Toward an Uncertain Modeling of Hypersonic Aerodynamic Forces

by

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ABSTRACT

The focus of this investigation is on the development of a surrogate model of hypersonic aerodynamic forces on structures to reduce the computational effort involved in the determination of the structural response. The application is more precisely focused on *uncertain* structures. Then, following an uncertainty management strategy, the surrogate may exhibit an error with respect to Computational Fluid Dynamics (CFD) reference data as long as that error does not significantly affect the uncertainty band of the structural response. Moreover, this error will be treated as an epistemic uncertainty introduced in the model thereby generating an uncertain surrogate. Given this second step, the aerodynamic surrogate is limited to those exhibiting simple analytic forms with parameters that can be identified from CFD data.

The first phase of the investigation focuses on the selection of an appropriate form for the surrogate for the 1-dimensional flow over a flat clamped-clamped. Following piston theory, the model search started with purely local models, linear and nonlinear of the local slope. A second set of models was considered that involve also the local displacement, curvature, and integral of displacement and an improvement was observed that can be attributed to a global effect of the pressure distribution. Various ways to involve such a global effect were next investigated eventually leading to a two-level composite model based on the sum of a local component represented as a cubic polynomial of the downwash and a global component represented by an auto-regressive moving average (ARMA) model driven nonlinearly by the local downwash. This composite model is applicable to both steady

pressure distributions with the downwash equal to the slope $\frac{\partial w}{\partial x}$ and to unsteady cases with

the downwash $\frac{\partial w}{\partial x} + \frac{1}{U_\infty} \frac{\partial w}{\partial t}$.

The second part of the investigation focused on the introduction of the epistemic uncertainty in the aerodynamic surrogate and it was recognized that it could be achieved by randomizing the coefficients of the local and/or the auto-regressive components of the model. In fact, the combination of the two effects provided an applicable strategy.

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CHAPTER 1

INTRODUCTION

Deterministic methods have been a norm for solving engineering problems but, in real world scenarios, the results that can be drawn from them only form a subset of all possible results. For instance, the dynamic response of a structure depends mathematically on its physical properties, geometry, and loading conditions but deviations in properties and geometry due to manufacturing processes and variations in operating loads while in service are inevitable. These deviations will be globally referred to here as *uncertainties*. They have been well understood for a very long time and have first led to the introduction of safety factors which typically lead to suboptimum designs, i.e., too heavy, too expensive, etc.

A gradual change in perspective on how to include uncertainties in design/analysis started in the middle of the 20th century with the modeling of forces induced by natural processes (e.g., earthquakes, wind, ocean waves) as stochastic processes [1,2]. That is, the uncertainty was *modeled* in a probabilistic manner. While this modeling seems very convenient, it triggers many more questions as the *characterization* of a stochastic process requires the specifications of all its probability density functions. This task is however formidable for most processes except for a small class of them, e.g., Gaussian and Markov processes. This issue has usually been resolved by adopting a Gaussian model.

In the mid 1980's, a new step in the modeling of uncertainties was undertaken by seeking to model the variations of the *structure*, not of its loading, see [3] for a key contribution and early references. These efforts, which are still ongoing, have led to the introduction of new approaches, e.g., the stochastic finite element method [3]. In such analyses, the need

for a proper modeling of the uncertainty is even more critical than it is in the case of uncertain forces to avoid violating physical properties of the system, e.g., a mass matrix cannot have negative eigenvalues, similarly for a stiffness matrix unless the system may be unstable, etc. This issue has led to the formulation of the maximum entropy-based nonparametric approach [4,5].

It is important to recognize that performing an uncertainty analysis will not lead to a single point prediction of the response of the structure but rather to the estimation of a *band* in which the response lies with a specified probability. Moreover, such analyses are typically achieved through a repetitive, and thus potentially expensive, process referred to as a Monte Carlo simulation. Specifically, an ensemble of samples of the properties of the structure are first simulated. Then, a series of deterministic analysis of these structure samples to the specified loading are carried out and a statistical analysis of the population of responses is performed to estimate the uncertainty band on the predicted response and/or characterize the response probabilistic structure, e.g., mean, standard deviations, probability density function, etc. In this regard, note that the deterministic analyses of each structure samples can be run in parallel leading to a dramatic reduction in wall CPU time when an appropriately large cluster is available.

The consideration of uncertainties in a structure naturally leads to the differentiation between *aleatoric* and *epistemic* uncertainties. The former, also referred to as irreducible, represent the uncertainties originating generally from variability from one structure to another when a population of structures is being considered. This variability can be modeled but not eliminated. Epistemic, or reducible, uncertainty is typically attached to a lack of knowledge which can be improved by further measurements, an improved model,

etc. For example, the use of a beam/plate finite element model leads to an epistemic uncertainty which can be reduced using block elements, the epistemic uncertainty associated with a finite element mesh can be reduced by using a finer mesh. In the context of aerodynamics, assuming air as inviscid leads to an epistemic uncertainty which can again be decreased by proceeding with a viscous modeling.

At this point, it is critical to note that even the best computational prediction achieved for a realization of the structure and its excitation will not fully represent the “true” structural response, e.g. the results of a physical experiment. Rather, this prediction will be a single point within the uncertainty band in prediction that is induced by the uncertainty. Thus, using the most detailed computational models (e.g., very refined meshes) and/or the most complete physical models to evaluate each sample of the uncertainty analysis is not an efficient use of computational or time resources. Rather, an optimum resource management strategy would be to relax the computational and modeling accuracies to a level such that the resulting (epistemic) uncertainty does not lead to a significant increase in the uncertainty band on the prediction as compared to the one obtained with the inherent uncertainty in the system/its excitation.

The work carried out in this thesis fits within the above uncertainty management concept in the case of uncertain structures subjected to aerodynamic forces in the hypersonic regime. More specifically, this concept motivates the development of a computationally expedient, as compared to expensive computational fluid dynamics (CFD) tools, approach to evaluating the aerodynamic loads and their epistemic uncertainty. That is, it is not simply desired to construct an approximation of the aerodynamics loads but more comprehensively an aerodynamic load model in which its epistemic uncertainty is

modeled. Then, the uncertain structure and aerodynamics will form an uncertain system with uncertainty on both components.

This requirement of modeling the aerodynamic loads epistemic uncertainty favors the use of “analytic” models of these forces as output of a functional expression with the structural deformations as input vs. “interpolation-based” surrogates, e.g., as utilized in [6]. Indeed, the epistemic uncertainty can then be introduced in the functional expression, e.g., by randomizing its coefficients.

This thesis is a first effort toward the construction of such uncertain aerodynamic loads models. It focuses more specifically on functional forms rooted in the piston theory (e.g., see [7,8]) approximation and extensions thereof, see Chapter 2, in which the uncertainty can be introduced by randomizing the parameters, as discussed in Chapter 3. Finally, Chapter 4 is concerned with the application of the maximum entropy-based nonparametric approach to random matrices associated with a localized response such as those resulting from the piston theory approximation.

CHAPTER 2

PISTON THEORY BASED SURROGATES FOR HYPERSONIC FLOW: MEAN

MODEL

Under the assumption that the flow is inviscid and away from any shock, the piston theory introduced by Lighthill [7,8] demonstrates that the pressure at a point on the surface of coordinates x, y and time t is only a function of the normal surface velocity at that time

$$v_n = \frac{\partial Z(x,y,t)}{\partial t} + V_\infty \left[\frac{\partial Z(x,y,t)}{\partial x} + \alpha_s \right] \quad (1)$$

where α_s is the angle of attack, V_∞ the far field upstream flow velocity and $Z(x,y,t)$ is the position of the structural surface.

$$Z(x,y,t) = w_d(x,y,t) + Z_{str}(x,y) \quad (2)$$

In Eq. (2), $Z_{str}(x,y)$ is the mean surface position and $w_d(x,y,t)$ its displacement. Then, the coefficient of pressure can be obtained as

$$C_p = \frac{p - p_\infty}{p_\infty} = \frac{2}{M^2} \left[c_1 \left(\frac{w}{a_0} \right) + c_2 \left(\frac{w}{a_0} \right)^2 + c_3 \left(\frac{w}{a_0} \right)^3 \right] \quad (3)$$

where, $w = v_n$ is the piston upwash, $c_1 = 1$, $c_2 = (\gamma + 1)/4$, and $c_3 = (\gamma + 1)/12$ and γ is specific heat ratio (= 1.4 for air)

In this chapter, piston theory-based surrogates are explored to represent the steady and unsteady C_p resulting from a hypersonic viscous flow over a one-dimensional panel represented as a clamped-clamped beam. The CFD computations were carried out by Prof. J. McNamara's group at Ohio State University using the NASA code CFL3D. The beam of length 1.5m was discretized by finite elements using a uniform mesh of 1001 nodes. The beam was deformed according to only one of its first four normal modes at various

amplitudes and for different Mach numbers of the fluid flow over the beam, see Table 1 for details. Positive amplitudes refer to beam displacements into the fluid flow. From the perspective of generating mode independent models later, note that the range of amplitudes for modes 1 and 2 is larger than for modes 3 and 4, hence a wider distribution of data is used for the former modes, note as well that there is an increasing complexity of the pressure with mode number that counterbalance the lower amplitudes at higher modes.

Table 1 CFD solution (steady C_p) data for model generation and validation

mode	Amplitudes (mm)	Mach
1	$\pm 8, \pm 6, \pm 4, \pm 3, \pm 2, \pm 1$	12, 10, 8, 7, 5
2	$\pm 7, \pm 6, \pm 4, \pm 3, \pm 2, \pm 1$	12, 10, 8, 7, 5
3	$\pm 3, \pm 2.5, \pm 2, \pm 1.5, \pm 1, \pm 0.5$	12, 10, 8, 7, 5
4	$\pm 1.5, \pm 1, \pm 0.5$	12, 10, 8, 7, 5

The steady data can be split into four groups to visualize the effect of each parameter (mode number, amplitude of displacement, polarity/sign of amplitude and Mach number). Figure 1 shows the variation for mode 1 for which the coefficient of pressure increases non-linearly with the amplitude of displacement and decreases with increasing Mach number a slight shift of the peak, i.e., a phase shift with respect to the structural displacements is also observed as the C_p becomes larger. Figure 2 shows the variations for mode 3, the coefficient of pressure increases linearly with amplitude (unlike mode 1 most likely because of the smaller range of amplitudes as compared to mode 1) and decreases with increasing Mach number, again generating a phase shift.

