( \dot{i}_{ci} - \dot{i} \right) + \sum_c s^V_c s^L_c \left( \dot{i}_c - \dot{i} \right), \tag{B.2}
\]

where

\[
s^L_c = \left( \frac{\sum_i s^V_{ci} s^L_{ci}}{s^V_c} \right), \quad \text{and} \quad s^V_c = \sum_i s^V_{ci}. \tag{B.3}
\]

Equation (B.2) splits up the labor misallocation terms into two parts: within-country misallocation of labor which is the first term on the RHS, and across-country component which is the second term. A positive within-country misallocation of labor states that hours are growing faster in industries that on average have higher labor share and contribute more to the country GDP. Higher labor share means that the wages are on average higher in these industries which indicates higher marginal product of labor. Hence, a positive term means that there are productivity gains from changes in the misallocation of labor within the country.

Similarly, a positive across-country misallocation means that hours are growing faster in countries with higher labor share and contribute more to world GDP. This will result in less misallocation of labor and contribute positively to world TFP growth. The capital misallocation term can be split up in a similar way.
B.2 Growth accounting with labor skill levels

Let \( \tau \in \{L, M, H\} \) denote the three labor inputs based on skill. Our raw accounting identity is the following (equation (2.11) in the main text):

\[
\dot{v} = \sum_{i} \frac{1}{(1 + \mu_i)} s_i^D \dot{z}_i + \sum_{i} s_i^V s_i^K \dot{k}_i + \sum_{i} s_i^V s_i^L \dot{l}_i + \sum_{i} s_i^D \frac{\mu_i}{(1 + \mu_i)} \dot{y}_i. \tag{B.4}
\]

Before rearranging this equation to get equation (2.12), we can manipulate the labor term to reflect labor quality. Assuming we have three categories for labor (Low, Medium, and High skilled), the above equation would be:

\[
\dot{v} = \sum_{i} \frac{1}{(1 + \mu_i)} s_i^D \dot{z}_i + \sum_{i} s_i^V s_i^K \dot{k}_i + \sum_{i} \sum_{\tau \in \{L,M,H\}} s_i^V s_i^{L\tau} \dot{l}_\tau + \sum_{i} s_i^D \frac{\mu_i}{(1 + \mu_i)} \dot{y}_i. \tag{B.5}
\]

We now add and subtract aggregate share-weighted factor growth to this equation. For labor, there are three types of aggregate workers, so we add and subtract \( \sum_{\tau \in \{L,M,H\}} \sum_{i} s_i^V s_i^{L\tau} \dot{l}_\tau \). We arrive at the modified version of the main equation:

\[
\dot{v} = \sum_{i} \frac{1}{(1 + \mu_i)} s_i^D \dot{z}_i + \sum_{i} \sum_{\tau \in \{L,M,H\}} s_i^V s_i^{L\tau} \dot{l}_\tau + \sum_{i} \sum_{\tau \in \{L,M,H\}} s_i^V s_i^{L\tau} (\dot{l}_\tau - \bar{l}_\tau). \tag{B.6}
\]

The final term is the change in labor reallocation. It is now the weighted average of labor reallocation across the three types of labor. Aggregate and industry TFP also change, because we now allow for shifts in the contribution of aggregate labor quality. For aggregate TFP, these shifts show up in the share-weighted growth in labor input in the final term on the first line. For industry TFP, we were previously attributing to technology a part of each industry’s growth that is due to labor shifting among education groups.

To see the contribution of labor quality more explicitly, note that the aggregate labor share, \( s^L \), is the sum of the labor shares across the three types of labor, \( \sum_{\tau \in \{L,M,H\}} s_i^{L\tau} \). Hence, following Jorgenson et al. (1987), we can write the contribution of aggregate labor term in the first line as the sum of share-weighted hours growth plus the change in aggregate labor quality:

\[
\sum_{\tau \in \{L,M,H\}} s_i^{L\tau} \dot{i}_\tau = s^L \dot{i} + \sum_{\tau \in \{L,M,H\}} s_i^{L\tau} (\dot{i}_\tau - \bar{i}_\tau). \tag{B.7}
\]
Returning to the labor reallocation term, it will be useful for intuition to express it a different way. First, define the average wage for each type of worker as $W^\tau = \left( \sum_i W^\tau_i L^\tau_i \right) / L^\tau$. Second, note that growth in hours of type $\tau$ is

$$\dot{L}^\tau = \sum_i \left( \frac{L^\tau_i}{L^\tau} \right) \dot{l}^\tau_i = \sum_i \left( \frac{W^\tau_i L^\tau_i}{W^\tau L^\tau} \right) \dot{l}^\tau_i. \quad (B.8)$$

We can now now return to the definition of the labor reallocation term, and substitute in for $\dot{L}^\tau$. We find:

$$\sum_{\tau \in \{L,M,H\}} \left( \left( \sum_i s^V_i s^L_i \dot{l}^\tau_i \right) - s^\tau \dot{l}^\tau \right) = \sum_{\tau \in \{L,M,H\}} \left( \sum_i \frac{W^\tau_i L^\tau_i}{PV} \dot{l}^\tau_i - \sum_i \frac{W^\tau L^\tau_i}{PV} \dot{l}^\tau_i \right). \quad (B.9)$$

$$= \sum_{\tau \in \{L,M,H\}} \sum_i \left( \frac{(W^\tau_i - W^\tau) L_i}{PV} \right) \dot{l}^\tau_i \quad (B.10)$$

Our earlier intuition for labor reallocation was that, if labor grows faster in country-industries where it has a higher than average wage, then this is an improvement in reallocation. Other things equal, that shift boosts growth in output and aggregate TFP. With multiple types of labor, the nuance is that the shift has to take place within a given type of labor. This difference may matter in the data. For example, suppose we see a shift in the data from labor in advanced economies to labor in emerging markets. Some of the cross-country wage differential in our earlier equation presumably reflects differences in the mix of skills across countries—so we need to compare the shifts within skill groups.\footnote{The same intuition holds for capital reallocation. Capital reallocation reflects differential user costs across country-industries for computers, or for machine tools, or for office buildings. The reason we think the capital-reallocation term should be small with an external user cost is that the user cost differences should presumably be small. Of course, there could still be differences to the extent we treat the capital-gains term as country-industry specific, or if there are differential tax wedges.}

### B.3 Detailed results and data

#### B.3.1 Detailed results

##### B.3.1.1 Comparison with World-Bank aggregates

Figure 42 shows how nominal GDP in our data, measured in current US$, lines up with world GDP. The short-dashed line shows the level of nominal GDP in our
sample countries in the 2013 vintage of the data. The other dashed line is the 2016 vintage of the data. Both of these lines are below the World GDP solid line, reflecting that our sample of countries covers about 80 percent of global economic activity (in dollars). The 2016 vintage is a bit higher in the overlapping period because of the inclusion of Croatia, Norway, and Switzerland.

Our time series for PPP-deflated world GDP growth lines up closely with that published by the World Bank in World Bank (2018). This is evident in Figures 43 and 44, which show the World GDP-PPP and its growth in our data versus that of the World Bank.

B.3.1.2 Value-added and factor shares by country and industry

Dollar-denominated value-added shares for the different periods by country and industry are reported in Tables 34 and 36, respectively. Similar PPP-weighted shares are listed in Tables 35 and 37, respectively. Profit shares by industry are reported in Table 38.

B.3.1.3 Detailed contributions to world ALP and TFP growth

The contributions of country-industry TFP growth, $\dot{z}_i$, by country/region for calculations based on dollar-weighted world GDP without taking into account markups are listed in 39, while these contributions with markups are in 40. The contribution of shifts in misallocation due to markups by region is reported in Table ?? while the same contribution by industry can be found in Table ??.

