Effect of Various Holomorphic Embeddings on Convergence Rate and Condition Number as Applied to the Power Flow Problem

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Outline

1. Introduction
   - Traditional Power Flow (PF) methods
   - Non-iterative PF methods
2. Basic Holomorphic Embedding (HE) Algorithm
   - Holomorphic Embedding
   - Analytic Continuation
   - Padé Approximation
   - Holomorphically Embedded Power Balance Equations
3. Proposed Modified HE Algorithm
   - Model of Three–Winding Transformer
   - Model of Phase-shifting Transformer
   - Modified Holomorphically Embedded Power Balance Equations (PBE’s)
4. Simulation Results
5. Conclusions and Future Work

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Introduction

- The power-flow (PF) study is used for power system operations, planning and expansion.
- The PF study algorithms are used to find the bus voltages and branch flows in an electric power system.
- The traditional iterative methods (such as Gauss-Seidel method, Newton-Raphson method, and Fast Decoupled Load Flow method) are widely used to solve the power flow problems, but sometimes are unreliable.

Introduction

Iterative methods (GS, NR, FDLF methods)	Non-iterative methods	Holomorphic Embedding Method
• They may produce voltage iterates that oscillate or diverge • Numerical performance is dependent on the choice of the initial voltage guess [1]-[3]	• Initial operating point is obtained through fixed-point numerical iteration process (Series Load Flow Method) [4] • Long execution time because of computational Complexity [5]	• It eliminates the uncertainty of solution existence • It's guaranteed to converge to the high-voltage solution when it exists • It unequivocally signals when no solution exists (precision limitations notwithstanding) [6]-[8]

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HE Algorithm: Holomorphic Embedding

- The Maclaurin series is generated when the Taylor series is expanded about zero:

$$f(\alpha) = \sum_{i=0}^{\infty} c[i]\alpha^i = \frac{f^{(i)}(\alpha)}{i!}\alpha^i, when\ |\alpha| < r$$

- Assume the voltage function is holomorphic, it can be expanded in a power series:

$$V(\alpha) = \sum_{i=0}^{\infty} V[i]\alpha^i = V[0] + V[1]\alpha + \cdots V[n]\alpha^n, when\ |\alpha| < r$$

- The complex conjugate of the voltage function *V*(*α*) can be expressed by two forms:

Form1: $V^*(\alpha^*) = V^*[0] + V^*[1]\alpha + \cdots V^*[n]\alpha^n$

Form2: $V^*(\alpha) = V^*[0] + V^*[1]\alpha^* + \cdots V^*[n](\alpha^*)^n$

- Form1 is used in the HE algorithm, as it is holomorphic.

HE Algorithm: Analytic Continuation

- Analytic continuation is used to extend the analytic domain of a function (in our case of interest) outside of the convergence region of the original analytic expression in the form of another analytic (holomorphic) function.
- Two examples are provided to illustrate this concept.

HE Algorithm: Analytic Continuation (con'd)

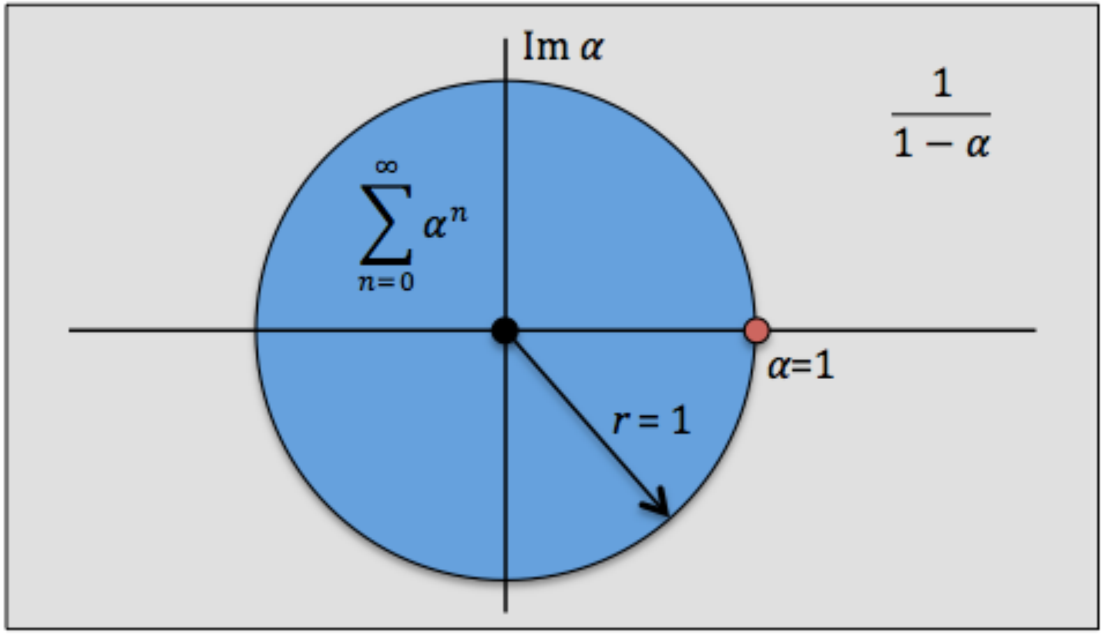
Ex1: The summation of a geometrical series is given as:

$$f_1(\alpha) = 1 + \alpha + \alpha^2 + \cdots \alpha^n + \cdots = \sum_{i=0}^{\infty} \alpha^i = \lim_{n\to\infty} \frac{1-\alpha^{n+1}}{1-\alpha}$$

When $|\alpha| < 1$, $\alpha^{n+1} \approx 0$. The equivalent function to $f_1(\alpha)$ is:

$$\tilde{f}_1(\alpha) = \frac{1}{1-\alpha}$$

The convergence radius for $f_1(\alpha)$ is the blue area. Whereas the converge radius for $\tilde{f}_1(\alpha)$ is the whole complex plane except the red point where α = 1.



HE Algorithm: Analytic Continuation (con'd)

Ex2: The integral of an exponential function is given as:

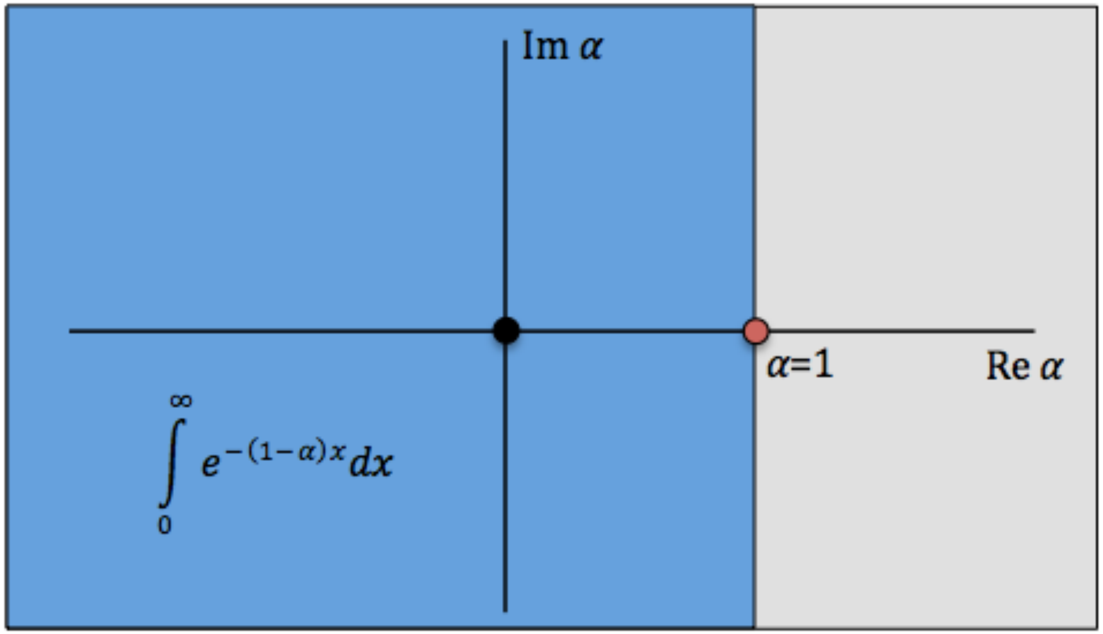
$$f_2(\alpha) = \int_0^{\infty} e^{-(1-\alpha)x} dx = \lim_{A\to\infty} \int_0^{A} e^{-(1-\alpha)x} dx = \lim_{A\to\infty} \frac{e^{-(1-\alpha)x}}{-(1-\alpha)}\Bigg|_0^A$$

$\lim_{x\to\infty} e^{-(1-\alpha)x} \approx 0$, when $\alpha < 1$.

The equivalent function to $f_2(\alpha)$ is:

$$\tilde{f}_2(\alpha) = \frac{1}{1-\alpha}$$

The convergence radius for $f_2(\alpha)$ is the blue area. Whereas the converge radius for $\tilde{f}_2(\alpha)$ is the whole complex plane except the red point where $\alpha = 1$.



HE Algorithm: Padé Approximation

- The maximal analytic continuation of a power series can be achieved by calculating its diagonal and near-diagonal Padé approximant [9].
- Two proposed approaches to calculate the Padé approximant are: direct matrix method and Viskovatov method (also known as the continued fraction method).

HE Algorithm: Padé Approximation (con'd)

- Direct Matrix Method [10]: the Padé approximant can be written as a rational function, which is a fraction of two polynomials:

$$V(\alpha) = [L/M]_\alpha = \frac{a[0] + a[1]\alpha + \cdots a[L]\alpha^L}{b[0] + b[1]\alpha + \cdots b[M]\alpha^M} + o(\alpha^{L+M+1}) = \sum_{n=0}^{\infty} V[n]\alpha^n$$

- By cross-multiplying the equation above and equating the coefficients of the same order of α^0, α^1,…, α^L, we get:

$$b[0]V[0] = a[0],$$
$$b[0]V[1] + b[1]V[0] = a[1],$$
$$b[0]V[2] + b[1]V[1] + b[2]V[0] = a[2]$$
$$\vdots$$
$$\sum_{i=0}^{L} b[i]V[L-i] = a[L]$$

HE Algorithm: Padé Approximation (con'd)

- By equating the coefficients of the α^{L+1}, α^{L+2},…, α^{L+M} to zero, we get:

$$b[M]V[L-M+1] + b[M-1]V[L-M+2] + \cdots b[0]V[L+1] = 0$$
$$b[M]V[L-M+2] + b[M-1]V[L-M+3] + \cdots b[0]V[L+1] = 0$$
$$\vdots$$
$$b[M]V[L] + b[M-1]V[L-M+2] + \cdots + b[0]V[L+M] = 0$$

- In matrix form:

$$\begin{bmatrix} V[L-M+1] & V[L-M+2] & V[L-M+3] & \cdots & V[L] \\ V[L-M+2] & V[L-M+3] & V[L-M+4] & \cdots & V[L+1] \\ V[L-M+3] & V[L-M+4] & V[L-M+5] & \cdots & V[L+2] \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ V[L] & V[L+1] & V[L+2] & \cdots & V[L+M] \end{bmatrix} \begin{bmatrix} b[M] \\ b[M-1] \\ b[M-2] \\ \vdots \\ b[1] \end{bmatrix} = - \begin{bmatrix} V[L+1] \\ V[L+2] \\ V[L+3] \\ \vdots \\ V[L+M] \end{bmatrix}$$

The coefficient matrix on the LHS is called the Padé matrix.

HE Algorithm: Holomorphically embedded PBE's

- Load bus:

$$\sum_{k=0}^{N} Y_{ik_{trans}} V_k(\alpha) = \frac{\alpha S_i^*}{V_i^*(\alpha^*)} - \alpha Y_{i_{shunt}} V_i(\alpha), i \in PQ$$

- Generator bus:

$$\sum_{k=0}^{N} Y_{ik_{trans}} V_k(\alpha) = \frac{\alpha P_i - jQ_i(\alpha)}{V_i^*(\alpha^*)} - \alpha Y_{i_{shunt}} V_i(\alpha), i \in PV$$

$$V_i(\alpha) V_i^*(\alpha^*) = 1 + \alpha\left[\left(V_i^{sp}\right)^2 - 1\right]$$

- Slack bus:

$$V_{slack}(\alpha) = 1 + \alpha\left(V_i^{sp} - 1\right), i \in slack$$

HE Algorithm: Germ Solution

The germ solution is obtained when $\alpha = 0$.

$$\begin{cases} \sum_{k=0}^{N} Y_{ik_{\downarrow}trans} V_k[0] = 0, i \in PQ \\ \sum_{k=0}^{N} Y_{ik_{\downarrow}trans} V_k[0] = -jQ_i[0]W_i^*[0], i \in PV \\ V_i[0]V_i^*[0] = 1, i \in PV \\ V_{slack}[0] = 1, i \in slack \end{cases}$$

$$\xrightarrow{\text{yields}} \begin{cases} V_i[0] = 1, i \in PQ \cup PV \cup slack \\ W_j[0] = 1, j \in PQ \cup PV \\ Q_k[0] = 0, k \epsilon PV \end{cases}$$

HE Algorithm: General Recursive Relation

- Load bus:

$$\sum_{k=0}^{N} Y_{ik_{trans}} V_k[n] = S_i^* W_i^*[n-1] - Y_{i_{shunt}} V_i[n-1], i \in PQ$$

- Slack bus:

$$V_{slack}[n] = \begin{cases} 1, n = 0 \\ V_i^{sp} - 1, n = 1, i \in slack \\ 0, n = 2,3,4, \ldots \end{cases}$$

HE Algorithm: General Recursive Relation (con'd)

- Generator bus:

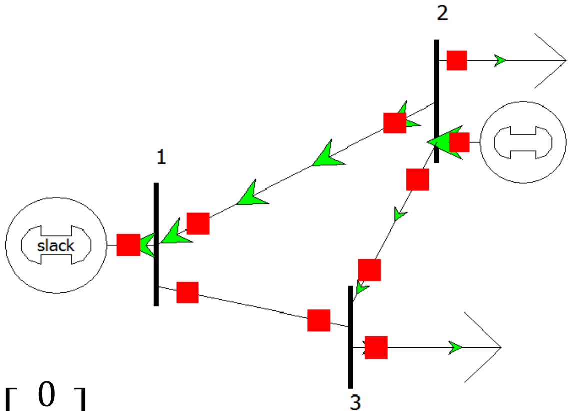
$$\sum_{k=1}^{N} Y_{ik_{trans}} V_k[n] = P_i W_i^*[n-1] - j\left(\sum_{l=0}^{n} Q_i[l] W_i^*[n-l]\right) - Y_{i_{shunt}} V_i[n-1]$$

$$V_{i_{\downarrow}real}[n] = \begin{cases} 1, n = 0 \\ \dfrac{\left(V_i^{sp}\right)^2 - 1}{2}, n = 1 \\ -\dfrac{1}{2}\displaystyle\sum_{l=1}^{n-1} V_k[l] V_k^*[n-l], n = 2,3,4,\ldots \end{cases}, i \in PV$$

HE Algorithm: Matrix Equation

- Take the three-bus system as an example, the matrix equation to calculate the coefficients for the n^{th} (n = 0, 1, 2, 3,…) term is:

$$
\begin{bmatrix}
1 & 0 & 0 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 & 0 & 0 \\
G_{21} & -B_{21} & 0 & -B_{22} & G_{23} & -B_{23} \\
B_{21} & G_{21} & 1 & G_{22} & B_{23} & G_{23} \\
G_{31} & -B_{31} & 0 & -B_{32} & G_{33} & -B_{33} \\
B_{31} & G_{31} & 0 & G_{32} & B_{33} & G_{33}
\end{bmatrix}
\begin{bmatrix}
V_{1r}[n] \\ V_{1i}[n] \\ Q_2[n] \\ V_{2i}[n] \\ V_{3r}[n] \\ V_{3i}[n]
\end{bmatrix}
=
\begin{bmatrix}
\delta_{n0} + \delta_{n1}(V_{slack} - 1) \\
0 \\
re\left\{P_2 W_2^*[n-1] - j\left(\sum_{l=1}^{n-1} Q_2[l] W_2^*[n-l]\right) - Y_{2_{shunt}} V_2[n-1]\right\} \\
im\left\{P_2 W_2^*[n-1] - j\left(\sum_{l=1}^{n-1} Q_2[l] W_2^*[n-l]\right) - Y_{2_{shunt}} V_2[n-1]\right\} \\
re\{S_3^* W_3^*[n-1] - Y_{3_{shunt}} V_3[n-1]\} \\
im\{S_3^* W_3^*[n-1] - Y_{3_{shunt}} V_3[n-1]\}
\end{bmatrix}
-
\begin{bmatrix}
0 \\ 0 \\ G_{22} \\ B_{22} \\ G_{32} \\ B_{32}
\end{bmatrix}
V_{2_{real}}[n]
$$



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Modified HE Algorithm: Holomorphically Embedded PBE's

- The basic embedding formula is:

$$\sum_{k=0}^{N} Y_{ik_{trans}} V_k(\alpha) = \frac{\alpha S_i^*}{V_i^*(\alpha^*)} - \alpha Y_{i_{shunt}} V_i(\alpha), i \in PQ$$

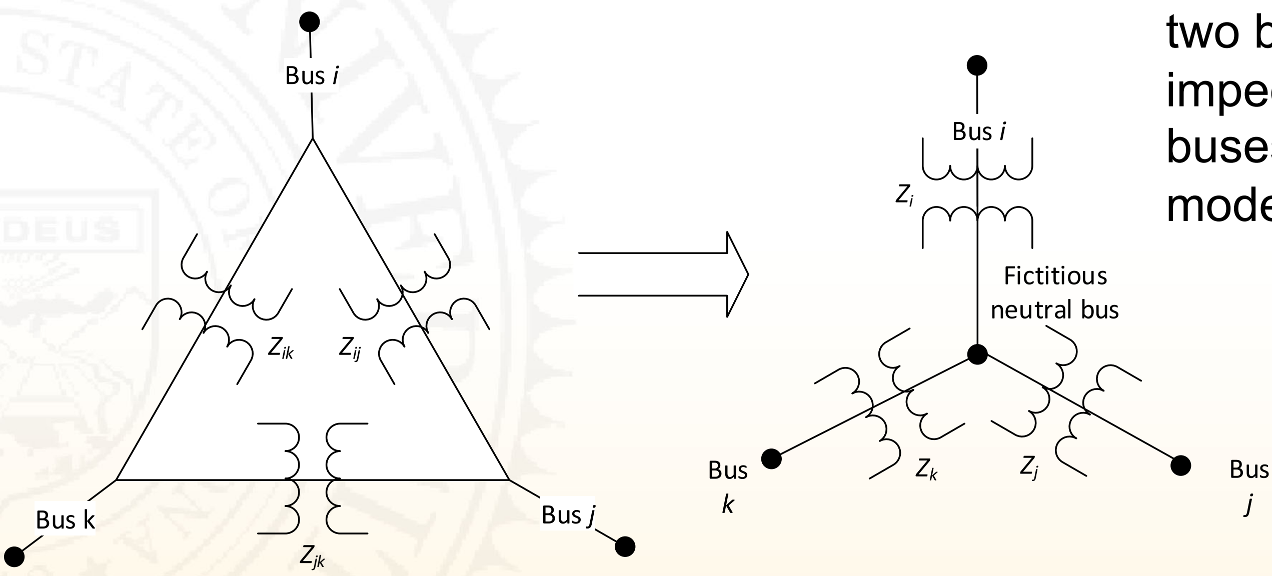
- What if the embedding formula becomes:

$$\sum_{k=0}^{N} Y_{ik_{trans}} V_k(\alpha) = \frac{\alpha^2 S_i^*}{V_i^*(\alpha^*)} - \alpha^2 Y_{i_{shunt}} V_i(\alpha), i \in PQ$$

- What will happen if we have components of both α and α^2?
- To examine if the HE algorithm can get a better numerical performance by including the component α^2, a modified formula $\beta\alpha+(1- \beta)\alpha^2$ is used to replace α in the basic embedded PBE's.
- The study parameter β ranges from 0 to 1.0.