Figure 1. Variation of Steady C_p for Mode 1 with (a) Amplitude for Mach 12 (b) Mach Number for Amplitude = +1 mm

Figure 2. Variation of Steady C_p for Mode 3 with (a) Amplitude for Mach 12 (b) Mach Number for Amplitude = +1 mm

For mode 4, as for mode 3, the C_p increases linearly with amplitude and decreases (with smaller phase difference) with Mach number (refer Figure 3). Due to the non-linearity in the data, C_p corresponding to negative amplitudes have different features from their positive counter parts. As the amplitude or Mach number is increased asymmetry in C_p increases, see Figure 4 and C_p for positive and negative amplitude are closely symmetric about offset in ordinate axis for smaller amplitudes (refer Figure 4(a)).

Figure 3. Variation of Steady C_p for Mode 4 with (a) Amplitude for Mach 12 (b) Mach Number for Amplitude = +1 mm

Figure 4. Variation of C_p for Mode 1 and Mach 12 (a) Amplitude = ± 1 mm (b) Amplitude = ± 8 mm

unsteady data was also generated, for 2.25 cycles of harmonic displacements, of amplitudes and frequencies stated in Table 2 with 400 time steps per cycle. Figure 5 displays the C_p vs. position along the beam at various time steps for a representative case. In comparison to steady C_p of increasing amplitudes (both polarity) there is generation of phase difference across the time step data (refer Figure 1(a) and 5). Note that the C_p is periodic through the excitation cycles as shown in Figure 6 which compares the distribution of C_p at various corresponding time steps of cycles 1 and 2.

Table 2. CFD solution (unsteady Cp) data for model generation and validation

mode	Amplitude (mm)	Frequency (Hz)	Mach number
1	6, 4, 3, 2, 1	160, 140, 100	12, 10, 8, 5
2	4, 3, 2, 1	160, 140, 100	12, 10, 8, 5
3	3, 2.5, 2, 1.5, 1, 0.5	160, 140, 100	12, 10, 8, 5
4	1.5, 1, 0.5	160, 140, 100	12, 10, 8, 5

Figure 5. Unsteady Cp for Mode 1 Mach 12 Frequency of 160hz Amplitude +6 mm, for Different Time Steps During First Cycle

It is important to recognize that the extreme snapshots of the unsteady Cp have similar shapes as the steady Cp distributions at matching amplitudes as confirmed in Figure 7. This observation is consistent with Eq. (1): when $w_d(x,y,t)$ reaches a maximum then its time derivative vanishes. As shown in Figure 8, the effect of frequency is more visible when the displacement (and thus the Cp) is smaller but the distributions of Cp corresponding to

different frequencies are quite similar with the largest differences in the downstream portion of the beam.

Figure 6. Comparison of Unsteady C_p Distributions for Mode 1 Displacement, Mach 12, Frequency of 160hz, Amplitude +6 mm for Corresponding Time Steps During First (C1) and Second (C2) Cycle

Figure 7. Comparison of Max. Unsteady C_p for Mode 1, Mach 12, Frequency of 160 Hz, Amplitude +6 mm and Steady C_p for Mode 1, Mach12, Amplitude +6mm

Figure 8. Comparison of Unsteady C_p for Mode 1, Mach12, Amplitude +6mm
(a) Maximum Displacement of Beam (b) Small Displacement of Beam for Various
Excitation Frequencies

The ensuing modeling efforts will focus on representing the steady C_p in terms of the static panel deflection. Then the applicability of this model to the unsteady situation will be assessed. Both mode dependent and mode independent models are considered below. Mode dependent models would be applicable when the displacement is only along one specific normal mode (as is the current data) or modal superposition is applicable for the C_p . This rigorously requires that the C_p be a linear function of the displacement. A mode independent model is one that is applicable irrespective of the imposed displacement and depends only on physical parameters. The performance of each model approximation is gauged with relative fitting-error given by

$$\text{relative error} = \frac{\|C_p^{CFL3D}(x) - C_p^{\text{mod}}(x)\|}{\|C_p^{CFL3D}(x)\|} \quad (4)$$

The following sections define the various models and discuss the characteristics of the C_p approximations.

2.1 Local Linear and Cubic Piston Theory

Linear piston theory model, $C_p = (a_1M + b_1M^2 + c_1) + (a_2M + b_2M^2 + c_2)\frac{dw}{dx}$ (5)

Cubic piston theory model, $C_p = (a_1M + b_1M^2 + c_1) + (a_2M + b_2M^2 + c_2)\frac{dw}{dx} + (a_3M + b_3M^2 + c_3)\left(\frac{dw}{dx}\right)^2 + (a_4M + b_4M^2 + c_4)\left(\frac{dw}{dx}\right)^3$ (6)

C_p is approximated by the equations shown above as described below –

- 1) The coefficient multiplying each parameter eg., $\left(\frac{dw}{dx}\right)$, etc. is identified by least squares for each Mach number independently.
- 2) The variables a_i, b_i, c_i of each coefficient are then obtained by second least square fit of the coefficients corresponding to the different Mach numbers.
- 3) The relative fitting error is obtained from Eq (4) with variables in Step 2

2.1.1 Mode independent model

As the Mach number increases in the hypersonic range, it was found that linear piston theory becomes increasingly inaccurate in modelling the pressure coefficient. This finding is clearly seen in Figure 9, the approximations obtained for mode 1 are similar in nature for different Mach numbers but have increasing errors. Accordingly, only the worst case, Mach 12, results are reported here.

The peak values of the fitting error occur for both positive and negative smallest amplitude (1mm and -1mm) due to an offset between the CFD C_p and its approximation of Eq. (5).

This offset is mitigated by introducing higher order terms of the slope as can be seen in

Figures 10-17. The second prominent feature of the local linear slope fit, Eqs (4) is the phase difference/peak shift of the C_p approximation with respect to the CFL3D C_p , see Figures 11, 13, 15, and 17. Unfortunately, this issue is not mitigated by adding higher order terms of slope. These two features, which are observed for all modes, are so prominent that nothing else was concluded about the fit removing these anomalies.

Figure 9. Fitting Error Comparison for All Mach Nos. with Local Linear Slope, Mode Independent Model

Figure 10. Fitting Error Comparison for Mode 1, Mach12 with Local Linear and Cubic Slope, Mode Independent Model

Figure 11. Comparison of CFL3D Cp and Approximated Cp with Local Linear Slope and Local Cubic Slope, Mode Independent Model (a) Mode 1, Mach 12, Amplitude of +1mm (b) Mode 1, Mach 12, Amplitude +3mm

Figure 12. Fitting Error Comparison for Mode 2, Mach 12 with Local Linear And Cubic Slope, Mode Independent Model

Figure 13. Comparison of CFL3D Cp and Approximated Cp with Local Linear Slope and Local Cubic Slope, Mode Independent Model (a) Mode 2, Mach 12, Amplitude +1 mm (b) Mode 2, Mach 12, Amplitude -2 mm

Figure 14. Fitting Error Comparison for Mode 3, Mach 12 with Local Linear and Cubic Slope, Mode Independent Model

Figure 15. Comparison of CFL3D Cp and Approximated Cp with Local Linear Slope and Local Cubic Slope, Mode Independent Model (a) Mode 3, Mach 12, Amplitude -0.5mm (b) Mode 3, Mach 12, Amplitude +3mm

Figure 16. Fitting Error Comparison for Mode 4, Mach 12 with Local Linear and Cubic Slope, Mode Independent Model

Figure 17. Comparison of CFL3D Cp and Approximated Cp with Local Linear Slope and Local Cubic Slope, Mode Independent Model (a) Mode 4 Mach 12 Amplitude +0.5mm (b) Mode 4, Mach 12, Amplitude -1.5mm

2.1.2 Modelling with Inverse Power of M

The standard piston theory approximations [7,8] see Eq. (3), yields the C_p as proportional to $\frac{1}{M^2}$, at the contrary of Eqs. (5) and (6) in which a quadratic dependence of M was assumed. Accordingly, in this subsection the C_p will be approximated by following equation -

$$C_p = \frac{(a_1M + b_1M^2 + c_1)}{M^2} + \frac{(a_2M + b_2M^2 + c_2)}{M^2} \frac{dw}{dx} + \frac{(a_3M + b_3M^2 + c_3)}{M^2} \left(\frac{dw}{dx}\right)^2 + \frac{(a_4M + b_4M^2 + c_4)}{M^2} \left(\frac{dw}{dx}\right)^3 \quad (7)$$

Figure 18. Comparison of the Modelled Coefficients of the Slope with Quad Functions of M and $1/M$

The difference between this approach and the one of the previous section lies in the secondary least square fit, see step 2 of page 12, which is constructed with polynomial of $1/M$. This representation is indeed a better fit of the coefficients, see Figure 18 for the coefficients of the slope in cubic slope models. While this approximation is consistent with aerodynamic theory and provide better fit of the coefficients, it does not provide an enhanced modelling of CFD Cp as can be seen by comparing Figure 19 with Figure 10, 12, 14 and 16. This issue will be revisited in Section 2.5 when the fitting error are significantly reduced.

