

Monte Carlo simulations of radiative heat exchange in a street canyon with trees

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1 **Abstract**

2 Land surface energy balance in a built environment is widely modelled using urban
3 canopy models with representation of building arrays as a big street canyon. Modification
4 of this simplified geometric representation, on the other hand, leads to challenging
5 numerical difficulties in improving physical parameterization schemes that are
6 deterministic in nature. In this paper, we develop a stochastic algorithm to estimate view
7 factors between canyon facets in the presence of shade trees based on Monte Carlo
8 simulation, where an analytical formulation is inhibited by the complex geometry. The
9 model is validated against analytical solutions of benchmark radiative problems as well as
10 field measurements in real street canyons. In conjunction with the matrix method resolving
11 infinite number of reflections, the proposed model is capable of predicting the radiative
12 exchange inside the street canyon with good accuracy. Modeling of transient evolution of
13 thermal field inside the street canyon using the proposed method demonstrate the potential
14 of shade trees in mitigating canyon surface temperatures as well as saving of building
15 energy use. This new numerical framework also deepens our insight into the fundamental
16 physics of radiative heat transfer and surface energy balance for urban climate modeling.

17

18 **Keywords:** *Building energy consumption; Monte Carlo method; Radiative heat transfer;*
19 *View factors*

20

21 **1. Introduction**

22 Today, urban areas are home to more than half of the world’s population, with a
23 projected urban population of 6.3 billion (68% total global population) in 2050 (United
24 Nations, 2012). Complex landscape characteristics presented in a built environment has led
25 to significant modification of surface partitioning of solar energy. Urban areas therefore
26 have higher environmental temperatures than their rural surroundings, a well-known
27 phenomenon as the “urban heat island” (UHI) (Oke, 1982; Taha, 1997; Arnfield, 2003). As
28 a consequence, urban climate, as largely dictated by functions of manmade infrastructure
29 and human stressors, has paramount effect on energy consumption in cities (Santamouris et
30 al., 2001; Kikegawa et al., 2003). The past decade has seen increasing effort in reducing
31 the impact of UHI on energy use with various mitigation strategies such as the use of cool
32 pavements, green roofs, and shade trees (Akbari et al., 2001; Ouldboukhitine et al., 2014;
33 Santamouris, 2014). Understanding the fundamental physics governing the working
34 mechanisms of these strategies, especially on how they change the surface energy balance
35 in urban canopies, is becoming increasingly pressing to researchers.

36 In addition to thermal and optical properties of pavement materials (Sailor et al.,
37 2006; Synnefa et al., 2007), urban morphology plays a critical role in dictating UHI
38 intensity and has a significant impact on building energy consumption (Wong et al., 2011).
39 In particular, the geometry and density of building arrays are important contributors to the
40 surface energy balance of built environments through radiative trapping and shading
41 effects (Harman et al., 2004; Wang et al., 2011b). Radiosity algorithms have been
42 developed to predict surface irradiance and interior illumination in urban environments
43 (Robinson and Stone, 2005; 2006). Contributions of radiance from discretized patches,

44 partially obscured by the canyon geometry and presence of obstructions, are predicted
45 using ray-tracing methods, and the associated view factors can be estimated. Radiosity
46 methods exhibit good accuracy in predicting radiative transfer at building-revolving scales
47 (with spatial resolutions < 10 km), as compared to other radiation models (Robinson and
48 Stone, 2004).

49 This study, on the other hand, focuses on the development of a radiative transfer
50 model in urban canopies that will later be incorporated into numerical weather predictions
51 of urban areas at city scales (with spatial resolutions $\sim 10 - 100$ km). At these large scales,
52 numerical urban land surface models do not resolve detailed building and street canyon
53 geometries, but rather resort to simplified representations. Currently, two broad types of
54 representations of a “generic” urban area are adopted, viz. as a two dimensional (2D) street
55 canyon (Nunez and Oke, 1976), or a three dimensional (3D) rectangular block (Aoyagi and
56 Takahash, 2012). With these simplified geometric representations, building arrays are
57 usually resolved by normalized roof, wall and road dimensions for 2D canyons (Kusaka et
58 al., 2001), or by roof and frontal areas for 3D blocks (Grimmond and Oke, 1999).
59 Currently, most urban surface energy models are based on the 2D street canyon
60 representation of urban areas, e.g. the urban canopy models (UCM) adopted in the widely-
61 used Weather Research and Forecasting (WRF) platform (Chen et al., 2011). With this
62 geometric simplification, radiative heat exchange in urban areas can be analytically
63 resolved based on the view factors among urban facets (sky, ground, and walls). While the
64 2D street canyon representation of a built terrain is attractive due to its geometric
65 simplicity, any incremental modification to the geometry can lead to laborious effort or
66 even formidable challenges in modifying physical parameterizations of UCMs.

67 Of particular importance to this study, recent advances in urban climate modeling
68 demonstrate that it is critical to include urban vegetation and the associated hydrological
69 processes for UCMs to realistically capture the surface energy budgets, especially the
70 latent heat (Grimmond et al., 2010, 2011). This requirement has led to new urban
71 parameterization schemes that integrate urban vegetation, grass or trees, in street canyons
72 to enable direct soil-vegetation-atmosphere interactions (Lemonsu et al., 2012; Wang et al.,
73 2013). On the other hand, these changes necessarily bring up new modeling challenges,
74 such as how the shading effect of trees in a street canyon can be realistically represented?
75 Analytical formulation of view factors with the presence of trees in a street canyon will be
76 extremely difficult, if not impossible, given a variety of geometry of trees, needless to
77 mention their spatial locations and sizes. In addition, degraded air quality in urban areas
78 can modify the optical and radiative properties in the canopy layer (Prabhakar et al., 2014)
79 and challenge the assumption of air as a non-participating (transparent, non-scattering, and
80 non-absorbing) medium for radiative transfer in urban areas. This is particularly concerned
81 for cities with severe pollution, e.g. heavy PM 2.5, PM 10 and aerosol loads in megacities
82 in northern China (Sun et al., 2006; Li et al., 2007).

83 To address these new challenges, one naturally resorts to stochastic approaches based
84 on random sampling, e.g. a Monte Carlo method. Monte Carlo simulations of radiative
85 heat transfer have a long history of development (Howell, 1968, 1998; Yang et al. 1995).
86 While its advantage may not be obvious for problems with simple geometries and ideal
87 transmitting media, Monte Carlo is an excellent technique for modeling complex terrains
88 and anthropogenic sources of emissions presented in highly urbanized environments. The
89 main advantage of Monte Carlo is that when the problem complexity increases, the

90 numerical expense of analytical methods involving mathematical integration of radiative
91 transport equation increases exponentially, while that of Monte Carlo procedures only
92 increases linearly (Howell, 1968). In the literature, only a handful number of Monte Carlo
93 methods were available for radiative heat transfer in street canyons. The most recent model
94 by Krayenhoff et al. (2014) is probably the only one that takes into account the presence of
95 trees in urban canopies. However, their work was developed for a multi-layer UCM with
96 probabilistic distribution of building heights, and there was a lack of comparison to field
97 measurements.