Modified HE Algorithm: Three-winding Transformer

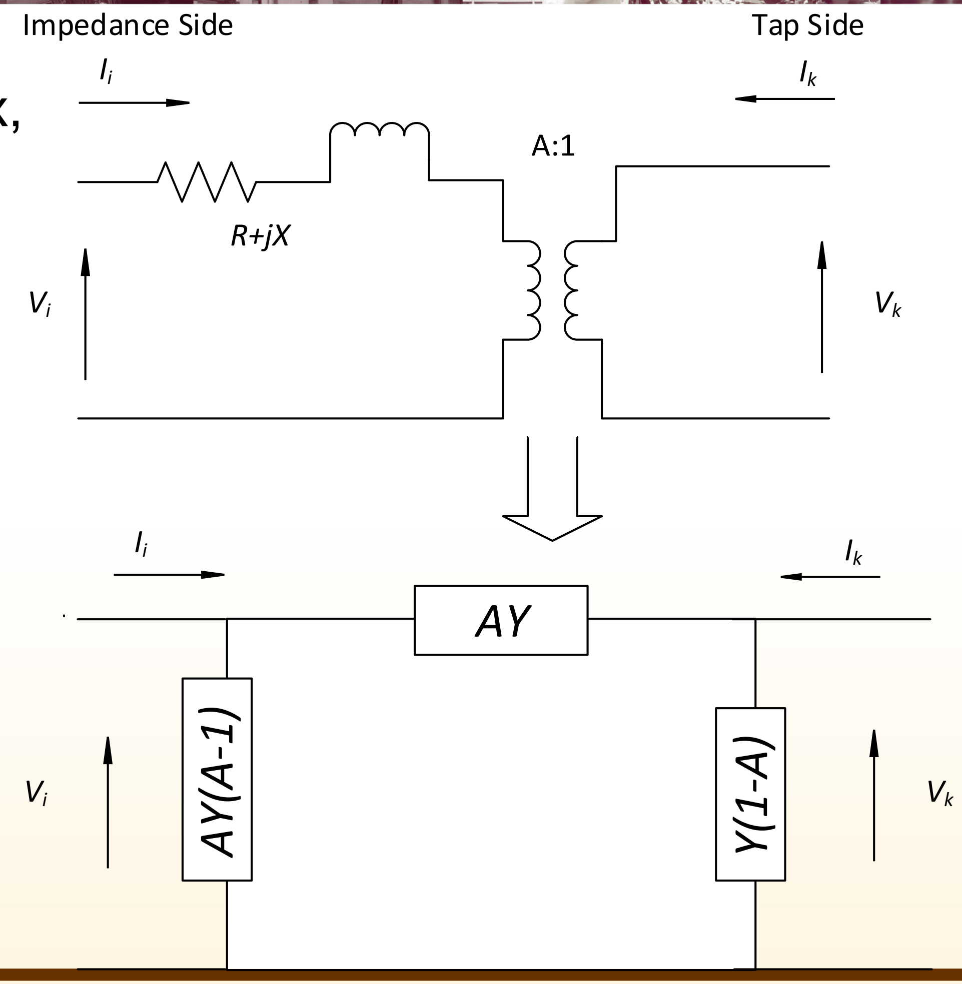
- The conversion of the equivalent circuit of three-winding transformer is:



Given impedances between any two buses (Z_{ij}, Z_{jk}, Z_{ik}), the impedances between every two buses in the converted wye model can be calculated as:

$$\begin{cases} Z_i = \dfrac{Z_{ij} + Z_{ik} - Z_{jk}}{2} \\ Z_j = \dfrac{Z_{ij} + Z_{jk} - Z_{ik}}{2} \\ Z_k = \dfrac{Z_{ik} + Z_{jk} - Z_{ij}}{2} \end{cases}$$

Modified HE Algorithm: Phase-shifting Transformer



The modified bus admittance submatrix, $\tilde{Y}'_{ik_{trans}}$, becomes:

$$\tilde{Y}'_{ik_{trans}} = \begin{bmatrix} Ay & -Ay\angle\theta_{shift} \\ -Ay\angle-\theta_{shift} & Ay \end{bmatrix}$$

In the modified HE algorithm, $\tilde{Y}'_{ik_{trans}}$ is divided into two parts as follows:

$$\tilde{Y}'_{ik_{trans}} = \tilde{Y}_{ik_{trans}} + \tilde{Y}_{ik_{unsym}}$$

where

$$\tilde{Y}_{ik_{unsym}} = \begin{bmatrix} 0 & Ay - Ay\angle\theta_{shift} \\ Ay - Ay\angle-\theta_{shift} & 0 \end{bmatrix}$$

$$\tilde{Y}'_{ik} = \begin{bmatrix} A^2y & -Ay\angle\theta_{shift} \\ -Ay\angle-\theta_{shift} & y \end{bmatrix}$$

Modified HE Algorithm: Holomorphically Embedded PBE's

- Load bus:

$$\sum_{k=0}^{N} Y_{ik_{trans}} V_k(\alpha) = \frac{[\beta\alpha + (1-\beta)\alpha^2] S_i^*}{V_i^*(\alpha^*)} - [\beta\alpha + (1-\beta)\alpha^2] Y_{i_{shunt}} V_i(\alpha) - [\beta\alpha + (1-\beta)\alpha^2] \sum_{k=0}^{N} Y_{ik_{unsym}} V_k(\alpha), i \in PQ$$

- Generator bus:

$$\sum_{k=0}^{N} Y_{ik_{trans}} V_k(\alpha) = \frac{[\beta\alpha + (1-\beta)\alpha^2] P_i - jQ_i(\alpha)}{V_i^*(\alpha^*)} - [\beta\alpha + (1-\beta)\alpha^2] Y_{i_{shunt}} V_i(\alpha) - [\beta\alpha + (1-\beta)\alpha^2] \sum_{k=0}^{N} Y_{ik_{unsym}} V_k(\alpha)$$

$$V_i(\alpha) V_i^*(\alpha^*) = 1 + [\beta\alpha + (1-\beta)\alpha^2] \left(\left| V_i^{sp} \right|^2 - 1 \right), i \in PV$$

- Slack bus:

$$V_{slack}(\alpha) = 1 + [\beta\alpha + (1-\beta)\alpha^2] \left(V_i^{sp} - 1 \right), i \in slack$$

- The germ solution is the same as that of the basic HE algorithm.

Modified HE Algorithm: General Recursive Relation

- Load bus:

$$\sum_{k=0}^{N} Y_{ik_{trans}} V_k[n]$$

$$= \beta S_i^* W_i^*[n-1] + (1-\beta) S_i^* W_i^*[n-2] - \beta Y_{i_{shunt}} V_i[n-1] - (1-\beta) Y_{i_{shunt}} V_i[n-2]$$

$$- (1-\beta) \boxed{\sum_{k=0}^{N} Y_{ik_{unsym}} V_k[n-2]} - \beta \boxed{\sum_{k=0}^{N} Y_{ik_{unsym}} V_k[n-1]}, i \in PQ$$

- Slack bus:

$$V_{slack}[n] = \begin{cases} 1, n = 0 \\ \beta\left(V_i^{sp} - 1\right), n = 1 \\ (1-\beta)\left(V_i^{sp} - 1\right), n = 2 \\ 0, n = 3,4,5,\ldots \end{cases}, i \in slack$$

Modified HE Algorithm: General Recursive Relation (con'd)

