Design and Analysis of Auto-parametrically Excited Platform for Active Vibration

Control

by

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ABSTRACT

Recent research and study have showed the potential of auto-parametric system in controlling stability and parametric resonance. In this project, two different designs for auto-parametrically excited mass-spring-damper systems were studied. The theoretical models were developed to describe the behavior of the systems, and simulation models were constructed to validate the analytical results. The error between simulation and theoretical results was within 2%. Both theoretical and simulation results showed that the implementation of auto-parametric system could help reduce or amplify the resonance significantly.

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CHAPTER 1: INTRODUCTION

1.1 Auto-parametric Systems

Auto-parametric system is a system that consists of at least two subsystems: the primary subsystem that is self-excited or is vibrated by external excitations, the secondary subsystem that is nonlinearly coupled with the primary subsystem. The system is said to have semi-trivial solutions when the secondary subsystem stays in equilibrium while the primary subsystem is in vibration. The system undergoes auto-parametric resonance or auto-parametric excitation when the frequency of the system falls in a certain range, which causes the secondary subsystem to become unstable and can even vibrate with larger amplitude than the primary subsystem. Auto-parametric system is a nonlinear dynamical system [1].

1.2 Motivation

An auto-parametric system can be used for two main purposes: reducing the resonance or amplifying the resonance. There have been many researches focused on controlling the dynamic stability of the system by reducing the resonance as well as maximize the system response by amplifying the resonance. In the first case, the purpose of implementing auto-parametric system is to keep the secondary subsystem stable under any external disturbance by controlling the vibration of the primary subsystem. Meanwhile, in the second case, the purpose is to amplify the vibration of the secondary subsystem using the primary system as a parametric excitation.

Depending on the system designs, the resonance is caused when the excitation frequency match with the natural frequency or with a multiple of natural frequency of

the system. For example, the Ben-Ahin and the Wandre bridges in Belgium failed because of the vibrations of the cables with large amplitude. One of the explanations for these failures was that due to the nonlinear effect of the parametric excitation, if the oscillations of the cables fell in a certain range of frequency, they could become unstable and oscillates with large amplitude regardless the damping [2]. Another research by Gonzalez-Buelga, Neild, Wagg, and Macdonald also showed that the cables of structures, such as cable-stay bridge, would swing with significantly large amplitude if the excitation frequency of the deck was twice the natural frequency of the cables [3]. In their article in the *Sensors and Actuators A: Physical Journal*, Jia and Seshia addressed the potential role of auto-parametric resonance for vibration energy harvesting. They showed that by with the ratio of 2:1 for longitudinal and transverse frequencies, the initiation threshold amplitude could be significantly reduced and the power output was also enhanced [4]. Not only with bridges, but the phenomenon of parametric resonance can also occur in car suspensions, buildings, turbines, rotating machines, and delay generators. Thus, many recent study and researches focused on the implementation of auto-parametric systems either to reduce the resonance and stabilize the system or amplify the resonance and increase the magnitude of the output.

1.3 Past Research on Auto-parametric Systems

Auto-parametric system has broad applications and is used widely in many field such as civil engineering, automotive, aerospace, etc.

A research from the *Journal of Intelligent Material Systems* discussed the failure of the bridges caused by auto-parametric resonance in the cable, and proposed the

parametrically excited system with active control to tackle this failure [5]. A vertical cantilever steel beam (as shown in Figure 1) subjected to a time-periodic load was used to model the system, based on which the stiffness and the frequency of the system were controlled.

Figure 1. Model of a Cantilever Beam Subject to a Harmonic Axial Load [5].

The motion of the beam was described by a linear single-degree-of-freedom equation with time-varying coefficient. The time-varying coefficient represented the parametric characteristic of the system, and the effective stiffness of the system changed depending on this coefficient, which would affect the stability of the system. The equation was in the form of the Mathieu equation, so the response of the beam was approximated using Fourier series, and from that the transition curves, which showed the regions where the motion of the beam was stable and unstable, were generated for both undamped and underdamped situations. It was shown in the curves that the system became unstable

and auto-parametric resonance occurred when the excitation frequency was twice the natural frequency. Understanding how the frequency affected the transition curve helped stabilizing the system. The damping ratio, stiffness, and frequency of the system were adjusted so that the transition curve would shift in a way that the system would eventually become stable regardless its initial conditions. The paper proposed several ways to relocate the transition curve such as using the velocity feedback, displacement feedback, and pole placement. The analytical model was then validated using numerical simulation and actual experiment.

In their research on automotive suspensions, Reina and Rose applied the concept of auto-parametric system to design the suspensions with active vibration absorbers [6]. A system of mass and damper was represented by a three-degree-of-freedom quarter-car model as shown in Figure 2 below.

Figure 2. Quarter-car Model [6].

By applying the linear quadratic regulation (LQR) control law on the theoretical model, the values for damping and stiffness of the system were tuned so that amplitude of the motion of the sprung mass was minimized. Numerical simulation results indicated that compared to the conventional system and passive system, the performance index J of active vibration absorber was improved by 20% and 15% respectively. Furthermore, active vibration absorber had the advantages in power consumption, simplicity, and cost, which made it outperformed fully active suspension. Both theoretical model and simulation were in good agreement with each other, actual experiment would still be required in future work to validate the results.