98 In this paper, we derive a Monte Carlo algorithm for radiative exchange in 2D street
99 canyons, incorporating the presence of trees (or generic obstacles alike) and their shading
100 effect. In combination with matrix method for infinite radiative reflections as well as
101 analytical method for heat conduction through the building envelope, a new modeling
102 framework is developed for capturing energy balance inside a street canyon with realistic
103 representation of radiative exchange based on stochastic procedures. The proposed method
104 is validated against benchmark radiative transfer problems using analytical method, as well
105 as in-situ measurements in urban areas. The validated model is then applied to study the
106 effect of various canyon and tree geometries on the radiative exchange in a street canyon.
107 Shading effect of trees lead to reduced surface temperature of canyon facets (walls and
108 roads), as well as potential saving of building energy.

109 The proposed method for radiative exchange is developed for a simplified 2D “big
110 canyon” with particular applications to numerical weather prediction of urban areas at
111 large (neighborhood to city) scales, via incorporation into the widely used WRF-UCM
112 platform. With improved model accuracy of resolving radiative heat fluxes as well as

113 surface temperatures at each canyon facet, the proposed method will enhance the
114 predictability of the overall numerical framework on other surface energy budgets, viz.
115 sensible and latent heat and thermal storage in built environments. Future development and
116 applications of this numerical framework will also help to provide useful guidelines for
117 urban landscape management and sustainable urban planning in terms of, e.g. solar energy
118 harvest, heat island mitigation, and/or building energy efficiency.

119

120 **2. Model algorithms**

121 In this section, we present the detailed algorithms and formulation of the proposed
122 numerical framework, including the Monte Carlo method for estimating view factors in a
123 street canyon with shade trees, and the matrix inversion for resolution of infinite
124 reflections among canyon facets. Note that the proposed method is developed for longwave
125 (diffuse) thermal radiation, which is appropriate for street canyons with direct solar
126 irradiance shaded by obstructions and trees.

127

128 **2.1. Monte Carlo method for radiative view factors**

129 Consider an energy bundle (radiative “ray”) between two generic surfaces, as shown
130 in Fig. 1, emitting from surface 1 and received by surface 2. The radiative view factors F_{12} ,
131 with a ray radiated from a generic area A_1 and incident on another generic area A_2 , is given
132 by,

$$133 \quad F_{12} = \frac{1}{A_1} \int_{A_1} \int_{A_2} \frac{\cos \eta_1 \cos \eta_2}{\pi S^2} dA_2 dA_1, \quad (1)$$

134 where η_1 and η_2 are the angle between the ray and the surface normal of A_1 and A_2 ,
 135 respectively; and S is the path length of the ray. Properties that must be satisfied by view
 136 factors matrix include: self-view factor for a flat facet must be zero, and no radiant energy
 137 can be lost, i.e.

$$138 \quad F_{ii} = 0, \text{ no summation over } i; \sum_{j=1}^N F_{ij} = 1. \quad (2)$$

139 In addition, the reciprocal relation holds, i.e.

$$140 \quad A_1 F_{12} = A_2 F_{21}. \quad (3)$$

141 In particular, the view factors between the four urban facets of the 2D street canyon
 142 (the ‘‘sky’’, two walls, and the road, without trees) can be solved by analytical integration,
 143 and are given by (Sparrow and Cess, 1978)

$$144 \quad F_{SG} = F_{GS} = \sqrt{1 + \left(\frac{H}{W}\right)^2} - \frac{H}{W}, \quad (4)$$

$$145 \quad F_{WW} = \sqrt{1 + \left(\frac{W}{H}\right)^2} - \frac{W}{H}, \quad (5)$$

$$146 \quad F_{SW} = F_{GW} = \frac{1}{2} \left[1 - \sqrt{1 + \left(\frac{H}{W}\right)^2} + \frac{H}{W} \right], \quad (6)$$

$$147 \quad F_{WS} = F_{WG} = \frac{1}{2} \left[1 - \sqrt{1 + \left(\frac{W}{H}\right)^2} + \frac{W}{H} \right], \quad (7)$$

148 where subscripts S , G , and W denote sky, ground, and wall, respectively, H is the building
 149 height, and W is the width of canyon, as shown in Figure 2.

150 It is straightforward to verify that the analytical formulas observe the properties of
 151 view factors in Eqs. (2)-(3). Analytical formulas of view factors, such as Eqs. (4)-(7), are

152 handy to use but may become difficult to formulate in more complex problems. For
 153 example, analytical computation of view factors for a street canyon with trees, as shown in
 154 Figure 2, is hitherto absent (Krayenhoff et al., 2014). The Monte Carlo method, on the
 155 other hand, invokes a probabilistic sampling of all rays emitted from surface by taking a
 156 “random walk”, and avoids the difficulty inherent in the integration process of Eq. (1) for
 157 complex geometry (Howell, 1968). To randomize the radiative exchange process, the
 158 direction of the emitted bundle can be determined by the polar angle θ_1 and the azimuthal
 159 angle η_1 , each associated with a random number R_θ and R_η as:

$$160 \quad R_\theta = \frac{\theta_1}{2\pi}, \quad (8)$$

$$161 \quad \sqrt{R_\eta} = \sin \eta_1. \quad (9)$$

162 The emitting coordinates of all four canyon facets of a 2D street canyon, are given by

$$163 \quad x_e = WR_x; \quad z_e = HR_z, \quad (10)$$

164 where R_x and R_z are the random numbers associated with emitting coordinates x_e and z_e
 165 from a given canyon facet in x and z directions, respectively, W the canyon width, and H
 166 the wall height. To track the incident location of a ray transfer between two parallel
 167 surfaces, only one coordinate will be involved. From the geometry, it is straightforward to
 168 show that between ground and sky, and the two parallel walls

$$169 \quad x_i = x_e + H \tan \eta_1 \cos \theta_1, \text{ between sky and ground,} \quad (11)$$

$$170 \quad z_i = z_e + W \tan \eta_1 \cos \theta_1, \text{ between walls,} \quad (12)$$

171 The incident coordinates of a ray transferring between two perpendicular surfaces are
 172 slightly more complicated, as given by

173
$$x_i = \frac{z_e}{\tan \eta_1 \cos \theta_1}, \text{ from walls to sky/ground,} \quad (13)$$

174
$$z_i = \frac{x_e}{\tan \eta_1 \cos \theta_1}, \text{ from sky/ground to walls,} \quad (14)$$

175 Tracing a ray emitting from surface A_1 with random motion, it is relatively straightforward
 176 to see if it actually absorbed by surface A_2 using Monte Carlo algorithm, by checking the
 177 incident coordinates. For example, if the incidental horizontal coordinate falls within the
 178 spatial location of ground, i.e. $0 \leq x_i \leq W$, the emitted ray is considered as received by the
 179 ground; it is “missed” by the ground otherwise.