Figure 19. Fitting Error Comparison for Mach 12 with Cubic Slope, Mode Independent Model and Coefficients as Quadratic Expression of $1/M$

2.1.3 Mode dependent model

The aim of this chapter is to generate an aerodynamic load model which only depends on the geometry of the beam within given ranges of the displacements and Mach numbers. The results presented in the previous section have shown that the fitting error is higher for higher modes, compare Figures 14 and 16 to Figures 10 and 12. This effect may result from (i) a higher complexity of the C_p corresponding to higher order modes or (ii) the least squares process which favors the larger C_p values obtained at the higher displacements amplitudes of lower modes. The effects of the complexity associated with higher mode numbers can be assessed by constructing and analyzing mode dependent models. Clearly, such models would also be of interest if either the displacement is comprised of one mode only or modal superposition is applicable for C_p estimation.

Figure 20. Fitting Error Comparison for Mode 1, Mach12 with Local Linear and Cubic Slope, Mode Dependent Model

The assessment of applicability of modal superposition is out of the scope of current work. Using a mode dependent model for mode 1 improves the fit, reducing the error, compare Figures 10 and 17.

Figure 21. Comparison of CFL3D Cp and Approximated Cp with Local Linear Slope and Local Cubic Slope, Mode Independent Model (a) Mode 1, Mach 12, Amplitude -1mm (b) Mode 1, Mach 12, Amplitude +3mm

Figure 22. Fitting Error Comparison for Mode 2, Mach 12 with Local Linear and Cubic Slope, Mode Independent Model

The prominent features of phase difference and offset at lower amplitudes still exist, see Figure 20 and 21 just as in the mode independent models. It is thus concluded that these features do not result from the mix of data from different modes at different amplitudes. In analyzing Figures 20-26, note first that mode 2 data have a unique feature that it has higher complexity as compared to that of mode 1 but the range of displacement amplitude is similar (for mode 3 and mode 4, the amplitude are lower). Hence the mode dependent modelling error increases for mode 2 (see Figure 22) as compared to mode 1. Moreover, the previously mentioned features of phase difference and offset are also present, see Figure 23.

Figure 23. Comparison of CFL3D Cp and Approximated Cp with Local Linear Slope and Local Cubic Slope, Mode Independent Model (a) Mode 2, Mach 12, Amplitude +1mm (b) Mode 2, Mach 12, Amplitude -3mm

In contrast to mode 2, the mode 3 peak error at lower amplitudes is not significantly larger than at higher amplitudes. This is believed to be due to the limited range of displacement amplitudes used and thus to the larger importance of the lower amplitude cases in the least squares optimization. But similar issues, phase difference and offset, persist with these

results as before (see Figures 24 and 25). The approximation for mode 4 leads to similar observations (Figure 26 and 27).

Figure 24. Fitting Error Comparison for Mode 3, Mach12 with Local Linear and Cubic Slope, Mode Independent Model

Figure 25. Comparison of CFL3D Cp and Approximated Cp with Local Linear Slope and Local Cubic Slope, Mode Independent Model (a) Mode 3, Mach 12, Amplitude -0.5mm (b) Mode 3, Mach 12, Amplitude +1.5mm

Figure 26. Fitting Error Comparison for Mode 4, Mach 12 with Local Linear and Cubic Slope, Mode Independent Model

Figure 27. Comparison of CFL3D Cp and Approximated Cp with Local Linear Slope and Local Cubic Slope, Mode Independent Model (a) Mode 4, Mach 12, Amplitude -1.5mm (b) Mode 4, Mach 12, Amplitude -0.5mm

2.2 Global Linear and Cubic Slope Models

2.2.1 Formulation

The critical feature of phase difference between the CFD solution and the local piston theory approximation observed in section 2.1 lead to the thought that the upstream flow may have an impact on the pressure developed downstream, i.e., that the model is no longer perfectly local. In this section, the contribution of global effects is studied by including slopes of upstream nodes in the model, see Eqs (8) and (9). As will be seen below, an important aspect of this new modeling is to involve the “right” number of nodes in the model, this number should correspond to the “information length” of the pressure i.e., the length along the panel over which the pressure are related. Considering too high number of nodes may lead to numerical conditioning problems while on the other hand too small a number of nodes may not lead to the best possible fitting.

$$\begin{aligned} Cp(i) = & (a_1M + b_1M^2 + c_1) + (a_{2,0}M + b_{2,0}M^2 + c_{2,0})\left(\frac{dw(i)}{dx}\right) + \dots \\ \text{Global linear slope,} & \\ & + (a_{2,n}M + b_{2,n}M^2 + c_{2,n})\left(\frac{dw(i-n)}{dx}\right) \end{aligned} \quad (8)$$

$$\begin{aligned} \text{Global cubic slope,} & \\ Cp(i) = & (a_1M + b_1M^2 + c_1) + \\ & (a_{2,0}M + b_{2,0}M^2 + c_{2,0})\left(\frac{dw(i)}{dx}\right) + \dots + (a_{2,n}M + b_{2,n}M^2 + c_{2,n})\left(\frac{dw(i-n)}{dx}\right) + \\ & (a_{3,0}M + b_{3,0}M^2 + c_{3,0})\left(\frac{dw(i)}{dx}\right)^2 + \dots + (a_{3,n}M + b_{3,n}M^2 + c_{3,n})\left(\frac{dw(i-n)}{dx}\right)^2 + \\ & (a_{4,0}M + b_{4,0}M^2 + c_{4,0})\left(\frac{dw(i)}{dx}\right)^3 + \dots + (a_{4,n}M + b_{4,n}M^2 + c_{4,n})\left(\frac{dw(i-n)}{dx}\right)^3 \end{aligned} \quad (9)$$

In Eqs. (8) and (9), $Cp(i)$ denotes the Cp at node i and n quantifies the *information length* which is $n\Delta x$, where Δx is spacing between nodes. A global formulation, i.e., including

all nodes, for the linear and cubic piston theory leads to ill-conditioned matrices for the parameters, hence the effect of global terms cannot be studied. The issue is due to the fine mesh which leads to almost equal values of displacement on adjacent nodes. The importance of global terms can however be assessed by examining (i) the fitting error vs. the distance of a single “global” node (to measure the information length), i.e., a node upstream of the one at which the C_p is computed and (ii) the fitting error vs. the number of such global nodes. These two analysis are conducted in the following two subsections.

2.2.2 Approximation with single global node

The modelling process was first performed using a single global node located upstream of the current point (referred to as node 1 below) by a 10% information length, i.e., 100 nodes away. Comparison of the fitting errors obtained over the nodes 101 to 1001 (the approximation can only start from node 101 onwards) for the linear and cubic global models, Eqs. (8) and (9), and their local counterparts are shown in Figures 28 and 29 for mode 1. It is first seen that the fitting error of the linear model is improved over higher amplitudes but not at lower ones. This situation results again from an offset of the approximate and CFL3D C_p 's which is not mitigated by the global term.

This offset is once again resolved with the cubic model for which the phase difference is improved although not completely corrected. As for mode 1 the distance of the global node is increased, the fitting improves but this change is minimal for mode 1 (refer to Figures 29 and 30). In case of mode 2, Figures 31 and 32, the C_p for large positive amplitudes is highly nonlinear hence the approximation worsens as amplitude increases. Observations

and findings for modes 3 (Figures 33 and 34) and 4 (Figures 35 and 36) are similar to those of mode 1. As observed with the local model, the fitting error increases with mode number.

Figure 28. Fitting Error Comparison for Mode 1, Mach12 with Local Linear and Global Linear Slope, Mode Independent Model

Figure 29. Fitting Error Comparison for Mode 1, Mach 12 with Local Cubic Slope and Global Cubic Slope (Various Configurations), Mode Independent Model

In the remainder of this section, the addition of global terms will be analyzed only for the cubic slope model (Eq. (9)).

Figure 30. Comparison of CFL3D Cp and Approximated Cp with Local Cubic Slope and Global Cubic Slope, Mode Independent Model (a) Mode 1, Mach 12, Amplitude -2mm (b) Mode 1, Mach 12, Amplitude +1mm

Figure 31. Fitting Error Comparison for Mode 2, Mach12 with Local Cubic Slope and Global Cubic Slope (Various Configurations), Mode Independent Model

Figure 32. Comparison of CFL3D Cp and Approximated Cp with Local Cubic Slope and Global Cubic Slope, Mode Independent Model (a) Mode 2, Mach 12, Amplitude -2mm (b) Mode 1, Mach 12, Amplitude +7mm

Figure 33. Fitting Error Comparison for Mode 3, Mach12 with Local Cubic Slope and Global Cubic Slope (Various Configurations), Mode Independent Model

Figure 34. Comparison of CFL3D Cp and Approximated Cp with Local Cubic Slope and Global Cubic Slope, Mode Independent Model (a) Mode 3, Mach 12, Amplitude -1.5mm (b) Mode 3, Mach 12, Amplitude +3mm

Figure 35. Fitting Error Comparison for Mode 4, Mach 12 with Local Cubic Slope and Global Cubic Slope (Various Configurations), Mode Independent Model

Figure 36. Comparison of CFL3D C_p and Approximated C_p with Local Cubic Slope and Global Cubic Slope, Mode Independent Model (a) Mode 4, Mach 12, Amplitude +1.5mm (b) Mode 4, Mach 12, Amplitude -1.5mm

2.2.3 Approximation with multiple global nodes

In this subsection, the effect of the number of global terms considered in the model will be analyzed by including in it the nodes 25, 50 and 75 nodes away from the current node in addition to the one 100 nodes away. In general, see Figures 37-42, the fitting error decreases with the number of global terms involved in the formulation except for mode 4 where the best fit is attained by only adding the term associated to node 50 to the default global model incorporating the current node and node 100. Approximate and CFL3D C_p 's are compared in Figure 40 for mode 4 which shows that the final approximation is distorted. Note that all computations were well conditioned.