In Dohnal's research published for the *Journal of Mechanical Engineering Science*, he performed theoretical calculations and experiments on three different systems

employing electromagnetic variable-stiffness actuators [7]. All of the three systems were represented by linear differential equations with time-periodic coefficients, and the paper mentioned several techniques to approximate the solution of these equations such as harmonic balance, Poincare-Lindstedt method, averaging method, etc. The first system included two rigid bodies connected by helical springs to each other and to the ground, and one of the two bodies was also connected to a long-stroke electromagnetic actuator consisting of an electromagnet. The second system was based on a flexible cantilever under transversal force with an electromagnetic mount. The third system addressed a flexible rotor shaft that attached to three rigid disks and two active magnetic bearings. In all three systems, the magnetic field strength and magnetic force could be adjusted by varying the amount of current provided to the electromagnet, which led to the control over the mechanical stiffness of the system. Both theoretical and experimental results showed that by introducing the parametric excitation, the stability of the system was controlled and the damping properties were enhanced significantly. The study by Jia and Seshia showed the potential of using auto-parametric resonance in vibration energy harvesting [4]. It has been known that even though it required a minimum threshold amplitude to be activated, parametric resonance helped magnify the power output significantly compared to the conventional direct excitation. The purpose of this paper is to study how auto-parametric resonance could help minimize the initiation threshold amplitude. In the paper, four systems of resonator were studied and compared: direct resonator which responded to the direct excitation; plain parametric resonator which responded to the parametric excitation; parametric

resonator which had an initial spring, but the frequency of the spring was not twice the natural frequency of the system; and auto-parametric resonator which had an initial spring, and the frequency of the spring is twice the natural frequency of the system. The parametric and auto-parametric resonators were represented by a model consisted of a vertically cantilever beam attached to a horizontal initial spring at the bottom as shown in Figure 3 below.

Figure 3. Vertical Cantilever Auto-parametric Resonator [4].

The beam was excited by a vertical driving force. Mathematical model was developed to describe the motion of the beam, and the natural frequency could be calculated in terms of the elastic modulus, the mass, and the dimensions of the beam. The theoretical, simulation, and experimental results all showed that compared to the plain parametric resonator and the parametric resonator, the initiation threshold amplitude required for the auto-parametric resonator was reduced significantly.

There are many more works focused on implementing the auto-parametric system to control the system stability such as: a study by Boeing Company focused on developing a model to define the locations of sensors and structural actuators for adaptive noise cancellation [8], and a research project at the Institute of Thermomechanics employed parametrically excited system into thin structures and performed experiments on a cantilever forced vibrated beam as a representative model [9]. With its broad applications, the role auto-parametric system has become more important in system control and stabilization.

1.4 Mathematical Methods

Auto-parametric systems are usually modeled by nonlinear differential equations, and sometimes they are challenging to solve. Some methods that have been used to analyze auto-parametric systems are Poincare-Lindstedt method, Mathieu equation, harmonic balance, bifurcation theory, and Floquet theory [1]. Below are a brief description and representative equations for each method. More details on the methods including examples can be found in reference [1].

The Poincare-Lindstedt method is efficient to find the approximation of periodic solutions for nonlinear equations. The method is applied for systems that have the form:

$$\dot{x} = Ax + \varepsilon B(t)x \quad (1.3.1)$$

Equation (1.3.1) can be solved using Floquet theory, which will be discussed later in this section.

Mathieu equation is a linear differential equation with time-varying periodic coefficients, which is usually presented in the form:

$$\ddot{x} + \kappa \dot{x} + [a + \varepsilon \cos(\omega t)]x = 0 \quad (1.3.2)$$

The periodic solution for Mathieu equation can also be obtained using Floquet theory. Mathieu equation has been used widely not only in auto-parametric systems, but also in wave motion, quantum pendulum, rotating electric dipole, etc.

Harmonic balance can only be used to find periodic solutions. The unknowns in the equation can be determined by substituting the solution that has the form of Fourier expansion into the equations and equating the coefficients of similar terms. The most well-known form for the Fourier expansion is:

$$x(t) = \sum_{n=0}^{\infty} [a_n \cos(n\omega t) + b_n \sin(n\omega t)] \quad (1.3.3)$$

Bifurcation theory studies the qualitative change depending on the control parameter μ of the dynamical system that has the general form of:

$$\dot{x} = f(x, t, \mu) \quad (1.3.4)$$

The goal is to describe the system stability around the critical value μ_c . The system is structurally unstable at the critical value, and a small change of μ around this value can significantly change the system's stability. There are two types of bifurcations: local bifurcations, and global bifurcations. Local bifurcations are more common studying dynamical systems because they can be used to analyze the stability of the systems in the neighborhood of the equilibrium. Some cases of local bifurcations are saddle-node bifurcation, transcritical bifurcation, pitchfork bifurcation, and Hopf bifurcation.

The general idea of the Floquet theory is to transform a linear time-variant system into a linear time-invariant system. Consider the linear time-variant system and its corresponding fundamental matrix as shown respectively in Equations (1.3.5) and (1.3.6) below:

$$\dot{x} = A(t)x \quad (1.3.5)$$

$$\Phi(t) = P(t)e^{Bt} \quad (1.3.6)$$

Using the Lyapunov-Floquet transformation in Equation (1.3.7), the linear time-invariant system can be obtained, which makes it easier to determine the stability of the system.

$$x(t) = P(t)z(t) \quad (1.3.7)$$

$$\dot{z} = Bz \quad (1.3.8)$$

Although it is quite challenging to obtain exact solutions for auto-parametric systems, there are many different methods to approximate the solutions and transform the equations into a simpler form to analyze the stability of the systems.

1.5 Problem Statement

The goal of this project is to design, analyze, and simulate auto-parametrically excited platforms for an active vibration control. In this project, two different auto-parametric systems that consisted of mass, spring, and damper were studied. These systems were considered as classical examples of auto-parametric system. In each system, the spring was attached to a motor that can change the orientation of the spring, making the system become auto-parametrical. By controlling the stiffness of the spring, the motion of the mass can be predicted, and the amplitude of the mass' vibration can be minimized or

maximized depending on the applications. Deliverables of the project include: two different designs for auto-parametrically excited platform, theoretical models that describe the dynamics of the systems, analysis on systems' stability, and simulation models of the two systems.

