Optimization Model and Algorithm for the Design of Connected and Compact Conservation Reserves

by

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ABSTRACT

Conservation planning is fundamental to guarantee the survival of endangered species and to preserve the ecological values of some ecosystems. Planning land acquisitions increasingly requires a landscape approach to mitigate the negative impacts of spatial threats such as urbanization, agricultural development, and climate change. In this context, landscape connectivity and compactness are vital characteristics for the effective functionality of conservation reserves. Connectivity allows species to travel across landscapes, facilitating the flow of genes across populations from different protected areas. Compactness measures the spatial dispersion of protected sites, which can be used to mitigate risk factors associated with species leaving and re-entering the reserve. This research proposes an optimization model to identify areas to protect while enforcing connectivity and compactness. In the suggested projected area, this research builds upon existing methods and develops an alternative metric of compactness that penalizes the selection of patches of land with few protected neighbors. The new metric is referred as leaf because it intends to minimize the number of selected areas with 1 neighboring protected area. The model includes budget and minimum selected area constraints to reflect realistic financial and ecological requirements. Using a lexicographic approach, the model can improve the compactness of conservation reserves obtained by other methods. The use of the model is illustrated by solving instances of up to 1100 patches.

DEDICATION

To,

My parents, R Ravishankar and Sumathi Ravishankar, my sister, Harini Ravishankar, my grandparents and other family members for their constant support, love and encouragement

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Chapter 1

INTRODUCTION

Conservation planning is fundamental to guarantee the survival of endangered species and preserve the ecological value of some ecosystems. Such planning increasingly requires a landscape approach, integrating information across sites to inform government and non-government organizations on the best sites to protect. This approach is essential when accounting for rapidly changing environments, such as urbanization, agricultural development, and climate change. The importance of conservation planning has been documented in the literature. Sanderson *et al.* (2002) propose a conceptual model for conservation planning in which the authors focus on a few species which have a large impact on the ecology for conservation and believe that meeting the requirements of these species will result in the conservation of other species and the of the landscape as a whole. Fleishman *et al.* (2002) discuss the importance of a concept of nestedness analysis in conservation planning, which implies that, the nestedness of birds was not affected by the difference in area and topology, where as the nestedness of butterflies were greatly affected by the difference in area and topology. This throws light on the difficulties of a conservation planning problem and the importance of understanding how important nestedness is for conservation planning. Haight and Travis (1997) propose a stochastic optimization method for wildlife conservation planning that focuses on building a model with a changing habitat size of species over time. They consider all the other attributes to be constants.The objective of this problem was to determine how much of the existing habitat has to be preserved and how much should be used for economic development.

A key component that makes landscape approaches challenging is inducing connectivity and compactness in protected areas. Connectivity allows species to travel across protected areas whereas compactness measures how close the protected sites are to each other. Williams and Snyder (2005) discuss about a shortest-path algorithm to induce connectivity between fragmented landscapes. Jafari and Hearne (2013), Wang *et al.* (2017), Jafari *et al.* (2017), Önal and Briers (2005) propose multiple models based on mathematical programming to induce connectivity in conservation planning problems.

Although a more difficult metric to express mathematically, compactness quantifies the spread of the conserved areas and allows decision makers to identify landscape configurations that, although connected, are undesirable. For instance, low-diameter (bulky) circular areas are preferred over long and thin areas. Young (1988) reviews eight metrics of compactness which can be potentially used in the conservation planning problems. Williams *et al.* (2005) discuss the importance of the spatial characteristics of a reserve and reviews a few of the formulations. Emphasizing on reserve size, reserve shape, reserve connectivity, number of reserves, reserve proximity and buffer zones. Önal and Briers (2003), Jafari and Hearne (2013), Wang and Önal (2016) and Önal *et al.* (2016) propose a metric of compactness based on the boundary, perimeter, total distance and distance to center respectively. We incorporate Önal *et al.* (2016) in our research because of its computational performance, for comparison and modeling.

The mathematical programming models available in the literature aim to design single-component protected areas, where a component is a group of connected patches that facilitate movement of wildlife. That is, models that can produce protected areas with more than one connected component are ignored, although they may be desirable given budget or landscape constraints (e.g., roads). Moreover, there are no available models to effectively induce connectivity and compactness at the same time for more than one component. Although a few models exist in the literature that claim to solve this variant of the conservation planning problem, we have produced counter examples in which they fail. Because both connectivity and compactness matter in landscape conservation, the focus of this research is to survey the existing methods and to build an improved model that incorporates both criteria under multi-component landscape conservation problems. We compare our proposed compactness metric with existing metrics.

This paper is organized as follows, section 1.1 presents the review of the available models for connectivity and compactness, while section 2.1 defines connectivity and illustrates with an example what connectivity is. Section 2.2 defines compactness and also provides visual examples to illustrate existing and new metrics. Chapter 3 introduces the mathematical programming model, we then introduce mathematical models to solve the problem. Chapter 4 describes the proposed solution algorithm. Chapter 5 presents our results and visual examples of the solution to the problem. Finally chapter 6 discusses the future scope of our research.

