

System Identification of Linear and Switching Regulators Using  
Switched Capacitor Correlator

by

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A Thesis Presented in Partial Fulfillment  
of the Requirements for the Degree  
Master of Science

Approved May 2015 by the  
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August 2015

## ABSTRACT

Power Management circuits are employed in almost all electronic equipment and they have energy storage elements (capacitors and inductors) as building blocks along with other active circuitry. Power management circuits employ feedback to achieve good load and line regulation. The feedback loop is designed at an operating point and component values are chosen to meet that design requirements. But the capacitors and inductors are subject to variations due to temperature, aging and load stress. Due to these variations, the feedback loop can cross its robustness margins and can lead to degraded performance and potential instability. Another issue in power management circuits is the measurement of their frequency response for stability assessment. The standard techniques used in production test environment require expensive measurement equipment (Network Analyzer) and time. These two issues of component variations and frequency response measurement can be addressed if the frequency response of the power converter is used as measure of component (capacitor and inductor) variations. So, a single solution of frequency response measurement solves both the issues. This work examines system identification (frequency response measurement) of power management circuits based on cross correlation technique and proposes the use of switched capacitor correlator for this purpose. A switched capacitor correlator has been designed and used in the system identification of Linear and Switching regulators. The obtained results are compared with the standard frequency response measurement methods of power converters.

## ACKNOWLEDGMENTS

I sincerely thank Prof. Bertan Bakkaloglu for giving me the opportunity to work with him and also for his guidance and support. Working with a patient and knowledgeable person like him has been a great learning experience.

I would like to thank Prof. Jennifer Kitchen for her encouragement and support of my interest in RF circuits. I also thank her for her time to be on my defense committee.

I sincerely thank Prof. Umit Ogras for taking time to be on my defense committee.

I also thank Prof. Armando Rodriguez for teaching two of the best courses I have taken. His courses were quite useful in completing this work.

I thank my friend Karthik Pappu for his encouragement.

I would like to thank Abhiram Mummareddy for his help with Verilog-A. I also thank all my colleagues at ASU.

I thank James Laux for his prompt help with computer issues.

I thank Jet Propulsion Laboratory, NASA for funding the research projects I worked for.

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## Chapter 1

### INTRODUCTION AND MOTIVATION

Power management circuits are employed almost in every electronic application ranging from avionics to microprocessors. These systems employ a variety of DC-DC converters. This work is focused on linear and switching regulators and proposes an integrated circuit to measure their frequency response (System Identification) in order to assess the changes in stability of these converters.

#### Power Management Circuits:

##### Linear regulators:

These are power converters which step down the voltage from that of the power input using a simple feedback loop employing a reference voltage and resistive feedback. Low drop out regulators (LDO) are one such class which are widely used in high efficiency, low noise, high PSR applications which demand decent line and load regulation. There are various types of LDOs and one of the classification that has become popular off late is that of LDOs with external capacitors and capacitor less LDOs. Though capacitor less LDOs are well suited for low power, battery powered and System on Chip (SoC) applications, higher power applications still require LDOs with external capacitors. This work describes system identification methods for LDOs with external capacitors.

##### Switching regulators (DC-DC):

Switching regulators (DC-DC) can either step up or step down the voltage and supply the required current while being highly efficient compared to the linear regulators. The drawback of switching converters over LDOs is that they are bulkier and noisy. These

too generally employ feedback techniques to achieve good load and line regulation and immunity from disturbances. The classification of switching converters can be made on various factors but the primary of them are buck (voltage step down), boost (voltage step up) and buck-boost (voltage step up and down). Buck converters are further classified based on the control loop they employ. This work discusses the system identification procedures for a buck converter employing voltage mode feedback.

#### Motivation for Health monitoring of power management systems:

Power management systems employ energy storage elements of capacitors and inductors many of which are subject to harsh load transients and variations due to temperature and aging.

Power management systems in general employ feedback loops for good line and load regulation and immunity from disturbances. This feedback loops are designed to operate at a given operating point and so designed using the nominal component values of inductors and capacitors with some parasitics taken to consideration. But the variation in the components can lead to degraded performance (lower efficiency, increased noise, lower disturbance rejection etc.), instability and even failure of the system.

This entails a need for online health monitoring of the power management systems.

#### Motivation for BIST in Power Management Circuits:

Built-In-Self-Test (BIST) circuits are employed to save production test time apart from being used in calibration procedures after the system is deployed. They can be very economic solutions if employed with DC-DC converters as the test procedures for them requires expensive equipment and is time consuming.

In this work a cross correlation based system identification technique is examined as a solution for health monitoring and BIST for power management circuits. This work uses an integrated analog correlator for this purpose. The power management circuits considered for illustrating the technique are a voltage mode buck converter and a low drop-out regulator (LDO)

#### System Identification as a technique for health monitoring and BIST:

System Identification is the procedure of extracting the characteristics of a system. Linear Time Invariant (LTI) systems can be characterized by their output responses and widely used of them are:

1. Impulse Response
2. Step Response
3. Frequency Response

So, by identifying the changes in the any of the above characteristics from the initial value, it can be inferred that the system properties have changed. The information regarding these observed changes can be quantified as a life time indicator (health monitoring) of the system and can be used to tune the system properties back to give the original intended performance.

System identification also serves the purpose of BIST as it gives the information regarding the system's performance.

Applications of Power Management Circuits requiring Health Monitoring:

Power management and distribution networks are crucial in many applications like server farms, aerospace applications and high performance GPUs and CPUs.

A typical server farm's DC power distribution network looks as follows [1]:

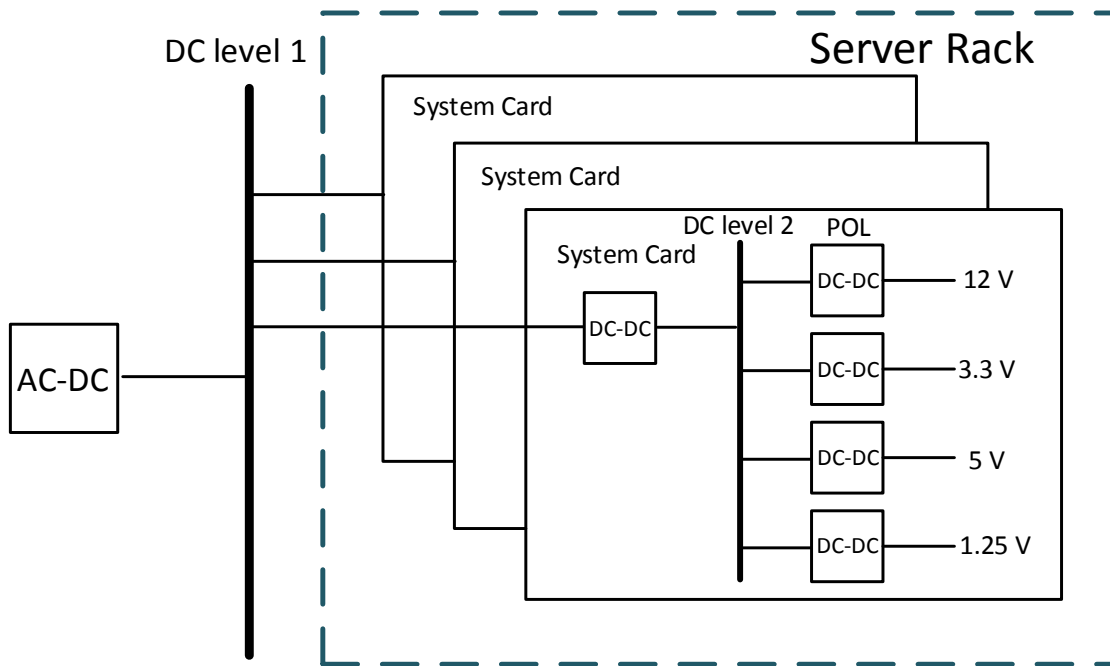


Figure 1: Power distribution network of a server farm [1]

As can be seen in the above figure, there are several system cards each employing quite a few Point of load (POL) DC-DC converters. These are buck converters supplying power locally.



The input filter could be the output of a DC-DC converter due to which the component values cannot be chosen arbitrarily, thereby constraining the whole design. The interaction between  $Z_o$  and  $Z_i$  should be appropriate while the switching occurs for stability. An example of a buck converter can be seen below:

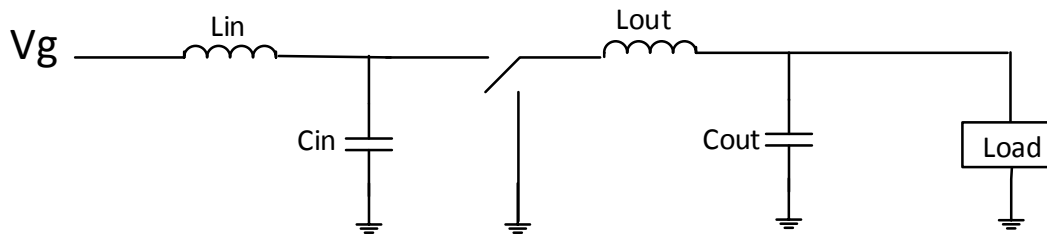


Figure 4: Input filter interaction in a buck converter [4] (controller and PWM not shown)

Here the interaction between the input filter and the output filter alters the stability criteria which in ideal case is only a function of the output filter.

So, the variation of the filter components can change the operating point for which the feedback loop is optimally designed. Monitoring of these component variations is helpful in predicting a failure due to instability.

Variations in the components of Power management circuits:

The reasons for variations in the components include temperature changes, aging effects and load stress. In space applications radiation effects also cause variations. These variations are observed primarily in capacitors as changes in their external series resistance (ESR) and capacitance itself. Both linear and switching regulators are sensitive to output capacitor variations

The component (inductors, capacitors, switching MOSFETs) variations either in the power stage or input filter of the DC-DC converter can degrade the performance, cause

instability leading to probable failure of the system. Typical inductors and capacitors have a nominal variation of +/- 10% in value. The other important issue is that of aging of the capacitors. This aging causes significant change in the value of the capacitance and the external series resistance (ESR). Typical variations due to aging are of on the order of 30% for capacitance [5], up to 200 % for ESR [6]. Temperature dependency of inductance and capacitance also causes variations. An overall variation of 20% in inductance, 50% in capacitance and 200% in ESR is possible [7]. There is also significant uncertainty due to decoupling capacitors as it is very likely that the values of the decoupling capacitors and the associated ESR is unknown during the converter design [7]. This uncertainty together with their variation also causes trouble. The variation in capacitance value and ESR affect the performance of LDO degrading its PSR, stability and noise immunity. The variations are typical numbers similar to the ones in the above section.

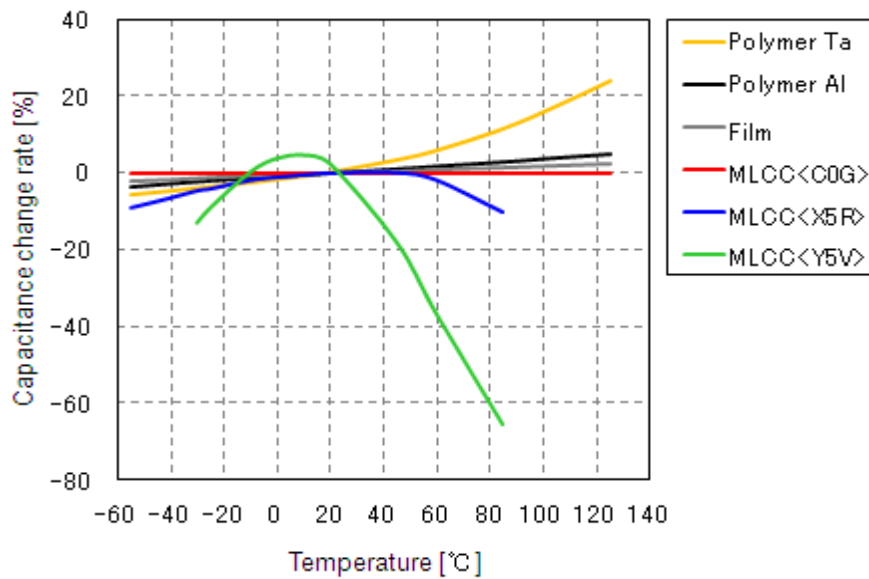


Figure 5: Effects of temperature on the capacitance value [8]

The variation of ESR can be found in various product data sheets [9].

The DC voltage at which the capacitor is being used also cause significant amount of variation [10]. So, it will be beneficial if the effect of these variations on the system stability are monitored and the necessary action of either replacing the part/circuit or tune the circuit to the new value. An analog correlator can serve this purpose with little overhead.

#### Requirement of BIST in Power management circuits:

Present methods in design, test or calibration setups for frequency response measurements of switching converters require a significant amount of hardware and time. The principle behind these measurements is Middlebrooks's technique of injecting a small signal sinusoidal perturbation into the loop without disturbing the DC operating point. This requires a transformer to be coupled at the appropriate place in the loop.

The method can be implemented relatively easily and accurately if a network analyzer is available [11]. If not a signal generator and oscilloscope are required [12].



Setup for loop gain measurement:

In a DC-DC converter Plant represents the switching FETS & Output filter.

Controller includes the Pulse width modulator and the compensator network.

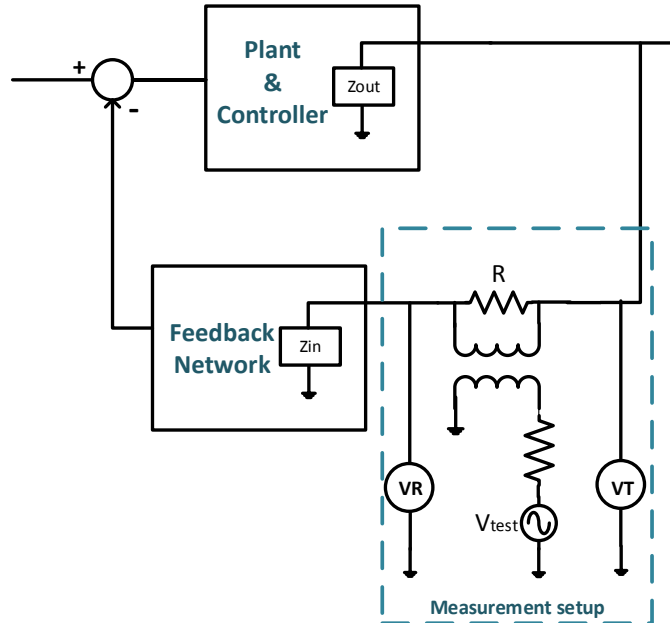


Figure 6: Loop gain measurement setup [11]

The principle is to sweep an injected sine wave's frequency and measure the corresponding output at the appropriate point. The ratio between output and input amplitudes gives the magnitude plot while the phase difference between them gives the phase plot. When plotted on a log scale these are bode frequency response plots revealing stability margins.

$V_{test}$  is the sine voltage sweep.  $VR$  and  $VT$  are the measured voltages. The injection of  $V_{test}$  and measurement of  $VR$  and  $VT$  can be done using a network analyzer. If network analyzer is not available the sinusoidal signal generator and an oscilloscope can be used.

$$\text{Loop Gain} = \left| \frac{VR}{VT} \right|$$

$$\text{Loop Phase} = \angle VR - \angle VT$$

If the hardware and time for measurement can be reduced by including a BIST circuit along with the converter or even having a simple integrated circuit that can measure the frequency responses with lesser overhead is a definite advantage. Towards this end an integrated solution using analog correlator is detailed in this work.

#### State of the Art in System Identification of Power management systems:

Over the past two decades system identification circuits for power management have been researched well and various techniques have been proposed. Few of them are presented to give an overview of the state of the art.

As per the author's knowledge no research has been published in the area of health monitoring or BIST for linear regulators. So, the rest of this chapter is focused on switching converters.

#### System identification techniques for switching converters:

Using Cross Correlation techniques:

In [13], cross correlation techniques have been used for system identification with the purpose of extracting open loop control to output transfer function and thereby monitor the stability of the system. Modeling the converter as a sampled data system and assuming linearity for small perturbations in steady state, white noise is injected into the duty cycle after opening the loop and the output of the converter is sampled using

an ADC. This sampled output is correlated with the time shifted PRBS to obtain the impulse response of the converter which when applied with DFT gives the frequency response,  $G(z)$  of control to output transfer function.