CHAPTER 2: MATHEMATICAL ANALYSIS

2.1 System Designs

In this project, two different auto-parametric systems that consisted of mass, spring, and damper were studied. These systems were considered as one of the classical examples of auto-parametric systems. In each system, the spring was attached to a motor that could change the orientation of the spring, making the system become auto-parametric. Diagrams of the two systems are shown in Figure 4 below, where m , k , and c represents the mass, the spring stiffness, and the damping coefficient respectively.

Figure 4. Mass-spring-damper Systems Controlled by Rotational Motion of the Spring (a) and by Sliding Motion of the Spring (b).

In both systems shown above, the mass could only move in the vertical direction. The external displacement and the mass' displacement were defined as y and x respectively. As shown in Figure 4a, the first system had a rod of fixed length r that had one end attached to the spring and the other end attached to a motor, which generated torque to rotate the rod and led to the change in orientation of the spring. The rotation of the

rod was described by the function $\phi(t)$. In the second system shown in Figure 4b, one end of the spring was attached directly to a motor, which could slide the spring back and forth. The sliding motion of the bottom end of the spring could be described by the function $u(t)$.

2.2 Equations of Motion

2.2.1 Mass-Spring-Damper System Controlled by Rotational Motion of the Spring

Assume that $x=0$ is defined at the equilibrium position of the mass when $\phi(t) = 0$. Then the following equation can be obtained from Newton's second law of motion:

$$mg + k(l_1 - l_0) = 0 \rightarrow l_1 = \frac{-mg}{k} + l_0 \quad (2.2.1)$$

where g is the gravitational acceleration, l_0 is the original length of the spring, and l_1 is the length of the spring when the mass is at the equilibrium position. The kinetic energy (KE) and the potential energy (PE) of the system are determined by the following equations:

$$KE = \frac{1}{2} m \dot{x}^2 \quad (2.2.2)$$

$$PE = mg(x + l_1 + r - y) + \frac{1}{2} k \left[\sqrt{(x + l_1 + r - r \cos \phi - y)^2 + (r \sin \phi)^2} - l_0 \right]^2 \quad (2.2.3)$$

Then, the equation that describes the motion of the mass can be obtained using Lagrangian method:

$$L = KE - PE \quad (2.2.4)$$

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{x}}\right) - \frac{\partial L}{\partial x} + c(\dot{x} - \dot{y}) = 0 \quad (2.2.5)$$

Substituting Equations (2.2.1), (2.2.2), and (2.2.3) into Equation (2.2.5) gives:

$$m\ddot{x} + c(\dot{x} - \dot{y}) + k(x + l_0 + r - r \cos \phi - y) - kl_0 \left[1 + \left(\frac{r \sin \phi}{x + l_1 + r - r \cos \phi - y} \right)^2 \right]^{\frac{1}{2}} = 0 \quad (2.2.6)$$

Applying binomial expansion twice on the nonlinear term, Equation (2.2.6) can be approximated by the following linear equation:

$$m\ddot{x} + c\dot{x} + \left[k - \frac{kl_0 r^2 \sin^2 \phi}{(l_1 + r - r \cos \phi - y)^3} \right] x = -k(r - r \cos \phi - y) - \frac{kl_0 r^2 \sin^2 \phi}{2(l_1 + r - r \cos \phi - y)^2} + c\dot{y} \quad (2.2.7)$$

2.2.2 Mass-Spring-Damper System Controlled by Sliding Motion of the Spring

Similar to the system controlled by rotational motion of the spring, for the system controlled by sliding motion of the spring, $x = 0$ is also defined as the equilibrium position of the mass when $u(t) = 0$. The length of the spring when the mass is at the equilibrium position can also be determined by Equation (2.2.1). The kinetic energy (KE) and the potential energy (PE) of the system are determined by the following equations:

$$KE = \frac{1}{2} m \dot{x}^2 \quad (2.2.8)$$

$$PE = mg(x + l_1 - y) + \frac{1}{2} k \left[\sqrt{(x + l_1 - y)^2 + u^2} - l_0 \right]^2 \quad (2.2.9)$$

Then, applying Lagrangian method as shown in Equations (2.2.4) and (2.2.5) gives the following equation that describes the motion of the mass:

$$m\ddot{x} + c(\dot{x} - \dot{y}) + k(x + l_0 - y) - kl_0 \left[1 + \left(\frac{u}{x + l_1 - y} \right)^2 \right]^{\frac{1}{2}} = 0 \quad (2.2.10)$$

Applying binomial expansion twice on the nonlinear term, Equation (2.2.10) can be approximated by the following linear equation:

$$m\ddot{x} + c\dot{x} + \left[k - \frac{kl_0 u^2}{(l_1 - y)^3} \right] x = ky - \frac{kl_0 u^2}{2(l_1 - y)^2} + c\dot{y} \quad (2.2.11)$$

2.3 Stability

In this section, the stability of linear homogeneous systems with no external displacement ($y = 0$) was studied. Equations (2.3.1) and (2.3.2) below were used to study the stability of the systems controlled by rotational motion and sliding motion of the spring respectively.

$$m\ddot{x} + c\dot{x} + \left[k - \frac{kl_0 r^2 \sin^2 \phi}{(l_1 + r - r \cos \phi)^3} \right] x = 0 \quad (2.3.1)$$

$$m\ddot{x} + c\dot{x} + \left[k - \frac{kl_0 u^2}{l_1^3} \right] x = 0 \quad (2.3.2)$$

Based on the systems' design, the control parameters were chosen as $\phi(t) = \gamma \cos(\omega_0 t + \alpha)$ for the system control by rotational motion of the spring, and $u(t) = \gamma \cos(\omega_0 t + \alpha)$ for the system controlled by the sliding motion of the spring, where γ is the control amplitude, ω_0 is the control frequency, and α is the phase

shift. The stability charts were generated for both systems to study how the stability of the systems changed depending on the spring stiffness and the control parameter $\phi(t)$ and $u(t)$. Thus, the spring stiffness (k) and the control amplitude (γ) were varied, while the other parameter were kept as constant and are shown in Table 1 below.

