Small-Scale Hybrid Rocket Test Stand & Characterization of Swirl Injectors

By

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ABSTRACT

Derived from the necessity to increase testing capabilities of hybrid rocket motor (HRM) propulsion systems for Daedalus Astronautics at Arizona State University, a small-scale motor and test stand were designed and developed to characterize all components of the system. The motor is designed for simple integration and setup, such that both the forward-end enclosure and end cap can be easily removed for rapid integration of components during testing. Each of the components of the motor is removable allowing for a broad range of testing capabilities. While examining injectors and their potential it is thought ideal to obtain the highest regression rates and overall motor performance possible. The oxidizer and fuel are N_2O and hydroxyl-terminated polybutadiene (HTPB), respectively, due to previous experience and simplicity. The injector designs, selected for the same reasons, are designed such that they vary only in the swirl angle. This system provides the platform for characterizing the effects of varying said swirl angle on HRM performance.

DEDICATION

I would like to dedicate this work my wife, Sarah Summers, for her continued support and encouragement, especially in those times when she was 9 months pregnant or caring for our new born son, Parker Summers.

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NOMENCLATURE

$\dot{r}$	= regression rate
$\dot{m}_n$	= mass flow through nozzle
G_{ox}	= oxidizer mass flux
P_c	= combustion chamber pressure
T	= temperature
A^*	= nozzle throat area
γ	= specific heat ratio
M_e	= exit Mach number
M	= molar mass
P_a	= atmospheric pressure
$\dot{m}_{ox}$	= oxidizer mass flow
v_e	= exit velocity
$\dot{m}_f$	= fuel mass flow
F	= thrust
R	= specific gas constant
v_0	= chamber volume
C^*	= characteristic exhaust velocity
ρ_0	= chamber gas density
T_e	= exit temperature
ρ^*	= density at nozzle throat
I_{sp}	= specific impulse
v^*	= velocity at nozzle throat

σ = stress

ρ = density of HTPB

t = thickness

A_{port} = combustion port area

r = radius

1 Introduction

1.1 Background

Hybrid rocket propulsion systems are defined as a propulsion system in which “one propellant component is stored in liquid phase while the other is stored in the solid phase”(Sutton & Biblarz, 2010). The two separate components of the propellant are most commonly split into the categories of an oxidizer and a fuel. The difference between the two is quite intuitive, where the oxidizer provides the oxygen required to sustain a combustion process, while the fuel operates as some medium to be burned away throughout the combustion process. Because the definition of a hybrid rocket propulsion system is so general, this allows for a large number of combinations of oxidizers and fuels, where both can range from states of solid to liquid and liquid to solid, respectively.

Some of the common oxidizers that have been employed throughout the development of hybrid rocket propulsion systems are liquid oxygen, hydrogen peroxide, and nitrous oxide. Liquid oxygen, or O_2 , is most often considered for large hybrid rocket motor applications. “Liquid oxygen is a widely used oxidizer in the space launch industry, is relatively safe, and delivers high performance at low cost,” (Sutton & Biblarz, 2010). Hydrogen peroxide, or H_2O_2 , is an alternative to liquid oxygen that offers some benefits such as a much higher boiling point, increased density, and non-cryogenic storage designs. However, hydrogen peroxide does not solve the high complexity or cost issues that are similarly inherent with liquid oxygen. Though both liquid oxygen and hydrogen peroxide provide benefits of higher performance, they introduce additional hurdles that can complicate a design that is meant to remain low-cost. The final mentioned oxidizer

choice that is often employed in these hybrid rocket propulsion systems is nitrous oxide. Nitrous oxide does require the sacrifice of theoretical I_{sp} (also known as I_s), as can be seen in Figure 1. Theoretical Vacuum-Specific Impulse of Selected Oxidizers reacted with Hydroxyl-Terminated Polybutadiene vs O/F Mixture Ratio.

Figure 1. Theoretical Vacuum-Specific Impulse of Selected Oxidizers reacted with Hydroxyl-Terminated Polybutadiene vs O/F Mixture Ratio

Some of the common fuels that have been employed throughout the development of hybrid rocket propulsion systems are hydroxyl-terminated polybutadiene and paraffin. Paraffin is a solid form fuel that is most often used in designs where a higher regression rate of the fuel is desired. This is because of the fact that paraffin will transition into a gel or liquid form relatively rapidly once the combustion chamber begins to heat up. However, for most applications paraffin regresses too quickly, and the more temperature-stable hydroxyl-terminated polybutadiene is preferred. Similar to “solid rocket motors, HTPB is used as the binder to consolidate the aluminum fuel and ammonium perchlorate oxidizer matrix. In a hybrid, HTPB becomes the entire fuel constituent. HTPB is low cost,

processes easily, and will not self-deflagrate under any condition,” (Sutton & Biblarz, 2010).

1.2 Advantages of Hybrid Rocket Motors

Hybrid rocket motor propulsion systems are becoming more common now as they continue to be identified as the ideal solution to all the overall system requirements. The main advantages of hybrid rocket motor propulsion systems, as described in Sutton’s 8th Edition of Rocket Propulsion Elements, are: (1) enhanced safety from explosion or detonation during fabrication, storage, and operation; (2) start-stop-restart capabilities; (3) relative simplicity which may translate into low overall system cost compared to liquids; (4) higher specific impulse than solid rocket motors and higher density-specific impulse than liquid bipropellant engines; and (5) the ability to smoothly change thrust over a wide range on demand.

1.3 Disadvantages of Hybrid Rocket Motors

Hybrid rocket motor propulsion systems have their setbacks like any other propulsion technology. The main disadvantages of hybrid rocket motor propulsion systems, as described in Sutton’s 8th Edition of Rocket Propulsion Elements, are: (1) mixture ratio and hence specific impulse may vary during steady-state operation (as well as during throttling); (2) relatively complicated fuel geometries with significant unavoidable fuel residues (slivers) at end of burn, which somewhat reduces the mass fraction and can vary if there is random throttling; (3) prone to large-amplitude, low-frequency pressure fluctuations (termed chugging); and (4) relatively complicated internal motor ballistics resulting in incomplete descriptions.

Of these four disadvantages, this research project sought to improve upon the incomplete descriptions of complicated internal motor ballistics. Of the different key variables that influence the internal motor ballistics, this research project focuses on the method of N_2O oxidizer injection into the combustion chamber, and characterizing how varying the angle of the radial, or swirl, injection ports effects the regression rate of the solid fuel HTPB grain.

As this research project progressed, it was found that a better understanding of the large-amplitude, low-frequency pressure fluctuations could be accomplished as well. As a result, this research project and results briefly discuss how these injection methods can be adopted to either reduce or eliminate these combustion instabilities.

1.4 Previous Research on Swirl Injectors

Some previous research has been completed in the area of swirl injectors in these hybrid rocket motor applications. The first of which was by Justin Pucci at Arizona State University in 2002. Pucci investigated the effects of these swirl injector designs on the hybrid flame-holding instability, and at the same time took a step towards better understanding the effects of these injectors on motor performance.

For Pucci's investigation four injector designs were employed, an axial injector, a radial injector, a 30-degree-swirl injector, and a 60-degree-swirl injector. Pucci was able to create conditions during the experiments which indicated that the combustion instabilities that had previously plagued hybrid rocket motor propulsion systems is something that is limited to certain types of injector designs. In fact, he concluded "that a super-critical swirl flow does produce stable combustion possibly due to the establishment of a central toroidal recirculation zone (CTRZ). This pre-heats the

incoming oxidizer and stabilizes the flame sheet, thus preventing the existence of the flame-holding instability,” (Pucci, 2002). Additionally, these same super-critical swirl flow injector design types were “found to increase regression rate by 182% over the comparatively stable axial flow case. This is due to the increase in effective mass flux and, therefore, convective heat transfer,” (Pucci, 2002). These results were additionally confirmed by the continued use of 60° super-critical injectors by Dennis at Arizona State University, (Dennis, Shark, & Hernandez, 2009).

Additional research focusing on the effects of swirl injectors was completed by a group at the University of Padua in Padova, Italy (Bellomo, et al., 2012). Their research focused primarily on tangential vortex injection in a hybrid rocket motor. To compare a tangential injection method to a swirl injector as this experiment considers it, the injector design would have an injection angle of 90°. Using these injectors they were able to show similar results to that previously completed by Pucci and Dennis where “vortex injection lower[ed] the chamber pressure oscillations [with] respect to [the] axial case from more than 7% down to 4%. Moreover, regression rate [was] increased [by] 41% and the a coefficient of its law up to 67% from axial,” (Bellomo, et al., 2012). After an extensive study, consisting of 66 tests, they proposed the following statement. “It can be said, then, that for the regression rate performance parameter the most important factor is the injection type,” (Bellomo, et al., 2012).

Though not the same application, swirl injectors are of a growing interest in injection systems used for gas turbines, where again the interest is primarily in making us of super-critical swirl flows, or higher swirl injection angles (Littlejohn & Cheng, 2010). One of the key things similarities between the results presented by Littlejohn and Cheng

in their particle image velocimetry (PIV) plots is that they too were able to identify a distinct recirculation zone. This is noteworthy because it further confirms the importance of employing these swirl injection methods in combustion applications, where the generation of a recirculation zone can help mitigate instabilities thus improving performance and reliability of the combustion system.

The growing interest in swirl injectors for both the use in hybrid rocket motors and other applications implies the need for continued research in this area. Once fully characterized, swirl injectors could be used to tailor hybrid rocket motor designs by adjusting swirl injection angle to provide variable regression rate, oxidizer mass flux, which in turn provides variable thrust and performance. Test Setup & Design Process

1.5 Background

Derived from the necessity to increase testing capabilities of hybrid rocket motor (HRM) propulsion systems for Daedalus Astronautics at Arizona State University, a small-scale motor and test stand were designed and developed to characterize all components of the system. The motor is designed for simple integration and setup, such that both the forward-end enclosure and end cap can be unscrewed and removed for rapid integration of components during testing. Each of the components of the motor is removable allowing for a broad range of testing capabilities. While examining injectors and their potential it was thought ideal to obtain the highest regression rates and overall motor performance possible. The oxidizer and fuel were selected to be N_2O and hydroxyl-terminated polybutadiene (HTPB), respectively due to previous experience and simplicity. The oxidizer/fuel (O/F) ratio for the motor was selected to be 8.1, for a motor using this oxidizer/fuel combination, as is determined from previous work.² Shown in Fig.

1, all components of the motor except for the motor grain and nozzle are made of aluminum, and designed to withstand loads during testing with a minimum margin of safety (M.S.) of 0.5-1. The nozzle was designed from Super Fine Isomold Graphite to reduce throat erosion, while maintaining structural integrity.

Figure 2. Small-Scale N₂O/HTPB Hybrid Rocket Motor

The test stand for the small-scale hybrid rocket motor is equipped with three pressure transducers, a venturi, pressure gauges, load cell, scale, and a DATAQ DI-718B data acquisition system. Two of the pressure transducers and the venturi operate to provide calibration results for the test stand as a means of measuring the mass flow rate into the injector via a measurement of the difference in pressures. In order to double check the initial calibration values obtained from the venturi, a scale will be used to measure the amount of mass leaving the tank per second and from this determine an approximate mass flow rate. The third pressure transducer operates as the measuring tool for providing real-time combustion chamber pressure which will be displayed simultaneously with real-time thrust from the load cell. Finally, the DATAQ DI-718B is used to consolidate the acquired data from the experiments and build up a useful series of data sets.

The first performance characteristic to be investigated is swirl injectors and their potential to increase the efficiency of hybrid rocket motors. This investigation was influenced by the past work of both Justin Pucci and Jacob Dennis at ASU on hybrid rocket motor propulsion systems.^{1,2} Preliminary investigation consisted of performing computational fluid dynamic (CFD) analysis on the swirl injectors to identify and characterize flow structures, such as the central toroidal recirculation zone (CTRZ). Further investigation provided complete characterization of sub-critical and super-critical swirl flows, and their effects on stability and performance. For reference, swirl numbers and swirl injectors types are classified shown in Figure 3 and Equation 1.

Figure 3. Definition and Classification of Swirl Injectors

$$S = \frac{2}{3} \tan \phi, \text{ where } \phi = \text{swirl angle}$$

Equation 1. Swirl Number Equation

Equation 1 solves for the established value, S , that characterizes swirl flow. Similar to the values shown in Figure 3 the following classifications can be made. If $S < 0.60$, then the injector is classified as having sub-critical swirl flow. If $S = 0.60$, then the injector is classified as having critical swirl flow. If $S > 0.60$, then the injector is classified as having super-critical swirl flow. These classifications help the designer to better understand the threshold for inherent injection-related combustion instabilities.

1.6 Objective / Intent

The objective is to characterize the effects of the swirl angle on the regression rate of hybrid rocket motor fuel grains. This study is focused primarily on the effects of N_2O injected into a combustion chamber with a HTPB solid fuel grain.

HTPB and N_2O were selected primarily because that is what has been used by Daedalus Astronautics at ASU in the past, and ideally the results of this research would directly correlate to the current hybrid rocket motor designs under way by the organization, thus further enabling their design and development capabilities as an organization. That being said, HTPB was initially selected “for manufacturing ease and availability. Daedalus Astronautics at ASU has a well established Solid Rocket Motor (SRM) program which also utilizes HTPB. General familiarity with the material eliminates the learning curve which would be present with a new fuel. N_2O is commonly used in both hobby hybrid rocket engines as well as the performance vehicle market. Performance associated with N_2O is not as high as other possible [oxidizer] choices such as H_2O_2 , but it has the benefit of being self pressurized which simplifies the overall engine [and test stand] design. HTPB and N_2O are both inert when separate and therefore require no special licenses or storage considerations,” (Dennis, Shark, & Hernandez, 2009).

1.7 Commercial Off-The-Shelf Components

In order to reduce cost, time, and overall effort associated with designing and fabricating custom components, many of the parts used in this design were purchased as commercial off-the-shelf components. In fact, the only parts that required significant

machining or processing were the motor casing, pre- and post-combustion chambers, forward end enclosure, and the fuel grains. All other components were identified as key candidates that met the established design requirements, and were then assembled or installed.

1.8 Sizing of Components

Using much of the work performed by Jacob Dennis, Steven Shark, Felipe Hernandez, and James Villarreal at Arizona State University as a baseline, some of the components were designed to be a half-scale version of motor they designed. That being said, a lot of design work was still completed to verify the theoretical performance of this specific hybrid rocket engine, because the performance of these motors isn't always so straight forward from one configuration to the next. A good summary of this difficulty is: “[d]imensional analysis and similarity theories are designed to generalize experimental results by casting the variables in suitable dimensionless forms but the most satisfactory approach is the one where theory and experiment go hand in hand,” (Sutton & Biblarz, 2010). This has been a key point of interest in the hybrid research community as researchers work to design scalable models for these combustion processes (Chelaru, Vasile, Florin, & Ion, 2011).

The physical dimensions of this motor were initially driven by the desire to maintain a half-scale version of the flight motor. The purpose of this was to minimize any scaling or design variations between the two configurations. This way motor improvements and components could be tested and characterized on this small-scale motor, and then related back to the larger motor.

The baseline dimensions of the large hybrid rocket motor were selected based on the readily available standard tube size with a 4 inches internal diameter, and a 0.25 inches wall thickness. “Length of the final motor casing was dictated by oxidizer mass flow rate and material availability. Initial calculations showed that a long casing and in turn solid fuel grain would result in an O/F which was well below ideal. A shorter grain has less surface area in the port and thus a higher O/F with a constant oxidizer mass flow rate. These constraints were used to find geometries to be used in the analytical design of the engine,” (Dennis, Shark, & Hernandez, 2009).

Continuing on this previous work, the following design method was followed to design a new smaller scale motor that uses similar key combustion parameters, and can provide results that are applicable or scalable to the larger motor. The first of these key combustion parameters are provide in Table 1.

Table 1. HTPB/_{N2O} Combustion Parameters (Dennis, Shark, & Hernandez, 2009)

Parameter	Value
Combustion Temperature (K)	3301
Combustion Pressure (psi)	500
Ambient Pressure (psi)	14.7
C* (ft/s)	16878.9
γ	1.244
O/F Ratio	8.1

Using these key combustion parameters, and selecting many of the internal diameter dimensions based on what commercial off-the-shelf components are available, the following Equation 2 to Equation 5 were used to size the minimum required material wall thickness for the combustion chamber.

$$\sigma_{von\ Mises} = \sqrt{0.5[(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2]}$$

Equation 2. Von Mises Failure Criterion Equation

$$\sigma_1 = \frac{P_c r}{t}$$

Equation 3. Hoop Stress Equation for Tubular Bodies

$$\sigma_2 = \frac{P_c r}{2t}$$

Equation 4. Axial Stress Equation for Tubular Bodies

$$\sigma_3 = 0$$

Equation 5. Perpendicular Stress for Tubular Bodies (Zero for This Case)

The Von Mises Failure Criterion of and its associated equations were used to calculate the minimum wall thickness because rocket motor combustion chambers require their design to meet requirements of the failure condition plus some amount of margin. In fact, it's partly because of the fact that the initial combustion chamber designs for this small-scale test stand didn't have a significant margin per this criterion that the combustion chamber was redesigned to fit securely in the steel collars on the moveable thrust sled. The final calculated stress calculations can be found summarized in Table 2.