B.3.2 Data

B.3.2.1 Countries and industries

The countries in each of the vintages as well as in the sample for PPP results are listed in Table 43. Throughout, we present these results for a set of regions that are the same across both vintages. The regions are listed in Table 44. The industries were classified into major categories, listed in Table 45, in order to be consistent with the North American Industry Classification System (NAICS).
B.3.2.2 Main variables used for our analysis

- **Gross Value Added**: This is the gross value added at current basic prices (in millions of national currency). The volume index which is normalized to 100 in 1995 and the price level normalized to 100 in 1995 are provided in the tables. The volume index of gross value added is the foundation of GDP growth calculation. We use the exchange rates provided in WIOD to express the nominal values in current U.S. Dollars. These exchange rates, however, are not PPP adjusted.

- **Labor**: Number of employees (thousands) and total hours worked by persons engaged (millions) provide information on the growth in hours along with misallocation of labor across countries and industries. It should be mentioned that the data on hours worked in China were imputed for the period 2008-2014 from the International Labor Organization (ILO). In SEA 2013, data on labor compensation (in millions of national currency) and total hours worked are decomposed based on skill level of the labor into three broad groups: low-, medium- and high-skill. Labor skill types are classified on the basis of educational attainment levels as defined in the International Standard Classification of Education (ISCED): low-skilled (ISCED categories 1 and 2), medium-skilled (ISCED 3 and 4) and high-skilled (ISCED 5 and 6). This decomposition, however, is absent in SEA 2016.

- **Capital**: Data on the current cost replacement value of the capital stock (in millions of national currency) and nominal gross fixed capital formation (in millions of national currency) along with the volume and price index of the latter is used to calculate capital deepening and misallocation of capital across countries and industries. For the 2013 vintage gross fixed capital formation and its associated volume index are used to calculate the implicit capital price deflator which is then used to construct a volume index for the real capital stock. For the 2016 vintage, the current cost replacement value of the capital stock by country-industry is deflated by a constructed capital price deflator. For country-industry combinations for which these deflators are available in OECD (2017b), these deflators are taken from the STAN database for the industry at the lowest level of aggregation that contains the industry in our data. For country-industry combinations for which the capital price deflator is not available in STAN, we use the implicit capital price deflator from the closest corresponding industry in the 2013 vintage and then extrapolate it assuming a constant growth rate for the years 2008-2014.

- **Profits**: Profits are calculated as value added minus compensation minus capital service flows. The latter are calculated assuming an external rate of return equal to the U.S. corporate 10-yr BBB rate. We use the exchange rate to express the capital price deflator in each country in U.S. dollars. This allows us to calculate the capital price inflation in U.S. dollars, i.e. $\pi_{USD}^{C}$. Capital service flows for
each country-industry combination are then calculated as

\[ \left( i_{BBB} - \pi_{USD}^K + \delta_i \right) P_i^K K_i \] (B.11)

Here, \( i_{BBB} \) is the nominal BBB 10-yr corporate bond rate and \( \delta_i \) is the average capital depreciation rate implied by the 2013 vintage capital data. In addition, \( P_i^K K_i \) is the nominal replacement value of the capital stock. For the empirical implementation we have smoothed out fluctuations in \( \pi_{USD}^K \) by using the average over vintage sample.

B.3.2.3 Construction of capital deflators for 2016 vintage

A major source of discrepancies between the 2013 and 2016 vintages is differences in the nominal replacement value of the capital stocks. For the 2013 vintage, when available, they are taken from EU and US KLEMS data. For the 2016 vintage, when available, they are taken from the OECD STAN database. Other values are imputed. However, even those that are taken from these two data sources seem to be very different.

We have merged the the capital deflators from STAN into our data for the 2016 vintage. They are consistent with the nominal replacement values used and, for the countries for which we can obtain them, make our growth rate of the capital stock consistent with OECD STAN. For the other countries, we extrapolated the capital deflators from the 2013 vintage for the years we have missing data.

Depreciation rates are calculated by industry for the 2013 and applied to both the 2013 and 2016 vintages of the data.

B.3.2.4 Construction of PPP-deflated value-added

In this section, we explain in more detail how we constructed a measure of PPP-deflated value added by double-deflating the benchmark PPP relative prices constructed by Timmer et al. (2007) and Inklaar and Timmer (2014).

PPP benchmark prices

The PPP benchmark tables report relative prices of industry gross output for industries and countries in the dataset. The numeraire good is US GDP in 2005, i.e. the relative price of US GDP in the benchmark table is 1. This means the relative price reported, \( P_{i,t} \), is the number of U.S. dollars in 2005 per unit of output.
in country-industry $i$ in 2005 relative to the number of U.S. dollars in 2005 per unit of U.S. GDP. It is useful to consider this in mathematical form

$$P_{i,t} = \frac{\$/GO_{i,t}}{\$/USGDP_t} = \frac{USGDP_t}{GO_{i,t}} \quad \text{for } t = 2005.$$  

(B.12)

The first step is to calculate a time series for $P_{i,t}$ for $t \neq 2005$. This can be done by using the time series for the price index for gross output in country-industry $i$ in year $t$, i.e. $P_{i,t}$, as well as the U.S. GDP deflator, $P_t$.

Using these two time series, we can construct

$$P_{i,t} = P_{i,2005} \frac{P_{i,t}/P_{i,2005}}{P_t/P_{2005}}.$$  

(B.13)

This gives us a time series of PPP conversion rates of the real gross output values into U.S. GDP.

Dollars to PPP, denominated in US GDP

The conversion factor derived above then allows us to convert nominal gross output in country-industry $i$ in year $t$, i.e. $P_{i,t} Y_{i,t}$, into units of U.S. GDP. Let $Y_{i,t}^*$ be output in country-industry $i$ in year $t$ measured in PPP units of U.S. GDP in the same period, then we can calculate it through

$$Y_{i,t}^* = \frac{P_{i,t} Y_{i,t}}{P_{i,t}} = P_{i,t} Y_{i,t}^*, \quad \text{where } P_{i,t} = \frac{P_{i,t}}{P_t}. \quad \text{(B.14)}$$

This equation means the following. The inverse of $P_{i,t}$ converts dollars of nominal gross output of country-industry $i$ in year $t$ into dollars of nominal U.S. GDP in year $t$ according to the PPP adjustment. Dividing these dollars by the U.S. GDP deflator then gives the quantity of U.S. GDP produced in the sector.

Now, this allows us to calculate PPP adjusted gross output. However, what we really want to calculate is PPP adjusted value added. To obtain this, we need to do an additional calculation.

Value added in terms of PPP

To PPP adjust value added, we basically PPP adjust the nominal gross output and intermediate inputs terms in the definition of value added. That is, nominal value added of country-industry $i$ in year $t$ is the difference between nominal gross output and the nominal value of intermediate inputs.

$$P_{i,t} V_{i,t} = P_{i,t} Y_{i,t} - \sum_{i'} P_{i',t} M_{i',t}. \quad \text{(B.15)}$$
Now PPP adjusted value added of sector $i$ during year $t$, i.e. $V_{i,t}^*$, is obtained by PPP adjusting each of the individual nominal components. That is,

$$V_{i,t}^* = \frac{P_{i,t}Y_{i,t}^*}{P_{i,t}^*} - \sum_{i'} \frac{P_{i',t}M_{i',j',t}}{P_{j',t}^*}. \quad \text{(B.16)}$$

The implicit PPP deflator of value added of sector $i$ in year $t$ is then given by

$$P_{i,t}^{V^*} = \frac{P_{i,t}^*V_{i,t}}{V_{i,t}^*}. \quad \text{(B.17)}$$

The calculation of (B.16) involves figuring out the intermediate inputs from all over the world using the WIOT and this requires using the input-output tables.