Table 1. Systems' Parameters for Stability Chart.

Parameter	Value
m	0.1 kg
c	1 N·s/m
l_0	0.25 m
r	0.1 m
ω_0	π rad/s
α	0 rad

The stability charts were generated using numerical solver in MATLAB. The code is attached in APPENDIX A. In order to verify the results, both trace condition and eigenvalues condition were used to plot the stability charts. Figure 5 and Figure 6 below show all the results.

Figure 5. Stability Charts for System Controlled by Rotational Motion of the Spring
Plotted based on Trace Condition (a) and based on Eigenvalues Condition (b).

Figure 6. Stability Charts for System Controlled by Sliding Motion of the Spring
Plotted based on Trace Condition (a) and based on Eigenvalues Condition (b).

In the plots above, the green dot represents the stability of the system, and the red dot represents the instability of the system. Due to the limitation of numerical methods, there were more data points generated based on the trace condition than based on the

eigenvalues condition. Despite that, within the generated domain, the stability charts based on the two conditions were identical for both systems. Based on Figure 5, it can be concluded that without external disturbance, the linearized system controlled by rotational motion of the spring is stable for the spring stiffness larger than 8 N/m and for any control amplitude from 0 rad to $\pi/2$ rad. Meanwhile, Figure 6 indicated that without external disturbance, the linearized system controlled by sliding motion of the spring is stable for the spring stiffness larger than 20 N/m and for any control amplitude from 0 m to 0.25 m.

2.4 System Analysis

To analyze the nonhomogeneous systems, let the external displacement be $y = Y \cos(\omega t)$, where Y is the forcing amplitude, and ω is the forcing frequency. Periodic functions were chosen to represent the control parameters (as shown in Section 2.3) as well as the external displacement for two main reasons. First, there has been many methods developed to solve linear and nonlinear systems with periodic time-varying coefficients and periodic boundary conditions. Thus, this choice made the systems become simpler and easier to solve. The second reason was that it made the systems more realistic because the motions of many external disturbances such as bumps on the road, wind, or earthquake have been modeled using periodic function. As discussed in Chapter 1, the relationship between the frequency of the control parameters and the frequency of the external displacement played an important role in reducing or amplifying the parametric resonance. The relationship between the two

frequencies could be determined based on the systems' design. Figure 7 and Figure 8 below show both systems with different orientations of the spring.

Figure 7. Different Orientations of the Spring for the System Controlled by Rotational Motion of the Spring.

Figure 8. Different Orientations of the Spring for the System Controlled by Sliding Motion of the Spring.

Since the mass only moved in the vertical direction, it can be seen from Figure 7 and Figure 8 above that for both systems, the behaviors of the mass when the spring moves from position A to B and when the spring moves from position C to B are identical. In addition, to reduce the resonance, the spring must be at position B when the external displacement is minimum and at either position A or C when the external displacement is maximum. On the other hand, to amplify the resonance, the spring must be at position B when the external displacement is maximum and at either position A or C when the external displacement is minimum. Based on this analysis, the plots for external displacement ($y / y_{\max}$) and control parameters ($\phi / \phi_{\max}$ and $u / u_{\max}$) were generated and shown in Figure 9 below.

Figure 9. Plots for External Displacement (a) and Control Parameters when Resonance is Reduced (b) and when Resonance is Amplified (c).

Recall that the external displacement was represented as $y = Y \cos(\omega t)$ and the control parameters were represented as $\phi(t) = \gamma \cos(\omega_0 t + \alpha)$ and $u(t) = \gamma \cos(\omega_0 t + \alpha)$. From Figure 9, it can be determined that in both cases of reducing and amplifying the resonance, the forcing frequency (ω) is twice the control frequency (ω_0). Also, when the phase shift (α) is 0 the resonance is reduce, and when it is $-\pi/2$, the resonance is amplified.

Theoretically, the control amplitude could range from 0 rad to 2π rad for system controlled by rotational motion and could be unbounded for the system controlled by sliding motion of the spring. However, in order for the mass to stay above the ground, there were some constraints implied in the systems' design that needed to be taken into account. If the mass must stay above the ground, then the design shown in Figure 4a implied that the system controlled by the rotational motion of the spring had a constraint of $x + l_1 + r - y \geq 0$, which meant that the difference between the mass' displacement (x) and the external displacement (y) had to be larger than the additive

inverse of the sum of the rod (r) and the length of the spring when the mass at equilibrium without any forcing (11). Let $y = 0.1 \cos(2\pi t)$ and $\phi(t) = \gamma \cos(\pi t)$, which gives the forcing frequency as twice as the control frequency as discussed previously. Let the spring stiffness (k) be 50 N/m, which would make the system stable as shown in Figure 5. With the chosen parameters as shown in

Table 1 and the chosen spring stiffness value, the sum of r and l_1 was calculated to be about 0.33 m, which meant that $x - y \geq -0.33$. The difference between x and y were obtained by solving the Equation (2.2.6) numerically using MATLAB. The code is attached in APPENDIX B. Below is the plot of minimum values of the difference between the mass' displacement and the external displacement ($x - y$) vs. the control amplitude (γ).

Figure 10. Minimum Values of the Difference between the Mass' Displacement and the External Displacement vs. the Control Amplitude for System Controlled by Rotational Motion of the Spring.