1.1 Literature Review

We first reviewed Literature on Conservation Planning. Fleishman *et al.* (2002) discuss the importance of a concept of nestedness analysis in conservation planning, nestedness is a concept which states that the population density of species keeps decreasing as the distance from the reserve center, which implies that, the nestedness of birds were not affected by the difference in area and topology, where as the nestedness of butterflies were greatly affected by the difference in area and topology. This throws light on the difficulties of a conservation planning problem and the importance of understanding how important nestedness is, for conservation planning, Sanderson *et al.* (2002) propose a conceptual model for conservation planning in which the authors focus on a few species which have a large impact on the ecology for conservation and believe that meeting the requirements of these species will result in the conservation of other species and the landscape as a whole, Önal and Briers (2005) discuss a method of modeling a conservation reserve by minimizing the fragmentation(gap) between two selected patches. Önal and Briers (2005) use the concept of graph to implement the model proposed. Snyder *et al.* (2004) discuss a two-objective optimization model to solve the habitat reserve planning problem. This literature give us insight on conservation planning and how problems on conservation planning can be tackled. There are many heuristic methods to solve a conservation planning problem, Csuti *et al.* (1997) compares five types of heuristic formulations which are four variations of richness-based heuristics and two variations of rarity-based heuristics, eleven variations of progressive rarity-based heuristic, simulated annealing and a linear programming-based branch and bound algorithm. Nalle *et al.* (2002) propose a model whose objective function is to minimize the difference between weighted sum of compactness and connectivity. The proposed model is solved using three heuristics which are simulated annealing, tabu search with short-term memory and genetic algorithm. Pressey *et al.* (1996) discuss and compare exact solution algorithms and heuristics, when is it advisable to use them, their advantages and disadvantages. Pressey *et al.* (1997) discuss two types of heuristic algorithms, Presence-absence algorithm and Proportional area algorithm and present a comparative study of the heuristics.

We then reviewed literature on connectivity, to see how it is modeled, and we came across the following models: Jafari and Hearne (2013) discuss a flow based network model to connectivity of reserves. Önal *et al.* (2016) discuss a method of modeling connectivity using constraints that enforce selection of patches to form a path from one selected patch to another selected patch. Önal and Briers (2005) discuss modeling connectivity by using an objective function to minimize the sum of gaps between selected patches. Jafari *et al.* (2017) discuss a multi-period connected reserve selection model for a cost-effective conservation problem. Carvajal *et al.* (2013) discuss the importance of connectivity (contiguity) in landscapes and how integer programming can be used to achieve it. We decided to use models Jafari and Hearne (2013) and Önal *et al.* (2016).

We then reviewed literature on compactness. Young (1988) discuss 8 metrics of compactness which can be used in conservation planning. There were a few other papers with dealt with compactness, they were, Nalle *et al.* (2002) propose a model whose objective function is to minimize the difference between weighted sum of compactness and connectivity. The proposed model is solved using three heuristics which are simulated annealing, tabu search with short-term memory and genetic algorithm. Vanegas *et al.* (2010) reviews spatial optimization approaches using techniques as heuristics, meta heuristics and mathematical programming and discuss the critical issues in identifying contiguous and compact areas in digital geographical information. Önal *et al.* (2016) and Wang and Önal (2016) propose a model that minimizes the sum of distance between patches and centers of the reserves. Önal and Briers (2003) discuss a metric of compactness, boundary, the smaller the value of the boundary, the more compact is the landscape. Their mathematical program involves minimizing the sum of parcel boundaries and the boundaries of the patches that are part of the landscape boundaries. Intuitively, packing more patches, while minimizing the boundary length produces more compact shape. Jafari and Hearne (2013) defined compactness as the perimeter of the reserve. They calculate the perimeter by adding the perimeter of the selected patches and subtracting the length of the shared edges between selected patches. The objective is to minimize the perimeter in order achieve better compactness. Intuitively, minimizing the perimeter also maximizes the length of shared edges between selected patches, which results in better compactness.

The motivation for our research is the inability of the existing metrics to resolve situations of ties between landscapes. Existing metrics result in same values for compactness measure for two landscapes with different configurations. In such situations, selecting the better of the landscapes becomes a concern, visual judgment cannot be quantified and will not be consistent. To tackle this issue, we design a metric, along with the existing metrics, that is capable of resolving ties and improving compactness of the landscape.