The other problem is that we cannot PPP adjust all intermediate inputs. One way of dealing with it is to use the same PPP deflator for the intermediate inputs for which we have no data compared to those for which we have data. The PPP deflator of the intermediate inputs that are covered is calculated using

$$P_{i,t}^{M^*} = \sum_{i'} \frac{P_{i',t}M_{i',j',t}}{P_{i',t}^*}. \quad \text{(B.18)}$$

where $i'$ and $j'$ cover the intermediate inputs for which PPP adjusted deflators are measured. We then use this to deflate all the nominal intermediate inputs.

So, practically, we calculate $P_{i,t}^{M^*}$ for each sector $i$ and year $t$ for all the intermediate inputs for which we have PPP adjusted gross output deflators. We then deflate all nominal intermediate inputs by this deflator to calculate PPP adjusted value added. We then calculate the implied PPP adjusted value-added deflator, (B.17).

This then allows us to calculate all the PPP adjusted data that we need for our analysis.
Table 34. Dollar-denominated value-added shares, by country/region: 1996-2014

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<th>SEA vintage</th>
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<th>2016</th>
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<tr>
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</table>
Table 35. PPP-denominated value-added shares, by country/region: 1996-2014

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Note: Reported are contributions by country/region in percentage points over various subperiods.
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Note: Reported are contributions by industry in percentage points over various subperiods.
Table 37. PPP-denominated value-added shares, by industry: 1996-2014

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<td>0.08</td>
<td>0.08</td>
<td>0.07</td>
<td>0.06</td>
<td>0.06</td>
<td>0.06</td>
<td>0.06</td>
<td>0.07</td>
</tr>
<tr>
<td>Hospitality</td>
<td>0.03</td>
<td>0.03</td>
<td>0.02</td>
<td>0.03</td>
<td>0.03</td>
<td>0.02</td>
<td>0.02</td>
<td>0.02</td>
<td>0.02</td>
<td>0.02</td>
<td>0.02</td>
</tr>
<tr>
<td>Personal services</td>
<td>0.04</td>
<td>0.04</td>
<td>0.03</td>
<td>0.04</td>
<td>0.03</td>
<td>0.02</td>
<td>0.02</td>
<td>0.02</td>
<td>0.02</td>
<td>0.02</td>
<td>0.02</td>
</tr>
<tr>
<td>Government</td>
<td>0.06</td>
<td>0.06</td>
<td>0.06</td>
<td>0.06</td>
<td>0.07</td>
<td>0.06</td>
<td>0.06</td>
<td>0.06</td>
<td>0.06</td>
<td>0.06</td>
<td>0.06</td>
</tr>
<tr>
<td>Households</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
</tr>
<tr>
<td>Total</td>
<td>1.00</td>
<td>1.00</td>
<td>1.00</td>
<td>1.00</td>
<td>1.00</td>
<td>1.00</td>
<td>1.00</td>
<td>1.00</td>
<td>1.00</td>
<td>1.00</td>
<td>1.00</td>
</tr>
</tbody>
</table>

Note: Reported are contributions by industry in percentage points over various subperiods.
Table 38. Profits as a percentage of world GDP, by industry: 1996-2014

| SEA vintage | 2013 | | | 2016 | | |
|------------|------|--|--|--|--|--|--|--|--|
| Agriculture | 0.63 | 0.85 | 1.40 | 1.06 | 1.20 | 1.86 | 1.86 | 2.49 | 2.02 |
| Construction | 0.55 | 0.71 | 1.02 | 0.82 | 0.67 | 0.99 | 0.84 | 1.04 | 0.97 |
| Nondurables manuf | 1.83 | 2.17 | 3.02 | 2.72 | 2.49 | 2.98 | 2.47 | 2.80 | 3.06 |
| Durables manuf | 0.49 | 0.35 | 0.74 | 0.67 | 0.68 | 1.05 | 0.63 | 0.70 | 0.95 |
| Trade Trans Utilities | 2.33 | 3.01 | 4.01 | 3.58 | 3.82 | 4.50 | 3.91 | 4.86 | 4.88 |
| FIRE | -2.10 | 1.23 | 3.65 | 1.47 | 4.49 | 5.46 | 4.52 | 6.96 | 6.27 |
| Business services | 0.73 | 0.79 | 1.19 | 1.08 | 1.57 | 1.79 | 1.44 | 1.57 | 1.97 |
| Education Healthcare | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| Hospitality | 0.39 | 0.49 | 0.55 | 0.52 | 0.59 | 0.63 | 0.48 | 0.48 | 0.61 |
| Personal services | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| Government | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| Households | -0.00 | -0.00 | -0.00 | -0.00 | 0.01 | 0.01 | 0.01 | 0.01 | 0.01 |
| Total | 4.96 | 9.72 | 15.96 | 12.25 | 15.53 | 19.24 | 16.16 | 20.92 | 20.74 |

Note: Reported are contributions by industry in percentage points over various subperiods. Profits in Education/Healthcare, Personal care, Government, and Households are set to zero by construction.
Table 39. Contribution of country-industry specific TFP growth, by country/region: 1996-2014

<table>
<thead>
<tr>
<th>Country/region</th>
<th>SEA vintage</th>
<th>2013</th>
<th>2016</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>2004</td>
<td>2007</td>
</tr>
<tr>
<td>Advanced</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>USA</td>
<td></td>
<td>0.22</td>
<td>0.51</td>
</tr>
<tr>
<td>UK</td>
<td></td>
<td>0.03</td>
<td>0.06</td>
</tr>
<tr>
<td>Japan</td>
<td></td>
<td>0.06</td>
<td>0.12</td>
</tr>
<tr>
<td>India</td>
<td></td>
<td>0.18</td>
<td>0.07</td>
</tr>
<tr>
<td>Other Advanced</td>
<td></td>
<td>0.15</td>
<td>0.06</td>
</tr>
<tr>
<td>Brazil</td>
<td></td>
<td>0.26</td>
<td>0.35</td>
</tr>
<tr>
<td>China</td>
<td></td>
<td>0.21</td>
<td>-0.00</td>
</tr>
<tr>
<td>India</td>
<td></td>
<td>0.03</td>
<td>0.01</td>
</tr>
<tr>
<td>Russia</td>
<td></td>
<td>-0.03</td>
<td>0.03</td>
</tr>
<tr>
<td>Other Emerging</td>
<td></td>
<td>0.04</td>
<td>0.02</td>
</tr>
<tr>
<td><strong>Total</strong></td>
<td></td>
<td>0.91</td>
<td>1.18</td>
</tr>
</tbody>
</table>

Note: Reported are contributions by country/region to line 10 in Table 4 in percentage points over various subperiods. Results without markups.
Table 40. Contribution of country-industry specific TFP growth, by country/region: 1996-2014