The plot above shows that as the control amplitude changed from 0 rad to 2π rad, the minimum difference between x and y decreased, but it was always less than -0.33 m. Thus, the system could operate properly for any value of the control amplitude ranging from 0 rad to 2π rad. For the system controlled by sliding motion of the spring, the design design shown in Figure 4b implied a constraint of $x + l_1 - y \geq 0$. Let $y = 0.1\cos(2\pi t)$ and $u(t) = \gamma \cos(\pi t)$, which gives the forcing frequency as twice as the control frequency as discussed previously. Let the spring stiffness (k) be 50 N/m, which would ensure the system's stability as indicated in Figure 6. With this spring stiffness value and the chosen parameters as shown in

Table 1, l_1 was calculated to be about 0.23 m, which meant that $x - y \geq -0.23$.

MATLAB code that was used to compute the difference between x and y is shown in APPENDIX B. Figure 11 below is the plot of minimum values of the difference between the mass' displacement and the external displacement ($x - y$) vs. the control amplitude (γ).

Figure 11. Minimum Values of the Difference between the Mass' Displacement and the External Displacement vs. the Control Amplitude for System Controlled by Sliding Motion of the Spring.

The plot above shows that the minimum of the difference between x and y reached -0.23 m when the control amplitude was about 0.22 m. Therefore, in simulation model and actual experiment, the control amplitude must be constrained in order for the system to operate properly.

2.5 Theoretical Results

Based on the analysis in Sections 2.3 and 2.4, the parameters for both systems were chosen as shown in

Table 2 below.

Table 2. Systems' Parameters

Parameter	Value for system controlled by rotational motion of the spring	Value for system controlled by sliding motion of the spring
m	0.1 kg	0.1 kg
c	1 N·s/m	1 N·s/m
k	50 N/m	50 N/m
l_0	0.25 m	0.25 m
r	0.1 m	N/A
γ	0 rad (no control parameter) $\pi/3$ rad (apply control parameter)	0 m (no control parameter) 0.15 m (apply control parameter)
ω_0	π rad/s	π rad/s
α	0 rad (to reduce the resonance) $-\pi/2$ rad (to amplify the resonance)	0 rad (to reduce the resonance) $-\pi/2$ rad (to amplify the resonance)
Y	0.1 m	0.1 m
ω	2π rad/s	2π rad/s

Both of the systems were studied under three cases: when there was no control parameters, when the control parameter was applied to reduce the resonance, and when the control parameter was applied to amplify the resonance. With the chosen parameters, Equations (2.2.6) and (2.2.7) could be used to obtain the displacement

of the mass for both nonlinear and linearized systems controlled by the rotational motion of the spring for each case. The MATLAB codes were used to numerically solve the equations are attach in APPENDIX B and APPENDIX C. The responses for all three cases are shown in the plots below.

Figure 12. Responses for the Nonlinear System (a) and the Linearized System (b)
Controlled by Rotational Motion of the Spring when the Control Parameter was not
Applied.

Figure 13. Responses for the Nonlinear System (a) and the Linearized System (b)
Controlled by Rotational Motion of the Spring when the Control Parameter was
Applied to Reduce the Resonance.

Figure 14. Responses for the Nonlinear System (a) and the Linearized System (b)

Controlled by Rotational Motion of the Spring when the Control Parameter was
Applied to Amplify the Resonance.

It can be seen from the plots above that the responses obtained from the nonlinear system and the linearized system were similar, which proved that the linearized system could be used to represent the nonlinear system. Also in all of the plots, it can be approximated that the frequency of the mass' vibration was about 1 Hz. By linearizing the system, Equation (2.2.7) was obtained, and it showed that the frequency of the response was determined by the frequency of the term $\sin^2 \phi$. Since $\sin^2 \phi = [1 - \cos(2\phi)] / 2$, the frequency of the response would be twice the frequency of the control parameter, which was π rad/s or 0.5 Hz. The numerical results got from the plot matched with the analytical result. Thus, the mathematical model was verified.

To analyze how effective the control parameter in reducing or amplifying the resonance, the amplitudes of the mass' vibration for all of the cases were computed from the numerical results and summarized in Table 3 below.

Table 3. Amplitudes of the Responses for the System Controlled by Rotational Motion of the Spring

	Amplitude obtained from nonlinear system	Amplitude obtained from linearized system
Control parameter was not applied	0.109 m	0.109 m
Control parameter was applied to reduce the resonance	0.073 m	0.073 m
Control parameter was applied to amplify the resonance	0.146 m	0.143 m

Compared to the amplitudes of the mass' vibration obtained from the nonlinear system, the results obtained from the linearized system was within 2% of error, which again proved that the linearized system was a good approximation to the nonlinear system. The results from Table 3 also indicates that by applying the control parameter, the resonance could be reduced by 33% or amplified by 34%.

The same study was conducted for the system controlled by the sliding motion of the spring. With the chosen parameters listed in

Table 2, the displacement of the mass for both nonlinear and linearized systems was obtained using Equations (2.2.10) and (2.2.11). The MATLAB codes were used to solve the equations are attach in APPENDIX B and APPENDIX C. The responses for all three cases are shown in the plots below.

Figure 15. Responses for the Nonlinear System (a) and the Linearized System (b)
Controlled by Sliding Motion of the Spring when the Control Parameter was not
Applied.

Figure 16. Responses for the Nonlinear System (a) and the Linearized System (b)
Controlled by Sliding Motion of the Spring when the Control Parameter was Applied
to Reduce the Resonance.

Figure 17. Responses for the Nonlinear System (a) and the Linearized System (b)
Controlled by Sliding Motion of the Spring when the Control Parameter was Applied
to Amplify the Resonance.