Table 2. Summarized Combustion Chamber Stress Calculations

Parameter	σ_1 (psi)	σ_2 (psi)	σ_3 (psi)	$\sigma_{von\ Mises}$ (psi)	$S_y, Al6061$ (psi)	F.O.S.
Value	2625	1312.5	0	2273.3	3.7×10^4	16.4

All of the listed equations are function of stagnation pressure, maximum radius, wall thickness, and are valid for $\frac{r}{t}$ ratios of less than 10, and the ratio is equal to 5.25 for this design. As is shown in Table 2, the final calculated Factor Of Safety (F.O.S.) was 16.4, which more than met the minimum safety requirements of 0.5-1.0 for using this hybrid rocket motor on the ASU campus for preliminary testing and experiments.

The oxidizer mass flow rate was estimate to be 0.5 lbf/s based on some preliminary testing. Using this estimated value and knowing the fuel grain geometry, the oxidizer mass flux can be calculated using Equation 14. Using this newly calculated oxidizer mass flux and an estimated optimum O/F ratio of 8.1, the baseline regression rate for the hybrid rocket motor design can be calculated using Equation 14.

$$\dot{r} = a \left(G_{ox} (1 + O/F) \right)^n$$

Equation 6. Baseline Regression Rate Equation

With the newly calculated baseline regression rate for the design, and again the fuel grain geometry the fuel mass flow rate can be calculated using Equation 7.

$$\dot{m}_f = A_{port} N \rho_f \dot{r}$$

Equation 7. Fuel Mass Flow Rate Equation

Using Equation 7 and Equation 13 the total mass flow rate of the system can be calculated by adding both the mass flow rate of the fuel and the mass flow rate of the oxidizer, as shown in Equation 8.

$$\dot{m} = \dot{m}_f + \dot{m}_{ox}$$

Equation 8. Total Mass Flow Rate Equation

The theoretical thrust of the overall hybrid rocket engine was calculated using the following Equation 9, where the exit Mach number comes from Equation 10.

$$F = \dot{m} \left(M_e \sqrt{\gamma R T} \right)$$

Equation 9. Thrust Equation

$$M_e = \left[\frac{2}{\gamma - 1} \left[\frac{P_c}{P_a}^{\frac{\gamma-1}{\gamma}} - 1 \right] \right]^{\frac{1}{2}}$$

Equation 10. Exit Mach Number Equation

1.9 Design Validation

Aided by the finite-element numerical methods of Solidworks Simulation, CFD analysis provided the exploitation of the transient, Central Toroidal Recirculation Zone (CTRZ), and combustion states of oxidizer flow. Transient startup of the HRM initiated the construction of the flow structures, where the radius and strength of the vortices induced a further acceleration of the flow as well as an increase in oxidizer interaction with the grain geometry. The establishing flow structure rapidly develops into a CTRZ, where pre-heating and atomization of the oxidizer occur, as shown in Fig. 4. This analysis showed that injectors with higher swirl numbers produce vortices with higher rates of rotation. From this, we can say that higher rates of rotation produced from super-critical swirl flow injectors is directly related to the increase in regression rate of the motor grain. This correlation between higher rotation rates and regression rates provides some critical insight into the driving variables behind atomization processes within the active combustion zone, shown in Fig. 3.

Figure 4. Description of Active Combustion Zone & General O/F Interactions

Similarly, by comparing the theoretical analysis of flow and combustion structures to numerical and experimental results, detailed characteristics can be made about the relationship between swirl numbers or injection angles and their effects on different components of the HRM. Once each aspect of these vital parts of hybrid rocket motor and its mechanics is better understood it will be possible to synthesize this understanding into the design of future hybrid rocket motors with the best of each integrated component for a particular mission. For example, military applications require rapid ascent and response of their missiles, but currently HRM technologies can't support these mission requirements. By truly understanding how each component of the motor works, future developments in HRM technologies could lead the way for providing more efficient and effective propulsion systems than even current solid rocket motor systems.

The following Figure 5 are some plotted numerical results from a basic and low-level model within the Solidworks flow simulation package. The purpose of this low-level modeling was to investigate whether or not the CTRZ flow structure discussed in literature could be easily and quickly identified.

Figure 5. (Left) Density Distribution in CTRZ Swirl Flow Structure, (Right) Constant Pressure Distribution in HRM

An additional plot, shown here as Figure 6, that was generated from the output of the same simulation was that of the temperature distribution down the full length of the

internal combustion volume. An interesting feature that was identified early on during these simulations was the uneven temperature distribution along the fuel grain. If this simulation is correct, then later experimental testing would require measurements of regression rate down the length of each fuel grain to identify whether or not uneven burning and non-constant regression rate down the length of the grain is actually occurring during the combustion process.

Figure 6. Temperature Distribution of CTRZ and Flow Inside HRM

1.10 Test Stand Overview

The test stand was originally designed by Robert Lanphear, another ASU Graduate Student, and I at Arizona State University. However, the test stand and a few other components were later modified to make the overall system more robust for large amounts of consecutive tests. The modifications included a more robust combustion chamber, a timer relay, stainless steel pipe, Swagelok fittings, and the design of a standard initiator. These modifications resulted in the final test stand design that is shown in Figure 7.

Figure 7. Final Small-Scale Hybrid Test Stand Setup

The more robust combustion chamber with thicker walls was built to reduce the heat transfer across the walls, and enable to person setting up in between static fires to handle the hardware. The timer relay, shown in the upper-most right corner of the test stand in Figure 7, is accurate to the thousandth of a second for controlling power to a given circuit. It was used for controlling the power to the solenoid, shown in the bottom-most right corner of the test stand in Figure 7, and thus the actuation of oxidizer flow start and stop, with great precision. The stainless steel pipe and Swagelok fittings were added to the test stand to reduce the need to make any adjustments to fittings, and make the entire design more rigid. Conversely, a stainless steel braided hose was still used for the connection between the line leaving the venturi meter and the hybrid rocket motor. This allows the motor to remain mobile along it's guide tracks during motor fire, and removed the concern of preventing any force from being applied to the load cell, which is located just to the right of the hybrid rocket motor sled in Figure 7.

1.11 Electrical Setup

The electrical setup for this system was designed specifically for this application, and enables the user to test remotely if necessary. All of the electrical subcomponents are powered by the two car batteries wired in series with the exception of the DATAQ DI-718 and the laptop. However, because only these two subcomponents required AC power, a converter located in a car was used to provide power during testing.

Typically a test setup like this can be accommodated by a single 12 VDC car battery; however, because of the high current and voltage draw in channel 3, as indicated in Figure 10, a second battery was required. During the early electrical system design phases, actual power supplies were considered; however, the low current ratings and dependency on another AC power outlet made them poor candidates for the needs of this test setup. The two combined batteries provide a 24 VDC power supply, capable of meeting any current needs for the test setup, and the setup can be seen below in Figure 8.

Figure 8. Side View of Final Small-Scale Hybrid Test Stand Setup

The Power/Circuit Box as shown in Figure 11 provided power to components that are integrated into three independent circuits that can be switched on and off via physical switches located on the Manual Control Box. This manual control box was designed with a 200 foot Ethernet wire connection, allowing for the user to control the electronics of the test stand from a safe and remote location. The green and white wires shown in the figure represent Signal Positive and Signal Negative wire connections, respectively, between the individual sensors and the data acquisition system. The red and black wires in the figure represent the Power Positive and Power Negative connections, respectively, between each of the powered components.

Another key feature worth mentioning is the series of stationary wire connections that are screwed down to the surface of the test stand, as shown in Figure 9 below.

Figure 9. Zoom-In Image of Figure 6, Wire Connections, Analog Pressure Sensor/N₂O Inlet from Tank, and Timer Relay (from left to right)

Figure 10. Top-Level Overview of Electrical Setup

In order to record and save the data, the data acquisition system had to be connected to a dedicated laptop via a USB cable. This setup and the commercially available DATAQ software known as WinDaq operated as the user-interface for recording all data. As indicated in Figure 10 this data acquisition setup was the only sublet of the overall test setup that required an AC power outlet.

It's important to note that the Power/Circuit Box has a relatively high number of input/output lines, and this is because this was the central power hub for all components. As seen in Figure 11, this box provided all of the power inputs and outputs to the test setup. For the purposes of this experiment, 24 V power was supplied to the indicated 12 V Power outlet on the left. This additional power allowed the use of more power-demanding circuits on the different indicated Igniter Channels. More specifically,

Channel 1 provided the 24 V power to the actual igniter to begin the pre-heating of the hybrid rocket motor, while providing a steady flame for ignition of the motor. For additional safety, this channel had a built-in switch that only held the open position when the user applied a constant upward force to hold the switch open. Channel 2 provided the 24 V power to the array of pressure transducers for excitation voltage. Because this channel wasn't wired to any energetic components it was not designed with a switch that switched by default was on the "off" position. Instead, the switch for channel two was simpler in that it held either open or close position after it was pushed into place. Channel 3 provided the 24 V power to the Timer Relay and Oxidizer Solenoid circuit. Similar to channel 1, this channel had a built-in switch that only held the open position when the user applied a constant upward force to hold the switch open. The Timer Relay was pre-programmed for a timed open position to allow the solenoid to energize.

Figure 11. Power/Circuit Box Overview (Dennis & Villarreal, 2010)

Tethered from the Power/Circuit Box is again the Manual Control Box, which operated as just what the name implies, it provided manual control of the entire test. This manual control box only energized the circuits when the user wanted them to, and had a series of safety controls built into the design for robustness.

Figure 12. Manual Control Box Overview (Dennis & Villarreal, 2010)

1.12 Sensor Selection

The sensors for this experiment were selected according to what were both readily available within the Daedalus Astronautics at ASU lab, and met the requirements of the planned tests. The Daedalus Astronautics at ASU student organization did have a group of sensors readily available to support this effort. Among those sensors, the following table consists of which of those particular sensors met the test requirements.

Table 3. Sensor Descriptions

Sensor Type	Brand / Model	Sensor Application	Measurement Capability	Quantity
-------------	---------------	--------------------	------------------------	----------

Data Acquisition	DATAQ / DI-718	Various	Various	1
Load Cell	FUTEK / LTH400	Compression Load	0 – 500 lbf	1
Pressure Transducer	Omegadyne / PX-309	Pressure	0 – 2000 psi	3

There were some concerns that using all of the pressure transducers on the same excitation power circuit might drop the excitation voltage across all pressure transducers enough to invalidate the results during testing. However, these concerns were addressed during initial test stand testing and checks.

For additional information or to review the specifications of the sensors used for this experiment, please see Appendix B.

2 Preliminary Testing

The series of experimental tests were performed at a secure test location in Tucson, AZ. This was made possible by the design of the test stand, since the hybrid rocket motor and all the necessary components for the tests were highly mobile.

2.1 Subsystem Testing

Preliminary subsystem testing began long before a full hot fire test was performed. Key subsystems were identified for the testing, and included cold flow testing, oxidizer flow rate checks, leak checks, sensor calibration, and electrical checks. This was critical for minimizing any future issues and schedule delays caused by an isolated subsystem failure.

The following test matrix describes what tests that were performed, what component was tested, what criteria determined a successful test, and the result of the test.

Table 4. Subsystem Test Matrix

Test Type	Component Tested	*Pass Criteria	Pass/Fail
Electrical Check	Solenoid	Actuate on Command	Pass
Electrical Check	Ignition System	Ignite on Command	Pass
Electrical Check	Sensors, DAQ	Read Pressure in Lines	Pass
Cold Flow Test	Injector System	Hold Seal, Provide Oxidizer Flow at Expected Mass Flow Rate	Pass
Cold Flow Test	Entire Motor	Hold Seal	Pass
Cold Flow Test	Venturi	Read Pressure, Measure In-Line Mass Flow	Pass
Mechanical Fit Check	Enclosures, Nozzles, etc.	All Components have as-designed interfaces	Pass
Mechanical Fit Check	Sensors	All Sensors interface as-designed	Pass
Data Check	DAQ	Verify the DAQ can record Data as-designed	Pass
Hot Fire Test	Entire Test Stand	Provide thrust, record data, sustain combustion, verify as-designed performance	Pass

2.1.1 Cold Flow Testing

The following table describes the initial cold flow testing that was performed.

Table 5. Cold Flow Test Matrix

Test No.	Description	$P_{\text{Tank},0}$ (psi)	Weight _{Tank,0} (lb)	Weight _{Tank,f} (lb)	t_{flow} (s)	$\dot{m}_{\text{ox}}$ (lb/s)	Comments
1	Axial Injector Test	690	170.8	170.6	4	0.05	Tank Closed, No Test
2	Axial Injector Test	690	170.6	169.8	1.8	0.45	Good Test
3	Axial Injector Test	675	169.8	169.0	2.8	0.3	Good Test
4	Axial Injector Test	660	169.0	168.4	2.3	<0.25	Leaks at Fittings, Unknown $\dot{m}_{\text{ox}}$
5	Axial Injector Test	665	168.0	167.4	2.5	~0.2	Small Leak at Fitting, Good Test
6	Swirl Injector Test	670	167.4	167.0	2.1	0.2	Good Test

Initial cold flow testing provided the opportunity to verify the design of the injector, forward enclosure, seal interface, snap-ring retention, and oxidizer flow rate

capability of the subsystem. By the end of the cold flow tests, each of these components was deemed acceptable for future testing.

As is common with N_2O , the flow was shown to be both temperature and pressure dependent across the series of test runs. Because it wasn't the goal to determine an exact mass flow rate capability, varying initial pressures and test times were acceptable. The two figures below are from this initial testing, and provide a good general sense of what the two different flow types look like. Shown in Figure 13 is the flow of the axial injector, which has all 16 injection ports located on the face of the injector. Shown in Figure 14 is the flow of the 60 degree swirl injector, which has 11 injection ports located on the face, and 5 additional injection ports equally spaced around the circumference of the injector.

Figure 13: Flow of Axial Injector Test

Figure 14: Flow of 60 deg. Swirl Injector

2.1.2 Venturi Testing

As previously stated, initial cold flow testing provided the opportunity to verify a number of integral components to the hybrid rocket motor, one of which is the venturi. The venturi is a carefully designed apparatus that accepts two pressure transducers for take in-line oxidizer pressure measurements, as shown in Figure 15. Each of these measurements is taken at pre-determined cross-sectional areas in the flow, thus allowing for the calculation of oxidizer mass flow rate.

Figure 15. Section View of Venturi Model

In order to calculate oxidizer mass flow rate, a few key equations were used. From what we know about fluid dynamics in control volumes, mass flow rate is a function of the fluid density, velocity, the cross-sectional area of pipe it is flowing through, and this relationship can be shown by the following equation, Equation 11 (Moran, Shapiro, Munson, & DeWitt, 2003).

$$\dot{m} = \rho AV = \rho Q$$

Equation 11. Equation for One-Dimensional Mass Flow Rate

Since both the areas and velocities of the flow cannot be assumed constant as it flows through a venturi, the mechanical energy equation was used to account for this. However since we can assume there are no losses to friction, no work is done to the control volume, both measurement points are in-line, the flow is steady, and density is constant, the mechanical energy equation simplifies to the following version of the Bernoulli equation.

$$p_1 - p_2 = \frac{\rho}{2}(v_2^2 - v_1^2)$$

Equation 12. Simplified Mechanical Energy Equation

By substituting Equation 11 in Equation 12, and then algebraically solving for the oxidizer mass flow rate, the following Equation 13 can be obtained. It's important to note that this equation solves for the oxidizer mass flow rate using only three input parameters of density, pressure, and cross-sectional area. Since we can calculate the density, measure the pressure, and know the cross-sectional areas from the design, this form of the oxidizer mass flow rate equation is ideal for this experiment.

$$\dot{m}_{ox} = A_1 \sqrt{\frac{2\rho(p_1 - p_2)}{\left(\left(\frac{A_1}{A_2}\right)^2 - 1\right)}}$$

Equation 13. Oxidizer Mass Flow Rate Equation Used with Venturi

To test both the oxidizer mass flow rate capability of the system, and the assumptions made for the previously mentioned equation to calculate oxidizer mass flow rate, cold flow testing of the system was completed. The following plot shows the rise transient, steady-state, and decay transient of the test. However, only the steady-state region between approximately 3 seconds and 6 seconds were used for any oxidizer mass flow rate measurements.