- Generator bus:

$$\sum_{k=1}^{N} Y_{ik_{trans}} V_k[n]$$
$$= \beta P_i W_i^*[n-1] - (1-\beta) P_i W_i^*[n-2] - \beta Y_{i_{shunt}} V_i[n-1] - (1-\beta) Y_{i_{shunt}} V_i[n-2]$$
$$- j\left(\sum_{l=0}^{n} Q_i[l] W_i^*[n-l]\right) - (1-\beta) \boxed{\sum_{k=0}^{N} Y_{ik_{unsym}} V_k[n-2]} - \beta \boxed{\sum_{k=0}^{N} Y_{ik_{unsym}} V_k[n-1]}, i \in PV$$

$$V_{i_{\downarrow} real}[n] = \begin{cases} 1, n = 0 \\ \dfrac{\beta(\left|V_i^{sp}\right|^2 - 1)}{2}, n = 1 \\ \dfrac{(1-\beta)(\left|V_i^{sp}\right|^2 - 1) - V_i[1] V_i^*[1]}{2}, n = 2 \\ -\dfrac{1}{2} \sum_{l=1}^{n-1} V_i[l] V_i^*[n-l], n = 3,4,5,\ldots \end{cases}, i \in PV$$

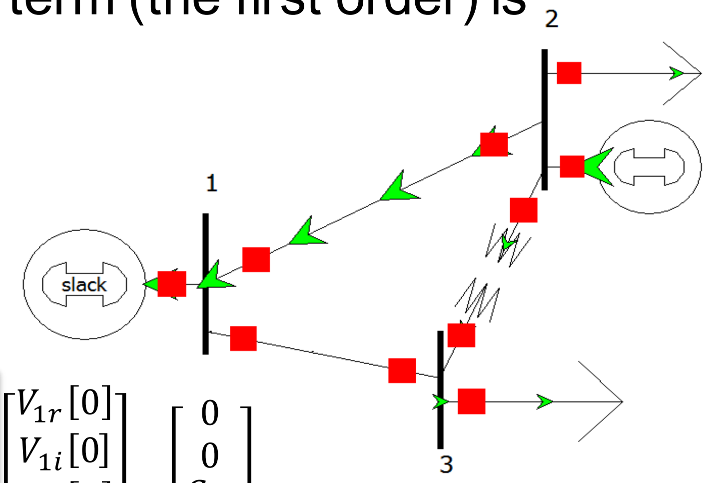
Modified HE Algorithm: Matrix Equation

The equation to compute the coefficients of α^1 term (the first order) is given as:

$$\begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ G_{21} & -B_{21} & 0 & -B_{22} & G_{23} & -B_{23} \\ B_{21} & G_{21} & 1 & G_{22} & B_{23} & G_{23} \\ G_{31} & -B_{31} & 0 & -B_{32} & G_{33} & -B_{33} \\ B_{31} & G_{31} & 0 & G_{32} & B_{33} & G_{33} \end{bmatrix} \begin{bmatrix} V_{1r}[1] \\ V_{1i}[1] \\ Q_2[1] \\ V_{2i}[1] \\ V_{3r}[1] \\ V_{3i}[1] \end{bmatrix}$$

$$= -\beta \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & G_{23_{unsym}} & -B_{23_{unsym}} \\ 0 & 0 & 0 & 0 & B_{23_{unsym}} & G_{23_{unsym}} \\ 0 & 0 & G_{32_{unsym}} & -B_{23_{unsym}} & 0 & 0 \\ 0 & 0 & B_{23_{unsym}} & G_{32_{unsym}} & 0 & 0 \end{bmatrix} \begin{bmatrix} V_{1r}[0] \\ V_{1i}[0] \\ V_{2r}[0] \\ V_{2i}[0] \\ V_{3r}[0] \\ V_{3i}[0] \end{bmatrix} - \begin{bmatrix} 0 \\ 0 \\ G_{22} \\ B_{22} \\ G_{32} \\ B_{32} \end{bmatrix} V_{2_{\downarrow real}}[0]$$

$$+ \begin{bmatrix} \beta(V_{slack} - 1) \\ 0 \\ re\{\beta P_2 W_2^*[0] - \beta Y_{2_{shunt}} V_2[0]\} \\ im\{\beta P_2 W_2^*[0] - \beta Y_{2_{shunt}} V_2[0]\} \\ re\{\beta S_3^* W_3^*[0] - \beta Y_{3_{shunt}} V_3[0]\} \\ im\{\beta S_3^* W_3^*[0] - \beta Y_{3_{shunt}} V_3[0]\} \end{bmatrix}, n = 1$$



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Simulation

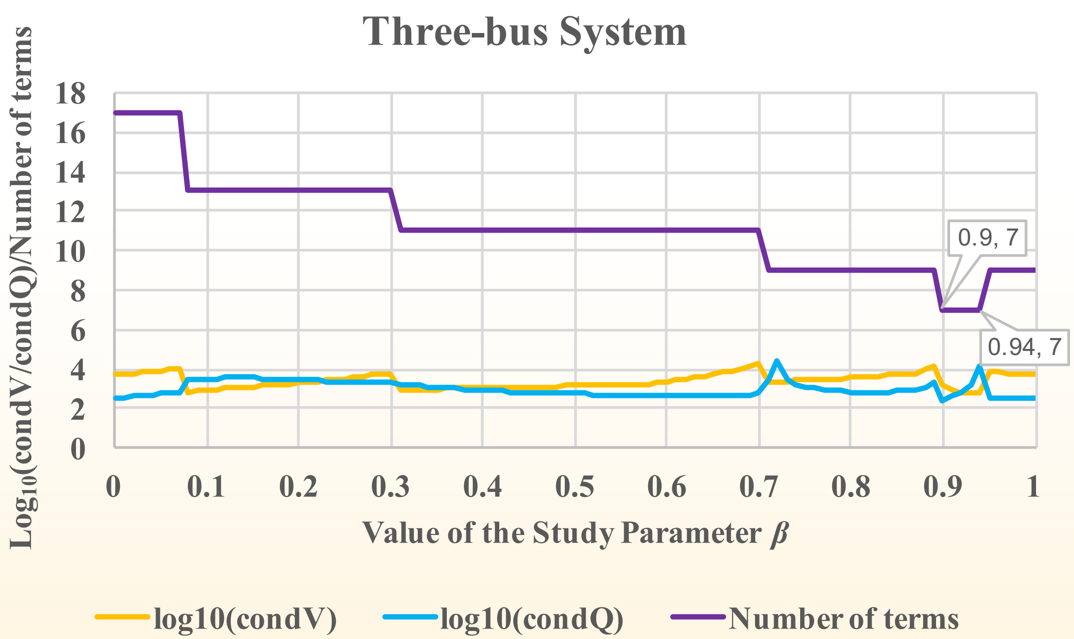
- Simulation tests are run by varying the value of the study parameter β in $\beta\alpha+(1-\beta)\alpha^2$ on (0,1] on the three-bus, the IEEE 14-bus, 118-bus, 300-bus and the ERCOT systems.
- The metrics for numerical performance are:
  - Number of terms required to get a converged solution.
  - Condition numbers of the Padé matrices built from the coefficients of the *V* and *Q* power series.

Simulation: Convergence Criteria

- The tolerance of the largest deviation of consecutive Padé approximant values for the bus voltage magnitude (namely $\Delta|V|$) is empirically chosen as 10^{-4} per unit value.
- The tolerance of the largest power mismatch among all buses is 0.1 MW/MVAR or 10^{-3} per unit value for ΔP and ΔQ, respectively.