180

181 **2.2. Matrix solution of net radiation**

182 Given i -th facet in a street canyon, with $1 \leq i \leq N$ and N the total number of facets, it
 183 is associated with a range of radiative fluxes, namely the irradiance (i.e. the total incoming
 184 radiation) I_i , the radiosity (the total outgoing) J_i , the emittance (the total emitted) M_i , and
 185 the net radiative flux Q_i , respectively. Assuming all facets are opaque, these fluxes are not
 186 independent but related by

187
$$I_i = \sum_{j=1}^N J_j F_{ji}, \quad (15)$$

188
$$J_i = M_i + (1 - \varepsilon_i) I_i, \quad (16)$$

189
$$Q_i = M_i - J_i, \quad (17)$$

190 where subscripts ‘ i ’ and ‘ j ’ are facet indices, ε is the emissivity and F_{ji} are the view factors
 191 for radiation transfer from j -th to i -th surface, as defined in Eq. (1).

192 The quantity of interest is the net radiation flux, which involves the radiosity from
 193 *other* facets incident on the surface of interest. Combining Eqs. (15) and (16), we have

194
$$J_i = M_i + (1 - \varepsilon_i) \sum_{j=1}^N J_j F_{ji}. \quad (18)$$

195 Clearly the solution of the problem involves recurrence of radiosity at a generic surface J_i .
 196 Exact solution therefore invokes solving the geometric series associated with multiple
 197 (infinite) radiative reflections. Rewrite Eq. (18) as

198
$$M_i = J_i - (1 - \varepsilon_i) \sum_{j=1}^N J_j F_{ji} = \sum_{j=1}^N J_j \Gamma_{ji}, \quad (19)$$

199 where $\Gamma_{ij} = \delta_{ij} - (1 - \varepsilon_i) F_{ij}$. The matrix Γ_{ij} always has an inverse, which is denoted as

200 $[\Psi_{ij}] = [\Gamma_{ij}]^{-1}$. Thus for each facet, we have

201
$$J_i = \sum_{j=1}^N M_j \Psi_{ji}, I_i = \frac{J_i - M_i}{1 - \varepsilon_i}, \quad (20)$$

202 and

203
$$Q_i = \begin{cases} \sum_{j=1}^N F_{ji} M_j - M_i & \text{if } \varepsilon_i = 1 \\ \frac{\varepsilon_i \sum_{j=1}^N \Psi_{ji} M_j - M_i}{1 - \varepsilon_i} & \text{if } \varepsilon_i \neq 1 \end{cases}. \quad (21)$$

204 For each facet, the material emissivity and temperature are known quantities. For diffusive
 205 thermal radiation, the emittance is diffuse and longwave in nature, and can thus be
 206 expressed using Boltzmann's law:

207
$$M_i = \varepsilon_i \sigma T_i^4, \quad (22)$$

208 where $\sigma = 5.67 \times 10^{-8} \text{ W m}^{-2} \text{ K}^{-4}$ is the Stephen-Boltzmann constant. Note that Eqs. (20)-
 209 (21) represent the matrix solution of radiative heat exchange between canyon facets. When
 210 the view factor matrix F_{ij} is analytically determined, these solutions are hereafter referred

211 to as “exact” for they analytically resolves *infinite* number of reflections between canyon
212 surfaces through matrix inversion.

213

214 **3. Model Validation**

215 In this section, we first examine that if the radiative view factors predicted by Monte
216 Carlo algorithm agree with the analytical values for a bare street canyon absent of trees.
217 Next, the predicted view factors are used in the matrix method for prediction of net
218 radiation arising from canyon facets under thermal equilibrium. Lastly, the validated
219 Monte Carlo algorithm will be applied to estimate ground and wall temperatures in a real
220 street canyon during a night cooling episode.

221

222 **3.1. Estimation of radiative view factors**

223 The numerical algorithm for estimating view factors using Monte Carlo simulations
224 (MCS) is outlined in Eqs. (8)-(14). Using random samples, the accuracy of Monte Carlo
225 method, as expected, improves with the sample size. Taking the view factor between sky
226 and ground F_{SG} as example, Figure 3 shows the model accuracy as a function of number of
227 samples. With a sample size of 1,000, MCS is capable of predicting the view factor with
228 reasonable accuracy, as compared to the analytical formulation, while MCS with a sample
229 size of 10,000 yields results with negligible discrepancy. For subsequent simulations, we
230 will use the sample size of 10,000. Predictions of other street canyon view factors exhibit
231 similar trend with respect to sample size. The comparisons of all four view factors between
232 urban facets by Monte Carlo and analytical methods are shown in Figure 4. The

233 discrepancy between predictions by the two methods is nearly indiscernible, for canyon
234 aspect ratio H/W ranging from 0.01 to 100. Also note that the most drastic change of all
235 four view factors occur around $H/W \sim 1.0$, and covers the practical range of actual street
236 canyon dimensions around 0.2 to 10. This observation highlights the importance of
237 accurate prediction of radiative view factors for real street canyons.

238

239 **3.2. Net radiation of canyon facets in thermal equilibrium**

240 With the view factors being accurately estimated by the Monte Carlo method, we
241 then apply the method to predict the net radiation from each urban facet under thermal
242 equilibrium, in conjunction with the matrix method outlined in Section 2.2. Emissivity is
243 set to be 1.0 for sky (canyon top) $\varepsilon_W = \varepsilon_G = 0.95$, where subscripts W and G denote
244 properties of walls and the ground, respectively. The surfaces enclosing the street canyon
245 (c.f. Figure 2) are set to be in constant temperatures as: sky $T_a = 300$ K, ground $T_G = 290$ K,
246 east wall $T_{W1} = 290$ K, and west wall $T_{W2} = 295$ K. Note that these values are chosen rather
247 arbitrarily for demonstration purpose, and they do not affect the accuracy of the model
248 predictions. The results of comparison between the Monte Carlo and the exact methods, as
249 functions of the canyon aspect ratio, are shown in Figure 5: here the exact solution refers to
250 the combination of analytical formulation of view factors in Eqs. (4)-(7) and the matrix
251 method for net radiation with infinite reflections in Section 2.2. It is clear that the MCS
252 predictions are in good agreement with the exact solution. As a function of canyon aspect
253 ratio, the most significant variation happens again around $H/W \sim 1.0$, indicating the view
254 factors are dominating the radiative energy distribution among different street canyon
255 facets. It is also noteworthy that the model is capable of resolving differences in surface

256 temperatures of two opposite walls, given that their net radiation can be accurately
 257 determined.

258

259 3.3. Transient nocturnal cooling episode

260 Given that the proposed model is capable of predicting both view factor and net
 261 radiation with good accuracy as compared to the analytical method, here we further test the
 262 model for its capability of predicting surface temperatures, in conjunction with numerical
 263 procedures for heat conduction. In this study, we adopt a spatially-analytical scheme for
 264 solving heat conduction through solid ground and walls, based on the Green's function
 265 approach (Wang et al., 2011a). The temperature distribution for a finite wall with one-
 266 dimensional (1D) spatial domain $0 \leq x \leq d$ where d is the wall thickness, is given by a
 267 convolution integral equation as (Carslaw and Jaeger 1959; Cole et al., 2011):

$$268 \quad T_w(x, t) = T_{i,w} + \int_0^t q_1(t - \tau) dG(x, \tau) - \int_0^t q_2(t - \tau) dG(d - x, \tau), \quad (23)$$

269 where q_1 and q_2 are the heat fluxes at the two surfaces of the wall; and G is the Green's
 270 function (fundamental) solution of a homogeneous heat conduction problem. For a finite
 271 wall with thickness d , the Green's function solution is given by

$$272 \quad G(x, t) = \frac{2\sqrt{(\alpha t / \pi)}}{k} \sum_{n=-\infty}^{\infty} \exp\left[-\frac{(x - 2nd)^2}{4\alpha t}\right] - \frac{1}{k} \sum_{n=-\infty}^{\infty} |x - 2nd| \operatorname{erfc}\left(\frac{|x - 2nd|}{2\sqrt{\alpha t}}\right) \quad (24)$$

273 where k and α are the thermal conductivity and diffusivity, respectively; and $\operatorname{erfc}(\cdot)$ is the
 274 complimentary error function. Equations (23)-(24) can be readily evaluated using

275 numerical integration, given the knowledge of boundary conditions q_1 and q_2 , and the
276 initial condition T_i . A detailed solution procedure for Green's function approach can be
277 found in Wang et al. (2011a).