Figure 16. Sample Data from Preliminary Venturi Testing (Lugo, Bowerman, & Summers, 2012)

This initial test provided an average oxidizer mass flow rate measurement of 0.23 lb/s. For the purposes of this test the varying differences in pressure between the two pressure transducers readings wasn't critical; however, it was corrected prior to completing the final hot-fire tests to increase the accuracy and reliability of the results. Additionally, this test was performed at lower tank pressure and fill quantity conditions than is anticipated for the later experiment tests, and was only used as an initial validation test.

3 Experimental Design & Approach

3.1 Design of Experiment

Injectors selected for the sub-critical and super-critical swirl flows were selected in a way that will ensure the greatest performance from the HRM. Sub-critical injectors were shown to provide significantly lower performance than the super-critical injectors, because of the significant instabilities present during combustion. By attempting to alleviate said instabilities, it was found that super-critical flow is only slightly less stable than pure axial flow, and it still provides the large increases in overall performance. Previous work in the topic shows that the transition from an axial oxidizer injector to a super-critical, 60°-swirl injector, as shown in Figure 17, has the potential to increase the regression rate of the fuel grain by up to 182% (Pucci, 2002).

Figure 17: Rendered 60 Degree Swirl Injector

A more detailed and thorough investigation obtained using this small-scale HRM provided the following regression rates for axial, radial, sub-critical, critical, and super-critical injectors in Table 6. From these results it can be said that super-critical injectors provide higher regression rates, and increase motor performance accordingly by up to 63%. It's important to note that different fuels were used for this experiment than that for

Pucci's testing. HTPB is commonly known to have a higher regression rate than the High-Density Polyethelynes (HDPEs) used in his testing across the board; however, the important thing is that the same general effects of swirl injection on regression rate are independent of the fuel used in the configuration.

As discussed in previous works, combustion instabilities are often present amongst range of the injectors used during characterization testing. The most significant instabilities tend to be consistent with sub-critical injectors, while super-critical injectors are much more stable and still maintain high efficiencies (Pucci, 2002). Sub-critical injectors can be seen as unnecessary for future integrations since they not only destabilize combustion of the motor, but also don't show significant increases in regression rates, and likewise, rocket performance. Yet, in order to fully characterize the effects of the swirl injection angle and its effects on regression rate, it was pertinent to keep these sub-critical injector designs as part of the overall test matrix.

Future application of the test stand will consist of characterization of motor grain configurations, fuel types, and nozzles. As previously mentioned, the motor design has been equipped with simple integration methods to allow for the rapid changing of each component, thus speeding up turnaround time during testing. These additional sets will enable the completion of multiple research investigations describing their effects on the performance and stability of the HRM.

3.1.1 Test Matrix

Provide a filled out test matrix that was generated per the justifications of design selection discussed previously.

Table 6: Hot Fire Test Matrix

Test No.	Swirl Injector Angle (deg)	$P_{\text{Tank},0}$ (psi)	$P_{\text{C},\text{avg}}$ (psi)	$\dot{m}_{\text{ox, avg}}$ (lb/s)	$G_{\text{ox},0}$ (lb/s-in ²)	$\dot{r}$ (in/s)
1	60	950	373.949	0.548	Low	0.087
2	15	950	470.946	0.597	Low	0.070
3	15	950	470.946	0.597	Low	0.066
4	0	1000	520.946	0.597	Low	0.063
5	0	970	490.946	0.597	Low	0.068
6	30	980	500.946	0.597	Low	0.082
7	30	970	490.946	0.597	Low	0.078
8	60	1000	520.946	0.597	High	0.106
9	60	1010	530.946	0.597	High	0.092
10	60	980	500.946	0.597	Low	0.104
11	60	975	495.946	0.597	Low	0.074
12	42	950	470.946	0.597	Low	0.079
13	42	950	421.186	0.664	High	0.081
14	42	950	470.946	0.597	High	0.085
15	42	955	475.946	0.597	Low	0.077
16	0	950	470.946	0.597	High	0.078
17	0	950	470.946	0.597	High	0.066
18	15	950	470.946	0.597	High	0.080

For this experiment, 5 unique injector designs were selected. Those injectors were physically identical in all aspects, except the angle of radial injection. The five different injectors were designed with injector angles of 0 degrees, which is perpendicular to the outer surface or straight in the radial direction, 15 degrees, 30 degrees, 42 degrees, and 60 degrees. The stored pressure of the N₂O in the tank was kept at a constant 950 psi, with a few minor exceptions. The average combustion chamber pressure was kept relatively constant, with the exception of slight variations that are inherent variability among experimental data of this sort.

As was previously mentioned, two unique conditions for oxidizer mass flux were selected for this experiment. The two conditions are simply labeled here in Table 6 as “Low” and “High”. In order to calculate the oxidizer mass flux for each condition the follow equation is used, Equation 14.

$$G_0 = \frac{\dot{m}_{ox}}{A_{port}}$$

Equation 14. Oxidizer Mass Flux Equation

Of the two variables in this equation, combustion port area was easily controlled by casting grains with different inside diameters of their combustion ports. The low oxidizer mass flow rate condition was tested with HTPB fuel grains that have a combustion port diameter that measures 1.0 inch on the internal diameter. The high oxidizer mass flow rate condition was tested with HTPB fuel grains that have a combustion port diameter that measures 1.25 inch on the internal diameter. The oxidizer mass flow rate was more difficult to control since the density of the N_2O is both temperature and pressure dependent, and both of these variables were subject to the availability of a heating blanket and the ambient test conditions, which are discussed later in Section 3.3.4. Nonetheless, the oxidizer mass flow rate was held at a relatively constant 0.59 lb/s throughout the experiment. The following table shows the calculated initial oxidizer mass flux values for each test condition.

Table 7. Oxidizer Mass Flux Calculations

Test Condition	D_{port,i} (in)	A_{port} (in²)	$\dot{m}_{ox, avg}$ (lb/s)	G_{ox,0} (lb/s-in²)
Low	1.25	3.927	0.59	0.150
High	1.00	3.063	0.59	0.193

3.1.2 Test Order

In order to characterize the effects of the combustion chamber pressure and swirl injector angle independently later on, it is critical that the test runs described in the design of the experiment stay random (Montgomery, 2013). This explains the random order of test numbers in the test matrix.

3.1.3 Experiment Replication

In order to reduce statistical variability throughout the results, replication of the test series was necessary. However, due to the average cost per test run, schedule limitations, and the overall added value of the additional test runs, it was deemed unnecessary for the purpose of this investigation to perform additional replicated tests for all test cases. Instead, replication was only employed when there was a desire to better understand a specific relationship among the test cases. More specifically, experiments for at both oxidizer mass flux conditions were replicated for both the 0 degree injector case and the 60 degree injector case.

3.2 Blocking

Throughout the experiment blocking was employed to have only one operator of the tests. However, as previously mentioned, the advantage of having a single test bench and operator is that any error that would have occurred from multiple test benches or operators is eliminated. Even though we will minimize the measurement error by having the same person measuring each time, we do expect there will be some measurement error occurring.

3.3 Randomization

As stated by Montgomery, “[r]andomization is the cornerstone underlying the use of statistical methods in experimental design. By randomization we mean that both the allocation of the experimental material and the order in which the individual runs the experiment are to be performed are randomly determined.” It is because of this that it was so critical to have my test runs within the overall experiment randomized. Also, by

randomizing the individual test runs and the distribution of raw material “...we also assist in ‘averaging out’ the effects of extraneous factors that may be present” (Montgomery, 2013). Some of the key extraneous factors that may be present in this experiment are briefly discussed in the following subsections, along with the attempts that were taken in addition to randomization to reduce any effects these variables would have on the experimental results.

In order to complete the designed test matrix, 18 individual hybrid rocket motor test fires had to be completed. To keep cost and schedule reduced, a fully-instrumented small-scale hybrid rocket motor was employed for the series of tests. A typical rocket motor test fire can be seen in Figure 22, where the motor is in steady-state combustion when the picture was taken. As is typical of analyzing results of rocket motor tests that study regression rate, the transient start and stop, or tail-off, of the tests were not studied extensively. Because the transients associated with this test occur in the milliseconds, and these tests had a total burn time length of either 2 or 3 seconds, we can safely assume that the transients are negligible to the overall results of each test case.

3.3.1 Fuel Grain Mixing

To properly prepare for hot fire tests, the grain development has to be manufactured with precision. An advantage of hybrid grains like HTPB is that it is safe to handle during development, safe for storing and it will not ignite or detonate simultaneously³. When developing the grain three main chemicals and additives are used in the process; liquid HTPB, Isonate 143L, carbon black, and silicon oil. Isonate 143L is the curative which bonds the liquid HTPB into solid form. Silicon oil is only used when formulating large amounts of hybrid grains. It helps lubricate the grain in

order for ease of vacuuming to force the air out of the mixture⁵. In the case for this small scale hybrid grain the silicon is not needed to properly force the air out of the mixture. Carbon black is used in order for distinct coloration of the fuel for proper measurements. Table 1 shows the formulation for the hybrid grain.

Table 8. Hybrid Grain Formulation

Amount	Chemical/Additive
85%	HTPB
15%	Isonate 143L
1 teaspoon/lb _{fuel}	Carbon Black
1 drop/ lb _{fuel}	Silicon Oil

As shown in Table 8, silicon oil was only required to be added to the formula when mixing this particular HTPB hybrid fuel grain in very small amounts. This is because previous mixing experiments have shown that when silicon oil is added in any amount more than 1 drop per pound of total propellant, the grain or grains will not cure properly, and often must be disposed of. Because of this previous knowledge, and the fact that the grains only required so much propellant, silicon oil was not included in the batches used for this experiment. That being said, this is acceptable and will not have any effect on the performance of the motor.

Once the proper formulation has been measured for the correct fuel volume the mixing process begins. The HTPB, carbon black and silicon oil are added first into a mixing bowl of a Kitchen Aid 5-Quart Kv25 Stand Mixer and is mixed at low speeds until homogenous and no particles are present as shown in Figure 18. Then the Isonate 143L is added last due to its quick curing property and is mixed at medium speeds for 10 min. While mixing is taking place the Ultra-High Density Polyethylene (UHDPE) plastic casting rod is assembled to its associated casting base and prepped for final casting. The

specific plastic of UHDPE was used for its ability to easily release molds, such as these HTPB grains, after curing is complete. The UHDPE casting rods were precision machined to be considered identical, and this ensures a smooth surface for the grain to cure on and will result in a fine smooth solid hybrid grain. To further ensure the grains remove easily from the casting setup after cure is complete, mold release is sprayed evenly on the UHDPE rod and is let dry for 5 minutes. This additional step could be skipped in the process if a self-lubricating material was used like that of Teflon. However, UHDPE was more readily available and was more cost effective.

Figure 18. Mixing of the HTPB Fuel

Once mixing is finished the next step is to force out all of the trapped air in the fuel in order to prevent air bubbles while the fuel is being cured. The bowl has to be removed from the mixer and the outer edges of the bowl are lined with Teflon tape to ensure proper seal while vacuuming. A Plexiglas vacuum cover plate, which is connected to the vacuum pump, is placed on top of the bowl as shown in Figure 19.

Figure 19. Vacuum Process Setup

The vacuum pump process continues until no additional air bubbles are seen outgassing from the fuel. This process is important to maintain consistent grain quality from one to the next by mitigating the opportunity for voids to appear within the grain during the later cure process. Once the outgassing has finished, the mix is acceptable, and ready to be poured into the casting setup.

3.3.2 Fuel Grain Curing

Controlling the curing process across all of the individual HTPB fuel grains was also critical to maintaining consistency between the different grains. Variations in the ambient temperatures during the curing process were mitigated by the use of a controlled volume, known as the heat box. Figure 20 shows the HTPB-mixture-filled casting assembly placed inside of the heat box.

Figure 20. Individual Hybrid Casting Assembly in Heat Box

The air temperature inside of the heat box is measured using a thermometer and should maintain a temperature reading of 120 ± 20 degrees Fahrenheit for the full duration of the cure. The hybrid grain is left inside the heat box for approximately 24 hours for proper curing. Though it wasn't the case for the grains used for this experiment, sometimes if sufficient hardness is achieved prior to the 24 hour period, then the casting assembly is removed from the heat box. Figure 21 is an example of how a perfectly mixed and casted hybrid grain appears.

Figure 21. Typical Cured Hybrid Grain

3.3.3 Fuel Grain Density

Again, as previously mentioned there were some concerns that in the construction of the fuel grains for the experiments there may be some possible variability between HTPB fuel grains utilized during each trial. In order to address this, the fuel grains were measured, cut to length, and weighed to determine the density of each grain prior to each test. The final calculated density of each 1.25 inch internal diameter fuel grain was within 0.046 lbm/in³, while the final calculate density of each 1.00 inch internal diameter fuel grain was 0.057 lbm/in³. For the purposes of this experiment, this range of densities was deemed acceptable, and the method of data normalization is discusses in greater detail in the results section of this paper.

Later investigations into the density variability discovered that small variations in Isonate 143L and cure conditions accelerated the cure of the fuel grains and reduced expansion of the mixture as it cured. Though both of these are considered to be process improvements, for the purposes of maintaining consistency across the different test cases the data required normalization. After the data was normalized, the trends were compared to historical results, and analyzed for correlations.

3.3.4 Extraneous Test Conditions

As previously mentioned, blocking had to be employed for the statistical analysis of this experiment because it was not possible to complete all of the test runs on the same day. Because of this, ambient test conditions were monitored to better understand any impact on the results by uncontrollable variables like that of ambient test conditions. A

table of the of the monitored extraneous and/or uncontrollable ambient test conditions is provide below as Table 9.

Table 9. Example of Monitored Extraneous Test Conditions

<u>State</u>	<u>Pre</u>	<u>Post</u>
T_a (F)	A1	A2
Wind Speed (ft/s)	B1	B2
Wind Direction	C1	C2
P_{N2O} (psi)	D1	D2
Nozzle Throat Dia. (in)	E1	E2
T_{Case Wall} (F)	F1	F2

Table 9 and subsequent tables in Section 4 like it were populated from a number of sources. Values A1-C2 were collected from the online database owned by The Weather Channel. To measure any influence that wind direction and velocity might have on the experiment results, it's important to note that the test stand was always positioned such that wind direction is normal to the direction of thrust. Values D1, D2, F1, and F2 were collected during testing, and were recorded before and after each individual test. Lastly, values E1 and E2 were only collected before and after the total set of tests because previous research showed these effects would likely be negligible, and this was later confirmed.

3.3.5 N₂O Test Conditions

Though it wasn't mentioned previously in the report, another technical issue was encountered during the experiment, where as the number of tests increased, the pressure of the N₂O storage tank began to decrease. In order to counteract this effect and ensure that each test was able to attain its designed combustion chamber pressure as prescribed by the designed experiment, a controlled and approved heating method was implemented

to slowly and equally distribute heat the tank. Previous experience has shown this to be a more than sufficient method for increasing the pressure inside of the storage tank. Also, because the storage tank is designed for pressures up to 5000 psi, and even when heated up, the N₂O never exceeded 1010 psi, a conservative Factor of Safety (FoS) of 4.95 was always maintained throughout the experiment.

3.3.6 Nozzle Erosion

Another small concern that was briefly discussed previously is the erosion of the graphite nozzle throat diameters over the course of the experiment. Initial predictions stated that this effect should be small (on the order of thousandths of an inch), but it will be measured, recorded, and remain a consideration during the experimental design planning.

As testing proceeded the commercial nozzle did start to erode at the throat from the high temperature and abrasive mass flux at that location of the nozzle. The average erosion of the nozzle was approximately 0.005 inches on the diameter of the throat per test, which if we assume equivalent thrust from each test of 136 lbf, and assume equivalent thrust coefficient for each test, then the increase in throat size will cause the combustion pressure to drop by 0.015%, as can be calculated using Equation 15.

$$F = P_0 A_t C_F$$

Equation 15. Basic Equation for Thrust as a Function of Pressure, Throat Area, and Thrust Coefficient

Because a 0.015% drop in performance is within the noise of the inherent variation within the experiment, this is considered negligible for this testing. In order to verify this, the relative standard deviation of the combustion chamber pressure across all

of the test runs was compared to the percent losses of the combustion chamber pressure. Using Equation 16 the relative standard deviation of the combustion chamber pressure was calculated to be 0.076%. Based on this calculation, the losses associated with nozzle erosion and performance are less than one fifth of the noise inherent with the experiment design.

$$\sigma_{REL} = \frac{\sqrt{\frac{1}{N} \sum_{i=1}^N (x_i - \mu)^2}}{\mu} \text{ where } \mu = \frac{1}{N} \sum_{i=1}^N x_i$$

Equation 16. Relative Standard Deviation Equation

For reference, the calculated percent losses with respect to nozzle erosion and performance are tabulated in Table 10.