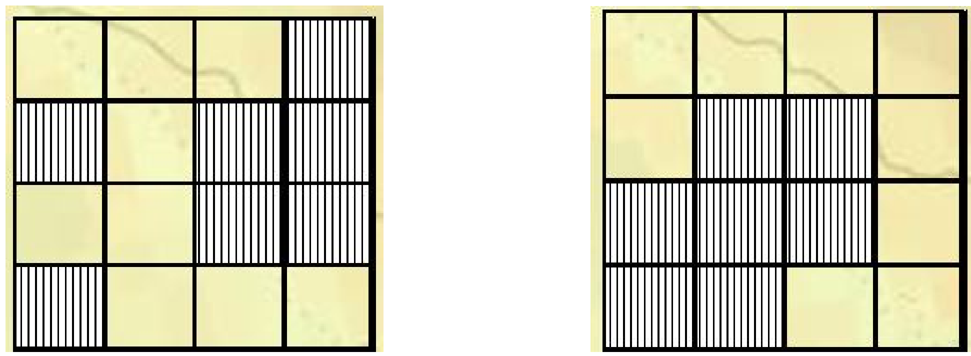
Chapter 2

LANDSCAPE CONNECTIVITY AND COMPACTNESS

2.1 Connectivity

The importance of connectivity has been emphasized in the literature of landscape conservation and habitat restoration. Carvajal *et al.* (2013) discusses the importance of connectivity (contiguity) in landscapes and how integer programming can be used to achieve it. Connectivity can be defined, in a simply way, as the existence of a path from every patch to every other patch in the protected landscape. Connectivity ensures the movement of animals from one patch to another in the reserve, which allows short and long term dynamics due to change in weather, seasons, and other ecological processes. Önal *et al.* (2016) mentions two types of connectivity, structural connectivity and functional connectivity. Structural connectivity is when one can travel from one patch to another patch in the protected landscape by only moving on the protected patches. Functional connectivity is when the species can traverse between one patch to another patch in the protected landscape without necessarily traversing the protected patches physically. Connectivity has been implemented using various techniques for a single connected reserve. Williams and Snyder (2005) uses multiple shortest path method to enforce connectivity whereas, Jafari and Hearne (2013) suggests a method using a network flow approach to impose connectivity for n, where n is the number of protected reserves to be selected. This multi-component structure is also achieved by Önal *et al.* (2016) whose model enforces the selection of at least one neighbor of a selected patch which is closer to the center, which induces the existence of a path from the selected patch to the center of the protected area.

In this paper, we use the connectivity model in Önal *et al.* (2016) because of its computational performance.



(a) Disconnected/Fragmented Landscape (b) Connected/contiguous Landscape

Figure 2.1: Landscape Connectivity

Pattern	Meaning
	Unselected Patch
	Selected Patch
	Center Patch of the selected Reserve
	Leaf Patch for the reserve

Figure 2.2: Patterns and their meaning

Figure 2.1(a) illustrates a landscape that is not connected, and where there is no path from one patch to any other patch using only selected patches. Figure 2.1(b) represents a landscape which is connected, containing a path from each patch to every other patch in the selected area. Figure 2.2 depicts the meaning of the patterns used in the figures to depict the landscape.

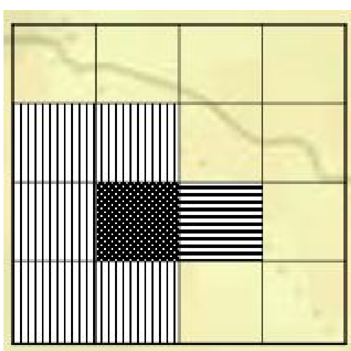
2.2 Compactness

The efficient design of a reserve not only depends on how well it is connected, but also on other spatial attributes. Williams *et al.* (2005) discuss the importance of the spatial attributes like size, proximity, number of reserves, connectivity and shape. Our focus in this paper is on reserve shape and in particular on compactness. Young (1988) lists eight measures of compactness, which are:

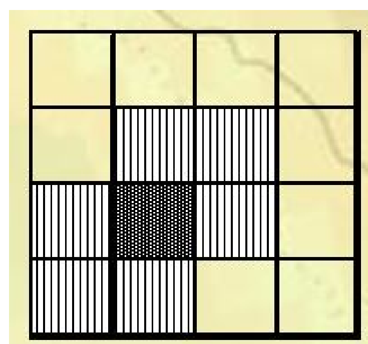
- Visual test, which greatly depends on an individuals perspective of compactness.
- Roeck test, which focuses on the ratio of the area selected to the area of the smallest circle enclosing the selected area, closer the ratio is to 1, more is its compactness.
- Schwartzberg test, defined as the ratio of adjusted perimeter to the perimeter of a circle whose area is equal to the area of the selected landscape.
- Length-width test, which is the ratio of the length and width of the largest rectangle that can be draw such that it touches all the 4 sides of the selected landscape, the closer the ratio is to 1, the more compact the landscape is.
- Taylor's test, which is very similar to Schwartzbern test, but it considers the ratio of the difference in the number of reflexive and non reflexive angles to the sum of the total number of angles.
- Moment of Inertia test, describing the geographical center and the value of the moment of inertia at that point, smaller the value of the moment of inertia at the geographical center, the better is its compactness.
- Boyce-Clark test, which is the average percentage difference between the radial distance and the average distance of the edges from the center of gravity, the closer it is to 0, the more compact is the landscape.
- Perimeter test, as the name suggests, the smaller the sum of the perimeter of all the selected areas, the better is its compactness.