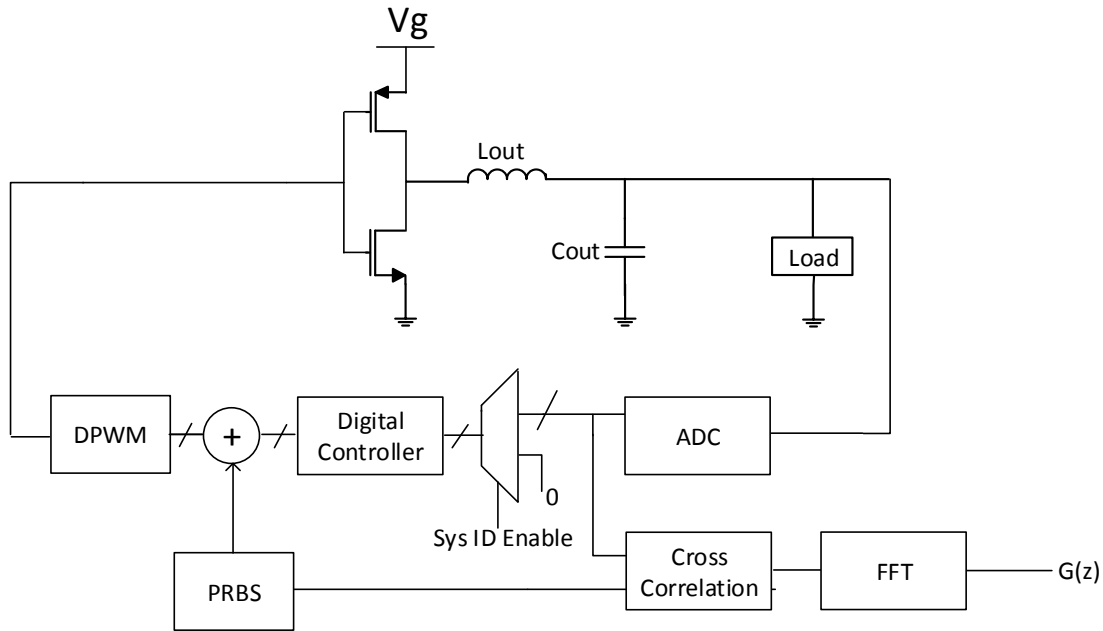


Figure 7: System Identification using cross correlation and FFT [13]

This method requires that the converter is in open loop when the system identification is being performed thereby imposing a limitation on the load and line regulations. If the application cannot tolerate open loop converter operation due to varying load profile, the converter needs to be brought off-line. Also, a high resolution ADC is required to sample the output of the noise (PRBS) excited converter. High resolution is required because the input PRBS amplitude can't be too, otherwise the assumption of small signal perturbation will not be valid and linearity is not hold. Another disadvantage is that owing to the sampled data nature and aliasing effects of ADC, the frequency response obtained is limited to  $1/10^{\text{th}}$  of the switching frequency. This will not be an issue if the loop bandwidth is within  $1/10^{\text{th}}$  of the switching frequency.

In [14], few improvements to the above methods were proposed to alleviate the issues of the non-idealities of the PRBS signal and phase uncertainty. Additionally estimation of stability without opening the loop was also proposed but this assumes steady state operation.

Using stability margin monitor:

In [15], a method for estimating the cross over frequency and phase margin of a switching converter while operating in closed loop was proposed. This method is based on Middlebrooks's approach for loop gain measurements but implemented digitally. This method requires sinusoidal signal generation and hence is more suited for converters with digital controller implementation. This method doesn't require the steady state operation for monitoring purpose.

## Chapter 2

### SYSTEM IDENTIFICATION USING ANALOG CROSS CORRELATOR

#### Impulse Response Extraction Using Cross Correlation:

A Linear Time Invariant [LTI] system's impulse response characterizes the system completely and when applied with Fourier transform it gives the frequency response of the system.

But as a delta function is difficult to generate and difficult to apply, an alternate method can be applied which gives the impulse response without actually applying an impulse/delta function. This is based on the similarity between the spectra of an impulse and white noise. Though they have same spectra they differ widely in time domain. So, some signal processing is required to extract an impulse by using white noise which is equivalent to an impulse in frequency domain but not in time domain. Correlation is that signal processing operation which serves the purpose. The math is as follows:

Let the system have an impulse response  $h(t)$ . If the system is excited by an input  $x(t)$ , then the output of the system,  $y(t)$  is given by:

$$y(t) = x(t) * h(t)$$

Now, if the output is correlated with the time delayed input,

$$g(\tau) = y(t) \otimes x(t-\tau)$$

$$\text{So, } g(\tau) = (x(t) \otimes x(t-\tau)) * h(t)$$

$$g(\tau) = x_{\tau} * h(t)$$

$$\text{If } x(t) = w(t), \text{ then } x_{\tau} = \delta(t)$$

Where  $w(t)$  is white noise and so has auto-correlation as a delta function/impulse

$g(\tau) = \delta(t) * h(t) = h(t)$ , by sifting property of delta functions

Now, by taking the Fourier transform of  $g(\tau)$ ,  $H(s)$  is obtained. The same theory applies to Linear Shift Invariant [LSI] systems from sampled data systems theory. Also the white noise can be approximated by a Pseudo Random Binary Sequence (PRBS), which can be easily generated using a few logic gates.

Cross correlation based impulse response extraction is a well-known method of system identification and it has been used in many fields of engineering, for example, acoustics [16]. It was used for switching converters in [13], [14], [15].

Most of the methods using cross correlation techniques first digitize the output of the system, and perform correlation digitally. But this may require a high resolution ADC, if the noise perturbation is to be kept small so as to operate the system in its linear range.

#### Motivation for Analog Correlator:

Instead of digitizing the output of the white noise excited system and then doing correlation digitally, if the digitization is performed after correlation the resolution requirements of the ADC can be relaxed as the correlator's output of impulse response has higher voltage levels in the regions of interest.

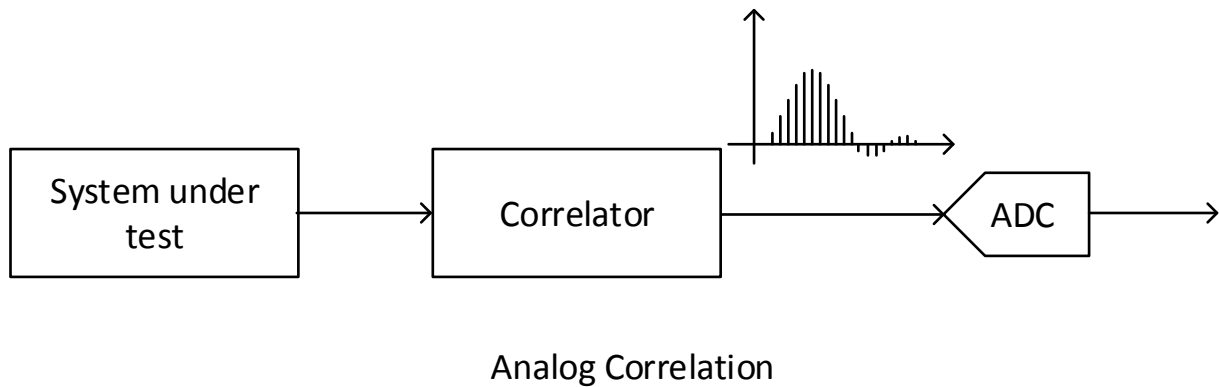
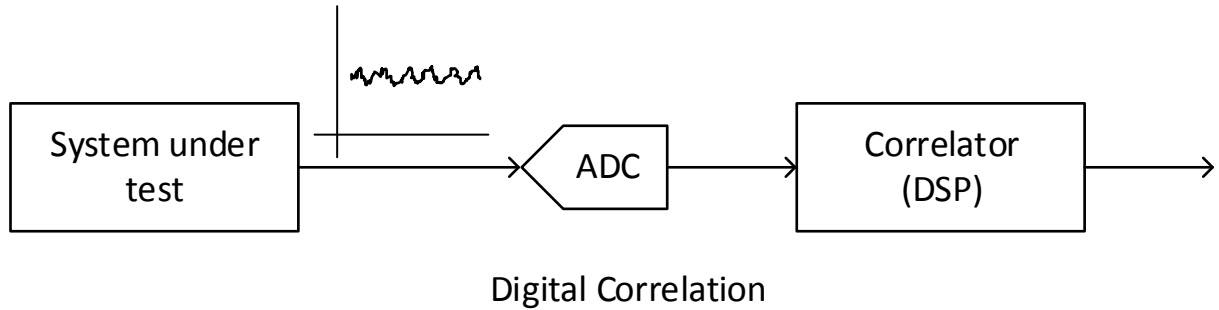


Figure 8: Comparison of Analog and Digital Correlation

Also, the resolution requirements are reduced as it need not sample the noise too accurately.

Implementation of Analog Correlator:

Analog correlator has been implemented using a resettable switched capacitor integrator. These type of circuits have been used in DS-CDMA applications [17]. The multiplication with PRBS can be implemented by manipulating the switch phases by directly controlling switches in the signal path, but this increases the number of switches seen the analog signal, thereby increasing the noise.

So, the multiplication is implemented by altering the phases of the switches using a multiplexer. Integration is implemented by the operational amplifier in negative feedback.

By choosing the inverting and non-inverting integrators based on the logic level of the PRBS signal, the multiplication with +/-1 is implemented.

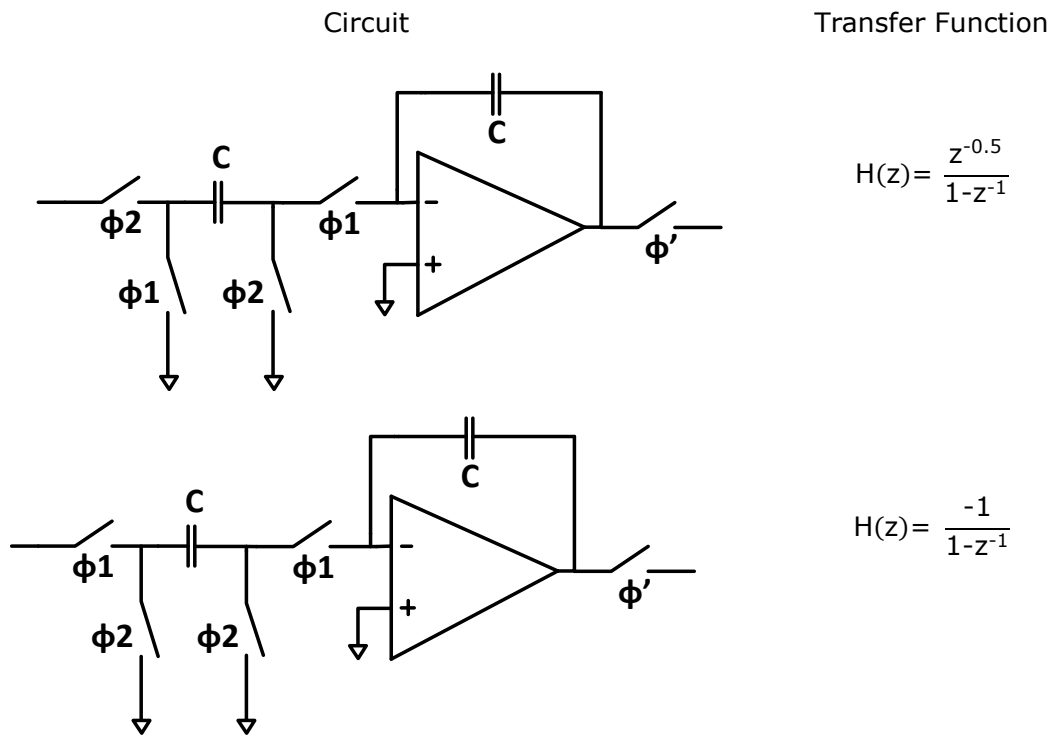


Figure 9: Switched capacitor integrators and their transfer functions [18]

As the output of the correlator is valid only on the falling edge of PRBS signal, the inverting integrator's transfer functions will also have a half cycle delay, thereby making the transfer functions differ only in sign.

The clock phases can be controlled by a multiplexer with select signal as the PRBS.



The implementation is shown below:

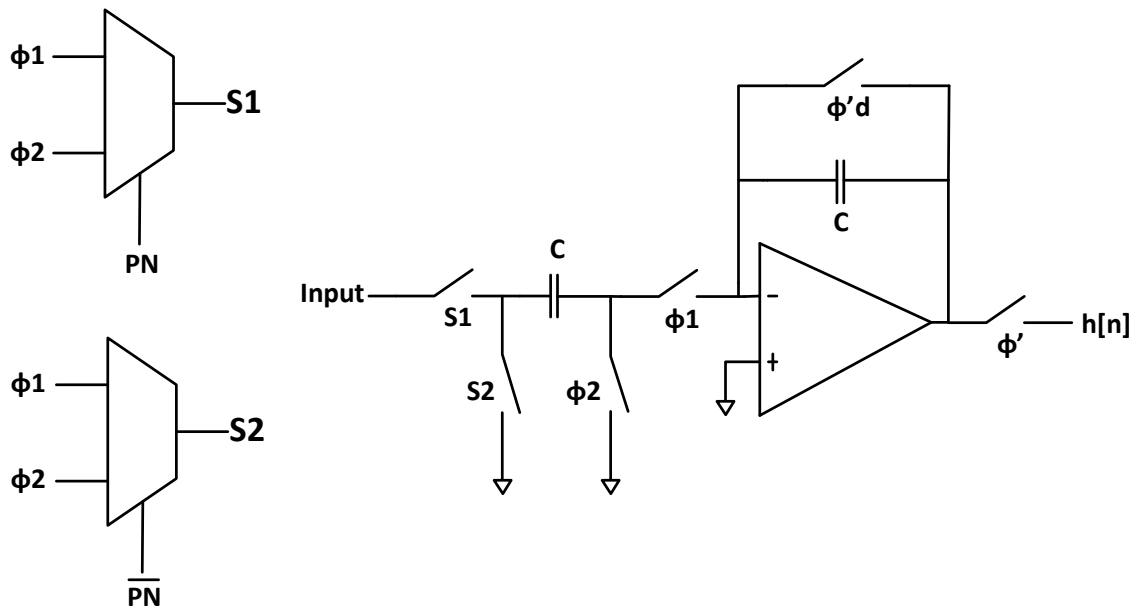


Figure 10: Implementation of basic analog correlator

- $\phi 1$  and  $\phi 2$  are non-overlapping clocks
- $\phi'$  is the phase that samples the output of correlator,  $h(t)$
- $\phi'd$  is the phase that resets the correlator after obtaining each sample of  $h(t)$

Specifications to consider while designing the correlator:

Unity gain frequency (UGF) of the operational amplifier: UGF of the OpAmp needs to support the transients due to switching while settling to the required precision within a specified time instant of the sampling cycle. As a rule of thumb, a UGF of 4-5 times the sampling frequency can be chosen.

Sampling frequency: This needs to be equal to twice pseudo noise sequence (PRBS) frequency as one cycle of sampling and one cycle of integration needs to be completed for each PN sequence cycle.

Noise: The noise floor of the correlator circuit should be lower than that of the minimum input signal, but this specification could easily be met as generally expected perturbations range above 1mV to give useful levels of impulse response.

Linearity: Linearity of the OpAmp is an important specification as output voltage can vary from values lower than common mode (undershoot in the impulse) to near supply voltage (peak of the impulse response) and the integration operation should be linear across this range.

DC offset: DC offset should be minimized so as to avoid the integrator saturating quickly, so that more dynamic range is obtained. This is achieved by using correlated double sampling technique [19]

Gain: Significantly high gain OpAmp is required for this application as a high DC gain translates to an ideal integration operation.

Power consumption: As this circuit is primarily a BIST circuit, lower power consumption is recommended, but as System ID mode is activated only occasionally, too much compromise need not be made as the linearity trades with power consumption.