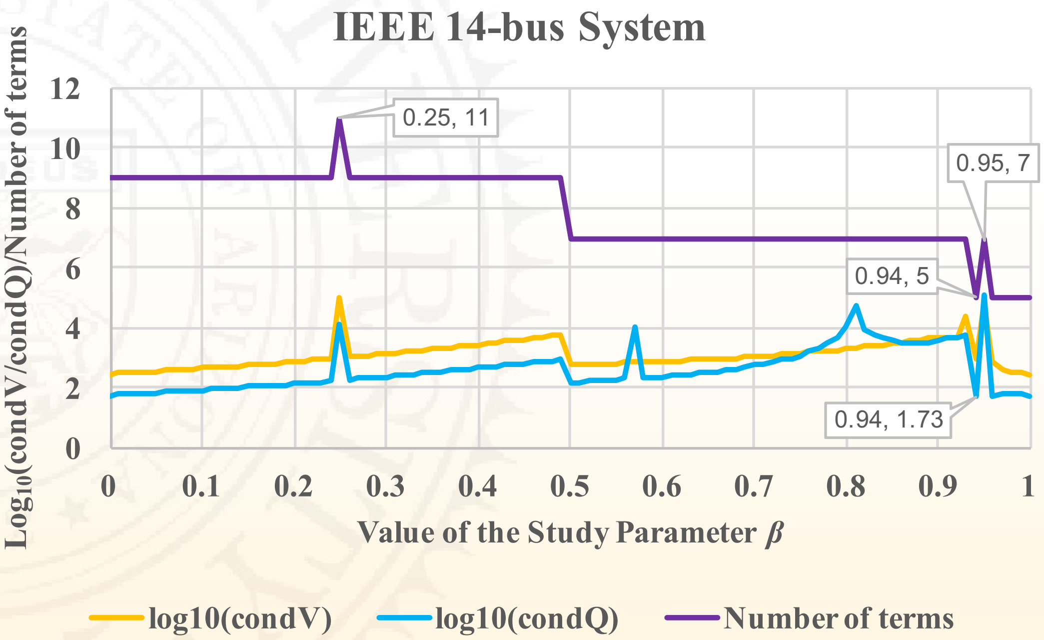
Simulation Results: Three-bus System

- The required number of terms to get a converged solution decreases from 17 to 9 terms as the embedding formula changes from α^2 to α.
- The discontinuities on the orange and blue curves correspond to those on the purple curve.
- When the value of β is between 0.90~0.94, the number of terms needed decreases to 7, which is fewer than the number required at $\beta = 1.0$.



Simulation Results: IEEE 14-bus System

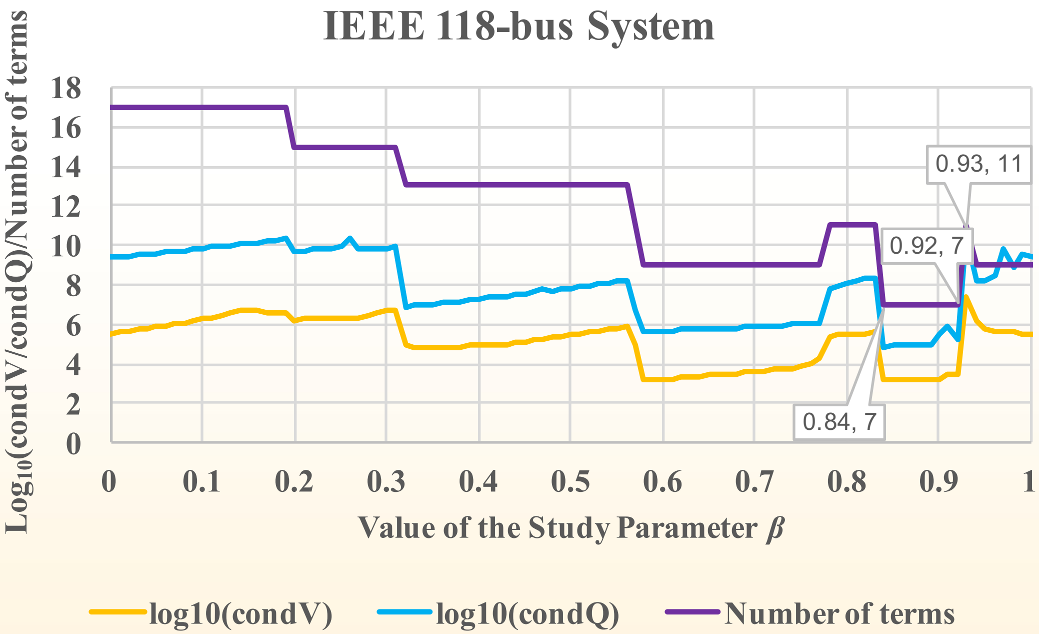
- The number of terms needed to converge decreases as the value of the study parameter β increases.
- The embedding formula α gives the optimal numerical performance.



- The flat segment on the purple curve is at β = 0.96~1.0.
- There are several jump points on the “number of terms” curve which correspond to jumps on the condition numbers curves.

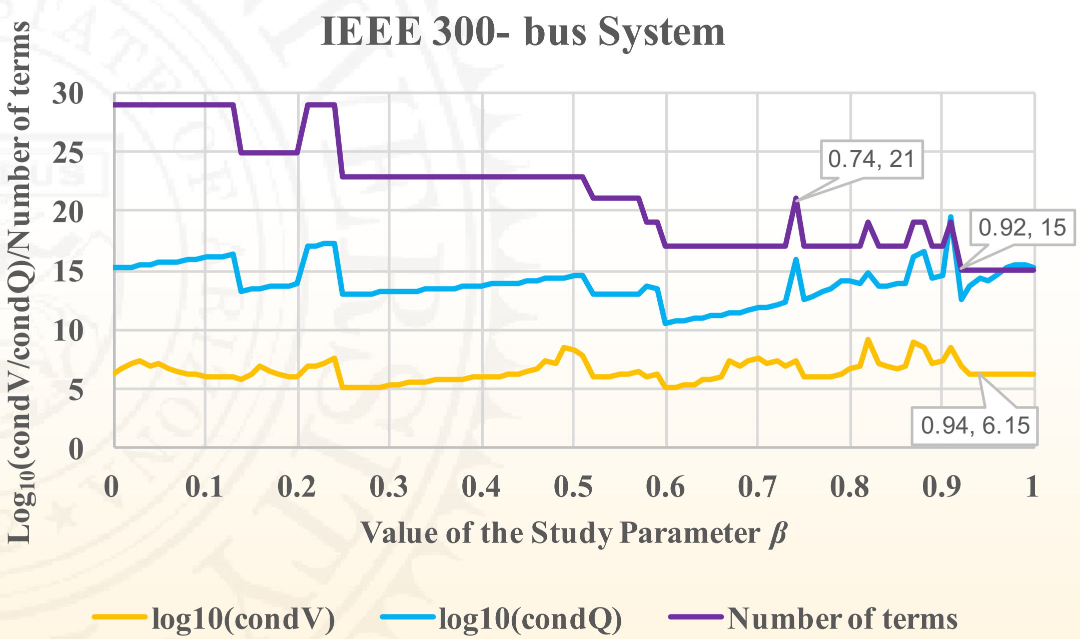
Simulation Results: IEEE 118-bus System

- The overall trend is: the numerical performance becomes better with the increase of β value.
- It can be observed that cond$V$ and cond$Q$ tend to decrease as the number of terms required decreases, and increases as the number of terms required increases.
- Compared to the numerical performance at β = 1, the numerical performance is better when the value of the study parameter β = 0.84~0.92.