It shows in the plots above that the responses obtained from the nonlinear system and the linearized system were different for the second case and similar for the other two cases. This indicates that the linearized system could not represent all of the characteristics of the nonlinear system. The control parameter was the factor that made the system become nonlinear, and to fully understand the effect of this factor on the system, more research needs to be conducted including the study of normal form and bifurcation of the system Equation (2.2.11) showed that the frequency of the response for the linearized system was determined by the frequency of the term u^2 . Since $u^2 = [\gamma \cos(\omega_0 t + \alpha)]^2 = \gamma^2 [1 + \cos(2\omega_0 t + 2\alpha)]/2$, the frequency of the response

would be twice the frequency of the control parameter (ω_0), which was π rad/s or 0.5 Hz. In all of the plots above, the frequency of the mass' vibration was about 1 Hz, which match with the result got from the linearized equation. This shows that even though the linearized system could not approximate the nonlinear system in some cases, it could still be used to predict some aspects of the response such as the frequency. From the numerical results, the amplitudes of the mass' vibration for all of the cases obtained from the nonlinear system were calculated and summarized in Table 4 below.

Table 4. Amplitudes of the Responses for the System Controlled by Sliding Motion of the Spring

	Amplitude obtained from nonlinear system
Control parameter was not applied	0.109 m
Control parameter was applied to reduce the resonance	0.078 m
Control parameter was applied to amplify the resonance	0.146 m

It can be seen from Table 4 that by applying the control parameter, the resonance could be reduced by 28% or amplified by 34%. Therefore, for both systems, the implementation of the control parameter, which could change the orientation of the spring and made the system become auto-parametric, could significantly help reducing or amplifying the resonance.

CHAPTER 3: SIMULATION VALIDATION

3.1 Introduction to Working Model 2D

Working Model 2D is a popular CAE tool and the 2D motion simulator. The software allows users develop the simulation model of real-life mechanical systems and analyze the stability as well as the response of the systems, which will help users to optimize the designs and prevent any potential problems before running actual experiments. The simulation objects, such as slot elements, pin joints, slot joints, motors, actuators, springs, dampers, ropes, rods, pulleys, and gears, allow user to quickly and easily construct the simulation models. Working Model has features such as collision detection, backlash in gears, the rolling and slipping of wheels, friction, and contact between objects, which can make the simulation environment closely resembles the real environment. The simulation results can be represented in many ways including animations, graphs, and numerical data. The quantity that can be measured and analyzed includes force, torque, distance, velocity, acceleration, and much more. Users can also import the 2D CAD drawings in DXF format or input values from MATLAB and Excel [10].

3.2 Simulation Models

Based on the system designs shown in Figure 4, the simulation models were built using Working Model 2D. Below are the simulation models for both systems controlled by rotational motion and by sliding motion of the spring.

Figure 18. Simulation Models for Systems Controlled by Rotational Motion (a) and by Sliding Motion (b) of the Spring.

In both systems above, the mass was represented by the light blue square which was connected with the spring and the damper, and the platform was represented by the blue rectangle where the external displacement was applied. The mass and the platform were

attached to a vertical slot using keyed slot joints so that they could only move in the vertical direction. The damper was attached vertically from the mass to the platform using rigid joints. The top end of the spring was also attached to the mass using rigid joint. An actuator was attached to the platform to provide external displacement. In the first system, the bottom end of the spring was attached to the perimeter of a circle. The radius of the circle was the same as the length of the rod in the design. In other words, the distance between the center of the circle and the bottom end of the spring represented the rod. In the center of the circle was a motor that would generate torque and change the orientation of the spring. The control parameter $\phi(t)$ was measured as the angle of rotation from the vertical axis going through the center of the mass to the axis going through the center of the circle and the bottom end of the spring. In the second system, the bottom end of the spring was attached to a circle that could only move in the horizontal direction. The circle was connected to the platform using a keyed slot joints. An actuator was attached to the center of the circle to provide the sliding motion and change the orientation of the spring. The control parameter $u(t)$ was measured as the horizontal distance from the center of the platform to the bottom end of the spring. The parameters implemented for the systems were listed in

Table 2. For both systems, the mass of the platform, the mass of the circle, and friction were assumed as negligible.

3.3 Simulation Results and Comparison with Theoretical Results

To validate the theoretical results, the simulation models for both systems were run for all three cases when there was no control parameter applied, when the control parameter was applied to reduce the resonance, and when the control parameter was applied to amplify the resonance. Figure 19 and Figure 20 below shows the simulation results for the system controlled by rotational motion of the spring.

(a)

(b)

(c)

Figure 19. Responses for the System Controlled by Rotational Motion of the Spring when the Control Parameter was not Applied (a), when the Control Parameter was

Applied to Reduce the Resonance (b), and when the Control Parameter was Applied
to Amplify the Resonance (c).

(a)

(b)

Figure 20. Responses for the System Controlled by Sliding Motion of the Spring when the Control Parameter was not Applied (a), when the Control Parameter was Applied to Reduce the Resonance (b), and when the Control Parameter was Applied to Amplify the Resonance (c).

The plots in Figure 19 and Figure 20 show that the frequency of the responses for both systems was about 1 Hz, which matched with the theoretical results. For the system controlled by rotational motion of the spring, the amplitudes of the mass' vibration in the three cases (a), (b), and (c) were 0.109 m, 0.072 m, and 0.146 m respectively, which were within 2% of error compared to the theoretical results for both nonlinear and linearized system. For the system controlled by sliding motion of the spring, the amplitudes of the mass' vibration in the three cases (a), (b), and (c) were 0.109 m, 0.078 m, and 0.146 m respectively, which matched with the theoretical results for the nonlinear system.