### Design of Switched Capacitor Correlator:

An analog correlator with above considerations in design has been designed in Jazz 0.6  $\mu\text{m}$  process as follows:

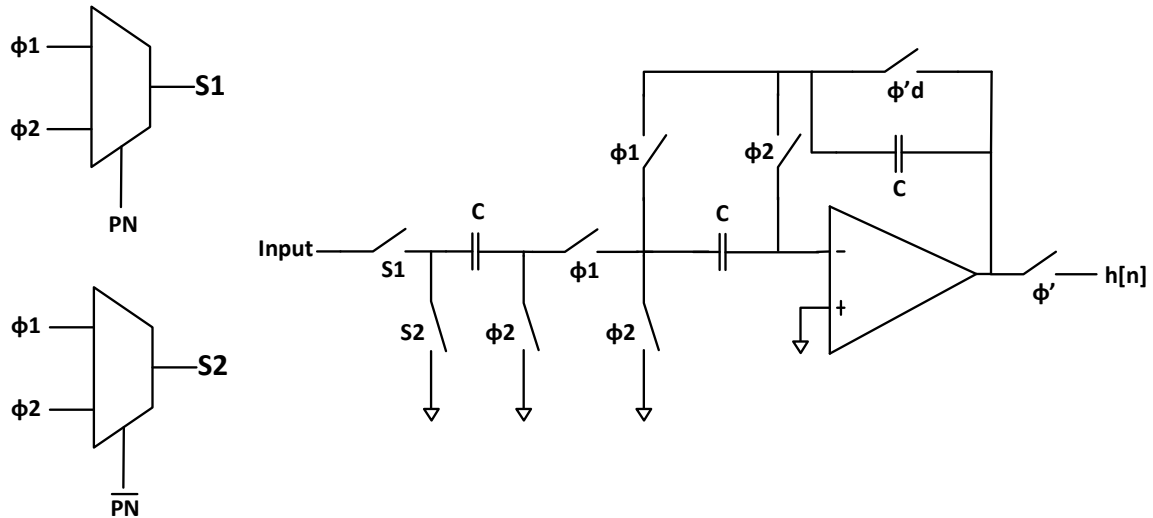


Figure 11: Analog Correlator employing Correlated Double Sampling [19]

Operational Amplifier specifications:

D.C Gain	97 dB
IM3	65.9 dB
Unity gain frequency	22 MHz
Quiescent Current at 3.3 V	170 $\mu\text{A}$

A symmetric OTA was used in the design:

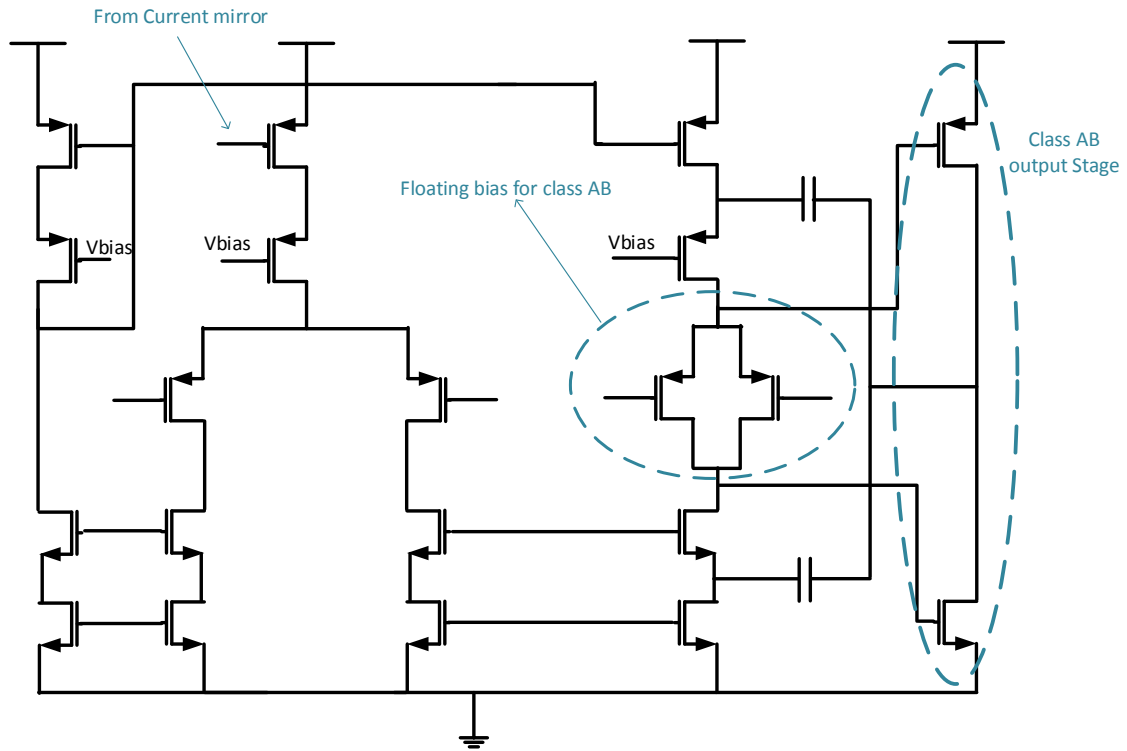


Figure 12: Operational amplifier used in the analog correlator

Layout of Correlator:

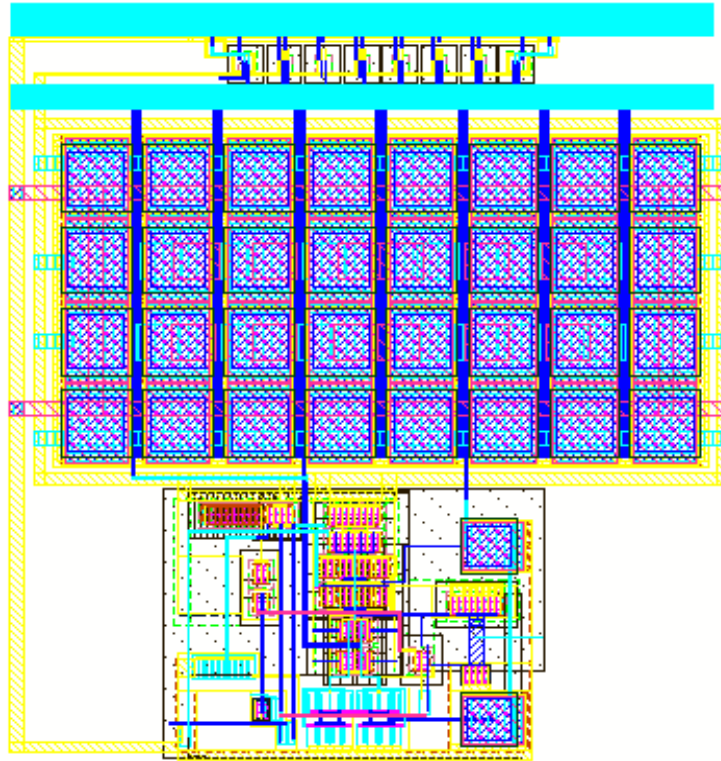


Figure 13: Layout of the analog correlator

### Application of Analog Correlator to an RLC Circuit:

An RLC filter was taken as a test circuit and correlation results on it were compared with AC analysis:

Test bench for Analog Correlator:

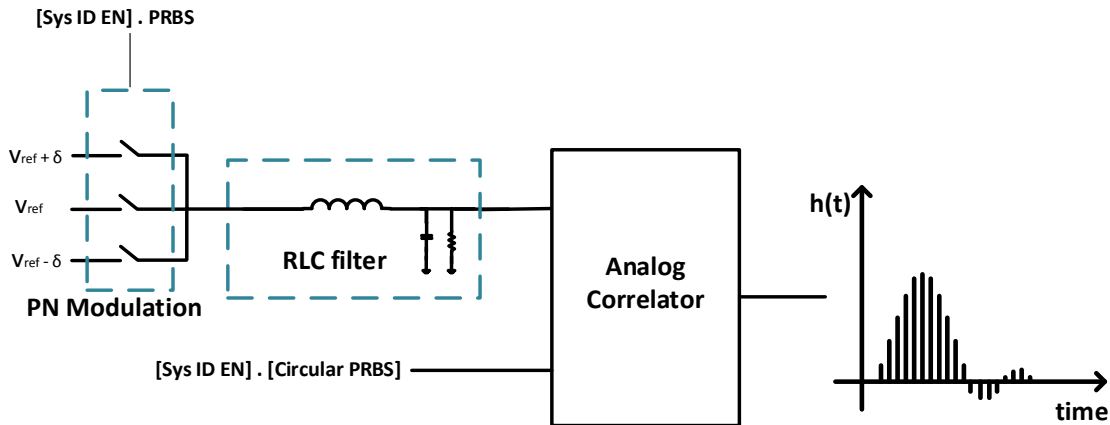


Figure 14: Test bench for Correlation Based Frequency Response Measurement of an RLC Filter

Output of the Correlator:

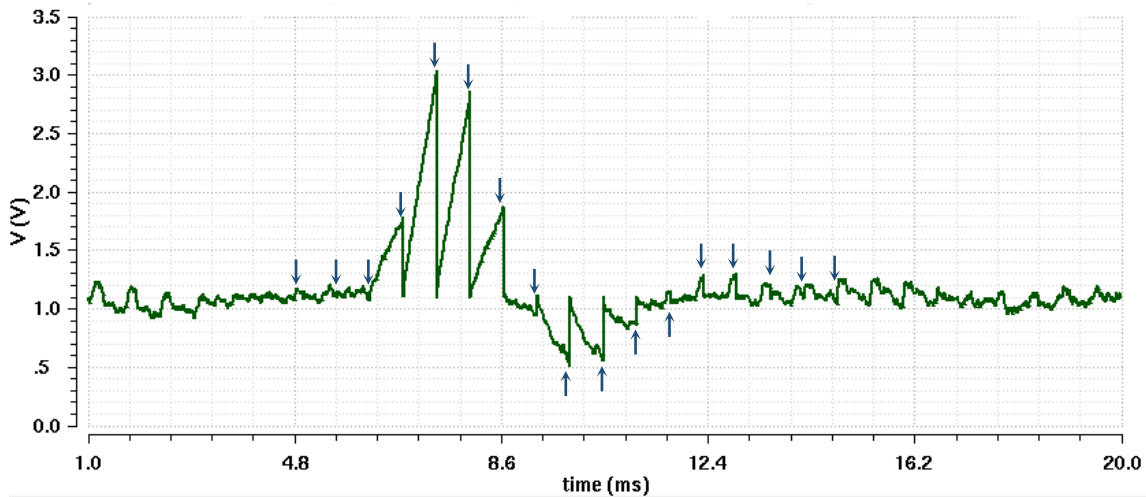


Figure 15: Output of the Analog Correlator

The output of the correlator needs to be sampled every  $f_{PRBS}^{-1} * N$  seconds,

Where  $f_{PRBS}^{-1}$  is time period of the PRBS and N is the number of bits in PRBS

Sampling instants are shown by the arrows on the on the figure.

This sampling is to ensure that N bits of PRBS are injected into the system. For N bits at the rate of  $f_{PRBS}^{-1}$  seconds, the total time will  $f_{PRBS}^{-1} * N$  seconds.

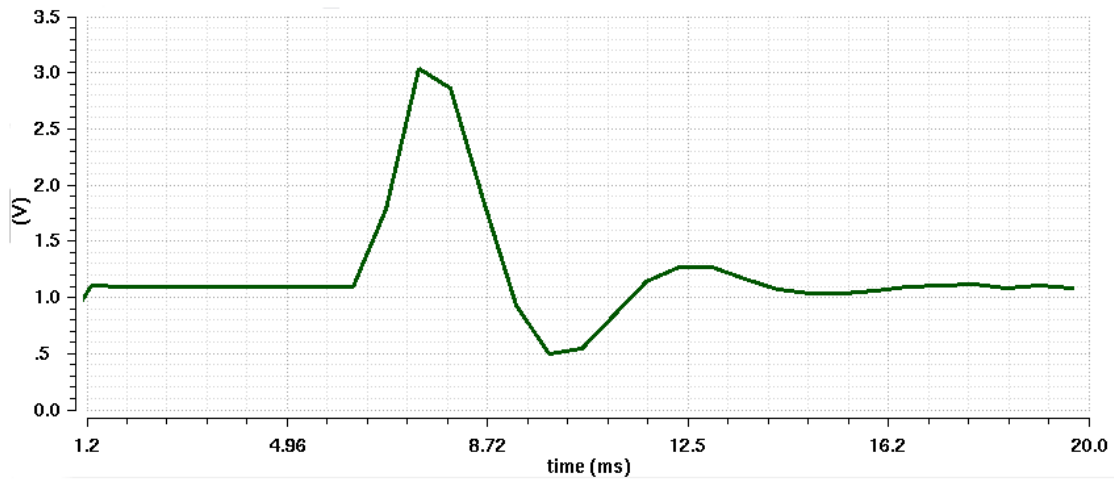


Figure 16: sampled (& reconstructed) output of the correlator

These samples are to be time compressed or frequency expanded before doing FFT to get the frequency response.

Comparison of AC analyses and Cross Correlation based methods:

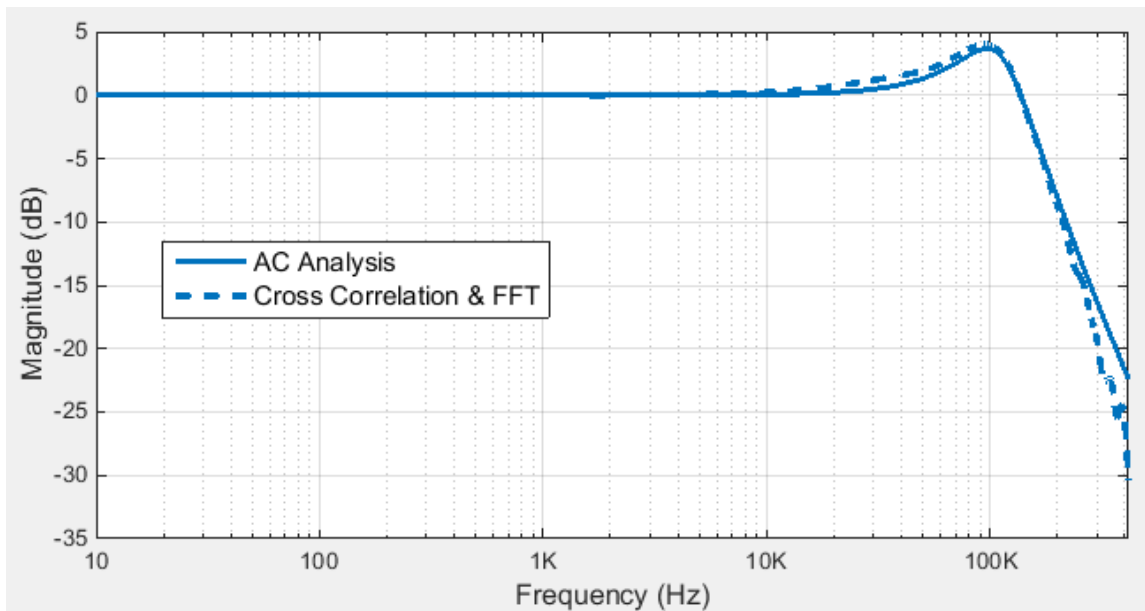


Figure 17: Frequency response of an RLC filter and comparisons

### Stability Monitoring of Feedback Systems using Impulse Response:

Though the above example was shown for a simple RLC filter the setup will be similar for closed loop systems. The difference is in the interpretation and analysis that follows the impulse and frequency response extractions. For a filter it the quality factor and cut-off are of most important parameters to be extracted.

On the other hand closed loop systems are generally characterized by their open loop parameters like open loop gain, phase margin, unity gain bandwidth etc. Extracting these from the obtained closed loop responses is difficult and sometimes not possible. But if the stability is the interest of extraction, then a characteristic of the closed loop response can used as a measure and it is the amount of peaking in the closed loop transfer. This peaking is quantified as Quality factor (proportional to the inverse of damping ratio) which is the difference between the peak magnitude in the frequency response plot and DC gain of the system. For second order systems, there is a direct relation between the open loop phase margin and either the closed loop quality factor or the damping ratio of the system [20].

For a closed loop second order system defined as,

$$H(s) = \frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2} = \frac{1}{\frac{s^2}{\omega_n^2} + \frac{1}{Q\omega_n} s + 1}$$

$\zeta$ -Damping ratio

$\omega_n$ - Undamped natural frequency

and, Quality factor,  $Q = \frac{1}{2\zeta}$



$$\text{Phase margin, PM} = \tan^{-1} \left[ \frac{2\zeta}{\sqrt{-2\zeta^2 + \sqrt{1+4\zeta^4}}} \right]$$

$$\text{In terms of } Q, \text{ PM} = \tan^{-1} \left[ \frac{\sqrt{2}}{\sqrt{(1+4Q^2)-1}} \right]$$

Plot of Phase margin, PM vs Damping factor,  $\zeta$  :

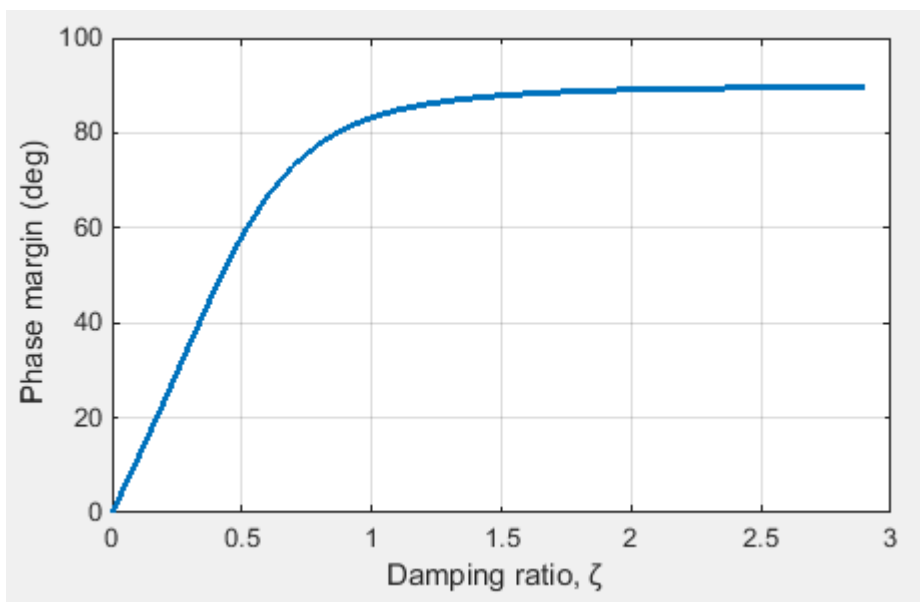


Figure 18: Phase margin vs damping ratio for second order systems

For higher order systems, such direct relations are difficult to derive, but the fact that peaking in the closed loop response increases as the stability decreases is still valid. So, the quality factor or the damping ratio can be used a measure of stability directly from the obtained closed loop response data thereby avoiding the problem of opening the loop.