Simulation Results: IEEE 300-bus System

- The condition numbers of the Padé matrix constructed from *Q* power series are larger than those of the *V* power series.
- The number of terms required to get the converged solution remains the same in the range: β = 0.92~1.0.



- This shows the existence of an alternative embedding formula, which is capable of giving a converged solution for the system with the same number of terms of the power series of $V(\alpha)$.

Simulation Results: ERCOT System

- The solution obtained from the HE algorithm with embedding formula α alone was also validated against the result from PowerWorld.

Table 1 Result Comparison between HE Method with α embedding and NR Method Using MATPOWER and PowerWorld

Tools	Using MATPOWER		Using PowerWorld	
The largest absolute difference on	PU Volt	Angle (Deg)	PU Volt	Angle (Deg)
	4.45E-06	5.34E-04	4.44E-04	6.65E-01

- However, the HE algorithm did not obtain a converged solution when a nonzero α^2 was introduced into the embedding formula.
- Instead, significant oscillations occurred on the maximum real and reactive bus power mismatches as well as on the largest voltage magnitude deviation.

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   - Traditional Power Flow (PF) methods
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2. Basic Holomorphic Embedding (HE) Algorithm
   - Holomorphic Embedding
   - Analytic Continuation
   - Padé Approximation
   - Holomorphically Embedded Power Balance Equations
3. Proposed Modified HE Algorithm
   - Model of Three–Winding Transformer
   - Model of Phase-shifting Transformer
   - Modified Holomorphically Embedded Power Balance Equations
4. Simulation Results
5. Conclusions and Future Work

Conclusions

- As the value of β approaches 1.0, fewer terms are generally required for convergence.
- As the number of power series terms required to converge to a solution increases, cond$V$ and cond$Q$ also increase.
- The Padé matrix built from Q power series coefficients is more poorly conditioned for larger systems.
- Even though the optimal value of β is system dependent, it seems to lie between 0.8~1.0 based on the observation on the three-bus, the IEEE 14-bus, 118-bus and 300-bus systems.
- Nevertheless, the modified HE algorithm presented here, when incorporating a nonzero α^2 component in the embedding formula, was unable to obtain a converged solution for the ERCOT system.
- The original α-only embedding formula presented in this work, seems to be a good choice for the embedding formulation when numerical performance and simplicity are considered.

Contribution

- Generated sparse-based MATLAB code capable of reading PSS/E format data.
- The sparsity-based code was further generalized to include a study parameter β to study α-embeddings of the form $\beta\alpha + (1-\beta)\alpha^2$.
- Implemented models of three-winding transformer and phase-shifting transformer.
- Tested the algorithm on ERCOT system.

Future Work

- Detection of islands and isolated (or out-of-service) bus in the data file
- DC line Model
- Further improvement on the bus-type switching subroutine
- Multiple-precision based algorithm in *C* programming

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Thank you

Questions?

Execution Time Comparison

Table of Execution Time Comparison Among Different Systems with Various Embedding Formulae

System	Case	value of β	Embedding formula	Number of terms needed	Execution time(s)
Three-bus System	base	1.00	α	9	0.0045
	optimal	0.93	$0.93\alpha+0.07\alpha^2$	7	0.0032
	worst	0.001	α^2	17	0.0252
IEEE 14-bus System	base	1.00	α	7	0.0079
	optimal	0.94	$0.94\alpha+0.06\alpha^2$	7	0.0078
	worst	0.25	$0.25\alpha+0.75\alpha^2$	11	0.0262
IEEE 118-bus System	base	1.00	α	9	0.1570
	optimal	0.85	$0.85\alpha+0.15\alpha^2$	7	0.0978
	worst	0.09	$0.09\alpha+0.91\alpha^2$	17	0.5166
IEEE 300-bus System	base	1.00	α	15	1.0093
	optimal	0.95	$0.95\alpha+0.05\alpha^2$	15	0.8178
	worst	0.22	$0.22\alpha+0.78\alpha^2$	29	3.3610

HE Algorithm: Holomorphic Function

- The complex conjugate of the voltage function $V(\alpha)$ can be expressed by two forms:

  Form1: $V^*(\alpha^*) = V^*[0] + V^*[1]\alpha + \cdots V^*[n]\alpha^n$

  Form2: $V^*(\alpha) = V^*[0] + V^*[1]\alpha^* + \cdots V^*[n](\alpha^*)^n$
- A holomorphic function must satisfy Cauchy-Riemann conditions
- An equivalent condition is that the Wirtinger derivative of the function $f(\alpha)$ with respect to the complex conjugate of α is zero, which is expressed as:

$$\frac{\partial f(\alpha)}{\partial \alpha^*} = 0$$

- Thus Form1 is used in the HE algorithm.

Go back

HE Algorithm: Padé Approximation

- The original power series $c^{(0)}(\alpha)$ is represented by the continued fractions:

$$c^{(0)}(\alpha) = c^{(0)}[0] + \cfrac{\alpha}{c^{(1)}[0] + \cfrac{\alpha}{c^{(2)}[0] + \cfrac{\alpha}{c^{(3)}[0] + \cdots}}}$$

HE Algorithm: Padé Approximation (con'd)

- The rational function is generated using a three-term recursion relation [11] as follows:

$$A_0(\alpha) = c^{(0)}[0],$$
$$A_1(\alpha) = c^{(0)}[0]c^{(1)}[0] + \alpha,$$
$$A_i(\alpha) = c^{(i)}[0]A_{i-1}(\alpha) + \alpha A_{i-2}(\alpha), i = 2,3,4,\ldots$$
$$B_0(\alpha) = 1,$$
$$B_1(\alpha) = c^{(1)}[0],$$
$$B_j(\alpha) = c^{(j)}[0]B_{j-1}(\alpha) + \alpha B_{i-2}(\alpha), j = 2,3,4,\ldots$$

- Then the Padé Approximant can be expressed as:

$$f(\alpha)_{[M/M]} = \frac{A_{2M}(\alpha)}{B_{2M}(\alpha)}, f(\alpha)_{[(M+1)/M]} = \frac{A_{2M+1}(\alpha)}{B_{2M+1}(\alpha)}$$

Go back

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Modified HE Algorithm: Matrix Equation (con’d)