278 Note that canyon ground, unlike walls bounded by two (building interior and exterior)
279 boundaries, can be treated as a 1D semi-infinite solid domain, bounded only at the upper
280 surface with an effective adiabatic (zero flux) condition at the lower boundary (in deep
281 soil). Thus the solution of surface temperature of the ground can be approximated by a
282 closed-form formula (Nunez et al., 1976; Wang et al., 2011a), as

$$283 \quad T_{s,G}(t) = T_{i,G} + Q_G \frac{2\sqrt{\alpha_G t / \pi}}{k_G}, \quad (25)$$

284 where T_s is the surface temperature; and Q_G is the net radiation received at the ground
285 surface, as predicted using matrix method with infinite reflections by Eq. (21).

286 The proposed method is tested for a nocturnal cooling event in the Grand-view
287 district of Vancouver, measured by Nunez and Oke (1976) during September 9-11, 1973.
288 The street canyon dimensions are $d = 0.3$ m, $H = 7.31$ m, and $W = 7.54$ m. Thermal
289 properties of walls and the ground are: $(\rho c_p)_W = 2.09 \times 10^6$ J K⁻¹ m⁻³, $k_W = 1.6$ W m⁻¹ K⁻¹,
290 $(\rho c_p)_G = 1.88 \times 10^6$ J K⁻¹ m⁻³, $k_G = 1.6$ W m⁻¹ K⁻¹, where ρ and c_p are the density and
291 specific heat; and $\varepsilon_W = \varepsilon_G = 0.95$. The nocturnal cooling episode was measured after sunset
292 with calm winds, so the short wave radiation, sensible and latent heat fluxes are neglected
293 in both the measurement and modelling. The initial longwave radiation is measured as 339
294 W m⁻². Surface temperatures of the canyon walls and ground predicted by the combined
295 numerical framework (Monte Carlo simulation of view factors, matrix method for net
296 radiation, and Green's function approach for heat conduction), are compared with field

297 measurements, as shown in Figure 6. The overall agreement between model predictions
298 and observations is reasonably good, with temperature discrepancy less than 1 °C in
299 general.

300

301 **4. Model applications and discussion**

302 With the proposed numerical framework validated against benchmark radiative
303 transfer problems and in-situ measurements, we proceed to apply the model to street
304 canyons with shade trees. We first test the effect of tree crown sizes on view factors
305 between canyon facets, followed by its implications to surface temperature evolution and
306 building energy consumption given diurnal atmospheric forcing. Some of the assumptions
307 made in the proposed methods and future model extensions are also discussed.

308

309 **4.1. Effect of tree sizes on view factors**

310 For simplicity, we ignore the size of tree trunks due to its relative small dimension as
311 compared to tree crowns. Further, in this study, tree crowns assume circular cross-sectional
312 shapes, as shown in Figure 2, with a radius of R_t . As the vertical variability is not explicitly
313 resolved in the 2D urban canyon, and subsequently in the single layer UCM adopted in
314 WRF, we do not account complex tree geometries, e.g. roof top shading and probabilistic
315 distribution of tree heights in this paper, such as those developed in multi-layer UCMs by
316 Krayerhoff et al (2014). With presence of trees in the street canyon, radiative exchange
317 between canyon facets will be partially “blocked” by tree crowns. Thus, trees will
318 effectively shade canyon facets by intercepting radiative rays, with their actual shading

319 effect depending on the size of the tree crowns. Figure 7 demonstrates this shading effect
320 as a function of canyon aspect ratio. Note that even with a very small tree crown size (R_t/W
321 = 0.1), all radiative view factors are effectively reduced. As tree crown size increases,
322 more radiation will be intercepted by trees and view factors further decrease. In addition,
323 the shading effect is more significant for shallower canyons (with smaller H/W ratios).
324 This is because for deep canyons, walls in the street canyon are already presented an
325 important factor for shading, and the additional shading by trees are less prominent.

326

327 **4.2. Canyon temperature and building energy consumption**

328 Next, we apply the combined numerical framework to test the effect of tree sizes on
329 diurnal evolution of canyon temperatures and building energy use. The model is driven by
330 in-situ measurement of atmospheric and radiative forcings at Maryvale, Phoenix, Arizona
331 on 04 June 2012 (clear day), measured by an eddy covariance flux tower. More details on
332 the instrumentation and data quality control of the field measurement can be found in
333 Chow et al. (2014). Relevant models parameters are given by measurement or previous
334 model calibration as: $d = 0.3$ m, $H = 15$ m, $W = 20$ m, $k_G = 1.6$ W m⁻¹ K⁻¹,
335 $k_W = 1.3$ W m⁻¹ K⁻¹, $(\rho c_p)_W = 1.26 \times 10^6$ J K⁻¹ m⁻³, $(\rho c_p)_G = 2.00 \times 10^6$ J K⁻¹ m⁻³, and $\varepsilon_W =$
336 $\varepsilon_G = 0.95$. Diurnal variation of the atmospheric temperature and net (shortwave +
337 longwave) downwelling radiation at the canyon top is plotted in Figure 8(a).

338 Note that this paper is focused on the radiative exchange in a street canyon, so
339 turbulent (sensible and latent) heat fluxes are not accounted in energy transport. For an arid
340 city like Phoenix, latent heat during a clear day is usually very small (<10% of the

341 irradiance), while the sensible heat flux can be significant (maximum daily sensible heat is
342 300 W m^{-2} on June 04, 2012 in Phoenix). So the negligence of turbulent heat is a crude
343 assumption. To include sensible and latent heat in surface energy balance, it requires
344 sophisticated physical parameterization schemes involving wind velocity, surface
345 roughness, atmospheric stability, humidity, soil moisture, and complex hydrological
346 processes (precipitation, infiltration, and surface runoff) (see Wang et al., 2013). It remains
347 a challenging task to build a complete land surface model based on stochastic simulations
348 including all physical processes. Furthermore, the Monte Carlo algorithm assumes
349 completely random emission angles of a ray (see Eqs. (8)-(9)), i.e. canyon facets are
350 Lambertian and opaque, and radiative rays are diffusive. This is not the case when direct
351 solar radiation is first impinged on a canyon facet. A sun-lit wall when receiving
352 directional solar radiation, for example, is certainly at higher temperature than a shaded
353 wall. One way to include that effect is to estimate a “shadow length” in a street canyon as a
354 function of city location, canyon orientation, and time of the day (Kusaka et al., 2001).
355 Nevertheless, the assumption of diffusive radiation is valid for subsequent reflections using
356 the matrix method.