<table>
<thead>
<tr>
<th>Country/region</th>
<th>2013</th>
<th>2016</th>
</tr>
</thead>
<tbody>
<tr>
<td>Advanced</td>
<td></td>
<td></td>
</tr>
<tr>
<td>United States</td>
<td>0.17 0.08 0.10 0.12</td>
<td>0.28 0.06 0.16 -0.09 0.10</td>
</tr>
<tr>
<td>Great Britain</td>
<td>0.20 0.09 0.17 0.16</td>
<td>0.08 0.04 -0.03 0.03 0.03</td>
</tr>
<tr>
<td>Japan</td>
<td>0.23 0.20 0.21 0.22</td>
<td>0.05 0.20 -0.15 0.09 0.05</td>
</tr>
<tr>
<td>Euro Area</td>
<td>0.05 0.07 0.06 0.06</td>
<td>0.06 0.09 -0.16 0.08 0.03</td>
</tr>
<tr>
<td>Other Advanced</td>
<td>0.27 0.55 0.09 0.32</td>
<td>0.05 0.03 -0.04 -0.01 0.01</td>
</tr>
<tr>
<td>Emerging</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Brazil</td>
<td>0.19 0.25 0.54 0.31</td>
<td>0.17 0.49 0.25 0.06 0.23</td>
</tr>
<tr>
<td>China</td>
<td>-0.02 0.02 0.05 0.01</td>
<td>0.11 0.37 0.15 0.28 0.22</td>
</tr>
<tr>
<td>India</td>
<td>0.03 0.01 0.08 0.04</td>
<td>0.02 0.04 0.04 -0.08 0.00</td>
</tr>
<tr>
<td>Russia</td>
<td>0.19 0.21 0.31 0.23</td>
<td>0.02 0.05 0.03 -0.02 0.02</td>
</tr>
<tr>
<td>Other Emerging</td>
<td>0.00 -0.01 -0.01 -0.01</td>
<td>0.03 0.07 0.01 -0.03 0.02</td>
</tr>
<tr>
<td>Total</td>
<td>1.13 1.25 1.17 1.18</td>
<td>0.70 0.91 0.03 0.18 0.45</td>
</tr>
</tbody>
</table>

Note: Reported are contributions by country/region to line 10 in Table 5 in percentage points over various subperiods. Results with markups.
Table 43. List of countries in each vintage of SEA and the ones that have PPP data

<table>
<thead>
<tr>
<th>Country</th>
<th>SEA 2013</th>
<th>SEA 2016</th>
<th>PPP</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. Australia</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
</tr>
<tr>
<td>2. Austria</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
</tr>
<tr>
<td>3. Belgium</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
</tr>
<tr>
<td>4. Bulgaria</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
</tr>
<tr>
<td>5. Brazil</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
</tr>
<tr>
<td>6. Canada</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
</tr>
<tr>
<td>7. Switzerland</td>
<td>✓</td>
<td></td>
<td>✓</td>
</tr>
<tr>
<td>8. China</td>
<td></td>
<td>✓</td>
<td>✓</td>
</tr>
<tr>
<td>9. Cyprus</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
</tr>
<tr>
<td>10. Czech Republic</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
</tr>
<tr>
<td>11. Germany</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
</tr>
<tr>
<td>12. Denmark</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
</tr>
<tr>
<td>13. Spain</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
</tr>
<tr>
<td>14. Estonia</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
</tr>
<tr>
<td>15. Finland</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
</tr>
<tr>
<td>16. France</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
</tr>
<tr>
<td>17. United Kingdom</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
</tr>
<tr>
<td>18. Greece</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
</tr>
<tr>
<td>19. Croatia</td>
<td></td>
<td></td>
<td>✓</td>
</tr>
<tr>
<td>20. Hungary</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
</tr>
<tr>
<td>21. Indonesia</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
</tr>
<tr>
<td>22. India</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
</tr>
<tr>
<td>23. Ireland</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
</tr>
<tr>
<td>24. Italy</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
</tr>
<tr>
<td>25. Japan</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
</tr>
<tr>
<td>26. South Korea</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
</tr>
<tr>
<td>27. Lithuania</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
</tr>
<tr>
<td>28. Luxembourg</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
</tr>
<tr>
<td>29. Latvia</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
</tr>
<tr>
<td>30. Mexico</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
</tr>
<tr>
<td>31. Malta</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
</tr>
<tr>
<td>32. Netherlands</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
</tr>
<tr>
<td>33. Norway</td>
<td></td>
<td></td>
<td>✓</td>
</tr>
<tr>
<td>34. Poland</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
</tr>
<tr>
<td>35. Portugal</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
</tr>
<tr>
<td>36. Romania</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
</tr>
<tr>
<td>37. Russia</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
</tr>
<tr>
<td>38. Slovakia</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
</tr>
<tr>
<td>39. Slovenia</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
</tr>
<tr>
<td>40. United States</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
</tr>
<tr>
<td>41. Turkey</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
</tr>
<tr>
<td>42. Taiwan</td>
<td>✓</td>
<td></td>
<td>✓</td>
</tr>
<tr>
<td>43. United States</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
</tr>
</tbody>
</table>
### Table 44. Country Classification

<table>
<thead>
<tr>
<th>Region</th>
<th>Country</th>
</tr>
</thead>
<tbody>
<tr>
<td>Euro Area</td>
<td>Germany, France, Austria, Italy, Belgium, Cyprus, Spain, Estonia, Finland, Greece, Ireland, Lithuania, Luxembourg, Latvia, Malta, Netherlands, Portugal, Slovakia, Slovenia</td>
</tr>
<tr>
<td>Other Advanced</td>
<td>Canada, South Korea, Taiwan, Australia, Switzerland, Denmark, Sweden, Norway, Bulgaria, Czech Republic, Croatia, Hungary, Poland, Romania</td>
</tr>
<tr>
<td>Other Emerging</td>
<td>Indonesia, Turkey, Mexico</td>
</tr>
</tbody>
</table>

### Table 45. Industry Classification

<table>
<thead>
<tr>
<th>Major sector</th>
<th>ISIC v3 industries included(^1)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Agriculture</td>
<td>Agriculture, Forestry, Fishing and Hunting, Mining</td>
</tr>
<tr>
<td>Construction</td>
<td>Construction</td>
</tr>
<tr>
<td>Nondurable manufacturing</td>
<td>Manufacturing</td>
</tr>
<tr>
<td>Durable manufacturing</td>
<td>Manufacturing</td>
</tr>
<tr>
<td>Trade, transportation and utilities</td>
<td>Wholesale Trade, Retail Trade, Transportation and Warehousing, Utilities</td>
</tr>
<tr>
<td>Finance, insurance and real estate (FIRE)</td>
<td>Finance and Insurance, Real Estate Rental and Leasing</td>
</tr>
<tr>
<td>Business services</td>
<td>Information, Professional, Scientific, and Technical Services, Management of Companies and Enterprises</td>
</tr>
<tr>
<td>Education and healthcare</td>
<td>Educational Services, Health Care and Social Assistance</td>
</tr>
<tr>
<td>Hospitality</td>
<td>Accommodation and Food Services</td>
</tr>
<tr>
<td>Personal services</td>
<td>Arts, Entertainment, and Recreation, Other Services, Administrative and Support and Waste Management and Remediation Services</td>
</tr>
<tr>
<td>Government</td>
<td>Public Administration</td>
</tr>
<tr>
<td>Households</td>
<td></td>
</tr>
</tbody>
</table>

\(^1\) For WIOD vintage 2016 ISIC v4 industries are aggregated to ISIC v3 using the crosswalk provided in the data documentation (Gouma et al., 2018).
Figure 42. Nominal world GDP in WIOD-SEA and World Development Indicators


Note: SEA data is total nominal value added for all industries and countries in both vintages of the WIOD. All measures are reported in current U.S. $.
Figure 43. World GDP PPP in WIOD-SEA and World Development Indicators

Source: Timmer (2012), and World Bank (2018), and authors’ calculations.