Table 10. Nozzle Erosion and Performance Losses

	D_t (in)	A_t (in^2)	F, avg (lbf)	C_f	P_c, avg (psi)
Initial	0.547	0.235	136.000	1.300	445.379
Final	0.551	0.238	136.000	1.300	438.735
% Losses	0.008	0.015	-	-	0.015

3.3.7 Erosion of Internal Components

Because of the high combustion temperature inherent with the rocket motor combustion processes, the integrity of internal components was monitored. However, the concern that internal metal components would erode over the duration of the motor burns was found to be relatively negligible. Initial predictions stated that this effect should be small (on the order of thousandths of an inch), but it was measured, recorded, and remained a consideration during the series of tests. As testing proceeded the wall thickness of the injector housing, pre-combustion chamber, and post-combustion chamber were selected as key characteristics for monitoring any erosion of internal metal

components. The total erosion of the injector housing wall was found to be at most 0.002 inches. The total erosion of the pre-combustion chamber was found to be at most 0.002 inches. The total erosion of the pre-combustion chamber was found to be at most 0.002 inches. If we assume equivalent thrust and combustion characteristics from each test then the average erosion per test is one eighteenth of each of the measured total erosion values provided previously, or 1.1×10^{-4} inches. Given the fact that these measured and calculated erosion values are near or less than the accuracy of available measurement tools, all erosion of internal metal components used for this experiment is to be considered negligible.

3.3.8 Sensor Calibration

All of the sensors were purchased from either FUTEK or Omegadyne. Because of this, all the sensors were professionally calibrated prior to use to meet their designed specifications, see Appendix B for additional sensor calibration and specification information.

An additional calibration of the data acquisition and sensor setup was required prior to testing to verify all sensor signal outputs, excitation voltages, and the internal DATAQ modules are nominal. During this process, the values from the calibration sheets were entered into the DATAQ software to provide accurate correlations of the raw voltage data to engineering units such as psi and lbf. This calibration was then saved to the laptop, and was used for each subsequent test.

3.3.9 Motor Assembly

Torque to install components in the forward end enclosure was monitored from test to test to ensure that everything was installed safely and correctly. Snap ring location and seating was verified to be correct during each motor assembly. If the torque or seating of these components was not verified prior to each motor firing, then it could result in a small change in motor performance by changing the internal volume that is pressurized during combustion. If the pressure of the combustion chamber changes in an unplanned way, then it would result in a change in regression rate accordingly, thus removing that test run from the group of acceptable results.

4 Results & Discussion

4.1 Test Runs

Shown in the following figure is a picture of a typical static fire of this rocket motor, and was taken during the preliminary testing that was performed at Arizona State University.

Figure 22. Preliminary Static Fire Testing of Rocket Motor

4.1.1 Test Run 1

Test run number 1 resulted in a 2.125 second burn that provided 124.1 lbf of average thrust. During this hot fire test there were no anomalies, and all results were deemed acceptable for analysis. As is shown in Table 11, this test consisted of a 60 degree swirl injector. As shown in Table 12, the 60 degree swirl injector results in a regression rate of 0.087 in/s.

Table 11. Summary of Test Conditions for Test Run 1

<u>Variable</u>	<u>Value</u>
Swirl Angle (deg.)	60
Initial Port Dia. (in)	1.25
Burn Time (s)	2.125

Table 12. Summary of Motor Performance for Test Run 1

<u>Performance Metric</u>	<u>Value</u>
$P_{c,avg}$ (psi)	373.9
$Thrust_{avg}$ (lbf)	124.1
Regression Rate (in/s)	0.087

The extraneous test conditions for test run number 1 are within family of the other test runs. Because of this, these test conditions are considered negligible for test number 1.

Table 13. Monitored Extraneous Test Conditions for Test Run 1

<u>State</u>	<u>Pre</u>	<u>Post</u>
T_a (F)	51	51
Wind Speed (ft/s)	23.5	23.5
Wind Direction	W	W
P_{N2O} (psi)	950	950
Nozzle Throat Dia. (in)	0.546	0.546
$T_{Case\ Wall}$ (F)	51	100

4.1.2 Test Run 2

Test run number 2 resulted in a 3.242 second burn that provided 143.5 lbf of average thrust. During this hot fire test there were no anomalies, and all results were deemed acceptable for analysis. As is shown in Table 14, this test consisted of a 15 degree swirl injector. As shown in Table 15Table 12, the 15 degree swirl injector results in a regression rate of 0.070 in/s.

Table 14. Summary of Test Conditions for Test Run 2

<u>Variable</u>	<u>Value</u>
Swirl Angle (deg.)	15
Initial Port Dia. (in)	1.25
Burn Time (s)	3.242

Table 15. Summary of Motor Performance for Test Run 2

<u>Performance Metric</u>	<u>Value</u>
$P_{c,avg}$ (psi)	470.9
$Thrust_{avg}$ (lbf)	143.5
Regression Rate (in/s)	0.070

The extraneous test conditions for test run number 2 are within family of the other test runs. Because of this, these test conditions are considered negligible for test number 2.

Table 16. Monitored Extraneous Test Conditions for Test Run 2

<u>State</u>	<u>Pre</u>	<u>Post</u>
T_a (F)	51	51
Wind Speed (ft/s)	23.5	23.5
Wind Direction	W	W
P_{N2O} (psi)	950	950
Nozzle Throat Dia. (in)	0.546	0.546
$T_{Case\ Wall}$ (F)	51	100

4.1.3 Test Run 3

Test run number 3 resulted in a 3.233 second burn that provided 143.6 lbf of average thrust. During this hot fire test there were no anomalies, and all results were

deemed acceptable for analysis. As is shown in Table 17, this test consisted of a 15 degree swirl injector. As shown in Table 18Table 12, the 15 degree swirl injector results in a regression rate of 0.066 in/s.

Table 17. Summary of Test Conditions for Test Run 3

<u>Variable</u>	<u>Value</u>
Swirl Angle (deg.)	15
Initial Port Dia. (in)	1.25
Burn Time (s)	3.233

Table 18. Summary of Motor Performance for Test Run 3

<u>Performance Metric</u>	<u>Value</u>
$P_{c,avg}$ (psi)	470.9
$Thrust_{avg}$ (lbf)	143.6
Regression Rate (in/s)	0.066

The extraneous test conditions for test run number 3 are within family of the other test runs. Because of this, these test conditions are considered negligible for test number 3.

Table 19. Monitored Extraneous Test Conditions for Test Run 3

<u>State</u>	<u>Pre</u>	<u>Post</u>
T_a (F)	51	51
Wind Speed (ft/s)	23.5	23.5
Wind Direction	W	W
P_{N2O} (psi)	950	950
Nozzle Throat Dia. (in)	0.547	0.547
$T_{Case\ Wall}$ (F)	51	100

4.1.4 Test Run 4

Test run number 4 resulted in a 3.333 second burn that provided 159.1 lbf of average thrust. During this hot fire test there were no anomalies, and all results were deemed acceptable for analysis. As is shown in Table 20, this test consisted of a 0 degree swirl injector. As shown in Table 21, the 0 degree swirl injector results in a regression rate of 0.063 in/s.

Table 20. Summary of Test Conditions for Test Run 4

<u>Variable</u>	<u>Value</u>
Swirl Angle (deg.)	0
Initial Port Dia. (in)	1.25
Burn Time (s)	3.333

Table 21. Summary of Motor Performance for Test Run 4

<u>Performance Metric</u>	<u>Value</u>
$P_{c,avg}$ (psi)	520.9
Thrust _{avg} (lbf)	159.1
Regression Rate (in/s)	0.063

The extraneous test conditions for test run number 4 are within family of the other test runs. Because of this, these test conditions are considered negligible for test number 4.

Table 22. Monitored Extraneous Test Conditions for Test Run 4

<u>State</u>	<u>Pre</u>	<u>Post</u>
T_a (F)	92	92
Wind Speed (ft/s)	23.5	23.5

Wind Direction	SE	SE
P_{N2O} (psi)	1000	1000
Nozzle Throat Dia. (in)	0.547	0.547
T_{Case Wall} (F)	92	120

4.1.5 Test Run 5

Test run number 5 resulted in a 2.933 second burn that provided 150.1 lbf of average thrust. During this hot fire test there were no anomalies, and all results were deemed acceptable for analysis. As is shown in Table 23, this test consisted of a 0 degree swirl injector. As shown in Table 24, the 0 degree swirl injector results in a regression rate of 0.068 in/s.

Table 23. Summary of Test Conditions for Test Run 5

<u>Variable</u>	<u>Value</u>
Swirl Angle (deg.)	0
Initial Port Dia. (in)	1.25
Burn Time (s)	2.933

Table 24. Summary of Motor Performance for Test Run 5

<u>Performance Metric</u>	<u>Value</u>
P_{c,avg} (psi)	490.9
Thrust_{avg} (lbf)	150.1
Regression Rate (in/s)	0.068

The extraneous test conditions for test run number 5 are within family of the other test runs. Because of this, these test conditions are considered negligible for test number 5.

Table 25. Monitored Extraneous Test Conditions for Test Run 5

<u>State</u>	<u>Pre</u>	<u>Post</u>
T_a (F)	92	92
Wind Speed (ft/s)	23.5	23.5
Wind Direction	SE	SE
P_{N2O} (psi)	970	970
Nozzle Throat Dia. (in)	0.547	0.547
T_{Case Wall} (F)	92	120

4.1.6 Test Run 6

Test run number 6 resulted in a 3.258 second burn that provided 153.4 lbf of average thrust. During this hot fire test there were no anomalies, and all results were deemed acceptable for analysis. As is shown in Table 26, this test consisted of a 30 degree swirl injector. As shown in Table 27, the 30 degree swirl injector results in a regression rate of 0.082 in/s.

Table 26. Summary of Test Conditions for Test Run 6

<u>Variable</u>	<u>Value</u>
Swirl Angle (deg.)	30
Initial Port Dia. (in)	1.25
Burn Time (s)	3.258

Table 27. Summary of Motor Performance for Test Run 6

<u>Performance Metric</u>	<u>Value</u>
P_{c,avg} (psi)	500.9
Thrust_{avg} (lbf)	153.4

Regression Rate (in/s)	0.082
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The extraneous test conditions for test run number 6 are within family of the other test runs. Because of this, these test conditions are considered negligible for test number 6.

Table 28. Monitored Extraneous Test Conditions for Test Run 6

<u>State</u>	<u>Pre</u>	<u>Post</u>
T_a (F)	92	92
Wind Speed (ft/s)	23.5	23.5
Wind Direction	SE	SE
P_{N2O} (psi)	980	980
Nozzle Throat Dia. (in)	0.548	0.548
T_{Case Wall} (F)	92	120

4.1.7 Test Run 7

Test run number 7 resulted in a 3.117 second burn that provided 150.5 lbf of average thrust. During this hot fire test there were no anomalies, and all results were deemed acceptable for analysis. As is shown in Table 29, this test consisted of a 30 degree swirl injector. As shown in Table 12, the 30 degree swirl injector results in a regression rate of 0.078 in/s.

Table 29. Summary of Test Conditions for Test Run 7

<u>Variable</u>	<u>Value</u>
Swirl Angle (deg.)	30
Initial Port Dia. (in)	1.25
Burn Time (s)	3.117

Table 30. Summary of Motor Performance for Test Run 7

<u>Performance Metric</u>	<u>Value</u>
$P_{c,avg}$ (psi)	490.9
$Thrust_{avg}$ (lbf)	150.5
Regression Rate (in/s)	0.078

The extraneous test conditions for test run number 7 are within family of the other test runs. Because of this, these test conditions are considered negligible for test number 7.

Table 31. Monitored Extraneous Test Conditions for Test Run 7

<u>State</u>	<u>Pre</u>	<u>Post</u>
T_a (F)	92	92
Wind Speed (ft/s)	23.5	23.5
Wind Direction	SE	SE
P_{N2O} (psi)	970	970
Nozzle Throat Dia. (in)	0.548	0.548
$T_{Case\ Wall}$ (F)	92	120

4.1.8 Test Run 8

Test run number 8 resulted in a 2.475 second burn that provided 159.8 lbf of average thrust. During this hot fire test there were no anomalies, and all results were deemed acceptable for analysis. As is shown in Table 32, this test consisted of a 60 degree swirl injector. As shown in Table 33, the 60 degree swirl injector results in a regression rate of 0.106 in/s.

Table 32. Summary of Test Conditions for Test Run 8

<u>Variable</u>	<u>Value</u>
Swirl Angle (deg.)	60
Initial Port Dia. (in)	1.00
Burn Time (s)	2.475

Table 33. Summary of Motor Performance for Test Run 8

<u>Performance Metric</u>	<u>Value</u>
$P_{c,avg}$ (psi)	520.9
$Thrust_{avg}$ (lbf)	159.8
Regression Rate (in/s)	0.106

The extraneous test conditions for test run number 8 are within family of the other test runs. Because of this, these test conditions are considered negligible for test number 8.

Table 34. Monitored Extraneous Test Conditions for Test Run 8

<u>State</u>	<u>Pre</u>	<u>Post</u>
T_a (F)	82	82
Wind Speed (ft/s)	23.5	23.5
Wind Direction	NW	NW
P_{N2O} (psi)	1000	1000
Nozzle Throat Dia. (in)	0.548	0.548
$T_{Case\ Wall}$ (F)	82	110

4.1.9 Test Run 9

Test run number 9 resulted in a 2.092 second burn that provided 163.1 lbf of average thrust. During this hot fire test there were no anomalies, and all results were

deemed acceptable for analysis. As is shown in Table 35, this test consisted of a 60 degree swirl injector. As shown in Table 36, the 60 degree swirl injector results in a regression rate of 0.092 in/s.

Table 35. Summary of Test Conditions for Test Run 9

<u>Variable</u>	<u>Value</u>
Swirl Angle (deg.)	60
Initial Port Dia. (in)	1.00
Burn Time (s)	2.092

Table 36. Summary of Motor Performance for Test Run 9

<u>Performance Metric</u>	<u>Value</u>
$P_{c,avg}$ (psi)	530.9
$Thrust_{avg}$ (lbf)	163.1
Regression Rate (in/s)	0.092

The extraneous test conditions for test run number 9 are within family of the other test runs. Because of this, these test conditions are considered negligible for test number 9.

Table 37. Monitored Extraneous Test Conditions for Test Run 9

<u>State</u>	<u>Pre</u>	<u>Post</u>
T_a (F)	82	82
Wind Speed (ft/s)	23.5	23.5
Wind Direction	NW	NW
P_{N2O} (psi)	1010	1010
Nozzle Throat Dia. (in)	0.549	0.549
$T_{Case\ Wall}$ (F)	82	110

4.1.10 Test Run 10

Test run number 10 resulted in a 1.867 second burn that provided 154.1 lbf of average thrust. During this hot fire test there were no anomalies, and all results were deemed acceptable for analysis. As is shown in Table 38, this test consisted of a 60 degree swirl injector. As shown in Table 39, the 60 degree swirl injector results in a regression rate of 0.104 in/s.

Table 38. Summary of Test Conditions for Test Run 10

<u>Variable</u>	<u>Value</u>
Swirl Angle (deg.)	60
Initial Port Dia. (in)	1.25
Burn Time (s)	1.867

Table 39. Summary of Motor Performance for Test Run 10

<u>Performance Metric</u>	<u>Value</u>
$P_{c,avg}$ (psi)	500.9
$Thrust_{avg}$ (lbf)	154.1
Regression Rate (in/s)	0.104

The extraneous test conditions for test run number 10 are within family of the other test runs. Because of this, these test conditions are considered negligible for test number 10.

Table 40. Monitored Extraneous Test Conditions for Test Run 10

<u>State</u>	<u>Pre</u>	<u>Post</u>
T_a (F)	82	82
Wind Speed (ft/s)	23.5	23.5

Wind Direction	NW	NW
P_{N2O} (psi)	980	980
Nozzle Throat Dia. (in)	0.549	0.549
T_{Case Wall} (F)	82	110

4.1.11 Test Run 11

Test run number 11 resulted in a 2.709 second burn that provided 152.7 lbf of average thrust. During this hot fire test there were no anomalies, and all results were deemed acceptable for analysis. As is shown in Table 41, this test consisted of a 60 degree swirl injector. As shown in Table 42, the 60 degree swirl injector results in a regression rate of 0.074 in/s.