Extraction of stability measure directly from time domain:

For a closed loop second order system as in Eq. 3.1, as the damping ratio,  $\zeta$  is directly related to the maximum peak,  $M_p$  in the step response, phase margin can be related to  $M_p$ .

An important point to note is that for a critically damped system, the overshoot is zero

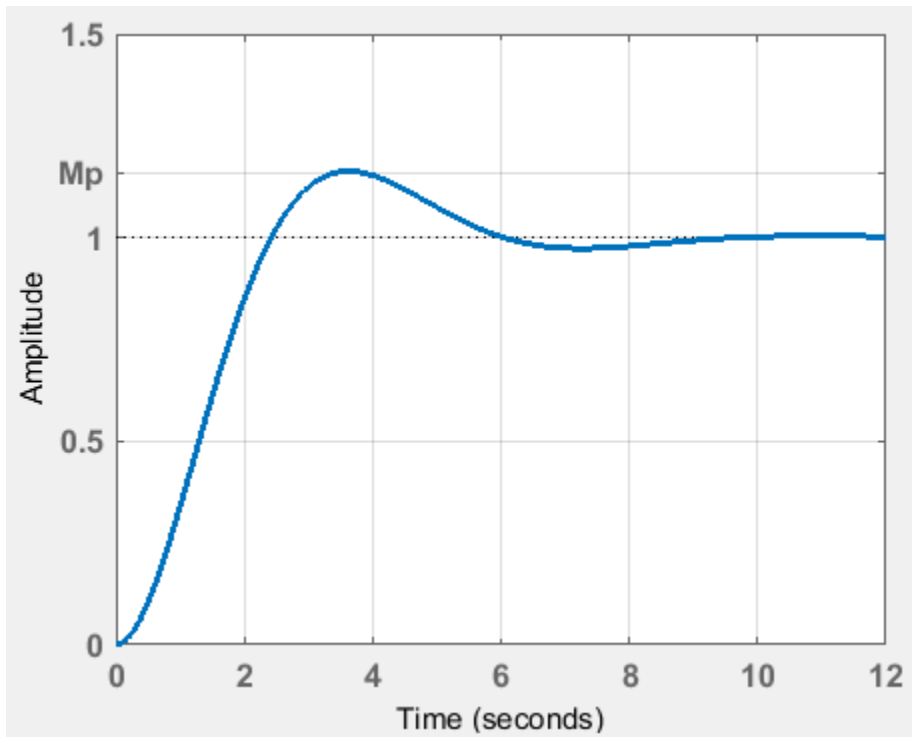


Figure 19: Step response of a second order underdamped system

Relation between damping ratio and peak overshoot,

$$\% M_p, \zeta = \frac{-\ln\left(\frac{\%M_p}{100}\right)}{\sqrt{\pi^2 + \ln^2\left(\frac{\%M_p}{100}\right)}}$$

If only the impulse response is known, the  $M_p$  can be obtained taking the area under the first lobe of the impulse response [20] as shown below:

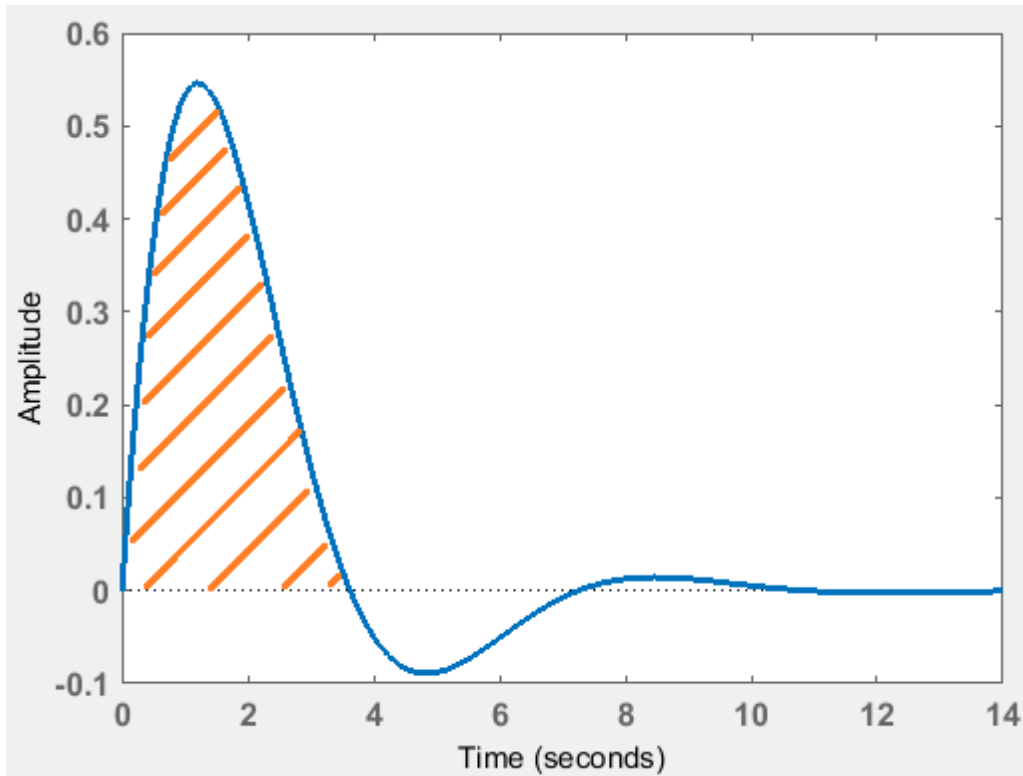


Figure 20: Impulse response and method to extract  $M_p$  [20]

Area shaded =  $1 + M_p$ , where it is assumed that the  $M_p$  is defined for unit step input and unity gain system.

Also, if the impulse response (when referenced to zero) doesn't change sign, the 2<sup>nd</sup> order system is a critically damped system. This is evident from the fact that no sign change in the impulse response indicates no overshoot in the step response.

Though these analysis and equations hold only for a second order system with no zeros, the concept of closed loop response peaking as an indicator of stability is valid at-least for most of the low pass systems.

The systems considered in this work are both low pass systems (Linear and Switching regulators) and quality factor/damping ratio is used as an indicator of system stability.

## Chapter 3

### SYSTEM IDENTIFICATION OF LINEAR REGULATORS USING ANALOG CORRELATOR

Linear regulators employ negative feedback to provide a stable supply voltage with good noise immunity, load and line regulation. As the loop will be designed at a given operating point, changes in the component values can degrade the performance and even cause instability. The widely used PMOS pass device based Low drop-out regulators are compensated using the ESR of the output capacitor (some use an explicit series resistor). In this case the variations of interest are the ESR of the output capacitor (or the series resistor) and the capacitance itself.

So monitoring of these variations would be helpful in predicting a failure and preventing it by either switching to a new component or tuning the loop to adjust with the variations.

For illustrating the process of system identification, an LDO has been designed as follows [21], [22] using an ESR zero based compensation:

## Design of Low Drop-Out Regulator:

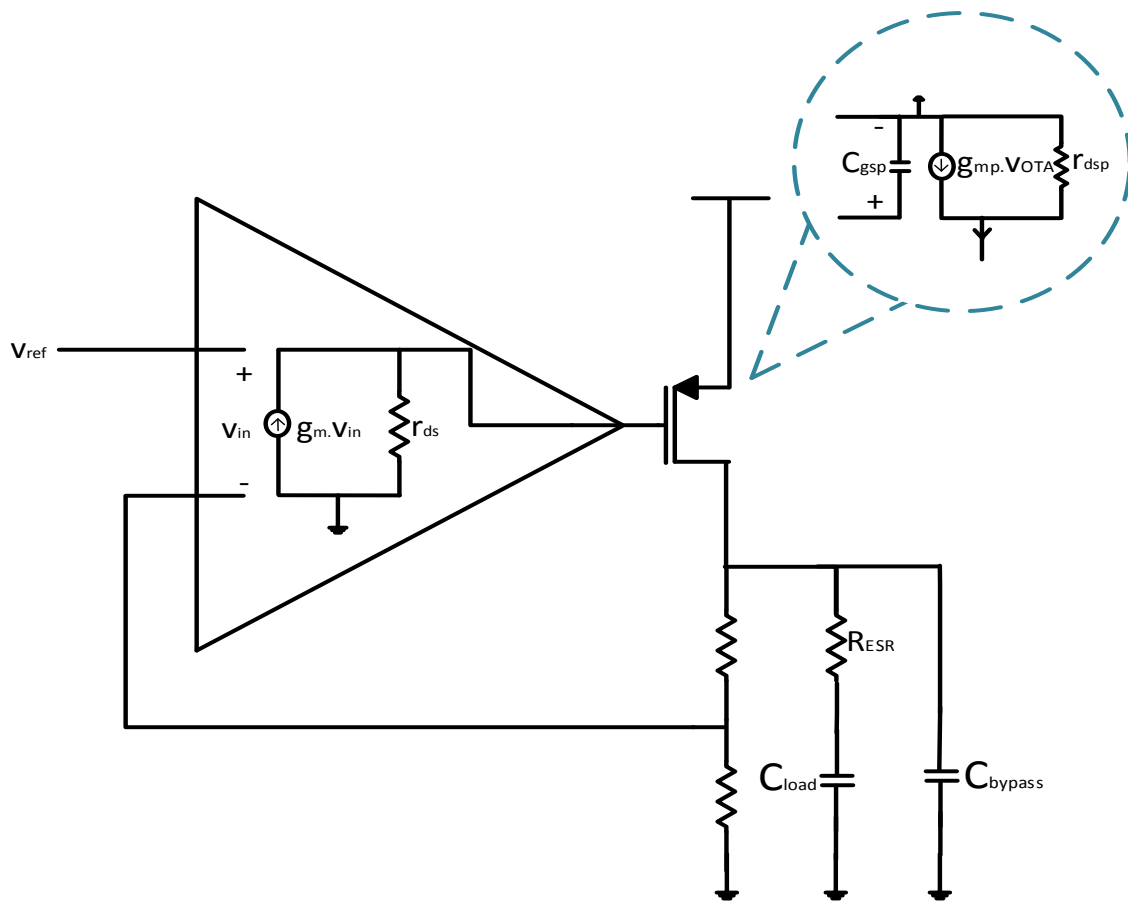


Figure 21: Low Drop-Out Regulator

A typical pole zero configuration has been chosen and ESR zero based compensation is used.

Gain of the OTA =  $g_m * r_{ds} = 35.56$  dB

PMOS pass device has  $g_{mp} = 123$  mS and  $r_{dsp} = 65$   $\Omega$

Capacitor,  $C_{gsp}$  of the pass device and Output capacitance of the OTA = 20 pF

Load capacitor,  $C_{load} = 1$   $\mu$ F and ESR = [0.5 $\Omega$  to 3.5 $\Omega$ ]

Bypass capacitor,  $C_{byp} = 50$  nF

$$\text{Pole due to load, } \omega_{p1} = \frac{1}{r_{dsp} * C_{load}} = 15.38 \text{ Krad/s}$$

$$\text{Pole due to OTA, } \omega_{p2} = \frac{1}{r_{ds} * C_{gsp}} = 166.7 \text{ Krad/s}$$

$$\text{Zero due to ESR, } \omega_{z1} = \frac{1}{R_{ESR} * C_{load}} = 500 \text{ Krad/s}$$

$$\text{Pole due to bypass capacitor and ESR, } \omega_{z1} = \frac{1}{R_{ESR} * C_{byp}} = 10 \text{ Mrad/s}$$

$$\text{Forward path gain} = (g_m * r_{ds}) * (g_{mp} * r_{dsp}) = (200\mu * 300K) = 479.7 \text{ V/V or } 53.61 \text{ dB}$$

$$\text{Feedback factor} = 0.33$$

#### Stability as a function of ESR:

The ESR zero based compensation has the limitation that the value of the ESR should lie within an upper and lower bound for stable operation of LDO. This is due to the fact that a lower ESR causes the pole due to the bypass capacitor to move in causing more phase shift before the gain goes to 0 dB and so lowering the phase margin, whereas a higher ESR can't avoid the phase drop due to the second pole (due to the OTA) as the ESR zero in this case is not close enough to this pole.

On the other hand the capacitor variations (due to temperature & aging) can also cause instability. The effects of these variations for the above LDO can be seen in the following plots:

The open loop phase margin as a function of ESR for the above LDO is plotted below:

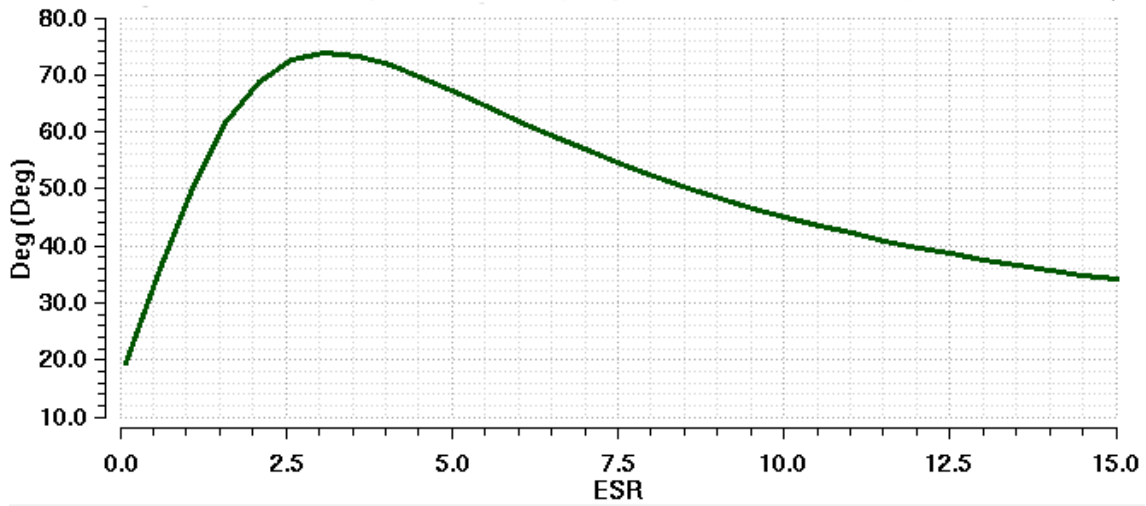


Figure 22: Phase margin vs ESR

The closed frequency responses for different ESRs are plotted below:

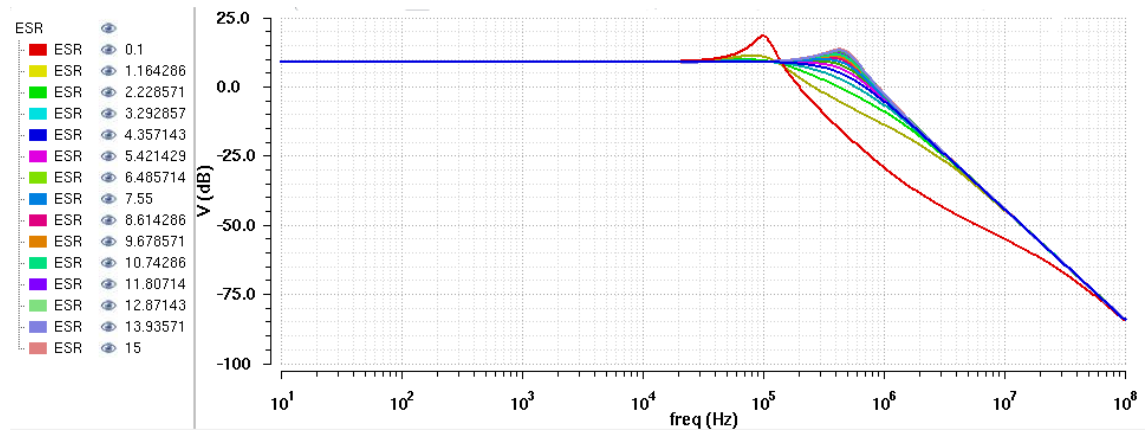


Figure 23: Closed loop frequency responses for various ESR values

The peaking (Q) in the closed loop response (Quality factor) is plotted below:

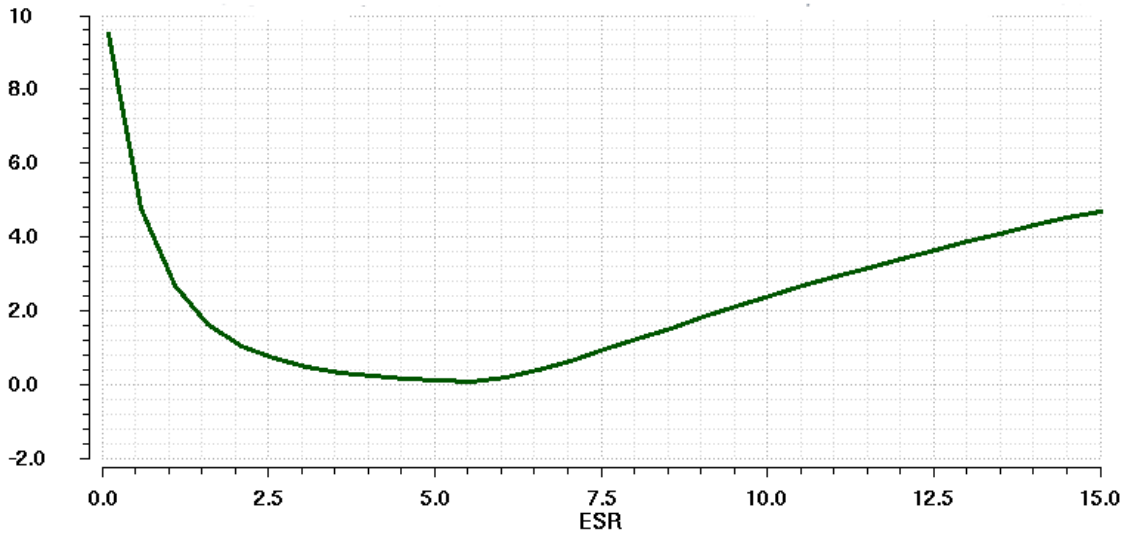


Figure 24: Quality factor Vs ESR

As can be seen from the above plots both the open loop phase margin and closed loop response's quality factor both indicate the potential instability. Thus, by detecting the increased peaking in the closed loop response the LDO or the load capacitor's life time can be predicted. This information of the closed loop frequency response can easily be obtained using cross correlation techniques.



Setup for performing cross correlation on LDO:

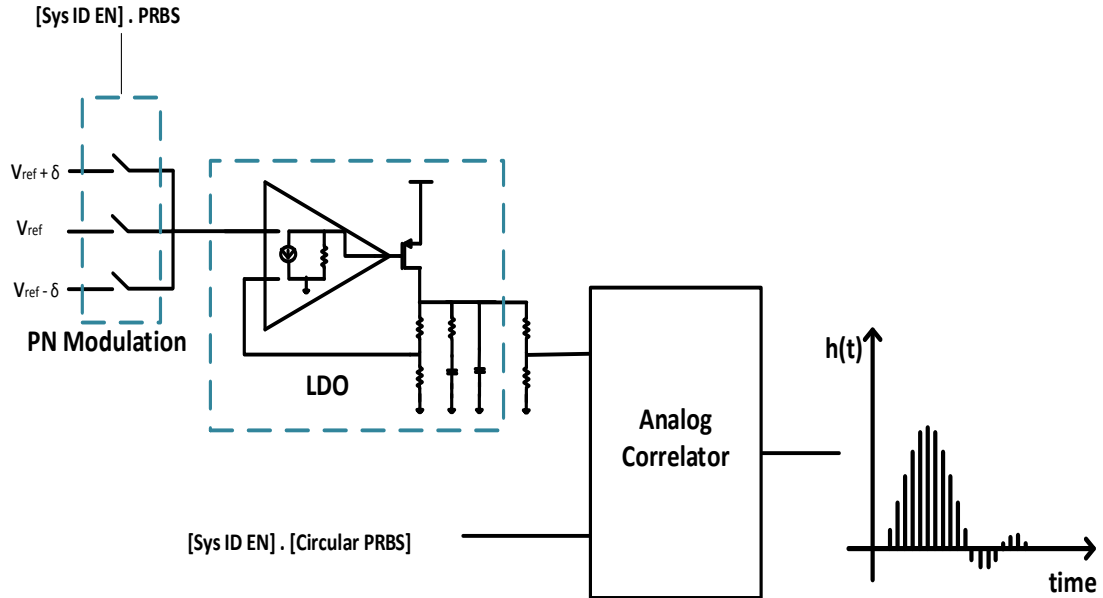


Figure 25: Cross correlation setup for an LDO

Now the ESR is varied and the corresponding closed loop responses are obtained using AC analysis in Spectre and using the cross correlation techniques.

### Results and Comparison:

ESR variation effects on the closed loop frequency response – Comparison of two methods:

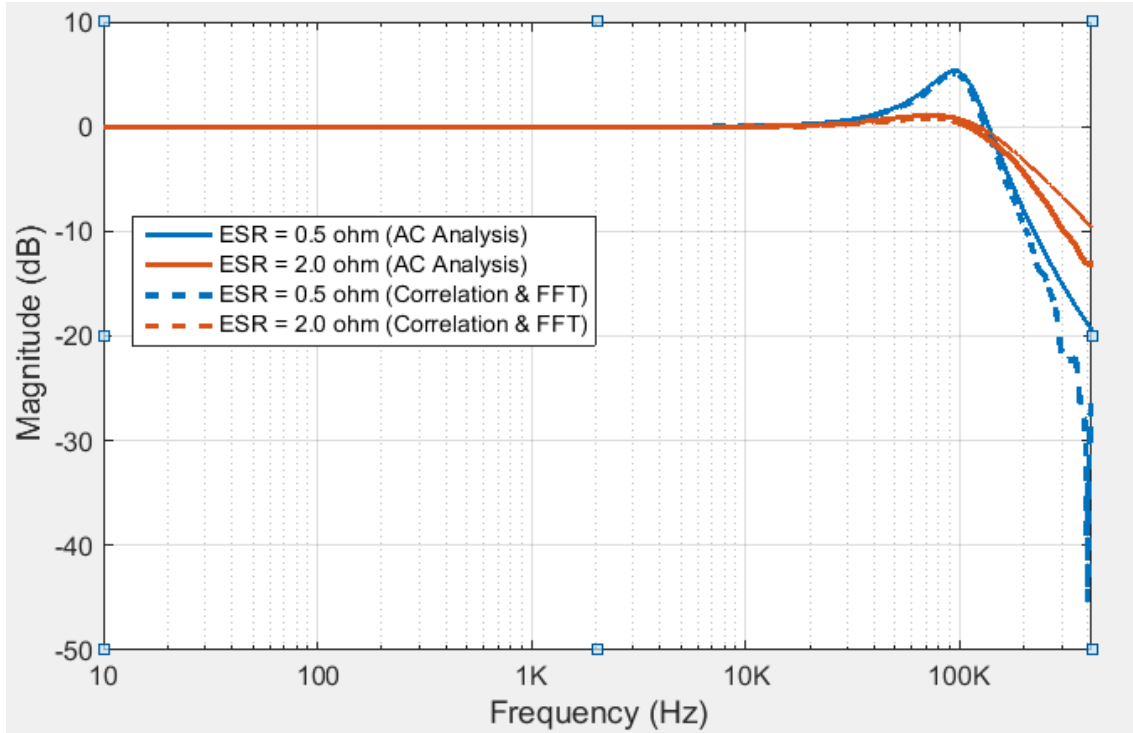


Figure 26: ESR variation effects on the closed loop frequency response-Comparison of Cross Correlation and AC Analysis

Output Capacitor variation also changes frequency response. The effect of bypass capacitor is usually negligible as it is generally chosen to be

$$C_{byp} \ll C_{output}$$

Variation of Capacitor for an ESR =  $2\Omega$  identified using both Cross Correlation and AC Analysis overlaid:

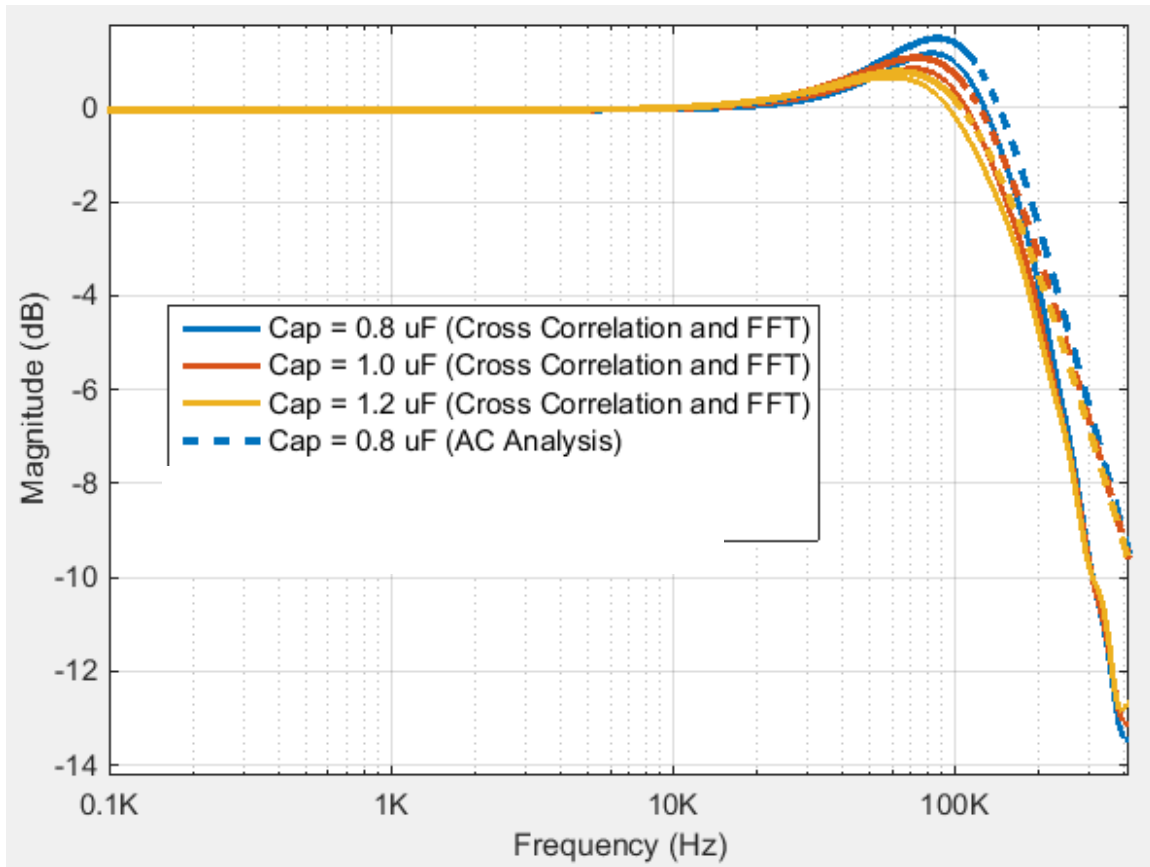


Figure 27: ESR variation effects on the closed loop frequency response

As can be seen in the above graphs frequency responses obtained using cross correlation methods followed by FFT are in close agreement with the actual frequency responses obtained using AC analysis.

So, these methods can be applied to monitor the component variations (ESR and Capacitance) thereby predicting a potential instability.

### Disadvantage of Cross Correlation Analysis in LDO:

The disadvantage of this method is that the noise floor of the system that is being supplied by the LDO under test will increase owing to the injected noise into the LDO. So, if the application cannot tolerate such noise penalty, LDO needs to be brought offline before performing system identification. On the other hand this disadvantage is present with the AC analysis too as there will be strong tone at the output of the LDO when being tested sinusoidally. So, correlation analysis is better in the sense that no energy at a single tone will be present affecting the circuit being supplied by the LDO. Another advantage of cross correlation technique is that while doing production testing the time consumed is lesser than that of sinusoidal testing.

## Chapter 4

### SYSTEM IDENTIFICATION OF DC-DC CONVERTERS USING ANALOG CORRELATOR

In this chapter the application of cross correlation techniques to Switch mode DC-DC converters is examined. System Identification of Switch mode DC-DC converters in closed loop configuration has already been dealt in [12], [13], [14] and they are based on digital implementations. This work applies the cross correlation techniques using an analog correlator instead of digital implementation which can be integrated along with the DC-DC converter.

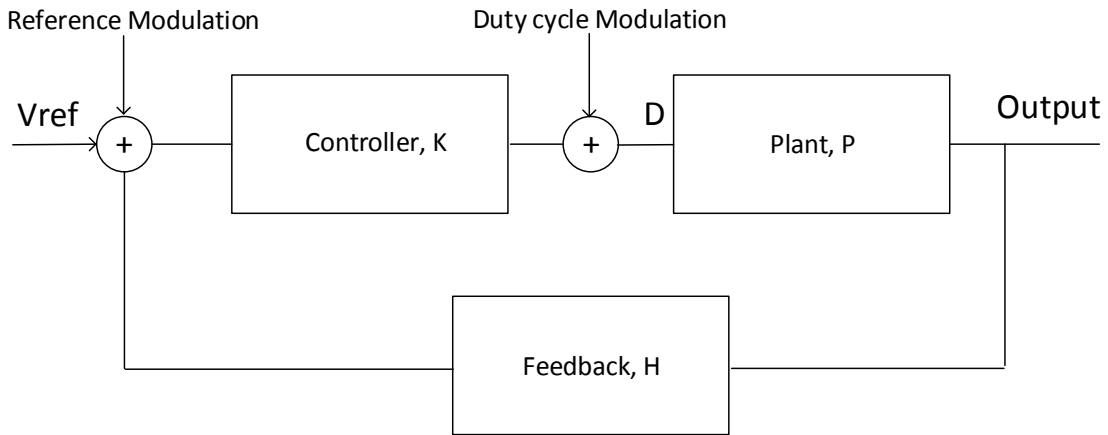
#### Application of Cross Correlation techniques to Switching Converters:

Switching converters cannot be termed as Linear Time Invariant [LTI] systems in general. Theoretically most topologies of Switch mode DC-DC converters belong to the class of Linear Periodically Time Varying [LPTV] Systems. This can be seen from the fact, that the control/modulating signal of a switching converter is a periodic signal (Pulse width modulated waveform) while the converter is working in steady state. .

For steady state operation and small signal perturbations, the Switching converter can be regarded as an LTI system [12]. The theory behind cross correlation operation has been given in [12], [13].

Transfer Function Analysis:

In steady state the converter can be modeled as follows:



Where 'D' denotes Duty Cycle

Figure 28: Block diagram of Switching Converter

The following transfer functions are of interest:

Reference to output transfer function

$$T_{ref\_out} = \frac{PK}{1+PKH} \text{ ---- (4.1)}$$

Duty cycle to output transfer function

$$T_{D\_out} = \frac{P}{1+PKH} \text{ ---- (4.2)}$$

Dividing the equations gives the controller transfer function, K. So, if H is known,

$$\frac{T_{D\_out}}{T_{ref\_out}} = K \text{ ---- (4.3)}$$

Also, Sensitivity of the system (assuming 'H' is known),

$$S = 1 - T_{ref\_out} * H = \frac{1}{1+PKH}$$

So,

$$\frac{T_{D\_out}}{S} = P \text{ ---- (4.4)}$$

So, theoretically using closed loop transfer functions, both the plant and controller transfer functions can be obtained and multiplying P and K when the H is known gives the loop gain. The practical implementations are limited by various noise sources particularly if the implementation is analog.

If the stability margins are the only information of interest but not the individual plant and controller transfer functions, obtaining equations 4.2 to 4.4 is not critical as the closed loop response's peaking can be used as an indicator of stability as mentioned in the earlier chapters. In this work too, only stability is considered as topic of interest but not variations in standalone plant and controller blocks.

#### Voltage Mode Buck Converter as Test Vehicle:

For a voltage mode buck converter, the plant 'P' is the LC filter and the controller is the pulse width modulator (PWM) along with the compensator employed.

In this work, a voltage mode DC-DC converter using a type III compensator is considered to illustrate the cross correlation technique using an analog correlator.