The work of Young (1988) illustrates that compactness is a measure that does not have an exact definition. There are many ways in which compactness can be interpreted. Using visual inspection, any landscape that is close to a perfect shape (circle, square) is more compact with respect to other which is not that close to a perfect shape (circle, square). Önal and Briers (2003) defined compactness using a matrix called boundary, the smaller the value of the boundary, the more compact is the landscape. Their mathematical program involves minimizing the sum of parcel boundaries and the boundaries of the patches that are part of the landscape boundaries. Intuitively, packing more patches, wile minimizing the boundary length produces more compact shape. Wang and Önal (2016) define another metric for compactness which is the total sum of the distances between selected patches and the center of the reserve. A center patch is the patch which has the least sum of distances to all the other patches in the reserve. They considered a grid landscape, and define two kinds of distances, one is structural and the other functional. For our research we considered the structural distance only because it is directly related to the landscape shape. The structural distance between two patches is the number of patches used to connect them *i.e.* number of patches between them. This makes the structural distance between two adjacent patches to be equal to zero. According to Önal *et al.* (2016) the smaller the sum of the total distance, the more compact is the landscape. Intuitively, having patches close to the reserve center is better for compactness. Jafari and Hearne (2013) defined compactness as the perimeter of the reserve. They calculate the perimeter by adding the perimeter of the selected patches and subtracting the length of the shared edges between selected patches. The objective is to minimize the perimeter in order achieve better compactness. Intuitively, minimizing the perimeter also maximizes the length of shared edges between selected patches, which results in better compactness. Önal *et al.* (2016) defined compactness in the same way as in Wang and Önal (2016), minimizing the sum of total distance from the selected patches to the reserve center.

Figures 2.3(a) and 2.3(b) illustrates two landscapes with seven selected patches. For these landscapes, Table 2.1(a) shows the compactness measures according to Önal and Briers (2003), Jafari and Hearne (2013), Wang and Önal (2016) and Önal *et al.* (2016). All the metrics result in the same value of compactness for both landscapes even though both the landscapes have different configurations. In this case, the distance to the center was calculated as the number of patches between the center and the patch under consideration. Moreover, since the landscapes are grids, we assume all patches are of the same dimensions (1×1 *unit*). This assumption is made to simplify the explanation of the different metrics and for the purpose of comparison. There are other cases where all metrics are the same. By visual inspection, the landscape in Figure 2.3(b) is more compact that the landscape in Figure 2.3(a) because of the presence of the patch with only one neighbor in the reserve, which we call leaf patch. In this case, individuals visiting that patch have a higher chance to leave the reserve compared to boundary patches in Figure 2.3(b).



(a) Landscape configuration 1



(b) Landscape configuration 2

Figure 2.3: Landscape Compactness

Figure 2.3(a), shows that the leaf patch is the reason for the landscape in Figure 2.3(a) not to have a perfect compact shape. Our definition of leaf patch is specific

Pattern	Figure 2.3(a)	Figure 2.3(b)
Boundary (Önal and Briers (2003))	12	12
Perimeter (Jafari and Hearne (2013))	20	20
Total Distance (Wang and Önal (2016))	2	2
Distance to center (Önal *et al.* (2016))	2	2
Leaf	1	0

Table 2.1: Metric comparison

to a grid discretization of the landscape, because a patch can have between two and four neighbors and all patches have identical dimensions. For any landscape, can be generalized to k neighbors.

Leaf is a metric to enhance the compactness attained by other metrics or to break the ties when there is no difference using other metrics (as in Table 2.1(a)). The reason of this is that the leaf metric by itself cannot induce compactness (for instance two rectangular shapes of size 2×3 and 3×3 patches have zero leaves but one is clearly more compact). We illustrate the use of our leaf metric by extending the formulation in Önal *et al.* (2016) because of its computational performance. As we will discuss later, any metric can be extended with our leaf metric.

Chapter 3

MATHEMATICAL PROGRAMMING MODELS

To model the landscape connectivity and compactness problem, we initially adopted the formulation from Jafari and Hearne (2013) for connectivity alone. After tests and further literature review, it was evident that the formulation for connectivity and compactness put together by Önal *et al.* (2016) performed better in terms of solution times and compactness metric, in comparison to connectivity formulation in Jafari and Hearne (2013). We first describe the formulation proposed in Önal *et al.* (2016) which we modify to focus only on its properties to induce structural (spatial) connectivity.