Table 41. Summary of Test Conditions for Test Run 11

<u>Variable</u>	<u>Value</u>
Swirl Angle (deg.)	60
Initial Port Dia. (in)	1.25
Burn Time (s)	2.709

Table 42. Summary of Motor Performance for Test Run 11

<u>Performance Metric</u>	<u>Value</u>
P_{c,avg} (psi)	495.9
Thrust_{avg} (lbf)	152.7
Regression Rate (in/s)	0.074

The extraneous test conditions for test run number 11 are within family of the other test runs. Because of this, these test conditions are considered negligible for test number 11.

Table 43. Monitored Extraneous Test Conditions for Test Run 11

<u>State</u>	<u>Pre</u>	<u>Post</u>
T_a (F)	82	82
Wind Speed (ft/s)	23.5	23.5
Wind Direction	NW	NW
P_{N2O} (psi)	975	975
Nozzle Throat Dia. (in)	0.549	0.549
T_{Case Wall} (F)	82	110

4.1.12 Test Run 12

Test run number 12 resulted in a 2.000 second burn that provided 146.5 lbf of average thrust. During this hot fire test there were no anomalies, and all results were deemed acceptable for analysis. As is shown in Table 44, this test consisted of a 42 degree swirl injector. As shown in Table 45, the 42 degree swirl injector results in a regression rate of 0.079 in/s.

Table 44. Summary of Test Conditions for Test Run 12

<u>Variable</u>	<u>Value</u>
Swirl Angle (deg.)	42
Initial Port Dia. (in)	1.25
Burn Time (s)	2.000

Table 45. Summary of Motor Performance for Test Run 12

<u>Performance Metric</u>	<u>Value</u>
P_{c,avg} (psi)	470.9
Thrust_{avg} (lbf)	146.5

Regression Rate (in/s)	0.079
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The extraneous test conditions for test run number 12 are within family of the other test runs. Because of this, these test conditions are considered negligible for test number 12.

Table 46. Monitored Extraneous Test Conditions for Test Run 12

<u>State</u>	<u>Pre</u>	<u>Post</u>
T_a (F)	82	82
Wind Speed (ft/s)	23.5	23.5
Wind Direction	NW	NW
P_{N2O} (psi)	950	950
Nozzle Throat Dia. (in)	0.549	0.549
T_{Case Wall} (F)	82	110

4.1.13 Test Run 13

Test run number 13 resulted in a 2.317 second burn that provided 139.7 lbf of average thrust. During this hot fire test there were no anomalies, and all results were deemed acceptable for analysis. As is shown in Table 47, this test consisted of a 42 degree swirl injector. As shown in Table 48, the 42 degree swirl injector results in a regression rate of 0.081 in/s.

Table 47. Summary of Test Conditions for Test Run 13

<u>Variable</u>	<u>Value</u>
Swirl Angle (deg.)	42
Initial Port Dia. (in)	1.00
Burn Time (s)	2.317

Table 48. Summary of Motor Performance for Test Run 13

<u>Performance Metric</u>	<u>Value</u>
$P_{c,avg}$ (psi)	421.2
$Thrust_{avg}$ (lbf)	139.7
Regression Rate (in/s)	0.081

The extraneous test conditions for test run number 13 are within family of the other test runs. Because of this, these test conditions are considered negligible for test number 13.

Table 49. Monitored Extraneous Test Conditions for Test Run 13

<u>State</u>	<u>Pre</u>	<u>Post</u>
T_a (F)	82	82
Wind Speed (ft/s)	23.5	23.5
Wind Direction	NW	NW
P_{N2O} (psi)	950	950
Nozzle Throat Dia. (in)	0.550	0.550
$T_{Case\ Wall}$ (F)	82	110

4.1.14 Test Run 14

Test run number 14 resulted in a 2.150 second burn that provided 145.5 lbf of average thrust. During this hot fire test there were no anomalies, and all results were deemed acceptable for analysis. As is shown in Table 50, this test consisted of a 42 degree swirl injector. As shown in Table 51, the 42 degree swirl injector results in a regression rate of 0.085 in/s.

Table 50. Summary of Test Conditions for Test Run 14

<u>Variable</u>	<u>Value</u>
Swirl Angle (deg.)	42
Initial Port Dia. (in)	1.00
Burn Time (s)	2.150

Table 51. Summary of Motor Performance for Test Run 14

<u>Performance Metric</u>	<u>Value</u>
$P_{c,avg}$ (psi)	470.9
$Thrust_{avg}$ (lbf)	145.5
Regression Rate (in/s)	0.085

The extraneous test conditions for test run number 14 are within family of the other test runs. Because of this, these test conditions are considered negligible for test number 14.

Table 52. Monitored Extraneous Test Conditions for Test Run 14

<u>State</u>	<u>Pre</u>	<u>Post</u>
T_a (F)	82	82
Wind Speed (ft/s)	23.5	23.5
Wind Direction	NW	NW
P_{N2O} (psi)	950	950
Nozzle Throat Dia. (in)	0.550	0.550
$T_{Case\ Wall}$ (F)	82	110

4.1.15 Test Run 15

Test run number 15 resulted in a 2.625 second burn that provided 147.2 lbf of average thrust. During this hot fire test there were no anomalies, and all results were

deemed acceptable for analysis. As is shown in Table 53, this test consisted of a 42 degree swirl injector. As shown in Table 54, the 42 degree swirl injector results in a regression rate of 0.077 in/s.

Table 53. Summary of Test Conditions for Test Run 15

<u>Variable</u>	<u>Value</u>
Swirl Angle (deg.)	42
Initial Port Dia. (in)	1.25
Burn Time (s)	2.625

Table 54. Summary of Motor Performance for Test Run 15

<u>Performance Metric</u>	<u>Value</u>
$P_{c,avg}$ (psi)	475.9
$Thrust_{avg}$ (lbf)	147.2
Regression Rate (in/s)	0.077

The extraneous test conditions for test run number 15 are within family of the other test runs. Because of this, these test conditions are considered negligible for test number 15.

Table 55. Monitored Extraneous Test Conditions for Test Run 15

<u>State</u>	<u>Pre</u>	<u>Post</u>
T_a (F)	82	82
Wind Speed (ft/s)	23.5	23.5
Wind Direction	NW	NW
P_{N2O} (psi)	955	955
Nozzle Throat Dia. (in)	0.550	0.550
$T_{Case\ Wall}$ (F)	82	110

4.1.16 Test Run 16

Test run number 16 resulted in a 2.050 second burn that provided 145.8 lbf of average thrust. During this hot fire test there were no anomalies, and all results were deemed acceptable for analysis. As is shown in Table 56, this test consisted of a 0 degree swirl injector. As shown in Table 12, the 0 degree swirl injector results in a regression rate of 0.078 in/s.

Table 56. Summary of Test Conditions for Test Run 16

<u>Variable</u>	<u>Value</u>
Swirl Angle (deg.)	60
Initial Port Dia. (in)	1.00
Burn Time (s)	2.050

Table 57. Summary of Motor Performance for Test Run 16

<u>Performance Metric</u>	<u>Value</u>
$P_{c,avg}$ (psi)	470.9
$Thrust_{avg}$ (lbf)	145.8
Regression Rate (in/s)	0.078

The extraneous test conditions for test run number 16 are within family of the other test runs. Because of this, these test conditions are considered negligible for test number 16.

Table 58. Monitored Extraneous Test Conditions for Test Run 16

<u>State</u>	<u>Pre</u>	<u>Post</u>
T_a (F)	82	82
Wind Speed (ft/s)	23.5	23.5

Wind Direction	NW	NW
P_{N2O} (psi)	950	950
Nozzle Throat Dia. (in)	0.551	0.551
T_{Case Wall} (F)	82	110

4.1.17 Test Run 17

Test run number 17 resulted in a 1.609 second burn that provided 150.0 lbf of average thrust. During this hot fire test there were no anomalies, and all results were deemed acceptable for analysis. As is shown in Table 59, this test consisted of a 0 degree swirl injector. As shown in Table 12, the 0 degree swirl injector results in a regression rate of 0.066 in/s.

Table 59. Summary of Test Conditions for Test Run 17

<u>Variable</u>	<u>Value</u>
Swirl Angle (deg.)	0
Initial Port Dia. (in)	1.00
Burn Time (s)	1.609

Table 60. Summary of Motor Performance for Test Run 17

<u>Performance Metric</u>	<u>Value</u>
P_{c,avg} (psi)	470.9
Thrust_{avg} (lbf)	150.0
Regression Rate (in/s)	0.066

The extraneous test conditions for test run number 17 are within family of the other test runs. Because of this, these test conditions are considered negligible for test number 17.

Table 61. Monitored Extraneous Test Conditions for Test Run 17

<u>State</u>	<u>Pre</u>	<u>Post</u>
T_a (F)	82	82
Wind Speed (ft/s)	23.5	23.5
Wind Direction	NW	NW
P_{N2O} (psi)	950	950
Nozzle Throat Dia. (in)	0.551	0.551
T_{Case Wall} (F)	82	110

4.1.18 Test Run 18

Test run number 18 resulted in a 3.325 second burn that provided 146.2 lbf of average thrust. During this hot fire test there were no anomalies, and all results were deemed acceptable for analysis. As is shown in Table 62, this test consisted of a 15 degree swirl injector. As shown in Table 63, the 15 degree swirl injector results in a regression rate of 0.080 in/s.

Table 62. Summary of Test Conditions for Test Run 18

<u>Variable</u>	<u>Value</u>
Swirl Angle (deg.)	15
Initial Port Dia. (in)	1.00
Burn Time (s)	3.325

Table 63. Summary of Motor Performance for Test Run 18

<u>Performance Metric</u>	<u>Value</u>
P_{c,avg} (psi)	470.9
Thrust_{avg} (lbf)	146.2

Regression Rate (in/s)	0.080
-------------------------------	-------

The extraneous test conditions for test run number 18 are within family of the other test runs. Because of this, these test conditions are considered negligible for test number 18.

Table 64. Monitored Extraneous Test Conditions for Test Run 18

<u>State</u>	<u>Pre</u>	<u>Post</u>
T_a (F)	82	82
Wind Speed (ft/s)	23.5	23.5
Wind Direction	NW	NW
P_{N2O} (psi)	950	950
Nozzle Throat Dia. (in)	0.551	0.551
T_{Case Wall} (F)	82	110

4.2 Discussion of Results

After all of the individual static-fire tests were complete, the data was compiled into a larger database for post-processing analysis. At the same time concrete molds were taken of the fuel grains to easily determine the internal shape and dimension of each individual grain after their respective test. Shown in Figure 23 is the concrete molds curing in place of the first batch of fuel grains. These concrete molds were made using a quick-setting concrete that can be found readily available at a Home Depot. This particular type of quick-setting concrete was selected over plaster of paris and other concretes because it starts curing within 15 minutes, completely cures within 1 hour after being mixed with water, and has an advertised compressive strength high enough to drive a car on it after the 1 hour cure time. These high-strength material properties were ideal because the concrete mold had to be strong enough to survive the fuel grain removal

process with as little deformation as possible to reduce any later variability in the mold measurements. In addition to these characteristics, there was an additional material property that made this brand of quick-setting concrete perfect for this application, of all the quick-setting concretes of its type it is claimed to have the smallest amount of shrinkage. Minimizing the change in size of these molds was critical to the final product, as these molds were measured using a precision set of digital calipers that are accurate to the ± 0.001 inches, and have a resolution of ± 0.0005 inches.

Figure 23. Concrete Molds Curing of First Batch

As previously mentioned, the molds were completely cured at the end of their 1 hour cure time. However, to compensate for any variability in water to concrete ratios, exposure time to air, minor variations in ambient air temperature, and other factors that may impact the cure time of the molds, all concrete molds were let sit for 2 hours, or twice the minimum required cure time.

After the 2 hour period, a utility knife was used to initially score the grains down the length, and then cut through the phenolic and fuel until a slight scraping of the concrete mold was felt through the knife. This process was then repeated on the opposing side, or 180 degrees from the first cut. Then a flathead screwdriver was used to gently and initially pry to two halves apart from the concrete mold. Finally the two fuel grain halves were separated enough to manually pull them apart from the mold. As shown in Figure 24, little to no material transfer was observed between the fuel grain and the concrete after the mold was removed. It's also worth noting that the molds and the fuel

grains required the flat-head screwdriver to initially pry the two halves apart from the mold because concrete mold maintained an intimate seal against the rubber fuel grain throughout the cure process. This further confirmed the idea that the quick-setting concrete wouldn't shrink during cure, and that an accurate representation of the fuel grains internal shape and dimension was captured.

Figure 24. Example Concrete Mold After Grain Is Sectioned, Test #1

During this analysis the following Figure 25 shows the different characteristic equations for regression rate under depending upon which injector design is selected. It is easy to see the step in regression rate characteristic functions as they progress from the injection angle of 0 degrees up to that of 60 degrees. In fact, an increase in regression rate as high as 45% was observed when comparing the highest performing test case of the 60 degree injector and the lowest performing test case of the 0 degree injector at the low oxidizer mass flux test condition. Meanwhile, an increase in regression rate as high as 63% was observed when comparing the highest performing test case of the 60 degree injector and the lowest performing test case of the 0 degree injector at the high oxidizer

mass flux test condition. Improvements in regression rate were seen at each increasing injection angle in between these data points, and can be again seen on the plots. The only characteristic regression rate equation that wasn't included in this plot was that of the 30 degree swirl injector. This was simply due to the fact that the experiment ran out of resources and time, and only one test was able to be completed at the low oxidizer mass flux test condition with this injector. That being said, the 30 degree injector did still show an improvement in performance over the 15 degree and 0 degree injectors, and that improvement can be found in Figure 25.

Figure 25. Regression Rate (Corrected) Characteristic Functions for Indicated Swirl Injector Angles

These characteristic equations shown in Figure 25 can be used to not only compare performance of a given hybrid rocket motor with different injectors, but can also be used to extrapolate additional trends about how swirl injection angle directly effects regression rate as a new variable. It's worth noting that while only one test was able to be

completed at the 30 degree swirl angle due to material and time constraints, the one test at the low-oxidizer mass flux test condition showed regression rates in between that of the 42 degree injector and the 60 degree injector. While no characteristic equation could be empirically derived for this 30 degree injector, it can clearly be seen that additional testing needs to be completed to better understand not only the effects of this injection angle, but the injection angles of the other injector designs as well.

The average regression rate values shown in Figure 25 were required to be corrected as a result of some lot-to-lot variation in raw materials of the fuel grains, which is discussed in more detail in Section 4.3. The correction that was applied to the high data was calculated based on the variation in fuel densities between the two batches used for this experiment. By providing this correction, it calculated what the regression rate should have been for the second batch of fuel grains. This was necessary in order to do any comparisons between tests since the fuel density has such a large impact on both the regression rate of the fuel grain, but also overall motor performance. This was accomplished by taking a series of measurements of the components of the fuel grains, and calculating average fuel density of the two different batches of cast grains. Once this was completed, the collected data was placed in the following table, Table 65. Using this tabulated data the average fuel mass flow rates and average oxidizer mass flow rates were calculated. With these two calculated values a ratio of O/F ratios was able to be calculated and was then inserted into the Equation 6.

Table 65. Fuel Density Measurement and Calculation Table

Variables	Calculated or Measured Values	
	<i>Low G_{ox}</i>	<i>High G_{ox}</i>
Grain Length (in)	7.000	7.000
Grain OD (in)	1.835	1.835
Avg Grain ID (in)	1.250	0.975

Grain Volume (in³)	9.922	13.286
Mass of Liner (oz)	1.100	1.100
Avg Grain Weight (lbm)	8.500	13.230
Avg Grain Mass (lbm)	0.463	0.758
Avg Grain Density (lbm/in³)	0.047	0.057
Density Ratio	0.817	1.000

In this application, injection angle can be used to either size a hybrid rocket motor with this new modified regression rate relationship, or vary the regression rate during motor operation to potentially throttle the motor. The latter of these two designs is likely the least popular application of these results, but nonetheless it is noteworthy.

During earlier testing a few interesting observations were made that merited additional research. The first of these observations was that the regression rate of the grains did not appear uniform down the length while using some injectors. Because of this, it was critical to develop a way to take regression rate measurements as a function of length for each of the tests. As can be seen in Figure 26 and Figure 27 the regression rate down the length of the fuel grains is rarely constant, and in fact requires a unique 60 degree injector configuration or design to accomplish a regression rate that is even close to constant down the length.