The converter has been designed with a plant LC filter of  $L = 8 \mu\text{H}$ ,  $C = 1.4 \mu\text{F}$ ,  $\text{ESR} = 100 \text{ m}\Omega$  as nominal values. The open loop gain of the converter is obtained as follows:

Open loop gain of the designed system:

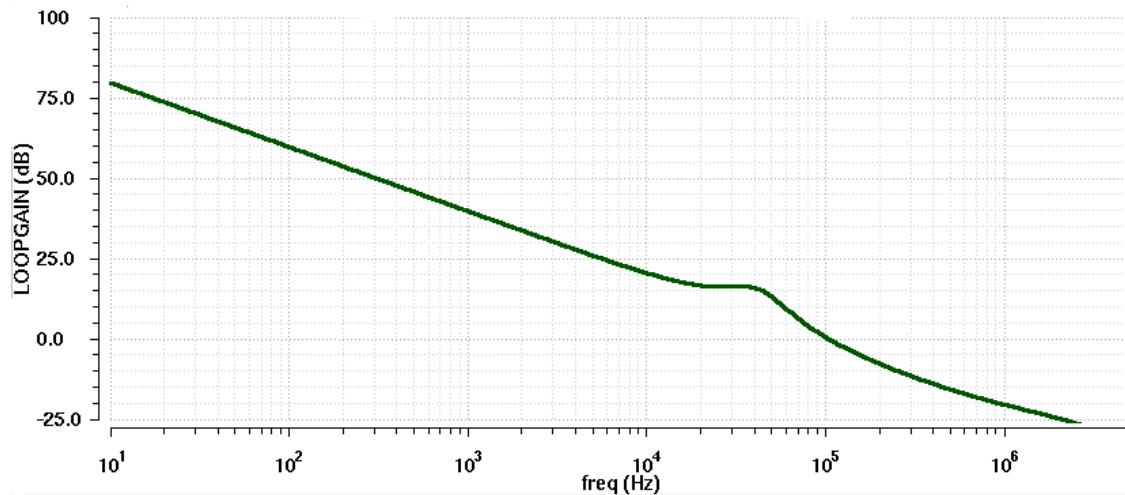


Figure 29: Open loop gain (open loop) of the DC-DC converter considered  
System identification is performed to obtain the transfer functions of interest are given by equations 4.1 and 4.2.

For this purpose, the points of PRBS injection need to be the reference voltage and the duty cycle. Reference voltage can be modulated with white noise using a single bit Digital to Analog Converter (DAC) as the PRBS considered is only a single bit signal. By injecting white noise (PRBS) at one injection point each time and cross correlating the output, the impulse response would be obtained.

Noise is injected either into the reference node or the duty cycle node depending on the transfer function required. The output for this noise excitation is fed to the analog correlator and the sampled output of it is Fourier transformed (DFT) to get the desired transfer function.



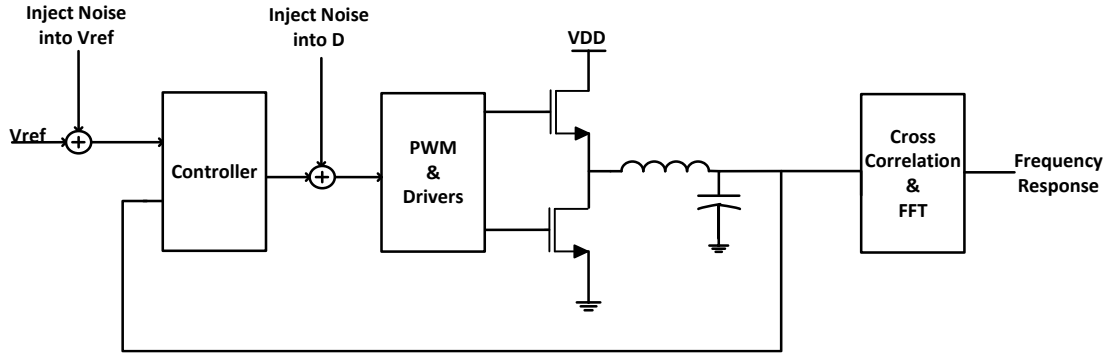


Figure 30: Block diagram of setup for cross correlation

For a switching converter the PRBS frequency needs to be chosen considering the fact that the PWM block can cause aliasing [23], [24]. The frequency of perturbation should be at-least less than half the PWM frequency to avoid aliasing. Also there should be sufficient energy in the PRBS waveform in the test circuit's bandwidth of interest. To simultaneously satisfy these conditions PRBS frequency is chosen as  $1/6^{\text{th}}$  of the switching frequency. Switching frequency of the converter is 5 MHz and so the PRBS frequency is chosen as 833.33 KHz

The output of the switching converter with and without white noise injection are shown in the figure below. The ripple voltage with the noise injection is increased to 29.74 mV when compared to the normal operation where the ripple was 4.42 mV.

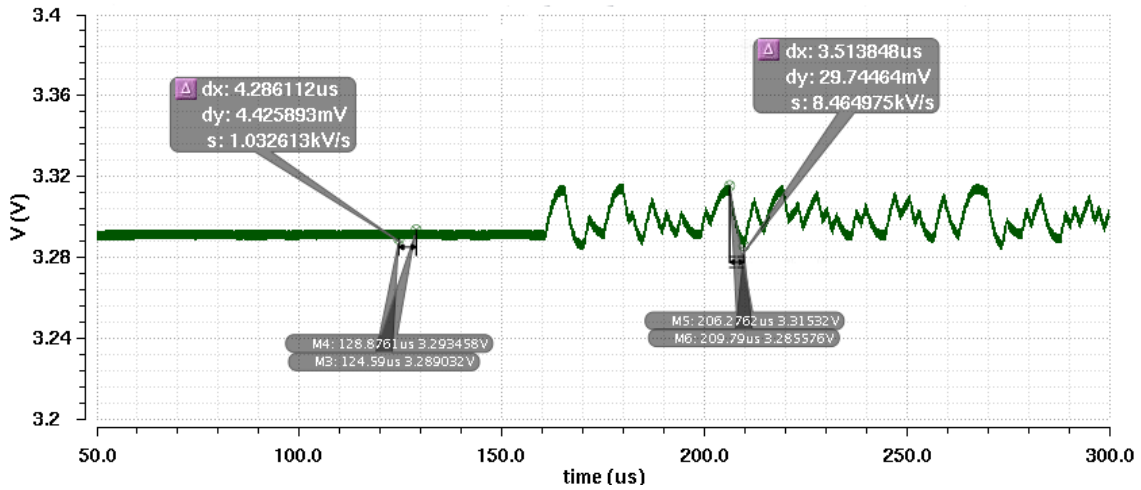


Figure 31: Output of the buck converter- with and without PRBS excitation

But the advantage of cross correlation technique over the normal sinusoidal measurements is visible in the frequency domain. The spectra of the output with and without white noise injection are shown below:

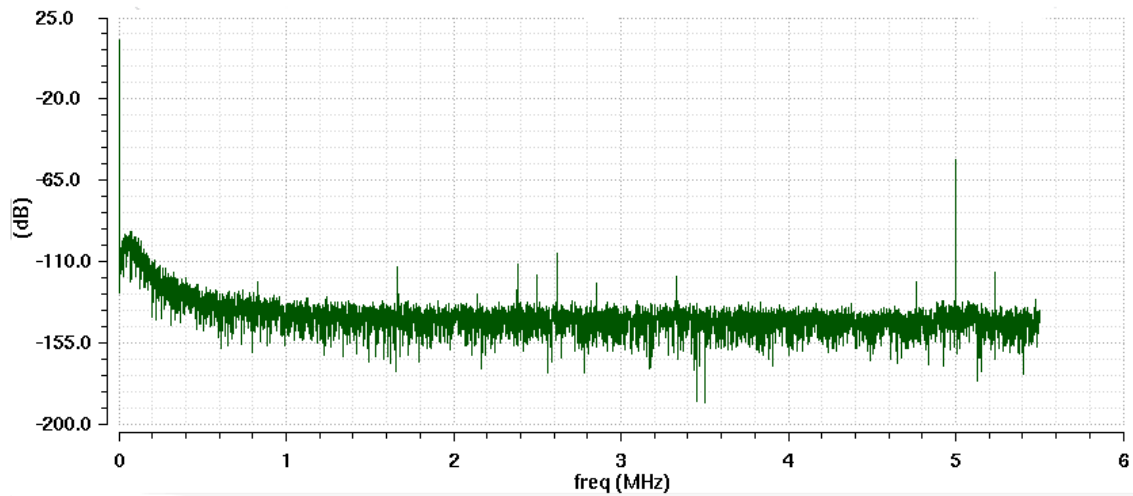


Figure 32: Output spectrum of the buck converter without noise injection

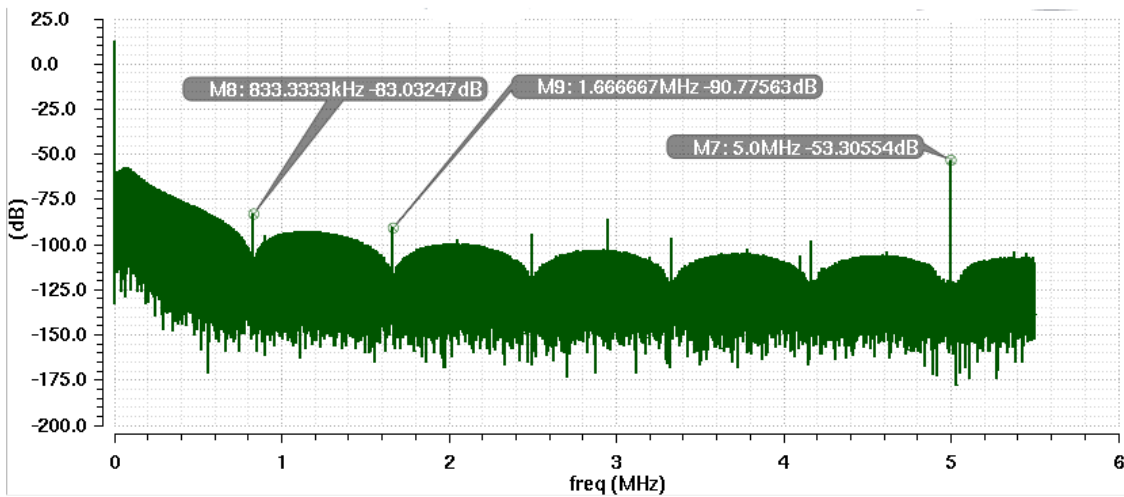


Figure 33: Spectrum of the DC-DC converter output with PRBS injection

It can be seen that though the noise floor has risen it is uniform across the spectrum unlike sinusoidal testing where energy is concentrated at a single tone at a given time. So, this method allows health monitoring while the converter is in operation. It also removes the need for a network analyzer/signal generator and oscilloscope for converter transfer function measurements.

The points of noise injection and output collection determine the transfer function that could be obtained. Closed loop transfer functions from reference to output nodes and plant (duty cycle node) to output nodes are obtained as shown in following sections.

In both the cases initially the setup of obtaining the transfer function is shown, followed by a top level view of the way to inject PRBS into the corresponding node. The amplitude of injection should be low enough not so that system's operation doesn't exceed the linear range but sufficient so the perturbation at the output is above the minimum detectable signal level of the correlator.

Obtaining Reference to Output transfer function:

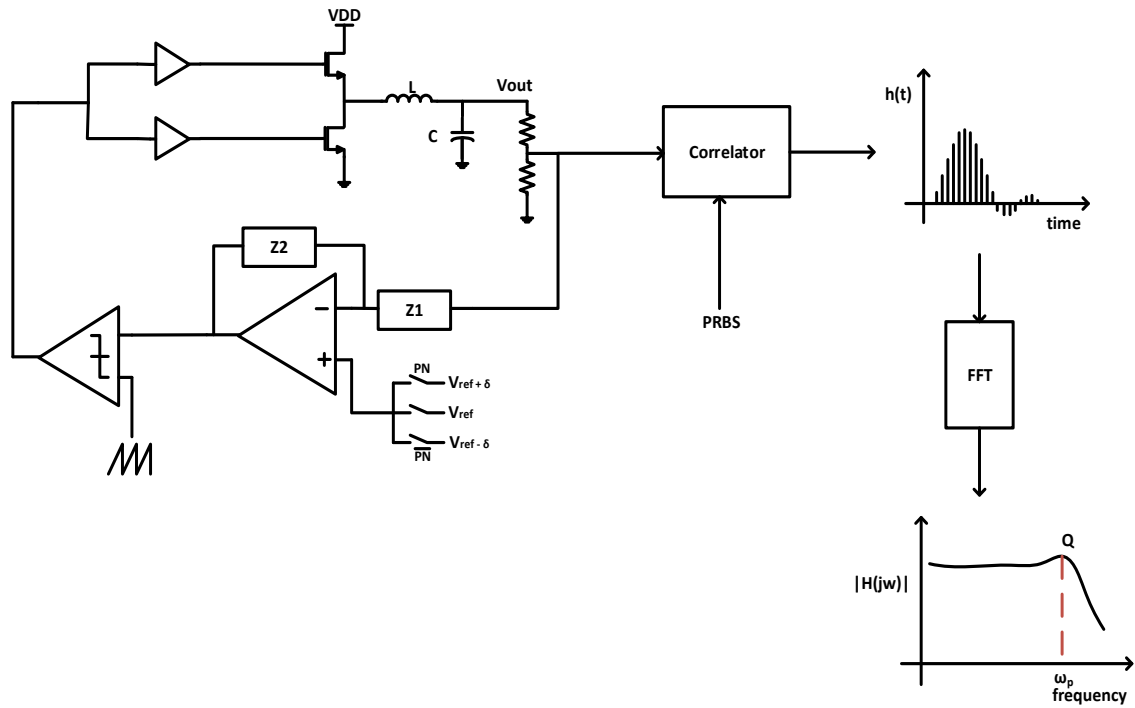


Figure 34: Obtaining reference to output transfer function

The PRBS signal is injected into the reference node using a single bit DAC which can be implemented as a simple resistive divider with two switches controlled by the PRBS and its complementary signal.

A block level view of this injection is shown in the following figure.

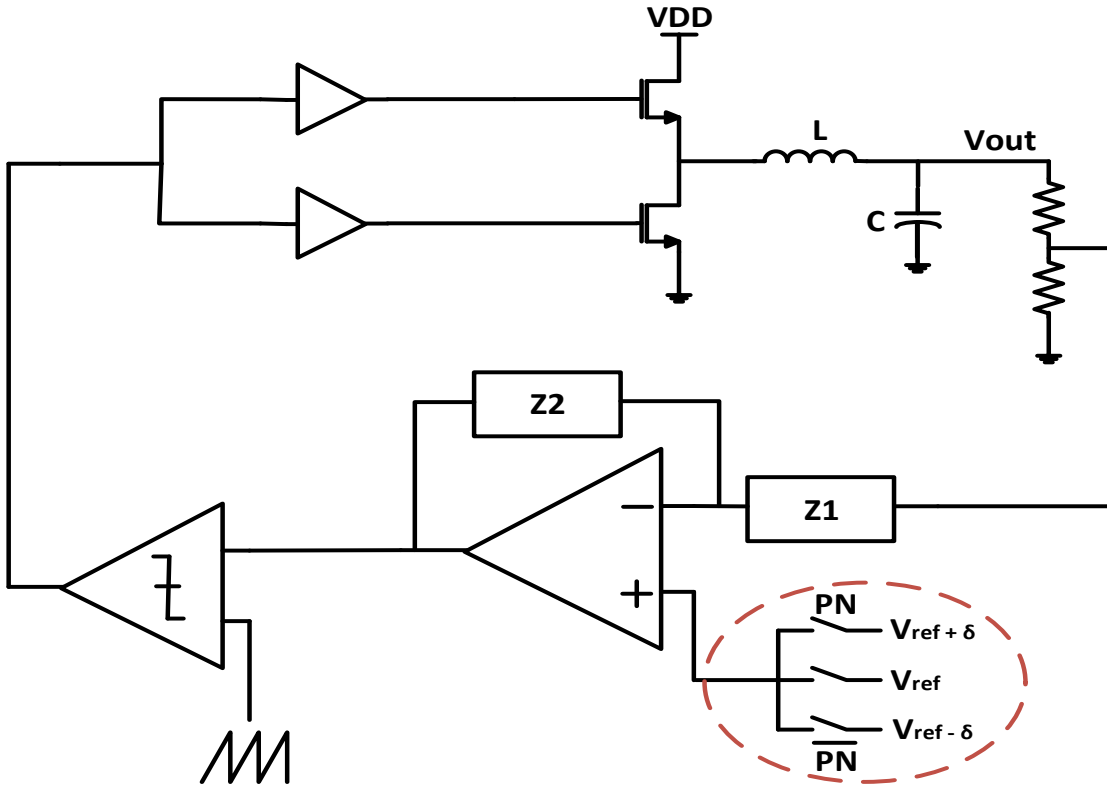


Figure 35: PRBS injection into the reference node

Typical output of the analog correlator would look as follows:

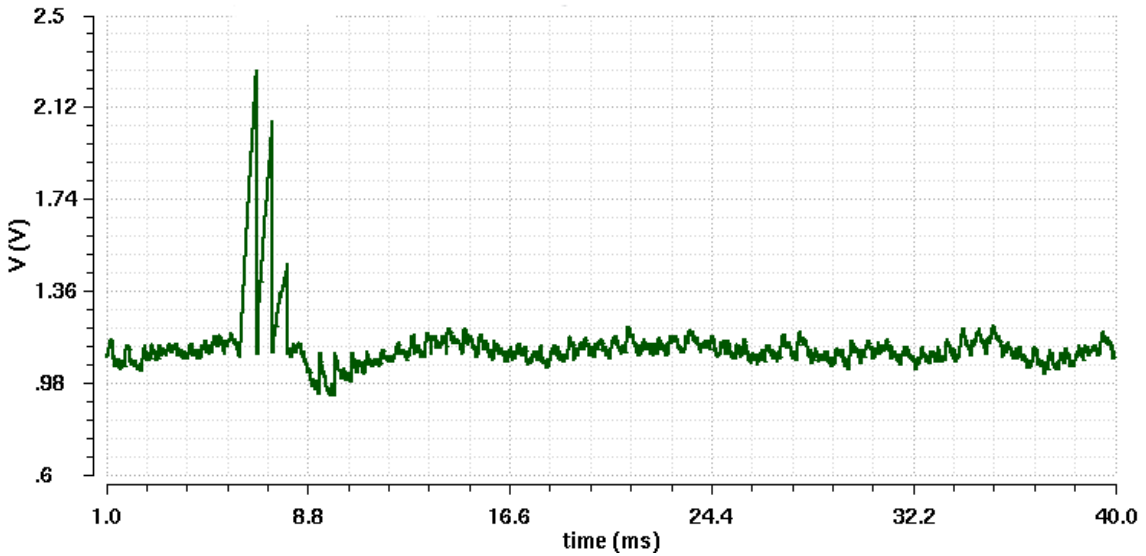


Figure 36: Output of the analog correlator

This output is sampled every  $N \cdot f_{PRBS}^{-1}$  seconds to get the sampled impulse response as explained in chapter 2.