3.1 Baseline Model (Önal *et al.* (2016))

The objective function in Önal *et al.* (2016) focuses on minimizing the sum of the total distance between selected patches and the center of the connected component. The original formulation included constraints to incorporate a minimum level of habitat quality in the resulting reserve. We decided not to use these constraints, as they are irrelevant when enforcing the spacial properties of the reserve, and only focus on the quality of the selected patches. Instead, we added a budget constraint to reflect the financial aspect in conservation planning decisions. This is similar to other works such as Beyer *et al.* (2016) state, where they minimize the total cost of planning units, which captures the capital investment factors involved in buying/protecting patches.

In our formulation, we define sets S as the set of all patches available, and N_j as the set of all neighboring patches of patch $j \in S$. The parameters in our formulation included d_{ki}, the distance between patch k and patch i for $k, i \in S$, and with $d_{kk} =$

0; n, the maximum number of connected components (reserves) allowed; m, the maximum number of nodes that can be attached to the center patch of each reserve, $\overline{a}$, the minimum area to be protected/selected; b, the budget available for land purchases; c_i, the cost of buying/protecting patch $i \in S$; and a_i, the area of patch $i \in S$. Our formulation has two sets of decision variables, x_{ki} is a binary variable which denotes the decision of buying/protecting a patch or not, note that x_{kk} also determines if patch k is a center. The indices of the x variables indicate whether patch i and patch k are bought/protected (*e.g.*, $x_{ki} = 1$). Moreover, they determine whether patch k is the center of the connected component where i is located. On the other hand if x_{ki} equals zero, it implies that at least one of the conditions mentioned is not satisfied (*i.e.*, i is not purchased, k is not purchased, or i is not in the connected component whose center is k. The mathematical model for the modified Önal *et al.* (2016) is presented in (3.1)-(3.9).

$$\min \sum_{i \in S} \sum_{k \in S} d_{ki} x_{ki} \tag{3.1}$$

$$s.t. \sum_{i \in S} \sum_{k \in S} c_i x_{ki} \leq b \tag{3.2}$$

$$\sum_{k \in S} x_{ki} \leq 1, \forall i \in S \tag{3.3}$$

$$\sum_{k \in S} x_{kk} \leq n \tag{3.4}$$

$$\sum_{\{i \in S,\, i \neq k\}} x_{ki} \leq m x_{kk}, \forall k \in S \tag{3.5}$$

$$\sum_{i \in S} \sum_{k \in S} a_i x_{ki} \geq \overline{a} \tag{3.6}$$

$$x_{ki} \leq x_{kk}, \forall i \in S, \forall k \in S \tag{3.7}$$

$$x_{kj} \leq \sum_{\{i \in N_j,\, d_{kj} > d_{ki}\}} x_{ki}, \forall j \in S, \forall k \in S, k \notin N_j,\ j \neq k \tag{3.8}$$

$$x_{ki} \in \{0, 1\}, \forall i \in S, \forall k \in S \tag{3.9}$$

The objective of this model is to minimize the sum of total distances between the patches and their respective center, which captured (3.1). This objective induces compactness in the selected landscape. Constraint (3.2) is the budget constraint, which imposes an upper bound on the total cost of buying/protecting patches.Constraints (3.3) enforce that a patch i can be selected and attached to at most one center. Constraint (3.4) reflects that not more than n reserves/components, are allowed in the solution. In other words, this constraint restricts the maximum number of center patches that can be selected to be no more than n. Constraints (3.5) enforce the condition that, if a patch k is selected as a center, then no more than m other patches can be attached to it. It also imposes the condition that if a patch is not selected as a center, then no patch should be attached to it. Constraint (3.6) imposes minimum area required to be protected. This is a very important constraint because, without Constraint (3.6) the problem will have an optimal objective function value of zero. For a grid landscape, this constraint reduces to a cardinality constraint that restricts the minimum number of patches to be selected. Constraints (3.7) enforce the condition that a patch i cannot be attached to a center patch k if k is not bought/protected. Imposing these constraints at the same time as constraints (3.5) produce a tighter formulation. Constraints (3.8) enforce connectivity, in each connected component. It states that if patch j is selected, then at least one neighboring patch i (*i.e.*, $i \in N_j$) should be selected such that the distance of patch i to the center k is less than the distance of patch j to the center k. Finally, constraints (3.9) impose the variable type constraint.

The modified version of the model in Önal *et al.* (2016) in (3.1)-(3.9) can produce landscapes with contiguous reserves/components, as shown in Figure 3.1. This is incorrect because both the components are connected to each other and then they cannot be treated as two different reserves. As a result, the model conveniently selects the centers and produces a very optimal value of the objective function. We point out that this issue is not due to the absence of the functional connectivity constraints, or the addition of the budget constraint because those are not required to enforce connectivity or compactness in the landscape. In order to fix this, we propose an additional set of constraints to the existing model, which we call separation constraints.