CHAPTER 4: CONCLUSION AND FUTURE WORK

In this project, two different designs for auto-parametric mass-spring-damper system were studied. The mathematical models were developed to describe the behavior of the systems. Stability charts were generated to study the stability of the systems. Theoretical results showed that the response for the system controlled by rotational motion of the spring could be approximated by the linearized equation. On the other hand, for the system controlled by the sliding motion of the spring, the linearized system could not represent all of the characteristics of the nonlinear system in some cases. Simulation models were developed for both systems, and the simulation results matched the theoretical results within 2% of error. Both theoretical and simulation results showed that the implementation of auto-parametric system could help reduce the resonance by up to 33% and amplify the resonance by up to 34%. Future work of the project includes: more research on the difference between the nonlinear and linearized systems controlled by the sliding motion of the spring, building prototypes for both systems, and performing experiment to validate theoretical and simulation results.

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APPENDIX A

MATLAB CODE FOR STABILITY CHART

For system controlled by rotational motion of the spring:

% Computation of State Transition Matrix STM

function dx=StabilityChartRotationalSpring(t,x,gamma,k)

[rows,cols]=size(x);
dx=zeros(rows,cols);

m=0.1;
g=9.81;
c=1;
l0=0.25;
r=0.1;
omega=pi;

Ao=[0 1;-k/m -c/m]; **%Constant matrix**

At=[0 0;(k*l0*r^2*(sin(gamma*cos(omega*t)))^2/((k*l0-m*g)/k+r-
r*cos(gamma*cos(omega*t)))^3)/m 0]; **% Time varying matrix**

dx(1)=x(2); **% x_dot**

dx(2)=Ao(2,1)*x(1)+At(2,1)*x(1)+Ao(2,2)*x(2)+At(2,2)*x(2); **% x_dot_dot**

% Script to run computeSTM (for trace condition)

gamma_terms=1.6;
k_terms=100;
ei1=[];

'Please Be Patient'

for j=0:1:k_terms

k=j;

tspan=0:1;

for i=0:0.016:gamma_terms

gamma=i;

[t,x1]=ode45(@StabilityChartRotationalSpring,tspan,[1,0],[],gamma,k);

[m,n]=size(x1);

x1=x1(m,:);

[t,x2]=ode45(@StabilityChartRotationalSpring,tspan,[0,1],[],gamma,k);

[m,n]=size(x2);

x2=x2(m,:);

FTM=[x1',x2']

```

tr=trace(FTM)

format long;
FTM;
abs(trace(FTM))
save('ftm','tr','-append')
pause(0.25)

figure(1)
xlabel('k')
ylabel('gamma')
title('Stability chart of SSF')
hold on
if (abs(trace(FTM)))<=2
    plot(k,gamma,'g.')
else
    plot(k,gamma,'r.')
end

end

end

% Script to run computeSTM (for eigenvalues condition)
gamma_terms=1.6;
k_terms=100;
ei1=[];

'Please Be Patient'

for j=0:1:k_terms
    k=j;
    tspan=0:1;
    for i=0:0.016:gamma_terms
        gamma=i;

        [t,x1]=ode45(@StabilityChartRotationalSpring,tspan,[1,0],[],gamma,k);
        [m,n]=size(x1);
        x1=x1(m,:);

        [t,x2]=ode45(@StabilityChartRotationalSpring,tspan,[0,1],[],gamma,k);

        [m,n]=size(x2);
        x2=x2(m,:);

```



```

        FTM=[x1',x2']
        e=eig(FTM)
        abs(e)

        format long;
        FTM;
        abs(eig(FTM))
        R=logm(FTM); % Exponent matrix
        eig(R); % -Real part stable, 0 real part simply stable
        ei1 = [ei1,e];
        save('ftm','e','-append')
        pause(0.25)

        figure(1)
        xlabel('k')
        ylabel('gamma')
        title('Stability chart of SSF')
        hold on
        if (abs(eig(FTM)))<=1
            plot(k,gamma,'g.')
        else
            plot(k,gamma,'r.')
        end

    end

end

```

For system controlled by sliding motion of the spring:

```

% Computation of State Transition Matrix STM
function dx=StabilityChartSlidingSpring(t,x,gamma,k)

[rows,cols]=size(x);
dx=zeros(rows,cols);

m=0.1;
g=9.81;
c=1;
l0=0.25;
omega=pi;
l1=(k*l0-m*g)/k;

```

```

Ao=[0 1;-k/m -c/m]; %Constant matrix
At=[0 0;(k*10*(gamma*cos(omega*t))^2/((k*10-m*g)/k)^3)/m 0]; % Time varying
matrix
dx(1)=x(2);% x_dot
dx(2)=Ao(2,1)*x(1)+At(2,1)*x(1)+Ao(2,2)*x(2)+At(2,2)*x(2); % x_dot_dot

% Script to run computeSTM (for trace condition)
gamma_terms=0.25;
k_terms=100;
ei1=[];

'Please Be Patient'

for j=0:1:k_terms
    k =j;
    tspan=0:1;
    for i=0:0.0025:gamma_terms
        gamma=i;

        [t,x1]=ode45(@StabilityChartSlidingSpring,tspan,[1,0],[],gamma,k);
        [m,n]=size(x1);
        x1=x1(m,:);

        [t,x2]=ode45(@StabilityChartSlidingSpring,tspan,[0,1],[],gamma,k);

        [m,n]=size(x2);
        x2=x2(m,:);

        FTM=[x1',x2']
        tr=trace(FTM)

        format long;
        FTM;
        abs(trace(FTM))
        save('ftm','tr','-append')
        pause(0.25)

        figure(1)
        xlabel('k')
        ylabel('gamma')
        title('Stability chart of SSF')
        hold on
        if (abs(trace(FTM)))<=2