Note: SEA data is total value added PPP for all industries and countries in both vintages of the WIOD. All measures are reported in U.S. $ of 2005 U.S. GDP.
Figure 44. Growth in world GDP PPP in WIOD-SEA and World Development Indicators

Source: Timmer (2012), and World Bank (2018), and authors' calculations.
Note: World GDP PPP growth is constructed as real PPP-adjusted value-added share weighted average of nominal GDP or real country-industry value-added PPP growth.
APPENDIX C

INEQUALITY OVER THE LIFE-CYCLE: US VS EUROPE
C.1 Data Details

C.1.1 U.S. Data

The Current Population Survey (CPS) is the source of statistics on labor market for the U.S. government. It was designed to be representative of the civilian non-institutional population. There is a supplement applied in March called the Annual Social and Economic Supplement (ASEC) that extends the survey with detailed questions on income.

I follow the sample selection that is customary in the macroeconomic literature. These criteria for sample selection are basically eliminating the extreme observations that are not precise enough especially for distributional studies such as the one carried out in this paper.

The age group is always 20-60, and they are grouped in 5-year age bin. All observations where a crucial value such as age, gender and education is missing are dropped. Also, if an observation reports positive labor earnings but zero or missing hours, it is dropped. The following criteria is used to further restrict the samples.

- Labor earnings: total wage and salary reported in CPS
- All earnings data are deflated by the price deflator reported in Flood et al. (2018).
- Agriculture workers, family workers and armed forces are excluded.
- Hours are restricted to at least 260 annual hours for age 30 and below, and 520 for above 30.
- Hourly earnings less than half of federal minimum wage are dropped.
- Mean, variance, and percentiles are calculated based on earnings levels and sample weights.

One important note is that the observations are not corrected for top coding. The reason is that this correction is not available for the European data since the top coding scheme changes from year to year, it is not uniform across countries, and the way it changes is not reported in the data documentation. Hence, the data is treated as is and not corrected for any top coding.

[^113]: See for example Storesletten et al. (2004); Huggett et al. (2006); Guvenen and Kuruscu (2007); Heathcote et al. (2010a); Huggett et al. (2011), among others.

[^114]: See Tormalehto (2017) for more detail about top-coding issue in Eu-SILC.
The main source of data for the European countries is EU-SILC (2016) which stands for the European Union Statistics on Income and Living Conditions. The general criteria for earnings and hours are the same as for the U.S. Observations with annual earnings less than half the effective minimum wage which are reported by EuroStat are excluded.

EU-SILC is an annual survey conducted by Eurostat in cooperation with the National Statistical Institutes (NSIs) of the European Union, European Free Trade Association (EFTA) and candidate countries. The goal of the survey is to collect comparable and reliable data on income, poverty, socio-economic and living conditions. This survey is the primary source of indicators on income by the Eurostat to evaluate progress towards EU policy objectives.

The EU-SILC is collected and harmonized under the coordination of Eurostat and NSIs. Harmonization means that Eurostat defines a set of target variables and defines a number of quality criteria regarding data collection. In most countries, the data collection is done via a survey, except for a few countries where administrative records are used.\textsuperscript{115}

EU-SILC data is seldom used in a cross-country study since it is a collection of surveys in different countries with potentially different form. However, Eurostat harmonizes most of the variables of interest for macroeconomists which include income data, education, and occupation data. For example, Hlasny and Verme (2018) uses EU-SILC to study variations in gini coefficient under different top-coding correction schemes.

C.1.3 Tax Data

In most countries, the unit of observation for tax purposes is the household. There is usually a primary earner and a secondary one and in some instances, there are more members of the household who draw an income. The main differences in the tax codes across countries are the tax brackets, marginal rates for each bracket, tax credits and their basis, and how the number of children affect tax liabilities of the household.

There are a few countries such as the United Kingdom where taxation is completely individual-based. There are other countries with individual tax system, but their tax code contain components such as tax credits that is based on the household, which makes their tax system hybrid. Netherlands is an example of that.

In order to estimate the parameters of the tax function in the main text, I used the tax system for each country and treated all of them as household based, even for the United Kingdom. In order to figure out the tax liabilities of each household, one needs to consult the tax code. This is done in a series of MATLAB programs by Bick et al. (2019). Therefore, for every household in the EU-SILC, I can recover the tax liabilities.

These tax liabilities take into account the structure of the household. For simplification, the household is disaggregated into three components: the principal earner, the secondary (spouse) earner, and the number of children. Given this structure, the programs look for the tax liabilities of the household in the tax code of the particular country and year. Of course, for the United Kingdom, the program calculates the tax liabilities separately for principal and secondary earner.

These tax liabilities include federal, state, local, and social security taxes for the U.S. and federal, and social security for European countries. I pool all the tax liabilities of the household together, and call that the taxes owed by the household. Total pre-tax income of the household is the sum of the income of principal and secondary earners. The income data constitutes only the wages, and I abstracted from business, asset, or other sources of income.

The estimation of the tax function is then a GMM estimator which chooses parameters of the tax function so that the difference between the tax liabilities for each household in the data and the one generated by the tax function is minimized. The standard errors are calculated by drawing 10% samples from the data and repeating for 1000 times. I also do a robustness check for the United Kingdom and focus on the principal’s income and his tax liabilities, and re-estimate the parameters. In terms of progressivity, this does not change the tax function for the U.K. as much. The only major difference is that the average taxes are higher. These estimates are presented in table 46.

<table>
<thead>
<tr>
<th>United Kingdom</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \lambda )</td>
</tr>
<tr>
<td>(0.0018)</td>
</tr>
<tr>
<td>( \tau )</td>
</tr>
<tr>
<td>(0.0005)</td>
</tr>
</tbody>
</table>

Note: Standard errors are in parenthesis.
C.1.4 Higher education in Europe

In order to define levels of education uniformly across all countries, United Nations Educational, Scientific, and Cultural Organization (UNESCO) developed terms that have been agreed upon by all participating countries to address different levels of educational attainment. These levels, called the International Standard Classification of Education (ISCED) levels, are used to compile internationally comparable statistics on education.

The classification distinguishes between seven levels of education ranging from pre-primary to tertiary. International definitions of pre-primary, primary, and tertiary education are similar to the definitions used in the United States; however, lower and upper secondary education have slightly different meanings.

Pre-primary education (level 0), also called early childhood education, usually includes education for children aged 3-5, although in some countries, it starts as early as age 2 and in others continues through age 6. In the United States, pre-primary education includes kindergarten. Primary education (level 1) runs from about ages 6-11, or about first through sixth grades in the United States. Specialization rarely occurs in any country before secondary education.

Secondary education covers ages 11 or 12 through 18 or 19 and is divided into two levels: lower and upper secondary (levels 2 and 3). For the purposes of statistical comparability, the United States has defined lower secondary education as grades 7 through 9 and upper secondary as grades 10 through 12. In the United States, lower secondary education is the loose equivalent of intermediate school, middle school, or junior high school; however, in many other countries lower secondary education ends with an examination and constitutes the completion of compulsory education. Upper secondary education immediately follows lower secondary education and includes general (academic), technical, and vocational education, or any combination thereof, depending on the country. An upper secondary attainment level is roughly equivalent to a U.S. high school diploma.

Higher education, also referred to as tertiary education, includes three ISCED levels and is the equivalent of post-secondary education in the United States. Non-university higher education includes education beyond the secondary school level involving programs (e.g., vocational, community college, and junior college programs) that terminate in less than a 4-year degree. This type of education is at ISCED level 5. ISCED level 6 comprises education programs that lead to a 4-year undergraduate degree. These programs are typically located in universities and other 4-year institutions. The highest level, ISCED level 7, includes graduate and professional degree programs.116

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116For the attainment indicators, a person is classified in the highest level for which they completed
Based on OECD (2007), post-secondary non-tertiary education straddles the boundary between upper secondary and post-secondary education from an international point of view, even though it might clearly be considered upper secondary or post-secondary programs in a national context. Although their content may not be significantly more advanced than upper secondary programs, they serve to broaden the knowledge of participants who have already gained an upper secondary qualification. The students tend to be older than those enrolled at the upper secondary level.