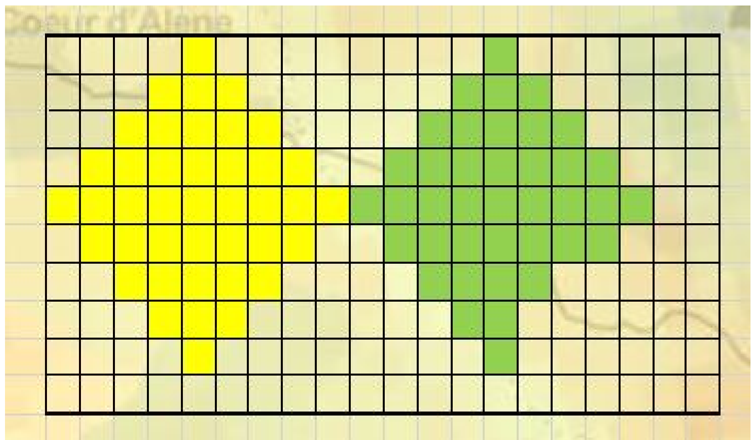


Figure 3.1: Landscape Configuration

3.2 Baseline Model with Separation Constraints

This model consists of (3.1) to (3.9) and the following constraints.

$$x_{ki} + x_{lj} \leq 1, \forall j \in S, \forall i \in S, \forall k \in S, \forall l \in S, k \neq l, i \in N_j, i \neq j \tag{3.10}$$

Constraints (3.10) state that, two neighboring patches i and j (*i.e.*, $i \in N_j$) cannot be purchased and connected to two different center patches k and l respectively. This ensures that if two neighboring patches i and j are bought/protected then they will be attached to the same center patch k. This fixes the issue identified in the Baseline Model. However, the number of constraints (3.10) may be prohibitively large. To overcome this situation, we propose a cutting plane algorithm that only adds constraints (3.10) when needed.

3.3 Leaf

The motivation for the Leaf metric is the inability of other metrics to detect compact configurations like those in Figure 2.4 and Figure 2.5. From Table 2.1 we have that, the only metric that identifies the more compact landscape is the Leaf. We implement the leaf metric as a mathematical programming model, to incorporate the results from other compactness models, that uses an epsilon constraint approach (Chankong and Haimes (2008)). We represent the leaf metric in the objective function with other metrics used as constraints. We first solve the modified model in Section 3.2, then use the objective function (3.1) as a constraint with an upper bound of δ^*, the optimal objective value obtained. Note that this strategy prevents any deterioration in the total distance to center patches. Although here we focus on improving the distance to center compactness metric, our models can be extended to other metrics. We define the leaf with a decision variable w_i where $i \in S$. If patch i is a leaf, then w_i equals one. If patch i is not a leaf, then w_i equals zero. The formulation for the Leaf Model is the following.

$$\min \sum_{i \in S} w_i \tag{3.11}$$

$$\sum_{i \in S} \sum_{k \in S} d_{ki} x_{ki} \leq \delta^* \tag{3.12}$$

$$2 \sum_{k \in S} x_{ki} - \sum_{j \in N_i} \sum_{k \in S} x_{kj} \leq w_i, \forall i \in S \tag{3.13}$$

$$w_i \leq \sum_{k \in S} x_{ki}, \forall i \in S \tag{3.14}$$

$$w_i \geq 0, \forall i \in S \tag{3.15}$$

$$x_{ki} \in \{0,1\}, \forall i \in S, \forall k \in S \tag{3.16}$$

Equation (3.11) is the objective function of the Leaf Model, which focuses on minimizing the total number of leaves in the landscape. Constraint (3.12) avoids the deterioration of the objective function value from Section 3.2. Constraints (3.13) determine if a patch is a leaf of not. If a patch i is protected/bought (*i.e.* $\sum_{k\in S} x_{ki} = 1$) and only one of its neighbor is also protected (*i.e.*, $\sum_{j\in N_i} \sum_{k\in S} x_{kj} = 1$), then patch i is a leaf (*i.e.* $w_i = 1$). If patch i is not selected or if more than one neighbors are selected, then the left hand side is non-positive and $w_i = 0$. This is specific to a grid landscape. Constraints (3.14) state that a patch cannot be a leaf if it is not connected to any center patch. This constraint is necessary because, from computational experience, it tightens the formulation and helps in better bounds for the linear relaxation of the problem. Constraints (3.15) are the variable type constraints, which state that w_i is non-negative. Note that because of (3.14) and (3.15), w-variables will take only binary values although they are allowed to be continuous.

The advantage of using an epsilon-constraint approach is that, a weight can be added to the value of δ^* to generate various scenarios to balance the leaf and the total distance to center metrics.

Chapter 4

SOLUTION APPROACH

We followed a lexicographic approach to induce compactness into a landscape. There are two objectives, the first one being minimizing the total distance to the center from the modified Önal *et al.* (2016). The second objective to minimize the number of leaves. In our solution approach we first solve the modified Önal *et al.* (2016) model, then its optimal objective function is added as a constraint to the leaf problem. Then, we solve the leaf problem.