Figure 37. Fitting Error Comparison for Mode1, Mach12 with Local Cubic Slope and Global Cubic Slope (Various Configurations), Mode Independent Model

Figure 38. Comparison of CFL3D C_p and Approximated C_p with Global Cubic Slope (Node 1, 50 & 100) and Global Cubic Slope (Node 1, 25, 50, 75 & 100), Mode Independent Model for Mode 1, Mach 12, Amplitude -1mm

Figure 39. Fitting Error Comparison for Mode 2, Mach 12 with Local Cubic Slope and Global Cubic Slope (Various Configurations), Mode Independent Model

Figure 40. Fitting Error Comparison for Mode 3, Mach 12 with Local Cubic Slope and Global Cubic Slope (Various Configurations), Mode Independent Model

Figure 41. Fitting Error Comparison for Mode 4, Mach12 with Local Cubic Slope and Global Cubic Slope (Various Configurations), Mode Independent

Figure 42. Comparison of CFL3d Cp and Approximated Cp with Global Cubic Slope (Node 1, 50 & 100) and Global Cubic Slope (Node 1, 25, 50, 75 & 100), Mode Independent Model for Mode 4, Mach 12, Amplitude -0.5mm

2.3 Local and global linear dsci model

The results presented in the previous section suggest that introducing a global component in the Cp modeling does improve the matching. However, the issues of numerical conditioning limit the previous formulation. Another strategy to include a global effect is considered here by introducing in the model the local displacement (“d”), curvature (“c”), and displacement-integral (“i”) in addition with linear slope (“s”). This model, referred to as local dsci, is defined as

Local linear dsci model,

$$Cp = (a_1M + b_1M^2 + c_1) + (a_2M + b_2M^2 + c_2)w + (a_3M + b_3M^2 + c_3)\left(\frac{dw}{dx}\right) + (a_4M + b_4M^2 + c_4)\left(\frac{d^2w}{dx^2}\right) + (a_5M + b_5M^2 + c_5)\left(\int_0^x w dx\right) \quad (10)$$

Global linear dsci model,

$$Cp(i) = (a_1M + b_1M^2 + c_1) + (a_{2,0}M + b_{2,0}M^2 + c_{2,0})w(i) + \dots + (a_{2,n}M + b_{2,n}M^2 + c_{2,n})w(i-n) + (a_{3,0}M + b_{3,0}M^2 + c_{3,0})\left(\frac{dw(i)}{dx}\right) + \dots + (a_{3,n}M + b_{3,n}M^2 + c_{3,n})\left(\frac{dw(i-n)}{dx}\right) + (a_{4,0}M + b_{4,0}M^2 + c_{4,0})\left(\frac{d^2w(i)}{dx^2}\right) + \dots + (a_{4,n}M + b_{4,n}M^2 + c_{4,n})\left(\frac{d^2w(i-n)}{dx^2}\right) + (a_{5,0}M + b_{5,0}M^2 + c_{5,0})\left(\int_0^x w dx\right) + \dots + (a_{5,n}M + b_{5,n}M^2 + c_{5,n})\left(\int_0^x w dx\right) \quad (11)$$

Local and global linear dsci models have similar characteristics as the linear slope models investigated above, e.g., the presence of an offset in the approximate Cp for the smaller amplitudes. However, the phase difference observed in prior linear slope models is improved, see Figures 43-50.

Figure 43. Fitting Error Comparison for Mode 1, Mach 12 with Local Linear Slope, Local Linear dsci and Global Linear dsci (1 & 100 Node), Mode Independent Model

It is observed in Figures 43, 45, 47, and 49 that the addition of the displacement, curvature, and displacement integral does reduce the fitting error as compared to the slope only models.

Figure 44. Comparison of CFL3D Cp and Approximated Cp with Local Linear dsci, Global Linear dsci (Node 1 & 100), Local Linear Slope and Global Linear Slope (Node 1 & 100), Mode Independent Model (a) Mode 1, Mach 12, Amplitude -1mm and (b) Mode 1, Mach 12, Amplitude +6mm

Figure 45. Fitting Error Comparison for Mode 2, Mach12 with Local Linear Slope, Local Linear dsci and Global Linear dsci (1 & 100 Node), Mode Independent Model

Further, the addition of the global term of node 100 appears to improve only little the fit for this dsci model except for mode 4. This finding would suggest that the dci terms contain the same global information as is in the slopes at upstream nodes.

Figure 46. Comparison of CFL3D Cp and Approximated Cp with Local Linear dsci, Global Linear dsci (Node 1& 100), Local Linear Slope and Global Linear Slope (Node 1 & 100), Mode Independent Model (a) Mode 2, Mach 12, Amplitude +1mm and (b) Mode 2, Mach 12, Amplitude -2mm

Figure 47. Fitting Error Comparison for Mode 3, Mach 12 with Local Linear Slope, Local Linear dsci and Global Linear dsci (1 & 100 Node), Mode Independent

Figure 48. Comparison of CFL3D Cp and Approximated Cp with Local Linear dsci, Global Linear dsci (Node 1 & 100), Local Linear Slope and Global Linear Slope (Node 1 & 100), Mode Independent Model (a) Mode 3, Mach 12, Amplitude -0.5mm and (b) Mode 3, Mach 12, Amplitude +2.5mm

Figure 49. Fitting Error Comparison for Mode 4, Mach12 with Local Linear Slope, Local Linear dsci and Global Linear dsci (1 & 100 Node), Mode Independent Model

Figure 50. Comparison of CFL3D Cp and Approximated Cp with Local Linear dsci, Global Linear dsci (Node 1 & 100), Local Linear Slope and Global Linear Slope (Node 1 & 100), Mode Independent Model (a) Mode 1, Mach 12, Amplitude -1mm and (b) Mode 1, Mach 12, Amplitude +6mm

2.4 Local and Global Non-linear dsci model

The issue of the offset between approximate and CFL3D Cp's at lower amplitudes was solved earlier, in the slope only models, by including in it nonlinear terms. This section accordingly focuses on local and global nonlinear dsci formulations. Specifically, only select linear and quadratic terms in the dsci variables leads to the model of Eq. (12). A global nonlinear dsci model is built from Eq. (12) by adding similar terms associated with prior nodes. There are 14 groups of terms or factors in Eq. (12) but not all of them contribute meaningfully to the approximation. To decide on the elimination of weakly contributing factors from the above formulation, the relative contribution of each factor in the approximation was determined. This task was achieved by removing one factor at a time and comparing the change in the fitting error as compared to the full configuration, i.e., with all 14 factors. The results of this analysis are shown in Figure 51.

Local non-linear dsci model,

$$\begin{aligned}
 Cp = & (a_1M + b_1M^2 + c_1) + (a_2M + b_2M^2 + c_2)w + (a_3M + b_3M^2 + c_3)\left(\frac{dw}{dx}\right) + \\
 & (a_4M + b_4M^2 + c_4)\left(\frac{d^2w}{dx^2}\right) + (a_5M + b_5M^2 + c_5)\left(\int_0^x w dx\right) + (a_6M + b_6M^2 + c_6)w^2 + \\
 & (a_7M + b_7M^2 + c_7)w\left(\frac{dw}{dx}\right) + (a_8M + b_8M^2 + c_8)w\left(\frac{d^2w}{dx^2}\right) + \\
 & (a_9M + b_9M^2 + c_9)w\left(\int_0^x w dx\right) + (a_{10}M + b_{10}M^2 + c_{10})\left(\frac{dw}{dx}\right)^2 + \\
 & (a_{11}M + b_{11}M^2 + c_{11})\left(\frac{dw}{dx}\right)\left(\frac{d^2w}{dx^2}\right) + (a_{12}M + b_{12}M^2 + c_{12})\left(\frac{dw}{dx}\right)\left(\int_0^x w dx\right) + \\
 & (a_{13}M + b_{13}M^2 + c_{13})\left(\frac{d^2w}{dx^2}\right)^2 + (a_{14}M + b_{14}M^2 + c_{14})\left(\frac{d^2w}{dx^2}\right)\left(\int_0^x w dx\right) + \\
 & (a_{15}M + b_{15}M^2 + c_{15})\left(\int_0^x w dx\right)^2
 \end{aligned} \tag{12}$$

Figure 51. Comparison of Contribution of Each Term of Non-linear dsci Formulation to Fitting Error

Based on these results, it is concluded that the dominating factors are d , s , s^2 , c , i , $d*s$, $d*c$, $s*i$ and constant. The ensuing analyses will thus be conducted on this formulation only. Similar to Section 2.2 for the global linear and cubic slope model, not all nodes can be used for a global formulation without leading to an ill conditioning of the least squares problem. Thus, the effects of global terms will be analyzed only for the cases yielding a good conditioning.

As expected from the analysis of prior models, the offset of the approximate C_p for small amplitudes is eradicated by the introduction of non-linear terms. Moreover, a larger decrease in fitting error with respect to all previous models at all amplitudes is observed,

see Figures 52-55. Considering finally the effects of global terms, it is seen that they do not contribute largely for mode 1 (Figure 52) or mode 2 (Figure 53).

Figure 52. Fitting Error Comparison for Mode 1, Mach 12 with Local Non-linear dsci and Global Non-linear dsci (Only 1 Global Term - Various Configurations of Distance), Mode Independent Model

As the distance between nodes increases the fitting error reduces but the improvement is not significant possibly because the fitting error corresponding to the local formulation is already small. In contrary to mode 1 and 2, for mode 3 (Figure 54) and mode 4 (Figure 55) the approximation improves with (i) the addition of global nodes and (ii) increasing distance of the global node. This result indicates that using the 100th global node would give the best fit across all modes.