Figure 26. Regression Rate (Uncorrected) vs Length of Nominal Tests at the Low-Oxidizer Flux Test Condition

One key different between the plotted data in Figure 26 and that of Figure 27 is that little to no injector configurations were shown to provide a constant regression rate as a function of length at the high oxidizer mass flux test case, unlike the low oxidizer test case. Inspection of the casting molds revealed that the core rod itself that is used to cast the 1.00 inch internal diameter fuel grains has a varying diameter down its length. Because of this, the increasing trend of all the regression rate plots is really only again true of the lower injection angles. The higher injection angles of 42 degrees and 60 degrees, once corrected maintain a relationship that is more similar to that of the low oxidizer mass flux plots.

Figure 27. Regression Rate (Uncorrected) vs Length of Nominal Tests at the High-Oxidizer Flux Test Condition

To further process these experimental results, a commercially available software package specifically designed for the design and analysis of experiments was used, JMP 10. This software package was used over other packages because it is one of the more accurate and user-friendly software packages for this specific type of analysis. The summarized details from Table 6 were input into the software package, and an analysis was performed to estimate the effects of these factors on the measured response variable of regression rate. The following Figure 28 shows the accuracy of the JMP-generated prediction model as the actual regression rate values are plotted against the predicted regression rate values calculated from the model. With the exception of a few points, all of the predicted points follow the model's estimation relatively close.

Figure 28. Actual vs Predicted Plot of Whole JMP Model

The following Figure 29 and Figure 30 show the predicted effect of swirl injection angle and oxidizer mass flux on the regression rate, respectively. When comparing these two figures side by side, it's easy to see that the slope of the swirl injector angle effect plot is higher than that of the oxidizer mass flux effect plot. Since both of the factors are plotted against the regression rate response variable, it can be said that the swirl injector angle appears to have a larger effect on the regression rate over the range of factor values tested at for this experiment.

Figure 29. Regression Rate Plot vs Swirl Injection Angle

Figure 30. Regression Rate Plot vs Oxidizer Mass Flux

Since all of the factors are quantitative, the effect estimates could be used to generate a regression model of the regression rate response variable. The reduced linear regression model for the fuel grain regression rate can be shown below as Equation 17. For analysis purposes of regression rate for this experiment, the following equation has been shown to provide a reasonably accurate prediction of the regression rate response variable. It is understood, that a more accurate regression model would be of the

exponential or power type so that effects of the flux exponent could be properly captured in the regression model.

$$\hat{y} = 0.03908 + 0.00039((Swirl\ Injector\ Angle - 29.4)/30) + 0.16888((G_{ox} - 0.024)0.024)$$

Equation 17. Reduced Linear Regression Model for HTPB/N₂O Regression Rate

The previously mentioned effect estimates were calculated, and an ANOVA was performed to determine how significant each effect was. Based on the results in Figure 42, if a standard 5% significant level of type I error is used, it can be seen from analysis of variance that the swirl injector angle, oxidizer mass flux, swirl injector angle multiplied by a squared combustion chamber pressure, and oxidizer mass flux multiplied by a squared combustion chamber pressure are all significant. However, it is worth considering that since we have the effect of the combustion chamber pressure as the next closest significant term, and it was considered significant once squared, that the combustion chamber pressure be monitored during future experiments closely as it obviously has some influence on the results. Yet, for the purpose of this experiment and the sake of simplicity, combustion chamber pressure was not included in the reduced linear regression model.

4.3 Discussion of Test Anomalies

Despite the efforts of having controlled procedures, mixing environments, and mixing tooling, some significant variability was seen between the two batches of fuel grains. After reviewing mixing notes, the cause of this variability isn't clear. I do know that it wasn't a result of contamination because the ingredients used to manufacture the grains were well controlled, and stored properly in a fire cabinet.

One thing that I was able to identify was a slight variation in the amount of Isonate 143L curative that was added to the formulation while mixing. The difference in Isonate 143L was 0.020 grams higher in the low density grains than high density grains. This higher amount of Isonate 143L curative, if all other cure controls were kept the same, would decrease the cure time such that the propellant couldn't be properly out-gassed prior to casting. This incomplete out-gassing process has the potential to decrease the density of the fuel grain making it prone to voids in the fuel grain. Because this variability in fuel density can influence the measured regression rate of the fuel grains, a correction of the data was required and is described below. A secondary leading theory, though it hasn't been investigated thoroughly, is that lot-to-lot variation between raw ingredients from the supplier may have been at least a contributing factor to the variability between cast propellant grains. Because of the way Daedalus standardizes its raw ingredients prior to the mixing and casting processes, it isn't impossible that lot-to-lot variation slipped through the screening process. Going forward it is suggested that standardization processes used by the organization more closely represent that of industry methods.

During the experiment some test anomalies did occur. During a few of the tests, including Test # 13, misfires were experienced. For the purposes of this test setup and experiment, a misfire is defined as applying power to Channel 1 to initiate the igniter, and then power was applied to Channel 3 to open the solenoid and begin the combustion process by initiating the N_2O as it entered the combustion chamber, but the N_2O never ignited. Looking back through the notes and test videos, it was found that the root cause of the misfires were human error, and the direct cause was that the user waited too long in

between switching the Channel 1 and Channel 3 switches. Fortunately, no impact to the results was seen as a result of these misfires.

Lastly, during the experiment there were a few cases where the solenoid physically locked in an open position, even after the power was removed. For two tests, this resulted in a brief, and considered to be negligible, amount of un-choked combustion in the chamber. The oxidizer flow was finally cut off by closing the manual safety shut-off valve located upstream in the flow of the solenoid. For one additional test, the solenoid again locked in a slightly open position; however, I actuated the solenoid

4.4 Large Impact of Results

For the lower injector angles these results show a positive slope trend of increasing regression rate as a function of combustion port length down the fuel grain. Because of this, future designs should be designed such that they have a similar initial positive slope down the length of the combustion port length. This would provide the forward initial port diameter to be large than the aft initial port diameter, and would require cast grains to be removed only in the aft direction from the molds. However, it would enable the hybrid rocket motor to maintain a constant burn time down the length of the combustion port, and reduce any concerns of local heating of the combustion chamber during motor operation that could be caused from the motor continuing to burn away propellant in the forward section of the fuel grain, while there is no propellant left in the aft section of the fuel grain.

For the high injector angles, and especially the case of the 60 degree injector, the results show a much more constant burn rate as a function of combustion port length. Because of this, the average hybrid rocket motor designs should employ these super-

critical angle type injectors. This would maintain a constant burn rate as a function of combustion port length, as well as maintain a constant burn surface area and uniform combustion of the fuel as a function of combustion port length. Also, previous research has shown that these high injector angles reduce combustion instabilities, which would allow for an acceptable reduced margin in the structural design of the hybrid rocket motor assembly, while maintaining the same performance.

As discussed in detail earlier in this paper, much of the initial driving design parameters for the motor were based on being able to extrapolate these results to the larger HRM used by Daedalus Astronautics at ASU. However, the long-term goal is that these results may be applied to much larger scale motors as well through a scale factor. In order to accomplish this additional research into the effects of swirl injection angle on both the macro and micro-combustion processes is required.

4.5 Confidence in Results

The results from this experiment show trends in the data and baseline characteristic equations to describe the effects of the different injection angles from the injectors; however, there are still some gaps in the results that could be addressed through additional testing. Additional tests would increase the fidelity of the models by providing a more accurate prediction of the regression rate response variable. Additional testing could be done at other injection angles, including those above 60 degrees to characterize the effects of swirl injection angles all the way up to that of 90 degrees or tangential injection, which some researchers are already adopting for their designs.

Additional testing aside, the results that were obtained in this experiment are quite accurate. Controlled processes were used for the mixing and casting processes, and short

of the few difficulties with minor variabilities from batch to batch, the controls worked quite well. Controlled test procedures were followed, and a large number of extraneous variables were monitored to mitigate any additional error that could affect the final results. There were even controlled mix processes and cast processes were used for the concrete molds, which provided a highly reliable mold of the fuel grains for post-processing and measurement.

5 Conclusions

The 18 tests performed as part of this experiment showed that, as Bellomo said, “...for the regression rate performance parameter the most important factor is the injection type,” (Bellomo, et al., 2012). The ANOVA results discussed in Section 4.2 show that, in contrast to what the basic theoretical equation suggests, oxidizer mass flux is the number two driving factor of regression rate, and this parameter is dependent upon the injector as well. The ANOVA results show that the number one most significant driving factor of regression rate was the swirl injection angle, which as the title of the parameter suggests, is again a dependent upon the injector itself. There are some influences of other test conditions and controlled parameters that have an impact on the regression rate of the fuel; however, the former two parameters are by far the most critical to regression rate and motor performance. It’s important to note that these results suggest that the current theoretical equation used across literature may require revisiting.

The results from this experiment not only showed the effects of the swirl injector angle on regression rate of the fuel grain, but It also identified a few characteristics that are unique to particular injector angles. For example, at the lower swirl injector angles it was found that the regression rate becomes less constant down the length of the fuel

grain. More specifically, the regression rate at the aft end of the fuel grains tends to be much higher than that at the forward end of the grains for the lower swirl injector angles. Because of this, super-critical swirl injector angles are recommended for even burns down the length of the grain. Also, to ensure accurate measurement of these uneven regression rates is properly being captured during this type of testing it is urged that regression rate should always be measured down the full length of the grain and represented as discrete regression rates. Replacing discrete regression rates with average regression rates for the overall fuel grain can be misleading.

As has been previously discussed, different fuels have been studied to better understand the effects swirl injectors on regression rates. It is noteworthy that across previous works and now this experiment, the same general effects of swirl injection on regression rate have been found to not be limited to a particular fuel type or motor configuration.

6 Future Work

The ideal end result of the research endeavors is to perform an up-scaled hybrid motor flight test for dynamic testing of all the previously investigated components of the motor, where the up-scaled hybrid motor is designed using both previous injection results and a well understood scale factor. This flight-ready HRM project has already begun in parallel to the work with the small-scale HRM such that the results from the small-scale can feed into the flight-ready hybrid rocket motor design, which can be seen in Figure 31.

Figure 31. Flight-Ready Hybrid Rocket Motor Design with TVA Assembly

Many of the components you see in this assembly were designed around the research results found using the small-scale hybrid rocket motor and test stand setup. One great example of this would be the variable orifice regulator, which isn't depicted in this model, but can be seen as part of the larger assembled rocket motor assembly in mid-test in Figure 32.

Figure 32. Flight-Ready Hybrid Rocket Motor Design during Successful Static Fire Test

Another great example of a component that will be tested using this test setup will be a new aerospoke nozzle configuration, which has been designed for this small-scale

hybrid rocket motor, as shown in Figure 33, so that its analytical thermal and structural properties can be verified via static fire testing.

Figure 33. (Left) Aerospike Integrated into Small-Scale Hybrid Rocket Motor (Right) 3D Model of Aerospike Nozzle Design

As previously mentioned in Section 4.2, additional testing is required to better understand the effects of these varying oxidizer injection angles on the regression rate of the fuel grain. Also, since this experiment focused only on the regression rate effects of N_2O and HTPB, additional testing to better understand these effects on different oxidizer and fuel combinations is suggested. Especially since the regression rates are highly subjective to what oxidizer and fuel are selected for use in the hybrid rocket motor.

A last and final suggested point of future investigation is the restart capability of a hybrid rocket engine, and the effects of restarting the motor on combustion instabilities. During one of the tests performed, the solenoid locked in a slightly open position, and for safety reasons the solenoid was given a second set of open and close electrical commands. When the solenoid locked in a slightly open position a second time, the same operation was repeated to provide a third set of open and close electrical commands. After the third attempt to close, the solenoid finally sealed into the closed position and we were able to proceed with standard post-test procedures. The interesting thing about this test anomaly was that each time the solenoid was opened back up the motor restarted

instantly and provided nominal pressure and thrust levels. For reference, the sequence of events went as follows: $t_{\text{burn},1} = 2.083\text{s}$, $t_{\text{off},1} = 1.100\text{s}$, $t_{\text{burn},2} = 0.283\text{ s}$, $t_{\text{off},2} = 0.875\text{s}$, $t_{\text{burn},3} = 0.259\text{s}$. Though unplanned as part of the test matrix, this testing was able to show that a successful restart of this small-scale hybrid rocket motor can occur successfully after a 1.100 second period is possible without the use of a second igniter.

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APPENDIX A

DRAWINGS

Figure 34. 0 Degree Injector Drawing

Figure 35. 15 Degree Injector Drawing

Figure 36. 30 Degree Injector Drawing

Figure 38. 60 Degree Injector Drawing

Figure 39. Injector Housing Drawing

Figure 40. Venturi Drawing

APPENDIX B

SENSOR CALIBRATIONS AND SPECIFICATIONS

GAGE & ABSOLUTE PRESSURE TRANSDUCER														
PX309, PX319, PX329 & PX359 0 TO 5 VDC														
CERTIFICATE OF CONFORMANCE DOCUMENT A012503														
MODEL NUMBER INTERPRETATION														
PX	ELECTRICAL CONNECTOR			RANGE CODE			GAGE	ABSOLUTE	0 TO 5 VDC OUTPUT	PRESSURE RANGE	SUPPLY VOLTAGE	OUTPUT	COMPENSATED TEMPERATURE	THERMAL EFFECTS
	X1	X2	X3	X4	X5	X6								
CABLE	3	0	9	0	0	1	G	N/A	5V	1	0 TO 30	0 TO 5	0 TO 80	4.5
DIN	3	1	9	0	0	2	G	N/A	5V	2	0 TO 30	0 TO 5	0 TO 80	5.0
EENDIX	3	2	9	0	0	5	G	A	3V	5	0 TO 30	0 TO 5	0 TO 80	1.5
M12	3	5	9	0	1	5	C	A	5V	15	0 TO 30	0 TO 5	-20 TO 80	1.0
				0	3	0	G	A	5V	30	0 TO 30	0 TO 5	-20 TO 80	1.0
				0	5	0	G	A	5V	50	0 TO 30	0 TO 5	-20 TO 80	1.0
				1	0	0	N/A	A	5V	100	0 TO 30	0 TO 5	-20 TO 80	1.0
				2	0	0	N/A	A	5V	200	0 TO 30	0 TO 5	-20 TO 80	1.0
				3	0	0	N/A	A	5V	300	0 TO 30	0 TO 5	-20 TO 80	1.0
				1	0	0	G	N/A	5V	100	0 TO 30	0 TO 5	-20 TO 80	1.0
				1	5	0	G	N/A	5V	150	0 TO 30	0 TO 5	-20 TO 80	1.0
				2	0	0	G	N/A	5V	200	0 TO 30	0 TO 5	-20 TO 80	1.0
				3	0	0	G	N/A	5V	300	0 TO 30	0 TO 5	-20 TO 80	1.0
				5	0	0	G	N/A	5V	500	0 TO 30	0 TO 5	-20 TO 80	1.0
				1	K	0	G	N/A	5V	1000	0 TO 30	0 TO 5	-20 TO 80	1.0
				2	K	0	G	N/A	5V	2000	0 TO 30	0 TO 5	-20 TO 80	1.0
				3	K	0	G	N/A	5V	3000	0 TO 30	0 TO 5	-20 TO 80	1.0
				5	K	0	G	N/A	5V	5000	0 TO 30	0 TO 5	-20 TO 80	1.0
				7	5	K	G	N/A	5V	7500	0 TO 30	0 TO 5	-20 TO 80	1.0
				1	0	K	G	N/A	5V	10000	0 TO 30	0 TO 5	-20 TO 80	1.0

GENERAL SPECIFICATION	
INPUT	VDC SEE TABLE
OUTPUT	0 TO 5 VDC SEE TABLE
OVER PRESSURE	1 TO 50 PSI GAGE & 5 TO 300 PSI ABSOLUTE: 3 TIMES RATED OR 20 PSI WHICHEVER IS GREATER
BURST PRESSURE	OR 100 TO 10,000 PSIG: 2 TIMES RATED PRESSURE
ACCURACY	OR 1 TO 50 PSI GAGE & 5 TO 300 PSI ABSOLUTE: 5 TIMES RATED OR 25 PSI WHICHEVER IS GREATER
SETTING ACCURACY ZERO	OR 100 TO 10,000 PSIG: 2 TIMES RATED PRESSURE
SETTING ACCURACY SPAN	COMBINED LINEARITY, HYSTERESIS AND REPEATABILITY $\pm 0.25\%$ BSL
TOTAL ERROR FOR 1 OR 2 PSI	$\pm 2.0\%$ FS (EXCLUDING 1 & 2 PSI)
LONG TERM STABILITY	TOTAL ERROR BAND INCLUDING SETTING ACCURACY
DURABILITY	($\pm 4.0\%$ FS FOR 1 PSI & $\pm 3.0\%$ FS FOR 2 PSI)
MEDIA COMPATIBILITY	CALIBRATION IN VERTICAL DIRECTION WITH FITTING DOWN
	10 MILLION MINIMUM
	PRESSURE RANGES 80 PSI GAGE & BELOW AND 300 PSI ABSOLUTE & BELOW: 316 SST
	OR PRESSURE RANGES 100 PSI GAGE TO 10K GAGE: 17-4 SST

0 TO 5 VDC EXTERNAL WIRING CODES					
	CABLE PX309	NOTE:	MINI DIN PX319	TWIST LOCK PX329	M12 PX359
SUPPLY (+)	RED	DO NOT ALLOW	PIN 1	PIN A	
SUPPLY (-)	BLACK	CABLE TO ROTATE	PIN 2	PIN B	
SIGNAL	WHITE	IN CABLE GLAND OR	PIN 3	PIN C	
NC*		PERMANENT DAMAGE	PIN 4	PIN D	
SPARE	SEE NOTE:	WILL OCCUR.		PIN E	
VENT		KEEP NUT ON		PIN F	
		CABLE GLAND TIGHT.			