The equation to compute the coefficients of higher-order ($n = 2, 3, 4, \ldots$) terms is given as:

$$
\begin{bmatrix}
1 & 0 & 0 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 & 0 & 0 \\
G_{21} & -B_{21} & 0 & -B_{22} & G_{23} & -B_{23} \\
B_{21} & G_{21} & 1 & G_{22} & B_{23} & G_{23} \\
G_{31} & -B_{31} & 0 & -B_{32} & G_{33} & -B_{33} \\
B_{31} & G_{31} & 0 & G_{32} & B_{33} & G_{33}
\end{bmatrix}
\begin{bmatrix}
V_{1r}[n] \\ V_{1i}[n] \\ Q_2[n] \\ V_{2i}[n] \\ V_{3r}[n] \\ V_{3i}[n]
\end{bmatrix}
$$

$$
= -\beta
\begin{bmatrix}
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & G_{23_{unsym}} & -B_{23_{unsym}} \\
0 & 0 & 0 & 0 & B_{23_{unsym}} & G_{23_{unsym}} \\
0 & 0 & G_{32_{unsym}} & -B_{23_{unsym}} & 0 & 0 \\
0 & 0 & B_{23_{unsym}} & G_{32_{unsym}} & 0 & 0
\end{bmatrix}
\begin{bmatrix}
V_{1r}[n-1] \\ V_{1i}[n-1] \\ V_{2r}[n-1] \\ V_{2i}[n-1] \\ V_{3r}[n-1] \\ V_{3i}[n-1]
\end{bmatrix}
- (1 - \beta)
\begin{bmatrix}
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & G_{23_{unsym}} & -B_{23_{unsym}} \\
0 & 0 & 0 & 0 & B_{23_{unsym}} & G_{23_{unsym}} \\
0 & 0 & G_{32_{unsym}} & -B_{23_{unsym}} & 0 & 0 \\
0 & 0 & B_{23_{unsym}} & G_{32_{unsym}} & 0 & 0
\end{bmatrix}
\begin{bmatrix}
V_{1r}[n-2] \\ V_{1i}[n-2] \\ V_{2r}[n-2] \\ V_{2i}[n-2] \\ V_{3r}[n-2] \\ V_{3i}[n-2]
\end{bmatrix}
-
\begin{bmatrix}
0 \\ 0 \\ G_{22} \\ B_{22} \\ G_{32} \\ B_{32}
\end{bmatrix}
V_{2r}[n] +
\begin{bmatrix}
\delta_{n2}(1-\beta)(V_{slack}-1) \\ 0 \\ re\{known1[n]\} \\ im\{known1[n]\} \\ re\{known2[n]\} \\ im\{known2[n]\}
\end{bmatrix}
, n = 2, 3, 4, \ldots
$$

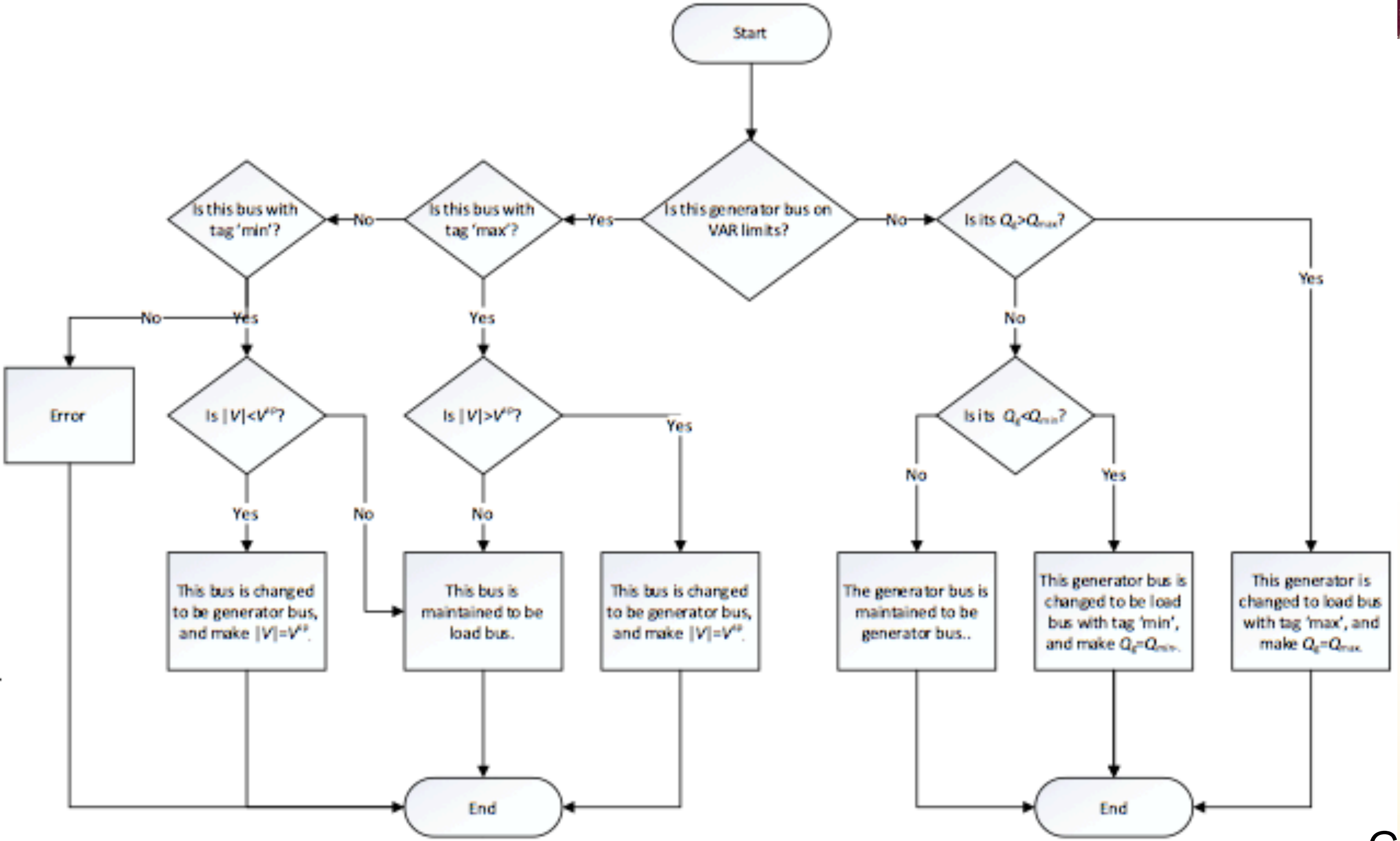
$$
\begin{aligned}
known1[n] &= \beta P_2 W_2^*[n-1] + (1-\beta) P_2 W_2^*[n-2] \\
&\quad - \beta Y_{2_{shunt}} V_2[n-1] - (1-\beta) Y_{2_{shunt}} V_2[n-2] \\
&\quad - j \left(\sum_{l=1}^{n-1} Q_2[l] W_2^*[n-l] \right), n = 2,3,4, \ldots
\end{aligned}
$$

$$
\begin{aligned}
known2[n] &= \beta S_3^* W_3^*[n-1] + (1-\beta) S_3^* W_3^*[n-2] \\
&\quad - \beta Y_{3_{shunt}} V_3[n-1] - (1-\beta) Y_{3_{shunt}} V_3[n-2], n = 2,3,4, \ldots
\end{aligned}
$$

Go back

$$
W[n] = -\frac{\sum_{l=1}^{n} V[l] W[n-l]}{V[0]}
$$

Bus-type Switching



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