Tertiary-type A programs (ISCED 5A) are largely theory-based and are designed to provide sufficient qualifications for entry to advanced research programs and professions with high skill requirements, such as medicine, dentistry or architecture. Tertiary-type A programs have a minimum cumulative theoretical duration (at tertiary level) of three years’ full-time equivalent, although they typically last four or more years. These programs are not exclusively offered at universities.

Conversely, not all programs nationally recognized as university programs fulfill the criteria to be classified as tertiary-type A. Tertiary-type A programs include second degree programs like the American Master. First and second programs are sub-classified by the cumulative duration of the programs, i.e., the total study time needed at the tertiary level to complete the degree.

Table 47 presents a summary of the cross-walk between ISCED levels and their U.S. equivalents. For the purpose of sample selection and educational expenditure statistics, level 6 and 7 are defined as college graduate.

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the last grade or degree for the level. For example, a U.S. student must complete grade 9 in order to attain a lower secondary education and 2 years of higher education (associate’s degree) in order to attain a non-university higher education.
Table 47. Cross-walk between ISCED levels and U.S. equivalent

<table>
<thead>
<tr>
<th>ISCED level</th>
<th>Definition</th>
<th>U.S. equivalent</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>Preprimary</td>
<td>Kindergarten and below</td>
</tr>
<tr>
<td>1</td>
<td>Primary</td>
<td>1st-6th grades</td>
</tr>
<tr>
<td>2</td>
<td>Lower secondary</td>
<td>7th-9th grades</td>
</tr>
<tr>
<td>3</td>
<td>Upper secondary</td>
<td>10th-12th grades or first 3 years of vocational education</td>
</tr>
<tr>
<td>5</td>
<td>Higher education</td>
<td>Community or junior colleges or vocational technical institutes (non-university) leading to an associate’s degree</td>
</tr>
<tr>
<td>6</td>
<td>Higher education</td>
<td>University or other 4-year education institution leading to a bachelor’s degree</td>
</tr>
<tr>
<td>7</td>
<td>Higher education</td>
<td>A University or professional institute leading to leading to a master’s or doctor’s degree</td>
</tr>
</tbody>
</table>

Note: In order to define levels of education uniformly across all countries, this publication uses terms that were developed by the United Nations Educational, Scientific, and Cultural Organization (UNESCO) and have been agreed upon by all participating countries, but which might be unfamiliar to readers from the United States. These levels, called the International Standard Classification of Education (ISCED) levels, are used to compile internationally comparable statistics on education.

Source: NCES (2020).

C.1.4.1 United Kingdom

Tuition fee loans are available to cover the full cost of tuition fees and are paid directly to the institution. They are non-income assessed loans available to both full-time and part-time students, but part-time students must be studying for a minimum of 25% of their time to be eligible.

Maintenance loans are available to help with the cost of accommodation and other living expenses for full-time and part-time undergraduate students. The exact amount which can be borrowed varies, but the loan includes a non-financially assessed portion which all students who are eligible for the loan receive. It also includes a financially assessed portion which depends on household income (i.e. the combined total income of the student and his / her parents, or the student and the partner they live with); and a portion based on the student’s place of residence (the family home, or away from home).

Repayment arrangements are the same for both tuition fee loans and maintenance loans. The threshold for when borrowers are required to start making repayments depends on when they studied their course. Any loan remaining after 30 years will be canceled. Payment is collected through the tax system. Student loans accrue interest from the date they are paid out up until they are repaid in full.
C.1.4.2 Netherlands

The Dutch government provides public aid. This covers student finance and benefits like healthcare and housing allowances. Student finance, or studiefinanciering in Dutch, is a 3-part financial aid package intended to help students with paying their tuition fees and student life. There are requirements you need to meet, with some students being eligible for all 3 components and some maybe one or two. The first is the loan or the tuition fee loan; the second is the supplementary grant, and the third is the student travel product.

Allowances are sums of money gifted to low-income citizens, or students, to aid with some of their living costs. Healthcare allowance is a monthly sum provided by the Dutch government to help cover your monthly health insurance bill. Similarly, the housing allowance is a sum to help with your monthly rent. As with student finance, there are specific requirements you need to meet.

Aside from student finance, there is the option of applying for a scholarship. A scholarship is like financial aid but it comes in the form of an award. Scholarships are usually given out by universities or other donors or institutions. Scholarships are also awarded based on specific criteria, like having certain grades or possessing certain qualities. Unlike a loan, scholarship money does not have to be paid back.

C.1.4.3 France

Grants are provided based on family or individual resources. Most students are entitled to a minimum grant. Any student receiving a state grant is automatically entitled to 100% reduction in tuition fees at state universities. Students are eligible for state-guaranteed loans of up to 15000 euros at a low interest rate. 70% of the amount loaned is guaranteed 10 years by the state. The loans are granted by commercial banks and require a further guarantee for the remaining 30%.

Fees for undergraduate studies are determined annually by the Education Ministry. Since 2007 universities may opt for an autonomous status. Autonomous universities have the ability to determine certain tuition fees. While undergraduate fees are capped at the level set by the Ministry of Education, post-graduate and doctorate studies may be set freely by the universities.
C.2 Robustness Checks

C.2.1 Mean and Variance Profiles Statistically Differ Across Countries

In this section, I show that the differences in mean earnings and variance of log earnings over the life-cycle are statistically significant using ANOVA analysis. This is also true for the college individuals. The profiles for non-college individuals for mean earnings are not different from a statistical standpoint (I reject the null hypothesis of similar profiles for all groups except for the mean earnings of the non-college individuals).

Table 48. ANOVA table for mean earnings

<table>
<thead>
<tr>
<th>sum of squared errors</th>
<th>degrees of freedom</th>
<th>F statistic</th>
<th>PR(&gt; F)</th>
</tr>
</thead>
<tbody>
<tr>
<td>number of countries</td>
<td>0.593</td>
<td>3</td>
<td>3.605</td>
</tr>
<tr>
<td>Residual</td>
<td>1.096</td>
<td>20</td>
<td>-</td>
</tr>
</tbody>
</table>

Table 49. ANOVA table for mean earnings, non-college individuals

<table>
<thead>
<tr>
<th>sum of squared errors</th>
<th>degrees of freedom</th>
<th>F statistic</th>
<th>PR(&gt; F)</th>
</tr>
</thead>
<tbody>
<tr>
<td>number of countries</td>
<td>0.015</td>
<td>3.0</td>
<td>0.226</td>
</tr>
<tr>
<td>Residual</td>
<td>0.435</td>
<td>20.0</td>
<td>-</td>
</tr>
</tbody>
</table>

Table 50. ANOVA table for mean earnings, college individuals

<table>
<thead>
<tr>
<th>sum of squared errors</th>
<th>degrees of freedom</th>
<th>F statistic</th>
<th>PR(&gt; F)</th>
</tr>
</thead>
<tbody>
<tr>
<td>number of countries</td>
<td>3.056</td>
<td>3</td>
<td>5.287</td>
</tr>
<tr>
<td>Residual</td>
<td>3.853</td>
<td>20</td>
<td>-</td>
</tr>
</tbody>
</table>
### Table 51. ANOVA table for variance of log earnings