* FACTORY PIN: DO NOT CONNECTION TO PIN

OMEGADYNE INC.
An Affiliate of Omega Engineering, Inc.

PRESSURE TRANSDUCER
FINAL CALIBRATION

0.00 - 2000.00 PSIG
Excitation 28.000 Vdc

Job: TN2927
Model: PX309-2KG5V
Date: 10/24/2012
Calibrated: 0.00 - 2000.00 PSIG
Serial: 050512D623
Tested By: JP
Temperature Range: -20 to +85 C
Specfile: PX309-5V+=100G

Pressure PSIG	Unit Data Vdc
0.00	0.001
1000.00	2.504
2000.00	4.995
1000.00	2.506
0.00	0.003

Balance 0.001 Vdc
Sensitivity 4.994 Vdc

ELECTRICAL LEAKAGE: PASS
PRESSURE CONNECTION/PIPING: 1/4-18 NPT Male
ELECTRICAL WIRING/CONNECTOR: BLACK = -EXC
WHITE = SIG
RED = +EXC

This Calibration was performed using Instruments and Standards that are
Traceable to the United States National Institute of Standards Technology.

S/N	Description	Range	Reference	Cal Cert
11568	Ametek 15K	0 - 2000.00 PSIG	C-2505	C-2505
MY41038674	AGILENT 34970A DMM	Unit Under Test	C-2473	N/A

Q.A. Representative : *John Picard*

Date: 10/24/2012

This transducer is tested to & meets published specifications. After final
calibration our products are stored in a controlled stock room & considered in
bonded storage. Depending on environment & severity of use factory calibration
is recommended every one to three years after initial service installation date.

Omegadyne Inc., 149 Steilzer Court, Sunbury, OH 43074 (740) 565-9340
http://www.omegadyne.com email: info@omegadyne.com (800) USA-DYNE

GAGE & ABSOLUTE PRESSURE TRANSDUCER														
PX309, PX319, PX329 & PX359 0 TO 5 VDC														
CERTIFICATE OF CONFORMANCE DOCUMENT A012503														
MODEL NUMBER INTERPRETATION														
PX	ELECTRICAL CONNECTOR			RANGE CODE			GAGE	ABSOLUTE	0 TO 5 VDC OUTPUT	PRESSURE RANGE	SUPPLY VOLTAGE	OUTPUT	COMPENSATION TEMPERATURE	THERMAL EFFECTS
	X1	X2	X3	X4	X5	X6								
CABLE	3	0	9	0	0	1	G	N/A	5V	1	0 TO 30	0 TO 5	0 TO 50	4.5
DIA	3	1	9	0	0	2	G	N/A	5V	2	0 TO 30	0 TO 5	0 TO 50	3.0
BENDIX	3	2	9	0	0	5	G	A	5V	5	0 TO 30	0 TO 5	0 TO 50	1.5
M12	3	5	9	0	1	5	G	A	5V	15	0 TO 30	0 TO 5	-20 TO 85	1.0
				0	3	0	G	A	5V	30	0 TO 30	0 TO 5	-20 TO 85	1.0
				0	5	0	G	A	5V	50	0 TO 30	0 TO 5	-20 TO 85	1.0
				1	0	0	N/A	A	5V	130	0 TO 30	0 TO 5	-20 TO 85	1.0
				2	0	0	N/A	A	5V	200	0 TO 30	0 TO 5	-20 TO 85	1.0
				3	0	0	N/A	A	5V	300	0 TO 30	0 TO 5	-20 TO 85	1.0
				1	0	0	G	N/A	5V	100	0 TO 30	0 TO 5	-20 TO 85	1.0
				1	5	0	G	N/A	5V	150	0 TO 30	0 TO 5	-20 TO 85	1.0
				2	0	0	G	N/A	5V	200	0 TO 30	0 TO 5	-20 TO 85	1.0
				3	0	0	G	N/A	5V	300	0 TO 30	0 TO 5	-20 TO 85	1.0
				5	0	0	G	N/A	5V	500	0 TO 30	0 TO 5	-20 TO 85	1.0
				1	K	0	G	N/A	5V	1000	0 TO 30	0 TO 5	-20 TO 85	1.0
				2	K	0	G	N/A	5V	2000	0 TO 30	0 TO 5	-20 TO 85	1.0
				3	K	0	G	N/A	5V	3000	0 TO 30	0 TO 5	-20 TO 85	1.0
				5	K	0	G	N/A	5V	5000	0 TO 30	0 TO 5	-20 TO 85	1.0
				7	.5	K	G	N/A	5V	7500	0 TO 30	0 TO 5	-20 TO 85	1.0
				1	0	K	G	N/A	5V	10000	0 TO 30	0 TO 5	-20 TO 85	1.0

GENERAL SPECIFICATION	
INPUT	VDC SEE TABLE
OUTPUT	0 TO 5 VDC SEE TABLE
OVER PRESSURE	1 TO 50 PSI GAGE & 5 TO 300 PSI ABSOLUTE: 3 TIMES RATED OR 20 PSI WHICHEVER IS GREATER
BURST PRESSURE	OR 100 TO 10,000 PSIG: 2 TIMES RATED PRESSURE
	OR 1 TO 50 PSI GAGE & 5 TO 300 PSI ABSOLUTE: 5 TIMES RATED OR 25 PSI WHICHEVER IS GREATER
ACCURACY	OR 100 TO 10,000 PSIG: 2 TIMES RATED PRESSURE
SETTING ACCURACY ZERO	COMBINED LINEARITY, HYSTERESIS AND REPEATABILITY $\pm 0.25\%$ FS
SETTING ACCURACY SPAN	$\pm 2.0\%$ FS (EXCLUDING 1 & 2 PSI)
TOTAL ERROR FOR 1 OR 2 PSI	$\pm 2.2\%$ FS (EXCLUDING 1 & 2 PSI)
LONG TERM STABILITY	TOTAL ERROR BAND INCLUDING SETTING ACCURACY
DURABILITY	$\pm 4.5\%$ FS FOR 1 PSI & $\pm 3.0\%$ FS FOR 2 PSI
MEDIA COMPATIBILITY	CALIBRATION IN VERTICAL DIRECTION WITH FITTING DOWN
	$\pm 0.25\%$ FS TYPICAL (1 YEAR)
	10 MILLION MINIMUM
	PRESSURE RANGES 50 PSI GAGE & BELOW AND 300 PSI ABSOLUTE & BELOW: 316 SST
	OR PRESSURE RANGES 100 PSI GAGE TO 10K GAGE: 17-4 SST

0 TO 5 VDC EXTERNAL WIRING CODES					
	CABLE PX309	NOTE:	MINI DIN PX319	TWIST LOCK PX329	M12 PX359
SUPPLY (+)	RED	DO NOT ALLOW	PIN 1	PIN A	
SUPPLY (-)	BLACK	CABLE TO ROTATE	PIN 2	PIN B	
SIGNAL	WHITE	IN CABLE GLAND OR	PIN 3	PIN C	
NC*		PERMANENT DAMAGE	PIN 4	PIN D	
SPARE	SEE NOTE:	WILL OCCUR		PIN E	
VENT		KEEP NUT ON CABLE GLAND TIGHT.		PIN F	

* FACTORY PIN DO NOT CONNECTION TO PIN

OMEGADYNE INC.
An Affiliate of Omega Engineering, Inc.

PRESSURE TRANSDUCER
FINAL CALIBRATION

0.00 - 2000.00 PSIG
Excitation 28.000 Vdc

Job: TN2927
Model: PX309-2KG5V
Date: 10/24/2012
Calibrated: 0.00 - 2000.00 PSIG
Serial: 0501120378
Tested By: JF
Temperature Range: -20 to +85 C
Specfile: PX309-5V+-100G

Pressure PSIG	Unit Data Vdc
0.00	- 0.001
1000.00	2.505
2000.00	4.997
1000.00	2.508
0.00	- 0.001

Balance - 0.001 Vdc
Sensitivity 4.998 Vdc

ELECTRICAL LEAKAGE: PASS
PRESSURE CONNECTION/FITTING: 1/4-18 NPT Male
ELECTRICAL WIRING/CONNECTOR: BLACK = -EXC
WHITE = SIG
RED = +FXC

This Calibration was performed using Instruments and Standards that are traceable to the United States National Institute of Standards Technology.

S/N	Description	Range	Reference	Cal Cert
11568	Ametek 15K	0 - 2000.00 PSIG	C-2505	C-2505
MY41038674	AGILENT 34970A DMM	Unit Under Test	C-2473	N/A

O.A. Representative : *John Picard*

Date: 10/24/2012

This transducer is tested to & meets published specifications. After final calibration our products are stored in a controlled stock room & considered in bonded storage. Depending on environment & severity of use factory calibration is recommended every one to three years after initial service installation date.

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http://www.omegadyne.com email: info@omegadyne.com (800) USA-DYNE

GAGE & ABSOLUTE PRESSURE TRANSDUCER														
PX309, PX319, PX329 & PX359 0 TO 5 VDC														
CERTIFICATE OF CONFORMANCE DOCUMENT A012503														
MODEL NUMBER INTERPRETATION										PRESSURE RANGE	SUPPLY VOLTAGE	OUTPUT	COMPENSATED TEMPERATURE	THERMAL EFFECTS
PX	ELECTRICAL CONNECTOR			RANGE CODE			GAGE	ABSOLUTE	0 TO 5 VDC OUTPUT					
	X1	X2	X3	X4	X5	X6	X7	X8	PSI	VDC	VDC	°C	±%FS	
CABLE	3	0	9	0	0	1	G	N/A	5V	1	0 TO 30	0 TO 5	0 TO 50	4.5
DIN	3	1	9	0	0	2	G	N/A	5V	2	0 TO 30	0 TO 5	0 TO 50	3.0
EENDIX	3	2	9	0	0	5	G	A	5V	5	0 TO 30	0 TO 5	0 TO 50	1.5
M12	3	5	9	0	1	5	G	A	5V	15	0 TO 30	0 TO 5	-20 TO 85	1.0
				0	3	0	G	A	5V	30	0 TO 30	0 TO 5	-20 TO 85	1.0
				0	5	0	G	A	5V	50	0 TO 30	0 TO 5	-20 TO 85	1.0
				1	0	0	N/A	A	5V	100	0 TO 30	0 TO 5	-20 TO 85	1.0
				2	0	0	N/A	A	5V	200	0 TO 30	0 TO 5	-20 TO 85	1.0
				3	0	0	N/A	A	5V	300	0 TO 30	0 TO 5	-20 TO 85	1.0
				1	0	0	G	N/A	5V	100	0 TO 30	0 TO 5	-20 TO 85	1.0
				1	5	0	G	N/A	5V	150	0 TO 30	0 TO 5	-20 TO 85	1.0
				2	0	0	G	N/A	5V	200	0 TO 30	0 TO 5	-20 TO 85	1.0
				3	0	0	G	N/A	5V	300	0 TO 30	0 TO 5	-20 TO 85	1.0
				5	0	0	G	N/A	5V	500	0 TO 30	0 TO 5	-20 TO 85	1.0
					1	K	G	N/A	5V	1000	0 TO 30	0 TO 5	-20 TO 85	1.0
					2	K	G	N/A	5V	2000	0 TO 30	0 TO 5	-20 TO 85	1.0
					3	K	G	N/A	5V	3000	0 TO 30	0 TO 5	-20 TO 85	1.0
					5	K	G	N/A	5V	5000	0 TO 30	0 TO 5	-20 TO 85	1.0
				7	5	K	G	N/A	5V	7500	0 TO 30	0 TO 5	-20 TO 85	1.0
				1	0	K	G	N/A	5V	10000	0 TO 30	0 TO 5	-20 TO 85	1.0
GENERAL SPECIFICATION														
INPUT	VDC SEE TABLE													
OUTPUT	0 TO 5 VDC SEE TABLE													
OVER PRESSURE	1 TO 30 PSI GAGE & 0 TO 300 PSI ABSOLUTE: 5 TIMES RATED OR 20 PSI WHICHEVER IS GREATER													
BURST PRESSURE	OR	100 TO 10,000 PSIG: 2 TIMES RATED PRESSURE												
	OR	1 TO 50 PSI GAGE & 0 TO 300 PSI ABSOLUTE: 5 TIMES RATED OR 25 PSI WHICHEVER IS GREATER												
ACCURACY	OR	100 TO 10,000 PSIG: 2 TIMES RATED PRESSURE												
SETTING ACCURACY ZERO	COMBINED LINEARITY, HYSTERESIS AND REPEATABILITY ±0.25% BSL													
SETTING ACCURACY SPAN	±2.0% FS (EXCLUDING 1 & 2 PSI)													
TOTAL ERROR FOR 1 OR 2 PSI	±2.0% FS (EXCLUDING 1 & 2 PSI)													
	TOTAL ERROR BAND INCLUDING SETTING ACCURACY													
	(+4.0% FS FOR 1 PSI & ±3.0% FS FOR 2 PSI)													
LONG TERM STABILITY	CALIBRATION IN VERTICAL DIRECTION WITH FITTING DOWN													
DURABILITY	±0.25% FS TYPICAL (1 YEAR)													
MEDIA COMPATIBILITY	10 MILLION MINIMUM													
	PRESSURE RANGES 80 PSI GAGE & BELOW AND 300 PSI ABSOLUTE & BELOW: 316 SST													
	OR	PRESSURE RANGES 100 PSI GAGE TO 10K GAGE: 17-4 SST												
0 TO 5 VDC EXTERNAL WIRING CODES														
	CABLE PX309	NOTE:	MINI DIN PX319	TWIST LOCK PX329	M12 PX359									
SUPPLY (+)	RED	DO NOT ALLOW	PIN 1	PIN A										
SUPPLY (-)	BLACK	CABLE TO ROTATE	PIN 2	PIN B										
SIGNAL	WHITE	IN CABLE GLAND OR	PIN 3	PIN C										
NC*		PERMANENT DAMAGE	PIN 4	PIN D										
SPARE	SEE NOTE:	WILL OCCUR		PIN E										
VENT		KEEP NUT ON		PIN F										
		CABLE GLAND TIGHT.												
* FACTORY PIN DO NOT CONNECTION TO PIN														

ALL STAINLESS STEEL TRANSDUCER MULTIMEDIA COMPATIBILITY HIGH-PERFORMANCE SILICON TECHNOLOGY

Engineered from
1 to 10,000 psi.

PX309 Series
Gage/Absolute Pressure
0 to 5 Vdc Output
0-1 to 0-10,000 psi
0-70 mbar to 0-690 bar

Starts at
\$225

- ✓ 1, 2, and 5 psi Low Pressure Ranges
- ✓ Rugged, Solid State Design
- ✓ All Stainless Steel
- ✓ High Stability, Low Drift
- ✓ 0.25% Accuracy
- ✓ RoHS Compliant
- ✓ IP65 Protection Class

SPECIFICATIONS

Excitation: 9 to 30 Vdc (<10 mA)
(reverse polarity and overvoltage protected)

Output: 0 to 5 Vdc

Accuracy: $\pm 0.25\%$ FS BSL at 25°C;
includes linearity, hysteresis and repeatability

Zero Offset: $\pm 2\%$ FSO;
 $\pm 4\%$ for 1 and 2 psi ranges

Span Setting: $\pm 2\%$ FSO;
 $\pm 4\%$ for 1 and 2 psi ranges

Total Error Band: $\pm 2\%$ FSO, includes linearity, hysteresis, repeatability, thermal hysteresis and thermal errors (except 1 psi = $\pm 4.5\%$ and 2 psi = $\pm 3\%$)

Long-Term Stability (1 Year):
 $\pm 0.25\%$ typical

Typical Life: 10 million cycles
Operating Temperature: -40 to 85°C
(-40 to 185°F)

Compensated Temperature:
 >5 psi Range: -20 to 85°C (-4 to 185°F)
 ≤5 psi Range: 0 to 50°C (32 to 122°F)

Proof Pressure:
 psia and ≤50 psig: 3x capacity
 or 20 psi, whichever is greater
 ≥100 psi: 2x capacity

Burst Pressure: 5x capacity or 25 psi, whichever is greater

Response Time: <1 ms

Shock: 50 g, 11 ms half-sine shock

Vibration: ± 20 g

Wetted Parts: 316 SS for all psia ranges and 1 to 50 psig; 17-4 PH SS for ranges 100 to 10,000 psig

Pressure Port: 1/8-18 MNPT

Electrical Connections:

PX309: 1.5 m (5') 3-conductor cable

PX319: mini DIN connector with mating connector included

PX329: Twist-lock connector, mating connector sold separately (PT06V-10-6S)

Weight: 155 g (5.4 oz) max

Note: Units 100 psig and above may be subjected to vacuum on the pressure port without damage

For alternative performance specifications contact Engineering.