```

```

        plot(k,gamma,'g.')
    else
        plot(k,gamma,'r.')
    end

end

end

end

% Script to run computeSTM (for eigenvalues condition)
gamma_terms=0.11;
k_terms=100;
ei1=[];

'Please Be Patient'

for j=0:1:k_terms
    k=j;
    tspan=0:1;
    for i=0:0.0011:gamma_terms
        gamma=i;

        [t,x1]=ode45(@StabilityChartSlidingSpring,tspan,[1,0],[],gamma,k);
        [m,n]=size(x1);
        x1=x1(m,:);

        [t,x2]=ode45(@StabilityChartSlidingSpring,tspan,[0,1],[],gamma,k);

        [m,n]=size(x2);
        x2=x2(m,:);

        FTM=[x1',x2']
        e=eig(FTM)
        abs(e)

        format long;
        FTM;
        abs(eig(FTM))
        R=logm(FTM); % Exponent matrix
        eig(R); % -Real part stable, 0 real part simply stable
        ei1 = [ei1,e];
        save('ftm','e','-append')
        pause(0.25)
    end
end

```

```
figure(1)
xlabel('k')
ylabel('gamma')
title('Stability chart of SSF')
hold on
if (abs(eig(FTM)))<=1.01
    plot(k,gamma,'g.')
else
    plot(k,gamma,'r.')
end

end

end
```

APPENDIX B

MATLAB CODE FOR SOLVING NONLINEAR SYSTEM

For system controlled by rotational motion of the spring:

```
function Untitled
[t,x]=ode45(@(t,x) f(t,x),[0 3*pi],[0 0]);
LW = 'linewidth';
figure
plot(t,x(:,1),LW,2)
xlabel('t (s)')
ylabel('x (m)')

function dx=f(t,x)
% dx=[dx1;dx2] and x=[x1;x2] and x=x1 and x'=x1'=x2
m=0.1;
g=9.81;
k=50;
c=1;
r=0.1;
l0=0.25;
gamma=pi/3;
alpha=-pi/2;
omega=pi;
l1=(k*l0-m*g)/k;
Y=0.1; w=2*pi;

dx=zeros(2,1);
dx(1)=x(2);
dx(2)=(-k*(x(1)+l0+r*cos(gamma*cos(omega*t+alpha))-
Y*cos(w*t))+k*l0*(1+((r*sin(gamma*cos(omega*t+alpha)))/(x(1)+l1+r-
r*cos(gamma*cos(omega*t+alpha))-Y*cos(w*t)))^2)^(-1/2))/m-
c*(x(2)+w*Y*sin(w*t))/m;
```

For system controlled by siding motion of the spring:

```
function Untitled1
[t,x]=ode45(@(t,x) f(t,x),[0 3*pi],[0 0]);
LW = 'linewidth';
figure
plot(t,x(:,1),LW,2)
xlabel('t (s)')
ylabel('x (m)')

function dx=f(t,x)
% dx=[dx1;dx2] and x=[x1;x2] and x=x1 and x'=x1'=x2
m=0.1;
```

```

g=9.81;
k=50;
c=1;
l0=0.25;
gamma=0.15;
omega=pi;
alpha=-pi/2;
l1=(k*l0-m*g)/k;
Y=0.1; w=2*pi;

dx=zeros(2,1);
dx(1)=x(2);
dx(2)=(-k*(x(1)+l0-Y*cos(w*t))+k*l0*(1+(gamma*cos(omega*t+alpha)/(x(1)+l1-
Y*cos(w*t)))^2)^(-1/2))/m-c*(x(2)+w*Y*sin(w*t))/m;

```

APPENDIX C

MATLAB CODE FOR SOLVING LINEARIZED SYSTEM

For system controlled by rotational motion of the spring:

```
function odeRotationalSpring
[t,x]=ode45(@(t,x) f(t,x),[0 3*pi],[0 0]);
figure
plot(t,x(:,1),'linewidth',2)
xlabel('t (s)')
ylabel('x (m)')

function dx=f(t,x)
% dx=[dx1;dx2] and x=[x1;x2] and x=x1 and x'=x1'=x2
m=0.1;
g=9.81;
k=50;
c=1;
r=0.1;
l0=0.25;
gamma=pi/3;
omega=pi;
alpha=-pi/2;
l1=(k*l0-m*g)/k;
Y=0.1; w=2*pi;

dx=zeros(2,1);
dx(1)=x(2);
dx(2)=-c/m*x(2)-(k-k*l0*r^2*(sin(gamma*cos(omega*t+alpha)))^2/(l1+r-
r*cos(gamma*cos(omega*t+alpha))-Y*cos(w*t))^3)/m*x(1)+(-
k*r+k*r*cos(gamma*cos(omega*t+alpha))+k*Y*cos(w*t)-
k*l0*r^2*(sin(gamma*cos(omega*t+alpha)))^2/(2*(l1+r-
r*cos(gamma*cos(omega*t+alpha))-Y*cos(w*t))^2)-c*Y*w*sin(w*t))/m;
```

For system controlled by sliding motion of the spring:

```
function odeSlidingSpring
[t,x]=ode45(@(t,x) f(t,x),[0 3*pi],[0 0]);
LW = 'linewidth';
figure
plot(t,x(:,1),LW,2)
xlabel('t (s)')
ylabel('x (m)')

function dx=f(t,x)
% dx=[dx1;dx2] and x=[x1;x2] and x=x1 and x'=x1'=x2
m=0.1;
```

```

g=9.81;
k=50;
c=1.5;
l0=0.25;
gamma=0.15;
omega=pi;
alpha=-pi/2;
l1=(k*l0-m*g)/k;
Y=0.1; w=2*pi;

dx=zeros(2,1);
dx(1)=x(2);
dx(2)=-c/m*x(2)-(k-k*l0*(gamma*cos(omega*t+alpha))^2/(l1-
Y*cos(w*t))^3)/m*x(1)+(k*Y*cos(w*t)-k*l0*(gamma*cos(omega*t+alpha))^2/(2*(l1-
Y*cos(w*t))^2)-c*Y*w*sin(w*t))/m;

```