Figure 53. Fitting Error Comparison for Mode 2, Mach 12 with Local Non-linear dsci and Global Non-linear dsci (Only 1 Global Term - Various Configurations of Distance), Mode Independent Model

Figure 54. Fitting Error Comparison for Mode 3, Mach 12 with Local Non-linear dsci and Global Non-linear dsci (Only 1 Global Term - Various Configurations of Distance), Mode Independent Model

Figure 55. Fitting Error Comparison for Mode 4, Mach 12 with Local Non-linear dsci and Global Non-linear dsci (Only 1 Global Term - Various Configurations of Distance), Mode Independent Model

Continuing from the previous section, the next step in the modelling effort is to include more global nodes to the model than the 100th one while monitoring the conditioning of the system. It was found that similar fits to the ones above are achieved for mode 1 (Figure 56) and mode 2 (Figure 57). However, once again, for modes 3 (Figure 58) and 4 (Figure 59) improvements are achieved, possibly due to the higher base error of the approximation as compared to lower mode numbers.

With global nodes 25, 50 and 100 in addition to the local node the fitting error is reduced considerably for mode 4 and somewhat for mode 3. With this global non-linear dsci model, the average error across the modes is 16%.

Figure 56. Fitting Error Comparison for Mode 1, Mach 12 with Local Non-linear dsci and Global Non-linear dsci (Various Configurations of Number of Nodes), Mode Independent Model

Figure 57. Fitting Error Comparison for Mode 2, Mach 12 with Local Non-linear dsci and Global Non-linear dsci (Various Configurations of Number of Nodes), Mode Independent Model

Figure 58. Fitting Error Comparison for Mode 3, Mach 12 with Local Non-linear dsci and Global Non-linear dsci (Various Configurations of Number of Nodes), Mode Independent Model

Figure 59. Fitting Error Comparison for Mode 4, Mach 12 with Local Non-linear dsci and Global Non-linear dsci (Various Configurations of Number of Nodes), Mode Independent Model

2.5 Auto -regressive Moving-average (Nonlinear dsci) model

In their piston theory analysis and revision, Liu et al. [7] obtained an excellent match of the pressure coefficient in inviscid supersonic flows with a model that expresses the C_p at one node in terms of the local downwash but also in terms of the C_p at neighboring nodes. In fact, this second contribution provides a global effect in the approximation, although different than the one introduced in the above sections. Taking inspiration from that formulation, the pressure coefficient C_p is here represented as the auto-regressive moving average model driven by quadratic dsci terms, i.e.,

$$\begin{aligned}
C_p(i) &+ \left(\frac{a_{0,0}M + b_{0,0}M^2 + c_{0,0}}{M^2} \right) C_p(i-1) + \dots + \left(\frac{a_{0,n}M + b_{0,n}M^2 + c_{0,n}}{M^2} \right) C_p(i-n-1) = \\
&\left(\frac{a_{1,0}M + b_{1,0}M^2 + c_{1,0}}{M^2} \right) + \left(\frac{a_{2,0}M + b_{2,0}M^2 + c_{2,0}}{M^2} \right) w(i) + \dots + \left(\frac{a_{2,n}M + b_{2,n}M^2 + c_{2,n}}{M^2} \right) w(i-n) + \\
&\left(\frac{a_{3,0}M + b_{3,0}M^2 + c_{3,0}}{M^2} \right) \left(\frac{dw(i)}{dx} \right) + \dots + \left(\frac{a_{3,n}M + b_{3,n}M^2 + c_{3,n}}{M^2} \right) \left(\frac{dw(i-n)}{dx} \right) + \\
&\left(\frac{a_{4,0}M + b_{4,0}M^2 + c_{4,0}}{M^2} \right) \left(\frac{d^2w(i)}{dx^2} \right) + \dots + \left(\frac{a_{4,n}M + b_{4,n}M^2 + c_{4,n}}{M^2} \right) \left(\frac{d^2w(i-n)}{dx^2} \right) + \\
&\left(\frac{a_{5,0}M + b_{5,0}M^2 + c_{5,0}}{M^2} \right) \left(\int_0^{x(i)} w dx \right) + \dots + \left(\frac{a_{5,n}M + b_{5,n}M^2 + c_{5,n}}{M^2} \right) \left(\int_0^{x(i-n)} w dx \right) + \\
&\left(\frac{a_{7,0}M + b_{7,0}M^2 + c_{7,0}}{M^2} \right) w(i) \left(\frac{dw(i)}{dx} \right) + \dots + \left(\frac{a_{7,n}M + b_{7,n}M^2 + c_{7,n}}{M^2} \right) w(i-n) \left(\frac{dw(i-n)}{dx} \right) + \\
&\left(\frac{a_{8,0}M + b_{8,0}M^2 + c_{8,0}}{M^2} \right) w(i) \left(\frac{d^2w(i)}{dx^2} \right) + \dots + \left(\frac{a_{8,n}M + b_{8,n}M^2 + c_{8,n}}{M^2} \right) w(i-n) \left(\frac{d^2w(i-n)}{dx^2} \right) + \\
&\left(\frac{a_{10,0}M + b_{10,0}M^2 + c_{10,0}}{M^2} \right) \left(\frac{dw(i)}{dx} \right)^2 + \dots + \left(\frac{a_{10,n}M + b_{10,n}M^2 + c_{10,n}}{M^2} \right) \left(\frac{dw(i-n)}{dx} \right)^2 + \\
&\left(\frac{a_{12,0}M + b_{12,0}M^2 + c_{12,0}}{M^2} \right) \left(\frac{dw(i)}{dx} \right) \left(\int_0^{x(i)} w dx \right) + \dots + \left(\frac{a_{12,n}M + b_{12,n}M^2 + c_{12,n}}{M^2} \right) \left(\frac{dw(i-n)}{dx} \right) \left(\int_0^{x(i-n)} w dx \right)
\end{aligned} \tag{13}$$

Equation (13) is using all of the consecutive n nodes upstream of the calculated node, this leads to ill-conditioned system (as discussed in Section 2.2). Configurations of discrete global nodes instead of consecutive ones will also be tested for performance in the section. While the least squares problems appear to be ill-conditioned ($RCOND=5 \cdot 10^{26}$), the model provides a good fit of the CFL3D pressure coefficients when using last 10 nodes. This situation is akin to the one encountered with the global model with 10 previous nodes but this observation doesn't guarantee that this methodology will be stable for any system. It is thus important to assess whether we need all 10 prior nodes because spacing the global nodes used does improve the conditioning number. Using the same strategy as in the prior sections, only 1 global node is first considered for both auto regressive and moving average terms. This effort is carried out using the 5th, 10th, 20th, 50th and 100th node upstream of the current (local) node, referred to as 1. The impact of using global terms is seen clearly in Figures 60-63, higher modes are better approximated and the fitting error reduces as the distance between global and current nodes increases.

It was also assessed whether the global node component could be used only in the autoregressive (curves labeled "AR node 1&50, MA node 1") or only in the moving average (curves labeled "AR node 1, MA node 1&50") components of the model. Comparing the fitting errors with those obtained with the global term present on both sides (curves labeled "1&50") shows a definite worsening of the error demonstrating that the global terms should be included on both sides of the model equation (13).

Figure 60. Fitting Error Comparison for Different Configuration of Mode Independent ARMA Model with Moving Average of Nonlinear Dsci for Mode 1, Mach 12

Figure 61. Fitting Error Comparison for Different Configuration of Mode Independent ARMA Model with Moving Average of Nonlinear Dsci for Mode 2, Mach 12

Figure 62. Fitting Error Comparison for Different Configuration of Mode Independent ARMA Model with Moving Average of Nonlinear dsci for Mode 3, Mach 12

Figure 63. Fitting Error Comparison for Different Configuration of Mode Independent ARMA Model with Moving Average of Nonlinear dsci for Mode 4, Mach 12

2.6 Auto -regressive Moving-average (Cubic Slope) model

While the model of the previous section provided fairly low fitting errors, the disadvantages of a nonlinear dsci model are that (i) it involves a large number of terms and (ii) the integration of displacement would need to be carefully formulated in a 2-dimensional problem (vs. the 1-D flow considered here). It is thus desired to assess whether a simpler cubic slope model constructed similarly would provide similar results. Another advantage of such a model over a global nonlinear dsci one is that more closely spaced global terms could be used which improves the fit. Figure 64-67 show the fitting errors for each mode with mode independent ARMA (global cubic slope) model relying on the 20th, 50th, and 100th global node in addition to local slope, i.e., of the form

$$\begin{aligned}
C_p(i) + \left(\frac{a_{0,0}M + b_{0,0}M^2 + c_{0,0}}{M^2} \right) C_p(i-1) + \dots + \left(\frac{a_{0,n}M + b_{0,n}M^2 + c_{0,n}}{M^2} \right) C_p(i-n-1) = \\
\left(\frac{a_1M + b_1M^2 + c_1}{M^2} \right) + \\
\left(\frac{a_{2,0}M + b_{2,0}M^2 + c_{2,0}}{M^2} \right) \left(\frac{dw(i)}{dx} \right) + \dots + \left(\frac{a_{2,n}M + b_{2,n}M^2 + c_{2,n}}{M^2} \right) \left(\frac{dw(i-n)}{dx} \right) + \\
\left(\frac{a_{3,0}M + b_{3,0}M^2 + c_{3,0}}{M^2} \right) \left(\frac{dw(i)}{dx} \right)^2 + \dots + \left(\frac{a_{3,n}M + b_{3,n}M^2 + c_{3,n}}{M^2} \right) \left(\frac{dw(i-n)}{dx} \right)^2 + \\
\left(\frac{a_{4,0}M + b_{4,0}M^2 + c_{4,0}}{M^2} \right) \left(\frac{dw(i)}{dx} \right)^3 + \dots + \left(\frac{a_{4,n}M + b_{4,n}M^2 + c_{4,n}}{M^2} \right) \left(\frac{dw(i-n)}{dx} \right)^3
\end{aligned} \tag{14}$$

Note that the coefficients were represented as powers of 1/M as suggested by the improved results of section 2.1.2.

The fit is enhanced with respect to the one shown in the previous section owing to the addition of closely spaced global nodes while keeping the system well-conditioned.