We also use a cutting plane algorithm while solving the leaf problem given the large number of Constraints (3.10). We implemented out algorithms using Julia 1.0.3, because our solution efficiency lies greatly on the callback abilities of the platform. Julia has the capability to implement a callback function to modify the problem while its being solved without having to re-start the problem after the addition of a new constraint. The cutting plane algorithm proves to be very useful in our case because of the large number of constraints (3.10) to be added, thus adding them all at once will slow down the solution algorithm. To avoid this, constraints are added only in case of a violation.

4.1 Solution Algorithms

Algorithm 1 describes a cutting plane algorithm to dynamically impose the separation constraint in (3.10). Line 1 calculates the distance between patches k and i which is needed for (3.8). Line 2 solves model (3.1)-(3.9) and obtains an optimal solution. Line 3 defines set α, which is an index set containing the variable indexes for all patches violation (3.10). Line 4 defines a while loop which runs as long as the set patches violating the separation condition is non-empty. Line 5 contains an inner loop which runs for every element in set α. Line 6 indicates the addition of the separation constraint.Line 8 indicates the model to be solved with the added constraints and obtain a solution to the decision variables. Line 9 recalculates set α. Line 11 returns the optimal solution values for the decision variables.

Algorithm 1: Cutting Plane Algorithm for model (3.1)-(3.10)

1 Pre-calculate d_{ki};

2 Solve model (3.1)-(3.9) and obtain a solution $\mathbf{x} \triangleq [x_{ki}]$;

3 Calculate $\alpha = \{(i,j,k,l),\ j \in N_i,\ x_{ji} = x_{lj} = 1,\ k \neq l\}$;

4 **while** $\alpha \neq \{\emptyset\}$ **do**

5   **forall** $(i,j,k,l \in \alpha)$ **do**

6   Add $x_{ki} + x_{lj} \leq 1$;

7   **end**

8   Solve model (3.1)-(3.10) and obtain a solution $\mathbf{x} \triangleq [x_{ki}]$;

9   Calculate $\alpha = \{(i,j,k,l),\ j \in N_i,\ x_{ji} = x_{lj} = 1,\ k \neq l\}$;

10 **end**

11 Return $\mathbf{x} \triangleq [x_{ki}]$

Chapter 5

COMPUTATIONAL RESULTS

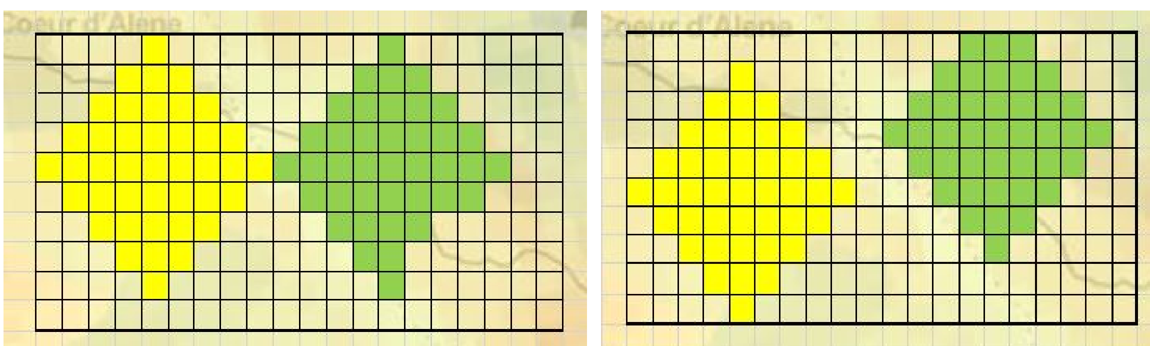
In this chapter, we present the results the proposed model on landscapes of up to 1100 patches, with randomly generated attributes. Section 5.1 presents our instance generation procedure. Section 5.2 illustrates the results that can be obtained with our models as well as a comparison of the performance with other metrics.All of our models were implemented using Julia 1.0.3 and Gurobi 8.1 solver on a intel i5 2.50 GHz processor and 8.0 GB RAM.

5.1 Instance Generation

We solve instances ranging from 100 patches (10×10 grid) to 1100 patches (25×44 grid). We considered only grid landscapes in order to be able to obtain comparable results with the Önal *et al.* (2016). We generate the cost of buying a patch using a uniform random function ranging between 10 to 15 (random values). The distance (adjacency) matrix was pre-generated, where the distance between patches k and i is the number of patches between them. The budget was randomly given using a predetermined 30% of the cost of purchasing the minimum area of protected landscape required, the area of the patches were considered to be 100 $units^2$ each (10×10 $units$). We first illustrate the need of the separation constraints (3.10) by comparing results of the model (3.1)-(3.9) with the model (3.1)-(3.10) and then describe the results for an instance minimizing the number of leaves.