<table>
<thead>
<tr>
<th></th>
<th>Sum of Squared Errors</th>
<th>Degrees of Freedom</th>
<th>F Statistic</th>
<th>PR(&gt; F)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of countries</td>
<td>0.644</td>
<td>3</td>
<td>17.523</td>
<td>0.0</td>
</tr>
<tr>
<td>Residual</td>
<td>0.294</td>
<td>24</td>
<td>-</td>
<td>-</td>
</tr>
</tbody>
</table>

### Table 52. ANOVA table for variance of log earnings, non-college individuals

<table>
<thead>
<tr>
<th></th>
<th>Sum of Squared Errors</th>
<th>Degrees of Freedom</th>
<th>F Statistic</th>
<th>PR(&gt; F)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of countries</td>
<td>0.564</td>
<td>3.0</td>
<td>89.193</td>
<td>0.0</td>
</tr>
<tr>
<td>Residual</td>
<td>0.051</td>
<td>24.0</td>
<td>-</td>
<td>-</td>
</tr>
</tbody>
</table>

### Table 53. ANOVA table for variance of log earnings, college individuals

<table>
<thead>
<tr>
<th></th>
<th>Sum of Squared Errors</th>
<th>Degrees of Freedom</th>
<th>F Statistic</th>
<th>PR(&gt; F)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of countries</td>
<td>0.465</td>
<td>3</td>
<td>15.486</td>
<td>0.0</td>
</tr>
<tr>
<td>Residual</td>
<td>0.240</td>
<td>24</td>
<td>-</td>
<td>-</td>
</tr>
</tbody>
</table>
C.3 Mathematical derivations

C.3.1 A simple model of college choice

Consider an individual who lives for two periods. The individual can potentially work in both periods, or she can go to college in period one and accumulate human capital while only work in the second period. There is no channel to increase human capital besides college and there are no human capital depreciation. I assume for simplicity that the only taxation system is a flat rate labor tax $\tau$. Individuals discount future at rate $r$, which equals the real interest rate. This implies a discount factor $\beta = \frac{1}{1+r}$. There is a rental rate per unit of human capital so that the labor earnings of an individual with human capital $h$ equals $wh$.

If the individual goes to college, she will have access to a technology to increase her human capital. This technology uses her initial human capital and investments in terms of consumption goods to produce new human capital. There is complementarity between initial human capital and goods investments in this technology as follows:

$$h_1 = h_0 + ah_0^\phi d^\nu, \quad \phi, \nu > 0, \quad \phi + \nu < 1,$$

where $d$ is the amount of investments in terms of consumption goods. The individual has to borrow this amount in order to fund her college education. College expenditure is subsidized at the rate $gd$, and the government transfers a fixed college grant $\bar{d}$ to the individual during college.

The individual has three state variables at the start of period one: learning ability $a$, initial human capital $h_0$ and disutility for college $\eta$. The last one is the utility cost of attending college that the individual incurs only if she chooses college in the first period. This cost captures the psychological cost of exerting effort during college. The individual compares the present discounted value of going to college $V^C(a, h_0, \eta)$ and not going to college $V^{NC}(a, h_0, \eta)$ and if $V^C > V^{NC}$, she chooses college. Otherwise, she does not choose college.

The problem of the individual if she chooses not to go to college is as follows:

$$V^{NC}(a, h_0, \eta) = \max_{c_0^{NC}, c_1^{NC}} \log(c_0^{NC}) + \beta \log(c_1^{NC})$$

s.t. $$c_0^{NC} + c_1^{NC} \frac{1}{1+r} = wh_0(1-\tau) + \frac{wh_0(1-\tau)}{1+r}.$$  

The problem of an individual if she chooses to go to college is as follows:

$$V^C(a, h_0, \eta) = \max_{c_0^C, c_1^C, d} \log(c_0^C) + \beta \log(c_1^C) - \eta$$

s.t. $$c_0^C + c_1^C \frac{1}{1+r} + (1-gd)d = \frac{wh_1(1-\tau)}{1+r} + \bar{d}.$$
\[ h_1 = h_0 + ah_0^\phi d^v \]  

(C.4)

Solving the above problem yields:

\[
V_{NC}^{NC} = 2 \log(c_{NC}^{NC}) \\
V^C = 2 \log(c_C^0) - \eta \\
d = \left( \frac{awv h_0^\phi}{(1+r)(1-gd)} \right)^{\frac{1}{1-\nu}} (1-\tau)^{\frac{1}{1-\nu}}.
\]  

(C.5)  
(C.6)  
(C.7)

I can now find the cutoff for disutility \( \eta^* \) in terms of ability \( a \) and initial human capital \( h_0 \) that makes the individual indifferent for going to college. From the solution to both problems I know

\[ 2 \log \left( \frac{c_C^0}{c_{NC}^{NC}} \right) = \eta^*. \]  

(C.8)

This means that if the individual satisfies \( \eta < \eta^* \), she chooses college and otherwise, she starts working from the start of the first period. Therefore, the cutoff value depends on the ratio of consumption in the first period during college relative to no-college path. Solving for consumption functions, I get:

\[
\frac{c_C^0}{c_{NC}^{NC}} = \left( \frac{1-\nu}{2+r} \right) \left( 1 + \left( \frac{wav}{(1+r)(1-gd)} \right)^{\frac{\nu}{1-\nu}} h_0^{\frac{\nu+\phi-1}{1-\nu}} (1-\tau)^{\frac{\nu}{1-\nu}} \right) + \frac{\bar{d}(1+r)}{(2+r)wh_0(1-\tau)}. 
\]  

(C.9)

Combining with equation (C.8) yields the expression in the main text.

C.3.2 Solution of the Model

The algorithm to compute a steady-state equilibrium for the model with a taxation system \((\lambda, \tau)\), given all model parameters, is outlined below.

1. Guess \((K/L, Tr)\). Calculate \( r = F_1(K/L, 1) - \delta \), and \( w = F_2(K/L, 1) \).
2. Solve the decision rules for every grid point for all stages of the life-cycle, both for college and non-college.
3. Simulate 10000 shock histories for every tuple of initial conditions \((a, h_0, \eta)\) using the decision rules calculated in step 2.
4. Calculate \((K'/L', Tr')\) implied by the simulation. If \( K/L = K'/L' \) and \( Tr' = Tr \) up to a tolerance, then stop. Otherwise, update the guess and repeat 1-3.
C.3.2.1 Solving the decision rules for the working period

For the working period, the problem of an individual with or without college is as follows. The state variables are age, learning ability, human capital, assets, and the earnings shock. Let the vector of state variables be $\Theta = (j, a, h, x, z)$.

$$V(\Theta) = \max_{c,l,s,h,x} \left\{ \log c - B(l + s) \frac{(1 + \frac{1}{\gamma})}{1 + \frac{1}{\gamma}} + \beta \mathbb{E}[V(\Theta')|z] \right\}$$  \tag{C.10}

$$c = (1 + r)x - x' + w(zh)l$$  \tag{C.11}

$$h' = (1 - \delta_h)h + a(hs)^\phi$$  \tag{C.12}

The associated Lagrangian is

$$\mathcal{L} = \log c - B(l + s) \frac{(1 + \frac{1}{\gamma})}{1 + \frac{1}{\gamma}} + \beta \mathbb{E}[V(\Theta')|z] + \lambda[(1 + r)x - x' + w(zh)l - c] + \mu[(1 - \delta_h)h + a(hs)^\phi - h']$$  \tag{C.13}