357 A comparison of model estimate of ground surface temperature and field
358 measurements is shown in Figure 8(b), with no tree shading in the model. Despite the
359 above-mentioned limitation of the model, its prediction is comparable with the
360 measurement (with a $R^2 = 0.945$). Next, we include shade trees in the canyon with different
361 crown sizes. The result of model predictions for diurnal evolution of canyon surface
362 temperatures is shown in Figure 9(a). The shading effect is clearly demonstrated in that
363 when the tree crown size increases, surface temperatures of wall and ground decrease. A

364 increase of crown size from 0.5 m to 1.0 m leads to the reduction of surface temperatures
365 up to 6-7 °C around noon.

366 Given the temperature profile through the wall is calculated using the Green's
367 function approach in Eq. (23), the conductive heat flux entering the building can be
368 computed using Fourier's law,

$$369 \quad q_w(x,t) = -k \frac{dT_w}{dx} = -k \left[\int_0^t q_1(t-\tau) dG'(x,\tau) - \int_0^t q_2(t-\tau) dG'(d-x,\tau) \right], \quad (26)$$

370 at $x = d$. This flux is a good indicator for energy consumption inside the building to offset
371 the heat inflow/outflow through building envelop and to maintain the interior thermal
372 comfort through operation of heating, ventilation, and air-conditioning systems. The model
373 predicted heat flux entering the building through wall is presented in Figure 9(b), for
374 various tree sizes. Again, it is clear that in the absence of trees, building interiors receives
375 large heat inflow (positive) through the wall in a clear summer day (June 04). When trees
376 are presented, the magnitude of heat inflow decreases significantly with the tree size,
377 indicating the potential of shade trees for building energy saving.

378

379 **5. Concluding remarks**

380 A new numerical framework is developed for radiative heat exchange in street canyons
381 with shade trees, by combining the Monte Carlo simulation of view factors and matrix
382 method for infinite reflections. The model is validated against analytical solutions of
383 benchmark radiative transfer problems as well as field measurements in real street canyons.
384 Results of comparison show that the model is of capable of predicting radiative view

385 factors, surface temperatures, and net radiation of canyon facets with good accuracy, in
386 both steady state and transient cases. We then apply the model to study the effect of shade
387 trees and their sizes on the diurnal evolution of canyon surface temperatures in conjunction
388 with a Green's function approach for heat conduction. It is manifested that shade trees are
389 effective in reducing canyon surface temperatures, with the shading effect enhanced by
390 increasing tree sizes. The presence of trees in a street canyon demonstrates good potential
391 in reducing cooling energy consumption as it mitigates the heat inflow into the building
392 through walls.

393 In addition, Monte Carlo method is also a powerful tool in computing absorption and
394 scattering of radiation if complex participating media (e.g. dust, soot, pollen, etc.) are
395 presented in street canyons. By randomizing radiation using energy bundles, scattering
396 deflects a ray's direction, and absorption causes a ray to be intercepted. The frequency,
397 direction, and fraction of attenuation due to either scattering or absorption can be simulated
398 by random numbers and as functions of scattering or absorption coefficients. This
399 treatment is particularly useful for cities with heavy atmospheric pollution, either caused
400 by natural sources with seasonal occurrence or by constant industrial sources. By inclusion
401 of participating media, numerical models will improve accuracy in simulating radiative
402 exchange and thermal field in an urban canopy layer, which will subsequently enhance
403 numerical capacity in, e.g. building energy model or land-atmosphere interactions.

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409

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523

524 **Caption of Figures:**

525 **Figure 1.** Schematic of radiative transfer between two generic surfaces

526 **Figure 2.** Cross sectional view of 2D street canyon with trees. The cross section of tree
527 crowns is simplified as circles with radius R_t .

528 **Figure 3.** Effect of sample size on Monte Carlo prediction of view factor F_{SG} : (a) as a
529 function of canyon aspect ratio, and (b) as a function of sample sizes at $H/W = 1.0$.

530 **Figure 4.** View factors of radiative heat exchange between canyon facets as functions of
531 the canyon aspect ratio H/W ; subscripts S , G , and W denote sky, ground, and wall,
532 respectively. The sample size is 10,000 for the Monte Carlo simulations.

533 **Figure 5.** Comparison of net radiation of different canyon facets at thermal equilibrium, as
534 predicted by the exact and the proposed hybrid methods. Surface of the four enclosing
535 surfaces are set as $T_a = 300$ K, $T_G = 290$ K, $T_{W1} = 290$ K and $T_{W2} = 295$ K.

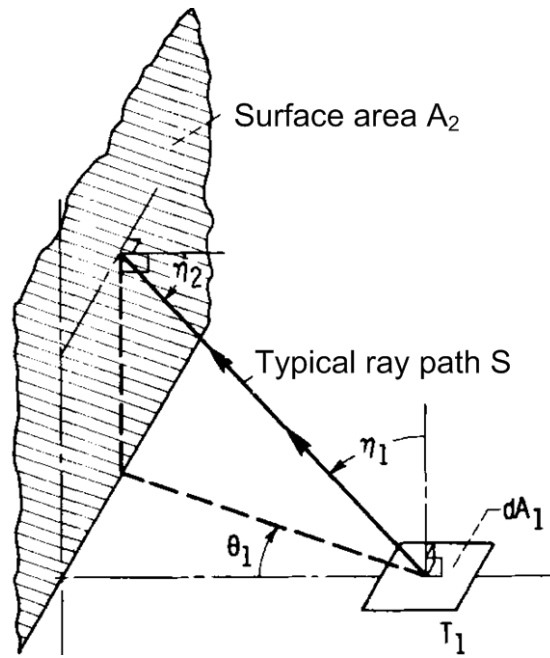
536 **Figure 6.** Comparison of averaged wall and ground temperatures predicted by the model
537 and field measurements in the canyon during the night cooling episode.

538 **Figure 7.** Monte Carlo simulation of view factors of radiative heat exchange between
539 canyon facets with trees, as functions of the canyon aspect ratio H/W . In this case, the
540 canyon width W is fixed as 20 m, and the centre of tree crown height is located at $H/2$.

541 **Figure 8.** Model application with (a) radiative forcing measured on 04 June 2012, Phoenix,
542 AZ, and (b) comparison between model prediction and measurement of ground surface
543 temperature.

544 **Figure 9.** Model prediction of diurnal variation of (a) surface temperatures, and (b) heat
545 conducted into building through walls. Diurnal radiative forcing is the same as in Figure 8.