5.2 Illustrative Examples

This section presents the results of our models on a few randomly generated instances. We use an instance with 200 patches and a minimum required area was 7500 sq. *units*. The budget is set to \$1000 and the maximum number of allowed reserves to two. The solution obtained for models (3.1)-(3.9) and (3.1)-(3.10) on this instance are depicted in Figures 5.1(a) and 5.1(b). The coloured patches (yellow and green) are the selected patches. Both problems led to an objective function value of 212 *units*. The models in (3.1)-(3.9) is solved in 3.2 seconds, whereas the model (3.1)-(3.10) required 102 seconds and 2686 Constraints (3.10) added to solve. Figure 5.1(a) shows that the reserves are connected to each other meaning that the distance to the center should have been calculated using only one center. To avoid this, the separation constraints have to be added to the problem. Figure 5.1(a) represents the solution of the model (3.1)-(3.10). It is clear that the reserves are not connected to each other, solving the issue.

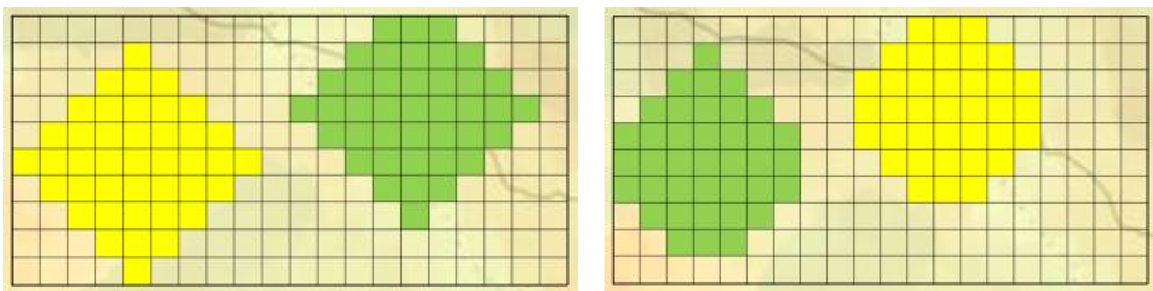


(a) Baseline Model

(b) Baseline Model with Separation Constraints

Figure 5.1: Landscapes I

Next we solved an instance to illustrate the use of the Leaf Model. We considered the same instance as before, with 200 patches and a minimum area to be purchased/protected as 7500 sq. *units*, with a budget \$1000 and the maximum number of allowed reserves is two. The optimal objective to model (3.1)-(3.10) is 212 *units* and the selected patches are shown in Figure 5.2. The optimal objective value obtained for the model (3.11)-(3.15) with $\delta^* = 212$ is equal to one, whereas the model in (3.1)-(3.10) produced seven leaves. The results are depicted in the Figures 5.2 (a) and 5.2 (b).



(a) Baseline Model with Separation Constraints

(b) Leaf Model

Figure 5.2: Landscapes II

It is also seen from Figure 5.2 (a) and 5.2 (b) that, visually landscape in Figure 5.2 (b) is more compact than the one that is in the Figure 5.2 (a). According to Young (1988), the better the landscape resembles a perfect figure, more compact it is. This is the case with the landscape in Figure 5.2 (b) when compared to the landscape in 5.2 (a).

Instance No.	No. Of Patches	Budget	Area	Time to Solve(sec)	
				Baseline Model with Separation Constraints	Leaf Model
1	200	600	4000	5	35
2	400	800	8000	24	3065
3	500	1000	10000	61.97	1615
4	625	1400	12500	189.9	1753.93
5	800	1900	16000	345.1	2855
6	900	2200	18000	391.53	4030.53
7	1000	2500	20000	450.72	5800.93
8	1100	2500	21000	400.18	5153.59

Table 5.1: Results

Table 5.1 depicts the results of the solution times for various instances of the model (3.1)-(3.10) and model (3.11)-(3.15). Instances beyond 1100 patches consumed more than 7200 *sec* to solve, and thus we did not report their results.

Chapter 6

FINAL REMARK AND FUTURE WORK

This thesis proposes a framework to solve the problem of landscape connectivity and compactness in conservation planning. This problem was formulated as a mixed integer problem, incorporating techniques like lexicographic approach, epsilon constraints and using the callback feature of Julia. We have built a new metric of measurement for compactness which we call by the name Leaf, and also to fix the issue with the modified version of the model in Önal *et al.* (2016). We built an algorithm to implement this model and tested it for real-life size instances and solved it in a reasonable time. In the future, better algorithms can be developed to solve larger instances in reasonable times. Also we came up with an idea for extended the concept of compactness to irregularly shaped landscape with patches which do not have a regular shape. This we call as the density metric. In the future we plan to develop this metric and be able to implement it to solve larger instances of irregularly shaped landscapes. We have done some literature on this, Elzinga and Hearn (1972) talks about method which uses the concept of minimum covering sphere to model compactness with the help of decomposition algorithm, we believe will be of a great help to us in modeling the density metric.

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