The set of first order conditions are:

$$\frac{1}{c} = \lambda$$  \tag{C.14}

$$B(l + s)^\frac{1}{\gamma} = \lambda w(zh)$$  \tag{C.15}

$$\beta \mathbb{E}[V_x(\Theta')|z] = \lambda$$  \tag{C.16}

$$\beta \mathbb{E}[V_h(\Theta')|z] = \mu$$  \tag{C.17}

$$B(l + s)^\frac{1}{\gamma} = \mu a\phi h(hs)^{\phi - 1}$$  \tag{C.18}

The Envelope conditions are:

$$V_x(\Theta) = \lambda(1 + r)$$  \tag{C.19}

$$V_h(\Theta) = \lambda wzl + \mu[(1 - \delta_h) + a(hs)^{\phi - 1}]$$  \tag{C.20}

Combining (C.14), (C.16), and (C.19), we get the consumption Euler equation:

$$\frac{1}{c} = \beta(1 + r)\mathbb{E}\left[\frac{1}{c(\Theta')}|z]\right]$$  \tag{C.21}

Combining (C.14) and (C.15) gives the intratemporal labor supply equation:

$$B(l + s)^\frac{1}{\gamma} = \frac{w(zh)}{c}$$  \tag{C.22}
From (C.15) and (C.17) we get

\[
\lambda = \frac{B(l + s)^{\frac{1}{\gamma}}}{w(zh)} \quad \text{(C.23)}
\]

\[
\mu = \frac{B(l + s)^{\frac{1}{\gamma}}}{a\phi h(hs)^{\phi-1}} \quad \text{(C.24)}
\]

Combining (C.18) and (C.20) with (C.23) and (C.24) we get

\[
\frac{B(l + s)^{\frac{1}{\gamma}}}{a\phi h(hs)^{\phi-1}} = \beta E\left[ \frac{wz'l'}{c'}|z \right] + \beta E\left[ \frac{B(l' + s')^{\frac{1}{\gamma}}}{a\phi h'(h's')^{\phi-1}}[(1 - \delta_h) + as'(h's')^{\phi-1}]|z \right] \quad \text{(C.25)}
\]

For solving for the optimal choices, one can use the following algorithm:

- **Step 1:** Choose a value for \( s \). That gives a value for \( h' \).
- **Step 2:** Choose a value for \( x' \). Together with a choice for \( h' \), we know \( \Theta' \). Using (C.21), we know \( c \).
- **Step 3:** Knowing \( \Theta' \), we can calculate the RHS of (C.25). Let’s call this value \( \Gamma \). This gives one equation in one unknown which is \( l \):

\[
l = \min \left\{ \max \left\{ \left( \frac{\Gamma a\phi h(hs)^{\phi-1}}{B} \right)^{\gamma} - s, 0 \right\}, 1 \right\} \quad \text{(C.26)}
\]

- **Step 4:** Using the budget constraint, we can update the choice of \( x' = (1 + r)x - w(zh)l - c \).
- **Step 5:** Using (C.22), we can update the choice for \( s \):

\[
s = \min \left\{ \max \left\{ \left( \frac{wzh}{B} \right)^{\gamma} - l, 0 \right\}, 1 \right\} \quad \text{(C.27)}
\]

With the updated choices in hand, we go to step 1, until all the equations are satisfied.
C.3.3 A Stationary Competitive Equilibrium

The economy has an overlapping generations structure. The fraction \((\mu_j)\) of age \((j)\) individuals in the economy satisfies \(\mu_{j+1} = \mu_j/(1+n)\), where \((n)\) is the population growth.

At a point in time, individuals are heterogeneous in their age \((j)\) and their individual state \(\theta\). The distribution of age \((j)\) individuals across individual states \(\theta\) is represented by a probability measure \(\lambda_j\) defined on subsets of the individual state space \(\Theta\). The individual state at age \((j)\) is defined as:

\[
\theta = \left( \frac{\theta_1}{\theta_2}, h, x, z; a, \eta \right)
\]

where \(\mathbb{1}^c\) is an indicator function which equals 1 if the individual chooses the college path, and 0 otherwise. \(\theta_1\) consists of college choice, human capital stock, asset holding, and shock to the stock of human capital, and \(\theta_2\) consists of learning ability and disutility for college. \(\theta_1\) evolves based on optimal choices, while \(\theta_2\) is constant over the life-cycle.

Let \((\Theta, \Gamma(\Theta), \psi_j)\) be a probability space where \(\Theta = \{0,1\} \times [0, \infty] \times (-\mathcal{X}, \infty) \times Z \times (0, \infty), [0, \infty]\) is the state-space, \(Z\) is the support of shocks, \(\mathcal{X}\) is the absolute value of the lower bound of natural borrowing constraint, and \(\Gamma(\Theta)\) is the Borel \(\sigma\)-algebra on \(\Theta\). Thus, for each set \(\Gamma\) in \(\Gamma(\Theta)\), \(\psi_j(\Gamma)\) represents the fraction of age \((j)\) individuals whose states lie in \(\Gamma\) as a proportion of all age \((j)\) individuals. These agents then make up a fraction \(\mu_j \psi_j(\Gamma)\) of all agents in the economy. The distribution of age \(j_h\) individuals is determined by the initial distribution over learning ability, initial human capital, and disutility for college. The distribution of age \(\{j_h+1, \ldots, j_d\}\) individuals are then given recursively as follows:

\[
\psi_{j+1}(\Gamma) = \int_{\Theta} P(\theta, j, \Gamma) \ d\psi_j.
\]

The function \(P(\theta, j, \Gamma)\) is a transition function which gives the probability that an age \((j)\) individual transits to the set \(\Gamma\) next period, given that the individual’s current state is \(\theta\). The transition function is determined by the optimal decisions.

The variables \((K, L, C, T, SS, D)\) are aggregate quantities of capital, labor, consumption, taxes, social security transfers, and total subsidies/transfers for college. Finally, \(T_j(\theta)\) is the total taxes paid by individuals at state \(\theta\) at age \((j)\).

Aggregate variables are calculated using individuals’ choices:

\[
K = \sum_{j=j_h}^{j_d} \mu_j \int_{\Theta} x_j(\theta)d\psi_j
\]
\[ L = \sum_{j=j_h}^{j_r-1} \mu_j \int_\Theta z_j(\theta) h_j(\theta) l_j(\theta) d\psi_j \]

\[ C = \sum_{j=j_h}^{j_d} \mu_j \int_\Theta c_j(\theta) d\psi_j, \quad T = \sum_{j=j_h}^{j_d} \mu_j \int_\Theta T_j(\theta) d\psi \]

\[ SS = ss \sum_{j=j_r}^{j_d} \mu_j \int_\Theta d\psi_j, \quad D = \sum_{j=j_h}^{j_w-1} \mu_j \int_\Theta (g_d d_j(\theta) + \bar{d}) \psi_j \]

**Competitive equilibrium.** A steady-state stationary competitive equilibrium is a collection of decisions \( \{c_j, l_j, s_j, h_j, x_j, d_j\}_{j=j_h}^{j_d} \), factor prices \( \{w, r\} \), government spending, taxes, social security transfers, and college subsidies/transfers \( \{G, T, SS, D\} \), and distributions \( (\psi_{j_h}, \ldots, \psi_{j_d}) \) such that

1. Agent decisions are optimal, given factor prices.
2. Distributions are consistent with individual behavior:
   \[ \psi_{j+1}(\Gamma) = \int_\Theta P(\theta, j, \Gamma) d\psi_j, \quad \forall j \in \{j_h, \ldots, j_d\}, \quad \forall \Gamma \in \Gamma(\Theta). \]
3. Competitive factor prices: \( r = AF_1(K, L) - \delta \), \( w = AF_2(K, L) \).
5. Resource Feasibility: \( C + (n + \delta)K + G = F(K, L) \).