Figure 64. Fitting Error Comparison for Different Configuration of Mode Independent ARMA Model with Moving Average of Cubic Slope for Mode 1, Mach 12

Figure 65. Fitting Error Comparison for Different Configuration of Mode Independent ARMA Model with Moving Average of Cubic Slope for Mode 2, Mach 12

Figure 66. Fitting Error Comparison for Different Configuration of Mode Independent ARMA Model with Moving Average of Cubic Slope for Mode 3, Mach 12

Figure 67. Fitting Error Comparison for Different Configuration of Mode Independent ARMA Model with Moving Average of Cubic Slope for Mode 4, Mach 12

It is concluded from the above figures that a cubic slope based ARMA model with 1st (local), 25th, 50th, 75th and 100th node contributions in the auto-regressive and moving average terms can give comparable results to the nonlinear dsci based ARMA formulation with a significantly lower computation effort. As the Mach number reduces, C_p becomes increasing local which can be observed by fitting error of the local linear slope model (Figure 9). Approximations of C_p with discrete terms in the ARMA model (nonlinear dsci or cubic slope) leads to unstable recursion (i.e., with poles outside of unit circle). To counter this instability, number of global terms is reduced for Mach 5 to – only the 50th and 100th nodes.

2.7 Composite modelling with local nonlinear dsci and ARMA cubic slope

Sections 2.5 and 2.6 have focused on the fit for nodes 101 to 1001 only as values of the C_p prior to the local point are needed. Given the very positive results obtained in these sections, it is now necessary to focus on the estimation of the C_p for the nodes 1-100 and to assess the effects of this estimation on the accuracy of the above methods. The simplest models of local cubic slope (Section 2.1) and local nonlinear dsci (Section 2.4) were found to provide acceptable fits of the C_p for the most upstream nodes of the beam with small phase difference until the first peak.

On this basis, the coefficient of pressure at nodes 1 to 100 is here first modelled with a local nonlinear dsci formulation from Section 2.4 and then further used to initiate the recursion of the cubic slope based ARMA model (discussed in section 2.6) to approximate the CFL3D C_p over the entire beam. Figure 68 shows that the local nonlinear dsci model provides an adequate set of C_p values on the first 100 nodes. Moreover, the use of these values for the initiation of the recursion of the ARMA model leads to fitting errors that are

consistent with those obtained earlier assuming the exact C_p values for nodes 1 to 100, compare Figure 69 and Figures 64-66 except for larger amplitudes for mode 2. Overall, the approximation of the CFL3D C_p values is adequate, see Figure 70.

Figure 68. Fitting Error Comparison for Node 1 to 100 Approximated with Mode Independent Local Nonlinear dsci Model (Note the reduced range of the ordinate axis as compared to prior error plots)

Figure 69. Fitting Error Comparison for Approximated with Mode Independent Local Nonlinear Dsci Model (1 to 100 Node) and Cubic Slope Based ARMA Model (101 to 1001 Node). (Note the reduced range of the ordinate axis as compared to prior error plots)

(a)

(b)

Figure 70. Comparison of C_p from CFL3d (Blue) and Approximated C_p with Composite Model (Red) for Mode 1, Mach 12, Amplitude +8mm, Amplitude +1mm; Mode 2, Mach 12, Amplitude +7mm, Amplitude +1mm; Mode 3, Mach 12, Amplitude +3mm, Amplitude +0.5mm; Mode 4, Mach 12, Amplitude +1.5mm, Amplitude +0.5mm

2.8 Two-level composite model

The fitting errors shown in Figure 69 clearly suggest that the corresponding C_p approximations worsen as the amplitude of displacement increases. To curb this trend, it is proposed here to combine the ARMA-based modeling with another approximation that behaved well as a function of this amplitude. Reviewing earlier models suggests the use for this purpose of the local cubic slope model.

That is, it is proposed to seek an approximation of the CFL3D coefficients of pressure as

$$C_p = C_p^L(s, s^2, s^3) + C_p^G = \alpha_1 \left(\frac{dw}{dx} \right) + \alpha_2 \left(\frac{dw}{dx} \right)^2 + \alpha_3 \left(\frac{dw}{dx} \right)^3 + C_p^G \quad (14)$$

where s is the local slope, α_i are quadratic function of $1/M$, C_p^L the local component of the coefficient of pressure, modeled by the cubic slope model, and C_p^G the global component which is represented as a cubic slope based ARMA model, i.e. in the form of Eq (13).

As a first step in validating this model, shown in Figures 71-74 are the C_p residuals after fitting the mode independent local cubic slope model. These residuals become increasing linear with respect to the amplitude, but differently for positive and negative amplitudes as higher modes are considered. This feature negates the increasing complexity of the higher order mode data.

Each level optimization was then carried out separately to obtain the coefficients of the local cubic slope model as well as the parameters of a global cubic slope ARMA model encompassing nodes 1, 25, 50, 75, and 100. In this model, the C_p for nodes 1 to 100 were obtained from a local cubic slope model only. The fitting error of the resulting model is

shown in Figure 75 and some typical comparisons of CFL3D and approximate C_p distributions are shown in Figure 76. Compared to the previous composite model, the fitting error reduces for large amplitudes of mode 2 and others remains in the same range. It should be noted that a simultaneous optimization of the local and global model parameters has also been conducted and essentially recover above results obtained with separate optimizations.

Figure 71. Residual of C_p from Fitting Mode Independent Local Cubic Slope Model for Mode 1, Mach 12

Figure 72. Residual of C_p from Fitting Mode Independent Local Cubic Slope Model for Mode 2, Mach 12

Figure 73. Residual of C_p from Fitting Mode Independent Local Cubic Slope Model for Mode 3, Mach 12

Figure 74. Residual of C_p from Fitting Mode Independent Local Cubic Slope Model for Mode 4, Mach 12

Figure 75. Fitting Error Comparison for C_p Approximated with Mode Independent Local Cubic Slope Model (1 to 1001 Node) and Residual with Cubic Slope Based ARMA Model (101 to 1001 Node)

(a)

(b)

(c)

(d)

(e)

(f)

Figure 76. Comparison of C_p from CFL3D (Blue) and Approximated C_p with Composite Model (Red) for Mode 1, Mach 12, Amplitude +8mm, Amplitude +1mm; Mode 2, Mach 12, Amplitude +7mm, Amplitude +1mm; Mode 3, Mach 12, Amplitude +3mm, Amplitude +0.5mm; Mode 4, Mach 12, Amplitude +1.5mm, Amplitude +0.5mm

The stability of the auto-regressive part of model is essential in obtaining a smooth finite approximation of C_p . This stability requires that the poles of the AR transfer function be within the unit circle in the complex plane, shown in Figure 77 are the corresponding poles for the various Mach number for the model with discrete nodes 1, 25, 50, 75 and 100. With the exception of the Mach 5 approximation, all auto-regressive components are stable. As shown in Figure 77(b), the Mach 5 model leads to a pole of magnitude 1.008, slightly outside the unit circle and thus this model is unstable.

Figure 77. (a) Poles of Auto-regressive Part of ARMA Model for All Mach (b) Zoomed View of 77(a)

This issue was addressed here by reducing the number of global nodes. As shown in Figure 78, using 1st, 50th and 100th node gives the best approximation as reducing the nodes further increases the error. Moreover, the corresponding auto-regressive part was found to be stable. Note as well on this figure that the large error of the unstable model using 1, 25, 50, 75 and 100 nodes.

Figure 78. Comparison of Approximation Error with Different Number of Global Nodes for Mach 5 (a) Mode 1 (b) Mode 2 (c) Mode 3 (d) Mode 4

2.9 Modelling Unsteady C_p with Two-Level Composite Model

Following the discussion of section 2.1, see Figures 5-8, on the similarities of the steady and unsteady C_p distributions, it is proposed here to apply the two-level composite model of section 2.8, **with the same coefficients**, to the unsteady situation based on the material

derivative, i.e., $\frac{1}{U_\infty} \frac{Dw}{Dt} (= \frac{\partial w}{\partial x} + \frac{1}{U_\infty} \frac{\partial w}{\partial t})$ instead of $\frac{dw}{dx}$. The approximate C_p

distributions exhibit an offset and a phase difference for smaller displacement (see Figure

(a)

(b)

(c)

(d)

Figure 79. Comparison of CFD Unsteady Cp (Blue) and Approximated with Two-level Model (Red) for Mode 1, Mach 12, Amplitude +6mm, Frequency of 160 Hz
(a) $t = 2.5e-05$ s (b) $t = 0.00025$ s (c) $t = 0.00052$ s (d) $t = 0.00077$ s

79 (a),(c)) but as the displacement level increases the fitting improves. In contrast to the steady case where the fitting error increased with increasing amplitude, it appears here that the error decreases (see Figure 79(b),(d)). Since the same coefficients are utilized to generate the unsteady Cp this is due to the nature of $\frac{Dw}{Dt}$ over time. Similar observations can also be drawn from Figure 80 corresponding to mode 3 displacement. The

improvement at higher negative displacements, Figure 80 (c), (d) is more pronounced than on the positive, Figure 80 (a), (b).

Figure 80. Comparison of Cfd Unsteady Cp (Blue) and Approximated with Two-level Model (Red) for Mode 3, Mach 12, Amplitude +3mm, Frequency of 160 Hz
(a) $t = 2.5 \times 10^{-5} \text{ s}$ (b) $t = 0.00025 \text{ s}$ (c) $t = 0.00052 \text{ s}$ (d) $t = 0.00077 \text{ s}$

CHAPTER 3

PISTON THEORY BASED SURROGATE MODEL FOR HYPERSONIC FLOW:

STOCHASTIC MODEL

From Chapter 2, it is concluded that the coefficient of pressure C_p can be closely represented with the two-level composite model combining both a local cubic slope component and a global ARMA-based component. This model is the simplest model satisfactory among the tested options. This model also gives the flexibility of modelling the epistemic uncertainty by randomizing the parameters of either local, ARMA or both components. Another approach to account for epistemic uncertainty would have been to introduce a random additive term to the C_p obtained from Chapter 2. The challenge in this approach is to model this term so that all generated samples of the C_p are smooth and exhibit features similar to those of the mean model. The introduction of uncertainty in the system, i.e., in the parameters of the local and global models avoid this difficulty is thus performed here.