VOLTAGE OUTPUT
PRESSURE TRANSDUCERS
B

RUGGED, GENERAL PURPOSE TRANSDUCER

LOW-PRESSURE RANGES HIGHLIGHTED

To Order (Specify Model Number)

RANGE		1.5 m CABLE CONNECTION	PRICE	MINI DIN CONNECTION	PRICE	TWIST-LOCK CONNECTION	PRICE
bar	psi						
ABSOLUTE PRESSURE							
0 to 0.34	0 to 5	PX309-005A5V	\$325	PX319-005A5V	\$325	PX329-005A5V	\$350
0 to 1	0 to 15	PX309-015A5V	245	PX319-015A5V	245	PX329-015A5V	295
0 to 2.1	0 to 30	PX309-030A5V	245	PX319-030A5V	245	PX329-030A5V	295
0 to 3.4	0 to 50	PX309-050A5V	245	PX319-050A5V	245	PX329-050A5V	295
0 to 6.9	0 to 100	PX309-100A5V	245	PX319-100A5V	245	PX329-100A5V	295
0 to 14	0 to 200	PX309-200A5V	245	PX319-200A5V	245	PX329-200A5V	295
0 to 21	0 to 300	PX309-300A5V	245	PX319-300A5V	245	PX329-300A5V	295
GAGE PRESSURE							
0 to 0.07	0 to 1	PX309-001G5V	\$345	PX319-001G5V	\$345	PX329-001G5V	\$370
0 to 0.14	0 to 2	PX309-002G5V	325	PX319-002G5V	325	PX329-002G5V	350
0 to 0.34	0 to 5	PX309-005G5V	300	PX319-005G5V	300	PX329-005G5V	325
0 to 1	0 to 15	PX309-015G5V	225	PX319-015G5V	225	PX329-015G5V	275
0 to 2.1	0 to 30	PX309-030G5V	225	PX319-030G5V	225	PX329-030G5V	275
0 to 3.4	0 to 50	PX309-050G5V	225	PX319-050G5V	225	PX329-050G5V	275
0 to 6.9	0 to 100	PX309-100G5V	225	PX319-100G5V	225	PX329-100G5V	275
0 to 10	0 to 150	PX309-150G5V	225	PX319-150G5V	225	PX329-150G5V	275
0 to 14	0 to 200	PX309-200G5V	225	PX319-200G5V	225	PX329-200G5V	275
0 to 21	0 to 300	PX309-300G5V	225	PX319-300G5V	225	PX329-300G5V	275
0 to 34	0 to 500	PX309-500G5V	225	PX319-500G5V	225	PX329-500G5V	275
0 to 69	0 to 1000	PX309-1KG5V	225	PX319-1KG5V	225	PX329-1KG5V	275
0 to 138	0 to 2000	PX309-2KG5V	225	PX319-2KG5V	225	PX329-2KG5V	275
0 to 207	0 to 3000	PX309-3KG5V	225	PX319-3KG5V	225	PX329-3KG5V	275
0 to 345	0 to 5000	PX309-5KG5V	225	PX319-5KG5V	225	PX329-5KG5V	275
0 to 517	0 to 7500	PX309-7.5KG5V	225	PX319-7.5KG5V	225	PX329-7.5KG5V	275
0 to 690	0 to 10,000	PX309-10KG5V	225	PX319-10KG5V	225	PX329-10KG5V	275

ACCESSORIES

MODEL NO.	PRICE	DESCRIPTION
CAL-3	150.00	Recalibration: 5-point NIST traceable
PT06V-10-6S	26.50	Mating twist-lock connector for PX329
CA-39-4PC22-5	90.00	Mating twist-lock connector with 1.5 m (5') cable for PX329
CX5302	15.00	Extra mating mini DIN connector for PX319

Comes complete with 5-point NIST-traceable calibration.

Ordering Examples: PX309-100G5V, 100 psi gage pressure transducer with 0 to 5 Vdc output and 1 m cable termination, \$225.

PX319-015A5V, 15 psi absolute pressure transducer with 0 to 5 Vdc output and mini DIN termination, \$300. PX329-3KG5V, 3000 psi gage pressure transducer with 0 to 5 Vdc output and twist-lock termination, \$275. Mating connector sold separately; order PT06V-10-6S, \$26.50.

B-97

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• Flow and Level

Air Velocity Indicators, Doppler Flowmeters, Level Measurement, Magnetic Flowmeters, Mass Flowmeters, Pitot Tubes, Pumps, Rotameters, Turbine and Paddle Wheel Flowmeters, Ultrasonic Flowmeters, Valves, Variable Area Flowmeters, Vortex Shedding Flowmeters

• pH and Conductivity

Conductivity Instrumentation, Dissolved Oxygen Instrumentation, Environmental Instrumentation, pH Electrodes and Instruments, Water and Soil Analysis Instrumentation

• Data Acquisition

Auto-Dialers and Alarm Monitoring Systems, Communication Products and Converters, Data Acquisition and Analysis Software, Data Loggers Plug-in Cards, Signal Conditioners, USB, RS232, RS485 and Parallel Port Data Acquisition Systems, Wireless Transmitters and Receivers

• Pressure, Strain and Force

Displacement Transducers, Dynamic Measurement Force Sensors, Instrumentation for Pressure and Strain Measurements, Load Cells, Pressure Gauges, Pressure Reference Section, Pressure Switches, Pressure Transducers, Proximity Transducers, Regulators, Strain Gages, Torque Transducers, Valves

• Heaters

Band Heaters, Cartridge Heaters, Circulation Heaters, Comfort Heaters, Controllers, Meters and Switching Devices, Flexible Heaters, General Test and Measurement Instruments, Heater Hook-up Wire, Heating Cable Systems, Immersion Heaters, Process Air and Duct, Heaters, Radiant Heaters, Strip Heaters, Tubular Heaters

• [click here to go to the omega.com home page](http://www.omega.com) •

EPG05

FUTEK MODEL LTH400

Drawing Number: F11004-B

INCH [mm] R.O. = Rated Output

WIRING CODE (MCS)			
+Excitation	-Excitation	+Signal	-Signal
RED	BLACK	GREEN	WHITE
Shield			
FLOATING			

DONUT / THRU HOLE LOAD CELL AVAILABLE WITH I.D.'S FROM 1/8 TO 5/8

+ OUTPUT
(COMPRESSION)

SPECIFICATIONS:

RATED OUTPUT	2 mV/V nom.
CAPACITY	250, 500, 1000, 2000, 3000, 5000, 7500, 10000*
WEIGHT	1112, 2224, 4448, 6672, 13344, 22240, 33360, 44480*
SAFE OVERLOAD	150% of R.O.
ZERO BALANCE	±1% of R.O.
EXCITATION (VDC OR VAC)	10-60VDC
BRIDGE RESISTANCE	700 Ω, 350Ω
NONLINEARITY	±0.5 % of R.O.
HYSTERESIS	±0.05 % of R.O.
NONREPEATABILITY	±0.05 % of R.O.
TEMP. SHIFT ZERO	±0.005 % of R.O./°F (±0.01 % of R.O./°C)
TEMP. SHIFT SPAN	±0.005 % of R.O./°F (±0.01 % of R.O./°C)
COMPENSATED TEMP.	80 to 160°F [16 to 72°C]
OPERATING TEMP.	-65 to 200°F [-60 to 93°C]
WEIGHT	8 oz [227 g]
MATERIAL	17-4PH S.S.
DEFLECTION	0.002 [0.05] mm.
CABLE: 604 AWG, 4 Conductor, Spiral Shielded Twisted Cable 16 ft [5 m] Long	
ACCESSORIES AND RELATED INSTRUMENTS AVAILABLE	
CALIBRATION (STD)	5 pt. COMPRESSION: 100K A. SHUNT CAL. VALUE
CALIBRATION TEST EXCITATION	10 VDC

* COMMONLY STOCKED CAPACITIES AND INNER DIAMETERS

ID	Inner Diameter	Inner Diameter
1/8	0.1250/0.124	[3.175/3.149]
3/16	0.1875/0.187	[4.763/4.751]
1/4	0.2500/0.250	[6.350/6.350]
5/16	0.3125/0.312	[7.938/7.928]
3/8	0.3750/0.374	[9.525/9.518]
7/16	0.4375/0.437	[11.113/11.105]
1/2	0.5000/0.500	[12.700/12.700]
9/16	0.5625/0.562	[14.288/14.283]
5/8	0.6250/0.624	[15.875/15.869]

NOTE:

WHILE TORQUING BOLTS, HIGH TENSION FORCES WILL OCCUR DURING INSTALLATION, WHICH COULD RESULT IN OVERLOADING THE SENSOR. FUTEK RECOMMENDS CONNECTING SENSOR TO A SYSTEM IN ORDER TO MONITOR THE APPLIED FORCES. FUTEK IS NOT LIABLE OR RESPONSIBLE FOR THE MISHANDLING OF THE SENSOR DURING INSTALLATION.

FUTEK
ADVANCED SENSOR TECHNOLOGY, INC.

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CAGE CODE # 12888

10 THOMAS
IRVINE, CA 92618 USA
1-800-23-FUTEK (38835)

INTERNET:
<http://www.futek.com>

APPENDIX C

MOLD MEASUREMENTS

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Therapiefeld Empfänger Name Pflanz Bewertung Erfolg	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40	41	42	43	44	45	46	47	48	49	50	51	52	53	54	55	56	57	58	59	60	61	62	63	64	65	66	67	68	69	70	71	72	73	74	75	76	77	78	79	80	81	82	83	84	85	86	87	88	89	90	91	92	93	94	95	96	97	98	99	100
	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40	41	42	43	44	45	46	47	48	49	50	51	52	53	54	55	56	57	58	59	60	61	62	63	64	65	66	67	68	69	70	71	72	73	74	75	76	77	78	79	80	81	82	83	84	85	86	87	88	89	90	91	92	93	94	95	96	97	98	99	100
	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40	41	42	43	44	45	46	47	48	49	50	51	52	53	54	55	56	57	58	59	60	61	62	63	64	65	66	67	68	69	70	71	72	73	74	75	76	77	78	79	80	81	82	83	84	85	86	87	88	89	90	91	92	93	94	95	96	97	98	99	100
	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40	41	42	43	44	45	46	47	48	49	50	51	52	53	54	55	56	57	58	59	60	61	62	63	64	65	66	67	68	69	70	71	72	73	74	75	76	77	78	79	80	81	82	83	84	85	86	87	88	89	90	91	92	93	94	95	96	97	98	99	100
	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40	41	42	43	44	45	46	47	48	49	50	51	52	53	54	55	56	57	58	59	60	61	62	63	64	65	66	67	68	69	70	71	72	73	74	75	76	77	78	79	80	81	82	83	84	85	86	87	88	89	90	91	92	93	94	95	96	97	98	99	100

APPENDIX D

ADDITIONAL JMP RESULTS

	Swirl Injector Angle (deg)	P_c, avg (psi)	G_ox (lb/s-in^2)	Regression Rate (in/s)
1	60	373.949	0.15	0.087
2	15	470.946	0.15	0.07
3	15	470.946	0.15	0.066
4	0	520.946	0.15	0.063
5	0	490.946	0.15	0.068
6	30	500.946	0.15	0.082
7	30	490.946	0.15	0.078
8	60	520.946	0.193	0.106
9	60	530.946	0.193	0.092
10	60	500.946	0.15	0.104
11	60	495.946	0.15	0.074
12	42	470.946	0.15	0.079
13	42	421.186	0.193	0.081
14	42	470.946	0.193	0.085
15	42	475.946	0.15	0.077
16	0	470.946	0.193	0.078
17	0	470.946	0.193	0.066
18	15	470.946	0.193	0.08

Figure 41. Data Table used for JMP Analysis Input

Screening for Regression Rate (in/s)						
Contrasts						
Term	Contrast		Lenth	Individual	Simultaneous	
			t-Ratio	p-Value	p-Value	
Swirl Injector Angle (deg)	0.008880		7.99	0.0004*	0.0031*	
G_ox (lb/s-in^2)	0.003540		3.18	0.0133*	0.1413	
P_c, avg (psi)	0.001750		1.57	0.1240	0.7930	
Swirl Injector Angle (deg)*Swirl Injector Angle (deg)	0.000732 *		0.66	0.5390	1.0000	
Swirl Injector Angle (deg)*G_ox (lb/s-in^2)	-0.000445 *		-0.40	0.7100	1.0000	
Swirl Injector Angle (deg)*P_c, avg (psi)	-0.000165 *		-0.15	0.8898	1.0000	
G_ox (lb/s-in^2)*P_c, avg (psi)	0.000958 *		0.86	0.3622	0.9999	
P_c, avg (psi)*P_c, avg (psi)	0.000741 *		0.67	0.5002	1.0000	
Swirl Injector Angle (deg)*Swirl Injector Angle (deg)*Swirl Injector Angle (deg)	0.000542 *		0.49	0.6483	1.0000	
Swirl Injector Angle (deg)*Swirl Injector Angle (deg)*G_ox (lb/s-in^2)	0.000104 *		0.09	0.9324	1.0000	
Swirl Injector Angle (deg)*Swirl Injector Angle (deg)*P_c, avg (psi)	-0.001813 *		-1.63	0.1140	0.7592	
Swirl Injector Angle (deg)*G_ox (lb/s-in^2)*P_c, avg (psi)	-0.000311 *		-0.28	0.7920	1.0000	
Swirl Injector Angle (deg)*P_c, avg (psi)*P_c, avg (psi)	0.003060 *		2.75	0.0233*	0.2274	
G_ox (lb/s-in^2)*P_c, avg (psi)*P_c, avg (psi)	-0.004653 *		-4.18	0.0036*	0.0489*	
P_c, avg (psi)*P_c, avg (psi)*P_c, avg (psi)	0.000693 *		0.61	0.5682	1.0000	
Null17	-0.001278		-1.15	0.2343	0.9825	
Null18	-0.001677		-1.51	0.1374	0.8333	

Figure 42. Calculated Contrasts from JMP Output

Figure 43. Calculated Half Normal Plot of the Contrasts from JMP Output

Summary of Fit

RSquare	0.670527
RSquare Adj	0.626597
Root Mean Square Error	0.00734
Mean of Response	0.079778
Observations (or Sum Wgts)	18

Analysis of Variance

Source	DF	Sum of Squares	Mean Square	F Ratio
Model	2	0.00164488	0.000822	15.2636
Error	15	0.00080823	0.000054	Prob > F
C. Total	17	0.00245311		0.0002*

Lack Of Fit

Source	DF	Sum of Squares	Mean Square	F Ratio
Lack Of Fit	6	0.00014707	0.000025	0.3337
Pure Error	9	0.00066117	0.000073	Prob > F
Total Error	15	0.00080823		0.9026

Max RSq

0.7305

Figure 44. Summary of Fit, Analysis of Variance, and Lack of Fit from JMP Output

Parameter Estimates				
Term	Estimate	Std Error	t Ratio	Prob> t
Intercept	0.0390831	0.014127	2.77	0.0144*
Swirl Injector Angle (deg)	0.0003939	7.617e-5	5.17	0.0001*
G_ox (lb/s-in^2)	0.1688845	0.082552	2.05	0.0587

Effect Tests					
Source	Nparm	DF	Sum of Squares	F Ratio	Prob > F
Swirl Injector Angle (deg)	1	1	0.00144067	26.7374	0.0001*
G_ox (lb/s-in^2)	1	1	0.00022551	4.1853	0.0587

Figure 45. Parameter Estimates and Effect Tests from JMP Output

Figure 46. Residual of Predicted Plot from JMP Output