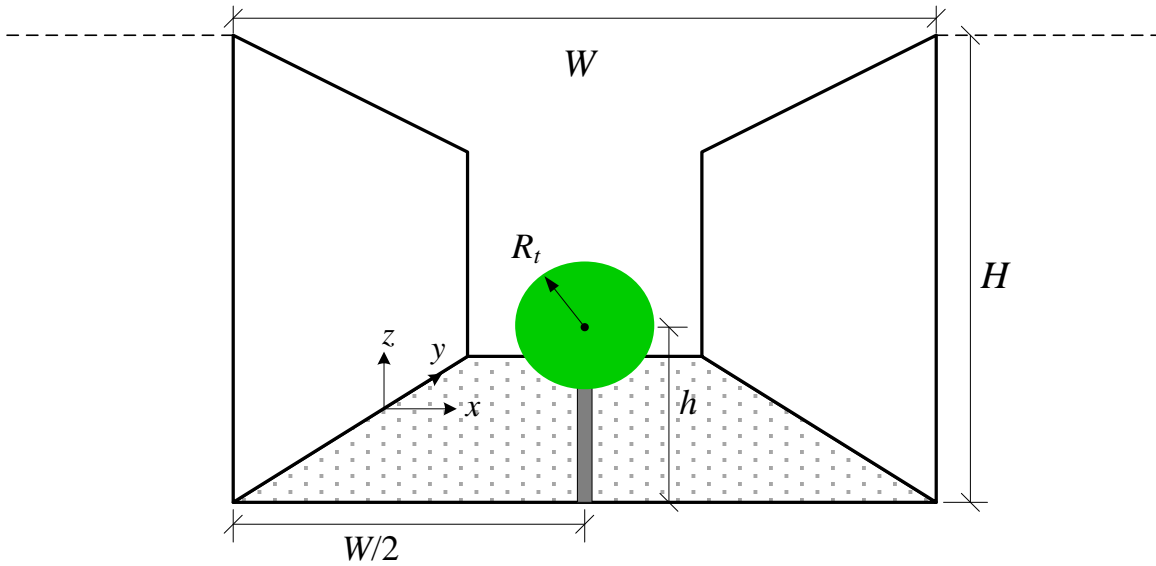
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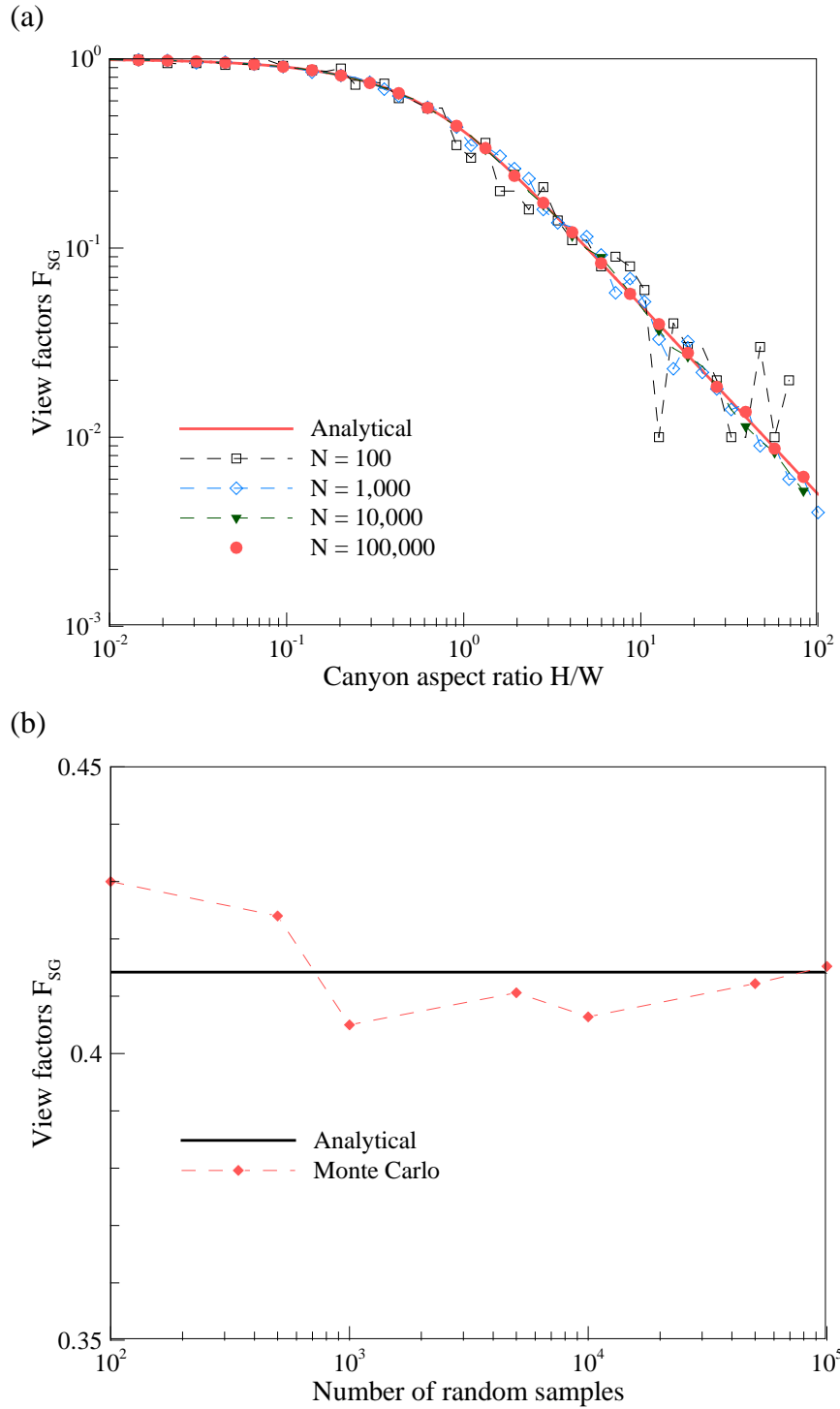
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552 **Figure 2.** Cross sectional view of 2D street canyon with trees. The cross section of tree

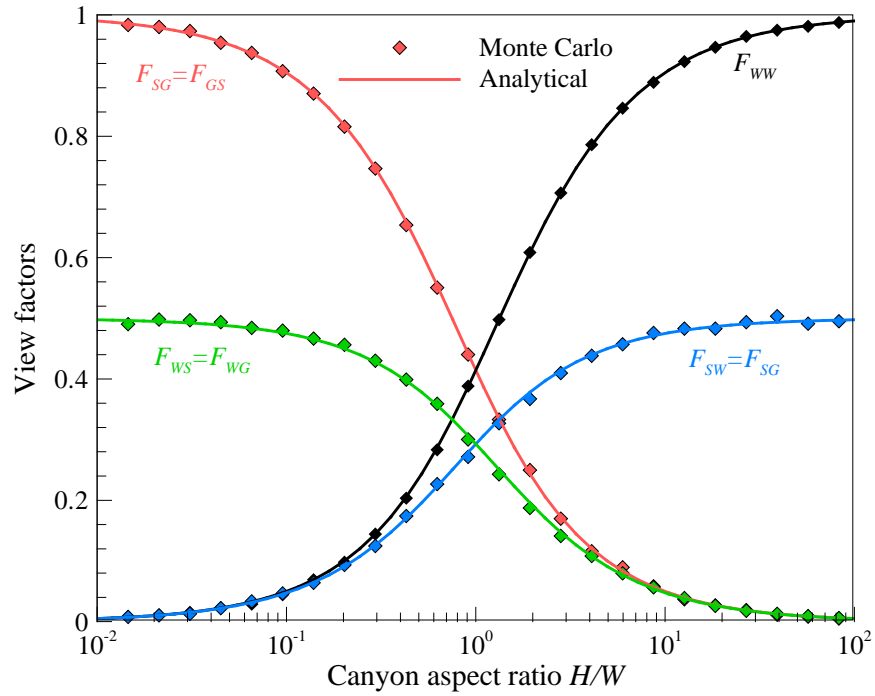
553 crowns is simplified as circles with radius R_t .