In the ensuing sections, uncertainty will be introduced in each of the components of the model. Section 3.2 discusses the randomization of the auto regressive coefficients of the composite model following the maximum entropy based method proposed by Wang et al. [9]. The randomization of the local model coefficients by two approaches and its related difficulties are covered in Section 3.3. Finally, Section 3.4 summarizes observations on random samples generated with random local and ARMA coefficients.

3.1 Contributions of the Local and Global component of the model

Before addressing the randomization aspects, it is worthwhile to quantify the relative magnitudes and behaviors of the two components of the composite model, i.e, the local and global parts. As shown in Figures 81-84, the local component is the major contributor to the approximation with the global component providing corrections to the phase difference and peak.

Figure 81. Comparative Plot of Local and Global Component of the Approximated C_p for Mode 1, Mach 12, Amplitude +4mm with Two-level Mode Independent Model

Figure 82. Comparative Plot of Local and Global Component of the Approximated Cp for Mode 2, Mach 12, Amplitude +4mm with Two-layer Mode Independent Model

Figure 83. Comparative Plot of Local and Global Component of the Approximated Cp for Mode 3, Mach 12, Amplitude +3mm with Two-layer Mode Independent Model

Figure 84. Comparative Plot of Local and Global Component of the Approximated Cp for Mode 4, Mach 12, Amplitude +1.5mm with Two-layer Mode Independent Model

3.2 Random Auto regressive coefficients

The randomization of the AR coefficients proceeds as follows consistent with the approach of [9]. First, the reflection coefficients are derived from the autoregressive parameters of the mean model (i.e., the model developed in Chapter 2). Their reflection coefficients are then randomized following the maximum entropy approach developed in [9] which ensures that the random AR component of the model is stable for all realizations. Finally each sample of the random AR model is reconstructed. Figure 85(a) shows the poles of 100 samples of random ARMA model at the uncertainty level $\delta = 0.1$. As expected all poles strictly lie within the unit circle ensuring stability.

Figure 85. (a) Poles of the Mean (Red) and Stochastic (Blue) ARMA Models (b) Autoregressive Coefficients of Mean (Red) and Stochastic (Blue) ARMA Models

Consider next the autoregressive coefficients. For the mean model, they are all zero except those associated with the local and global nodes, i.e., 1, 25, 50, 75 and 100. However, for the stochastic model all the coefficients are typically non-zero. Figure 85(b) shows the random auto-regressive coefficients corresponding to the non-zero mean ones. The random samples of C_p generated with the random AR coefficients are smooth and Figures 86– 89 show the 5th - 95th percentile band of 100 random C_p samples. For nodes 1 to 100, there is no band as those C_p values are constructed solely from the local component. The uncertainty band then grows fairly rapidly from node 101 onward remaining constant throughout most of the length of the beam. Note next that the CFL3D results do not belong in the band, they deviate most significantly from the mean model near the extremes

Figure 86. Uncertainty Band Obtained with Random Autoregressive Coefficients for Approximated C_p for Mode 1, Mach 12, Amplitude +4mm

Figure 87. Uncertainty Band for Reconstructed C_p with Random Autoregressive Coefficients for Mode 2, Mach 12, Amplitude +4mm

Figure 88. Uncertainty Band for Reconstructed C_p with Random Autoregressive Coefficients for Mode 3, Mach 12, Amplitude +3mm

Figure 89. Uncertainty Band for Reconstructed C_p with Random Autoregressive Coefficients for Mode 4, Mach 12, Amplitude +1.5mm

These observations suggest that this stochastic model is not well suited for the present problem, it should exhibit a larger band near the extreme to capture the CFL3D results but its band is constant.

3.3 Random local coefficient

The introduction of uncertainty on the local component is performed next. Two advantages of this approach are: (i) the local component resembles the shape of the CFL3D C_p , see Figures 81-84, and hence would be more effective in generating an uncertainty band that would capture the peaks/valleys and (ii) only the local component is used to construct the C_p of nodes 1 to 100 and thus the uncertainty would affect the C_p over the entire beam length. Following the maximum entropy concepts, each coefficient of the local component is considered as independent Gaussian random variable with mean equal to the value obtained in Chapter 2 and equal coefficient of variation, δ . Figures 90-93 shows the 5th-95th percentile uncertainty band constructed for each mode using 100 samples. It is seen from these figures that the uncertainty band width is proportional to the magnitude of the local approximation for the mean model. As a result, the band comes close to including the peak of the CFL3D C_p , not exactly due to the phase difference of the local component. The similarity of the uncertainty band and local model also leads to narrow or zero width uncertainty band at the nodes with small or zero values of the local component (for example, near the mid node of the beam for mode 1 in Figure 90). Several such instances are seen for higher modes due to the repeated crossing of the zero value by the local component. This behavior is clearly not acceptable.

Figure 90. Uncertainty Band for Reconstructed C_p with Random Local Coefficients for Mode 1, Mach 12, Amplitude +4mm

Figure 91. Uncertainty Band for Reconstructed C_p with Random Local Coefficients for Mode 2, Mach 12, Amplitude +4mm

Figure 92. Uncertainty Band for Reconstructed C_p with Random Local Coefficients for Mode 3, Mach 12, Amplitude +3mm

Figure 93. Uncertainty Band for Reconstructed C_p with Random Local Coefficients for Mode 4, Mach 12, Amplitude +1.5mm

An alternate approach to introducing the uncertainty in the local component can be formulated by first rewriting this approximation as a direct product of slope and coefficients, i.e., as

$$\begin{bmatrix} C_p^l(1) \\ \vdots \\ C_p^l(1001) \end{bmatrix} = \begin{bmatrix} a_1 & 0 & 0 & 0 \\ 0 & \cdot & 0 & 0 \\ 0 & 0 & \cdot & 0 \\ 0 & 0 & 0 & a_1 \end{bmatrix} \begin{bmatrix} \frac{dw}{dx} \end{bmatrix} + \begin{bmatrix} a_2 & 0 & 0 & 0 \\ 0 & \cdot & 0 & 0 \\ 0 & 0 & \cdot & 0 \\ 0 & 0 & 0 & a_2 \end{bmatrix} \begin{bmatrix} \left(\frac{dw}{dx}\right)^2 \end{bmatrix} + \begin{bmatrix} a_3 & 0 & 0 & 0 \\ 0 & \cdot & 0 & 0 \\ 0 & 0 & \cdot & 0 \\ 0 & 0 & 0 & a_3 \end{bmatrix} \begin{bmatrix} \left(\frac{dw}{dx}\right)^3 \end{bmatrix} + \begin{bmatrix} a_4 \\ \vdots \\ a_4 \end{bmatrix} \quad (15)$$

Next, note that the coefficient matrices are either positive or negative definite, symmetric, and their mean is prescribed. The maximum entropy nonparametric approach can then be applied to each of them in turn. Figure 94 shows the 5th - 95th percentile uncertainty band for this approach for mode 1, Mach 12, amplitude +8mm.

Figure 94. Uncertainty Band for Approximate C_p for Mode 1, Mach 12, Amplitude +8mm, Random Local Coefficient Matrices

Two undesirable features of this model are (i) the rather uniform band shown in Figure 94 but more important (ii) the fact that each sample of the generated C_p is “erratic”. Both effects are due to global coupling caused by non-zero non-diagonal elements in the random coefficient matrices. This behavior is clearly unacceptable physically. Note that global coupling in a local problem is further investigated in Chapter 4.

3.4 Random local and ARMA coefficients

The combination of the uncertainties on the local (the first approach of Figures 90-93) and auto-regressive components for an equal uncertainty level $\delta = 0.1$ appears to resolve the issues associated with each model separately as seen in Figures 95-98.

Figure 95. (a) Uncertainty Band for Reconstructed C_p with Random Local and AR Coefficients (b) Comparison of Random Samples with CFL3D C_p for Mode 1, Mach 12, Amplitude +4mm

Figure 96. (a) Uncertainty Band for Reconstructed C_p with Random Local and AR Coefficients (b) Comparison of Random Samples with CFL3D C_p for Mode 2, Mach 12, Amplitude +6mm

Figure 97. (a) Uncertainty Band for Reconstructed C_p with Random Local and AR Coefficients (b) Comparison of Random Samples with CFL3D C_p for Mode 3, Mach 12, Amplitude +3mm

Figure 98. (a) Uncertainty Band for Reconstructed C_p with Random Local and AR Coefficients (b) Comparison of Random Samples with CFL3D C_p for Mode 4, Mach 12, Amplitude +1.5mm

CHAPTER 4

ASSESSMENT OF NON-PARAMETRIC APPROACH ON LOCAL SYSTEM

In the previous chapter, the randomization of the local component of the two-level composite model by the maximum entropy nonparametric method was found to lead to global effects on the C_p . This behavior is similar to the one observed in [10].

To analyze this feature of the non-parametric approach, a one-dimensional beam in axial displacement discretized with 39 nodes is considered (see Figure 99). The corresponding stiffness matrix is tridiagonal and thus akin to the local component of the composite surrogate model which is diagonal. Following [4,5] the introduction of uncertainty is not carried out on the full stiffness matrix but rather on a reduced order model. Of interest, here are the questions:

- (1) Is the random response to a localized force still local as it is for the mean model?
- (2) Are the random reduced order models “consistent” with a (nearly) tridiagonal structure of corresponding random full order matrices?

Note that it is desired here to answer the above questions as functions of the size of the reduced order model (ROM).