Sampled impulse response:

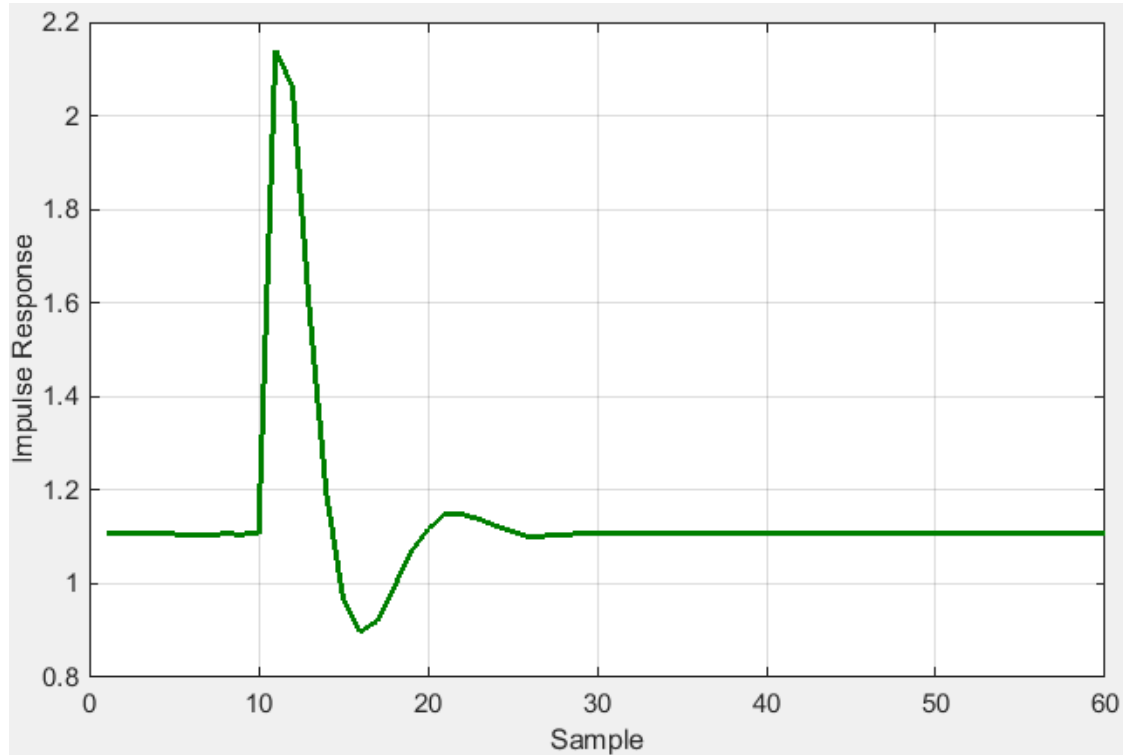


Figure 37: Sampled (and reconstructed) Impulse response of the reference to output transfer function

Frequency response is obtained by taking FFT on the above samples and is shown in the following sections.

The reference to output transfer functions are obtained using the above technique. Also the output capacitor and inductor values are varied and the results are compared with that obtained from a sinusoidal measurement based method.

In simulations frequency responses for a switching converter can be obtained using periodic steady state analysis (PSS) and Periodic AC analysis (PAC). This is possible because of the Linear periodically time varying nature of the DC-DC converters.

Results and Comparison:

Variation of Output Capacitor – AC Analysis and Cross Correlation methods overlaid:

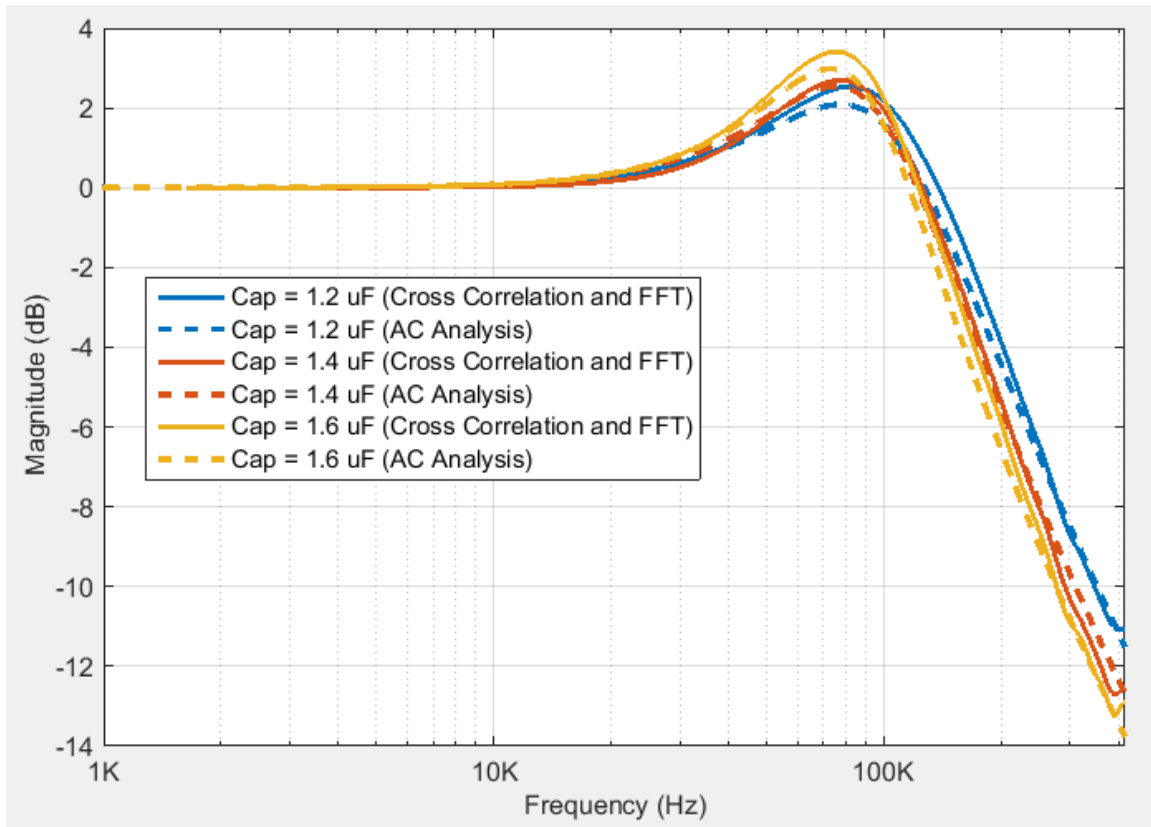


Figure 38: Reference to output transfer function –comparison of two methods

The following are the graphs showing results from each method individually.

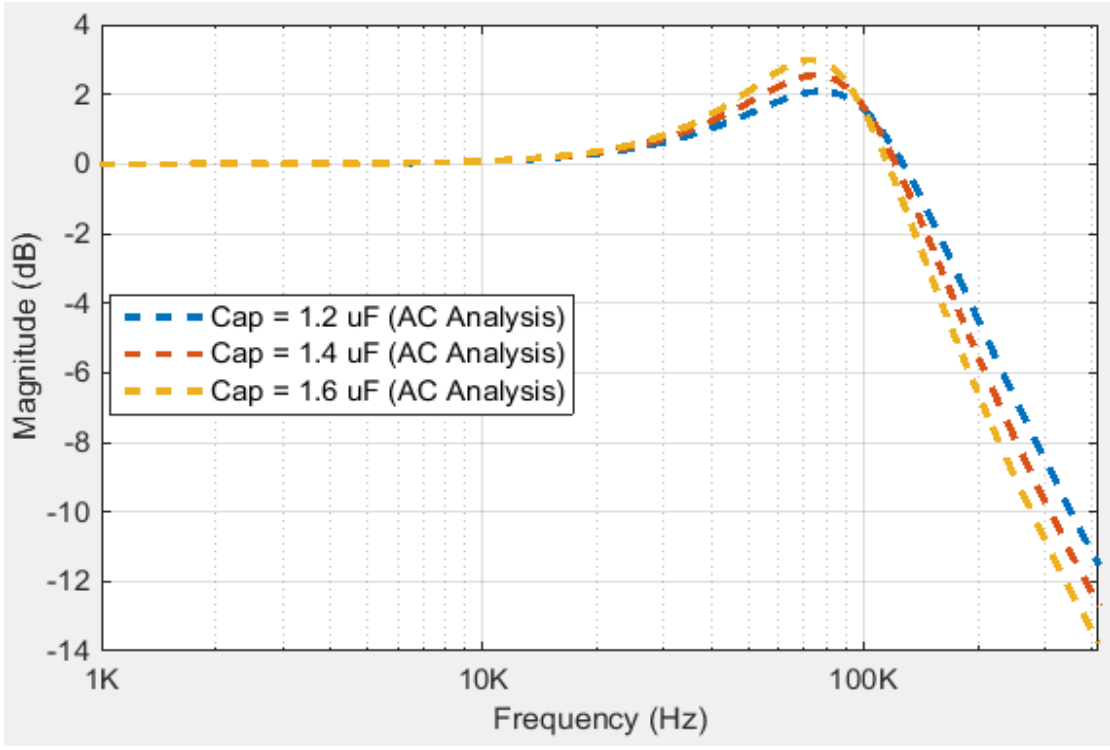


Figure 39: Reference to output transfer functions from AC Analysis

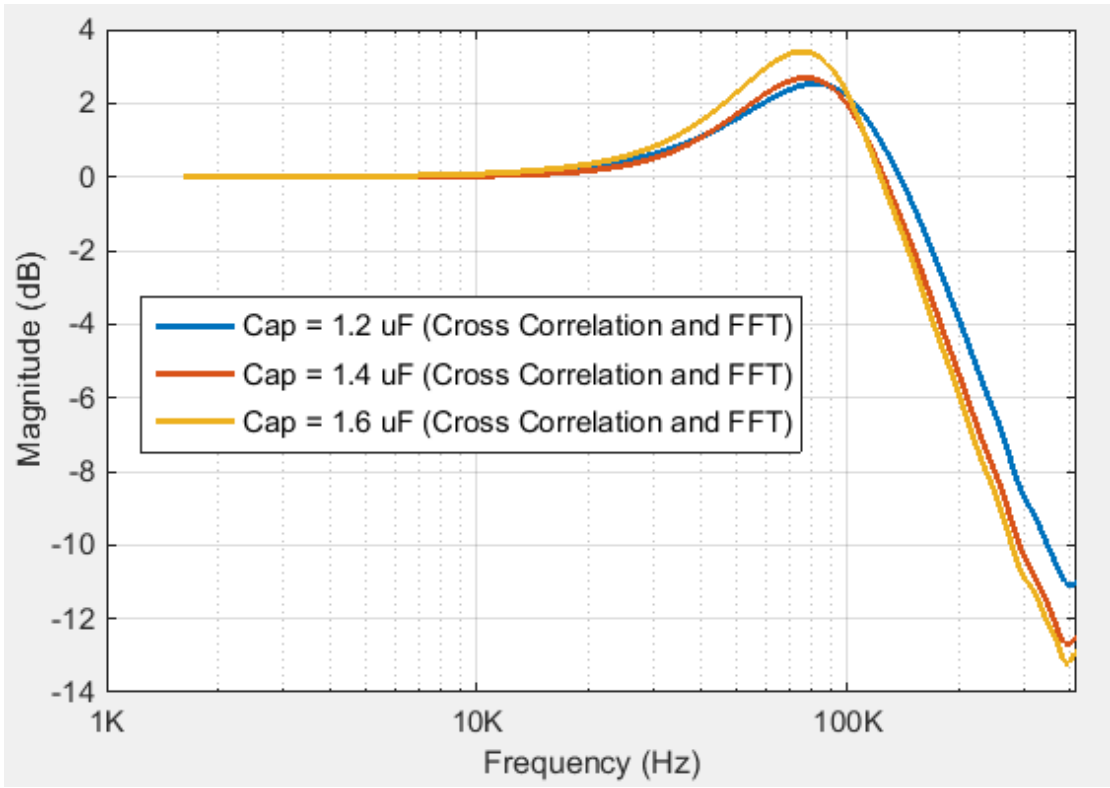


Figure 40: Control to output transfer functions using Cross Correlation



Plots showing the effect on frequency response due to variation in inductors

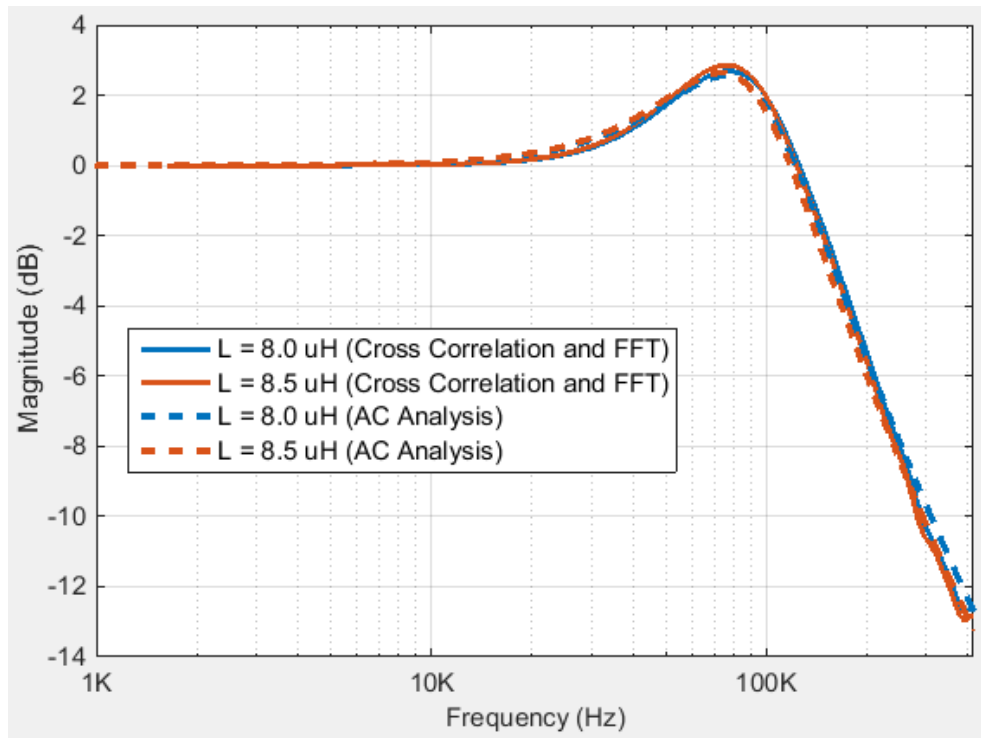


Figure 41: Reference to output transfer functions for various inductors

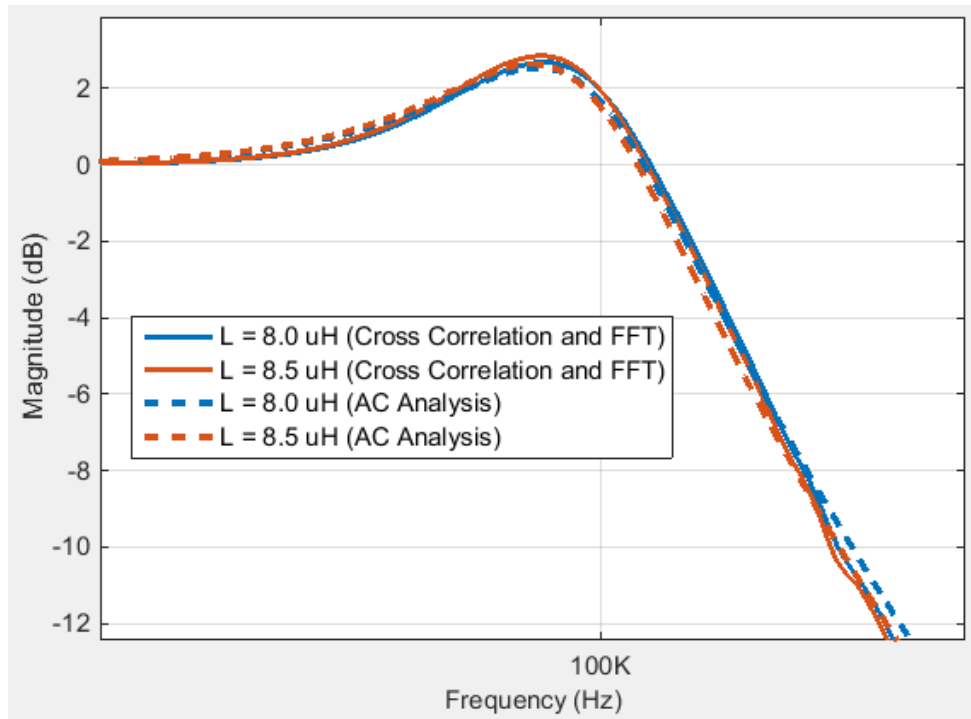


Figure 42: Magnified version of Figure 36

It can be seen that the trend of the variation is reflected the same way in both AC analysis and Cross Correlation technique. This proves that cross correlation can be used as an effective replacement to sinusoidal measurements either in health monitoring or during production testing similar to BIST techniques. Also, in the similar way as for L and C, the ESR variation of the capacitor can also be observed in the closed loop response of the DC-DC converter.