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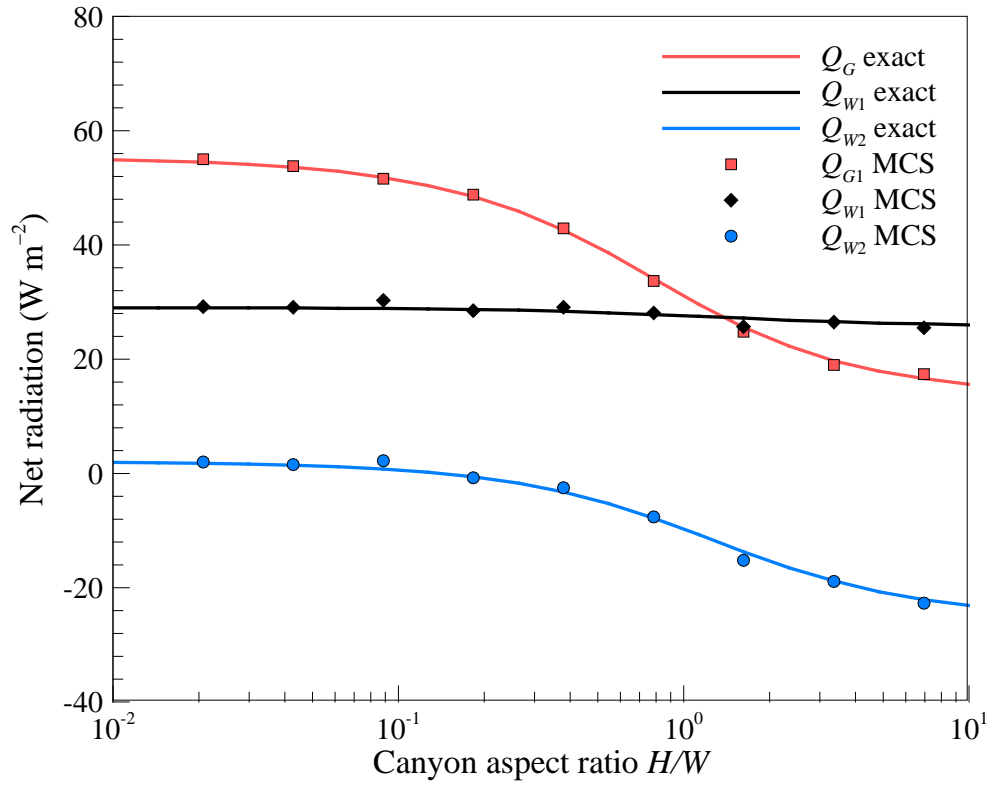
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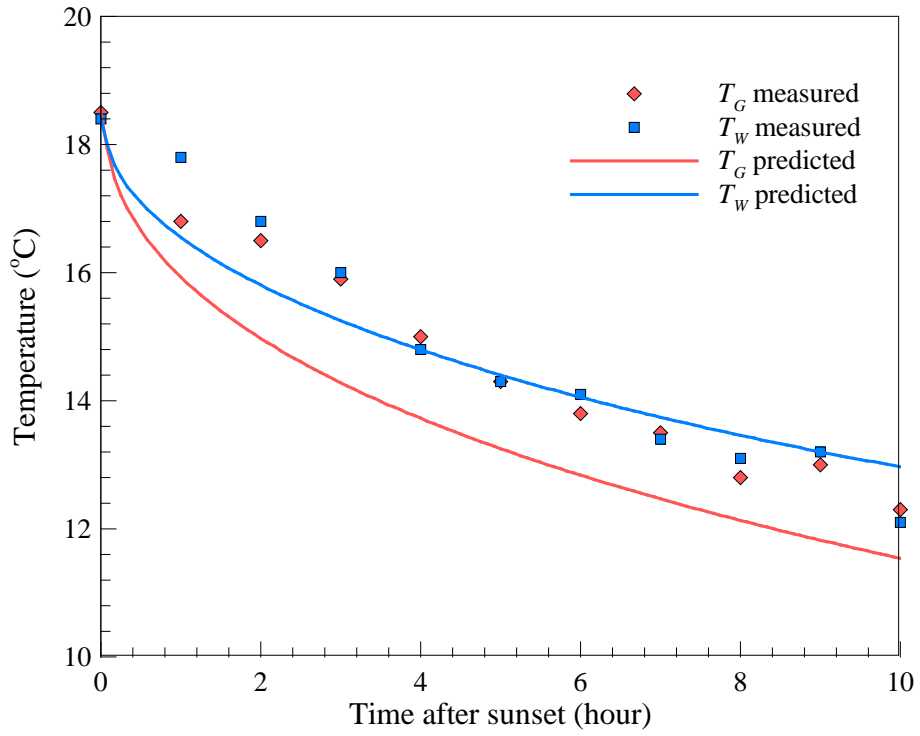


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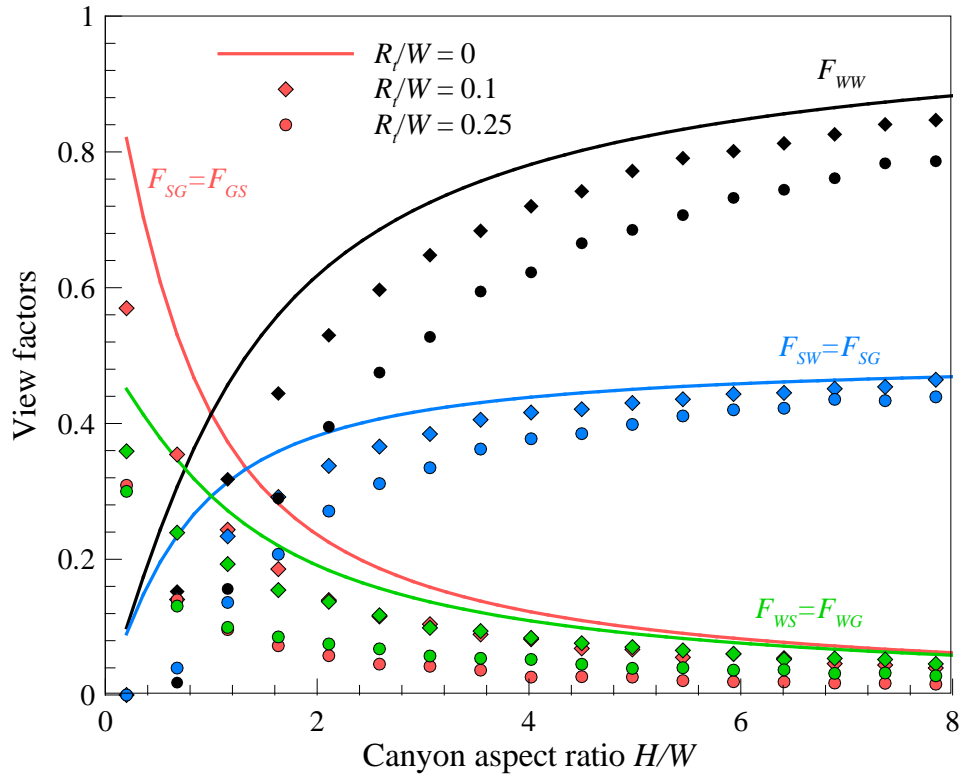
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565