Figure 99. System under Consideration (Length = 1m, $k_g = 40$ N/m, $k_c = 10$ N/m)

Consider first the second question. The reconstruction of a full order random stiffness matrix cannot be directly carried out because the modal matrix transforming full order to reduced order one is rectangular and hence not invertible. This non-uniqueness would appear at first to be an opportunity to impose the tridiagonal structure of the full order matrix. Denoting by N the size of this matrix, the number of its unknown nonzero elements is thus $2N - 1$ owing to symmetry and tridiagonal structure. Moreover, the number of known reduced order stiffness coefficients is $n(n+1)/2$ where n is the number of modes used in the reduction. So, as long as the latter number is smaller than the former one, it is potentially possible to identify a random full order tridiagonal matrix. Note however that the obtained stiffness values must be positive constraining further the problem.

Following is the procedure employed for constructing the “best” tridiagonal random model for an ensemble of the realizations of uncertain reduced order model

1. Calculate the mean reduced order stiffness matrix $\bar{K}_{ROM}$ for the desired ROM size,

i.e.,

$$\bar{K}_{ROM} = \phi' \bar{K} \phi$$

where ϕ is the matrix of the selected eigenvectors (in columns) of the mean full order model stiffness matrix $\bar{K}$

2. Randomize the reduced order matrix through the nonparametric approach, i.e.,

$$K_{ROM} = \bar{L}_{ROM} G \bar{L}_{ROM}^T$$

where $\bar{L}_{ROM}$ is obtained by Cholesky decomposition of $\bar{K}_{ROM}$, i.e.,

$$\bar{K}_{ROM} = \bar{L}_{ROM} \bar{L}_{ROM}^T$$

and G is a random symmetric positive definite matrix generated per the maximum entropy principle, see [4,5] for construction algorithm.

The aim is to reconstruct $N \times N$ matrices K such that $K_{ROM} = \phi' K \phi$ from the given samples of K_{ROM} . Denoting by k_g the 39-component vector of the ground spring values and by k_c the 38-component vector of the connecting spring values, the equation $K_{ROM} = \phi' K \phi$ can then be rewritten as $Ax = b$ where A is a matrix with $n(n+1)/2$ rows and 77 columns, x is the vector of the 77 variables k_g and k_c stacked, and b is the $n(n+1)/2$ vector of the lower triangular components of K_{ROM} .

3. Identify the matrix A from the eigenvector components in ϕ
4. Identify the vector b for each realization of K_{ROM}
5. Apply an optimization technique (discussed in Section 4.1) to find x
6. Generate the full order random matrix K from the identified vector x
7. Compute the constraint violation, $\varepsilon = \|Ax - b\|$ and the stiffness deviations,

$$\Delta = \|x - \bar{x}\| \text{ where } \bar{x} \text{ are the mean model values of } k_g \text{ and } k_c$$

4.1 Comparison of optimization techniques

In this section, the values of stiffnesses k_g and k_c corresponding to the “best” tridiagonal stiffness matrix are obtained by following optimization techniques:

1. Lagrange multiplier: The optimization is conducted to minimize the stiffness deviation Δ^2 while satisfying the constraint $Ax = b$. Using the Lagrange multiplier technique, the solution is found as

$$x = \bar{x} - A^T \lambda \quad (16)$$

where λ is vector of Lagrange multipliers, given by

$$A A^T \lambda = A \bar{x} - b \quad (17)$$

2. Lagrange multiplier + *fmincon* (MATLAB TOOLBOX): The negative optimum values obtained for k_g and k_c with the Lagrange multiplier approach are set to zero (positive values are unchanged) and given as initial conditions for an optimization with *fmincon* to minimize Δ with the constraint $Ax = b$ and positive values of x .
3. *fmincon*: This is the same as technique 2 except for the initial conditions which are here selected as the mean stiffnesses with positiveness imposed by representing the components of x as the square of search variables.
4. *fminsearch*: The constraint violation ε is minimized with the positiveness on stiffness values imposed as in technique 3.

The optimization was performed using various initial conditions for techniques 3 and 4; the best results (smallest constraint violation values) were obtained with the mean values of k_g and k_c as the starting point. In all cases, the same 100 samples of K_{ROM} were generated and used in the optimization process to obtain the corresponding full order stiffness matrices. Figures 100 and 101 give the 5th - 95th percentile uncertainty bands of the constraint violation errors (ε) and stiffness deviation (Δ) for the four methods for random ROM matrices with $\delta = 0.1$. The constraint violation for the 4 methods are

comparable with the Lagrange multiplier strategy providing a lower error for lower ROM sizes. However, a disadvantage of this method is that the constraint of positiveness of the elements of k_g and k_c cannot be applied. However, only few negative stiffnesses were found. Using either the *fmincon* or the *fminsearch* optimization method yielded very similar positive stiffness values with *fminsearch* being computationally much less demanding than *fmincon*.

Figure 100. Comparison of Constraint Violation (ε) 5th and 95th Percentile Band (Log-linear Scale) Vs. The Rom Size (“Nmodes”) for the Lagrange Multiplier (Yellow), Lagrange + Fmincon (Cyan), Fmincon (Red) and Fminsearch (Blue) Optimization Methods

The Lagrange multiplier methods also gives a lower stiffness deviation but with the above stated disadvantage of converging to negative (unfeasible) values (see Figure 101). Given

its computational efficiency and its overall lower constraint violation and stiffness deviation, the *fminsearch* optimization will be used in the sequel.

Figure 101. Comparison of Deviation of Elemental Stiffness (Δ) 5th and 95th Percentile Band (log-log Scale) with Lagrange Multiplier (Yellow), Lagrange + Fmincon (Cyan), Fmincon (Red) and Fminsearch (Blue)

4.2 Effects of Uncertainty Level

The tridiagonal structure of the desired full order stiffness matrix implies particular properties of the elements of the corresponding reduced order matrix which lead, when combined with the requirement of positive k_g and k_c spring values, to a series of inequalities they need to satisfy. For the mean model, the structure of the reduced order matrix is clear: it should be diagonal. For random configurations, the reduced order matrix structure is much more complex and it is clearly not met by the nonparametric approach. As the level of uncertainty is reduced, it may be expected that the deviations in stiffness from the mean model decrease thus permitting the satisfaction of the positiveness property

of the elements of k_g and k_c . To test this possibility, the value of δ , the uncertainty level in the nonparametric approach was set to 0.1, 0.001, and 10^{-5} . Shown in Figures 102-104 are the approximation error ($\|E\|_F$) vs. the ROM size for these values of δ .

Denoting by $A_{i,j}$ the i, j element of an arbitrary matrix A , the error matrix E is by its elements as

$$E_{i,j} = \frac{(\phi^T K \phi - K_{ROM})_{i,j}}{K_{ROM\ i,j}} \text{ for } i = j$$

(17)

$$E_{i,j} = \frac{(\phi^T K \phi - K_{ROM})_{i,j}}{\sqrt{K_{ROM\ i,i} K_{ROM\ j,j}}} \text{ for } i \neq j \quad (18)$$

The 3 curves are clearly very similar for ROM sizes larger than 6 with the error scaling with the value of δ . However, for smaller sizes some clear differences are visible. At the lowest value of δ , nearly zero errors are obtained for almost all samples for ROM sizes 2, 3, and 4 demonstrating the possibility to achieve the mapping back to the full order matrix at extremely low uncertainty levels. For the already low value of $\delta=0.001$, the lower part of the band (the 5th percentile) still gives zero error but most samples do not. Finally, for $\delta=0.1$, it is only for 2x2 ROM matrices that the mapping back is possible and only for very few samples, i.e., the 5th percentile.

The previous discussion demonstrates that, in general, the mapping back to the full order matrix cannot be done exactly, except at small/very small ROM sizes and very low uncertainty levels.

Figure 102. 5th - 95th Percentile Uncertainty Band of Approximation Error vs. ROM Size, $\delta = 0.1$

Figure 103. 5th - 95th Percentile Uncertainty Band of Approximation Error vs. ROM Size, $\delta = 0.001$

Figure 104. 5th - 95th Percentile Uncertainty Band of Approximation Error vs. ROM Size, $\delta = 0.00001$

4.3 Response Error

Since the random ROM matrices can be mapped back to full order matrices but those will not be tridiagonal, in general the coupling between masses that they imply extend further than to the direct neighbor. In fact, this coupling typically extend to all other masses so that a global coupling exists unlike in the mean full order system. Then, the response to a localized force on these random matrices is expected to produce a global response, i.e., even on nodes far away from the force at the contrary of the mean system. This expectation is fully confirmed by the responses shown in Figure 105 (b) and it may be observed that the global effect (i.e., the response away from the middle) is not decreasing as the ROM size is increasing.

Based on these observations, a response error that measures the non-local character of the response for a particular sample is defined here as deviation from response of mean system. That is, the

$$response\ error = \|\bar{u} - u\|$$

where $\bar{u}$ is the response calculated with mean-full stiffness matrix ($\bar{K}_{ROM}$)
and u is the response calculated with K_{ROM} samples

Then, shown in Figures 105(a)-107 are the uncertainty bands on the response error obtained for the values of δ used in the previous section, i.e., 0.1, 0.001, and 10^{-5} . These figures closely resemble Figures 102-104 indicating that the constraint violation error and the response error measure the same effects.

Figure 105. (a) Variation of Response Error 5th - 95th Percentile Band with Order of Reduction for Delta = 0.1 for Localized Force (Point Load at Mid Node) (b) Samples of Response with Random Reduced Order Stiffness Matrix

Figure 106. 5th - 95th Percentile Band of Response Error vs. ROM Size, $\delta = 0.001$ (Localized Force at the Middle Node)

Figure 107. 5th - 95th Percentile Band of Response Error vs. ROM Size, $\delta = 0.00001$
(Localized Force at the Middle Node)

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