The useful and important inference from the information in these graphs is the quality factor (peaking) as it is an indicator of the stability of the system.

If individual plant and controller transfer functions are of interest then one can proceed and extract the closed loop plant to output transfer function and using the equations 4.1 – 4.4, the required responses can be obtained.

#### Obtaining Control to Output Transfer Function:

Another point of injection is at the output of the controller or the input of the plant which effectively is duty cycle modulation.

This I achieved by modulating the saw-tooth waveform used to generate the pulse width modulated (PWM) waveform at the output of the comparator.

Setup to measure plant to output transfer function:

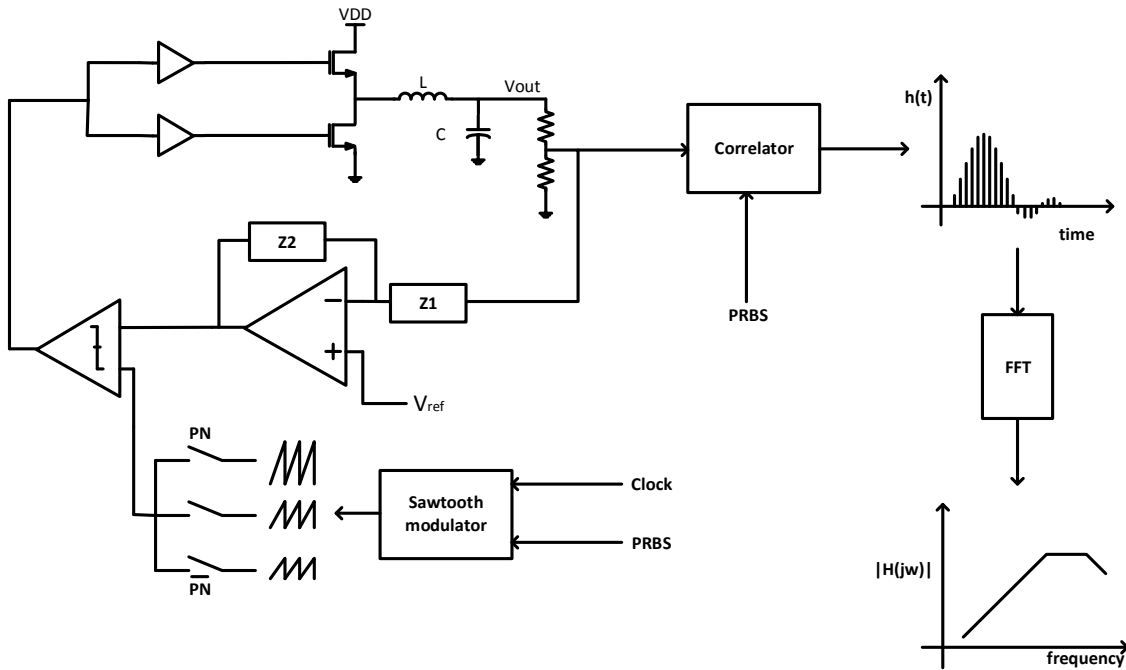


Figure 43: Obtaining Plant to output transfer function

PRBS can be injected into the saw-tooth waveform by modulating either the slope of the saw-tooth or the reference voltage of the comparator used to generate the PWM waveform. This in turn modulates the duty cycle of the PWM waveform. The following figure a block level view of saw-tooth waveform modulated with PRBS signal.

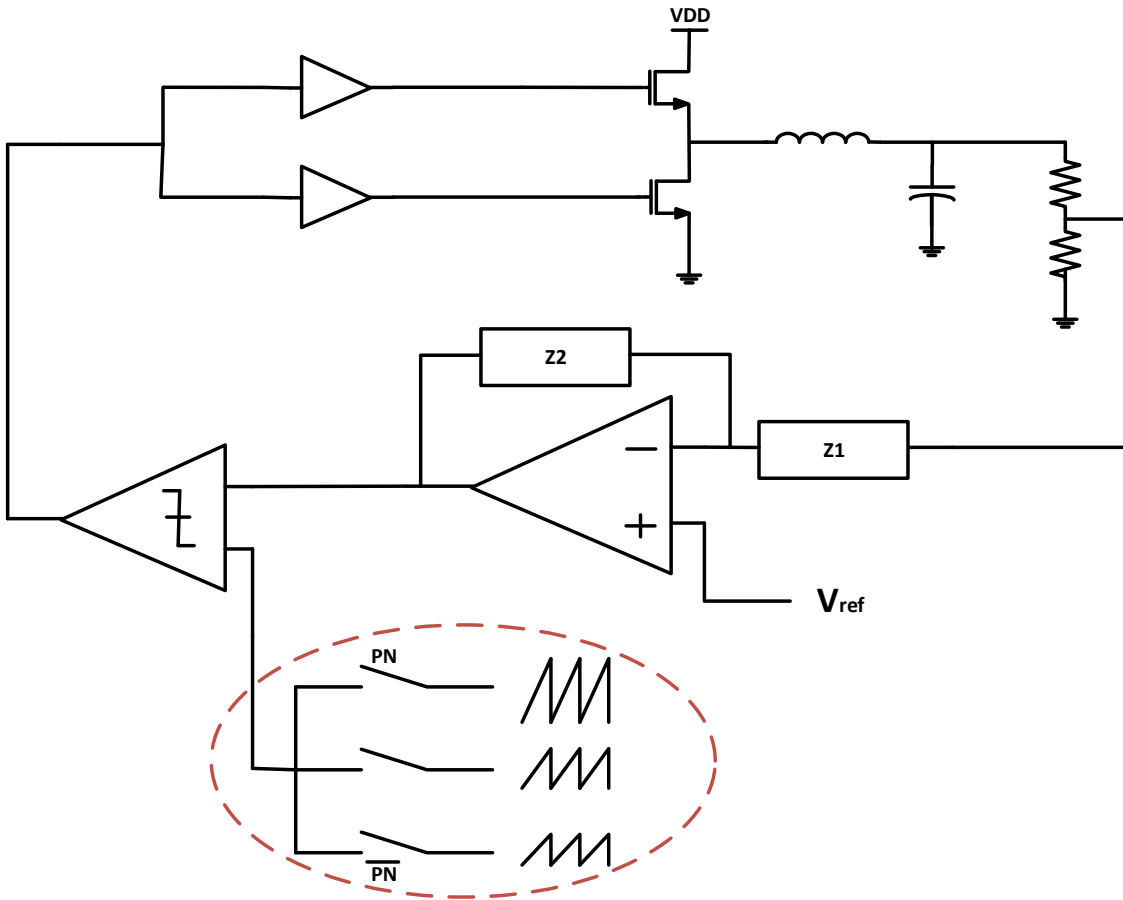


Figure 44: Injecting PRBS signal into the plant (Duty cycle modulation)

The output of the buck converter would look much the same way as for the reference to output method, but the impulse response's spectral content is now a completely different transfer function which in fact happens to be a band-pass response.

Results and Comparison:

The frequency responses of control to output transfer function obtained using AC Analysis and Cross Correlation analysis for capacitor variations are compared in the following figures.

Plots showing effects of capacitor variation on plant to output transfer function

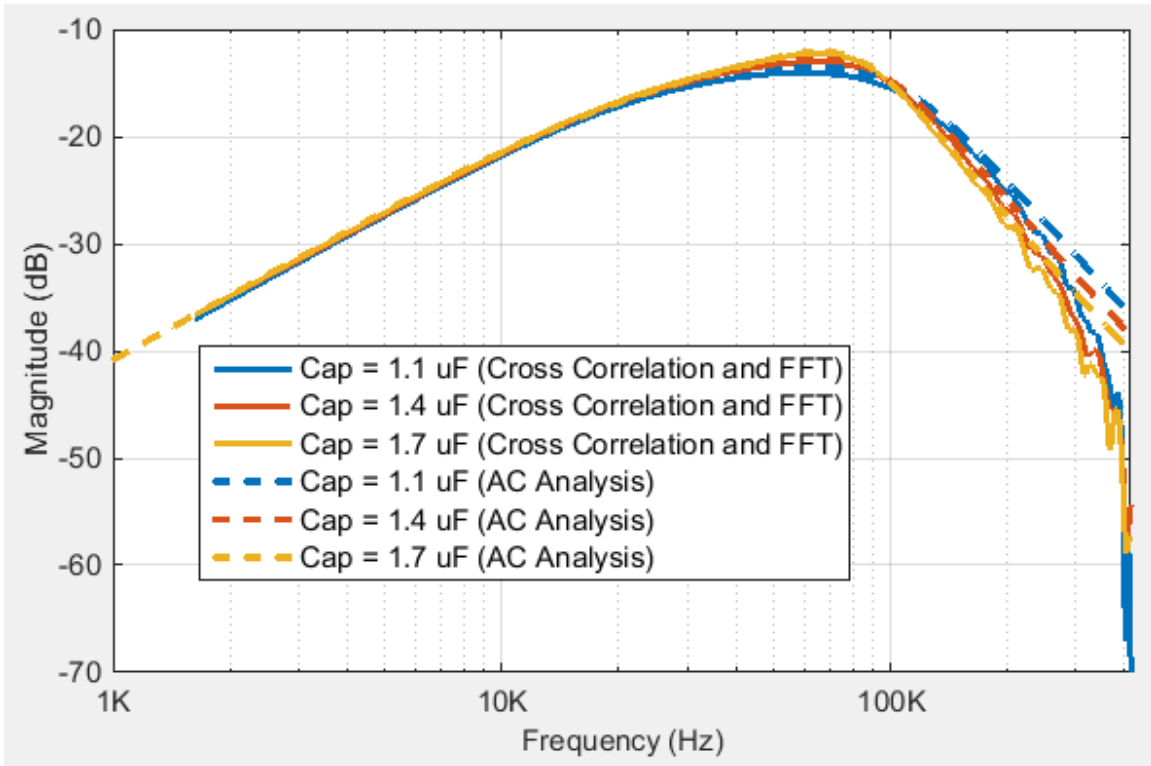


Figure 45: Control to output transfer functions for various output capacitors

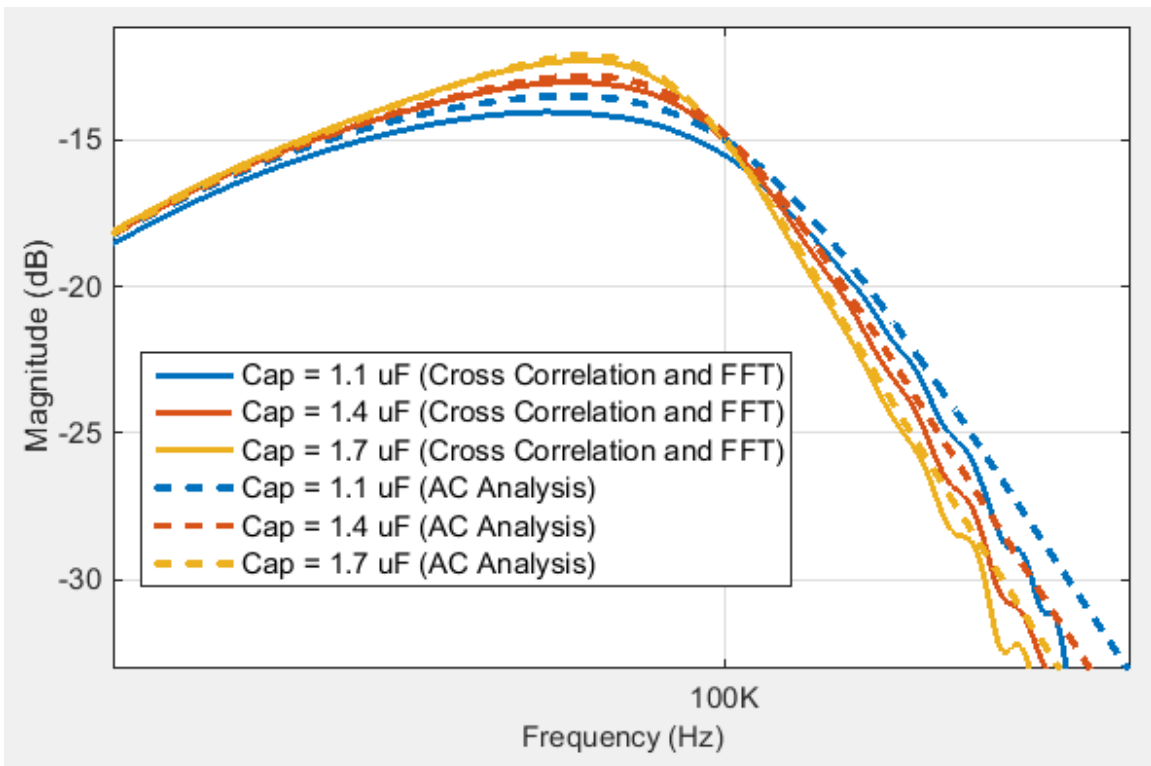


Figure 46: Magnified version of Figure 40

The above examples show that the variation of passive components employed in switching converters effect the closed loop transfer functions in a noticeable way. As explained in chapter 2, these effects can be used to monitor the stability of the converter. By obtaining both reference to output and control to output transfer functions plant and controller can also be identified.

Though not considered here the switching MOSFET's on-resistance  $R_{dson}$  also causes performance variations in switching converters. But the effect of this is hard to notice in closed loop frequency responses as they are a very weak function of  $R_{dson}$ .  $R_{dson}$  variation causes more pronounced effects in the efficiency. So, if effects of  $R_{dson}$  changes are to be detected efficiency monitoring techniques need to be employed [7].

## Chapter 5

### CONCLUSION

Linear and switching regulators are prone to performance degradation and instability due to their component variation with temperature, aging and other factors. A circuit to monitor these variations can help estimate the time to failure and thereby allowing for necessary actions like switching to a new circuit or tuning the circuit to retain the required performance.

Measurements of frequency response in power converters has high hardware overhead and is time consuming. A BIST circuit is quite useful in this regard if it can measure the frequency response characteristics.

Health monitoring of power management circuits can be performed based on their frequency response characteristics too. So, the BIST itself can serve this purpose.

Cross correlation techniques can be used to measure frequency responses of power converters without much hardware overhead and in less time. An integrated cross correlator based on switched capacitor integrator has been developed in this regard and tested on a low drop-out regulator and a voltage mode buck converter. The obtained results closely track that of the standard sinusoidal analysis and are useful in estimating the stability of the power converter.

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