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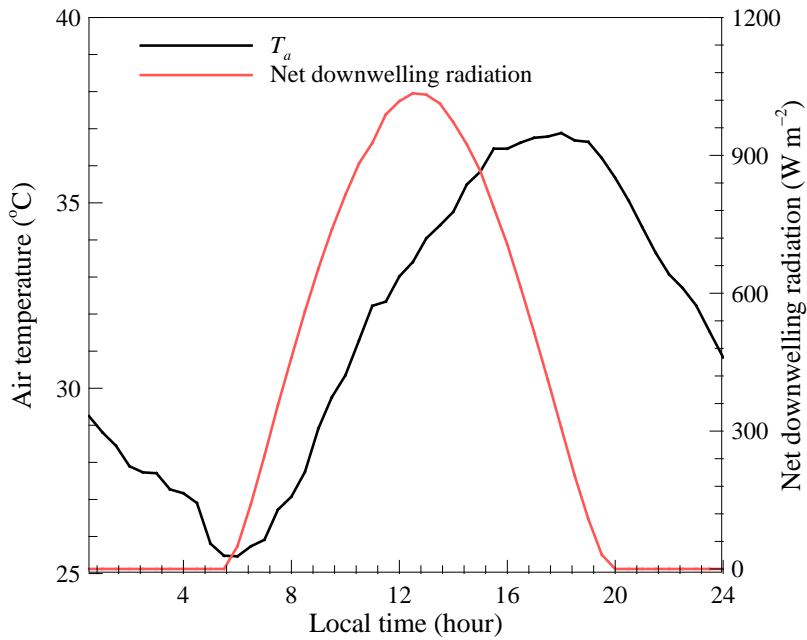
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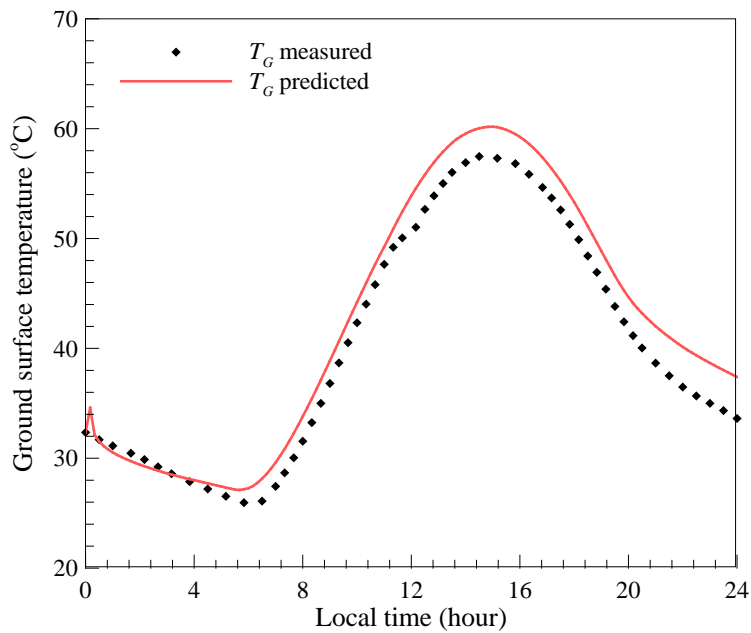
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(a)



(b)



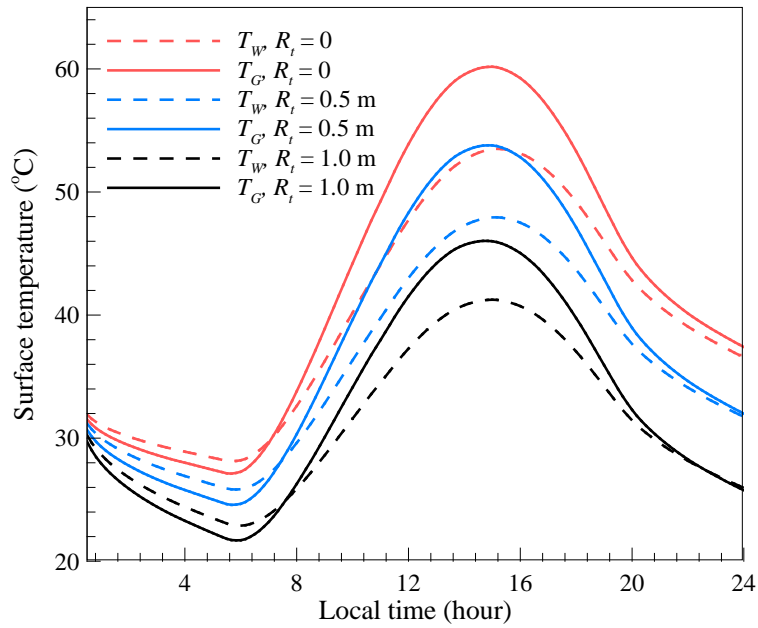
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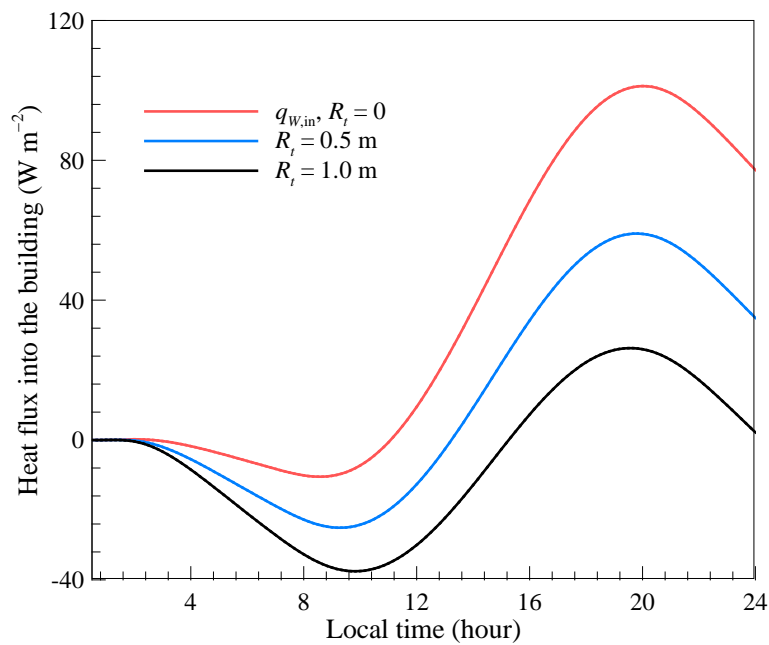
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(a)



(b)



576

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